The Network Origins of Inflation Stances in a Currency Union

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Abstract

This paper proposes a production network mechanism to explain divergent inflation stances within a currency union. I develop a heterogeneous-agent input–output (HAIO) model in which countries differ in labor supply, consumption, and ownership structures, and characterize the optimal monetary policy under arbitrary Pareto weights. The analysis shows that the inflation bias of optimal policy arises from redistributive incentives to manipulate the terms of trade whenever Pareto weights deviate from countries' income shares. Applying the model to the euro area, I find that a country's unilateral inflation stance in the union is inversely related to its upstreamness in the production network. Moreover, policy alignment loss increases with distance between a country's inflation stance and the union-wide consensus, implying that countries at either extreme of the production network, whether highly upstream or highly downstream, incur the greatest welfare costs from a one-size-fits-all monetary policy.

Keywords: production networks, optimal monetary policy, currency union, heterogeneous agents, inflation stances

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1 Introduction

A salient feature of monetary policymaking in the euro area is the persistent divergence in inflation stances across member states, often discussed as a North–South divide (see, e.g., Iversen et al., 2016). Countries such as Germany and the Netherlands are commonly associated with a stronger emphasis on price stability, while others, including Italy, Greece, and Portugal, are typically viewed as more supportive of accommodative policies. This enduring heterogeneity motivates a fundamental question for currency unions: what structural forces shape countries' preferred inflation outcomes under a common monetary regime, and what are the welfare consequences of imposing a common monetary policy across economies with systematically different economic structures?

This paper proposes a network explanation for divergent inflation stances within a currency union. It departs from the traditional New Open Economy Macroeconomics (NOEM) literature, which emphasizes cross-country asymmetries in shocks and nominal rigidities as the primary sources of policy divergence (see, e.g., Benigno, 2004), by highlighting the role of input-output linkages. I argue that asymmetries in production networks and ownership structures can endogenously generate divergent policy preferences through their impact on the propagation and incidence of shocks across member states.

Building on Baqaee and Farhi (2018, 2024), I construct a multisector general equilibrium model with heterogeneous agents, input–output linkages, and nominal rigidities, in which countries differ in labor supply, consumption baskets, and firm ownership structures. Within this framework, monetary policy induces first-order changes in country-level allocative efficiency through two distinct channels: a direct-incidence channel and a factoral terms-of-trade channel. In a setting without production networks, where firms are owned domestically, the direct-incidence channel coincides with the conventional notion of commodity terms of trade. More generally, it captures network-adjusted commodity terms of trade, which together with the factoral terms-of-trade channel determine a country's overall terms of trade. As a result, in the absence of initial distortions, changes in a country's allocative efficiency arise exclusively from changes in its overall terms of trade.

I then derive a second-order approximation to the utilitarian welfare loss function and characterize the Ramsey-optimal monetary policy under arbitrary Pareto weights. When Pareto weights coincide with countries' income shares, the optimal policy is a pure price-stabilization rule that places greater weight on industries that are larger, exhibit greater nominal rigidity, and belong to countries whose labor supply is more sensitive to monetary expansions. By contrast, when Pareto weights deviate from income shares—reflecting a misalignment between political influence and economic size—monetary policy faces a fundamental trade-off between aggregate stabilization and first-order redistribution. This tension generates a redistributive inflation bias, as the central bank has incentives to tilt the overall terms of trade in favor of certain member states.

Building on this result, the analysis turns to policy evaluation from the perspective of an individual country. When Pareto weights are unilateral—that is, when full weight is assigned to a single country—the corresponding unilateral optimal policy minimizes that country's welfare loss. While this policy implements the second-best allocation, it presumes that a single country has full control over the policy instruments—an arrangement ruled out by the union's common price-targeting regime. Once the price-index regime is taken as given and inflation must be non–state-contingent, the country faces a third-best problem. The unilateral inflation stance is therefore defined as the inflation rate that minimizes its expected welfare loss under this institutional constraint. The analysis shows that this stance is proportional to the inflation bias under the unilateral optimal policy, and thus summarizes how the country's overall terms of trade respond to monetary expansions: a positive value reflects an improvement, while a negative value reflects a deterioration.

How does a country's inflation stance relate to its position in the union's production network? The model predicts a negative relationship between inflation stance and upstreamness.[1] When the stance is primarily determined by the direct-incidence channel, it reflects the interaction between a country's sectoral export exposure and sectoral nominal rigidities through the production network. Since nominal rigidities compound along supply chains, downstream sectors exhibit lower price pass-through than upstream sectors. As a result, a monetary expansion compresses markups asymmetrically, with larger reductions in upstream sectors. For countries that are net producers of upstream goods and net users of downstream goods, this asymmetry leads to a deterioration in the overall terms of trade, generating a lower (more hawkish) inflation stance. Conversely, more downstream-oriented economies benefit from asymmetric markup compression and exhibit higher (more dovish) inflation stances. These mechanisms imply that production network position is a key determinant of systematic differences in inflation stances within a currency union.

These differences in inflation stances raise a further question: what are the welfare consequences of imposing a one-size-fits-all monetary policy across economies with divergent preferred inflation rates? To address this question, I introduce the policy-alignment loss

[1] Country upstreamness, following Antràs et al. (2012), captures how upstream a country is in the union's production network, measured as the value-added-weighted average distance of its industries from final consumption.

(PAL), defined as the welfare loss a country incurs when the union-wide stabilization policy deviates from its unilateral optimal policy. This measure offers a consistent benchmark for assessing the distributional consequences of common monetary policy in a structurally diverse currency union.

I then apply the theoretical framework to a quantitative analysis of the euro area. Using the World Input–Output Database (WIOD), I calibrate the full inter- and intra-country production network and match it with industry-level data on nominal rigidities. I compute the centralized optimal monetary policy and compare its resulting welfare losses to those under alternative price-stabilization rules, both at the union-wide and country-specific levels. The results demonstrate that moving from a CPI stabilization rule to the centralized optimal policy would generate welfare gains for almost all member states, with only a negligible deviation for one member state.

Next, I compute each country's unilateral optimal policy and identify its unilateral inflation stance and policy-alignment loss. The quantitative analysis reveals a robust negative relationship between inflation stance and production upstreamness. Among major euro-area economies, the Netherlands and Germany exhibit more hawkish stances than France, Italy, and Spain, closely mirroring the widely discussed North–South divide. Furthermore, I find that the distance between a country's unilateral inflation stance and the union-wide inflation consensus serves as a sufficient statistic for its policy-alignment loss: a one-percentage-point increase in this deviation is associated with an average loss of 0.54 percentage points of quarterly consumption. This finding suggests that countries located at either extreme of the production chain—those with highly upstream or highly downstream production structures—tend to exhibit more extreme inflation stances and incur larger welfare losses under a common monetary policy.

Related literature. This paper is primarily related to the NOEM literature on currency-union policy design (see e.g., Benigno, 2004; Galí, 2008; Ferrero, 2009; Farhi and Werning, 2017).[2] Foundational work in this field typically focuses on the centralized optimal policy, assuming Pareto weights equal to income shares in welfare aggregation. In contrast, this paper demonstrates that even in the absence of steady-state distortions, monetary policy faces a fundamental trade-off between aggregate stabilization and first-order redistribution when Pareto weights deviate from income shares. This trade-off results in an inflation

[2] Other related contributions in the HANK literature include Acharya et al. (2023); Bhandari et al. (2021); Dávila and Schaab (2023); La'O and Morrison (2024); Le Grand et al. (2022); McKay and Wolf (2023); Nuño and Thomas (2022), among others.

bias that reflects a policymaker's incentive to tilt the terms of trade between members. The source of this inflation stance is rooted in a strand of NOEM literature highlighting how the incentive to manipulate terms of trade can generate inflation bias. One of the first to formalize this mechanism is Corsetti and Pesenti (2001, 2005), who show that the central bank of a country with monopoly power in trade has an incentive to appreciate its currency to improve the terms of trade, resulting in a deflationary bias. Subsequent research shows that the direction of this bias, whether it is inflationary or deflationary, depends on model assumptions. In particular, it depends on the elasticity of substitution within and across countries, which determines the direction of expenditure switching (e.g., Tille, 2001; De Paoli, 2009), and on the price-setting regime, which influences the degree of exchange rate pass-through (e.g., Devereux and Engel, 2003).[3] Building on this insight, this paper contributes to the literature by demonstrating that a country's unilateral inflation stance in the union manifests itself as an interaction between its sectoral export exposure and nominal rigidities through the production network, and thus relates to its position in the supply chain.

This contribution also places the paper within the burgeoning literature on production networks, which studies how microeconomic shocks propagate through input–output linkages. For example, building on Long and Plosser (1983), a strand of work, including Acemoglu et al. (2012, 2013, 2017); Baqaee and Farhi (2019); Dew-Becker (2023) and Taschereau-Dumouchel (2025), examine how input–output linkages transmit micro-level productivity shocks into macroeconomic outcomes and shape the aggregate output distribution.[4] In contrast, Jones (2013); Liu (2019); Baqaee and Farhi (2020), Bigio and La'O (2020) and Baqaee and Sangani (2025) focus on the macroeconomic consequences of micro-level distortions. Extending this line, Baqaee and Farhi (2018, 2024) develop a general heterogeneous-agent framework, which serves as a framework for this paper.[5] Relative to their general setup, this paper considers a more tractable case with Cobb–Douglas production and log balanced-growth preferences.[6] This simplification enables explicit welfare analysis at the individual level, showing that cross-sector misallocation captures second-order changes in both the labor wedge and the factoral terms of trade.

This paper also contributes to the literature on monetary transmission in multi-sector

[3]Recent studies include Bergin and Corsetti (2023) and Bianchi and Coulibaly (2025).

[4]Other papers in this line of work include Gabaix (2011); Foerster et al. (2011); Atalay (2017); Baqaee (2018); Carvalho et al. (2021) and Acemoglu and Tahbaz-Salehi (2025), among others. Also see Carvalho (2014); Carvalho and Tahbaz-Salehi (2019) and Baqaee and Rubbo (2023) for surveys.

[5]This framework has also been extended in subsequent work; see, for example, Baqaee and Burstein (2025).

[6]This corresponds to the log-separable case of KPR preferences (King et al., 1988).

economies with input-output linkages, including Pastén et al. (2020, 2024); Wei and Xie (2020); Ghassibe (2021); Ozdagli and Weber (2025); La'O and Tahbaz-Salehi (2022, 2025); Rubbo (2023, 2025); Afrouzi and Bhattarai (2023); Ferrari and Ghassibe (2024); Xu and Yu (2025); Qiu et al. (2025) and Fang et al. (2025). Among these, a small but growing strand of the literature studies optimal monetary policy in production networks. La'O and Tahbaz-Salehi (2022) show that monetary policy cannot implement the first-best allocation in a multisector production network economy. Instead, the optimal policy should stabilize a price index with greater weights assigned to larger, stickier, and more upstream industries. In a parallel work, Rubbo (2023) emphasizes how nominal rigidities accumulate through production chains and flatten both the sectoral and aggregate Phillips curves, and derives a divine coincidence index that outperforms consumer price index in Phillips curve regressions. Following this line of research, Xu and Yu (2025) characterize optimal monetary policy in production networks with steady-state distortions, while Qiu et al. (2025) study optimal monetary policy in a small open economy with domestic and cross-border input-output linkages. This paper complements the literature by generalizing the characterization of optimal policy to a heterogeneous-agent economy with arbitrary Pareto weights in welfare aggregation.

The closest antecedent to this paper is Rubbo (2025), which also studies monetary policy in a HAIO economy. While both papers move beyond standard HANK models by emphasizing the role of production structures in shaping the incidence of monetary policy on labor income, this paper extends the analysis from first-order incidence to second-order welfare analysis, thereby facilitating a full characterization of the optimal policy.

Outline. The rest of the paper is organized as follows. Section 2 sets up the NK–HAIO environment and defines both the sticky-price and flexible-price equilibria. Section 3 log-linearizes the model around the efficient equilibrium and characterizes monetary transmission. Section 4 derives a closed-form characterization of the Ramsey-optimal monetary policy under arbitrary Pareto weights and introduces the concepts of unilateral inflation stance and policy-alignment loss. Section 5 presents the quantitative analysis. Section 6 discusses extensions of the basic framework to a global economy under dominant currency pricing. All proofs, derivations, and additional extensions are included in the online Appendix.

2 Framework

This section sets up a New Keynesian model with monopolistic competition and nominal rigidities in a currency union composed of C member countries and N industries. Each country $c \in \{1, ..., C\}$ is represented by a representative household that supplies a distinct type of labor and differs across countries in labor supply elasticity, sectoral labor input shares, ownership structures, and consumption baskets.

The model is formulated in a general environment that does not require geographic segmentation of labor inputs or ownership claims. In the quantitative analysis, such segmentation is imposed as a calibration baseline, under which each country c is associated with a subset of industries N_c, in order to map the model to a currency-union setting and to discipline the magnitude of cross-country asymmetries.

2.1 Firms

In each industry $i \in \{1, ..., N\}$, a continuum of monopolistically competitive firms (indexed by $k \in [0, 1]$) produces differentiated varieties using the same constant-returns-to-scale production function,

$$y_{ik} = z_i F_i(\{x_{ij,k}\}_{j=1}^N, \{L_{ic}\}_{c=1}^C) = z_i \varsigma_i \prod_{j=1}^N x_{ij,k}^{\omega_{ij}} \prod_{c=1}^C L_{ic,k}^{\alpha_{ic}}$$

where $\alpha_{ic} \geq 0$ denotes the share of labor supplied by agent c used by the firm; $\omega_{ij} \geq 0$ denotes the share of intermediate input j; and ς_i is a normalization constant independent of the shocks.

Differentiated varieties from all the firms k within an industry i are aggregated into an industry-level output using a CES aggregator:

$$y_i = (\int_0^1 y_{ik}^{\frac{\theta_i - 1}{\theta_i}} dk)^{\frac{\theta_i}{\theta_i - 1}}, \tag{1}$$

where $\theta_i > 1$ denotes the elasticity of substitution between varieties within industry i.

The implied sectoral price index is thus given by

$$p_i = (\int_0^1 p_{ik}^{1-\theta_i} dk)^{\frac{1}{1-\theta_i}}. \tag{2}$$

All firms in industry i solve the following cost-minimization problem:

$$mc_i = \min_{\{x_{ij,k}\}_{j=1}^{N},\{L_{ic,k}\}_{c=1}^{C}} \sum_{c=1}^{C} w_c L_{ic,k} + \sum_{j=1}^{N} p_j x_{ij,k}, \quad s.t.\ y_{ik} = 1$$

where w_c is the wage rate for labor supplied by agent c. Under constant returns to scale, all firms within an industry face the same marginal cost and employ inputs in the same proportions.

Nominal rigidities. Price rigidities are modeled à la Calvo (1983). After the realization of exogenous shocks, only a fraction $\delta_i \in (0, 1]$ of firms in sector i are allowed to adjust their prices after observing the money supply and productivity. Given industry level output y_i, price p_i, and marginal cost mc_i, the optimal reset price p_i^* maximizes profits:

$$\Pi_{ik} = \max_{p_{ik}} \left[p_{ik} - (1-\tau_i) mc_i\right] \left(\frac{p_{ik}}{p_i}\right)^{-\theta_i} y_i, \tag{3}$$

where τ_i is an input subsidy provided by the government. Throughout this paper, the subsidies τ_i are non-state-contingent and set to eliminate the distortions that arise under the CES demand structure, resulting in the profit-maximizing price being equal to pre-subsidy marginal costs:

$$1 - \tau_i = \frac{\theta_i - 1}{\theta_i}.$$

This assumption is standard in the New Keynesian literature (see, e.g., Woodford, 2003; Galí, 2008) and eliminates the incentive for monetary policy to use nominal adjustments to substitute for missing tax instruments, thereby isolating the welfare effects of nominal rigidities.

After productivity and monetary shocks are realized, firms within each industry charge different prices due to price rigidities. This implies that, to a first-order approximation, the vector of sectoral inflation rates is given by

$$\log \boldsymbol{p} = \boldsymbol{\delta} \cdot \log \boldsymbol{mc}$$

where $\log \boldsymbol{p}$ denotes the vector of sectoral inflation rates, $\log \boldsymbol{mc}$ is the vector of changes in sectoral marginal costs, and $\boldsymbol{\delta} = \mathrm{diag}(\delta_1, ..., \delta_N)$ is a diagonal matrix of industry-specific price adjustment probabilities. This then implies that sectoral markups are related to sectoral inflation through

$$\log \boldsymbol{\mu} = \log(p/mc) = -(\boldsymbol{\delta}^{-1} - I)\log \boldsymbol{p}. \tag{4}$$

2.2 Households

Each country is populated by a representative household that earns income from labor supply and from sectoral profits. Each household retains exclusive ownership of its labor income, so labor earnings are not shared across countries. Profit income, by contrast, is distributed according to an ownership structure that governs the allocation of firm profits across countries. Following Baqaee and Farhi (2024), I introduce an $N \times C$ ownership matrix Φ, where each entry Φ_{ic} denotes the share of profits generated in industry i that accrues to the representative household in country c. A block-diagonal structure of Φ (i.e., $\Phi_{ic} = \mathbf{1}\{i \in N_c\}$) corresponds to geographically segmented ownership, whereby the representative household in country c receives profits only from industries operating domestically. More general ownership structures allow profits to be distributed independently of production location, thereby altering the cross-country incidence of income and the redistribution effects induced by monetary policy.

The utility function of the representative household in country c is given by

$$U_c(C_c, L_c) = \log C_c - \psi_c \frac{L_c^{1+1/\eta_c}}{1 + 1/\eta_c},$$

where L_c denotes labor supply, η_c is the Frisch elasticity, and ψ_c is calibrated to match steady-state labor supply. Consumption is summarized by the Cobb–Douglas aggregator

$$C_c = \prod_{i=1}^{N} \left(\frac{c_{ci}}{\beta_{ci}}\right)^{\beta_{ci}},$$

where c_{ci} is consumption of good i by the household in country c, and $\beta_{ci} \geq 0$ denotes the expenditure share of good i in country c's consumption basket.

Each household maximizes utility subject to the budget constraint

$$\sum_{i=1}^{N} p_i c_{ci} \;\leq\; w_c L_c + \sum_{i=1}^{N} \Phi_{ic} \Pi_i - T_c, \tag{5}$$

where $\Pi_i = \int_0^1 \Pi_{i,k}\, dk$ denotes total profits generated in industry i, and T_c is a lump-sum tax. The budget constraint equates country c's nominal gross national expenditure (GNE_c)

with its nominal gross national income (GNI_c), given by labor and profit income accruing to domestic residents, net of international taxes and transfers.

2.3 Policy Instruments

The economy also features a government comprising fiscal and monetary authorities, each responsible for implementing its respective policy instruments.

To ensure budget balance, fiscal policy finances subsidies to firms via lump-sum taxes imposed on households in proportion to their ownership shares. In particular, the tax paid by the representative household in country c is given by

$$T_c = \sum_{i=1}^{N} \Phi_{ic} \tau_i mc_i \int_0^1 y_{ik} dk \tag{6}$$

where $\tau_i\, mc_i \int_0^1 y_{ik} dk$ denotes the total subsidy allocated to sector i.

This tax scheme renders the fiscal system neutral: in steady state, each country's nominal expenditure is fully financed by labor income, and trade is balanced across countries. For example, under geographically segmented ownership, equation (6) implies that subsidies to domestic firms are financed entirely by taxes raised domestically.

On the monetary side, I adopt the cash-in-advance setup of Pastén et al. (2020), in which the money supply m directly determines nominal aggregate demand:

$$\mathrm{GNE} = \sum_{c=1}^{C} \sum_{i=1}^{N} p_i c_{ci} = m.$$

This formulation abstracts from monetary micro-foundations and treats m as an intermediary policy instrument.

2.4 Equilibrium

I now formalize equilibrium in the model economy. The environment features nominal rigidities faced by firms, and clearing in both goods and factor markets. I define two equilibrium concepts: one with nominal rigidities and one with fully flexible prices. These serve as the basis for analyzing how monetary policy transmits through the economy and for quantifying the inefficiencies introduced by price rigidity.

Before presenting the formal definitions, note that the market-clearing conditions for goods and labor markets are given by

$$y_i = \sum_{c=1}^{C} c_{ci} + \sum_{j=1}^{N} \int_0^1 x_{jik} dk$$
$$L_c = \sum_{i=1}^{N} \int_0^1 L_{ic,k} dk$$

for all industries $i \in N$ and countries $c \in C$. The first condition requires that total output in each industry equals its use for final consumption across countries and as intermediate inputs across firms. The second condition equates total labor supplied by country c to the sum of labor services employed across industries.

Definition 1. For any realization of productivity shocks $\{z_i\}_{i=1}^{N}$ and a monetary shock m, a sticky price equilibrium consists of: a vector of prices $\{p_i\}_{i=1}^{N}$, a vector of sectoral output $\{y_i\}_{i=1}^{N}$, a vector of nominal wages $\{w_c\}_{c=1}^{C}$, a vector of labor supply $\{L_c\}_{c=1}^{C}$, a matrix of intermediate inputs $\{x_{ij}\}_{i,j \in N}$, a matrix of primary inputs $\{L_{ic}\}_{i \in N, c \in C}$ and a matrix of final use $\{c_{ci}\}_{c \in C, i \in N}$, such that: (i) firms optimally choose intermediate inputs and labor demand to minimize their costs, and optimally reset their prices when they have a chance to adjust; (ii) each representative household maximizes utility subject to the budget constraint; (iii) the government budget constraint is satisfied; (iv) all markets for goods and labor clear.

To isolate the effects of nominal rigidities, I define a benchmark equilibrium in which all prices are fully flexible.

Definition 2. The flexible-price equilibrium is defined by the same conditions as the sticky-price equilibrium, except that all firms are assumed to reset their prices optimally in response to realized shocks.

3 Log-Linearized Model

In this section, I approximate the model around the efficient equilibrium and characterize some important results of the log-linearized economy. I state these results in terms of changes in ex post markups.

3.1 Definitions and Notations

Throughout this paper, I express the log deviation of the variable x from the flexible-price equilibrium x^* as:

$$\hat{x} = \log x - \log x^*.$$

For example, the variable $\hat{c}_c = \log(C_c/C_c^*)$ denotes the consumption gap, representing the percentage deviation of country c's consumption in the sticky-price equilibrium relative to the first-best flexible-price benchmark.

Before presenting results for the log-linearized economy with input-output linkages, I introduce notation to streamline the expressions. Table 1 defines key input-output parameters, including the Leontief inverse and Domar weights.

Table 1: Input-output definitions.

Income share of country c	$\boldsymbol{\chi} \in \mathbb{R}^C$, $\chi_c = \frac{\text{GNE}_c}{\text{GNE}}$
Consumption basket of country c	$\beta_c \in \mathbb{R}^N$, $\beta_{ci} = \frac{p_i c_{ci}}{\text{GNE}_c}$
Union-wide consumption shares	$b \in \mathbb{R}^N$, $b_i = \sum_{c=1}^{C} \chi_c \beta_{ci}$
Input–output matrix	$\Omega \in \mathbb{R}^{N\times N}$, $\omega_{ij} = \frac{p_j x_{ij}}{mc_i y_i}$
Labor input matrix	$\boldsymbol{\alpha} \in \mathbb{R}^{N\times C}$, $\alpha_{ic} = \frac{w_c L_{ic}}{mc_i y_i}$
Leontief inverse	$\Psi = (I - \Omega)^{-1}$
Leontief inverse of factors	$\Psi_{(f)} = \Psi \boldsymbol{\alpha}_{(f)}$
Pass-throughs of nominal wages into prices	$\varrho^w = (I - \boldsymbol{\delta}\Omega)^{-1}\boldsymbol{\delta}\boldsymbol{\alpha}$, $\varrho^w_{ic} = \frac{d\log p_i}{d\log w_c}$
Exposures of country c to sectors	$(\lambda^c)' = \beta_c'\Psi$
Domar weights	$\lambda' = b'\Psi$, $\lambda_i = \frac{p_i y_i}{\text{GNE}}$
Exposures of country c to factors	$(\Lambda^c)' = \beta_c'\Psi\boldsymbol{\alpha}$
Labor income shares	$\Lambda' = b'\Psi\boldsymbol{\alpha}$, $\Lambda_c = \frac{w_c L_c}{\text{GNE}}$

Input-output matrices. The entry ω_{ij} in the input–output matrix represents the direct elasticity of sector i's marginal cost with respect to the price of sector j. In contrast, the element Ψ_{ij} of the associated Leontief inverse captures the total exposure of sector i to sector j, incorporating both direct and indirect effects—namely, how price changes in sector j transmit through input-output linkages to affect the marginal cost of sector i. [7] By analogy, the element Ψ_{if} in the Leontief inverse of factor inputs captures the total exposure of sector i's marginal cost to country f's nominal wage, accounting for both direct and indirect

[7] This follows from the Neumann series expansion of the Leontief inverse: $\Psi = (I - \Omega)^{-1} = I + \Omega + \Omega^2 + \cdots$, which captures successive rounds of input propagation across sectors.

effects. Since value added in each sector is entirely composed of labor inputs from different countries, it follows that $\sum_{f=1}^{C} \Psi_{if} = 1$ holds for all sectors.

Domar weights. The Domar weight (or sales share) of sector i, denoted by $\lambda_i = p_i y_i/\text{GNE}$, captures the economy's total exposure to sector i, directly and indirectly. Formally, it aggregates the i-th column of the Leontief inverse Ψ using union-wide consumption shares β, i.e., $\lambda_i = \sum_{j=1}^{N} \beta_j \Psi_{ji}$. Similarly, the labor income share of country c, denoted by $\Lambda_c = w_c L_c/\text{GNE}$, measures country c's labor income as a share of total nominal output. It reflects the economy's total exposure to labor from country c, and relates to the Leontief structure via $\Lambda_c = \sum_{j=1}^{N} \beta_j \Psi_{jc}$. Note that the ratio $\Lambda_c/\chi_c = \frac{w_c L_c}{\text{GNE}_c}$ defines country c's labor wedge, measuring the share of labor income in country c's total expenditure. In the flexible-price (efficient) equilibrium, the fiscal setup implies $\Lambda_c = \chi_c$, so the labor wedge equals one in all countries.

Analogously, country-specific Domar weights are constructed using country c's own consumption shares β_c. The weight $\lambda_i^c = \sum_{j=1}^{N} \beta_{ci} \Psi_{ji}$ measures country c's consumption cost exposure to sector i, while $\Lambda_f^c = \sum_{j=1}^{N} \beta_{ci} \Psi_{jf}$ measures its exposure to labor from country f. These country-level weights provide a decomposition of aggregate Domar weights across consumers, satisfying the identities: $\lambda_i = \sum_{c=1}^{C} \chi_c \lambda_i^c$ and $\Lambda_f = \sum_{c=1}^{C} \chi_c \Lambda_f^c$.

Pass-throughs of nominal wages into prices. Due to nominal rigidities, sectors only partially transmit changes in marginal costs along the production chain. As a result, the pass-through of nominal wages to sectoral prices is dampened relative to the frictionless benchmark and is lower than sectors' total structural exposure to labor inputs.

Formally, the matrix of pass-throughs of nominal wages to sectoral prices is given by $\varrho^w = (I - \boldsymbol{\delta}\Omega)^{-1}\boldsymbol{\delta\alpha}$, where each element ϱ_{ic}^w quantifies the elasticity of the price in sector i with respect to the nominal wage in country c. The production network governs how cost shocks propagate across sectors, with nominal rigidities preventing full transmission. In particular, the effective pass-through to a sector's price is bounded above by the product of that sector's own price adjustment probability and its total (direct and indirect) exposure to labor from country c: $\varrho_{ic}^w \leq \delta_i \Psi_{ic}$.

3.2 Basic Results

In the presence of nominal rigidities, monetary policy generates distortions that reallocate resources across industries and countries. This section establishes that, to a first-order approximation, country-level employment gaps, income shares, and consumption gaps are all proportional to weighted sums of sectoral ex post markups, with weights determined by production networks, ownership structures, and labor supply elasticities.

Lemma 1. To a first-order approximation, the country-level employment gaps $\{\hat{l}_c\}_{c=1}^{C}$ and income share changes $\{\hat{\chi}_c\}_{c=1}^{C}$ are linear functions of sectoral ex post markups $\{\mu_i\}_{i=1}^{N}$

$$\hat{l}_c = \sum_{i=1}^{N} \ell_{ic}^{\mu} \log \mu_i, \quad \ell_{ic}^{\mu} \doteq \frac{d \log L_c}{d \log \mu_i} = -\frac{\eta_c}{1+\eta_c} \cdot \frac{\Phi_{ic}\lambda_i}{\chi_c} \tag{7}$$

$$\hat{\chi}_c = \sum_{i=1}^{N} \Gamma_{ic} \log \mu_i, \quad \Gamma_{ic} \doteq \frac{d \log \chi_c}{d \log \mu_i} = \chi_c^{-1} \lambda_i \sum_{f=1}^{C} Q_{cf}(\Phi_{if} - \Psi_{if}). \tag{8}$$

Here, $\ell^{\mu} \in \mathbb{R}^{N\times C}$ and $\Gamma \in \mathbb{R}^{N\times C}$ respectively record the elasticities of country-level labor supply and income shares with respect to sectoral markups. The matrix $Q = (I - \mathbf{\Lambda}')|_{\mathcal{S}}^{-1}$ denotes the inverse of $I - \mathbf{\Lambda}'$ restricted to the subspace $\mathcal{S} = \left\{\mathbf{x} \in \mathbb{R}^C : \mathbf{1}'\mathbf{x} = 0\right\}$, where $\mathbf{\Lambda} \in \mathbb{R}^{C\times C}$ has entries $\mathbf{\Lambda}(c, f) = \Lambda_f^c$, representing country c's consumption cost exposure to labor from country f.

The first equation in Lemma 1 shows that each sector i's contribution to country c's employment gap is proportional to its sales share (λ_i) and to country c's ownership share Φ_{ic}. This result provides a significant departure from representative-agent frameworks, such as Rubbo (2023), where sectoral contributions to the aggregate employment gap are governed solely by the Domar weights. The intuition rests on endogenous fluctuations in the country-specific labor wedge. An increase in sectoral markups acts as an implicit tax on production, yet its impact is mediated by income composition. Countries that hold larger ownership shares in highly distorted sectors receive a greater fraction of income in the form of profits relative to total expenditure and therefore exhibit labor wedges that are more sensitive to changes in ex post markups. As ex post markups rise and labor wedges widen, the associated wealth effects are stronger in these countries, leading to larger reductions in labor supply. Consequently, employment responses are inherently shaped by a country's position within the economy's value chain.

The second equation in Lemma 1 characterizes how country c's income share responds to sectoral markup changes. The structure of the elasticity Γ_{ic} reflects three forces: (i) the sector's centrality in nominal expenditure (λ_i); (ii) the differential between country f's ownership claims on sector i (Φ_{if}) and sector i's exposure to the labor from country f (Ψ_{if}); and (iii) the country-income network Q, which governs the general equilibrium propagation of income changes across countries.

Structurally, the matrix Q functions as a country-level Leontief inverse or a global income multiplier. In an economy characterized by deep interdependence—where countries consume baskets that embody the labor of others—an initial shift in sectoral profits or labor demand triggers a cascade of expenditure–income feedback. As a country's income share adjusts, its endogenous expenditure response alters the labor demand and income of all other countries in the union. The matrix Q aggregates these infinite rounds of expenditure-income feedback, mapping primitive profit-vs-factor shocks into final equilibrium adjustments.

The economic intuition behind the income redistribution mechanism is as follows. An increase in the markup of sector i simultaneously raises sectoral profits and depresses output demand. The resulting direct redistribution of income across countries is governed by the vector of differences between ownership claims and labor exposure, $\{\Phi_{if} - \Psi_{if}\}_{f=1}^{C}$: country f benefits from higher markups through its ownership claims on sector i's profits, captured by Φ_{if}. At the same time, higher markups reduce demand for sector i's output and, through sector i's exposure to labor from country f, Ψ_{if}, lead to a contraction in labor demand and labor income in that country. These primitive incidence effects are subsequently processed through the linear operator Q, which accounts for the general-equilibrium feedback across the country network. Since both the primitive incidence and the network propagation are linear, the resulting elasticity of country c's income share is a linear combination of all country-level differentials $(\Phi_{if} - \Psi_{if})$, as expressed in equation (8). This expression highlights that income-share adjustments are entirely driven by asymmetries between sectoral profit ownership and sectoral labor exposure across industries and countries.

In the special case with two countries—say, country h and r—equation (8) admits an explicit solution. The change in country h's income share is given by

$$\hat{\chi}_h = \chi_h^{-1}(\Lambda_h^r + \Lambda_r^h)^{-1} \sum_{i=1}^{N} \lambda_i(\Phi_{ih} - \Psi_{ih}) \log \mu_i.$$

This formula makes transparent how markup changes redistribute income across countries. An increase in $\log \mu_i$ raises profits in sector i, benefiting country h's profit income in proportion to its ownership claims Φ_{ih}. At the same time, higher markups reduce sector i's demand for labor across countries. Since sector i's exposure to labor from country h is given by Ψ_{ih}, the contraction in labor demand lowers the labor income accruing to country h in proportion to Ψ_{ih}. These opposing forces partially offset each other, with the net effect scaled by the sector's Domar weight λ_i. The resulting expression summarizes the total impact of markup changes on country h's income share, incorporating both profit incidence and factor-demand effects.

Remark 1. Combining equation (7) with the relationship between sectoral markups and prices in equation (4), employment gaps can be related to sectoral price changes as follows,

$$(1 + 1/\eta_c)\chi_c \hat{\ell}_c = \lambda'(\boldsymbol{\delta}^{-1} - I)\,\mathrm{diag}(\Phi(:,c))\log \boldsymbol{p}.$$

This equation reveals that the transmission of monetary policy to the employment of country c breaks down under each of the following limiting cases: (i) there are no nominal rigidities across sectors, i.e., $\delta_i = 1$ for all i; (ii) labor supply is perfectly inelastic, $\eta_c = 0$; or (iii) country c has no profit claims across sectors, $\Phi_{ic} = 0$ for all i.

Remark 2. A key implication of equation (8) is that markup shocks have no incidence on country income shares under *labor-equivalent ownership*, defined by $\Phi_{if} = \Psi_{if}$ for all sectors i and countries f. Under this condition, each country receives profits from a sector in exactly the same proportion as it supplies direct and indirect labor services to that sector, so profit-income and labor-income effects exactly offset one another. This eliminates the structural asymmetry that drives income-share redistribution, ensuring that monetary shocks—while still generating aggregate distortions—do not induce any ex post redistribution of income across countries. Consequently, $\hat{\chi}_c = 0$ for all countries.

I now turn to country-level consumption gaps and show that, in a heterogeneous-agent economy, consumption and employment gaps generally do not coincide. Proposition 1 establishes that the difference between consumption and employment gaps is proportional to a sum of sectoral ex post markups.

Proposition 1 (Country-Level Allocative Efficiency)**.** To a first-order (log-linear) approximation, the allocative efficiency of country c, measured by the difference between its consumption and employment gaps, can be decomposed into a direct incidence effect and a Viner's factor terms-of-trade effect,

$$\hat{c}_c - \hat{\ell}_c = \underbrace{\sum_{i=1}^{N}\left(\frac{\lambda_i \Phi_{ic}}{\chi_c} - \lambda_i^c\right)\log \mu_i}_{\text{direct incidence (DI)}} + \underbrace{d\log w_c - \sum_{f=1}^{C}\Lambda_f^c\, d\log w_f}_{\text{factoral terms-of-trade (FToT)}}. \tag{9}$$

Furthermore, the FToT component can be rewritten in terms of sectoral markups as

$$\Delta\text{FToT} = \sum_{i=1}^{N}\left[\left(\Gamma_{ic} + \frac{1}{\eta_c}\ell_{ic}^{\mu}\right) - \sum_{f=1}^{C}\Lambda_f^c\left(\Gamma_{if} + \frac{1}{\eta_f}\ell_{if}^{\mu}\right)\right]\log \mu_i. \tag{10}$$

Combining both components, country-level allocative efficiency can be written compactly as a linear exposure to sectoral markups

$$\hat{c}_c - \hat{\ell}_c = \sum_{i=1}^{N} \mathcal{J}_{ic} \, \log \mu_i = \mathcal{J}_c' \log \boldsymbol{\mu},$$

where $\mathcal{J}_c \in \mathbb{R}^N$ collects the total markup-exposure coefficients relevant for the allocative efficiency of country c. For completeness, I also construct the direct-incidence index $\mathcal{J}_c^{\mathrm{DI}}$ and the factoral terms-of-trade index $\mathcal{J}_c^{\mathrm{FToT}}$, which together decompose $\mathcal{J}_c$ into its direct-incidence and terms-of-trade components.[8]

The direct incidence (DI) effect represents a generalized terms-of-trade effect that captures how changes in sectoral markups redistribute real income across countries while holding factor prices fixed. In the classical international macroeconomics literature, a country's terms of trade improve when the price of its exports rises relative to its imports, shifting rents from foreign consumers to domestic producers. The direct incidence effect extends this logic to a production-network environment by recognizing each country's dual role as both a consumer of sectoral goods and a residual claimant on sectoral profits.

Specifically, a rise in the markup $\log \mu_i$ increases the cost of country c's consumption basket by λ_i^c representing a deterioration in the country's terms of trade on the consumption side. At the same time, it increases country c's income by $\lambda_i \Phi_{ic}/\chi_c$ through its ownership claims on sectoral profits, corresponding to a terms-of-trade improvement on the income side. The direct-incidence index $\mathcal{J}_{ic}^{\mathrm{DI}}$ summarizes the net effect of these opposing forces. It is positive when country c is a net owner of good i, so that profit income more than offsets the increased consumption cost, and negative when country c is a net user, bearing higher user cost without sufficient profit compensation.

In a currency union without input-output linkages, and assuming ownership segmentation such that firms are owned domestically, the direct incidence effect maps precisely to the classical changes in terms-of-trade found in the international macroeconomics literature

$$\Delta \mathrm{DI} = \mathrm{Cov}(\mathcal{J}_c^{\mathrm{DI}}, \log \boldsymbol{\mu}) = \underbrace{\sum_{i \in N_c} \sum_{f \neq c} \chi_f \beta_{fi} / \chi_c \log \mu_i}_{\text{change in export price}} - \underbrace{\sum_{i \notin N_c} \beta_{ci} \log \mu_i}_{\text{change in import price}} .$$

[8] The coefficients appearing in the decomposition are $\mathcal{J}_{ic}^{\mathrm{DI}} = \frac{\lambda_i \Phi_{ic}}{\chi_c} - \lambda_i^c$, $\mathcal{J}_{ic}^{\mathrm{FToT}} = \Gamma_{ic} + \frac{1}{\eta_c} \ell_{ic}^{\mu} - \sum_{f=1}^{C} \Lambda_f^c (\Gamma_{if} + \frac{1}{\eta_f} \ell_{if}^{\mu})$, and they sum to the total coefficient $\mathcal{J}_{ic} = \mathcal{J}_{ic}^{\mathrm{DI}} + \mathcal{J}_{ic}^{\mathrm{FToT}}$.

In this limiting case, the index $\mathcal{J}_{ic}^{\mathrm{DI}}$ captures export exposure. It is positive for domestic industries (exports) and negative for foreign industries (imports), summing to zero across all sectors. Thus, in a general HAIO economy, the direct-incidence effect can be understood as changes in a country's network-adjusted commodity terms of trade.

The factoral terms-of-trade (FToT) effect, by contrast, captures the redistribution arising from endogenous changes in relative factor prices. It measures the change in relative nominal wages between the labor supplied by country c and the labor to which its consumption basket is exposed. Specifically, it reflects the difference between the change in country c's own wage, $d\log w_c$, and the exposure-weighted average wage change across all labor types f embodied in its consumption basket, $\sum_{f\in C}\Lambda_f^c\, d\log w_f$. Since the weights satisfy $\sum_{f\in C}\Lambda_f^c = 1$, this component isolates a pure relative-wage change between labor types across countries. Equation (10) then maps these wage movements to the underlying structural distortions, expressing the FToT effect in terms of sectoral markups.

By integrating the direct-incidence and factoral terms-of-trade effects, country-level allocative efficiency provides a comprehensive measure of changes in a country's **overall terms of trade**. Monetary policy influences these relative terms of trade by shifting sectoral markups, thereby redistributing allocative efficiency across countries. However, as established in the following corollary, such redistribution is neutral from an aggregate efficiency perspective: when evaluated using income-share-weighted aggregation, these first-order gains and losses perfectly cancel out, reflecting the zero-sum nature of terms-of-trade shifts in a closed economy.

Corollary 1. Country-level allocative efficiencies, while generally nonzero, exactly offset at first order when aggregated using income shares

$$\sum_{c=1}^{C}\chi_c(\hat{c}_c - \hat{l}_c) = \sum_{c=1}^{C}\chi_c\mathcal{J}_c'\log\boldsymbol{\mu} = 0. \tag{11}$$

This aggregate neutrality reflects a macro-level envelope theorem. As shown by Negishi (1960), the competitive equilibrium allocation can be replicated by a Negishi planner who maximizes a weighted sum of country utilities.[9] As a result, the economy-wide allocative efficiency corresponds to the first-order variation in the planner's welfare function. Since the equilibrium allocation is optimal from the planner's perspective, any marginal reallocation—such as that induced by a monetary shock—yields no first-order improvement

[9] Lemma 3 shows that when utility is logarithmic in consumption, the Negishi welfare weights coincide with income shares.

in aggregate welfare. Thus, while monetary policy redistributes income across countries, it does not improve allocative efficiency at the union-wide level.

Examples 1 below illustrate the role of input-output linkages in shaping redistributive effects. For simplicity, I shut down the factoral terms-of-trade channel by assuming labor-equivalent ownership ($\Phi_{if} = \Psi_{if}$) and perfectly elastic labor supply in all countries.

Example 1 (Two-Stage Vertical Chain)**.** Consider an economy featuring a vertical production chain (Figure 1). The upstream producer (sector 2) uses labor from country 2 to manufacture an intermediate input, which is then purchased by the downstream producer (sector 1). The downstream sector combines this input with labor from country 1 to produce the final good, with a labor share of α_1. Proposition 1 implies that the allocative efficiencies across countries are determined by the ex post markup of the upstream sector μ_2:

$$\hat{c}_1 - \hat{l}_1 = -(1-\alpha_1)\log\mu_2 \quad \text{and} \quad \hat{c}_2 - \hat{l}_2 = \alpha_1 \log\mu_2.$$

In this setting, an expansionary monetary policy that reduces the upstream sector's markup improves country 1's terms of trade but deteriorates those of country 2. This is because, under labor-equivalent ownership, both countries' terms of trade are independent of changes in downstream markups.[10] Since country 2 is a net supplier of the upstream good while country 1 is a net user, a reduction in the upstream sector's markup benefits users at the expense of suppliers.

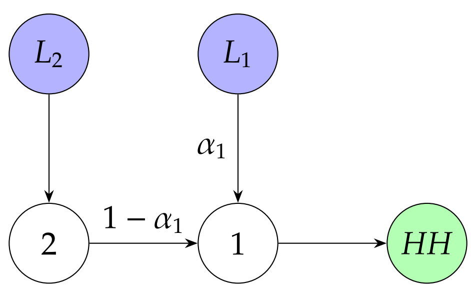

Figure 1: Two-stage vertical chain

3.3 Monetary Transmission

I now characterize the transmission of monetary policy within the HAIO framework. The following proposition derives the general equilibrium elasticities of sectoral prices and

[10]Given the assumed labor-equivalent ownership structure, country 2 owns sector 2 and holds a $1-\alpha_1$ share of sector 1.

country-level employment with respect to a monetary shock, accounting for the endogenous feedback between nominal rigidities and cross-country heterogeneity.

Proposition 2 (Monetary Transmission in HAIO)**.** Assume that the matrix $I - \varrho^w(\Gamma + \ell^\mu \boldsymbol{\eta}^{-1})'(I - \boldsymbol{\delta}^{-1})$ is nonsingular. Then, in response to a change in the money supply, the elasticities of sectoral prices with respect to nominal aggregate demand are given by

$$\varrho^m \equiv \frac{d\log p}{d\log m} = [I - \varrho^w(\Gamma + \ell^\mu \boldsymbol{\eta}^{-1})'(I - \boldsymbol{\delta}^{-1})]^{-1}\varrho^w \mathbf{1},$$

where $\boldsymbol{\eta} = \mathrm{diag}(\eta_1, \ldots, \eta_C)$ denotes the diagonal matrix of household-specific labor supply elasticities. The corresponding effects on household employment are:

$$\boldsymbol{\ell}^m \equiv \frac{d\log L}{d\log m} = \ell^{\mu'}(I - \boldsymbol{\delta}^{-1})\varrho^m.$$

Proposition 2 describes how a monetary expansion is transmitted through the HAIO economy. Sectoral prices respond through two mechanisms. First, a numéraire effect raises nominal wages uniformly, which, in the presence of nominal price rigidities, increases sectoral prices mechanically. Second, monetary shocks alter ex post markups, which redistribute income and employment across countries and thereby change relative nominal wages through the factoral terms-of-trade channel. These wage adjustments feed back into marginal costs and amplify or dampen price responses across sectors. The matrix inverse in ϱ^m captures this general-equilibrium feedback loop, while the associated employment elasticities $\boldsymbol{\ell}^m$ summarize the resulting reallocation of labor across countries. Together, the proposition shows that monetary policy affects prices and employment not only through the aggregate numéraire effect, but also through redistribution across countries embedded in the production network.

To build intuition, the following example shows how production networks affect the sectoral price pass-through of monetary shocks.

Example 2 (General Vertical Economy)**.** This example extends the two-stage vertical chain in Example 1 to a general setting with C countries and C sectors. As before, I assume labor-equivalent ownership ($\Phi_{if} = \Psi_{if}$) and perfectly elastic labor supply in all countries, which shuts down feedback from nominal wages to the money supply arising from endogenous changes in ex post markups. As illustrated in Figure 2, the most upstream sector C uses labor from country C to produce intermediate good C. Each downstream sector k then combines intermediate input $k + 1$ from its immediate upstream sector with labor supplied by the representative household in country k to produce good k. Only the final good produced by sector 1 is consumed by households across countries.

Given this structure, Proposition 2 shows that the elasticity of sector i's price with respect to the money supply satisfies

$$\varrho_i^m = \alpha_i \delta_i + (1 - \alpha_i)\delta_i \varrho_{i+1}^m,$$

with terminal condition $\varrho_C^m = \delta_C$. This recursion reveals that nominal rigidities accumulate along the supply chain. In particular, given the fact that ϱ_i^m lies between 0 and 1, when a sector relies more heavily on upstream inputs (that is, when α_i is small), it effectively imports the stickiness of the entire upstream network, causing its price to respond more sluggishly to monetary shocks and thereby echoing the insight in Basu (1995).

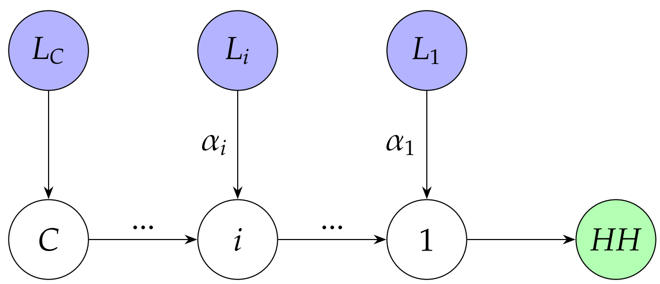

Figure 2: General vertical economy

Remark 3. In an economy with input–output linkages, price rigidities compound along the production chain (as illustrated in Example 2), so an expansionary monetary policy generates a larger reduction in upstream markups than in downstream markups. Because the direct-incidence index is positive when a country is a net supplier of a sector and negative when it is a net user, a decline in markups improves country-level allocative efficiency through the user channel but worsens it through the supplier channel. In response to monetary expansions, countries that are primarily upstream suppliers experience a larger reduction in their supplier rents than the gain they obtain as downstream users. By contrast, countries that are predominantly downstream face the reverse imbalance: the reduction in supplier rents is smaller than the gain they receive as downstream users. As a result, expansionary monetary policy generates larger improvements in allocative efficiency in downstream-oriented countries.

4 Optimal Monetary Policy and Inflation Stance

In multi-sector economies with input–output linkages and nominal rigidities, monetary policy cannot attain the first-best allocation and instead trades off competing welfare objectives (see, e.g., La'O and Tahbaz-Salehi, 2022). This section formulates a Ramsey problem for a heterogeneous-agent input–output economy under a utilitarian Bergson–Samuelson welfare criterion with arbitrary Pareto weights. The resulting characterization of optimal policy motivates the definitions of unilateral inflation stances and policy-alignment losses, which are used to analyze the heterogeneous incidence of a common monetary policy across countries.

4.1 Welfare Loss and Policy Objective

A central question in welfare measurement is how to aggregate country utilities into an economy-wide metric. Throughout the paper, I adopt a utilitarian Bergson–Samuelson welfare function,

$$W(\{\kappa_c\}_{c=1}^{C}) = \sum_{c=1}^{C} \kappa_c U_c,$$

where $\{\kappa_c\}_{c=1}^{C}$ denotes an arbitrary set of non-negative Pareto weights, which reflect the political influence of each country. Let $\mathbf{e}_c$ denote the unilateral Pareto weights, where $\kappa_c = 1$ and $\kappa_{c'} = 0$ for all $c' \neq c$. This corresponds to a scenario in which the monetary authority place full weight on country c's welfare. Under this formulation, the aggregate welfare criterion collapses to that country's utility: $W(\mathbf{e}_c) = U_c$. The welfare analysis begins with this polar case of policy weighting and then generalizes to arbitrary Pareto weights, leading to the following result.

Lemma 2 (Unilateral Welfare Loss)**.** Up to a second-order approximation, the welfare loss under unilateral Pareto weights for country c is given by:

$$W(\mathbf{e}_c) - W^{*}(\mathbf{e}_c) \;=\; \mathcal{J}_c' \log \boldsymbol{\mu} \;-\; \tfrac{1}{2} \log \boldsymbol{\mu}' \, \mathcal{L}_c \, \log \boldsymbol{\mu}, \tag{12}$$

where the matrix $\mathcal{L}_c$ summarizes second-order welfare losses and is decomposed as

$$\mathcal{L}_c \equiv \mathcal{L}_c^{\text{e.g.}} + \mathcal{L}_c^{\text{within}} + \mathcal{L}_c^{\text{across}},$$

with the following components:

$$\mathcal{L}_c^{\text{e.g.}} = (1+1/\eta_c)\ell^{\mu}_{(:,c)}[\ell^{\mu}_{(:,c)}]',$$
$$\mathcal{L}_c^{\text{within}} = \text{diag}(\lambda^c \circ \theta)(\boldsymbol{\delta}^{-1} - I)^{-1},$$
$$\mathcal{L}_c^{\text{across}} = 2\left(\frac{\eta_c}{1+\eta_c}\Upsilon^c + \sum_{f=1}^{C}\frac{1}{1+\eta_f}\Lambda_f^c\Upsilon^f - \Xi^c + \sum_{f=1}^{C}\Lambda_f^c\Xi^f\right),$$

where the matrices Υ^c, $\Xi^c \in \mathbb{R}^{N\times N}$ have entries $\Upsilon^c_{ij} = \frac{\partial^2 \log(\Lambda_c/\chi_c)}{\partial \log\mu_i \partial\log\mu_j}$ and $\Xi^c_{ij} = \frac{\partial^2 \log\chi_c}{\partial\log\mu_i\partial\log\mu_j}$. [11]

Lemma 2 provides a closed-form second-order approximation of country-level welfare loss in a HAIO economy. Specifically, equation (12) illustrates that the welfare loss function for country c consists of a first-order allocative efficiency specific to that country, along with a second-order loss comprising three distinct components.

The first two components parallel those in standard New Keynesian model (e.g., Galí, 2008), reflecting the classical trade-off between employment-gap stabilization and price stability. The first term, $\log\boldsymbol{\mu}'\mathcal{L}_c^{\text{e.g.}}\log\boldsymbol{\mu} = (1+1/\eta_c)\hat{l}_c^2$, captures welfare loss from the volatility in the employment gap of country c. This term vanishes when labor supply is perfectly inelastic ($\eta_c = 0$).

The second term, $\log\boldsymbol{\mu}'\mathcal{L}_c^{\text{within}}\log\boldsymbol{\mu} = \sum_{i=1}^{N}\lambda_i^c\theta_i\frac{\delta_i}{1-\delta_i}\log\mu_i^2$ measures welfare loss from price dispersion within industries. This loss is strictly increasing with the within-sector elasticity of substitution, θ_i, since greater substitutability amplifies resource misallocation arising from relative price distortions. The key distinction from La'O and Tahbaz-Salehi (2022) lies in the weighting of sectoral misallocation. In this framework, price dispersion in sector i is weighted by country c's consumption cost exposure, λ_i^c, rather than the sector i's Domar weight, λ_i. This refinement highlights how heterogeneity in consumption baskets across countries generates unequal exposure to sectoral inefficiencies.

The third term, $\log\boldsymbol{\mu}'\mathcal{L}_c^{\text{across}}\log\boldsymbol{\mu}$, arises exclusively in multisector economies and characterizes the misallocation of resources across industries. Building on second-order

[11] Specifically, Lemma 4 in the Appendix formalizes the reduced-form expressions

$$\Xi^c_{ij} = -\frac{1}{2}\Gamma_{jc}\Gamma_{ic} + \chi_c^{-1}\sum_{f=1}^{C} Q_{cf}(\Psi_{if} - \Phi_{if})\left(\sum_{g=1}^{C}\lambda_i^g\chi_g\Gamma_{jg} + \frac{1}{2}\lambda_i\iota_{ij} - \lambda_j\Psi_{ji}\right),$$
$$\Upsilon^c_{ij} = \frac{\Phi_{ic}}{\chi_c}\left(\lambda_j\Psi_{ji} - \sum_{f=1}^{C}\lambda_i^f\chi_f\Gamma_{jf} + \lambda_i\Gamma_{jc} - \frac{1}{2}\lambda_i\iota_{ij} - \frac{1}{2}\lambda_i\lambda_j\frac{\Phi_{jc}}{\chi_c}\right),$$

where ι_{ij} denotes the (i,j)-th entry of the identity matrix.

approximations of labor wedges and income shares, it extends cross-sector misallocation term in Baqaee and Farhi (2024) to the country level. In a standard representative-agent production network economy, such a term typically captures the second-order approximation of the aggregate labor wedge (see e.g., Baqaee and Farhi, 2020; La'O and Tahbaz-Salehi, 2022). In this heterogeneous-agent framework, however, the term is broader in scope: it integrates the second-order approximation of country c's labor wedge, Λ_c/χ_c, alongside the second-order approximation of the country's factoral terms-of-trade. Consequently, it captures not only the standard inefficiencies in sectoral labor allocation but also the distortionary effects of monetary shocks on the relative factor prices embedded in the country's consumption basket.

Building on Lemma 2, Proposition 3 establishes the general Bergson–Samuelson welfare loss by aggregating the unilateral welfare losses using arbitrary Pareto weights.

Proposition 3 (Bergson–Samuelson Welfare Loss)**.** The Bergson–Samuelson welfare loss function with arbitrary Pareto weights $\{\kappa_c\}_{c=1}^C$ can be expressed as the weighted sum of country-level welfare losses:

$$\begin{aligned} W(\{\kappa_c\}_{c=1}^C) - W^*(\{\kappa_c\}_{c=1}^C) &= \sum_{c=1}^C \kappa_c \Big[W(\mathbf{e}_c) - W^*(\mathbf{e}_c) \Big] \\ &= \sum_{c=1}^C \kappa_c \mathcal{J}_c' \log \boldsymbol{\mu} - \tfrac{1}{2} \log \boldsymbol{\mu}' \, \mathcal{L}(\{\kappa_c\}) \log \boldsymbol{\mu}, \end{aligned} \tag{13}$$

where the aggregate loss matrix $\mathcal{L}(\{\kappa_c\})$ is the Pareto-weighted average of country-level loss matrices,

$$\mathcal{L}(\{\kappa_c\}) \;\equiv\; \sum_{c=1}^C \kappa_c \mathcal{L}_c \;=\; \mathcal{L}^{\text{e.g.}}(\{\kappa_c\}) + \mathcal{L}^{\text{within}}(\{\kappa_c\}) + \mathcal{L}^{\text{across}}(\{\kappa_c\}) \tag{14}$$

with each component of $\mathcal{L}(\{\kappa_c\})$ defined as

$$\mathcal{L}^{\text{e.g.}}(\{\kappa_c\}) = \sum_{c=1}^C \kappa_c \mathcal{L}_c^{\text{e.g.}}, \quad \mathcal{L}^{\text{within}}(\{\kappa_c\}) = \sum_{c=1}^C \kappa_c \mathcal{L}_c^{\text{within}}, \quad \mathcal{L}^{\text{across}}(\{\kappa_c\}) = \sum_{c=1}^C \kappa_c \mathcal{L}_c^{\text{across}}.$$

Proposition 3 generalizes the welfare loss expression from a New Keynesian production network economy with a representative agent (e.g., La'O and Tahbaz-Salehi, 2022) to a fully heterogeneous-agent input–output (HAIO) economy. While Baqaee and Farhi (2024) similarly provide a second-order approximation of welfare in a HAIO framework, their analysis is restricted to the special case where Pareto weights are given by income shares. In contrast, Proposition 3 accommodates arbitrary welfare weights, facilitating a more flexible evaluation of the distributional incidence of a common monetary policy. This generalization is central to characterizing the policy-alignment loss (PAL) in Section 4.4, as it explicitly accounts for the potential misalignment between a country's unilateral preferences and the centralized objectives of the monetary authority.

In another comparison to Baqaee and Farhi (2024), the welfare function in equation (13) implies that monetary policy faces a fundamental trade-off between aggregate stabilization and first-order redistribution. These redistributive motives are silenced under the income-share weighting assumed in their framework, forcing the central bank to focus exclusively on aggregate efficiency. The following corollary formalizes this result, showing that the welfare objective collapses to a purely efficiency-based, quadratic measure, when Pareto weights coincide with countries' income shares.

Corollary 2 (Redistribution Neutrality under Income-Share Weighting)**.** When Pareto weights coincide with countries' income shares, such that $\kappa_c = \chi_c$ for all c, the optimal monetary policy is redistribution-neutral. In this case, the central bank cannot enhance aggregate efficiency through manipulating the countries' overall terms of trade. Consequently, the aggregate welfare loss becomes purely quadratic in ex post markups [12]

$$W(\{\chi_c\}_{c=1}^{C}) - W^*(\{\chi_c\}_{c=1}^{C}) = -\tfrac{1}{2}\log\boldsymbol{\mu}'\,\mathcal{L}(\{\chi_c\})\log\boldsymbol{\mu}.$$

4.2 Optimal Policy

To characterize optimal monetary policy, I model the monetary authority as a Ramsey planner who chooses the policy to minimize aggregate welfare loss, subject to the competitive-equilibrium constraints of the economy. The nominal policy instrument is the money supply $\log m(z)$, which affects equilibrium prices and markups through the cash-in-advance constraint.

While policy is implemented via $\log m(z)$, welfare depends only on the induced movements in prices and markups. It is therefore without loss of generality to represent monetary

[12] In this case, the across-sector misallocation matrix simplifies substantially to

$$\mathcal{L}^{\text{across}}_{ij}(\{\chi_c\}) = \lambda_i\Psi_{ij} + \lambda_j\Psi_{ji} - \lambda_i\iota_{ij} + \sum_{c=1}^{C}\Big[(\lambda_i\Phi_{ic} - \lambda_i^c\chi_c)\Gamma_{jc} + (\lambda_j\Phi_{jc} - \lambda_j^c\chi_c)\Gamma_{ic} - \chi_c^{-1}\lambda_i\lambda_j\Phi_{ic}\Phi_{jc}\Big].$$

policy by a sectoral price index target of the form

$$\zeta' \log \boldsymbol{p} = \pi,$$

for some $(\zeta, \pi) \in \mathcal{P} \subseteq \mathbb{R}^N \times \mathbb{R}$. This representation captures the reduced-form implications of monetary policy through its impact on the sectoral price.

Lemma 6 in the Appendix shows that, to a first-order approximation, any price-targeting rule of the form $\zeta' \log \boldsymbol{p} = \pi$ can be implemented by an appropriate choice of the money supply rule $\log m(z)$. In particular, for any admissible (ζ, π), there exists a policy of the form $\log m(z) = \varsigma_0 + \varsigma' \log z$ that reproduces the same equilibrium price allocation. This result justifies treating (ζ, π) as reduced-form policy instruments throughout the analysis.

Building on this result, the Ramsey-optimal policy under arbitrary Pareto weights is characterized by the optimal price-targeting index ζ^* and the associated inflation bias π^* that jointly minimize the aggregate welfare loss specified in equation (13).

Theorem 1. Given the Pareto weights $\{\kappa_c\}_{c=1}^{C}$, the optimal monetary policy solves for (ζ^*, π^*) that minimizes the welfare loss in equation (13). The resulting policy targets a sectoral price index of the form

$$\sum_{j=1}^{N} \zeta_j^*(\{\kappa_c\}) \log p_j = \pi^*(\{\kappa_c\}), \tag{15}$$

where

$$\pi^*(\{\kappa_c\}) = \sum_{c=1}^{C} \kappa_c \, \mathcal{J}_c'(I - \boldsymbol{\delta}^{-1})\varrho^m. \tag{16}$$

The optimal sectoral weight can be further broken down into three components,

$$\zeta_j^*(\{\kappa_c\}) = \zeta_j^{\text{e.g.}}(\{\kappa_c\}) + \zeta_j^{\text{within}}(\{\kappa_c\}) + \zeta_j^{\text{across}}(\{\kappa_c\}),$$

with

$$\zeta_j^{\text{e.g.}}(\{\kappa_c\}) = (\delta_j^{-1} - 1)\lambda_j \sum_{c=1}^{C} \kappa_c\, \Phi_{jc}\, \ell_c^m / \chi_c,$$

$$\zeta_j^{\text{within}}(\{\kappa_c\}) = (\delta_j^{-1} - 1)\theta_j \varrho_j^m \sum_{c=1}^{C} \kappa_c\, \lambda_j^c,$$

$$\zeta_j^{\text{across}}(\{\kappa_c\}) = (\delta_j^{-1} - 1) \sum_{i=1}^{N} (\delta_i^{-1} - 1)\varrho_i^m\, \mathcal{L}_{ij}^{\text{across}}(\{\kappa_c\}).$$

Theorem 1 characterizes the optimal policy as a function of sectoral nominal rigidities, the production network, and ownership structures. As implied by Corollary 2, the monetary authority faces a trade-off between aggregate stabilization and first-order redistribution whenever Pareto weights deviate from income shares. In such cases, an inflation bias arises on the right-hand side of equation (15), reflecting incentives to redistribute across countries by shifting the overall terms of trade in favor of certain groups. By contrast, when Pareto weights align with income shares, the optimal monetary policy reduces to a pure price-stabilization rule with zero inflation bias ($\pi^*(\{\chi_c\}) = 0$). This redistribution-neutral policy serves as the benchmark for centralized optimal monetary policy.[13]

The optimal industry weights in equation (15) determine which price index monetary policy should target to minimize second-order welfare losses arising from nominal rigidities. Recall that the monetary authority must balance between three distinct sources of misallocation: the aggregate volatility of employment gaps, price dispersion within sectors, and price dispersion across sectors. Accordingly, the optimal weights decompose into three components, each capturing the relative welfare importance of these distortions.

4.3 Unilateral Inflation Stance

This section studies monetary policy from the perspective of an individual member country. Taking the unilateral optimal policy as a benchmark, it characterizes the constrained unilateral inflation stance under the union's common price-index regime and examines its relationship to the production structure.

To facilitate the analysis, policy instruments are mapped into sectoral markups, the

[13] Corollary 4 in the Appendix shows that when cross-country asymmetries are shut down by homogeneous consumption baskets and ownership shares aligned with income shares (i.e., $\beta_{ci} = b_i$ and $\Phi_{ic} = \chi_c$ for all c and i), the centralized optimal policy coincides with the one derived in La'O and Tahbaz-Salehi (2022).

channel through which monetary policy affects real allocations. Given the realization of productivity shocks $\log z$, the vector of ex post markups is, to a first-order approximation, a differentiable function of the policy pair $(\zeta, \pi) \in \mathcal{P} \subseteq \mathbb{R}^N \times \mathbb{R}$:

$$\log \boldsymbol{\mu} \;=\; \mathcal{M}(\zeta, \pi; z),$$

where $\mathcal{M} : \mathcal{P} \to \mathbb{R}^N$ represents the reduced-form mapping from monetary policy instruments to sectoral markups.[14] For notational simplicity, the dependence on z is suppressed for notational simplicity.

Substituting this mapping into country c's unilateral welfare loss in equation (12) yields

$$\mathbb{L}_c(\zeta, \pi) \;\equiv\; -\mathcal{J}_c' \mathcal{M}(\zeta, \pi) + \tfrac{1}{2}\, \mathcal{M}(\zeta, \pi)' \mathcal{L}_c \mathcal{M}(\zeta, \pi).$$

As established in Theorem 1, assigning full weight to country c implies that the associated optimal policy solves

$$(\zeta^*(\mathbf{e}_c), \pi^*(\mathbf{e}_c)) \in \underset{(\zeta,\pi)\in\mathcal{P}}{\arg\min}\, \mathbb{L}_c(\zeta, \pi).$$

This policy therefore defines country c's unilateral optimal policy and implements its second-best allocation by minimizing welfare loss over the entire set of monetary instruments.

In practice, however, such an allocation is generally unattainable, as it requires the country to exercise sole authority over those instruments—an arrangement precluded by the union's common price-index regime ζ.[15] Once the price-index regime is taken as given and inflation is restricted to be non–state-contingent, the country's decision problem becomes a third-best one. The following definition formalizes the associated third-best inflation stance.

Definition 3. The unilateral inflation stance of country c is the inflation rate it selects to minimize its expected welfare loss, taking the union's target price-index regime ζ (normalized to sum to one) as given

$$\pi_c(\zeta) = \underset{\pi\in\mathbb{R}}{\arg\min}\, \mathbb{E}[\mathbb{L}_c(\zeta, \pi)].$$

Unlike the second-best unilateral optimal policy, which minimizes welfare loss for every realization of sectoral shocks, the inflation stance is defined in expectation. The expectation operator reflects the non–state-contingent inflation constraint, requiring the

[14] See Lemma 7 in the Appendix for the full expression.

[15] Lemma 8 shows that, under a common price-index regime, replicating the unilateral optimal policy requires state-contingent inflation.

country to choose a single union-wide inflation rate ex ante rather than adjust inflation state by state. Consequently, the third-best policy $(\zeta, \pi_c(\zeta))$ weakly underperforms the second-best unilateral optimal policy $(\zeta^*(\mathbf{e}_c), \pi^*(\mathbf{e}_c))$. Nevertheless, as the following propositions demonstrate, the two objects remain closely connected.

Proposition 4. Let $(\zeta^*(\mathbf{e}_c), \pi^*(\mathbf{e}_c))$ denote country c's unilateral optimal policy. Given the union's target price-index regime ζ (normalized to sum to one), the unilateral inflation stance of country c satisfies

$$\pi_c(\zeta) = \underbrace{\frac{\zeta' \varrho^m}{\zeta^*(\mathbf{e}_c)' \varrho^m}}_{\substack{\text{adjustment scalar due to} \\ \text{different target price indices}}} \pi^*(\mathbf{e_c}). \tag{17}$$

Equation (17) shows that the unilateral inflation stance is proportional to the inflation bias $\pi^*(\mathbf{e}_c)$ of the unilateral optimal policy, rescaled by a factor that accounts for the difference between the country's preferred target price index and union's common index ζ.

Proposition 5. The unilateral inflation stance replicates the expected first-order allocative efficiency of the unilateral optimal policy

$$\mathbb{E}[\mathcal{J}_c' \mathcal{M}(\zeta, \pi_c(\zeta))] = \mathbb{E}[\mathcal{J}_c' \mathcal{M}(\zeta^*(\mathbf{e}_c), \pi^*(\mathbf{e}_c))].$$

Despite the institutional constraints imposed by the union's price-index regime, Proposition 5 demonstrates that the unilateral inflation stance preserves the expected first-order effects of the unconstrained unilateral optimal policy.

The next step is to uncover the economic sources of this inflation stance. Proposition 1 suggests that a country's inflation stance reflects two distinct redistributive motives under unilateral monetary policy, both operating through nominal rigidities:

$$\pi_c(\zeta) = \frac{\zeta' \varrho^m}{[\zeta^*(\mathbf{e}_c)]' \varrho^m} [\underbrace{\mathcal{J}_c^{\mathrm{DI}\prime}(I - \delta^{-1}) \varrho^m}_{\text{inflation stance from direct incidence}} + \underbrace{\mathcal{J}_c^{\mathrm{FToT}\prime}(I - \delta^{-1}) \varrho^m}_{\text{inflation stance from factoral ToT}}]. \tag{18}$$

The first term captures the direct-incidence component, reflecting how a country's commodity terms of trade respond to monetary expansions through its dual role as a consumer and residual profit claimant across sectors. The second term captures the factoral terms-of-trade component, measuring how monetary policy alters the relative value of labor a country supplies versus the labor embedded in its consumption. Taken together, these two components determine the unilateral inflation stance, which summarizes how a country's overall terms of trade responds to monetary expansions in the presence of nominal rigidities. A positive stance indicates that a monetary expansion improves the country's terms of trade; a negative stance implies the opposite.

Assume that a country's inflation stance is primarily driven by the direct-incidence effect.[16] Recall that, in a currency union, the direct-incidence index $\mathcal{J}_{ic}^{\mathrm{DI}}$ captures country c's sectoral export exposure when firms are owned domestically. The index is positive when sector i serves as a net export sector for country c, indicating that the profits earned by domestic firms exceed the country's expenditure on that sector's output. In contrast, it is negative when sector i is a gross import, in which case country c bears high input costs without receiving corresponding profit income.[17] Therefore, a country's inflation stance reflects the interaction between its sectoral export exposure and nominal rigidities transmitted through the production network.

This interaction underscores the fundamental link between a country's production structure and its optimal inflation stance. Since nominal rigidities compound along supply chains, price pass-through is typically lower in downstream sectors than in upstream ones. An expansionary monetary shock therefore generates asymmetric markup compression, with sharper reductions in upstream industries. For an economy whose exports are concentrated upstream while its gross imports are concentrated downstream, such an expansion deteriorates the terms of trade: export revenues shrink significantly, while import costs remain relatively rigid. To mitigate this adverse effect, the country favors a more hawkish inflation stance. This mechanism is formalized in the following remark.

Remark 4 (Inflation Stance and Upstreamness)**.** Suppose the direct-incidence channel in equation (18) is the dominant driver of the unilateral inflation stance. If downstream sectors exhibit lower price pass-through than upstream sectors, then countries whose exports are relatively more upstream than their gross imports tend to exhibit a more hawkish inflation stance (i.e., a lower $\pi_c(\zeta)$). Conversely, countries whose exports are relatively more downstream tend to exhibit a more dovish inflation stance.

By shutting down input–output linkages, the following example illustrates how asymmetries in nominal rigidities between imports and exports can generate an inflation bias under unilateral monetary policy.

[16] This is supported by the quantitative results in Figure A.2.

[17] The term "gross" is used here to emphasize that a country may be a net user of a sector's output through indirect input chains, even if it does not directly import the good.

Example 3 (Horizontal Economy)**.** Consider an economy where N industries are distributed across C countries. Each industry $i \in N_c$ employs local labor and sells sells output on the world market. Under ownership segmentation, the direct-incidence index captures trade exposure, satisfying $\mathcal{J}_{ic}^{\mathrm{DI}} = \sum_{f \neq c} \chi_f \beta_{fi} / \chi_c > 0$ for domestic industries (exports) and $\mathcal{J}_{ic}^{\mathrm{DI}} = -\beta_{ci} < 0$ for foreign industries (imports).

Assuming labor supply is perfectly elastic across countries (shutting down the factoral terms-of-trade channel), the unilateral inflation stance is then exclusively driven by the direct incidence channel and is proportional to the covariance between a country's trade exposure and sectoral price flexibility:

$$\pi_c \propto \mathrm{Cov}(\mathcal{J}_c^{\mathrm{DI}}, \boldsymbol{\delta}).$$

This expression demonstrates that a country favors lower inflation when its export sectors are more flexible (lower δ_i) than its import sectors, as monetary expansions reduce export markups more than import prices, thereby deteriorating the country's terms of trade.

4.4 Policy-Alignment Loss

To assess how divergent policy preferences translate into welfare outcomes, this section introduces a measure of policy-alignment loss (PAL), defined as the welfare difference between a centralized monetary policy and a country's unilateral optimal policy. It quantifies the welfare cost incurred by a country when a common monetary policy is implemented, thereby providing a benchmark for evaluating the distributional consequences of centralized monetary policy in a heterogeneous-agent economy.

Definition 4. Let $(\zeta^*(\mathbf{e}_c), \pi^*(\mathbf{e}_c))$ denote country c's unilateral optimal monetary policy. Given a union-wide stabilization policy $(\zeta, 0)$, the *policy-alignment loss* (PAL) for country c is defined as

$$\mathbb{PAL}_c \;\equiv\; \mathbb{L}_c\Big(\zeta, 0\Big) - \mathbb{L}_c\Big(\zeta^*(\mathbf{e}_c), \pi^*(\mathbf{e}_c)\Big) \;\geq\; 0, \tag{19}$$

where the inequality holds because $(\zeta^*(\{\chi_c\}), 0)$ remains a feasible policy in country c's optimization problem, but may not be optimal from their individual perspective.

The policy-alignment loss admits a transparent decomposition that clarifies its sources:

$$\mathbb{PAL}_c = \underbrace{\mathbb{L}_c\Big(\zeta, 0\Big) - \mathbb{L}_c\Big(\zeta, \pi_c(\zeta)\Big)}_{\text{inflation misalignment}} + \underbrace{\mathbb{L}_c\Big(\zeta, \pi_c(\zeta)\Big) - \mathbb{L}_c\Big(\zeta^*(\mathbf{e}_c), \pi^*(\mathbf{e}_c)\Big)}_{\text{price-index misalignment}} \tag{20}$$

where $\pi_c(\zeta)$ denotes country c's unilateral inflation stance characterized in equation (17).

Holding the centralized optimal price index fixed, the first component isolates the welfare loss arising from inflation misalignment—that is, from setting union-wide inflation away from a country's unilateral inflation stance. The second component captures price-index misalignment, measuring the additional welfare loss from stabilizing a price index that differs from the one a country would optimally choose once inflation is aligned ex ante.

This decomposition highlights that policy-alignment losses need not arise from the design of the target price index itself. Instead, they reflect the inability of a common monetary policy to accommodate heterogeneity in preferred inflation rates across member states. When the inflation-misalignment component dominates, holding the price-index regime fixed and setting the union-wide inflation rate equal to a country's unilateral inflation stance delivers welfare outcomes close to those under its unilateral optimal policy.[18] This approximate optimality implies that the distance between a country's unilateral inflation stance and the union-wide inflation consensus acts as an approximate sufficient statistic for its policy-alignment loss.

5 Quantitative Analysis

This section applies the theoretical framework to the euro area. I first evaluate union-wide welfare by solving for the centralized optimal policy and comparing it to alternative price-stabilization rules. I then analyze countries' unilateral welfare, computing each country's preferred inflation stance and policy-alignment loss, and relate these outcomes to positions in the union's production network.

5.1 Calibration

Given the availability of rich cross-country input–output data, I focus on the euro area as the empirical context.[19] The model is calibrated at a quarterly frequency. To construct the

[18]The quantitative results in Figures 4 and A.3 show that, for most euro-area countries, inflation misalignment accounts for the bulk of policy-alignment losses.

[19]For example, interstate input–output linkages are not directly available in U.S. data.

input–output matrix Ω, I use data from the World Input–Output Database (WIOD) (Timmer et al., 2015), which provides annual IO tables for 44 countries, each with 56 industries, from 2000 to 2014. I extract the 20 euro area member countries and reconstruct their integrated production network, preserving all intra-Euro linkages at the country–industry level. Assuming that direct factor inputs are entirely domestically sourced, I set α_{ic} equal to the value-added share of industry i when it is located in country c, and zero otherwise. For each country c, I construct the vector of consumption shares, β_c, to replicate the distribution of final uses across industries within that country. Income shares (GNE shares), χ_c, are calibrated from each country's total nominal expenditure relative to the euro area aggregate.

Given the industry classification provided by the WIOD, I next calibrate nominal rigidities across sectors. Sectoral price flexibilities are inferred from the frequency of price adjustment (FPA) data of Pastén et al. (2020). Each WIOD industry is matched to the closest NAICS code with available FPA estimates, following the mapping procedure described in the Appendix. Nominal rigidities are assumed to be uniform across countries but vary across industries according to the matched FPA data. In addition, following standard practice in the production network literature (e.g., La'O and Tahbaz-Salehi, 2022), I incorporate nominal wage rigidities by introducing, for each country, a pseudo-industry that transforms domestic labor into labor services supplied to all other domestic industries. The degree of nominal wage rigidity is calibrated based on the empirical estimates of Barattieri et al. (2014) and Beraja et al. (2019), setting the price flexibility of the pseudo-industry to 0.30.

I then calibrate the distribution of sectoral productivity shocks using data from the WIOD Socio-Economic Accounts (2000–2014). I construct annual series of productivity growth rates through a Törnqvist decomposition based on observed inputs and outputs, and linearly interpolate them to quarterly frequency to match the model's time period. The log productivity shocks, $(\log z_1, \ldots, \log z_N)$, are assumed to be jointly normally distributed, with a variance–covariance matrix equal to that of the linearly detrended quarterly productivity series.

Finally, I set the within-industry elasticity of substitution to $\theta_i = 6$ for all industries and the Frisch elasticity of labor supply to $\eta_c = 2$ for all countries. These parameter values are consistent with standard calibrations in the New Keynesian literature. The baseline calibration also assumes ownership segmentation, under which firms are owned exclusively by their domestic countries.

5.2 Welfare Comparison of Alternative Monetary Policies

This section evaluates union-wide expected welfare losses under alternative monetary policy regimes in a currency union. The analysis adopts a welfare aggregation in which Pareto weights coincide with countries' income shares, implying that centralized monetary policy addresses a pure efficiency problem. This specification is commonly used in the literature and provides a natural benchmark for the euro area. Although the European Central Bank (ECB) does not explicitly assign welfare weights to member states, its institutional design—such as permanent membership on the Executive Board and the rotating voting system of the Governing Council—implicitly links policy influence to economic size.

Table 2 reports the expected welfare losses arising from nominal rigidities, expressed as a percentage of steady-state union-wide real consumption. Following the decomposition in equation (14), the total loss is partitioned into three distinct sources of misallocation. The first column presents results under the centralized optimal policy, which yields an expected welfare loss equivalent to 0.572% of quarterly consumption relative to the flexible-price equilibrium. The largest source of this welfare loss is within-industry misallocation, responsible for 0.322 percentage points of the total loss, followed by across-industry misallocation at 0.241 percentage points. In contrast, the welfare cost attributable to the aggregate volatility of employment gaps is an order of magnitude smaller.[20]

Table 2: Expected welfare losses under various policies.

	(1) Cent. Optimal	(2) EG Stabilization	(3) CPI Stabilization
Total welfare loss	0.572	0.581	0.592
Employment Gaps	0.009	0.006	0.008
Within-sector misallocation	0.322	0.335	0.344
Across-sector misallocation	0.241	0.239	0.239
Cosine similarity to optimal policy	1	0.976	0.122

Table 2 also compares the performance of the optimal policy with two alternative price-stabilization rules. The first is an employment-gap (EG) stabilization policy, which minimizes the aggregate volatility of employment gaps across countries.[21] This corresponds to a

[20]The ordering between within- and across-industry misallocations reflects the model's structure: the high elasticity of substitution within sectors ($\theta_i = 6$) makes within-sector price dispersion more damaging than the across-sector misallocation which controlled by the unit across-sector elasticity implied by the Cobb–Douglas production functions.

[21]Formally, the EG stabilization policy minimizes $\log \boldsymbol{\mu}' \mathcal{L}^{\text{e.g.}}(\{\chi_c\}) \log \boldsymbol{\mu} = \sum_{c=1}^{C} (1 + 1/\eta_c)\chi_c \hat{l}_c^2$.

price-stabilization rule of the form $\sum_{i=1}^{N}\zeta_i^{\text{e.g.}}(\{\chi_c\})\log p_i = 0$, where the industry weights are given by $\zeta_i^{\text{e.g.}}(\{\chi_c\}) = \lambda_i(\delta_i^{-1}-1)(\sum_{c=1}^{C}\Phi_{ic}\ell_c^m)$, which simplifies under ownership segmentation to $\zeta_i^{\text{e.g.}}(\{\chi_c\}) = \lambda_i(\delta_i^{-1}-1)\ell_c^m$ for $i \in N_c$. Intuitively, the EG stabilization policy targets a price index that assigns greater weight to industries that are larger (higher λ_i), stickier (lower δ_i), and located in countries with more pronounced employment responses to monetary shocks (higher ℓ_c^m).

As shown in the second column of the table, the EG stabilization policy generates a welfare loss equivalent to a 0.581% reduction in quarterly consumption, with just 0.009 percentage points higher than under the optimal policy. It achieves slightly better stabilization of employment gaps but at the cost of greater within-sector misallocation. The approximate optimality of the EG stabilization echoes prior results in La'O and Tahbaz-Salehi (2022) and Rubbo (2023), which show that stabilizing the output or employment gap can be approximately optimal. A key distinction here is that the heterogeneous-agent structure implies multiple employment gaps (one per country), so the EG policy must balance stabilization across member states. Importantly, the target price indices implied by the two policies are also very similar: their cosine similarity exceeds 97%. Thus, the centralized optimal policy implicitly shares the same logic as the EG stabilization, assigning greater weight to sectors that are larger, stickier, and belong to countries with more pronounced employment responses.

The third column examines a CPI stabilization rule, $\sum_{i=1}^{N} b_i \log p_i = 0$, which stabilizes the union-wide consumer price index. Relative to the other two policies, CPI stabilization performs substantially worse, generating larger welfare losses primarily due to within-sector misallocation. This underperformance reflects a pronounced misalignment between the CPI and the welfare-relevant price index targeted by the optimal policy, as indicated by a cosine similarity of only 12.2 percent. To illustrate this difference in underlying price indices, Figure A.1 in the Appendix aggregates industry weights to the country level and plots each country's weight in the target price index against its income share under both the optimal policy and CPI stabilization. Under CPI stabilization, country weights closely track income shares, with most observations near the 45-degree line. In contrast, under the optimal policy, while weights broadly reflect income shares, five countries—Belgium, Germany, Ireland, Luxembourg, and the Netherlands—receive systematically higher weights. These countries are often seen as occupying upstream positions in the union production network, consistent with our theoretical result that the optimal policy stabilizes a price index with greater weights assigned to more upstream industries.

5.3 Country-Level Welfare and Policy Alignment

The previous section evaluated alternative stabilization rules from a union-wide welfare perspective. This section shifts attention to country-level welfare in order to understand how a common monetary policy translates into heterogeneous welfare outcomes across member states.

Figure 3 provides a first step in this direction by comparing country-level welfare losses under alternative monetary policy regimes. Panel (a) shows that CPI stabilization leads to higher welfare losses than the centralized optimal policy for essentially all countries, as almost all observations lie above the 45-degree line. This suggests that a regime change from CPI stabilization to the optimal policy would deliver welfare gains across member states, with Ireland being the only exception, for which the welfare loss increases by a very small and economically negligible amount. Panel (b) compares welfare losses under the centralized optimal policy with those under each country's unilateral optimal policy. As expected, all observations lie below the 45-degree line, since the vertical distance to the frontier corresponds to the non-negative policy-alignment loss (PAL) defined in equation (19) when the union-wide stabilization policy is given by the centralized optimal policy. These results indicate that CPI stabilization is associated with uniformly larger policy-alignment losses across countries. Accordingly, the remainder of the analysis adopts the centralized optimal policy as the benchmark union-wide stabilization regime when evaluating country-level policy-alignment loss and unilateral inflation stance, with results under CPI regime reported in the Appendix.

To shed light on the sources of the policy-alignment losses documented above, Figure 4 provides two complementary decompositions. Panel (a) decomposes each country's PAL into first- and second-order components. The first-order term isolates the welfare gains arising from a country's ability to manipulate its overall terms of trade under unilateral optimal policy.[22] By plotting the total PAL against this first-order component, a striking pattern emerges: for 17 out of 20 countries, the first-order component exceeds the total policy-alignment loss, placing these observations above the 45-degree line. This outcome highlights a significant redistribution–stabilization trade-off under unilateral policymaking. Once a country gains unilateral control over monetary policy, it can aggressively pursue its own first-order redistributive objectives—even at the cost of larger second-order welfare losses due to price dispersion and employment volatility. By contrast, Panel (b) decomposes PAL into components attributable to inflation misalignment and to misalignment in the

[22]Formally, the first-order component of PAL is given by $\mathcal{J}_c'\mathcal{M}(\zeta^*(\mathbf{e}_c), \pi^*(\mathbf{e}_c)) - \mathcal{J}_c'\mathcal{M}(\zeta^*(\chi_c), 0)$.

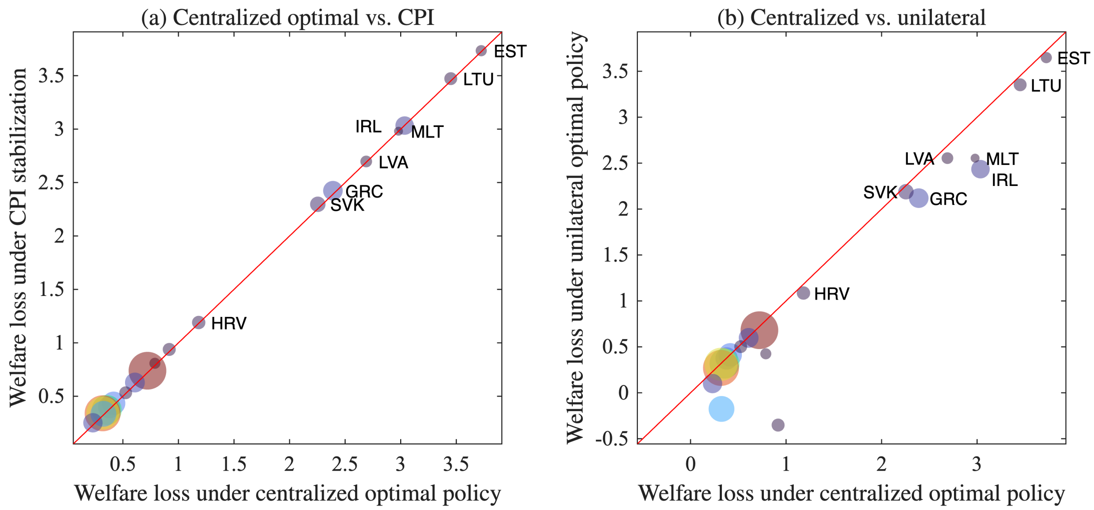

Figure 3: Welfare loss under alternative monetary policies. *Note:* Each dot represents a euro-area country, with size proportional to its income share. For each country, welfare loss is expressed as a percentage of its steady-state consumption. Panel (a) compares welfare losses under the centralized optimal policy and CPI stabilization. Panel (b) compares welfare losses under the centralized optimal policy and each country's unilateral optimal policy. The 45-degree line indicates identical welfare outcomes across policies.

target price index, as in equation (20). Observations cluster tightly around the 45-degree line, indicating that policy-alignment losses are driven primarily by inflation misalignment rather than by differences in the price-index regime. Accordingly, under the centralized optimal price-index regime, setting union-wide inflation equal to a country's unilateral inflation stance is approximately optimal from that country's unilateral perspective.

To better understand the sources of these welfare losses, I next zoom in on a representative member state and decompose its welfare losses across various policies using equation (12). Relative to the union-wide analysis in the previous section, country-level welfare loss includes an additional component arising from underutilized allocative efficiency. Using Spain as an illustrative case, Table 3 reports welfare losses under the unilateral optimal policy, the centralized optimal policy, CPI stabilization, as well as counterfactual policies that retain the union's price-targeting regime but implement a country's unilateral inflation stance. For completeness, Table A.1 in the Appendix reports the corresponding welfare losses for all 20 euro-area countries under the same set of policies.

The first column reports the unilateral optimal policy, in which the country fully internalizes monetary policy decisions at the union level and chooses both the target price index and the inflation rate to minimize its own welfare loss. The associated price-

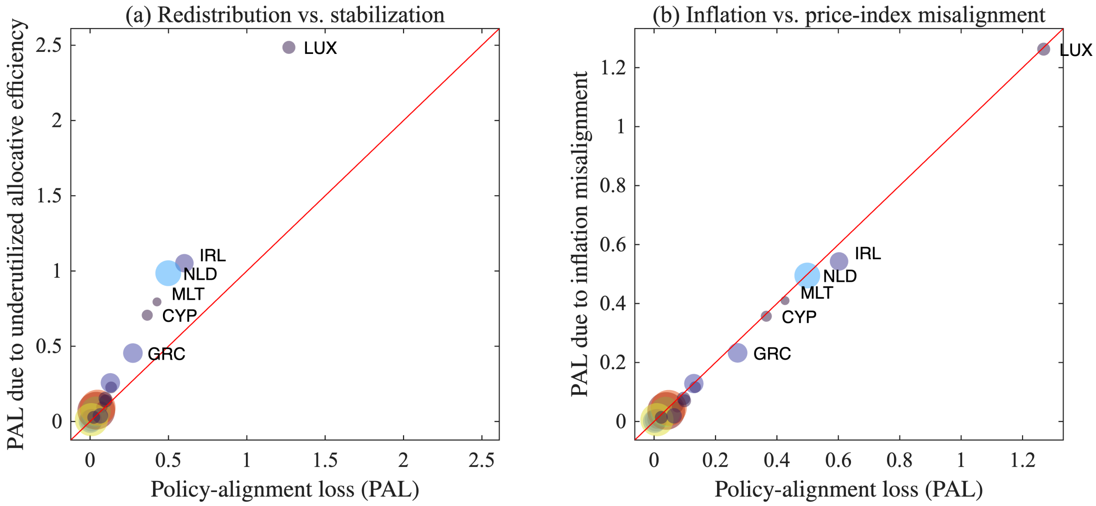

Figure 4: Decomposition of policy-alignment loss. *Note:* Each dot represents a euro-area country, with size proportional to its income share. Panels (a) and (b) plot total policy-alignment loss against its underutilized allocative-efficiency and inflation-misalignment components, respectively. The 45-degree line indicates full accounting.

targeting rule is $\sum_{i=1}^{N} \zeta_i^*(\mathbf{e}_c) \log p_i = \pi^*(\mathbf{e}_c)$. The second and fourth columns correspond to the centralized optimal policy and CPI stabilization, respectively, as introduced in the previous section. The third column keeps the centralized optimal price-index regime but sets inflation equal to Spain's unilateral inflation stance, $\sum_{i=1}^{N} \zeta_i^*(\{\chi_c\}) \log p_i = \pi_c(\zeta^*(\{\chi_c\}))$. Similarly, the fifth column retains CPI targeting while allowing Spain to choose its preferred inflation rate under that regime, $\sum_{i=1}^{N} b_i(\{\chi_c\}) \log p_i = \pi_c(b)$.

Table 3: Expected welfare loss for Spain under various polices.

	(1) Uni. Opt.	(2) Cent. Opt.	(3) Cent. Uni. π	(4) CPI Stab.	(5) CPI Uni. π
Total welfare loss	0.319	0.355	0.322	0.373	0.340
Allocative efficiency (FO)	-0.066	0	-0.066	0	-0.066
Employment gaps	0.010	0.004	0.013	0.006	0.015
Within-sector misallocation	0.264	0.241	0.264	0.256	0.280
Across-sector misallocation	0.110	0.110	0.111	0.110	0.110
Cosine similarity to unilateral optimal policy	1	0.315	0.315	0.036	0.036

Three results are worth highlighting. First, welfare losses due to underutilized allocative efficiency are quantitatively important under stabilization policies. Allowing Spain to set its preferred inflation rate (policies (1), (3), and (5)) yields substantially lower welfare losses than pure stabilization policies (policies (2) and (4)). These gains reflect improvements in allocative efficiency driven by a favorable adjustment of the overall terms of trade, albeit at the cost of larger second-order losses arising from employment volatility and within-sector price dispersion.

Second, policy-alignment losses are largely attributable to inflation misalignment. Under both the centralized optimal and CPI regimes, aligning the union-wide inflation rate with Spain's unilateral inflation stance defined under the corresponding price-index regime substantially reduces welfare losses toward the unilateral optimum. In particular, under the centralized optimal price-index regime, policy (3) delivers welfare outcomes very close to the unilateral optimal policy, with a remaining loss of only 0.003 percentage points of consumption.

Third, price-index misalignment is generally smaller under the centralized optimal regime than under CPI targeting. The price index targeted by the centralized optimal policy is considerably closer to that implied by the unilateral optimal policy (cosine similarity of 0.315) than is the CPI (cosine similarity of 0.036). As a result, policy (3) substantially outperforms policy (5). For the same reason, the centralized optimal policy (2) dominates CPI stabilization (4), consistent with Panel (a) of Figure 3, which shows uniformly lower welfare losses under the centralized optimal policy across countries.

5.4 Unilateral Inflation Stance and Policy-Alignment Loss

When policy-alignment losses are driven primarily by inflation misalignment, as shown in Panel (b) of Figure 4, a country's unilateral inflation stance becomes the key object summarizing how its structural characteristics translate into a preferred union-wide inflation rate. To study the determinants of these stances, Figure A.2 in the Appendix decomposes unilateral inflation stances into direct-incidence and factoral terms-of-trade components, as defined in equation (18). Panel (a) reports the decomposition under the centralized optimal price-index regime, while Panel (b) reports the same exercise under CPI targeting. In both cases, most observations lie close to the 45-degree line, indicating that unilateral inflation stances are predominantly driven by the direct-incidence channel.

The close correspondence between Figures 4 and A.2 suggests a tight link between unilateral inflation stances and policy-alignment losses. Countries with larger policy-alignment losses also exhibit more extreme unilateral inflation stances, including Cyprus (CYP), Greece (GRC), Ireland (IRL), Luxembourg (LUX), Malta (MLT), the Netherlands

(NLD), and Portugal (PRT). To formalize this relationship, Figure 5 plots each country's policy-alignment loss against the distance between its unilateral inflation stance and the union-wide inflation consensus. Under income-share weighting, the centralized optimal policy is inflation-neutral, so this distance equals the absolute unilateral inflation stance.

The figure reveals a strong positive relationship: a one-percentage-point increase in the absolute unilateral inflation stance is associated with a 0.54 percentage point increase in policy-alignment loss, and this single statistic explains about 87.5% of the cross-country variation in losses. These results indicate that the distance between a country's unilateral inflation stance and the union-wide consensus provides an empirically sufficient summary measure of the welfare costs of centralized monetary policy.

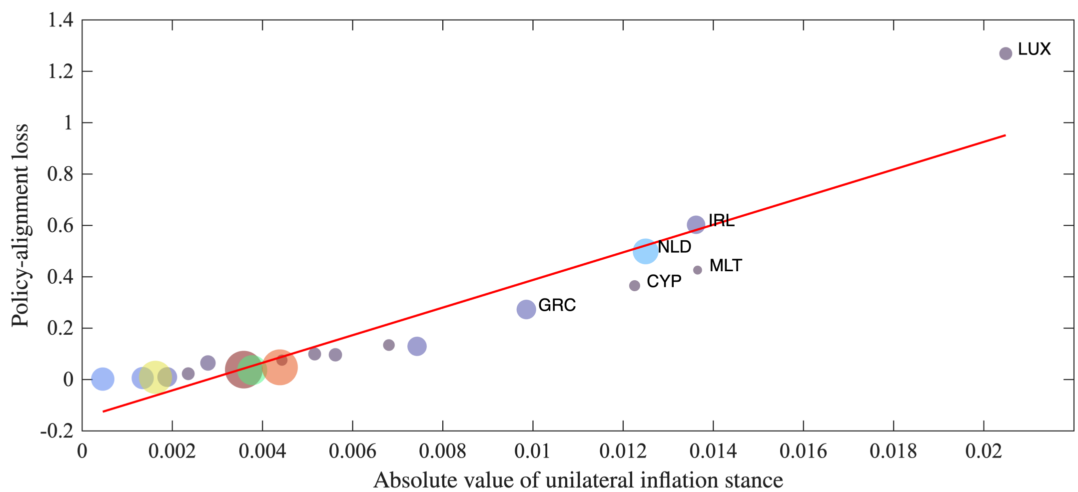

Figure 5: Inflation stance deviation and policy-alignment loss. *Note:* Each dot represents a euro-area country, with size proportional to its income share. The figure relates the absolute unilateral inflation stance to the corresponding policy-alignment loss, measured in percentage of steady-state consumption. The red line shows the fitted linear relationship.

5.5 Inflation Stance and Production Networks

How does a country's incidence of a common monetary policy, measured by its policy-alignment loss (PAL), relate to its position within the union's production network? As established in the previous section, the deviation of a country's unilateral inflation stance from the union-wide benchmark serves as a sufficient statistic for its policy-alignment loss. We therefore begin by examining how the structure of production networks shapes cross-country variation in inflation stances.

To quantify a country's position in the union's production network, I construct a measure of industry-level upstreamness following Antràs et al. (2012), which captures the average distance of an industry's output from final consumption. These industry-level upstreamness values are then aggregated to the country level using value-added shares as weights, yielding a measure of country-level upstreamness that reflects how far a country's production lies from final demand within the broader supply chain.

Figure 6 displays a clear negative relationship between upstreamness and a country's unilateral inflation stance: more upstream countries tend to favor lower inflation (hawkish), while more downstream countries favor higher inflation (dovish).[23] The pattern is systematic and not driven by outliers. This gradient mirrors the familiar North–South divide in the euro area, with core Northern economies occupying upstream positions and peripheral Southern economies located further downstream in the production network. Notably, Luxembourg (LUX), the Netherlands (NLD), and Ireland (IRL) exhibit highly upstream structures and hawkish stances, while Greece (GRC), Cyprus (CYP), and Spain (ESP) are more downstream and dovish. Germany (DEU), a major upstream producer, also leans hawkish.

This relationship is confirmed by a weighted least squares (WLS) regression that accounts for cross-country heterogeneity using income shares as weights. The estimated slope of the fitted red line in Figure 6 is –32.5 (standard error 4.27), highly statistically significant ($p < 0.001$), with an R-squared of 0.75. Quantitatively, the estimate implies that a one-unit increase in upstreamness is associated with an average 3.1 percentage point reduction in a country's preferred inflation rate. This highlights the strong explanatory power of production network position in accounting for cross-country variation in monetary preferences.

These quantitative patterns align with the theoretical prediction in Proposition 4, which links inflation preferences to the interaction between trade positions and sectoral price pass-throughs within production networks. When downstream sectors exhibit lower price pass-throughs than upstream ones, monetary expansions erode export markups more than they reduce import costs for upstream countries, creating a profit squeeze. To avoid this redistributive loss, upstream countries prefer tighter policy. In contrast, downstream countries benefit more from falling input costs and thus prefer looser policy. Production network structure thus plays a central role in shaping both the direction and intensity of inflation stances across member states.

[23] Figure A.5 in the Appendix shows that this pattern is robust when the unilateral inflation stance is computed under the CPI regime.

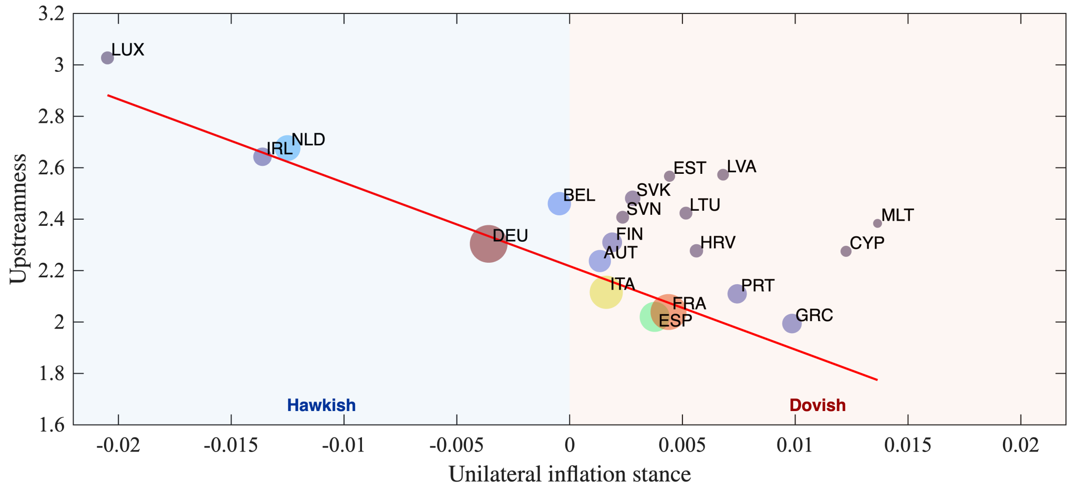

Figure 6: Inflation stance and upstreamness. Each dot represents a euro-area country, with size proportional to its income share. The figure plots unilateral inflation stances against upstreamness in the union production network. The red line shows the fitted linear relationship.

The results in Figures 5–6 together reveal a structural link between policy misalignment loss and a country's position in the production network. Countries at either end of the production chain—those with highly upstream or downstream structures—tend to exhibit more extreme inflation stances and suffer larger policy-alignment losses. Production network structure thus plays a systematic role in shaping both inflation preferences and the incidence of misalignment under a common monetary policy.

6 Extensions

Appendix E extends the analysis to a global economy under dominant currency pricing, characterizing the optimal monetary policy of the dominant-currency country and the resulting policy-alignment losses across non-dominant-currency economies. In this environment, with all goods priced in a dominant currency and nominal rigidities applying to dominant-currency prices, domestic exchange rate movements do not affect ex post markups or relative prices. As a result, from the perspective of the allocation of resources, non-dominant-currency economies are locally equivalent to members of a single-currency area. The key distinction relative to a monetary union is that the centralized benchmark is determined by the dominant-currency country's optimal monetary policy, rather than by a union-wide planner. Policy-alignment losses therefore increase with the distance between a country's inflation stance and the dominant-currency country's optimal policy.

Countries whose production network structures are closer to those of the dominant-currency economy experience smaller welfare losses, reflecting a tighter alignment with the policy that effectively anchors global nominal rigidities.

7 Conclusion

This paper studies the origins and welfare implications of divergent inflation preferences within a currency union using a heterogeneous-agent input–output framework. Monetary policy induces first-order changes in country-level allocative efficiency through shifts in overall terms of trade. When these allocative effects are not neutralized in welfare aggregation, the monetary authority faces a trade-off between aggregate stabilization and first-order redistribution. This trade-off generates an inflation bias, reflecting incentives to tilt the terms of trade in favor of certain member states.

Applying the model to the euro area, I find that countries with more upstream production structures prefer lower union-wide inflation. Moreover, the distance between a country's preferred inflation rate and the union-wide consensus strongly predicts its policy-alignment loss, indicating that countries located at either end of the production chain bear the largest welfare costs under a common monetary policy.

This framework provides a tractable approach for analyzing the structural foundations of policy divergence and its distributional consequences in integrated monetary unions. It can be extended to explore other dimensions of macroeconomic coordination, including fiscal transfers, institutional design, and political representation in heterogeneous economies.

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Online Appendix to " *The Network Origins of Inflation Stances in a Currency Union*"

Zhihao Xu[1]

A Additional Figures and Tables

Table A.1: Expected welfare losses by countries.

	Uni. Opt.	Cent. Opt.	Cent. Uni. π	CPI Stab.	CPI Uni. π
AUT	0.373	0.379	0.374	0.397	0.393
BEL	0.414	0.416	0.416	0.435	0.435
CYP	0.423	0.788	0.432	0.809	0.452
DEU	0.682	0.721	0.686	0.740	0.704
ESP	0.319	0.355	0.322	0.373	0.340
EST	3.649	3.725	3.676	3.735	3.685
FIN	0.598	0.609	0.600	0.628	0.620
FRA	0.270	0.318	0.272	0.342	0.296
GRC	2.118	2.390	2.157	2.423	2.190
HRV	1.085	1.181	1.103	1.189	1.111
IRL	2.434	3.036	2.493	3.033	2.490
ITA	0.311	0.320	0.314	0.343	0.337
LTU	3.351	3.451	3.381	3.472	3.402
LUX	-0.353	0.916	-0.346	0.937	-0.326
LVA	2.555	2.689	2.573	2.696	2.580
MLT	2.554	2.980	2.570	2.980	2.570
NLD	-0.175	0.324	-0.171	0.336	-0.159
PRT	0.100	0.229	0.101	0.251	0.122
SVK	2.190	2.255	2.235	2.297	2.278
SVN	0.502	0.525	0.511	0.533	0.519

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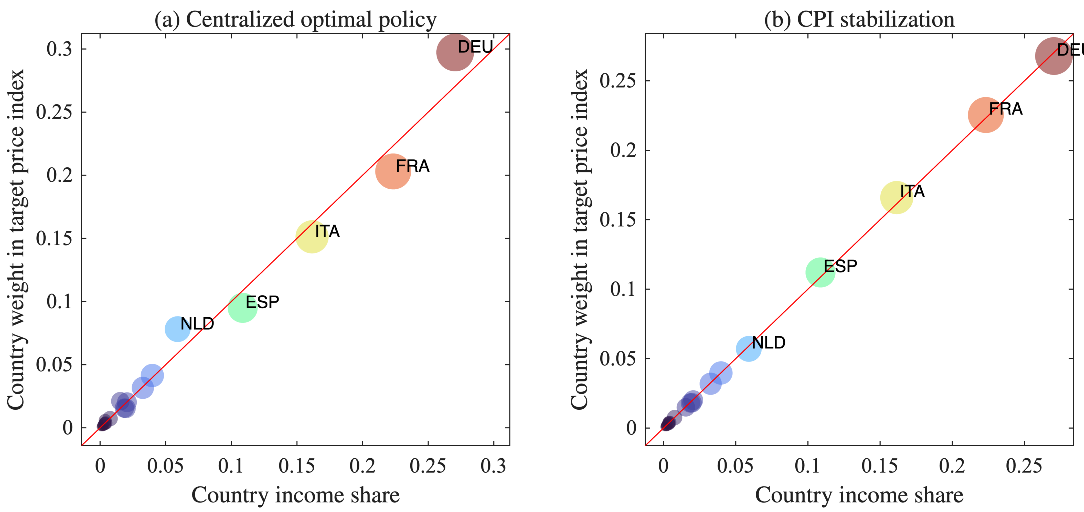

Figure A.1: Country weights in the target price index under alternative monetary policies. *Note:* Panels (a) and (b) compare country income shares with their weights in the target price indices under the centralized optimal policy and CPI stabilization, respectively. The red 45-degree line indicates perfect coincidence.

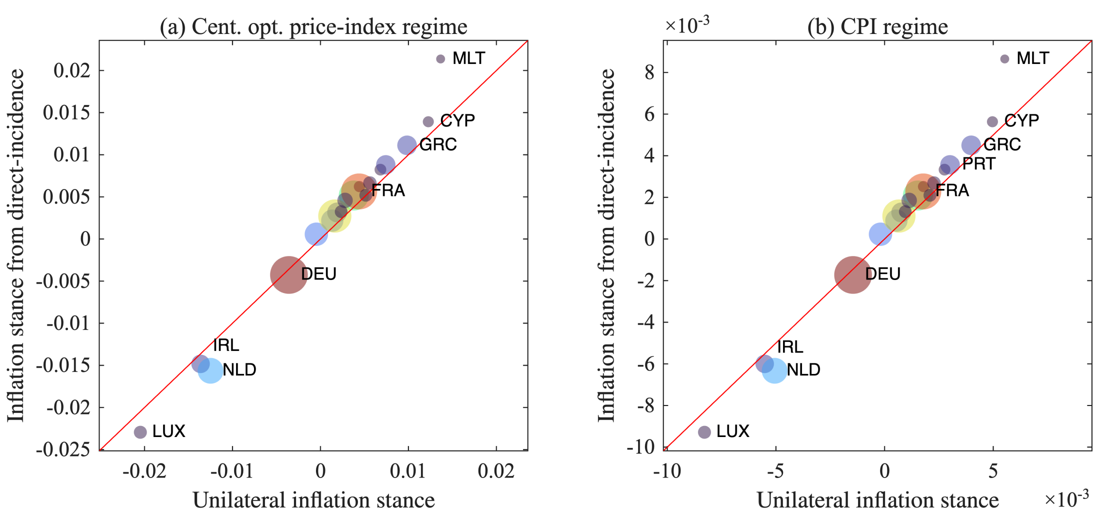

Figure A.2: Decomposition of inflation stance. *Note:* Panels (a) and (b) plot total unilateral inflation stances against their direct-incidence components under the centralized optimal price-index regime and CPI regime, respectively. The red 45-degree line denotes equality.

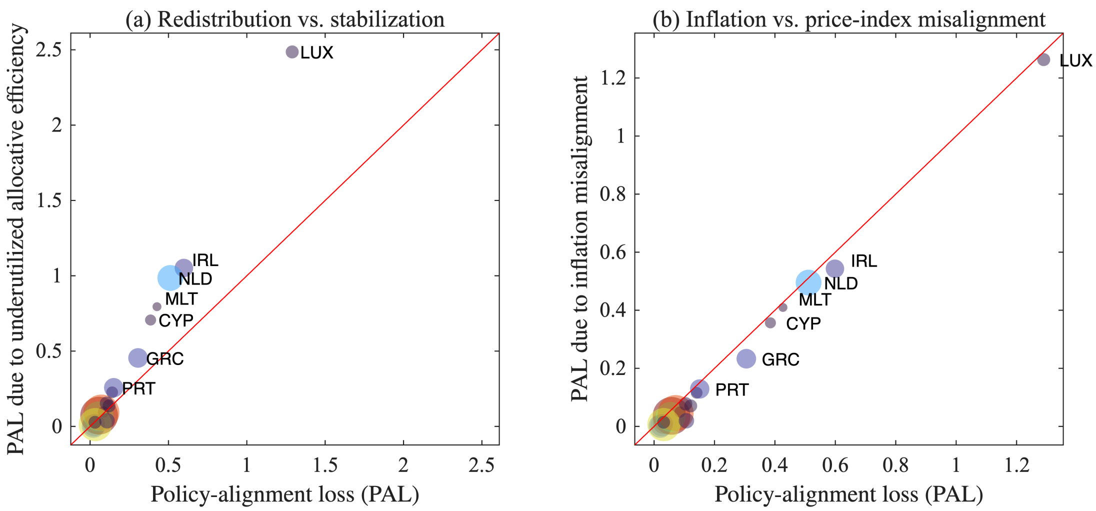

Figure A.3: Decomposition of policy-alignment loss under CPI regime. *Note:* Panel (a) plots total PAL against the component attributable to underutilized allocative efficiency. Panel (b) plots total PAL against the component attributable to inflation misalignment, as in equation (20). In each panel, the 45-degree line indicates that the component accounts for the full policy-alignment loss.

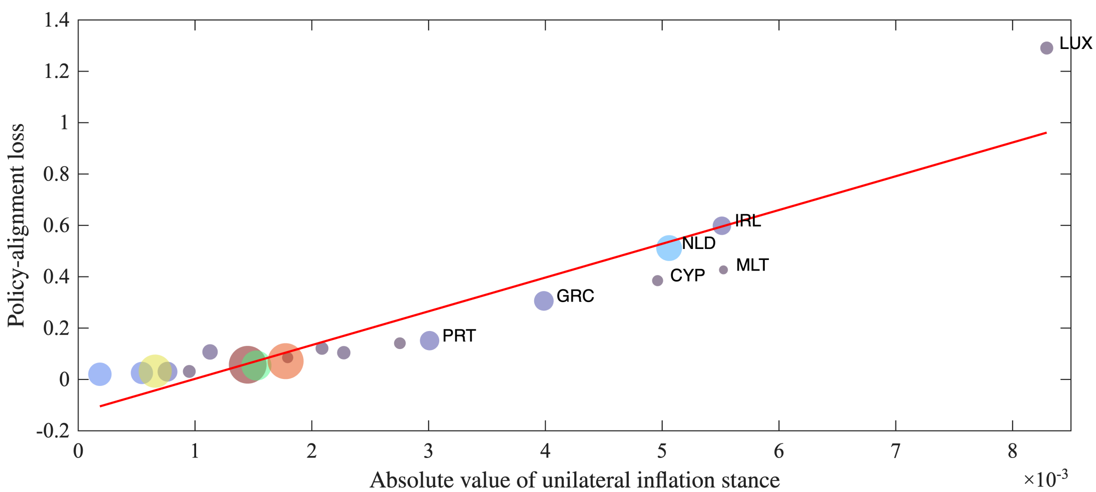

Figure A.4: Inflation stance deviation and policy-alignment loss under CPI regime. *Note:* Each dot represents a euro area country. Dot size reflects the country's income share. The horizontal axis measures the absolute value of the country's unilateral inflation stance. The vertical axis shows the corresponding policy-alignment loss as a percentage of the country's steady-state consumption. The red line is a fitted linear trend.

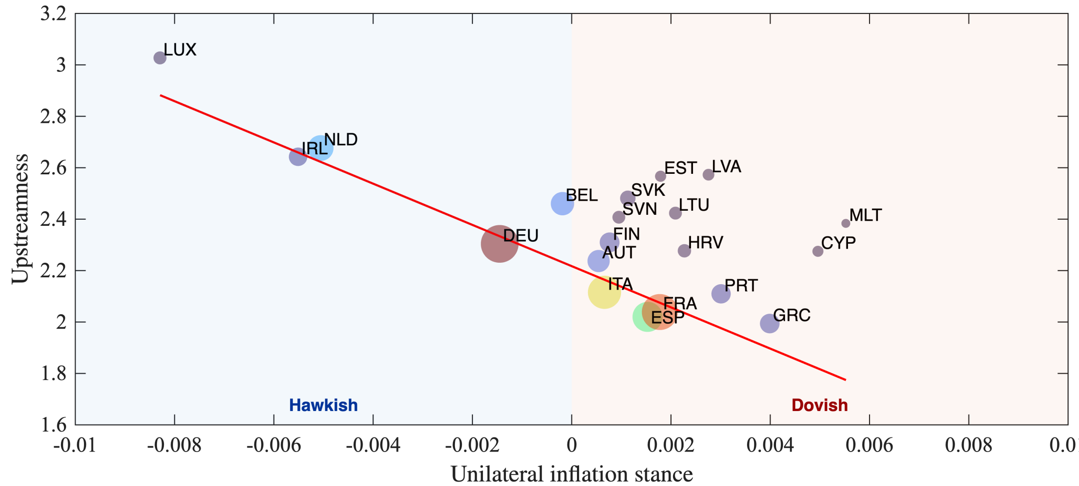

Figure A.5: Unilateral inflation stance and upstreamness under the CPI regime. *Note:* The horizontal axis shows each country's unilateral inflation stance, computed using the CPI as the common price index. The vertical axis reports country-level upstreamness in the union production network. The red line depicts a fitted linear trend.

B Related Proofs and Derivations

Throughout the appendix, I use a bar to denote the steady-state (initial) value of a variable, and a superscript * to denote its value in the flexible-price equilibrium. For example, $\bar{L}_c$ and L_c^* denote household c's labor supply in the steady state and under flexible prices, respectively. To simplify notation, I normalize the initial nominal aggregate demand to one ($\bar{m} = 1$), so that the monetary shock can be represented directly by $\log m$. Accordingly, I define the vector of exogenous shocks as $\xi \equiv (\log z', \log m)'$, which includes both sectoral productivity and monetary shocks.

I begin proofs by defining sectoral wedges following La'O and Tahbaz-Salehi (2022), a formulation that is convenient for analyzing within-sector distortions. Importantly, to a first-order approximation, the sectoral wedge is equivalent (up to a negative sign) to the sectoral markup introduced in equation (4). This equivalence is established formally in Remark 5.

Cost minimization implies that the demand of firm k in industry i for the good produced by industry j can be expressed as

$$x_{ij,k} = \omega_{ij} y_{ik} mc_i / p_j.$$

Firm k's output in industry i is

$$y_{ik} = y_i (p_{ik}/p_i)^{-\theta_i}$$

where the sectoral price index is defined as

$$p_i = (\int_0^1 p_{ik}^{1-\theta_i} dk)^{1/(1-\theta_i)}.$$

Aggregating over the unit mass of firms in industry i, the total demand for goods from industry j equals

$$\int_0^1 x_{ij,k} dk = \omega_{ij} p_i y_i \varepsilon_i / p_j$$

where the sectoral wedge ε_i is defined as

$$\varepsilon_i = \frac{mc_i}{p_i} \int_0^1 (p_{ik}/p_i)^{-\theta_i} dk. \tag{B-1}$$

In the flexible price equilibrium, firms in industry i set identical prices and charge no markups. As a result, equation (B-1) implies $\varepsilon_i^* = 1$ for all i.

Let $\varepsilon_{ik} = mc_i / p_{ik}$ denote the firm-level wedge. Using the representation above, the sectoral wedge aggregates firm-level wedges as

$$\log \varepsilon_i = \log \int_0^1 \varepsilon_{ik}^{\theta_i} dk - \log \int_0^1 \varepsilon_{ik}^{\theta_i - 1} dk$$

Expanding around the efficient allocation, I obtain

$$\varepsilon_{ik}^{\theta_i} = 1 + \theta_i \log \varepsilon_{ik} + o(\|\xi\|), \quad \text{and} \quad \varepsilon_{ik}^{\theta_i - 1} = 1 + (\theta_i - 1) \log \varepsilon_{ik} + o(\|\xi\|),$$

which yields

$$\begin{aligned}
\log \varepsilon_i &= \log\left(1 + \theta_i \int_0^1 \log \varepsilon_{ik} dk\right) - \log\left(1 + (\theta_i - 1) \int_0^1 \log \varepsilon_{ik} dk\right) + o(\|\xi\|) \\
&= \int_0^1 \log \varepsilon_{ik} dk + o(\|\xi\|).
\end{aligned}$$

Thus, to a first-order approximation, the sectoral wedge equals the cross-sectional average of firm-level wedges.

Moreover, the sectoral wedge is directly related to the sectoral markup. Up to a first-order approximation,

$$\log \varepsilon_i = \log mc_i - \log p_i + o(\|\xi\|) = -\log \mu_i + o(\|\xi\|)$$

Remark 5. Throughout the appendix, sectoral distortions are expressed using the sectoral wedge $\log \varepsilon_i$, defined in (B-1). In the main text, I instead work with the equivalent representation in terms of sectoral markups $\log \mu_i$. Since $\log \varepsilon_i = -\log \mu_i + o(\|\xi\|)$, the two notations are interchangeable to a first-order approximation.

Proof of Lemma 1. I first show (8) of Lemma 1. I begin by aggregating profits across the unit mass of firms in industry i. Using the firm-level profit expression (3), the total profit in industry i is

$$\begin{aligned}\int_0^1 \Pi_{ik} dk &= \int_0^1 [p_{ik} - (1-\tau_i) mc_i] y_{ik}\, dk \\ &= \int_0^1 p_{ik} y_{ik}\, dk - (1-\tau_i) mc_i \int_0^1 y_{ik}\, dk \\ &= p_i y_i - (1-\tau_i) mc_i \int_0^1 y_{ik}\, dk.\end{aligned}$$

Substituting this expression, together with the tax scheme (6), into the household budget constraint (5), yields

$$P_c C_c = w_c L_c + \sum_{i=1}^N \Phi_{ic} \left(p_i y_i - mc_i \int_0^1 y_{ik} dk \right) = w_c L_c + \sum_{i=1}^N \Phi_{ic} p_i y_i (1 - \varepsilon_i),$$

where the second equality uses the definition of the sectoral wedge (B-1).

Applying the input–output definitions yields a convenient representation of the labor wedge for household c:

$$\frac{\Lambda_c}{\chi_c} = \frac{w_c L_c}{P_c C_c} = 1 - \sum_{i=1}^N \frac{\lambda_i \Phi_{ic}}{\chi_c}(1 - \varepsilon_i). \tag{B-2}$$

This expression reveals that deviations of labor income from total consumption expenditure are driven exclusively by sectoral wedges, $1 - \varepsilon_i$, scaled by sectoral Domar weights, λ_i, and household-specific ownership exposures, Φ_{ic}. Thus, the labor wedge captures the transmission of production-network distortions into non-labor income for household c. In the undistorted flexible-price equilibrium ($\varepsilon_i^* = 1$ for all i), the wedge vanishes, implying $\Lambda_c^* = \chi_c^*$.

A first order approximation of the labor wedge implies

$$\log(\Lambda_c/\chi_c) = \log\Lambda_c - \log\chi_c = \sum_{i=1}^{N} \frac{\lambda_i \Phi_{ic}}{\chi_c} \log\varepsilon_i + o(\|\xi\|). \tag{B-3}$$

Market clearing for good i implies

$$\begin{aligned} p_i y_i &= p_i \sum_{c=1}^{C} c_{ci} + p_i \sum_{j=1}^{N} \int_0^1 x_{jik} dk \\ &= \sum_{c=1}^{C} p_i c_{ci} + \sum_{j=1}^{N} \omega_{ji} p_j y_j \varepsilon_j \end{aligned}$$

Using the input-output definitions, this is equivalent to

$$\lambda_i = \sum_{c=1}^{C} \chi_c \beta_{ci} + \sum_{j=1}^{N} \omega_{ji} \lambda_j \varepsilon_j$$

which in vector form gives

$$\lambda = (I - \Omega' \operatorname{diag}(\varepsilon))^{-1} \boldsymbol{\beta\chi} \tag{B-4}$$

where $\boldsymbol{\beta} = [\beta_1, ..., \beta_C] \in \mathbb{R}^{N\times C}$.

Thus, to a first order, we have

$$\begin{aligned} \lambda - \lambda^* &= (I - \Omega' \operatorname{diag}(\varepsilon))^{-1} \boldsymbol{\beta\chi} - (I - \Omega')^{-1} \boldsymbol{\beta\chi}^* \\ &= (I - \Omega' \operatorname{diag}(\varepsilon))^{-1} \Omega' (\operatorname{diag}(\varepsilon) - I) (I - \Omega')^{-1} \boldsymbol{\beta\chi} + (I - \Omega')^{-1} \boldsymbol{\beta}(\boldsymbol{\chi} - \boldsymbol{\chi}^*) \\ &= \Psi'\Omega' \operatorname{diag}(\log\varepsilon)\Psi'\boldsymbol{\beta\chi} + \Psi'\boldsymbol{\beta}(\boldsymbol{\chi} - \boldsymbol{\chi}^*) + o(\|\xi\|) \\ &= (\Psi' - I) \operatorname{diag}(\log\varepsilon)\lambda^* + \boldsymbol{\lambda}^*(\boldsymbol{\chi} - \boldsymbol{\chi}^*) + o(\|\xi\|) \end{aligned}$$

or componentwise,

$$\lambda_i = (1 - \log \varepsilon_i)\lambda_i^* + \sum_{j=1}^{N} \Psi_{ji}\lambda_j^* \log \varepsilon_j + \sum_{c=1}^{C} \lambda_i^{c*}(\chi_c - \chi_c^*) + o(\|\xi\|). \tag{B-5}$$

By analogy,

$$\Lambda_f = \Lambda_f^* + \sum_{j=1}^{N} \Psi_{jf}\lambda_j^* \log \varepsilon_j + \sum_{c=1}^{C} \Lambda_f^{c*}(\chi_c - \chi_c^*) + o(\|\xi\|) \tag{B-6}$$

Combining (B-3) and (B-6) yields the first-order approximation for household income-share changes in (8), as stated in Lemma 1,

$$\hat{\chi}_c = \log \chi_c - \log \chi_c^* = -\sum_{j=1}^{N} \underbrace{\sum_{f=1}^{C} (\chi_c^*)^{-1} Q_{cf} \lambda_j^* (\Phi_{jf} - \Psi_{jf})}_{\Gamma_{jc}} \log \varepsilon_j + o(\|\xi\|). \tag{B-7}$$

The matrix Q denotes the inverse of $I - \boldsymbol{\Lambda}'$ restricted to the subspace $\mathcal{S} = \{x \in \mathbb{R}^C : \mathbf{1}'x = 0\}$,[2] and can equivalently be written as

$$Q \;=\; Z\Big(Z'(I - \boldsymbol{\Lambda}')Z\Big)^{-1} Z',$$

where $Z \in \mathbb{R}^{C\times(C-1)}$ is any orthonormal basis for $\mathcal{S}$.

I now turn to the first-order approximation (7) for the household employment gap in Lemma 1.

The consumption-leisure optimality condition for household c implies

$$\psi_c L_c^{1/\eta_c} = C_c^{-1} w_c / P_c$$

where ψ_c is a household-specific preference parameter and $P_c = \prod_{i=1}^{N} p_i^{\beta_{ci}}$ is household c's consumer price index (CPI).

Combining this condition with the definition of the labor wedge in (B-2), I can express household consumption and labor supply directly in terms of the household's real wage

[2] Income shares satisfy $\sum_{c=1}^{C} \chi_c = 1$ in every equilibrium. Hence their level deviations must sum to zero, which means that perturbations of $\boldsymbol{\chi}$ lie in the $(C-1)$-dimensional subspace orthogonal to $\mathbf{1}$.

and labor wedge

$$C_c = (w_c/P_c)(\Lambda_c/\chi_c)^{-\frac{1}{1+\eta_c}}\psi_c^{\frac{1}{1+\eta_c}}, \tag{B-8}$$

$$L_c = (\Lambda_c/\chi_c)^{\frac{\eta_c}{1+\eta_c}}\psi_c^{-\frac{\eta_c}{1+\eta_c}}. \tag{B-9}$$

A direct implication of (B-9) is that natural (flexible-price) employment, $L_c^* = \psi_c^{-\frac{\eta_c}{1+\eta_c}}$, is invariant to both productivity and monetary shocks. Log-linearizing (B-9) yields the first-order approximation for employment gap (7) in Lemma 1:

$$\begin{aligned}
\hat{l}_c = \log L_c - \log L_c^* &= \frac{\eta_c}{1+\eta_c}\log(\Lambda_c/\chi_c) \\
&= \frac{\eta_c}{1+\eta_c}\sum_{i=1}^{N}\frac{\lambda_i\Phi_{ic}}{\chi_c}\log\varepsilon_i + o(\|\xi\|) \\
&= \sum_{i=1}^{N}\underbrace{-\frac{\eta_c}{1+\eta_c}\frac{\lambda_i\Phi_{ic}}{\chi_c}}_{\ell_{ic}^{\mu}}\log\mu_i + o(\|\xi\|),
\end{aligned}$$

where the second line substitutes the first-order expression for the labor wedge in (B-3). □

Lemma 3 (Income shares as Negishi weights)**.** The competitive equilibrium allocation can be replicated by a Negishi planner who maximizes a weighted sum of household utilities, where the Negishi weights coincide with households' income shares.

Proof of Lemma 3. The competitive equilibrium allocation solves a social planner's problem that maximizes a weighted sum of household utilities. Letting $\{\kappa_c\}$ denote undetermined welfare weights, the planner chooses allocations $\{c_{ci}, x_{ij}, L_{ic}, L_c\}$ to solve (omitting firm subscripts as firms are identical within each industry in the absence of nominal rigidities)

$$\begin{aligned}
\max \sum_{c=1}^{C}\kappa_c\left(\sum_{i=1}^{N}\beta_{ci}\log c_{ci} - \psi_c\frac{L_c^{1+1/\eta_c}}{1+1/\eta_c}\right) \\
+\sum_{i=1}^{N}\tilde{p}_i\left(z_i\varsigma_i\prod_{j=1}^{N}x_{ij}^{\omega_{ij}}\prod_{c=1}^{C}L_{ic}^{\alpha_{ic}} - \sum_{j=1}^{N}x_{ji} - \sum_{c=1}^{C}c_{ci}\right) + \sum_{c=1}^{C}\tilde{w}_c\left(L_c - \sum_{i=1}^{N}L_{ic}\right),
\end{aligned}$$

where $\{\tilde{p}_i\}_{i=1}^{N}$ and $\{\tilde{w}_c\}_{c=1}^{C}$ are the Lagrange multipliers on the goods-market and labor constraints, respectively.

The planner's first-order conditions with respect to intermediate inputs x_{ij} and consumption c_{ci} imply

$$\tilde{p}_i \omega_{ij} \frac{y_i}{x_{ij}} = \tilde{p}_j,$$
$$\tilde{p}_i c_{ci} = \kappa_c \beta_{ci}.$$

In a competitive equilibrium, optimality implies

$$p_i \omega_{ij} \frac{y_i}{x_{ij}} = p_j,$$
$$p_i c_{ci} = \beta_{ci} \chi_c m,$$

where $m = \sum_c \sum_i p_i c_{ci}$ is nominal aggregate expenditure.

Comparing planner and market allocations, it follows that $\tilde{p}_i = p_i/m$, $\tilde{w}_c = w_c/m$, and $\kappa_c = \chi_c$. Thus, the welfare weights that rationalize the planner's allocation coincide with the income shares realized in the competitive equilibrium.

To verify consistency on the labor margin, the planner's first-order conditions with respect to labor inputs L_{ci} and labor supply L_c imply

$$\tilde{p}_i \alpha_{ic} \frac{y_i}{L_{ic}} = \tilde{w}_c, \qquad \tilde{w}_c = \kappa_c \psi_c L_c^{1/\eta_c}.$$

Substituting $\kappa_c = \chi_c$, $\tilde{w}_c = w_c/m$, and using the competitive-equilibrium condition $w_c L_c = \chi_c m$, we obtain

$$L_c = \psi_c^{-\frac{\eta_c}{1+\eta_c}},$$

which coincides with labor supply in the flexible-price equilibrium.

Hence, the competitive allocation is exactly rationalized by a Negishi planner whose welfare weights equal households' income shares. □

Lemma 4 (Second-Order Approximations)**.** Up to a second-order approximation in the sectoral wedges $\{\varepsilon_i\}_{i=1}^N$, the income share χ_c and the labor wedge Λ_c/χ_c satisfy

$$\log \chi_c - \log \chi_c^* = -\sum_{j=1}^N \Gamma_{jc} \log \varepsilon_j + \sum_{i=1}^N \sum_{j=1}^N \Xi_{ij}^c \log \varepsilon_j \log \varepsilon_i + o(\|\xi\|^2)$$

and

$$\log(\Lambda_c/\chi_c) = \sum_{i=1}^{N} \frac{\lambda_i \Phi_{ic}}{\chi_c} \log \varepsilon_i + \sum_{i=1}^{N} \sum_{j=1}^{N} \Upsilon_{ij}^c \log \varepsilon_j \log \varepsilon_i + o(\|\xi\|^2)$$

respectively. The associated second-order coefficient matrices are

$$\Xi_{ij}^c = -\frac{1}{2}\Gamma_{jc}\Gamma_{ic} + \chi_c^{-1} \sum_{f=1}^{C} Q_{cf}(\Psi_{if} - \Phi_{if}) \left(\sum_{g=1}^{C} \lambda_i^g \chi_g \Gamma_{jg} + \frac{1}{2}\lambda_i \iota_{ij} - \lambda_j \Psi_{ji} \right),$$

$$\Upsilon_{ij}^c = \frac{\lambda_j \Phi_{ic}}{\chi_c} \Psi_{ji} - \sum_{f=1}^{C} \frac{\lambda_i^f \Phi_{ic}}{\chi_c} \chi_f \Gamma_{jf} + \frac{\lambda_i \Phi_{ic}}{\chi_c} \Gamma_{jc} - \frac{1}{2} \frac{\lambda_i \Phi_{ic}}{\chi_c} \iota_{ij} - \frac{1}{2} \frac{\lambda_i \Phi_{ic}}{\chi_c} \frac{\lambda_j \Phi_{jc}}{\chi_c}.$$

Proof of Lemma 4. To establish Lemma 4, I derive log-quadratic approximations to the labor wedges in (B-2) and the sectoral Domar weights in (B-4), expanding both expressions around the economy's steady state. The overall structure parallels the first-order analysis but extends the derivations to second order in the sectoral wedges.

I begin with the labor-wedge expression (B-2). A second-order expansion yields

$$\begin{aligned} \Lambda_c - \chi_c &= -\sum_{i=1}^{N} \lambda_i \Phi_{ic}(1 - \varepsilon_i) \\ &= \sum_{i=1}^{N} \lambda_i \Phi_{ic}(\log \varepsilon_i + \frac{1}{2}\log^2 \varepsilon_i + o(\|\xi\|^2)) \\ &= \sum_{i=1}^{N} \lambda_i \Phi_{ic} \log \varepsilon_i + \frac{1}{2} \sum_{i=1}^{N} \lambda_i \Phi_{ic} \log^2 \varepsilon_i + o(\|\xi\|^2). \end{aligned} \tag{B-10}$$

I then substitute the first-order expression for Domar weights, (B-5), into the above expansion and use $\Lambda_c^* = \chi_c^*$ to obtain

$$\begin{aligned} \Lambda_c - \Lambda_c^* - (\chi_c - \chi_c^*) = &\sum_{i=1}^{N} \lambda_i^* \Phi_{ic} \log \varepsilon_i + \sum_{i=1}^{N} \sum_{j=1}^{N} \lambda_j^* \Phi_{ic} \Psi_{ji} \log \varepsilon_j \log \varepsilon_i \\ &+ \sum_{i=1}^{N} \sum_{f=1}^{C} \lambda_i^{f*}(\chi_f - \chi_f^*) \Phi_{ic} \log \varepsilon_i - \frac{1}{2} \sum_{i=1}^{N} \lambda_i^* \Phi_{ic} \log^2 \varepsilon_i + o(\|\xi\|^2). \end{aligned} \tag{B-11}$$

Next, I derive a log–quadratic approximation to sectoral Domar weights using (B-4).

$$
\begin{aligned}
\lambda - \lambda^* &= (I - A' \operatorname{diag}(\varepsilon))^{-1} \boldsymbol{\beta}\boldsymbol{\chi} - (I - A')^{-1} \boldsymbol{\beta}\boldsymbol{\chi}^* \\
&= (I - A' \operatorname{diag}(\varepsilon))^{-1} A' (\operatorname{diag}(\varepsilon) - I) (I - A')^{-1} \boldsymbol{\beta}\boldsymbol{\chi} + (I - A')^{-1} \boldsymbol{\beta}(\boldsymbol{\chi} - \boldsymbol{\chi}^*) \\
&= \Psi' A' \operatorname{diag}(\log \varepsilon + \frac{1}{2} \log^2 \varepsilon) \Psi' \boldsymbol{\beta}\boldsymbol{\chi}^* + \Psi' A' \operatorname{diag}(\log \varepsilon) \Psi' A' \operatorname{diag}(\log \varepsilon) \Psi' \boldsymbol{\beta}\boldsymbol{\chi}^* \\
&\quad + \Psi' A' \operatorname{diag}(\log \varepsilon) \Psi' \boldsymbol{\beta}(\boldsymbol{\chi} - \boldsymbol{\chi}^*) + \Psi' \boldsymbol{\beta}(\boldsymbol{\chi} - \boldsymbol{\chi}^*) + o(\|\xi\|^2) \\
&= (\Psi' - I) \operatorname{diag}(\log \varepsilon + \frac{1}{2} \log^2 \varepsilon) \lambda^* + [(\Psi' - I) \operatorname{diag}(\log \varepsilon)]^2 \lambda^* \\
&\quad + (\Psi' - I) \operatorname{diag}(\log \varepsilon) \boldsymbol{\lambda}^*(\boldsymbol{\chi} - \boldsymbol{\chi}^*) + \boldsymbol{\lambda}^*(\boldsymbol{\chi} - \boldsymbol{\chi}^*) + o(\|\xi\|^2)
\end{aligned}
$$

or, componentwise,

$$
\begin{aligned}
\lambda_i - \lambda_i^* &= \sum_{c=1}^{C} \lambda_i^{c*}(\chi_c - \chi_c^*) - \lambda_i^* \log \varepsilon_i + \sum_{j=1}^{N} \lambda_j^* \Psi_{ji} \log \varepsilon_j + \sum_{j=1}^{N} \sum_{c=1}^{C} \lambda_j^{c*} (\Psi_{ji} - \iota_{ji})(\chi_c - \chi_c^*) \log \varepsilon_j \\
&\quad - \frac{1}{2} \lambda_i^* \log^2 \varepsilon_i + \frac{1}{2} \sum_{j=1}^{N} \lambda_j^* \Psi_{ji} \log^2 \varepsilon_j + \sum_{j=1}^{N} \sum_{k=1}^{N} \lambda_j^* (\Psi_{jk} - \iota_{jk})(\Psi_{ki} - \iota_{ki}) \log \varepsilon_j \log \varepsilon_k + o(\|\xi\|^2).
\end{aligned}
$$

By analogy, a second-order approximation of the labor income share Λ_f is given by

$$
\begin{aligned}
\Lambda_f - \Lambda_f^* &= \sum_{c=1}^{C} \Lambda_f^{c*}(\chi_c - \chi_c^*) + \sum_{j=1}^{N} \lambda_j^* \Psi_{jf} \log \varepsilon_j + \sum_{j=1}^{N} \sum_{c=1}^{C} \lambda_j^{c*} \Psi_{jf} (\chi_c - \chi_c^*) \log \varepsilon_j \\
&\quad + \frac{1}{2} \sum_{j=1}^{N} \lambda_j^* \Psi_{jf} \log^2 \varepsilon_j + \sum_{j=1}^{N} \sum_{k=1}^{N} \lambda_j^* (\Psi_{jk} - \iota_{jk}) \Psi_{kf} \log \varepsilon_j \log \varepsilon_k + o(\|\xi\|^2) \\
&= \sum_{c=1}^{C} \Lambda_f^{c*}(\chi_c - \chi_c^*) + \sum_{j=1}^{N} \lambda_j^* \Psi_{jf} \log \varepsilon_j + \sum_{j=1}^{N} \sum_{c=1}^{C} \lambda_j^{c*} \Psi_{jf} (\chi_c - \chi_c^*) \log \varepsilon_j \\
&\quad - \frac{1}{2} \sum_{j=1}^{N} \lambda_j^* \Psi_{jf} \log^2 \varepsilon_j + \sum_{j=1}^{N} \sum_{k=1}^{N} \lambda_j^* \Psi_{jk} \Psi_{kf} \log \varepsilon_j \log \varepsilon_k + o(\|\xi\|^2)
\end{aligned}
$$

Combining this expression with (B-11) and eliminating $\Lambda_f - \Lambda_f^*$ results in the following linear system

$$
\begin{aligned}
\chi_f - \chi_f^* = &\sum_{c=1}^{C} \Lambda_f^{c*}(\chi_c - \chi_c^*) - \sum_{j=1}^{N} \lambda_j^*(\Phi_{jf} - \Psi_{jf}) \log \varepsilon_j \\
&+ \frac{1}{2} \sum_{j=1}^{N} \lambda_j^*(\Phi_{jf} - \Psi_{jf}) \log^2 \varepsilon_j + \sum_{i=1}^{N} \sum_{j=1}^{N} \lambda_j^* \Psi_{ji}(\Psi_{if} - \Phi_{if}) \log \varepsilon_j \log \varepsilon_i \\
&+ \sum_{i=1}^{N} \sum_{c=1}^{C} \lambda_i^{c*}(\chi_c - \chi_c^*)(\Psi_{if} - \Phi_{if}) \log \varepsilon_i + o(\|\xi\|^2)
\end{aligned}
$$

together with the adding-up constraint $\sum_{f=1}^{C}(\chi_f - \chi_f^*) = 0$.

Solving this system using the restricted inverse Q of $I - \mathbf{\Lambda}'$ and collecting terms yields

$$
\begin{aligned}
\log \chi_c - \log \chi_c^* = &- \sum_{j=1}^{N} \Gamma_{jc} \log \varepsilon_j - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \Gamma_{jc} \Gamma_{ic} \log \varepsilon_j \log \varepsilon_i \\
&+ \frac{1}{2} (\chi_c^*)^{-1} \sum_{j=1}^{N} \sum_{f=1}^{C} Q_{cf} \lambda_j^* (\Phi_{jf} - \Psi_{jf}) \log^2 \varepsilon_j \\
&+ (\chi_c^*)^{-1} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{f=1}^{C} \sum_{g=1}^{C} Q_{cf} \lambda_i^{g*} \chi_g^* \Gamma_{jg} (\Phi_{if} - \Psi_{if}) \log \varepsilon_j \log \varepsilon_i \\
&+ (\chi_c^*)^{-1} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{f=1}^{C} Q_{cf} \lambda_j^* \Psi_{ji} (\Psi_{if} - \Phi_{if}) \log \varepsilon_j \log \varepsilon_i + o(\|\xi\|^2) \\
= &- \sum_{j=1}^{N} \Gamma_{jc} \log \varepsilon_j + \sum_{i=1}^{N} \sum_{j=1}^{N} \Xi_{ij}^c \log \varepsilon_j \log \varepsilon_i + o(\|\xi\|^2)
\end{aligned}
$$

where

$$
\begin{aligned}
\Xi_{ij}^c = &- \frac{1}{2} \Gamma_{jc} \Gamma_{ic} + \chi_c^{-1} \sum_{f=1}^{C} \sum_{g=1}^{C} Q_{cf} \lambda_i^g \chi_g \Gamma_{jg} (\Phi_{if} - \Psi_{if}) \\
&+ \frac{1}{2} \chi_c^{-1} \sum_{f=1}^{C} Q_{cf} \lambda_j (\Phi_{jf} - \Psi_{jf}) \iota_{ij} + \chi_c^{-1} \sum_{f=1}^{C} Q_{cf} \lambda_j \Psi_{ji} (\Psi_{if} - \Phi_{if}) \\
= &- \frac{1}{2} \Gamma_{jc} \Gamma_{ic} + \chi_c^{-1} \sum_{f=1}^{C} Q_{cf} (\Psi_{if} - \Phi_{if}) \left(\sum_{g=1}^{C} \lambda_i^g \chi_g \Gamma_{jg} + \frac{1}{2} \lambda_i \iota_{ij} - \lambda_j \Psi_{ji} \right),
\end{aligned}
$$

This establishes the claimed log-quadratic approximation for the income share χ_c.

To establish a second-order approximation for the labor wedge Λ_c/χ_c, I start from (B-2), which implies

$$\begin{aligned}
\Lambda_c/\chi_c &= 1 - \sum_{i=1}^{N} \frac{\lambda_i \Phi_{ic}}{\chi_c}(1-\varepsilon_i) \\
&= 1 + \sum_{i=1}^{N} \frac{\lambda_i \Phi_{ic}}{\chi_c}(\log \varepsilon_i + \frac{1}{2}\log^2 \varepsilon_i + o(\|\xi\|^2)) \\
&= 1 + \sum_{i=1}^{N} \frac{\lambda_i \Phi_{ic}}{\chi_c}\log \varepsilon_i + \frac{1}{2}\sum_{i=1}^{N} \frac{\lambda_i \Phi_{ic}}{\chi_c}\log^2 \varepsilon_i + o(\|\xi\|^2)
\end{aligned}$$

which then implies

$$\log(\Lambda_c/\chi_c) = \sum_{i=1}^{N} \frac{\lambda_i \Phi_{ic}}{\chi_c}\log \varepsilon_i + \frac{1}{2}\sum_{i=1}^{N} \frac{\lambda_i \Phi_{ic}}{\chi_c}\log^2 \varepsilon_i - \frac{1}{2}\left(\sum_{i=1}^{N} \frac{\lambda_i \Phi_{ic}}{\chi_c}\log \varepsilon_i\right)^2 + o(\|\xi\|^2). \quad \text{(B-12)}$$

Next, substitute the first-order expression for Domar weights (B-5) and the first-order expansion for the inverse income share,

$$\frac{1}{\chi_c} = \frac{1}{\chi_c^*} + \sum_{i=1}^{N} \frac{\Gamma_{ic}}{\chi_c^*}\log \varepsilon_i + o(\|\xi\|)$$

into (B-12) and collect terms up to second order. This yields

$$\begin{aligned}
\log(\Lambda_c/\chi_c) = &\sum_{i=1}^{N} \frac{\lambda_i^* \Phi_{ic}}{\chi_c^*}\log \varepsilon_i + \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{\lambda_j^* \Phi_{ic}}{\chi_c^*}\Psi_{ji}\log \varepsilon_j \log \varepsilon_i - \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{f=1}^{C} \frac{\lambda_i^{f*} \Phi_{ic}}{\chi_c^*}\chi_f^* \Gamma_{jf}\log \varepsilon_j \log \varepsilon_i \\
&+ \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{\lambda_i^* \Phi_{ic}}{\chi_c^*}\Gamma_{jc}\log \varepsilon_j \log \varepsilon_i - \frac{1}{2}\sum_{i=1}^{N} \frac{\lambda_i^* \Phi_{ic}}{\chi_c^*}\log^2 \varepsilon_i - \frac{1}{2}(\sum_{i=1}^{N} \frac{\lambda_i^* \Phi_{ic}}{\chi_c^*}\log \varepsilon_i)^2 + o(\|\xi\|^2) \\
= &\sum_{i=1}^{N} \frac{\lambda_i^* \Phi_{ic}}{\chi_c^*}\log \varepsilon_i + \sum_{i=1}^{N}\sum_{j=1}^{N} \Upsilon_{ij}^c \log \varepsilon_j \log \varepsilon_i + o(\|\xi\|^2)
\end{aligned}$$

where the second-order coefficients are given by

$$\Upsilon_{ij}^{c} = \frac{\lambda_j \Phi_{ic}}{\chi_c} \Psi_{ji} - \sum_{f=1}^{C} \frac{\lambda_i^f \Phi_{ic}}{\chi_c} \chi_f \Gamma_{jf} + \frac{\lambda_i \Phi_{ic}}{\chi_c} \Gamma_{jc} - \frac{1}{2} \frac{\lambda_i \Phi_{ic}}{\chi_c} \iota_{ij} - \frac{1}{2} \frac{\lambda_i \Phi_{ic}}{\chi_c} \frac{\lambda_j \Phi_{jc}}{\chi_c}.$$

This establishes the claimed log-quadratic approximation for the labor wedge Λ_c/χ_c.

□

Proof of Proposition 2. Cost minimization by firm k in industry i implies that

$$mc_i = \frac{1}{z_i \varsigma_i} \prod_{j=1}^{N} \left(\frac{p_j}{\omega_{ij}} \right)^{\omega_{ij}} \prod_{c=1}^{C} \left(\frac{w_c}{\alpha_{ic}} \right)^{\alpha_{ic}}, \tag{B-13}$$

where ς_i is a normalization constant chosen so that the steady-state price and marginal cost satisfy $\bar{p}_i = \bar{mc}_i = 1$ for all i. Since all prices are normalized in the steady state, each $\bar{w}_c$ represents the real wage of household c. The constant ς_i thus absorbs these steady-state real wages and is given by

$$\varsigma_i = \prod_{j=1}^{N} \omega_{ij}^{-\omega_{ij}} \prod_{c=1}^{C} \left(\frac{\bar{w}_c}{\alpha_{ic}} \right)^{\alpha_{ic}}.$$

Taking logs on (B-13) yields the marginal-cost system

$$\log mc_i = -\log z_i + \sum_{j=1}^{N} \omega_{ij} \log p_j + \sum_{c=1}^{C} \alpha_{ic} \Delta \log w_c \tag{B-14}$$

where $\Delta \log w_c \equiv \log w_c - \log \bar{w}_c$ denotes the deviation of household c's wage from its steady-state level.

Or in matrix notations,

$$\log mc = -\log z + \Omega \log p + \alpha \Delta \log w \tag{B-15}$$

To characterize the response of nominal wages to monetary policy, start from the definition of the labor income share:

$$\log w = \log m + \log \Lambda - \log L. \tag{B-16}$$

Applying the first-order approximation results from Lemma 1 yields

$$d\log w = d\log m + d\log\chi + \boldsymbol{\eta}^{-1}d\log L + o(\|\xi\|) \tag{B-17}$$

$$= \log m + \varrho^{w}(\Gamma + \ell^{\mu}\boldsymbol{\eta}^{-1})'\log\boldsymbol{\mu} + o(\|\xi\|) \tag{B-18}$$

Here, the term $\varrho^{w}(\Gamma + \ell^{\mu}\boldsymbol{\eta}^{-1})'\log\boldsymbol{\mu}$ captures the endogenous feedback of sectoral markups onto nominal wages. Note that sectoral markups are linked to sectoral prices through (4): $\log\boldsymbol{\mu} = (I - \boldsymbol{\delta}^{-1})\log\boldsymbol{p} + o(\|\xi\|)$.

Substituting these expressions into the marginal-cost system gives

$$\begin{aligned}\log\boldsymbol{p} &= \boldsymbol{\delta}(-\log z + \Omega\log p + \alpha\, d\log w) + o(\|\xi\|)\\ &= -(I-\boldsymbol{\delta}\Omega)^{-1}\boldsymbol{\delta}\log z + (I-\boldsymbol{\delta}\Omega)^{-1}\boldsymbol{\delta}\alpha\, d\log w + o(\|\xi\|)\\ &= -(I-\boldsymbol{\delta}\Omega)^{-1}\boldsymbol{\delta}\log z + \varrho^{w}\mathbf{1}\cdot\log m + \varrho^{w}(\Gamma + \ell^{\mu}\boldsymbol{\eta}^{-1})'(I-\boldsymbol{\delta}^{-1})\log\boldsymbol{p} + o(\|\xi\|)\end{aligned}$$

Rearranging terms gives the first-order equilibrium mapping from productivity and monetary shocks to sectoral prices,

$$\begin{aligned}\log\boldsymbol{p} = &-[I - \varrho^{w}(\Gamma + \ell^{\mu}\boldsymbol{\eta}^{-1})'(I-\boldsymbol{\delta}^{-1})]^{-1}(I-\boldsymbol{\delta}\Omega)^{-1}\boldsymbol{\delta}\log z\\ &+ [I - \varrho^{w}(\Gamma + \ell^{\mu}\boldsymbol{\eta}^{-1})'(I-\boldsymbol{\delta}^{-1})]^{-1}\varrho^{w}\mathbf{1}\cdot\log m + o(\|\xi\|).\end{aligned} \tag{B-19}$$

□

The next result summarizes the second-order approximation to sectoral wedges in terms of prices, nominal wages, productivities.

Lemma 5. *Up to a second-order approximation around the steady-state equilibrium, the sectoral wedge ε_i satisfies*

$$\log\varepsilon_i = \sum_{j=1}^{N}\omega_{ij}\log p_j - \log p_i + \sum_{c=1}^{C}\alpha_{ic}\Delta\log w_c - \log z_i + \frac{1}{2}\theta_i\vartheta_i + o(\|\xi\|^2) \tag{B-20}$$

where $\Delta\log w_c \equiv \log w_c - \log\bar{w}_c$ denotes the log-deviation of the wage of household c from its steady-state level, and

$$\vartheta_i = \mathrm{Var}(\log p_{ik}) = \int_0^1(\log p_{ik} - \log p_i)^2 dk - \left(\int_0^1(\log p_{ik} - \log p_i)dk\right)^2$$

is the cross-sectional dispersion of prices within sector i. Under Calvo pricing, this dispersion satisfies

$$\vartheta_i = \frac{1-\delta_i}{\delta_i}(\log p_i)^2 + o(\|\xi\|^2) = \frac{\delta_i}{1-\delta_i}\log^2 \varepsilon_i + o(\|\xi\|^2).$$

Proof of Lemma 5. First, consider the CES price dispersion expression $\int_0^1 (p_{ik}/p_i)^{-\theta_i} dk$. To a second-order approximation,

$$\log \int_0^1 (p_{ik}/p_i)^{-\theta_i} dk = -\theta_i \int_0^1 (\log p_{ik} - \log p_i) dk + \frac{1}{2}\theta_i^2 \vartheta_i + o(\|\xi\|^2).$$

The definition of the sectoral price index in (2) then implies

$$\begin{aligned} 0 &= \frac{1}{1-\theta_i}\log[\int_0^1 (p_{ik}/p_i)^{1-\theta_i} dk] \\ &= (1-\theta_i)\int_0^1 (\log p_{ik} - \log p_i) dk + \frac{1}{2}(1-\theta_i)^2 \vartheta_i + o(\|\xi\|^2), \end{aligned}$$

which yields

$$\int_0^1 (\log p_{ik} - \log p_i) dk = \frac{1}{2}(\theta_i - 1)\vartheta_i + o(\|\xi\|^2).$$

Thus, the CES price-dispersion term simplifies to

$$\log \int_0^1 (p_{ik}/p_i)^{-\theta_i} dk = -\theta_i \left(\frac{1}{2}(\theta_i - 1)\vartheta_i\right) + \frac{1}{2}\theta_i^2 \vartheta_i + o(\|\xi\|^2) = \frac{1}{2}\theta_i \vartheta_i + o(\|\xi\|^2)$$

This equation, together with (B-14), allows for a log-quadratic approximation of the sectoral wedges in terms of prices, nominal wages, and productivities:

$$\begin{aligned} \log \varepsilon_i &= \log mc_i - \log p_i + \log \int_0^1 (p_{ik}/p_i)^{-\theta_i} dk \\ &= \sum_{c=1}^{C} \alpha_{ic} \Delta \log w_c - \log z_i + \sum_{j=1}^{N} \omega_{ij} \log p_j - \log p_i + \frac{1}{2}\theta_i \vartheta_i + o(\|\xi\|^2) \end{aligned}$$

□

Proof of Lemma 2. The household's welfare loss relative to the flexible-price equilibrium takes the form

$$W(\mathbf{e}_c) - W^*(\mathbf{e}_c) = U_c - U_c^* = (\log C_c - \log C_c^*) - \psi_c\left(\frac{L_c^{1+1/\eta_c}}{1+1/\eta_c} - \frac{(L_c^*)^{1+1/\eta_c}}{1+1/\eta_c}\right).$$

A second-order approximation of the disutility from labor yields

$$\begin{aligned}\psi_c\left(\frac{L_c^{1+1/\eta_c}}{1+1/\eta_c} - \frac{(L_c^*)^{1+1/\eta_c}}{1+1/\eta_c}\right) &= \psi_c(L_c^*)^{1+1/\eta_c}(\hat{l}_c + \frac{1+1/\eta_c}{2}\hat{l}_c^2) + o(\|\xi\|^2) \\ &= \hat{l}_c + \frac{1+1/\eta_c}{2}\hat{l}_c^2 + o(\|\xi\|^2)\end{aligned}$$

where the second line uses the fact that $\psi_c(L_c^*)^{1+1/\eta_c} = \Lambda_c^*/\chi_c^* = 1$ as established in (B-9).

Substituting this approximation into the welfare loss expression gives

$$\begin{aligned}W(\mathbf{e}_c) - W^*(\mathbf{e}_c) &= \hat{c}_c - \left(\hat{l}_c + \frac{1+1/\eta_c}{2}\hat{l}_c^2\right) + o(\|\xi\|^2) \\ &= \hat{c}_c - \hat{l}_c - \frac{1+1/\eta_c}{2}\hat{l}_c^2 + o(\|\xi\|^2). \qquad \text{(B-21)}\end{aligned}$$

The next part of the proof is devoted to obtaining a second-order approximation of allocative efficiency, represented by the difference $\hat{c}_c - \hat{l}_c$.

First, combining equations (B-8) and (B-9), household c's consumption-labor ratio satisfies

$$\log C_c - \log L_c = \log w_c - \log P_c - \log(\Lambda_c/\chi_c) + \log \psi_c.$$

Second, applying Lemma 5 and aggregating sectoral wedges using household-level Domar weights $(\lambda_i^c)^*$ yields

$$\begin{aligned}\sum_{i=1}^N (\lambda_i^c)^* \log \varepsilon_i &= \sum_{i=1}^N \sum_{j=1}^N (\lambda_i^c)^* \omega_{ij} \log p_j - \sum_{i=1}^N (\lambda_i^c)^* \log p_i \\ &\quad + \sum_{i=1}^N \sum_{f=1}^C (\lambda_i^c)^* \alpha_{if} \Delta \log w_f - \sum_{i=1}^N (\lambda_i^c)^* \log z_i + \frac{1}{2}\sum_{i=1}^N (\lambda_i^c)^* \theta_i \vartheta_i + o(\|\xi\|^2) \\ &= -\sum_{i=1}^N \beta_i^c \log p_i + \sum_{f=1}^C (\Lambda_f^c)^* \Delta \log w_f - \sum_{i=1}^N (\lambda_i^c)^* \log z_i + \frac{1}{2}\sum_{i=1}^N (\lambda_i^c)^* \theta_i \vartheta_i + o(\|\xi\|^2)\end{aligned}$$

where I use the identities $\sum_{i=1}^N (\lambda_i^c)^* \omega_{ij} = (\lambda_j^c)^* - \beta_j^c$ and $\sum_{i=1}^N (\lambda_i^c)^* \alpha_{if} = (\Lambda_f^c)^*$.

Recall that the household-level price index satisfies $\log P_c = \sum_{i=1}^N \beta_i^c \log p_i$. Substituting the expression above into the consumption-labor ratio to obtain

$$\log C_c - \log L_c = \sum_{i=1}^N (\lambda_i^c)^* \log z_i + \sum_{i=1}^N (\lambda_i^c)^* \log \varepsilon_i - \frac{1}{2}\sum_{i=1}^N (\lambda_i^c)^* \theta_i \vartheta_i$$
$$+ \log w_c - \sum_{f=1}^C (\Lambda_f^c)^* \log w_f - \log(\Lambda_c/\chi_c) + F_c^1 + o(\|\xi\|^2) \tag{B-22}$$

where $F_c^1 = \log \psi_c + \sum_{f=1}^C (\Lambda_f^c)^* \log \varpi_f$ is a constant.

Using the expression for the labor income share in equation (B-16), the factoral terms-of-trade $\log w_c - \sum_{f=1}^C (\Lambda_f^c)^* \log w_f$ can be rewritten as

$$\begin{aligned}
\mathrm{FToT}_c =&(\log m + \log \Lambda_c - \log L_c) - \sum_{f=1}^C (\Lambda_f^c)^* (\log m + \log \Lambda_f - \log L_f) \\
=&(\log \Lambda_c - \log L_c) - \sum_{f=1}^C (\Lambda_f^c)^* (\log \Lambda_f - \log L_f) \\
=&\frac{1}{1+\eta_c}\log \Lambda_c + \frac{\eta_c}{1+\eta_c}\log \chi_c - \sum_{f=1}^C (\Lambda_f^c)^* (\frac{1}{1+\eta_f}\log \Lambda_f + \frac{\eta_f}{1+\eta_f}\log \chi_f) + F_c^2 \\
=&\frac{1}{1+\eta_c}\log(\Lambda_c/\chi_c) - \sum_{f=1}^C \frac{1}{1+\eta_f}(\Lambda_f^c)^* \log(\Lambda_f/\chi_f) + \log \chi_c - \sum_{f=1}^C (\Lambda_f^c)^* \log \chi_f + F_c^2
\end{aligned} \tag{B-23}$$

where the second equality uses the fact that $\sum_{f=1}^C (\Lambda_f^c)^* = 1$, and the third equality substitutes $\log L_c = \frac{\eta_c}{1+\eta_c}(\log \Lambda_c - \log \chi_c - \log \psi_c)$ with the resulting constant term $F_c^2 = \frac{\eta_c}{1+\eta_c}\log \psi_c - \sum_{f=1}^C (\Lambda_f^c)^* \frac{\eta_f}{1+\eta_f}\log \psi_f$.

Thus, substituting the expression for the terms-of-trade effect yields

$$\log C_c - \log L_c = \sum_{i=1}^N (\lambda_i^c)^* \log z_i + \sum_{i=1}^N (\lambda_i^c)^* \log \varepsilon_i - \frac{1}{2}\sum_{i=1}^N (\lambda_i^c)^* \theta_i \vartheta_i - \frac{\eta_c}{1+\eta_c}\log(\Lambda_c/\chi_c)$$
$$- \sum_{f=1}^C \frac{1}{1+\eta_f}(\Lambda_f^c)^* \log(\Lambda_f/\chi_f) + \log \chi_c - \sum_{f=1}^C (\Lambda_f^c)^* \log \chi_f + F_c + o(\|\xi\|^2),$$

where $F_c = F_c^1 + F_c^2$ is a constant.

In the flexible price equilibrium, where $\varepsilon_i^* = 1$, $\vartheta_i^* = 0$, and $\Lambda_c^* = \chi_c^*$, the household-level consumption-labor ratio simplifies to

$$\log C_c^* - \log L_c^* = \sum_{i=1}^{N} (\lambda_i^c)^* \log z_i + \log \chi_c^* - \sum_{f=1}^{C} (\Lambda_f^c)^* \log \chi_f^* + F_c + o(\|\xi\|^2).$$

Taking the difference between the two expressions gives the second-order approximation of allocative efficiency:

$$\begin{aligned}
\hat{c}_c - \hat{l}_c &= \log C_c - \log L_c - (\log C_c^* - \log L_c^*) \\
&= \sum_{i=1}^{N} (\lambda_i^c)^* \log \varepsilon_i - \frac{1}{2} \sum_{i=1}^{N} (\lambda_i^c)^* \theta_i \vartheta_i + \hat{\chi}_c - \sum_{f=1}^{C} (\Lambda_f^c)^* \hat{\chi}_f \\
&\quad - \frac{\eta_c}{1+\eta_c} \log(\Lambda_c/\chi_c) - \sum_{f=1}^{C} \frac{1}{1+\eta_f} (\Lambda_f^c)^* \log(\Lambda_f/\chi_f) + o(\|\xi\|^2)
\end{aligned} \tag{B-24}$$

Finally, applying Lemma 4 allows us to express the welfare loss in terms of sectoral wedges,

$$\begin{aligned}
W(\mathbf{e}_c) - W^*(\mathbf{e}_c) &= \sum_{i=1}^{N} \left[(\lambda_i^c)^* - \frac{\eta_c}{1+\eta_c} \frac{\lambda_i^* \Phi_{ic}}{\chi_c^*} - \sum_{f=1}^{C} \frac{1}{1+\eta_f} (\Lambda_f^c)^* \frac{\lambda_i^* \Phi_{if}}{\chi_f^*} - \Gamma_{ic} + \sum_{f=1}^{C} (\Lambda_f^c)^* \Gamma_{if} \right] \log \varepsilon_i \\
&\quad - \sum_{i=1}^{N} \sum_{j=1}^{N} \left[\frac{\eta_c}{1+\eta_c} \Upsilon_{ij}^c + \sum_{f=1}^{C} \frac{1}{1+\eta_f} \Lambda_f^c \Upsilon_{ij}^f - \Xi_{ij}^c + \sum_{f=1}^{C} \Lambda_f^c \Xi_{ij}^f \right] \log \varepsilon_j \log \varepsilon_i \\
&\quad - \frac{1}{2} \sum_{i=1}^{N} (\lambda_i^c)^* \theta_i \vartheta_i - \frac{1+1/\eta_c}{2} \hat{l}_c^2 + o(\|\xi\|^2).
\end{aligned} \tag{B-25}$$

Note that $\vartheta_i = \frac{\delta_i}{1-\delta_i} \log^2 \varepsilon_i + o(\|\xi\|^2)$ and $\hat{l}_c^2 = (\sum_{i=1}^{N} \ell_{ic}^{\mu} \log \varepsilon_i)^2 + o(\|\xi\|^2)$, so the last two terms are quadratic in $\log \varepsilon_i$. Grouping them accordingly completes the proof.

□

The second-order approximation of allocative efficiency derived in Lemma 2 serves as a key intermediate step for establishing Proposition 1, which provides a first-order characterization of household-level allocative efficiency.

Proof of Proposition 1. Starting from the second-order approximation to the consumption–labor ratio in equation (B-22), a first-order approximation yields:

$$\begin{aligned}
\log C_c - \log L_c &= \sum_{i=1}^{N} (\lambda_i^c)^* \log z_i + \sum_{i=1}^{N} (\lambda_i^c)^* \log \varepsilon_i - \log(\Lambda_c/\chi_c) \\
&\quad + \log w_c - \sum_{f=1}^{C} (\Lambda_f^c)^* \log w_f + F_c^1 + o(\|\xi\|) \\
&= \sum_{i=1}^{N} (\lambda_i^c)^* \log z_i - \sum_{i=1}^{N} \left(\frac{\lambda_i^* \Phi_{ic}}{\chi_c^*} - (\lambda_i^c)^* \right) \log \varepsilon_i \\
&\quad + \log w_c - \sum_{f=1}^{C} (\Lambda_f^c)^* \log w_f + F_c^1 + o(\|\xi\|)
\end{aligned}$$

where the second equality uses the first-order approximation of $\log(\Lambda_c/\chi_c)$ from equation (B-3).

In the flexible price equilibrium, where $\varepsilon_i^* = 1$, and $\log w_c = \log \varpi_c + \log m$ for all c, the consumption–labor ratio simplifies to

$$\log C_c^* - \log L_c^* = \sum_{i=1}^{N} (\lambda_i^c)^* \log z_i + \log \varpi_c - \sum_{f=1}^{C} (\Lambda_f^c)^* \log \varpi_f + F_c^1 + o(\|\xi\|)$$

Taking the difference between sticky-price and flexible-price outcomes yields the household-level allocative efficiency

$$\hat{c}_c - \hat{l}_c = -\sum_{i=1}^{N} \left(\frac{\lambda_i^* \Phi_{ic}}{\chi_c^*} - (\lambda_i^c)^* \right) \log \varepsilon_i + d \log w_c - \sum_{f=1}^{C} (\Lambda_f^c)^* d \log w_f + o(\|\xi\|).$$

Approximating the factoral terms-of-trade effect to a first order using equation (B-23) and Lemma 4 yields

$$\begin{aligned}
\Delta\text{FToT}_c &= d\log w_c - \sum_{f=1}^{C} (\Lambda_f^c)^* d\log w_f \\
&= \frac{1}{1+\eta_c} d\log(\Lambda_c/\chi_c) - \sum_{f=1}^{C} \frac{1}{1+\eta_f} (\Lambda_f^c)^* d\log(\Lambda_f/\chi_f) + d\log\chi_c - \sum_{f=1}^{C} (\Lambda_f^c)^* d\log\chi_f \\
&= -\sum_{i=1}^{N} \left[\left(\Gamma_{ic} - \frac{1}{1+\eta_c} \frac{\lambda_i^* \Phi_{ic}}{\chi_c^*} \right) - \sum_{f=1}^{C} (\Lambda_f^c)^* \left(\Gamma_{if} - \frac{1}{1+\eta_f} \frac{\lambda_i^* \Phi_{if}}{\chi_f^*} \right) \right] \log\varepsilon_i + o(\|\xi\|).
\end{aligned}$$

□

Proof of Proposition 3. The social welfare loss is defined as the Pareto-weighted sum of household-level losses:

$$W(\{\kappa_c\}) - W^*(\{\kappa_c\}) = \sum_{c=1}^{C} \kappa_c \left[W(\mathbf{e}_c) - W^*(\mathbf{e}_c) \right].$$

Applying Lemma 2 to each term, we have

$$W(\mathbf{e}_c) - W^*(\mathbf{e}_c) = \mathcal{J}_c' \log\boldsymbol{\mu} - \tfrac{1}{2} \log\boldsymbol{\mu}' \mathcal{L}_c \log\boldsymbol{\mu}.$$

Summing across households yields

$$W(\{\kappa_c\}) - W^*(\{\kappa_c\}) = \sum_{c=1}^{C} \kappa_c \mathcal{J}_c' \log\boldsymbol{\mu} - \tfrac{1}{2} \log\boldsymbol{\mu}' \mathcal{L}(\{\kappa_c\}) \log\boldsymbol{\mu},$$

where $\mathcal{L}(\{\kappa_c\}) = \sum_{c=1}^{C} \kappa_c \mathcal{L}_c$. □

Corollary 3 (Welfare Loss under Income-Share Weights)**.** When Pareto weights are set equal to households' income shares, i.e., $\kappa_c = \chi_c$ for all c, reallocations across households do not generate first-order welfare gains. This implies that monetary policy cannot improve aggregate efficiency through redistribution at the first order. In this case, the aggregate welfare loss simplifies to a purely quadratic form in ex post markups

$$W(\{\chi_c\}_{c=1}^{C}) - W^*(\{\chi_c\}_{c=1}^{C}) = -\tfrac{1}{2} \log\boldsymbol{\mu}' \, \mathcal{L}(\{\chi_c\}_{c=1}^{C}) \, \log\boldsymbol{\mu}.$$

The aggregate loss matrix is given by

$$\mathcal{L}(\{\chi_c\}_{c=1}^C) = \mathcal{L}^{\text{e.g.}}(\{\chi_c\}) + \mathcal{L}^{\text{within}}(\{\chi_c\}) + \mathcal{L}^{\text{across}}(\{\chi_c\}),$$

where

$$\begin{aligned}
\mathcal{L}_{ij}^{\text{e.g.}}(\{\chi_c\}) &= \sum_{c=1}^{C} \frac{\eta_c}{1+\eta_c}\, \chi_c^{-1}\, \lambda_i \lambda_j \Phi_{ic} \Phi_{jc}, \\
\mathcal{L}_{ij}^{\text{within}}(\{\chi_c\}) &= \lambda_i \theta_i \frac{\delta_i}{1-\delta_i}\, \iota_{ij}, \\
\mathcal{L}_{ij}^{\text{across}}(\{\chi_c\}) &= \lambda_i \Psi_{ij} + \lambda_j \Psi_{ji} - \lambda_i \iota_{ij} \\
&\quad + \sum_{c=1}^{C} \Big[(\lambda_i \Phi_{ic} - \lambda_i^c \chi_c) \Gamma_{jc} + (\lambda_j \Phi_{jc} - \lambda_j^c \chi_c) \Gamma_{ic} - \chi_c^{-1} \lambda_i \lambda_j \Phi_{ic} \Phi_{jc} \Big].
\end{aligned}$$

Proof of Corollary 3. From equation (B-21), the aggregate welfare loss under income-share Pareto weights $\{\chi_c\}$ is:

$$\begin{aligned}
W(\{\chi_c\}_{c=1}^C) - W^*(\{\chi_c\}_{c=1}^C) &= \sum_{c=1}^{C} \chi_c^* \left[W(\mathbf{e}_c) - W^*(\mathbf{e}_c) \right] \\
&= \sum_{c=1}^{C} \chi_c^* (\hat{c}_c - \hat{l}_c) - \sum_{c=1}^{C} \frac{1 + 1/\eta_c}{2} \chi_c^* \hat{l}_c^2 + o(\|\xi\|^2)
\end{aligned}$$

Using the second-order approximation of allocative efficiency in (B-24) and aggregating across households with weights χ_c^* yields

$$\sum_{c=1}^{C}\chi_c^*(\hat{c}_c-\hat{l}_c)=\sum_{c=1}^{C}\sum_{i=1}^{N}\chi_c^*(\lambda_i^c)^*\log\varepsilon_i-\frac{1}{2}\sum_{c=1}^{C}\sum_{i=1}^{N}\chi_c^*(\lambda_i^c)^*\theta_i\vartheta_i+\sum_{c=1}^{C}\chi_c^*\hat{\chi}_c-\sum_{c=1}^{C}\sum_{f=1}^{C}\chi_c^*(\Lambda_f^c)^*\hat{\chi}_f$$
$$-\sum_{c=1}^{C}\frac{\eta_c}{1+\eta_c}\chi_c^*\log(\Lambda_c/\chi_c)-\sum_{c=1}^{C}\sum_{f=1}^{C}\frac{1}{1+\eta_f}\chi_c^*(\Lambda_f^c)^*\log(\Lambda_f/\chi_f)+o(\|\xi\|^2)$$
$$=\sum_{i=1}^{N}\lambda_i^*\log\varepsilon_i-\frac{1}{2}\sum_{i=1}^{N}\lambda_i^*\theta_i\vartheta_i+\sum_{c=1}^{C}\chi_c^*\hat{\chi}_c-\sum_{f=1}^{C}\chi_f^*\hat{\chi}_f$$
$$-\sum_{c=1}^{C}\frac{\eta_c}{1+\eta_c}\chi_c^*\log(\Lambda_c/\chi_c)-\sum_{f=1}^{C}\frac{1}{1+\eta_f}\chi_f^*\log(\Lambda_f/\chi_f)+o(\|\xi\|^2)$$
$$=\sum_{i=1}^{N}\lambda_i^*\log\varepsilon_i-\frac{1}{2}\sum_{i=1}^{N}\lambda_i^*\theta_i\vartheta_i-\sum_{c=1}^{C}\chi_c^*\log(\Lambda_c/\chi_c)+o(\|\xi\|^2), \tag{B-26}$$

where the second equality uses the identities: $\sum_{c=1}^{C}\chi_c^*(\lambda_i^c)^*=\lambda_i^*$ and $\sum_{c=1}^{C}\chi_c^*(\Lambda_f^c)^*=\chi_f^*$.

Next, aggregating the second-order labor-wedge expansion (B-12) across households using income shares gives

$$\sum_{c=1}^{C}\chi_c^*\log(\Lambda_c/\chi_c)=\sum_{i=1}^{N}\lambda_i^*\log\varepsilon_i+\sum_{i=1}^{N}\sum_{j=1}^{N}\lambda_j^*\Psi_{ji}\log\varepsilon_j\log\varepsilon_i-\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{f=1}^{C}\lambda_i^f\chi_f^*\Gamma_{jf}\log\varepsilon_j\log\varepsilon_i$$
$$+\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{c=1}^{C}\lambda_i^*\Phi_{ic}\Gamma_{jc}\log\varepsilon_j\log\varepsilon_i-\frac{1}{2}\sum_{i=1}^{N}\lambda_i^*\log^2\varepsilon_i-\frac{1}{2}\sum_{c=1}^{C}\chi_c^*\Big(\sum_{i=1}^{N}\frac{\lambda_i^*\Phi_{ic}}{\chi_c^*}\log\varepsilon_i\Big)^2+o(\|\xi\|^2)$$
$$=\sum_{i=1}^{N}\lambda_i^*\log\varepsilon_i+\sum_{i=1}^{N}\sum_{j=1}^{N}\lambda_j^*\Psi_{ji}\log\varepsilon_j\log\varepsilon_i+\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{c=1}^{C}(\lambda_i^*\Phi_{ic}-\lambda_i^c\chi_c^*)\Gamma_{jc}\log\varepsilon_j\log\varepsilon_i$$
$$-\frac{1}{2}\sum_{i=1}^{N}\lambda_i^*\log^2\varepsilon_i-\frac{1}{2}\sum_{c=1}^{C}(\chi_c^*)^{-1}\Big(\sum_{i=1}^{N}\lambda_i^*\Phi_{ic}\log\varepsilon_i\Big)^2+o(\|\xi\|^2).$$

Substituting (B-26) into the aggregate welfare loss function and eliminating the intermediate term $\sum_c \chi_c^* \log(\Lambda_c/\chi_c)$ yields

$$
\begin{aligned}
W(\{\chi_c\}) - W^*(\{\chi_c\}) &= \sum_{c=1}^{C} \chi_c^*(\hat{c}_c - \hat{l}_c) - \sum_{c=1}^{C} \frac{1+1/\eta_c}{2} \chi_c^* \hat{l}_c^2 + o(\|\xi\|^2) \\
&= -\frac{1}{2}\Bigg[\sum_{c=1}^{C} \frac{\eta_c}{1+\eta_c} (\chi_c^*)^{-1} (\sum_{i=1}^{N} \lambda_i^* \Phi_{ic} \log \varepsilon_i)^2 + \sum_{i=1}^{N} \lambda_i^* \theta_i \vartheta_i \\
&\quad + 2 \sum_{i=1}^{N} \sum_{j=1}^{N} \lambda_i^* \Psi_{ij} \log \varepsilon_j \log \varepsilon_i + 2 \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{c=1}^{C} (\lambda_i^* \Phi_{ic} - \lambda_i^c \chi_c^*) \Gamma_{jc} \log \varepsilon_j \log \varepsilon_i \\
&\quad - \sum_{i=1}^{N} \lambda_i^* \log^2 \varepsilon_i - \sum_{c=1}^{C} (\chi_c^*)^{-1} (\sum_{i=1}^{N} \lambda_i^* \Phi_{ic} \log \varepsilon_i)^2 \Bigg] + o(\|\xi\|^2) \\
&= -\tfrac{1}{2} \log \boldsymbol{\mu}' \, \mathcal{L}(\{\chi_c\}) \, \log \boldsymbol{\mu} + o(\|\xi\|^2).
\end{aligned}
$$

where the aggregate loss matrix $\mathcal{L}(\{\chi_c\})$ admits the decomposition stated in the corollary.

□

Lemma 6 (Implementability of Price-Targeting Rules)**.** To a first-order approximation, any price-targeting rule of the form $\zeta' \log \boldsymbol{p} = \pi$, for some pair $(\zeta, \pi) \in \mathbb{R}^N \times \mathbb{R}$, can be implemented by a monetary policy rule for nominal aggregate demand of the form $\log m(z) = \varsigma_0 + \sum_{i=1}^{N} \varsigma_i \log z_i$, for a suitable choice of coefficients (ς_0, ς).

Proof of Lemma 6. From the first-order approximation in (B-19), the vector of sectoral prices satisfies

$$\log \boldsymbol{p} = -\varrho^z \log z + \varrho^m \cdot \log m + o(\|\xi\|),$$

where $\varrho^z \equiv \left[I - \varrho^w (\Gamma + \ell^\mu \boldsymbol{\eta}^{-1})'(I - \boldsymbol{\delta}^{-1}) \right]^{-1} (I - \boldsymbol{\delta}\Omega)^{-1} \boldsymbol{\delta}$ characterizes the elasticity of sectoral prices with respect to productivity shocks under price stickiness.

Taking inner products with ζ and applying the price-targeting condition gives

$$\pi = \zeta' \log \boldsymbol{p} = -\zeta' \varrho^z \log z + \zeta' \varrho^m \cdot \log m + o(\|\xi\|).$$

Solving for $\log m$ yields

$$\log m(z) = \frac{\pi}{\zeta' \varrho^m} + \frac{\zeta' \varrho^z}{\zeta' \varrho^m} \log z + o(\|\xi\|).$$

Thus, setting

$$\varsigma_0 = \frac{\pi}{\zeta' \varrho^m}, \quad \varsigma_i = \frac{\zeta' \varrho^z_{(:,i)}}{\zeta' \varrho^m}, \quad i = 1, \ldots, N,$$

defines a monetary policy rule that implements the price target to a first-order approximation. Note that since $\varrho^m = \varrho^z \boldsymbol{\alpha} \mathbf{1}$, price-targeting rules span only a subset of admissible monetary policies to first order. □

Proof of Theorem 1. The monetary authority chooses the policy instrument $\log m$ to minimize the aggregate welfare loss in (13). Since monetary policy affects welfare only through its impact on prices and markups, the first-order condition is obtained by differentiating the loss with respect to $\log m$:

$$\frac{d}{d\log m}\Big[W(\{\kappa_c\}) - W^*(\{\kappa_c\})\Big] = \frac{d\log \boldsymbol{\mu}'}{d\log m}\left(\sum_{c=1}^{C} \kappa_c \mathcal{J}_c - \mathcal{L}(\{\kappa_c\}) \log \boldsymbol{\mu}\right) = 0. \tag{B-27}$$

Under Calvo pricing, sectoral markups are related to sectoral prices through (4), $\log \boldsymbol{\mu} = -(\boldsymbol{\delta}^{-1} - I) \log \boldsymbol{p}$. Moreover, from Proposition 2, sectoral prices respond to the policy shock according to $\frac{d\log \boldsymbol{p}}{d\log m} = \varrho^m$. Combining the two expressions implies

$$\frac{d\log \boldsymbol{\mu}}{d\log m} = -(\boldsymbol{\delta}^{-1} - I)\varrho^m.$$

Substituting into the first-order condition (B-27) gives

$$(\varrho^m)'(I - \boldsymbol{\delta}^{-1})\mathcal{L}(\{\kappa_c\}) \log \boldsymbol{\mu} = \sum_{c=1}^{C} \kappa_c \mathcal{J}_c'(I - \boldsymbol{\delta}^{-1})\varrho^m.$$

Using (4) once more, the condition can be re-expressed as a target criterion for a weighted sectoral price index

$$\sum_{j=1}^{N} \zeta_j^*(\{\kappa_c\}) \log p_j = \pi^*(\{\kappa_c\}),$$

where the optimal weights and target level are given by

$$\zeta_j^*(\{\kappa_c\}) = (\delta_j^{-1} - 1) \sum_{i=1}^{N} (\delta_i^{-1} - 1)\varrho_i^m \mathcal{L}_{ij}(\{\kappa_c\}),$$

and

$$\pi^*(\{\kappa_c\}) = \sum_{c=1}^{C} \kappa_c \mathcal{J}_c'(I - \boldsymbol{\delta}^{-1})\varrho^m,$$

respectively.

Finally, using the decomposition

$$\mathcal{L}(\{\kappa_c\}) = \mathcal{L}^{\text{e.g.}}(\{\kappa_c\}) + \mathcal{L}^{\text{within}}(\{\kappa_c\}) + \mathcal{L}^{\text{across}}(\{\kappa_c\}),$$

the sectoral weights $\zeta_j^*(\{\kappa_c\})$ admit the corresponding decomposition:

$$\zeta_j^*(\{\kappa_c\}) = \zeta_j^{\text{e.g.}}(\{\kappa_c\}) + \zeta_j^{\text{within}}(\{\kappa_c\}) + \zeta_j^{\text{across}}(\{\kappa_c\}),$$

where each term reflects the contribution of employment-gap volatility, within-sector price dispersion, and cross-sector misallocation to the overall optimal sectoral weight.

□

Lemma 7. Given a realization of productivity shocks $\log z$, the vector of ex post markups is, to a first-order approximation, a function of the policy pair (ζ, π):

$$\log \boldsymbol{\mu} \;=\; \mathcal{M}(\zeta, \pi) = \frac{\pi}{\zeta' \varrho^m}(I - \boldsymbol{\delta}^{-1})\varrho^m + (I - \boldsymbol{\delta}^{-1})\left(\frac{\varrho^m \zeta'}{\zeta' \varrho^m} - I\right)\varrho^z \log z + o(\|\xi\|). \tag{B-28}$$

Proof of Lemma 7. By Lemma 6, under the policy instrument (ζ, π), sectoral prices satisfy

$$\begin{aligned}
\log \boldsymbol{p} &= -\varrho^z \log z + \varrho^m \log m(z) + o(\|\xi\|), \\
&= -\varrho^z \log z + \varrho^m \left(\frac{\pi}{\zeta' \varrho^m} + \frac{\zeta' \varrho^z}{\zeta' \varrho^m} \log z\right) + o(\|\xi\|), \\
&= \frac{\pi}{\zeta' \varrho^m}\varrho^m + \left(\frac{\varrho^m \zeta'}{\zeta' \varrho^m} - I\right)\varrho^z \log z + o(\|\xi\|)
\end{aligned}$$

Substituting this expression into (4) delivers the stated mapping $\log \boldsymbol{\mu} = \mathcal{M}(\zeta, \pi)$. □

Lemma 8. Under a given union-wide price-index regime ζ, the unilateral optimal policy can be implemented by a state-contingent inflation rule, $\tilde{\pi}_c(\zeta; z) = \arg\min_\pi \mathbb{L}(\zeta, \pi)$. To a first-order approximation, this rule is given by

$$\tilde{\pi}_c(\zeta; z) = \frac{\zeta' \varrho^m}{\zeta^*(\mathbf{e}_c)' \varrho^m}\pi^*(\mathbf{e_c}) + \zeta' \left[I - \frac{\varrho^m \zeta^*(\mathbf{e}_c)'}{\zeta^*(\mathbf{e}_c)' \varrho^m}\right]\varrho^z \log z + o(\|\xi\|).$$

Proof of Lemma 8. By Lemma 6, the unilateral optimal policy for country c can, to a first-order approximation, be implemented by a monetary rule of the form

$$\log m(z) = \frac{\pi^*(\mathbf{e}_c)}{\zeta^*(\mathbf{e}_c)'\varrho^m} + \frac{\zeta^*(\mathbf{e}_c)'\varrho^z}{\zeta^*(\mathbf{e}_c)'\varrho^m}\log z + o(\|\xi\|).$$

Under a given union-wide price-index regime ζ, any admissible price-targeting rule that delivers inflation π induces a monetary policy of the form

$$\log m(z) = \frac{\pi}{\zeta'\varrho^m} + \frac{\zeta'\varrho^z}{\zeta'\varrho^m}\log z + o(\|\xi\|).$$

To replicate the unilateral optimum under regime ζ, the induced monetary rule must coincide with the unilateral rule state by state, up to first order. Equating the two expressions yields

$$\frac{\pi}{\zeta'\varrho^m} + \frac{\zeta'\varrho^z}{\zeta'\varrho^m}\log z = \frac{\pi^*(\mathbf{e}_c)}{\zeta^*(\mathbf{e}_c)'\varrho^m} + \frac{\zeta^*(\mathbf{e}_c)'\varrho^z}{\zeta^*(\mathbf{e}_c)'\varrho^m}\log z + o(\|\xi\|).$$

Rearranging the equality establishes the stated inflation rule. □

Proof of Proposition 4. Assume productivity shocks are normally distributed, $\log z \sim N(\mathbf{0}, \Sigma_z)$. By Lemma 7, the expected welfare loss of country c can be written as

$$\begin{aligned}
\mathbb{E}[\mathbb{L}_c(\zeta,\pi)] =& \mathbb{E}\Big[-\mathcal{J}_c'\mathcal{M}(\zeta,\pi) + \tfrac{1}{2}\mathcal{M}(\zeta,\pi)'\mathcal{L}_c\mathcal{M}(\zeta,\pi)\Big] \\
=& -\frac{\pi}{\zeta'\varrho^m}\mathcal{J}_c'(I-\boldsymbol{\delta}^{-1})\varrho^m + \tfrac{1}{2}\left(\frac{\pi}{\zeta'\varrho^m}\right)^2(\varrho^m)'(I-\boldsymbol{\delta}^{-1})\mathcal{L}_c(I-\boldsymbol{\delta}^{-1})\varrho^m \\
&+ \tfrac{1}{2}\mathrm{tr}\left((\mu^z)'\mathcal{L}_c\mu^z\Sigma_z\right) \\
=& -\frac{\pi}{\zeta'\varrho^m}\pi^*(\mathbf{e}_c) + \tfrac{1}{2}\left(\frac{\pi}{\zeta'\varrho^m}\right)^2\zeta^*(\mathbf{e}_c)'\varrho^m + \tfrac{1}{2}\mathrm{tr}\left((\mu^z)'\mathcal{L}_c\mu^z\Sigma_z\right)
\end{aligned}$$

where $\mu^z = (I-\boldsymbol{\delta}^{-1})\left(\frac{\varrho^m\zeta'}{\zeta'\varrho^m} - I\right)\varrho^z$.

The last equality follows from Theorem 1, which characterizes the unilateral optimal policy

$$(\zeta^*(\mathbf{e}_c), \pi^*(\mathbf{e}_c)) = \left((I-\boldsymbol{\delta}^{-1})\mathcal{L}_c(I-\boldsymbol{\delta}^{-1})\varrho^m, \mathcal{J}_c'(I-\boldsymbol{\delta}^{-1})\varrho^m\right).$$

Note that the trace term is independent of π. Minimization with respect to π yields the unilateral inflation stance of country c:

$$\pi_c(\zeta) = \arg\min_{\pi\in\mathbb{R}} \mathbb{E}[\mathbb{L}_c(\zeta,\pi)] = \frac{\zeta' \varrho^m}{\zeta^*(\mathbf{e}_c)' \varrho^m} \pi^*(\mathbf{e}_\mathbf{c}).$$

□

Proof of Proposition 5. By Lemma 7 and using $\mathbb{E}[\log z] = \mathbf{0}$, the expected sectoral markup distortions satisfy

$$\mathbb{E}\Big[\mathcal{M}(\zeta,\pi)\Big] = \frac{\pi}{\zeta' \varrho^m}(I - \boldsymbol{\delta}^{-1})\varrho^m + o(\|\xi\|).$$

Evaluating at $\pi = \pi_c(\zeta) = \frac{\zeta' \varrho^m}{\zeta^*(\mathbf{e}_c)' \varrho^m} \pi^*(\mathbf{e}_\mathbf{c})$ yields

$$\mathbb{E}[\mathcal{M}(\zeta,\pi_c(\zeta))] = \frac{\pi^*(\mathbf{e}_c)}{\zeta^*(\mathbf{e}_c)' \varrho^m}(I - \boldsymbol{\delta}^{-1})\varrho^m + o(\|\xi\|) = \mathbb{E}[\mathcal{M}(\zeta^*(\mathbf{e}_c),\pi^*(\mathbf{e}_c))].$$

Premultiplying by $\mathcal{J}_c'$ yields the stated equality in expected allocative efficiency. □

C Optimal Monetary Policy in Example Economies

Corollary 4. When Pareto weights coincide with households' income shares and the economy features homogeneous consumption baskets and perfect risk sharing (i.e., $\beta_{ci} = b_i$ and $\Phi_{ic} = \chi_c$ for all c and i), the industry weights in the target price index under the optimal policy simplify to:

$$\begin{aligned}
\zeta_j^{\text{e.g.}}(\{\chi_c\}_{c=1}^C) &= (1/\delta_j - 1)\lambda_j, \\
\zeta_j^{\text{within}}(\{\chi_c\}_{c=1}^C) &= (1/\delta_j - 1)\lambda_j \theta_j \mathcal{K}_j, \\
\zeta_j^{\text{across}}(\{\chi_c\}_{c=1}^C) &= (1/\delta_j - 1)\sum_{i=1}^N (1/\delta_i - 1)\mathcal{K}_i(\lambda_i \Psi_{ij} + \lambda_j \Psi_{ji} - \lambda_i \iota_{ij} - \lambda_i \lambda_j),
\end{aligned}$$

where $\mathcal{K}_j = \varrho_j^m / (\sum_{c=1}^C \chi_c \ell_c^m)$ represents the slope of the Phillips curve for sector j. This result recovers the characterization in La'O and Tahbaz-Salehi (2022).

Example 4 (Roundabout economy)**.** Consider a roundabout economy where all sectors use the same input bundle, $\alpha_{ic} = \bar{\alpha}_c$ and $\omega_{ij} = \bar{\omega}_j$ for all i, while nominal rigidities are allowed to vary across sectors.

Proposition 2 then implies,

$$\varrho_i^m = \delta_i/\tilde{\delta} \quad \text{and} \quad \ell_c^m = \frac{\sum_{i=1}^N \Phi_{ic}\lambda_i(1-\delta_i)}{\tilde{\delta}\chi_c(1+1/\eta_c)},$$

where $\tilde{\delta} = \mathbb{E}_b(\delta) + \sum_{i=1}^N \sum_{c=1}^C \lambda_i(1-\delta_i)\frac{\Phi_{ic}}{1+1/\eta_c}$ is a constant. Notably, $\tilde{\delta}$ is an increasing function of the Domar weights (λ_i). As the economy becomes more roundabout—characterized by a higher intensity of intermediate input use (lower $\sum_c \bar{\alpha}_c$), the Domar weights rise, leading to a more sluggish aggregate price adjustment.[3] This confirms that greater production complexity acts as a structural amplifier of nominal rigidities.

When Pareto weights coincide with countries' income shares, the optimal industry weights in this economy are given by:

$$\begin{aligned}
\zeta_j^{\text{e.g.}}(\{\chi_c\}_{c=1}^C) &= (1/\delta_j - 1)\lambda_j \sum_{c=1}^C \Phi_{jc} \frac{\sum_{i=1}^N \Phi_{ic}\lambda_i(1-\delta_i)}{\chi_c(1+1/\eta_c)}, \\
\zeta_j^{\text{within}}(\{\chi_c\}_{c=1}^C) &= (1-\delta_j)\lambda_j\theta_j, \\
\zeta_j^{\text{across}}(\{\chi_c\}_{c=1}^C) &= (1/\delta_j - 1)\left[\lambda_j[\mathbb{E}_\beta(\delta) - \delta_j] + (2\lambda_j - \beta_j)\sum_{i=1}^N (1-\delta_i)\lambda_i\right] \\
&\quad + (1/\delta_j - 1)\sum_{i=1}^N \sum_{c=1}^C (1-\delta_i)(\chi_c^{-1}\lambda_i\lambda_j\Phi_{ic}\Phi_{jc} - \lambda_j\lambda_i^c\Phi_{jc} - \lambda_i\lambda_j^c\Phi_{ic}).
\end{aligned}$$

D Data Appendix

Input–output matrix. The input–output calibration relies on the World Input–Output Database (WIOD) (Timmer et al., 2015), which reports annual industry-level data for 44 countries (including a composite "rest of the world"), each with 56 industries, from 2000 to 2014.

I extract the 20 euro area countries and reconstruct their joint production network. All intra-Euro input–output linkages at the industry level are retained exactly as in the WIOD, while the remaining non-Euro countries are collapsed into 56 aggregate external sectors representing bilateral trade flows with the euro area. These external sectors transmit import and export linkages but do not engage in domestic production or trade among themselves, ensuring that the rest of the world enters the system only through its trade exposure to the

[3] In this example the Domar weights satisfy $\lambda_i = b_i + \frac{\bar{\omega}_i}{\sum_c \bar{\alpha}_c}$.

euro area.

The resulting dataset combines: (i) a fully detailed euro area production network with cross-country and cross-industry linkages, and (ii) a compact representation of external trade linkages aggregated by industry.

From the WIOD tables, I collect gross output, intermediate inputs, and value added, and aggregate final demand across destinations. Country income shares are then computed in proportion to their final-use shares and normalized to sum to one across the euro area. The final data are organized as an annual panel for 2000–2014 and forms the empirical basis for model calibration.

Price flexibility calibration. To calibrate the model's vector of price flexibilities, I use the industry-level frequency of price adjustment (FPA) data from Pastén et al. (2020), which provide monthly FPA estimates across NAICS industries ranging from broad 2-digit sectors to detailed 5-digit industries. Each WIOD industry is mapped to its closest NAICS code according to three principles: (i) if a WIOD industry corresponds to a single NAICS industry, it is directly matched to that NAICS code; if the exact code lacks FPA data, I use the next higher-level (truncated) NAICS classification; (ii) if a WIOD industry spans multiple NAICS industries that share a common parent classification, it is mapped to that parent (root) NAICS sector (e.g., Crop and animal production, hunting with NAICS 111/112/114 is assigned to 2-digit 11); and (iii) if a WIOD industry spans NAICS industries belonging to different parent sectors, it is matched to the single NAICS code with the largest sales share among its constituent industries (e.g., Warehousing and support activities for transportation with NAICS 488/493 is assigned to 488).

Sectoral productivity shocks. To estimate the variance–covariance matrix of sectoral productivity shocks, I construct annual series of sectoral total factor productivity (TFP) growth using the WIOD Socio-Economic Accounts (SEA, 2016 release) for euro-area economies over the period 2000–2014. The SEA provide industry-level data on nominal values, price indices, and real volume indices for gross output and intermediate inputs, as well as employment.

Nominal cost shares of labor and intermediate inputs in gross output are defined as

$$s_{i,t}^{L} \equiv \frac{\mathrm{VA}_{i,t}}{\mathrm{GO}_{i,t}}, \qquad s_{i,t}^{M} \equiv \frac{\mathrm{II}_{i,t}}{\mathrm{GO}_{i,t}},$$

where $\mathrm{VA}_{i,t}$ and $\mathrm{II}_{i,t}$ denote value added and intermediate inputs, respectively, and $\mathrm{GO}_{i,t}$ is gross output at current basic prices.

Let GO_QI and II_QI denote the real volume indices for gross output and intermediate inputs (2010 = 100), and EMP denote total employment. Annual growth rates of real variables are computed as log differences:

$$\begin{aligned}\Delta \log Y_{i,t} &\equiv \log \mathrm{GO\_QI}_{i,t} - \log \mathrm{GO\_QI}_{i,t-1},\\ \Delta \log L_{i,t} &\equiv \log \mathrm{EMP}_{i,t} - \log \mathrm{EMP}_{i,t-1},\\ \Delta \log M_{i,t} &\equiv \log \mathrm{II\_QI}_{i,t} - \log \mathrm{II\_QI}_{i,t-1}.\end{aligned}$$

Sectoral TFP growth is then obtained using a Törnqvist decomposition,

$$\Delta \log \mathrm{TFP}_{i,t} = \Delta \log Y_{i,t} - \tfrac{1}{2}\Big[(s^L_{i,t} + s^L_{i,t-1})\,\Delta \log L_{i,t} + (s^M_{i,t} + s^M_{i,t-1})\,\Delta \log M_{i,t}\Big].$$

which measures the Solow residual as output growth net of input growth weighted by average nominal cost shares. The Solow residuals are winsorized at the 1st and 99th percentiles to reduce the influence of extreme outliers.

These resulting series $\Delta \log \mathrm{TFP}_{i,t}$ form the basis for constructing the empirical variance–covariance matrix of sectoral productivity shocks, after interpolation to quarterly frequency and detrending.

E Global Economy under Dominant Currency Pricing

This section extends the analysis to a global economy under dominant currency pricing (DCP), with a quantitative focus on the optimal monetary policy of the dominant-currency country and the resulting policy-alignment losses across non-dominant-currency economies.

Irrelevance of exchange rates under dominant currency pricing. Assuming that all goods are tradable, DCP implies that nominal rigidities operate exclusively in dominant-currency prices. Domestic exchange rate movements then do not affect ex post markups or relative prices, rendering non-dominant-currency economies locally equivalent to members of a single-currency area in terms of real allocations. Exchange rates are thus irrelevant for resource allocation under this assumption.

Optimal monetary policy of the dominant-currency country. Given this allocation equivalence, the optimal monetary policy problem is effectively centralized at the level of the dominant-currency country, the United States. The optimal policy of the dominant-currency country, which is given by the price-index targeting rule $\sum_{i=1}^{N} \zeta_i^*(\mathbf{e}_{\mathrm{US}}) \log p_i = \pi^*(\mathbf{e}_{\mathrm{US}})$, governs global nominal rigidities and, through them, shapes real allocations worldwide. In this sense, it serves as a de facto global benchmark, analogous to the union-wide optimal policy in a monetary union. Given this benchmark price-index regime, a country's unilateral inflation stance is

$$\pi_c = \frac{\left[\zeta^*(\mathbf{e}_{\mathrm{US}})\right]' \varrho^m}{\left[\zeta^*(\mathbf{e}_c)\right]' \varrho^m} \pi^*(\mathbf{e}_c).$$

As in the currency-union case, Figure B.1 documents a negative relationship between unilateral inflation stances and upstreamness in the global production network. Consistent with its downstream position, the optimal monetary policy of the United States is inflationary. In addition, domestic sectors receive approximately 93.2% of the weight in the optimal price index, indicating that U.S. monetary policy primarily reflects domestic production structures while anchoring global nominal rigidities.

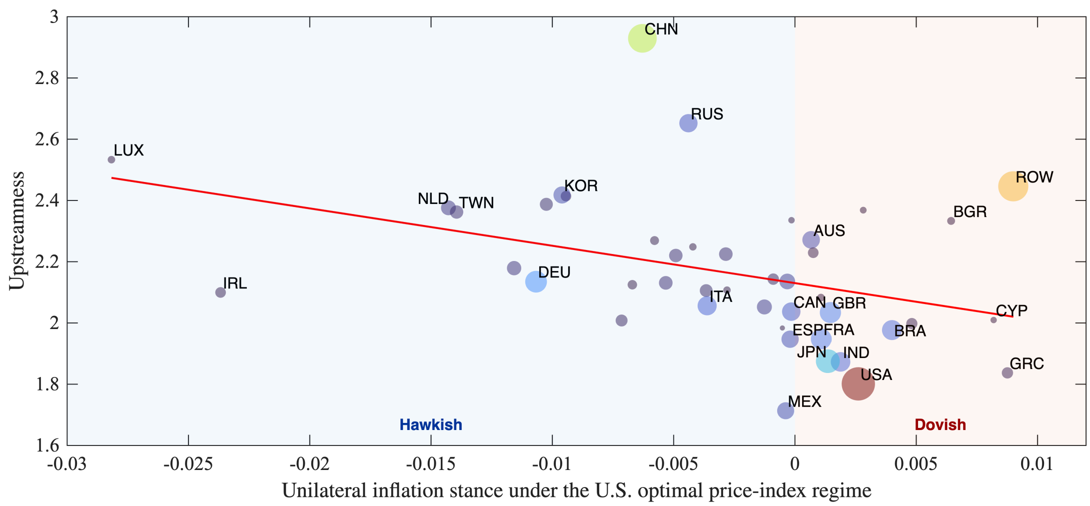

Figure B.1: Unilateral inflation stance and upstreamness in the global production network. *Note:* The horizontal axis reports each country's unilateral inflation stance computed under the price-index regime implied by the dominant-currency country's optimal policy. The vertical axis measures country-level upstreamness. The red line shows a fitted linear trend.

Policy-alignment losses across non-dominant-currency economies. Under DCP, policy-alignment loss is defined relative to the dominant-currency country's optimal monetary policy rather than a union-wide consensus. For country c,

$$\mathbb{PAL}_c \equiv \mathbb{L}_c(\zeta^*(\mathbf{e}_{\mathrm{US}}), \pi^*(\mathbf{e}_{\mathrm{US}})) - \mathbb{L}_c(\zeta^*(\mathbf{e}_c), \pi^*(\mathbf{e}_c)) \geq 0,$$

which captures the welfare cost of operating under the dominant-currency country's policy rather than the country's own unilateral optimum.

Figure B.2 shows that policy-alignment losses increase with the distance between a country's unilateral inflation stance and the dominant-currency country's optimal inflation rate. Countries such as Brazil (BRA), France (FRA), Hungary (HUN), Japan (JPN), and the United Kingdom (GBR), whose production network structures are closer to that of the dominant-currency economy, experience smaller policy-alignment losses.

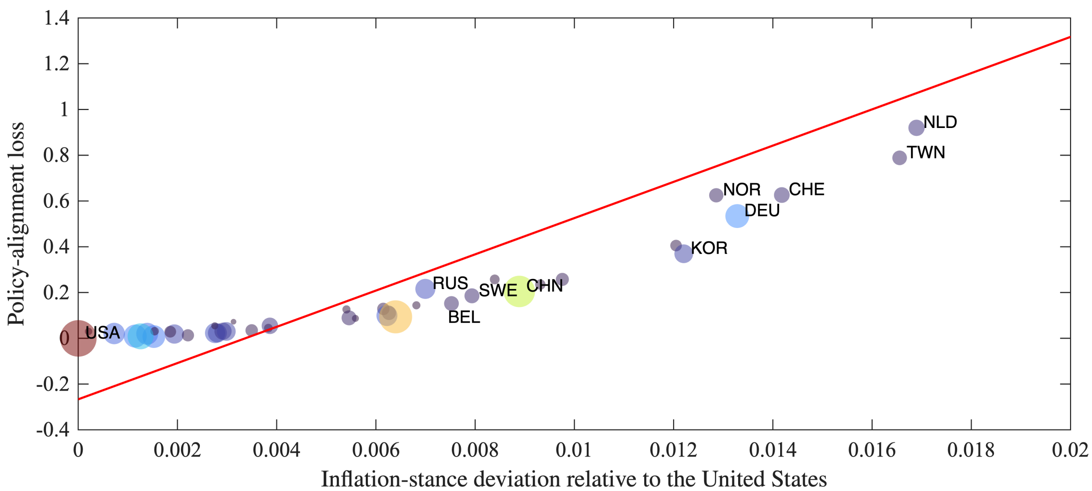

Figure B.2: Inflation-stance deviation and policy-alignment loss under dominant currency pricing. *Note:* The horizontal axis measures the distance between the country's unilateral inflation stance and the dominant-currency country's optimal inflation rate. The vertical axis reports the policy-alignment loss as a percentage of steady-state consumption. The red line shows a fitted linear trend.