

Industrial Policies Cascade in Production Networks

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ABSTRACT

This paper analyzes industrial policy in input–output networks with economies of scale and firm entry and exit. We show that the effect of a subsidy on aggregate productivity can be decomposed into two components: distortion centrality, following Liu (2019), which captures misallocations across sectors, and entry centrality, which captures the cascading effects of subsidies through firm entry decisions in the production network. We further demonstrate that a sector’s entry centrality reflects the complexity of the supply chains required for its output and grows stronger when substitution elasticities in production are high. Our quantitative results show that entry centrality complements distortion centrality and is particularly important in industries such as housing, construction, and motor vehicles, where the aggregate effects of subsidies are substantially larger once entry dynamics are incorporated. Consistent with this perspective, our reduced-form regression results indicate that entry centrality is as important as distortion centrality in predicting modern sectoral interventions in China.

KEYWORDS: production networks, industrial policy, entry and exit, economies of scale

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1 Introduction

One of the oldest questions in macroeconomics is whether and how industrial policies can impact aggregate efficiency and foster economic growth ([Hirschman, 1958](#)). From the perspective of an economy with input-output networks and market imperfections, [Liu \(2019\)](#) demonstrates that distortion centrality—a sector-level measure capturing the compounded effects of market imperfections through the production network—is a sufficient statistic for evaluating the aggregate impact of introducing subsidies in a non-intervention economy.

Recently, a growing body of literature, including [Baqae \(2018\)](#), [Elliott et al. \(2022\)](#), [Acemoglu and Tahbaz-Salehi \(2024\)](#), [Taschereau-Dumouchel \(forthcoming\)](#), has shown that extensive margin adjustments in production networks can significantly amplify the macroeconomic response to shocks. This implies that industrial policies, such as input subsidies aimed at reducing markup distortions, could also influence the macroeconomy through their impact on the extensive margin.

In light of this, this paper investigates how industrial policies, particularly input subsidies, propagate through the economy via both the intensive and extensive margins. This is achieved by incorporating external economies of scale, driven by firm entry and exit, into a standard input-output model à la [Baqae and Farhi \(2020b\)](#), which provides a framework for mapping microeconomic primitives to macroeconomic outcomes.

We characterize the response of aggregate output or productivity to changes in subsidies. Specifically, we decompose the elasticity of output with respect to a subsidy into two distinct components: *distortion centrality*, which captures the direct effect of the subsidy in reducing markup distortions while holding firm entry and exit fixed, and *entry centrality*, which reflects the output response driven by the love-of-variety effect through changes in the extensive margin. Together, these two centralities form a sufficient statistic for policy intervention.

Entry centrality arises from two main channels. First, a subsidy lowers markups within a sector, which raises its output and increases demand for inputs from its upstream sectors, thereby stimulating firm entry along the supply chain. Second, the subsidy induces relative price changes, altering the distribution of sales shares and, consequently, the number of firms across sectors through expenditure substitution.

We then show that entry centrality reflects the complexity of the supply chain required for a sector's output. Specifically, in a Cobb-Douglas economy with a Dixit-Stiglitz structure and identical love-of-variety effects, as the economy approaches its efficient frontier, entry centrality depends linearly on the love-of-variety effect, the sector's sales share, and total input requirements per unit sectoral output.

We further illustrate our decomposition through three examples. First, in a roundabout economy with a Dixit-Stiglitz demand system and production complementarity, distortion centrality is positive when firms have markups and use intermediate inputs. Entry centrality, however, can be either positive or negative, depending on the relative changes in sectoral sales share and firms' intensive margin. Additionally, distortion centrality increases proportionally with substitution elasticity, whereas entry centrality increases more than proportionally.

Second, in a horizontal economy where firms linearly transform labor into products, homogeneity in markup distortions eliminates distortion centrality across sectors, while entry centrality is determined by the direction of resource reallocation and the relative magnitude of the love-of-variety effect. For instance, under complementarity in consumption, entry centrality is positive when a sector exhibits a below-average love-of-variety effect. In such cases, a subsidy to this sector reallocates resources away from it toward sectors with stronger love-of-variety effects, thereby improving aggregate efficiency.

Third, in a vertical supply chain, resource allocation remains efficient regardless of wedges, making distortion centrality uniformly zero. However, an increase in subsidy can propagate backward through the production chain, driving significant firm entry in upstream sectors while causing firm exits within the subsidized sector. Entry centrality simply aggregates these changes in the extensive margin, weighted by the love-of-variety effect across sectors. Moreover, we show that changes in the mass of entrants are amplified by the endogenous response of markups to entry in intermediate sectors.

When applying our theoretical results to data and quantitatively examining the input-output structures of the United States, we find that nearly half of the variation in total centrality (the sum of distortion and entry centrality, both normalized by sectoral sizes) remains unexplained by distortion centrality alone. This highlights the limitations of relying solely on distortion centrality as a criterion for policy intervention. Specifically, in industries such as housing, construction, motor vehicles, and food and beverage, entry centrality plays a more significant role than distortion centrality in determining the macroeconomic impact of subsidies, indicating that subsidies to these industries have a greater impact along the extensive margin than the intensive margin. Additionally, after accounting for entry centrality, subsidies to industries such as professional services, administration and support, and wholesale trade could potentially be harmful.

In our quantitative calibration, entry centrality is insensitive to different markup specifications and is nearly perfectly correlated across all specifications of within-sector elasticities, which control the love-of-variety effect. However, it is significantly more sensitive to vari-

ations in cross-sector elasticities. In contrast, our theoretical results suggest that distortion centrality maintains a perfect correlation of one when all cross-sector elasticities change proportionally and is orthogonal to variations in within-sector elasticities.

We apply our theoretical results to modern-day China, where the government actively implements industrial policies to shape sectoral development and resource allocation. We find that non-state-owned firms in sectors with higher distortion centrality and entry centrality receive more favorable financial policies, including better access to loans, lower interest rates, and reduced tax burdens. While distortion centrality remains an important predictor of sectoral interventions, incorporating entry centrality allows our regressions to perform better, providing a more comprehensive explanation of industrial policies. This suggests that industrial policy is not only shaped by sectoral misallocations but also by a sector's ability to influence firm entry and generate spillover effects through the production network. These patterns persist even after controlling for alternative sectoral characteristics.

Related Literature. This paper nests the growing literature on the role of production networks like [Long and Plosser \(1983\)](#), [Gabaix \(2011\)](#), [Acemoglu et al. \(2012\)](#), [Baqae and Farhi \(2019\)](#) and [Bigio and La'o \(2020\)](#). The framework of this paper builds on [Baqae and Farhi \(2020b\)](#), which examines the macroeconomic output impact of microeconomic markup shocks. Our concept of distortion centrality aligns with their elasticity of output to markups in a nested CES economy with a single primary input. However, this paper departs by incorporating firm entry and exit, focusing on how changes in extensive margins alter the economy's response to markup/subsidy shocks.

This paper also contributes to the literature on industrial policies, including [Hirschman \(1958\)](#), [Song et al. \(2011\)](#), [Aghion et al. \(2015\)](#), and [Itskhoki and Moll \(2019\)](#). These studies, however, abstract from production networks, which are central to our analysis. A more closely related work is [Liu \(2019\)](#), who studies industrial policy in an input–output economy with constant returns to scale and preexisting imperfections. He shows that distortion centrality—a statistic shaped by the production network and distortions—serves as a sufficient statistic for predicting the aggregate impact of subsidies in a non-intervention economy. This paper departs from his analysis in two key ways. First, by incorporating firm entry and exit, we extend the framework to capture the extensive-margin effects of subsidies in addition to the intensive-margin effects emphasized in his study. Second, we assume increasing returns to scale in the aggregation across varieties within each industry, such that real output rises with firm entry through a return-to-variety effect.¹

¹Regarding the treatment of non-tax wedges, [Liu \(2019\)](#) treats them as deadweight losses not rebated to

This paper also contributes to an emerging literature like [Baqae \(2018\)](#), [Baqae and Farhi \(2020a\)](#), [Carvalho et al. \(2021\)](#), [Elliott et al. \(2022\)](#), [Acemoglu and Tahbaz-Salehi \(2024\)](#), [Baqae et al. \(2024\)](#) [Taschereau-Dumouchel \(forthcoming\)](#), which emphasizes that extensive margin adjustment within industries can have significant macroeconomic consequences through production networks.² The most related work among these studies is [Baqae \(2018\)](#), following which we model increasing returns via the love-of-variety effect and endogenize the number of firms within industries using a zero-profit condition. However, our study differs from [Baqae \(2018\)](#) in several ways. First, we relax the assumption of a uniform cross-sector elasticity of substitution, allowing for heterogeneous elasticities. Second, in their framework, gross operating surplus is owned by non-production labor (entrepreneurs), and nominal wages coincide with nominal GDP when labor (including both production and non-production labor) is inelastically supplied. In contrast, our framework assumes all labor is production labor, resulting in nominal wages deviating from nominal GDP. This adjustment enables the derivation of a distortion centrality concept similar to [Liu \(2019\)](#). Third, this paper abstracts to focus on the role of industrial policies in shaping misallocations, entry, and aggregate outcomes.

Outline. The rest of the paper is organized as follows. Section 2 introduces a multi-sector model with input-output linkages, increasing returns to scale, entry, and distortions. Section 3 establishes aggregation and propagation results, illustrating their insights through three example economies. Section 4 presents quantitative results, demonstrating that the impact of subsidies along the extensive margin can be significant. Section 5 concludes. Proofs and additional results are provided in the Appendix.

2 Framework

In this section, we introduce the model framework and formally define the key statistics of interest for our analysis.

2.1 Model Environment

Our static economy consists of a representative household, a government implementing industrial policies through subsidies on marginal costs, and firms that operate in different N

households. The object of interest in his analysis is therefore GDP net of these losses, making distortion centrality independent of cross-sector elasticities of substitution. By contrast, this paper follows [Baqae and Farhi \(2020b\)](#), where distortion centrality depends linearly on cross-sector elasticities within a nested CES framework.

²See also related works on endogenous adjustments in production networks: [Acemoglu and Azar \(2020\)](#), [Dhyne et al. \(2021\)](#).

sectors. Labor is the only primary factor input in this economy. The set of sectors is denoted by $\mathcal{N} \equiv \{1, 2, \dots, N\}$.

Firms. Within each sector k , firm v uses labor input $l_k(v)$ and sectoral products as intermediate inputs $\{x_{kh}(v)\}_{h \in \mathcal{N}}$ to produce a differentiated variety $y_k(v)$. The production function of firm v in sector k is

$$y_k(v) = z_k \left(\alpha_k^{\frac{1}{\sigma_k}} l_k(v)^{\frac{\sigma_k-1}{\sigma_k}} + \sum_{h \in \mathcal{N}} \omega_{kh}^{\frac{1}{\sigma_k}} x_{kh}(v)^{\frac{\sigma_k-1}{\sigma_k}} \right)^{\frac{\sigma_k}{\sigma_k-1}}$$

where z_k represents the sector-specific productivity shock that affects all firms within sector k . $l_k(v)$ denotes the labor input, while $x_{kh}(v)$ represents the intermediate input acquired from sector h . The parameter ω_{kh} captures the intensity of intermediate input from sector h , and α_k denotes the labor intensity. Lastly, σ_k is the elasticity of substitution among inputs.

Following [Baqae \(2018\)](#), the composite good produced by industry k is given by

$$y_k = \left(M_k^{-\varphi_k} \int_0^{M_k} y_k(v)^{\frac{\varepsilon_k-1}{\varepsilon_k}} dv \right)^{\frac{\varepsilon_k}{\varepsilon_k-1}}$$

where $\varepsilon_k > 1$ represents the elasticity of substitution across firms within industry k , M_k is the mass of firms, and the parameter φ_k controls the returns to product variety. Setting $\varphi_k = 0$ recovers the [Dixit and Stiglitz \(1977\)](#) demand system.

The corresponding price index p_k of industry k is then given by

$$p_k = \left(M_k^{-\varphi_k \varepsilon_k} \int_0^{M_k} p_k(v)^{1-\varepsilon_k} dv \right)^{\frac{1}{1-\varepsilon_k}}$$

where $p_k(v)$ is the price of firm v in industry k .

Conditional on entry, all firms optimally choose the combination of inputs to minimize the marginal cost,

$$mc_k = \min_{\{x_{kh}(v)\}, l_k(v)} wl_k(v) + \sum_{h \in \mathcal{N}} p_h x_{kh}(v),$$

subjective to a unit production constraint.

Firms incur a fixed overhead cost f_k to operate. The profits are expressed as

$$\pi_k(v) = p_k(v)y_k(v) - \tau_k^{-1} mc_i y_i - f_k$$

where τ_k is the effective subsidy rate provided by the government that discounts the marginal cost of production: $\tau_k > 1$ implies a subsidy reducing costs while $\tau_k < 1$ implies a tax increasing costs. Firms set prices by charging a desired markup over marginal cost, $p_k(v) = \mu_k mc_k$, where the markup μ_k depends on the effective subsidy rate and a markup function of the mass of entrants, $\mu_k = \tau_k^{-1} \tilde{\mu}_k(M_k)$. We assume markups decline as an industry experiences more entry, $\tilde{\mu}'_k(M_k) \leq 0$.

Households. The households in the model are homogeneous with a unit mass. The representative household maximizes the utility

$$U(\{c_k\}_{k=1}^N) = Y = \left(\sum_{k \in N} \beta_k^{\frac{1}{\sigma_0}} c_k^{\frac{\sigma_0-1}{\sigma_0}} \right)^{\frac{\sigma_0}{\sigma_0-1}},$$

where c_k is the sectoral consumption from industry k , the parameter σ_0 denotes the elasticity of substitution across industries, β_k determines household's preference for goods or services from industry k .

The household's budget constraint is

$$\sum_{k \in N} p_k c_k = wl + \sum_{k \in N} \int_0^{M_k} [\pi_k(v) + f_k] dv - T$$

where wl represents the total labor income (Labor l is inelastically supplied at $l = \bar{l} = 1$), $\sum_{k \in N} \int_0^{M_k} \pi_k(v) dv$ captures the total operating surplus from owning firms, while T represents a lump-sum tax.

Government. Policy interventions are modeled as sector-specific subsidies $\{\tau_i\}_{i=1}^N$ paid by the government. We assume that the government charges a lump-sum tax T on households to balance its budget,

$$T = \sum_{i=1}^N (1 - \tau_i^{-1}) mc_i y_i.$$

Equilibrium. Given productivities z_k , markup functions $\tilde{\mu}_k(M_k)$, overhead labor costs f_k and subsidies τ_k , a general equilibrium is a set of $p_k(v)$, wage w , intermediate input choices $x_{kh}(v)$, labor input choices $l_k(v)$, outputs $y_k(v)$, consumption choices c_k and masses of firms

M_k , such that: (i) each firm optimally chooses intermediate and labor inputs to minimize its costs; (ii) the representative household chooses consumption to maximize utility subject to its budget set; (iii) government budget constraint is satisfied; (iv) the markets for all goods and labor clear so that: $y_k = c_k + \sum_h \int_0^{M_h} x_{hk}(v)dv$ for all k and $\sum_{k=1}^N \int_0^{M_k} l_k(v)dv = \bar{l}$; (v) profits are zero.

2.2 Definitions and Notations

Input-Output Matrices. The cost-based (forward) input–output matrix is an $(N + 1) \times (N + 1)$ matrix whose kh -th element represents the elasticity of marginal cost for firm v in sector k with respect to the price of inputs from sector h

$$\tilde{\Omega}_{kh} = \frac{\partial \log mc_k}{\partial \log p_h} = \frac{p_h x_{kh}(v)}{mc_k y_k(v)},$$

where mc_k is the marginal cost given by $mc_k = \frac{1}{z_l} \left(\alpha_l w^{1-\sigma_k} + \sum_{h \in N} \omega_{lh} p_h^{1-\sigma_k} \right)^{\frac{1}{1-\sigma_k}}$. In this matrix, the first N rows and columns correspond to sectoral products, and the last row and column to labor.

While the input-output matrix captures the direct exposure between sectors, the Leontief inverse matrix, defined as

$$\tilde{\Psi} = (I - \tilde{\Omega})^{-1} = I + \tilde{\Omega} + \tilde{\Omega}^2 + \dots$$

captures both the direct and indirect exposures throughout the production network.

Analogously, we define the revenue-based (backward) input-output matrix and Leontief inverse to be

$$\Omega = \mu^{-1} \tilde{\Omega}, \quad \text{and} \quad \Psi = (I - \Omega)^{-1}$$

where μ denotes the diagonal matrix of wedges.

Sales Shares. The direct exposure of households to sectors is captured by an $(N + 1) \times 1$ vector of final expenditure shares

$$b_k = \frac{p_k c_k}{\sum_{j \in N} p_j c_j}.$$

The sales shares (or Domar weights) instead record the direct and indirect exposures of GDP to sectors:

$$\lambda' = b' \Psi$$

with the last element Λ_L denoting the income share of labor. By analogy, the cost-based Domar weights are defined by $\tilde{\lambda} = b' \tilde{\Psi}$.

Masses of Entrants. Since all firms within an industry have the same constant-returns-to-scale production, the profit function in equilibrium can be rewritten as

$$\pi_k(v) = \frac{1}{M_k} \left(1 - \frac{1}{\mu_k}\right) p_k y_k - f_k.$$

This, combined with the zero profit condition, determines the mass of entrants in each sector,

$$M_k = \left(1 - \frac{1}{\mu_k}\right) \frac{p_k y_k}{f_k}, \quad (1)$$

indicating that the equilibrium mass of firms increases with the markup and sales share, but decreases with overhead costs.

Additional Notations. To streamline the analysis, we introduce the following operation. Define the substitution matrix $\Phi(\sigma)$ as an $(N + 1) \times (N + 1)$ matrix that depends linearly on the set of parameters $\sigma = \{\sigma_j\}_{j=0}^N$. Its (l, k) -th element is given by:

$$[\Phi(\sigma)]_{l,k} = \sum_{j=0}^N \sigma_j \lambda_j \mu_j^{-1} \text{Cov}_{\tilde{\Omega}(j,:)} \left(\frac{\Psi_{(l)}}{\lambda_l}, \frac{\tilde{\Psi}_{(k)}}{\tilde{\lambda}_k} \right).$$

In particular, the (L, k) -th element of $\Phi(\sigma)$ directly captures distortion centrality (see more discussion in Proposition 3), following Liu (2019). In an efficient economy where labor is the only primary input, the total (direct and indirect) payment to labor equals each sector's sales share. This condition implies that $\Psi_{iL} = 1$ across all sectors. Consequently, the distortion centrality $\Phi(\sigma)_{L,k}$ is always zero, indicating that sectoral subsidies do not have first-order effects on aggregate efficiency through the traditional misallocation channel alone.

3 Sufficient Statistics

In this section, we first characterize changes in real output through endogenous but observable ex post changes in sectoral markups and the mass of sectoral entrants. We then relate these endogenous variables to exogenous sectoral subsidies, showing that subsidies generate cascading effects on firm entry. Finally, we express the response of real output to

sectoral subsidies in terms of microeconomic primitives and decompose it into two components: distortion centrality and entry centrality, corresponding to the intensive and extensive margins, respectively.

3.1 Ex Post Results

Proposition 1 describes the change in aggregate output in terms of ex post changes in markups and the mass of entrants.

Proposition 1. *The change in aggregate output in response to changes in subsidy can be written as*

$$d \log Y = \sum_{k \in N} \tilde{\lambda}_k [\Phi(\sigma)]_{L,k} d \log \mu_k + \sum_{k \in N} \eta_k \left(\lambda_k \frac{\Psi_{kL}}{\Lambda_L} - \tilde{\lambda}_k [\Phi(\sigma)]_{L,k} \right) d \log M_k \quad (2)$$

where $\eta_k = \frac{1-\varphi_k \varepsilon_k}{\varepsilon_k - 1}$ represents the love-of-variety in sector k .

Proposition 1 builds on the framework of Baqaee and Farhi (2020b), extending it to economies with fixed costs, increasing returns, and an extensive margin of products. The first term on the right-hand side of (2) is a direct application of Proposition 3 in Baqaee and Farhi (2020b) to a nested CES economy with a single primary input. The second term, by contrast, is novel to our paper: it captures the effect of changes in the mass of entrants on aggregate productivity while holding ex post markups fixed. When we shut down the love-of-variety channel ($\eta_k = 0$ implying $\varphi_k = 1/\varepsilon_k$), the aggregation across varieties within each sector degenerates to constant returns to scale. In this case, sectoral prices become unresponsive to changes in the mass of entrants, and our framework collapses to that of Baqaee and Farhi (2020b).

3.2 Firm Entry Cascades

We next relate ex post markups and the mass of entrants to exogenous subsidy shocks and demonstrate that sectoral subsidies may trigger a cascading response of firm entry across sectors.

First, recall that $\mu_k = \tau_k^{-1} \tilde{\mu}_k(M_k)$, implying that a sector's ex post markup declines with higher subsidy rate and greater firm entry:

$$d \log \mu_k = -d \log \tau_k - \xi_k d \log M_k, \quad (3)$$

where $\xi_k = -\tilde{\mu}'_k(M_k)M_k/\mu_k$ represents the elasticity of the markup with respect to the mass of entrants.

Second, from the zero-profit condition (equation (1)), the mass of firms in each sector is proportional to the sector's gross operating surplus, yielding:

$$d \log M_k = \mu_k^{-1} d \log \mu_k + d \log \lambda_k. \quad (4)$$

Finally, changes in ex post markups and the mass of entrants induce relative price changes and, in turn, resource reallocations across sectors, reshaping the distribution of sales shares in the network:

$$d \log \lambda = \underbrace{[\text{diag}(\lambda^{-1})\Psi' \text{diag}(\lambda) - I] d \log \mu}_{\text{Backward propagation}} + \underbrace{\Phi^n(\sigma - 1) \text{diag}(\tilde{\lambda})(\eta d \log M - d \log \mu)}_{\text{Forward propagation}} \quad (5)$$

This relationship also indicates that changes in ex post markups propagate both backward and forward through the network, whereas—holding markups fixed—changes in the number of firms propagate only forward via expenditure substitution across downstream sectors.

Taken together, equations (3)–(5) reveal a feedback loop between the mass of entrants and sectoral sales shares, indicating that entry decisions can cascade through the production network. A change in entry in one sector alters markups (via variable markups) and marginal costs (via product differentiation), leading to relative price changes that reallocate resources and reshape the distribution of sales and profits. These shifts, in turn, influence entry decisions across the network.

Proposition 2 consolidates these relationships and characterizes the response of the mass of entrants to subsidies.

Proposition 2 (The cascading effects of the subsidies). *The response of the mass of entrants to subsidies is characterized by:*

$$\frac{d \log M}{d \log \tau} = [I - \Lambda_M - (\Lambda_\mu - \mu^{-1})\xi]^{-1} (\Lambda_\mu - \mu^{-1}) \quad (6)$$

where $\Lambda_\mu = -\frac{\partial \log \lambda}{\partial \log \mu} = \text{diag}(\lambda^{-1})(\Psi^n)' \text{diag}(\lambda) - I + \Phi^n(\sigma - 1) \text{diag}(\tilde{\lambda})$ and $\Lambda_M = \frac{\partial \log \lambda}{\partial \log M} = \Phi^n(\sigma - 1) \text{diag}(\tilde{\lambda} \circ \eta)$.

3.3 Entry Centrality

Building on Proposition 1 and 2, Proposition 3 characterizes the output response to a subsidy shock in terms of the model's primitives including production networks, substitution elasticities, love-of-variety effects, and the responsiveness of markups to changes in the mass of entrants.

Proposition 3. *Following a change in subsidy, the output response is given by the following sufficient statistic*

$$\frac{d \log Y}{d \log \tau_k} = \underbrace{\frac{\partial \log Y}{\partial \log \tau_k}}_{\text{Distortion centrality}} + \underbrace{\sum_j \frac{\partial \log Y}{\partial \log M_j} \frac{\partial \log M_j}{\partial \log \tau_k}}_{\text{Entry centrality}} \quad (7)$$

where

$$\frac{\partial \log Y}{\partial \log \tau_k} = -\tilde{\lambda}_k[\Phi(\sigma)]_{L,k}, \quad \frac{\partial \log Y}{\partial \log M_k} = \eta_k \lambda_k \frac{\Psi_{kL}}{\Lambda_L} - (\eta_k + \xi_k) \tilde{\lambda}_k[\Phi(\sigma)]_{L,k}. \quad (8)$$

In response to a subsidy shock, the change in output can be decomposed into two components: (1) a direct effect along the intensive margin, driven by changes in distortions while keeping firm entry and exit constant, and (2) an indirect effect along the extensive margin, stemming from changes in the mass of entrants across sectors induced by equilibrium adjustments in sectoral sales shares.

Specifically, the direct effect is captured by the distortion centrality of sector k , $\tilde{\lambda}_k[\Phi(\sigma)]_{L,k}$, which, as in Liu (2019), reflects the aggregate degree of misallocation in each sector. This measure serves as a sufficient statistic for understanding the impact of a change in sectoral wedge on aggregate output. However, in economies with firm entry, distortion centrality alone is insufficient for guiding policy intervention. Subsidy changes can trigger entry across sectors, and these entry decisions cascade through the network, producing significant output effects. To account for this dynamic, the indirect effect—termed “entry centrality”—is introduced. It is defined as the inner product of the output response to sectoral firm entry and the equilibrium changes in the mass of entrants across sectors induced by the subsidy.

Taken together, distortion centrality and entry centrality provide a unified framework for evaluating the aggregate consequences of sectoral subsidies. Distortion centrality isolates the intensive-margin effect of reducing misallocation, while entry centrality captures the extensive-margin effect of reallocation through firm entry cascades. Importantly, entry centrality depends not only on technological primitives—such as substitution elasticities,

the complexity of production chains, and love-of-variety effects—but also on how variable markups adjust with entry. This complementarity implies that policies targeting sectors with high distortion centrality may have muted aggregate effects if entry margins are weak, whereas sectors with modest distortion centrality but strong entry linkages can generate disproportionately large productivity gains once subsidies alter firm entry patterns across the network.

3.4 Cobb-Douglas

In this section, we analyze a benchmark specification where production and consumption functions are Cobb-Douglas. This specification shuts down expenditure substitution across industries, resulting in sales shares unresponsive to changes in firm entry, i.e., $\Lambda_M = \mathbf{O}$. To simplify further, we assume a Dixit-Stiglitz market structure ($\varphi_k = 0$), ensuring that markups are constant ($\xi_k = 0$) and all effects arise purely from the love of variety. Under this setting, Proposition 2 and 3 imply that

$$\frac{\partial \log Y}{\partial \log \tau_k} = \tilde{\lambda}_k - \lambda_k \frac{\Psi_{kL}}{\Lambda_L}, \quad \frac{\partial \log Y}{\partial \log M_l} = \frac{1}{\varepsilon_l - 1} \tilde{\lambda}_l,$$

and

$$\frac{\partial \log M_l}{\partial \log \tau_k} = \lambda_k (\Psi_{kl} - \delta_{kl}) / \lambda_l - \mu_k^{-1} \delta_{kl}.$$

where $\delta_{kl} = \mathbf{1}(k = l)$.

In this example, a subsidy increase in sector k propagates backwardly, triggering higher demand from sector k for its upstream sectors, leading to higher sales shares and firm entry in sector l when $\Psi_{kl} > 0$. This effect arises because sector k 's production depends on the outputs of these upstream sectors, either directly or indirectly. However, the change in the number of firms in sector k is ambiguous: while the sales share of sector k may increase if $\Psi_{kk} > 1$, the Lerner index of sector k , $1 - \mu_k^{-1}$ decreases, indicating growth in the intensive margin. Consequently, the change in the extensive margin of sector k depends on the relative change between its sales share and its intensive margin.

Corollary 1. *In a Cobb-Douglas economy with Dixit-Stiglitz demand systems, when markups uniformly approach zero from above ($\mu \rightarrow \mathbf{1}^+$), and within-sector elasticities are constant across*

sectors ($\varepsilon_k = \varepsilon$),³ the response of aggregate productivity to a change in subsidy is given by:

$$\frac{d \log Y}{d \log \tau_k} = \underbrace{0}_{\text{Distortion centrality}} + \underbrace{\frac{1}{\varepsilon - 1} \tilde{\lambda}_k \left(\sum_{l \in N} \tilde{\Psi}_{kl} - 2 \right)}_{\text{Entry centrality}}.$$

Around efficient initial equilibrium, the cost-based definition aligns with the revenue-based definition ($\Psi_{kL} = \lambda_L = 1, \forall k$ and $\lambda_k = \tilde{\lambda}_k$), causing distortion centrality to be zero. This implies that the effect of a subsidy along the intensive margin is negligible. However, the entry centrality is generally non-zero and depends linearly on the love-of-variety, sales share, and total input requirements per unit sectoral output $\sum_{l \in N} \tilde{\Psi}_{kl}$. Due to the roundabout nature of production reflected in realistic input-output tables, $\sum_{l \in N} \tilde{\Psi}_{kl}$ is typically greater than 2 for all sectors. As a result, a subsidy increase generally has a positive effect on aggregate productivity through the extensive margin when all cross-sector elasticities are Cobb-Douglas.⁴

3.5 Illustrative Examples

In this section, we present three simple examples to illustrate the intuition behind Proposition 3.

1. Roundabout Economy with Dixit-Stiglitz Demand

The Cobb-Douglas example is mathematically simple and thus useful for gaining intuition; however, the Cobb-Douglas assumption is highly restrictive, as it holds the structure of the production network fixed. To address this limitation, we first explore a roundabout economy with a single sector that combines labor and its own sectoral output using a CES production function with non-unitary elasticity ($\sigma_1 \in (0, 1)$). Proposition 2 and 3 imply that

$$\frac{\partial \log Y}{\partial \log \tau_1} = \sigma_1 \lambda_1 (\tilde{\lambda}_1 - 1) (1 - \mu_1^{-1}), \quad \frac{\partial \log Y}{\partial \log M_1} = \eta_1 \lambda_1 [1 + \sigma_1 (\tilde{\lambda}_1 - 1) (1 - \mu_1^{-1})],$$

and

$$\frac{d \log M_1}{d \log \tau_1} = \frac{\sigma_1 (\lambda_1 - 1) - \mu_1^{-1}}{1 + \eta_1 (1 - \sigma_1) (\lambda_1 - 1)}.$$

In this example, distortion centrality is positive whenever the economy has a positive

³For simplicity, we assume that within-sector elasticities are identical across sectors ($\varepsilon_k = \varepsilon$) and that input subsidies are set to exactly cancel the monopolistic markups ($\tau_k = 1/\varepsilon$).

⁴Figure A.1 in Appendix A plots distortion and entry centrality in a Cobb-Douglas economy.

markup and uses intermediate inputs (i.e. $\mu_1 > 1$ and $\tilde{\lambda}_1 > 1$) . This indicates that input subsidies help reduce resource misallocation, thereby enhancing aggregate output and productivity.

For entry centrality, the output response to firm entry is positive, but the effect of the subsidy on the mass of entrants depends on the relative changes in the sectoral sales share and the profit margin. Specifically, when $\sigma_1(\lambda_1 - 1) > \mu_1^{-1}$, the increase in the sales share more than offsets the decline in the profit margin, leading to firm entry and a positive entry margin. The entry margin becomes more pronounced with greater roundaboutness (higher sales share λ_1). Furthermore, it increases with the elasticity of substitution, as higher elasticity amplifies both the output response to firm entry and the sensitivity of firm entry to the subsidy.

2. Horizontal Economy with Dixit-Stiglitz Demand

We then consider a case where substitution occurs solely on the demand side: all firms use labor as their only input, while the household combines sectoral outputs using a CES demand function. Specifically, Proposition 3 now implies that the distortion centrality is given by:

$$\frac{\partial \log Y}{\partial \log \tau_k} = \sigma_0 b_k \mu_k^{-1} (\mu_k - \bar{\mu}),$$

with $\bar{\mu} = [\mathbb{E}_b(\mu^{-1})]^{-1}$ denoting the harmonic average of markups, weighted by the expenditure shares.

The output response to changes in the mass of entrants is:

$$\frac{d \log Y}{d \log M_l} = \eta_l b_l + (\sigma_0 - 1) \eta_l b_l \mu_l^{-1} (\mu_l - \bar{\mu}).$$

and the effect of a subsidy on the mass of entrants is:

$$\frac{\partial \log M_l}{\partial \log \tau_k} = (1 - \sigma_0) \chi_k (1 - \eta_k \mu_k^{-1}) \left(b_k \frac{\chi_l}{\mathbb{E}_b(\chi)} - \delta_{kl} \right) - \mu_k^{-1} \delta_{kl}$$

where $\chi_k = [1 + (1 - \sigma_0) \eta_k]^{-1}$.

In this horizontal economy, the direct effect of a subsidy to sector k on aggregate output linearly depends on the elasticity of substitution in consumption (σ_0) and the sectoral expenditure share (b_k). It is further shaped by the relationship between the sector's markup (μ_k) and the economy-wide average markup ($\bar{\mu}$). Subsidizing a sector with an above-average

markup ($\mu_k > \bar{\mu}$) reduces distortions by reallocating resources from low-markup sectors to high-markup sectors, thereby enhancing aggregate output. In contrast, subsidizing a sector with a below-average markup ($\mu_k < \bar{\mu}$) reallocates resources inefficiently to a less-distorted sector, exacerbating misallocation and ultimately reducing output.

The subsidy to sector k also affects output along the extensive margin. Specifically, it reduces sector k 's price, causing relative price changes across sectors and driving firm entry through a substitution effect. The direction of resource reallocation depends on the deviation from the Cobb-Douglas case. Assuming $1 - \eta_k \mu_k^{-1} > 0$, under complementarity ($\sigma_0 < 1$), a subsidy to sector k reallocates resources away from sector k to other sectors, encouraging firm entry in those sectors. Conversely, with substitutability ($\sigma_0 > 1$), resources are reallocated toward sector k , leading to firm exits in other sectors.

Corollary 2. *In a horizontal economy with Dixit-Stiglitz demand systems and homogeneous markups, the output response to a change in subsidy is given by:*

$$\frac{d \log Y}{d \log \tau_k} = \underbrace{0}_{\text{Distortion centrality}} + \underbrace{(1 - \sigma_0) b_k \chi_k (1 - \eta_k \mu_k^{-1}) [\mathbb{E}_{b_X}(\eta) - \eta_k] - b_k \eta_k \mu_k^{-1}}_{\text{Entry centrality}}.$$

Corollary 2 illustrates that when there is no dispersion in markups, subsidies cannot improve efficiency through the intensive margin. However, they can still affect output via the extensive margin, as evidenced by non-zero entry centrality.

This entry centrality is highly sensitive to cross-sector elasticity. Suppose sector k has a below-average love-of-variety ($\eta_k < \mathbb{E}_{b_X}(\eta)$). Under complementarity ($\sigma_0 < 1$), the subsidy reallocates resources to sectors with higher love-of-variety, potentially enhancing aggregate efficiency. In contrast, under substitutability ($\sigma_0 > 1$), the subsidy shifts resources toward sector k , leading to inefficiency.

3. Vertical Economy

The vertical economy represents a production line where the most upstream sector, N , produces solely using labor. Each downstream sector i processes the output of the sector immediately upstream ($i + 1$) linearly, transforming it into its own output. The household consumes the final goods produced by the most downstream sector, sector 1. Proposition 2 and 3 now yield

$$\frac{\partial \log Y}{\partial \log \tau_k} = 0, \quad \frac{\partial \log Y}{\partial \log M_j} = \eta_j,$$

and

$$\frac{\partial \log M_j}{\partial \log \tau_k} = \begin{cases} \delta_j \delta_k \prod_{j < h < k} (1 + \xi_h \delta_h), & j > k, \\ -\frac{1}{\mu_k + \xi_k}, & j = k, \\ 0, & j < k. \end{cases}$$

where $\delta_k = \mu_k / (\mu_k + \xi_k)$.

In this economy, there is only one feasible allocation of resources for a given labor supply, so the distortion centrality is trivially zero, and aggregate output is unresponsive to changes in wedges. This implies that changes in subsidies do not have a first-order effect on aggregate productivity along the intensive margin. However, a subsidy to sector k can stimulate critical firm entry in all its upstream sectors, potentially leading to a significant output response along the extensive margin. This occurs because, similar to the Cobb-Douglas case, expenditure substitution (forward propagation) is also inactive in a vertical economy; therefore, subsidies propagate back through the supply chain. For example, when markups are exogenous (i.e., $\xi_k = 0$ for all k), a subsidy to a sector leads to a one-for-one increase in firm entry across all of its upstream sectors. When markups decline with the number of firms due to pro-competitive effects, variable markups serve as an amplification mechanism, with ξ_h in sectors positioned between j and k intensifying the response of the mass of entrants in upstream sector j to a subsidy in downstream sector k .

4 Quantitative Analysis

In this section, we apply our theoretical framework to quantify distortion centrality and entry centrality across industries, followed by comprehensive robustness checks to validate our findings.

4.1 Calibration

Our analysis draws on two key data sources. First, we utilize the 2006 input-output tables from the Bureau of Economic Analysis (BEA) to construct a cost-based input-output matrix, which captures input expenditure shares across industries and final expenditure shares by consumers.

Second, we incorporate markup distortion estimates from [Baqae and Farhi \(2020b\)](#), who derive markups using three methodologies: (i) the user-cost approach (UC), (ii) production

function estimation (PF), and (iii) the accounting profits approach (AP). We use their UC markups for our benchmark calibration.

For other parameters, we set the within-sector elasticity of substitution to 8 ($\varepsilon_k = 8, \forall k \in \mathcal{N}$), consistent with values commonly used in the New Keynesian literature. The elasticity of substitution in consumption is specified as ($\sigma_C = 0.9$), while the cross-sector elasticities of substitution in firms' production are set to 0.5 (i.e., $\sigma_{kj} = 0.5, \forall j \in \mathcal{N}$) aligning with estimates from [Atalay \(2017\)](#). For our baseline calibration, we assume $\varphi_k = 0$, recovering the standard [Dixit and Stiglitz \(1977\)](#) demand system and impose constant markups that do not respond to increased entry ($\xi_k = 0, \forall k \in \mathcal{N}$). We will relax these parameterizations in the robustness tests.

4.2 Entry Centrality: Complementary Insights Beyond Distortion Centrality

Figure 1 compares distortion centrality and entry centrality across various industries, revealing that manufacturing sectors tend to display higher distortion centrality, while some service sectors exhibit markedly negative entry centrality. Table A.1 lists the five industries with the lowest and highest entry centrality. In certain industries—such as housing, construction, motor vehicles, and food and beverage—entry centrality even surpasses distortion centrality, highlighting the limitations of using distortion centrality alone as a criterion for policy intervention. This observation is reinforced: after normalizing both measures by the cost of intervention (cost-based Domar weights), the correlation between distortion and entry centrality is -0.0878, and when regressing total centrality (the sum of distortion and entry centralities) on distortion centrality, the residual sum of squares (RSS) still accounts for 47.32% of the total sum of squares (TSS). In other words, nearly half of the variation in total centrality remains unexplained by distortion centrality, underscoring the importance of considering entry centrality as well. These findings suggest that entry centrality offers valuable complementary insights, potentially enhancing our understanding of how industrial policy fosters economic growth and informing more nuanced policy decisions.

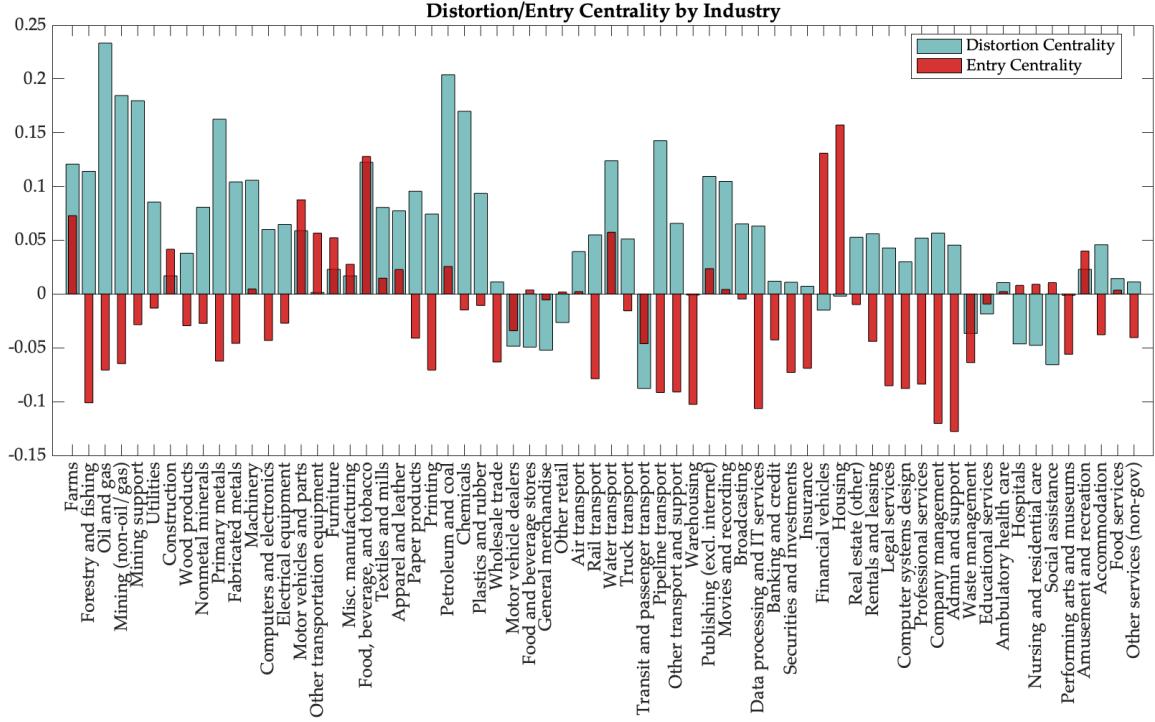


Figure 1: Distortion Centrality and Entry Centrality Across Industries. Notes: This figure presents numerical values for distortion and entry centrality across industries, normalized by the sectoral sizes. The wide green bars represent distortion centrality, emphasizing its primary significance, while the narrower red bars illustrate entry centrality.

Figure 2 shows the distributions of distortion and entry centrality. We investigate two alternative calibrations of markups: one based on estimated user-cost (UC) markups (averaging 5.06%) and another set to their CES monopolistic values ($\mu_k = \varepsilon_k / (\varepsilon_k - 1)$).

Comparing panels (a) and (b) reveals that industrial policies generally have a beneficial effect through the intensive margin by reducing markup distortions in the economy, while simultaneously exerting negative effects in many sectors along the extensive margin. This outcome arises because entry centrality is influenced by two main factors: first, a subsidy in a specific sector increases the sales share across all sectors, thereby encouraging firm entry into other sectors; second, it may lead to firm exits within the subsidized sector when the growth of the intensive margin surpasses that of the extensive margin. These findings are robust when using the monopolistic markups calibration, as evidenced by the comparison of panels (c) and (d). However, when employing CES markups, the economy moves farther away from the efficient frontier, and distortion centrality increases in magnitude, indicating more room for policy intervention. In contrast, the distribution of entry centrality remains

nearly unchanged, as evidenced by a high correlation of 0.996 between entry centrality under different markup calibrations. This suggests that entry centrality is insensitive to markup distortions.

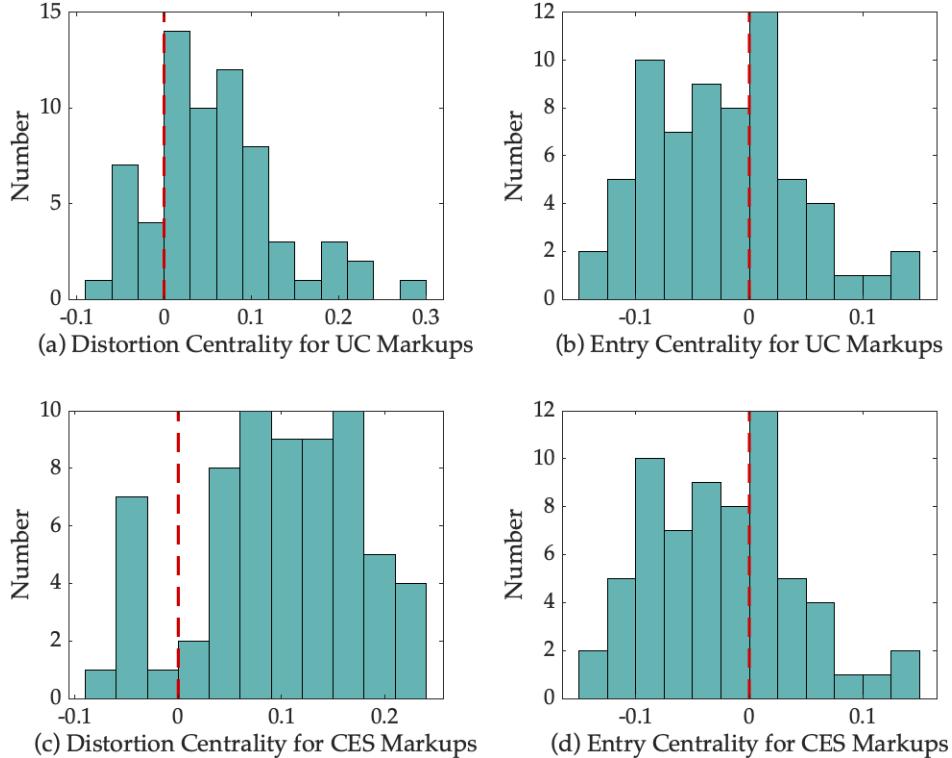


Figure 2: Distribution of Distortion and Entry Centrality. Notes: This figure shows the distribution of distortion and entry centrality that normalized by the cost of the intervention. The first row uses estimated UC markups, while the second row uses monopolistic markups.

4.3 Robustness Test

Figure 3 illustrates the correlation between entry centrality under various model parameters and that under the benchmark calibration. Panel (a) modifies the cross-sector elasticity in production, revealing a sharp decline in correlation as elasticity increases beyond the benchmark. Panel (b) adjusts the within-sector elasticity, which simultaneously controls the love-of-variety effect (returns to entry). Given that this elasticity is assumed to be homogeneous across sectors, the correlation remains exceptionally high (exceeding 0.998) when within-sector elasticity varies from 6 to 10. Collectively, these panels demonstrate that entry centrality is highly consistent across different parameterizations but is more sensitive to the

specification of cross-sector elasticities. In contrast, when elasticities in consumption and production are the same, distortion centrality maintains a perfect correlation of 1, as it is directly proportional to cross-sector elasticities. As elasticity increases, distortion centrality increases proportionally. Additionally, distortion centrality is independent of within-sector elasticities and is therefore orthogonal to their variations.

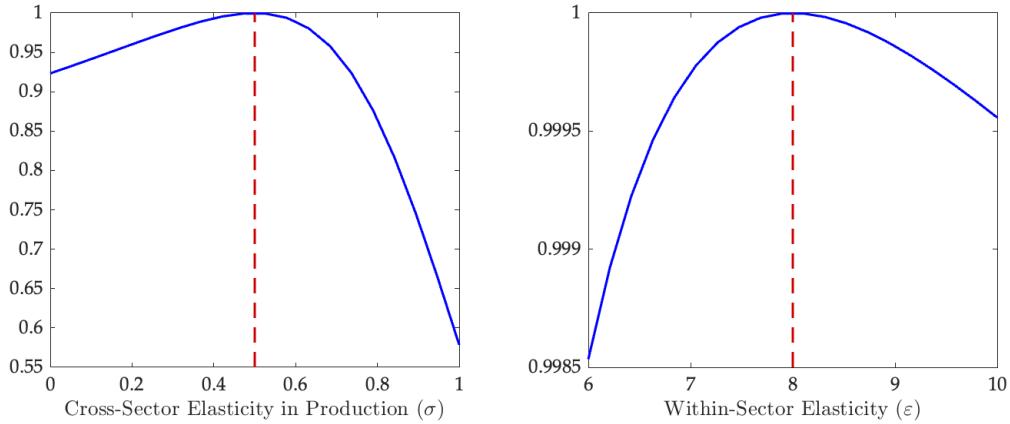


Figure 3: Correlation of Entry Centrality Across Specifications. Notes: This figure illustrates the correlation between entry centrality under various model parameters and entry centrality under the benchmark calibration. Entry centrality is normalized by the cost of intervention to control for size effects. The red dashed line in each panel identifies the benchmark calibration.

Figure A.2 in Appendix A presents distortion and entry centrality for the “Motor Vehicles, Bodies, Trailers, and Parts” sector across different specifications. It corroborates the aforementioned results by showing that distortion centrality is a linear function of cross-sector elasticity in production, whereas entry centrality is a convex function of elasticity. This indicates a diverging effect on entry centrality as elasticity increases across sectors. Additionally, Figure A.2 demonstrates that entry centrality in a sector is boosted by the presence of viable markups in other sectors.

4.4 Reduced-Form Evidence from Modern-Day China

We then apply our theoretical results to modern-day China, recalibrating the cost-based input-output matrix using China’s 2007 IO tables, disaggregated at 135 three-digit sector level. We construct distortion centrality and entry centrality under the assumption that initial markups are 10% across all sectors.⁵

⁵From Section 4.2, we know that both distortion centrality and entry centrality remain highly correlated across different specifications of initial markups.

Drawing from the 2007 edition of the Chinese Annual Survey of Manufacturing, Liu (2019) constructs several quantitative measures of sectoral interventions in China based on private firms' interest payments, debt obligations, corporate income taxes, and government subsidies. The dataset covers 79 three-digit manufacturing sectors, the finest partition that aligns with both the national input-output (IO) table and firm-level datasets. To ensure comparability with his results, we adopt his three key measures of sectoral interventions: (i) Effective interest rates paid by private manufacturing firms, (ii) Average debt-to-asset ratio, and (iii) Fraction of firms receiving tax incentives from the tax authority.

We now demonstrate that entry centrality complements distortion centrality and better explains variations in sectoral interventions in China. To do so, we estimate the following cross-sector regression:

$$\text{Outcome}_i = \alpha + \beta \text{ Distortion centrality}_i + \gamma \text{ Entry centrality}_i + \text{controls}_i + \epsilon_i$$

where each observation i represents a sector and is weighted by sectoral value-added in the regression. To ensure comparability with Liu (2019), we use the same set of control variables, including: capital intensity (fixed assets over output), Lerner index (operating profits over output), average log-fixed capital of firms in their first year of operation (a proxy for the minimum scale of operation), and export intensity (exports over output).

Our findings highlight the crucial role of entry centrality in explaining sectoral policy interventions in China. First, despite theoretical differences in our measure of distortion centrality, our regression results, particularly in terms of R-squared, closely align with those of Liu (2019). Second, we find that private firms in sectors with high distortion or high entry centrality tend to receive more favorable policies, suggesting that industrial policy is shaped not only by sectoral distortions but also by a sector's capacity to attract firm entry. Third, incorporating entry centrality significantly enhances the explanatory power of our model, with distortion and entry centrality together accounting for up to 49% of the variation in effective interest rates, even without control variables. Moreover, the coefficients on both measures remain stable after including controls, indicating that their effects are independent of standard sectoral characteristics. These results suggest that Chinese industrial policy prioritizes not only sectors with high upstreamness but also those with high entry centrality, where subsidies can generate greater overall demand through supply chain linkages.

Table 1: The Role of Entry Centrality in Explaining Sectoral Interventions

	Effective Interest Rate				Debt Ratio				Tax Break			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Distortion cent.	-4.353*** (1.198)	-4.590*** (0.992)	-5.366*** (1.155)	-4.963*** (1.043)	13.775*** (3.068)	14.068*** (2.971)	13.070*** (3.325)	12.410*** (3.250)	17.049** (6.952)	17.970*** (6.472)	22.831*** (7.117)	21.601*** (7.015)
Entry cent.		-9.617*** (1.592)		-7.839*** (1.837)		11.894** (4.770)		12.850** (5.724)		37.431*** (10.389)		23.949* (12.356)
Capital intensity			-0.355* (0.191)	-0.266 (0.173)			-0.673 (0.550)	-0.819 (0.539)			0.905 (1.177)	0.633 (1.164)
Lerner index			0.050 (0.174)	0.012 (0.156)			0.002 (0.500)	0.063 (0.487)			-1.028 (1.070)	-0.913 (1.052)
Log (fixed assets)			-0.013 (0.202)	-0.147 (0.184)			0.555 (0.581)	0.775 (0.574)			-0.944 (1.244)	-0.535 (1.239)
Export intensity			-0.819*** (0.173)	-0.450** (0.178)			0.639 (0.497)	0.034 (0.554)			3.318*** (1.064)	2.191* (1.195)
Constant	6.301*** (0.553)	10.714*** (0.862)	6.716*** (0.533)	10.044*** (0.916)	48.754*** (1.416)	43.297*** (2.582)	49.015*** (1.536)	43.559*** (2.853)	23.292** (3.207)	6.117 (5.624)	20.602*** (3.287)	10.434* (6.159)
Observations	79	79	79	79	79	79	79	79	79	79	79	79
R-squared	0.146	0.423	0.360	0.489	0.207	0.267	0.249	0.298	0.072	0.208	0.216	0.255

Note: This table is generated under a calibration in which the elasticity of substitution across sectors in production is uniformly set to 1.

5 Conclusion

This paper highlights the critical role of extensive margin adjustments in shaping sectoral interventions. We demonstrate that the allocation of subsidies across sectors can be understood through two key components: distortion centrality, which reflects intensive margin effects, and entry centrality, which captures extensive margin effects. While subsidies can be partly explained by high distortion centrality, our findings suggest that entry centrality plays an equally important role in determining subsidy allocation. A simple quantitative calibration indicates that sectors with high entry centrality receive more favorable policies, suggesting that industrial interventions are not solely driven by pre-existing distortions but also by a sector's potential to influence firm entry and broader economic activity.

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Appendix

A Additional Figures and Tables

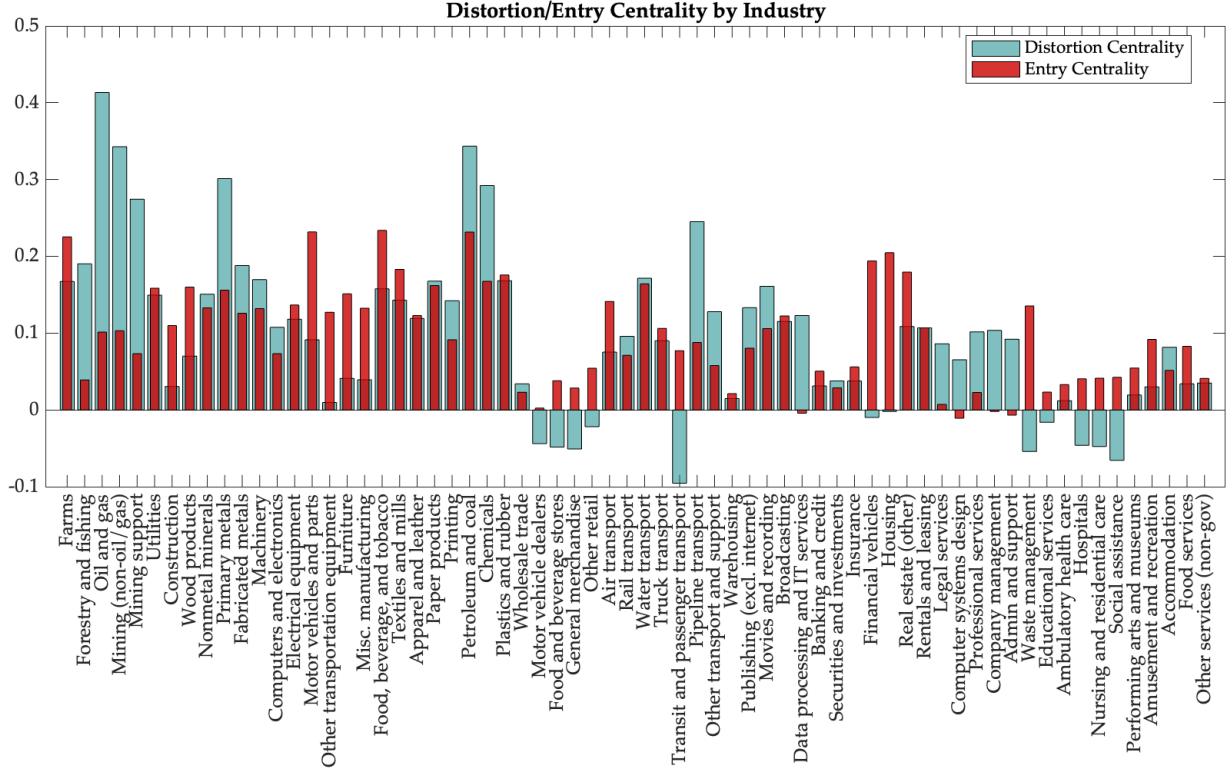


Figure A.1: Distortion Centrality and Entry Centrality Across Industries in a Cobb-Douglas Economy. Notes: This figure presents numerical values for distortion and entry centrality across industries. The wide green bars represent distortion centrality, emphasizing its primary significance, while the narrower red bars illustrate entry centrality.

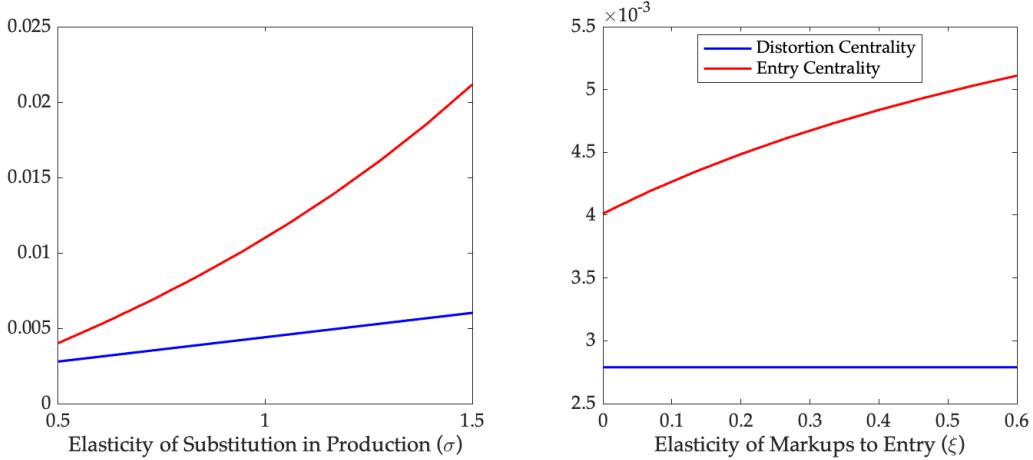


Figure A.2: Distortion and Entry Centrality: Motor Vehicles and Parts. Notes: This figure shows the distortion and entry centrality for the “Motor Vehicles, Bodies, Trailers, and Parts” sector under two scenarios. The left panel varies the elasticity of substitution in production (σ), while the right panel varies the elasticity of markups to entry (ξ) in other sectors, holding the sector’s own elasticity of markups to entry fixed. Blue lines represent distortion centrality, and red lines indicate entry centrality.

Table A.1: Industries with the Lowest and Highest Entry Centrality

US IO2006 Industry	Distortion Centrality	Entry Centrality
Housing	-0.021	1.710
Food and beverage and tobacco products	0.689	0.729
Construction	0.272	0.633
Motor vehicles, bodies and trailers, and parts	0.279	0.401
Farms	0.318	0.185
Professional, scientific, and technical services	0.931	-1.490
Administrative and support services	0.399	-1.120
Wholesale trade	0.164	-0.831
Management of companies and enterprises	0.340	-0.735
Oil and gas extraction	2.220	-0.703

Table A.2: The Role of Entry Centrality in Explaining Sectoral Interventions

	Effective Interest Rate				Debt Ratio				Tax Break			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Distortion cent.	-13.373*** (2.626)	-25.679*** (3.029)	-14.612*** (2.521)	-24.812*** (3.298)	38.338*** (6.786)	53.634*** (9.094)	36.179*** (7.349)	55.400*** (10.293)	42.969** (16.178)	71.428*** (22.011)	48.701*** (16.671)	49.837** (24.399)
Entry cent.		-12.412*** (2.109)		-10.804*** (2.535)		15.428** (6.334)		20.360** (7.913)		28.702* (15.329)		1.204 (18.757)
Capital intensity			-0.406** (0.180)	-0.333** (0.163)			-0.535 (0.525)	-0.673 (0.509)			0.826 (1.191)	0.818 (1.206)
Lerner index			0.094 (0.164)	0.066 (0.148)			-0.114 (0.478)	-0.062 (0.461)			-0.998 (1.084)	-0.995 (1.093)
Log (fixed assets)			0.005 (0.189)	-0.136 (0.173)			0.501 (0.550)	0.767 (0.540)			-0.761 (1.249)	-0.745 (1.281)
Export intensity				-0.716*** (0.162)	-0.332* (0.172)		0.386 (0.473)	-0.338 (0.535)			2.910*** (1.072)	2.867** (1.269)
Constant	7.085*** (0.552)	9.721*** (0.643)	7.305*** (0.531)	9.489*** (0.701)	47.079*** (1.427)	43.803*** (1.929)	47.460*** (1.548)	43.346*** (2.187)	22.124*** (3.402)	16.029*** (4.670)	20.834*** (3.512)	20.590*** (5.184)
Observations	79	79	79	79	79	79	79	79	79	79	79	79
R-squared	0.252	0.486	0.432	0.546	0.293	0.344	0.317	0.375	0.084	0.124	0.199	0.199

Note: This table is generated under a calibration in which the elasticity of substitution across sectors in production is uniformly set to 0.4.

Table A.3: The Role of Entry Centrality in Explaining Sectoral Interventions

	Effective Interest Rate				Debt Ratio				Tax Break			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Distortion cent.	-10.257*** (2.210)	-21.698*** (2.545)	-11.568*** (2.119)	-20.915*** (2.752)	30.210*** (5.686)	44.948*** (7.740)	28.511*** (6.154)	45.788*** (8.674)	34.858** (13.342)	65.207*** (18.360)	41.573*** (13.704)	49.370** (20.242)
Entry cent.		-13.333*** (2.099)		-11.721*** (2.529)		17.176*** (6.383)		21.666*** (7.971)		35.369** (15.142)		9.777 (18.602)
Capital intensity			-0.394** (0.184)	-0.313* (0.163)			-0.569 (0.533)	-0.718 (0.514)			0.866 (1.187)	0.799 (1.200)
Lerner index			0.082 (0.167)	0.050 (0.148)			-0.084 (0.485)	-0.024 (0.465)			-1.019 (1.079)	-0.992 (1.086)
Log(fixed assets)			0.004 (0.193)	-0.142 (0.173)			0.507 (0.560)	0.776 (0.546)			-0.830 (1.247)	-0.709 (1.274)
Export intensity			-0.752*** (0.165)	-0.329* (0.172)			0.477 (0.479)	-0.305 (0.542)			3.032*** (1.067)	2.680** (1.265)
Constant	6.864*** (0.557)	10.323*** (0.708)	7.150*** (0.535)	10.012*** (0.778)	47.519*** (1.433)	43.063*** (2.154)	47.876** (1.553)	42.586*** (2.451)	22.376*** (3.362)	13.199** (5.111)	20.616*** (3.459)	18.229*** (5.719)
R-squared	0.219	0.490	0.411	0.546	0.268	0.332	0.297	0.363	0.081	0.143	0.205	0.208
Observations	79	79	79	79	79	79	79	79	79	79	79	79

Note: This table is generated under a calibration in which the elasticity of substitution across sectors in production is uniformly set to 0.5.

Table A.4: The Role of Entry Centrality in Explaining Sectoral Interventions

	Effective Interest Rate				Debt Ratio				Tax Break			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Distortion cent.	-8.199*** (1.899)	-17.722*** (2.114)	-9.485*** (1.822)	-17.026*** (2.256)	24.657*** (4.874)	36.972*** (6.462)	23.295*** (5.275)	36.890*** (7.132)	29.066** (11.308)	57.087*** (15.020)	35.954*** (11.595)	46.398*** (16.361)
Entry cent.		-13.379*** (2.049)		-11.693*** (2.465)		17.302** (6.264)		21.077*** (7.792)		39.369*** (14.561)		16.192 (17.874)
Capital intensity		-0.383** (0.186)	-0.295* (0.164)			-0.598 (0.539)	-0.757 (0.520)			0.886 (1.184)	0.764 (1.193)	
Lerner index		0.073 (0.169)	0.035 (0.149)			-0.059 (0.490)	0.010 (0.470)			-1.027 (1.076)	-0.974 (1.079)	
Log (fixed assets)		0.001 (0.196)	-0.145 (0.175)			0.517 (0.567)	0.780 (0.552)			-0.873 (1.246)	-0.671 (1.267)	
Export intensity		-0.776*** (0.167)	-0.341* (0.173)			0.535 (0.484)	-0.249 (0.548)			3.123*** (1.065)	2.521** (1.256)	
Constant	6.692*** (0.558)	10.733*** (0.765)	7.023*** (0.536)	10.327*** (0.841)	47.881*** (1.432)	42.655*** (2.338)	48.212*** (1.552)	42.256*** (2.658)	22.621*** (3.322)	10.730* (5.435)	20.543*** (3.412)	15.967** (6.098)
Observations	79	79	79	79	79	79	79	79	79	79	79	79
R-squared	0.195	0.484	0.395	0.539	0.249	0.318	0.282	0.348	0.079	0.160	0.209	0.218

Note: This table is generated under a calibration in which the elasticity of substitution across sectors in production is uniformly set to 0.6.

Table A.5: The Role of Entry Centrality in Explaining Sectoral Interventions

	Effective Interest Rate				Debt Ratio				Tax Break			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Distortion cent.	-5.735*** (1.472)	-10.463*** (1.404)	-6.887*** (1.416)	-10.158*** (1.444)	17.774*** (3.773)	23.754*** (4.269)	16.832*** (4.085)	22.426*** (4.543)	21.578** (8.624)	37.877*** (9.565)	28.020*** (8.832)	35.483*** (10.079)
Entry cent.		-11.916*** (1.860)		-10.083*** (2.203)		15.071*** (5.656)		17.245** (6.930)		41.080*** (12.674)		23.005 (15.377)
Capital intensity		-0.366* (0.189)	-0.271 (0.169)			-0.643 (0.546)	-0.806 (0.531)			0.901 (1.180)	0.684 (1.179)	
Lerner index		0.059 (0.172)	0.015 (0.153)			-0.023 (0.496)	0.053 (0.480)			-1.031 (1.072)	-0.930 (1.065)	
Log (fixed assets)		-0.006 (0.200)	-0.148 (0.179)			0.538 (0.576)	0.779 (0.565)			-0.920 (1.245)	-0.598 (1.253)	
Export intensity		-0.804*** (0.171)	-0.390** (0.176)			0.602 (0.492)	-0.105 (0.554)			3.243*** (1.064)	2.300* (1.229)	
Constant	6.454*** (0.556)	11.005*** (0.841)	6.839*** (0.535)	10.419*** (0.915)	48.406*** (1.424)	42.650*** (2.559)	48.695*** (1.544)	42.572*** (2.878)	23.013*** (3.256)	7.323 (5.733)	20.545*** (3.339)	12.377* (6.385)
Observations	79	79	79	79	79	79	79	79	79	79	79	79
R-squared	0.165	0.458	0.374	0.515	0.224	0.290	0.262	0.320	0.075	0.188	0.214	0.237

Note: This table is generated under a calibration in which the elasticity of substitution across sectors in production is uniformly set to 0.8.

B Related Proofs

We first present an aggregation theorem that characterizes the change in real output in response to exogenous shocks in disaggregated economies with input-output linkages, distortions, and an extensive margin of products.

Theorem 1. *In response to shocks to productivities and subsidies, the first-order change in real output is:*

$$d \log Y = \sum_{k \in N} \tilde{\lambda}_k d \log z_k + \sum_{k \in N} \tilde{\lambda}_k \eta_k d \log M_k - \sum_{k \in N} \tilde{\lambda}_k d \log \mu_k - d \log \Lambda_L \quad (\text{B-1})$$

where $\eta_k = \frac{1-\varphi_k \epsilon_k}{\epsilon_k - 1}$ represents the love-of-variety in sector k .

Proof of Theorem 1. By Shepard's lemma, the change in the marginal cost is given by:

$$d \log mc_k = -d \log z_k + \sum_{h \in N} \tilde{\Omega}_{kh} d \log p_h + \tilde{\Omega}_{kL} d \log w \quad (\text{B-2})$$

where $\tilde{\Omega}_{kh} = \frac{p_h x_{kh}(\nu)}{mc_k y_k(\nu)} = z_k^{\sigma_k - 1} \omega_{kh} \left(\frac{p_h}{mc_k} \right)^{1-\sigma_k}$.

In equilibrium, the price index for sector k is:

$$p_k = \left(M_k^{-\varphi_k \epsilon_k} \int_0^{M_k} p_k(\nu)^{1-\epsilon_k} \right)^{\frac{1}{1-\epsilon_k}} = M_k^{\frac{1-\varphi_k \epsilon_k}{1-\epsilon_k}} p_k(\nu) = M_k^{\frac{1-\varphi_k \epsilon_k}{1-\epsilon_k}} \mu_k mc_k \quad (\text{B-3})$$

which implies that

$$d \log p_k = -\eta_k d \log M_k + d \log \mu_k + d \log mc_k \quad (\text{B-4})$$

$$= -\eta_k d \log M_k + d \log \mu_k - d \log z_k + \sum_{h \in N} \tilde{\Omega}_{kh} d \log p_h + \tilde{\Omega}_{kL} d \log w \quad (\text{B-5})$$

where $\eta_k = \frac{1-\varphi_k \epsilon_k}{\epsilon_k - 1}$.

This then implies that

$$d \log p = \sum_{k \in N} \tilde{\Psi}_{(k)} [-\eta_k d \log M_k - d \log z_k + d \log \mu_k] + d \log w \quad (\text{B-6})$$

and

$$d \log P^Y = b' d \log p = \sum_{k \in N} \tilde{\lambda}_k [-\eta_k d \log M_k - d \log z_k + d \log \mu_k] + d \log w. \quad (\text{B-7})$$

Since labor is inelastically supplied, the change in nominal wages is

$$d \log w = d \log \Lambda_L + d \log P^Y + d \log Y. \quad (\text{B-8})$$

Finally, combining these expressions yields

$$d \log Y = d \log w - d \log P^Y - d \log \Lambda_L \quad (\text{B-9})$$

$$= \sum_{k \in \mathcal{N}} \tilde{\lambda}_k (d \log z_k + \eta_k d \log M_k - d \log \mu_k) - d \log \Lambda_L \quad (\text{B-10})$$

□

Proof of Proposition 1. Since $\Omega_{ji} = \mu_j^{-1} \tilde{\Omega}_{ji} = \frac{p_i x_{ji}(\nu)}{\mu_j \sum_h p_h x_{jh}(\nu)}$,

$$d\Omega_{ji} = \Omega_{ji} \left[-d \log \mu_j + (1 - \sigma_j)(d \log p_i - \sum_l \tilde{\Omega}_{jl} d \log p_l) \right] \quad (\text{B-11})$$

$$= -\Omega_{ji} d \log \mu_j + (1 - \sigma_j) \mu_j^{-1} \text{Cov}_{\tilde{\Omega}(j,:)}(d \log p, I_{(\nu)}). \quad (\text{B-12})$$

Or equivalently,

$$d\Psi_{mn} = \sum_j \sum_i \Psi_{mj} d\Omega_{ji} \Psi_{in} \quad (\text{B-13})$$

$$= -\sum_j \sum_i \Psi_{mj} \Psi_{in} \Omega_{ji} d \log \mu_j + \sum_j \sum_i \Psi_{mj} \Psi_{in} \mu_j^{-1} (1 - \sigma_j) \text{Cov}_{\tilde{\Omega}(j,:)}(d \log p, I_{(\nu)}) \quad (\text{B-14})$$

$$= -\sum_j \Psi_{mj} d \log \mu_j \sum_i \Omega_{ji} \Psi_{in} + \sum_j \Psi_{mj} \mu_j^{-1} (1 - \sigma_j) \text{Cov}_{\tilde{\Omega}(j,:)}(d \log p, \Psi_{(n)}). \quad (\text{B-15})$$

Using $\Omega \Psi = \Psi - I$, we can rewrite the expression above as

$$d\Psi_{mn} = -\sum_j \Psi_{mj} (\Psi_{jn} - \delta_{jn}) d \log \mu_j + \sum_j \Psi_{mj} \mu_j^{-1} (1 - \sigma_j) \text{Cov}_{\tilde{\Omega}(j,:)}(d \log p, \Psi_{(n)}), \quad (\text{B-16})$$

where δ_{jn} is the jn -th element of the identity matrix.

This also implies that

$$\lambda_l d \log \lambda_l = -\sum_j \lambda_j (\Psi_{jl} - \delta_{jl}) d \log \mu_j + \sum_j (1 - \sigma_j) \lambda_j \mu_j^{-1} \text{Cov}_{\tilde{\Omega}(j,:)}(d \log p, \Psi_{(l)}) \quad (\text{B-17})$$

and

$$d \log \Lambda_L = -\sum_j \lambda_j \frac{\Psi_{jL}}{\Lambda_L} d \log \mu_j + \sum_j (1 - \sigma_j) \lambda_j \mu_j^{-1} \text{Cov}_{\tilde{\Omega}(j,:)}(d \log p, \frac{\Psi_{(L)}}{\Lambda_L}). \quad (\text{B-18})$$

Thus,

$$d \log Y = \sum_{k \in N} \tilde{\lambda}_k (d \log z_k + \eta_k d \log M_k - d \log \mu_k) - d \log \Lambda_L \quad (\text{B-19})$$

$$= \sum_{k \in N} \lambda_k \frac{\Psi_{kL}}{\Lambda_L} (d \log z_k + \eta_k d \log M_k) + \sum_j \sigma_j \lambda_j \mu_j^{-1} \text{Cov}_{\bar{\Omega}(j,:)} (d \log p, \frac{\Psi_{(L)}}{\Lambda_L}) \quad (\text{B-20})$$

Combined with equation (B-6), we have

$$\begin{aligned} d \log Y &= \sum_{k \in N} \lambda_k \frac{\Psi_{kL}}{\Lambda_L} (d \log z_k + \eta_k d \log M_k) \\ &\quad + \sum_j \sigma_j \lambda_j \mu_j^{-1} \text{Cov}_{\bar{\Omega}(j,:)} \left(\sum_{k \in N} \tilde{\Psi}_{(k)} [-\eta_k d \log M_k - d \log z_k + d \log \mu_k], \frac{\Psi_{(L)}}{\Lambda_L} \right) \end{aligned} \quad (\text{B-21})$$

which then simplifies to

$$\begin{aligned} d \log Y &= \sum_{k \in N} \left(\lambda_L \frac{\Psi_{kL}}{\Lambda_L} - \tilde{\lambda}_k [\Phi(\sigma)]_{L,k} \right) d \log z_k \\ &\quad + \sum_{k \in N} \tilde{\lambda}_k [\Phi(\sigma)]_{L,k} d \log \mu_k + \sum_{k \in N} \eta_k \left(\lambda_L \frac{\Psi_{kL}}{\Lambda_L} - \tilde{\lambda}_k [\Phi(\sigma)]_{L,k} \right) d \log M_k \end{aligned} \quad (\text{B-22})$$

□

Proof of Proposition 2. To derive the response of the mass of entrants to subsidies, we begin with the equilibrium sales share equation

$$d \log \lambda = -\Lambda_\mu d \log \mu + \Lambda_M d \log M \quad (\text{B-23})$$

with $[\Lambda_\mu]_{l,k} = -\frac{\partial \log \lambda_l}{\partial \log \mu_k} = \lambda_k (\Psi_{kl} - \delta_{kl}) / \lambda_l + [\Phi(\sigma - 1)]_{l,k} \tilde{\lambda}_k$ and $\Lambda_M = \Phi(\sigma - 1) \text{Diag}(\tilde{\lambda} \circ \eta)$.

Under the zero-profit condition, the mass of firms in each sector is proportional to its gross operating surplus

$$d \log M_k = \mu_k^{-1} d \log \mu_k + d \log \lambda_k. \quad (\text{B-24})$$

Substituting $d \log \lambda_k$ gives: sector is proportional to its gross operating surplus

$$(I - \Lambda_M) d \log M = (\mu^{-1} - \Lambda_\mu) d \log \mu \quad (\text{B-25})$$

Combined with the endogenous response of markups: $d \log \mu_k = -\xi_k d \log M_k - d \log \tau_k$, we derive the response of the mass of entrants to the subsidized sector is proportional to its

gross operating surplus

$$d \log M = \left[I - \Lambda_M + (\mu^{-1} - \Lambda_\mu) \xi \right]^{-1} (\Lambda_\mu - \mu^{-1}) d \log \tau \quad (\text{B-26})$$

□

Proof of Proposition 3. Given the endogenous response of markups to the mass of entrants, we have

$$d \log Y = \sum_{k \in N} \tilde{\lambda}_k [\Phi(\sigma)]_{L,k} (-d \log \tau_k - \xi_k d \log M_k) + \sum_{k \in N} \eta_k \left(\lambda_L \frac{\Psi_{kL}}{\Lambda_L} - \tilde{\lambda}_k [\Phi(\sigma)]_{L,k} \right) d \log M_k \quad (\text{B-27})$$

$$= - \sum_{k \in N} \tilde{\lambda}_k [\Phi(\sigma)]_{L,k} d \log \tau_k + \sum_{k \in N} \left[\eta_k \lambda_k \frac{\Psi_{kL}}{\Lambda_L} - (\eta_k + \xi_k) \tilde{\lambda}_k [\Phi(\sigma)]_{L,k} \right] d \log M_k \quad (\text{B-28})$$

which implies that

$$\frac{\partial \log Y}{\partial \log \tau_k} = -\tilde{\lambda}_k [\Phi(\sigma)]_{L,k} \quad (\text{B-29})$$

and

$$\frac{\partial \log Y}{\partial \log M_k} = \eta_k \lambda_k \frac{\Psi_{kL}}{\Lambda_L} - (\eta_k + \xi_k) \tilde{\lambda}_k [\Phi(\sigma)]_{L,k} \quad (\text{B-30})$$

□

Related Proof for Roundabout Economy.

$$\Phi(1) = \begin{bmatrix} \mu_1^{-1} \lambda_1 \tilde{\lambda}_1^{-1} (1 - \tilde{\lambda}_1^{-1}) & 0 \\ -\lambda_1 (1 - \tilde{\lambda}_1^{-1}) (1 - \mu_1^{-1}) & 0 \end{bmatrix} \quad (\text{B-31})$$

$$(1 - \Lambda_M)^{-1} = \left(1 - (\sigma_1 - 1) \Phi^1(1) \text{diag}(\tilde{\lambda} \circ \eta) \right)^{-1} = [1 - \eta_1 (\sigma_1 - 1) (\lambda_1 - 1)]^{-1} \quad (\text{B-32})$$

and

$$\Lambda_\mu = \sigma_1 (\lambda_1 - 1) \quad (\text{B-33})$$

□

Related Proof for Horizontal Economy. In a horizontal economy,

$$\frac{d \log Y}{d \log M_k} = \eta_k b_k \frac{\mu_k^{-1}}{\mathbb{E}_b(\mu^{-1})} - \sigma_0(\eta_k + \xi_k) b_k \left[\frac{\mu_k^{-1}}{\mathbb{E}_b(\mu^{-1})} - 1 \right] \quad (\text{B-34})$$

$$\xi_k \stackrel{=} 0 \quad \eta_k b_k + (1 - \sigma_0) \eta_k b_k \left[\frac{\mu_k^{-1}}{\mathbb{E}_b(\mu^{-1})} - 1 \right] \quad (\text{B-35})$$

Since $\Phi(\sigma - 1) = (1 - \sigma_0)(\mathbf{1}b' - I)$,

$$[I - \Lambda_M]^{-1} = \left[\underbrace{I + (1 - \sigma_0)\eta}_{\chi^{-1}} - (1 - \sigma_0)\mathbf{1}b'\eta \right]^{-1} \quad (\text{B-36})$$

$$= \chi + \frac{1 - \sigma_0}{1 - (1 - \sigma_0)b'\eta\chi} \chi \mathbf{1}b'\eta\chi \quad (\text{B-37})$$

$$= \chi + (1 - \sigma_0)[\mathbb{E}_b(\chi)]^{-1} \chi \mathbf{1}b'\eta\chi \quad (\text{B-38})$$

$$[I - \Lambda_M]^{-1} (\Lambda_\mu - \mu^{-1}) = [\chi + (1 - \sigma_0)[\mathbb{E}_b(\chi)]^{-1} \chi \mathbf{1}b'\eta\chi][(1 - \sigma_0)(\mathbf{1}b' - I) - \mu^{-1}] \quad (\text{B-39})$$

$$= (1 - \sigma_0)\chi \mathbf{1}b' + (1 - \sigma_0)[\mathbb{E}_b(\chi)]^{-1} \chi \underbrace{\mathbf{1}b'\eta\chi(1 - \sigma_0)\mathbf{1}b'}_{1 - \mathbb{E}_b(\chi)} \quad (\text{B-40})$$

$$- [I + (1 - \sigma_0)[\mathbb{E}_b(\chi)]^{-1} \chi \mathbf{1}b'\eta]\chi[(1 - \sigma_0)I + \mu^{-1}] \quad (\text{B-41})$$

$$= (1 - \sigma_0)[\mathbb{E}_b(\chi)]^{-1} \chi \mathbf{1}b' - [I + (1 - \sigma_0)[\mathbb{E}_b(\chi)]^{-1} \chi \mathbf{1}b'\eta][\mu^{-1} + (1 - \sigma_0)\chi(I - \eta\mu^{-1})] \quad (\text{B-42})$$

$$= -[\mu^{-1} + (1 - \sigma_0)\chi(I - \eta\mu^{-1})] + (1 - \sigma_0)[\mathbb{E}_b(\chi)]^{-1} \chi \mathbf{1}b' \underbrace{[I - (1 - \sigma_0)\eta\chi]}_{\chi}(I - \eta\mu^{-1}) \quad (\text{B-43})$$

$$= -\mu^{-1} + (1 - \sigma_0)[\mathbb{E}_b(\chi)]^{-1} \chi \mathbf{1}b' - I]\chi(I - \eta\mu^{-1}) \quad (\text{B-44})$$

□