Owings - Oral defense, April 29, 2011

My dissertation develops a new paracomplete logic, starting with a propositional logic, and then an extension to a modal logic. I develop a corresponding tableaux proof system, for which I prove soundness and completeness results.

'Gappy' objects

The system is called GO, for 'gappy objects'. The initial motivation comes from views that attribute some sort of metaphysical indeterminacy, or 'gappy behavior', to a class of simple objects, or 'atoms'.

An early incarnation of this is in 1944, with a succinct (if simple) argument by WHF Barnes relating to sense-data. Roughly, sense-data are supposed to have exactly the properties they appear to have. So if a sense-datum appears F, then it is F. Given that there is a case where a sense-datum appears neither F nor non-F, it follows that it *is* neither F nor non-F.

Though intended as a sort of reductio against sense-data, it suggests the philosophical position that the Law of Excluded Middle could be challenged as a result of the peculiar logical behavior of atoms. Perhaps, though, if it were not for these deviant entities, we might not otherwise have reason to accept the failure of LEM.

Background

A system is *paracomplete* if the Law of Excluded Middle fails:

$$B \nvdash A \vee \neg A$$

A review of *Strong Kleene* (K_3):

- Adding a middle value $\frac{1}{2}$, the set of values is $\{0, \frac{1}{2}, 1\}$.
- A valuation ν assigns each atomic parameter one of these values.
- $\nu(\neg A) = 1 - \nu(A)$
- $\nu(A \wedge B) = \mathsf{min}\{\nu(A), \nu(B)\}$
- $\nu(A \vee B) = \mathsf{max}\{\nu(A), \nu(B)\}$
- *Logical Consequence.* $B_1, \ldots, B_n \vdash A$ iff all valuations that assign each of $B_1, \ldots, B_n$ value 1 assign A value 1.

Truth tables for K_3:

$\neg$	
0	1
½	½
1	0

$\vee$	0	½	1
0	0	½	1
½	½	½	1
1	1	1	1

$\wedge$	0	½	1
0	0	0	0
½	0	½	½
1	0	½	1

GO

$\neg$	
0	1
½	½
1	0

$\wedge$	0	½	1
0	0	0	0
½	0	0	0
1	0	0	1

$\vee$	0	½	1
0	0	0	1
½	0	0	1
1	1	1	1

Some notable features

$$B \nvdash A \vee \neg A$$

$$B \vdash \neg(A \wedge \neg A)$$

$$\neg(A \vee \neg A) \nvdash A \wedge \neg A$$

$$\neg(A \vee B) \nvdash \neg A \wedge \neg B$$

$$\neg(A \wedge B) \nvdash \neg A \vee \neg B$$

Gap operator:

$$\circ A := \neg(A \vee \neg A)$$

Conditional:

$$A \rightarrow B := (A \supset B) \vee (\circ A \wedge \circ B)$$

The result is similar to the Łukasiewicz $\rightarrow$:

$\rightarrow$	0	½	1
0	1	1	1
½	0	1	1
1	0	0	1