Towards a Categorical Characteristica Universalis

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Abstract

We present a categorical logic interpretation and generalisation of Bealer's intensional logic.

The semantics of Bealer's logic[1] (and also Zalta's version) though having roots in the work of Tarski and Carnap is slightly suggestive of category theoretic notions such as simplicial sets, operads and hyperdoctrines. In this note we try to given a simple and precise categorical and type-theoretic interpretation of the semantics. We note that topos theory being a categorical version of set theory is essentially extensionality. The proper framework for intensional logic is merely a cartesian closed category with a distinguished object Ω representing not truth values or proofs, but propositions/meanings in the sense of Bealer.

Let C be a cartesian closed category with distinguished object Ω and consider the associated product multifunctors $T^n : C \times .... \times C \to C$ and the constant functor $\Omega^n : C \times ... \times C \to C$ given by $\Omega(A_1, ..., A_n) = \Omega$ with the obvious extension to morphisms. Ω corresponds to Bealer's D_0, the set of propositions which are the same as the Stoic lekta or sayables /statables. We are given a finite set of natural transformations $F : T^n \Rightarrow \Omega^n$ for $i \in I$ which correspond to Bealer's elementary abstracts. We call such natural transformations primitive lekta. In general natural transformation of such a forma is an unsaturated lekton (or just lekton). All lekta are to be built up from the elementary lekta through natural categorical operations which generalised Bealer's operations[1]. As usual we think of elements of $hom_C(1, A)$ as 'elements' of A.

Consider the following term in Bealer's logic $[F(x, [G(y, z)]_z)]_{xy}$. Let us take y, z as variables "in" (according to the standard philosophy of categorical logic) objects Y and Z. We have a morphism $G_{Y,Z} : Y \times Z \to \Omega$ which yields $\alpha_G = tr(G_{Y,Z}) : Y \to \Omega^Z$. Now consider $\beta_F = F_{X,\Omega^Z} : X \times \Omega^Z \to \Omega$. Then we interpret the above term as

$$\beta_F \circ < id_x, \alpha_G >: X \times Y \to \Omega$$

The same ideas can be easily transferred to Coq.

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Axiom F G : forall (A B : Type), A * B -> Prop.

Section A.

Variable X Y Z : Type.

Definition alpha ( y: Y) :=   fun ( z : Z) => (G Y Z) (y,z).
Definition beta := F X (Z -> Prop).
Definition BealerTerm ( w : X * Y ):= beta (fst w, alpha (snd w)).

End A.
(* BealerTerm
: forall X Y : Type, Type -> X * Y -> Prop *)
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References

[1] Protin, C. L. (2023). *On the Soundness of Bealer's Logics.* http://dx.doi.org/10.48550/arXiv.2012.09846.