Towards a Theory of Consciousness

Clarence Protin

October 31, 2021

Abstract

We begin by proposing a generalisation of cellular automata, inspired by the work of Joseph Goguen on sheaf models for concurrency and the use of partial differential equations in phyiscs. The main inspiration comes from trying to deal with formalising biochemistry and biological systems as well as complex sociological structures. After presenting two possible approaches to our model, the Eulerian and Lagrangian approaches, we show how our model is applied to the mind and consciousness and in particular to the representation of semantic or cognitive structures.

General Systems Theory

To construct a model of reality we must consider what are to be considered the basic elements. Postulating such elements is necessary even if they are seen as provisory or only approximative, to be analysed in terms of a more refined set of basic elements. A very general scheme for models involves distinguishing between *time* T and the possible states of reality S at a given time t. T is the set of possible moments of time. Thus our model is concerned with the Cartesian product $S \times T$[1]. It is our task to decompose or express elements of S in terms of a set of basic elements E and to use such a decomposition to study their temporal evolution.

The most general aspect of T is that it is endowed with an order of temporal precedence $\prec$ which is *transitive*. We may leave open question whether T with this order is linear (such as in the usual model of the real numbers) or branching. The most fundamental question regarding T concerns the density properties of $\prec$. Is time ultimately discrete (as might be suggested by quantum theory) or is it dense (between two instants we can always find a third) or does it satisfy some other property (such as the standard ordering of ordinals in set theory) ? The way we answer this question has profound consequences on our concept of *determinism.*

For a discrete time T we have a computational concept of determinism which we call *strong determinism.* Let t be a given instant of time and t' be the moment $t \prec t'$ immediately after t. Then given the state s of the universe at time t we should be able to compute the state s' at time t'. If this transition function (called the *state transition* function) is not computable how can we still have determinism regarding certain properties of s which we call *weak determinism.* Stochastic models also offer a weak form of determinism although a rigorous formalisation of this may be quite involved. A very weak statement of determinism would be simply postulating the non-branching nature of T.

We can also consider a determinism which involves not the state in the previous time but the entire past history of states and having an algorithm which determines not only the next state but the states for a fixed number of subsequent moments. For instance the procedure

[1] In modern physics we would require a more complex scheme in which T would be associated with a particular observer.

would analyse the past history and determine which short patterns most frequently occurred and then yield as output one of these which the system would then repeat as if by "habit".

The postulate of *memory* says that the all the necessary information about the past history is somehow codified in the state of the system in the previous time.

For a dense time T it is more difficult to elaborate a formal concept of determinism. In this case strong determinism is formulated as follows: given a t and a state s of the universe at t and a $t \prec t'$ which is in some sense sufficiently close to t we can compute the state s' at t'. Models based on the real numbers such as the various types of differential equations are problematic in two ways. First, obtaining strong determinism, even locally, is problematic and will depend on having solutions given by convergnet power series expansions with computable coeficients or on numerical approximation methods. Secondly, differential models are clearly only continuum-based approximations (idealisations) of more complex real systems having many aspects which are actually discrete. The determinism of differential models can be thus seen as based on *an approximation of an approximation.*

We now consider the states of the universe S. The most basic distinction that can be made is that between a *substrate* E and a space of qualities Q [2]. States of the universe are given by functions $\phi : E \times T \to Q$. We will see later that in fact it is quite natural to replace such a function by the more general mathematical structure of a "functor". To understand ϕ we must consider the two fundamental alternatives for E: the Lagrangian and Eulerian approaches (these terms are borrowed from fluid mechanics).

In the Lagrangian approach the elements of E represent different entities and beings whilst in the Eulerian approach they represent different regions of space or some medium[3]. This can be for instance points or small regions in standard Euclidean space. The difficulty with the Lagrangian approach is that our choice of the individual entities depends on the context and scale and in any case we have to deal with the problem of beings merging or becoming connected , coming to be or disappearing or the indiscernabiliy problem in quantum field theory. The Eulerian approach besides being more natural for physics is also very convenient in biochemistry and cellular biology where we wish to keep track of individual biomolecules or cells or nuclei of the brain. In computer science the Lagrangian approach could be seen in taking as basic elements the objects in an object-oriented programming language while the Eulerian approach would consider the variation in time of the content of a specific memory array.

We call the elements of E *cells* and $\phi : E \times T \to Q$ the *state function.* For now we do not say anything about the nature of Q. In the Eulerian approach E is endowed with a fundamental bordering or adjacency relation $\oplus$ which is not reflexive, that is, a cell is not adjacent to itself. The only axiom we postulate is that $\oplus$ is symmetric and each cell must have at least one adjacent cell. We have that $\oplus$ induces a graph structure on E. This graph may or not be planar, spatial or embeddable in n-dimensional space for some n.

We can impose a condition making E locally homogenous in such a way that each $e \in E$ has the same number of uniquely identified neighbours.

For the case of discrete T, the condition of *local causality* states that if we are in a deterministic scenario and at time t we have cell e with $\phi(e) = q$ then the procedure for determing $\phi(e)$ at the next instance t' will only need the information regarding the value of ϕ for e and its adjacent cells at the previous instant. Many variations of this definition are possible in which adjacent cells of adjacent cells may also be included. This axiom is seen clearly in the methods of numerical integration of partial differential equations.

[2]There is also an alternative approach such as the one of Takahara et al. based on the black box model in which for each system we consider the cartesian product $X \times Y$ of inputs X and outputs Y. In this model we are lead to derive the concept of internal state as well as that of the combination of various different systems. We can easily represent this scenario in our model by simulating the input and output signalling mechanism associated to a certain subset of E.

[3]such as mental or semantic space.

Now suppose that T is dicrete and that E is locally homogenous and that we indicate the neighbours of a cell e by $e \oplus_1 e_1, e \oplus_2 e_2, ...e \oplus_i e_i$. Then the condition for *homogenous local causality* can be expressed as follows. For any time t and cells e and e' such that $\phi(e,t) = \phi(e',t)$ and $\phi(f_i,t) = \phi(f'_i,t)$,where f_i and f'_i are the corresponding neighbours of e and e', we have that $\phi(e,t') = \phi(e',t')$ where t' is the instant after t.

An example in the conditions of the above definition is that of a propagating symbol according to a direction j. If a cell e is in state *on* and cell e' such that $e \oplus_j e'$ is in state *off* then in the next instant e is in state *off* and e' is in state *on*. Stochastic processes such as *diffusion* can easily be expressed in our model.

A major problem in the Eulerian approach is to define the notion of identity of a complex being. For instance how biological structures persist in their identity. despite the constant flux and exchange of matter, energy and information with their environment.

We clearly must have a nest hierarchy of levels of abstraction and levels approximation and this calls for a theory of approximation. Some kind of metric and topology on E, T and the functional space of functions ϕ is necessary.

Note that all the previous concepts carry over directly to the Lagrangian approach as well. In this approach a major problem involves formalising the way in which cells can combine with each other to form more complex being. If we consider the example of biochemistry then we see that complex beings made up from many cells have to be treated as units well and that their will have their own quality space Q' which will contain elements not possible to be realise by a single $e \in E$. This suggests that we need to add a new relation on E to account for the joining and combination of cells and to generalise the definition of $\phi : E \times T \to Q$. This is the topic of the next section.

Theory of Complex Systems

We take the Lagrangian approach. Let E and T be as in the previous section. We now add a *junction relation* J on E. When eJe' then e and e' are to be seen as forming an irreducible being whose state cannot be decomposed in terms of the states of e and e'. The state transition function must not only take into account all the neighbours of a cell e but all the cells that are joined to any of these neighbours.

Let J' be the transitive closure of J. Let $\mathcal{E}_J$ denote the set of subsets of E such that for each $S \in \mathcal{E}$ we have that if $e, e' \in S$ then $eJ'e'$. Inclusion induces a partial order on $\mathcal{E}$. Instead of Q we consider a set $\mathcal{Q}$ of different quality spaces Q,Q', Q'',...which represent the states of different possible combinations of cells. Let us assume that Q represents as previously the states for single cells. For instance a combination of three cells will have states which will not be found in the combination of two cells or a single cell. Suppose e and joined to e' and the conglomerate has state $q \in Q'$. Then we can consider e and e' individually and there is function which restricts q to states q_1 and q_2 of e and e'. In category theory there is an elegant way to combine all this information: the notion of presheaf.

To define the state functions for a given time t we must consider a presheaf:

$$\Phi_J : \mathcal{E}_J^{op} \to \mathcal{Q}$$

The state of the universe at given instant will be given by compatible *sections* of this presheaf. To define this we need to consider the category of elements $El(\mathcal{Q})$ associated to $\mathcal{Q}$ whose objects consists of pairs (Q, a) where $a \in Q$ and morphisms $f : (Q, a) \to (Q', a')$ are maps $f : Q \to Q'$ which preserve the second components $f(a) = a'$. Thus a state function at a given time is given by a functor:

$$\phi_J : \mathcal{E}_J \to El(\mathcal{Q})$$

But J can vary in time and we need a state transition function for J itself which will clearly also depend on ϕ_J for the previous moment. Thus the transition function will involve a functor:

$$\mathcal{J}_J : hom(\mathcal{E}_J, El(\mathcal{Q})) \to Rel(E)$$

and will yield a functor

$$\phi_{\mathcal{J}_J(\phi_J)} : \mathcal{E}_{\mathcal{J}_J(\phi_J)} \to El(\mathcal{Q})$$

Note that we could also consider a functor

$$\mathcal{E} : Rel(E) \to Pos$$

which associates $\mathcal{E}_J$ to each J.

The relation J is the basic form of junction. We can use it to define higher-level complex concepts of connectivity such as that which connects various regions of biological systems. We might define living systems as those systems that are *essentially connected*. These can be defined as systems in which the removal of any part results necessarily in the loss of some connection between two other parts. This can be given an abstract graph-theoretic formulation which poses interesting non-trivial questions. Finally we believe this model can be an adequate framework to study self-replicating systems.

Eulerian Theory of the Mind

What role does space or spatial intuition play in consciousness ? It is important to note that our intuition of spatial (and temporal) continuity and the mathematical definition of continuity are quite distinct. The intuition of spatial continuity is a fact of consciousness. The mathematical structure of the continuum is model which has highly useful and efficient in science. But we do not know how far physical reality itself conforms to this mathematical structure. The Eulerian approach to the mind certainly imposes itself from a neuroscientific point of view wherein various measurements are taken according to the spatial layout of the brain. Our conscious experience itself seems to be inherently spatial as it contains a representation of our immediate surroundings as well as our own body. Memory and recollection, imagination, the processing and organisation of meaning all seem to involve the intuition of spatial and temporal continuity. Another fundamental aspect of our conscious experience - which is reflected in neural architecture and physiology - is that of concurrency and superimposition. There are various processes running in parallel and each one has intuitively its separate space: we are aware of the space of sounds simultaneously with the space of visual perception. And the same time there is also superimposition, various processes can be layered over the same mental space as when we are walking and talking. Some of these layers can leave or not be in the spotlight of our focused awareness yet still be in some sense present. A good mathematical analogy is that of Fourier decomposition or the spectral decomposition of an operator[4]. Consciousness is a superimposition or integration of a selection of range of fundamental frequencies related to fundamental domains. If possible states of the mind are represented by elements of an algebra A then we can expect that A has a canoncial decomposition into subalgebras A_i for which we have projections $\pi_i : A \to A_i$. Thus a state $\phi \in A$ is uniquely determined by certain states $\phi_i \in A_i$. Now if a certain A_i has a structure like a commutative ring or in particular a commutative C^*-algebra then its elements can be represented as functions (or global sections of sheaves) over a certain

[4]The Fourier decomposition of a function and its inversion express a duality similar to the one between the Eulerian and Lagrangian approaches. Also we could consider the significance of generalised functions (distributions): in this case point-sources are the cells.

canonically determined space *spec* A_i. But we can consider also that there may some A_i that cannot be spatialised in this way. These would be the noncommutative subalgebras. Perhaps such elements are always part of our consciousness and in certain states the major part. One problem with the Eulerian approach is the same as in the case of biochemistry and biology. It is difficult to identify and define (for instance using approximation concepts) the individuality and identity of elements of consciousness: particular images, thoughts, concepts and their structure and dynamics which involves interaction, growth, combination, creation and dissolution. Thus another approach is called for. Also if we consider the structural layout of the brain and take an Eulerian approach based on the neurons as cells then if we wish to approximate differential models (obtained by taking limits) we see clearly that the local causality of standard partial differential equation models is violated and integro-differential equations are called for. The next state of each neuron may depend potentially on the present state of all other neurons in the brain. Cognition might be likened to the phenomenon of resonance or periodic solutions for forced oscillators.

Lagrangian Theory of the Mind

The cornerstone of the Lagrangian approach is the postulation of structural analogy between different domains of reality. Of particular importance is the analogy between the human mind and human sociological structures which are inherently dynamic and deal with problems of specialisation, synchronisation, coordination, organisation, adaptation, response and resource management. Good examples are found in business adminstration or political organisation in general. Plato's *Republic* seems to have been the first through exploration of this idea as the analogy between the ideal organisation of the utopia and the structure and development of individual soul is the foundation of the entire work[5]. Although based on virtual concurrency and time-sharing rather than genuine concurrency modern operating systems such as those of the Unix family offer us an extremely interesting formal model for complex forms of administration embodying many important paradigms and features. It is thus the design of operating systems which will guide us in our Lagrangian approach to the mind. The set E will consist of *processes*. The mind will be analysed in terms of concurrent interacting processes. It is also important to make the connection with neuroscience which in certain domains may play a role analogous to that of the hardware in operating systems. Thus various nuclei in the brain involves with processing motor and sensory input having functions quite analogous to the various drivers in operating system as well as the process synchronisation and system calls of the kernel. A major function of the mind consists in generating a simulation and representation of the world in a global meaninful and coherent narrative. On the other hand the mind can be seen itself as a world and a community of organised dynamic mind-processes analogous to individual persons. It would be interesting to compare this to the classical tradition of *ars memoriae*.

A few questions pose themselves: how do we distinguish between processes that are elementary (elements of E) and those that are combinations for some junction relation J ? What is Q for thought processes ? How do they interact with each other ? What does in mean for a process to be in the foreground (in our immediate awareness) or to run subconsciously like a daemon in operating systems ? How are concurrent processes synchronised ? What is the analogue of the kernel in operating system ?

Consider the emotive states which may accompany a given thought. These are not processes themselves but qualities, adjunctions, which may be present in a process, i.e., they are part of elements of Q. So Q will admit a general decomposition $\Pi_{i\in I}Q_i$. This encodes the vital stats and records of a process. Q will be like the trace function in operating system, it specifies the

[5]The text also suggest to us that here is much to explorered in the analogy between embryogenesis and the early genesis of state.

system calls it us using. Or using the sociological analogy it tells us about the character, mood, knowledge, skills and relationships of the process. If we switch temporarily back to the Eulerian perspective then it is quite legitimate to consider that Q can encode or help to encode the analogue (even if rather metaphorically) of many physical, chemical and biological properties. For instance the mind can be described as "luminous", "malleable", "calm", "energetic", "vast" or else "turgid", "inert", "agitated", "constrained", etc.

Our highest mental faculties pertaining to abstract thought, creativity, self-control, self-understanding and moral conscience will belong to a special process which is the analogue of the kernel, at least *potentially* with the proper habit and training. The kernel process, even if weakened and partially innactive will always *accompany* (to use the Kantian expression) our ordinary waking experience.

The mind has a profound unity and this suggests that in our case the relation J will link most active processes. A symmetric relation which links any two elements of a set is a particular case of a *groupoid*.

Theory of Meaning

We can apply the Lagrangian approach of the previous section to semantics (and semiotics). We will not enter here into the problem of the ontological status of abstract concepts nor into epistemological problems in general. Rather we simply try to describe the structure of a *universe of meaning* which the mind enters into some sort of relationship with. This semantic universe is inhabited by basic units of meaning called *semes* (which will be our space E). These semes can either be in an inert state or activated. We need a more complex version of J- Each seme has a series of *receptors* as well as an indeterminate number of *connectors*. We write $s \rightarrow_i s'$ to indicate the complex in which s connects to receptor i of s'. Individual semes which neither connect nor are connected to have only one possible active state in Q. But the active state q of the complex $s \rightarrow_i s'$ restricts to states q' and q'' on s and s' which are distinct from the isolated state of these cells. Semantically we interpret s' as being an unsaturated seme (as a function symbol) whose meaning becomes further refined and made concrete through its arguments (receptors) . On the other hand the argument s becomes also altered and enriched by entering into this relation. Thus in *John is walking* the abstract concept of walking becomes instantiated to the walking of John and the concept of John becomes enriched with the activity of walking.

We have various examples of development: embryogenesis (including plant development), the development of social structures, cognitive and linguistic development, the booting process of an operating system. But there is another important example: the story or narrative. We can find interesting structural analogies with the above examples. But also we can expect that a complete and meaningful narrative or story will present us with a representation or abridgment of the entire semantic universe and its genesis.

The Community of Minds

The previous Lagrangian model was concerned with the individual mind. But what about the community of minds ? Here the Lagrangian approach is also convenient. We consider a set of individuals $\mathcal{I}$. The transition functions for each individual will be as in the previous section but will also depend on various forms of interactions of other individuals with the given individual. This will be expressed by relations on $\mathcal{I}$ which will all vary over time and depend in turn on the global state function of all individuals.

Theory of Knowledge

Our model is based on ontological pluralism. We do not take for granted any hypothetical reductionism of one domain of reality to another. Thus the physical world , the domain of consciousness and the domain of concepts and propositions are considered to be irreducible to each other and ontologically primitive.

As we mentioned in a previous section, mind must contain a representation of the state of a subset of the physical and mental universe (including the the community of minds) of the previous section, the faithfulness of the representation naturally varying according to the extent of such a subset. We are faced with the dificulty that the mind must contain a *representation of itself* as it is in relationship to the world.

But the mind also contains countless propositions with different degrees of epistemic assent pertaining to all ontological domains. If knowledge involves a relationship between the mental and objective conceptual domains how are we to understand memory ?

The Mind and Physics

Physics is immeasurably more than just the foundational preoccupations of theoretical physics, such as the standard model or general relativity. Rather philosophically it is difficult not to see the whole domain of partial differential equations and dynamical systems as constituting the very essence of physics. Without a good mathematical (both geometric-topological and computational-numeric) understanding of the solution spaces of the various equations which may occur in nature in a variety of meaningful contexts the fact that we could write down the Lagrangian for some hypothetical unified theory.

In order to adequately be adapted to their environment living beings must incorporate within themselves the structure of the solutions of the differential equations of physics. The psyche of animals in general must be able to simulate or compute solutions in an immediate intuitive way (or rather in a way which interfaces with conditioned neuro-muscular pathways) and to possess a specific representation or simulation of the surrounding world as well as a representation of the body of the animal in this world.

If we take the Eulerian approach to the brain in which individual neurons are taken as cells then the brain can be seen as governed by integro-differential equations. If there is a universal topological-geometric theory of dynamical systems - in particular for those that occur in nature - then we can expect this theory to apply to such an Eulerian model of the brain and perhaps by extension to the mind itself. That is, the invariant structures that we find in the study of the partial differential equations and dynamical systems in physics may also be reflected in the structure of consciousness and in semantic spaces. This is plausible in particular because we would expect the simulation of specifically physical processes to play an important part both in animals and in the human mind.

Lately is has become clear that algebraic geometry and number theory play a fundamental role both theoretical physics and in the geometric theory of differential equations. So such pure mathematics is an investigation of the mind in two different ways. Pure mathematics reveals the structure of the mind and brain by the considerations above. But pure mathematics is a very remarkable capacity and activity of the human mind, the fact that such capacity and activity exists and is possible casts great light on what the mind ultimately is.

Consciousness and the World

We hold that what is valid in both the analytical and continental traditions in philosophy can be stated in very clear terms. We will address our approach to the logicist and formalist tradition in another paper. We close here with a brief remark on phenomenology. Phenomenology is actually based on some extremely simple and extremely obvious observations. We live in the world but almost always in an extremely localised and instrumentalised subsection of the world. We seldom consider this world in its totality and seek its structure. Likewise we live in our own consciousness, amidst our thoughts, intentions, perceptions and feelings, but almost always in a very localised subsection of our consciousness directed to a local subsection of the world. We seldom pause to consider the structure of our consciousness as a whole and how it relates to the world as a whole. We think, but most of the time we are drawn in and immersed passively in our thoughts. It is rare and difficult to maintain a clear awareness of the process of our own thought. This is only possible from a vantage point which considers consciousness in its totality.

References

[1] D.D. Hoffman and C. Prakash, Objects of Consciousness, Frontiers in Psychology, June 2014, Vol. 5, Article 577.

[2] G.Kato and D.C. Struppa, A Sheaf Theoretic Approach to Consciousness, Noetic Jounral V.2 No. 1, Jan 1999.

[3] M.A. Arbib, Theory of Abstract Automata, Prentice-Hall, 1969.

[4] M.D. Mesarovic, Y. Takahara, *General Systems Theory: A Mathematical Approach*, Academic Press, 1975.

[5] J.F. Sowa, Conceptual Structures, Addison-Wesley Publishing Company, 1984.

[6] A. Tannenbaum, A.S. Woodhull, Operating Systems: Design and Implementation, 3rd Ed., Pearson, 2006.

[7] M. J.Bach, The Design of the Unix Operating System, Prentice-Hall, 1986.

[8] M. Garzon, Models of Massive Parallelism: Analysis of Cellular Automata and Neural Networks, Springer, 1995.

[9] T. Nakagomi, Mathematical formulation of Leibnizian world: a theory of individual-whole or interior-exterior reflective systems, Biosystems 69 (2003) 15-26.

[10] L. F. Monteiro, F.C.N. Pereira, A Sheaf-Theoretic Model of Concurrency.

[11] J. Petitot, Sheaf mereology and Husserl's morphological ontology, Int. J. Human-Computer Studies (1995), 00, 741-763.

[12] R. Milner, Communicating and Mobile Systems: the π- calculus, Cambridge, 1999.

[13] A.J. Greimas, Sémantique Structurale, Libraire Larousse, 1996.

[14] F.E. De Sousa Webber, Semantic Folding: Theory and its Application in Semantic Fingerprinting, Vienna, 2016.

[15] A. Sernadas, C. Sernandas, Object Specification Logic, 1995.

[16] J.A. Goguen, Sheaf Semantics for Concurrent Interacting Objects.

[17] R. Thom, Modèles Mathématiques de la Morphogenèse, 1974.