Formalising Aristotle's Topics

Clarence Protin

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Abstract

In these notes we sketch a formalisation of the intensional natural language based logic present in Aristotle's *Topics*. In the introduction we discuss how the later development of the Organon represents an extensionalist turn (inspired by mathematics). We justify the statement that many elements of the theory of the syllogism are found implicitly in the Topics and the ontological categories therein shed additional light on their meaning and structure. We end the introduction with a brief discussion of Stoic logic as another non-extensionalist approach to logic. We then sketch an algebraic semantic model and a system of axioms which can be used to formalise the Topics and in which many of Aristotle's propositions become demonstrable. Such a model seems a promising candidate for a formalisation of natural language based reasoning.

Introduction

Quantifier logic is the extensional logic of mathematics, that is, it is in mathematics that it finds justification and utility. The logic of Aristotle's *Analytics* attempts to extrapolate such mathematical logic to other domains of human discourse. The Aristotelic concept of definition, of genus and species is seen to be adequate for specifically mathematical definitions. In modern mathematics most definitions are of the form $S = \{x \in G : D(x)\}$. A circle is a line (Genus) such that its points are equidistant from a given point (difference), an Abelian group is a group (Genus) such that the commutativity condition is satisfied (Difference). The division of genus into species reflects also a typical mathematical concern with classification, that is, with disjoint decompositions $G = G_1 \cup ... \cup G_n$. The *Topics* differs from the *Analytics* in not being yet fully extensionalist and in introducing ontological classifications (in modern language: we have a type theory) which were later dropped. The Topics seem closer to intensional natural language based discourse. We present some ideas on how the syllogistic of the *Analytics* might be reininterpreted in the framework of the *Topics* and justify that much of it is found therein implicitly. In Aristotle we have two kinds of entities: those relating to individual substances or sets of such substances (for instance Socrates or Man) and those which are attributes of substances, existing within sustance ,such as Whiteness and Knowledge. We call the first kind type 1 and the second kind type 2. The difference that distinguishes between species of a given genus of type 1 clearly must be of type 2: in Man is a rational animal, rationality is of type 2. Now what type are the differences that define the species of type 2 entities such as colours ? How do we define white and blue ? Is it legitimate to use type 1 entities as in: white is the colour of snow, blue is the colour of the sky ? Aristotle himself seems to have considered the necessitity for a proto-scientific theory of colour in order to effect such a definition. This is not a question for the lexicologist but for the physicist.

What are we make of *some* A *is* B ? One intepretation is that A is a genus and there is a species C of A which has difference B. For instance, some *mammals can fly*. But then what is the conversion of the *all* B *is* A to *some* A *is* B ? We may interpret *all* B *is* A as saying that A

is a species of B. In this case *some A is B* is saying that B is a species of A. If I say bats are mammals (or all bats are mammals) I am saying that some mammals are bats, but this time being a bat is not a difference but a species. Hence Aristotle should have made this important distinction between *some A is B* expressing i) that B is a difference of some species C of B or ii) that B is a species of C. This would agree with the general tendency of the *Topics* to make ontological distinctions and classifications. Also we note that in natural language the use of *some* seems to strict, that is, if I say *some A is B* I am implying that there is some B is not A.

If we have two species S_1 and S_2 and G is a genus of one but not the other, then S_1 and S_2 are clearly distinct, in particular not subordinate to each other. We could introduce a new notation $S_1|S_2$[1]. A could also be a difference. If bats fly but hedgehogs don't then bats are not a species of hedgehog nor hedgehogs a species of bat. Aristotle does not seem to hold that a single species could fall under two genuses which are not subordinate to each other. What are we then to make of the syllogism *A is B, A is C, therefore some B is A* ? One solution is that B is a genus but C is a difference. Thus *bats are mammals, bats can fly, therefore some mammals can fly.* There is still the question whether bats have been *defined* and not merely encompassed by, the definition *flying mammal.* The other reading is where both B and C are genuses. Thus bats are mammals, bats are animals, therefore either mammals are animals or animals are mammals.

It is possible that the Topics contain an older layer reflecting the practices of genuine Socratic and early Platonic dialectic. We recommend Slomkowski's book[2]. This book, based on the author's Oxford PhD thesis, is a rich contribution to the perennial quest of understanding Aristotle's Organon and clarifying its content in the light of modern formal logic and its historical development. The scholarship is meticulous and painstakingly accurate. Slomkowski reconstructs in detail the rules and procedures of the Aristotelic version of ancient Greek debate, particularly as practiced as a game between the questioner and answerer and sets the stage for the Topics. He then proceeds to determine exactly what a topic (topos) is and concludes that it is a form of syllogism, but a richer kind of syllogism than that of the main focus of the Prior Analytics. These syllogisms are hypothetical syllogisms, that is, they involve propositional implications, and also can involve relations.

We note that Aristotle seems to employ implicitly quantifier logic when establishing his theory of the syllogism. Most rules found in the Topics could be conceived as being expressed as non-trivial sentences in first-order logic. Also, in the process of such debate-games, the propositions which are to be unanimously accepted (we can think of this as the way a corpus is used in combinatorical approaches to linguistics wherein a word is defined by the company it keeps) are not listed and made clear, although this might not be relevant to the primary goal to inducing the answerer into self-contradiction.

Also it appears that the logic used in Euclid (attempts have been made to formalise it) is quite weak compared to full first-order logic and yet is an instrument for deriving non-trivial results. This is of course not the same as Hilbert's first-order formalisation of Euclid.

A close analysis of the *Analytics* reveals that there seems to be some propositional logic present. The discovery of propositional logic is usually associated with the Stoics.

It is a tragedy that the treatises of Stoic logic did not come down to us. Stoic logic was the intensional logic of discourse and debate in general and was the legitimate heir of Socratic dialectic and the true father of modern formal philosophical logic, that is, of a formal logic applicable not only to mathematics. We even find Frege's distinction between saturated and unsaturated symbols. Stoic logic and the phiosophical, semantic and grammatical problems it addresses is both sophisticated and completely modern.

It has been suggested that Stoic logic is more focused on the propositions that express events.

[1] or Boolean-wise $S_1 \wedge S_2 = 0$.

[2] Aristotle's Topics by Paul Slomkowski, Brill, 1997.

But sentences such as *It is raining* in themselves do not express well-defined occurrences within space and time. It must always be understood that a definite space and time is determined as well as an observer (relativistic principles can follow from uncertainty about relative motion). Indeed how can we give an objective definition of the location of Athens ? All cosmography is an abstraction both local and relative. Thus in *If it is raining there are clouds* let α be *it is raining* and β be *there are clouds*. Then $\alpha \to \beta$ is to be understood as needing to be saturated by a chosen place and time. Thus if p is *Athens* and t is *1st of March 400 B.C.* then we obtain the instantiation $\alpha pt \to \beta pt$. But what if we want to express temporal causality ? Perhaps we can use temporal modal operators or different kinds of implication operators.

We shall now proceed to sketch how the intensional, natural language based and ontologically rich logic of the Topics might be formalised.

Formalisation of the Topics

An *Aristotelic semantic algebra* $\mathcal{S}$ consists of a tuple $(O, <, \in, \{\circ_n\}_{n\in N}, (\ )^c, \sim, |, \mathfrak{I}, \triangleright)$ where:

1. O is a finite set whose elements are called *terms*.
2. $<$ is a transitive relation on O.
3. $\in, |$ and $\circ_n$ are partial binary operations on O for $n \in N$.
4. $(\ )^c$ is a monotone operator on $(O, <)$.
5. $\sim$ is antitone operator on $(O, <)$ (124b).
6. $\mathfrak{I}$ is a transitive relation on O.
7. $\triangleright$ is a distinguished element of O.

For $s, t \in O$ we will write $s \in t$ to express that the operation ϵ is defined for s and t. Given $s \in O$ the maximum length n such that $s \circ_n t$ is defined for some t is called the *arity* of s. If an element s has arity at least 1 then it is called *unsaturated*. The the value of the partial binary operation will be denoted by $[s \in t]$. We require that $\in$ be antitone on the first component and monotone in the second component for $<$. Likewise for $\circ$. We will denote $\circ$ by concatenation.

For $c \in O$ we denote by $S(c)$ the set of elements $s < c$ such that there is no $s' \neq s$ such that $s < s' < c$. We also write $s \prec c$. We call $S(c)$ the set of *species* of c.

The following axioms must be satisfied:

$$a < b \ \& \ a < c \to b < c \text{ or } c < b$$

$$a \in b \to \text{ neither } a < b \text{ nor } b < a$$

$$a \prec c \to a^c \prec c$$

$$a < b \to a \mathfrak{I} b$$

$$\text{if } a \prec b \text{ then there is a } d \text{ in } O \text{ such that } x < b \text{ and } x \in d \text{ iff } x = a$$

Given $a \in O$ we denote by $E(a)$ the set of all $x \in O$ such that $x \in a$ and by $Q(a)$ the set of all x in O such that $a \in q$.

1) 120b15 $\exists s. s \prec g_1 \ \& \ s \not< g_2 \to g_1 \not\prec g_2$
2) 121a20 $\exists s. s < g_1 \ \& \ s \not< g_2 \to g_1 \not\prec g_2$
3) 121a10 $g < s \to s \not\prec g$
4) 121a25 For $S(g) \neq \emptyset$, $(\forall s. s \prec g \to t \not< s) \to t \not< g$
5) 121b25 *This is an axiom*
6) 122a5 $g_1 \prec g_2 \prec ... \prec g_i \ \& \ s \not< g_i \to s \not\prec g_1$
7) 122a32 $g_1 \prec g_2 \prec ... \prec g_i \ \& \ g_1 \not< s \to g_i \not\prec s$

8) 123a20 $s > g^c \to s \not\prec g$
9) 123b Some genera g have no contraries: $g = g^c$
10) $g = g^c \ \& \ s \prec g \to s^c \prec g$
11) 125b5 $s \prec g \to s^c \prec g^c$
12) 125b8 $\sim \exists t : g \prec t \to \sim \exists t : g^c \prec t$

in 123b15 where some species in a genus are said to possess *intermediaries*, and this recalls a continuous line AB seen as G where $A = B^c$ and $B = A^c$ but we have also a $C = A \mid B$ *between* A and B with the additional axiom relating for $|$:

13) 123b22 $g = g^c \ \& \ s \prec g \to s \mid s^c \prec g$

in 124a we get that

14) $g = g^c \ \& \ s^c \prec g \to s \prec g$
15) $s \mid s^c \prec g \to s \prec g \ \& \ s^c \prec g$
16) $s^c \prec g^c \to s \prec g$
17) $a \in r \circ b \leftrightarrow b \in r^{-1} \circ a$
18) $a \in \triangleright cb \ \& \ a \in \sim c \to b \in \sim c$

Aristotle's concept of proper might be formalised as follows: p is called *proper* for a if $E(p) = \{x\}$.

The system above is only meant as a sketch. Further work and a more complete analysis of the Topics will no doubt reveal further refinements and simplifications.