Lösungen – Übungsblatt 2

1) a) $4 + 3i$

b) $(-7+8i) - (1-i) = -7+8i-1+i$
$= -8+9i$

c) $z_1 \cdot z_3 = (3+4i)\cdot(-7+8i) = -21+24i-28i+32i^2 \quad i^2 = -1$
$= -53-4i$

d) $|3+4i| = \sqrt{(3+4i)(3-4i)} = \sqrt{9+16} = 5$

e) $Re(z_2) = 1$

f) $Im(z_3) = 8$ (nicht $8i$)

g) $\overline{z_1} = \cancel{4}\, 3-4i$

h) $\frac{z_3}{z_1} = \frac{-7+8i}{3+4i} \cdot \frac{3-4i}{3-4i} = \frac{-21+28i+24i+32}{25}$

$= \frac{11+52i}{25}$

i) $z_2 \cdot \overline{z_2} = 1+1 = 2$

2) $\underline{\underline{z_1}}$ $\quad r = 5$ (aus 1d))

$\frac{z_1}{r} = \cancel{\frac{4}{5} + \frac{3}{5}i}\ \frac{3}{5} + \frac{4}{5}i$

$\alpha = \cos^{-1}\left(\frac{3}{5}\right)$ (oder $\sin^{-1}\left(\frac{4}{5}\right)$ oder $\tan^{-1}\left(\frac{4}{3}\right)$)

$= 0,$ $\qquad z_1 = 5(\cos(0,\ ) + i\cdot\sin(0,93)) = 5e^{i(0,93)}$

$\underline{z_2}$ $\quad r = \sqrt{z_2 \cdot \overline{z_2}} = \sqrt{2}$ (aus 1 i))

$\varphi = -\frac{\pi}{4}$ (aus der Vorlesung)

$z_2 = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\pi/4\right)\right)$

$z_3 = -1 + \sqrt{3} \cdot i$

$r = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$

$\frac{z_3}{2} = -\frac{1}{2} + \frac{\sqrt{3}}{2} \cdot i$

1. Quad.   4. Quad.   2. Quad.

$\frac{\pi}{3} \rightarrow -\frac{\pi}{3} \xrightarrow[-\frac{\pi}{3}+\pi]{} \frac{2\pi}{3}$

3. reelle Nullstellen: 4, 2, 0

echt komplexe Nullstellen: 0, 2, 4

komplexe Nullstellen: 4

4. $\sin(\alpha+\beta) = \sin\alpha \cos\beta + \sin\beta \cos\alpha$

$$\sin\alpha \cdot \cos\beta + \sin\beta \cos\alpha = \frac{(e^{i\alpha} - e^{-i\alpha})}{2i} \frac{(e^{i\beta} + e^{-i\beta})}{2} + \frac{(e^{i\beta} - e^{-i\beta})}{2i} \cdot \frac{(e^{i\alpha} + e^{-i\alpha})}{2}$$

$$= \Big(e^{i(\alpha+\beta)} + \cancel{e^{i(\alpha-\beta)}} - \cancel{e^{i(-\alpha+\beta)}} - e^{-i(\alpha+\beta)} + e^{i(\alpha+\beta)} + \cancel{e^{i(-\alpha+\beta)}} - \cancel{e^{i(\alpha-\beta)}} - e^{-i(\alpha+\beta)}\Big) / 4i$$

$$= \frac{2e^{i(\alpha+\beta)} - 2e^{-i(\alpha+\beta)}}{2 \cdot 2i}$$

$$= \frac{\cancel{2}(e^{i(\alpha+\beta)} - e^{-i(\alpha+\beta)})}{\cancel{2} \cdot 2i}$$

$$= \sin(\alpha+\beta)$$

5, $z_k = e^{\frac{2\pi k}{n} \cdot i}$, $n = 7$, $k \in \{0, 1, \dots, 6\}$.

$z_0 = e^{0 \cdot i} = 1$

$z_1 = e^{2\pi/7 \cdot i}$

$z_2 = e^{4\pi/7 \cdot i}$

$z_3 = e^{6\pi/7 \cdot i}$

$z_4 = e^{8\pi/7 \cdot i}$

$z_5 = e^{10\pi/7 \cdot i}$

$z_6 = e^{12\pi/7 \cdot i}$