School Bus Routing for Pupils with Special Needs Under Uncertainty: An Averaging Approach

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Abstract

School Bus Routing can be affected by significant delays, particularly when transporting students with special educational needs and disabilities (SEND). The associated uncertainties can disrupt pickup schedules, negatively impacting students' commuting experience and readiness to learn upon arrival at their schools. These observations were further corroborated by our findings from stakeholders, where delays and unpredictability were consistently identified as significant pain points within existing SBRP solutions. In response to these challenges, this study, conducted in collaboration with an industry partner in North-West England, explores averaging techniques as a noise-handling strategy to mitigate the effects of stochastic disruptions. Specifically, the research investigates implicit and explicit averaging techniques to assess their effectiveness in improving the robustness of bus routes under uncertainty. Our results and findings using real-world datasets suggest that simulating delays and applying averaging techniques throughout the search process helps improve the robustness of solutions. We also examine the computational cost of repeated evaluations and discuss the trade-off with increased robustness. The concluding findings demonstrate the potential of our approach to reduce the effects of random disruptions that can be commonplace when designing school bus routes for pupils with special needs.

CCS Concepts

• **Computer systems organization → Mathematics of computing**; *Combinatorial optimization*; Applied computing.

Keywords

School Bus Routing Problems, Uncertainties in Vehicle Routing, Averaging Techniques, Threshold Accepting Algorithm, Special Needs School Bus Routing, Vehicle Routing, Capacitated Vehicle Routing Problem

ACM Reference Format:
Ozioma Paul, Julia Handl, and Manuel López-Ibáñez. 2025. School Bus Routing for Pupils with Special Needs Under Uncertainty: An Averaging Approach. In *Genetic and Evolutionary Computation Conference (GECCO '25 Companion), July 14–18, 2025, Malaga, Spain.* ACM, New York, NY, USA, 4 pages. https://doi.org/10.1145/3712255.3734325

GECCO '25 Companion, July 14–18, 2025, Malaga, Spain

ACM ISBN 979-8-4007-1464-1/2025/07
https://doi.org/10.1145/3712255.3734325

1 Introduction

The transportation of people, goods, and services has long been a complex challenge, often studied under the Vehicle Routing Problem (VRP). While there is extensive research on VRPs, most formulations are deterministic, assuming fixed travel and service times. However, real-world applications like school bus routing for pupils with special needs involve uncertainties such as unpredictable delays, variable travel times, and irregular service durations.

Deterministic approaches often prove impractical in addressing the complexities of these scenarios. Additionally, school bus routing often involves time windows. Pickups and arrivals outside these windows can result in lateness to school, missed classes, or extended waiting times. Pickup time delays were recurring concerns in our stakeholder engagement involving key decision-makers such as members of the parent-teacher boards and local councils. This study is motivated by the need to design more robust school bus routing models to maintain stability and minimise disruptions such as extreme lateness in the face of unexpected delays.

Much work has been done in modelling optimisation problems under uncertainties, as broadly discussed in [4]. Specifically, in the domain of school bus routing, a few interesting studies have looked at routing under stochastic conditions, especially with respect to uncertainties in the pickup times, travel times and eventual arrival times at the school. However, to the best of our knowledge, no prior studies have explicitly addressed the mitigation of uncertainties in the context of school bus routing for special needs students. Routing for special needs pupils presents distinct challenges, as travel times are not only stochastic due to potential delays but also because pupil pickup times may vary, resulting in possible disruptions to predefined time windows both at pickup stops and upon arrival at school. The main contribution of this paper is to highlight this gap and explore averaging techniques for noise handling and the mitigation of stochastic disruptions that may arise in practical school bus routing scenarios. Averaging, an established noise-handling approach, assesses robustness by simulating different instantiations of the uncertainty when calculating the objective function, deriving an estimate for expected performance.

In this paper, we focus on two common averaging techniques, implicit and explicit averaging, and the effect they have on the robustness of solutions. The explicit averaging strategy evaluates solutions using the average estimates of the objective functions obtained over a specified number of simulations [3]. In contrast, the implicit averaging strategy uses a single simulation to estimate objective functions [5], relying on the repeated re-evaluation of solutions within meta-heuristics to indirectly account for solution robustness. While explicit averaging is expected to offer a more reliable signal to the optimizer, the computational cost is significantly higher than implicit averaging.

A threshold-accepting algorithm [2] is used as the optimization approach in this study. Stochastic delays are simulated for a subset of the bus stops that serve as pickup points to mirror real-life disruptions. We formulate both hard and soft constraints for the problem. The soft constraints allow for solutions that exceed the predefined time windows or spend more time in transit beyond the stipulated maximum riding time. Such solutions are considered feasible, in principle. However, when any constraints are broken, such as a bus arriving later or earlier than the time windows or a bus route requiring more time than the riding limit, a penalty is added to the objective value. As a result, even though solutions that break the soft constraints are allowed, they are discouraged during the search. The number of broken constraints and the total amount of delay / violation of time windows also serve as robustness indicators to evaluate the final solutions.

2 Methods

2.1 Problem Formulation

The mathematical model used in this study is based on the one proposed by Caceres et al. [1] and extended to handle travel assistants, a provision for pupils with special needs.

2.1.1 Objective Function. The objective function f (see Equation (1)) is a weighted sum of the number of buses used, maximum riding time, number of travel assistants and the total travel time across all used buses. The weights, ω_1, ω_2, ω_3, and ω_4, are assigned based on set priorities given to each of the four sub-objectives. In the context of this study's experiment, 0.3, 0.2, 0.2, and 0.1 are the weights assigned to the number of buses used, total travel time, number of travel assistants and the maximum riding time, respectively. Additionally, since the number of buses and travel assistants used have different units from the total travel time and maximum riding time, a sufficiently large scaling factor, M_{Time}, is introduced to avoid unit-based dominance during the search process.

$$\text{Minimize } f = \sum_{k \in B} (\omega_1 z_k + \omega_2 \mathrm{TA}_k + \omega_3 m + \omega_4 \sum_{(i,j) \in E} t_{ij} x_{ijk}) \quad (1)$$

where B is the set of available buses, $z_k \in \{0, 1\}$ is a binary variable indicating whether a bus $k \in B$ is used or not (1 if used, 0 otherwise), $\mathrm{TA}_k \in \{0, 1\}$ is a binary variable indicating whether a travel assistant is needed on bus $k \in B$ (1 if needed, 0 otherwise). m represents the maximum riding time of any student traveling on bus $k \in B$. Lastly, $x_{ijk} \in \{0, 1\}$ is a binary variable indicating whether bus $k \in B$ travels from location i to location j, with $i, j \in L$ (where L denotes the set of depots, stops and schools), and t_{ij} represents the time to travel between two locations i and j.

2.1.2 Generic Vehicle Routing Problem Constraints.

(1) All students picked up at stop i are taken to their respective schools.
(2) A school cannot be visited before the corresponding stop.
(3) All buses should originate from and return to their depots.
(4) Each bus departs from its depot at most once.
(5) Every stop is visited exactly once.
(6) If a bus enters a location, it should also exit from it.
(7) The time between when a student is picked up at a stop and when they are dropped off at the school cannot be less than 0 and should not exceed the allowed maximum riding time, T.

2.1.3 School Bus Routing-Relevant Constraints.

(1) Every travel assistant (TA) is only assigned to a single bus. It is assumed in this study that a TA is assigned to any bus where there is any student(s) with SEND. Once there is a TA on the bus, it is assumed that they can cater to the needs of all the SEND students on that bus.
(2) If a bus visits a stop, it must have enough capacity for the walker and wheelchair space requirements of all the students in that stop.

2.2 Threshold Accepting

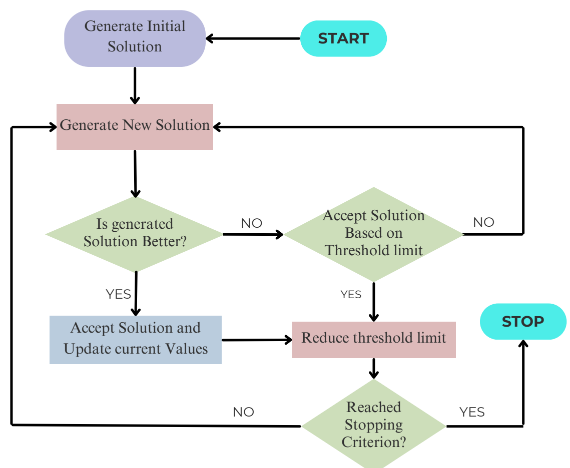

Figure 1: Threshold Accepting Algorithm

Threshold Accepting Algorithm (TAA)[2] is a variant of simulated annealing (SA) which seeks to solve the problem of oversensitivity of parameters in different instances. The major difference is that while the latter accepts a worse solution based on a probability function that decreases over time, TAA accepts worse solutions as long as they do not exceed a pre-defined threshold, which decreases over time. This change simplifies the acceptance criterion and avoids the challenge of finding the right temperature and parameters as the problem landscape evolves.

The initialisation process adapted in the TAA algorithm is based on the BUILDCHAINS algorithm [6] for generating initial solutions for school bus routes, which we extend to handle travelling assistants and the maximum riding time constraint.

Three major operators are employed for the search process:

Perturb_bus. This operator selects an occupied bus, picks a pupil, and perturbs the arrangement by moving the pupil to different positions. A "pupil" here represents multiple students from the same stop heading to the same school. There are two variants: one

Algorithm 1 *Perturb_bus* Operator

Start: Select a random pupil p from a bus B_i, based on the time loss function
Randomly perturb the bus arrangement by moving p to different positions
if Variant 1 (Best Solution) **then**
  Select the configuration with the least objective function for pupil p
else if Variant 2 (Feasible Solution) **then**
  Select the configuration with any (the first seen) feasible position for pupil p
  if Solution is feasible **then**
    Accept the new configuration and stop
  end if
end if
End: Return updated solutions

Algorithm 2 *Move_pupil* Operator

Start: Select a random pupil p from a bus B_i, based on the time loss function
Identify buses B_j $(j \neq i)$ going to the same school
if Variant 1 (Best Solution) **then**
  Move p to the best feasible position in B_j
else if Variant 2 (Feasible Solution) **then**
  Move p to any feasible position in B_j
end if
End: Return updated solutions

Algorithm 3 *Exchange_pupils* Operator

Start: Randomly select two different pupils a and b from different buses B_i and B_j
Attempt to fit pupil a into B_j and pupil b into B_i
End: Return updated solutions

evaluates all possible insertions and selects the best based on the objective function, while the other stops at the first feasible insertion. The time loss [6] of a pupil is given by (2), where ST_{school} is the school's latest start time, PT_i is the pupil's pickup time, $load\_time_i$ is the loading duration, and d_{ij} is the travel time from stop i to school j. The probability function $g_t(i)$ in (3) normalizes the time loss at stop i by the total time loss across all stops.

$$g(i) = ST_{school} - PT_i - d_{ij} - load\_time_i \tag{2}$$

$$g_t(i) = \frac{g(i)}{\sum_{i \in S} g(i)} \tag{3}$$

Move_pupil. randomly selects a pupil from a bus, using the time loss function defined above and attempts to move the pupil from its current bus to another bus going to the same school. This operator has two variants similar to the *Perturb_bus* operator. In the first variant, it seeks to move the pupil to the best position on the new bus, and in the second variant, it seeks only to move the pupil into a feasible position on the new bus.

Exchange_pupils. randomly selects two different pupils from different buses and attempts to exchange them. It does this by attempting to fit pupil a in the bus that originally contained pupil b and pupil b in the bus that originally contained pupil a.

Five possible operators are used in total and selected based on equal probabilities throughout the search. We implemented the TAA using Python, and 10 runs were done for each instance size, each run with a different random seed, to assess statistical significance of results.

2.3 Experiment

Three different approaches for handling delays were adopted for this experiment.

No Delays Simulated: This baseline scenario assumes deterministic travel and pickup times with no delays. It serves as a baseline to evaluate the effect of delays in other approaches.
Implicit Averaging: This approach involves a single simulation where delays are stochastically assigned to bus stops with SEND pupils, and the objective function is calculated accordingly. Two variants (A and B) are considered: in the first, the objective value from the initial evaluation of a candidate solution is used consistently throughout the search. In the second, delays are re-simulated, and the objective function is re-evaluated each time a solution is referenced. The latter is expected to be more beneficial, as it should help balance out stochastic fluctuations.
Explicit Averaging: Instead of a single delay simulation, simulations are repeated several times. We choose to perform 5 replications, and implement two variants, consistent with the Implicit Averaging approach to re-evaluation.

Delays were modelled for Implicit and Explicit Averaging using an exponential distribution with a mean of 10 minutes. Each algorithm was run for 100, 1,000, 5,000, and 10,000 iterations to assess their performance over varying time scales.

2.4 Dataset

The data used is an anonymised real-life dataset. It contains 623 pickup points, comprising 729 pupils spread across five schools, each with an early and late drop-off time window. Of these 729, 11% are pupils living with SEND. A 20-vehicle heterogeneous fleet, that is, a fleet of vehicles with different capacities, is used for this study. We randomly sample 10 instances for each stop size for experiment purposes: $\{10, 20, 30, \ldots, 100\}$. Each stop is randomly drawn from the 623 pickup points and has between 1 and 3 pupils. The random sampling ensures diverse scenarios, providing a robust basis for evaluating the routing performance under varying conditions.

3 Observation and Results

This study assessed robustness using two key soft constraints. The first evaluated whether buses arrived at stops or schools outside the designated time windows, representing potential pickup delays or late arrivals at school. The second constraint monitored whether the maximum riding time exceeded a given limit T. During the search process, solutions that violate these soft constraints are penalised to guide the optimisation toward feasible and high-quality outcomes. Specifically, when a violation occurs, the cumulative delay is raised to the fourth power and scaled by a large constant M. This non-linear penalty function imposes a disproportionately higher cost on solutions with greater violations, thereby prioritising solutions that minimise delay and adhere more closely to the time windows and riding limits.

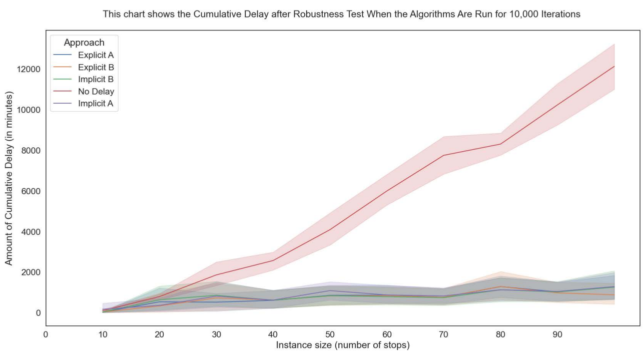

Figure 2: Cumulative Delays After 1000 Evaluations

A robustness test was conducted using 1,000 delay simulations to evaluate the solutions at the end of the search process. We assess the average number of soft constraint violations (e.g., arrivals outside time windows and exceeded riding times) and the cumulative delay times. More robust solutions should have fewer constraint violations and lower cumulative delay times.

The cumulative delays are recorded as shown in Figure 2. The Mann-Whitney U Test, suitable for comparing independent datasets with non-normal distributions, was used to analyse the performance of the five algorithms using their penalised objective function values. These results are presented in Table 1.

Table 1: P-Values from the Mann-Whitney U Test (< 0.05)

Iterations	Algorithm	Explicit B	Implicit A	Implicit B	No Delay
100	Explicit A	0.94	0.95	0.94	**0.00** *
	Explicit B		0.94	0.94	**0.00** *
	Implicit A			0.94	**0.00** *
	Implicit B				**0.00** *
1000	Explicit A	0.89	0.89	0.89	**0.00** *
	Explicit B		0.89	0.89	**0.00** *
	Implicit A			0.89	**0.00** *
	Implicit B				**0.00** *
5000	Explicit A	0.96	0.96	0.96	**0.00** *
	Explicit B		0.96	0.96	**0.00** *
	Implicit A			0.96	**0.00** *
	Implicit B				**0.00** *
10000	Explicit A	0.97	0.89	0.89	**0.00** *
	Explicit B		0.89	0.89	**0.00** *
	Implicit A			0.89	**0.00** *
	Implicit B				**0.00** *

Table 1 shows significant differences in the performance of the algorithms even when run for a few iterations. However, it is interesting that even at 10,000 iterations, there are no significant differences between the four averaging variants.

Comparing insights on significant differences with Figure 3 highlights the trade-offs of each algorithm. While the model without

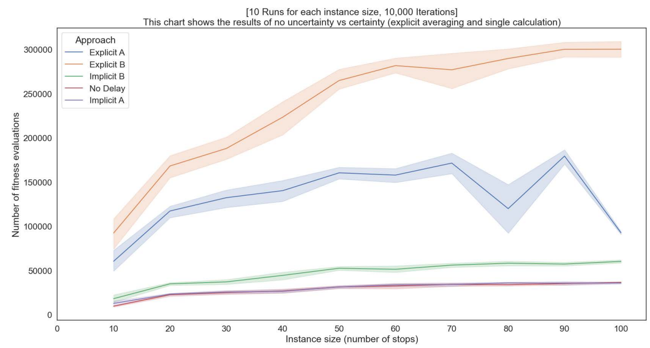

Figure 3: Fitness counts as a measure of computational costs

uncertainty is the least expensive, it lacks robustness in the face of delays. Explicit averaging increases computational cost without necessarily significantly improving robustness.

4 Discussion and Conclusions

Based on a real-world case study of school bus routing in North-West England, this paper offers valuable insights into the trade-offs between robustness, computational cost, and performance consistency in real-world applications.

We explored the use of implicit and explicit averaging techniques to enhance the robustness of school bus routing for pupils with special needs under stochastic uncertainty. Our findings indicate that differences in algorithmic performance are noticeable even with relatively few iterations, and these differences become more pronounced as the number of iterations increases. Furthermore, in the context of our problem, explicit averaging approaches incur higher computational costs without offering substantial robustness gains over implicit averaging. For future work, it would be useful to expand the experiments to consider different delay distributions.

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