CS 498 AML HW3

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We used R as the programming language for this homework.

Problem 1

a. After combined the training and testing batches, we treated each image as a vector in order of R,G,B which length is 3072 and run a self-defined function on the 60k images to compute the mean image for each of the 10 categories, and displayed the following mean images by grid.raster():

Airplane	Automobile	Bird	Cat	Deer

Dog	Frog	Horse	Ship	Truck
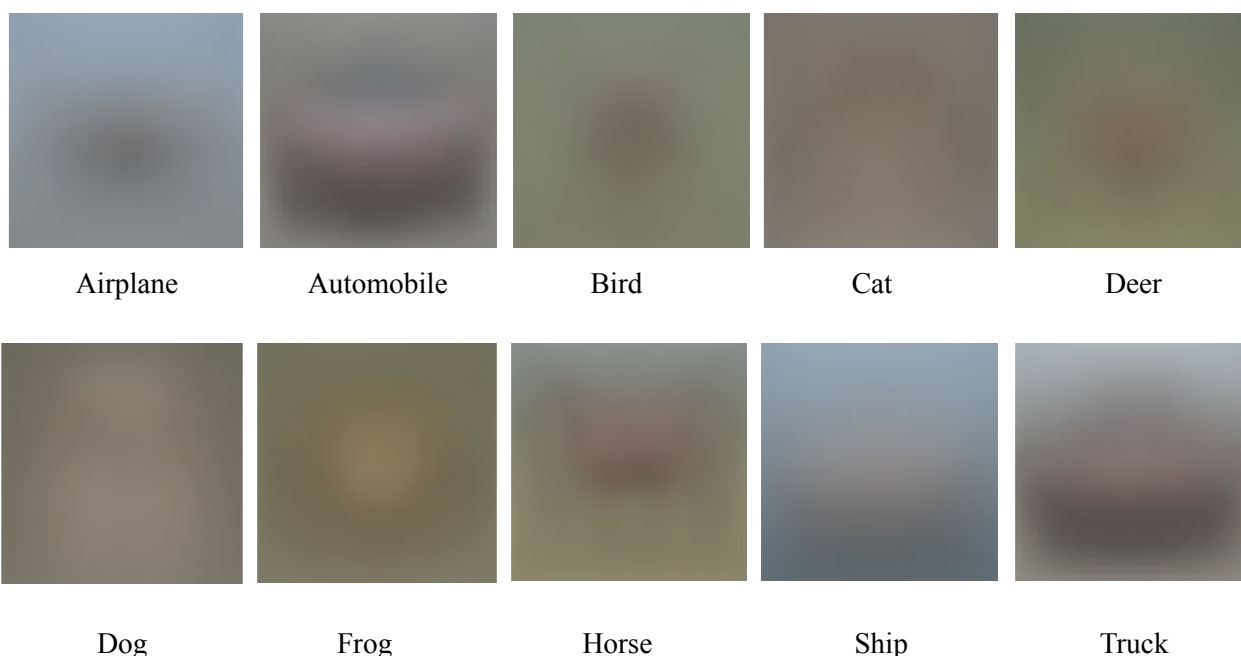
b. We used prcomp(X, scale=FALSE, rank. = 20) function to perform PCA on each category data, here X is the image data of each category. Then we took the rotation part of the PCA results as the first 20 principal components, finally applied head function to display the first six elements of ten results:

1) Airplane

```
            PC1         PC2         PC3        PC4         PC5         PC6         PC7        PC8          PC9        PC10
[1,] -0.02068911  0.02788187 -0.02425359 0.004419106 -0.01895053 0.003886998 -0.01866955 0.01710101 -0.005888020 0.002077501
[2,] -0.02053497  0.02811101 -0.02409987 0.004773921 -0.01879847 0.002826780 -0.01838065 0.01805040 -0.006563983 0.001938840
[3,] -0.02056297  0.02860207 -0.02401703 0.005042013 -0.01848467 0.002614288 -0.01739574 0.01911459 -0.006525232 0.003304947
[4,] -0.02056971  0.02895439 -0.02403830 0.005168508 -0.01861191 0.003203653 -0.01638382 0.01971177 -0.006281705 0.003966196
[5,] -0.02050475  0.02915305 -0.02397431 0.005201775 -0.01846351 0.003656244 -0.01583367 0.02051096 -0.004979019 0.004732703
[6,] -0.02043355  0.02935668 -0.02413323 0.005341477 -0.01823349 0.003312862 -0.01520611 0.02160634 -0.003843529 0.004391536
           PC11        PC12        PC13       PC14         PC15       PC16         PC17        PC18         PC19       PC20
[1,] -0.02970363  0.02360204 -0.01139254 0.02148906 -0.010299512 0.01277572 -0.010518062 0.001567350 -0.004296431 0.02183887
[2,] -0.02936638  0.02382900 -0.01191715 0.02027221 -0.009332297 0.01242704 -0.010280751 0.003183792 -0.003756182 0.02240948
[3,] -0.02984394  0.02411141 -0.01214503 0.01984159 -0.008853660 0.01301201 -0.009167301 0.004539272 -0.002686569 0.02247848
[4,] -0.02988990  0.02442999 -0.01294394 0.01845924 -0.008541656 0.01395495 -0.008025385 0.005505893 -0.001694098 0.02189724
[5,] -0.02988801  0.02453130 -0.01375172 0.01640848 -0.008449558 0.01543699 -0.007106297 0.007415383 -0.001348706 0.02051224
[6,] -0.02893033  0.02473911 -0.01465311 0.01447596 -0.007897811 0.01638715 -0.005572582 0.008895019 -0.001022974 0.01911038
```

2) Automobile

```
            PC1        PC2        PC3         PC4        PC5          PC6        PC7          PC8         PC9         PC10
[1,] -0.02945004 0.01115761 0.01085893 -0.02402192 0.02164088 -0.006372055 0.02007101 -0.0009986840 0.01562921 -0.01628013
[2,] -0.02928557 0.01095572 0.01123880 -0.02429383 0.02137693 -0.004874464 0.01991094 -0.0000617963 0.01635956 -0.01702291
[3,] -0.02930782 0.01122535 0.01179226 -0.02490294 0.02133607 -0.004036643 0.02013863  0.0013413993 0.01809753 -0.01713837
[4,] -0.02916729 0.01184407 0.01234402 -0.02521201 0.02134182 -0.003546123 0.01992054  0.0028227343 0.02009336 -0.01661480
[5,] -0.02891096 0.01233188 0.01270104 -0.02499167 0.02036268 -0.002859414 0.01931708  0.0041678146 0.02215081 -0.01676334
[6,] -0.02888108 0.01263665 0.01226070 -0.02489830 0.01891356 -0.003371735 0.01857973  0.0060354984 0.02366301 -0.01600013
           PC11        PC12       PC13         PC14       PC15         PC16       PC17         PC18        PC19          PC20
[1,] 0.02070980 -0.01343895 0.02029745 -0.004065248 0.007919082 -0.04094610 0.03459784 -0.017910839 0.008610453 -0.0001401041
[2,] 0.02210845 -0.01330412 0.01990027 -0.005917178 0.008640215 -0.04137303 0.03525720 -0.017812151 0.006738953 -0.0009543200
[3,] 0.02214426 -0.01345509 0.02010522 -0.008684118 0.009736663 -0.04213749 0.03564488 -0.017104837 0.005363231 -0.0052345215
[4,] 0.02151973 -0.01291711 0.01914837 -0.009559056 0.011272322 -0.04295015 0.03366707 -0.016133719 0.004979743 -0.0107516735
[5,] 0.02087340 -0.01379117 0.01835335 -0.010133506 0.012383960 -0.04269158 0.03220182 -0.012949339 0.006025110 -0.0141563504
[6,] 0.02030737 -0.01468246 0.01760431 -0.011311315 0.012843142 -0.04208665 0.02985122 -0.009086289 0.008625874 -0.0197056134
```

3) Bird

```
            PC1        PC2         PC3         PC4        PC5          PC6        PC7         PC8        PC9         PC10
[1,] -0.02249444 0.01669615 -0.02819171 -0.010817109 0.01245167 -0.009313448 0.01269640 -0.01793704 0.03200182 -0.02109555
[2,] -0.02241051 0.01711018 -0.02809842 -0.010473792 0.01255389 -0.009760174 0.01425544 -0.01834340 0.03015630 -0.02134082
[3,] -0.02229355 0.01786196 -0.02819798 -0.009719529 0.01273970 -0.010107245 0.01566253 -0.01873863 0.02921531 -0.02097413
[4,] -0.02211715 0.01843688 -0.02870674 -0.008495494 0.01261156 -0.010720611 0.01655675 -0.01897493 0.02804700 -0.02097808
[5,] -0.02200571 0.01879292 -0.02868548 -0.007691564 0.01215328 -0.011721597 0.01787037 -0.01867437 0.02659979 -0.02010517
[6,] -0.02190132 0.01952130 -0.02883957 -0.007408700 0.01191284 -0.012912945 0.01803705 -0.01890453 0.02502868 -0.01857440
            PC11         PC12        PC13         PC14        PC15        PC16        PC17        PC18       PC19        PC20
[1,] 0.003701451 -0.0018740725 0.007862537 -0.003445803 0.005360746 -0.03481999 0.003991971 -0.01874841 0.04701940 -0.01405170
[2,] 0.003188230 -0.0017353638 0.008128904 -0.001894412 0.007223106 -0.03437221 0.002028306 -0.02028738 0.04511472 -0.01546548
[3,] 0.002829575 -0.0009348562 0.007573435  0.000794869 0.008760418 -0.03447984 0.002331857 -0.02127931 0.04377275 -0.01592986
[4,] 0.002827735 -0.0011719999 0.006169488  0.003901490 0.010355691 -0.03523010 0.002011910 -0.02097106 0.03956064 -0.01572697
[5,] 0.002146735 -0.0020993559 0.005589662  0.007248235 0.010220377 -0.03531128 0.001896378 -0.02065049 0.03652067 -0.01635746
[6,] 0.002701930 -0.0031710599 0.004197409  0.009875468 0.010773683 -0.03561987 0.002521923 -0.02006878 0.03520025 -0.01759029
```

4) Cat

```
            PC1         PC2          PC3          PC4        PC5         PC6        PC7         PC8         PC9         PC10
[1,] -0.02529398 -0.02432488 -0.007303958 -0.008195793 0.01493347 -0.01981248 0.01062201 -0.03363649 0.010164270 -0.03011128
[2,] -0.02528944 -0.02476216 -0.007453607 -0.007637327 0.01550023 -0.02074457 0.01090578 -0.03393937 0.009785042 -0.02837912
[3,] -0.02558019 -0.02481837 -0.007816765 -0.007242858 0.01704631 -0.02069854 0.01152440 -0.03411726 0.008860447 -0.02548662
[4,] -0.02574797 -0.02486455 -0.008209747 -0.006891640 0.01768489 -0.02023362 0.01286790 -0.03302430 0.007457898 -0.02168863
[5,] -0.02566438 -0.02484332 -0.008016523 -0.006118236 0.01876221 -0.01954129 0.01373032 -0.03205951 0.005816048 -0.01848270
[6,] -0.02557390 -0.02447691 -0.007619548 -0.005058201 0.02074692 -0.01919009 0.01539518 -0.03122568 0.004806867 -0.01605358
            PC11         PC12        PC13         PC14         PC15       PC16         PC17         PC18        PC19         PC20
[1,] 0.005509773 -0.008768408 0.001202579 -0.02425261 -0.005471025 0.03676029 -0.016204939  0.010319754 -0.02591986  0.002759268
[2,] 0.004780919 -0.009771124 0.001918583 -0.02642815 -0.003352703 0.03712841 -0.012938001  0.007107075 -0.02108285  0.002982741
[3,] 0.004729690 -0.009943342 0.003942158 -0.02676374 -0.003267550 0.03586344 -0.009750980  0.003159753 -0.01617280  0.002115703
[4,] 0.005975463 -0.011503004 0.006877982 -0.02642221 -0.003262469 0.03212409 -0.008373423 -0.000287953 -0.01394186 -0.002389160
[5,] 0.007826343 -0.013261460 0.010687383 -0.02481567 -0.003526586 0.02694970 -0.008930604 -0.005111982 -0.01224427 -0.009098264
[6,] 0.010647708 -0.014947210 0.013584751 -0.02271458 -0.005470589 0.02240613 -0.011530741 -0.009701325 -0.01064904 -0.014052115
```

5) Deer

```
           PC1        PC2        PC3         PC4        PC5          PC6        PC7         PC8        PC9         PC10
[1,] 0.02664151 0.01580590 0.01796242 -0.02781832 0.01344686 -0.007934540 0.01852276 -0.03419960 0.02837997 -0.007711125
[2,] 0.02661630 0.01643477 0.01765525 -0.02784268 0.01247202 -0.008712808 0.01832421 -0.03420339 0.02655698 -0.007558445
[3,] 0.02663838 0.01702466 0.01753993 -0.02809737 0.01198645 -0.009844761 0.01757101 -0.03446059 0.02493832 -0.006962466
[4,] 0.02648024 0.01757982 0.01749862 -0.02809849 0.01135970 -0.011153622 0.01664564 -0.03406996 0.02361225 -0.006218435
[5,] 0.02661489 0.01828122 0.01748033 -0.02768328 0.01095009 -0.012016807 0.01584935 -0.03223659 0.02286367 -0.007257235
[6,] 0.02662736 0.01861904 0.01767622 -0.02717571 0.01051581 -0.013113030 0.01600883 -0.03048278 0.02171326 -0.006689820
           PC11         PC12        PC13        PC14       PC15        PC16       PC17        PC18       PC19        PC20
[1,] 0.005780250 -0.012694312 0.015344128 -0.01365032 0.02943453 -0.01356351 0.01792121 -0.01254939 0.02351599 -0.02980814
[2,] 0.006580580 -0.010399475 0.012622799 -0.01399224 0.02821914 -0.01547427 0.01554569 -0.01334916 0.02011278 -0.03022254
[3,] 0.005589859 -0.009462568 0.010909836 -0.01614568 0.02837511 -0.01716813 0.01551153 -0.01420599 0.01751253 -0.03053986
[4,] 0.005241232 -0.008717594 0.007676495 -0.01883277 0.02828093 -0.01895739 0.01592601 -0.01496220 0.01555465 -0.02849113
[5,] 0.004466186 -0.006043172 0.004849515 -0.01931206 0.02709402 -0.02155161 0.01650709 -0.01398660 0.01699339 -0.02669524
[6,] 0.005475698 -0.003937649 0.002079573 -0.01934751 0.02399608 -0.02262625 0.01762829 -0.01129744 0.01801466 -0.02517540
```

6) Dog

```
                PC1        PC2         PC3         PC4        PC5         PC6          PC7         PC8        PC9         PC10
[1,] 0.02555281 0.02157260 0.002807096 0.01365457 0.01079684 -0.02972872  0.0006541674 -0.02005796 0.02378289 -0.008717570
[2,] 0.02543965 0.02187737 0.003366477 0.01410919 0.01107482 -0.03086947 -0.0003875032 -0.02135790 0.02199080 -0.008725337
[3,] 0.02537582 0.02210392 0.004245158 0.01408541 0.01197413 -0.03164101 -0.0021662699 -0.02201840 0.02105320 -0.008807839
[4,] 0.02529409 0.02198552 0.004698792 0.01398706 0.01265050 -0.03216804 -0.0031940469 -0.02220663 0.01995350 -0.008216701
[5,] 0.02535890 0.02162855 0.005330255 0.01390570 0.01337201 -0.03197722 -0.0042843648 -0.02283328 0.01769483 -0.006956410
[6,] 0.02561133 0.02128469 0.006198432 0.01337814 0.01469045 -0.03108315 -0.0048508010 -0.02412452 0.01540629 -0.006038823
                PC11        PC12        PC13        PC14       PC15         PC16       PC17        PC18        PC19         PC20
[1,] 0.006256082 -0.01455049 0.006726211 -0.02382354 0.02503457 -0.014010425 0.01963539 -0.024493655 0.001268855 -0.007838521
[2,] 0.006318951 -0.01572160 0.006912933 -0.02479301 0.02377907 -0.012459032 0.02004043 -0.022159510 0.001429737 -0.008852802
[3,] 0.006064478 -0.01792173 0.007073909 -0.02507877 0.02346550 -0.010909488 0.01958842 -0.017781793 0.003372449 -0.008166020
[4,] 0.006412337 -0.02033133 0.006472657 -0.02425117 0.02230724 -0.008747066 0.01651974 -0.012762315 0.005134713 -0.006303886
[5,] 0.006197631 -0.02318576 0.005579642 -0.02256977 0.01994636 -0.006891360 0.01276010 -0.008994593 0.004834284 -0.004521814
[6,] 0.006556350 -0.02567198 0.004948861 -0.01986994 0.01711008 -0.006598137 0.01075294 -0.004619301 0.005523335 -0.002908635
```

7) Frog

```
                PC1         PC2          PC3          PC4         PC5        PC6         PC7         PC8         PC9        PC10
[1,] 0.02712535 -0.02092755 -0.009955310 -0.006824229 -0.02166630 0.02278245 -0.03180664 0.008847606 -0.02329419 0.004434627
[2,] 0.02715843 -0.02117561 -0.008908064 -0.006173190 -0.02166404 0.02377184 -0.03250565 0.007064732 -0.02110452 0.003773035
[3,] 0.02728446 -0.02167730 -0.008348932 -0.005553007 -0.02165213 0.02477909 -0.03260984 0.005887145 -0.01850456 0.003956160
[4,] 0.02712041 -0.02210511 -0.007938520 -0.004680990 -0.02165005 0.02577895 -0.03209957 0.005340343 -0.01714927 0.004579616
[5,] 0.02684222 -0.02256885 -0.007348239 -0.004015594 -0.02172619 0.02649754 -0.03282855 0.006044225 -0.01437196 0.004729657
[6,] 0.02674192 -0.02276709 -0.006342982 -0.003349020 -0.02146930 0.02581598 -0.03266796 0.007823375 -0.01131797 0.004186790
                PC11         PC12        PC13          PC14         PC15        PC16        PC17         PC18        PC19         PC20
[1,] -0.007280650 0.005412833 -0.02936108 9.126070e-03 -0.0048251539 0.03041073 -0.01197072 0.014112875 -0.014424710  0.0012455771
[2,] -0.007437456 0.006363642 -0.03142517 6.252086e-03 -0.0059440122 0.03195261 -0.01436002 0.012253600 -0.012580153  0.0005174976
[3,] -0.008392348 0.006848877 -0.03195696 3.463136e-03 -0.0045871660 0.03199630 -0.01579662 0.010791945 -0.011810509  0.0016549953
[4,] -0.009061787 0.009138610 -0.03159251 3.341407e-03 -0.0035927180 0.03232176 -0.01665358 0.009749268 -0.011228048 -0.0002519585
[5,] -0.008474433 0.009549959 -0.03161630 1.430725e-03 -0.0008248132 0.03218097 -0.01557751 0.009268871 -0.010792139 -0.0032103699
[6,] -0.008462494 0.010105588 -0.03028835 2.567655e-05  0.0020938592 0.03215223 -0.01424985 0.006205452 -0.009935531 -0.0057749618
```

8) Horse

```
                PC1         PC2          PC3          PC4         PC5         PC6        PC7         PC8         PC9        PC10
[1,] 0.03344285 -0.01187202 -0.01537709 -0.003615749 0.007316042 -0.01901769 0.02035277 -0.02613302 0.02587960 -0.01961791
[2,] 0.03343548 -0.01301205 -0.01478576 -0.003223704 0.008830740 -0.01998731 0.02130920 -0.02605388 0.02499399 -0.01894319
[3,] 0.03350003 -0.01385586 -0.01414486 -0.002453809 0.009648532 -0.02097653 0.02131413 -0.02577770 0.02419486 -0.01923943
[4,] 0.03367554 -0.01427018 -0.01372804 -0.001892965 0.009465962 -0.02122946 0.02215378 -0.02591013 0.02304853 -0.01986424
[5,] 0.03369349 -0.01440837 -0.01348628 -0.001841903 0.009837812 -0.02074786 0.02280135 -0.02635798 0.02171388 -0.01939208
[6,] 0.03376146 -0.01462194 -0.01363453 -0.001956022 0.010569279 -0.01954619 0.02245801 -0.02668288 0.01976700 -0.01951251
                PC11         PC12        PC13        PC14        PC15          PC16        PC17        PC18         PC19        PC20
[1,] 0.01430143 -0.02574546 0.019105232 0.004752440 -0.01131044  0.0051354766 -0.013286085 0.04060208 -0.02383874 0.01381223
[2,] 0.01206039 -0.02692543 0.016364648 0.005651203 -0.01369489  0.0022809662 -0.014683455 0.03674600 -0.02728880 0.01404901
[3,] 0.01081210 -0.02662250 0.013428665 0.006632592 -0.01545760 -0.0007337261 -0.014017283 0.03486551 -0.02857183 0.01564419
[4,] 0.01109192 -0.02606817 0.009432132 0.006466460 -0.01724809 -0.0023894679 -0.012988857 0.03320533 -0.02986728 0.01768773
[5,] 0.01146963 -0.02515029 0.007415249 0.007348625 -0.01748216 -0.0047967370 -0.010985226 0.03129977 -0.03047955 0.01802885
[6,] 0.01193718 -0.02431117 0.006914877 0.006175374 -0.01834593 -0.0081732093 -0.008015513 0.02895068 -0.03129637 0.01896460
```

9) Ship

```
                PC1         PC2          PC3         PC4          PC5        PC6          PC7        PC8         PC9        PC10
[1,] 0.02768825 0.007541319 -0.01640282 0.02208037 -0.004265141 0.03441742 -0.007754374 0.01149423 -0.01403254 0.02581721
[2,] 0.02777998 0.008056888 -0.01658560 0.02250261 -0.003594380 0.03422687 -0.007526455 0.01151093 -0.01384987 0.02552531
[3,] 0.02791284 0.008404780 -0.01674071 0.02340194 -0.002915398 0.03408735 -0.007229176 0.01228698 -0.01368797 0.02512147
[4,] 0.02795010 0.008606015 -0.01690438 0.02375196 -0.002301084 0.03429823 -0.006883581 0.01323288 -0.01345099 0.02462730
[5,] 0.02801159 0.008737870 -0.01719875 0.02441443 -0.001858797 0.03439103 -0.006636487 0.01367568 -0.01330229 0.02413233
[6,] 0.02805163 0.008833828 -0.01745509 0.02497410 -0.001212306 0.03421685 -0.005981981 0.01398198 -0.01249838 0.02315617
                PC11         PC12        PC13        PC14        PC15        PC16         PC17        PC18        PC19       PC20
[1,] -0.009597107 0.0026421467 -0.02950086 0.001360809 -0.02315732 0.016311981 -0.004511945 0.009890544 -0.01303323 0.02442788
[2,] -0.009939044 0.0015807278 -0.02910318 0.003275597 -0.02390138 0.014365565 -0.003643214 0.011688387 -0.01284084 0.02352273
[3,] -0.010349754 0.0012413746 -0.02952239 0.004333398 -0.02394265 0.012948464 -0.003329604 0.012247995 -0.01306554 0.02321477
[4,] -0.011024536 0.0005759759 -0.02939503 0.004956469 -0.02382347 0.012073295 -0.002507077 0.012774841 -0.01376485 0.02288827
[5,] -0.012057088 0.0003875383 -0.02904276 0.005567478 -0.02392585 0.010535042 -0.001452757 0.012450164 -0.01340538 0.02115465
[6,] -0.012864476 0.0003465015 -0.02897826 0.007002818 -0.02371247 0.007787832 -0.001198428 0.012170048 -0.01134783 0.01998417
```

10) Truck

```
            PC1         PC2         PC3        PC4         PC5        PC6         PC7        PC8          PC9        PC10
[1,]  0.03040126 -0.01430363 0.02062647 0.01416074 -0.02000790 0.01652912 -0.01637266 0.02732678 -0.008588854 0.02381742
[2,]  0.03057937 -0.01486679 0.02067006 0.01449388 -0.01885500 0.01831519 -0.01618267 0.02709813 -0.007749938 0.02300098
[3,]  0.03069863 -0.01516347 0.02096721 0.01479221 -0.01822711 0.01945094 -0.01605656 0.02723419 -0.007816568 0.02273112
[4,]  0.03064684 -0.01557991 0.02114275 0.01514110 -0.01718970 0.02013027 -0.01514368 0.02757762 -0.007111900 0.02258438
[5,]  0.03065825 -0.01611796 0.02117590 0.01487628 -0.01558748 0.02084535 -0.01468533 0.02781283 -0.006633274 0.02238230
[6,]  0.03074952 -0.01619696 0.02144945 0.01416732 -0.01389941 0.02140774 -0.01414809 0.02899301 -0.006504285 0.02197711
            PC11        PC12         PC13        PC14         PC15       PC16        PC17       PC18        PC19         PC20
[1,] -0.01576199 0.01958389 -0.009454163 0.012136720 -0.02935390 0.02447512 -0.03601407 0.01246654 -0.02525779  0.0051208568
[2,] -0.01618048 0.02006189 -0.010119003 0.011758000 -0.02783456 0.02440030 -0.03698857 0.01321037 -0.02496232  0.0052845960
[3,] -0.01667104 0.02117746 -0.011044473 0.010119814 -0.02763929 0.02376443 -0.03784303 0.01559447 -0.02342254  0.0033272564
[4,] -0.01623012 0.02098254 -0.013556073 0.008260073 -0.02659015 0.02305901 -0.03799668 0.01712477 -0.02260621  0.0010747507
[5,] -0.01493686 0.01998269 -0.017070703 0.006596842 -0.02446407 0.02193749 -0.03741865 0.01819930 -0.02271257 -0.0004184476
[6,] -0.01326562 0.01804150 -0.019858305 0.004891798 -0.02305580 0.02194893 -0.03570860 0.01820135 -0.02175572 -0.0024060525
```

c. We used option 1 to define the error and imported "factoextra" library then calculated the sum of the unused eigenvalues for each of the category using the get_eigenvalue() function as the errors, and plot the following bar chart:


Bar Chart of Error
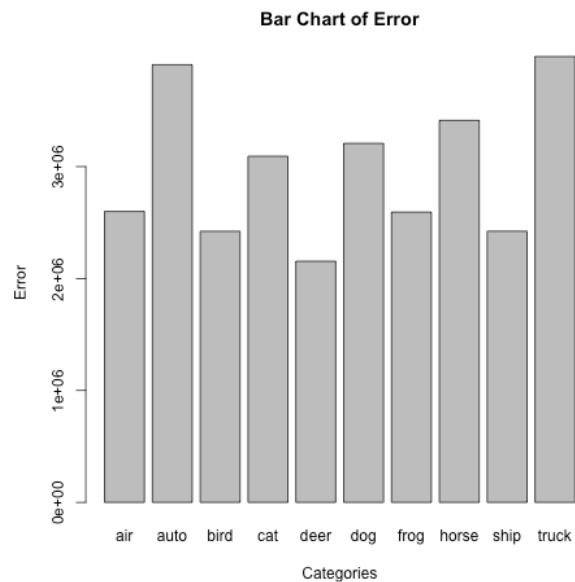
Problem 2

a. We treated the mean images as vectors and computed the distances between the mean images of each category by norm() function, then got the following Distance Matrix :

```
           airplane automobile    bird      cat     deer      dog     frog    horse     ship    truck
airplane     0.0000 1683.6354 1605.0243 1905.5353 2148.7634 1965.2215 2445.6797 1663.6459  945.5411 1449.0949
automobile 1683.6354    0.0000  886.2367 1027.6498 1143.0814 1216.0794 1191.1920  950.7861 1303.4665  949.9958
bird       1605.0243  886.2367    0.0000  517.3115  601.2503  701.4682  913.7475  418.2763 1557.7150 1416.6747
cat        1905.5353 1027.6498  517.3115    0.0000  469.7917  412.1817  677.4920  596.3767 1851.2145 1676.4679
deer       2148.7634 1143.0814  601.2503  469.7917    0.0000  617.6971  460.5109  684.3469 2065.6217 1830.7409
dog        1965.2215 1216.0794  701.4682  412.1817  617.6971    0.0000  828.5811  843.6721 1897.5918 1880.2438
frog       2445.6797 1191.1920  913.7475  677.4920  460.5109  828.5811    0.0000  948.7040 2249.1998 1913.2409
horse      1663.6459  950.7861  418.2763  596.3767  684.3469  843.6721  948.7040    0.0000 1660.2681 1347.3341
ship        945.5411 1303.4665 1557.7150 1851.2145 2065.6217 1897.5918 2249.1998 1660.2681    0.0000 1066.9416
truck      1449.0949  949.9958 1416.6747 1676.4679 1830.7409 1880.2438 1913.2409 1347.3341 1066.9416    0.0000
```

b. Then we used cmdscale() function with the distance matrix to perform MDS and made a 2D map of the means of each categories:

Map of The Means of Each Categories
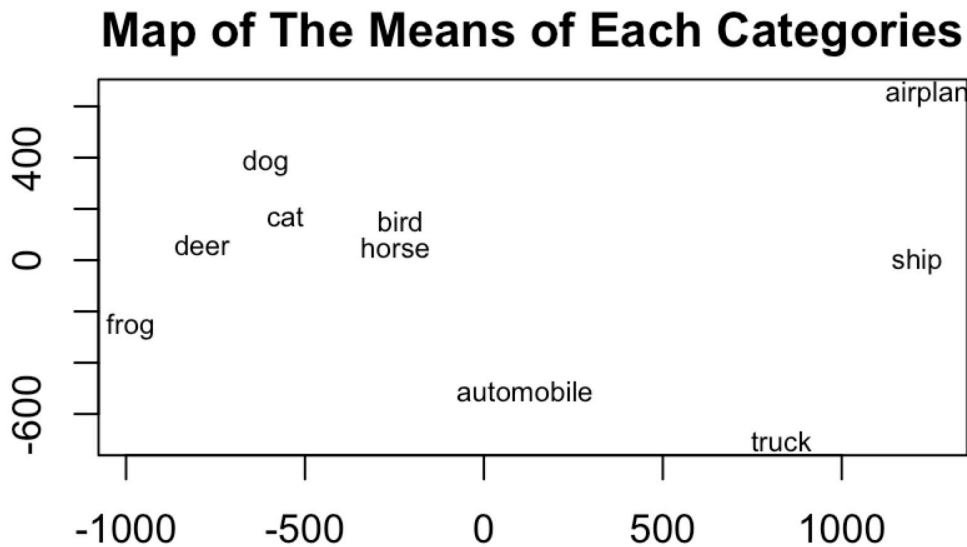
Problem 3

In this problem, we used a measurement of similarity that comes from swapping principal components between categories: For class A and class B, define E(A | B) to be the average error obtained by representing all the images of class A using the mean of class A and the first 20 principal components of class B, define the similarity between classes to be (1/2)(E(A | B) + E(B | A)).

Here we write the reconstructAwithB() function based on the following formula:

$$Xi' = mean(X) + \Sigma j[transpose(U(Bj)) * (Xi - mean(X)) * U(Bj)]$$

where Xi' is the reconstructed image, Xi is the original image, mean(X) is the mean of A's mean image, U(Bj) is jth column of the B's principal components.

Then for each pair of 10 categories:

First calculated the reconstructed image for B category with A category's first 20 principal components and the reconstructed image for A category with B category's first 20 principal components using the above function, and calculated the difference between A's original image with A's reconstructed image and the difference between B's original image with B's reconstructed image.

Then we sum the differences between original image and reconstructed image for A and B category separately during the previous step and calculate the average by dividing the total number of images in each class.

Finally we average E(A | B) and E(B | A) and filled in the output into the entry of distance matrix that corresponding to the pair between category A and category B.

a. We got the following Distance Matrix:

	airplane	automobile	bird	cat	deer	dog	frog	horse	ship	truck
airplane	1549.886		1870.264	1603.858	1758.120	1533.970	1781.874	1660.541	1783.061	1580.527	1888.538
automobile	1870.264		1956.705	1865.040	1963.991	1796.273	2006.779	1870.770	2016.977	1809.402	2000.456
bird	1603.858		1865.040	1501.921	1656.493	1494.635	1662.190	1582.759	1730.406	1616.030	1848.038
cat	1758.120		1963.991	1656.493	1716.879	1642.228	1756.582	1691.248	1838.051	1732.798	1927.807
deer	1533.970		1796.273	1494.635	1642.228	1421.152	1657.095	1544.100	1685.385	1538.018	1793.054
dog	1781.874		2006.779	1662.190	1756.582	1657.095	1749.422	1709.836	1854.936	1781.456	1972.020
frog	1660.541		1870.770	1582.759	1691.248	1544.100	1709.836	1577.911	1779.882	1643.946	1860.391
horse	1783.061		2016.977	1730.406	1838.051	1685.385	1854.936	1779.882	1816.133	1782.859	1993.850
ship	1580.527		1809.402	1616.030	1732.798	1538.018	1781.456	1643.946	1782.859	1502.719	1821.341
truck	1888.538		2000.456	1848.038	1927.807	1793.054	1972.020	1860.391	1993.850	1821.341	1971.600

b.Same as problem 2, we performed MDS by applying cmdscale() function on the distance matrix, and got the following plot:

Map of The Means of Each Categories
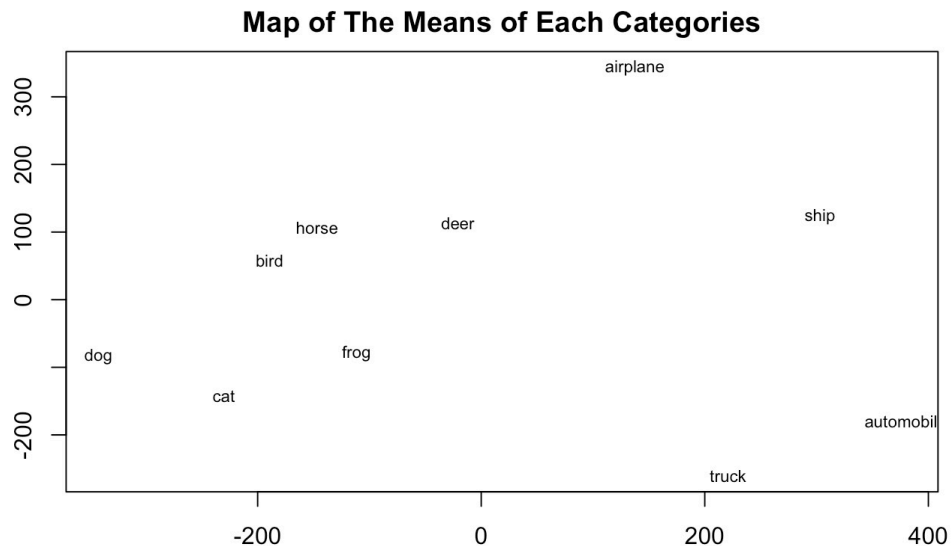
c. Comparation
Here we scaled the two plots in problem 2 (left) and problem 3 (right) into same scale to do the comparison:
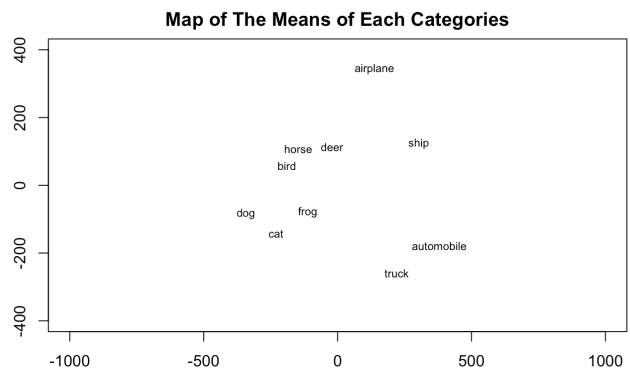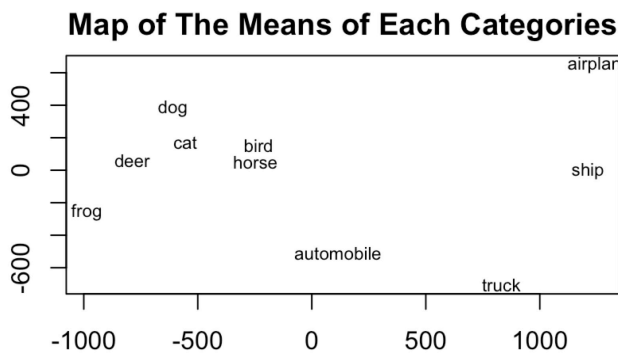
The two plots have some similarities, for example: horse and bird are closely positioned together, whereas truck and airplane seen by the large distance between those labels in both map. However since the later one only used first 20 principal components, it is reasonable to see some difference between the distance maps.The main difference is that the map of problem 3 is more clustered.