Solving the AFL Fixture Scheduling Problem

Daniel Olshansky, Leilah Taouk, Steven Nguyen, and William Batchelor

Scheduling and Optimisation

October 2023

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2.2 Objective

Our objective is made up of two metrics, with one being the attractiveness of the fixture (A), and the other being the inequality of the fixture (I).

Inherently, our aim is to maximise our fixture's attractiveness, while minimising its inequality.

Hence, for the attractiveness, our aim is to maximise the attractiveness of our AFL matches, with the attractiveness of each potential match based on the home and away teams playing ($i, j \in T$), as well as the stadium where the game is hosted ($s \in S$), and the time when it is played ($t \in TS$). Furthermore, the attractiveness also depends on the round in which the game, given that the first and last rounds are likely to generate higher attendances, as is shown by real-world data.

Hence, for the attractiveness, our aim is to find the following:

$$A = \max \sum_{i \in T} \sum_{j \in T} \sum_{s \in S} \sum_{t \in TS} \sum_{r \in R} \left(\text{reward}(i, j, s, t, r) \cdot F(i, j, s, t, r) \right). \tag{1}$$

Meanwhile, for the inequality, we aim to minimise the variance of the teams expected win numbers...

$$I = \sum_{r \in R} r \sum_{i \in T} \left(\left[\sum_{j \in T} \sum_{s \in S} \sum_{t \in TS} \sum_{r' \in \{1,\dots,r\}} \left(F(i, j, s, t, r') P(i, j, s) + F(i, j, s, t, r') P(j, i, s) \right) \right] - \frac{r}{2} \right)^2. \tag{2}$$

Hence, our final objective is to maximise the following:

$$\max \left[A - wI \right], \tag{3}$$

where w is the weight that we assign to the inequality of our fixture schedule.

3 Methodology

The first optimisation algorithm that we attempt is a mixed-integer linear program (MILP). Despite the lack of use of the MILP in other scheduling papers, we observe that our constraints are linear in nature, which we see in Section 2.1. This means that we can use a MILP that can find a feasible solution to our problem, something that is a challenging problem with numerous papers written about it, including (add citations).
Furthermore, as written in (include citation), we know that MILP are quite efficient solvers. This means that using a MILP can relatively quickly yield us a feasible AFL schedule.

An issue, however, with using a MILP for our problem is that our objective is non-linear, due to the function that we use to represent the inequality of our fixture, 2.
Hence, an MILP program is unlikely to give us our optimal schedule, however, its output can be used as an initial solution (or construction heuristic) for other algorithms, as well as a point of reference for the performance for our other optimisation algorithms that can handle the quadratic nature of our objective.
Therefore, we attempt to build an MILP program that can provide us with a good starting point for other algorithms, as well as a good point of comparison.

The variables included in Section 2 are included in our MILP, with

$$F_{i,j,s,t,r} \in \{0, 1\} \; \forall \, i, j \in T, s \in S, t \in TS, r \in R,$$

corresponding to the whether team i is hosting team j at stadium s at timeslot t, in round r. However, a new set of variables is also included in our MILP, given by

$$D_{i,r} \in \mathbb{R}^+ \; \forall \, i \in T, r \in R.$$

Similarly to our variables, all of the constraints listed in Section 2.1 are included within our MILP (with all of our constraints being hard constraints), with the inclusion of an additional set of constraints.

Our additional set of constraints is utilised along our new variables, and is given by

$$D_{i,r} \geq \sum_{j \in T} \sum_{s \in S} \sum_{t \in TS} \sum_{r' \in \{1,\ldots,r\}} [F(i,j,s,t,r')P(i,j,s) + F(i,j,s,t,r')P(j,i,s)] - \frac{r}{2}, \ \forall i \in T, r \in R,$$

(4)

and

$$D_{i,r} \geq \frac{r}{2} - \sum_{j \in T} \sum_{s \in S} \sum_{t \in TS} \sum_{r' \in \{1,\ldots,r\}} [F(i,j,s,t,r')P(i,j,s) + F(i,j,s,t,r')P(j,i,s)], \ \forall i \in T, r \in R.$$

(5)

These constraints, coupled with our new variables, ensure that the new variables correspond to the deviation of the expected number of wins for each team from the average, for each round, for both the cases when a team is expected to have less wins that the average (Equation 5), as well as when they are expected to have more than the average number of wins (Equation 4).

Thus, the objective of our MILP also changes to incorporate the values of our new variables, becoming

$$\max \sum_{i \in T} \sum_{r \in R} \left[\sum_{j \in T} \sum_{s \in S} \sum_{t \in TS} (\text{reward}(i,j,s,t,r) \cdot F(i,j,s,t,r)) - wr D_{i,r} \right].$$

(6)

Hence, our MILP objective includes Equation 1 from our problem formulation, which incorporates the attractiveness feature of our fixture.

However, given that our second problem objective (given in Equation 2), which is to minimise the sum of squares (or variance) of the teams' deviations from the average, is non-linear, it cannot be included in our MILP. Hence, in our MILP we seek to minimise the gaps between the AFL teams' expected number of wins - which preserves linearity.

While, clearly, this differs from the original problem objective, the solutions that we achieve using this method perform very well compared to our other methods, as we observe in our results section (Section 4).

4 Results

We run our MILP using Gurobi's solver software, through its Python programming language API. We run the program locally, using a personal computer with 16GB of RAM, featuring 4 cores with a base speed of 1.90 GHz, along with 8 logical processors. We set our program's accepted optimality gap to %1. Our complete code is included in Section Section 7.

Using our MILP, our fixture is outputted as the following:

Round 1

Thursday Night: Richmond Tigers vs. Geelong Cats at MCG

Friday Night: Collingwood Magpies vs. Melbourne Demons at MCG

Saturday Afternoon: Fremantle Dockers vs. West Coast Eagles at Optus

Saturday Evening: Port Adelaide Power vs. Brisbane Lions at Adelaide Oval

Saturday Evening: Western Bulldogs vs. Essendon Bombers at Marvel

Saturday Night: Carlton Blues vs. St Kilda Saints at MCG

Saturday Night: Sydney Swans vs. Greater Western Sydney Giants at SCG

Sunday Afternoon: Hawthorn Hawks vs. North Melbourne Kangaroos at MCG

Sunday Evening: Adelaide Crows vs. Gold Coast Suns at Adelaide Oval

Round 2

Thursday Night: Essendon Bombers vs. Western Bulldogs at MCG

Friday Night: Carlton Blues vs. Melbourne Demons at MCG

Saturday Afternoon: Port Adelaide Power vs. St Kilda Saints at Adelaide Oval

Saturday Evening: Greater Western Sydney Giants vs. Sydney Swans at Giants

Saturday Evening: North Melbourne Kangaroos vs. Hawthorn Hawks at Marvel

Saturday Night: Collingwood Magpies vs. Brisbane Lions at MCG

Saturday Night: Geelong Cats vs. Richmond Tigers at GMHBA

Sunday Afternoon: Adelaide Crows vs. Fremantle Dockers at Adelaide Oval

Sunday Evening: Gold Coast Suns vs. West Coast Eagles at HBS

Round 3

Thursday Night: Melbourne Demons vs. Greater Western Sydney Giants at MCG

Friday Night: Collingwood Magpies vs. Essendon Bombers at MCG

Saturday Afternoon: Hawthorn Hawks vs. Adelaide Crows at MCG

Saturday Evening: Brisbane Lions vs. Sydney Swans at Gabba

Saturday Evening: Port Adelaide Power vs. Western Bulldogs at Adelaide Oval

Saturday Night: Fremantle Dockers vs. Geelong Cats at Optus

Saturday Night: North Melbourne Kangaroos vs. Richmond Tigers at Marvel

Sunday Afternoon: St Kilda Saints vs. West Coast Eagles at Marvel

Sunday Evening: Gold Coast Suns vs. Carlton Blues at HBS

Round 4

Thursday Night: Richmond Tigers vs. Sydney Swans at MCG

Friday Night: Carlton Blues vs. Port Adelaide Power at MCG

Saturday Afternoon: Greater Western Sydney Giants vs. Collingwood Magpies at Giants

Saturday Evening: North Melbourne Kangaroos vs. Melbourne Demons at Marvel

Saturday Evening: West Coast Eagles vs. Essendon Bombers at Optus

Saturday Night: Adelaide Crows vs. Western Bulldogs at Adelaide Oval

Saturday Night: Hawthorn Hawks vs. Fremantle Dockers at MCG

Sunday Afternoon: Brisbane Lions vs. Geelong Cats at Gabba

Sunday Evening: Gold Coast Suns vs. St Kilda Saints at HBS

Round 5

Thursday Night: Western Bulldogs vs. Hawthorn Hawks at Marvel

Friday Night: St Kilda Saints vs. Essendon Bombers at Marvel

Saturday Afternoon: Geelong Cats vs. North Melbourne Kangaroos at GMHBA

Saturday Evening: Adelaide Crows vs. Greater Western Sydney Giants at Adelaide Oval

Saturday Evening: Fremantle Dockers vs. Richmond Tigers at Optus

Saturday Night: Brisbane Lions vs. Carlton Blues at Gabba

Saturday Night: Melbourne Demons vs. Port Adelaide Power at MCG

Sunday Afternoon: Sydney Swans vs. West Coast Eagles at SCG

Sunday Evening: Gold Coast Suns vs. Collingwood Magpies at HBS

Round 6

Thursday Night: Essendon Bombers vs. Hawthorn Hawks at MCG

Friday Night: St Kilda Saints vs. Melbourne Demons at Marvel

Saturday Afternoon: West Coast Eagles vs. Adelaide Crows at Optus

Saturday Evening: Brisbane Lions vs. Greater Western Sydney Giants at Gabba

Saturday Evening: Carlton Blues vs. North Melbourne Kangaroos at Marvel

Saturday Night: Collingwood Magpies vs. Richmond Tigers at MCG

Saturday Night: Geelong Cats vs. Western Bulldogs at GMHBA

Sunday Afternoon: Port Adelaide Power vs. Fremantle Dockers at Adelaide Oval

Sunday Evening: Sydney Swans vs. Gold Coast Suns at SCG

Round 7

Thursday Night: Carlton Blues vs. Brisbane Lions at MCG

Friday Night: Collingwood Magpies vs. Western Bulldogs at MCG

Saturday Afternoon: Fremantle Dockers vs. Melbourne Demons at Optus

Saturday Evening: Gold Coast Suns vs. Richmond Tigers at HBS

Saturday Evening: Greater Western Sydney Giants vs. West Coast Eagles at Giants

Saturday Night: Essendon Bombers vs. St Kilda Saints at MCG

Saturday Night: Geelong Cats vs. Sydney Swans at GMHBA

Sunday Afternoon: North Melbourne Kangaroos vs. Adelaide Crows at Marvel

Sunday Evening: Hawthorn Hawks vs. Port Adelaide Power at MCG

Round 8

Thursday Night: Sydney Swans vs. Melbourne Demons at SCG

Friday Night: Geelong Cats vs. Essendon Bombers at GMHBA

Saturday Afternoon: Greater Western Sydney Giants vs. Western Bulldogs at Giants

Saturday Evening: Adelaide Crows vs. St Kilda Saints at Adelaide Oval

Saturday Evening: Collingwood Magpies vs. North Melbourne Kangaroos at Marvel

Saturday Night: Brisbane Lions vs. Port Adelaide Power at Gabba

Saturday Night: Richmond Tigers vs. Fremantle Dockers at MCG

Sunday Afternoon: Carlton Blues vs. Hawthorn Hawks at MCG

Sunday Evening: West Coast Eagles vs. Gold Coast Suns at Optus

Round 9

Thursday Night: St Kilda Saints vs. Fremantle Dockers at Marvel

Friday Night: Collingwood Magpies vs. Carlton Blues at MCG

Saturday Afternoon: Brisbane Lions vs. Richmond Tigers at Gabba

Saturday Evening: Port Adelaide Power vs. Essendon Bombers at Adelaide Oval

Saturday Evening: Western Bulldogs vs. Greater Western Sydney Giants at Marvel

Saturday Night: Melbourne Demons vs. Adelaide Crows at MCG

Saturday Night: West Coast Eagles vs. Geelong Cats at Optus

Sunday Afternoon: Hawthorn Hawks vs. Sydney Swans at MCG

Sunday Evening: Gold Coast Suns vs. North Melbourne Kangaroos at HBS

Round 10

Thursday Night: St Kilda Saints vs. Greater Western Sydney Giants at Marvel

Friday Night: Richmond Tigers vs. Essendon Bombers at MCG

Saturday Afternoon: North Melbourne Kangaroos vs. Gold Coast Suns at Marvel

Saturday Evening: Geelong Cats vs. Hawthorn Hawks at GMHBA

Saturday Evening: Sydney Swans vs. Collingwood Magpies at SCG

Saturday Night: Adelaide Crows vs. Port Adelaide Power at Adelaide Oval

Saturday Night: Melbourne Demons vs. Western Bulldogs at MCG

Sunday Afternoon: Fremantle Dockers vs. Carlton Blues at Optus

Sunday Evening: Brisbane Lions vs. West Coast Eagles at Gabba

Round 11

Thursday Night: Essendon Bombers vs. North Melbourne Kangaroos at Marvel

Friday Night: Collingwood Magpies vs. Port Adelaide Power at MCG

Saturday Afternoon: Geelong Cats vs. West Coast Eagles at GMHBA

Saturday Evening: Adelaide Crows vs. Carlton Blues at Adelaide Oval

Saturday Evening: Fremantle Dockers vs. Sydney Swans at Optus

Saturday Night: Melbourne Demons vs. Richmond Tigers at MCG

Saturday Night: Western Bulldogs vs. St Kilda Saints at Marvel

Sunday Afternoon: Hawthorn Hawks vs. Brisbane Lions at MCG

Sunday Evening: Greater Western Sydney Giants vs. Gold Coast Suns at Giants

Round 12

Thursday Night: Essendon Bombers vs. Fremantle Dockers at MCG

Friday Night: Carlton Blues vs. Western Bulldogs at MCG

Saturday Afternoon: Greater Western Sydney Giants vs. Adelaide Crows at Giants

Saturday Evening: Sydney Swans vs. Brisbane Lions at SCG

Saturday Evening: West Coast Eagles vs. Collingwood Magpies at Optus

Saturday Night: Richmond Tigers vs. North Melbourne Kangaroos at MCG

Saturday Night: St Kilda Saints vs. Geelong Cats at Marvel

Sunday Afternoon: Melbourne Demons vs. Hawthorn Hawks at MCG

Sunday Evening: Gold Coast Suns vs. Port Adelaide Power at HBS

Round 13

Thursday Night: Richmond Tigers vs. Western Bulldogs at MCG

Friday Night: Hawthorn Hawks vs. West Coast Eagles at MCG

Saturday Afternoon: North Melbourne Kangaroos vs. Sydney Swans at Marvel

Saturday Evening: Brisbane Lions vs. Essendon Bombers at Gabba

Saturday Evening: Geelong Cats vs. Adelaide Crows at GMHBA

Saturday Night: Collingwood Magpies vs. St Kilda Saints at MCG

Saturday Night: Port Adelaide Power vs. Carlton Blues at Adelaide Oval

Sunday Afternoon: Greater Western Sydney Giants vs. Fremantle Dockers at Giants

Sunday Evening: Gold Coast Suns vs. Melbourne Demons at HBS

Round 14

Thursday Night: Sydney Swans vs. St Kilda Saints at SCG

Friday Night: Melbourne Demons vs. Brisbane Lions at MCG

Saturday Afternoon: Gold Coast Suns vs. Essendon Bombers at HBS

Saturday Evening: Fremantle Dockers vs. Collingwood Magpies at Optus

Saturday Evening: Western Bulldogs vs. North Melbourne Kangaroos at Marvel

Saturday Night: Port Adelaide Power vs. Geelong Cats at Adelaide Oval

Saturday Night: Richmond Tigers vs. Carlton Blues at MCG

Sunday Afternoon: Hawthorn Hawks vs. Greater Western Sydney Giants at MCG

Sunday Evening: Adelaide Crows vs. West Coast Eagles at Adelaide Oval

Round 15

Thursday Night: Western Bulldogs vs. Sydney Swans at Marvel

Friday Night: Essendon Bombers vs. Richmond Tigers at MCG

Saturday Afternoon: Geelong Cats vs. Gold Coast Suns at GMHBA

Saturday Evening: Adelaide Crows vs. Collingwood Magpies at Adelaide Oval

Saturday Evening: Fremantle Dockers vs. Hawthorn Hawks at Optus

Saturday Night: Carlton Blues vs. Greater Western Sydney Giants at MCG

Saturday Night: St Kilda Saints vs. Brisbane Lions at Marvel

Sunday Afternoon: West Coast Eagles vs. Melbourne Demons at Optus

Sunday Evening: North Melbourne Kangaroos vs. Port Adelaide Power at Marvel

Round 16

Thursday Night: Richmond Tigers vs. Gold Coast Suns at MCG

Friday Night: Carlton Blues vs. Collingwood Magpies at MCG

Saturday Afternoon: West Coast Eagles vs. Port Adelaide Power at Optus

Saturday Evening: St Kilda Saints vs. North Melbourne Kangaroos at Marvel

Saturday Evening: Sydney Swans vs. Adelaide Crows at SCG

Saturday Night: Brisbane Lions vs. Melbourne Demons at Gabba

Saturday Night: Hawthorn Hawks vs. Geelong Cats at MCG

Sunday Afternoon: Western Bulldogs vs. Fremantle Dockers at Marvel

Sunday Evening: Greater Western Sydney Giants vs. Essendon Bombers at Giants

Round 17

Thursday Night: Hawthorn Hawks vs. St Kilda Saints at MCG

Friday Night: Melbourne Demons vs. Carlton Blues at MCG

Saturday Afternoon: Gold Coast Suns vs. Fremantle Dockers at HBS

Saturday Evening: West Coast Eagles vs. Richmond Tigers at Optus

Saturday Evening: Western Bulldogs vs. Brisbane Lions at Marvel

Saturday Night: Collingwood Magpies vs. Geelong Cats at MCG

Saturday Night: Port Adelaide Power vs. Adelaide Crows at Adelaide Oval

Sunday Afternoon: Essendon Bombers vs. Sydney Swans at MCG

Sunday Evening: North Melbourne Kangaroos vs. Greater Western Sydney Giants at Marvel

Round 18

Thursday Night: Geelong Cats vs. Melbourne Demons at GMHBA

Friday Night: St Kilda Saints vs. Collingwood Magpies at Marvel

Saturday Afternoon: Brisbane Lions vs. Gold Coast Suns at Gabba

Saturday Evening: Sydney Swans vs. Carlton Blues at SCG

Saturday Evening: West Coast Eagles vs. Hawthorn Hawks at Optus

Saturday Night: Essendon Bombers vs. Greater Western Sydney Giants at MCG

Saturday Night: Western Bulldogs vs. Adelaide Crows at Marvel

Sunday Afternoon: Fremantle Dockers vs. North Melbourne Kangaroos at Optus

Sunday Evening: Port Adelaide Power vs. Richmond Tigers at Adelaide Oval

Round 19

Thursday Night: Collingwood Magpies vs. Hawthorn Hawks at MCG

Friday Night: Essendon Bombers vs. Melbourne Demons at MCG

Saturday Afternoon: West Coast Eagles vs. Carlton Blues at Optus

Saturday Evening: Geelong Cats vs. Fremantle Dockers at GMHBA

Saturday Evening: Western Bulldogs vs. Gold Coast Suns at Marvel

Saturday Night: Adelaide Crows vs. Sydney Swans at Adelaide Oval

Saturday Night: Richmond Tigers vs. St Kilda Saints at MCG

Sunday Afternoon: Greater Western Sydney Giants vs. Port Adelaide Power at Giants

Sunday Evening: North Melbourne Kangaroos vs. Brisbane Lions at Marvel

Round 20

Thursday Night: Essendon Bombers vs. Adelaide Crows at MCG

Friday Night: Melbourne Demons vs. St Kilda Saints at MCG

Saturday Afternoon: North Melbourne Kangaroos vs. Fremantle Dockers at Marvel

Saturday Evening: Sydney Swans vs. Port Adelaide Power at SCG

Saturday Night: Brisbane Lions vs. Collingwood Magpies at Gabba

Saturday Night: Carlton Blues vs. Geelong Cats at MCG

Sunday Afternoon: Hawthorn Hawks vs. Gold Coast Suns at MCG

Sunday Afternoon: Western Bulldogs vs. West Coast Eagles at Marvel

Sunday Evening: Greater Western Sydney Giants vs. Richmond Tigers at Giants

Round 21

Thursday Night: Melbourne Demons vs. Sydney Swans at MCG

Friday Night: Carlton Blues vs. Essendon Bombers at MCG

Saturday Afternoon: Gold Coast Suns vs. Hawthorn Hawks at HBS

Saturday Evening: Richmond Tigers vs. Adelaide Crows at MCG

Saturday Evening: West Coast Eagles vs. North Melbourne Kangaroos at Optus

Saturday Night: Port Adelaide Power vs. Collingwood Magpies at Adelaide Oval

Saturday Night: St Kilda Saints vs. Western Bulldogs at Marvel

Sunday Afternoon: Fremantle Dockers vs. Brisbane Lions at Optus

Sunday Evening: Greater Western Sydney Giants vs. Geelong Cats at Giants

Round 22

Thursday Night: Richmond Tigers vs. Hawthorn Hawks at MCG

Friday Night: Melbourne Demons vs. Collingwood Magpies at MCG

Saturday Afternoon: Port Adelaide Power vs. Greater Western Sydney Giants at Adelaide Oval

Saturday Evening: Fremantle Dockers vs. Gold Coast Suns at Optus

Saturday Evening: Sydney Swans vs. Western Bulldogs at SCG

Saturday Night: Essendon Bombers vs. Geelong Cats at MCG

Saturday Night: St Kilda Saints vs. Carlton Blues at Marvel

Sunday Afternoon: North Melbourne Kangaroos vs. West Coast Eagles at Marvel

Sunday Evening: Adelaide Crows vs. Brisbane Lions at Adelaide Oval

The runtime for our program is 3118 seconds, and the best objective function value is -73840, with a gap of 0.8217%. When we calculate the objective using the equation for our problem's objective, our objective value is -3076989.

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5   Discussion

6 Conclusion

Bibliography

7 Appendix

Figure 1: Part 1 of MILP code.

Figure 2: Part 2 of MILP code.