ORCA 4500: Foundation of Data Science Fall 2019

Assignment 4

Exercise 1: Suppose you flip n fair coins. let X be the number of heads

1. what is the probability of getting exactly i heads, for each i, ie. $P(X=i)$?

Solution: $P(X=i) = C_n^i (\frac{1}{2})^i (\frac{1}{2})^{n-i}$

$= \frac{n!}{i!(n-i)!2^n}$

2. What is the probability of getting at least i heads for each i, ie $P(X \geq i)$?

Solution: $P(X \geq i) = C_n^i (\frac{1}{2})^i (\frac{1}{2})^{n-i} + C_n^{i+1} (\frac{1}{2})^{i+1} (\frac{1}{2})^{n-i-1} + \cdots + C_n^n (\frac{1}{2})^n$

$= \sum_{k=i}^{n} C_n^k (\frac{1}{2})^k (\frac{1}{2})^{n-k}$

$= \sum_{k=i}^{n} C_n^k (\frac{1}{2})^n$

Exercise 2: Assume you have three fair five-face dice (ie: faces are 1, 2, 3, 4 and 5) what is the probability of an odd sum when you roll these three dice.

Solution: It can be two conditions:

① three odd numbers

② two even numbers add one odd number

$P(\text{odd sum}) = (\frac{3}{5})^3 + C_3^2 (\frac{2}{5})^2 \times (\frac{3}{5})$

$= \frac{27}{125} + 3 \times \frac{12}{125} = \frac{27+36}{125} = \frac{63}{125} = 50.4\%$

Exercise 3: Consider the experiment where you randomly draw 4 cards from a standard 52-card deck of poker cards. (note the desk is without the jokers)

1. what is the probabiliey that you have 2 or more kings in on run of the experiment.

Solution: $P(X \geq 2) = 1 - \frac{C_{48}^4}{C_{52}^4} - \frac{C_{48}^3 C_4^1}{C_{52}^4}$

$$= 1 - \frac{\frac{48\times47\times46\times45}{4\times3\times2} + \frac{48\times47\times46}{3\times2}\times4}{\frac{52\times51\times50\times49}{4\times3\times2}}$$

$$= 1 - \frac{48\times47\times46\times45 + 4\times4\times48\times47\times46}{52\times51\times50\times49}$$

$$= 1 - \frac{6330336}{6497400} = \frac{167064}{6497400} = 2.57144\%$$

2. If you run the experiment 9 times. What is the probability that in at least one of the experiment you have 2 or more kings.

Solution:

$$P = 1 - \left[1 - P(X\geq 2)\right]^9$$
$$= 1 - (1 - 2.57144\%)^9 = 0.208856 = 20.8856\%$$

Exercise 4: Which event is more likely:

(a) drawing an ace and a king, when you draw 2 cards from a 52-card deck.

(b) drawing an ace and a king, when you draw 2 cards from a 13-card deck consisting of only hearts.

please explain by calculating the probability of both events.

Solution: $P(a) = \frac{C_4^1 C_4^1}{C_{52}^2} = \frac{4\times4}{\frac{52\times51}{2}} = \frac{16\times2}{52\times51} = \frac{8}{663} = 1.20664\%$

$$P(b) = \frac{C_1^1 \times C_{13}^1}{C_{13}^2} = \frac{1\times1}{\frac{13\times12}{2}} = \frac{2}{13\times12} = \frac{1}{78} = 1.28205\%$$

$\because P(b) > P(a) \therefore$ event (b) is more likely.

Exercise 5:

1. Solution: frequency table:

| $t$ (minutes) | $0<t\leq 5$ | $5<t\leq 10$ | $10<t\leq 14$ | $14<t\leq 18$ | $18<t\leq 25$ | $25<t\leq 40$ |
|---|---|---|---|---|---|---|
| frequency | 0 | 10 | 16 | 24 | 35 | 15 |

2. Solution: $\text{Mean time} = \dfrac{2.5\times 0 + 7.5\times 10 + 12\times 16 + 16\times 24 + 21.5\times 35 + 32.5\times 15}{10+16+24+35+15}$

$= 18.91$

3. Solution: $P = 10\% + 16\% + 24\% + 35\% \times \dfrac{20-18}{25-18}$

$= 10\% + 16\% + 24\% + 10\% = 60\%$