Contents

1 PID controller

PID controller computes the control value $u(t)$ as a weighted sum of proportional, integral, and derivative error terms

$$u(t) = K_{\mathrm{p}} e(t) + K_{\mathrm{i}} \int_0^t e(\tau)\, d\tau + K_{\mathrm{d}} \frac{de(t)}{dt},$$

where K_{p}, K_{i}, and K_{d}, all non-negative, denote the coefficients for the proportional, integral, and derivative terms.

2 Initial parameters tuning

The initial parameter estimation was made with a heuristic Ziegler–Nichols method

Control Type	K_p	K_i	K_d
P	$0.50K_u$	-	-
PI	$0.45K_u$	$0.54K_u/T_u$	-
PID	$0.60K_u$	$1.2K_u/T_u$	$3K_uT_u/40$

The ultimate gain K_u is obtained via bisecting K_p parameter finding an approximate value between stable and unstable oscillation

K_u	result
0	straight
1.	unstable oscillations
0.5	stable oscillations
0.75	unstable oscillations
0.625	unstable oscillations
0.5625	unstable oscillations
0.52	K_u

The final results are $K_u = -0.052$ and the oscillation period $T_u = 18$ seconds.

Control Type	K_p	K_i	K_d
PID	0.312	3.466×10^{-2}	0.702

After some manual parameters adjustments I chose the following values $K_p = 0.2$, $K_i = 0.004$, and $K_d = 2.5$.

3 Parameters twiddling

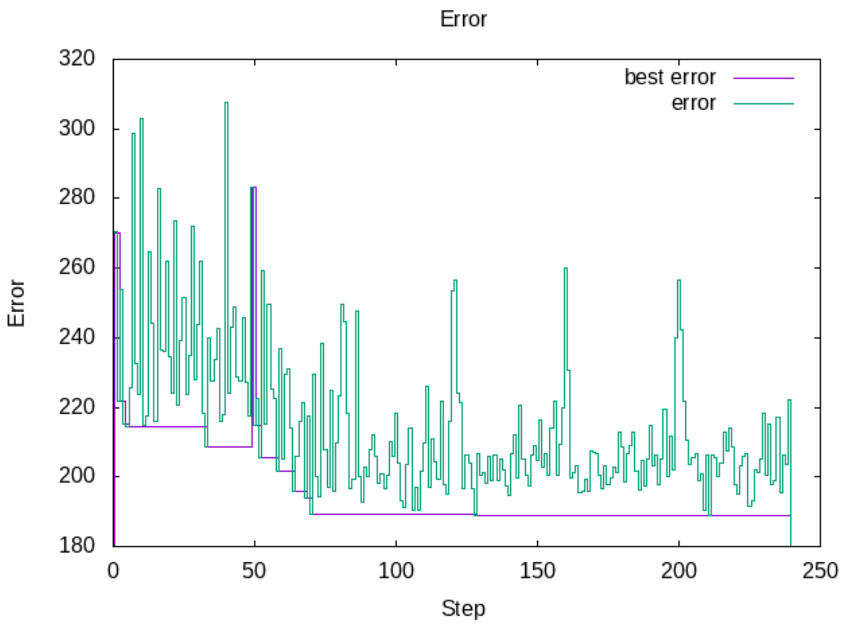
These values I used as a starting point for the parameters twiddling procedure explained in the lecture. The twiddling procedure is implemented in the PID::Twiddle() method and represents a variant of the bisection method for finding minimum of the error function that I defined as

$$E = \int_0^T e_{\mathrm{cte}}(t)^2 dt \approx \sum_{i=1}^{1400} {e_{\mathrm{cte}}}_i^2$$

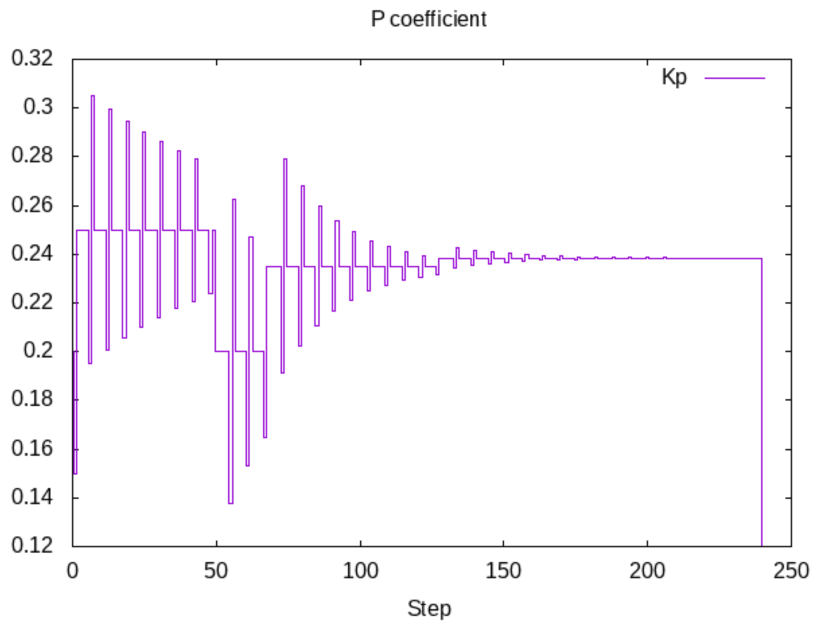
The twiddling method iterates over K parameters and modifies one parameter per step as $K \pm dK$ while keeping other parameters constant. If the error per period for the modified parameter is better than the current best one then the parameter value is saved and dK is increased by the factor 1.25. If both errors for $K \pm dK$ are worse then the current best error then dK is decreased by the factor 1.25.

After finding the local minimum with $E = 208.765$ I have restarted twiddling with initial parameters $K_p = 0.25$, $K_i = 5e - 3$, and $K_d = 2$. The second local minimum $K_p = 0.238358$, $K_i = 0.00788281$, and $K_d = 2$ with $E = 188.776$.

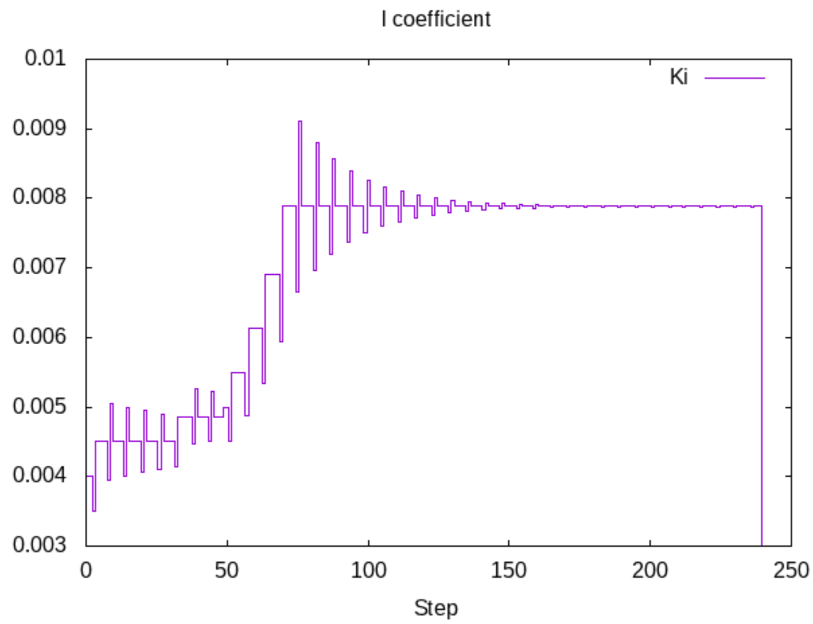
```
reset

set title "Error"
set xlabel "Step"
set ylabel "Error"

plot 'out.dat' using 0:1 with histeps title 'best error', 'out.dat' using 0:2 with hist
```



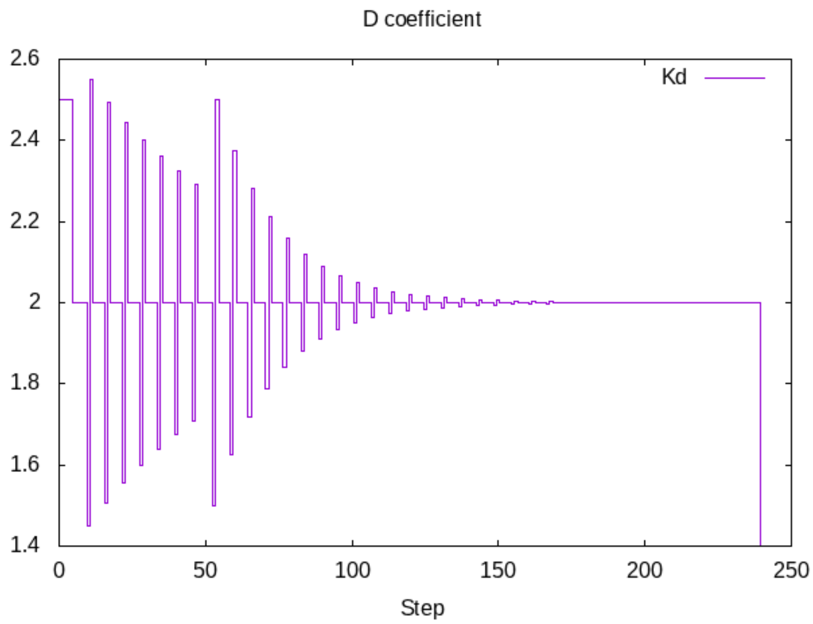

```
reset
set title "P coefficient"
set xlabel "Step"
plot 'out.dat' using 0:3 with histeps title 'Kp'
```




```
reset
set title "I coefficient"
set xlabel "Step"
plot 'out.dat' using 0:4 with histeps title 'Ki'
```




```
reset
set title "D coefficient"
set xlabel "Step"
plot 'out.dat' using 0:5 with histeps title 'Kd'
```



This procedure is very simple to implement but very sensitive to local minima,