T-2

d) $\frac{1}{n}\sum_{i=1}^{n} X_i \sim \underbrace{p(x)}_{?}$

По ЦПТ Ляпунова:

$$\frac{\frac{1}{n}\sum_{i=1}^{n} \xi_i - M[\xi]}{\sqrt{D[\xi]}}\sqrt{n} \rightsquigarrow N(\underset{\tilde{x}}{0}, \underset{\tilde{\sigma}^2}{1})$$

$$\frac{\tilde{\alpha} - \alpha}{\sqrt{\tilde{D}}}\sqrt{n} \rightsquigarrow N(0, 1)$$

$D = 1$

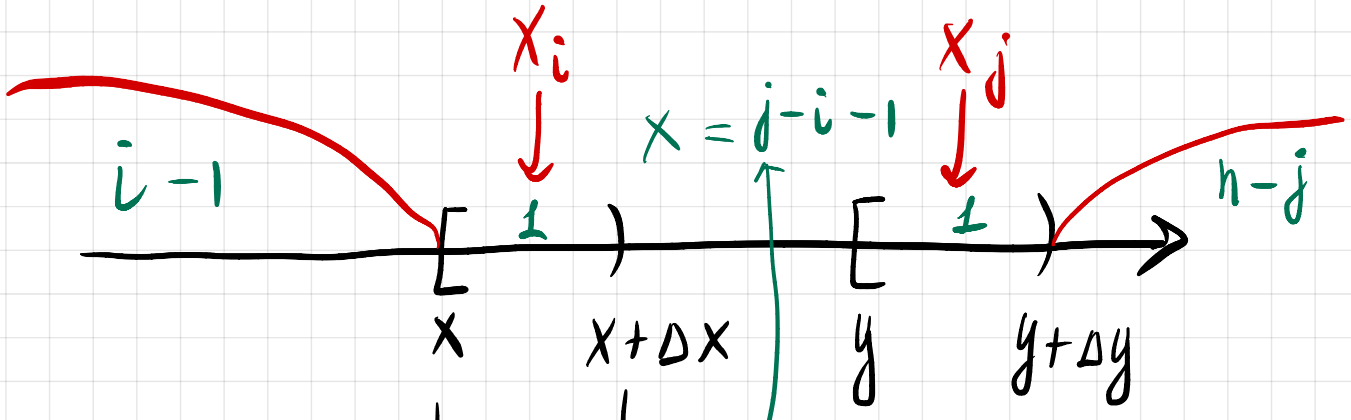
$$\left.\begin{aligned} D &= 1 \\ D' &= \frac{n}{\tilde{D}} D \end{aligned}\right\} \Rightarrow D = \frac{\tilde{D}}{n} D$$

$$\tilde{\alpha} - \alpha \rightsquigarrow N\left(0, \frac{\tilde{D}}{n}\right)$$

$$\alpha - \tilde{\alpha} \rightsquigarrow N\left(0, \frac{\tilde{D}}{n}\right)$$

$$\boxed{\alpha \rightsquigarrow N\left(\tilde{\alpha}, \frac{\tilde{D}}{n}\right)}$$

f) плотн совм. распр. i, j – статистики:



$$\cancel{n} = i - \cancel{1} + \cancel{1} + x + 1 + \cancel{n} - j$$

$$x = j - i - 1$$

(1) (2) (3) (4) (5)

$$(2):\ C_n^1 \, P(\xi \in [x, x+\Delta x)) = n \cdot f(x)\Delta x$$

$$(4):\ C_{n-1}^1 \, P(\xi \in [y, y+\Delta x)) = (n-1)\cdot f(y)\Delta y$$

$$(1):\ C_{n-2}^{i-1} \, P^{i-1}(\xi < x) = C_{n-2}^{i-1} \, F(x)^{i-1}$$

$$(5):\ C_{n-i-1}^{n-j} \, P^{n-j}(\xi \geqslant y+\Delta y) = C_{n-i-1}^{n-j}(1-F(y))^{n-j}$$

$$(3):\ \underbrace{C_{j-i-1}^{j-i-1}}_{=1} \, P^{j-i-1}(x+\Delta x \leqslant \xi < y) =$$

$$= 1\cdot\left(F(y)-F(x)\right)^{j-i-1}$$

после предельного перехода получим:

$$\boxed{n\, f(x)\,(n-1)\, f(y)\; C_{n-2}^{i-1} F(x)^{i-1} C_{n-j-1}^{n-j}(1-F(y))^{n-j} \cdot (F(y)-F(x))^{j-i-1}}$$