

Introduction

It has been a continual surprise, that simple combinations of embeddings perform so well for a variety of tasks in natural language processing. At first glance, such simple methods capturing only unordered word use should have little capacity in representing the rich and highly structured human language. However at a second glance, similar surface information has been used in information retrieval with great success since the inception of the field (Maron 1961). Linear combinations of embeddings can be considered as a dimensionality reduction of a bag of words, with a particular weighting scheme. Dimensionality reduction can be characterised as finding the best low dimensional representation of a high dimensional input according to some quality criterion. In the case of word embeddings, that quality criterion is generally relating to the ability to predict the co-occurring words – a salient quality of lexical semantics. As such, linear combinations of embeddings take as input a very sparse high dimensional bag of words (which is itself a strong surface form representation), then reduce it to a dense representation that captures lexical semantics.

When we discuss linear combinations of word embeddings (LCOWE), we are considering various forms of weighted sums of vector word representations. These models are equivalent to representing bags of words (BOW), and are sometimes called *bags of vectors*, or *embedding-BOW* or similar. The primary focus of this work has been on sums of word embeddings (SOWE), i.e. a linear combination with unit weight. Closely related to this is a mean of word embeddings (MOWE), which is a SOWE weighted such that it normalizes over the size of the bag of words. More complicated weightings, such as using probabilities, or term significance are also options for constructing LCOWEs.

The mechanism behind the functioning of the addition of word embeddings capturing there combined meaning, was partially explained in one of the pioneering works on word embeddings Mikolov et al. (2013). As shown below, for w and u being words, C being an embedding matrix, and $P(\mathbb{V} \mid a)$ being the set of probabilities for each word in the vocabulary \mathbb{V} co-occurring with the word a .

$$C_{:,w} \propto \log P(\mathbb{V} \mid w) \tag{1}$$

$$C_{:,u} \propto \log P(\mathbb{V} \mid u) \tag{2}$$

$$\therefore C_{:,w} + C_{:,u} \propto \log P(\mathbb{V} \mid w) + \log P(\mathbb{V} \mid u) \tag{3}$$

$$= \log P(\mathbb{V} \mid w) \cdot P(\mathbb{V} \mid u) \tag{4}$$

$$\propto \log P(\mathbb{V} \mid w \cap u) \tag{5}$$

They note that under the skip-gram model, there is a linear relationship between a word embedding and the logarithm of the probability distribution over co-occurring words.¹ Thus there is a linear relationship between the sum of two (or more) embeddings, and the product of the probability distribution over co-occurring words. Which is roughly proportional to the probability distribution over words co-occurring with that two-word bigram (or n-gram).² Which is to say it is proportional to the distribution estimate that would have been found had that bigram (or n-gram) been replaced

¹The log in the relationship explains why summing embeddings works well, but taking their product does not. While the sum of two log-likelihoods is a log of the product of likelihoods, the product of two log likelihoods does not correspond to anything with intuitive meaning.

²This is only a rough relationship as it depends on the assumption of independence.

with a single token. By the distributional hypothesis, similarity of meaning is characterised by the distribution of words that may co-occur. This is how skip-gram-like word embeddings function, and this relationship explains why it generalised to sums of the words in short phrases. If one considers this for large structures than phrases, giving a larger bag-of-words, it can be considered that a sum of word embeddings, is proportional to the distribution over other worlds of the likelihood to co-occur with the entire bag of words. Which interestingly, is a distribution over the vocabulary, such that words that could have been present and included in the BOW have high likelihood.

Throughout the last three years that we have been research this problem, others have also found, often to their own surprise, the strength of simple linear combinations of embeddings.

Arora, Liang, and Ma (2017)’s work describes a “A simple but tough-to-beat baseline for sentence embeddings”, which is a linear combination of word embeddings. There proposed model is a more complicated combination than considered here. But nevertheless, is primarily a weighted sum of embeddings, with small adjustments based on linear dimensionality reduction methods. In particular when using the word embeddings of Wieting et al. (2016), they find this to be very competitive when compared with more complex models which take into account word order.

Cífka and Bojar (2018) found that taking a mean of word embeddings outperformed almost all their more sophisticated machine-translation-based sentence representations when used on classification and paraphrase detection tasks. This is not to say that linear combinations of embeddings are ideal models for all tasks. They clearly cannot truly handle all the complexities of language. But rather that the occurrence of the complexities they cannot handle is rarer in practice in many tasks than is often expected.

Conneau et al. (2018) constructed 10 probing tasks to isolate some of the information captured by sentence representations. They found the strong performance of the mean of word embeddings on sentence level tasks to be striking. They attribute it to the sentence level information being redundantly encoded in the word-forms: the surface level information is surprisingly useful for tasks which at first look very sophisticated. With the exception of their word-content task, they did find more sophisticated models able to perform better than the mean of word embeddings. However, when correlating the performance of their probing task against real world tasks, they found that the word-content probing task was by far the most positively correlated with the real word tasks. This makes it clear how valuable this surface information is in practical tasks.

In the work presented in this dissertation, we find that even in tasks where it would seem that non-surface information incorporating word-order is required, in practice other issues cause the more powerful models that are (theoretically) able to handle these situations correctly to be never-the-less outperformed. This is particularly the case where the theoretical improvement from incorporating this information is small, relative to the practical complexity of the techniques required to leverage it. Such a case where word order matters but the error from ignoring it is small, is particular illustrated in ??.

At a high-level the success of these techniques comes down to most human language being easy to understand and simple. This expectation of language being easily understood is highlighted by Grice (1975), which claims that the communication is conducted following a cooperative principle. The overall supermaxim for Grice’s cooperative principle is the speakers are expected to “be perspicuous” or more perspicuously, to use speech that is clearly expressed and easily understood. The particular relevant maxims within the principle are: the *maxim of quantity*, that speakers are expected to make contributions that are no more nor less informative than required; and the *maxim of manner*: that speakers are expected to avoid ambiguity and obscurity of expression, and to make contributions that are brief and orderly. While Grice originally proposed these are exceptions upon conversation, the general principle applies more broadly to natural language communication. This general principle being

that language used is normally expected to be understood easily – thus fulfilling the goal of communicating.

Adversarial examples are reasonably easy to construct. An adversarial example to a linear combination of word embeddings is any text where the word order significantly affects that meaning; and where multiple possible word orders exist. For such an adversary to be significant, both word orders must be reasonably likely to occur. However; such cases are rarer than one might expect as is demonstrated in ???. Particularly when punctuation tokens are included in the embeddings. As such, while these cases certainly exist, we find that for real applications they are sufficiently rare that the simplicity of the linear combinations of embeddings approach can work very well.

This when applied in sentence or phrase representation contexts, such as discussed in ??, and ?? which gives support to the notion that word order is often not a very significant feature in determining meaning. While it seems clear that word order, and other factors of linguistic structure must contribute significantly to the meaning of the phrase. However, our result suggest that it is often in a minor way, and that for many tasks these linear combinations are superior due to their simplicity and effectiveness. While taking into account greater linguistic structure may be the key to bridging the gap between “almost perfect” and “perfect”, the current state of the field for many tasks has not reached “almost perfect”, and as such simpler methods still form an important part. The successes of the sums of word embeddings discussed in ?? and ??, for sentence and phrase embeddings, leads us to consider other uses of linear combinations for representation. ?? and ?? consider tasks well outside of phrase representation where the order clearly does not matter: namely word-sense representation, and context of named entity usage across a document.

To further understand the relationship between SOWE and BOW, and the extent to which word order matters the capacity to reverse the conversion from phrase to SOWE is investigated in ?? and ??. The results in ?? show that it is indeed largely possible to reconstruct bags of words from SOWE, suggesting that when considered as a dimensionality reduction technique SOWE does not lose much information. This is extended in ?? to order those bags of words back to sentences via a simple tri-gram language model. This had some success at outright reconstructing the sentences. This highlights the idea that for many bags of words (which can be reconstructed from a sum of word embeddings) there may truly be only one reasonable sentence from which they might have come. This would explain why SOWE, and BOW, ignorance of word order does not prevent them from being useful representations of sentences.

One of the attractive features of these linear combinations is their simplicity. This is true both in an implementation sense, and in the sense of gradient descent. For example, the vanishing gradient problem in deep networks, especially RNNs (Bengio, Simard, and Frasconi 1994) and RvNNs (Socher 2014), simply does not exist for a sum of word embeddings. A sum of word embeddings is not a deep input structure – it is only one hidden layer. This is in contrast to recurrent neural networks (RNNs) which are deep in time: having effective depth $O(n)$ where n is the number of terms. Similarly, recursive neural networks (RvNNs) are deep in structure: having effective depth $O(\log n)$. Information does not have to propagate as far when a SOWE is used as an input representation. Thus it is easier to attribute changes during gradient descent. This is not to say that SOWE can only be used in a shallow network – it is simply an input representation subnetwork. Just like for RNNs and RvNNs, a deep network can be placed on top of the SOWE.

0.1 Thesis Outline and Contributions

This research tackles a number of natural language understanding problems, and in the solutions draws conclusions on the capacity of linear combinations of embeddings. The dissertation is organised into two parts.

?? contains a detailed discussion of the established methods for input representation in natural language understanding tasks. This literature review does not focus on

linear combinations of embeddings, which we develop upon through-out the rest of this dissertation. Rather it focuses upon the techniques we build upon, and the alternatives to our methods. ?? was originally published as the main content of our book *Neural Representations of Natural Language* (White et al. 2018a). It excludes the introductory chapters on machine learning and recurrent neural networks which was present in that book. ?? contains the investigations on how LCOWE perform in key NLP tasks. These investigations constitute the bulk of this research effort. Further to the literature review section of this dissertation, each chapter in ?? includes a background or related works section with particularly relevant works to that paper discussed, as is usual for a thesis by publication.

?? Word Representations

We begin by introducing word embeddings in ?. Word embeddings form the basis of the work in this dissertation, and more the basis of many of the advancements in the field more generally. The chapter begins with the consideration from a language modelling perspective, where word embeddings are equivalent to one-hot input representations in a neural network being employed for a language modelling task. Then expands towards the considerations of word embeddings as more general purpose representations. This chapter also includes detailed tutorials explaining the details of hierarchical softmax and negative sampling.

0.1.1 Word Sense Representations

Word sense representations are discussed in ?. These are of particular relevance to the work discussed in ?. More generally the considerations of words having multiple senses informs the discussion of meaning representation more broadly.

0.1.2 Sentence Representations and Beyond

? contains an overview of methods used for representing structures large than just words. In particular this section focuses on sentences, but also discusses techniques relevant to shorted phrases. This chapter contains some discussion of the sums of word embeddings that are the focus of this work, but contains primarily discussion of the alternatives which we contrast with.

0.1.3 Overview of Novel Contributions

An overview of tasks investigated in this work is shown in Table 1. The representation of *sentences* is investigated in ?, through a paraphrase grouping tasks. Similarly, the representation of *phrases* is investigated in ? through a color understanding (estimation) task. Given the observed properties found by sums of word embeddings, this leads to the investigation of if weighted sums of word sense embeddings might better represent a particular usage of a word in ?. The capacity also lends to the investigation of using a sum of word embeddings to represent the contexts of all usages of a named entity, for the point of view character detection task investigated in ?. We conclude with a complementary pair of works in ???? , which investigate the ability to recover bags of words and sentences, from sums of word embeddings representing sentences. These final works illustrate some of the reasons why the linear combinations work so well.

?: (White et al. 2015) “How Well Sentence Embeddings Capture Meaning”

We begin by examining methods for representing sentences. Sentences are a fundamental unit of communication – a sentence is a single complete idea. The core goal

Chapter	Structure	Task	Embeddings
??	Sentences	Paraphrase grouping	Word2Vec (Mikolov et al. 2013)
??	Short Phrases	Color understanding	FastText (Bojanowski et al. 2017)
??	Word Senses	Similarity with context & Word sense disambiguation	AdaGram (Bartunov et al. 2015) & Bespoke greedy sense embeddings
??	Adj. Contexts	POV character detection	FastText (Bojanowski et al. 2017)
??	Sentences	Recovering bags of words	GLoVE (Pennington, Socher, and Manning 2014)
??	Sentences	Recovering sentences	GLoVE (Pennington, Socher, and Manning 2014)

Table 1: Summary of the investigations published within this dissertation.

is to determine if different sentence embedding methods clearly separate the different ideas.

Paraphrases are defined by a bidirectional entailment relationship between two sentences. This is an equivalence relationship, it thus gives rise to a partitioning of all sentences in the space of natural language. If a sentence embedding is of high quality, it will be easy to define a corresponding partitioning of the embedding space. One way to determine how easy it is to define the corresponding partitioning is to attempt to do just that as a supervised classification task using a weak classifier. A weak classifier, (namely a linear support vector machine (SVM)) was used as a more powerful classifier (such as a deep neural network) could learn arbitrary transforms. The classification task is to take in a sentence embedding and predict which group of paraphrases it belongs to. The target paraphrase group is defined using other paraphrases with the same meaning as the candidate.

Under this course of evaluation it was found that the sum and mean of word embeddings performed very well as a sentence representation. These LCOWEs were the best performing models under evaluation. They were closely followed by the bag of words, which is advantaged by being much higher dimensionality than other models. The LCOWEs outperform the bag of words as they also capture synonyms and other features of lexical relatedness. Slightly worse than the bag of words was the bag of words with PCA dimensionality reduction to 300 dimensions. This confirms our expectation that LCOWEs are a better form of dimensionality reduction for preserving meaning from a bag of words than PCA.

The poor results of the paragraph vector models (Le and Mikolov 2014) is in line with the observation in the footnotes of the less well-known follow up work of Mesnil et al. (2014). Which found that the performance reported in Le and Mikolov (2014) cannot be reliably repeated on other tasks, or even the same tasks with a slightly different implementation.

A limitation of our investigation is that it does not include the examination of any encoder-decoder based methods, such as Skip-Thought (Kiros et al. 2015), or machine translation models. Another limitation of the work is that the unfolding recursive autoencoder (Socher et al. 2011) use a pretrained model with only 200 dimensions, rather than 300 dimensions as was used in the other evaluations.

The **key contribution** of this work was to evaluate the properties of sentence representations using an abstract task. This is in-contrast to most prior evaluations, which use less abstract real-world tasks. While real world task has its own important value, it is harder to judge the generalisation ability for example of a sentence representation that works well for sentiment analysis. This idea of using an abstract probing task to evaluate sentence representations has been significantly advanced and generalised

to a battery of such tasks in later works such as Adi et al. (2017) and Conneau et al. (2018). The interesting finding in our work, which significantly contributed to the direction of this dissertation, was that the LCOWEs (SOWE/MOWE) were notably the best performing on this task to separate meaning. Different word content, particularly with lexical similarity features, effectively gives a much stronger separability of the meaning space than any of the more complex methods considered.

Paraphrases provide one source of grounding for evaluation of sentences. Color names are a subset of short phrases which also have a ground truth for meaning – the color. They are thus useful for evaluating the performance of LCOWE on short phrases.

?: (White et al. 2018)

“Learning of Colors from Color Names: Distribution and Point Estimation”

To evaluate the performance of input representations for short phrases, we considered a color understanding task. Color understanding is considered a grounded microcosm of natural language understanding (Monroe, Goodman, and Potts 2016). It appears as a complicated sub-domain, with many of the same issues that plague natural language understanding in general: it features a lot of ambiguity, substantial morphological and syntax structure, and depends significantly on context that is not made available to the natural language understanding algorithms. Unlike natural language more generally, it has a comparatively small vocabulary, and it has grounded meaning. The meaning of a particular utterance, say **bluish green**, can be grounded to a point in color space, say in HSV (192°, 93%, 72%), based on the questioning the speaker. The general meaning of the a color phrase can be grounded to a distribution over color space, based on surveying the population of speakers.

Models were thus created to learn a mapping from the natural language space, to points or distributions in the color space. Three input representations were considered: a sum of word embeddings (SOWE), a convolutional neural network (CNN), and a recurrent neural network (RNN). The SOWE corresponds to a bag of words – no knowledge of order. The CNN corresponds to a bag of ngrams – it includes features of all length, thus can encode order. The RNN is a fully sequential model – all inputs are processed in order and it must remember previous inputs.

It was expected that this task would benefit significantly from a knowledge of word order. For example, **bluish green** and **greenish blue** are visibly different colors. The former being greener than the later. However, it was found that the SOWE was the best performing input representation, followed closely by the CNN, with the RNN performing much worse. This was even the case when the test set was restricted to only contain colors names for which multiple different word orders (representing different colors) were found in the training set. This can be attributed to the difficulty in training the more complicated models. In contrast to a simple feed-forward SOWE, in a RNN the gradient must propagate further from the output, and there are more weights to be learned in the gates. This difficulty dominated over the limitation in being able to model the color names correctly. We note that while **bluish green** and **greenish blue** are different colors, they are still very similar colors. As such, the error from treating them as the same, is less than the error caused by training difficulties.

The solving problem of color estimation from natural language color name, has pragmatic uses. Color estimation from description has utility as a tool for improving human-computer interaction. For example allowing free(-er) text for specifying colors in plotting software, using point estimation. It also has utility as an education tool: people from different cultures, especially non-native English speakers, may not know exactly what color range is described by **dark salmon**, and our model allows for tools to be created to answer such queries using distribution estimation.

A limitation of this study is the metrics used. For distribution estimation, the perplexity of the discretized distributions in color space is reported. It would be preferable to use Kullback–Leibler divergence, which would allow comparisons to future works

that output truly continuous distributions. Kullback–Leibler divergence is monotonically related to the discretized perplexity, however. For point estimation, using an evaluation metric such as a Delta-E, which is controlled for the varying sensitivity of human perception for different hues. Neither limitation has direct bearing on the assessment of the input representations.

The **key contribution** of this work was to evaluate the properties of short phrase representations using a grounded task of color understanding. Secondary contributions include creating the a neural network based method for color distribution estimation, which itself has practical use as a teaching tool; and demonstrating a novel method for point estimation of angular data, such as HSV colors. Again, we found surprisingly that SOWE was the most effective representation.

???: (White et al. 2018)

“Finding Word Sense Embeddings Of Known Meaning”

With the demonstrated utility of linear combinations of embeddings for representing the meanings of larger structures made from words, it is worth investigating their utility for representing the possible different meanings of words. When it comes to representing word senses, it may be desirable to find a representation for the exact sense of a word being used in a particular example. A very fine grained word sense for just that one use. If one has a collection of induced word senses, it seems reasonable to believe that the ideal word sense for a particular use, would lie somewhere between them in the embedding space. Further more, if one knows the probability of each of the coarse induced senses being the correct sense for this use, then it makes sense that the location of the fine grained sense embedding would be closer to the more likely coarse sense, and further from the less likely coarse sense. As such we propose a method to define these specific case word senses based on a probability weighted sum of coarser word sense embeddings. We say that we *refit* the original sense embeddings, using the single example sentence to induce the fine grained sense embedding.

Using this we define a similarity measure which we call RefittedSim, which we find to work better than AvgSimC (Reisinger and Mooney 2010). AvgSimC is a probability-weighted average of all the pairwise similarity scores for each sense embedding. In contrast RefittedSim is a single similarity score as measured between the two refitted vectors – which are the probability weighted averages of the coarser sense vectors. On the embeddings used in our evaluations this gave a solid improvement over AvgSimC. It is also asymptotically faster to evaluate.

We also evaluated using refitting for word sense disambiguation (WSD). Normally, induced senses cannot be used for word sense disambiguation, as they do not correspond to standard dictionary word senses. By using the WordNet gloss (definition) as an example sentence, we are able to use refitting to create a new set of sense embeddings suitable for WSD. Using this we can use the skip-gram formulation for probability of the context given the refitted sense, and so apply Baye’s theorem to find the most-likely sense. However, we found that the results were only marginally better than the baseline. Though it was notably better than the results of the method presented by Agirre et al. (2006); which, to the best of our knowledge, is the only prior method for leveraging induced senses for WSD with only a limited number of examples. Nearly unsupervised WSD is a very difficult problem; with a strong baseline of simply reporting the most-common sense. Our results do suggest that our refitting method does not learn features that are antithetical to WSD. However, they do incorporate the most frequent sense as a prior and seem to provide little benefit beyond that.

A limitation of this study was that it did not perform the evaluation on state-of-the-art word-sense embeddings. As such, while it’s comparisons between these embeddings are valid they cannot be readily compared to the current state-of-the-art on the tasks. It is thus not entirely clear that improvements when our method is applied to better performing models would be proportionate.

The **key contribution** of this work was to define a method for specializing word sense embeddings for a single use case. In doing an important property of embeddings from

skip-gram like formulations was demonstrated. We showed that a good representation can be found by linearly interpolating between less ideal representation according to how likely they are to be correct. Important secondary contributions include the method for smoothing the probability of correctness; and RefittedSim, a new similarity measure using this refitting to evaluated the similarity of word in context.

?: (White et al. 2018b)

“NovelPerspective: Identifying Point of View Characters”

Given the success of LCOWEs for representing meaningful linguistic structures (sentences and phrases), a natural follow up question is on their capacity to represent combinations of words that do not feature this natural kind of structure. These would be more arbitrary bags of words; that never-the-less may be useful features for a particular task. The task investigated in this work was about identifying point of view characters in a novel.

Given some literary text written in third person limited point of view, such as Robert Jordan’s popular “*Wheel of Time*” series of novels, it is useful to a reader (or person analysing the text), to identify which sections are from the perspective of which character. That is to say, we would like to classify the chapters of a book according to which character they are from the perspective of. This at first looks like a multiclass classification problem; however it is in-fact an information extraction problem. The set of possible classes for any given chapter is the set of all named entities in the book. Different books have different characters, thus the set of named entities in the training data will not match that of an arbitrary book selected by a user. As such, the named entity tokens themselves cannot be used in training for this task. Instead, it must be determined whether or not a named entity is the point of view character, based on how the named entity token is used. To do this, a representation of the context of use is needed.

The task can be treated as a binary classification problem. Given some feature vector representing how a particular named entity token was used throughout a chapter, find the probability of that named entity being the point of view character. We considered two possible feature sets to use to generate the feature vectors for named entity token use. Both feature sets consider the context primarily in terms of the token (word) immediately prior to, and the token (word) immediately after the named entity. We define a 200 dimensional hand-crafted *classical feature set* in terms of the counts of adjacent part of speech tags, position in the text, and token frequency. We define a *mean of word embedding based feature set* as the concatenation of the mean of the word embedding for the words occurring immediately prior, to the mean of the word occurring immediately after. As this was using 300 dimensional embeddings, this gives a 600 dimensional feature vector.

It was found that the two feature sets performed similarly, with both working very well. It seems like the primary difficulty was with the high dimensionality of the word embedding based feature set. Without sufficient training data, it over-fit quiet easily. It’s performance dropped sharply on the test set, compared to it’s oracle performance if trained on the testset, when the largest book series was removed. This likely could have been ameliorated by using lower dimensional embeddings.

The good performance of the word embedding based feature set is surprising here as it does not include any frequency information. We used a mean, rather than a sum, of word embeddings to represent the context of named entity token use. In the classical feature set, we found that by far the most important feature was how often that named entity token was used. Indeed just reporting the most frequently mentioned named entity gave a very strong baseline. The lexical information captured by the MOWE is clearly similarly useful to the part of speech tag counts, and almost certainly makes more fine grained information available to the classifier. Thus allowing it to define good decisions boundaries for if the feature vector represents a point of view character or not.

A limitation of this study is that different binary classifiers were used for the two

feature sets. Ideally, the performance using a range of classifiers for both would have been reported. Our preliminary results, not including in the study, suggested that the classifier choice was not significant. With logistic regression, SVM, and decision trees giving similarly high results for both feature sets.

The **key contribution** of this work was produce a system to identify the point of view characters from the context of the named entity tokens being used. In doing so it was demonstrated that a mean of word embeddings can perform similarly well to a hand engineered feature set. The system produced was deployed, and is openly available for public use at <https://white.ucc.asn.au/tools/np>.

?: (White et al. 2016a)

“Generating Bags of Words from the Sums of their Word Embeddings”

Given the consideration of a sum of word embeddings as dimensionality reduced form of a bag of words a important question is to how recoverable the bag of words is from the sum. A practical way to lower bound the loss of information is to demonstrate a deterministic method that can recover a portion of the bag of words.

We propose as method to extract the original bag of words from a sum of word embeddings. Thus placing a bound on the information lost in the transformation of BOW to SOWE. This is done via a simple greedy algorithm with a correction step. The core of this method functions by iteratively searching the vocabulary of word embeddings for the nearest embedding to the sum, adding it's word to the bag of words and subtracting its embedding from the sum. It is thus only computationally viable with reasonably small vocabularies. This method works as each component word in the sum has a unique directional contribution in the high dimensional space. As one would expect, this works better the higher dimensional the embeddings are, and with fewer words. Even with relatively low dimensions it works quiet well. This shows that embeddings are not for example constantly cancelling each other in the sum.

An interesting alternative to this deterministic method would be to train a supervised model to project from SOWE to a fuzzy bag of words. This is similar to the word-content task considered by Adi et al. (2017). In that task a binary classifier was trained to take a sentence representation and a word embedding for a single word that may or may not appear in the sentence.

The **key contribution** of this work was produce a system which attempts to convert from a SOWE to the BOW which defined it. In doing so it was demonstrated that one can largely recover the bag of words from the sum of word embeddings, thus showing that word content information was effectively maintained.

?: (White et al. 2016b)

“Modelling Sentence Generation from Sum of Word Embedding Vectors as a Mixed Integer Programming Problem”

Given that it was demonstrated that the bag of word can be recovered, the obvious follow up question is if we can recover the the sentence. This ?? is a small supplement to ??.

The word ordering problem tackled is given a bag of words, and a trigram language model determine the most-likely order for words. This allows bags of words to be turned into the most likely sentences. We define a deterministic algorithm to solve this using linear mixed integer programming. Using this algorithm we can use the partially recovered bags of words from ?? and determine how frequently they can be correctly ordered to find the original sentence.

We find that surprisingly often they can. The majority of sentences of length up to 18 can be successfully recovered from a SOWE. With, as expected, the longer the sentence the more difficult the recovery. This suggests that the number of likely

Can I comment that "This method would not work as well on a mean of word embeddings", is there something interesting to say about that

possible orderings for the words in a sentence is much lower than it may at first seem. Particularly since this method is based on a simple trigram language does so well. There is no doubt that a more sophisticated language model perform significantly better.

The algorithm used in our method is a minor extension of that of Horvat and Byrne (2014). We take advantage of the slight differences between the word ordering problem and the generalised asymmetric travelling sales man problem. We can eliminate some branches that would not be possibly for a travelling salesman solver; by directly defining it as a mixed integer linear programming problem.

The **key contribution** of this work was produce a system which attempts order bags of words recovered from sums of word embeddings into the sentences from which they came. The capacity to do this places a lower bound on how well sentences can be represented. If a correctly sentence can be fully recovered from a sum of word embeddings using just a language model; then a SOWE is effectively sending sentences to unique areas of the representation space. The use of the methods of ?? and ??, together with a system trained to output a approximation to a SOWE, is an interesting, though not really practical, method for natural language generation.

0.1.4 ?? Appendix Tooling

Beyond the main content of this dissertation, included is an appendix detailing software contributions. These tools created do not directly contribute towards the main content of this thesis. However, they were created as a result of of this research; and have facilitated several of the experiments involved. They are presented in the form of short software papers. The detail collaborations on improving the Julia (Bezanson et al. 2014) data-science ecosystem, in particular in the area of reproducibility and machine learning.