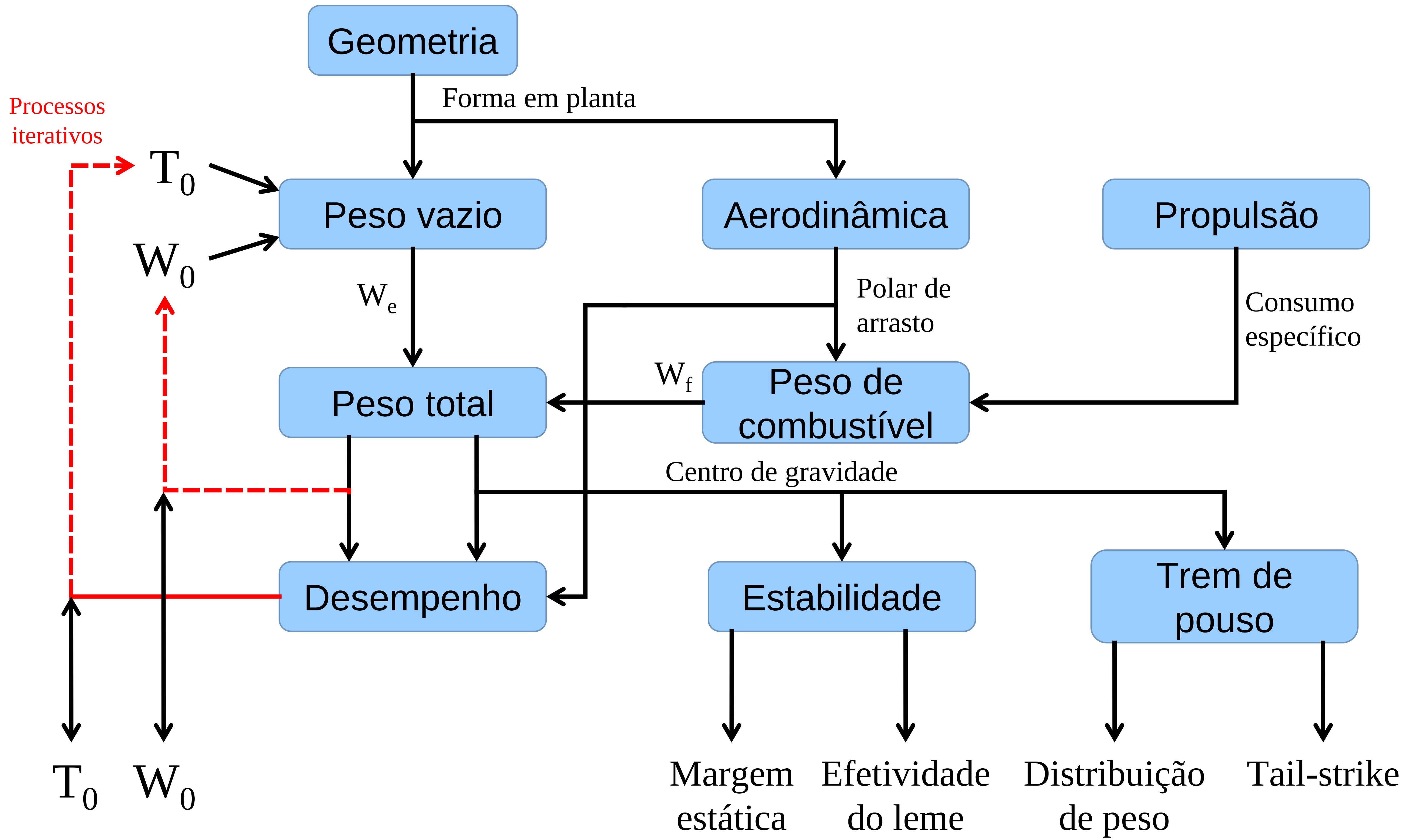
Instituto Tecnológico de Aeronáutica
Mestrado Profissional em Engenharia Aeronáutica

AP-701
Fundamentos do Projeto de Aeronaves

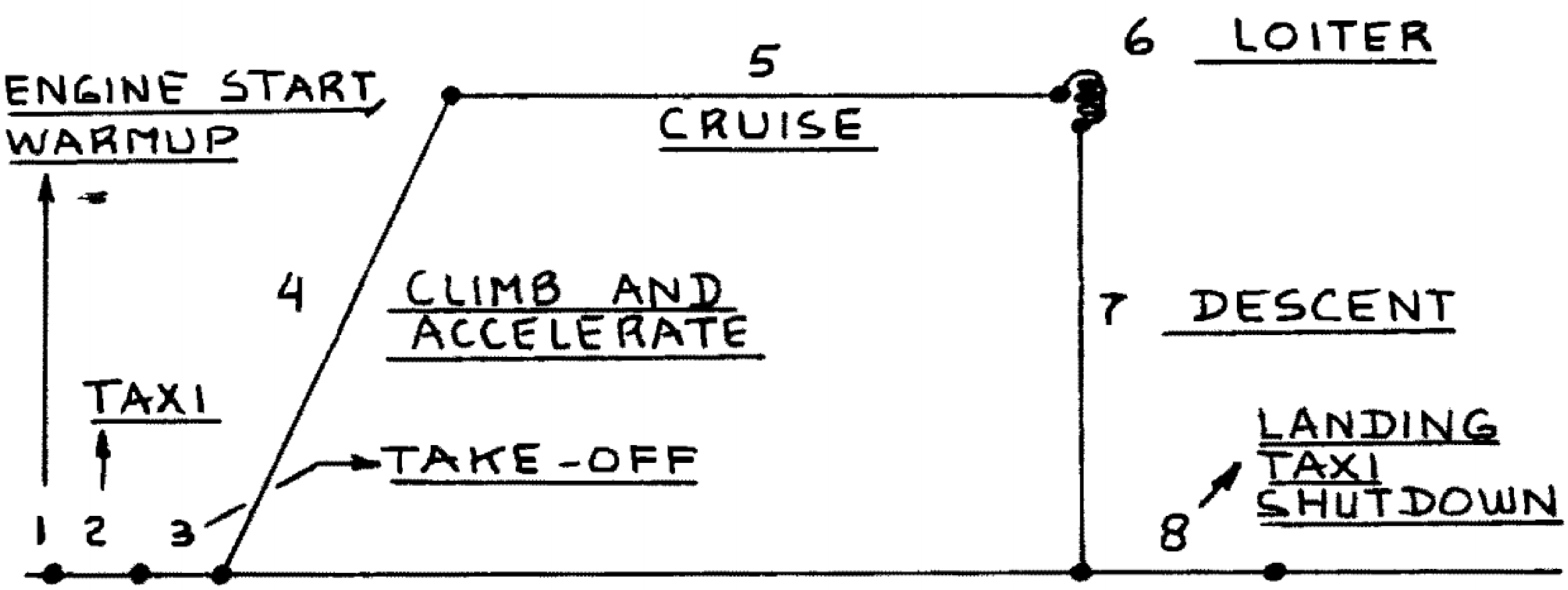
Aula 5 – Desempenho

Cap. Ney Sêcco

Fluxo de análise

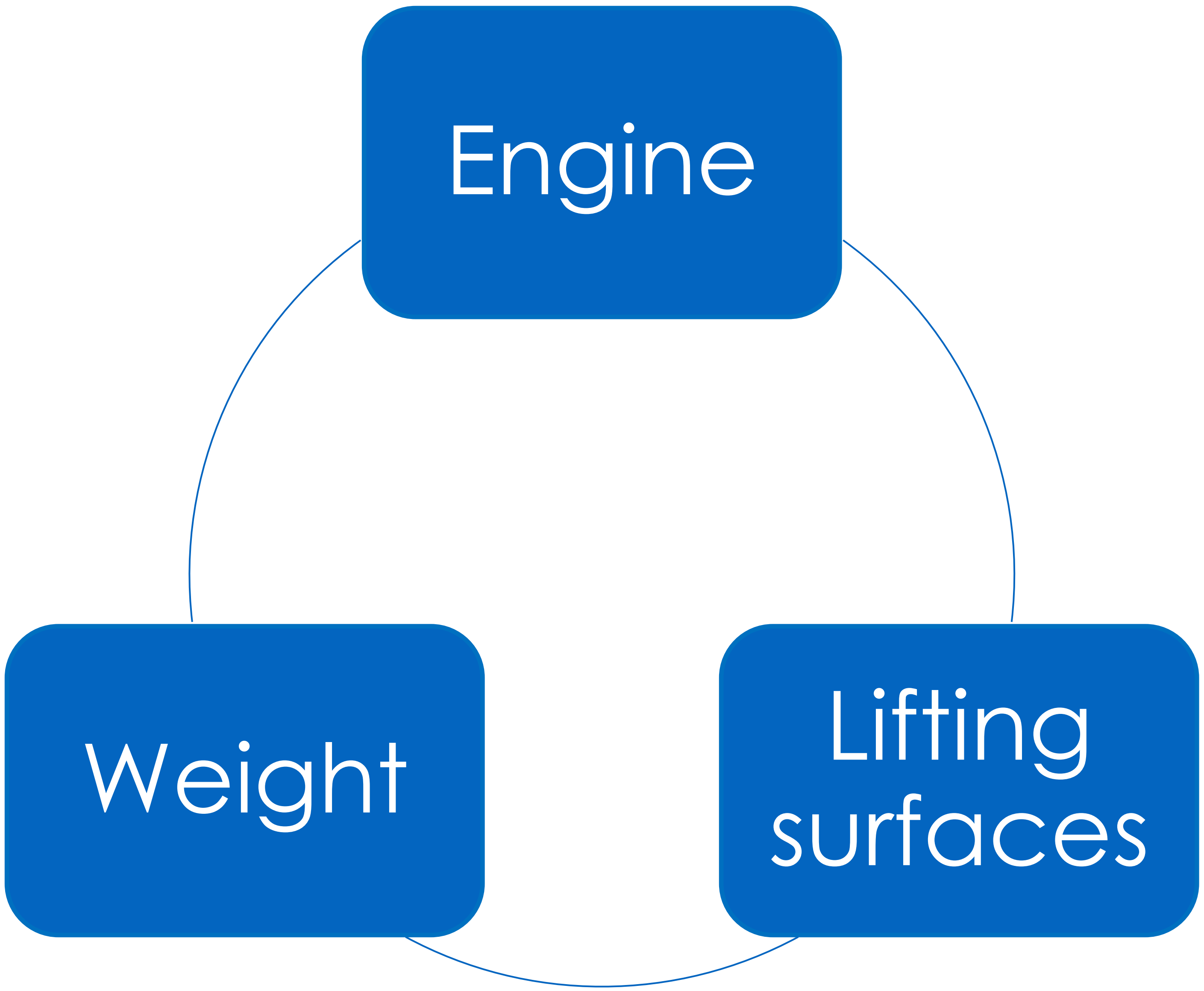


Precisamos averiguar se a aeronave consegue atender a todos os requisitos da missão

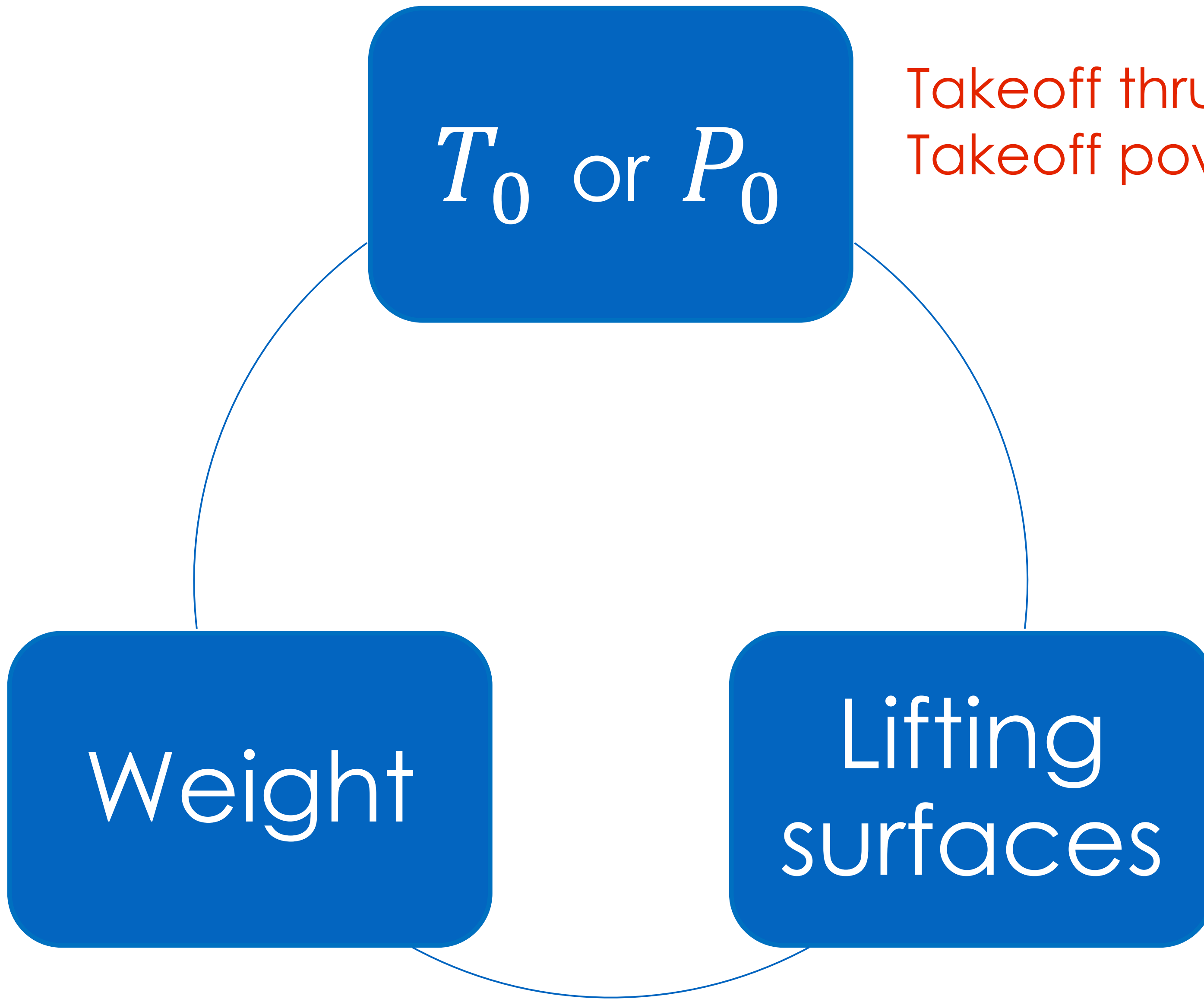


Roskam

- There should be a balance between:

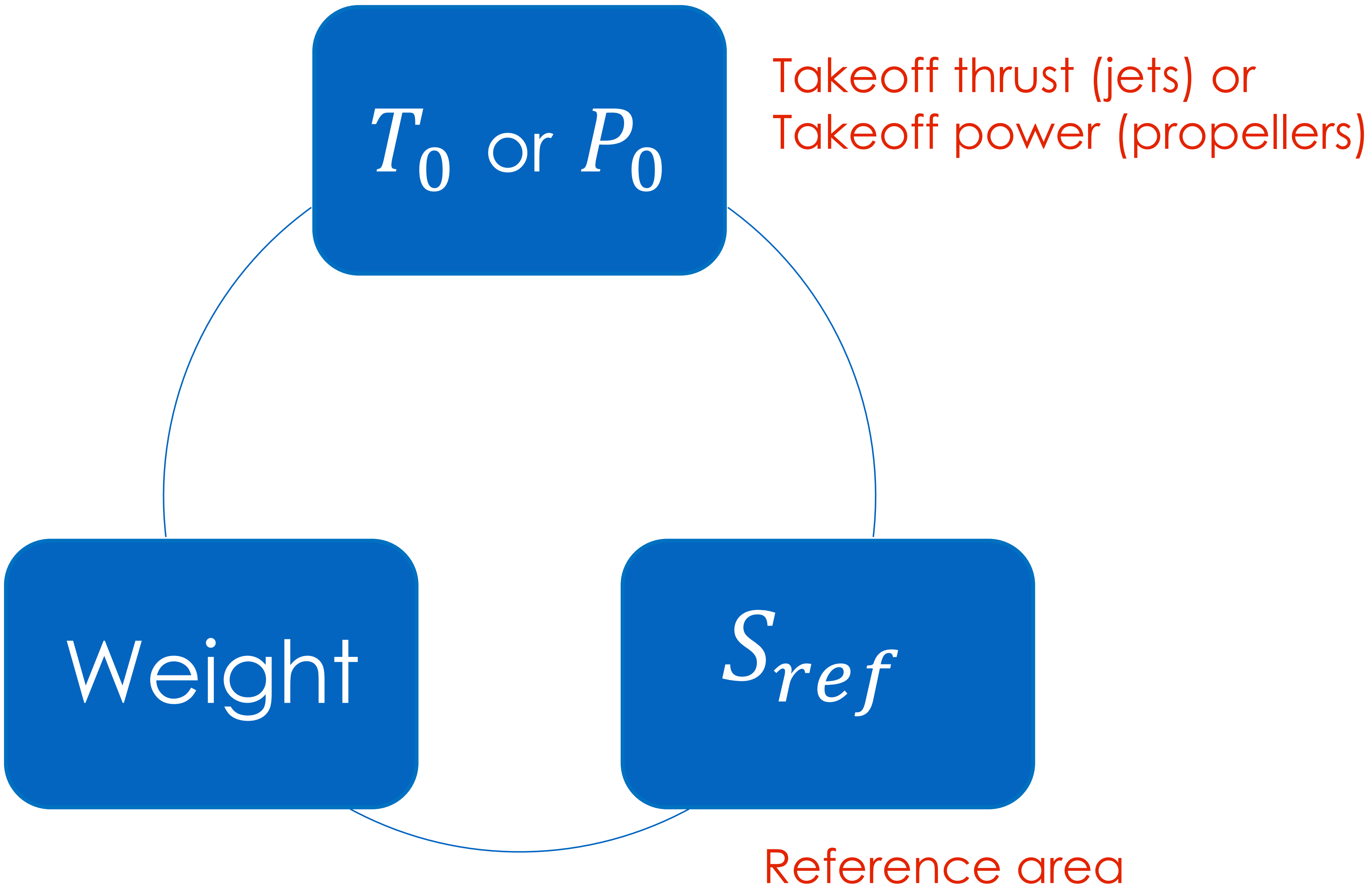


- There should be a balance between:

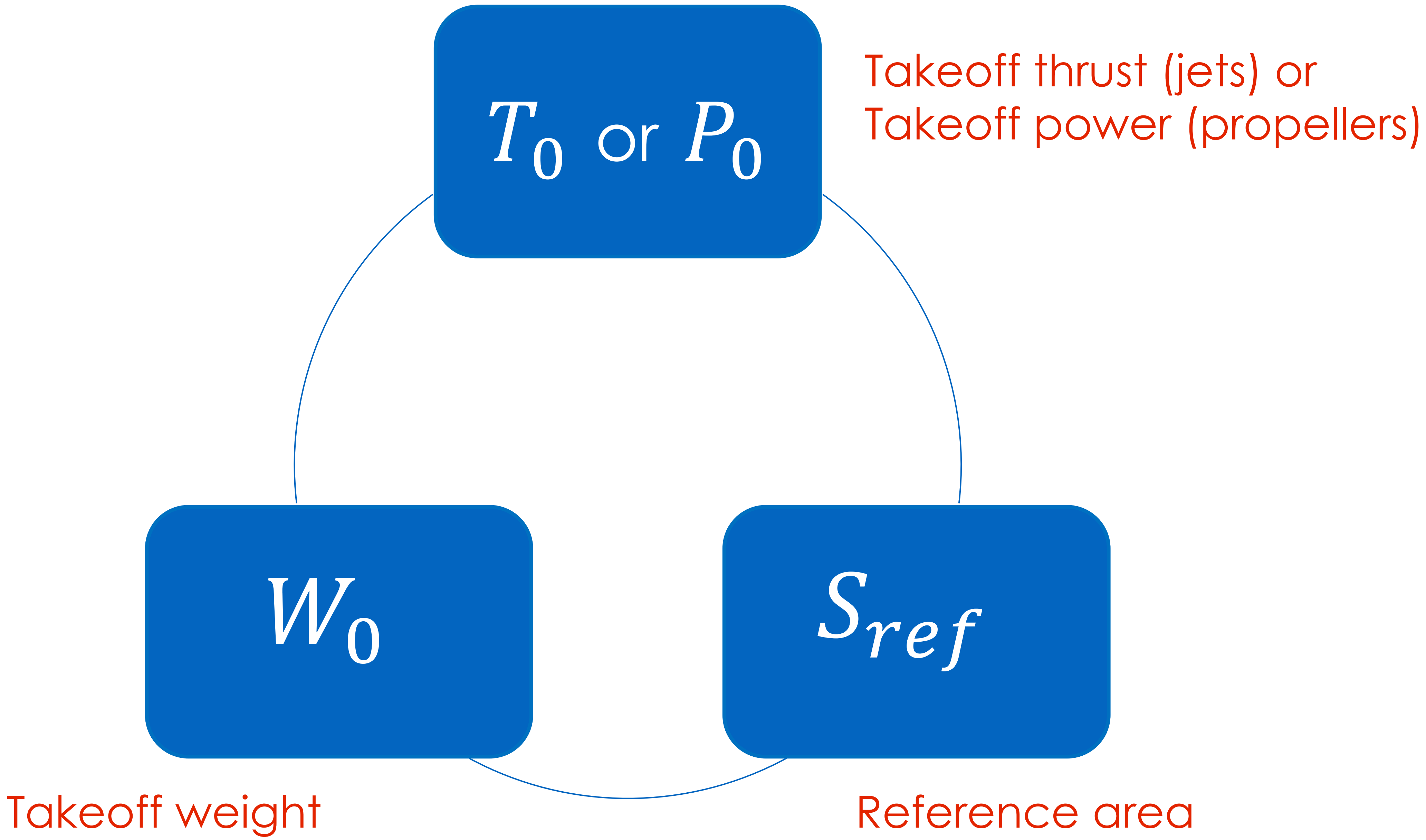


Takeoff thrust (jets) or
Takeoff power (propellers)

- There should be a balance between:

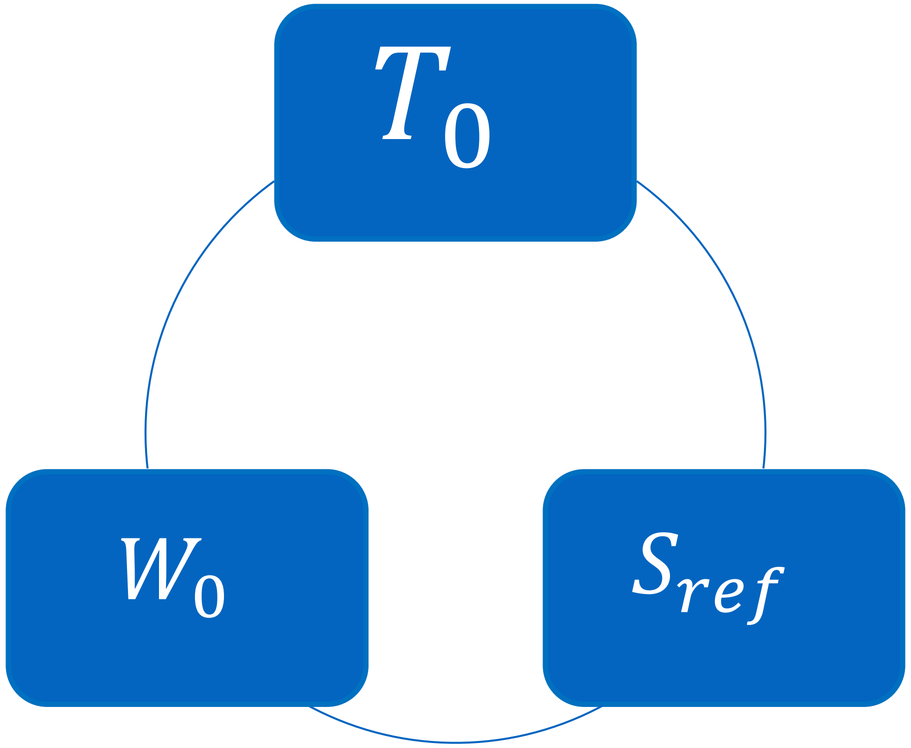


- There should be a balance between:



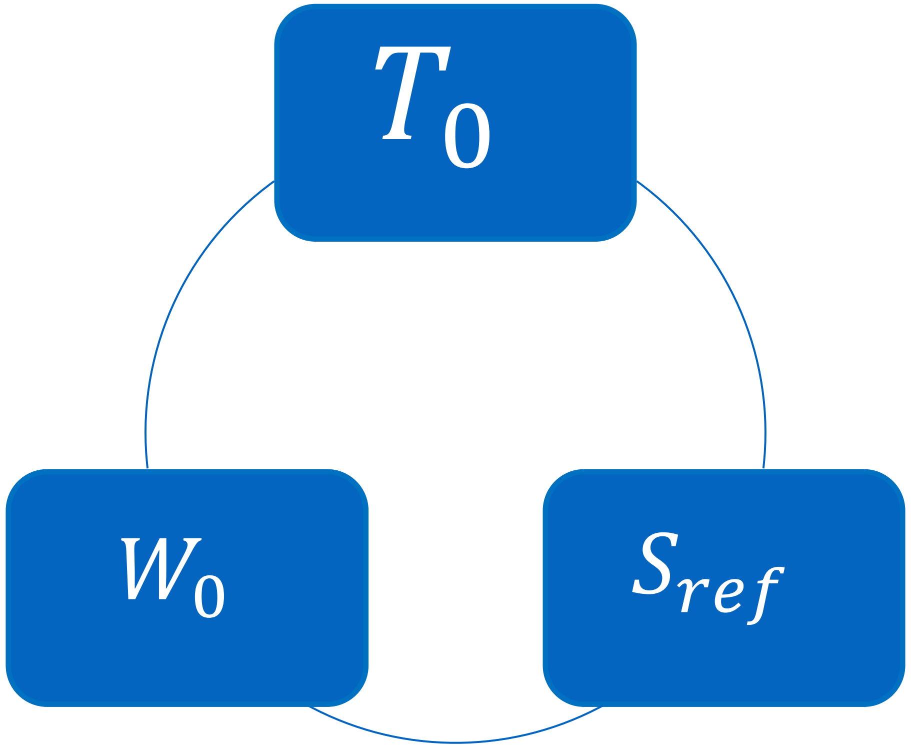
What is Sizing?

- Sizing means finding the right combination of W_0, T_0, and S_{ref} that satisfies:
  - Mission
  - Performance



What is Sizing?

- Sizing means finding the right combination of W_0, T_0, and S_{ref} that satisfies:
  - Mission → Client
  - Performance → Client / Regulations



Example: Cruise Speed

- During cruise: $L = W$

Example: Cruise Speed

- During cruise: $L = W$

$$\frac{\rho V_{cruise}^2}{2} S_{ref} C_L = W$$

$$V_{cruise} = \sqrt{\frac{2}{\rho C_L} \cdot \frac{W}{S_{ref}}}$$

Example: Cruise Speed

- During cruise: $L = W$

$$\frac{\rho V_{cruise}^2}{2} S_{ref} C_L = W$$

$$V_{cruise} = \sqrt{\frac{2}{\rho C_L} \cdot \boxed{\frac{W}{S_{ref}}}}$$

Wing loading
[kg/m²]
[lb/ft²]

Example: Cruise Speed

- During cruise: $L = W$

$$\frac{\rho V_{cruise}^2}{2} S_{ref} C_L = W$$

$$V_{cruise} = \sqrt{\frac{2}{\rho C_L} \cdot \boxed{\frac{W}{S_{ref}}}} \rightarrow \text{Wing loading}$$

Wing loading
[kg/m^2]
[lb/ft^2]

- For low speeds
- For heavy loads

Example: Cruise Speed

- During cruise: $L = W$

$$\frac{\rho V_{cruise}^2}{2} S_{ref} C_L = W$$

$$V_{cruise} = \sqrt{\frac{2}{\rho C_L} \cdot \boxed{\frac{W}{S_{ref}}}}$$

Wing loading
[kg/m^2]
[lb/ft^2]

- For low speeds → Low wing loading
- For heavy loads

Example: Cruise Speed

- During cruise: $L = W$

$$\frac{\rho V_{cruise}^2}{2} S_{ref} C_L = W$$

$$V_{cruise} = \sqrt{\frac{2}{\rho C_L} \cdot \boxed{\frac{W}{S_{ref}}}} \rightarrow \text{Wing loading}$$

Wing loading
[kg/m²]
[lb/ft²]

- For low speeds → Low wing loading
- For heavy loads → Higher cruise speed to keep low C_L

Example: Cruise speed

wikipedia.org

wikipedia.org

wikipedia.org

	Cessna 150	Beechcraft Skipper	Cessna 208 Caravan	
MTOW	7122	7456	39191	N
S	15	12.1	25.9	m^2
W/S	475	616	1513	N/m^2
Cruise spd	152	195	343	km/h

Example: Cruise speed

wikipedia.org

wikipedia.org

wikipedia.org

	Cessna 150	Beechcraft Skipper	Cessna 208 Caravan	
MTOW	1600	1675	8807	lb
S	160	129.8	279	ft^2
W/S	10	12.9	31.6	lb/ft^2
Cruise spd	82	105	185	kts

Performance parameters

- Wing loading: $\dfrac{W_0}{S_{ref}}$
- Thrust-to-Weight ratio: $\dfrac{T_0}{W_0}$
- Power-to-Weight ratio: $\dfrac{P_0}{W_0}$
- Power loading: $\dfrac{W_0}{P_0}$

Performance parameters

- Wing loading: $\dfrac{W_0}{S_{ref}}$
- Thrust-to-Weight ratio: $\dfrac{T_0}{W_0}$
- Power-to-Weight ratio: $\dfrac{P_0}{W_0}$
- Power loading: $\dfrac{W_0}{P_0}$

Reference for weights and thrusts:

- Takeoff configuration
- Standard-day conditions
- Sea-level static (SLS)
- Maximum throttle
- Afterburners on (if available)

Performance parameters

- Wing loading: $\frac{W_0}{S_{ref}}$
- Thrust-to-Weight ratio: $\frac{T_0}{W_0}$
- Power-to-Weight ratio: $\frac{P_0}{W_0}$
- Power loading: $\frac{W_0}{P_0}$

Reference for weights and thrusts:
- Takeoff configuration
- Standard-day conditions
- Sea-level static (SLS)
- Maximum throttle
- Afterburners on (if available)

Converting from thrust to power:

$$\frac{P}{W} = \frac{V}{\eta_p} \cdot \frac{T}{W}$$

Performance parameters

- Wing loading: $\frac{W_0}{S_{ref}}$
- Thrust-to-Weight ratio: $\frac{T_0}{W_0}$
- Power-to-Weight ratio: $\frac{P_0}{W_0}$
- Power loading: $\frac{W_0}{P_0}$

Reference for weights and thrusts:
- Takeoff configuration
- Standard-day conditions
- Sea-level static (SLS)
- Maximum throttle
- Afterburners on (if available)

Converting from thrust to power:

$$\frac{P}{W} = \frac{V}{\eta_p} \cdot \frac{T}{W}$$

Why do we use these fractions?

Performance parameters

- Wing loading: $\frac{W_0}{S_{ref}}$
- Thrust-to-Weight ratio: $\frac{T_0}{W_0}$
- Power-to-Weight ratio: $\frac{P_0}{W_0}$
- Power loading: $\frac{W_0}{P_0}$

Reference for weights and thrusts:
- Takeoff configuration
- Standard-day conditions
- Sea-level static (SLS)
- Maximum throttle
- Afterburners on (if available)

Converting from thrust to power:

$$\frac{P}{W} = \frac{V}{\eta_p} \cdot \frac{T}{W}$$

Why do we use these fractions?
- Easy to compare different airplanes

Historical trends: Wing loading

lb/ft²

Dominant Mission Requirement	$(W/S)_{TO}$	Example
High-altitude, long-endurance solar-powered ISR[a]	0.5–3.0	Helios
Competition sailplanes	7–12	ASW 17
Light civil aircraft with short range and field length	10–30	C-172
High-altitude, long-endurance hydrocarbon-powered ISR	25–50	RQ-4A
STOL[b] and utility transports	40–90	C-130
Short or intermediate range with moderate field length	50–90	Learjet 35
Long-range transports and bombers (>3000 n mile)	110–150	B 747
Fighter, high-altitude	30–60	F-106
Fighter, air-to-air	50–80	F-15A
Fighter, close air support	65–90	A-10A
Fighter, strike interdiction	90–130	F-4E
Fighter, interceptor	120–150	F-104G
Low-altitude subsonic cruise missiles	200–240	AGM-109

[a]Intelligence, surveillance, and reconnaissance.
[b]Short takeoff and landing.

[Nicolai]

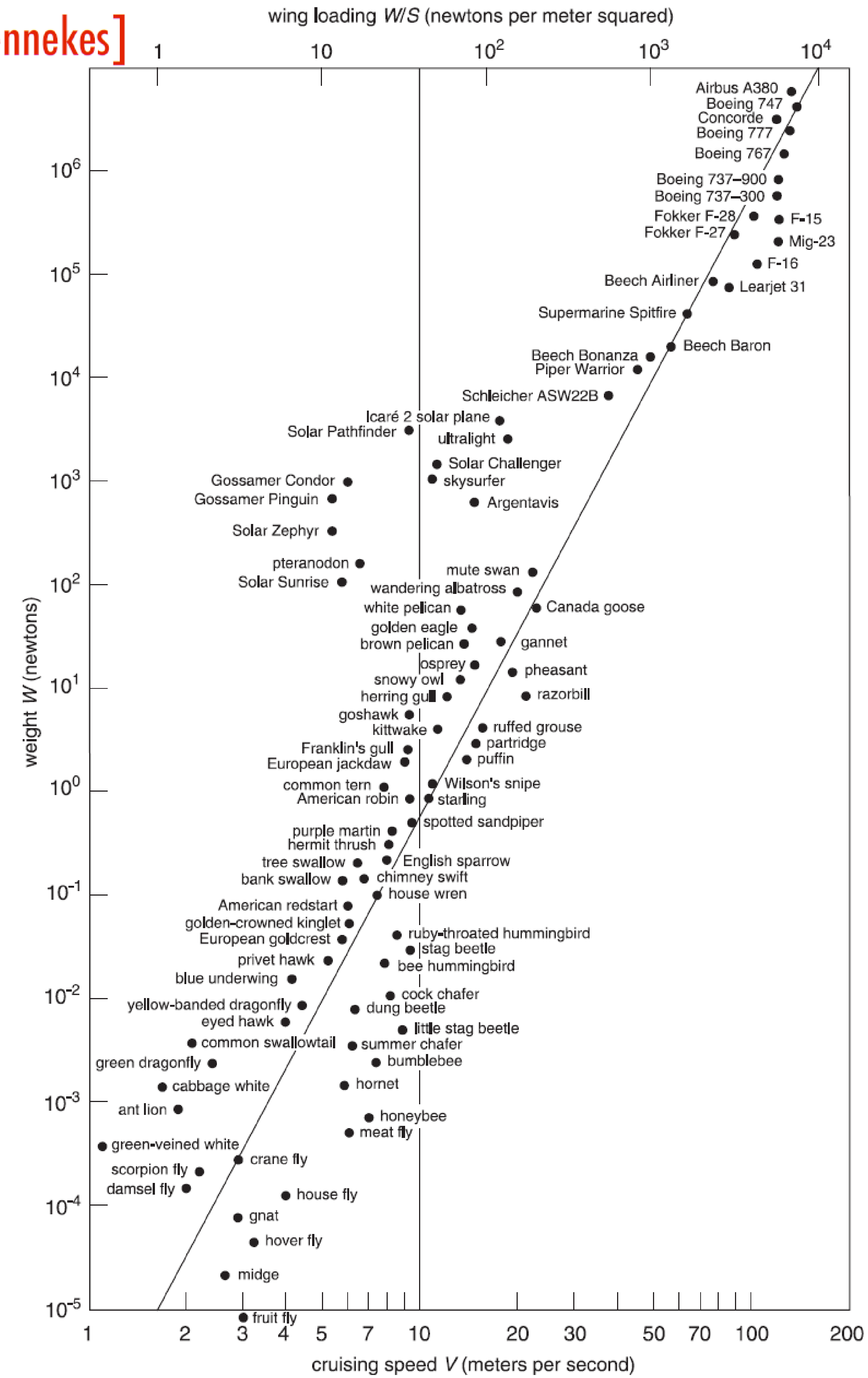
Wing-Loading: Nature use it as well!

$$W \propto b^3 \qquad S \propto b^2$$

$$\frac{W}{S} \propto \frac{b^3}{b^2} = b$$

$$\frac{W}{S} \propto W^{1/3}$$

[Tennekes]



Historical Trends: T/W and P/W

Airplane type	T_0/W_0	P_0/W_0 [hp/lb]
Powered Sailplane	—	0.04
Homebuilt	—	0.08
Single-engine GA	—	0.07
Twin-engine GA	—	0.17
Agricultural	—	0.09
Twin Turboprop	—	0.2
Flying boat	—	0.1
Jet trainer	0.4	—
Jet Fighter	0.6–0.9	—
Military cargo/bomber	0.25	—
Jet transport	0.25–0.4	—

Performance parameters

- Wing loading: $\dfrac{W_0}{S_{ref}}$
- Thrust-to-Weight ratio: $\dfrac{T_0}{W_0}$
- Power-to-Weight ratio: $\dfrac{P_0}{W_0}$
- Power loading: $\dfrac{W_0}{P_0}$

Reference for weights and thrusts:
- Takeoff configuration
- Standard-day conditions
- Sea-level static (SLS)
- Maximum throttle
- Afterburners on (if available)

Converting from thrust to power:

$$\frac{P}{W} = \frac{V}{\eta_p} \cdot \frac{T}{W}$$

Why do we use these fractions?
- Easy to compare different airplanes

Performance parameters

- Wing loading: $\dfrac{W_0}{S_{ref}}$

- Thrust-to-Weight ratio: $\dfrac{T_0}{W_0}$

- Power-to-Weight ratio: $\dfrac{P_0}{W_0}$

- Power loading: $\dfrac{W_0}{P_0}$

Reference for weights and thrusts:
- Takeoff configuration
- Standard-day conditions
- Sea-level static (SLS)
- Maximum throttle
- Afterburners on (if available)

Converting from thrust to power:

$$\frac{P}{W} = \frac{V}{\eta_p} \cdot \frac{T}{W}$$

Why do we use these fractions?
- Easy to compare different airplanes
- They show up in almost every performance equation

Performance Requirements

- Stall speed
- Takeoff
- Landing
- Climb
- Ceiling
- Maneuver (load factor)
- Cruise speed
- Sink ratio

komar.org

Performance Requirements

- Stall speed
- Takeoff
- Landing
- Climb
- Ceiling
- Maneuver (load factor)
- Cruise speed
- Sink ratio

komar.org

Stall speed

- 1-g definition: minimum airspeed such that the airplane lift can balance its own weight

Stall speed

- 1-g definition: minimum airspeed such that the airplane lift can balance its own weight:

$$L = W_0$$

Takeoff weight is the most critical case

$$\frac{\rho V^2}{2} S_{ref} C_L = W_0$$

$$V = \sqrt{\frac{2}{\rho C_L} \cdot \frac{W_0}{S_{ref}}}$$

- To minimize speed, we need to maximize C_L

$$V_{stall} = \sqrt{\frac{2}{\rho C_{Lmax}} \cdot \frac{W_0}{S_{ref}}} \qquad \frac{W_0}{S_{ref}} = \frac{\rho V_{stall}^2}{2} C_{Lmax}$$

Stall speed

- Stall speed is one of the most important performance parameters, as other requirements are based on this airspeed.
- Stall speed depends on wing loading, but not T/W.
- Want to fly slow? →Increase C_{Lmax} or reduce W/S.
- Remember to use the correct flap setting for C_{Lmax}. We can have requirements for flaps-up and/or flaps-down.
- FAR 23: Maximum stall speed allowed is 61 kts.
- Regulations may have different definitions for stall speed. The definition we are using is suitable for preliminary design.

$$V_{stall} = \sqrt{\frac{2}{\rho C_{Lmax}} \cdot \frac{W_0}{S_{ref}}} \qquad \frac{W_0}{S_{ref}} = \frac{\rho V_{stall}^2}{2} C_{Lmax}$$

Example: Stall speed

wikipedia.org

wikipedia.org

wikipedia.org

	Cessna 150	Beechcraft Skipper	Cessna 208 Caravan	
MTOW	7122	7456	39191	N
S	15	12.1	25.9	m^2
W/S	475	616	1513	N/m^2
Cruise spd	152	195	343	km/h
Stall spd	79	87	113	km/h

Example: Stall speed

wikipedia.org

wikipedia.org

wikipedia.org

	Cessna 150	Beechcraft Skipper	Cessna 208 Caravan	
MTOW	1600	1675	8807	lb
S	160	129.8	279	ft^2
W/S	10	12.9	31.6	lb/ft^2
Cruise spd	82	105	185	kts
Stall spd	42	47	61	kts

Performance Requirements

- Stall speed
- Takeoff
- Landing
- Climb
- Ceiling
- Maneuver (load factor)
- Cruise speed
- Sink ratio

komar.org

Performance Requirements

- Stall speed
- Takeoff
- Landing
- Climb
- Ceiling
- Maneuver (load factor)
- Cruise speed
- Sink ratio

komar.org

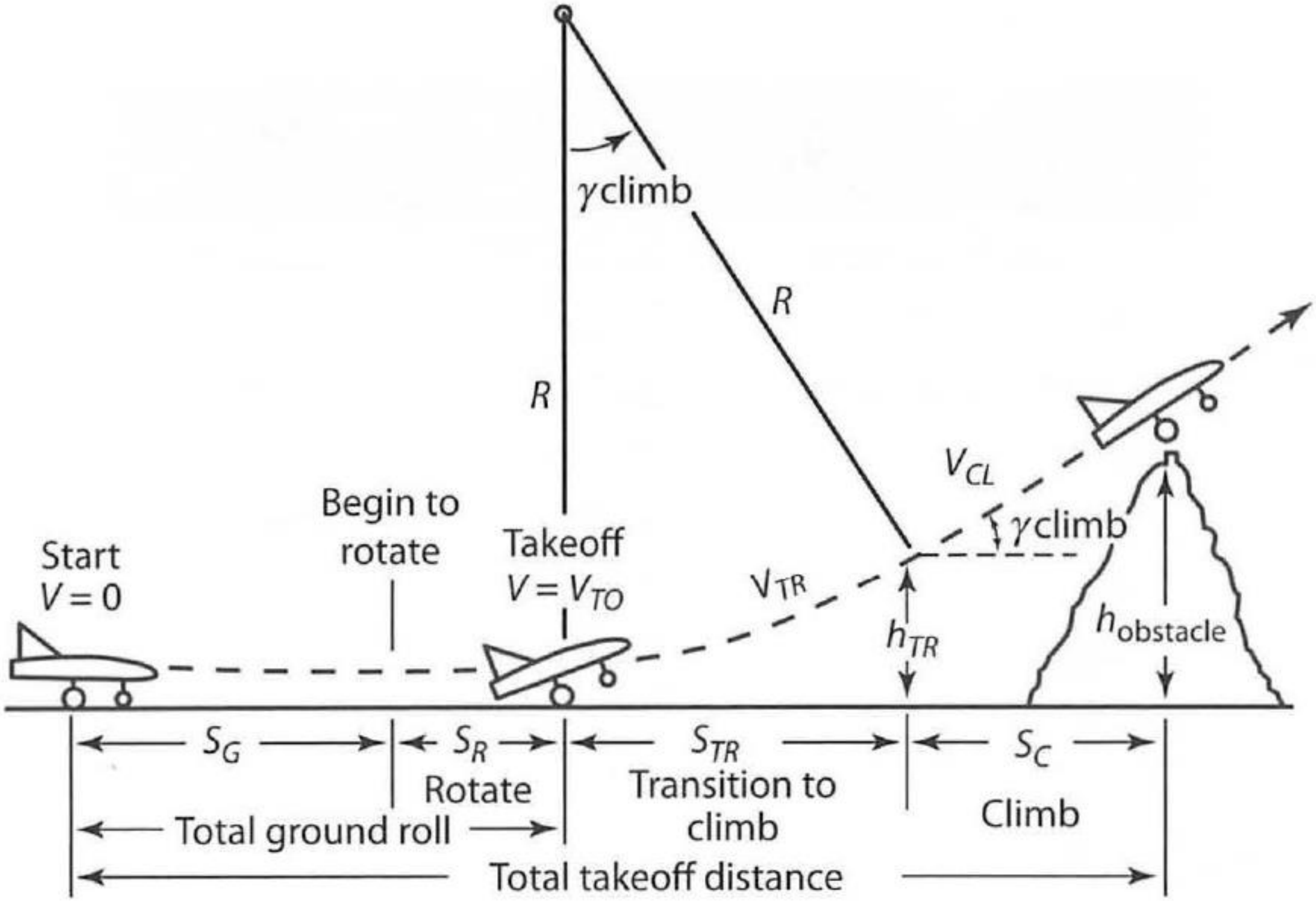
Sizing to Takeoff Distance Requirements

- Takeoff is complicated
  - Airplane is accelerating in x
  - Airplane is rotating in pitch
  - Airplane is accelerating in z once it lifts off
  - Lift is modified by ground effect
  - Ground is exerting a force in x and z
- Takeoff affected by

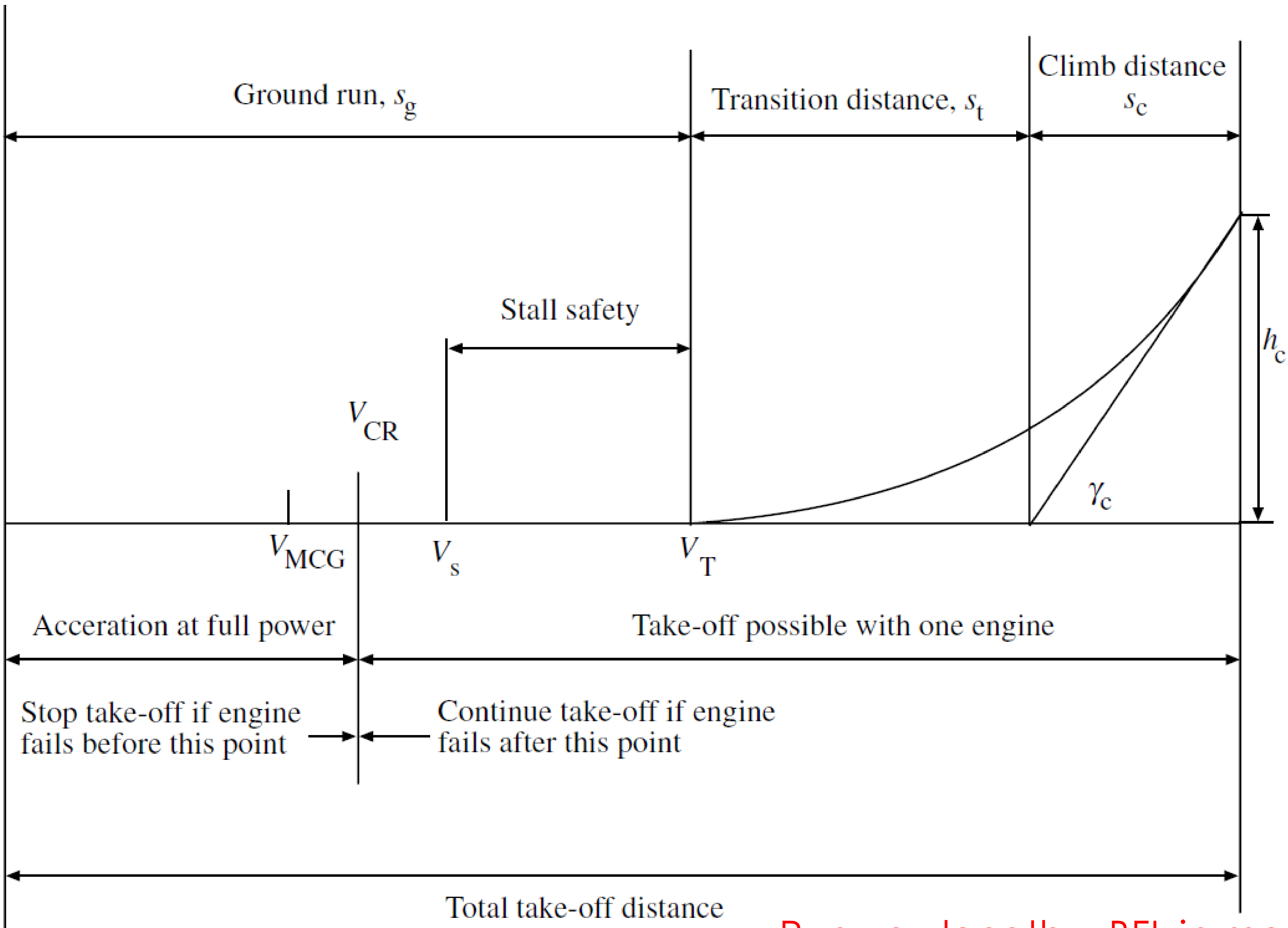
Ground friction coefficient

$$W_{TO},\ V_{TO},\ T/W \text{ or } P/W, D, \mu_G$$

Takeoff Dynamics



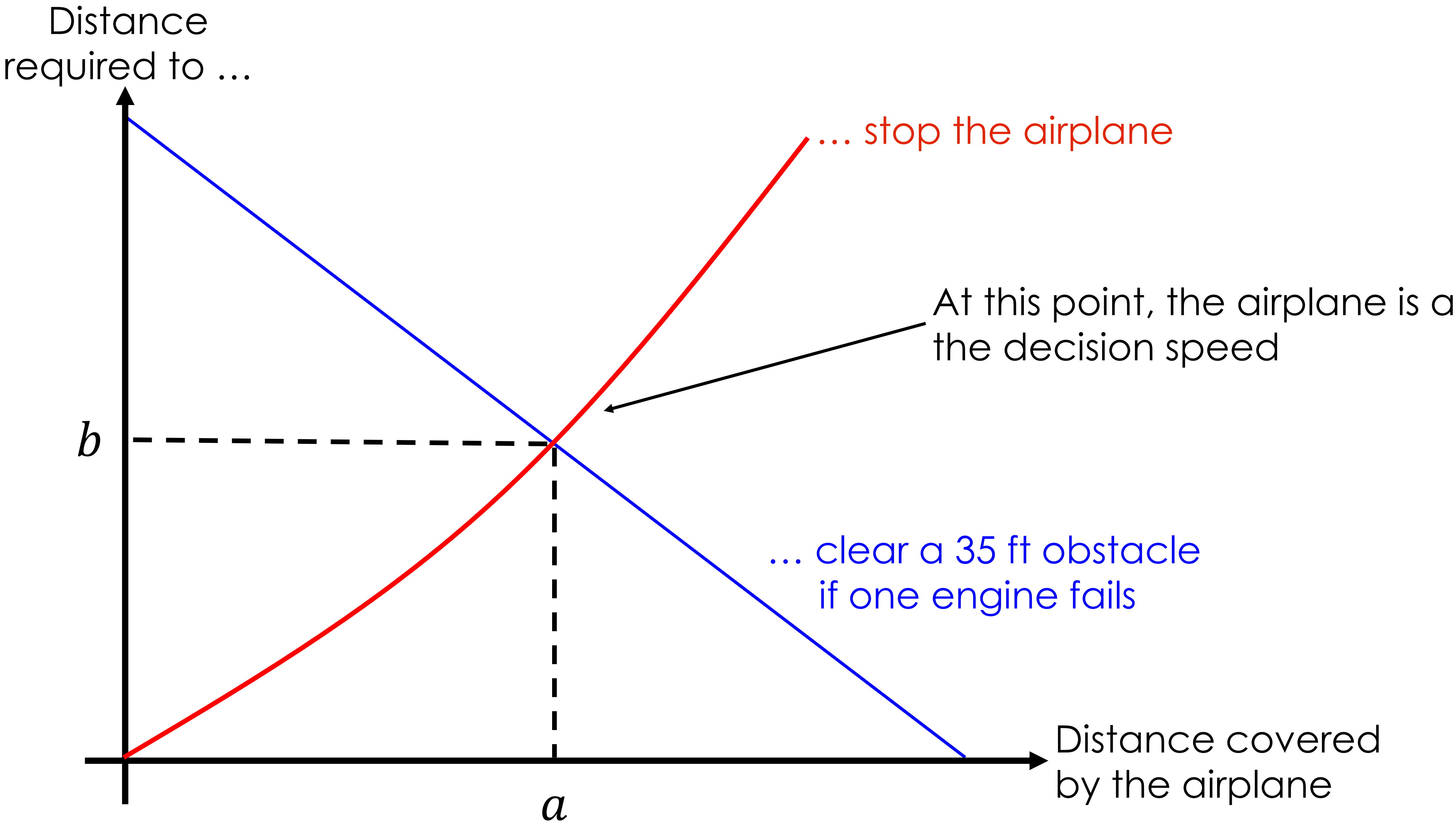
Balanced Field Length (BFL)



35 ft for civilian
50 ft for military

= Runway length = BFL in most cases

How to find decision speed and BFL?



$$BFL = a + b$$

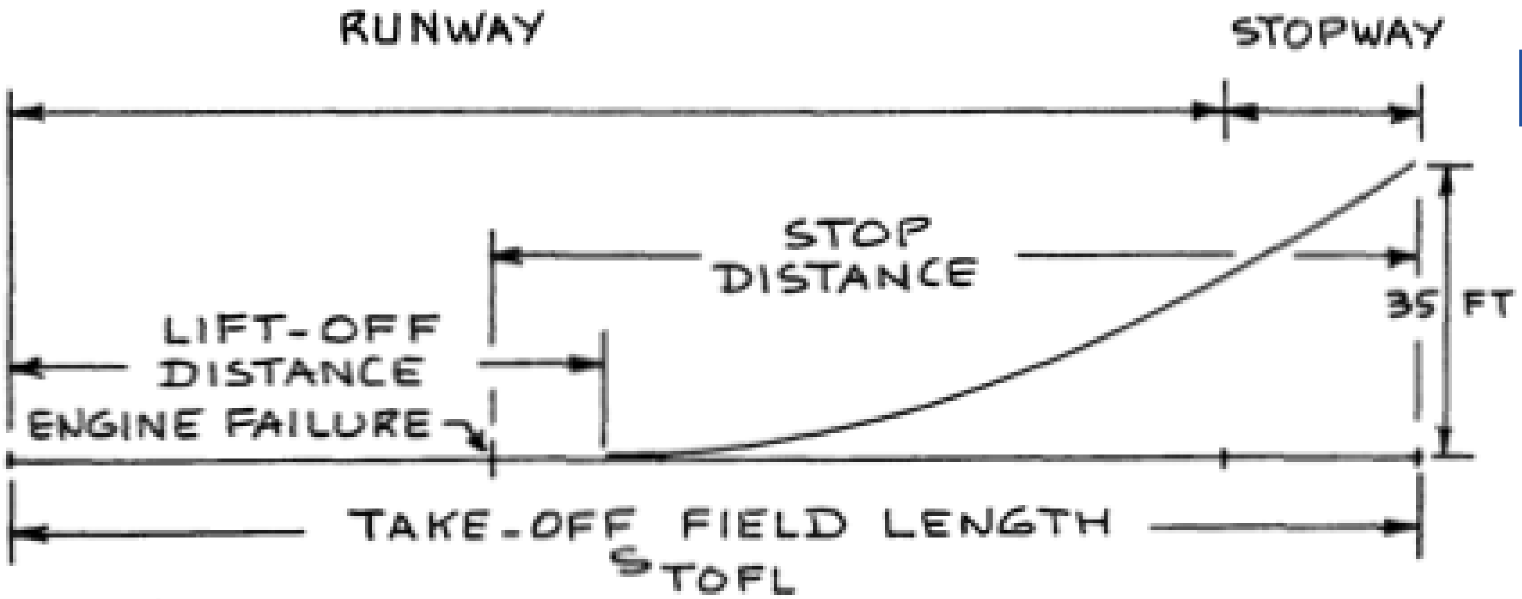
Takeoff Distance Definitions

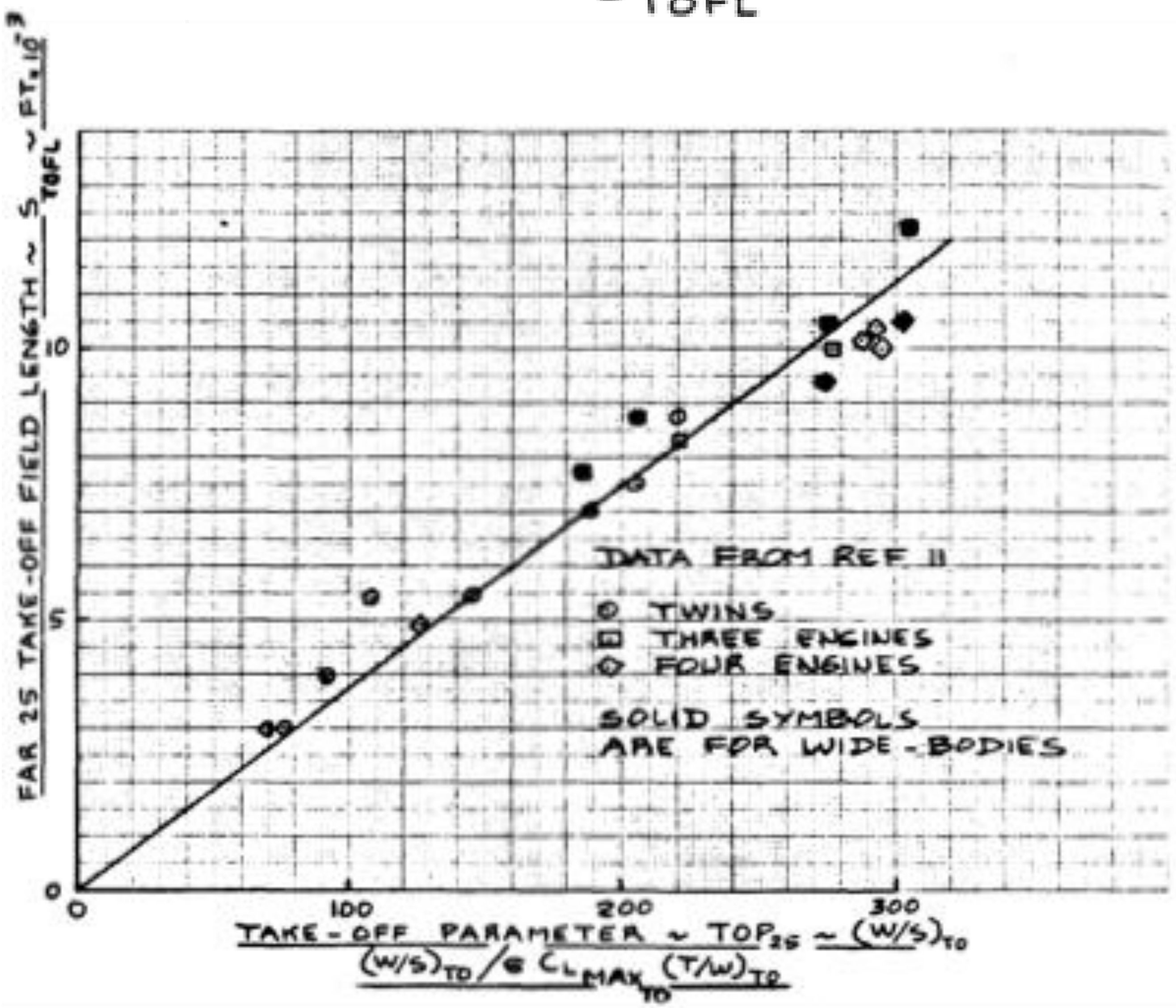
- Ground Roll: Actual ground distance until liftoff
- Obstacle clearance distance: Distance required (from brake release) to reach a certain altitude
- Decision speed (V1): Speed at which distance to stop after an engine failure is equal to the distance to continue the takeoff with remaining engines
- Balanced field length: Takeoff distance when one engine fails at decision speed

09/08/2015
LAS airport
British airways flight 2276

FAR 25 Takeoff Requirements

[Roskam, Part I, Figs. 3.6, 3.7]





$$TOP = \frac{W_{TO}/S_{ref}}{(\rho/\rho_{SL}) \cdot C_{Lmax,TO} \cdot T_{TO}/W_{TO}}$$

$$BFL = 37.5 \cdot TOP$$

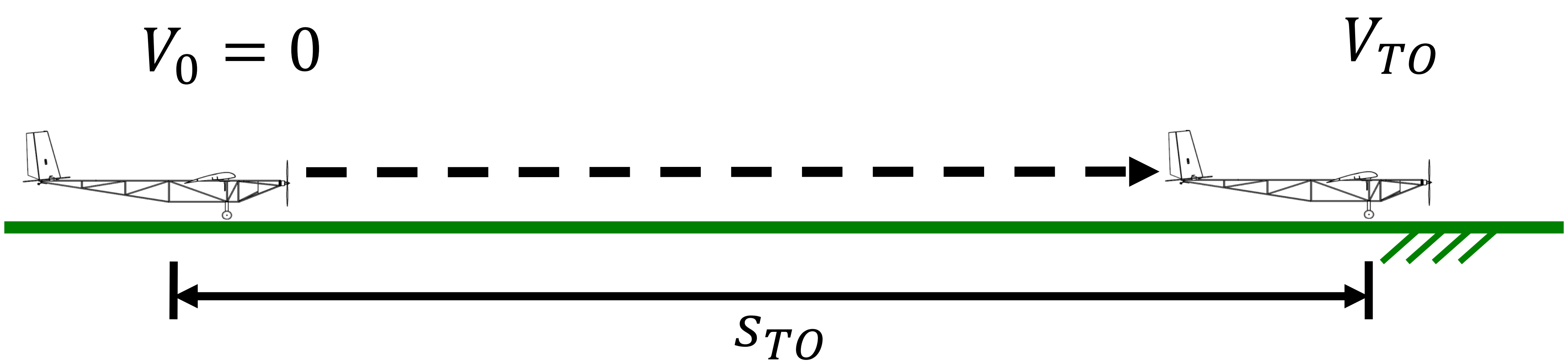
English units!

$$C_{Lmax,TO} \sim 0.80 \cdot C_{Lmax,landing}$$

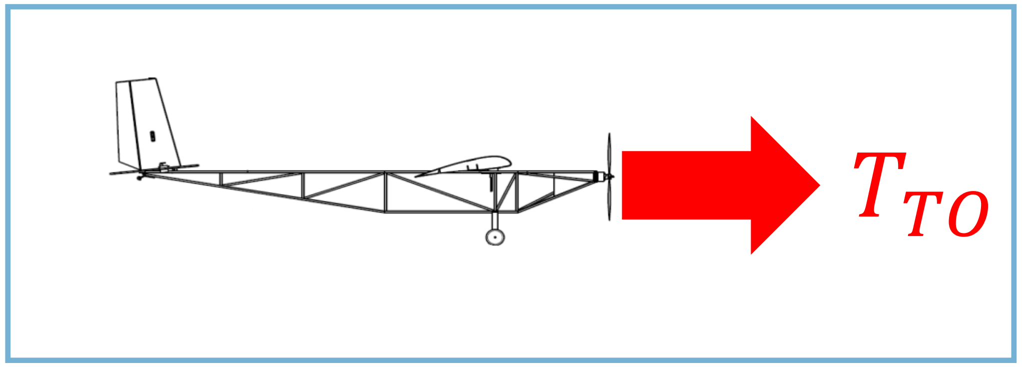
Takeoff Parameter (TOP)

- Consider only thrust during ground run. Also neglect the rotation phase.
- The airplane should accelerate from $V_0 = 0$ until the takeoff speed V_{TO}.
- We will define the takeoff speed as a function of the stall speed:

$$V_{TO} = k_s V_{stall} \quad \text{with} \quad k_s > 1.0$$



Takeoff Parameter (TOP)



$$\vec{F} = m\vec{a}$$

$$T_{TO} = \frac{W_{TO}}{g} \cdot \frac{dV}{dt}$$

$$T_{TO} = \frac{W_{TO}}{g} \cdot \frac{dV}{dx} \cdot \frac{dx}{dt}$$

$$T_{TO} = \frac{W_{TO}}{g} \cdot \frac{dV}{dx} \cdot V$$

$$\frac{g \cdot T_{TO}}{W_{TO}} dx = V dV$$

$$\int_0^{s_{TO}} \frac{g \cdot T_{TO}}{W_{TO}} dx = \int_0^{V_{TO}} V dV$$

$$\frac{g \cdot T_{TO} \cdot s_{TO}}{W_{TO}} = \frac{V_{TO}^2}{2} = \frac{k_s^2 V_{stall}^2}{2}$$

$$\frac{g \cdot T_{TO} \cdot s_{TO}}{W_{TO}} = \frac{k_s^2}{2} \cdot \frac{2}{\rho C_{Lmax,TO}} \cdot \frac{W_{TO}}{S_{ref}}$$

$$s_{TO} = \frac{k_s^2}{g} \cdot \frac{(W_{TO}/S_{ref})}{\rho C_{Lmax,TO}(T_{TO}/W_{TO})}$$

Takeoff Parameter (TOP)

$$s_{TO} = \frac{k_s^2}{g} \cdot \frac{(W_{TO}/S_{ref})}{\rho C_{Lmax,TO}(T_{TO}/W_{TO})}$$

$$s_{TO} = \frac{k_s^2}{g\rho_{SL}} \cdot \underbrace{\boxed{\frac{(W_{TO}/S_{ref})}{(\rho/\rho_{SL}) \cdot C_{Lmax,TO} \cdot (T_{TO}/W_{TO})}}}_{TOP}$$

$$s_{TO} = \frac{k_s^2}{g\rho_{SL}} \cdot TOP$$

As $BFL \propto s_{TO}$, then $\boxed{BFL \propto TOP}$

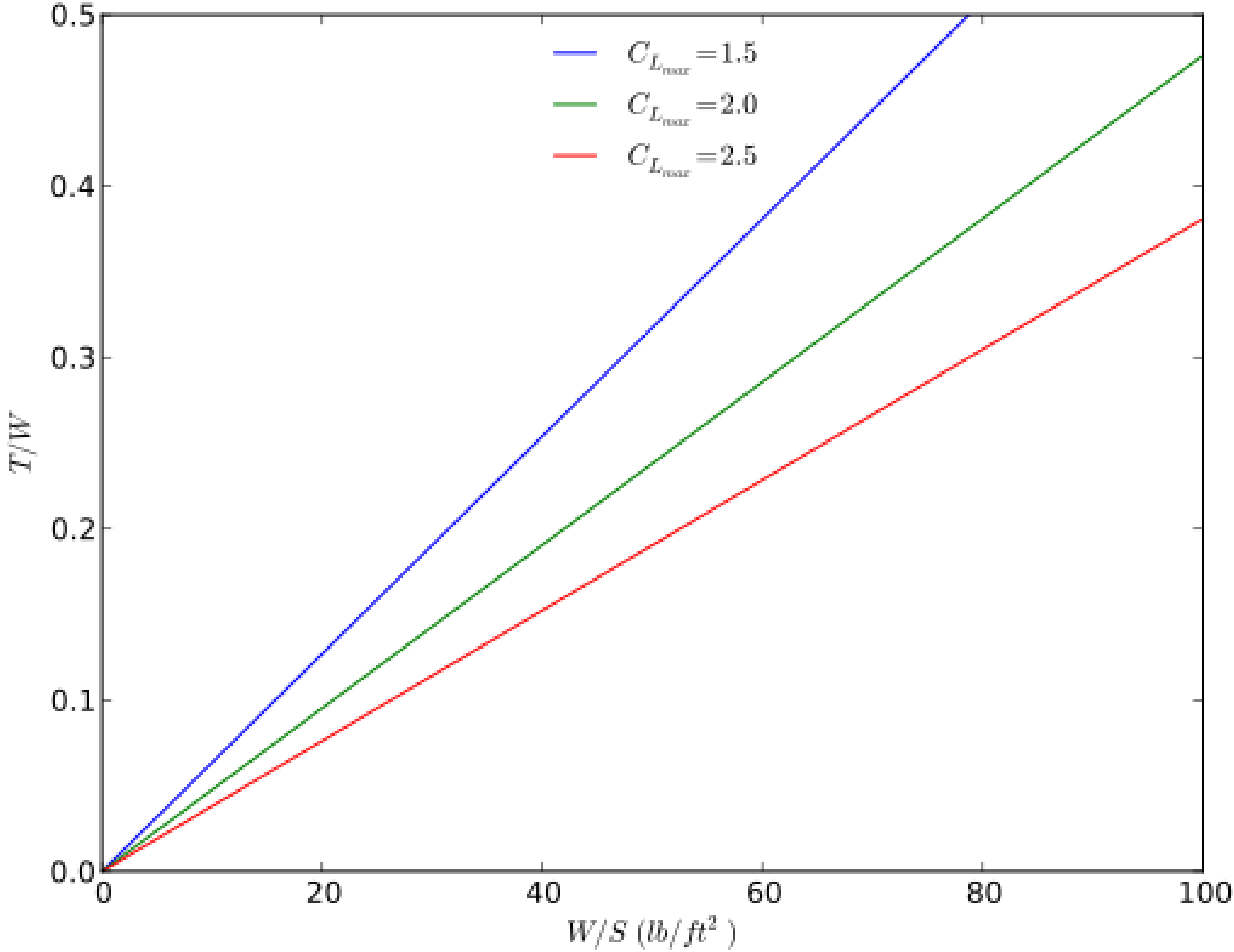
Roskam showed that for FAR 25: $BFL = 37.5 \cdot TOP$

FAR 25 Takeoff Sizing Example

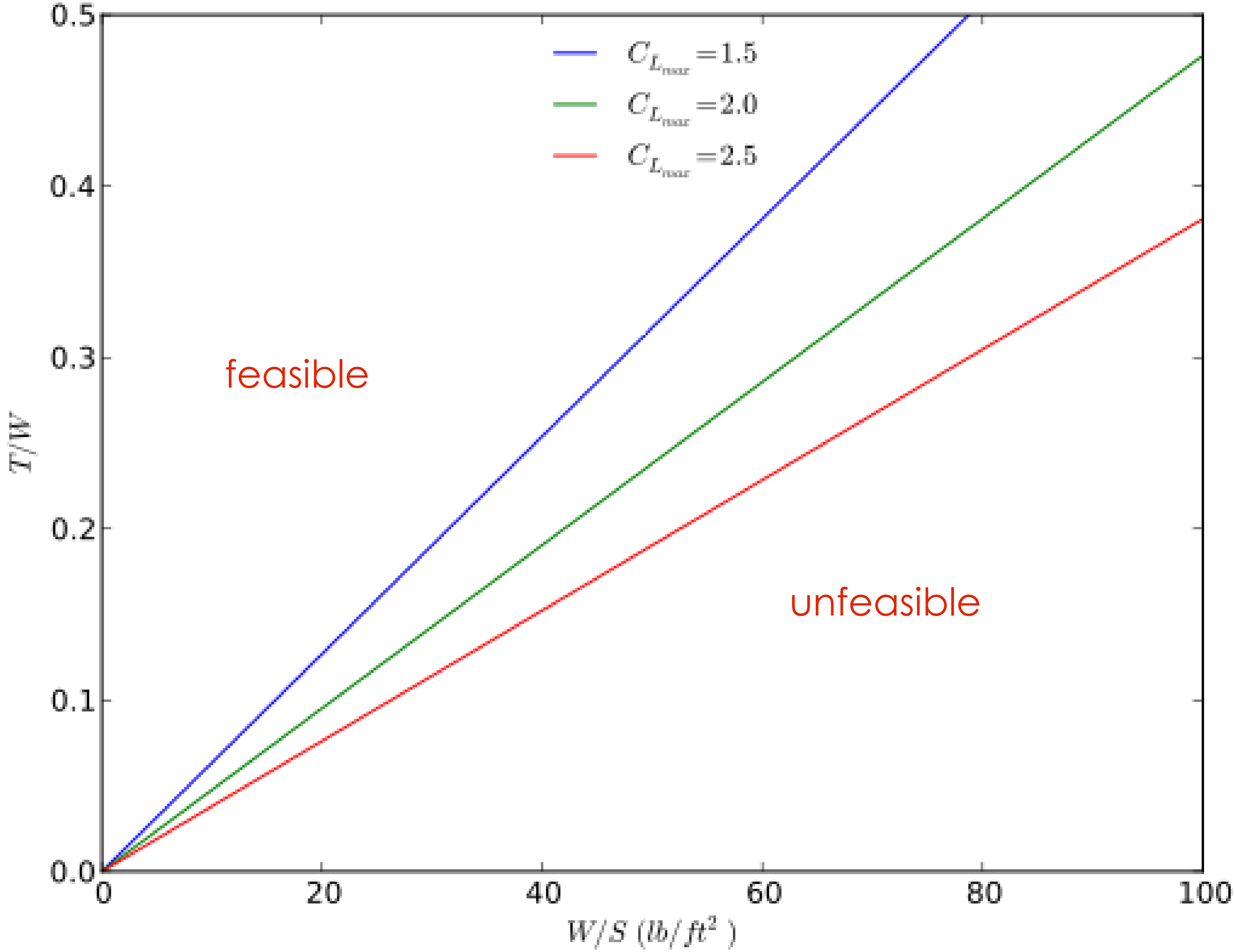
Size a passenger jet so that the takeoff field length is less than 5,000 ft at 8,000 ft altitude

$$
\begin{aligned}
s_{TOFL} &= 37.5\ TOP_{25} \\
&= 37.5 \times \frac{\left(\frac{W}{S}\right)_{TO}}{\sigma \left(C_{L_{max}}\right)_{TO} \left(\frac{T}{W}\right)_{TO}} \\
5000/37.5 &> \frac{\left(\frac{W}{S}\right)_{TO}}{\sigma \left(C_{L_{max}}\right)_{TO} \left(\frac{T}{W}\right)_{TO}} \\
\frac{\left(\frac{W}{S}\right)_{TO}}{\left(C_{L_{max}}\right)_{TO} \left(\frac{T}{W}\right)_{TO}} &< \sigma \times 5000/37.5 \\
&< 0.786 \times 5000/37.5 \\
&< 104.8 \text{ psf}
\end{aligned}
$$

Representing the Constraint Graphically

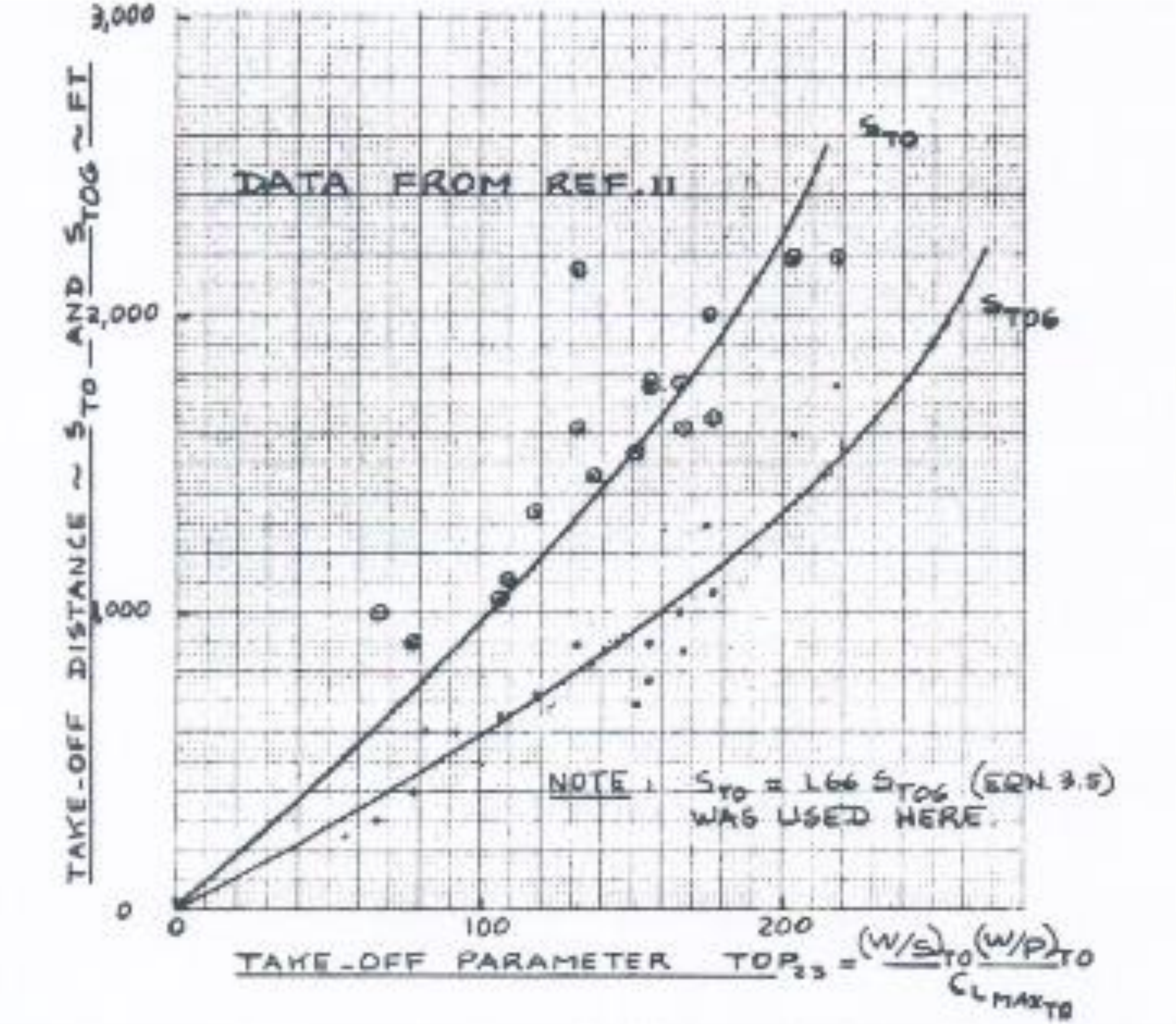


Representing the Constraint Graphically



Relating the Takeoff Parameter to Distance (FAR 23)

[Roskam, Part I, Fig 3.3]



$$s_{TOG} = 4.9\ TOP_{23} + 0.009\ TOP_{23}^2 = \text{Total Takeoff Distance}$$

$$s_{TO} = 1.66\ s_{TOG} = \text{Total Ground Roll}$$

Example: Takeoff Sizing Using Roskam's Formula

- Size a propeller-driven airplane to meet, at an altitude of 5,000 ft,

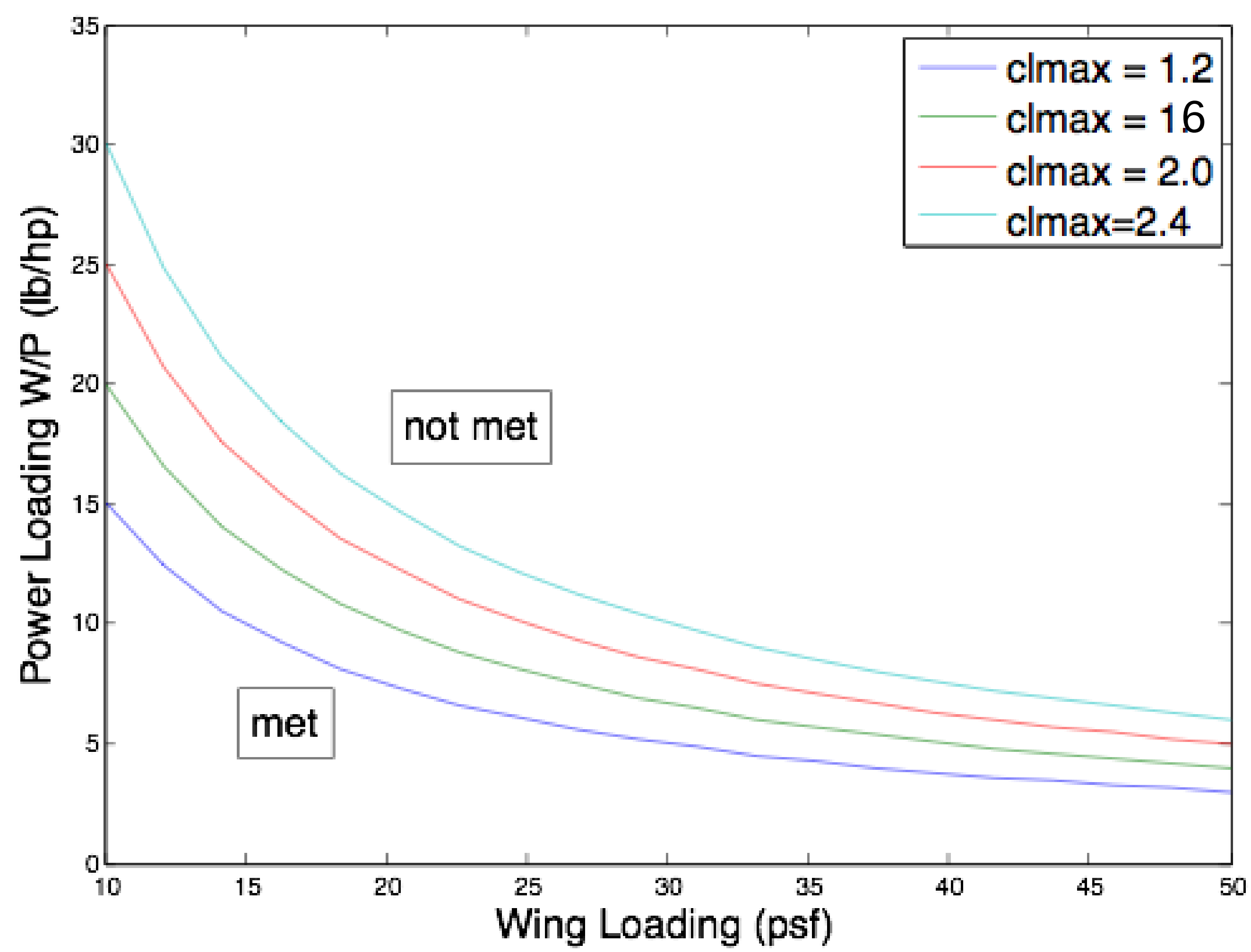
$$\begin{aligned} s_{TOG} &< 1000 \text{ ft} \\ s_{TO} &< 1500 \text{ ft} \end{aligned}$$

- Since $s_{TO} = 1.6 s_{TOG}$, the second requirement is the more restrictive one
- From Roskam's formula:

$$\begin{aligned} 1500 \text{ ft} &> 1.66 \left(4.9\, TOP_{23} + 0.009\, TOP_{23}^2\right) \\ TOP_{23} &< 145.6 \frac{\text{lb}^2}{\text{ft}^2\text{hp}} \\ \frac{(W/S)_{TO}}{(P/W)_{TO}\, \sigma\, (C_{L_{max}})_{TO}} &< 145.6 \frac{\text{lb}^2}{\text{ft}^2\text{hp}} \\ \frac{(W/S)_{TO}}{(P/W)_{TO}\, (C_{L_{max}})_{TO}} &< 145.6 \times 0.8616 \frac{\text{lb}^2}{\text{ft}^2\text{hp}} \end{aligned}$$

Representing the Constraint Graphically

Write a simple MATLAB code to plot power loading (*W*/*P*) as a function of wing loading (*W*/*S*) for several lift coefficients, using the derived constraint formula




```
>> wingLoading=linspace(10,50,20);
>> clmax=1.2;
>> wp1 = clmax*125.4./wingLoading;
>> clmax=1.6;
>> wp2 = clmax*125.4./wingLoading;
>> clmax=2.0;
>> wp3 = clmax*125.4./wingLoading;
>> clmax=2.4;
>> wp4 = clmax*125.4./wingLoading;
>> plot(wingLoading,wp1,wingLoading,wp2,wingLoading,wp3,wingLoading,wp4);
```

Performance Requirements

- Stall speed
- Takeoff
- Landing
- Climb
- Cruise speed
- Ceiling
- Maneuver (load factor)

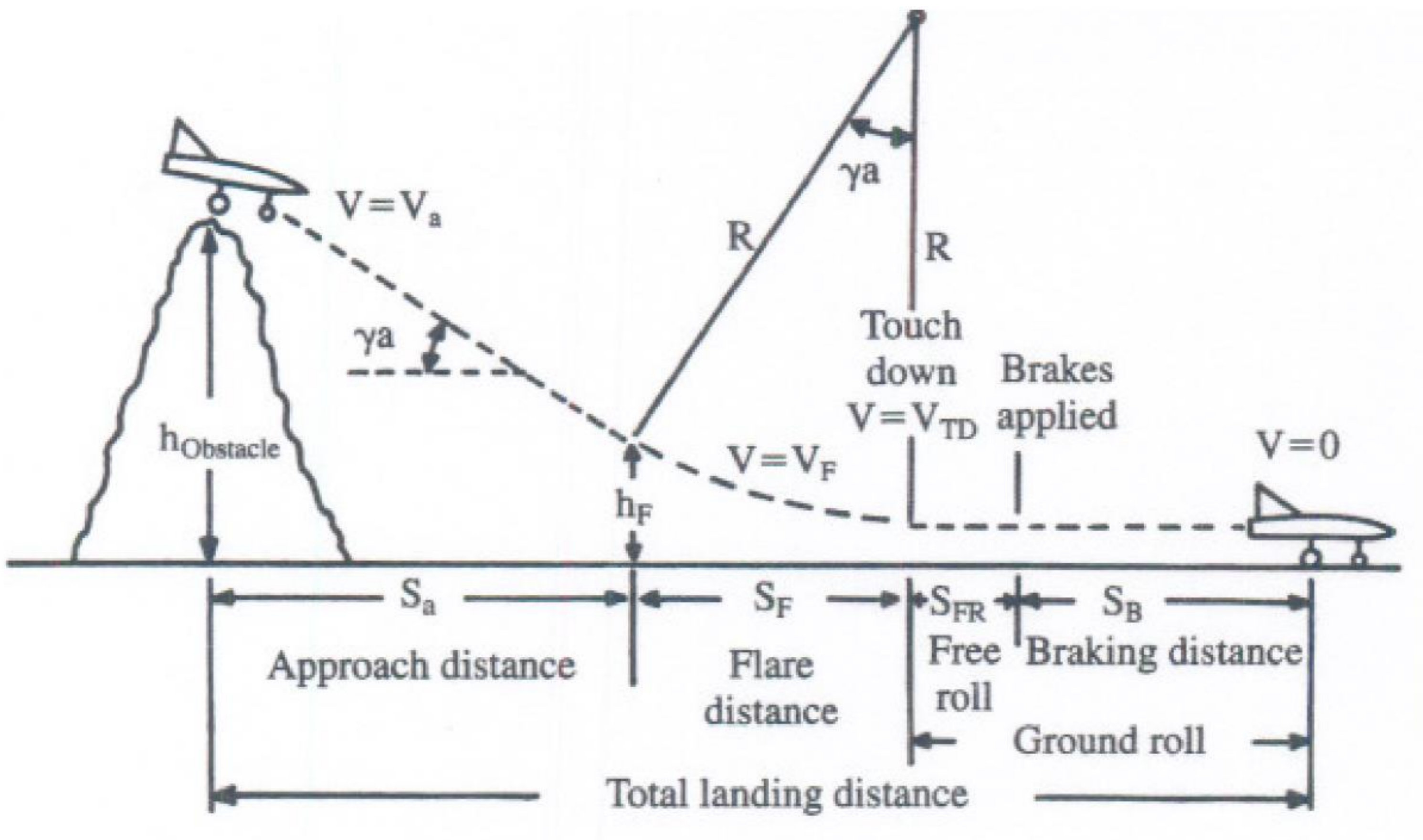
komar.org

Performance Requirements

- Stall speed
- Takeoff
- Landing
- Climb
- Cruise speed
- Ceiling
- Maneuver (load factor)

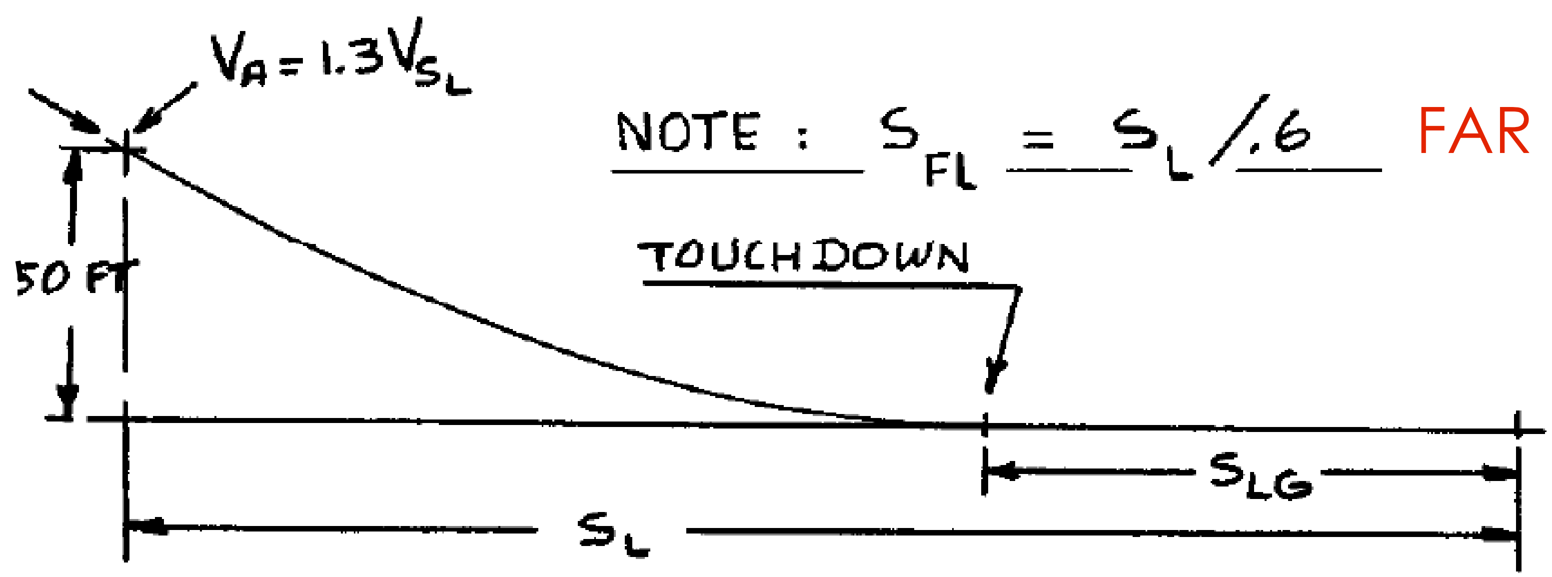
komar.org

Analyzing Landing



[Raymer]

[Roskam]



FAR 25

Landing Field Lengths

- FAR 23: Clear a 50-foot object at approach speed (1.3 times stall speed) and on approach glide path (3 degrees). Touchdown at 1.15 times stall speed.
- FAR 25: Clear a 50-foot object at approach speed (1.3 times stall speed) and on approach glide path (3 degrees). Apply a safety factor of 1.66 (=1/0.6) to the landing distance.

$$s_{FL} = s_L/0.6$$

Sizing to Landing Distance Requirements

- Landing distance depends on landing weight, approach speed and deceleration method used
- Deceleration methods include
  - Brakes
  - Thrust reversers
  - Parachutes
  - Arresting systems
  - Crash barriers (ouch!)

FAR says these can not be used to meet landing specifications!

- Simple energy analysis suggests that landing distance scales with square of approach speed
- Wing loading greatly affects landing distance

Because approach speed is set by stall speed, and stall speed depends on wing loading

Landing-Distance Formulas

- The ground run distance (s_{LG}) is proportional to the kinetic energy of airplane, and the kinetic energy is proportional to the square of approach speed:

$$s_{LG} \propto V_A^2$$

- As the approach speed is 1.3 times the stall speed of the landing configuration, we can also say that:

$$s_{LG} \propto V_{stall,L}^2$$

$$s_{LG} \propto \frac{(W_L/S_{ref})}{\rho C_{Lmax,L}}$$

- W_L should be the Maximum Landing Weight (MLW):

Landing-Distance Formulas

We need to consider an additional safety margin for FAR 25 airplanes:

$$s_{FL} = s_L/0.6$$

This should be compared with the runway length

$$V_a = 1.701\sqrt{s_{FL}} \quad \text{(m)}$$

[Roskam]

Now we can compute the stall speed with

$$V_s = \frac{V_a}{1.3}$$

The corresponding wing loading is

$$\frac{W_L}{S_{ref}} = \frac{\rho V_s^2}{2} C_{Lmax}$$

We also need to convert the weights back to takeoff weight:

$$\frac{W_{TO}}{S_{ref}} = \frac{W_L/S_{ref}}{W_L/W_{TO}}$$

W_L should be the Maximum Landing Weight (MLW)

FAR Regulations for Landing Weight

- FAR 25.1001: The airplane should reach its maximum landing weight 15 minutes after takeoff. You can consider climb, go-around, and descent for this computation.
- If the airplane cannot meet this requirement by fuel-consumption alone, then the manufacturer should install fuel-dumping systems.

A340-600

wikipedia.org

IMPORTANT:
Do NOT use the landing weight from the fuel-fraction computation!

Fuel jettison system

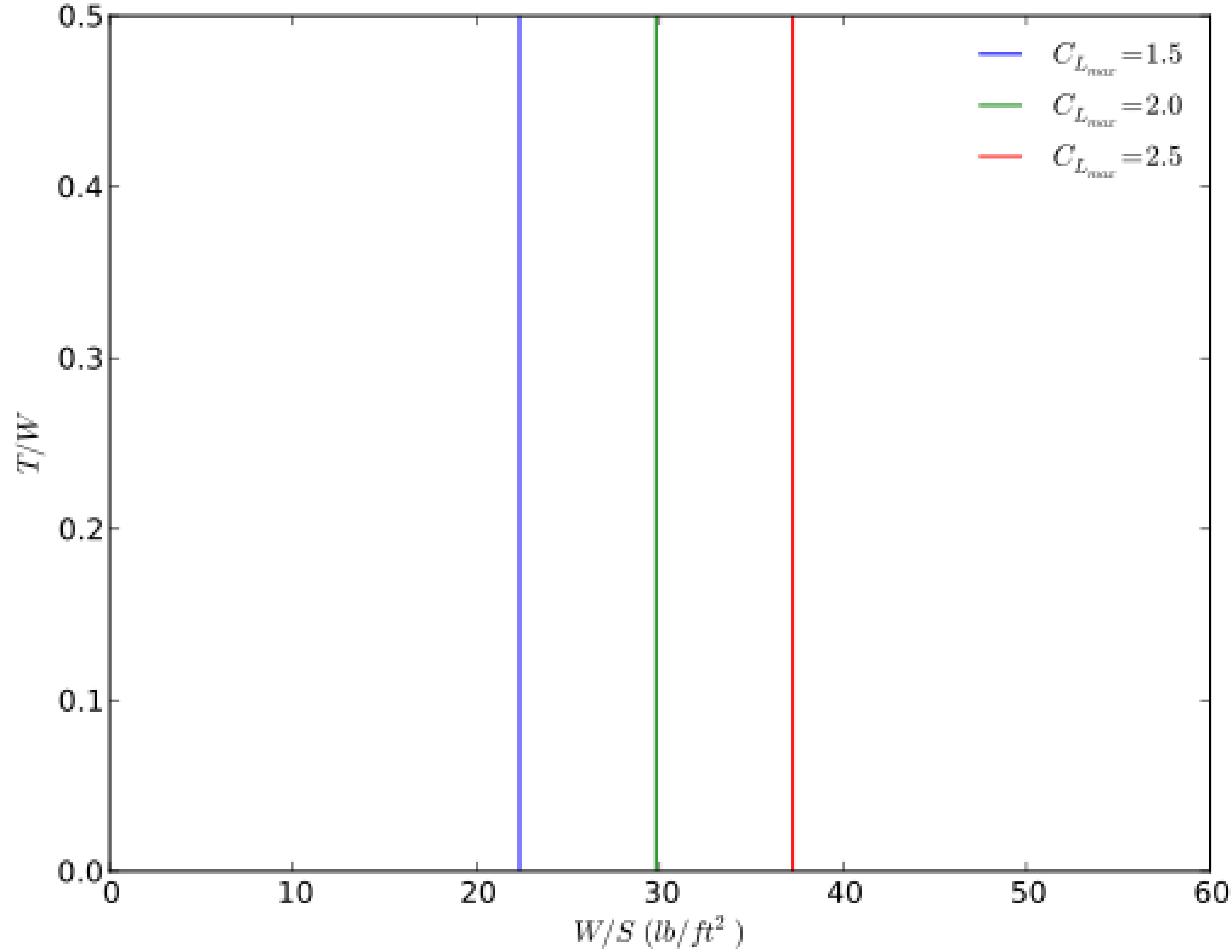
F-111

Landing Weight to Takeoff Weight: History/Statistics

[Roskam, Part I, Table 3.3]

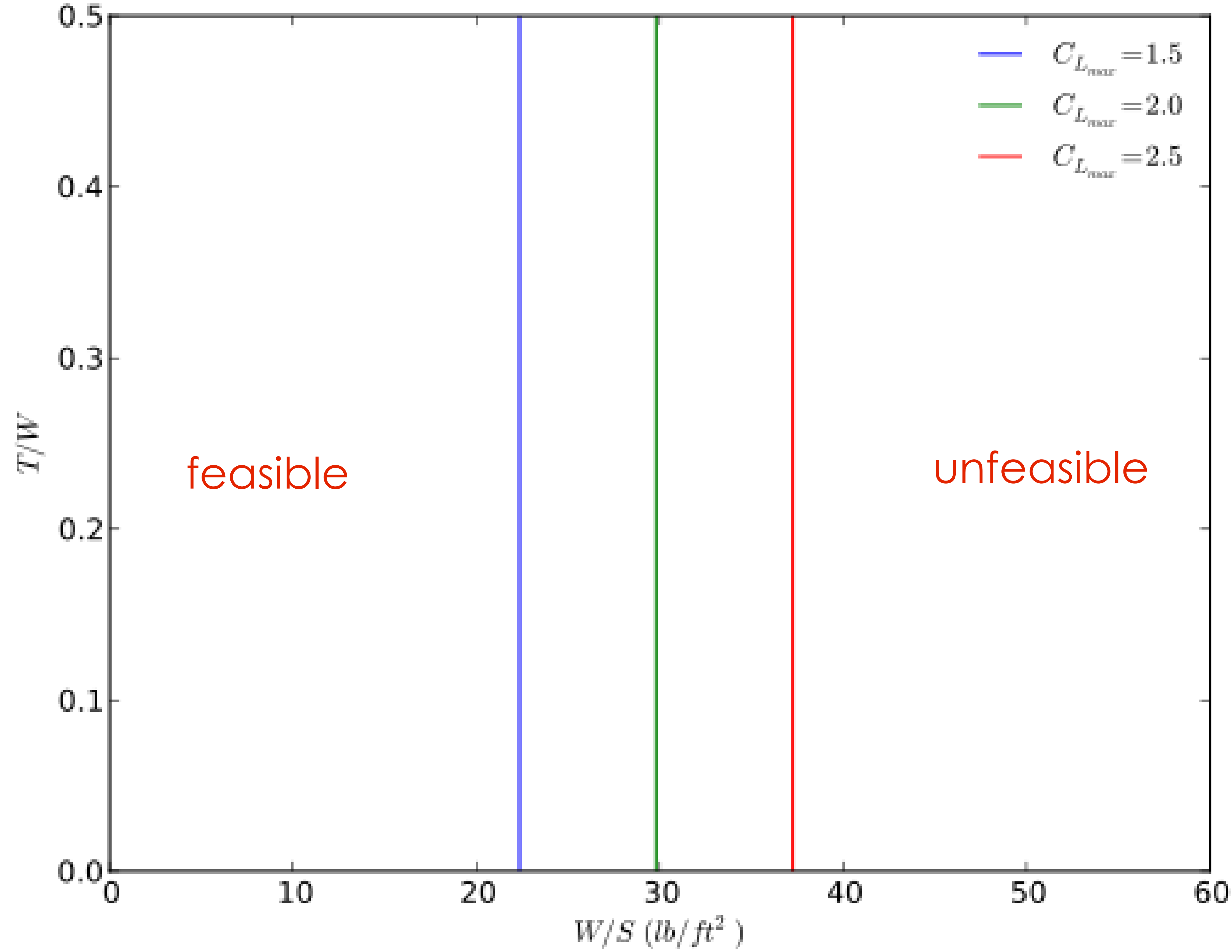
Plane type	min W_L/W_{TO}	ave W_L/W_{TO}	max W_L/W_{TO}
Homebuilt	0.96	1	1
Single-engine GA	0.95	0.997	1
Twin-engine GA	0.88	0.99	1
Agricultural	0.7	0.94	1
Business Jet	0.69	0.88	0.96
Twin Turboprop	0.92	0.98	1
Transport Jet	0.65	0.84	1
Jet trainer	0.87	0.99	1
Jet Fighter	0.78	?	1
Military cargo jet	0.68	0.76	0.83
Flying boat	0.98	?	1
Supersonic cruise	0.63	0.75	0.88

Graphical Representation of Constraint



Landing config values

Graphical Representation of Constraint



Landing config values

Performance Requirements

- Stall speed
- Takeoff
- Landing
- Climb
- Cruise speed
- Ceiling
- Maneuver (load factor)

komar.org

Performance Requirements

- Stall speed
- Takeoff
- Landing
- Climb
- Cruise speed
- Ceiling
- Maneuver (load factor)

komar.org

Climb considerations

- Climb starts after takeoff and finishes when the aircraft is 1500 ft above the runway.
- During this phase the aircraft will have different configurations:
  - Flaps down and landing gear down
  - Flaps down and landing gear up
  - Flaps up and landing gear up
- We can use the same equations to analyze balked landing.
- Requirements specify the climb gradient (G) and the climb speed as a function of stall speed (k_s factor).

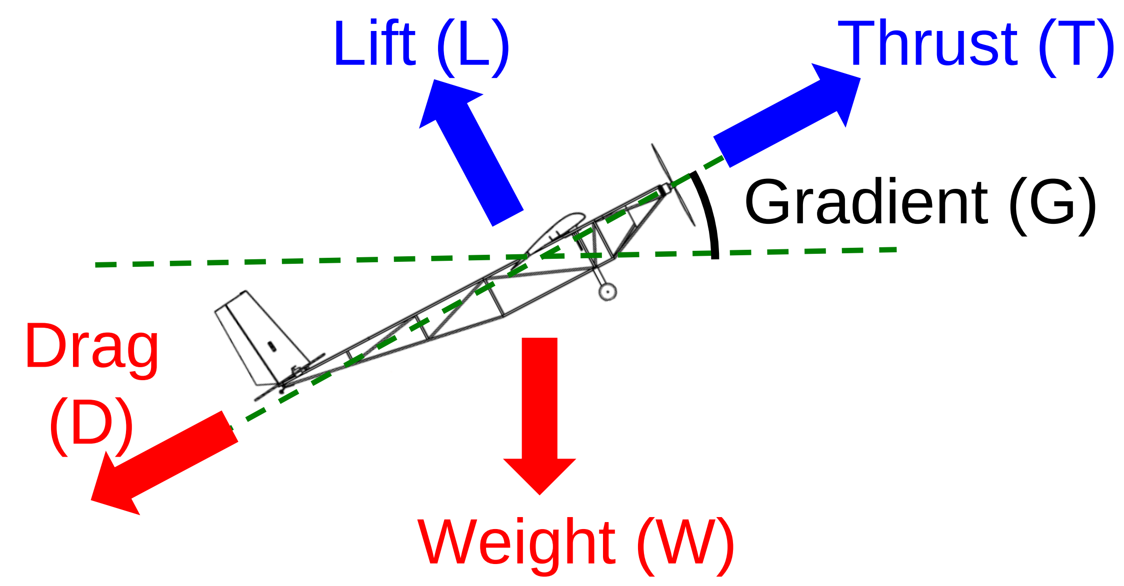
Climb equations

From equilibrium in both aerodynamic axes:

$$T = D + W \cdot \sin(G)$$
$$L = W \cdot \cos(\mathrm{G})$$

For small values of G:

$$T = D + W \cdot G$$
$$L = W$$



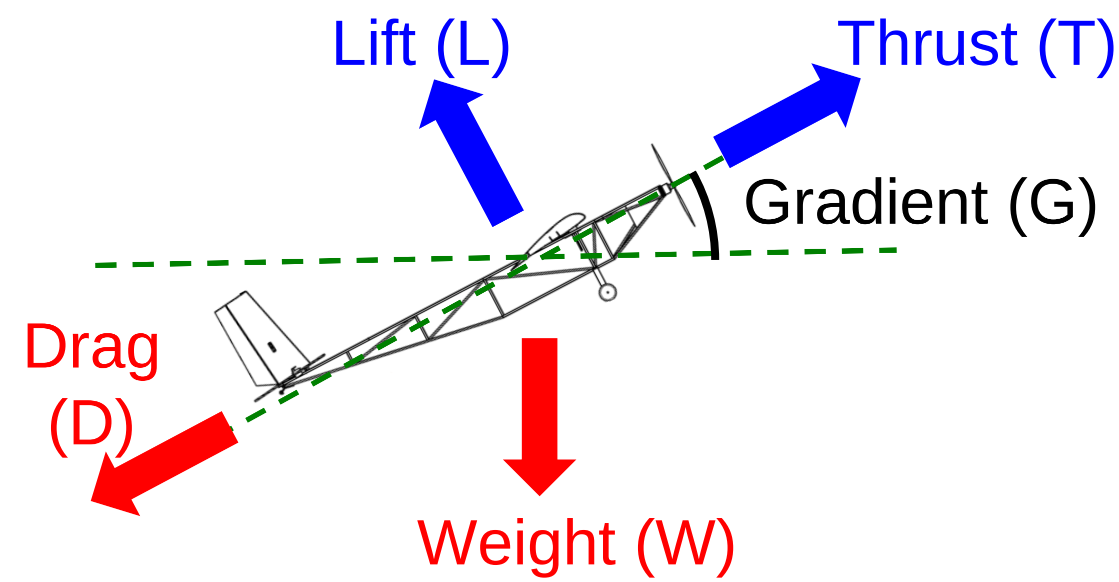
Climb equations

From equilibrium in both aerodynamic axes:

$$T = D + W \cdot \sin(G)$$
$$L = W \cdot \cos(\mathrm{G})$$

For small values of G:

$$T = D + W \cdot G$$
$$L = W$$



Let's start with the lift equation:

$$L = W$$

$$\frac{\rho V^2}{2} \cdot S_{ref} \cdot C_L = W$$

$$C_L = \frac{2W}{\rho S_{ref} V^2}$$

The requirement states that:

$$V = k_s V_{stall} = k_s \sqrt{\frac{2W}{\rho S_{ref} C_{Lmax,CL}}}$$

Then the climb lift coefficient is:

$$C_L = \frac{C_{Lmax,CL}}{k_s^2}$$

Climb equations

Now let's go back to the first equation of the system:

$$T = D + W \cdot G$$

$$\frac{T}{W} = \frac{D}{W} + G$$

$$\frac{T}{W} = \frac{D}{L} + G$$

$$\frac{T}{W} = \frac{C_D}{C_L} + G$$

$$\frac{T}{W} = \frac{C_{D0} + KC_L^2}{C_L} + G$$

$$\frac{T}{W} = \frac{C_{D0}}{C_L} + KC_L + G$$

$$\boxed{\frac{T}{W} = \frac{k_s^2}{C_{Lmax,CL}} C_{D0} + \frac{C_{Lmax,CL}}{k_s^2} K + G}$$

IMPORTANT:

- Aerodynamic coefficients may change from one climb phase to another.
- We need to correct T/W to compare with takeoff values.

Thrust corrections

$$\frac{T_{TO}}{W_{TO}} = \frac{1}{0.94} \cdot \frac{N}{N-1} \cdot \frac{W}{W_{TO}} \cdot \frac{T}{W}$$

Calculated with the climb equation

Factor to take into account weight changes.
Important for balked landing climb phases, when it should be MLW/MTOW

Ratio between maximum thrust and maximum continuous thrust.
Use this ratio only if the climb phase requires maximum continuous thrust.

Factor for one engine inoperative condition (OEI). N is the total number of engines of the airplane.
Only use this factor if the climb phase requires OEI.

FAR 25 Requirements

- FAR 25.111 (takeoff climb):
  - OEI (One engine inoperative). Remaining engines at takeoff thrust or power (T_{TO}).
  - $G > 1.2\%$ (two-engine airplanes)
  - $G > 1.5\%$ (three-engine airplanes)
  - $G > 1.7\%$ (four-engine airplanes)
  - Takeoff flaps, landing gear retracted
  - $k_s = 1.2$
  - Maximum takeoff weight

This changes the aerodynamic coefficients!

FAR 25 Requirements

- FAR 25.121 (transition segment climb):
  - OEI (One engine inoperative). Remaining engines at takeoff thrust or power (T_{TO}).
  - G > 0.0% (two-engine airplanes)
  - G > 0.3% (three-engine airplanes)
  - G > 0.5% (four-engine airplanes)
  - Takeoff flaps, landing gear down
  - $1.1 < k_s < 1.2$
  - Maximum takeoff weight

FAR 25 Requirements

- FAR 25.121 (second segment climb):
  - OEI (One engine inoperative). Remaining engines at takeoff thrust or power (T_{TO}).
  - G > 2.4% (two-engine airplanes)
  - G > 2.7% (three-engine airplanes)
  - G > 3.0% (four-engine airplanes)
  - Takeoff flaps, landing gear retracted
  - $k_s = 1.2$
  - Maximum takeoff weight

FAR 25 Requirements

- FAR 25.121 (en-route climb):
  - OEI (One engine inoperative). Remaining engines at maximum continuous thrust or power (~ $0.94 \cdot T_{TO}$).
  - G > 1.2% (two-engine airplanes)
  - G > 1.5% (three-engine airplanes)
  - G > 1.7% (four-engine airplanes)
  - Flaps retracted, landing gear retracted
  - $k_s = 1.25$
  - Maximum takeoff weight

FAR 25 Requirements

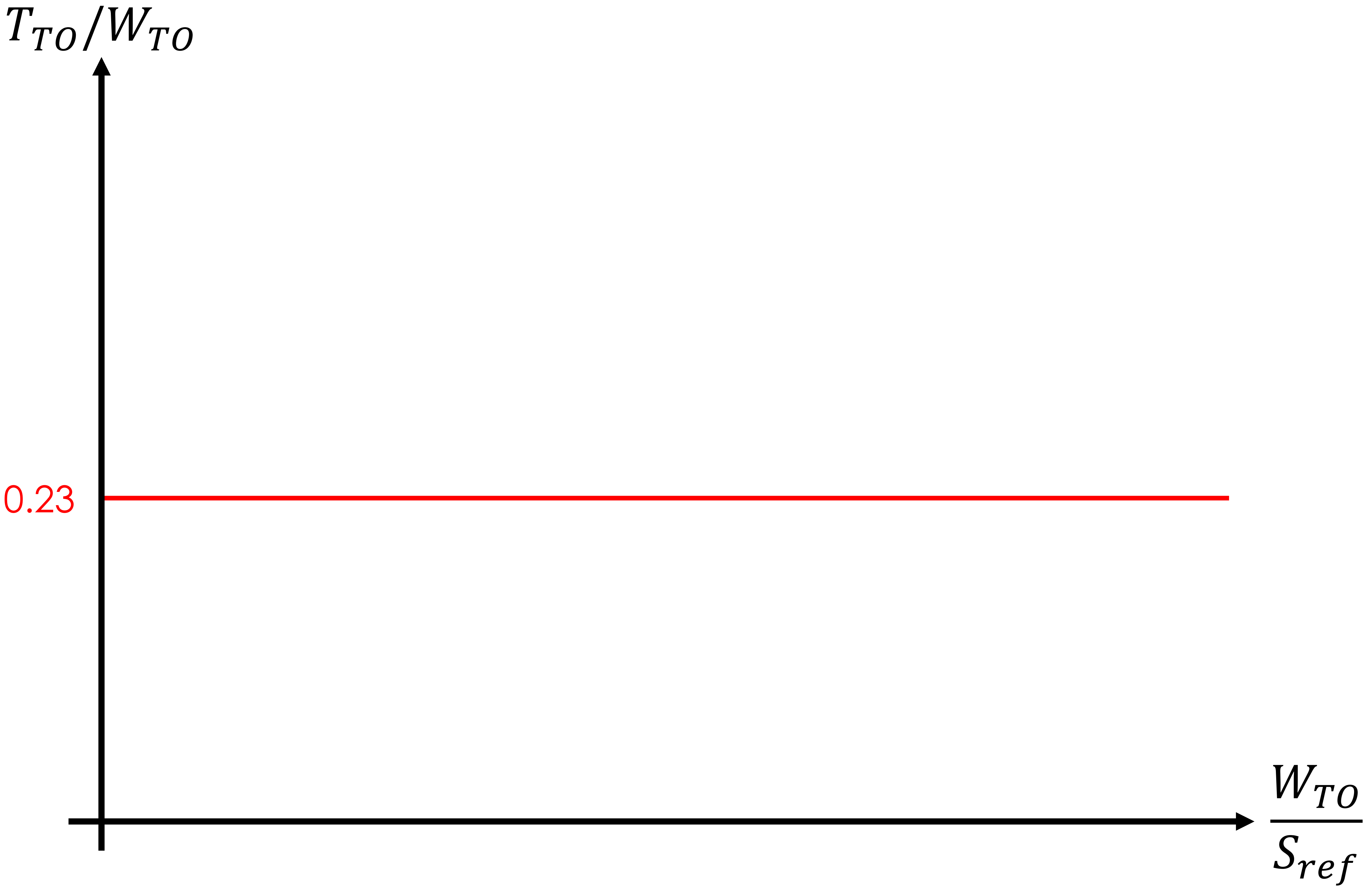
- FAR 25.119 (balked landing climb):
  - AEO (All engines operative).
  - $G > 3.2\%$
  - Landing flaps, landing gear down
  - $k_s = 1.3$
  - Maximum landing weight

FAR 25 Requirements

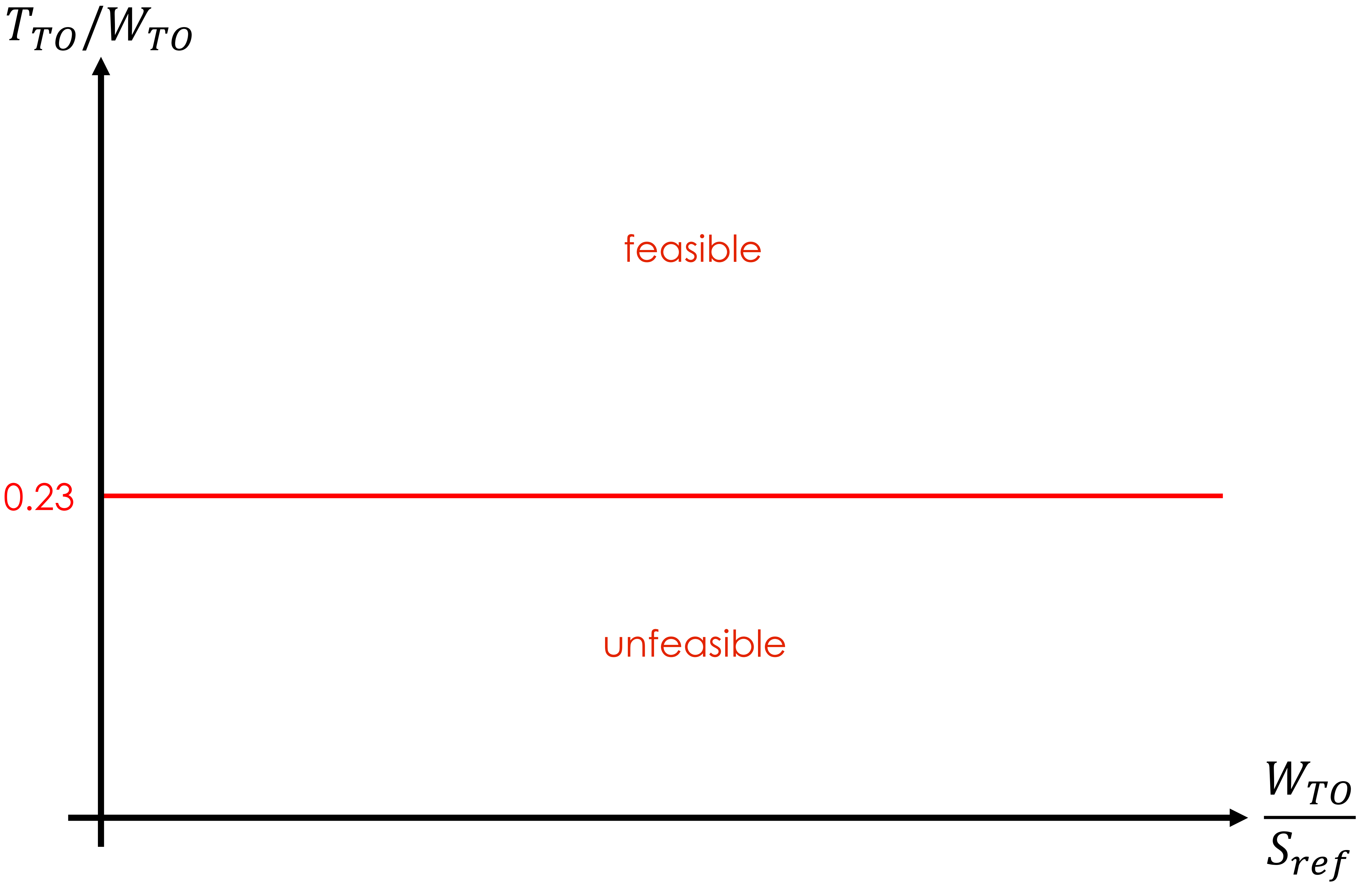
- FAR 25.121 (balked landing climb):
  - OEI (One engine inoperative). Remaining engines at takeoff thrust or power (T_{TO}).
  - G > 2.1% (two-engine airplanes)
  - G > 2.4% (three-engine airplanes)
  - G > 2.7% (four-engine airplanes)
  - Approach flaps, landing gear down
  - $k_s = 1.5$
  - Maximum landing weight

You can use $C_{Lmax,approach} \sim 0.85 C_{Lmax,L}$

Climb example



Climb example



Climb example [Roskam]

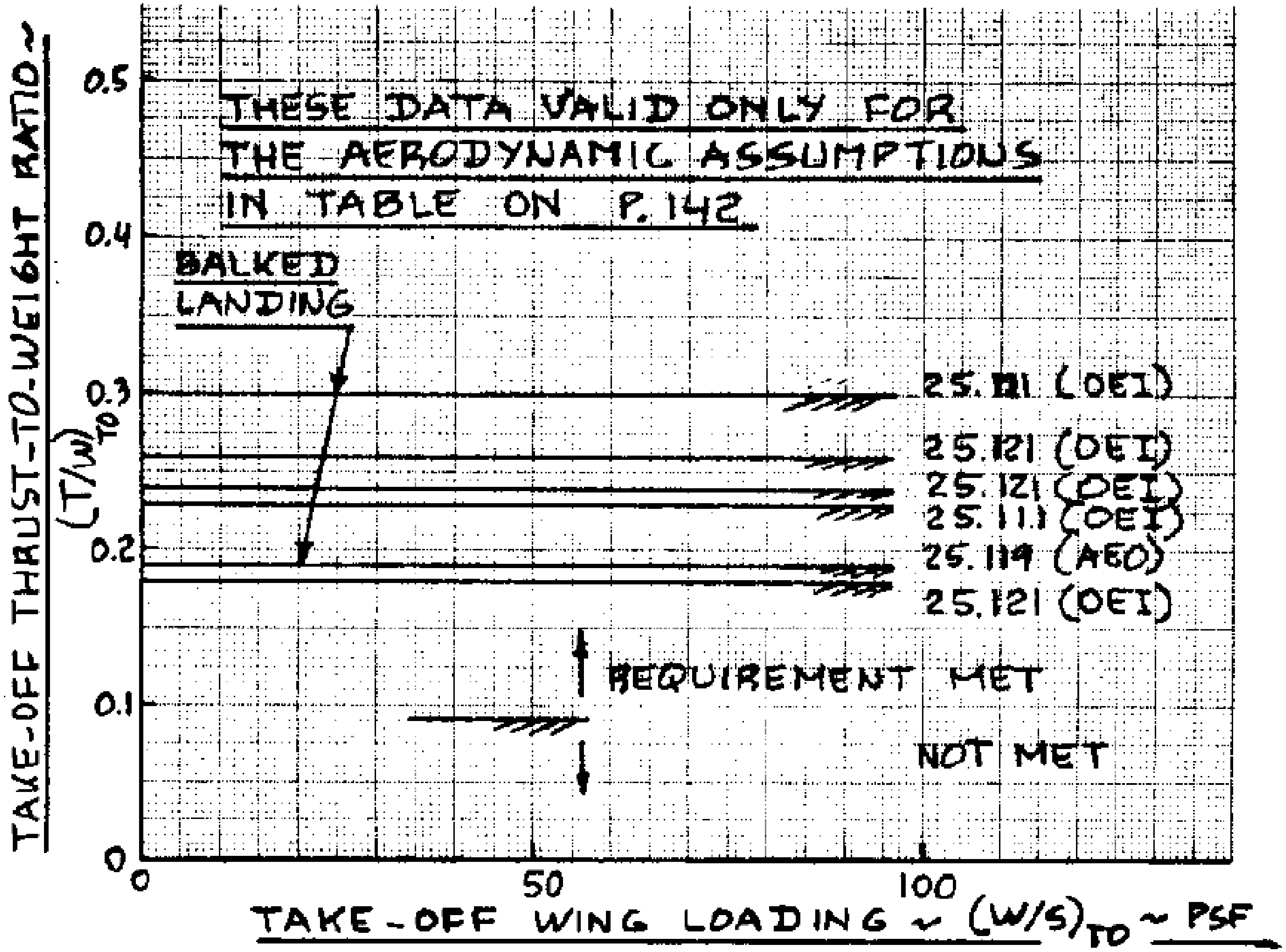


Figure 3.25 Effect of FAR 25 Climb Requirements on the Allowable Values of Take-off Thrust-to-Weight Ratio and Take-off Wing Loading

Performance Requirements

- Stall speed
- Takeoff
- Landing
- Climb
- Cruise speed
- Ceiling
- Maneuver (load factor)

komar.org

Performance Requirements

- Stall speed
- Takeoff
- Landing
- Climb
- Cruise speed
- Ceiling
- Maneuver (load factor)

komar.org

Cruise equations

- Assume that cruise altitude and cruise airspeed are given. We can compute the dynamic pressure as:

$$q = \frac{\rho V_{CR}^2}{2}$$

- For level flight:

$$T_{CR} = D$$
$$L = W_{CR}$$

- From the lift equation:

$$L = W_{CR} \quad \rightarrow \quad qSC_L = W_{CR} \quad \rightarrow \quad C_L = \frac{1}{q} \cdot \frac{W_{CR}}{S_{ref}}$$

- Dividing the equilibrium equations:

$$\frac{T_{CR}}{W_{CR}} = \frac{D}{L} = \frac{C_D}{C_L} = \frac{C_{D0}}{C_L} + KC_L \quad \rightarrow \quad \boxed{\frac{T_{CR}}{W_{CR}} = \frac{q}{W_{CR}/S_{ref}} C_{D0} + \frac{W_{CR}/S_{ref}}{q} K}$$

Cruise equations

- Assume that cruise altitude and cruise airspeed are given. We can compute the dynamic pressure as:

$$q = \frac{\rho V_{CR}^2}{2}$$

- For level flight:

$$T_{CR} = D$$
$$L = W_{CR}$$

- From the lift equation:

$$L = W_{CR} \quad \rightarrow \quad qSC_L = W_{CR} \quad \rightarrow \quad C_L = \frac{1}{q} \cdot \frac{W_{CR}}{S_{ref}}$$

- Dividing the equilibrium equations:

$$\frac{T_{CR}}{W_{CR}} = \frac{D}{L} = \frac{C_D}{C_L} = \frac{C_{D0}}{C_L} + KC_L \quad \rightarrow \quad \boxed{\frac{T_{CR}}{W_{CR}} = \frac{q}{W_{CR}/S_{ref}} C_{D0} + \frac{W_{CR}/S_{ref}}{q} K}$$

Compare this value with the one you used for the weight estimation!

Cruise corrections

- We need to adjust cruise thrust and weight.

$$\frac{W_{TO}}{S_{ref}} = \frac{1}{W_{CR}/W_{TO}} \cdot \frac{W_{CR}}{S_{ref}}$$

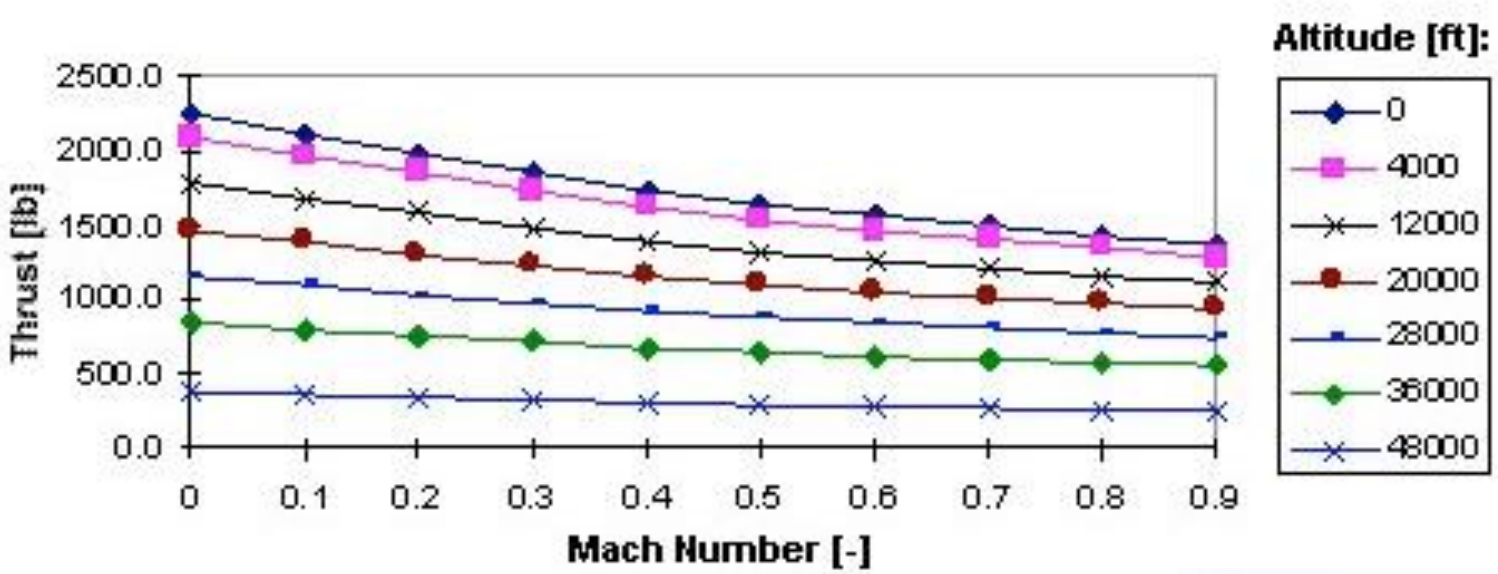
$$\frac{T_{TO}}{W_{TO}} = \frac{W_{CR}/W_{TO}}{T_{CR}/T_{TO}} \cdot \frac{T_{CR}}{W_{CR}}$$

- The weight correction comes from the fuel fractions you obtained previously.

$$\frac{W_{CR}}{W_{TO}} = \frac{W_{CL}}{W_{TO}} \cdot \frac{W_{CR}}{W_{CL}}$$

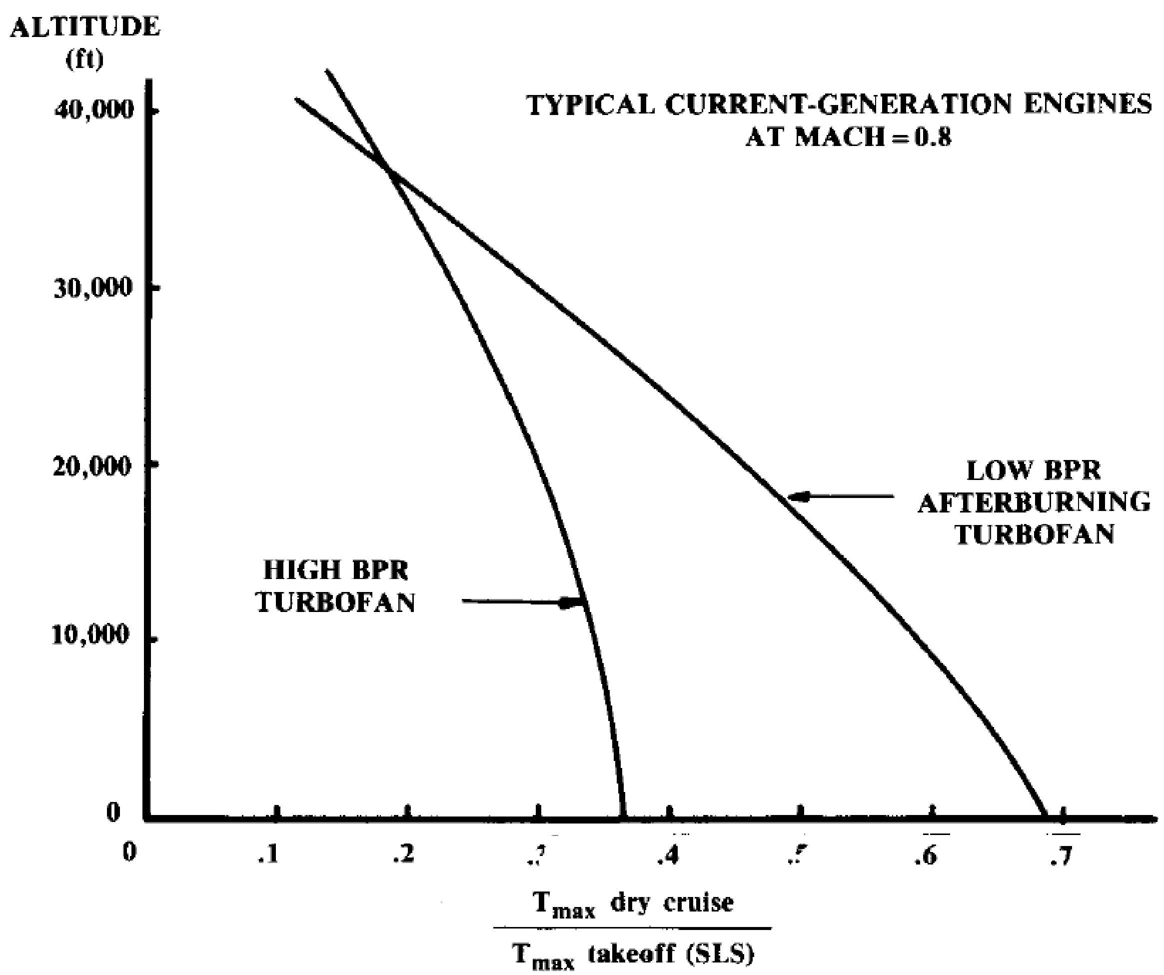
- Engine data should be used for thrust correction.

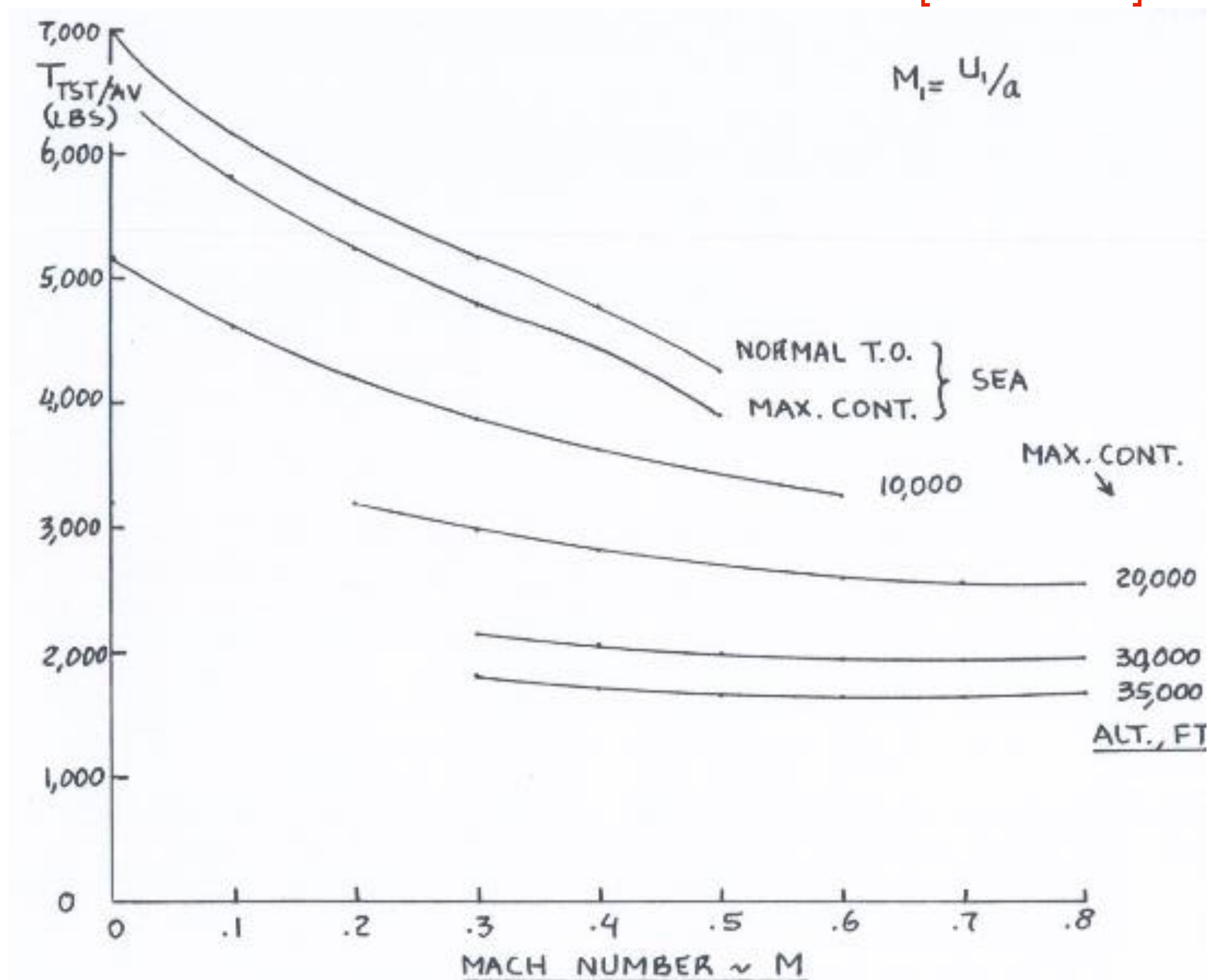
Thrust Variation with Altitude and Mach Number



Williams FJ44-2A, from [Aerospaceweb]

Generic turbofan [Roskam]

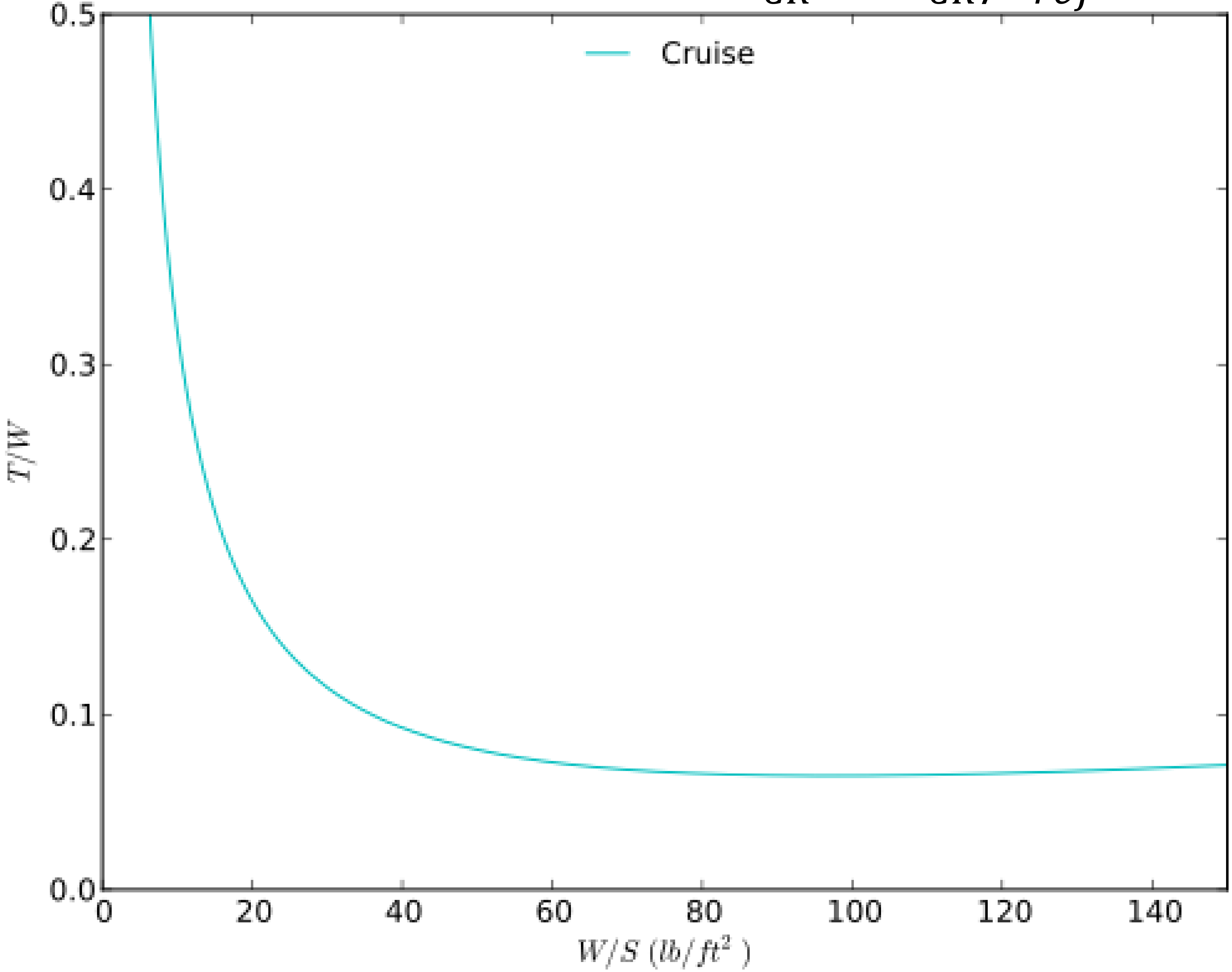




Generic turbofan [Raymer]

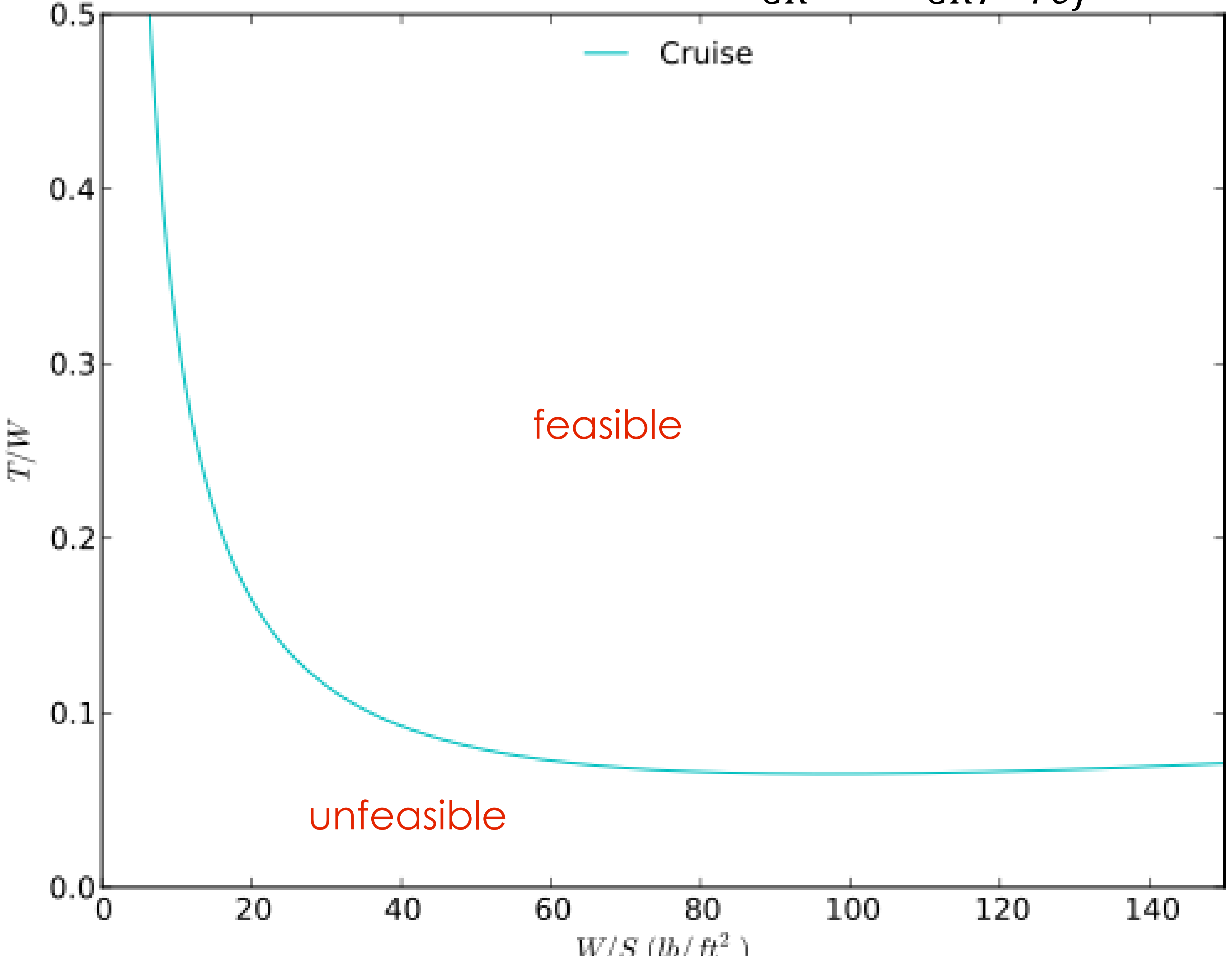
Sizing to Cruise Speed Requirements

$$\frac{T_{CR}}{W_{CR}} = \frac{q}{W_{CR}/S_{ref}} C_{D0} + \frac{W_{CR}/S_{ref}}{q} K$$



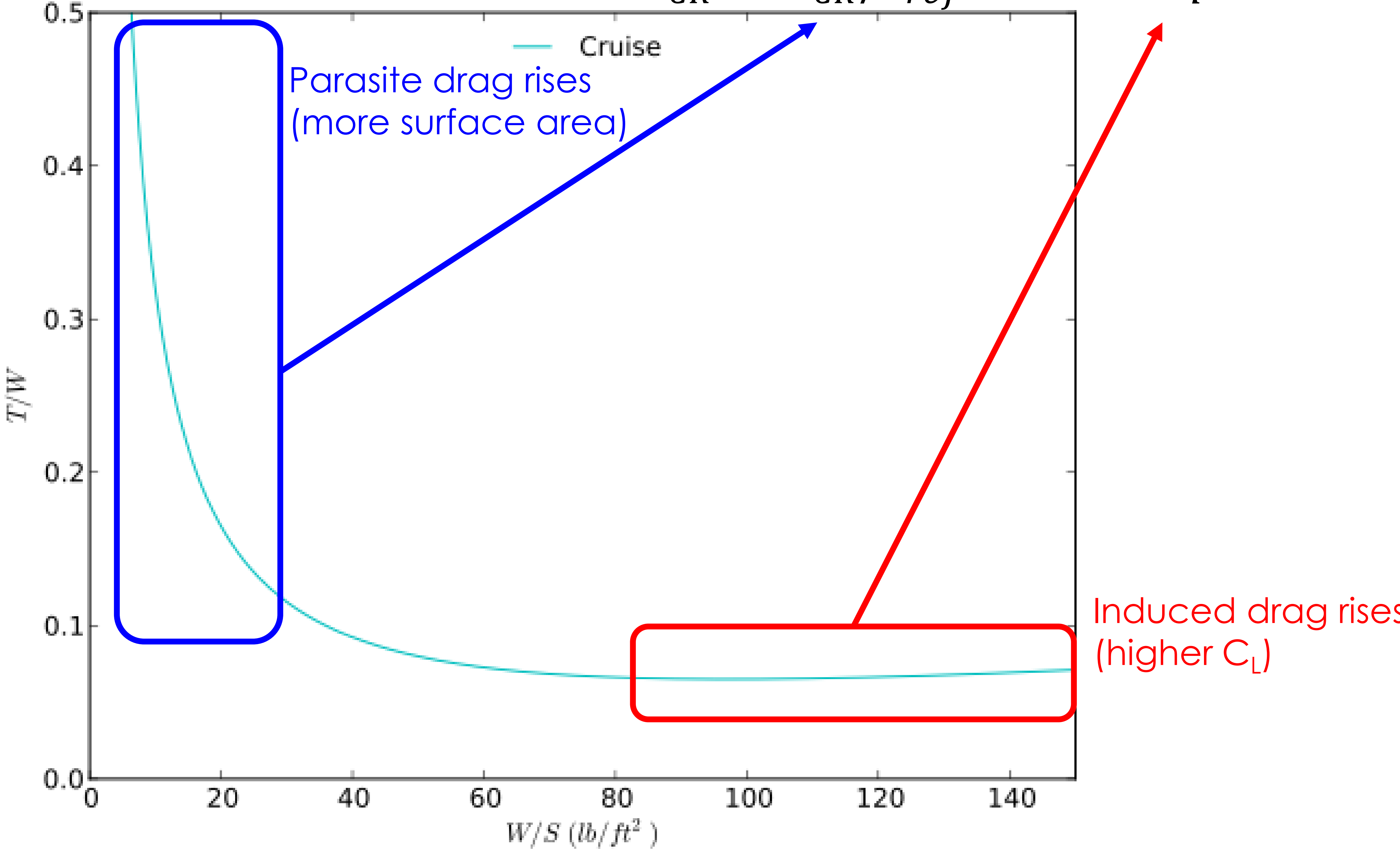
Sizing to Cruise Speed Requirements

$$\frac{T_{CR}}{W_{CR}} = \frac{q}{W_{CR}/S_{ref}} C_{D0} + \frac{W_{CR}/S_{ref}}{q} K$$



Sizing to Cruise Speed Requirements

$$\frac{T_{CR}}{W_{CR}} = \frac{q}{W_{CR}/S_{ref}} C_{D0} + \frac{W_{CR}/S_{ref}}{q} K$$



Performance Requirements

- Stall speed
- Takeoff
- Landing
- Climb
- Cruise speed
- Ceiling
- Maneuver (load factor)

komar.org

Performance Requirements

- Stall speed
- Takeoff
- Landing
- Climb
- Cruise speed
- Ceiling
- Maneuver (load factor)

komar.org

Ceiling equation

- Absolute ceiling is the maximum altitude that the aircraft can sustain a level flight.
- At the absolute ceiling, we have that G=0 (level flight). Therefore we can use the cruise results for this case too:

$$\frac{T_{CE}}{W_{CE}} = \frac{q}{W_{CE}/S_{ref}} C_{D0} + \frac{W_{CE}/S_{ref}}{q} K$$

- However, we can select an airspeed that minimizes the thrust requirement. If you derive the equation above with respect to dynamic pressure you will get the following optimum:

$$q = \frac{W_{CE}}{S_{ref}} \sqrt{\frac{K}{C_{D0}}}$$

Ceiling equation

- If we substitute the dynamic pressure back into the first equation we get:

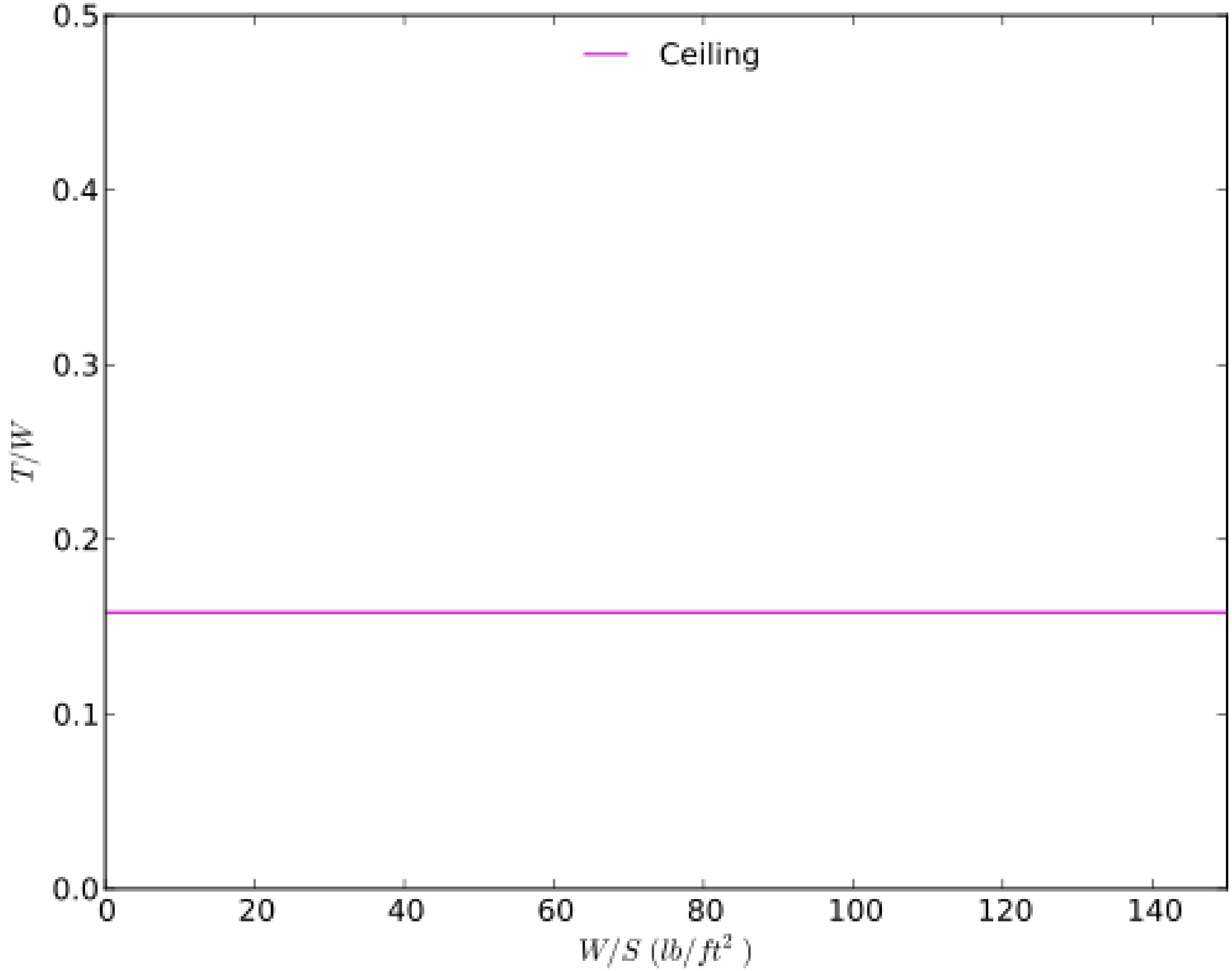
$$\frac{T_{CE}}{W_{CE}} = 2\sqrt{C_{D0} \cdot K}$$

- You may want to keep a small climb gradient (G=0.001) as a safety margin. Otherwise, it may be difficult to even reach the desired ceiling. In this case the equation becomes:

$$\frac{T_{CE}}{W_{CE}} = G + 2\sqrt{C_{D0} \cdot K}$$

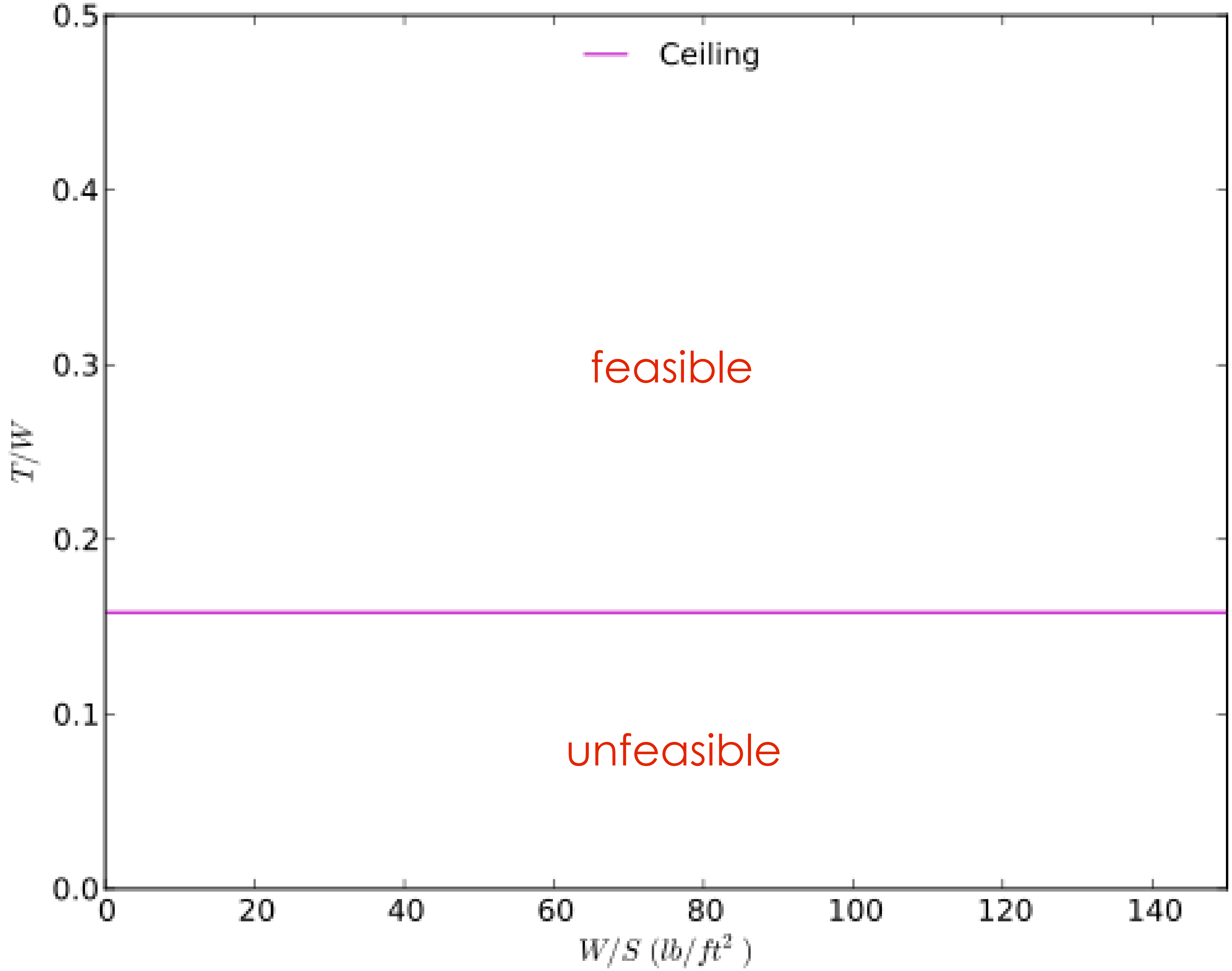
Sizing to Ceiling Requirements

$$\frac{T_{CE}}{W_{CE}} = G + 2\sqrt{C_{D0} \cdot K}$$



Sizing to Ceiling Requirements

$$\frac{T_{CE}}{W_{CE}} = G + 2\sqrt{C_{D0} \cdot K}$$



Performance Requirements

- Stall speed
- Takeoff
- Landing
- Climb
- Cruise speed
- Ceiling
- Maneuver (load factor)

komar.org

Performance Requirements

- Stall speed
- Takeoff
- Landing
- Climb
- Cruise speed
- Ceiling
- Maneuver (load factor)

komar.org

Maneuver (load factor)

- Load factor is approximated as:

$$n_z = \frac{L}{W_M}$$

- The maneuver equations are:

$$T_{MN} = D$$
$$L = n_z W_{MN}$$

- If we use the same procedures used for cruise we get:

$$\frac{T_{MN}}{W_{MN}} = \frac{q}{W_{MN}/S_{ref}} C_{D0} + n_z^2 \frac{W_{MN}/S_{ref}}{q} K$$

- It is important to check if the airplane is not exceeding its aerodynamic limit:

$$C_L = \frac{n_z}{q} \cdot \frac{W_{MN}}{S_{ref}} < C_{Lmax,MN}$$

Maneuver (load factor)

- During a sustained turn, the aircraft should maintain its airspeed and altitude. If the sustained turn rate $\dot{\psi}$ is given, we can compute the corresponding load factor:

$$n_z = \sqrt{\left(\frac{\dot{\psi}V}{g}\right)^2 + 1}$$

- If we have an instantaneous turn requirement, we can neglect the thrust-drag equation. The aircraft will be limited by its maximum lift capability in this case. Using the lift-weight equation for maximum C_L gives:

$$\frac{W_{MN}}{S_{ref}} = \frac{qC_{Lmax,MN}}{n_z}$$

Performance Requirements

- Stall speed
- Takeoff
- Landing
- Climb
- Cruise speed
- Ceiling
- Maneuver (load factor)

komar.org

Takeaways

- Higher wing-loading (i.e. smaller wing) means
  - Higher stall speed (bad)
  - Longer take-off and landing distances (bad)
  - Poorer maneuvering performance (bad)
  - Reduced drag, weight and cost (very, very good)

Typically, smallest wing (i.e. highest wing loading) that meets the requirements is best

- Higher thrust-to-weight ratio (i.e. bigger engine) means:
  - Better takeoff, climb, and cruise performance (good)
  - Higher engine weight (bad)
  - Higher engine cost and fuel consumption (very, very bad)

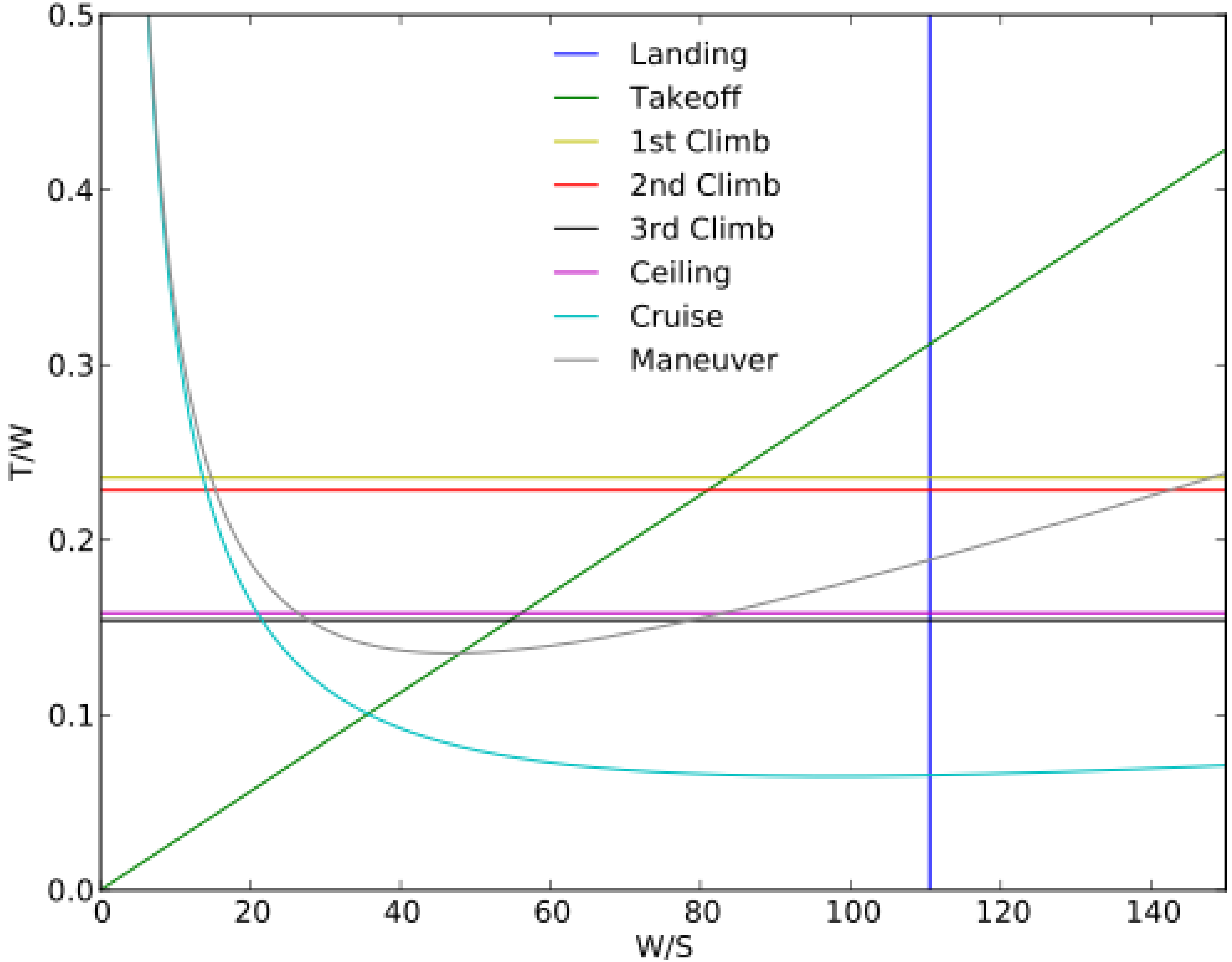
Typically, smallest engine (i.e. lowest T/W) that does the job is best

Matching all the results

1. Specify a range of wing loading at takeoff condition (W_{TO}/S_{ref}).
2. For each mission requirement:
   1. Apply correction factors to the takeoff wing loading to reflect weight changes, if necessary.
   2. Use the requirement equations to get the corresponding thrust-to-weight ratio.
   3. Apply corrections to the thrust-to-weight ratio to take it back to takeoff condition.
3. Plot the results in a W/S x T/W chart.
4. Identify feasible region.

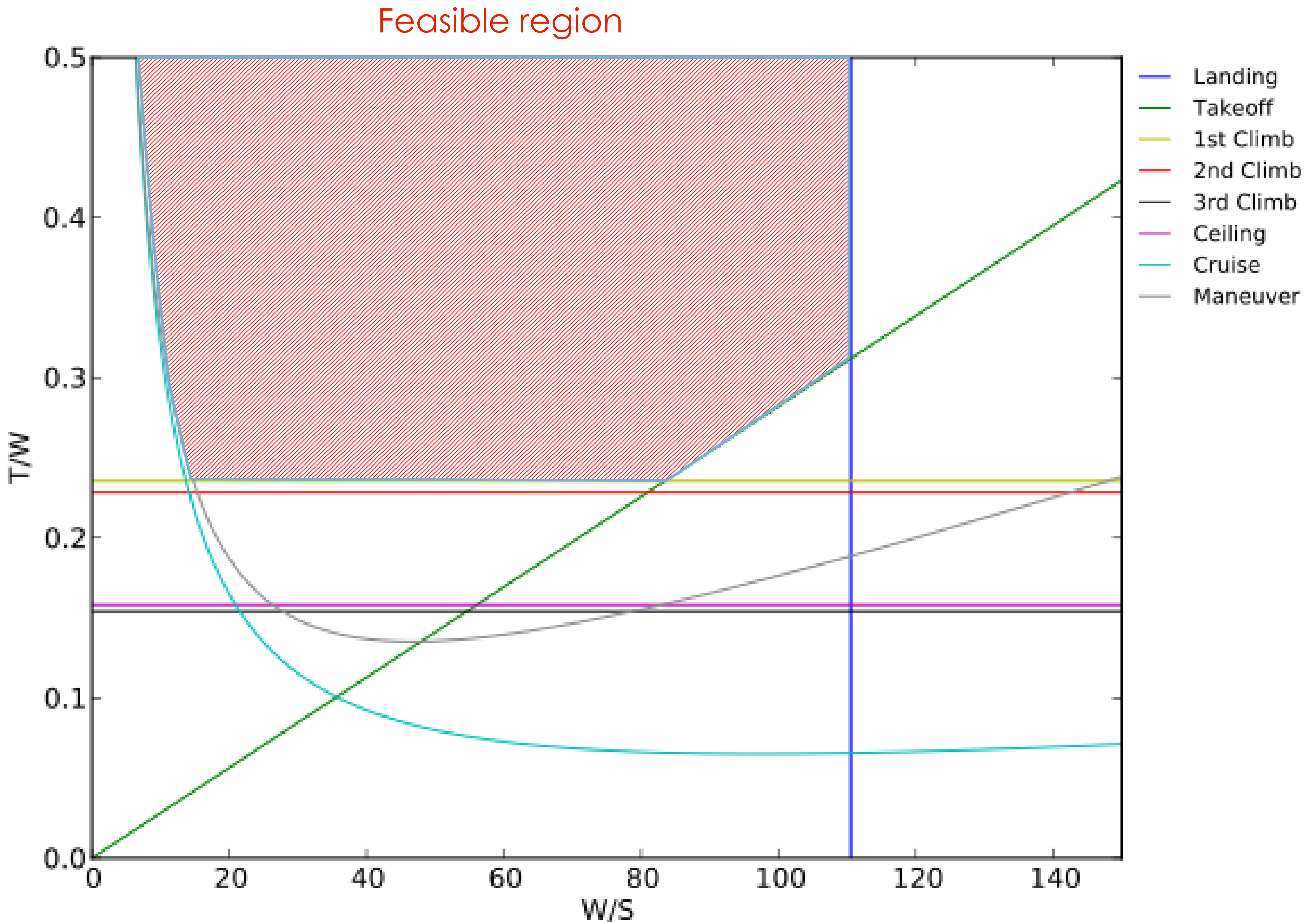
Some requirements depend only on T/W or W/S (e.g. climb or stall speed), so you only need to compute one value and draw a straight line.

Example



Landing
Takeoff
1st Climb
2nd Climb
3rd Climb
Ceiling
Cruise
Maneuver
T/W
W/S
0.0
0.1
0.2
0.3
0.4
0.5
0
20
40
60
80
100
120
140

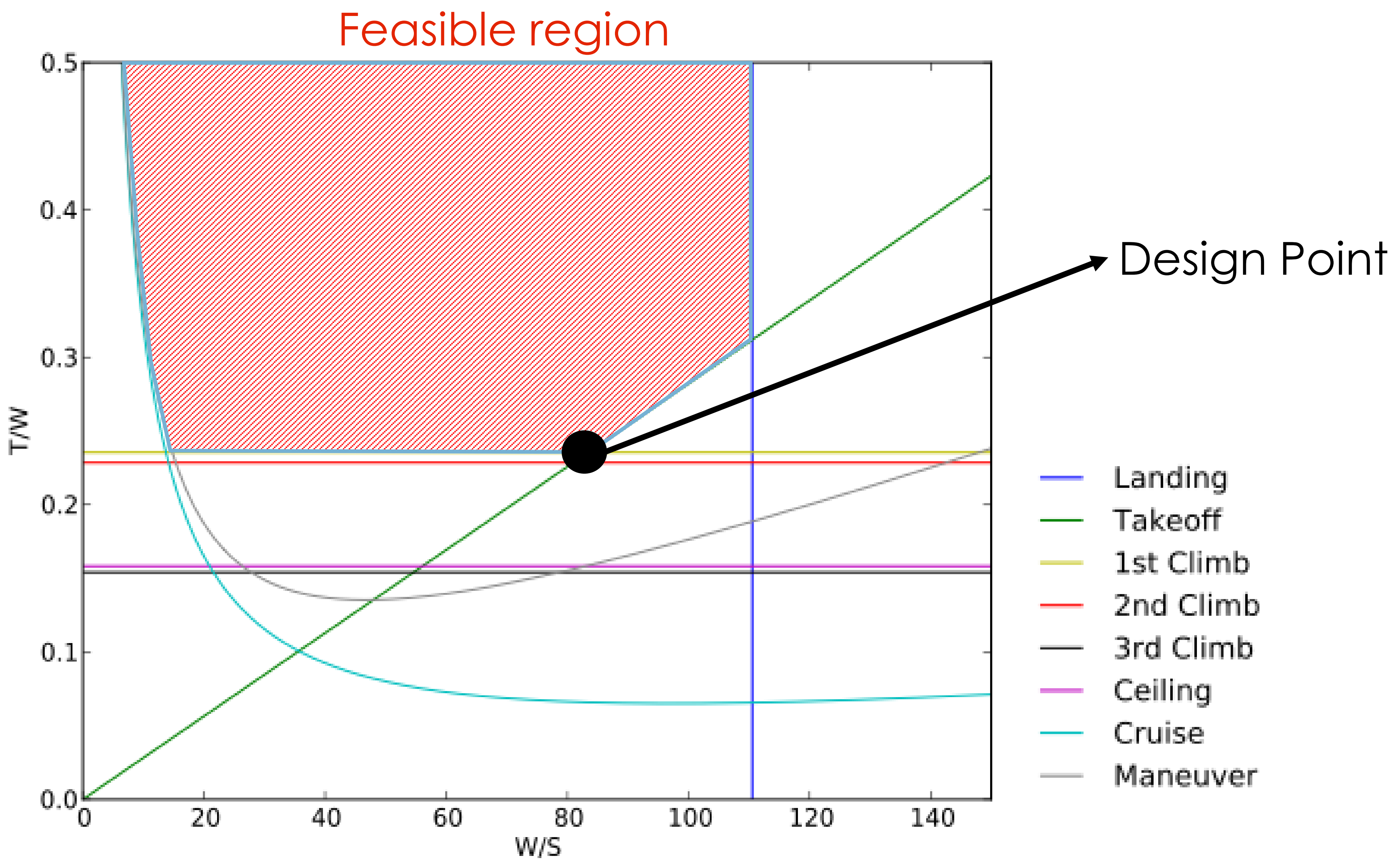
Example



Feasible region
0.5
0.4
0.3
0.2
0.1
0.0
T/W
0
20
40
60
80
100
120
140
W/S
Landing
Takeoff
1st Climb
2nd Climb
3rd Climb
Ceiling
Cruise
Maneuver

Selecting a Design Point

- You usually want the lowest T/W (smaller engine) and the greatest W/S (smaller airplane) within the feasible region.



Sizing airplane and engine

- Now that we have the values of wing loading and thrust-to-weight ratio, we can finally compute the reference area and the engine thrust for our airplane:

$$S_{ref} = \frac{W_{TO}}{W_{TO}/S_{ref}}$$

$$T_{TO} = W_{TO} \cdot \frac{T_{TO}}{W_{TO}}$$

Examples

Which one has a higher W/S?

Canada goose

Pelican

Which one has a higher W/S?

Canada goose
5.8 lb/ft^2

Pelican
1.3 lb/ft^2

Which one has a higher W/S?

Cessna 152

B747-400

Which one has a higher W/S?

Cessna 152
10 lb/ft^2

B747-400
151 lb/ft^2

Which one has the lowest W/S?

Transall C-160
(Utility transport)

F-104G Starfighter
(Supersonic interceptor)

F-15C Eagle
(Air superiority)

Which one has the lowest W/S?

Transall C-160
(Utility transport)
65 lb/ft^2

F-104G Starfighter
(Supersonic interceptor)
148 lb/ft^2

F-15C Eagle
(Air superiority)
111 lb/ft^2

Which one has a higher T/W?

B787
(Jet transport)

F-22
(Fighter)

Which one has a higher T/W?

B787
(Jet transport)

F-22
(Fighter)

T/W=0.28

T/W=1.14

Which one has a higher T/W?

BAe-146
(Regional Transport)

E-190
(Regional Transport)

Which one has a higher T/W?

BAe-146
(Regional Transport)

T/W=0.30

E-190
(Regional Transport)

T/W=0.35

More engines → OEI conditions may become easier

Which one has a higher T/W?

Concorde

B767-300

Which one has a higher T/W?

Concorde	B767-300
	
T/W=0.37	T/W=0.28

Cruise speed requirement
Aerodynamic efficiency

Direct Operating Cost (DOC)

- Figure of merit for transport airplanes.
  - Cost/Distance or Cost/(PAX.Distance).

- Method developed by Mentzer and Nourse of United Airlines (1944) and improved by ATA (Air Transport Association of America) in 1967.

- Roskam updated the method for 1989’s costs.

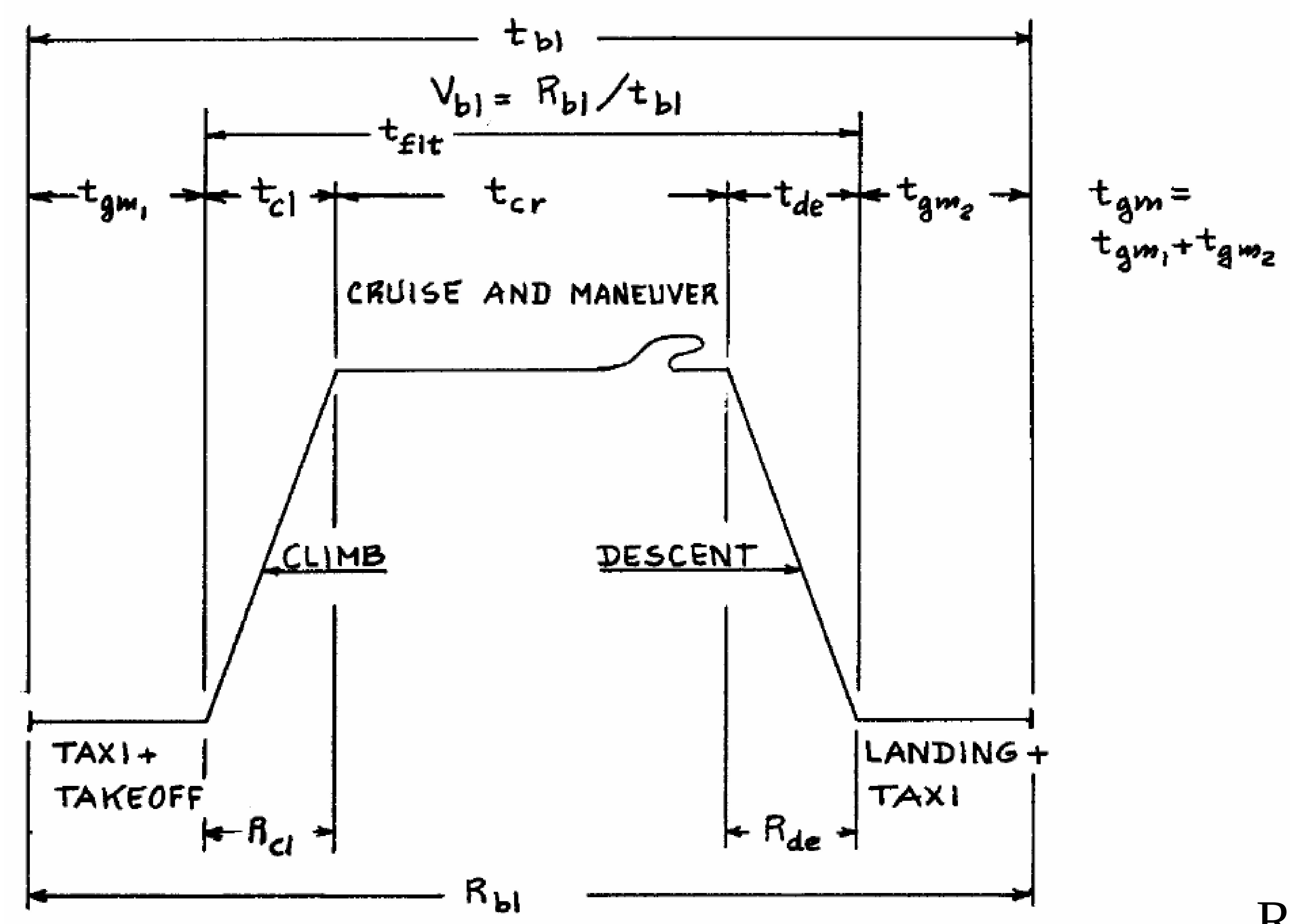
Direct Operating Cost Breakdown

- Flying
  - Crew, Fuel, and Insurance
- Maintenance
  - Labor Cost and Spare Parts
- Depreciation
  - Airframe, Engine, and Systems
- Landing and Navigation Fees
- Financing

Important factors for DOC estimation

- Weights
  - MTOW, Fuel Weight, Empty Weight
- Engine
  - Thrust, Weight
- Prices
  - Fuel Price, Labor Cost
- Mission Profile
  - Block time, Block range

Mission Profile



Roskam

Examples

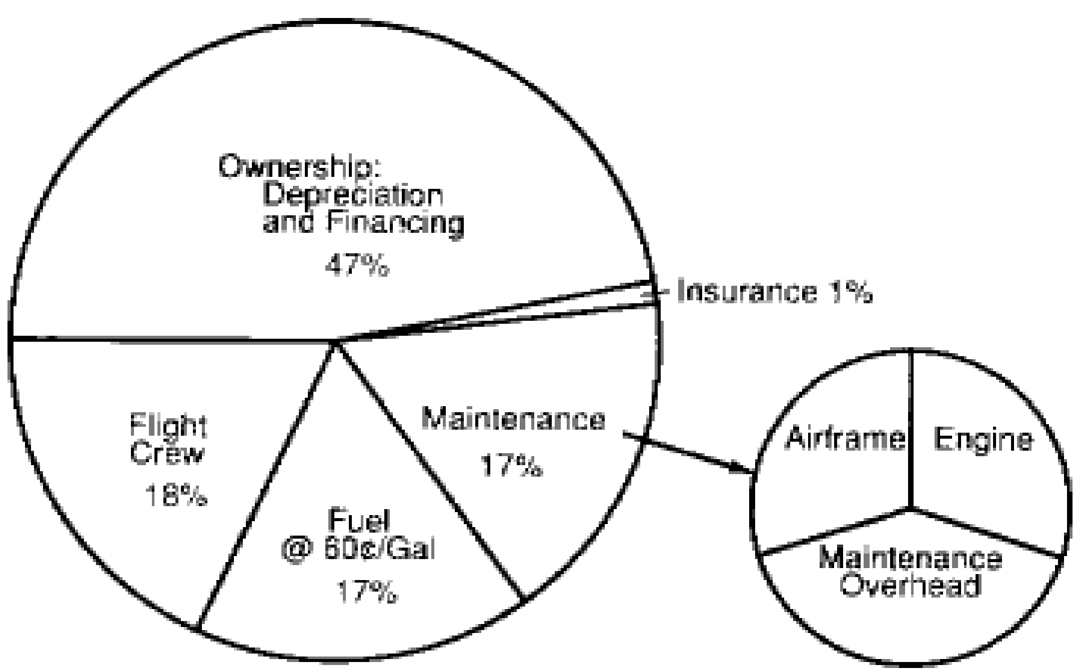
Direct Operating Cost Comparison
In Year 2000 $

Airplane (passengers)	DOC (ct/seat n.mi)	$/n.mi.
MD-81 at 500 n.mi.	6.15	8.79
MD-11 at 3000 n.mi.	5.81	17.03
747-400 at 3000 n.mi.	5.43	22.58

737-300

500 nmi

Direct Operating Cost
U. S. Domestic Rules



http://aerodesign.stanford.edu/aircraftdesign/cost/cost.html

Tarefa

- Seções 3.7 e 3.8 do roteiro