exercise1-newton (Score: 13.0 / 13.0)

1. Test cell (Score: 1.0 / 1.0)
2. Test cell (Score: 1.0 / 1.0)
3. Test cell (Score: 1.0 / 1.0)
4. Written response (Score: 1.0 / 1.0)
5. Test cell (Score: 1.0 / 1.0)
6. Coding free-response (Score: 2.0 / 2.0)
7. Test cell (Score: 1.0 / 1.0)
8. Coding free-response (Score: 2.0 / 2.0)
9. Written response (Score: 3.0 / 3.0)

Lab 2

1. 提交作業之前，建議可以先點選上方工具列的**Kernel**，再選擇**Restart & Run All**，檢查一下是否程式跑起來都沒有問題，最後記得儲存。
2. 請先填上下方的姓名(name)及學號(stduent_id)再開始作答，例如：

```
name = "我的名字"
student_id= "B06201000"
```

3. 四個求根演算法的實作可以參考lab-2 (https://yuanyuyuan.github.io/itcm/lab-2.html)，裡面有教學影片也有範例程式可以套用。
4. **Deadline: 10/9(Wed.)**

In [1]:

```
name = "歐陽秉志"
student_id = "B05201012"
```

Exercise 1 - Newton

Use the Newton's method to find roots of

$$f(x) = cosh(x) + cos(x) - c, \text{ for } c = 1, 2, 3,$$

Import libraries

In [2]:

```
import matplotlib.pyplot as plt
import numpy as np
```

1. Define the function $g(c)(x) = f(x) = cosh(x) + cos(x) - c$ with parameter $c = 1, 2, 3$ and its derivative df.

In [3]:

(Top)

```
def g(c):
    assert c == 1 or c == 2 or c == 3
    def f(x):
        return np.cosh(x) + np.cos(x) - c
    return f

def df(x):
    return np.sinh(x) - np.sin(x)
```

Pass the following assertion.

In [4]:

cell-b59c94b754b1fc9e

(Top)

```
assert g(1)(0) == np.cosh(0) + np.cos(0) - 1
assert df(0) == 0
### BEGIN HIDDEN TESTS
assert g(2)(0) == np.cosh(0) + np.cos(0) - 2
assert g(3)(0) == np.cosh(0) + np.cos(0) - 3
assert df(1) == np.sinh(1) - np.sin(1)
### END HIDDEN TESTS
```

2. Implement the algorithm

In [5]:

(Top)

```python
def newton(
    func,
    d_func,
    x_0,
    tolerance=1e-7,
    max_iterations=5,
    report_history=False,
    report_xn = False
):
    '''
    Parameters
    ----------
    func : function
        The target function.
    d_func : function
        The derivative of the target function.
    x_0 : float
        Initial guess point for a solution f(x)=0.
    tolerance : float
        One of the termination conditions. Error tolerance.
    max_iterations : int
        One of the termination conditions. The amount of iterations allowed.
    report_history: bool
        Whether to return history.
    report_xn: bool
        Whether to return hostory of x_n.

    Returns
    -------
    solution : float
        Approximation of the root.
    history: dict
        Return history of the solving process if report_history is True.
    '''

    x_n = x_0
    num_iter = 0

    if report_history:
        history = {'estimation': [], 'error': []}
    if report_xn:
        history['lxn'] = []

    while True:
        fxn = func(x_n)

        error = abs(fxn)

        if report_history:
            history['estimation'].append(x_n)
            history['error'].append(error)
        if report_xn:
            history['lxn'].append(x_n)

        if error < tolerance:
            print('Found solution after',num_iter,'iterations.')
            return (x_n, history) if report_history else x_n

        dfxn = d_func(x_n)
        if dfxn == 0:
            print("f'({}) is 0, no solution found.".format(x_n))
            return (None, history) if report_history else None

        if num_iter < max_iterations:
            num_iter += 1
            x_n = x_n - fxn/dfxn
        else:
            print('max iteration achieved.')
            return (x_n, history) if report_history else x_n


    # ====================
```

Test your implementation with the assertion below.

In [6]:

cell-4d88293f2527c82d (Top)

```
root = newton(
    lambda x: x**2 - x - 1,
    lambda x: 2*x - 1,
    1.2,
    max_iterations=100,
    tolerance=1e-7,
    report_history=False
)
assert abs(root - ((1 + np.sqrt(5)) / 2)) < 1e-7
```

```
Found solution after 4 iterations.
```

3. Answer the following questions under the case $c = 1$.

Plot the function to find an interval that contains the zero of f if possible.

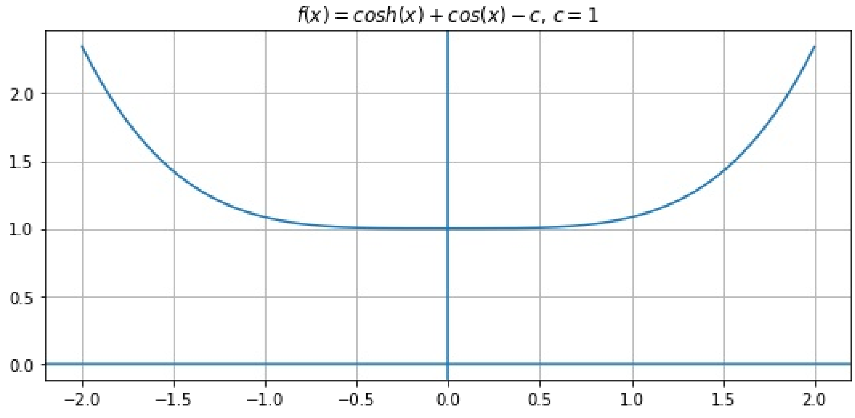
In [7]:

(Top)

```
c = 1
f = g(c)

search_range = np.arange(-2.0, 2.0, 0.001)
# ===================

fig, ax = plt.subplots(figsize=(9, 4))
ax.plot(search_range, f(search_range))
ax.set_title(r'$f(x)=cosh(x)+cos(x)-c$, $c=$%d' % c)
ax.grid(True)
ax.axhline(y=0)
ax.axvline(x=0)
plt.show()
```



According to the figure above, estimate the zero of f.

For example,

```
root = 3        # 單根
root = -2, 1    # 多根
root = None     # 無解
```

In [8]:

(Top)

```
# Hint: root = ?
root = None
```

In [9]:

cell-d872c7c57f11c968

(Top)

```
print('My estimation of root:', root)
### BEGIN HIDDEN TESTS
if root == None:
    print('Right answer!')
else:
    raise AssertionError('Wrong answer!')
### END HIDDEN TESTS
```

```
My estimation of root: None
Right answer!
```

Try to find the zero with a tolerance of 10^{-10}. If it works, plot the error and estimation of each step. Otherwise, state the reason why the method failed on this case.

(Top)

root, history = newton(g(c), df, 1, 1e-10, 5, True, True) print(root)

the estimation failed since the function $cosh(x) + cos(x) - 1$ is always positive. It has no root. But the function returned **nan**, it means some things go wrong, we print our history:

In [10]:

```
print(history)
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
<ipython-input-10-0b220396248c> in <module>
----> 1 print(history)

NameError: name 'history' is not defined
```

我們發現找到第四次的時候，x_n 跑到8000多，所以我們把前一次的 x_n 的值代到g(c)和df看看，然後看他們的商

In [11]:

```
print(g(c)(-0.070283343782922), df(-0.070283343782922))
print('quotient:', g(c)(-0.070283343782922) / df(-0.070283343782922))
```

```
1.0000020334262283 -0.00011572734867298928
quotient: -8641.017399024095
```

所以 x_n 跑到8000多，這是因為微分後的值太小，而這導致g(c)(8640)算不出來（numpy.cosh 算不出來），所以導致nan。

---

4. Answer the following questions under the case $c = 2$.

Plot the function to find an interval that contains the zero of f if possible.

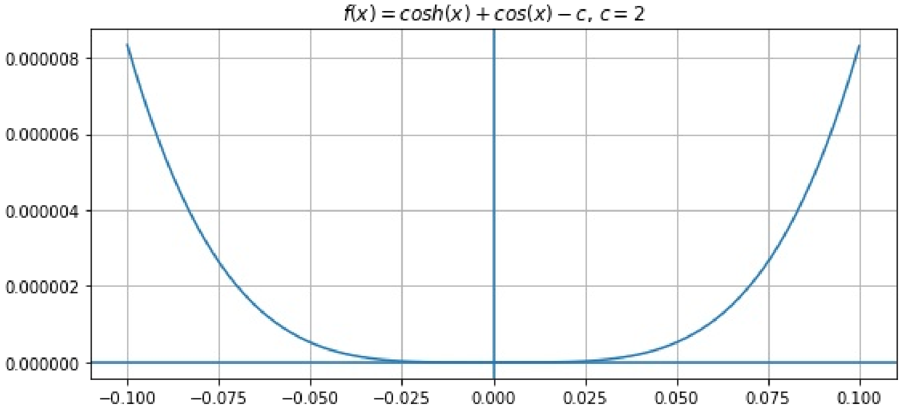
In [12]:

(Top)

```
c = 2
f = g(c)

# Hint: search_range = np.arange(左端點, 右端點, 點與點之間距),
# e.g. search_range = np.arange(0.0, 1.0, 0.01)
# ===== 請實做程式 =====
search_range = np.arange(-0.1, 0.1, 0.0001)
# ====================

fig, ax = plt.subplots(figsize=(9, 4))
ax.plot(search_range, f(search_range))
ax.set_title(r'$f(x)=cosh(x)+cos(x)-c$, $c=$%d' % c)
ax.grid(True)
ax.axhline(y=0)
ax.axvline(x=0)
plt.show()
```



According to the figure above, estimate the zero of f.

For example,

```
root = 3        # 單根
root = -2, 1    # 多根
root = None     # 無解
```

In [13]:

(Top)

```
# Hint: root = ?
# ===== 請實做程式 =====
root = 0
# ====================
```

In [14]:

cell-20fddbe6fa4c437b (Top)

```
print('My estimation of root:', root)

### BEGIN HIDDEN TESTS
assert type(root) is float or int, 'Wrong type!'
### END HIDDEN TESTS
```

```
My estimation of root: 0
```

Try to find the zero with a tolerance of 10^{-10}. If it works, plot the error and estimation of each step. Otherwise, state the reason why the method failed on this case.

In [15]:

```
root, history = newton(g(c), df, 1, 1e-10, 20, True, True)
print(root)
```

```
Found solution after 18 iterations.
0.005639347364278358
```

In [16]:

(Top)

```
找到根使得誤差小於 $10^{-10}$
```

```
  File "<ipython-input-16-8dc055813269>", line 1
    找到根使得誤差小於 $10^{-10}$
             ^
SyntaxError: invalid syntax
```

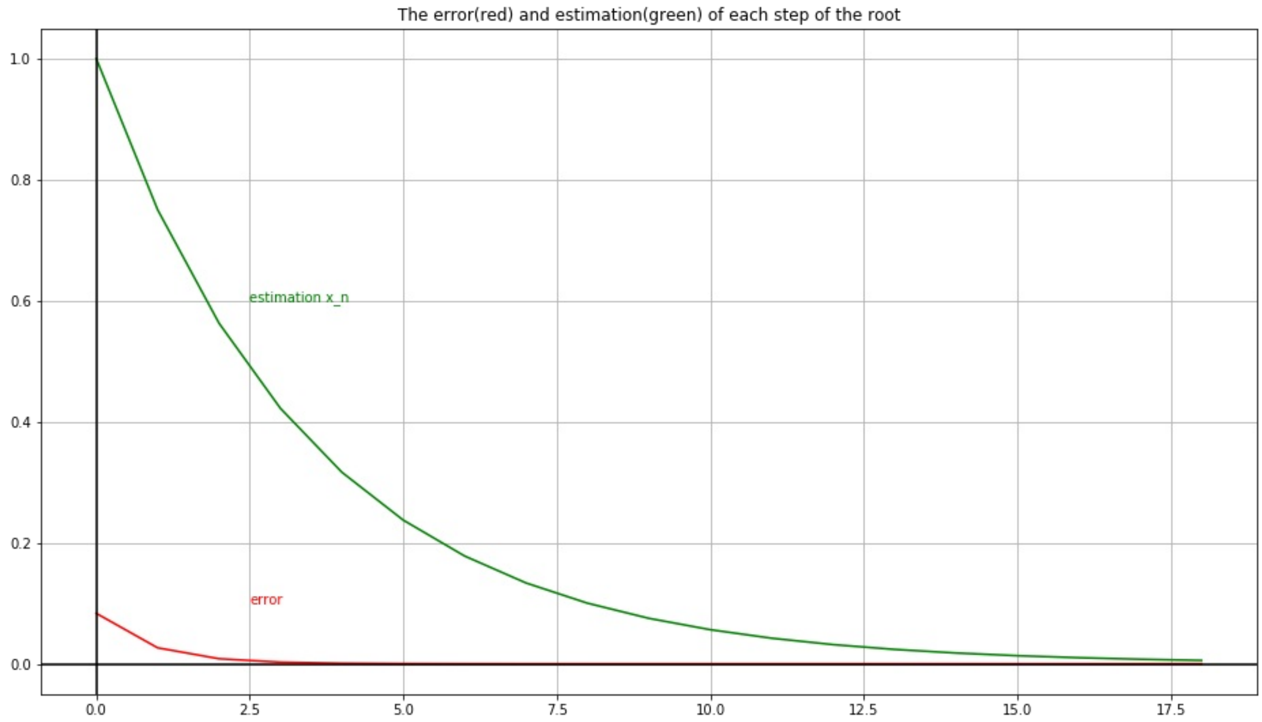
In [17]:

```
fig, ax = plt.subplots(figsize=(16, 9))

ax.plot(range(len(history['error'])), history['error'], 'r')
ax.plot(range(len(history['estimation'])), history['estimation'], 'g')
ax.set_title("The error(red) and estimation(green) of each step of the root")
ax.annotate('error', (2.5, 0.1), c='r')
ax.annotate('estimation x_n', (2.5, 0.6), c='g')
ax.grid(True)
ax.axhline(y=0, color='k')
ax.axvline(x=0, color='k')
```

Out[17]:

```
<matplotlib.lines.Line2D at 0x7f9da2c25c88>
```



5. Answer the following questions under the case $c = 3$.

Plot the function to find an interval that contains the zeros of f if possible.

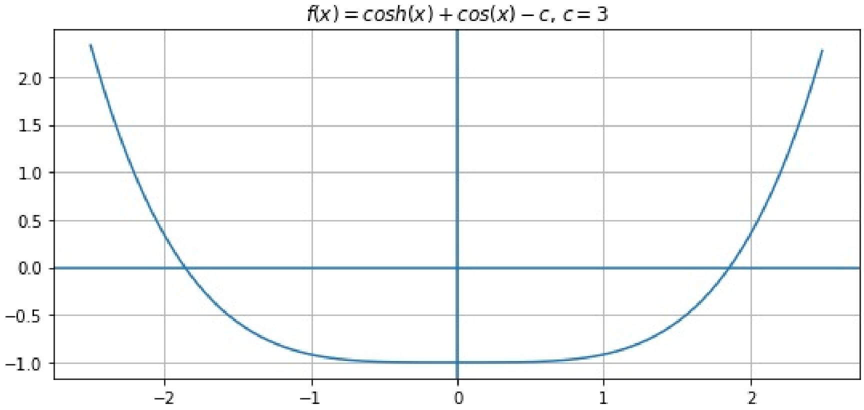
In [18]:

(Top)

```
c = 3
f = g(c)

# Hint: search_range = np.arange(左端點, 右端點, 點與點之間距),
# e.g. search_range = np.arange(0.0, 1.0, 0.01)
# ===== 請實做程式 =====
search_range = np.arange(-2.5, 2.5, 0.01)
# ====================

fig, ax = plt.subplots(figsize=(9, 4))
ax.plot(search_range, f(search_range))
ax.set_title(r'$f(x)=cosh(x)+cos(x)-c$, $c=$%d' % c)
ax.grid(True)
ax.axhline(y=0)
ax.axvline(x=0)
plt.show()
```



According to the figure above, estimate the zero of f.

For example,

```
root = 3        # 單根
root = -2, 1    # 多根
root = None     # 無解
```

In [19]:

(Top)

```
# Hint: root = ?
# ===== 請實做程式 =====
root = -1.8, 1.8
# ====================
```

In [20]:

cell-06ec0b20844075c7

(Top)

```
print('My estimation of root:', root)

### BEGIN HIDDEN TESTS
assert type(root) == tuple, 'Should be multiple roots!'
### END HIDDEN TESTS
```

```
My estimation of root: (-1.8, 1.8)
```

Try to find the zero with a tolerance of 10^{-10}. If it works, plot the error and estimation of each step. Otherwise, state the reason why the method failed on this case.

In [21]:

```
root1, history1 = newton(g(c), df, 2, 1e-10, 20, True, True)
root2, history2 = newton(g(c), df, -2, 1e-10, 20, True, True)
print("the roots are:", root1, root2)
```

```
Found solution after 4 iterations.
Found solution after 4 iterations.
the roots are: 1.8579208291501987 -1.8579208291501987
```

In [22]:

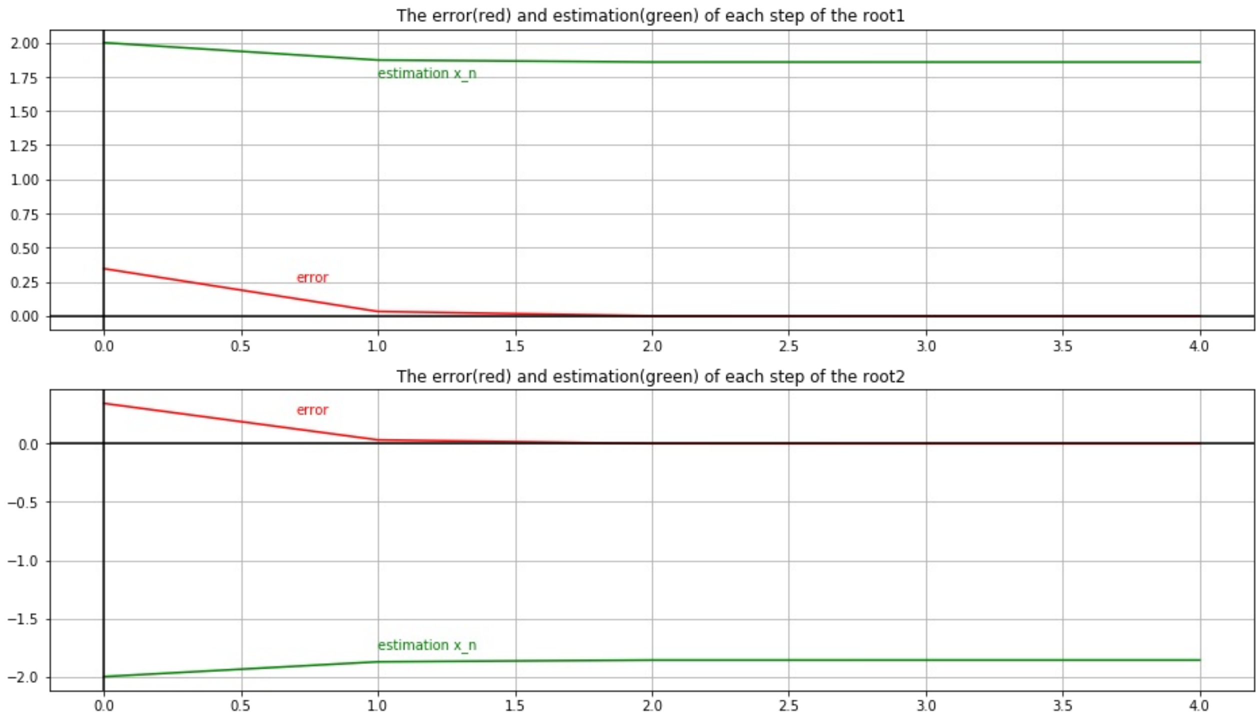
(Top)

```
fig, axes = plt.subplots(2, 1, figsize=(16, 9))
ax1, ax2 = axes

ax1.plot(range(len(history1['error'])), history1['error'], 'r')
ax1.plot(range(len(history1['estimation'])), history1['estimation'], 'g')
ax1.set_title("The error(red) and estimation(green) of each step of the root1")
ax1.annotate('error', (0.7, 0.25), c='r')
ax1.annotate('estimation x_n', (1, 1.75), c='g')
ax1.grid(True)
ax1.axhline(y=0, color='k')
ax1.axvline(x=0, color='k')

ax2.plot(range(len(history2['error'])), history2['error'], 'r')
ax2.plot(range(len(history2['estimation'])), history2['estimation'], 'g')
ax2.set_title("The error(red) and estimation(green) of each step of the root2")
ax2.annotate('error', (0.7, 0.25), c='r')
ax2.annotate('estimation x_n', (1, -1.75), c='g')
ax2.grid(True)
ax2.axhline(y=0, color='k')
ax2.axvline(x=0, color='k')

plt.show()
```



Discussion

For all cases above(c=1,2,3), do the results(e.g. error behaviors, estimations, etc) agree with the theoretical analysis?

(Top)

c=1 時因為在0點附近的導數會跑到0，g(c)(-0.070283343782922) / df(-0.070283343782922) 是 -8641 左右，這讓 x_n 從0附近跑到8000 多，所以找不到解

c=2跟c=3都work