exercise1-secant (Score: 14.0 / 14.0)

1. Test cell (Score: 1.0 / 1.0)
2. Test cell (Score: 1.0 / 1.0)
3. Test cell (Score: 1.0 / 1.0)
4. Written response (Score: 1.0 / 1.0)
5. Test cell (Score: 1.0 / 1.0)
6. Written response (Score: 1.0 / 1.0)
7. Test cell (Score: 1.0 / 1.0)
8. Coding free-response (Score: 4.0 / 4.0)
9. Written response (Score: 3.0 / 3.0)

Lab 2

1. 提交作業之前，建議可以先點選上方工具列的**Kernel**，再選擇**Restart & Run All**，檢查一下是否程式跑起來都沒有問題，最後記得儲存。
2. 請先填上下方的姓名(name)及學號(stduent_id)再開始作答，例如：

```
name = "我的名字"
student_id= "B06201000"
```

3. 四個求根演算法的實作可以參考lab-2 (https://yuanyuyuan.github.io/itcm/lab-2.html)，裡面有教學影片也有範例程式可以套用。
4. **Deadline: 10/9(Wed.)**

In [1]:

```
name = "歐陽秉志"
student_id = "B05201012"
```

Exercise 1 - Secant

Use the secant method to find roots of

$$f(x) = cosh(x) + cos(x) - c, \text{ for } c = 1, 2, 3,$$

Import libraries

In [2]:

```
import matplotlib.pyplot as plt
import numpy as np
```

1. Define a function $g(c)(x) = f(x) = cosh(x) + cos(x) - c$ with parameter $c = 1, 2, 3$.

In [3]:

(Top)

```
def g(c):
    assert c == 1 or c == 2 or c == 3
    def f(x):
        # Hint: return ...
        # ===== 請實做程式 =====
        return np.cosh(x) + np.cos(x) - c
        # ====================
    return f
```

Pass the following assertion.

In [4]:

cell-b59c94b754b1fc9e (Top)

```
assert g(1)(0) == np.cosh(0) + np.cos(0) - 1
### BEGIN HIDDEN TESTS
assert g(2)(0) == np.cosh(0) + np.cos(0) - 2
assert g(3)(0) == np.cosh(0) + np.cos(0) - 3
### END HIDDEN TESTS
```

2. Implement the algorithm

In [5]:

```
def xx(x):
    return 1, 2 if x else 1
```

In [6]:

(Top)

```python
def secant(
    func,
    interval,
    max_iterations=5,
    tolerance=1e-7,
    report_history=False,
):
    '''Approximate solution of f(x)=0 on interval [a,b] by the secant method.

    Parameters
    ----------
    func : function
        The target function.
    interval: list
        The initial interval to search
    max_iterations : (positive) integer
        One of the termination conditions. The amount of iterations allowed.
    tolerance: float
        One of the termination conditions. Error tolerance.
    report_history: bool
        Whether to return history.

    Returns
    -------
    result: float
        Approximation of the root.
    history: dict
        Return history of the solving process if report_history is True.
    '''

    # ===== 請實做程式 =====
    a, b = interval
    assert func(a) * func(b) < 0, 'This initial interval does not satisfied the prerequisites!'

    num_iterations = 0
    a_next, b_next = a, b

    if report_history:
        history = {'estimation': [], 'x_error': [], 'y_error': []}

    while True:
        d_x = -1*( func(a_next) * (b_next-a_next) / (func(b_next) - func(a_next)) )
        c = a_next + d_x

        x_error = abs(d_x)
        y_error = abs(func(c))

        if report_history:
            history['estimation'].append(c)
            history['x_error'].append(x_error)
            history['y_error'].append(y_error)

        # Satisfy the criterion and stop
        if x_error < tolerance or y_error < tolerance:
            print('Found solution after', num_iterations,'iterations.')
            return (c, history) if report_history else c

        if num_iterations < max_iterations:
            num_iterations += 1

            # Find the next interval
            value_of_func_c = func(c)
            if func(a_next) * value_of_func_c < 0:
                a_next = a_next
                b_next = c
            elif value_of_func_c * func(b_next) < 0:
                a_next = c
                b_next = b_next
            else:
                return (c, history) if report_history else c

        else:
            print('Terminate since reached the maximum iterations.')
            return (c, history) if report_history else c

    # ====================
```

Test your implementation with the assertion below.

In [7]:

```
secant(lambda x: x**2 - x - 1, [1.0, 2.0], max_iterations=100, tolerance=1e-7, report_history=False)
```

```
Found solution after 8 iterations.
```

Out[7]:

```
1.6180339631667067
```

In [8]:

cell-4d88293f2527c82d (Top)

```
root = secant(lambda x: x**2 - x - 1, [1.0, 2.0], max_iterations=100, tolerance=1e-7, report_history=False)
assert abs(root - ((1 + np.sqrt(5)) / 2)) < 1e-7
```

```
Found solution after 8 iterations.
```

3. Answer the following questions under the case $c = 1$.

Plot the function to find an interval that contains the zero of f if possible.

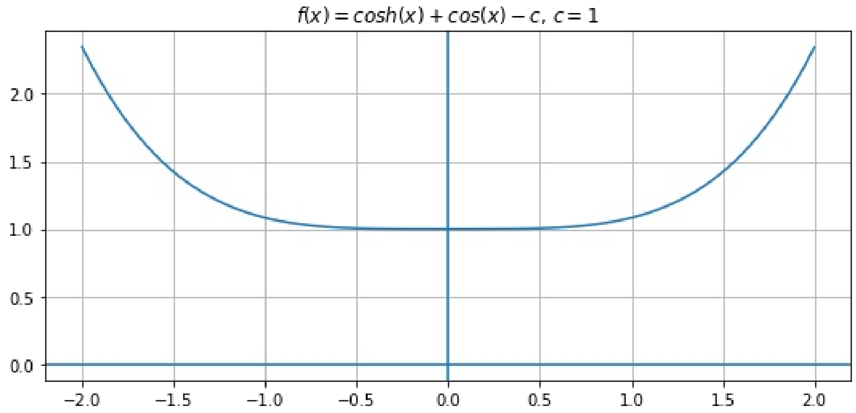
In [9]:

(Top)

```
c = 1
f = g(c)

# Hint: search_range = np.arange(左端點, 右端點, 點與點之間距),
# e.g. search_range = np.arange(0.0, 1.0, 0.01)
# ===== 請實做程式 =====
search_range = np.arange(-2.0, 2.0, 0.001)
# ====================

fig, ax = plt.subplots(figsize=(9, 4))
ax.plot(search_range, f(search_range))
ax.set_title(r'$f(x)=cosh(x)+cos(x)-c$, $c=$%d' % c)
ax.grid(True)
ax.axhline(y=0)
ax.axvline(x=0)
plt.show()
```



According to the figure above, estimate the zero of f.

For example,

```
root = 3         # 單根
root = -2, 1     # 多根
root = None      # 無解
```

In [10]:

(Top)

```
# Hint: root = ?
# ===== 請實做程式 =====
root = None
# ====================
```

In [11]:

cell-d872c7c57f11c968

(Top)

```
print('My estimation of root:', root)
### BEGIN HIDDEN TESTS
if root == None:
    print('Right answer!')
else:
    raise AssertionError('Wrong answer!')
### END HIDDEN TESTS
```

```
My estimation of root: None
Right answer!
```

Try to find the zero with a tolerance of 10^{-10}.I f it works, plot the error and estimation of each step. Otherwise, state the reason why the method failed on this case.

In [12]:

```
root = secant(g(c), [-1.0, 1.0], max_iterations=100, tolerance=1e-10, report_history=False)
```

```
---------------------------------------------------------------------------
AssertionError                            Traceback (most recent call last)
<ipython-input-12-745075b318fd> in <module>
----> 1 root = secant(g(c), [-1.0, 1.0], max_iterations=100, tolerance=1e-10, report_history=
False)

<ipython-input-6-434edf429c5b> in secant(func, interval, max_iterations, tolerance, report_hi
story)
     31     # ===== 請實做程式 =====
     32     a, b = interval
---> 33     assert func(a) * func(b) < 0, 'This initial interval does not satisfied the prere
quisites!'
     34 
     35     num_iterations = 0

AssertionError: This initial interval does not satisfied the prerequisites!
```

(Top)

the estimation failed since the function $cosh(x) + cos(x) - 1$ is always positive. It is not possible to find an interval [a, b]

s.t. f(a) * f(b) < 0

4. Answer the following questions under the case $c = 2$.

Plot the function to find an interval that contains the zero of f if possible.

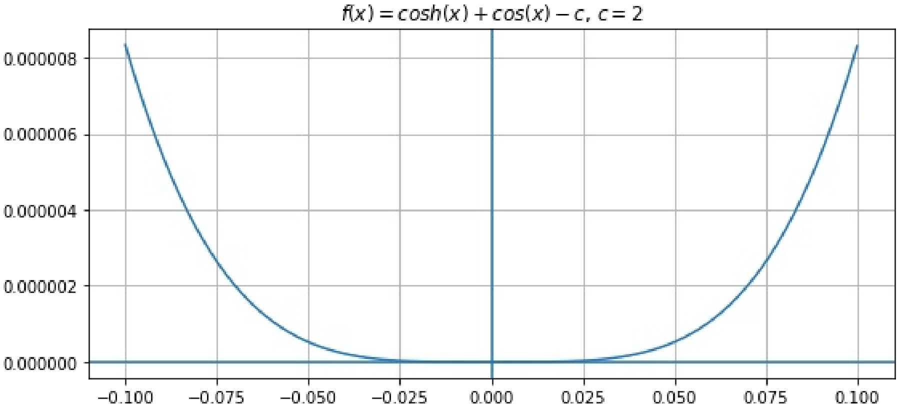
In [13]:

(Top)

```
c = 2
f = g(c)

# Hint: search_range = np.arange(左端點, 右端點, 點與點之間距),
# e.g. search_range = np.arange(0.0, 1.0, 0.01)
# ===== 請實做程式 =====
search_range = np.arange(-0.1, 0.1, 0.0001)
# ====================

fig, ax = plt.subplots(figsize=(9, 4))
ax.plot(search_range, f(search_range))
ax.set_title(r'$f(x)=cosh(x)+cos(x)-c$, $c=$%d' % c)
ax.grid(True)
ax.axhline(y=0)
ax.axvline(x=0)
plt.show()
```



According to the figure above, estimate the zero of f.

For example,

```
root = 3        # 單根
root = -2, 1    # 多根
root = None     # 無解
```

In [14]:

(Top)

```
# Hint: root = ?
# ===== 請實做程式 =====
root = 0
# ====================
```

In [15]:

```
          cell-20fddbe6fa4c437b                                                                              (Top)
```

```
print('My estimation of root:', root)

### BEGIN HIDDEN TESTS
assert type(root) is float or int, 'Wrong type!'
### END HIDDEN TESTS
```

```
My estimation of root: 0
```

Try to find the zero with a tolerance of 10^{-10}. If it works, plot the error and estimation of each step.Otherwise, state the reason why the method failed on this case.

In [16]:

```
root = secant(g(c), [-1.0, 1.0], max_iterations=100, tolerance=1e-10, report_history=False)
```

```
---------------------------------------------------------------------------
AssertionError                            Traceback (most recent call last)
<ipython-input-16-745075b318fd> in <module>
----> 1 root = secant(g(c), [-1.0, 1.0], max_iterations=100, tolerance=1e-10, report_history=
False)

<ipython-input-6-434edf429c5b> in secant(func, interval, max_iterations, tolerance, report_hi
story)
     31     # ===== 請實做程式 =====
     32     a, b = interval
---> 33     assert func(a) * func(b) < 0, 'This initial interval does not satisfied the prere
quisites!'
     34 
     35     num_iterations = 0

AssertionError: This initial interval does not satisfied the prerequisites!
```

(Top)

the estimation failed since the function $cosh(x) + cos(x) - 2$ is always greater than or equal to 0. It is not possible to find an interval [a, b]

s.t. f(a) * f(b) < 0

5. Answer the following questions under the case $c = 3$.

Plot the function to find an interval that contains the zeros of f if possible.

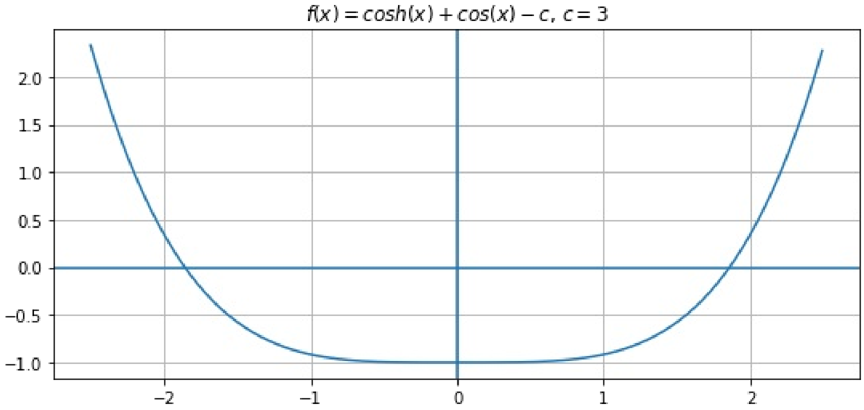
In [17]:

(Top)

```
c = 3
f = g(c)

# Hint: search_range = np.arange(左端點, 右端點, 點與點之間距),
# e.g. search_range = np.arange(0.0, 1.0, 0.01)
# ===== 請實做程式 =====
search_range = np.arange(-2.5, 2.5, 0.01)
# ====================

fig, ax = plt.subplots(figsize=(9, 4))
ax.plot(search_range, f(search_range))
ax.set_title(r'$f(x)=cosh(x)+cos(x)-c$, $c=$%d' % c)
ax.grid(True)
ax.axhline(y=0)
ax.axvline(x=0)
plt.show()
```



According to the figure above, estimate the zero of f.

For example,

```
root = 3        # 單根
root = -2, 1    # 多根
root = None     # 無解
```

In [18]:

(Top)

```
# Hint: root = ?
# ===== 請實做程式 =====
root = -1.8, 1.8
# ====================
```

In [19]:

cell-06ec0b20844075c7 (Top)

```
print('My estimation of root:', root)

### BEGIN HIDDEN TESTS
assert type(root) == tuple, 'Should be multiple roots!'
### END HIDDEN TESTS
```

```
My estimation of root: (-1.8, 1.8)
```

Try to find the zero with a tolerance of 10^{-10}. If it works, plot the error and estimation of each step. Otherwise, state the reason why the method failed on this case.

In [20]:

```
root1 = secant(g(c), [-2, 0], max_iterations=100, tolerance=1e-10, report_history=True)
root2 = secant(g(c), [0, 2], max_iterations=100, tolerance=1e-10, report_history=True)
print("The roots are {} and {}.".format(root1[0], root2[0]))
```

```
Found solution after 11 iterations.
Found solution after 11 iterations.
The roots are -1.8579208291378282 and 1.857920829137828.
```

In [21]:

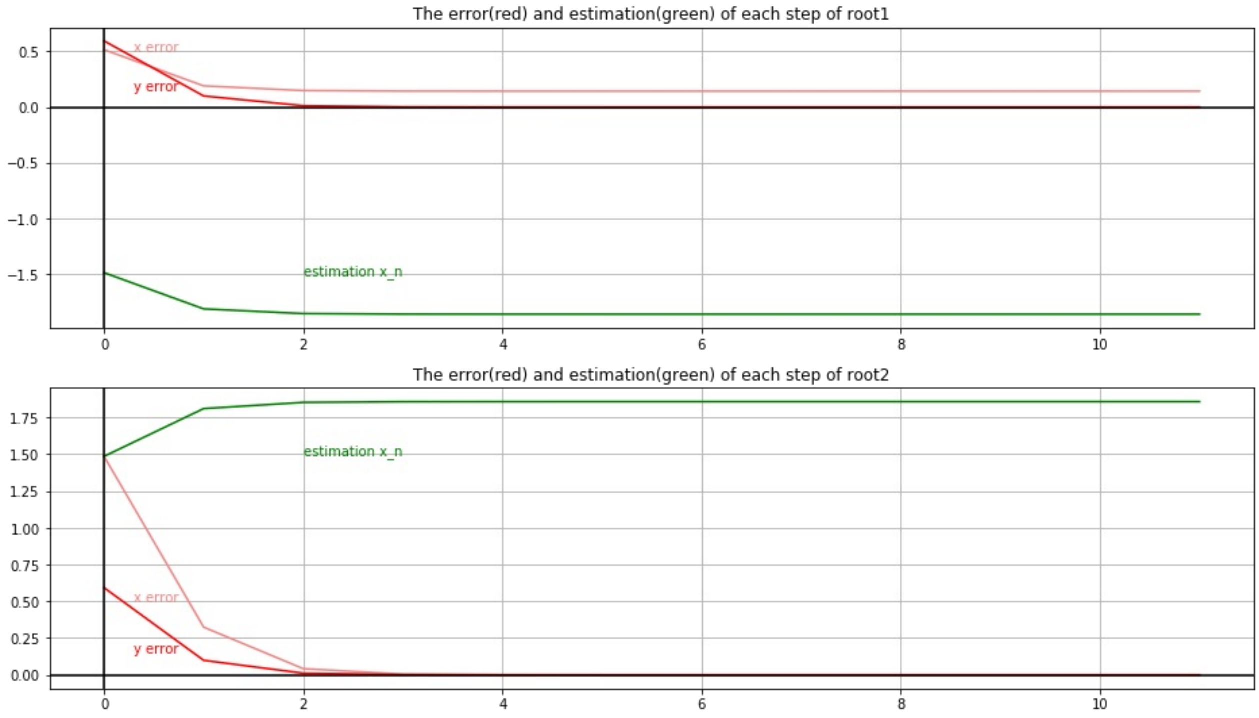
(Top)

```
fig, axes = plt.subplots(2, 1, figsize=(16, 9))
ax1, ax2 = axes

ax1.plot(range(len(root1[1]['x_error'])), root1[1]['x_error'], '#F08787')
ax1.plot(range(len(root1[1]['y_error'])), root1[1]['y_error'], 'r')
ax1.plot(range(len(root1[1]['estimation'])), root1[1]['estimation'], 'g')
ax1.set_title("The error(red) and estimation(green) of each step of root1")
ax1.annotate('x error', (0.3, 0.5), c='#F08787')
ax1.annotate('y error', (0.3, 0.15), c='r')
ax1.annotate('estimation x_n', (2, -1.5), c='g')
ax1.grid(True)
ax1.axhline(y=0, color='k')
ax1.axvline(x=0, color='k')

ax2.plot(range(len(root2[1]['x_error'])), root2[1]['x_error'], '#F08787')
ax2.plot(range(len(root2[1]['y_error'])), root2[1]['y_error'], 'r')
ax2.plot(range(len(root2[1]['estimation'])), root2[1]['estimation'], 'g')
ax2.set_title("The error(red) and estimation(green) of each step of root2")
ax2.annotate('x error', (0.3, 0.5), c='#F08787')
ax2.annotate('y error', (0.3, 0.15), c='r')
ax2.annotate('estimation x_n', (2, 1.5), c='g')
ax2.grid(True)
ax2.axhline(y=0, color='k')
ax2.axvline(x=0, color='k')

plt.show()
```



Discussion

For all cases above(c=1,2,3), do the results(e.g. error behaviors, estimations, etc) agree with the theoretical analysis?

(Top)

c=1 時函數恆正，無根 c=2 時函數只有在原點時值為0，其他地方都大於0，無法使用此方法 c=3 的情況有兩個根，所以各自給相對應的區間 [-2, 0], [0, 2]