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Дисциплина:

«Simulation of Robotic System»

ОТЧЕТ ПО ПРАКТИЧЕСКОЙ РАБОТЕ №1

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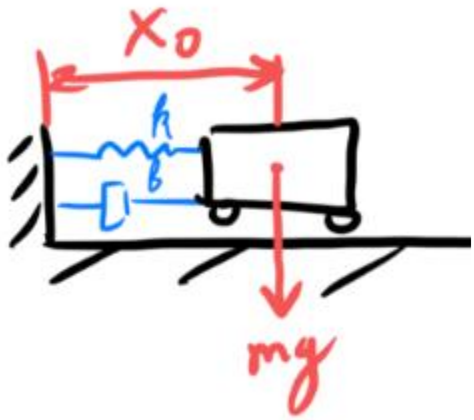
1. Introduction

This report details the mathematical modeling and analysis of the mechanical system designated "Variant 2," a horizontal mass-spring-damper cart. The primary objective is to derive the governing Ordinary Differential Equation (ODE) for this system using the Lagrangian analysis method, as outlined in the provided lecture materials.

Following the derivation, a specific analytical solution to the ODE will be found using the given system parameters. Finally, this report will discuss the nature of this solution and its role in comparing and validating numerical simulation methods.



System Parameters:

- Mass (m): 0.8 kg
- Spring Constant (k): 11.2 N/m
- Damping Coefficient (b): 0.05 N·s/m
- Initial Displacement (x_0): 0.87 m
- Initial Velocity ($\dot{x_0}$): 0.0 m/s

The equation of motion is derived using the Euler-Lagrange equation, which requires defining the system's kinetic and potential energies. The generalized coordinate for this 1D system is x.

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \frac{\partial \mathcal{L}}{\partial x} = Q$$

Kinetic Energy ($\mathcal{K}$)

The kinetic energy is purely translational:

$$\mathcal{K} = \frac{1}{2} m \dot{x}^2$$

Potential Energy ($\mathcal{P}$)

The potential energy is stored in the spring. Unlike the vertical example in the lecture (which includes mgx), our system is horizontal. The gravitational force is balanced by the normal force and does not contribute to the potential energy in the x-direction.

$$\mathcal{P} = \frac{1}{2} k x^2$$

Lagrangian ($\mathcal{L}$)

The Lagrangian $\mathcal{L} is defined as \mathcal{K} - \mathcal{P}$:

$$\mathcal{L} = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} k x^2$$

Non-Conservative Force (Q)

The external force Q is the damping force, which opposes velocity:

$$Q = -b\dot{x}$$

Derivation

We compute the partial derivatives for the Euler-Lagrange equation:

- $\frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x}$
- $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = m\ddot{x}$
- $\frac{\partial \mathcal{L}}{\partial x} = -kx$

Substituting these terms into the Euler-Lagrange equation yields:

$$[m\ddot{x}] - [-kx] = [-b\dot{x}]$$

$$m\ddot{x} + kx = -b\dot{x}$$

Rearranging into the standard form, the governing ODE for Variant 2 is:

$$m\ddot{x} + b\dot{x} + kx = 0$$

2. Analytical Solution

The derived ODE is a 2nd-order, linear, homogeneous equation with constant coefficients, which is analytically solvable.

Substituting the given parameters ($m = 0.8, b = 0.05, k = 11.2$):

$$0.8\ddot{x} + 0.05\dot{x} + 11.2x = 0$$

3.1. System Characterization

To find the form of the solution, we first calculate the system's damping ratio (ζ):

1. Natural Frequency (ω_n):

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{11.2}{0.8}} = \sqrt{14} \approx 3.742 \text{ rad/s}$$

2. Damping Ratio (ζ):

$$\zeta = \frac{b}{2\sqrt{mk}} = \frac{0.05}{2\sqrt{0.8 \times 11.2}} \approx \frac{0.05}{5.987} \approx 0.00835$$

Since $\zeta < 1$, the system is underdamped. The solution will be a decaying oscillation.

3.2. Specific Solution

The general solution for an underdamped system is:

$$x(t) = e^{-\zeta\omega_n t} \cdot [A\cos(\omega_d t) + B\sin(\omega_d t)]$$

We solve for the specific constants using our parameters and initial conditions ($x(0) = 0.87, \dot{x}(0) = 0$):

- Decay Rate ($\zeta\omega_n$):

$$\frac{b}{2m} = \frac{0.05}{2 \times 0.8} = 0.03125 \text{ s}^{-1}$$

- Damped Frequency (ω_d):

$$\omega_d = \omega_n\sqrt{1 - \zeta^2} = 3.742\sqrt{1 - (0.00835)^2} \approx 3.742 \text{ rad/s}$$

- Constants of Integration (A, B):

$$A = x(0) = 0.87$$

$$B = \frac{\dot{x}(0) + \zeta\omega_n A}{\omega_d} = \frac{0 + (0.03125)(0.87)}{3.742} \approx 0.00726$$

Substituting these values, the final analytical solution for Variant 2 is:

$$x(t) = e^{-0.03125t} \cdot [0.87\cos(3.742t) + 0.00726\sin(3.742t)]$$

3. Conclusion

The analytical solution provides an exact, closed-form equation describing the position of the mass x at any time t. The solution consists of two parts:

1. Decaying Envelope ($e^{-0.03125t}$): The damping coefficient $b=0.05$ is very small, resulting in a very small decay rate. This indicates the oscillations will persist for a long time.
2. Oscillatory Part ($[A\cos(\dots) + B\sin(\dots)]$): This describes the cart's back-and-forth motion, which occurs at a frequency of 3.742 rad/s.

4.2. Comparison of Methods and Conclusion The task requires a comparison of methods. For Variant 2, the system is linear. This is the key takeaway.

Because the system is linear, our derived analytical solution is exact. It is the "ground truth" that perfectly describes the system's behavior.

When comparing this result to a numerical method (such as a Simulink model or a Python solver [cite: modeling_n_simulation.ipynb]), the numerical simulation's output should perfectly match the plot of this analytical solution.

For this linear system, the analytical solution serves as the primary tool for validation. Its existence confirms the predictability of the system, and its value is in verifying that any numerical simulation is implemented correctly.