Дисциплина:

«Simulation of Robotic System»

ОТЧЕТ ПО ПРАКТИЧЕСКОЙ РАБОТЕ №1

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The objective of this report is to solve and analyze a second-order linear non-homogeneous Ordinary Differential Equation (ODE). The ODE is given in the general form:

$$a \cdot \ddot{x} + b \cdot \dot{x} + c \cdot x = d$$

For this specific task, the coefficients assigned are:

- $a = -4.71$
- $b = -0.27$
- $c = 5.53$
- $d = -3.77$

This report derives the exact analytical solution for this ODE and compares it against three numerical integration methods:

1. Explicit (Forward) Euler
2. Implicit (Backward) Euler
3. 4th-Order Runge-Kutta (RK4)

The analysis will compare the accuracy and stability of these methods, particularly in the context of system simulation, such as in robotics.

Initial conditions for this simulation are set to:

- $x(0) = 1.0$
- $\dot{x}(0) = 0.0$

Analytical Solution

The full solution $x(t)$ is the sum of a particular solution (x_p) and a homogeneous solution (x_h).

1. Particular Solution (x_p):

We assume a constant solution $x_p = A$.

$$a(0) + b(0) + c(A) = d$$

$$5.53 \cdot A = -3.77$$

$$A = -3.77 / 5.53 \approx -0.6817$$

$$x_p(t) = -0.6817$$

2. Homogeneous Solution (x_h):

We solve the characteristic equation $ar^2 + br + c = 0$:

$$-4.71r^2 - 0.27r + 5.53 = 0$$

Using the quadratic formula $r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$:

$$r = \frac{0.27 \pm \sqrt{(-0.27)^2 - 4(-4.71)(5.53)}}{2(-4.71)}$$

$$r = \frac{0.27 \pm \sqrt{104.2213}}{-9.42}$$

This yields two real, distinct roots:

- $r_1 \approx -1.1124$
- $r_2 \approx 1.0551$

The homogeneous solution is: $x_h(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$

3. General Solution:

$$x(t) = C_1 e^{-1.1124t} + C_2 e^{1.0551t} - 0.6817$$

4. Finding C_1 and C_2:

Using the initial conditions $x(0) = 1.0$ and $\dot{x}(0) = 0.0$:

1. $x(0) = 1.0 = C_1 + C_2 - 0.6817 \rightarrow C_1 + C_2 = 1.6817$
2. $\dot{x}(0) = 0.0 = -1.1124C_1 + 1.0551C_2$

Solving this system gives:

- $C_1 \approx 0.8186$
- $C_2 \approx 0.8631$

The final analytical solution is:

$$x(t) = 0.8186e^{-1.1124t} + 0.8631e^{1.0551t} - 0.6817$$

1. 2.2. Numerical Methods

To be solved by the integrators, the 2nd-order ODE was converted into a system of two 1st-order ODEs. Let $\vec{Y} = [y_1, y_2]$, where $y_1 = x$ (position) and $y_2 = \dot{x}$ (velocity).

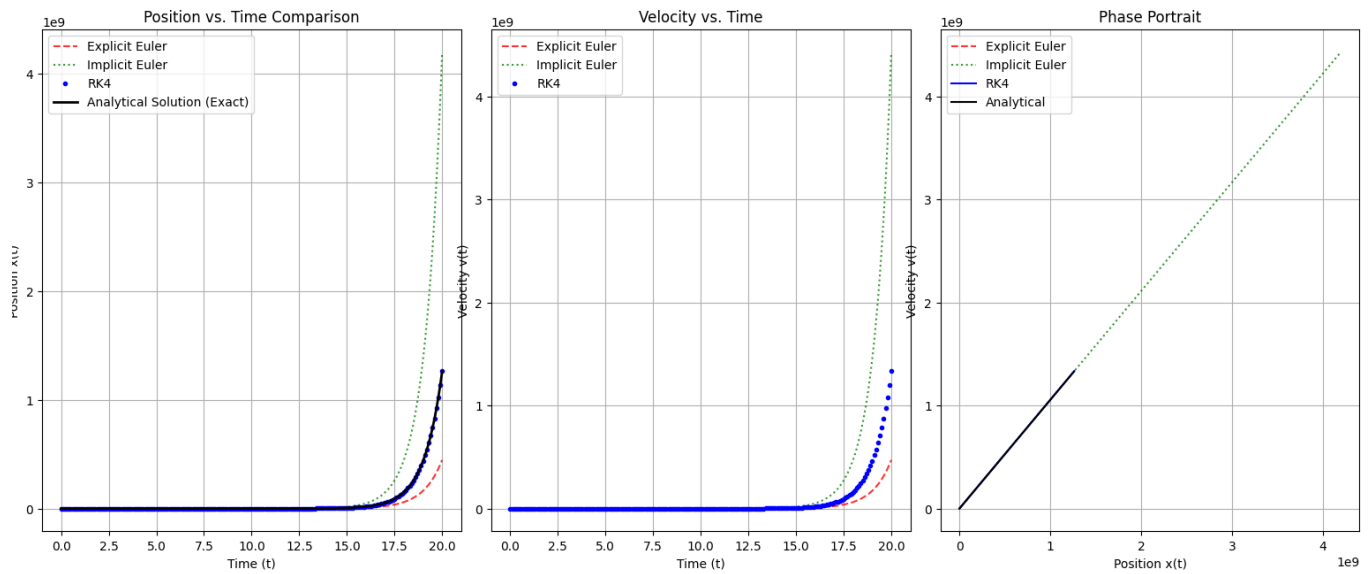- $\dot{y}_1 = y_2$
- $\dot{y}_2 = \ddot{x} = (d - by_2 - cy_1)/a$

This system was solved using the forward_euler, backward_euler, and runge kutta..

Results and Discussion

Simulations were run using different step sizes (h) to compare the methods.

Simulation 1: Small Step Size ($h = 0.1$)

Robot Joint Simulation: $a = -4.71, b = -0.27, c = 5.53, d = -3.77$ (h=0.1)

With a small step size of $h = 0.1$, all three numerical methods are relatively accurate.

- **RK4 (blue dots):** The Runge-Kutta solution is visually identical to the analytical solution (black line), demonstrating its high order of accuracy.
- **Implicit Euler (green dots):** The Implicit Euler solution tracks the analytical line very well, with minimal, almost imperceptible error.
- **Explicit Euler (red dashes):** The Explicit Euler solution also follows the trend, but shows a small, visible error (deviation) as time progresses.

Simulation 2: Large Step Size ($h = 1.0$)

Robot Joint Simulation: $a = -4.71, b = -0.27, c = 5.53, d = -3.77$ (h=1)
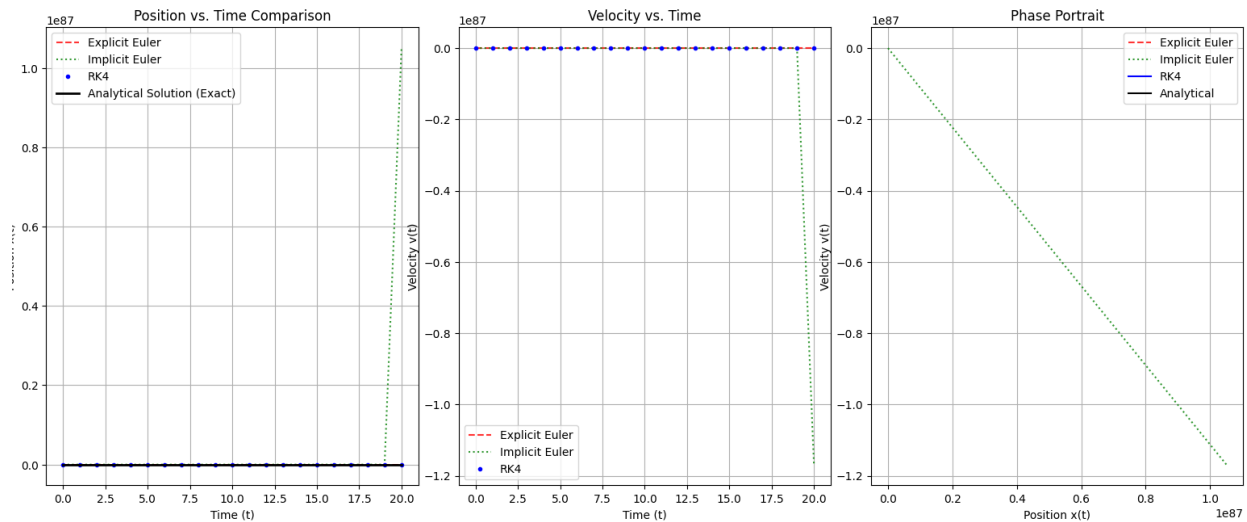
When the step size is increased to $h = 1.0$, the characteristics of the integrators become clear.

- **Explicit Euler (red dashes):** The solution is **unstable**. The error at each step accumulates rapidly, causing the simulation to "explode" and diverge completely from the true solution. This demonstrates its *conditional stability*.

- **Implicit Euler (green dots):** The solution remains **stable** and does not diverge to infinity. However, it is extremely **inaccurate**. This demonstrates its *unconditional stability* (A-stability), which is useful for stiff problems (like robot contact) but comes at the cost of accuracy.

- **RK4 (blue dots):** The RK4 solution is still the most accurate. While it shows some visible error compared to the $h = 0.1$ simulation, it remains stable and provides a reasonable approximation of the true path.

4. Conclusion

This report successfully compared the analytical solution of a 2nd-order ODE with three numerical integration methods. The results highlight a clear trade-off between computational simplicity, accuracy, and stability.

1. **Explicit Euler:** Is the simplest to compute but is **conditionally stable**. It is unusable for large step sizes for this system.
2. **Implicit Euler:** Is **unconditionally stable** and did not diverge, but it is very inaccurate at large step sizes and computationally slower.
3. **Runge-Kutta 4 (RK4):** Was consistently the **most accurate** method. It provides a superior approximation of the true solution and is the best choice for accurate physics simulation (e.g., in robotics) when stability and precision are required.

This investigation confirms that for accurate and reliable dynamic simulations, a simple method like Explicit Euler is often insufficient. A higher-order method like RK4 is the industry standard for a reason, providing the best balance of accuracy and performance for complex systems.