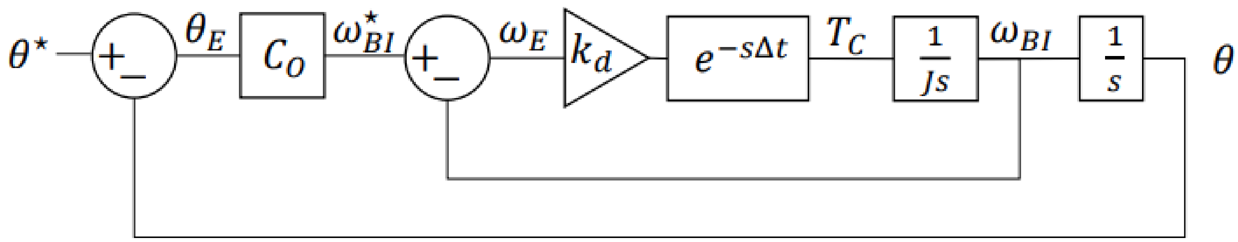

Figure 1: Inner/Outer Loop Control Design

Attitude Control

1. **Steady State Error to Ramp Input**

   Derive the steady state error of the system to a ramp input.

   Solution. Given the system in figure 1, and our outer loop controller as shown in Equation 1:

$$C_O = K\frac{s+z}{s(s+p)} \tag{1}$$

where $s = j\omega$, K is the gain of the controller, z is the zero location, and p is the pole location, and we have the following equations to derive the steady state error

$$C_I = K_d e^{-s\Delta t} \tag{2}$$

$$G_I = \frac{1}{Js} \tag{3}$$

$$G_O = \frac{1}{s} \tag{4}$$

$$\theta_E = \theta^* - \theta = \theta^* - \omega_{BI}G_O \tag{5}$$

$$\omega_{BI} = \omega_{BI}^*\frac{C_I G_I}{1 + C_I G_I} \tag{6}$$

$$\omega_{BI}^* = \theta_E C_O. \tag{7}$$

Therefore:

$$\theta_E = \theta^* - \theta_E C_O\frac{C_I G_I}{1 + C_I G_I}G_O \tag{8}$$

$$\theta_E = \frac{1 + C_I G_I}{1 + C_I G_I + C_O C_I G_I G_O}\theta^* \tag{9}$$

$$\theta_E = \frac{1 + K_d e^{-s\Delta t}\frac{1}{Js}}{1 + K_d e^{-s\Delta t}\frac{1}{Js} + K\frac{s+z}{s(s+p)}K_d e^{-s\Delta t}\frac{1}{Js}\frac{1}{s}}\theta^* \tag{10}$$

Using the final value theorem and a ramp input, $\theta^* = \frac{m}{s^2}$, where m is the slope of the ramp input:

$$\theta_E(\infty) = \lim_{s\to 0} s\theta_E(s) \tag{11}$$

$$\theta_E(s) = \frac{1 + K_d e^{-s\Delta t}\frac{1}{Js}}{1 + K_d e^{-s\Delta t}\frac{1}{Js} + K\frac{s+z}{s(s+p)}K_d e^{-s\Delta t}\frac{1}{Js}\frac{1}{s}}\frac{m}{s^2} \tag{12}$$

$$\theta_E(s) = \frac{Jms + K_d m e^{-s\Delta t}}{Js^3 + K_d s^2 e^{-s\Delta t} + K\frac{s+z}{s+p}K_d e^{-s\Delta t}} \tag{13}$$

$$\theta_E(\infty) = (0)\frac{Jm(0) + K_d m e^{-(0)\Delta t}}{J(0)^3 + K_d(0)^2 e^{-(0)\Delta t} + K\frac{0+z}{0+p}K_d e^{-(0)\Delta t}} = (0)\frac{0 + K_d m}{0 + 0 + K\frac{z}{p}K_d} = 0 \tag{14}$$

Therefore the steady state error of the system to a ramp input is zero.

■

2. **Disturbance Torque**

Derive the transfer function between a disturbance torque input and the angle output.

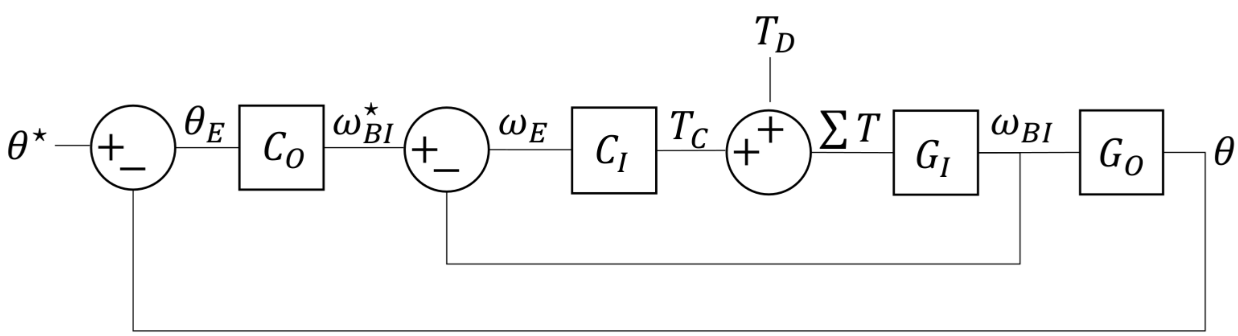

Figure 2: Inner/Outer Loop Control System With Disturbance Torque Applied

Solution. From the system shown in Figure 2, the following equations are derived:

$$\theta = \omega_{BI} G_O \tag{15}$$

$$\omega_{BI} = (\omega_E C_I + T_D)G_I \tag{16}$$

$$\omega_E = \theta_E C_O - \omega_{BI} \tag{17}$$

$$\theta_E = \theta^* - \theta. \tag{18}$$

Therefore

$$\omega_E = \theta_E C_O - (\omega_E C_I + T_D) G_I \tag{19}$$

$$\omega_E + \omega_E C_I G_I = \omega_E (1 + C_I G_I) = \theta_E C_O - T_D G_I \tag{20}$$

$$\omega_E = \frac{\theta_E C_O - T_D G_I}{1 + C_I G_I}. \tag{21}$$

Using substitution for ω_{BI} gives

$$\theta = (\omega_E C_I + T_D) G_I G_O = \left(\frac{\theta_E C_O C_I - T_D C_I G_I}{1 + C_I G_I} + T_D \right) G_I G_O. \tag{22}$$

This can be simplified to

$$\theta = \left(\frac{(\theta^* - \theta) C_O C_I - T_D C_I G_I}{1 + C_I G_I} + T_D \right) G_I G_O. \tag{23}$$

Finding a common denominator in the largest parenthesis gives

$$\left(\frac{(\theta^* - \theta) C_O C_I - T_D C_I G_I}{1 + C_I G_I} + \frac{T_D + T_D C_I G_I}{1 + C_I G_I} \right) G_I G_O = \frac{((\theta^* - \theta) C_O C_I + T_D) G_I G_O}{1 + C_I G_I}. \tag{24}$$

Expanding terms then gives

$$\theta = \frac{\theta^* C_O C_I G_I G_O - \theta C_O C_I G_I G_O + T_D G_I G_O}{1 + C_I G_I}. \tag{25}$$

$$\therefore \theta + \theta \frac{C_O C_I G_I G_O}{1 + C_I G_I} = \frac{\theta^* C_O C_I G_I G_O + T_D G_I G_O}{1 + C_I G_I} \tag{26}$$

$$\theta (1 + C_I G_I + C_O C_I G_I G_O) = \theta^* C_O C_I G_I G_O + T_D G_I G_O \tag{27}$$

$$\theta = \frac{C_O C_I G_I G_O}{1 + C_I G_I + C_O C_I G_I G_O} \theta^* + \frac{G_I G_O}{1 + C_I G_I + C_O C_I G_I G_O} T_D. \tag{28}$$

The principle of superposition holds for this system because it is linear. Therefore

$$\frac{\theta}{T_D} = \frac{G_I G_O}{1 + C_I G_I + C_O C_I G_I G_O} \tag{29}$$

which is the transfer function between T_D and θ. ■