https://hao-ai-lab.github.io/cse234-w25/

CSE 234: Data Systems for Machine Learning
Winter 2025

LLMSys

Optimizations and Parallelization

MLSys Basics

Logistics Update

- Enrollment:
  - We have roughly 40 vacant now, CSE will process enrollments soon
  - Will enroll waitlisted students near the end of this week

Last week

- We summarized our workload
  - Matmul + softmax + …
- Computational graphs
  - Nodes, edges
- Programming
  - Imperative vs. symbolic
  - Static vs. dynamic
  - JIT and its bottleneck

Today

A repr that expresses the computation using primitives

✅ A repr that expresses the forward computation using primitives

❓ A repr that expresses the backward computation using primitives

Today’s learning goals

- Autodiff
- MLSys architecture overview
  - Optimization opportunities

Recap: how to take derivative?

- Given $f(\theta)$, what is $\frac{\partial f}{\partial \theta}$?

$$\frac{\partial f}{\partial \theta} = \lim_{\epsilon \to 0} \frac{f(\theta + \epsilon) - f(\theta)}{\epsilon}$$
$$\approx \frac{f(\theta + \epsilon) - f(\theta - \epsilon)}{2\epsilon} + o(\epsilon^2)$$

Problem:
slow: evaluate f twice to get
one gradient
Error: approximal and floating point has errors

Instead, Symbolic Differentiation

- Write down the formula, derive the gradient following rules

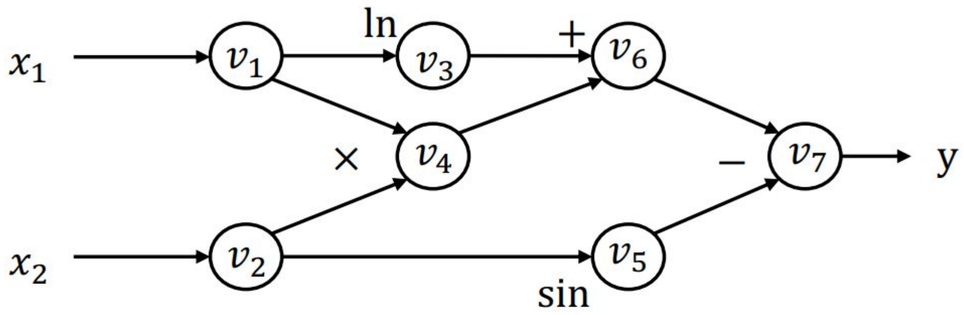
$$\frac{\partial(f(\theta)+g(\theta))}{\partial\theta}=\frac{\partial f(\theta)}{\partial\theta}+\frac{\partial g(\theta)}{\partial\theta}$$

$$\frac{\partial(f(\theta)g(\theta))}{\partial\theta}=g(\theta)\frac{\partial f(\theta)}{\partial\theta}+f(\theta)\frac{\partial g(\theta)}{\partial\theta}$$

$$\frac{\partial(f(g(\theta))}{\partial\theta}=\frac{\partial f(g(\theta))}{\partial g(\theta)}\frac{\partial g(\theta)}{\partial\theta}$$

Map autodiff rules to computational graph

$$y = f(x_1, x_2) = \ln(x_1) + x_1 x_2 - \sin x_2$$



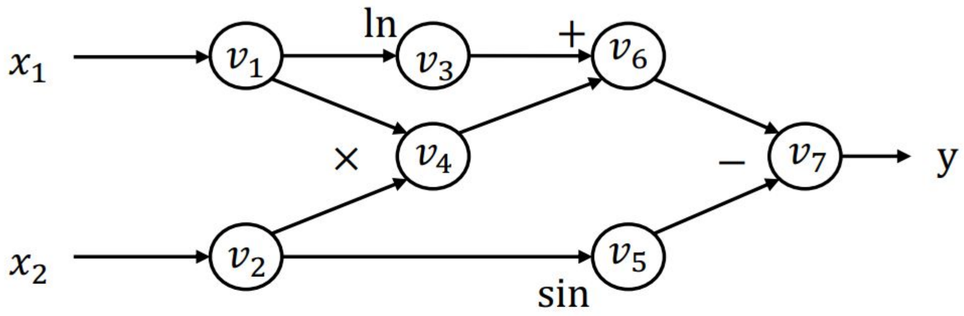
Forward evaluation trace

$v_1 = x_1 = 2$
$v_2 = x_2 = 5$
$v_3 = \ln v_1 = \ln 2 = 0.693$
$v_4 = v_1 \times v_2 = 10$
$v_5 = \sin v_2 = \sin 5 = -0.959$
$v_6 = v_3 + v_4 = 10.693$
$v_7 = v_6 - v_5 = 10.693 + 0.959 = 11.652$
$y = v_7 = 11.652$

- Q: Calculate the value of $\frac{\partial y}{\partial x_1}$
  - A: use chain rules
- There are two ways of applying chain rules
  - Forward: from left (inside) to right (outside)
  - Backward: from right (outside) to left (inside)
  - Which one fits with deep learning?

Forward Mode Autodiff

$$y = f(x_1, x_2) = \ln(x_1) + x_1 x_2 - \sin x_2$$



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$$v_7 = v_6 - v_5 = 10.693 + 0.959 = 11.652$$
$$y = v_7 = 11.652$$

- Define $\dot{v}_i = \frac{\partial \mathrm{v}_i}{\partial x_i}$
- We then compute each $\dot{v}_i$ following the forward order of the graph

$$\dot{v}_1 = 1$$
$$\dot{v}_2 = 0$$
$$\dot{v}_3 = \dot{v}_1 / v_1 = 0.5$$
$$\dot{v}_4 = \dot{v}_1 v_2 + \dot{v}_2 v_1 = 1 \times 5 + 0 \times 2 = 5$$
$$\dot{v}_5 = \dot{v}_2 \cos v_2 = 0 \times \cos 5 = 0$$
$$\dot{v}_6 = \dot{v}_3 + \dot{v}_4 = 0.5 + 5 = 5.5$$
$$\dot{v}_7 = \dot{v}_6 - \dot{v}_5 = 5.5 - 0 = 5.5$$

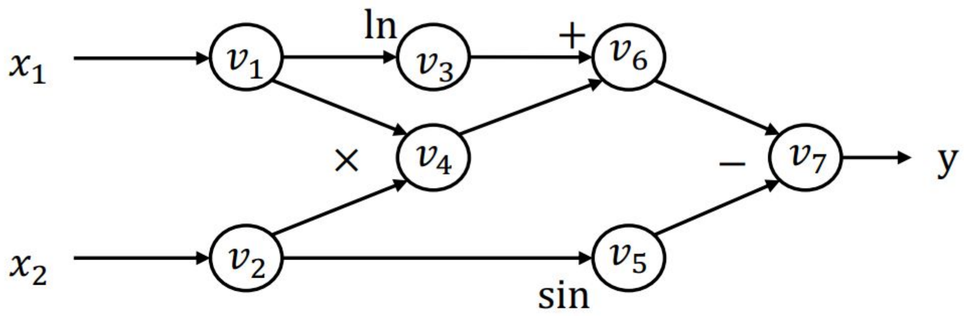
- Finally: $\frac{\partial y}{\partial x_1} = \dot{v}_7 = 5.5$

Summary: Forward Mode Autodiff

- Start from the input nodes
- Derive gradient all the way to the output nodes
- Pros and Cons of FM Autodiff?
  - For $f: R^n \rightarrow R^k$, we need n forward passes to get the grad w.r.t. each input
  - However, in ML: $k = 1$ mostly, and n is very large

Reverse Mode Autodiff

$$y = f(x_1, x_2) = \ln(x_1) + x_1 x_2 - \sin x_2$$



Forward evaluation trace

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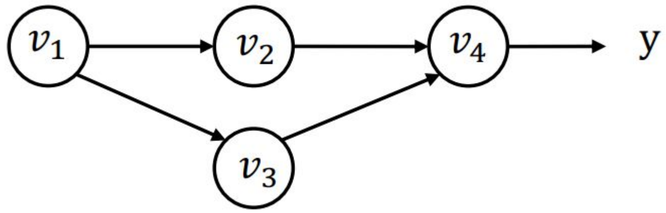
$v_6 = v_3 + v_4 = 10.693$
$v_7 = v_6 - v_5 = 10.693 + 0.959 = 11.652$
$y = v_7 = 11.652$

- Define adjoint $\bar{v}_i = \frac{\partial y}{\partial v_i}$
- We then compute each $\bar{v}_i$ in the reserve topological order of the graph

$$\overline{v_7} = \frac{\partial y}{\partial v_7} = 1$$
$$\overline{v_6} = \overline{v_7}\frac{\partial v_7}{\partial v_6} = \overline{v_7} \times 1 = 1$$
$$\overline{v_5} = \overline{v_7}\frac{\partial v_7}{\partial v_5} = \overline{v_7} \times (-1) = -1$$
$$\overline{v_4} = \overline{v_6}\frac{\partial v_6}{\partial v_4} = \overline{v_6} \times 1 = 1$$
$$\overline{v_3} = \overline{v_6}\frac{\partial v_6}{\partial v_3} = \overline{v_6} \times 1 = 1$$
$$\overline{v_2} = \overline{v_5}\frac{\partial v_5}{\partial v_2} + \overline{v_4}\frac{\partial v_4}{\partial v_2} = \overline{v_5} \times \cos v_2 + \overline{v_4} \times v_1 = -0.284 + 2 = 1.716$$
$$\overline{v_1} = \overline{v_4}\frac{\partial v_4}{\partial v_1} + \overline{v_3}\frac{\partial v_3}{\partial v_1} = \overline{v_4} \times v_2 + \overline{v_3}\ \frac{1}{v_1} = 5 + \frac{1}{2} = 5.5$$

- Finally: $\frac{\partial y}{\partial x_1} = \bar{v}_1 = 5.5$

Case Study



- How to derive the gradient of v_1

$$\overline{v_1} = \frac{\partial y}{\partial v_1} = \frac{\partial f(v_2,v_3)}{\partial v_2}\frac{\partial v_2}{\partial v_1} + \frac{\partial f(v_2,v_3)}{\partial v_3}\frac{\partial v_3}{\partial v_1} = \overline{v_2}\frac{\partial v_2}{\partial v_1} + \overline{v_3}\frac{\partial v_3}{\partial v_1}$$

For a v_i used by multiple consumers:

$$\overline{v_i} = \sum_{j \in next(i)} \overline{v_{i\to j}} \quad \text{, where} \quad \overline{v_{i\to j}} = \overline{v_j}\frac{\partial v_j}{\partial v_i}$$

Summary: Backward Mode Autodiff

- Start from the output nodes
- Derive gradient all the way back to the input nodes
- Discussion: Pros and Cons of FM Autodiff?
  - For $f: R^n \rightarrow R^k$, we need k backward passes to get the grad w.r.t. each input
  - in ML: $k = 1$ and n is very large
  - How about other areas?

Back to Our Question

A repr that expresses the computation using primitives

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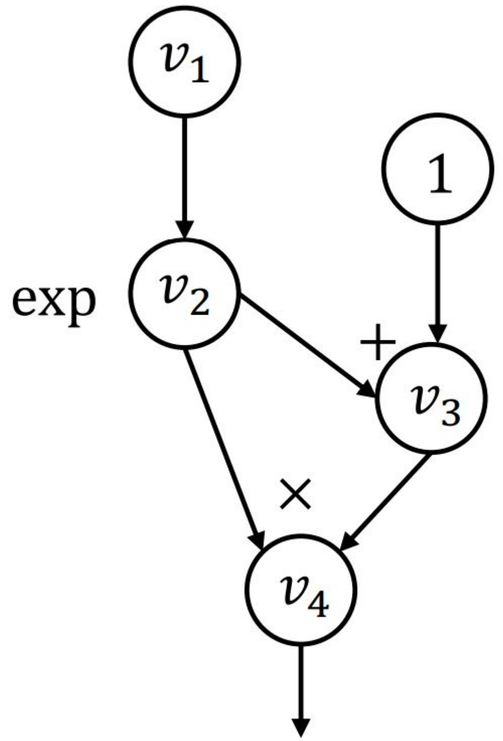
Back to our question: Construct the Backward Graph

- How can we construct a computational graph that calculates the adjoint value?

```
def gradient(out):
  node_to_grad = {out:  [1]}
  for i in reverse_topo_order(out):
    v̄_i = ∑_j v̄_{i→j} = sum(node_to_grad[i])
    for k ∈ inputs(i):
        compute v̄_{k→i} = v̄_i ∂v_i/∂v_k
        append v̄_{k→i} to node_to_grad[k]
    return adjoint of input v̄_input
```



f: $(\exp(v_1) + 1)\exp(v_1)$

How to implement reverse Autodiff (aka. BP)

```
def gradient(out):
  node_to_grad = {out:  [1]}
  for i in reverse_topo_order(out):
    v̄_i = Σ_j v̄_{i→j} = sum(node_to_grad[i])
    for k ∈ inputs(i):
        compute v̄_{k→i} = v̄_i ∂v_i/∂v_k
        append v̄_{k→i} to node_to_grad[k]
    return adjoint of input v̄_input
```

Record all partial adjoints of a node

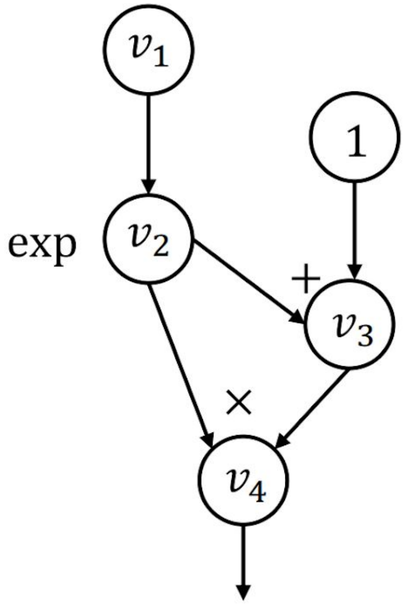
Sum up all partial adjoints to get the gradient

Compute and propagates partial adjoints to its inputs.

Start from v_4

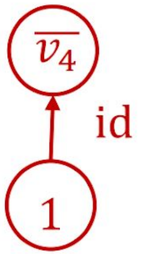
i = 4: $v_4 = sum([1]) = 1$

```
def gradient(out):
  node_to_grad = {out:  [1]}
  for i in reverse_topo_order(out):
    v̄_i = Σ_j v̄_{i→j} = sum(node_to_grad[i])
    for k ∈ inputs(i):
        compute v̄_{k→i} = v̄_i ∂v_i/∂v_k
        append v̄_{k→i} to node_to_grad[k]
    return adjoint of input v̄_input
```



$i = 4$

node_to_grad: {
  4: [$\overline{v_4}$]
}



v_4: Inspect (v_2, v_4) and (v_3, v_4)

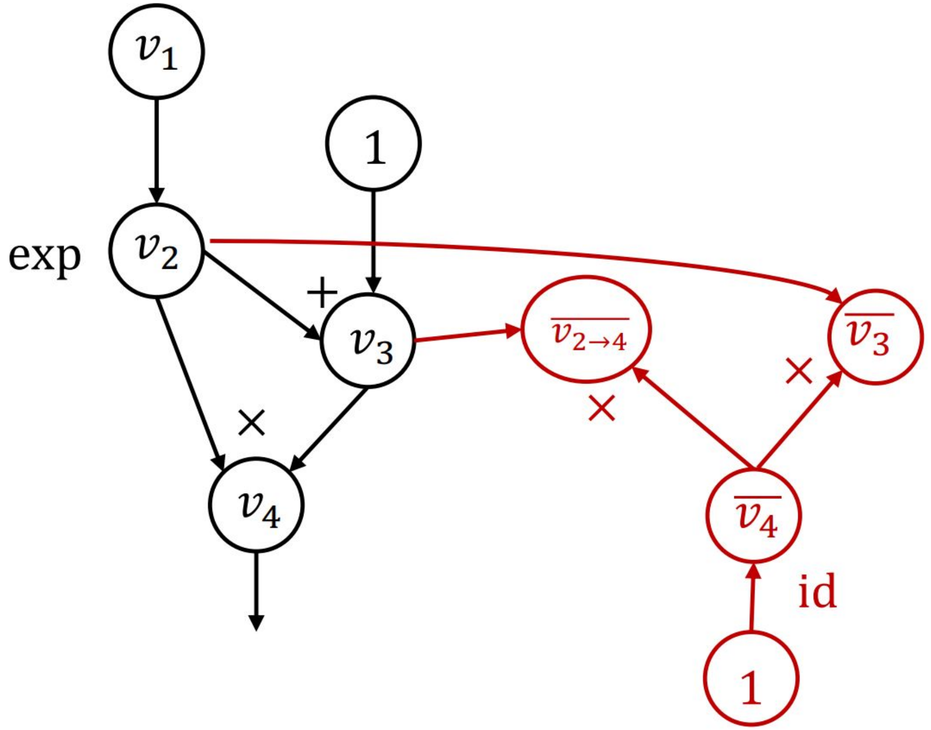
```
def gradient(out):
  node_to_grad = {out:  [1]}
  for i in reverse_topo_order(out):
    v̄_i = Σ_j v̄_{i→j} = sum(node_to_grad[i])
    for k ∈ inputs(i):
        compute v̄_{k→i} = v̄_i ∂v_i/∂v_k
        append v̄_{k→i} to node_to_grad[k]
    return adjoint of input v̄_input
```

$i = 4$
node_to_grad: {
  2: $[\overline{v_{2\to4}}]$
  3: $[\overline{v_3}]$
  4: $[\overline{v_4}]$
}

i=4: $\bar{v}_4 = sum([1]) = 1$

k=2: $\overline{v_{2\to4}} = \bar{v}_4 \frac{\partial v_4}{\partial v_2} = \bar{v}_4 v_3$

k=3: $\overline{v_{3\to4}} = \bar{v}_4 \frac{\partial v_4}{\partial v_3} = \bar{v}_4 v_2, \overline{v_{3\to4}} = \bar{v}_3$



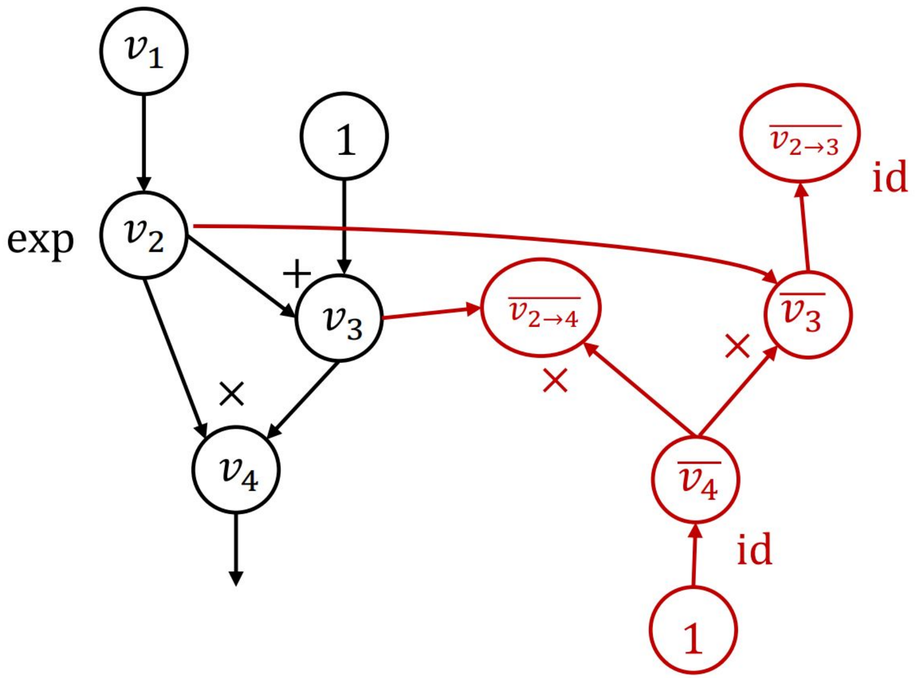
Inspect v_3

```
def gradient(out):
  node_to_grad = {out:  [1]}
  for i in reverse_topo_order(out):
    v̄_i = Σ_j v̄_{i→j} = sum(node_to_grad[i])
    for k ∈ inputs(i):
        compute v̄_{k→i} = v̄_i ∂v_i/∂v_k
        append v̄_{k→i} to node_to_grad[k]
    return adjoint of input v̄_input
```

$i = 3$
node_to_grad: {
  2: $[\overline{v_{2\to4}}, \overline{v_{2\to3}}]$
  3: $[\overline{v_3}]$
  4: $[\overline{v_4}]$
}

i=3: $\overline{v_3}$ done!

k=2: $\overline{v_{2\to3}} = \overline{v_3}\frac{\partial v_3}{\partial v_2} = \overline{v_3}$



i=2: $\overline{v_2} = \overline{v_{2\to3}} + \overline{v_{2\to4}}$

Inspect v_2

```
def gradient(out):
  node_to_grad = {out:  [1]}
  for i in reverse_topo_order(out):
    v̄_i = Σ_j v̄_{i→j} = sum(node_to_grad[i])
    for k ∈ inputs(i):
        compute v̄_{k→i} = v̄_i ∂v_i/∂v_k
        append v̄_{k→i} to node_to_grad[k]
    return adjoint of input v̄_input
```

$i = 2$

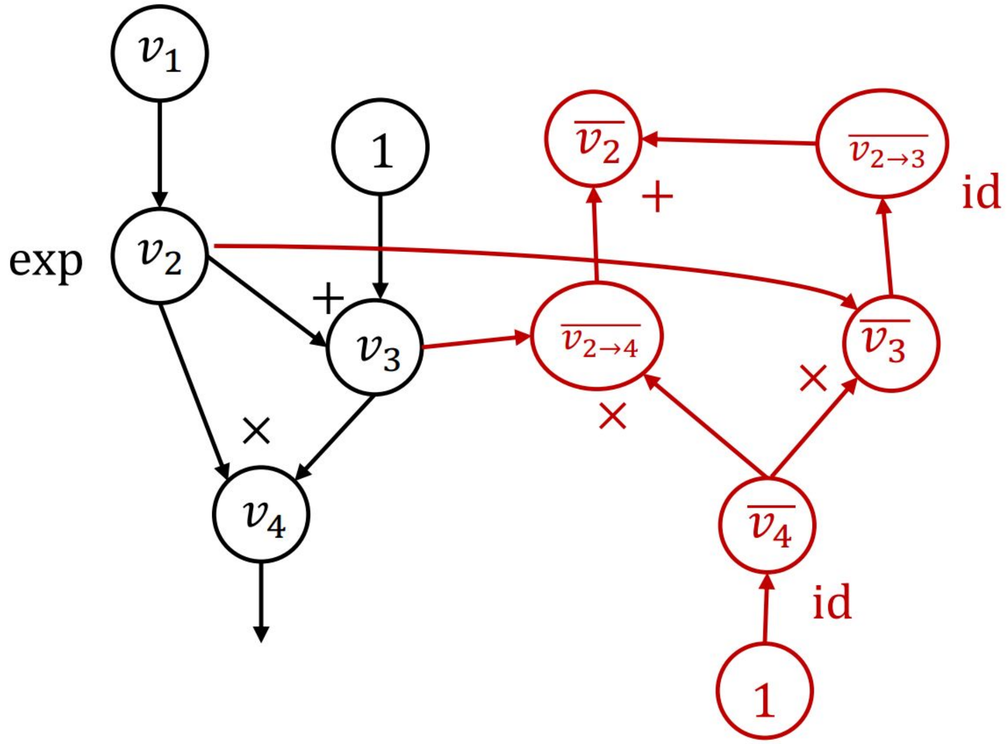
node_to_grad: {

  2: $[\overline{v_{2\to4}}, \overline{v_{2\to3}}]$

  3: $[\overline{v_3}]$

  4: $[\overline{v_4}]$

}



Inspect (v_1, v_2)

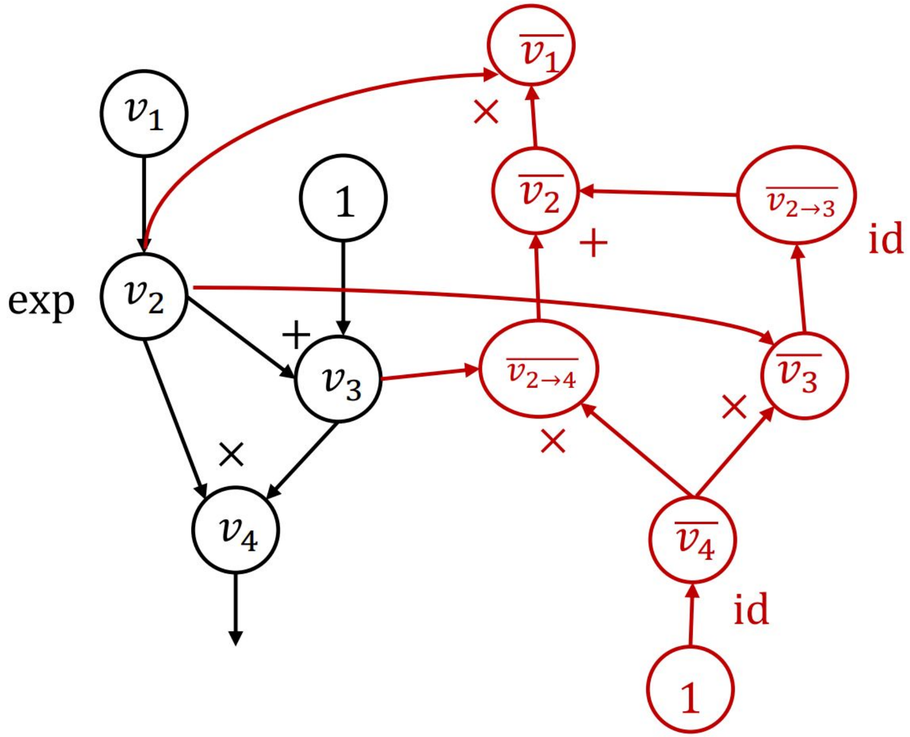
```
def gradient(out):
  node_to_grad = {out:  [1]}
  for i in reverse_topo_order(out):
    v̄_i = Σ_j v̄_{i→j} = sum(node_to_grad[i])
    for k ∈ inputs(i):
        compute v̄_{k→i} = v̄_i ∂v_i/∂v_k
        append v̄_{k→i} to node_to_grad[k]
    return adjoint of input v̄_input
```

$i = 2$
node_to_grad: {
  1: $[\overline{v_1}]$
  2: $[\overline{v_{2\to4}}, \overline{v_{2\to3}}]$
  3: $[\overline{v_3}]$
  4: $[\overline{v_4}]$
}

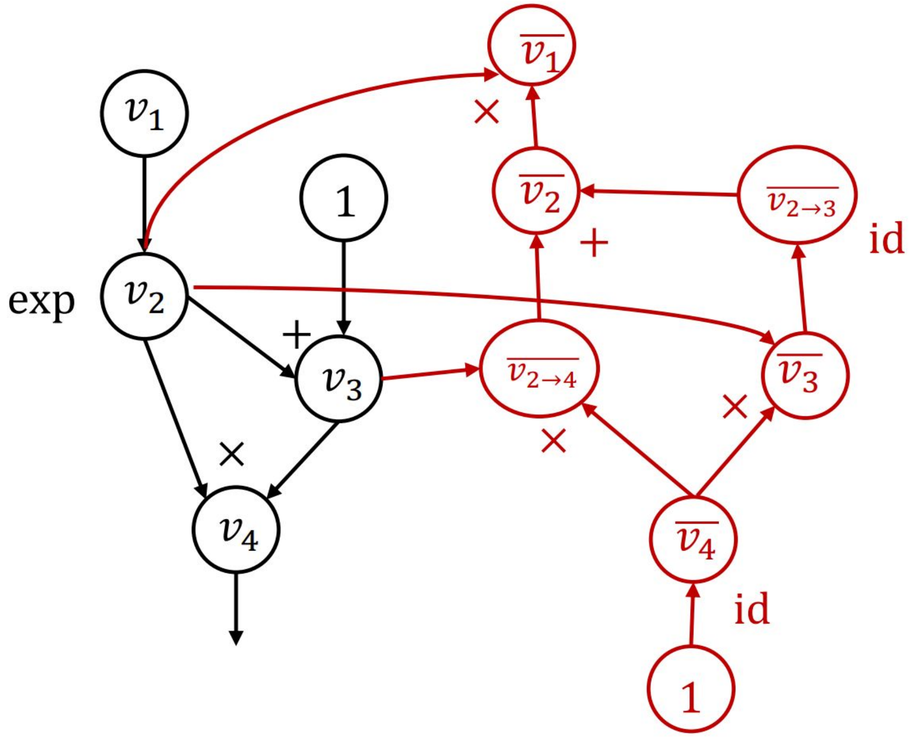
i=2: $\bar{v}_2 = \overline{v_{2\to3}} + \overline{v_{2\to4}}$

k=1: $\overline{v_{1\to2}} = \bar{v}_2 \frac{\partial v_2}{\partial v_1} = \bar{v}_2 \exp(v_1)$,

$\bar{v}_1 = \overline{v_{1\to2}}$

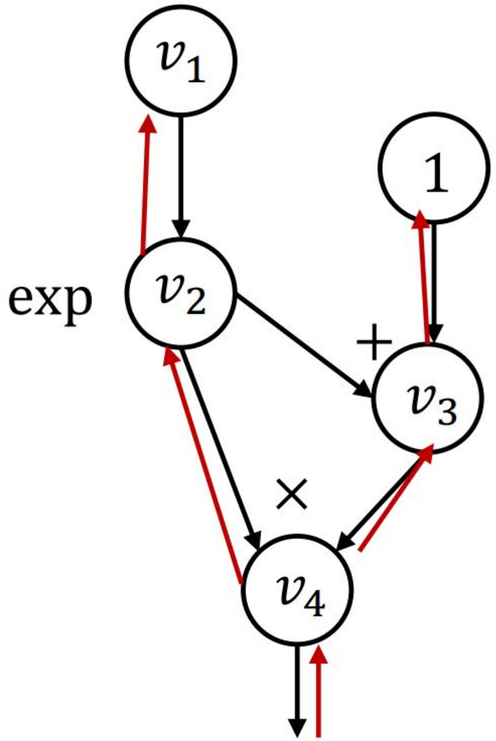


Summary: Backward AD

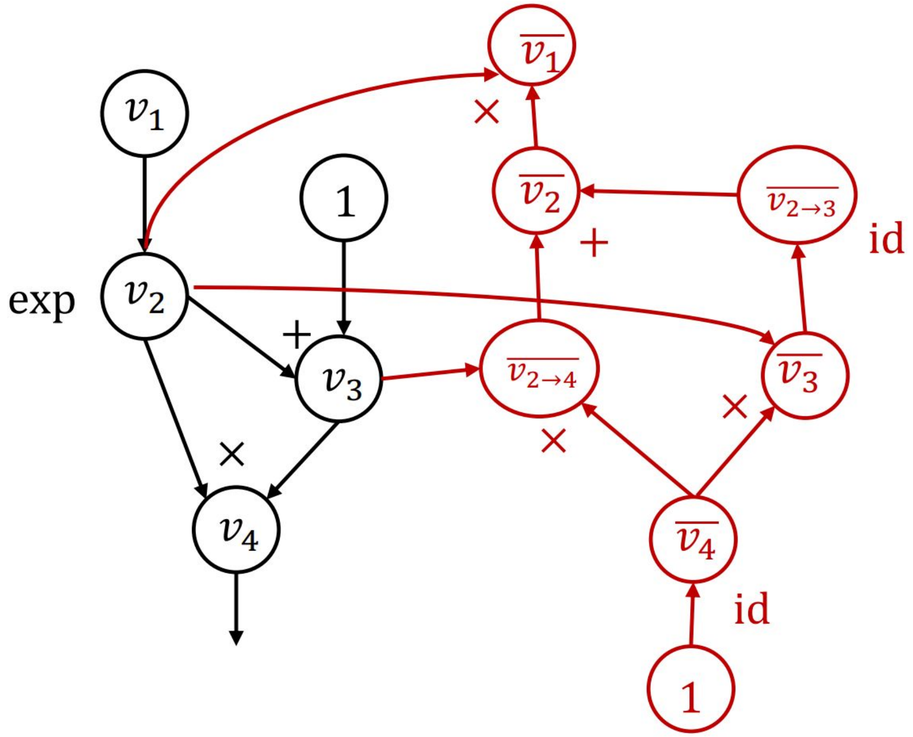


- Construct backward graph in a symbolic way (instead of concrete values)
- This graph can be reused by different input values

Backpropagation vs. Reverse-mode AD



VS.

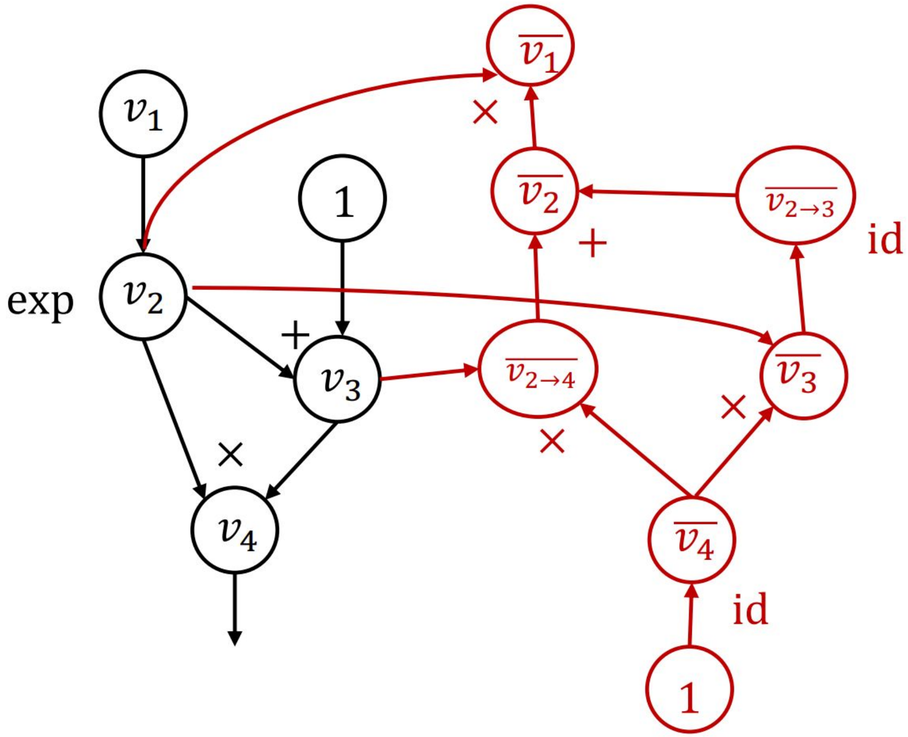


- Run backward through the forward graph
- Caffe/cuda-convnet

- Construct backward graph
- Used by TensorFlow, PyTorch

Incomplete yet?

- What is the missing from the following graph for ML training?



Recall Our Master Equation

$$\theta^{(t+1)} = f\left(\theta^{(t)}, \nabla_L\left(\theta^{(t)}, D^{(t)}\right)\right)$$

$$L = \text{MSE}(w_2 \cdot \text{ReLU}(w_1 x), y) \quad \theta = \{w_1, w_2\}, D = \{(x, y)\}$$

$$f(\theta, \nabla_L) = \theta - \nabla_L$$

Forward

$L(\cdot)$

Backward

$\nabla_L(\cdot)$

Weight update

$f(\cdot)$

Put in Practice

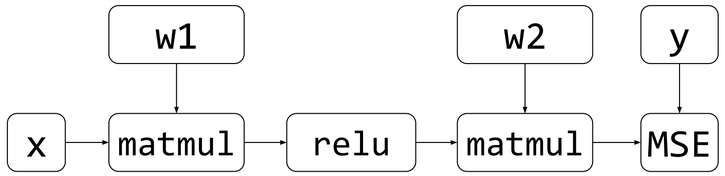
$$\theta^{(t+1)} = f\left(\theta^{(t)}, \nabla_L\left(\theta^{(t)}, D^{(t)}\right)\right)$$

$$L = \mathrm{MSE}(w_2 \cdot \mathrm{ReLU}(w_1 x),\, y) \quad \theta = \{w_1, w_2\},\, D = \{(x, y)\}$$

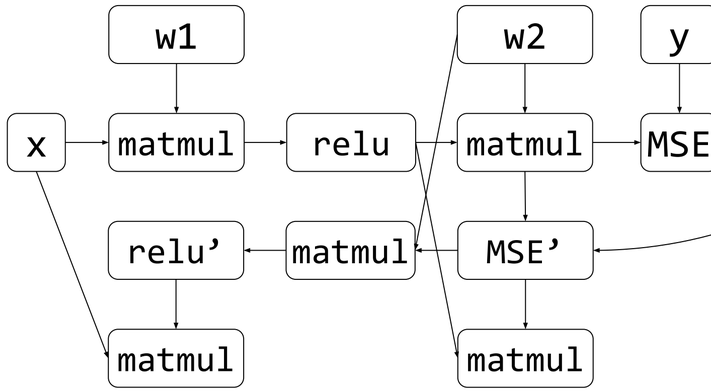
$$f(\theta, \nabla_L) = \theta - \nabla_L$$

Operator / its output tensor Data flowing direction

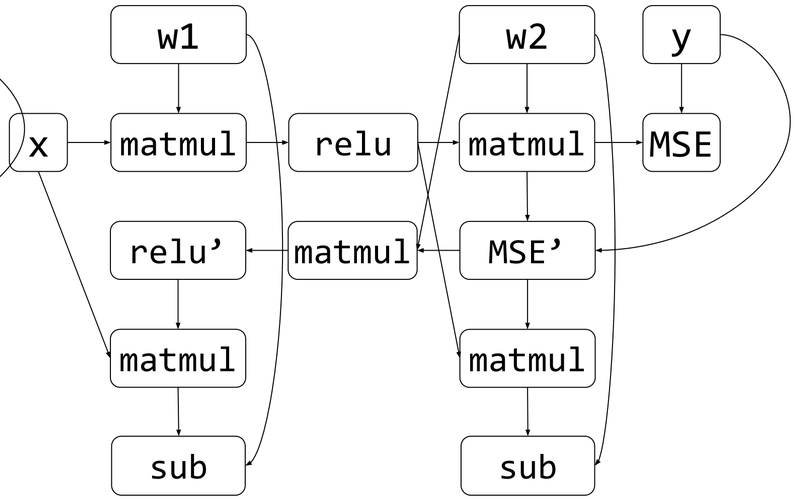
Forward



+Backward



+Weight update



Homework: How to derive gradients for

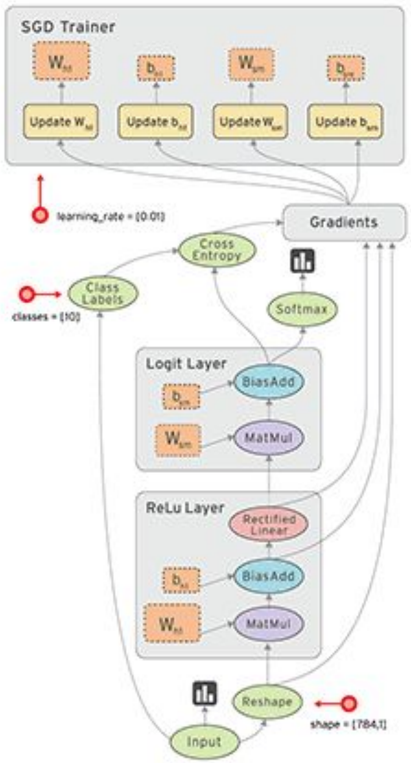
- **Softmax cross entropy:**

$$L = -\sum t_i \log(y_i), y_i = softmax(\boldsymbol{x})_i = \frac{e^{x_i}}{\sum e^{x_d}}$$

Today

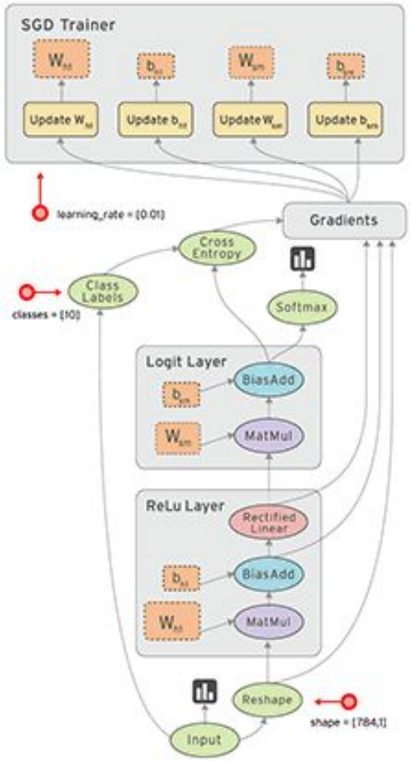
- Autodiff
- **Architecture Overview**

MLSys' Grand problem



- Our system goals:
  - Fast
  - Scale
  - Memory-efficient
  - Run on diverse hardware
  - Energy-efficient
  - Easy to program/debug/deploy

ML System Overview



Dataflow Graph

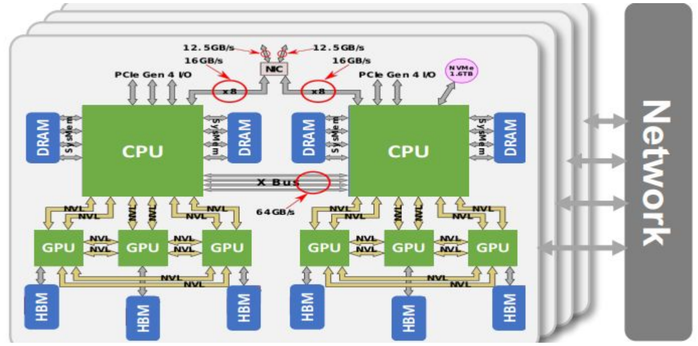
Autodiff

Graph Optimization

Parallelization

Runtime: schedule / memory

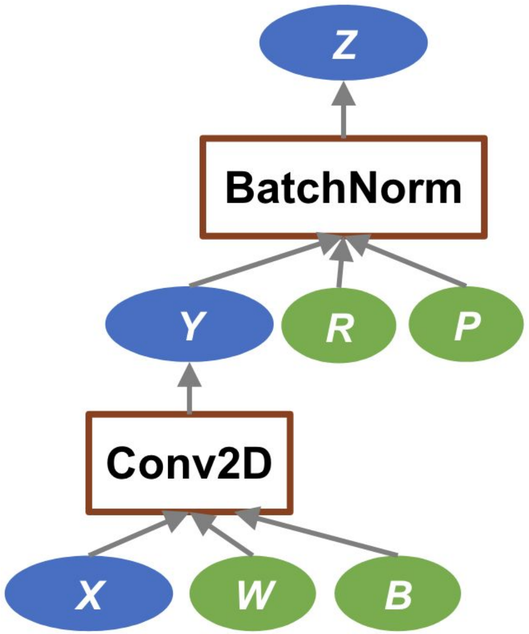
Operator optimization/compilation



Graph Optimization

- Goal:
  - Rewrite the original Graph G to G’
  - G’ runs faster than G

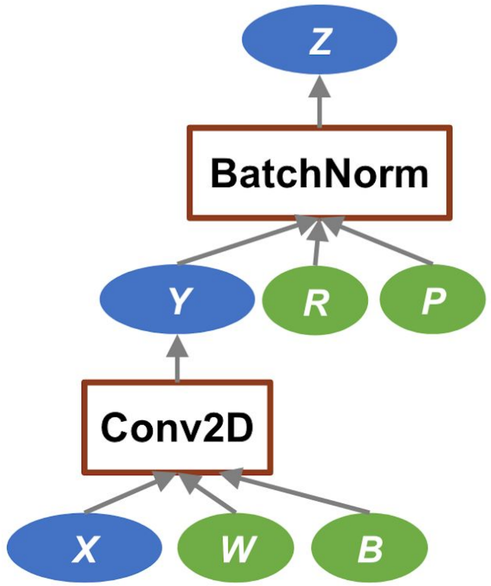
Motivating Example: ResNet



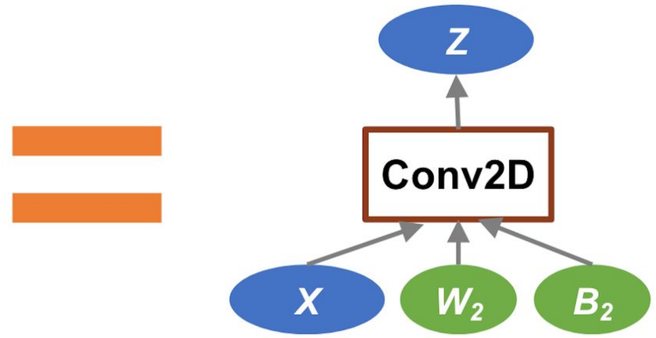
$$Z(n,c,h,w) = Y(n,c,h,w) * R(c) + P(c)$$

$$Y(n,c,h,w) = \left(\sum_{d,u,v} X(n,d,h+u,w+v) * W(c,d,u,v) \right) + B(n,c,h,w)$$

Motivating Example: ResNet



$$Z(n,c,h,w) = \left(\sum_{d,u,v} X(n,d,h+u,w+v) * W_2(c,d,u,v) \right) + B_2(n,c,h,w)$$



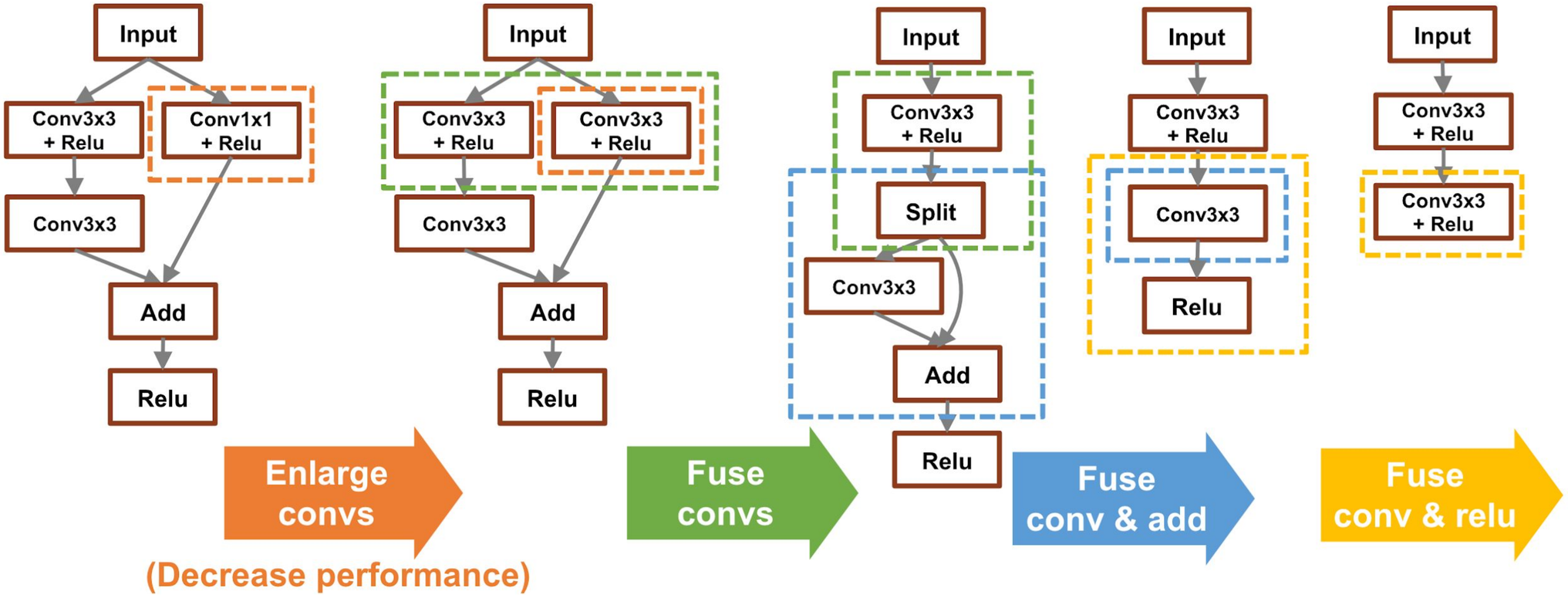
$$W_2(n,c,h,w) = W(n,c,h,w) * R(c)$$

$$B_2(n,c,h,w) = B(n,c,h,w) * R(c) + P(c)$$

- Why the fusion of conv2d & batchnorm is faster?

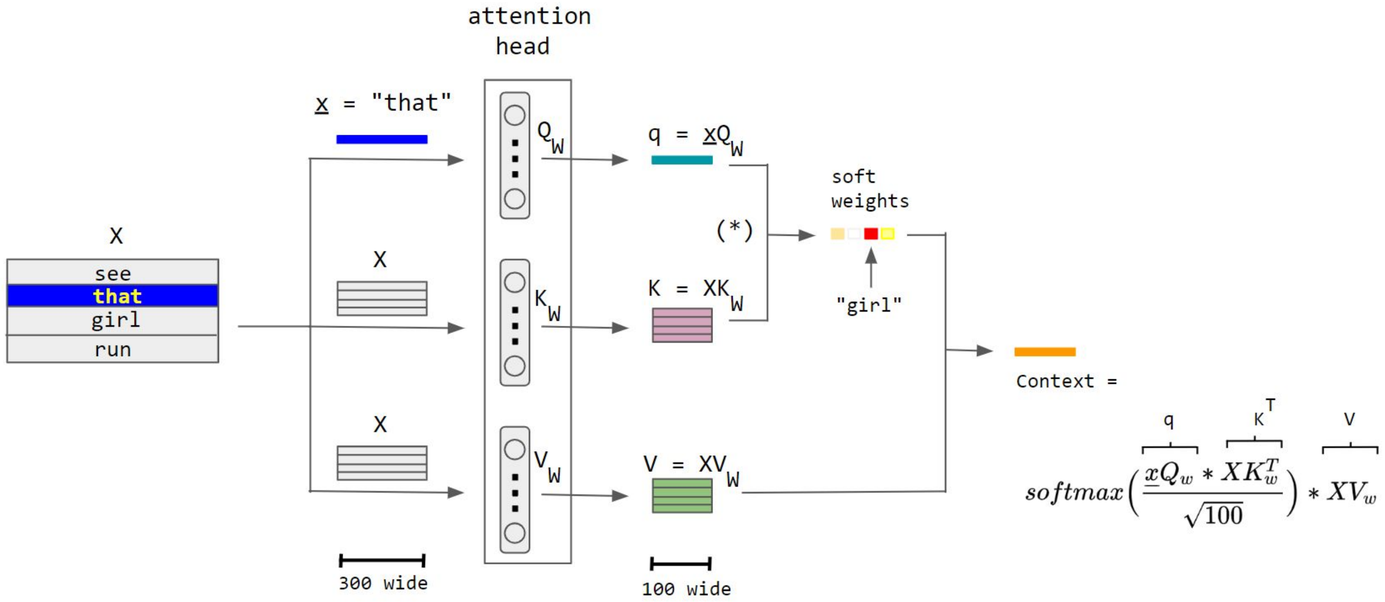
Dataflow Graph
Autodiff
Graph Optimization
Parallelization
Runtime: schedule / memory
Operator

Motivating Example: we can go further



- Does each step become faster than previous step?
- How does it perf on different hardware?

Motivating Example 2: Attention




```
# Original
Q = matmul(W_q, h)
K = matmul(W_k, h)
V = matmul(W_v, h)

# Merged QKV
QKV = matmul(concat(W_q, W_k, W_v), h)
```

- Why merged QKV is faster?

Arithmetic Intensity

$$AI = \#ops / \#bytes$$

Arithmetic intensity

```
void add(int n, float* A, float* B, float* C){
    for (int i=0; i<n; i++)
        C[i] = A[i] + B[i];
}
```

Two loads, one store per math op
(arithmetic intensity = 1/3)

1. Read A[i]
2. Read B[i]
3. Add A[i]+B[i]
4. Store C[i]

Which program performs better? Program 1

```
void add(int n, float* A, float* B, float* C){
  for (int i=0; i<n; i++)
    C[i] = A[i] + B[i];
}



void mul(int n, float* A, float* B, float* C) {
  for (int i=0; i<n; i++)
    C[i] = A[i] * B[i];
}



float* A, *B, *C, *D, *E, *tmp1, *tmp2;
// assume arrays are allocated here
// compute E = D + ((A + B) * C)
add(n, A, B, tmp1);
mul(n, tmp1, C, tmp2);
```

Two loads, one store per math op
(arithmetic intensity = 1/3)

Two loads, one store per math op
(arithmetic intensity = 1/3)

Overall arithmetic intensity = 1/3

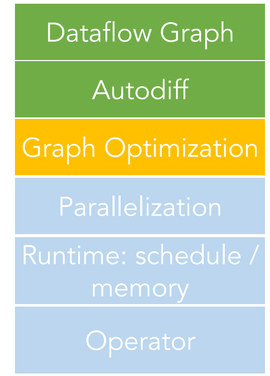
Which program performs better? Program 2

```
float*  A,*B,  *C,  *D,  *E,  *tmp1,   *tmp2;
// assume  arrays are  allocated here
// computeE = D + ((A   + B)*  C)
add(n, A,B, tmp1);
mul(n, tmp1, C,tmp2);
add(n, tmp2, D,    E);
```

```
void    fused(int n, float*  A,float*  B, float*  C,float*  D,
   float*  E){
   for   (int  i=0; i<n; i++)
      E[i]  = D[i]  + (A[i] + B[i]) *  C[i];
```

Overall arithmetic intensity = 1/3

Four loads, one store per 3 math ops
arithmetic intensity = 3/5



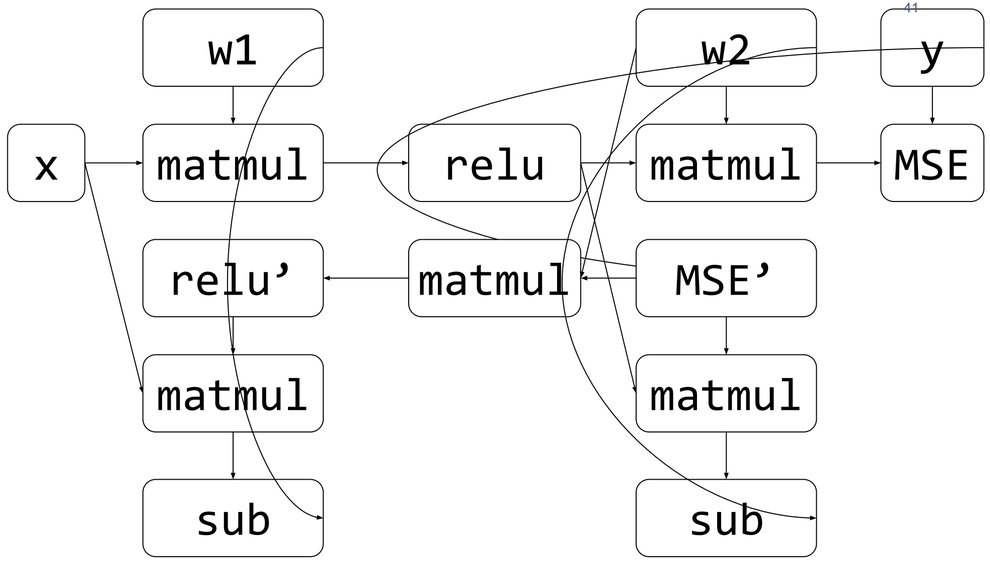
How to perform graph optimization?

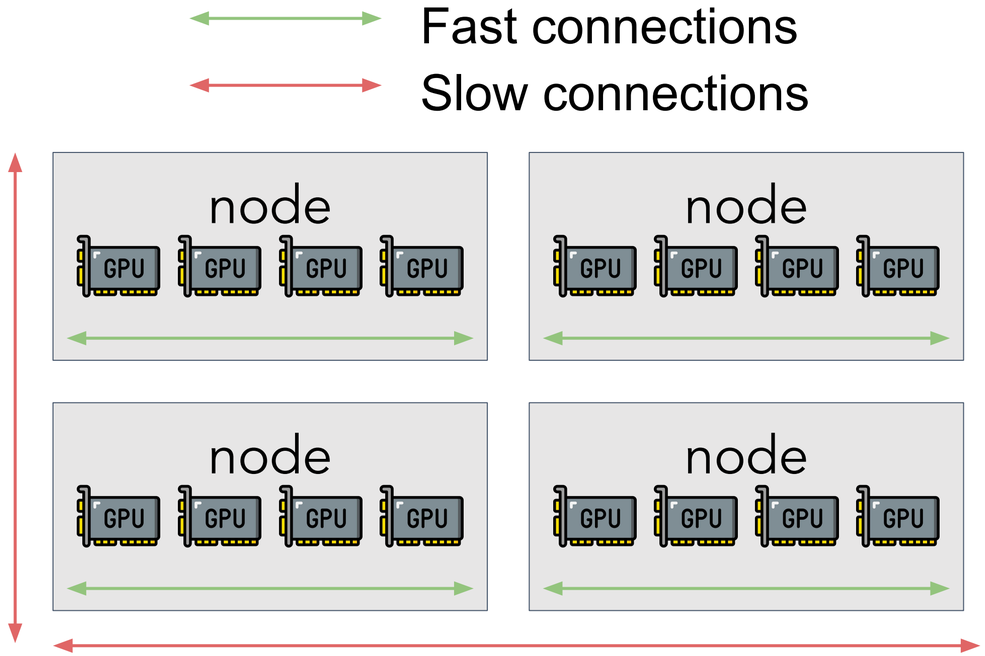
- Writing rules / template
- Auto discovery

Parallelization

- Goal: parallelize the graph compute over multiple devices

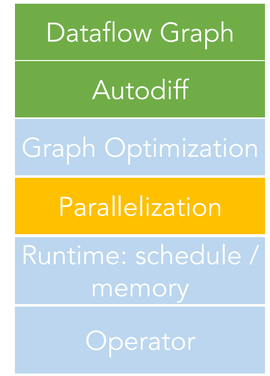
How to partition the computational graph on the device cluster?

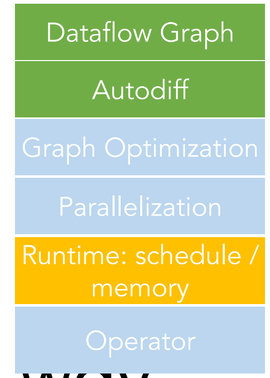




Parallelization Problems

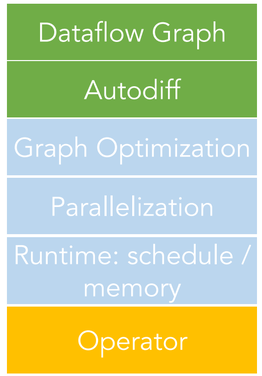
- How to partition
- How to communicate
- How to schedule
- Consistency
- How to auto-parallelize?





Runtime and Scheduling

- Goal: schedule the compute/communication/memory in a way that
  - As fast as possible
  - Overlap communication with compute
  - Subject to memory constraints



Operator Implementation

- Goal: get the fastest possible implementation of
  - Matmul
  - Conv2d?
- For different hardware: V100, A100, H100, phone, TPU
- For different precision: fp32, fp16, fp8, fp4
- For different shape: conv2d_3x3, conv2d_5x5, matmul2D, 3D, attention

High-level Picture

Data

Model

Comput
e

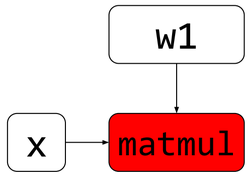
❓ $\{x_i\}^n{}_{i=1}$

✅ Math primitives (mostly matmul)

✅ A repr that expresses the computation using primitives

❓ Make them run on (clusters of) different kinds of hardware

Next: How to make operators run (fast) on devices?



Dataflow Graph

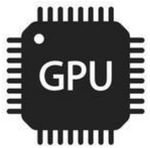
Autodiff

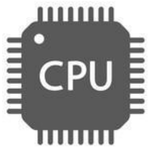
Graph Optimization

Parallelization

Runtime: schedule / memory

Operator optimization/compilation





Our Goal in This Layer: Maximize Arithmetic Intensity

$$\max \text{AI} = \#\text{ops} / \#\text{bytes}$$

Next

- How we can make operator fast in general
- Case study: Matmul
- GPU architecture and programming
- Roofline model

How we can make operator fast in general

- Vectorization
- Data layout
- Parallelization
- Matmul-specific
  - Tiling

Using vectorized operations: array add

Why vectorized is faster than unvectorized?

```
Float A[256], B[256], C[256]
For (int i = 0; i < 256; ++i) {
  C[i] = A[i] + B[i]
}
```

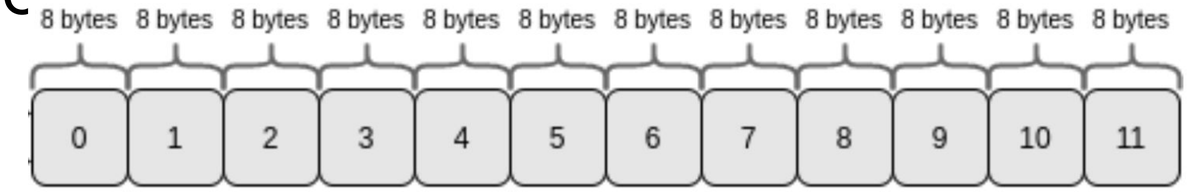
unvectorized

```
for (int i = 0; i < 64; ++i) {
      float4 a = load_float4(A + i*4);
      float4 b = load_float4(B + i*4);
      float4 c = add_float4(a, b);
      store_float4(C + i* 4, c);
```

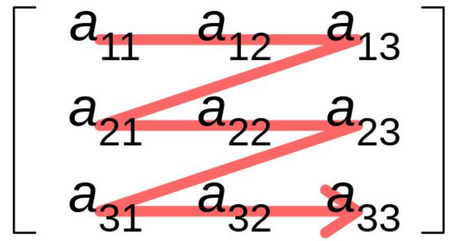
vectorized

Data Layout: make read/write faster

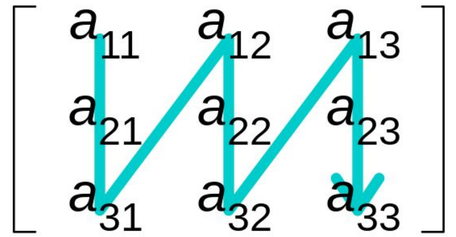
- How to store a matrix in memory
  - Data in memory are stored sequentially (no tensor awareness)
- Row Major: A[i, j] = A.data[i*A.shape[1] + j]
- Column major: A[i, j] = A.data[i*A.shape[0] + j]



Row-major order



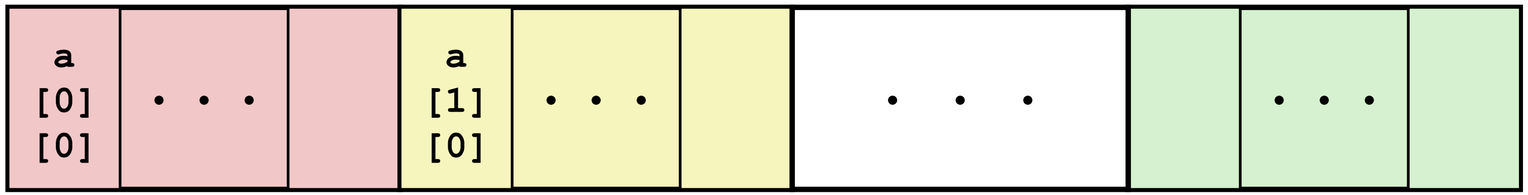
Column-major order



Be aware of your data layout

Assuming row-major array

```
int sum_array_rows(int a[M][N])
{
    int i, j, sum = 0;



    for (j = 0; j < N; j++)
        for (i = 0; i < M; i++)
            sum += a[i][j];
    return sum;
}
```



How to improve the above program?

High-level Picture: MCQ Time

Data

❓ $\{x_i\}^n{}_{i=1}$

ML Systems Store Data in:

A. Row major

B. Col major

C. Strides format: A[i, j] = A.data[offset + i*A.strides[0] + j * A.strides[1]]

Strides in High-dimension

Offset: the offset of the tensor relative to the underlying storage

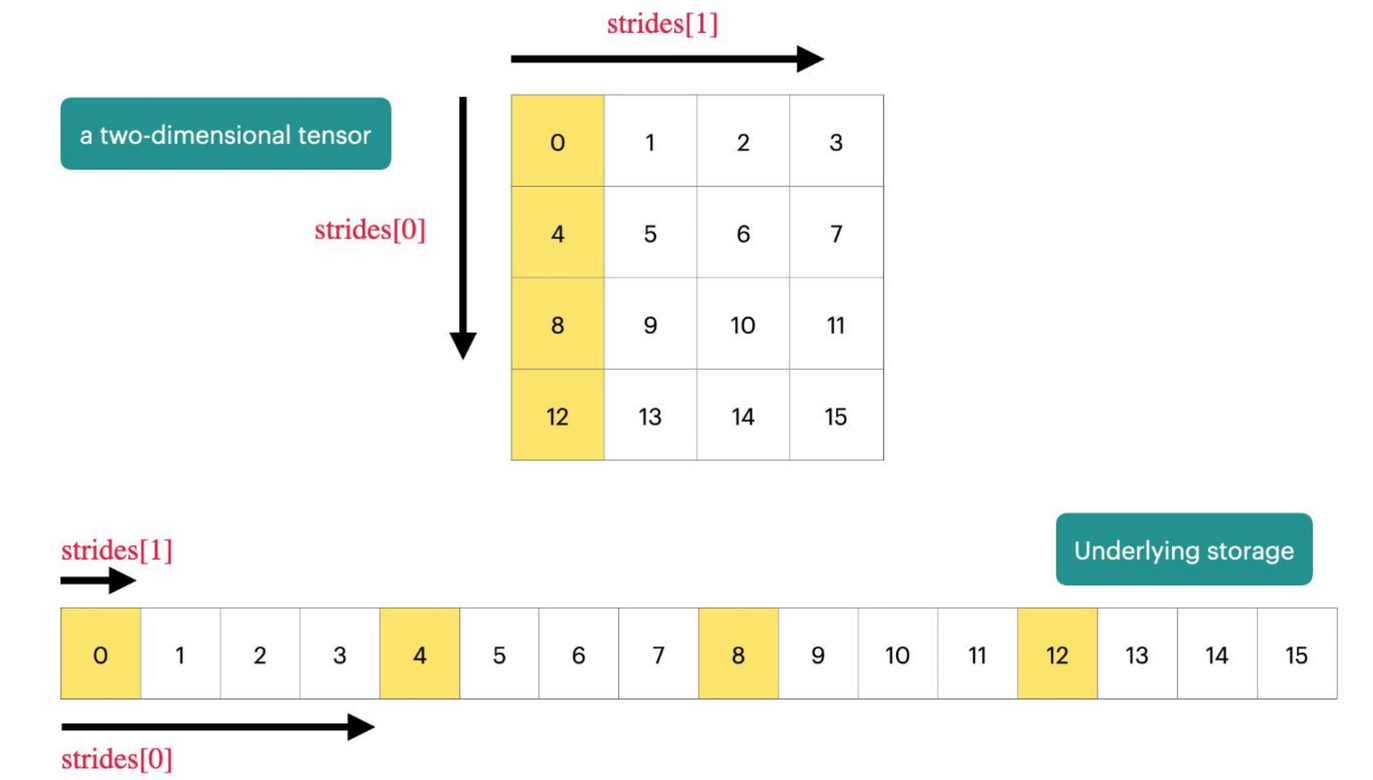
Strides: strides[i] indicates how many “elements” need to be skipped in memory to move “one unit” in the iii-th dimension of the tensor

Python

```
A[i0][i1][i2]... = A_internal[
    stride_offset
    + i0 * A.strides[0]
    + i1 * A.strides[1]
    + i2 * A.strides[2]
    + ...
    + in-1 * A.strides[n-1]
]
```

Strides format

- What we have when:
  - A.strides[0] = 1,
  - A.strides[1] = A.shape[0]?
- What we have when:
  - A.strides[0] = A.shape[1]
  - A.strides[1] = 1,
- Strides offers more flexibility



Questions

- If a tensor of shape [1, 2, 3, 4] is stored contiguous in memory following row Major, write down its strides?

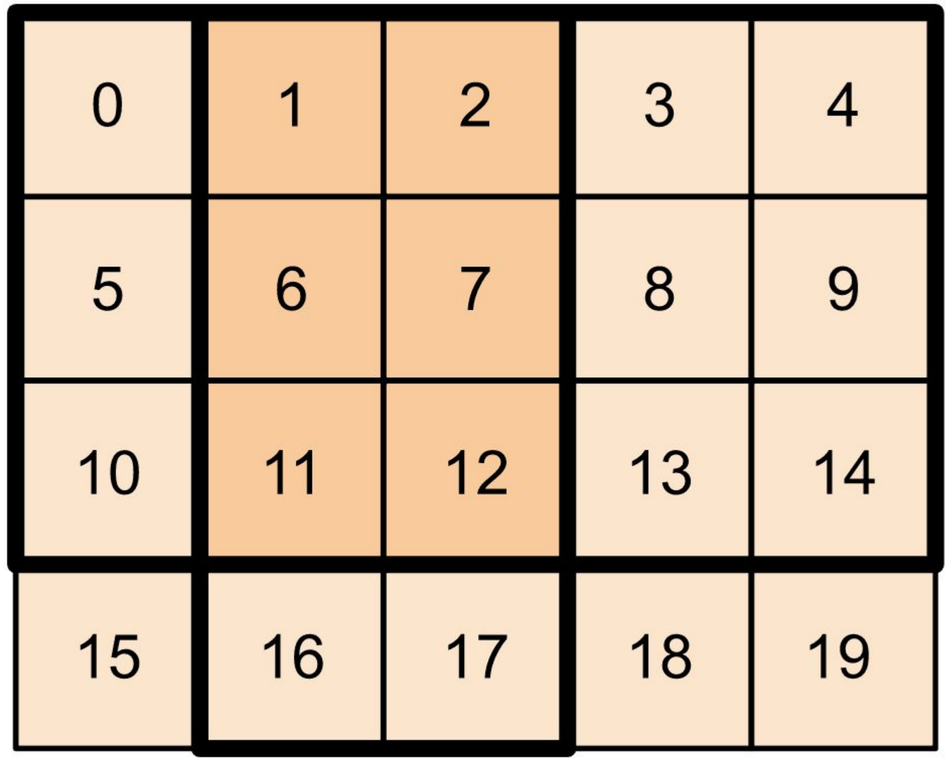
```
torch.arange(0, 24).reshape(1, 2, 3, 4)
print(t)
# tensor([[[[ 0,  1,  2,  3],
#           [ 4,  5,  6,  7],
#           [ 8,  9, 10, 11]],

#          [[12, 13, 14, 15],
#           [16, 17, 18, 19],
#           [20, 21, 22, 23]]]])

print(t.stride())
# (24, 12, 4, 1)
```

Why we bother saving “strides” when saving tensors

- Strides can separate the underlying storage and the view of the tensor -> Enable zero-copy of some very frequently used ops (recall “tensor.view” in pytorch)
  - Slice
  - Transpose (or reshape)
  - Broadcast

How to do Slice with strides

- Change the offset by +1
- Reduce the shape to [3, 2]
- Note: zero copy



Transpose

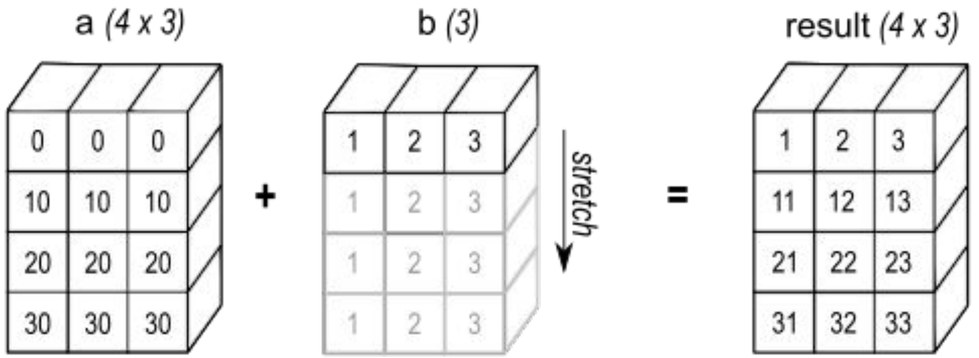
- How to do Transpose?
- Note: it is zero copy
- Note: What are the underlying Storage looking like?

Python

```python
print(t.stride())
# (24, 12, 4, 1)

print(t.permute((1, 2, 3, 0)).is_contiguous())
# True

print(t.permute((1, 2, 3, 0)).stride())
# (12, 4, 1, 24)

print_internal(t.permute((1, 2, 3, 0)))
# tensor([0,  1,  2,  3,
#         4,  5,  6,  7,
#         8,  9, 10, 11,
#         12, 13, 14, 15,
#         16, 17, 18, 19,
#         20, 21, 22, 23])
```

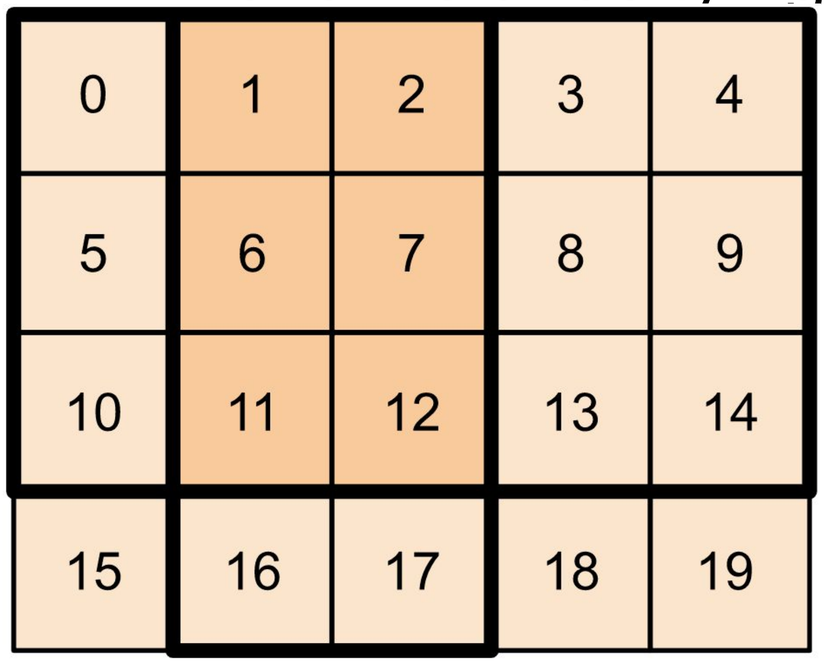
Broadcast

- Question: how to do broadcast?



Problems of Strides

- Memory Access may become not continuous
  - Many vectorized ops requires continuous storage
  - What's the underlying storage after slice?



TORCH.TENSOR.CONTIGUOUS

Tensor.contiguous(*memory_format=torch.contiguous_format*) → Tensor

Returns a contiguous in memory tensor containing the same data as `self` tensor. If `self` tensor is already in the specified memory format, this function returns the `self` tensor.

Parameters

memory_format (`torch.memory_format` , optional) – the desired memory format of returned Tensor. Default: `torch.contiguous_format` .

Parallelization (on CPUs)

- How to parallelize the loop?

```
for (int i = 0; i < 64; ++i) {
      float4 a = load_float4(A + i*4);
      float4 b = load_float4(B + i*4);
      float4 c = add_float4(a, b);
      store_float4(C + i* 4, c);
```

vectorized

```
#pragma omp parallel for
for (int i = 0; i < 64; ++i) {
      float4 a = load_float4(A + i*4);
      float4 b = load_float4(B + i*4);
      float4 c = add_float4(a, b);
      store_float4(C * 4, c);
}
```

Vectorized & parallelized

Summary

- Vectorization
  - reduce seek time
- Data layout
  - Stride format
  - Convenient
  - Zero copy
- Parallelization on CPUs