Final Project: A Mission on Mars

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1 Introduction

In this project, we simulate the complete trajectory of a rocket beginning from Earth's surface and ultimately reaching Mars. The primary objectives are to (1) model the ascent from Earth's surface to an initial low Earth orbit (LEO)-like orbit while accounting for gravitational and thrust effects, (2) perform an orbital maneuver to circularize what initially becomes an elliptical orbit, (3) execute the required burn to escape Earth's sphere of influence, and (4) compute and carry out an interplanetary transfer to Mars using a Lambert solver. Through these steps, we gain insights into the required delta-v, fuel consumption, and orbital mechanics involved in an interplanetary mission design. The approach relies on a set of Python scripts:

- `launch.py`: Models the rocket's ascent from Earth's surface using a gravity turn profile. Initially, the rocket is given a small tilt, about 0.25 degrees off-vertical, and thrust is always aligned with the rocket's velocity vector. Through trial and error, this initial angle was chosen to help the rocket gradually transition from vertical ascent to a more horizontal trajectory. Eventually, the rocket achieves an elliptical orbit with a perigee of about 150 km altitude and an apogee of about 1000 km.

- `tocircular.py`: At apogee (1000 km), a burn is applied to circularize the orbit. Since the velocity at apogee is nearly tangential and has no radial component, we simply add the delta-v needed to reach the circular orbital speed at that radius. The magnitude of the delta-v and the fuel required were computed using the rocket equation, assuming an instantaneous burn.

- `tomars.py`: Once the spacecraft is in a circular orbit around Earth, we compute Earth's position and Mars' position at a chosen epoch. We select a departure time where Earth leads Mars by about 40 to 45 degrees as an approximation of a suitable transfer window. We then perform an Earth escape burn, accelerating parallel to Earth's orbital motion to increase the spacecraft's heliocentric energy. After approximately 8 days of travel (based on trial and error to estimate when the spacecraft leaves

Earth's sphere of influence at about 900,000 km), we switch to a Sun-centered frame. In this frame, we use the Lambert solver to determine the heliocentric transfer orbit from Earth to Mars. The solver provides the initial heliocentric velocity vector needed. Implementing this burn sets the spacecraft on a trajectory that intercepts Mars.

These phases together show the step-by-step progression from launch, through elliptical orbit insertion, then to circularization, Earth escape, and finally interplanetary transfer. We have produced various plots to visualize radial and tangential velocities, radius versus angle to understand orbit geometry, and trajectory plots in both Earth-centered and heliocentric frames.

2 Methodology and Key Steps

For the launch phase, the gravity turn was implemented by starting with a small initial inclination angle (about 0.25 degrees) above the vertical. This angle was not analytically derived but found through repeated simulations and adjustments. The goal was to achieve a final orbit with significant tangential velocity, minimizing the required corrections later. By carefully choosing this initial angle through trial and error, the rocket was guided into a suitable elliptical orbit.

At apogee (1000 km altitude), a short burn is performed to circularize the orbit. Since we have the radius and know the desired circular orbit speed, the delta-v is computed straightforwardly. The exact mass of fuel burned is then calculated using the rocket equation. This approach ensures a stable orbit with no radial velocity component, making it easier to plan the subsequent escape maneuver.

For the interplanetary stage, we pick a time when Earth leads Mars by about 40 to 45 degrees, representing a rough transfer window. We align the rocket's Earth-escape burn direction with Earth's motion. After about 8 days of travel outward from Earth, determined by repetitive testing, the spacecraft leaves Earth's SOI. At this point, we switch to a heliocentric frame, treating the Sun as the central body. Solving the Lambert problem gives us the velocity vector required to reach Mars. While some approximations and iterative methods are involved, it seems to be providing a close match to the intended interplanetary trajectory.

3 Results and Discussion

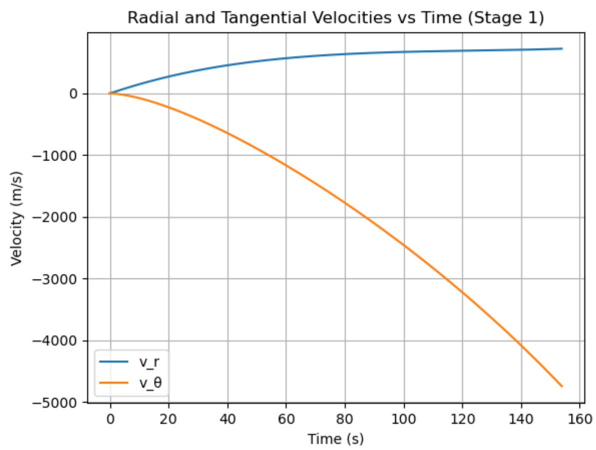

Figure 1: Radial and Tangential Velocities vs Time (Stage 1)

In Figure 1, we observe that the tangential velocity reaches approximately 5000 m/s, while the radial velocity remains just below 1000 m/s after about 150 seconds. This corresponds to the duration of Stage 1, demonstrating how the rocket's speed and direction evolve as it ascends through the initial phase of flight.

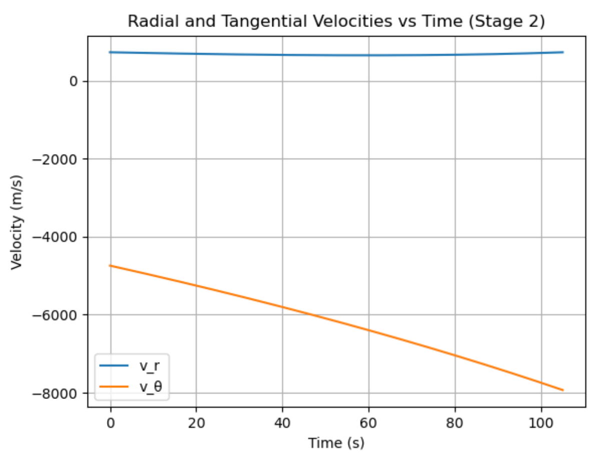

Figure 2: Radial and Tangential Velocities vs Time (Stage 2)

In Figure 2, we see that the tangential velocity reaches approximately the orbital velocity required for maintaining a low Earth orbit, while the radial velocity is about 1500 m/s. This suggests that the spacecraft is capable of achieving a stable orbit at this stage, rather than falling back to Earth.

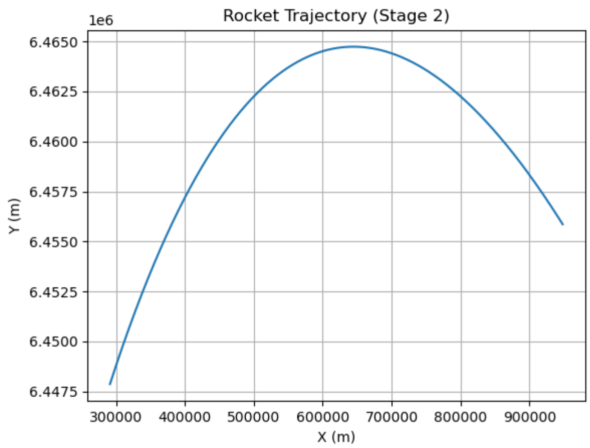

Figure 3: Rocket Trajectory (Stage 2) with respect to Earth's center

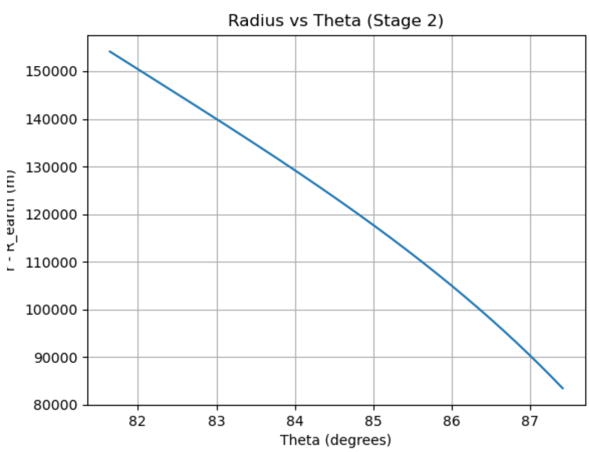

Figure 4: Radius vs Theta (Stage 2) with respect to Earth's center

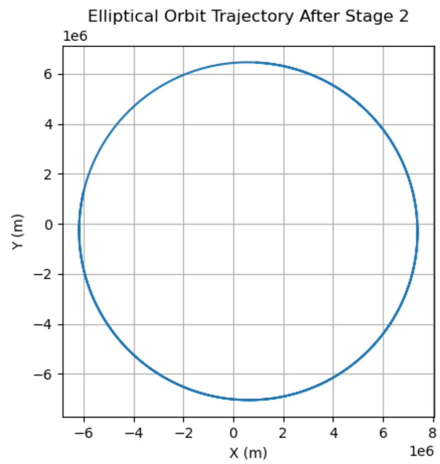

Figure 5: Elliptical Orbit Trajectory After Stage 2

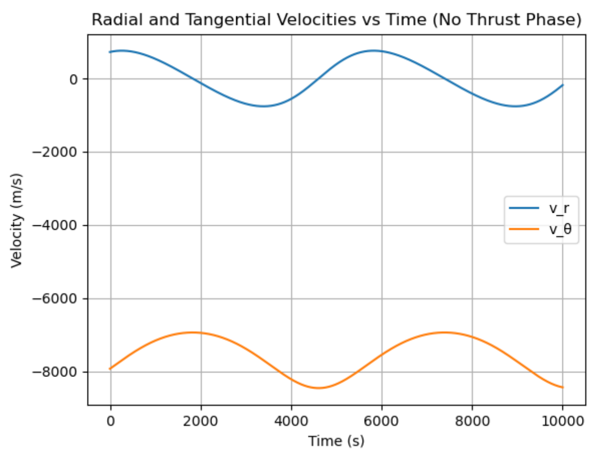

Figure 6: Radial and Tangential Velocities vs Time (No Thrust Phase)

As shown in Figures 5 and 6, the elliptical orbit has a perigee of about 150 km altitude and an apogee of approximately 1000 km. By examining the velocity plots and observing when the tangential velocity (v_theta) passes through zero and changes sign, we can precisely determine the times at which the spacecraft is at apogee or perigee. This information was crucial in identifying the optimal moment to perform the burn needed to circularize the orbit, ensuring a stable orbit rather than allowing the spacecraft to fall back to Earth.

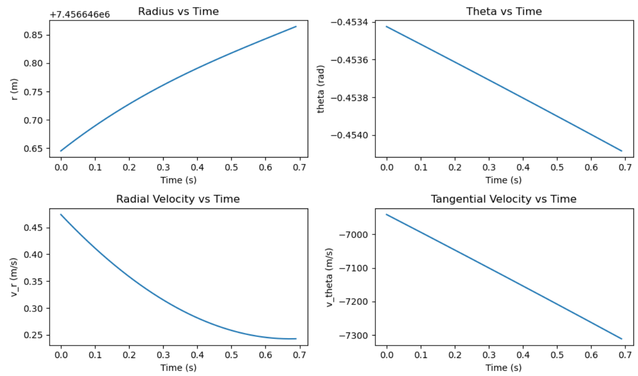

Figure 7: This figure shows the 0.7 s transfer from elliptical to circular orbit. This burn is unrealistically fast, involving a very high mass flow rate. For subsequent maneuvers, we assume instantaneous burns for convenience, eliminating trajectory deviations during the acceleration period.

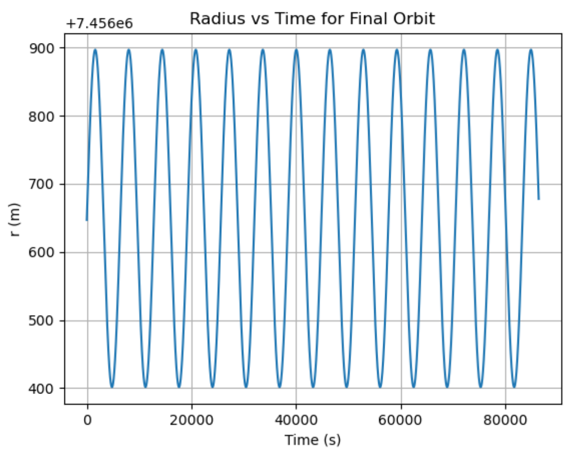

Figure 8: Radius vs Time for the Final Orbit. The stable radius (approximately Earth's radius plus 1000 km) indicates that the desired circular orbit was successfully achieved.

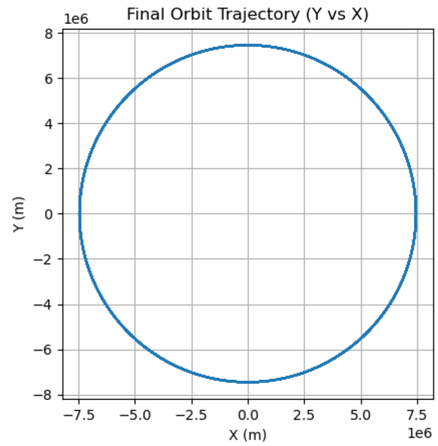

Figure 9: Final Orbit Trajectory (Y vs X). This figure confirms that the spacecraft maintains the intended circular orbit with the chosen altitude, verifying the mission's orbit shaping success.

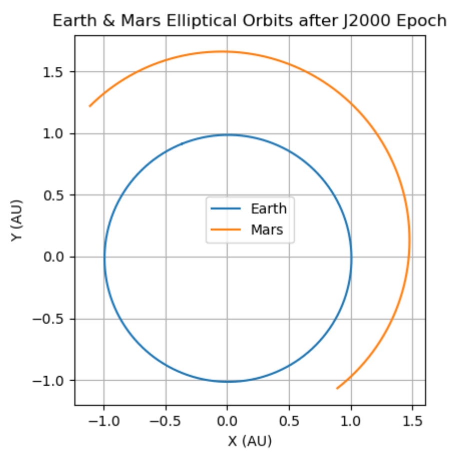

Figure 10: Earth & Mars Elliptical Orbits after J2000 Epoch

As shown in Figure 10, we determined the positions of Earth and Mars at various times over one year using their orbital elements. Starting from the epoch (January 1, 2000), we used a Kepler solver (using a Newton-Raphson root finding) to convert mean anomaly to eccentric anomaly and then to true anomaly. With the semi-major axis, eccentricity, and argument of perihelion known, we applied the `position_from_elements` function and the `ellipse_to_xy` transformation to compute the Cartesian coordinates (x, y) of each planet. The position_from_elements function converts the mean anomaly that we have obtained from NASA Database for the given epoch to a true anomaly and we add the argument of perihelion to get the angle with respect to our coordinate system. Then, we convert the elliptical coordinates to Cartesian. This process allows us to "synchronize" the orbits, the relative locations of Earth and Mars. Afterwards, we choose a time span to produce the elliptical paths for Earth and Mars as shown in the figure. We used the orbital parameters $a = 1.0$ AU, $e = 0.0167$ for Earth and $a = 1.523679$ AU, $e = 0.0934$ for Mars along with known angles omega and mean anomalies from NASA data at the time of the epoch to accurately reconstruct their orbits in a Sun-centered frame.

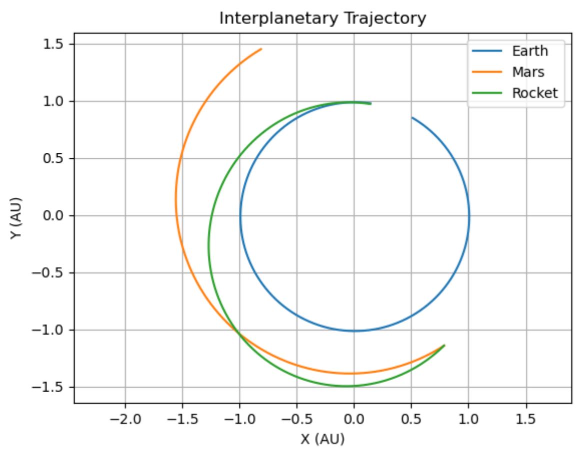

Figure 11: Interplanetary Trajectory Considering Only the Sun's Gravity

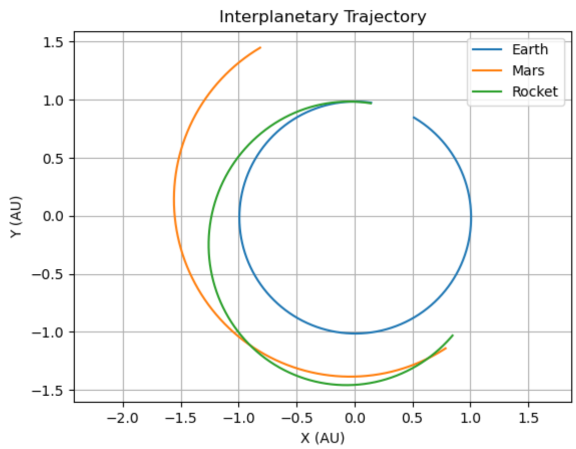

Figure 12: Interplanetary Trajectory Including Perturbations from Other Bodies

We used the Lambert solver from the `pykep` library to determine the interplanetary transfer trajectory after the spacecraft had escaped Earth to about 900,000 km. Note that after escaping earth we switched to the heliocentric frame of reference. When we consider only the Sun's gravity, as shown in Figure 11, the rocket follows a neat path that intercepts Mars as intended. However, once we include additional gravitational influences from Earth, Mars, and even Jupiter, perturbations naturally arise (Figure 12). As expected, these perturbations cause slight deviations from the planned trajectory.

Intermediary correction burns could be applied to compensate for these perturbations and optimize the path to Mars, but in our study, we did not attempt these. We also created a movie of these trajectories to visualize how the spacecraft evolves under both conditions. Careful analysis of these paths could help understanding how to reverse the perturbations and burn fuel to achieve a stable orbit around Mars. Optimizing a stable orbit around Mars and analyzing/testing landing strategies would be ideal for further studies in this project. Note that when the simulation ends the remaining mass is about 60 tons, which is about 98-percent fuel. So, a safe landing on Mars could be achieved. Of course in our simulation we had a very powerful engine in our rocket (with an Isp value of 800) and the paths we took were merely rough estimates, not even close to being as effective as actual methods used in real missions.