Conditional Prob.

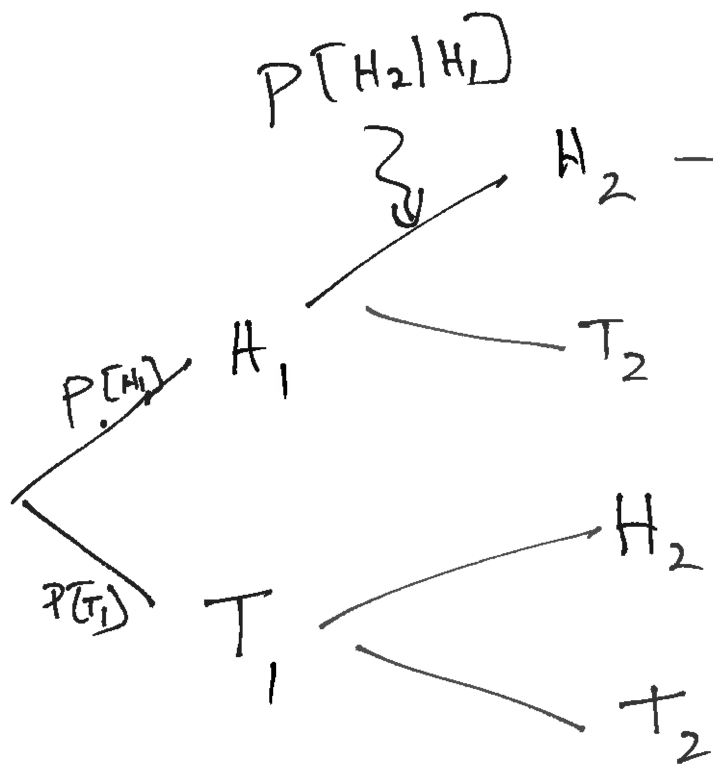
$$P[A|B] = \frac{P[AB]}{P[B]}$$

$$P[A] = \sum_i P[A|B_i]\, P[B_i]$$

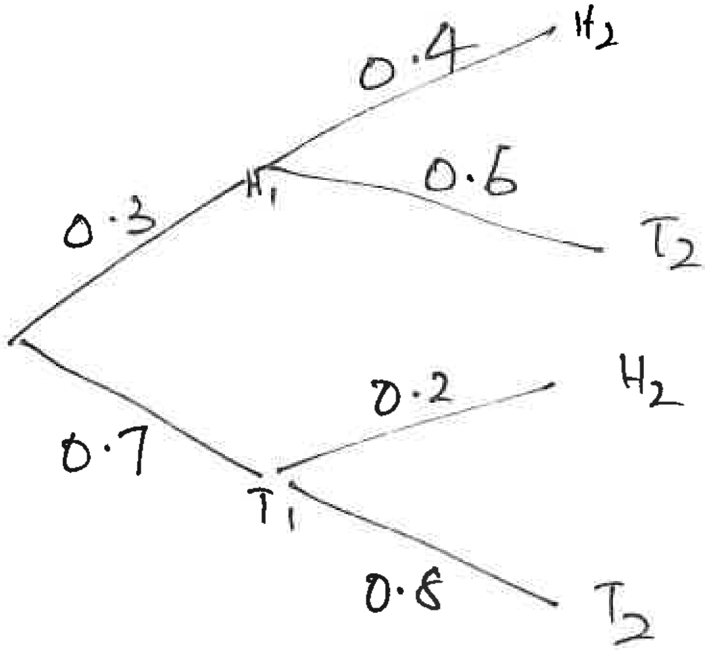
Sequential Expts.

↓ Can use a tree diagram

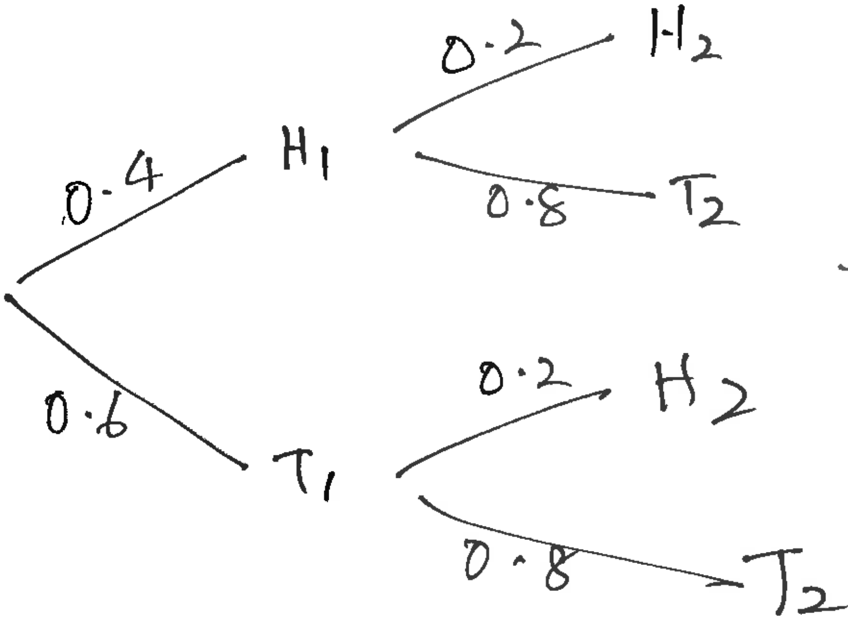
Prob. tree



$\rightarrow P[H_1 H_2] = P[H_1]\, P[H_2|H_1]$



$\rightarrow$ 1st & second Tosses are <u>not</u> independent



$\rightarrow$ 1st & second Tosses are independent

<u>HW</u> $P[T_2 \mid H_1]$ & $P[T_2 / T_1]$

$P[\text{No. of heads} \geq 1]$

EX 2·3 (3rd), 1·27 (2nd)

2 Coins → C_1, C_2 → fair

C_1 → Biased

$P[H|C_2] = 1/2$

$P[T|C_2] = 1/2$

$\leftarrow P[H] = 3/4$

$P[T] = 1/4$

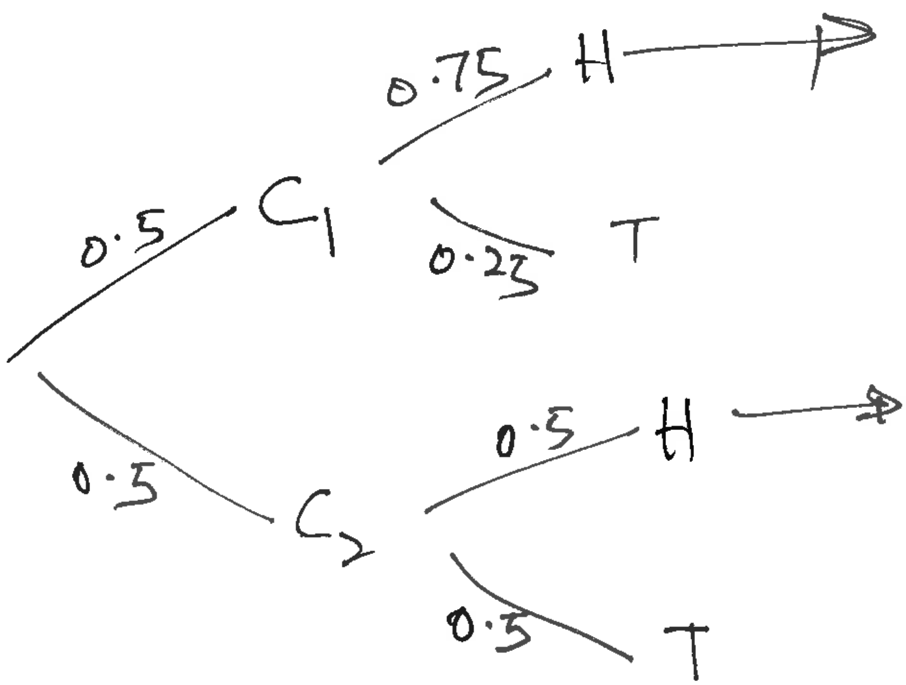
$P[H|C_1] = 3/4$

$P[T|C_1] = 1/4$

$$P[C_1] = 1/2$$
$$P[C_2] = 1/2$$

→ randomly select a coin

$$P[C_1|H] = ?$$



Mutually Exclusive

$$P[C_1|H] = \frac{P[C_1,H]}{P[H]}$$

$$P[H] = 0.5 \times 0.75 + 0.5 \times 0.5$$

$$P[C_1,H] = 0.5 \times 0.75$$

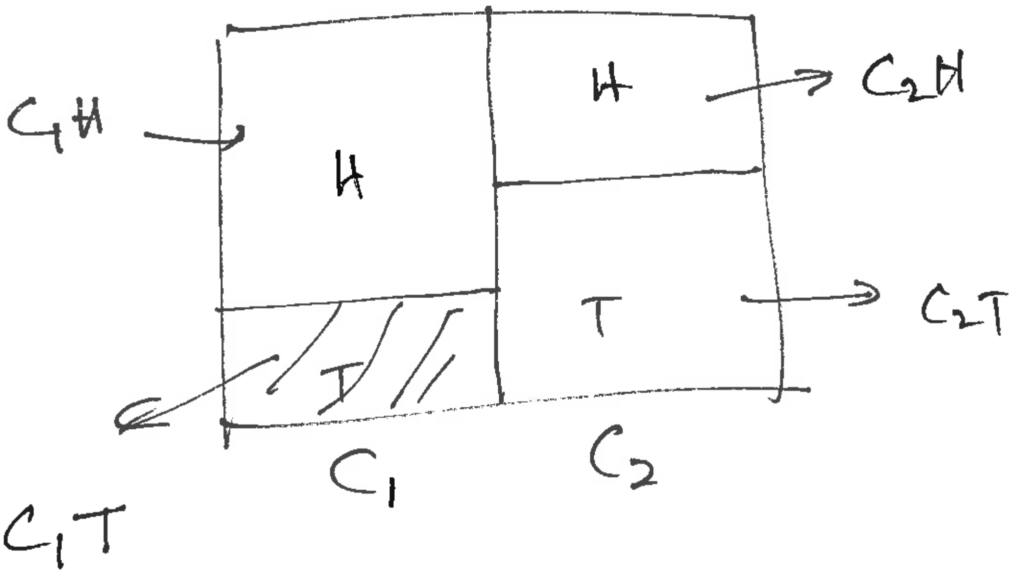
$$\therefore P_r[C_1|H] =$$

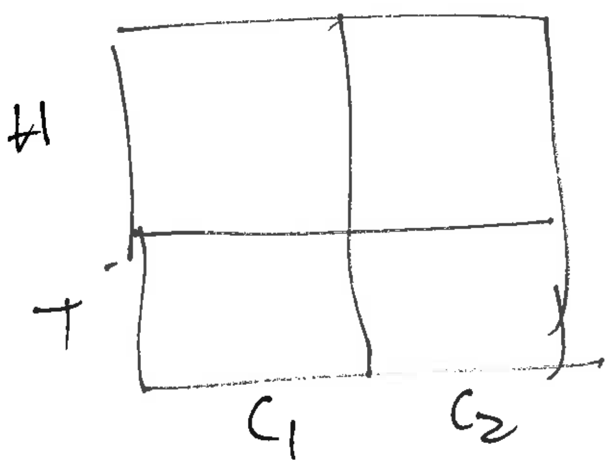
without drawing the tree

$$P[C_1,H] = P[H|C_1]P[C_1]$$
$$= 0.75 \times 0.5$$

$$P[H] = P[H|C_1]P[C_1] + P[H|C_2]P[C_2]$$

$=$ Same as before.





Ex Store $\longrightarrow$ products from A, B, C companies

20% are from A

30% . . . B

50% . . . C

1% of the products from A

2% . . . from B

5% . . . from C are defective

A, B, C

D → Defective

G → Good

$P[A] = 0.2$, $P[B] = 0.3$, $P[C] = 0.5$

$P[D|A] = 0.01$, $P[D|B] = 0.02$

$P[D|C] = 0.05$

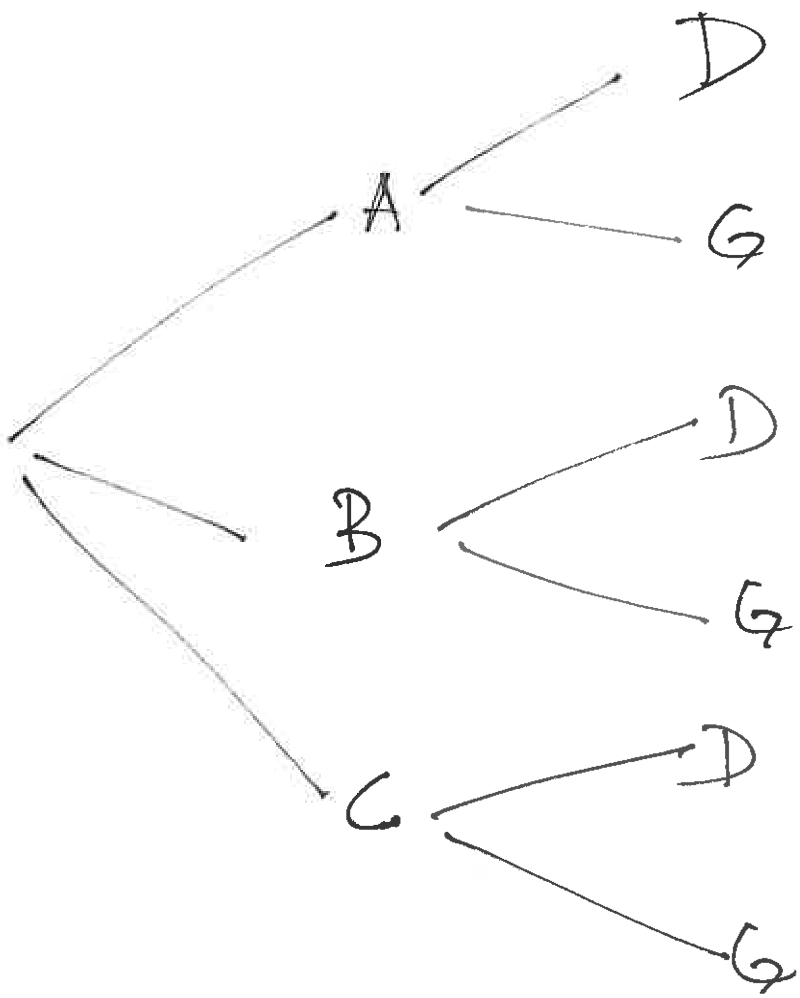
If a randomly selected product was defective, find the prob. that it came from company C

$$P[C|D] = \frac{P[CD]}{P[D]}$$

$$P[CD] = P[D|C]\,P[C] = 0.05 \times 0.5$$

$$P[D] = P[D|A]\,P[A] + P[D|B]\,P[B] + P[D|C]\,P[C]$$

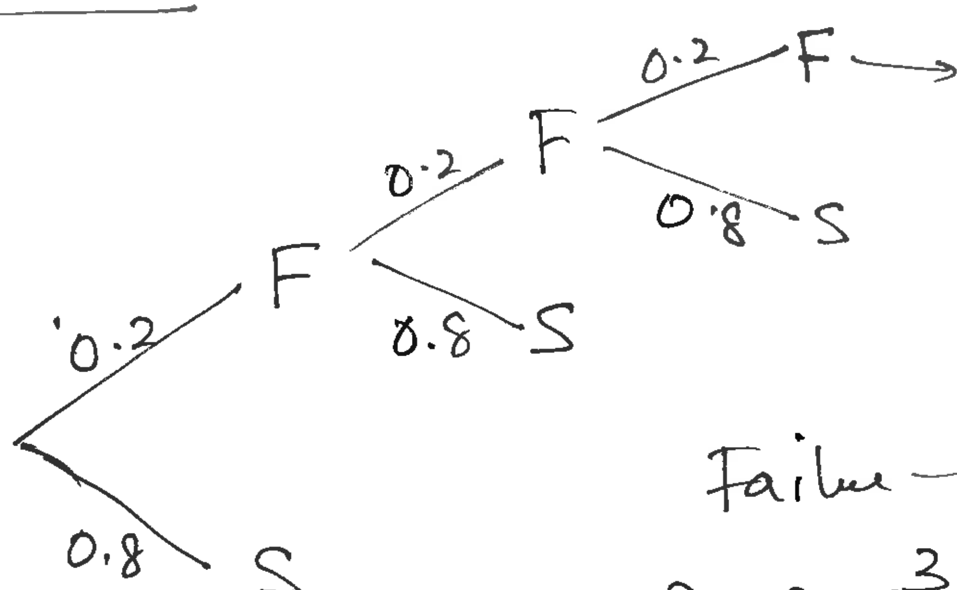
$=$



→ Solve using the Tree AW

Q 2.1 (3rd)
1.7 (2nd)

F – Failure
S – Success



Failure → FFF

$P[FFF] = (0.2)^3 = 0.008$

→ 0.8%

Counting Methods

ex :- Deck of cards → 52

Expt: Draw cards one at a time.

No. of possible ways =

$52 \times 51 \times 50 \cdot \cdot \cdot \times 4 \times 3 \times 2 \times 1$

$= 52!$

Also the no. of tree terminals

eg:- Draw 4 cards.

No. of ways $= 52 \times 51 \times 50 \times 49$

No. of tree terminals $= \dfrac{52!}{48!}$

n objects

↳ n distinguishable objects

draw k objects

No. of ways to draw k distinguishable objects from n object

$$= \frac{n!}{(n-k)!}$$

→ Order matters

Each way to draw k objects is called a k-permutation

Order matters

In permutations

eg:-

10 hearts

3 Spades

Ace Diamond

4 hearts

One permutation

No. of k-permutations out of n distinguishable objects

$$(n)_k = \frac{n!}{(n-k)!}$$

When the order does not matter

Each way (selection) is called a k-combination

No. of k combinations out of n distinguishable objects

(n choose k) $\rightarrow$ $\binom{n}{k}$

order does not matter

$$\binom{n}{k} = ?$$

Every k-combination has k! permutations

$$\therefore \binom{n}{k} k! = (n)_k = \frac{n!}{(n-k)!}$$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

n objects.

Draw (select) k objects

if the order matters

No. of k-permutations $(n)_k = \dfrac{n!}{(n-k)!}$

If the order does not matter

No. of k-combinations

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

eg:- $\binom{100}{2} = \dfrac{100!}{2!\,(98!)} = \dfrac{100 \times 99}{1 \times 2}$

$$\binom{80}{3} = \frac{80 \times 79 \times 78}{1 \times 2 \times 3}$$

Notes on $\binom{n}{k}$

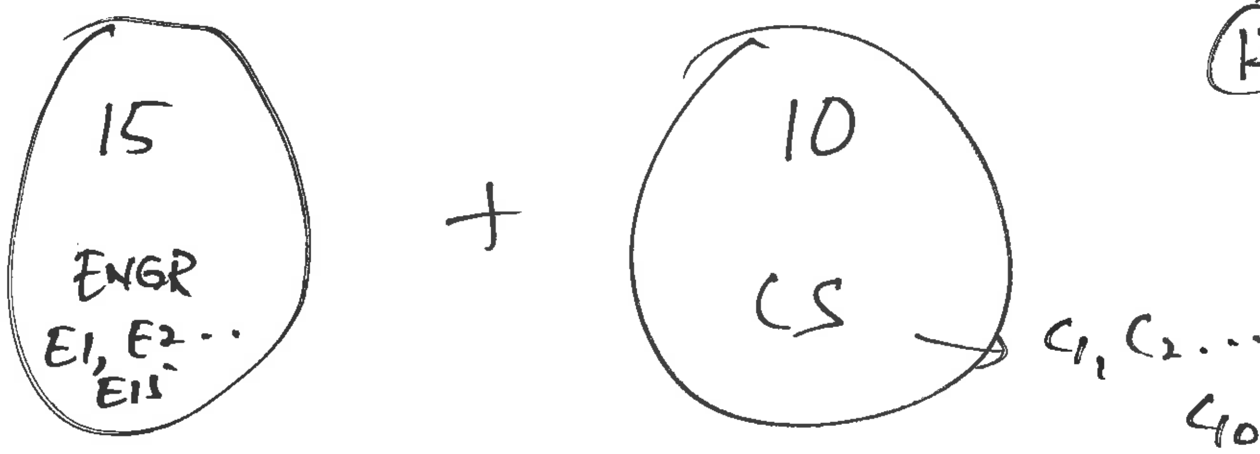
1. $$\binom{n}{k} = \binom{n}{n-k}$$

eg:- $$\binom{100}{98} = \binom{100}{2} = \frac{100 \times 99}{1 \times 2}$$

2. $$\binom{n}{1} = n = \binom{n}{n-1}$$

3. $$\binom{n}{n} = \binom{n}{0} = 1$$

Ex:-



Form a committee of 5 students

$\downarrow$

3 from ENGR

+ 2 from CS

How many Committees can be formed?

No. of 3-combinations from ENGR student

$$N_1 = \binom{15}{3} = \frac{15 \times 14 \times 13}{1 \times 2 \times 3}$$

No of 2-combinations from 10 CS stude

$$N_2 = \binom{10}{2} = \frac{10 \times 9}{1 \times 2}$$

$\therefore$ No. of committees

$$N = N_1 N_2 = \frac{15 \times 14 \times 13}{1 \times 2 \times 3} \times \frac{10 \times 9}{1 \times 2}$$

If every committee is equally likely

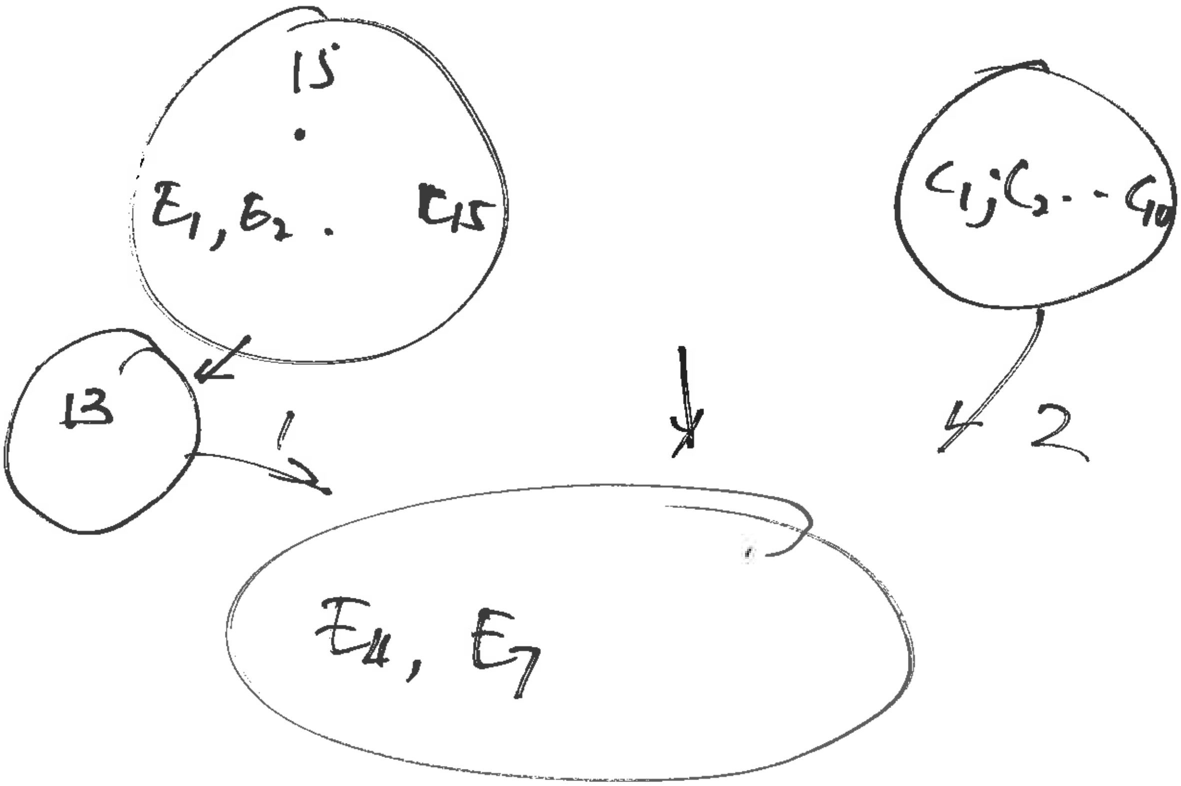
find the prob. that the following committee $(E_9, E_{12}, E_{15}, C_4, C_6)$ is selected

$$P[E_9, E_{12}, E_{15}, C_4, C_6] = \frac{1}{N_1 N_2}$$

eg:- Same prob.

Find the prob. that E_4 & E_7 gets selected.

First find the no. of committees that can be formed with E_4 & E_7 on it



No. of ways to select 1 ENGR stud from 13 ENGR stude

$$= \binom{13}{1} = 13$$

" " " " " 2 CS students from 10 CS students

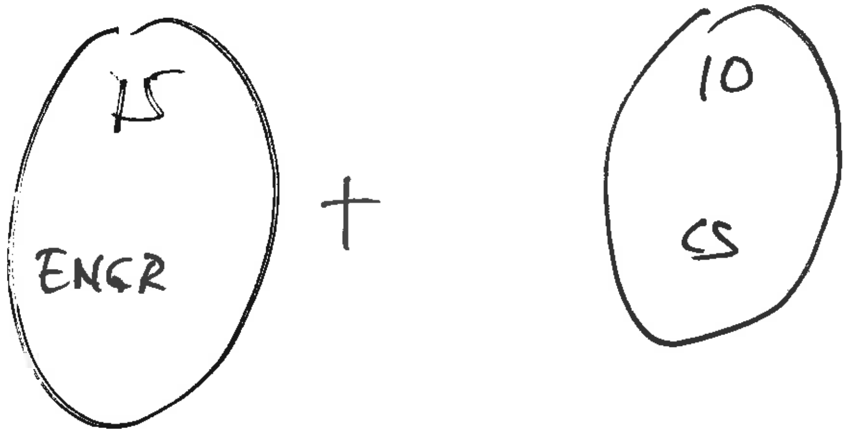
$$= \binom{10}{2}$$

∴ No. of committees that can be formed with E_4 & E_7 on it

$$= 13 \times \frac{10 \times 9}{1 \times 2} = N'$$

∴ $P[E_4 \ \& \ E_7 \text{ gets selected}] = \frac{N'}{N}$

eg:-



Committee of 5

At least 2 must come from Each group

Total No. of committees ?

2 Cases

Case 1: 3 ENGR + 2 CS $\longrightarrow$ Previous example

$$\frac{15\times14\times13}{1\times2\times3}\times\frac{10\times9}{1\times2} = N = N_1 N_2$$

Case 2: 2 ENGR + 3 CS

No. of Committees

$$= \binom{15}{2}\times\binom{10}{3}$$

$$= \frac{15\times14}{1\times2}\times\frac{10\times9\times8}{1\times2\times3} \Rightarrow \bar{N}$$

Total No. of committees

$$= N + \bar{N}$$

HW: $P[E_4 \,\&\, E_7$ get selected$]$

15 10

$$\binom{15}{2}\binom{10}{2}\binom{21}{1} \; ?. \; \times$$

$E_1 E_2$ ~~E~~ E_3

$E_1 E_3$ ~~E~~ E_2

Same combination is counted twice

Repeated Trials

eg:-

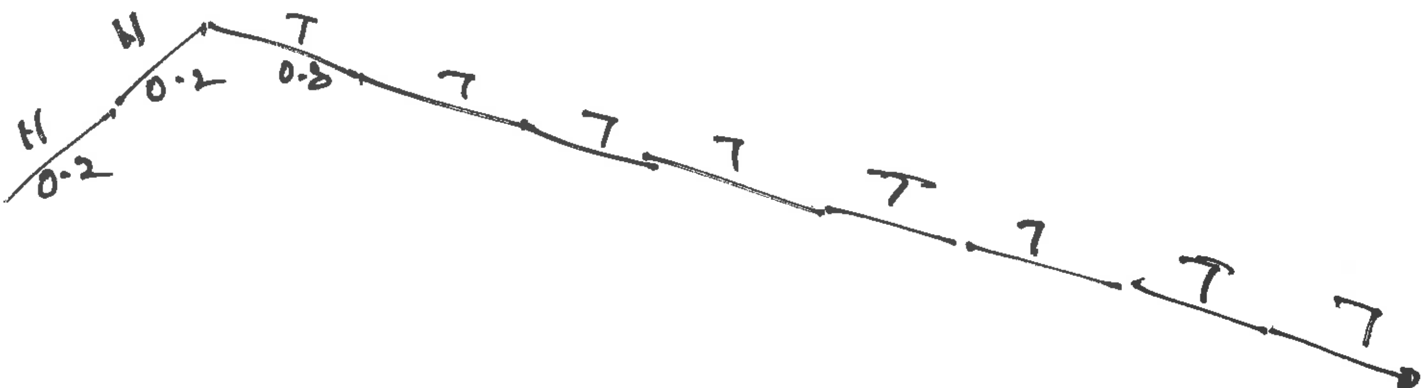
Toss a coin 10 times

tosses are independent

$$P[H] = 0.2$$

in each toss

Find the prob. of getting exactly 2 heads in 10 tosses.



$$(0.2)^2 (0.8)^8$$

$$P[2 \text{ Heads} + 8 \text{ Tails}] = \binom{10}{2} (0.2)^2 (0.8)^8$$

$$= \frac{10 \times 9}{1 \times 2} (0.2)^2 (0.8)^8$$

Find the prob. of getting

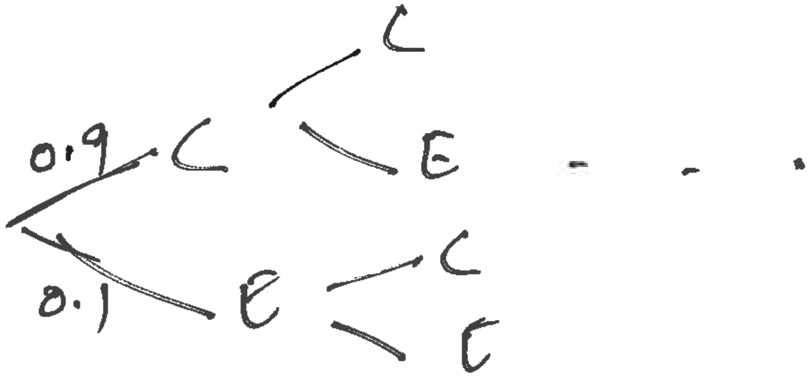
$$P[H\,H\,T\,T\,T\,H\,T\,H\,T\,T] = (0.2)^4 (0.8)^6$$

only one tree termination

eg:- Consider transmission of 5 binary bits.

Each bit can be received in error with prob. 0.1

Find the prob. of receiving more than 3 bits in error



2 Cases

Case 1 4 Errors + 1 Correct → eg: EECEE

$P[\;] = (0.1)^4$ 0.9

$P[4 \text{ Errors} + 1 \text{ Correct}]$

$= \binom{5}{4}(0.1)^4(0.9) = 5 \times (0.1)^4(0.9)$

$\binom{5}{1} = 5$

Case 2 All 5 in Error

⇓

$P[EEEEE] = (0.1)^5$

⇓

One tree terminal

$\therefore P[\text{More than 3 errors}] = 5 \times (0.1)^4(0.9) + (0.1)^5$