**Name:** **Quiz #5** **11/08/15**
**UTD ID:**

[1] The joint probability mass function of two discrete random variables X and Y is

$$P_{X,Y}(x,y) = \begin{cases} c x^2 y, & x = -1, 1, 2 \text{ and } y = 1, 2 \\ 0, & \text{otherwise} \end{cases}$$

Find the value of c and P[X>Y].

| | $x=-1$ | $x=1$ | $x=2$ |
|---|---|---|---|
| $y=1$ | $c$ | $c$ | $4c$ |
| $y=2$ | $2c$ | $2c$ | $8c$ |

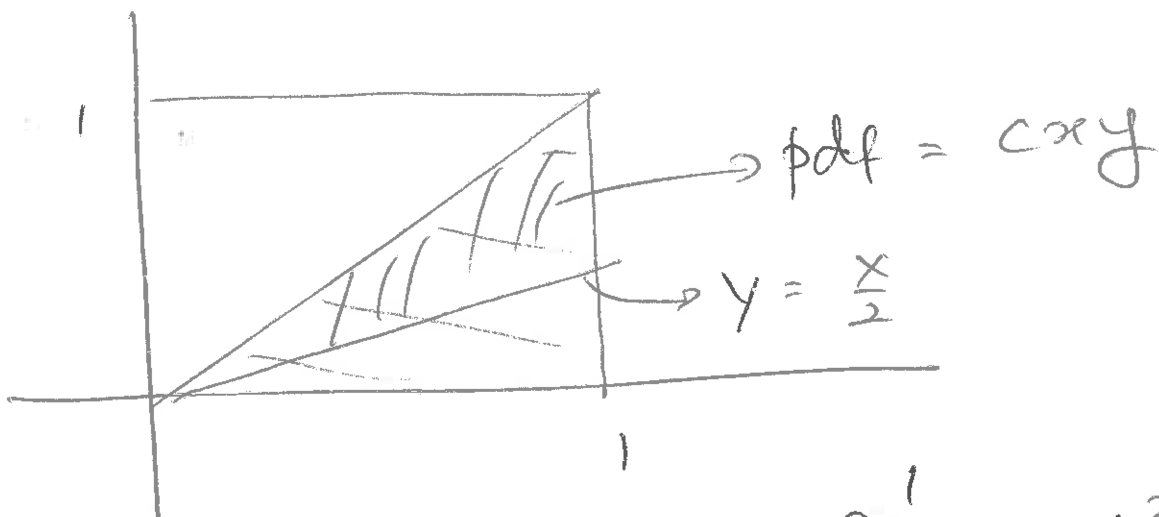
$18c = 1 \longrightarrow c = \frac{1}{18}$

$P[X>Y] = 4c = \frac{4}{18} = \frac{2}{9}$

[2] The joint probability density function of two continuous random variables X and Y is

$$f_{X,Y}(x,y) = \begin{cases} cxy, & 0 \le y \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the value of c and the P[2Y>X]




$$c\int_0^1\int_0^x xy\,dy\,dx = 1 \rightarrow c\int_0^1 x\cdot \frac{y^2}{2}\Big|_0^x dx$$

$$= \frac{c}{2}\int_0^1 x^3\,dx = 1$$

$$= \frac{c}{2}\,\frac{x^4}{4}\Big|_0^1 = 1 \rightarrow \frac{c}{8} = 1$$

$c = 8$

$$P\left[Y > \frac{1}{2}X\right] = c\int_0^1 x\int_{\frac{x}{2}}^x y\,dy\,dx$$

$$= c\int_0^1 x\,\frac{y^2}{2}\Big|_{x/2}^x dx$$

$$= \frac{c}{2}\int_0^1 x\cdot\left[x^2 - \frac{x^2}{4}\right]dx = \frac{c}{8}\int_0^1 3x^3\,dx$$

$$= \frac{3c}{8}\ \frac{x^4}{4}\Big|_0^1 = \frac{3c}{32} = \frac{3}{4}$$

$$P[2y > x] = \frac{3}{4}$$