Discrete RV ✓

Ch. 4 → 3^{rd} Edition

" 3 → 2^{nd} Edition

<u>Continuous</u> RVs

Perform the Expt → Based on the Outcome Assig a value for X

↓

Continuous

eg:- Duration of a call
Mass of an apple.

Real

eg:- Randomly select a real number betn 0 & 10

→ infinite no. of possibilities

Space of X is infinitely large.

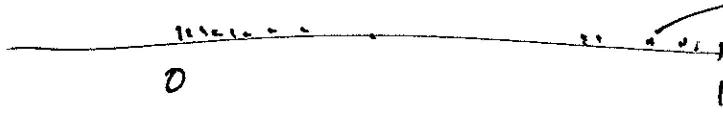
Recall: when x is discrete ②

$\hookrightarrow$ $P_X(x)$ & $F_X(x)$

$P_X(x) = P[X = x]$ $\quad$ $F_X(x) = P[X \leq x]$

eg:- X is a continuous RV, that can take any real value bet^n 0 & 10 with <u>same prob.</u>

$P_X(x)$



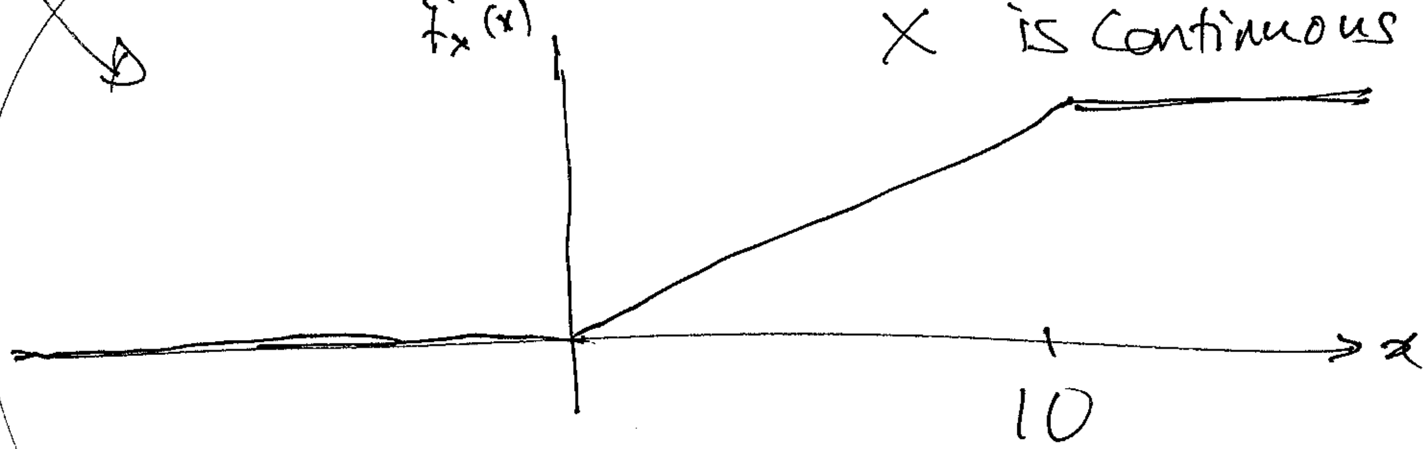
$\rightarrow$ infinite no. of them

$\therefore$ Height $\rightarrow 0$

$\Downarrow$

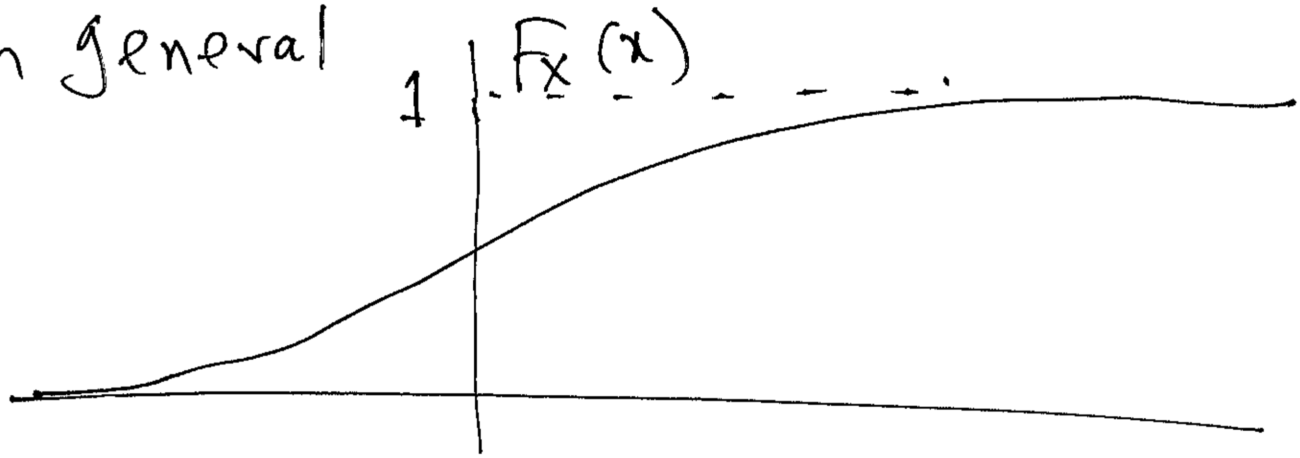
PMF is not really useful when X is continuous

$F_X(x)$



$\Rightarrow$ is continuous, can use $F_X(x)$

In general



Properties of $F_X(x)$:

1. $F_X(-\infty) = 0, \quad F_X(+\infty) = 1$

2. $F_X(x)$ is a non-decreasing func^n

3. $P[a < X \leq b] = F_X(b) - F_X(a)$

If X is continuous,

$P[X = a] = 0$ — Prob. that the continuous RV X is exactly equal to some value $= 0$.

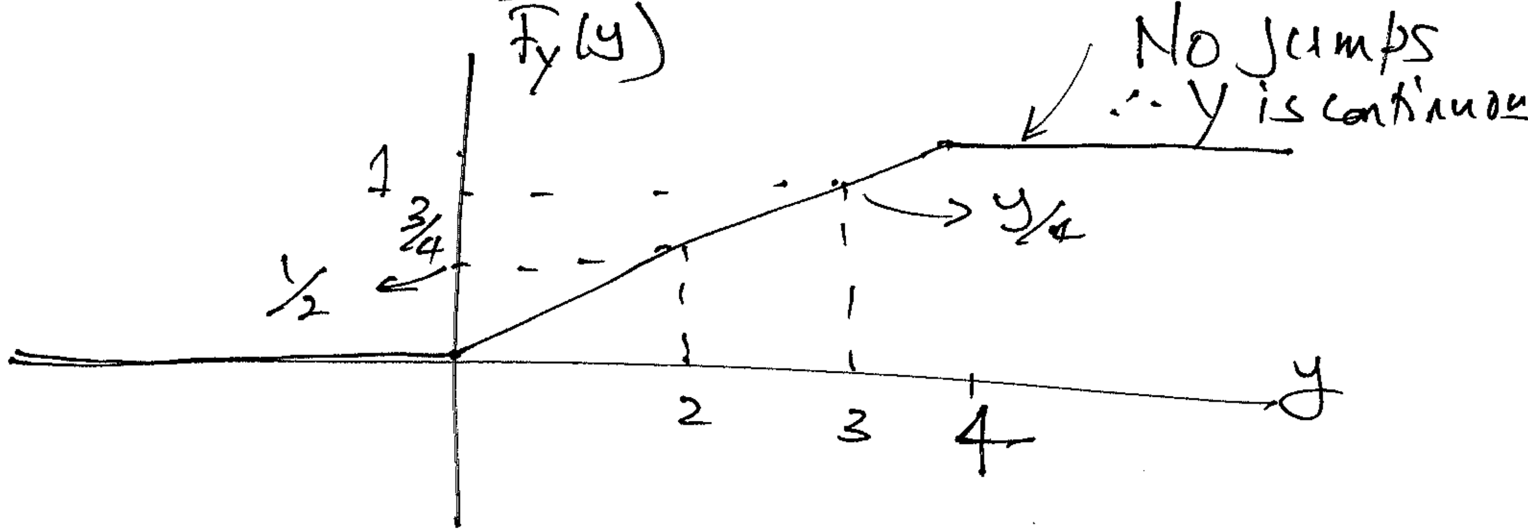
$P[a < X \leq b] = P[a < X < b] = P[a \leq X \leq b]$

Q 4.2 (Q 3.1)

$$F_Y(y) = \begin{cases} 0, & y < 0 \\ \frac{y}{4}, & 0 \le y < 4 \\ 1, & y \ge 4 \end{cases}$$

$P[2 < X < 4]$

$P[Y \le -1]$



$P[Y \le -1] = 0$

$\to F_Y(-1)$

$P[2 < Y < 4] = P[2 < Y \le 4]$

$(P[Y=4]=0)$

$= F_Y(4) - F_Y(2) = \frac{1}{2}$

4.3 (3.2) Prob. Density Func^n (pdf) (5)

Recall: we discarded $P_X(x)$

(pdf) takes its place

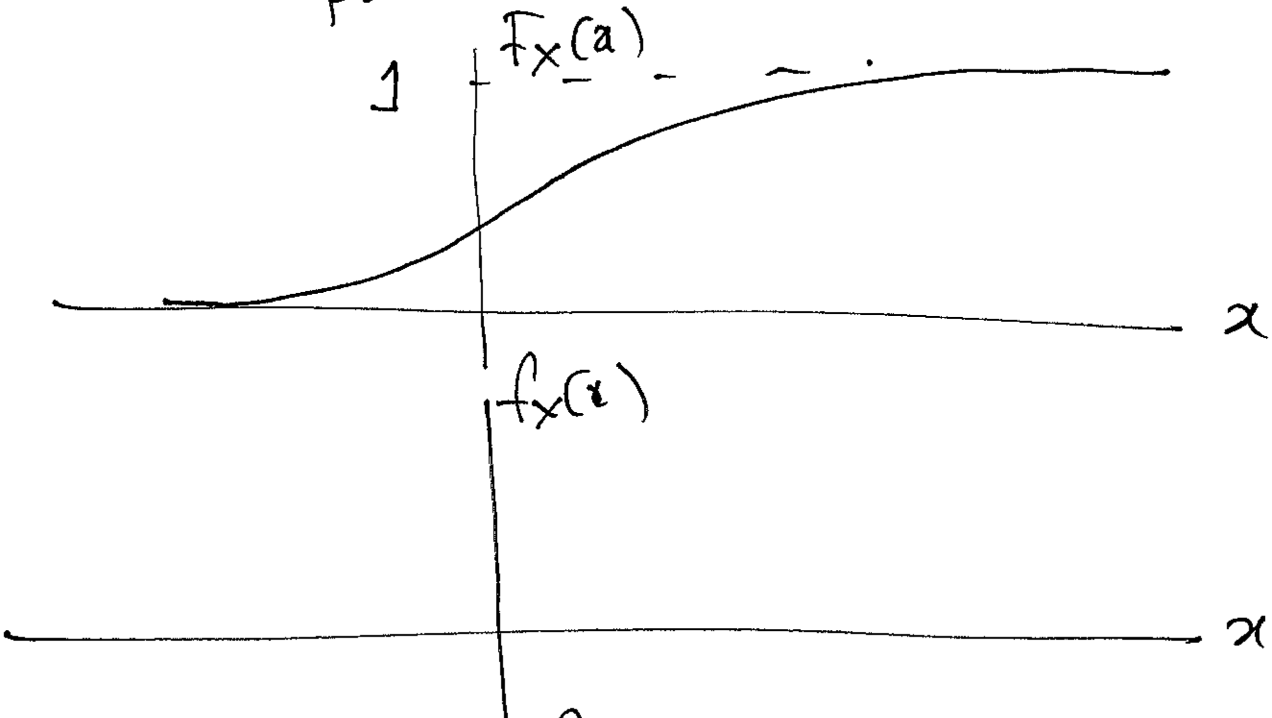
Not^n: $f_X(x)$

Def^n: $f_X(x) = \frac{d}{dx} F_X(x)$

$f_X(x)$: pdf of X

$\frac{d}{dx} F_X(x)$: slope of the CDF

$F_X(x)$: CDF of X

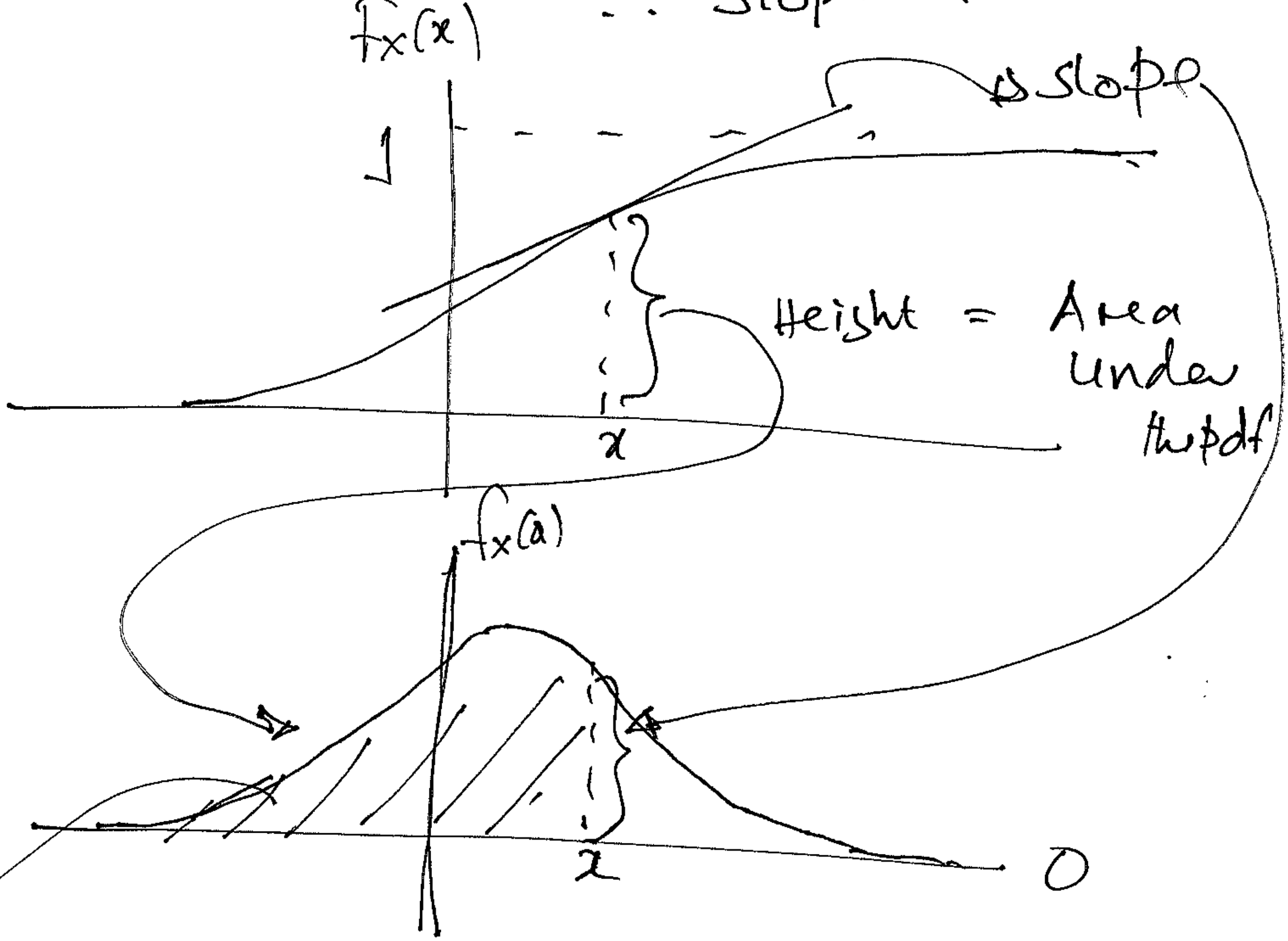


$f_X(-\infty) = 0 \quad f_X(+\infty) = 0, \quad f_X(x) \geq 0$

$f_X(x) \geq 0 \rightarrow$ why? (6)

$F_X(x)$ is Non-decreas

$\therefore$ Slope of $F_X(x) \geq 0$



$$f_X(x) = \frac{d}{dx} F_X(x)$$

$$F_X(x) = \underbrace{\int_{-\infty}^{x} f_X(u)\, du}_{\text{Result is a func}^n \text{ of } x}$$

$\rightarrow u$ is a dummy Variab

(7)

$$\int_{-\infty}^{\infty} f_X(u)\,du = F_X(+\infty) = 1$$

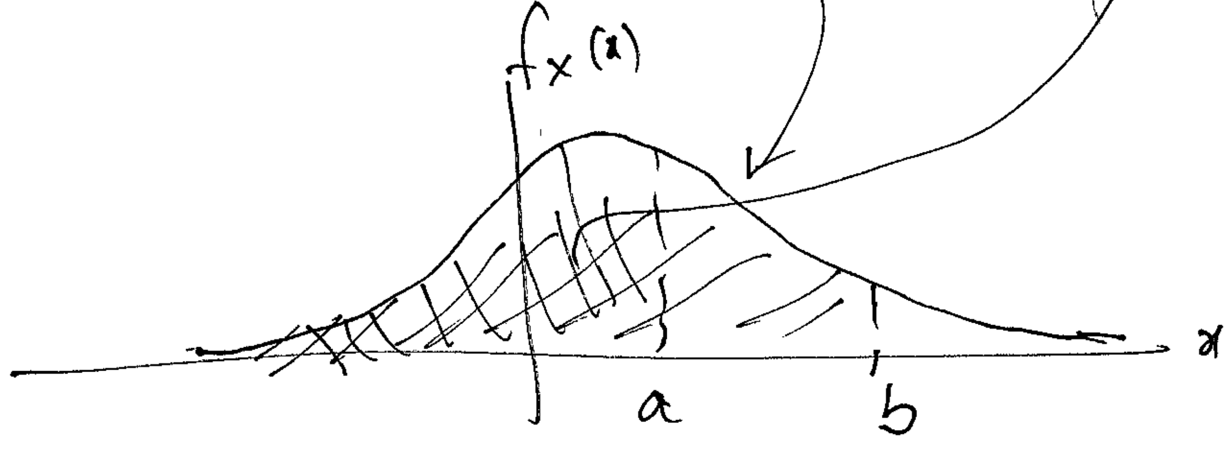
$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

Total Area under the pdf = 1.

$$P[a < X \le b] = F_X(b) - F_X(a)$$

$\underline{b > a}$

$$= \int_{-\infty}^{b} f_X(u)\,du - \int_{-\infty}^{a} f_X(u)\,du$$



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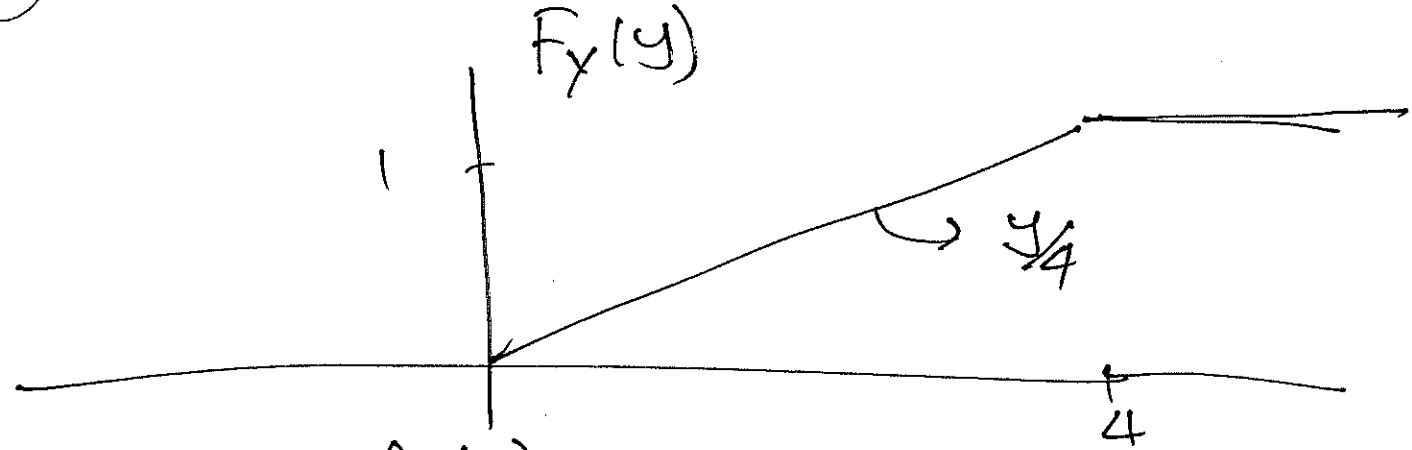
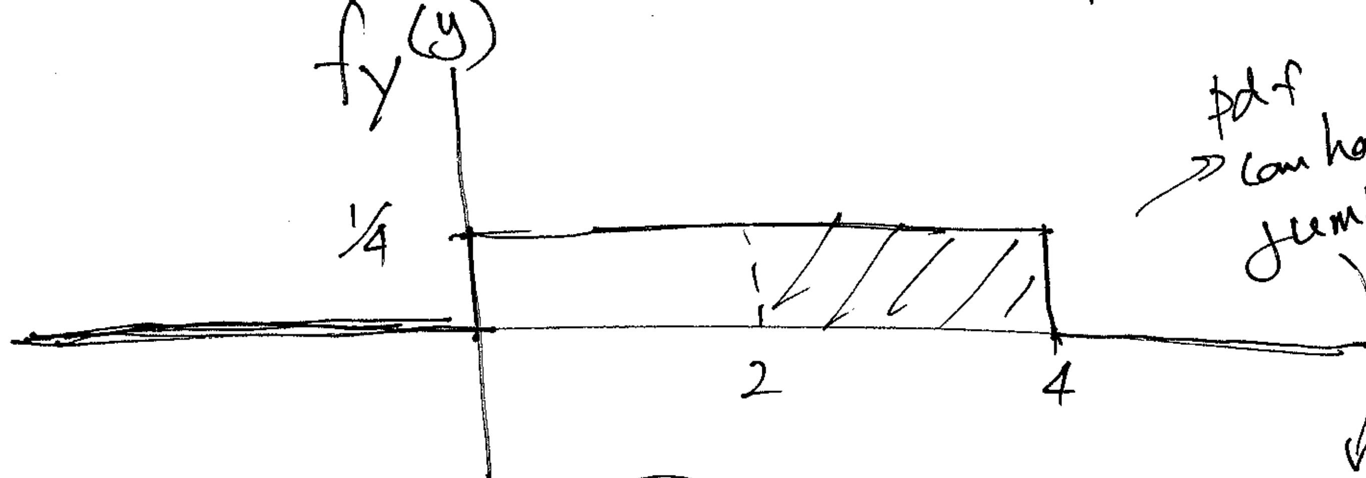
$$P[a < X \leq b] = \int_a^b f_X(u)\,du$$

eg:- Q 4.2 (Q 3.1)





pdf can have jumps when the RV is continuous

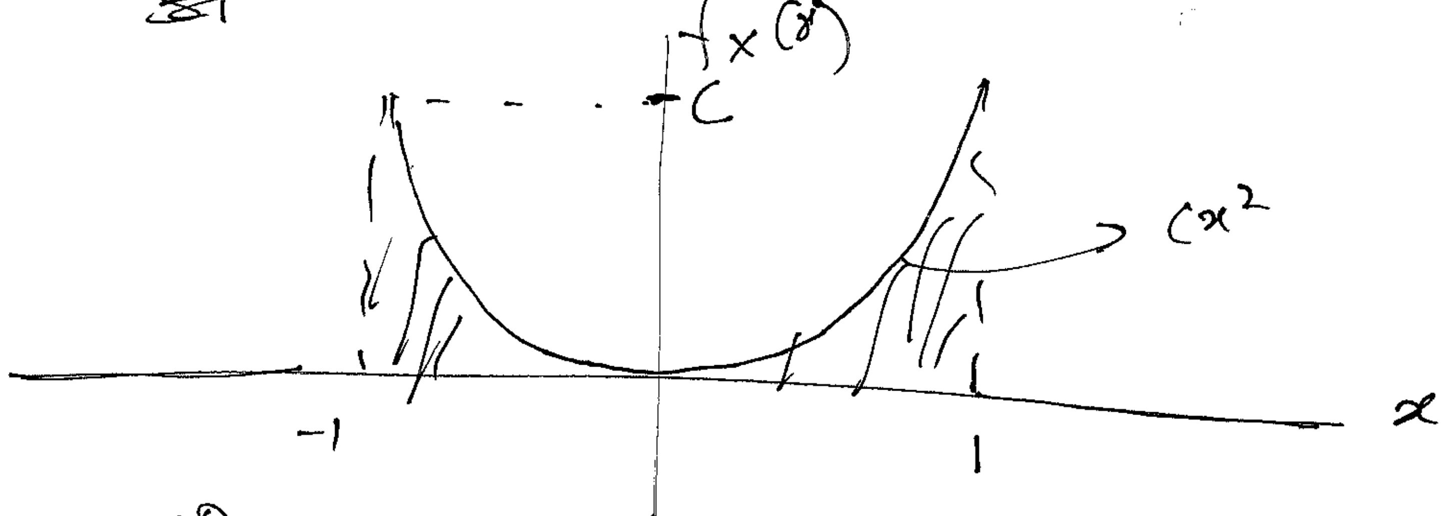
$$P[2 < Y < 4] = \int_2^4 f_Y(y)\,dy = \tfrac{1}{4}(2) = \tfrac{1}{2}$$

eg:- $f_X(x) = \begin{cases} cx^2, & |x| \le 1 \\ 0, & \text{otherwise} \end{cases}$

$-1 \le x \le 1$

Find c, $P[X > \frac{1}{2}]$

Sketch the CDF



$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

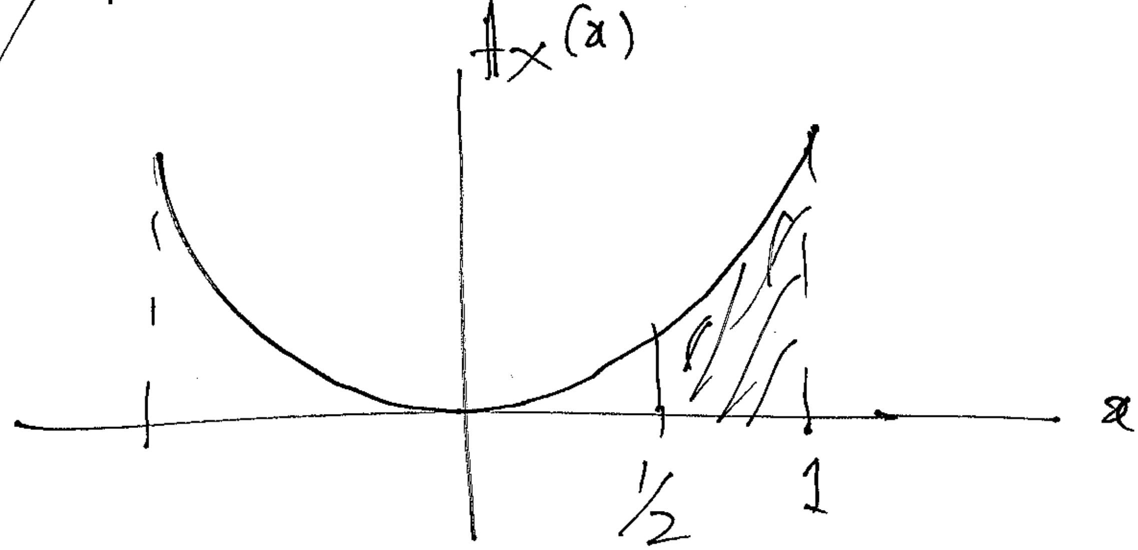
$$\int_{-1}^{1} \underbrace{cx^2}_{\text{even}}\,dx = 1$$

$$= 2\int_{0}^{1} cx^2\,dx = 1$$

$$\therefore\ 2c \left.\frac{x^3}{3}\right|_0^1 = 1$$

$$\frac{2c}{3} = 1 \longrightarrow c = \frac{3}{2}$$

$$P[X > 1/2] = \ ?$$



$$= \int_{1/2}^{1} cx^2\,dx = c\left.\frac{x^3}{3}\right|_{1/2}^{1} = \checkmark$$

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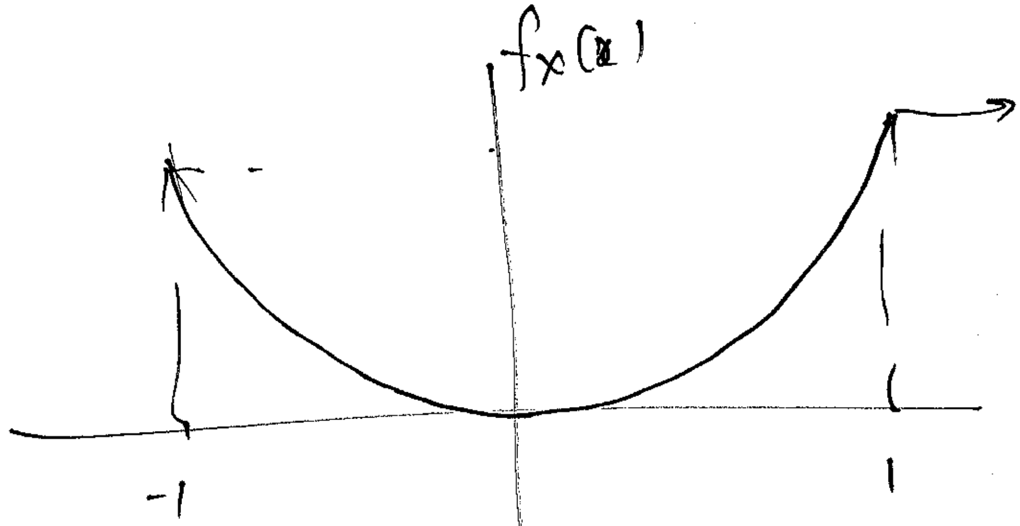
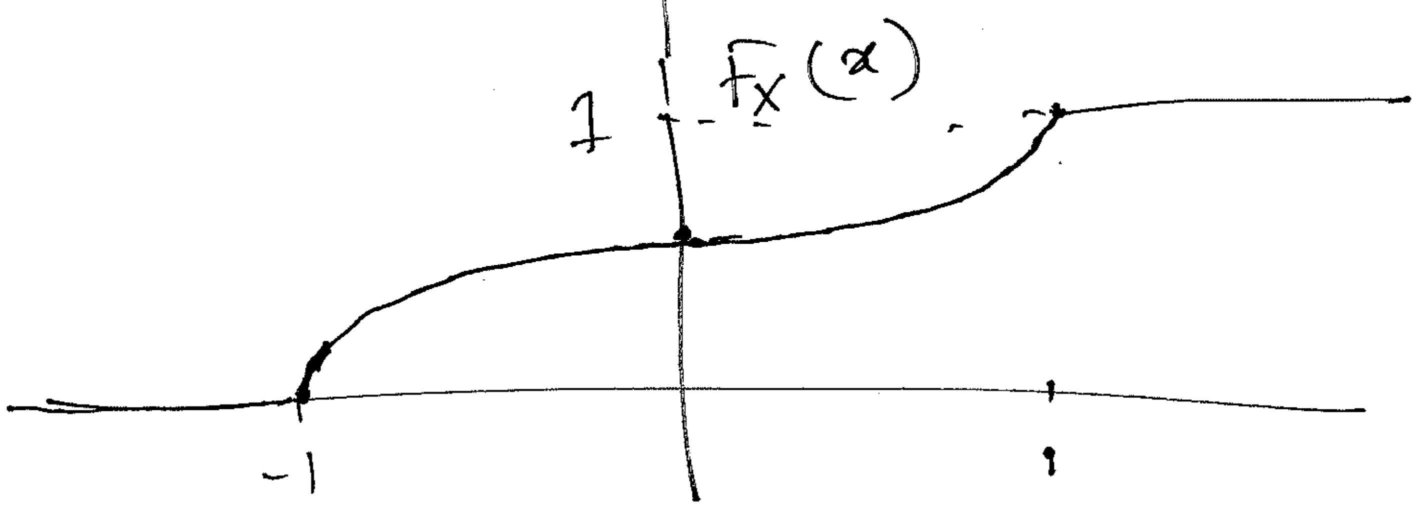


Slope
$2cx$
$= 2\left(\frac{3}{2}\right)(1)$
$=$

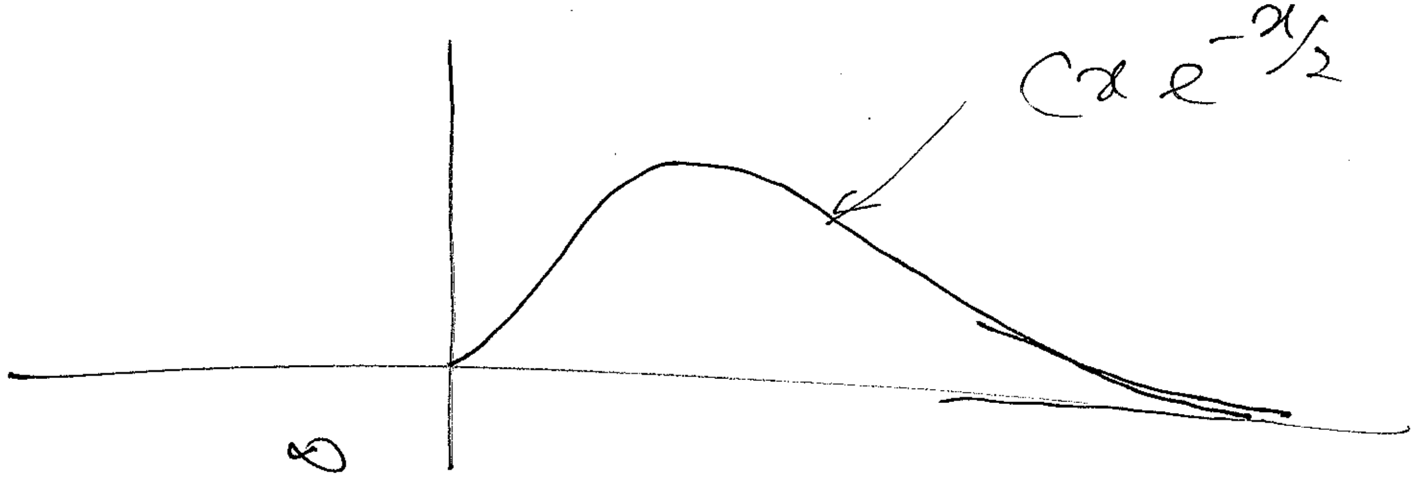


<u>HW</u> Find an expression for $F_X(x)$.

$$P\left[X > \tfrac{1}{2}\right] = F_X(1) - F_X(1/2)$$

Q 4.3 (3.2)

$$f_X(x) = \begin{cases} c x e^{-\frac{x}{2}}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$



$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

$$\int_{0}^{\infty} c x e^{-x/2}\,dx = 1$$

use integration by parts.

HW

$\therefore \quad c = \checkmark$

$c = 1/4$

$$F_X(x) = \int_{-\infty}^{x} f_X(u)\,du$$

For $x \geq 0$ $$= \int_{0}^{x} cu\, e^{-\frac{u}{2}}\,du$$

For $x < 0$ $$F_X(x) = 0$$

$$= c\int_{0}^{x} u\, e^{-u/2}\,du.$$

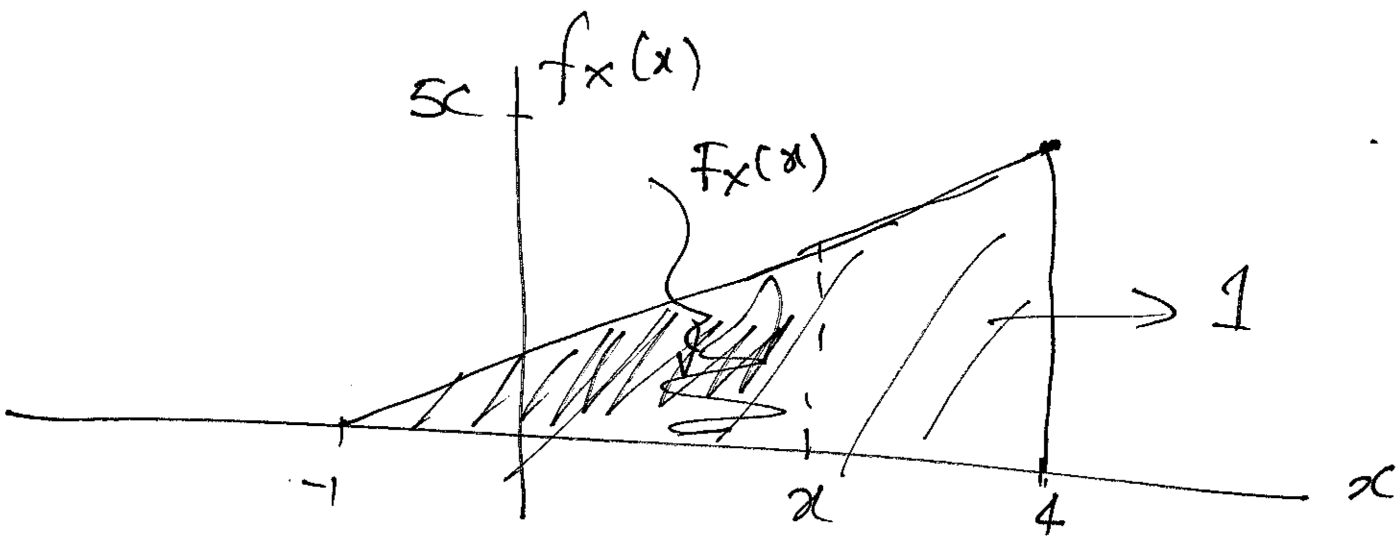
same integration by parts

$\vdots$

eg:-

$$f_X(x) = \begin{cases} c(x+1), & -1 \le x \le 4 \\ 0, & \text{otherwise} \end{cases}$$

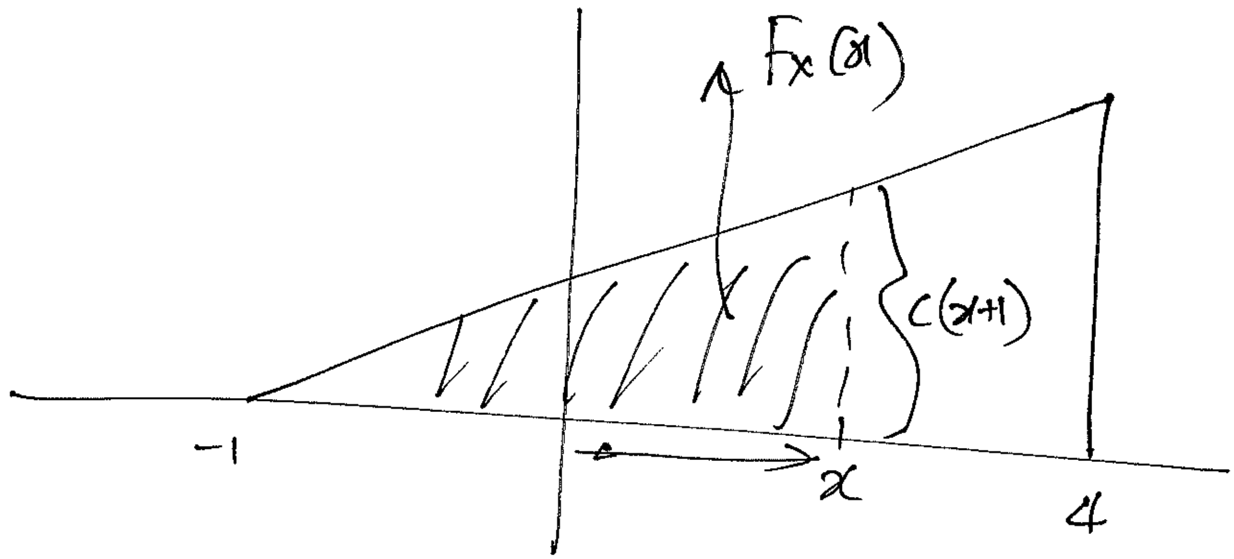
Find c, $F_X(x)$, $P[X > 1]$



$$\frac{1}{2}(5)(5c) = 1$$

$$c = \frac{2}{25}$$

$$F_X(x) = \int_{-\infty}^{x} f_X(u)\,du$$



$F_X(x) = 0, \quad$ for $\quad x < -1$

for $\underline{-1 \leq x \leq 4}$

$$F_X(x) = \frac{1}{2}(x+1)\,c\,(x+1)$$

$$= \frac{c}{2}(x+1)^2$$

For $\underline{x > 4}$ $\quad F_X(x) = 1$

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$$F_X(x) = \begin{cases} 0, & x < -1 \\ \frac{c}{2}(x+1)^2, & -1 \le x \le 4 \\ 1, & x > 4 \end{cases}$$

$$P[X > 1] = 1 - \underbrace{P[X \le 1]}_{F_X(1)}$$

$$= \left[1 - \frac{c}{2}(2)^2\right]$$