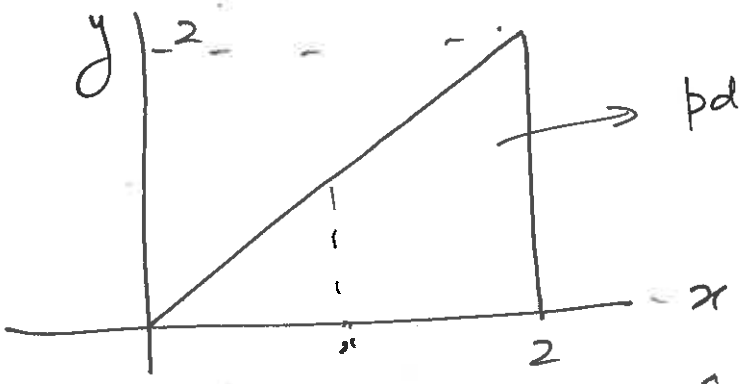
Name: **Quiz #7** **12/07/15**

[1] The joint pdf of X and Y is

$$f_{X,Y}(x,y) = \begin{cases} c, & 0 \le x \le 2, 0 \le y \le |x| \\ 0, & \text{otherwise} \end{cases}$$

Find the mean and variance of Y when X=1.

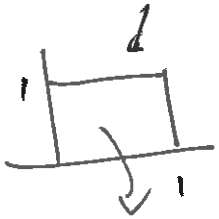


$$c \times \frac{1}{2}(2)(2) = 1 \longrightarrow c = 1/2$$

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

$$f_X(x) = \int_0^x c\,dy = \begin{cases} cx, & 0 \le x \le 2 \\ 0, & \text{o/w} \end{cases}$$

$$f_{Y|X}(y|1) = \begin{cases} \frac{c}{c} = 1, & 0 < y < 1 \\ 0, & \text{o/w} \end{cases}$$



Uniform.

$$\mu_{Y|X}(y|1) = \frac{0+1}{2} = 1/2$$

$$\text{Var}[Y|x=1] = \frac{(1-0)^2}{12} = \frac{1}{12}$$

$$\sigma_{Y|x=1} = \frac{1}{\sqrt{12}}$$

[2] X and Y are jointly Gaussian with the joint pdf given by N(2,3;1,2;-0.5).

Find $P[X^2>9]$ when Y=2.

$f_{X|Y}(x|2)$ is $N(\tilde{\mu}_1, \tilde{\sigma}_1)$

$\tilde{\mu}_1 = \mu_1 + \frac{\rho\sigma_1}{\sigma_2}(y-\mu_2) = 2 + \frac{(-0.5)(1)}{(2)}(2-3)$

$= 2 + 1/4 = \frac{9}{4}$

$\tilde{\sigma}_1^2 = \sigma_1^2(1-\rho^2)$

$= 1(1-1/4) = 3/4$

$\tilde{\sigma}_1 = \frac{\sqrt{3}}{2}$

$f_{X|Y}(x|2)$ is $N\left(\frac{9}{4}, \frac{\sqrt{3}}{2}\right)$

$P[x^2>9|Y=2] = P[X>3|Y=2] + P[X<-3|Y=2]$

$= P\left[Z > \frac{3-9/4}{\sqrt{3}/2}\right] + P\left[Z < \frac{-3-9/4}{\sqrt{3}/2}\right]$

$= \Phi\left(\frac{3/4}{\sqrt{3}/2}\right) + \Phi\left(\frac{21/4}{\sqrt{3}/2}\right)$

Name: **Quiz #8** **12/07/15**

[1] In an experiment the variable Q was observed for different values of P. The following (P,Q) observations were found:

(-3,1), (-2,1.7), (-1,3.1), (0,3.9), (1,4.9), (2,6).

Find a linear estimate of P in terms of Q.

P:	-3	-2	-1	0	1	2
Q:	1	1.7	3.1	3.9	4.9	6

$$\mu_P = \frac{-3-2-1+0+1+2}{6} = -\frac{1}{2}$$

$$\mu_Q = \frac{1+1.7+3.1+3.9+4.9+6}{6} =$$

$$E[P^2] = \frac{(-3)^2+(-2)^2+(-1)^2+(1)^2+(2)^2}{6} =$$

$$E[Q^2] = \frac{(1)^2+(1.7)^2+(3.1)^2+(3.9)^2+(4.9)^2+6^2}{6}$$

$$\sigma_P = \sqrt{E[P^2]-\mu_P^2}, \quad \sigma_Q = \sqrt{E[Q^2]-\mu_Q^2}$$

$$\rho_{PQ} = \frac{E[PQ]-\mu_P\mu_Q}{\sigma_P\sigma_Q}$$

$$E[PQ] = \frac{(-3)(1)+(-2)(1.7)+(-1)(3.1)+(1)(4.9)+(2)(6)}{6}$$

Name: **Quiz #1** **09/16/15**
UTD ID:

[1] In a gathering of 50 students, 20 major in Engineering and the remaining 30 major in CS. If four students are selected randomly from the gathering, find the probability that all four of them major in Engineering.

(1) $\hat{P}_L(q) = \mu_p = \dfrac{\rho_{pq}\,\sigma_p}{\sigma_q}(q - \mu_q)$

(2) X is $N(\mu, 0.5)$

$$P[|M_n(X) - \mu_X| \geq c] \leq \frac{Var[X]}{nc^2} = \alpha$$

Confidence Coeff. $= 1 - \alpha = 1 - \dfrac{Var[X]}{nc^2} \geq 0.99$

$$\frac{Var[X]}{nc^2} \leq 0.01$$

$$n \geq \frac{100\,Var[X]}{c^2}$$

$$n \geq \frac{100\,(0.5)^2}{(0.01)^2} = 0.25 \times 10^6 = 2.5 \times 10^5$$