EE 3341 : Probability Theory and Statistics

Homework Solutions #01.

Problem 1.3.5

The sample space of the experiment is ;

$$S = \{LF, BF, LW, BW\}$$

From the problem statement, we know that $P[LF] = 0.5$, $P[BF] = 0.2$ and $P[BW] = 0.2$. This implies $P[LW] = 1 - 0.5 - 0.2 - 0.2 = 0.1$

The questions can be answered using Theorem 1.5.

(a) The probability that a program is slow is

$$P[W] = P[LW] + P[BW] = 0.1 + 0.2 = \underline{0.3}.$$

(b) The probability that a program is big is

$$P[B] = P[BF] + P[BW] = 0.2 + 0.2 = \underline{0.4}$$

(c) The probability that a program is slow or big is

$$P[W \cup B] = P[W] + P[B] - P[BW]$$

$$= 0.3 + 0.4 - 0.2 = \underline{0.5}.$$

Problem 1.3.6

The sample space is ;

$$S = \{HF, HW, MF, MW\}.$$

The problem statement tells us that $P[HF] = 0.2$, $P[MW] = 0.1$ and $P[F] = 0.5$. We can use these facts to find the probabilities of other outcomes. In particular,

$$P[F] = P[HF] + P[MF].$$

This implies

$$P[MF] = P[F] - P[HF] = 0.5 - 0.2 = 0.3$$

Also, since the probabilities must sum to 1,

$$P[HW] = 1 - P[HF] - P[MF] - P[MW]$$

$$= 1 - 0.2 - 0.3 - 0.1 = 0.4$$

Now, that we have found the probabilities of the out comes, finding any other probability is easy.

(a) The probability a cell phone is slow is

$$P[W] = P[HW] + P[MW] = 0.4 + 0.1 = \underline{\underline{0.5}}$$

(b) The probability that a cell phone is mobile and fast is

$$P[MF] = \underline{\underline{0.3}}$$

(c) The probability tha a cell phone is handheld is,

$$P[H] = P[HF] + P[HW]$$

$$= 0.2 + 0.4 = \underline{\underline{0.6}}.$$

Problem 1.5.2

(a) From the given probability distribution of billed minutes M, the probability that a call is billed for more than 3 minutes is,

$$P[L] = 1 - P[3 \text{ or fewer billed minutes}]$$
$$= 1 - P[B_1] - P[B_2] - P[B_3]$$
$$= 1 - \alpha - \alpha(1-\alpha) - \alpha(1-\alpha)^2$$
$$= (1-\alpha)^3 = \underline{0.57}$$

(b) The probability that a call will be billed for 9 minutes or less is

$$P[9 \text{ minutes or less}] = \sum_{i=1}^{9} \alpha(1-\alpha)^{i-1} = \underline{1-(0.57)^3}$$

Problem 1.4.2

Let S_i denote the outcome that the roll is i, so for $1 \le i \le 6$, $R_i = \{S_i\}$. Similarly, $G_j = \{S_{j+1}, \ldots, S_6\}$.

(a) Since $G_1 = \{S_2, S_3, S_4, S_5, S_6\}$ and all outcomes have probability $1/6$, $P[G_1] = 5/6$. The event $R_3G_1 = \{S_3\}$ and $P[R_3G_1] = 1/6$ so that

$$P[R_3/G_1] = \frac{P[R_3G_1]}{P[G_1]} = \frac{1}{5}$$

(b) The conditional probability that 6 is rolled given that the roll is greater than 3 is,

$$P[R_6/G_3] = \frac{P[R_6 G_3]}{P[G_3]} = \frac{P[S_6]}{P[S_4, S_5, S_6]} = \frac{1/6}{3/6} = \frac{1}{3}.$$

(c) The event E that the roll is even is $E = \{S_2, S_4, S_6\}$ and has probability $3/6$. The joint probability of G_3 and E is

$$P[G_3 E] = P[S_4, S_6] = 1/3.$$

The conditional probabilities of G_3 given E is,

$$P[G_3/E] = \frac{P[G_3 E]}{P[E]} = \frac{1/3}{1/2} = 2/3.$$

Problem 1.4.3

Since the 2 of clubs is an even numbered card, $C_2 \subset E$ so that $P[C_2 E] = P[C_2] = 1/3$. Since $P[E] = 2/3$,

$$P[C_2/E] = \frac{P[C_2 E]}{P[E]} = \frac{1/3}{2/3} = 1/2.$$

The probability that an even numbered card is picked given that the 2 is picked is

$$P[E/C_2] = \frac{P[C_2 E]}{P[C_2]} = \frac{1/3}{1/3} = 1.$$

Problem 1.6.7

(a) Since $A \cap B = \phi$, $P[A \cap B] = 0$. To find $P[B]$, we can write

$$P[A \cup B] = P[A] + P[B] - P[A \cap B]$$
$$5/8 = 3/8 + P[B] - 0$$

Thus, $P[B] = 1/4$. Since A is a subset of B^c, $P[A \cap B^c] = P[A] = 3/8$. Furthermore, since A is a subset of B^c, $P[A \cup B^c] = P[B^c] = 3/4$.

(b) The events A and B are dependent because

$$P[AB] = 0 \neq 3/32 = P[A] \cdot P[B].$$

Problem 1.6.8

(a) Since C and D are independent $P[CD] = P[C] \cdot P[D]$.

So, $$P[D] = \frac{P[CD]}{P[C]} = \frac{1/3}{1/2} = 2/3.$$

In addition, $P[C \cap D^c] = P[C] - P[C \cap D] = 1/2 - 1/3 = 1/6$.

To find $P[C^c \cap D^c]$; we first observe that

$$P[C \cup D] = P[C] + P[D] - P[C \cap D] = 1/2 + 2/3 - 1/3 = \underline{5/6}.$$

By De Morgan's Law, $C^c \cap D^c = (C \cup D)^c$. This implies

$$P[C^c \cap D^c] \cancel{=} \cancel{P[C^c]} = P[(C \cup D)^c] = 1 - P[C \cup D] = \underline{\underline{1/6}}$$

Note that a second way to find $P[C^c \cap D^c]$ is to use the fact that if C and D are independent, then C^c and D^c are independent. Thus

$$P[C^c \cap D^c] = P[C^c] \cdot P[D^c] = (1 - P[C])(1 - P[D]) = \underline{\underline{1/6}}$$

Finally, since C and D are independent events, $P[C|D] = P[C] = \underline{\underline{1/2}}$.

(b) Note that we found $P[C \cup D] = 5/6$. We can also use the earlier results to show:

$$P[C \cup D^c] = P[C] + P[D] - P[C \cap D^c]$$
$$= 1/2 + (1 - 2/3) - 1/6 = 2/3.$$

(c) By Definition 1.7, events C and D^c are independent because

$$P[C \cap D^c] = 1/6 = (1/2)(1/3) = P[C] \cdot P[D^c]$$

Problem 1.6.9

For a sample space $S = \{1, 2, 3, 4\}$ with equiprobable outcomes consider the events $A_1 = \{1, 2\}$, $A_2 = \{2, 3\}$, $A_3 = \{3, 1\}$.

Each event A_i has probability $1/2$. Moreover, each pair of events is independent since

$$P[A_1 A_2] = P[A_2 A_3] = P[A_3 A_1] = 1/4.$$

However, the three events A_1, A_2, A_3 are not independent since

$$P[A_1 A_2 A_3 A_4] = 0 \neq P[A_1] P[A_2] P[A_3]$$