Equation Sheet of Exam #2

$$E[g(X)] = \sum_x g(x) P_X(x)$$

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx$$

$$P_X(x) = \sum_y P_{X,Y}(x, y)$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x, y) dy$$

$$\int_{-\infty}^{\infty} f(x) \delta(x - a) dx = f(a)$$

Families of Continuous Random Variables

If X is **Exponential(λ),**

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & , \quad x \geq 0 \\ 0 & , \ therwise \end{cases} \qquad F_X(x) = \begin{cases} 1 - e^{-\lambda x} & , \quad x \geq 0 \\ 0 & , \ therwise \end{cases}$$

$$\mu_X = \frac{1}{\lambda} \qquad Var[X] = \frac{1}{\lambda^2}$$

If X is **Uniform(a,b)**

$$f_X(x) = \begin{cases} \frac{1}{b-a} & , \quad a \leq x < b \\ 0 & , \ therwise \end{cases} \qquad F_X(x) = \begin{cases} 0 & , \quad x \leq a \\ \frac{x-a}{b-a} & , \quad a < x \leq b \\ 1 & , \ x > b \end{cases}$$

$$\mu_X = \frac{a+b}{2} \qquad Var[X] = \frac{(b-a)^2}{12}$$

If X is **Gaussian** (μ, σ),

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(X-\mu)^2}{2\sigma^2}} \qquad Z = \frac{X - \mu}{\sigma}$$

$$P_{X|B}(x) = \begin{cases} \dfrac{P_X(x)}{P[B]}, x \in B \\ 0, otherwise \end{cases}$$

$$f_{X|B}(x) = \begin{cases} \dfrac{f_X(x)}{P[B]}, x \in B \\ 0, otherwise \end{cases}$$

$$P_X(x) = \sum_{i=1}^{N} P_{X|B_i}(x) P[B_i]$$

$$f_X(x) = \sum_{i=1}^{N} f_{X|B_i}(x) P[B_i]$$

If $Y = aX + b$, then

$$\mu_Y = a\mu_X + b, Var[Y] = a^2 Var[X]$$

$$f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right)$$