Name:

Quiz #8

12/07/15

[1] In an experiment the variable Q was observed for different values of P. The following (P,Q) observations were found:

(-3,1), (-2,1.7), (-1,3.1), (0,3.9), (1,4.9), (2,6).

Find a linear estimate of P in terms of Q.

P : -3 , -2, -1 , 0, 1, 2

Q : 1 , 1.7 , 3.1, 3.9, 4.9, 6

$$\mu_P = \frac{-3-2-1+0+1+2}{6} = -\frac{1}{2} \qquad \mu_Q = \frac{1+1.7+3.1+3.9+4.9+6}{6} = \frac{103}{30}$$

$$E\{P^2\} = \frac{(-3)^2+(-2)^2+(-1)^2+0^2+(1)^2+(2)^2}{6} = \frac{19}{6}$$

$$E\{Q^2\} = \frac{(1)^2+(1.7)^2+(3.1)^2+(3.9)^2+(4.9)^2+6^2}{6} \cong 70.7$$

$$\sigma_P = \sqrt{E[P^2]-\mu_P^2} = 1.7$$

$$\sigma_Q = \sqrt{E[Q^2]-\mu_Q^2} = 7.67$$

$$E\{PQ\} = \frac{(-3)(1)+(-2)(1.7)+(-1)(3.1)+(1)(4.9)+(2)(6)}{6} = \frac{37}{}$$

$$\rho_{PQ} = \frac{E[PQ]-\mu_P\mu_Q}{\sigma_P\sigma_Q} \qquad \underbrace{\hat{P}_L(Q)+\tfrac{1}{2}}_{\hat{P}_L(Q)-\mu_P} = \frac{\rho_{PQ}\sigma_P}{\sigma_Q}(Q-\mu_Q)$$

[2] X is a Gaussian random variable with standard deviation 0.5. The mean of X is estimated by taking the sample mean of independent samples of X. If the mean needs to be estimated within 0.01 from the actual mean with a confidence coefficient of 0.99, find the minimum number of samples required in the estimation.

$$X \sim N(N, 0.5)$$

$$P[|M_n(X) - N_X| \geq c] \leq \frac{Var[X]}{nc^2} = \alpha$$

$$\text{Confid.. Coef} = 1-\alpha = 1 - \frac{Var[X]}{nc^2} \geq 0.99$$

$$\Rightarrow \frac{Var[X]}{nc^2} \leq 0.01 \Rightarrow n \geq \frac{100\, Var[X]}{c^2}$$

$$\Rightarrow n \geq \frac{100 \cdot (0.5)^2}{(0.01)^2} \Rightarrow n \geq 2.5 \times 10^5$$

minimum $n = 2.5 \times 10^5$