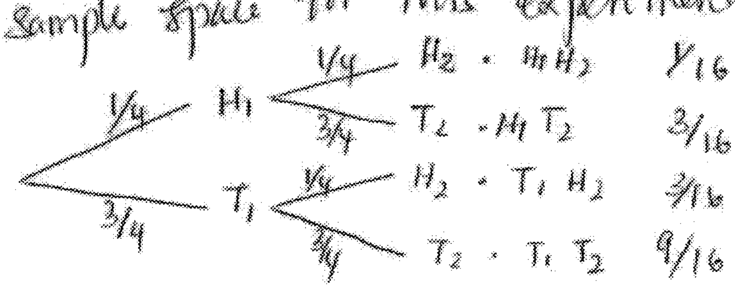
Problem 2.1.1

A sequential sample space for this experiment is



(a) From the tree, we observe

$$P[H_2] = P[H_1H_2] + P[T_1H_2] = 1/4.$$

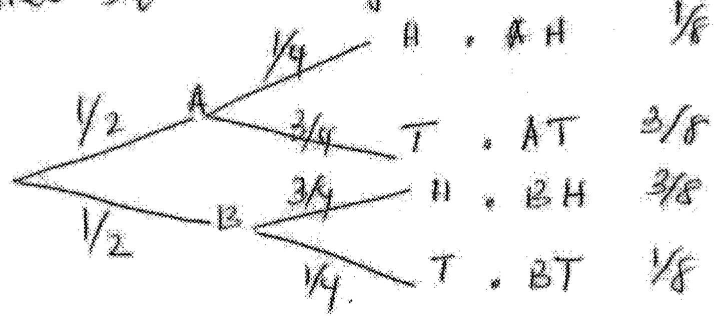
This implies

$$P[H_1 / H_2] = \frac{P[H_1 H_2]}{P[H_2]} = \frac{1/16}{1/4} = \underline{\underline{1/4}}.$$

(b) The probability that the first flip is heads and the second flip is tails is $P[H_1 T_2] = \underline{\underline{3/16}}$.

Problem 2.1.4

The tree for this experiment is,

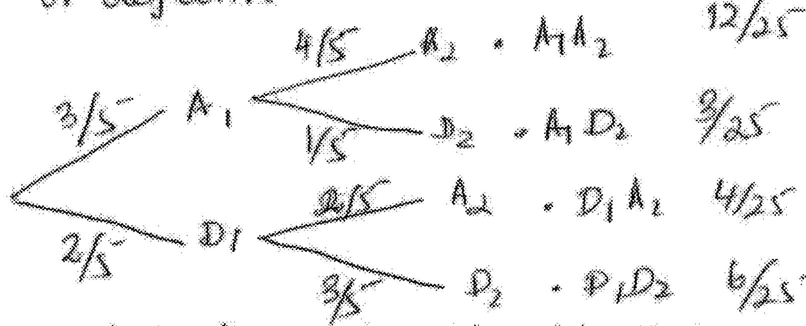


The probability that you guess correctly is,

$$P[C] = P[AT] + P[BH] = 3/8 + 3/8 = \underline{\underline{3/4}}.$$

Problem 2.1.6

Let A_i and D_i indicate whether the ith photodetector is acceptable or defective.



(a) We wish to find the probability $P[E_1]$ that exactly one photodetector is acceptable. From the tree we have,

$$P[E_1] = P[A_1D_2] + P[D_1A_2] = 3/25 + 4/25 = \underline{\underline{7/25}}$$

(b) The probability that both photodetectors are defective is $P[D_1 D_2] = \underline{\underline{6/25}}$.

Problem 2.2.8

Since each letter can ~~take~~ ~~only~~ any one of the 4 possible letters in the alphabet, the number of 3 letter words that can be formed is $4^3 = 64$. If we allow each letter only once then we have 4 choices for the first letter and 3 choices for the second and two choices for the third letter. Therefore, there are a total of $4 \cdot 3 \cdot 2 = 24$ possible codes.

Problem 2.2.9

We can break down the experiment of choosing a starting lineup into a sequence of subexperiments:

1. Choose 1 of the 10 pitchers. There are $N_1 = \binom{10}{1} = 10$ ways to do this
2. Choose 1 of the 15 field players to be designated hitter (DH). There are $N_2 = \binom{15}{1} = 15$ ways to do this.
3. Of the remaining 14 field players, choose 8 for the remaining field positions. There are $N_3 = \binom{14}{8}$ to do this
4. For the 9 batters (consisting of the 8 field players and the designated hitter), choose a batting lineup. There are $N_4 = 9!$ ways to do this.

So the total number of different starting lineups when the DH is selected among the field players is

$$N = N_1 N_2 N_3 N_4 = (10)(15)\binom{14}{8} \cdot 9!$$

$$= 163,459,296,000.$$

Note that this overestimates the number of combinations the manager must really consider because most field players can play only one or two positions. Although these constraints on the manager reduce the number of possible lineups, it typically makes the manager's job more difficult. As for the counting, we note that our count did not need to specify the positions played by the field players. Although this is an important consideration for the manager, it is not part of our counting of different lineups. In fact, the 8 nonpitching field players are allowed to switch positions at any time of the field. For example, the short stop and second baseman could trade positions in the middle of an inning. Although the DH can go play the field, there are some complicated rules about this. Here is an excerpt from Major League Baseball Rule 6.10:

The Designated Hitter may be used defensively, continuing to bat in the same position in the batting order, but the pitcher must then bat in the place of the substituted defensive player, unless more than one substitution is made, and the manager then must designate their spots in the batting order.

If you're curious, you can find the complete rule on the web.

Problem 2.2.12

(a) We can find the number of valid starting line ups by noticing that the swingman presents three situations: (1) the swingman plays guard, (2) the swingman plays forward, (3) the swingman doesn't play. The first situation is when the swingman can be chosen to play the guard position, and the second where the swingman can only be chosen to play the forward position.

Let N_i denote the number of lineups corresponding to case i. Then we can write the total number of line ups as $N_1 + N_2 + N_3$. In the first situation, we have to choose 1 out of 3 centers, 2 out of 4 forwards, and 1 out of 4 guards so that

$$N_1 = \binom{3}{1}\binom{4}{2}\binom{4}{1} = 72$$

In the second case, we need to choose 1 out of 3 centers, 1 out of 4 forwards and 2 out of 4 guards, yielding

$$N_2 = \binom{3}{1}\binom{4}{1}\binom{4}{2} = 72$$

Finally, with the swingman on the bench, we choose 1 out of 3 centers, 2 out of 4 forward, and 2 out of four guards. This implies

$$N_3 = \binom{3}{1}\binom{4}{2}\binom{4}{2} = 108,$$

$\therefore$ and the total number of line ups is $N_1 + N_2 + N_3 = \underline{\underline{252}}$.