Prob. Mass.

Conditional Prob.

A, B $\rightarrow$ 2 events

$$P[A|B] = \frac{P[AB]}{P[B]} \qquad P[AB] = P[A|B]P[B]$$

if B has occurred, prob. that A occurs too

$$P[B|A] = \frac{P[AB]}{P[A]}$$

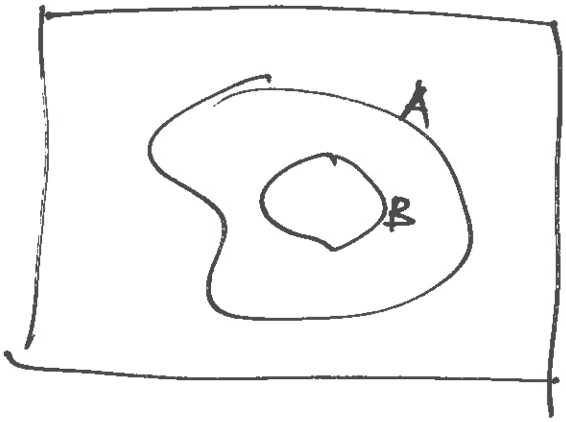
$$P[A|B]P[B] = P[B|A]P[A]$$

$$P[D|L] = \frac{P[DL]}{P[L]} = \frac{0.25}{0.6}$$

In Q1.3

$$P[L|B] = 0$$

eg:-



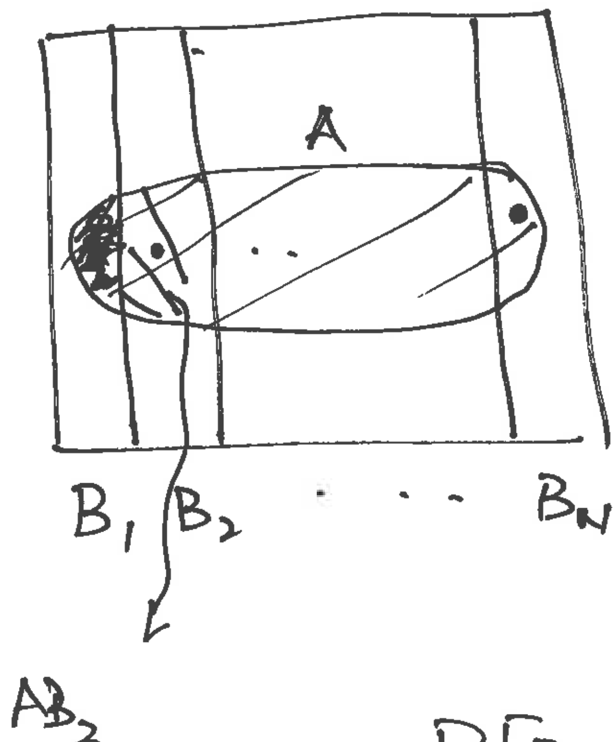
$B \subset A$

$AB = B$

$$P[A|B] = 1$$

$$= \frac{P[AB]}{P[B]} = \frac{P[B]}{P[B]} = 1$$

$$P[B|A] = \frac{P[AB]}{P[A]} = \frac{P[B]}{P[A]}$$



$B_i \cap B_j = \phi,\ i \neq j$

$\bigcup_{i=1}^{N} B_i = S$

$P[B_i B_j] = 0$

$P[B_1] + P[B_2] \cdots + P[B_N] = 1$

eg:- B_i → students in row i

A → " major in EE

Given: $P[B_i]$, $i = 1, 2 \ldots N$

$P[A|B_i]$, $i = 1, 2, \ldots N$

$i = 1, \ldots N$

$P[AB_i]$ ✓

$P[A] = ?$

4

$$P[A] = P[AB_1] + P[AB_2] + \cdots + P[AB_N]$$
$$= P[A|B_1]P[B_1] + \cdots + P[A|B_N]P[B_N]$$

$$P[A] = \sum_{i=1}^{N} P[A|B_i]\,P[B_i]$$

Total Prob. Eq^n

* B_is must be mutually Exclusive
* All B_is combined must form the space

Collectively Exhaustive

B_is must be mutually Exclusive & collectively exhaustive.

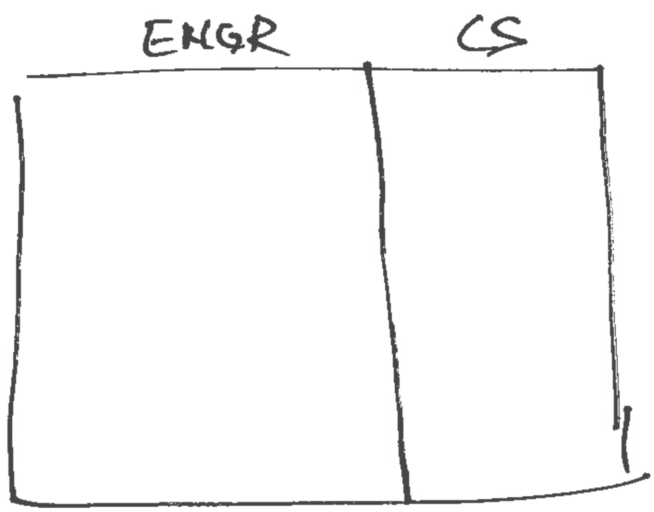
eg:- Gallery of ~~EE~~ ENGR & CS students.

$\downarrow$

ENGR & CS are mutually exclusive & collectively exhaustive.

60% are ENGR

$P[ENGR] = 0.6$



Among ENGR students, 30% are $\overbrace{\text{female}}^{F}$

" CS " 40% . . .

$P[F \mid ENGR] = 0.3$ $\quad P[M \mid ENGR] = 0.7$

$P[F \mid CS] = 0.4,$ $\quad P[M \mid CS] = 0.6$

If a randomly selected student from the gathering is a female, find the prob. that she majors in CS.

$$P[CS|F] = ?$$

$$= \frac{P[CS \& F]}{P[F]}$$

$$\cancel{P[CS] = \frac{P[CS|F]}{P[F]}}$$

$$P[F|CS]\,P[CS]$$

$$P[CS \& F] = P[F|CS]P[CS]$$

$$= 0.4 \times 0.4 = 0.16$$

$$P[F] = P[F|ENGR]\,P[ENGR] + P[F|CS]\,P[CS]$$

$$= 0.3 \times 0.6 + 0.4 \times 0.4$$

$$P[CS|F] = \frac{0.4 \times 0.4}{0.3 \times 0.6 + 0.4 \times 0.4}$$

eg:- Binary Transmission

$X \to$ Transmitter

$Y \to$ Receiver

0

0

1

1

60% of the bits transmitted are '0's

$\rightarrow$ Event A: $X = 0$

Event B: $X = 1$

$\downarrow$

A & B are mutually Exclusive & collectively Exhaustive.

When a '0' is transmitted there is a 1% of error in the detect

Event C: $Y = 0$

Event D: $Y = 1$

$P[D \mid A] = 0.01$

$\hookrightarrow P[C/A] = 0.99$

C&D → Mutually Exclusive & collectively Exhaustive

When a '1' is transmitted, there is a 2% chace of error

$P[C \mid B] = 0.02$, $P[D \mid B] = 0.98$

If a '0' is received find the prob. a '0' was transmitted.

$P[A \mid C] = ?$

$$P[A \mid C] = \frac{P[AC]}{P[C]} = \frac{P[C/A] \cdot P[A]}{?}$$

$P[C] = P[C/A]\,P[A] + P[C/B]\,P[B]$

Independence.

Recall: $P[A|B] = \dfrac{P[AB]}{P[B]}$

If A & B are independent,

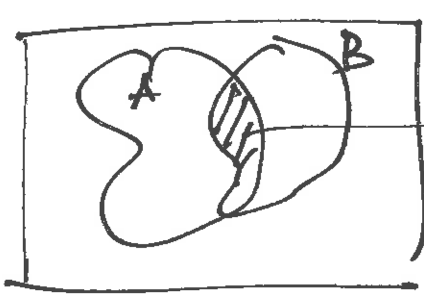
$$P[A|B] = P[A]$$

occurance of B does not chage the chance A occurs or not.

$$P[B|A] = P[B]$$

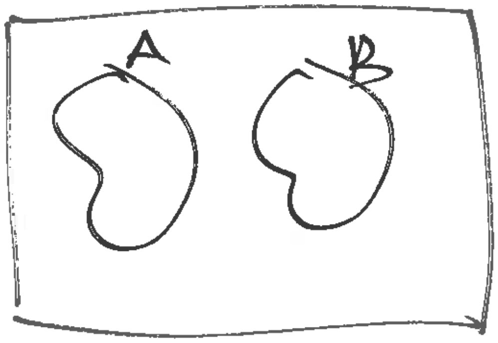
$$\frac{P[AB]}{P[B]} = P[A]$$

& $P[AB] = P[A]P[B]$



Mass $= P[A]\,P[B]$

Recall:



$\rightarrow$ Mutually Exclusive

$\therefore$ If A & B are mutually Exclusive they are not independent.

eg:- A, B, C are 3 events

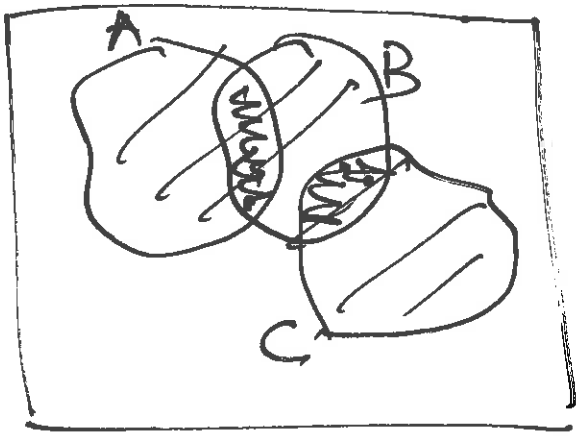
A & B are independent

B & C " "

A & C are mutually exclusive

$P[A] = 1/2$, $P[B] = 1/4$, $P[C] = 3/8$

$P[A+B+C] = ?$



$$P[ABC] = 0$$

$$P[A+B+C] = P[A] + P[B] + P[C] - \underbrace{P[AB]}_{P[A]P[B]} - \underbrace{P[BC]}_{P[B]P[C]}$$

$$= \left[\frac{1}{2} + \frac{1}{4} + \frac{3}{8} - \frac{1}{2}\left(\frac{1}{4}\right) - \frac{1}{4}\left(\frac{3}{8}\right)\right]$$

If A & B are independent

1. $P[A|B] = P(A)$
2. $P[B/A] = P(B)$
3. $P[AB] = P[A]P[B]$

If $P[A|B] = P[A]$

$$\frac{P[AB]}{P[B]} = P[A]$$

$$P[B/A] = \frac{P[AB]}{P[A]} = \frac{P[A]\,P[B]}{P[A]}$$

$$= P[B]$$

If any of the above 3 conditions is true the other 2 are automatically true.

To prove A & B are independent → can select any condition.

Q 1.6, ~~1.5~~ ~~2nd Edition~~

~~3rd Edition~~

$C_1 \rightarrow$ 1st packet

$C_2 \rightarrow$ 2nd "

C_1 & C_2 are independent

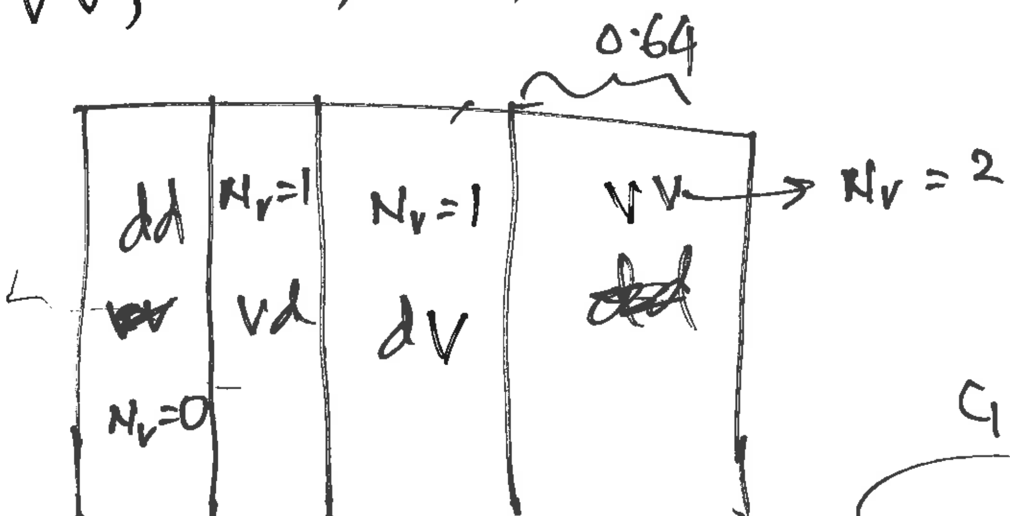
$P[C_i = v] = 0.8$

$P[C_i = d] = 0.2$

v & d are mutually Exclusive & collectively Exhaustive.

Expt: Monitor 2 packets.

vv, vd, dv, dd



C_1 & C_2 are indep.

$P[vv] = P[C_1 = v \text{ \& } C_2 = v] = P[C_1 = v]\,P[C_2 = v]$

$= 0.8 \times 0.8 = 0.64$

2 ~~$N_v=1$~~

$P[vd] = 0.8 \times 0.2 = 0.16$

$P[dv] = 0.2 \times 0.8 = 0.16$

$P[dd] = 0.2 \times 0.2 = 0.04$

$P[N_v = 2] = 0.64$

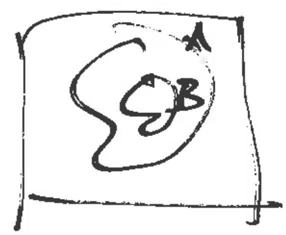
$P[N_v = 1] = 0.16 + 0.16 = 0.32$

$P[N_v = 0] = 0.04$

Are events $N_v = 2$ & $N_v \geq 1$ independent

$N_v = 2$ is a subset of $N_v \geq 1$

$P[N_v \geq 1 \mid N_v = 2] = 1$



$P[A|B] = 1$

If $N_v \geq 1$ & $N_v = 2$

are independent

must be $P[N_v \geq 1]$ too.

$\therefore$ they are not independent 0.96

Are $\{N_V \geq 1$ & $C_1 = V\}$ independent

$$P[N_V \geq 1 \mid C_1 = V] = 1$$

is this equal $P[N_V \geq 1]$ → No

$\therefore$ $N_V \geq 1$ & $C_1 = V$ are not independent.

$C_2 = d$ & $C_1 = V$ → Yes. Given

Already used it to calculate $P[Vd]$

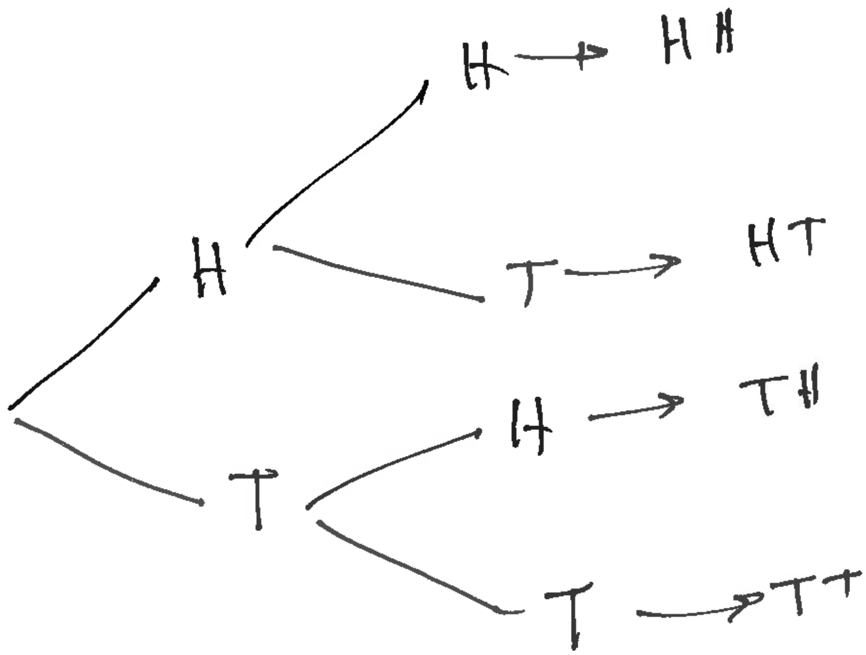
Sequential Experiments

eg :- Toss a coin 2 times

multiple smaller Experiments

— can draw a tree diagram → prob. tree

Each Toss → H
→ T



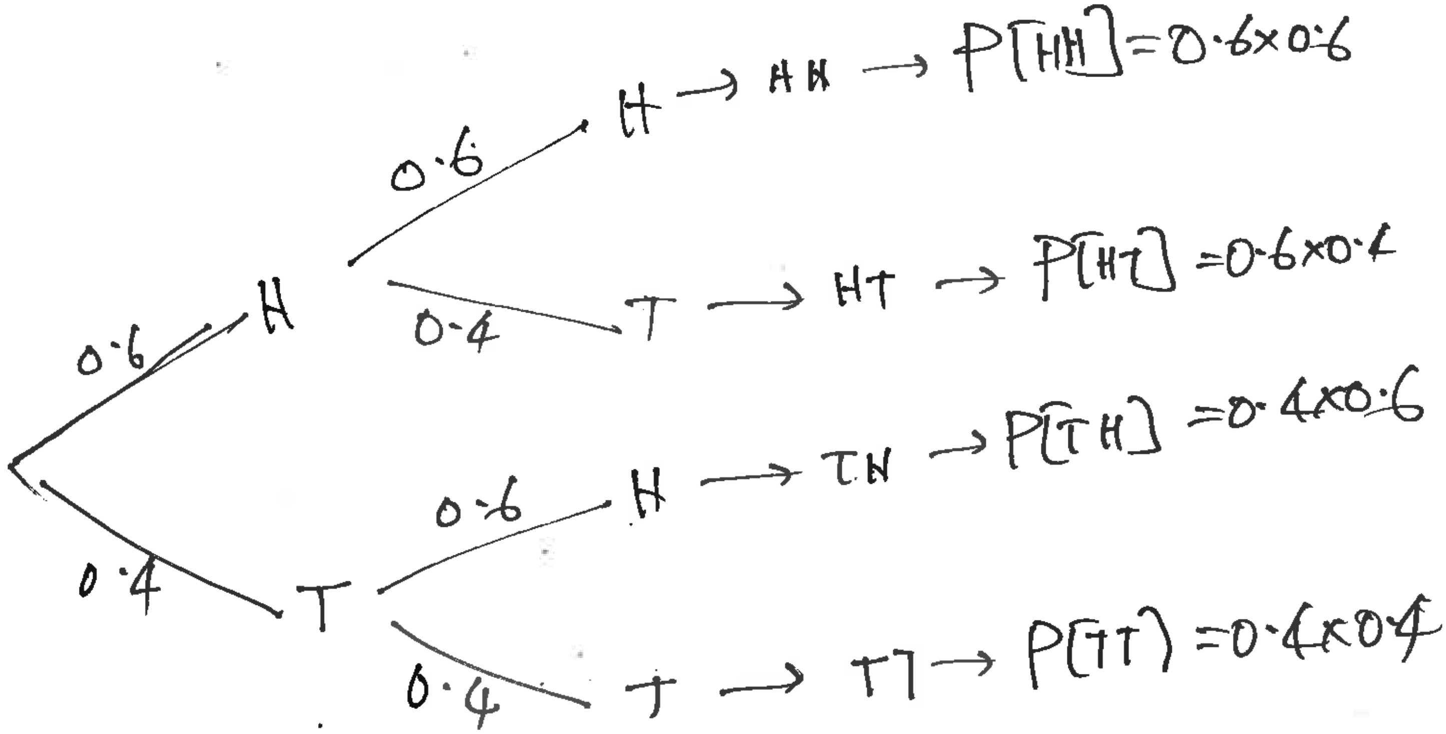
- Can use the tree to get all possible experimental outcomes → they are mutually Exclusive & collectively Exhaustive

Tree can be used to calculate the probabilities of the experimental outcomes too.

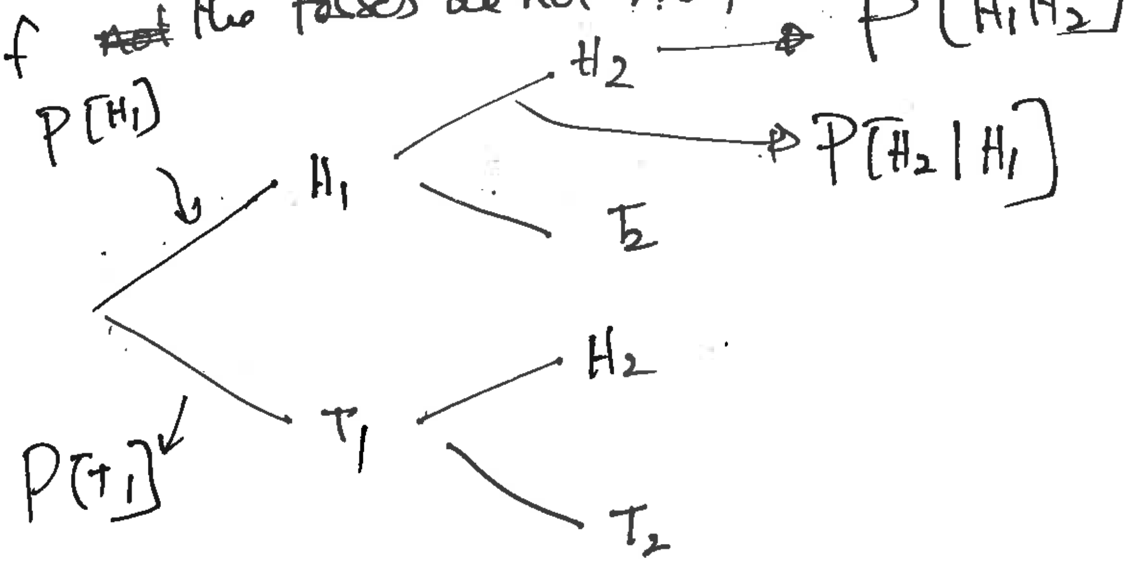
eg:- Toss a biased coin

$P[H] = 0.6, \quad P[T] = 0.4$

Tosses are independent.

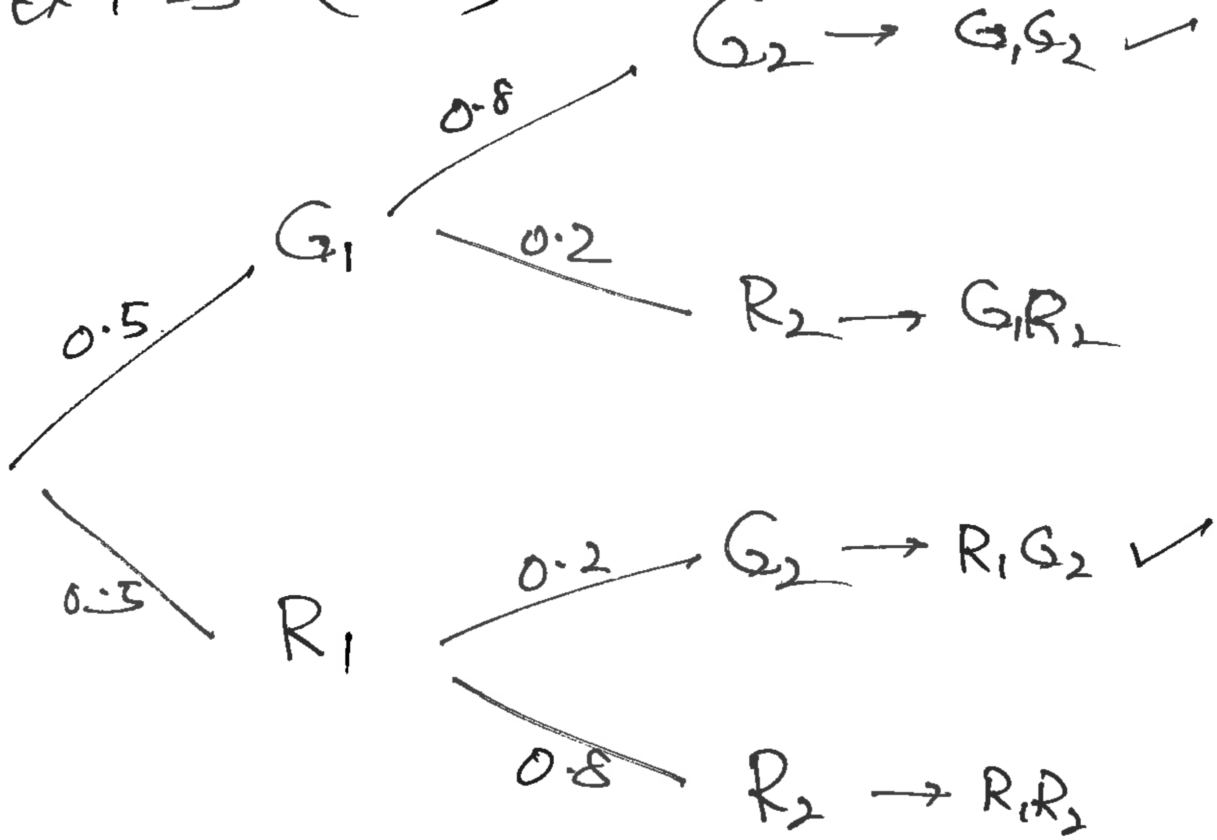


If ~~not~~ the tosses are not independent



Ex 2.2 (3rd)

↳ Ex 1.25 (2nd)



$$P[G_2] = P[G_1G_2] + P[R_1G_2]$$

$$= \overset{0.5\times 0.8}{\cancel{0.8\times 0.5}} + 0.5\times 0.2$$

$$= 0.5$$

$$P[W] = 1 - P[G_1G_2] = 1 - 0.5\times 0.8$$

$$= 0.6$$

↓

wait at atleast one light

$$P[G_1 | R_2] = \frac{P[G_1 R_2]}{P[R_2]} = \frac{0.5 \times 0.2}{1 - 0.5}$$

$$P[G_1 R_2] + P[R_1 R_2] \quad \text{or} \quad 1 - P[\overset{0.5}{G_2}]$$

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Teaching