Problem 3.2.2

(a) We must make the PMF of V sum to one.

$$\sum_{v=1}^{4} P_V(v) = c(1^2+2^2+3^2+4^2) = 30c = 1$$

$$\Rightarrow \quad c = \underline{\underline{1/30}}.$$

(b) Let $U = \{u^2 \mid u = 1, 2, \ldots\}$ so that

$$P[V \in U] = P_V(1) + P_V(4) = \frac{1}{30} + \frac{4^2}{30} = \underline{\underline{\frac{17}{30}}}.$$

(c) The probability that V is even is

$$P[V \text{ is even}] = P_V(2) + P_V(4) = \frac{2^2}{30} + \frac{4^2}{30} = \underline{\underline{\frac{2}{3}}}$$

(d) The probability that $V > 2$ is

$$P[V > 2] = P_V(3) + P_V(4) = \frac{3^2}{30} + \frac{4^2}{30} = \underline{\underline{\frac{5}{6}}}$$

Problem 3.2.3

(a) We choose c so that the PMF sums to one.

$$\sum_x P_X(x) = \frac{c}{2} + \frac{c}{4} + \frac{c}{8} = \frac{7c}{8} = 1.$$

$$\Rightarrow \quad c = \underline{\underline{8/7}}$$

(b) $$P[X=4] = P_X(4) = \frac{8}{7} \cdot \frac{1}{4} = \underline{\underline{2/7}}$$

(c) $$P[X<4] = P_X(2) = \frac{8}{7} \cdot \frac{1}{2} = \underline{\underline{4/7}}.$$

(d) $$P[3 \le X \le 9] = P_X(4) + P_X(8) = \frac{8}{7\cdot 4} + \frac{8}{7\cdot 8} = \underline{\underline{3/7}}$$

Problem 3.2.10

From the problem statement, a single is twice as likely as double, which is twice as likely as a triple, which is twice as likely as a home-run. If p is the probability of a home run, then

$$P_B(4) = p \quad , P_B(3) = 2p \; , P_B(2) = 4p \; ; \; P_B(1) = 8p$$

Since a hit of any kind occurs with probability of 0.3, $p + 2p + 4p + 8p = 0.300$ which implies $p = 0.02$

Hence, the PMF of B is

$$P_B(b) = \begin{cases} 0.70 & , b = 0 \\ 0.16 & b = 1 \\ 0.08 & b = 2 \\ 0.04 & b = 3 \\ 0.02 & b = 2 \\ 0 & \text{otherwise} \end{cases}$$

Problem 3.3.1

(a) If it is indeed true that Y, the number of yellow M&Ms in a package, is uniformly distributed between 5 and 15, then the PMF of Y, is

$$P_Y(y) = \begin{cases} 1/11 & y = 5, 6, 7, \ldots, 15 \\ 0 & \text{otherwise} \end{cases}$$

(b) $P[Y < 10] = P_Y[5] + P_Y[6] + P_Y[7] + P_Y[8] + P_Y[9] + \cancel{P_Y}$

$= \underline{\underline{5/11}}$

(c) $P[Y > 12] = P_Y[13] + P_Y[14] + P_Y[15] = \underline{\underline{3/11}}$

(d) $P[8 \leq Y \leq 12] = P_Y[8] + P_Y[9] + P_Y[10] + P_Y[11] + P_Y[12]$

$= \underline{\underline{5/11}}$

Problem 3.3.5

Whether a hook catches a fish is an independent trial with success probability h. The number of fish hooked, K, has a binomial PMF

$$P_K(k) = \begin{cases} \binom{m}{k} h^k (1-h)^{m-k} & k = 0, 1, \ldots, m. \\ 0 & \text{otherwise.} \end{cases}$$

Problem 3.3.10

Since an average of $T/5$ buses arrive in an interval of T minutes, buses arrive at the bus stop at a rate of $1/5$ buses per minute.

(a) From the definition of the Poisson PMF, the PMF of B, the number of buses in T minutes,

$$P_B(b) = \begin{cases} (T/5)^b e^{-T/5}/b! & b = 0, 1, \ldots \\ 0 & \text{otherwise} \end{cases}$$

(b) Choosing $T=2$ minutes, the probability that three buses arrive in a two minute interval is

$$P_B(3) = (2/5)^3 e^{-2/5}/3! \approx \underline{0.00715}.$$

(c) By choosing $T=10$ minutes, the prob. of zero buses arriving in a ten minute interval is

$$P_B(0) = e^{-10/5}/0! = e^{-2} \approx \underline{0.135}$$

(d) The probability that at least one bus arrive in T minutes is $P[B \geq 1] = 1 - P[B=0] = 1 - e^{-T/5} \geq 0.99$

$$\Rightarrow 0.01 \geq e^{-T/5} \Rightarrow e^{T/5} \geq 100.$$

$$\therefore T/5 \geq \ln 100 \Rightarrow T \geq 5 \ln 100$$

$$\approx \underline{23.0} \text{ mins} \quad (23.026 \text{ min})$$

Problem 3.3.11

(a) If each message is transmitted 8 times the probability of a successful transmission is p, then the PMF of N, the number of successful transmissions has the binomial PMF

$$P_N(n) = \begin{cases} \binom{8}{n} p^n (1-p)^{8-n} & n = 0, 1, \ldots 8 \\ 0 & \text{otherwise} \end{cases}$$

(b) The indicator random variable I equals zero if and only if $N = 0$, Hence,

$$P[I=0] = P[N=0] = 1 - P[I=1].$$

Thus, the complete expression for the PMF of I is,

$$P_I(i) = \begin{cases} (1-p)^8 & i = 0 \\ 1-(1-p)^8 & i = 1 \\ 0 & \text{otherwise} \end{cases}$$

Problem 3.4.1

Using the CDF given in the problem statement we find that

(a) $P[Y<1] = 0$

(b) $P[Y \le 1] = 1/4$.

(c) $P[Y>2] = 1 - P[Y \le 2] = 1 - 1/2 = 1/2$

(d) $P[Y \ge 2] = 1 - P[Y < 2] = 1 - 1/4 = 3/4$.

(e) $P[Y=1] = 1/4$.

(f) $P[Y=3] = 1/2$.

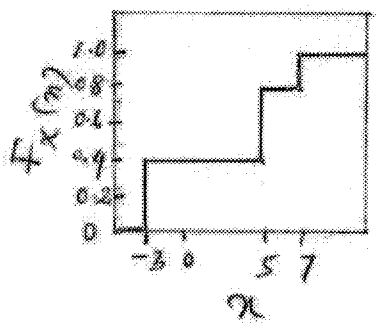
(g) From the staircase CDF of Problem, we see that Y is a discrete random variable. The jumps in CDF occur at the values Y can take on. The height of each jump equals the prob. of that value. The PMF of Y is,

$$P_Y(y) = \begin{cases} 1/4 & y=1 \\ 1/4 & y=2 \\ 1/2 & y=3 \\ 0 & \text{otherwise.} \end{cases}$$ //

Problem 3.4.3

(a) The graph of the CDF ;

$$F_X(x) = \begin{cases} 0 & x < -3 \\ 0.4 & -3 \le x < 5 \\ 0.8 & 5 \le x < 7 \\ 1 & x \ge 7. \end{cases}$$



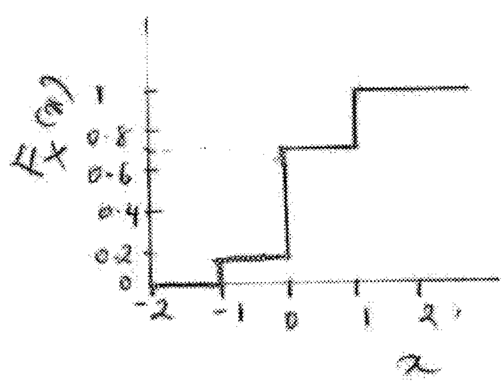
(b) The corresponding PMF of X is

$$P_X(x) = \begin{cases} 0.4 & x=-3 \\ 0.4 & x=5 \\ 0.2 & x=7 \\ 0 & \text{otherwise} \end{cases}$$ .//.

Problem 3.4.2

(a) The CDF is ;

$$F_X(x) = \begin{cases} 0 & x < -1 \\ 0.2 & -1 \le x < 0 \\ 0.7 & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$



(b) The corresponding PMF of X is

$$P_X(x) = \begin{cases} 0.2 & x=-1 \\ 0.5 & x=0 \\ 0.3 & x=1 \\ 0 & \text{otherwise}. \end{cases}$$

Problem 3.5.15

In this "double-or-nothing" type game, there are only two possible payoffs. The first is zero dollars, which happens when we lose 6 straight bets, and the second payoff is 64 dollars which happens unless we loose 6 straight bets. So the PMF of Y is

$$P_Y(y) = \begin{cases} (1/2)^6 = 1/64 & y=0 \\ 1-(1/2)^6 = 63/64. & y=64. \\ 0 & \text{otherwise}. \end{cases}$$

The expected amount you take home is

$$E[Y] = 0\cdot(1/64) + 64\cdot(63/64) = 63.$$

So, on the average, we can expect to break even, which is not a very exciting proposition.

Problem 3.6.2

From the sol^n to Problem 2.4.2, the PMF of X;

$$P_X(x) = \begin{cases} 0.2 & x=-1 \\ 0.5 & x=0 \\ 0.3 & x=1 \\ 0 & \text{otherwise}. \end{cases}$$

(a) The PMF of $V = |X|$ satisfies

$$P_V(v) = P[\,|X| = v\,] = P_X(v) + P_X(-v).$$

In particular, $P_V(0) = P_X(0) = 0.5$, $P_V(1) = P_X(-1) + P_X(1) = 0.5$

The complete expression for the PMF of V is

$$P_V(v) = \begin{cases} 0.5 & v = 0 \\ 0.5 & v = 1 \\ 0 & \text{otherwise} \end{cases}$$

(b) From the PMF, we can construct the staircase CDF of V

$$F_V(v) = \begin{cases} 0 & v < 0 \\ 0.5 & 0 \le v < 1 \\ 1 & v \ge 1 \end{cases}$$

(c) From the PMF $P_V(v)$, the expected value of V is

$$E[V] = \sum_v P_V(v) = 0 \cdot (1/2) + 1\,(1/2) = \underline{1/2}.$$

You can also compute $E[V]$ directly by using Theorem 2.10.

Problem 3.6.4

A tree for the experiment is

$\frac{1}{3}$ $D = 99.75$ • $C = 10074.75$

$\frac{1}{3}$ $D = 100$ • $C = 10100$

$\frac{1}{3}$ $D = 100.25$ • $C = 10125.125$

Thus C has 3 equally likely outcomes.
The PMF of C is

$$P_C(c) = \begin{cases} 1/3 & c = 10074.75,\ 10100,\ 10125.125 \\ 0 & \text{otherwise.} \end{cases}$$

Problem 3.6.6

The cellular calling plan charges a flat rate of \$20 per month up to and including 30 th minute, and an additional 50 cents for each minute over 30 minutes. Knowing that time you spend on the phone is a geometric random variable M with mean $1/p = 30$, the PMF of M is,

$$P_M(m) = \begin{cases} (1-p)^{m-1} p & m = 1, 2, \ldots \\ 0 & \text{otherwise.} \end{cases}$$

The monthly cost, C obeys.

$$P_C(20) = P[M \leq 30] = \sum_{m=1}^{30} (1-p)^{m-1} p = 1 - (1-p)^{30}$$

when $M \geq 30$, $C = 20 + (M-30)/2$ or $M = 2C - 10$

Thus, $P_C(c) = P_M(2c - 10) \qquad c = 20.5, 21, 21.5, \ldots$

The complete PMF of C is

$$P_C(c) = \begin{cases} 1 - (1-p)^{30} & ; c = 20. \\ (1-p)^{2c-10-1} p & ; c = 20.5, 21, 21.5, \ldots \end{cases}$$

Problem 3.7.4 Let X denote the number of points the shooter scores. If the shot is uncontested, the expected number of points scored is

$$E[X] = (0.6)2 = 1.2.$$

If we foul the shooter, then X is a binomial random variable with mean $E[X] = 2p$. If $2p > 1.2$, then we should not foul the shooter. Generally, p will exceed 0.6 since a free throw is usually easier than an uncontested shot taken during the action of the game. Furthermore, fouling the shooter ultimately leads to the detriment of players possibly fouling out. This suggests that fouling a player is not a good idea. The only real exception occurs when facing a player like

Shaquille O'Neal whose free throw probability p is lower than his goal percentage during a game.

Problem 3.7.5

(a) $\mu_D = E[D] = \sum_{d=1}^{4} d \cdot P_D(d) = 1.(0.2) + 2(0.4) + 3(0.3) + 4(0.1) = \underline{\underline{2.3}}$ days.

(b) $E[D-\mu_D] = E[D] - E[\mu_D] = \mu_D - \mu_D = \underline{\underline{0}}$.

(c)
$$C(D) = \begin{cases} 90 & D=1 \\ 70 & D=2 \\ 40 & D=3 \\ 40 & D=4. \end{cases}$$

(d) $E[C] = \sum_{d=1}^{4} c \cdot P_d(d) = 90 \times 0.2 + 70 \times 0.4 + 40 \times 0.3 + 40 \times 0.1 = \underline{\underline{62 \text{ dollars}}}$.

Problem 3.8.5

$$P_X(x) = \binom{4}{x} (1/2)^4.$$

(a)
$$E[X] = \sum_{0}^{4} x P_X(x) = 0\binom{4}{0}\frac{1}{2^4} + 1\cdot\binom{4}{1}\frac{1}{2^4} + 2\binom{4}{2}\frac{1}{2^4} + 3\cdot\binom{4}{3}\frac{1}{2^4} + 4\binom{4}{4} 1/2^4$$
$$= [4 + 12 + 12 + 4]/2^4 = \underline{\underline{2}}$$

Then
$$E[X^2] = \sum_{x=0}^{4} x^2 P_X(x) = 0^2\binom{4}{0} 1/2^4 + 1^2\binom{4}{1} 1/2^4 + 2^2\binom{4}{2}\frac{1}{2^4} + 3^2\binom{4}{3} 1/2^4 + 4^2\binom{4}{4} 1/2^4$$
$$= [4 + 24 + 36 + 16]/2^4 = 5$$

$$Var[X] = E[X^2] - (E[X])^2 = 5 - 2^2 = 1$$

$\therefore$ standard deviation $\sigma_X = \sqrt{Var(X)} = 1$ //.

(b) $P[\mu_x - \sigma_x \leq x \leq \mu_x + \sigma_x] = P[2-1 \leq x \leq 2+1]$

$= P[1 \leq x \leq 3]$.

using the PMF of X;

$P[1 \leq x \leq 3] = P_x(1) + P_x(2) + P_x(3) = \underline{7/8}$.