Equation Sheet of Exam #3

$$E[g(X)] = \sum_x g(x)P_X(x)$$

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x)dx$$

$$\mu_X = E[X], \qquad Var[X] = E[X^2] - \mu_X^2$$

$$P_{X|B}(x) = \begin{cases} \frac{P_X(x)}{P[B]}, x \in B \\ 0, otherwise \end{cases}$$

$$f_{X|B}(x) = \begin{cases} \frac{f_X(x)}{P[B]}, x \in B \\ 0, otherwise \end{cases}$$

If $Y = aX + b$, then

$$\mu_Y = a\mu_X + b, Var[Y] = a^2 Var[X]$$

$$[\hat{x}_L(y) - \mu_x] = \frac{\rho_{X,Y}\sigma_X}{\sigma_Y}(y - \mu_Y)$$

$$P[|M_n(X) - \mu_X| \geq c] \leq \alpha$$

$$\alpha = \frac{Var[X]}{nc^2}$$

$$P_X(x) = \sum_y P_{X,Y}(x, y)$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x, y)d_y$$

$$E[g(x, y)] = \sum_x \sum_y g(x, y)P_{X,Y}(x, y)$$

$$E[g(x, y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x, y) f_{X,Y}(x, y)d_y d_x$$

$$P_{X|Y}(x | y) = \frac{P_{X,Y}(x, y)}{P_Y(y)}$$

$$f_{X|Y}(x | y) = \frac{f_{X,Y}(x, y)}{f_Y(y)}$$

$$r_{X,Y} = E[XY]$$

$$Cov[X, Y] = E[XY] - \mu_X \mu_Y$$

$$\rho_{X,Y} = \frac{Cov[X, Y]}{\sigma_X \sigma_Y}$$

If X and Y are $N(\mu_X, \mu_Y; \sigma_X, \sigma_Y; \rho)$**,**

1. X is $N(\mu_X, \sigma_X)$**, Y is** $N(\mu_Y, \sigma_Y)$

2. $f_{X|Y}(x | y)$ **is** $N(\tilde{\mu}_X(y), \tilde{\sigma}_X)$

$$\tilde{\mu}_X(y) = \mu_X + \rho\frac{\sigma_X}{\sigma_Y}(y - \mu_Y), \tilde{\sigma}_X^{\,2} = \sigma_X^{\,2}(1 - \rho^2)$$

$$Var[X + Y] = Var[X] + Var[Y] + 2Cov[X, Y]$$

If $X_i s, i = 1, \ldots, n,$ **are iids, then**

$W = \sum_{i=1}^{n} X_i$ **is approximately Gaussian with**

$\mu_W = n\mu_X$ **and** $Var[W] = nVar[X]$

Table 1: Families of Discrete Random Variables

If X is a **Bernoulli(p) RV,** $\mu_X = p \qquad Var[X] = p(1-p)$ $P_X(x) = \begin{cases} 1-p & , \ x = 0 \\ \text{p} & , \ \text{x} = 1 \\ 0, & \text{otherwise} \end{cases}$	If X is a **Geometric(p) RV,** $\mu_X = \frac{1}{p} \qquad Var[X] = \frac{1-p}{p^2}$ $P_X(x) = \begin{cases} p(1-p)^{x-1} & , \ x = 1,2,\dots \\ 0 & , \ otherwise \end{cases}$ $F_X(x) = \begin{cases} 1-(1-p)^{x} & , \ x = 1,2,\dots \\ 0 & , \ otherwise \end{cases}$
If X is a **Poisson(α) RV**, $\mu_X = \alpha \qquad Var[X] = \alpha$ $P_X(x) = \begin{cases} \frac{\alpha^x e^{-\alpha}}{x!} & , \ x = 0,1,2,\dots \\ 0 & , \ otherwise \end{cases}$	If X is a **Binomial (n, p) RV,** $\mu_X = np \qquad Var[X] = np(1-p)$ $P_X(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{(n-x)}, & x = 0,1,2,\dots,n \\ 0 & , \ otherwise \end{cases}$
If X is a **discrete Uniform (** k **,** l **) RV**, $\mu_X = (k+l)/2 \quad Var[X] = (l-k)(l-k+2)/12$ $P_X(x) = \begin{cases} \frac{1}{l-k+1} & , \ x = k, k+1, \dots, l \\ 0 & , \ therwise \end{cases}$	If X is a **Pascal (k, p) RV,** $\mu_X = k/p \quad Var[X] = k(1-p)/p^2$ $P_X(x) = \begin{cases} \binom{x-1}{k-1} p^k (1-p)^{(x-k)}, & x = k, k+1, \dots \\ 0 & , \ otherwise \end{cases}$

Table 2: Families of Continuous Random Variables

If X is a **Uniform(a, b) RV,** $\mu_X = \frac{a+b}{2} \qquad Var[X] = \frac{(b-a)^2}{12}$ $f_X(x) = \begin{cases} \frac{1}{b-a} & , \ a \le x < b \\ 0 & , \ therwise \end{cases}$ $F_X(x) = \begin{cases} 0 & , \ x \le a \\ \frac{x-a}{b-a} & , \ a < x \le b \\ 1 & , \ x > b \end{cases}$	If X is a **Gaussian** (μ, σ) RV, $\mu = \mu, Var[X] = \sigma^2$ $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(X-\mu)^2}{2\sigma^2}}$ $Z = \frac{X-\mu}{\sigma}$
If X is **Exponential(λ),** $\mu_X = \frac{1}{\lambda} \qquad Var[X] = \frac{1}{\lambda^2}$ $f_X(x) = \begin{cases} \lambda e^{-\lambda x} & , \ x \ge 0 \\ 0 & , \ therwise \end{cases}$ $F_X(x) = \begin{cases} 1 - e^{-\lambda x} & , \ x \ge 0 \\ 0 & , \ therwise \end{cases}$	