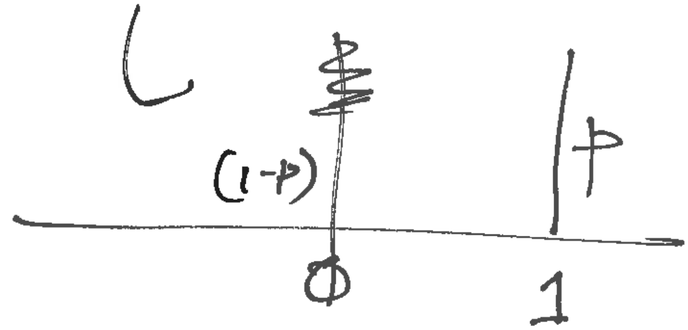
CDF ✓ $F_X(x)$

Mean

$$\mu_X = \sum_x x \cdot P_X(x)$$

ex:- μ_X of a Bernoulli (p)



$$\mu_X = (0)(1-p) + (1)(p)$$

$$= p$$

On the average, value of x is p

ex:- Mean of Geometric (p)

$$P_X(x) = \begin{cases} p(1-p)^{x-1}, & x = 1, 2, \dots \\ 0, & o/w. \end{cases}$$

$$\mu_X = \sum_{x=1}^{\infty} x \cdot p(1-p)^{x-1}$$

(2)

$$\mu_x = p \sum_{x=1}^{\infty} x \underbrace{(1-p)}_{a}^{x-1}$$

$$\sum_{x=0}^{\infty} a^x = \frac{1}{1-a}$$

Note: $\frac{d}{da}\left[a^x\right] = x a^{x-1}$

$$\sum_{x=0}^{\infty} x a^{x-1} = \frac{d}{da}\left(\frac{1}{1-a}\right)$$

$$= \frac{1}{(1-a)^2}$$

$$\frac{d}{da}\left[(1-a)^{-1}\right] = (-1)(1-a)^{-2}(-1)$$

$$\sum_{x=1}^{\infty} x a^{x-1} = \frac{1}{(1-a)^2}$$

$$\therefore \mu_x = p \cdot \frac{1}{\left[1-(1-p)\right]^2} = \frac{1}{p}$$

eg:- $p = 0.1 \rightarrow \frac{1}{p} = 10$

It can be shown

If X is Binomial (n, p)

$$\mu_X = np$$

If X is Pascal (k, p)

$$\mu_X = \frac{k}{p}$$

ex:- If X is Poisson (α)

is Av. no. of calls

$$P_X(x) = \frac{e^{-\alpha} \cdot \alpha^x}{x!}, \quad x = 0, 1, 2 \ldots$$

$$\mu_X = \sum_{x=0}^{\infty} x \cdot \frac{e^{-\alpha} \cdot \alpha^x}{x!}$$

$$= \cancel{e^{-\alpha}} \sum_{x=1}^{\infty} \frac{e^{-\alpha} \alpha^x}{(x-1)!} = \alpha \sum_{x=1}^{\infty} \frac{e^{-\alpha} \cdot \alpha^{x-1}}{(x-1)!}$$

Let $y = x - 1$

$$\mu_x = \alpha \sum_{y=0}^{\infty} \frac{e^{-\alpha} \cdot \alpha^y}{y!} = 1$$

Since $P_X(x) = \begin{cases} \dfrac{e^{-\alpha} \cdot \alpha^x}{x!}, & x = 1, \dots \\ 0, & o.w. \end{cases}$

$$\sum_{x=0}^{\infty} \frac{e^{-\alpha} \cdot \alpha^x}{x!} = 1$$

$\therefore \mu_x = \alpha \rightarrow$ Mean of $X = \alpha$.

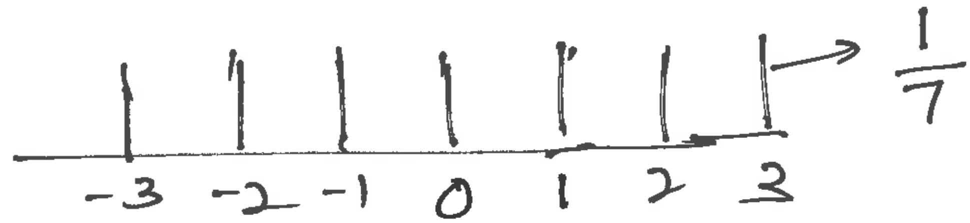
α is the average no. of calls

(as stated before)

If X is Discrete Uniform (k, l)

$$\mu_X = \frac{k+l}{2}$$

eg:-



$$\mu_X = 0$$

Q 3.5 (Q 2-5 2nd Edition)

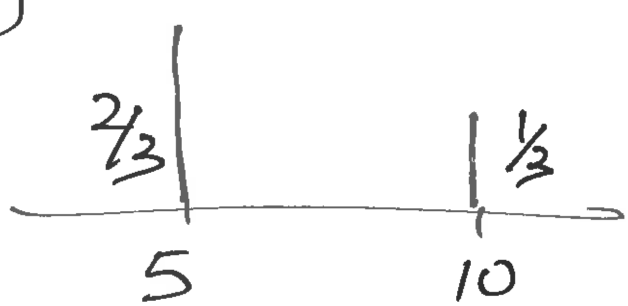
Sending → prob. 1/3 → 10 cents/text

Receiving → " 2/3 → 5 cents/text.

PMF of the cost

C per text ↳ can be 5 or 10

$P_C(c)$



$P[C=5] = P[\text{Receiving}]$
$= 2/3$

~~Mean~~ Average Cost of a Text

$$\mu_C = E[C] = \sum_c c \cdot P_C(c)$$

$$= \left[(5)\left(\tfrac{2}{3}\right) + (10)\left(\tfrac{1}{3}\right)\right]$$

Func^n of a Discrete RV → In 3rd Edition → 6.1

2nd Edit → 2.6

X is a discrete RV

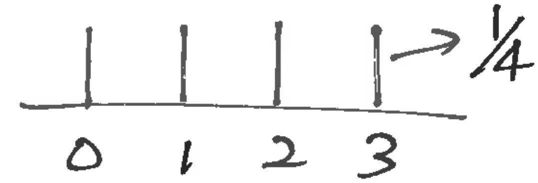
$P_X(x)$ is given

let $Y = g(X)$

→ Generally, Y is also a discrete RV

$P_Y(y)$?

eg:- X is discrete uniform from 0 to 3 → $P_X(x)$



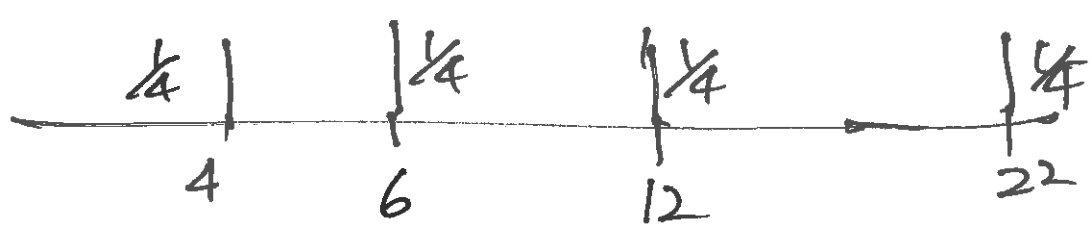
$$Y = 2x^2 + 4$$

$P_Y(y)$?

Prepare a Table.

X	Y	Prob.
0	4	1/4
1	6	1/4
2	12	1/4
3	22	1/4

$P_Y(y)$



$$\mu_x = \frac{0+3}{2} = 1.5$$

$$\mu_y = \sum_y y \cdot P_Y(y)$$

$$= (4)(1/4) + (6)(1/4) + (12)(1/4) + (22)(1/4)$$