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## 5LN445 Mathematics for language technologists

A two-part written examination. **You can not retake a part you have already passed. Give clear and easy to read solutions on a separate paper!** Calculator can be used. Grading criterion (course): Pass: at least 60% correct on each test. Pass with distinction: at least 80% correct on the two tests added.

# Part 1 – solutions

12 numbered tasks/questions, each worth 2 points. Short explanations sufficient – no essays necessary.

**(1–8) Give an example of the following as described here or explain why the request cannot be satisfied (i.e. why an example is impossible)...**

1. ... two different sets, $A$ and $B$,

   such that $|A \times B| = 13$.

$A = \{1,2,3,4,5,6,7,8,9,10,11,12,13\}$; $B = \{1\}$. (Some of you were "fooled" by the fact that 13 is prime, but here multiplication with 1 is relevant.)

2. ... three different sets, $A$, $B$, and $C$,

   such that $A \subseteq B$, $A \subseteq C$, and $A \nsubseteq (B \cap C)$.

All elements in $A$ are also elements in $B$ and $C$, and so, also in $B \cap C$, i.e. $A \subseteq (B \cap C)$ (so impossible).

3. ... two different sets, $A$ and $B$,

   such that $|(A \times B)| \neq |(B \times A)|$.

$|(A \times B)| = (|A| \times |B|) = (|B| \times |A|) = |(B \times A)|$ (so impossible).

4. ... two different sets, $A$ and $B$,

   such that $\{\emptyset\} = \mathcal{P}(A) \cap \mathcal{P}(B)$

True for any disjoint $A$ and $B$.

5. ... a directed graph $G_1$, such that

   $G_1$ is a directed tree, $|V(G_1)| = \{1,2,3\}$, and $|E(G_1)| = 5$.

Impossible, two edges would attach the two non-root nodes.

6. … a directed graph $G_1$, such that
   $G_1$ is a DAG (directed acyclic graph), $|V(G_1)| = \{1,2,3\}$, and $|E(G_1)| = 5$.

Impossible.

7. … a relation $R : A \to A$, such that
   $|\mathrm{gr}(R)| = |A|$ and $R$ is not reflexive ("gr" = graph).

$A = \{1,2\}$. $\mathrm{gr}(R) = \{\langle 1 \to 2\rangle, \langle 2 \to 1\rangle\}$.

8. … an injective function $f : [0,100] \to [0,1000]$.

E.g. $f(x) = x$ or $f(x) = 5x$ or $f(x) = x + 17\ldots$

**(9–12) "Self-contained" tasks/questions.**

9. Say which set this is by listing its elements: $\mathcal{P}(\{1\}) \cup \mathcal{P}(\{1,2\})$!

$\{\emptyset, \{1\}, \{2\}, \{1,2\}\}$.

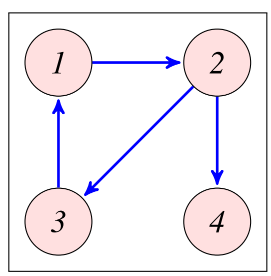
10. Consider this definition of a function $f : \mathbb{N} \to \mathbb{N}$:

$$f(n) \;=\; \begin{cases} n+1 & \text{if } n \leq 2 \\ f(n-2)\cdot f(n-1) & \text{if } n > 2 \end{cases}$$

   Which are the values of $f(4)$ and $f(5)$?

18 and 108.

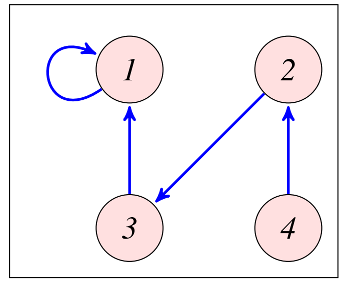
11. How many different DAGs (directed acyclic graphs) have the same vertices as the graph $G_1$ (to the right) and a subset of the edges of $G_1$ as their edges? Explain your answer briefly.




($G_1$)

All answered like this concerned trees, as in previous test. DAGs include all acyclic subsets of $\{(1,2),(2,3),(2,4),(3,1)\}$, i.e $\{(1,2),(2,3),(2,4)\}$, $\{(1,2),(2,4),(3,1)\}$, $\{(2,3),(2,4),(3,1)\}$, $\{(1,2),(2,3)\}$, $\{(1,2),(2,4)\}$, $\{(1,2),(3,1)\}$, $\{(2,3),(2,4)\}$, $\{(2,3),(3,1)\}$, $\{(2,4),(3,1)\}$, $\{(1,2)\}$, $\{(2,3)\}$, $\{(2,4)\}$, $\{(3,1)\}$, and $\emptyset$. Thirteen DAGs, if I counted correctly.

12. Is the relation $R_1$ (to the right) transitive?
Give a brief argument supporting your answer.




($R_1$)

No, counterexample: (4,2), (2,3), but (4,3) missing.