07/08
1

Joint pdf $\rightarrow$ 2 RVs $\hookrightarrow$ both continuous

$$f_{X,Y}(x,y) = \frac{\partial^2}{\partial x \partial y} F_{X,Y}(x,y)$$

1. $f_{X,Y}(x,y) \geq 0$

2. $\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy\,dx = 1$

3. $P[X,Y \in R] = \displaystyle\iint_R f_{X,Y}(x,y)\,dy\,dx$

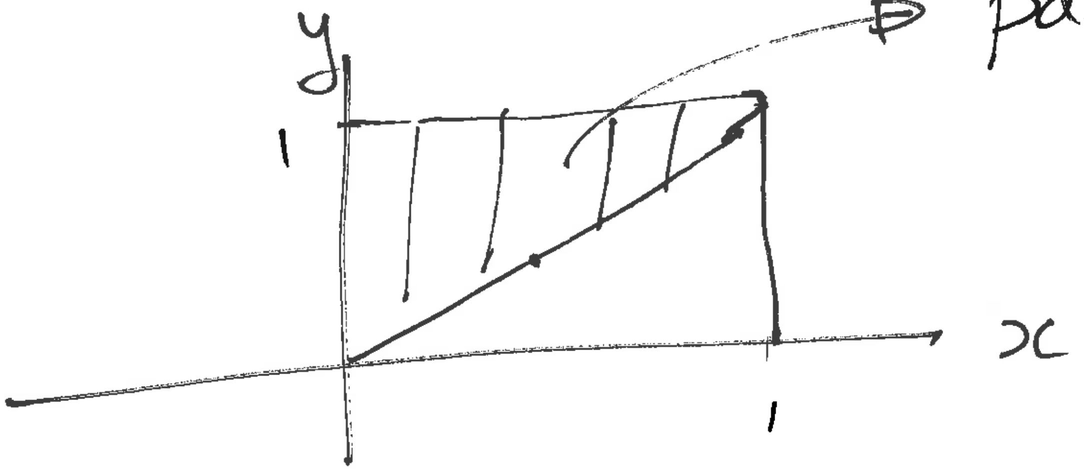
eg :- Joint pdf of X, & Y is ②

$$f_{X,Y}(x,y) = \begin{cases} c, & 0 \le x \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find c & $P[Y > 2X]$

Draw → Region

pdf = c



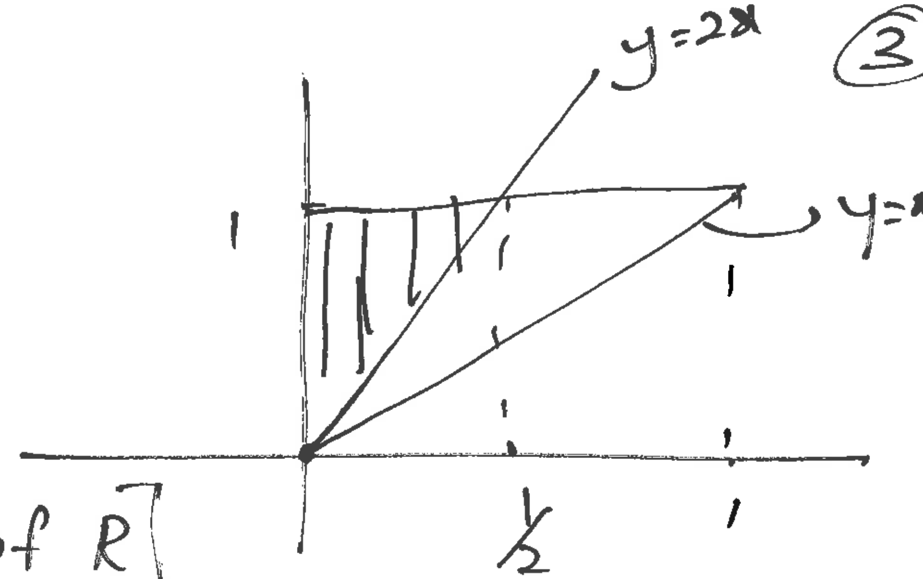
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underbrace{f_{X,Y}(x,y)}_{\text{constant} = c}\, dy\, dx = 1$$

$$c \times \text{Area} = c \times \frac{1}{2}(1)(1) = 1$$

$$c = 2$$

<u>Or</u> $\int_{0}^{1}\int_{x}^{1} c \, dy\, dx = 1$

$P[Y>2X]$



$= C\,[$~~New~~ Area of $R]$

$= \cancel{C}\; 2\left[\frac{1}{2}\times\frac{1}{2}\times 1\right] = \frac{1}{2}$

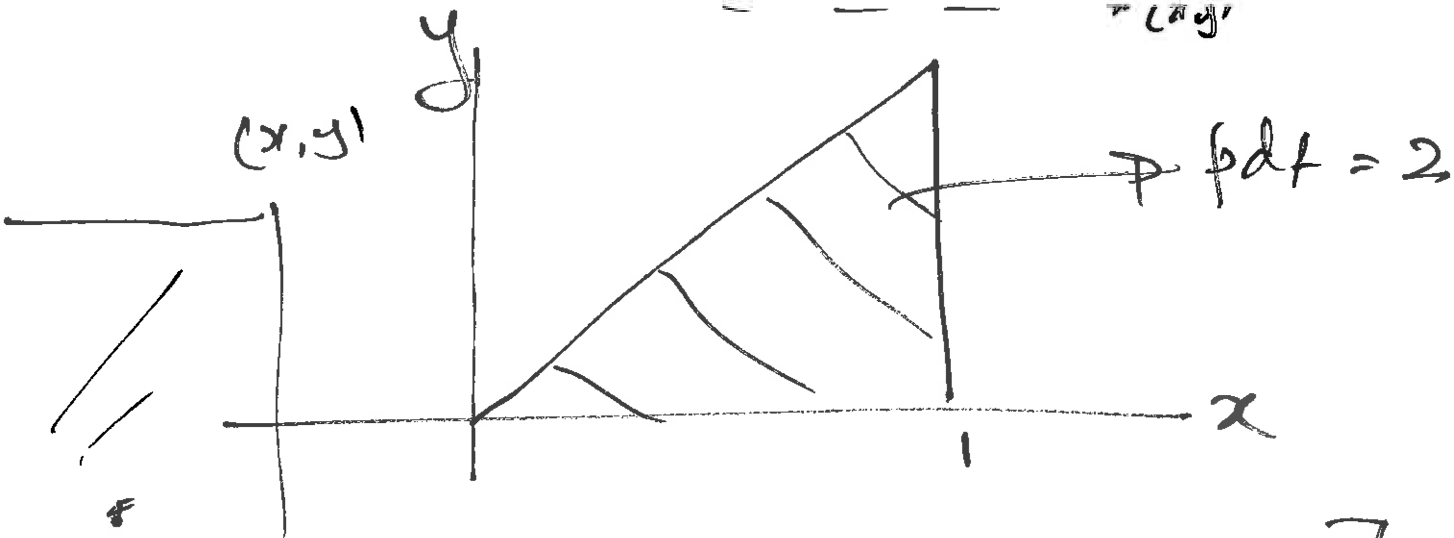
Finding the joint CDF $\longrightarrow$ given the joint pdf

Ex 5.8 (Ex 4.5)

$$f_{X,Y}(x,y) = \begin{cases} 2, & 0 \le y \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find $F_{X,Y}(x,y)$

④



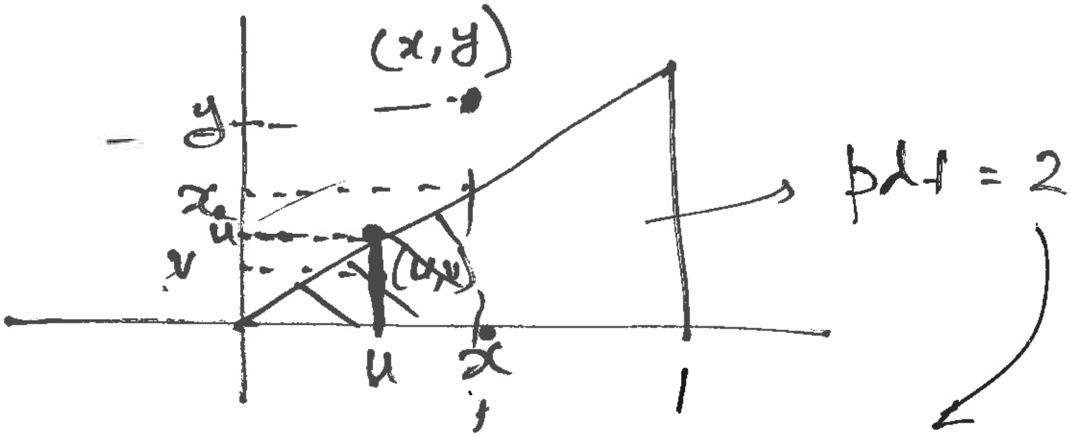
$F_{X,Y}(x,y) = P[X \le x \ \& \ Y \le y]$

2^{nd}, 3^{rd}, 4^{th} Quadrants $F_{X,Y}(x,y) = 0$

1^{st} Quadrant

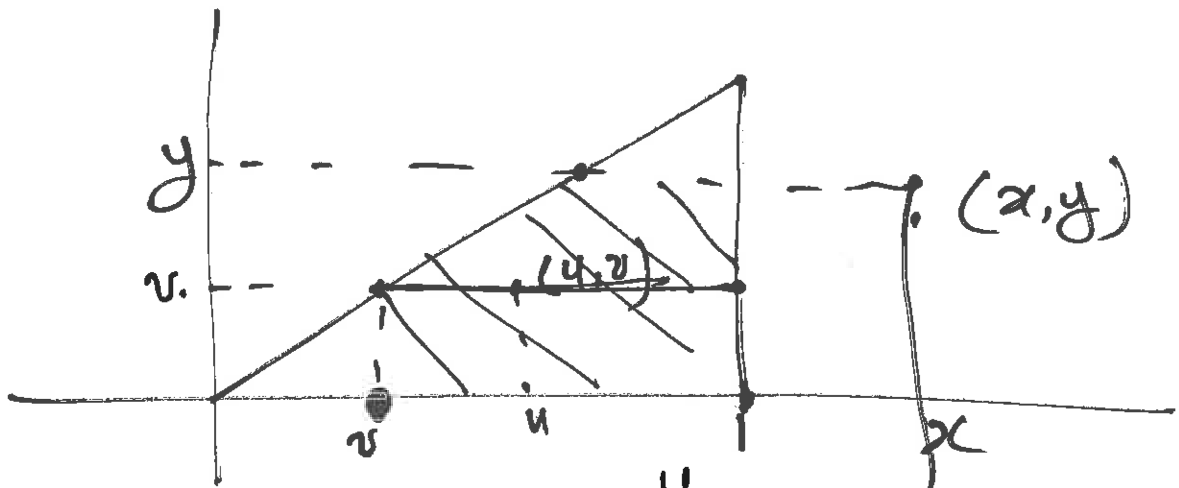
$x \geqslant 1, \ y \geqslant 1 \longrightarrow$

$0 \le x \le 1, \qquad y > x$



$F_{X,Y}(x,y) = \underbrace{\frac{1}{2}(x)(x)}_{\text{Area}} \times 2$

$$F_{X,Y}(x,y) = \int_0^x \int_0^u 2 \, dv \, du$$



$$F_{X,Y}(x,y) = \int_0^y \int_v^1 2 \, du \, dv$$

Look at the book for the other cases

5.5 (4.5) Marginal PDFs

$X, Y \to$ 2 continuous RVs

$f_{X,Y}(x,y)$ is given

$f_X(x) \longrightarrow$ Marginal pdf of X

$f_Y(y) \longrightarrow$ . . . . . . Y

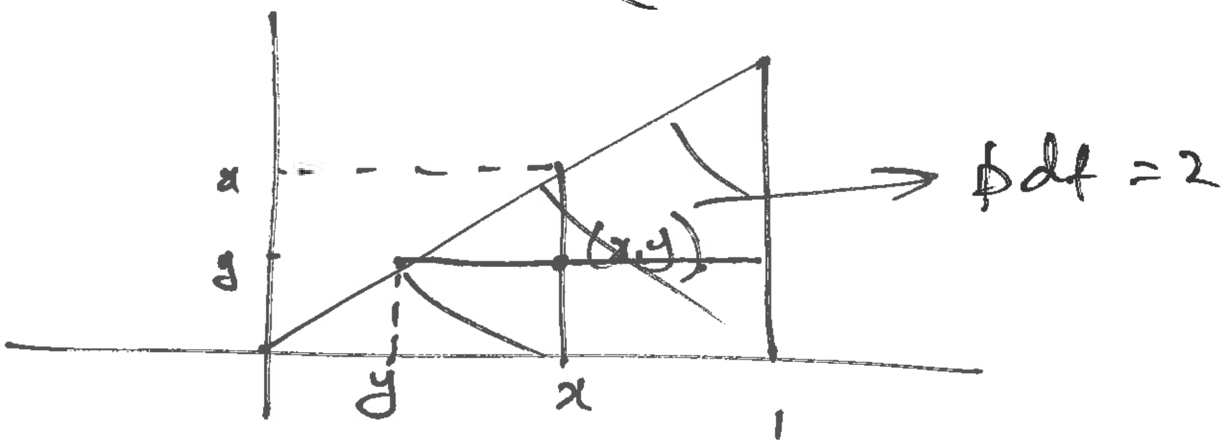
Recall in the Discrete

$$P_X(x) = \sum_y P_{X,Y}(x,y)$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dy$$

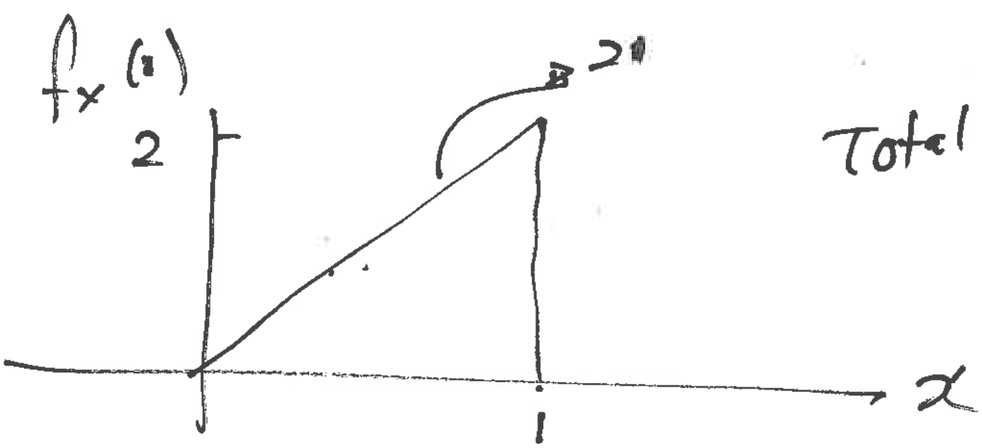
$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dx$$

Ex :- $f_{X,Y}(x,y) = \begin{cases} 2, & 0 \le y \le x \le 1 \\ 0, & \text{o/w.} \end{cases}$



$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy \quad , \; 0 \le x \le 1$$

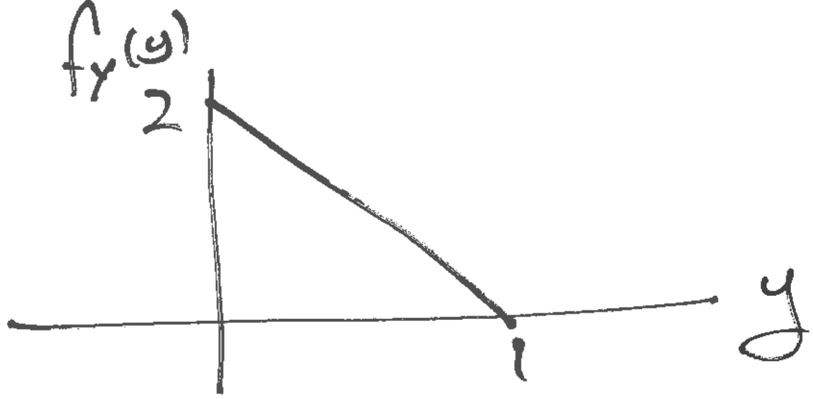
$$= \int_{0}^{x} 2 \; dy = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & \text{o/w.} \end{cases}$$



Total Area = 1

8

$$f_Y(y) = \int_y^1 2 \, dx = \begin{cases} 2(1-y), & 0 \le y \le 1 \\ 0, & o/w \end{cases}$$



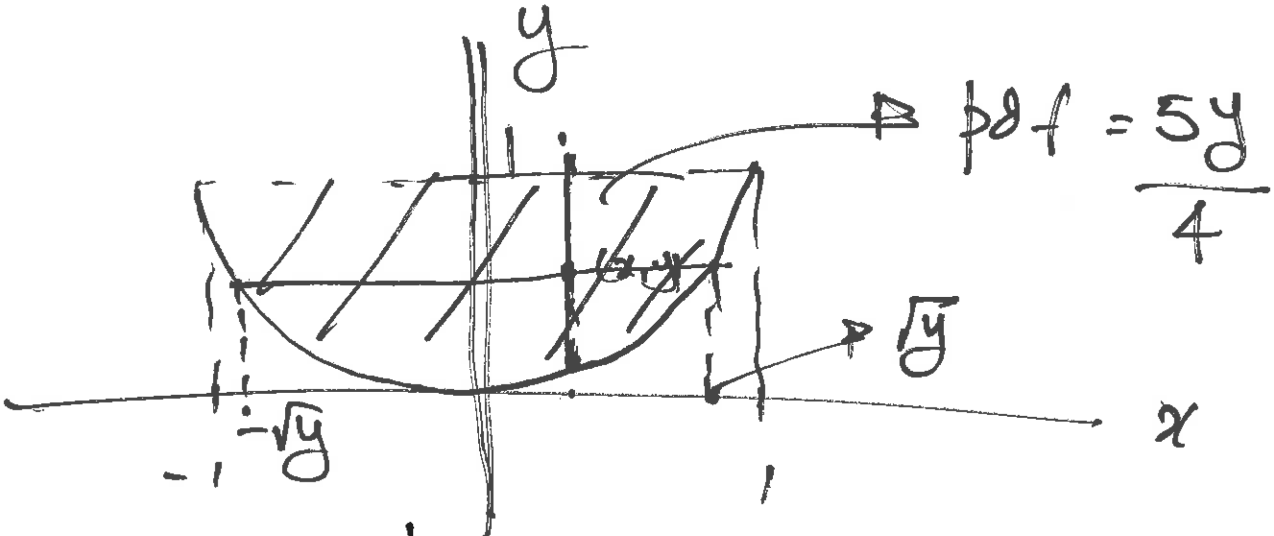
t/w Find $f_X(x)$ & $f_Y(y)$

$$f_{X,Y}(x,y) = \begin{cases} cxy, & 0 \le x \le 5, 0 \le y \le 3 \\ 0, & o/w. \end{cases}$$

Ex ~~5~~.10 (Ex 4.7)

$$f_{X,Y}(x,y) = \begin{cases} \frac{5y}{4}, & -1 \le x \le 1, x^2 \le y \le 1 \\ 0, & o/w \end{cases}$$

$f_X(x)$ & $f_Y(y)$?



$$f_X(x) = \int_{x^2}^{1} \frac{5y}{4}\, dy = \frac{5}{4}\, \frac{y^2}{2}\Big|_{x^2}^{1}$$

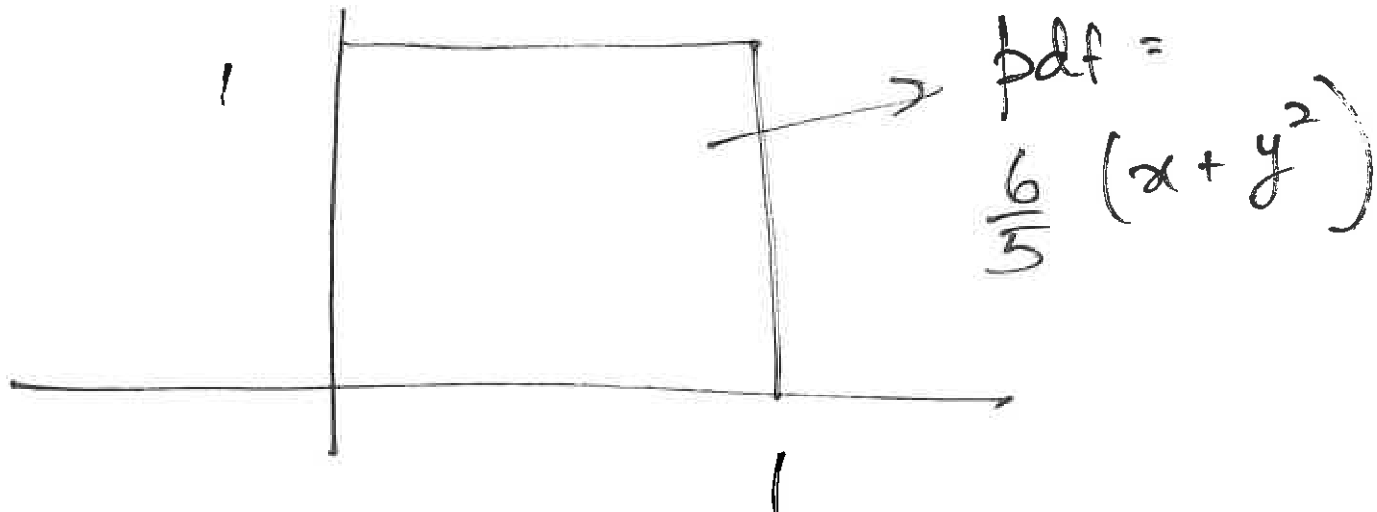
$$= \begin{cases} \frac{5}{8}\left(1 - x^4\right), & -1 \le x \le 1 \\ 0, & \text{o/w.} \end{cases}$$

$$f_Y(y) = \int_{-\sqrt{y}}^{\sqrt{y}} \frac{5y}{4}\, dx = \frac{5y}{4}\, x\Big|_{-\sqrt{y}}^{\sqrt{y}}$$

$$= \begin{cases} \frac{5}{4}\, y\left(2\sqrt{y}\right), & 0 \le y \le 1 \\ 0, & \text{o/w.} \end{cases}$$

Q 5.5 (Q 4.5)

$$f_{X,Y}(x,y) = \begin{cases} \dfrac{6(x+y^2)}{5}, & 0 \le x \le 1 \\ & 0 \le y \le 1 \\ 0, & \text{oth.} \end{cases}$$



$$f_X(x) = \int_0^1 \frac{6}{5}(x+y^2)\,dy$$

$$= \frac{6}{5}\left[xy + \frac{y^3}{3}\right]_0^1$$

$$= \begin{cases} \dfrac{6}{5}\left(x + \dfrac{1}{3}\right), & 0 \le x \le 1 \\ 0, & \text{othe.} \end{cases}$$

$$f_Y(y) = \int_0^1 \frac{6}{5}(x+y^2)\,dx$$

$$= \frac{6}{5}\left[\frac{x^2}{2} + xy^2\right]_0^1$$

$$= \begin{cases} \frac{6}{5}\left(\frac{1}{2} + y^2\right), & 0 \le y \le 1 \\ 0, & \text{oth.} \end{cases}$$

6.1, 6.4 (4.6)

Func$^{\underline{n}}$ of 2 RVs.

$X, Y \rightarrow$ 2 RVS

$\downarrow$

both Discrete

$W = g(X, Y) \longrightarrow$ given

$P_{X,Y}(x, y)$ is given

$P_W(w)$?

Use a Table

$P_{X,Y}(x,y) \longrightarrow$ Comes in a Table

eg :- $P_{X,Y}(x,y) = \begin{cases} cx^2y, & x = -1, 1, 2 \\ & y = 1, 2 \\ 0, & \text{oth} \end{cases}$

Find c & the PMF of $W = X^2Y$

	$y=1$	$y=2$
$x=-1$	c $\boxed{1}$	$2c$ $\boxed{2} \longrightarrow w=2$
$x=1$	c $\boxed{1}$	$2c$ $\boxed{2}$
$x=2$	$4c$ $\boxed{4}$	$8c$ $\boxed{8}$

$$c + 2c + c + 2c + 4c + 8c = 1 \longrightarrow c = \frac{1}{18}$$

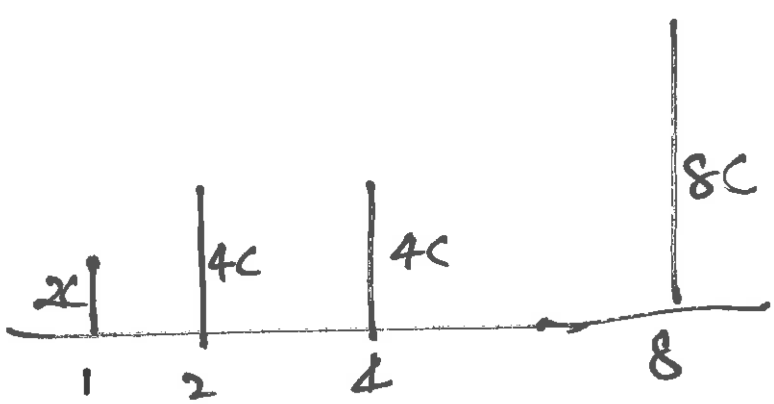
Space of w is $\{1, 2, 4, 8\}$

$$P_W(w) \longrightarrow P_W(1) = 2c$$

$$P_W(2) = 4c$$

$$P_W(4) = 4c$$

$$P_W(8) = 8c$$

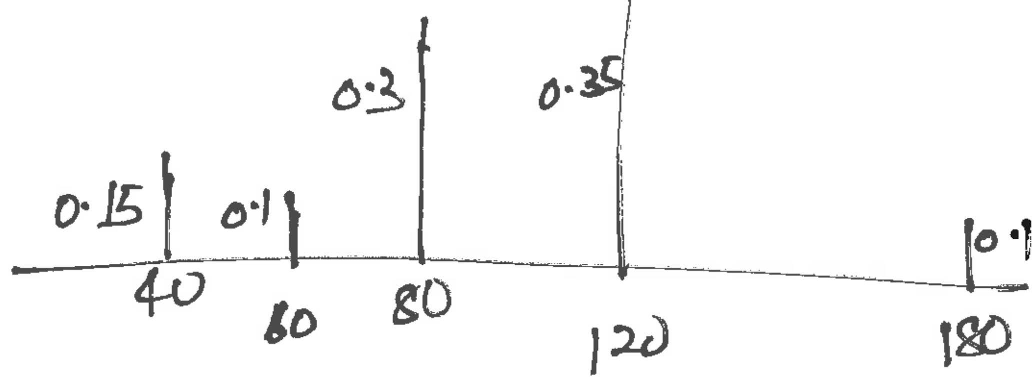


Ex 6.1 (Ex 4.8)

	$x=40$	$x=60$
$l=1$	0.15 $\boxed{40}$	0.1 $\boxed{60}$
$l=2$	0.3 $\boxed{80}$	0.2 $\boxed{120}$
$l=3$	0.15 $\boxed{120}$	0.1 $\boxed{180}$

$W = LX$

$P_W(w)$



6.4 When X & Y are continuous

$f_{X,Y}(x,y) \longrightarrow$ given

$W = g(X,Y) \longrightarrow$ given

$f_W(w) = \,?$

Process

*. Start with the CDF of W

$$F_W(w) = P[W \le w]$$
$$= P[g(x,y) \le w]$$
$$= P[x,y \in R]$$

can depend on w

$$= \iint_R f_{x,y}(x,y)\,dy\,dx$$

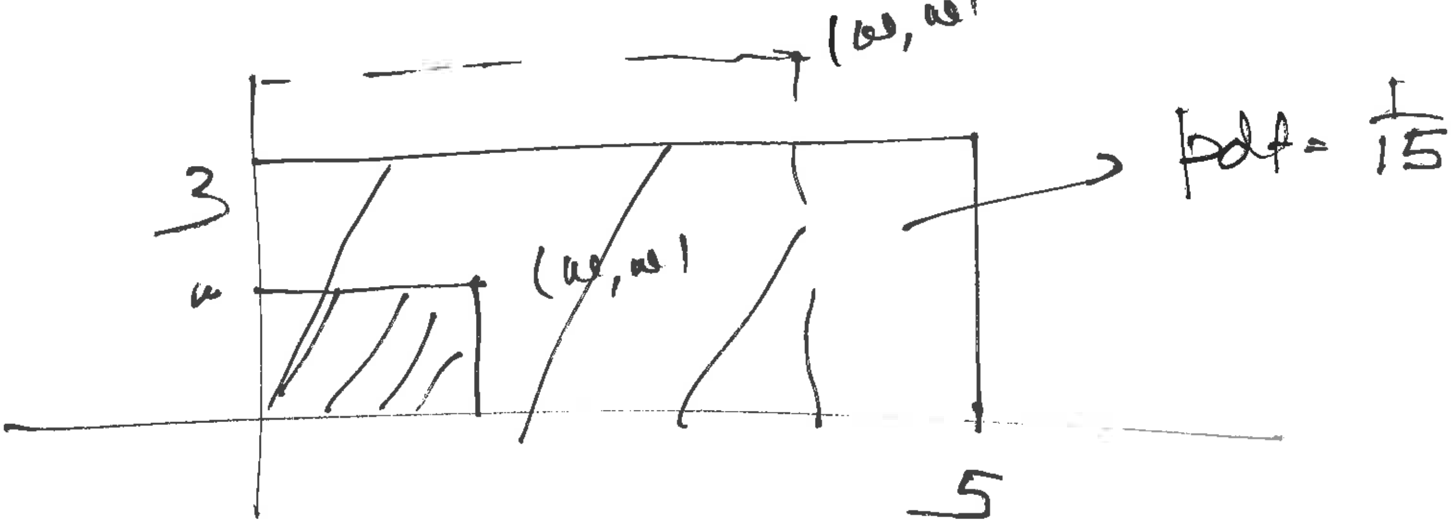
as a func^n of w

$$f_W(w) = \frac{d}{dw} F_W(w)$$

Ex 6.9 (Ex 4.9)

Given $f_{X,Y}(x,y) = \begin{cases} \frac{1}{15}, & 0 \le x \le 5, \ 0 \le y \le 3 \\ 0, & \text{o/w} \end{cases}$

$$W = \text{Max}\{X, Y\}$$



$$F_W(w) = P[W \le w]$$

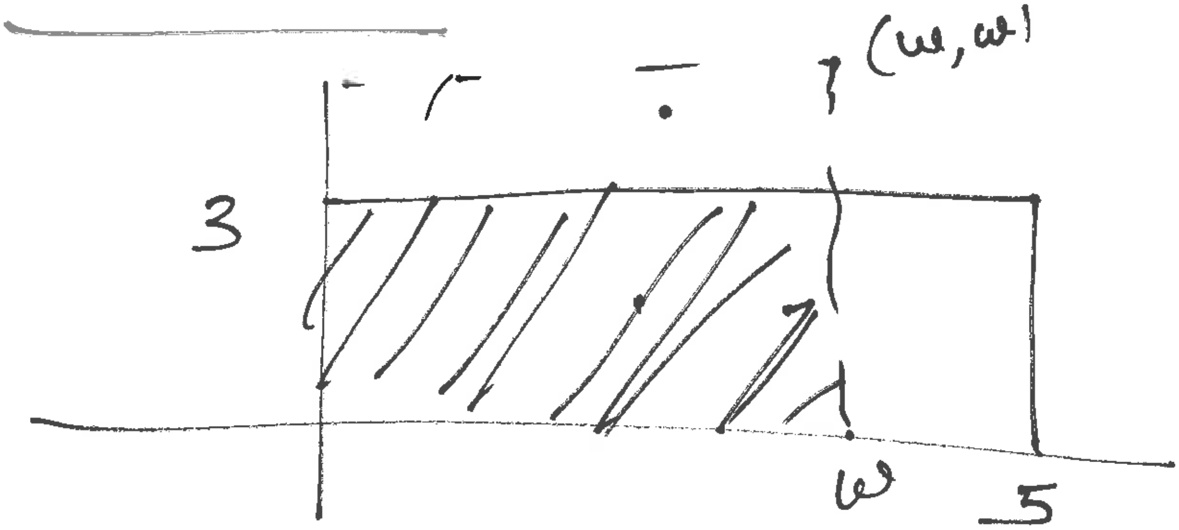
varies from 0 to 5

$$= P[\text{Max}(X,Y) \le w]$$

$$= P[X \le w \ \& \ Y \le w]$$

$$= w^2 \cdot \frac{1}{15}, \quad 0 \le w \le 3$$

$3 < w \leq 5$



$$F_W(w) = 3w \times \frac{1}{15}$$

$$= \frac{w}{5}$$

$$F_W(w) = \begin{cases} 0, & w < 0 \\ \frac{w^2}{15}, & 0 \leq w < 3 \\ \frac{w}{5}, & 3 \leq w < 5 \\ 1, & w \geq 5 \end{cases}$$

$$f_W(w) = \begin{cases} \frac{2w}{15}, & 0 \le w < 3 \\ \frac{1}{5}, & 3 \le w \le 5 \\ 0, & \text{otherwise} \end{cases}$$

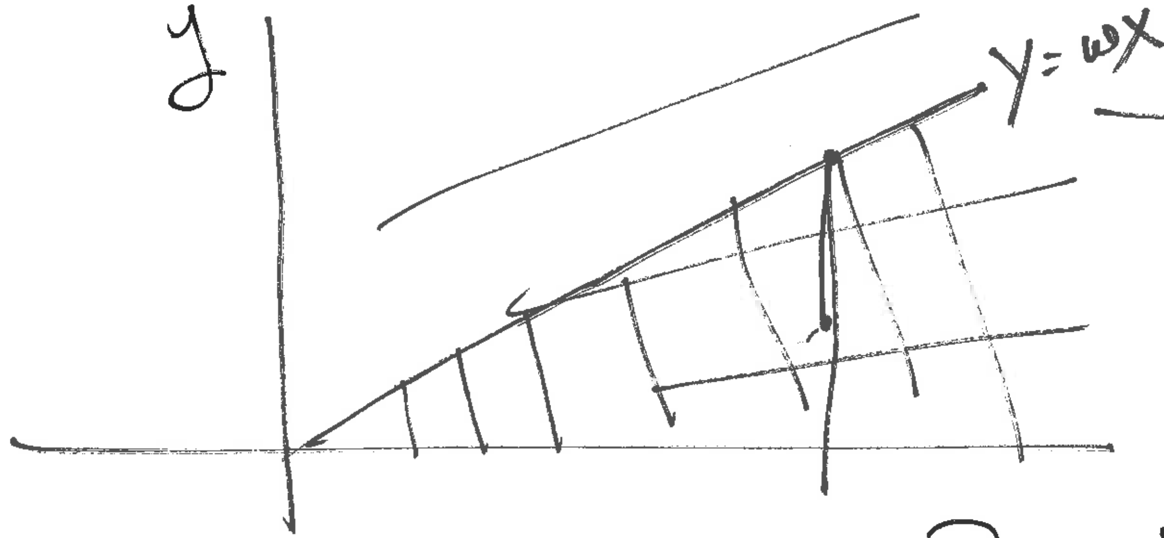
Ex 6.10 (Ex 4.10)

$$f_{X,Y}(x,y) = \begin{cases} \lambda\mu e^{-(\lambda x + \mu y)}, & x > 0,\ y > x \\ 0, & \text{o/w} \end{cases}$$

$\lambda, \mu > 0$

pdf of $W = \frac{Y}{X}$

HW: Find $f_X(x)$ & $f_Y(y)$



$\rightarrow$ pdf $\lambda\mu e^{-(\lambda x + \mu y)}$

W varies from 0 to ∞

$$F_W(w) = P\left[\frac{Y}{X} \le w\right]$$

$$= P[Y \le wX]$$

$$= \int_0^\infty \int_0^{wx} \lambda\mu e^{-(\lambda x + \mu y)}\, dy\, dx$$

$\vdots$