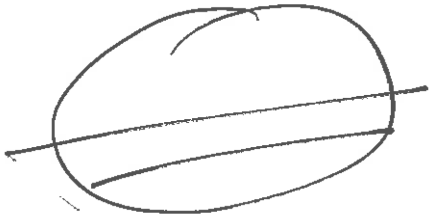
6/8
1



HW Prob.

Ex::

3 Centers (C)

5 Forwards (F)

5 Guards (G)

1 Swingman $\longrightarrow$ can play as a F or a G

Coach: $1C + 2F + 2G$

No. of possible stating line-ups $\rightarrow$ ?

3 Cases:

Case 1: Swingman not playing

$$N_1 = \binom{3}{1}\binom{5}{2}\binom{5}{2} = 3 \times \frac{5\times4}{1\times2} \times \frac{5\times4}{1\times2}$$

Case 2: Swingman playing as a F

$$N_2 = \binom{3}{1}\binom{5}{1}\binom{5}{2} = 3\times5\times10$$

Case 3: Swingman playing as a G

$$N_3 = \binom{3}{1}\binom{5}{2}\binom{5}{1} = 3\times10\times5$$

$\therefore$ Total $N = N_1 + N_2 + \underline{N_3}$

If all starting line-ups are equally likely, find the prob. Swingman plays in the starting lineup

$$= \frac{N_2+N_3}{N_1+N_2+N_3}$$

---

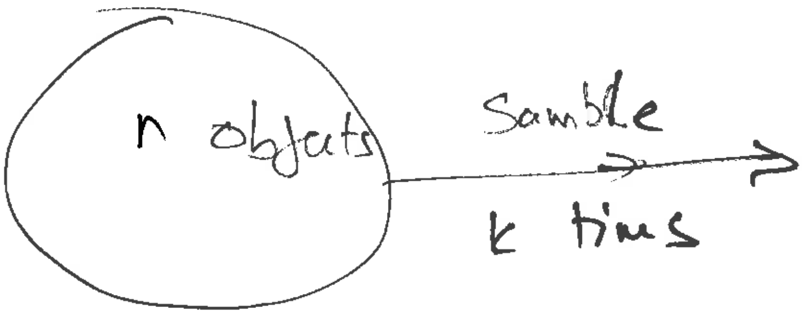
Sampling with replacement → what was selected is put back

So far: Sampling without replacement

Recall: $(n)_k = \dfrac{n!}{(n-k)!}$, $\dbinom{n}{k} = \dfrac{n!}{k!\,(n-k)!}$

with replacement: Ex:- Binary transmission 

if a zero is selected in the 1st interval zero is available in the 2nd interval too




No. of ways

$n^k$

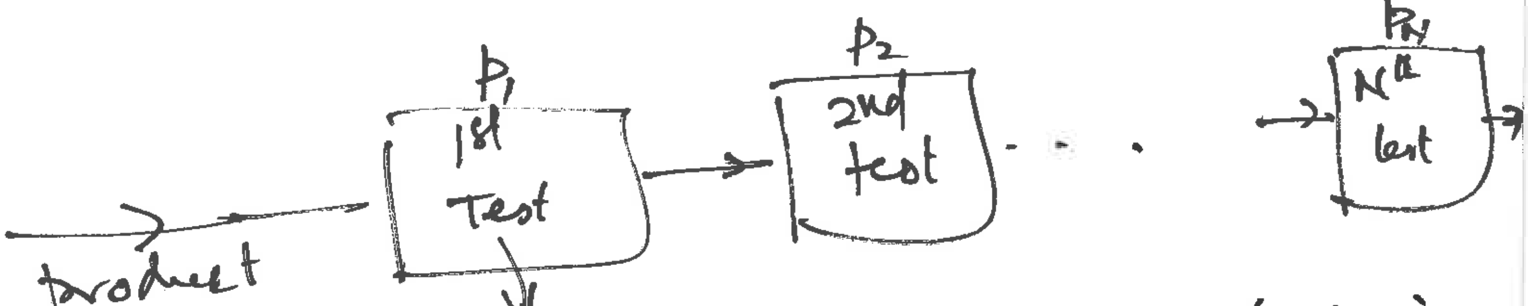
---

ex:-

1.10 (2nd)

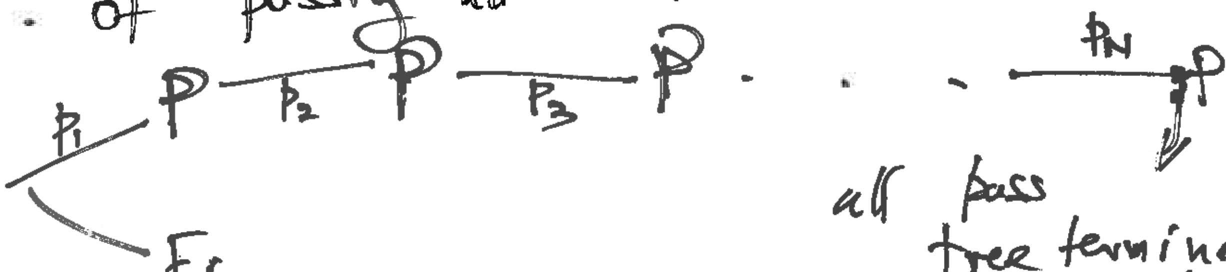
last

2.4 (3rd) → Reliability Analysis




prob. that the product passes test $i$ successfully $= p_i$

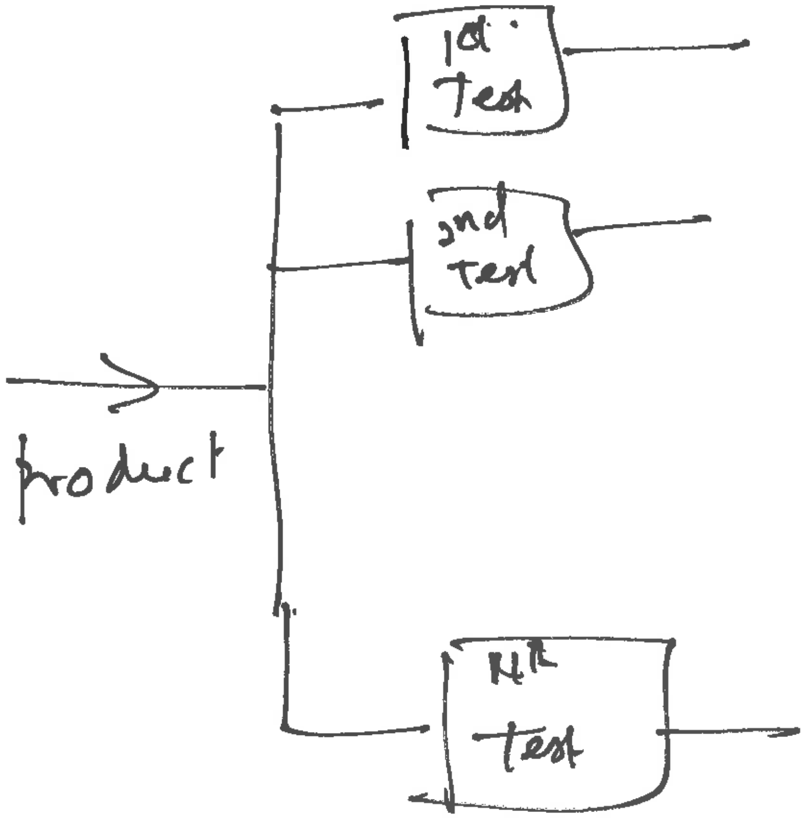
Prob. of passing all N tests =




all pass

tree terminates

$P[\text{All pass}] = p_1 p_2 \cdots p_N$




P[Passing at least one test]

$$= ?$$

$$= 1 - P[\text{Failing all tests}]$$

$$= 1 - (1-P_1)(1-P_2) \cdots (1-P_N)$$

$$= 1 - \prod_{i=1}^{N} (1-P_i)$$

Ch. 3 $\longrightarrow$ 3rd

Ch. 2 $\longrightarrow$ 2nd

# Discrete <u>Random</u> Variables (RVs)

Set Th. $\xrightarrow{\text{Exp.}}$ Prob. Th.

Perform an Expt.

- Based on the experimental outcome assign a value to a variable $X$

eg:- Toss a coin

$H \longrightarrow X = 1$

$T \longrightarrow X = 0$

$X$ depends on the experimental outcome.

$X$ is called a RV

Some experimental outcomes are already values → ~~can~~ can easily define a RV

eg:- Roll a dice
outcome $X$ is a RV
↓
~~def~~ denoted by upper case letters

eg:- $X$ is a RV
it can take values $\underbrace{\{x_1, x_2, \dots x_n\}}$
Space of $X$
↓
$S'$

If $X$ can take only discrete values
$X$ is called Discrete RV.

# Probability Mass Function (PMF)

PMF of a RV X is denoted by

$$P_X(x)$$

Def$^n$: $P_X(x) = P[X = x]$

eg:- Toss of a coin

$P[H] = 0.6$ $P[T] = 0.4$

$$X = \begin{cases} 1, & \rightarrow H \\ 0 & \rightarrow T \end{cases}$$

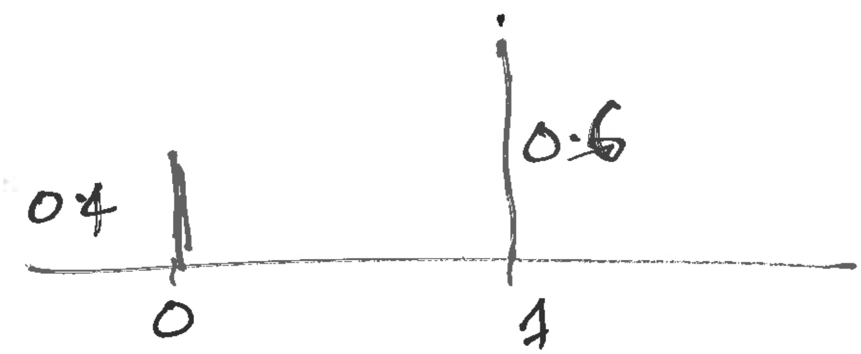
$P_X(x)$ ?

$P_X(0) = P[T] = 0.4$

$P_X(1) = P[H] = 0.6$

$$P_X(x) = \begin{cases} 0.6, & x = 1 \\ 0.4, & x = 0 \\ 0, & \text{otherwise} \end{cases}$$

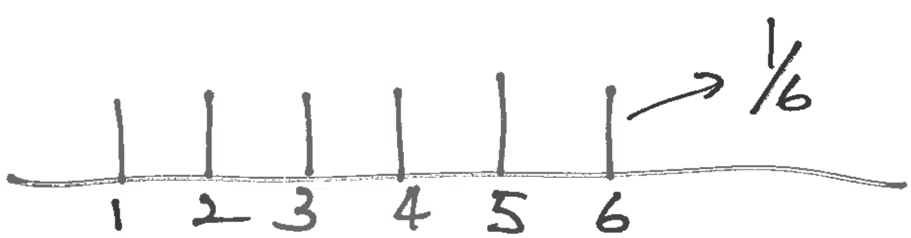
Graphical Representation of $P_X(x)$




eg:- Roll a fair dice

$X \rightarrow$ outcome

$P_X(x)$



In general:

$P_X(x)$


X can take value $x_i$ with prob. $p_i$

$i = 1, 2, \ldots N$

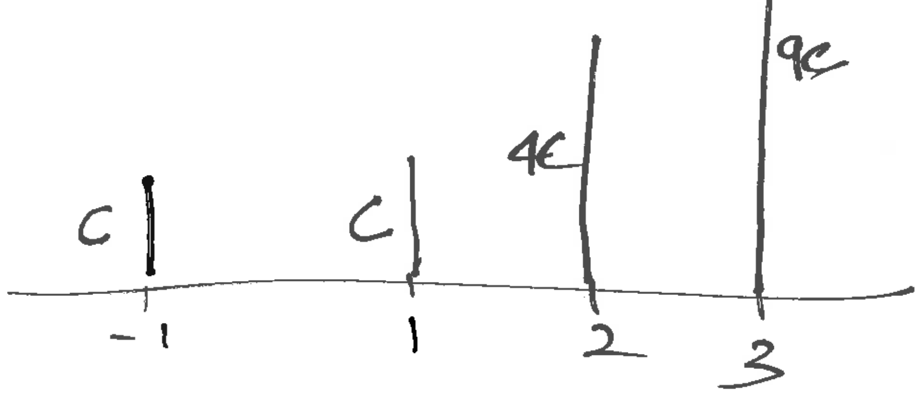
* $p_i$ Height of a line on the PMF $\geq 0$ $\downarrow$ 1st Axiom

* Sum of the Heights $= 1 \rightarrow$ 2nd Axiom

eg:- PMF of a RV $X$ is

$$P_X(x) = \begin{cases} cx^2, & x = -1, 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$$

Find $c$ & $P[X > 1]$




$$c + c + 4c + 9c = 1$$

$$c = \frac{1}{15}$$

$$P[X > 1] = P[X = 2] + P[X = 3]$$

$$= 4c + 9c = 13c = \frac{13}{15}$$

~~Classi~~

# Classes of Discrete RV

1. Bernoulli RV → ~~has~~ can take only 2 Val

↓

Previous Example

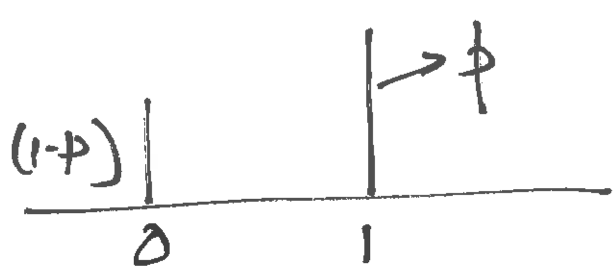
Toss of a coin

$$P[H] = p$$
$$P[T] = (1-p)$$

Expt: Toss the coin once

$$X = \begin{cases} 1, & H \\ 0, & T \end{cases}$$

$P_X(x)$



2. Geometric RV

Same coin

Expt: Keep Tossing the coin until we ~~a~~ see a Head

$X$ = No. of Tosses.

$\hookrightarrow$ space $\{1, 2, 3, \ldots\}$ $\hookrightarrow$ all ~~+ve~~ positive integer

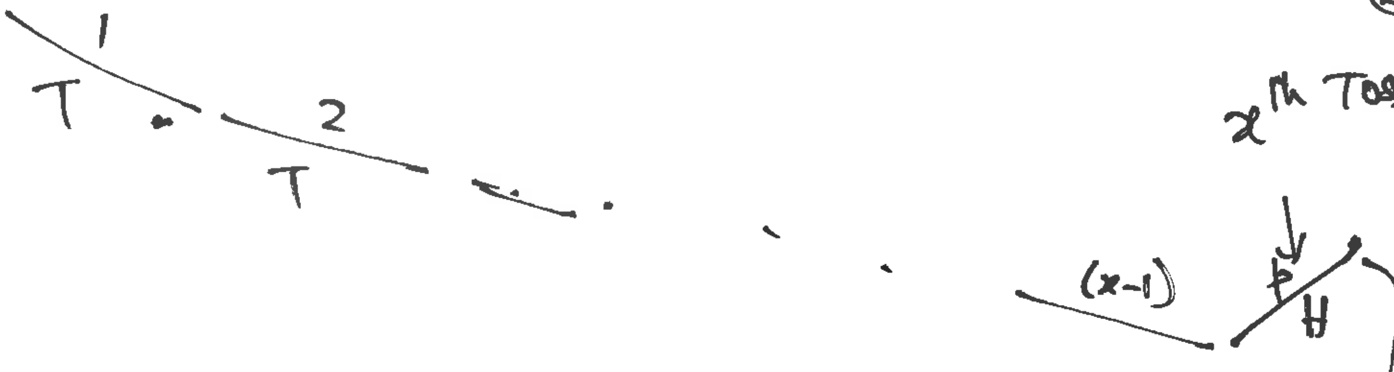
$P_X(x) = ?$

$\hookrightarrow P_X(x) = P[X = x]$

$= P[\text{we get the } 1^{st} \text{ head in the } x^{th} \text{ Toss}]$

All Tosses: 1 thru $(x-1)$ Must be Tails

$x^{th}$ Toss is a Head




$$P[\underbrace{T\,T\,T\,\cdots\,T}_{(x-1)}\,H] = (1-p)^{(x-1)}\,p$$

$$P_X(x) = \begin{cases} p(1-p)^{x-1}, & x = 1, 2, \cdots \\ 0, & \text{otherwise} \end{cases}$$

eg:- Prob. of a defective product $= 0.1$

Find the prob. that on a given day the ~~s~~ defective product manufactured is the $16^{th}$ product

D — Defective ⇝ H ⟶ $p = P[D] = 0.1$

G — Good ⇝ T $(1-p) = P[G] = 0.9$

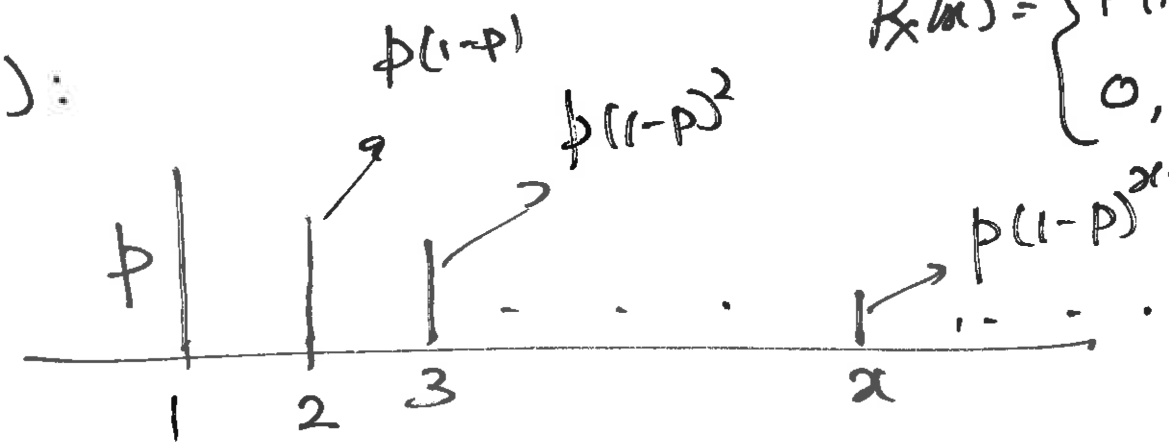
$$P_X(16) = ?$$

$$= p(1-p)^{x-1}$$

$$= (0.1)(0.9)^{15}$$

$$P_X(x) = \begin{cases} p(1-p)^{x-1}, & x = 1, 2, \dots \\ 0, & o/w \end{cases}$$

$P_X(x)$:




$$\sum_{x=1}^{\infty} p(1-p)^{x-1} = 1$$

~~Proof:~~
Recall

$$\sum_{n=0}^{\infty} a^n = 1 + a + a^2 \cdots = \frac{1}{1-a}$$

$|a| < 1$

$$\sum_{x=1}^{\infty} p(1-p)^{x-1} = p \sum_{y=0}^{\infty} a^y$$

$x-1=y$

$1-p=a$

$$= p \frac{1}{1-a}$$

$$= p \frac{1}{1-(1-p)}$$

$$= p \frac{1}{p} = 1$$

1. Bernoulli ($p$)
2. Geometric ($p$)
3. Binomial RV

Same coin

Expt: TOSS $n$ times

$X$ = No. of Heads

$\hookrightarrow$ Space of $X$ $\{0, 1, 2, 3 \cdots n\}$

If $X$ is Binomial$(n, p)$.

$X$ = No. of Heads in $n$ tosses.

$n$: No. of tosses.

$P_X(x) = ?$

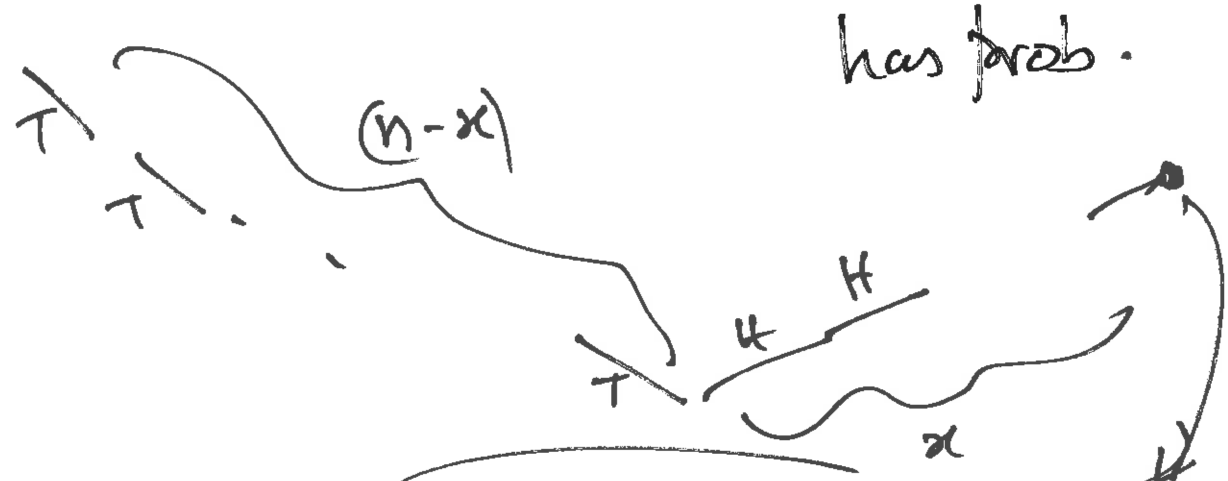
$= P[X = x]$

$= P[\text{Getting } x \text{ Heads in } n \text{ Tosses}]$

→ $2^n$ Tree termin

Any tree terminal with $x$ heads & $(n-x)$ tails

→ has prob.

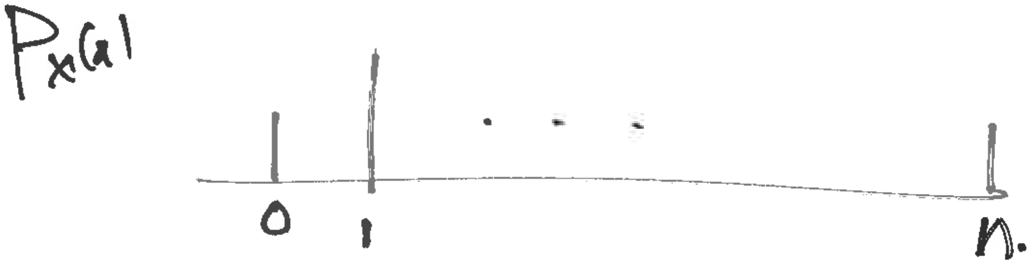



$p^x (1-p)^{n-x}$

No. of Tree terminals with
$x$ Heads & $(n-x)$ Tails

$$= \binom{n}{x}$$

$$\therefore P_X(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x = 0, 1, 2 \ldots n \\ 0, & \text{otherwise} \end{cases}$$




eg:- 10% of the products are defective.
Find the Prob.
(a) 8 out of 160 are defective
(b) 61st product is defective.
(c) 2 or more defective products out of 10.

(a) Binomial (160, 0.1)

Product → Toss (18)

~~H~~ D → H

$P[D] = p = 0.1$

$P_X(8) = ?$

$$= \binom{160}{8} (0.1)^8 (1-0.1)^{152}$$

$$= \frac{160 \times 159 \times 158 \times 157 \times 156 \times 155 \times 154 \times 153}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8} \times (0.1)^8 (0.9)^{152}$$

(b) Single Toss → Bernoulli ~~Geometric~~

$P[8^{th} \text{ is defective}]$

$= P[D] = 0.1$

(C) $n = 10$, $p = 0.1$

$X = 2, 3, 4 \cdots 10$

each is a Binomial

not 0, 1,

$$P[\text{2 or more Defective out of 10}]$$

$$= 1 - P[\text{No defective out of 10}] - P[\text{1 Defective out of 10}]$$

$$= 1 - P_X(0) - P_X(1)$$

X is Binomial (10, 0.1)

$$= 1 - \binom{10}{0}(0.1)^0(0.9)^{10} - \binom{10}{1}(0.1)^1(0.9)^9$$

$$= 1 - (0.9)^{10} - 10(0.1)(0.9)^9$$

If $X$ is Binomial $(n, p)$

$$P_X(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x = 0, 1 \dots n \\ 0, & \text{otherwise} \end{cases}$$

$$\sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x} = 1$$

Recall:
Binomial Expansion →

$$(a+b)^n = \sum_{x=0}^{n} \binom{n}{x} a^x b^{n-x}$$

Set $a = p$, $b = (1-p)$

$$LHS = [p + (1-p)]^n = 1^n = 1$$

Quiz #1 & Quiz #2 will be on 6/10