Problem 6.3.1

From Problem 3.6.1, random variable X has CDF,

$$F_X(x) = \begin{cases} 0 & x < -1 \\ x/3 + 1/3 & -1 \le x < 0 \\ x/3 + 2/3 & 0 \le x < 1 \\ 1 & 1 \le x \end{cases}$$

(a) we can find the CDF of Y, $F_Y(y)$ by noting that Y can only take on two possible values, 0 and 100. And the probability that Y takes on these two values depends on the probability that $X < 0$ and $X \geq 0$, respectively.

Therefore,

$$F_Y(y) = P[Y \le y] = \begin{cases} 0 & y < 0 \\ P[X < 0] & 0 \le y < 100 \\ 1 & y \geq 100. \end{cases}$$

The probabilities concerned with X can be found from the given CDF $F_X(x)$. This is the general strategy for solving problems of this type: to express the CDF of Y in terms of the CDF of X.
Since $P[X < 0] = F_X(0^-) = 1/3$, the CDF of Y is

$$F_Y(y) = P[Y \le y] = \begin{cases} 0 & , y < 0 \\ 1/3 & 0 \le y < 100 \\ 1 & y \geq 100 \end{cases}$$

(b) The CDF $F_Y(y)$ has jumps of $1/3$ at $y=0$ and $2/3$ at $y=100$. The corresponding pdf of Y is

$$f_Y(y) = \delta(y)/3 + 2\delta(y-100)/3$$

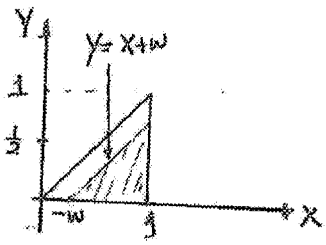
(c) The expected value of Y is

$$E[Y] = \int_{-\infty}^{\infty} y f_Y(y)\,dy = 0 \cdot 1/3 + 100 \cdot 2/3 = 66.6$$

Problem 6.4.5

(a) Since the joint PDF $f_{XY}(x,y)$ is nonzero only for $0 \le y \le x \le$ we observe that $W = Y - X \le 0$ since $Y \le X$. In additio the most negative value of W occurs when $Y = 0$ and $X = 1$ and $W = -1$.
Hence the range of W is $S_W = \{w \mid -1 \le w \le 0\}$

(b) For, $w < -1$, $F_W(w) = 0$.
For $w > 0$, $F_W(w) = 1$
For $-1 \le w \le 0$, the CDF of W is



$$F_W(w) = P[Y - X \le w]$$
$$= \int_{-w}^{1} \int_{0}^{x+w} 6y \, dy \, dx$$
$$= \int_{-w}^{1} 3(x+w)^2 \, dx$$
$$= (x+w)^3 \Big|_{-w}^{1} = (1+w)^3$$

Therefore, the complete CDF of W is

$$F_W(w) = \begin{cases} 0 & , w < -1 \\ (1+w)^3 & , -1 \le w \le 0 \\ 1 & , w > 0 \end{cases}$$

By taking the derivative of $f_W(w)$ with respect to w, we obtain the PDF as follows:

$$f_W(w) = \begin{cases} 3(w+1)^2 & -1 \le w \le 0 \\ 0 & \text{otherwise.} \end{cases}$$