Types of Discrete RVs

Bernoulli (p) $\rightarrow$ Toss a coin once

$H \rightarrow X = 1$

$T \rightarrow X = 0$

PMF

Geometric (p) $\rightarrow$ X = No. of Tosses

$\rightarrow$ keep tossing until we see a H

$p = P[H]$

Binomial (n, p) $\quad n \rightarrow$ No. of Toss

$\rightarrow$ ~~To~~ X = No. of Heads

$$P_X(x) = P[X = x]$$

Pascal (k, p) RV

Same coin :-

Expt: Keep Tossing until we see the k^{th} Head

$k = 1 \longrightarrow$ Geometric X = No. of Tosses

$P_X(x)$ of a Pascal (k, p) RV

$$P_X(x) = P[X = x]$$

$= P[$Getting the k^{th} Head in the x^{th} Toss$]$

x^{th} Toss $\rightarrow$ H

Tosses 1 through $(x-1) \longrightarrow$ Must give me $(k-1)$ heads

$= P[\underbrace{\text{Getting } (k-1) \text{ Heads in Tosses 1 through } (x-1)}_{\text{Binomial } (x-1,\, p)}$ &

$\underbrace{\text{Getting a H in the } x^{th} \text{ Toss}}_{\text{Bernoulli}}]$

Pascal $\rightarrow$ is a combination of a Binomial & a Bernoulli

Recall: If X is Binomial (n, p)

$$P_X(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x = 0, 1, 2, \ldots \\ 0, & \text{o.w.} \end{cases}$$

$X \to$ pascal

$$P_X(x) = \underbrace{\binom{x-1}{k-1} p^{(k-1)} (1-p)^{(x-1)-(k-1)}}_{\text{Binomial}} \times p \qquad ③$$

$n \rightsquigarrow (x-1)$

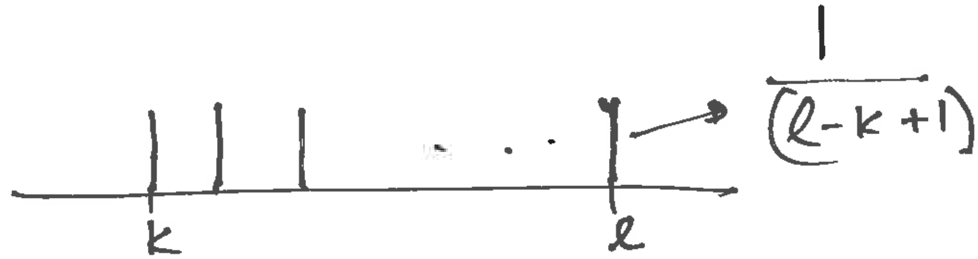
$x \rightsquigarrow (k-1)$

$$= \begin{cases} \binom{x-1}{k-1} (1-p)^{x-k} p^{k}, & x = k, k+1, \cdots \\ 0, & \text{oth.} \end{cases}$$

④ Discrete Uniform (k, l)

PMF is ~~constant~~ the same for every integer from k to l

$P_X(x)$

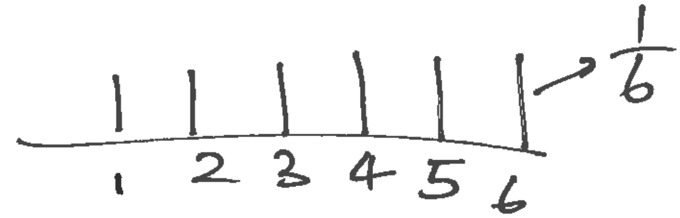


space of X is $\{k, k+1, k+2, \cdots, l\}$

$$P_X(x) = \begin{cases} \dfrac{1}{l-k+1}, & x = k, k+1, \dots, l \\ 0, & \text{otherwise} \end{cases}$$

Ex:- Roll a fair dice

X is discrete Uniform $(1, 6)$



6 Poisson RV (α)

eg:- No. of calls received in an hour

" . . hits on a website.

space → 0 to ∞ (integer)

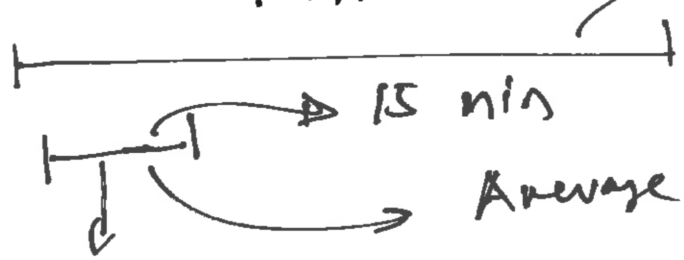
* If X is Poisson (α)

$$P_X(x) = \frac{e^{-\alpha} \cdot \alpha^{x}}{x!}, \quad x = 0, 1, \dots$$

α is the average no. of calls received over the time period

eg:- On the average an office receives 10 calls in an hour. Find the prob. that it receives 2 calls in 15 mins.

1 hr → Avg. = 10 calls



15 min

Average over 15 mins

observation Period

$$= \frac{1\cancel{0}}{6\cancel{0}} \times 15$$

$$= \frac{5}{2} = \alpha$$

$$P_X(x) = \frac{e^{-\alpha} \cdot \alpha^{x}}{x!}, \quad x = 0, 1, \ldots$$

$$P[X=2] = P_X(2) = \frac{e^{-\frac{5}{2}} \cdot \left(\frac{5}{2}\right)^{2}}{2!}, \quad 2! = 2$$

ex:- Same Problem.

Find the Prob. that the company receives more than 2 calls in 15 mins $\rightarrow$ Same α.

$\alpha = \frac{5}{2}$

$$P[X > 2]$$

$$= 1 - P[X = 0] - P[X = 1] - P[X = 2]$$

$$= 1 - e^{-5/2} - \frac{5}{2} e^{-5/2} - \frac{e^{-5/2} \left(\frac{5}{2}\right)^2}{2}$$

~~Prob.~~ Cummulative Distribution Function (CDF)

X is Discrete RV

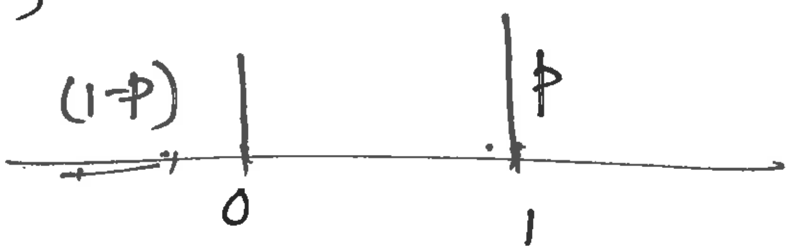
Recall: $P_X(x) = P[X = x]$

Not^n: $F_X(x)$

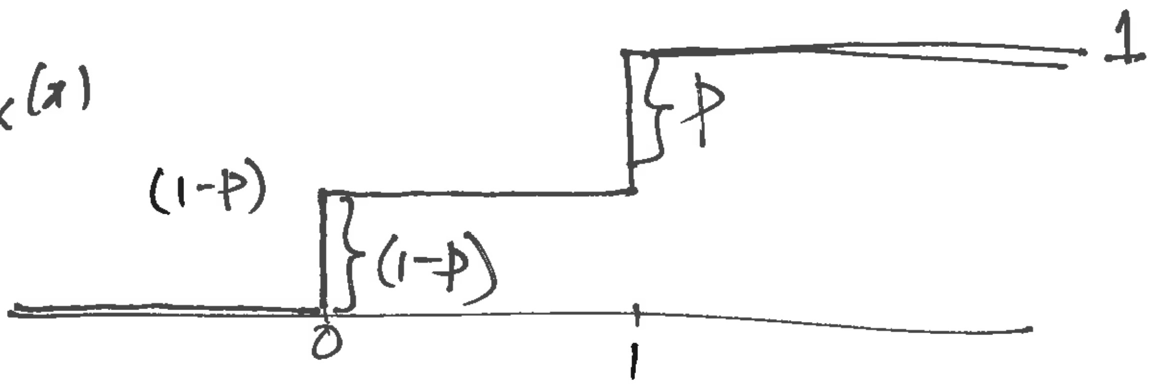
Def^n: $F_X(x) = P[X \leq x]$

ex:- X is Bernoulli (p)

$P_X(x)$



$F_X(x)$



For $x < 0$, $P[X \le x] = F_X(x) =$

For $0 \le x < 1$ $\quad F_X(x) = P[X \le x]$

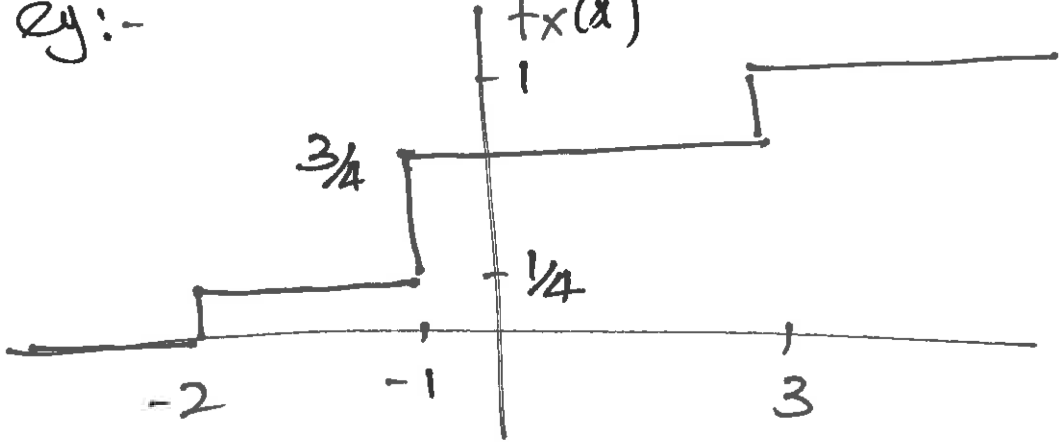
$$= P[X=0] = (1-p)$$

For $x \geq 1$ $\quad F_X(x) = P[X \le x]$

$$= P[X=0] + P[X=1]$$

$$= 1$$

eg:-



Find the PMF & $P[X<0]$, $P[X>-1]$

$$P[X<0] = \tfrac{1}{4} + \tfrac{1}{2} = \tfrac{3}{4}$$

$$P[X>-1] = \tfrac{1}{4}$$

$$P[X \geqslant -1] = \tfrac{3}{4}$$

Properties of $F_X(x)$

1. $F_X(-\infty) = P[\underbrace{X \le -\infty}_{\phi}] = 0.$

$\cancel{P_X(}$ $F_X(+\infty) = P[\underbrace{X \le +\infty}_{S}] = 1$

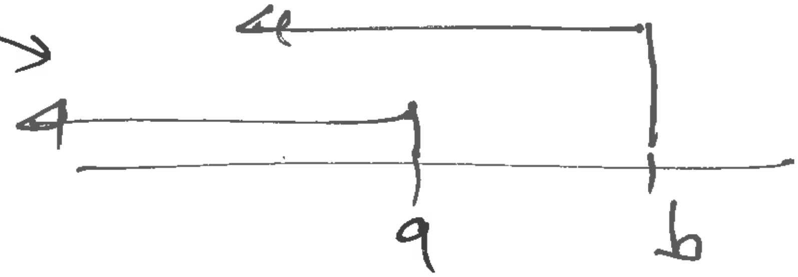
CDF starts from 0 and ends at 1 (as x increases)

2. If $\cancel{a > b}$ $b > a$

$F_X(a)$ $\quad$ $F_X(b) \rightarrow P[X \le b]$

$F_X(b) \geq F_X(a)$

$= P[X \le a]$



$P[X \le b] = P[X \le a] + P[a < X \le b]$

$\therefore\ P[a < X \le b] = P[X \le b] - P[X \le a]$

$$= \underbrace{F_X(b) - F_X(a)}_{\geq 0}$$

$\therefore\ F_X(b) \geq F_X(a)$

$\therefore\ F_X(x)$ is a Non-decreasing func^n

Exam I → Next Monday (6/15)
1:20 mins.

No calculators
No need to simply
Cannot leave answer ~~st~~ in Combinations, Permutations & factorials
$\binom{20}{2}$
Show your work

$P[A|B] = \frac{P[AB]}{P[B]}$

EXAM I : $\rightarrow$

2nd Edition: Chapter 1
+ 2.1 $\rightarrow$ 2.4

3rd Edition: Chapters 1 & 2
+ 3.1 $\rightarrow$ 3.4

3.5 Averages

$\hookrightarrow$ Read section 3.5 (2.5 Edition 2)

X is a Discrete RV

Expt. $\longrightarrow$ value ~~of~~ for X

Space of X is $\{x_1, x_2 \ldots x_n\}$

Variation

$F_X(x) \rightarrow$ Stair - Case

$P_X(x)$

Experiment No	$Y \rightarrow$ RV
1 $\longrightarrow$	y_1
	y_2
2	
$\vdots$	
N	y_N
$\vdots$	

Average value of $Y = \dfrac{y_1 + y_2 \cdots + y_N}{N}$

$$\lim_{N \to \infty} \left[\dfrac{y_1 + y_2 \cdots + y_N}{N} \right] \longrightarrow \mu_Y$$

$\mu_Y \downarrow$ Mean value of Y

Mean of Y is also called the expected value of Y

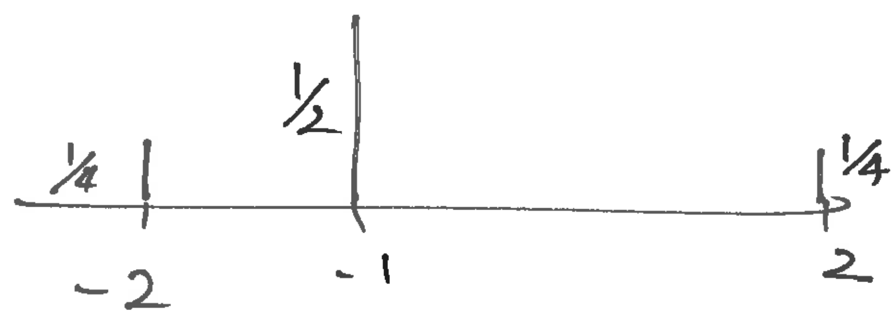
$$\mu_Y = E[Y]$$

If Y is discrete & the PMF of Y is $P_Y(y)$

then $$\mu_Y = \sum_{y} y \cdot P_Y(y)$$

$y \to$ sample space.

eg:- $P_Y(y)$



$$\mu_Y = \sum y \, P_Y(y)$$

$$= (-2)\tfrac{1}{4} + (-1)\tfrac{1}{2} + (2)(\tfrac{1}{4})$$