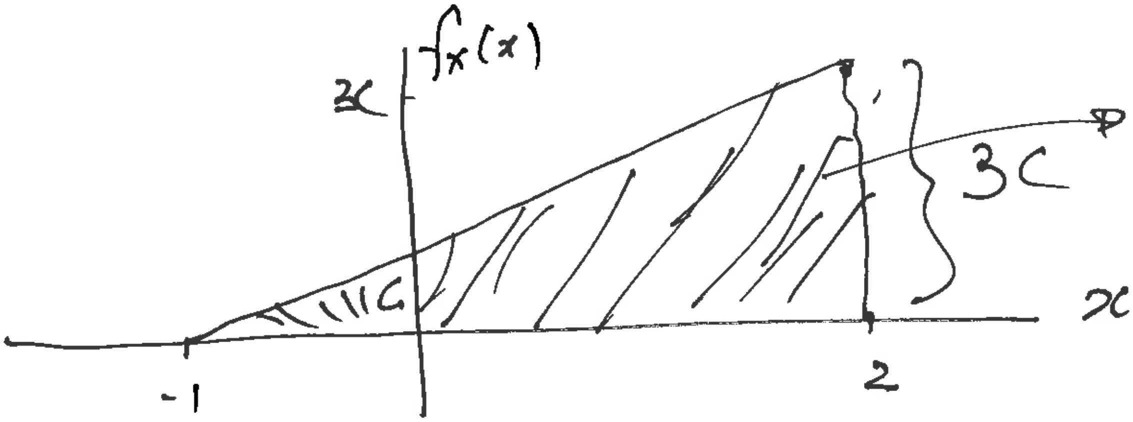
$$f_X(x) = \frac{d}{dx} F_X(x)$$

$$F_X(x) = \int_{-\infty}^{x} f_X(u)\, du$$

eg:- $$f_X(x) = \begin{cases} c(x+1), & -1 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

Find c, $P[X>0]$



$\frac{1}{2}[c+3c]2$

$$\int_{-\infty}^{\infty} f_X(x)\, dx = 1 \longrightarrow \text{Area} = 1$$

$$\frac{1}{2}(3)(3c) = 1$$

$$c = \frac{2}{9}$$

$$P[X>0] = 1 - P[X<0]$$
$$= 1 - \frac{1}{2}(1)(c)$$

4.4 (3.3) Expected value of $g(X)$

Expt. No	X	$g(X)$
1	x_1	$g(x_1)$
⋮		
N	x_N	$g(x_N)$

$$\lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} g(x_i)$$

$$E[g(X)] = \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} g(x_i)$$

Recall $E[g(x)] = \sum g(x) P_X(x)$ $\rightarrow$ when X is discrete ③

When X is continuous

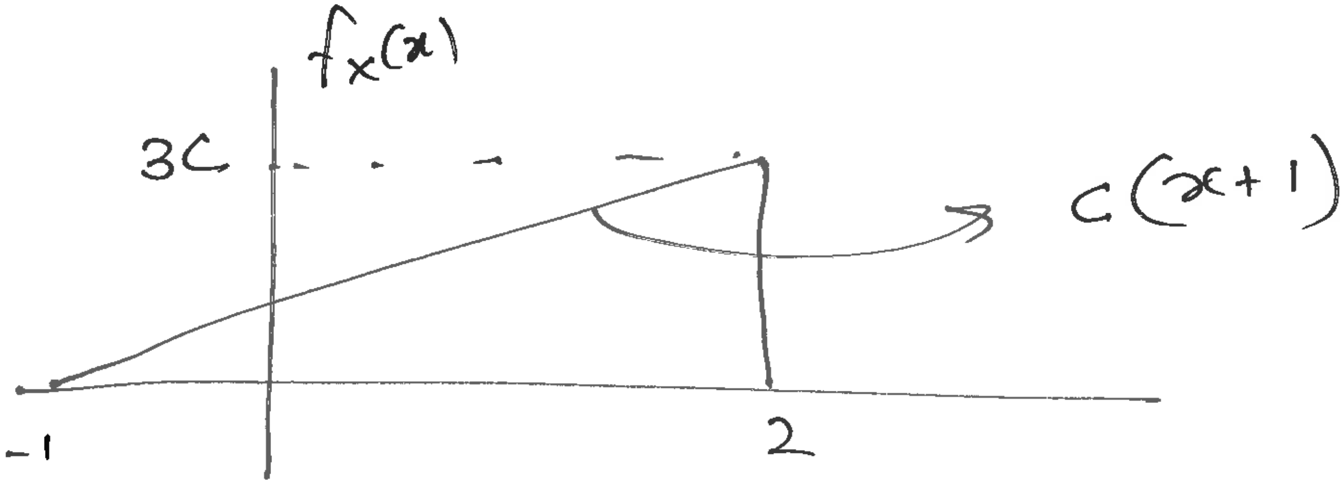
$$E[g(x)] = \int_{-\infty}^{\infty} \cancel{f_X(x)}\, g(x)\, f_X(x)\, dx$$

$$\mu_X = E[X] = \int_{-\infty}^{\infty} x f_X(x)\, dx$$

$$Var[X] = E[X^2] - \mu_X^2$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx$$

Ex:- Go back to the previous Exa



Find mean & variance of X

$$\mu_x = \int_{-1}^{2} x \cdot c(x+1)\,dx =$$

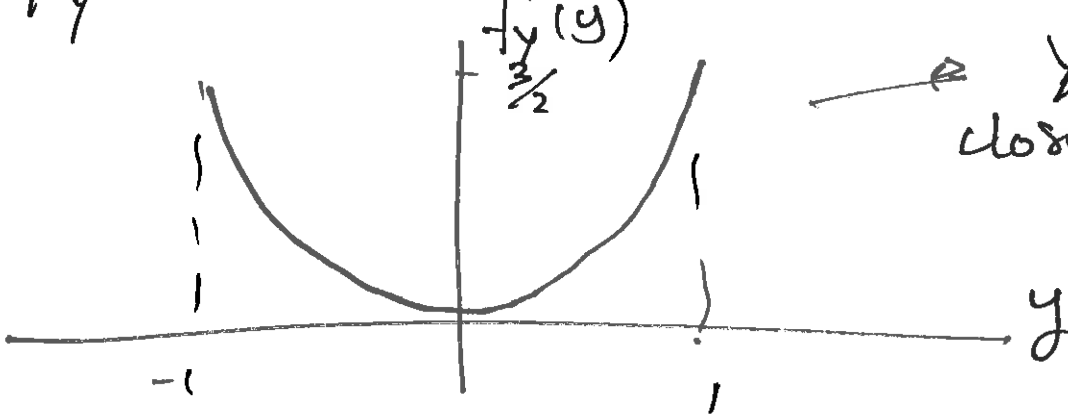
$$E[X^2] = \int_{-1}^{2} x^2 \cdot c(x+1)\,dx =$$

$$Var[x] = E[x^2] - \mu_x^2$$

Φ 4·4 (Φ 3·3)

$$f_Y(y) = \begin{cases} \frac{3y^2}{2}, & -1 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

μ_Y & $Var[Y]$?



Y values closer to +1 & -1 are more likely

$f_Y(y)$ is symmetric around $y = 0$

↓

μ_Y should be zero

as expected

$$\mu_Y = \int_{-1}^{1} \underbrace{y \cdot \frac{3y^2}{2}}_{\text{odd}} \, dy = 0$$

$$Var[Y] = E[Y^2] - \cancel{\mu_Y^2} \to 0$$

$$E[Y^2] = \int_{-1}^{1} y^2 \cdot \frac{3y^2}{2} \, dy \longrightarrow = \frac{3(2)}{2} \int_{0}^{1} y^4 \, dy$$

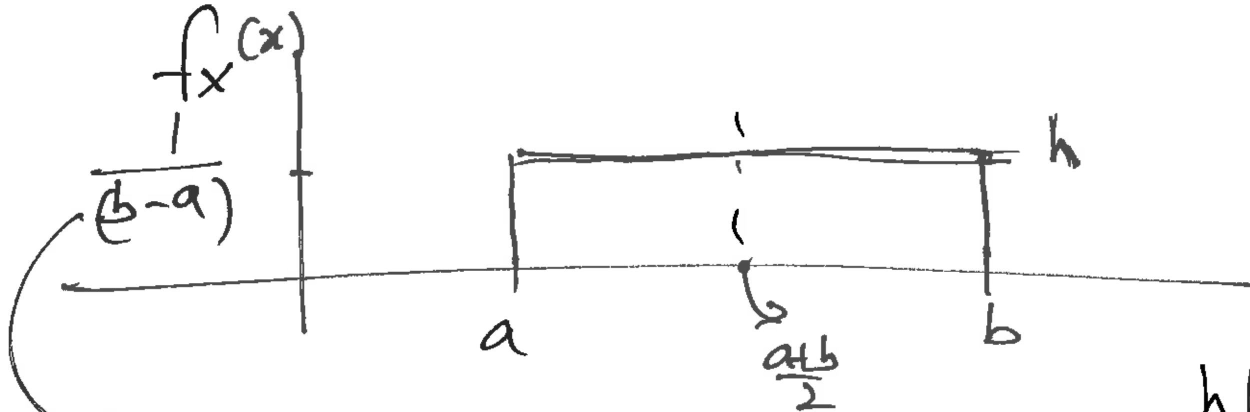
$$= 3 \cdot \frac{y^5}{5}\Big|_0^1 = \frac{3}{5}$$

4.5 (3.4) Families of continuous RVs

1. ~~E~~ Continuous Uniform (a, b)

$f_X(x)$ is flat from a to b

$b > a$



Total Area = 1

$h(b-a) = 1$

$h = \frac{1}{(b-a)}$

$$\mu_X = \frac{a+b}{2} \longrightarrow \int_a^b x \cdot \frac{1}{b-a}\, dx$$

It can be shown

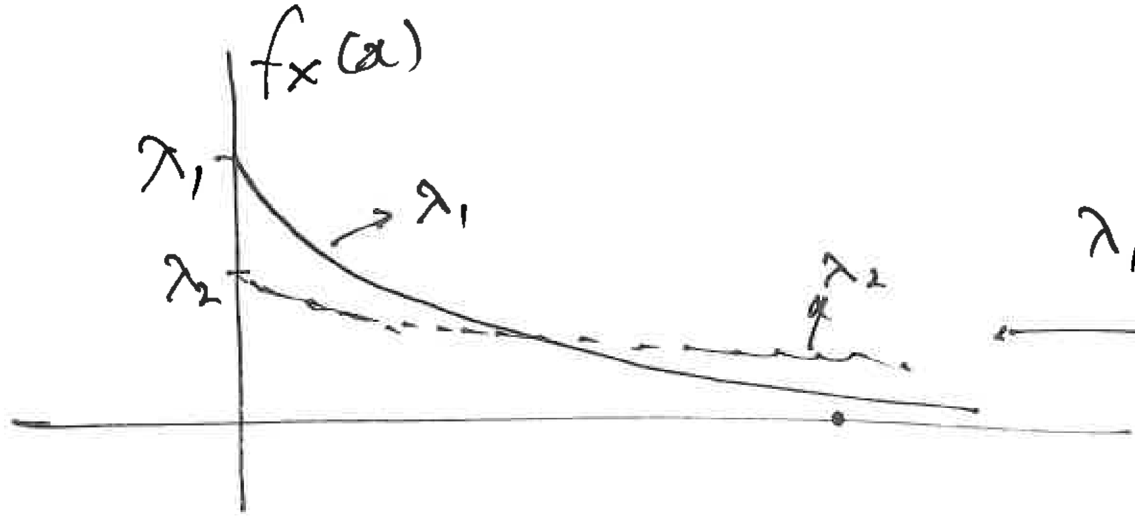
$$\text{Var}[X] = \frac{(b-a)^2}{12} \longrightarrow \text{Prove at home}$$

2. Exponential RV $\rightarrow$ One parameter $\lambda > 0$

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\int_0^{\infty} \lambda e^{-\lambda x}\, dx = \lambda \cdot \frac{e^{-\lambda x}}{-\lambda}\Big|_0^{\infty}$$

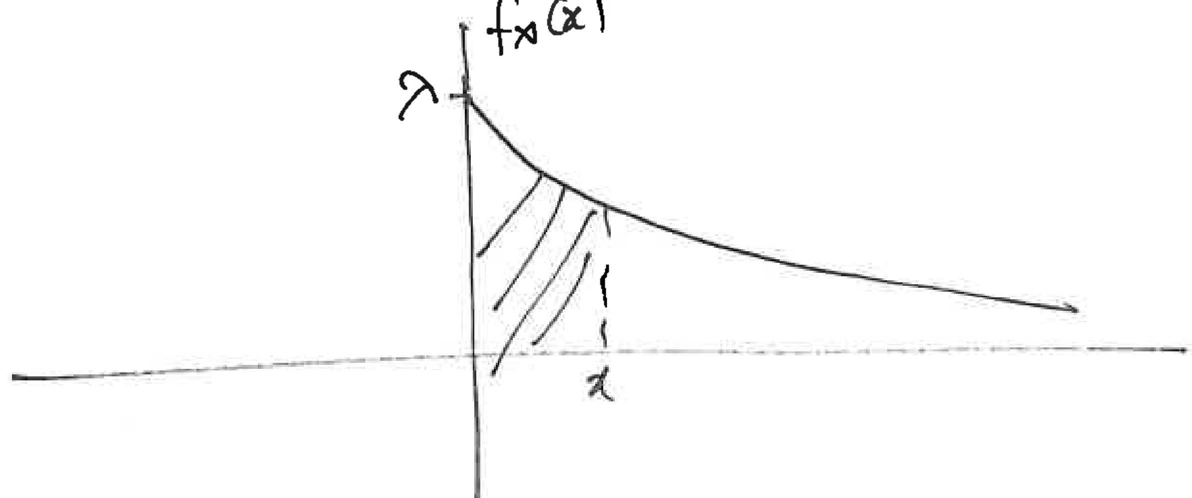
$= 1 \rightarrow$ Total Area under the pdf.

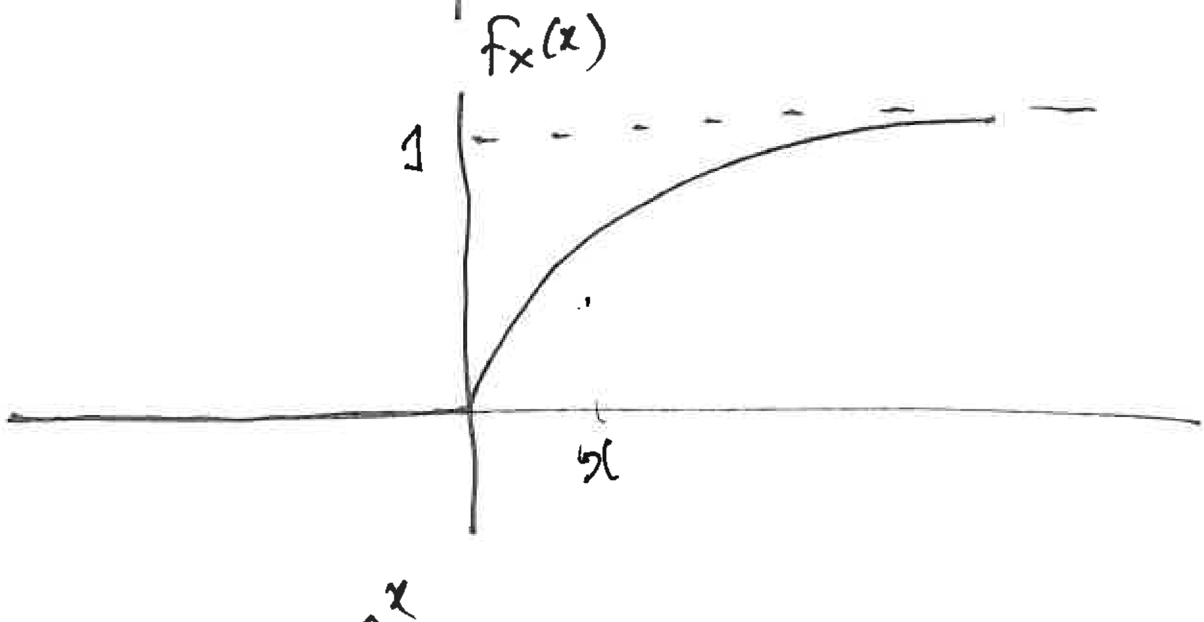


λ_1, λ_2

$\lambda_1 > \lambda_2$

$F_X(x) = ?$





$$F_X(x) = \int_{-\infty}^{x} f_X(u)\,du$$

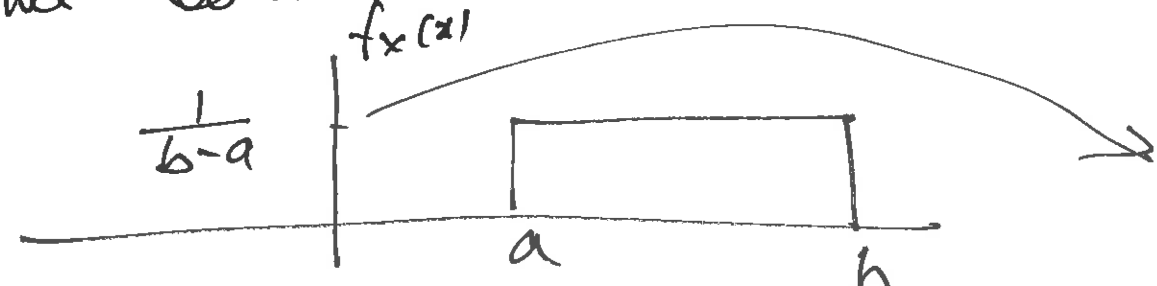
For $x>0$

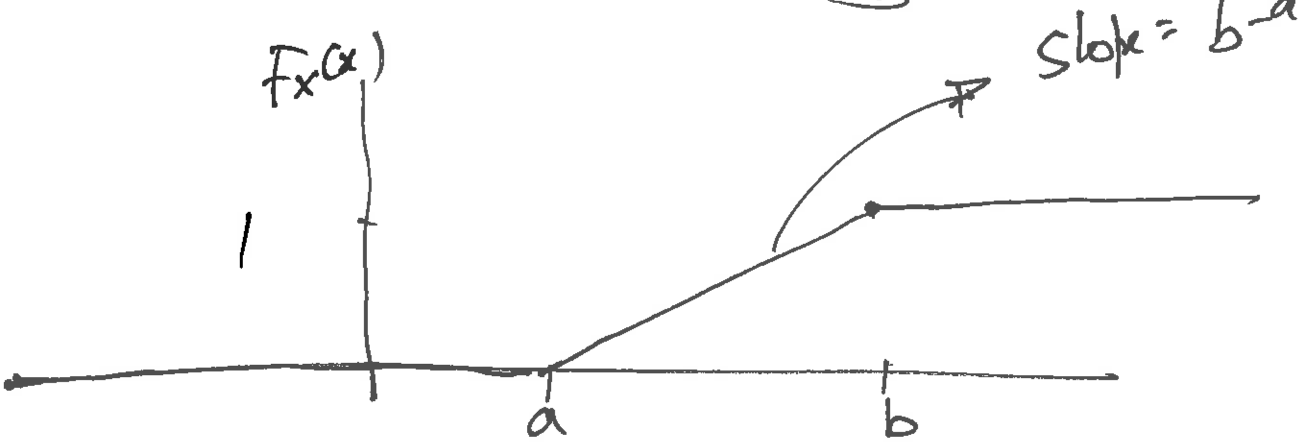
$$F_X(x) = \int_0^x \lambda e^{-\lambda u}\, du$$

$$= \lambda \left. \frac{e^{-\lambda u}}{-\lambda} \right|_0^x$$

$$F_X(x) = \begin{cases} [1-e^{-\lambda x}], & x \geq 0 \\ 0, & \text{other.} \end{cases}$$

In the continuous uniform ca





HW: Show that if X is exponential (λ) (10)

$$\mu_x = \frac{1}{\lambda}$$

$$Var[x] = \frac{1}{\lambda^2}$$

eg:- X is exponential

$$P[X > 2] = e^{-4}$$

Find the mean & variance of X

$$= 1 - P[x \leq 2]$$

$$= 1 - F_X(2)$$

$$= 1 - \left[1 - e^{-\lambda(2)}\right] = e^{-4}$$

$$e^{-2\lambda} = e^{-4} \rightarrow \lambda = 2$$

$$\therefore \mu_x, \ Var[x] = \checkmark$$

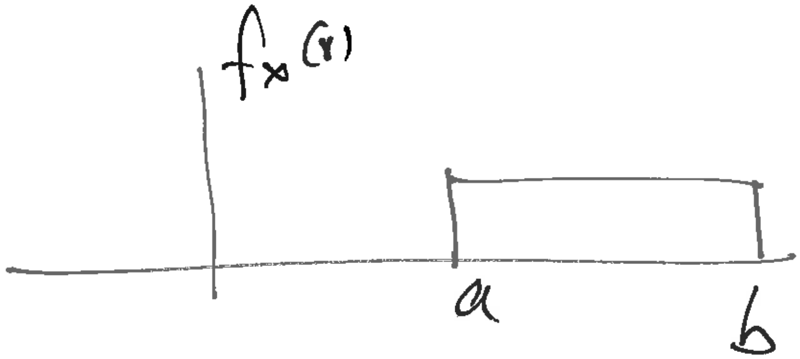
$\mu_X = 3$, $var[X] = 9$

X is exponential $f_X(x)$?

$\frac{1}{\lambda} = 3 \rightarrow \lambda = 1/3$

$\therefore f_X(x) = \begin{cases} \frac{1}{3} e^{-\frac{x}{3}}, & x \geq 0 \\ 0, & o/w \end{cases}$

When X is continuous uniform



Need a & b

$b > a$

$\mu_X = \frac{a+b}{2} = 3$, $var[X] = \frac{(b-a)^2}{12} = 9$

$(b-a)^2 = 12 \times 9$

$\therefore (b-a) = \sqrt{12 \times 9}$

$a + b = 6$

$b - a = \sqrt{12 \times 9}$

[1] A manufacturing company manufactures 40% of its products by process A, the rest by process B. It is known that 2% of products manufactured by process A are defective. It is also known that among all products manufactured by the company, 3% are defective.

Find

(a) [13 pts] the probability of finding a defective product among those manufactured by process B,

(b) [12 pts] the probability of finding a product that is either defective or manufactured by process B among all products manufactured by the company.

$P[A] = 0.4, \quad P[B] = 0.6$ D - Defective

$P[D|A] = 0.02$

$P[D] = 0.3$

$P[D] = P[D/A]\,P[A] + P[D/B]\,P[B]$

$0.3 = 0.02 \times 0.4 + P[D|B] \times 0.6$

(a) $P[D|B] = \dfrac{0.3 - 0.02 \times 0.4}{0.6}$

(b) $P[D+B] = P[D] + P[B] - P[DB]$

$= 0.3 + 0.6 - P[D|B]\,P[B]$

$P[D+B] = 0.9 - \left[\dfrac{0.3 - 0.02 \times 0.4}{0.6}\right] \times 0.6$

b

[2] [25 pts] A basketball team has three designated centers, four designated forwards, four designated guards, and a swingman who can play either as a forward or as a guard. The coach wants to select a starting a lineup with a center, two forwards and two guards. If all valid lineups are equally likely, find the probability that the swingman gets selected as a guard in the starting lineup.

Swingman Not playing

$$N_1 = \binom{3}{1}\binom{4}{2}\binom{4}{2} = 3 \times \frac{4\times3}{1\times2} \times \frac{4\times3}{1\times2}$$
$$= 3\times6\times6$$

Swingman as a Guard

$$N_2 = \binom{3}{1}\binom{4}{1}\binom{4}{2} = 3\times4\times\frac{4\times3}{1\times2}$$
$$= 9\times8 = 72$$

Swingman as a Forward

$$N_3 = \binom{3}{1}\binom{4}{2}\binom{4}{1} = 3\times\frac{4\times3}{1\times2}\times4$$
$$= 72$$

$$P[\text{swingman playing as a Guard}] = \frac{N_2}{N_1+N_2+N_3}$$

$$= \frac{72}{3\times36+72+72}$$

[3] [25 pts] It is known that 10% of a population carries a virus V. A test T is developed to check if a person carries the virus V or not. When tested on a person who actually carries the virus V, 98% of the time the test T correctly shows a positive result. Similarly, when tested on person who does not carry the virus V, 4% of the time it incorrectly shows a positive result. If a randomly selected person from the population showed a positive result from test T, find the probability he/she actually carries the virus V.

$P[v]=0.1$, $P[\bar{v}]=0.9$

P - Positive

N - Negative

$P[P/v]=0.98$, $P[N|v]=0.02$

$P[P|\bar{v}]=0.04$, $P[N|\bar{v}]=0.96$

$$P[v|P]=\frac{P[P|v]\,P[v]}{P[P]}$$

$$P[P]=P[P|v]\,P[v]+P[P|\bar{v}]\,P[\bar{v}]$$

$$=0.98\times 0.1+0.04\times 0.9$$

$$=$$

$$P[v/P]=\frac{0.98\times 0.1}{0.98\times 0.1+0.04\times 0.9}$$

[4] **(Parts (a) and (b) are separate)**

(a) [13 pts] On the average an office receives 10 calls per hour. If the number of calls received by the office is modeled by a Poisson random variable, find the probability that the office receives more than 2 calls in 20 minutes.

(b) [12 pts] A manufacturing company manufactures 100 products a day. It is known that 5% of the products manufactured by the company are defective. Find the probability that, on a given day, the company manufactures 3 defective products in the first 50 products manufactured **and** 2 defective products in the second 50 products manufactured.

(a) Average no. of calls in 20 mins $= \frac{10}{60} \times 20$

$$= \frac{10}{3} = \lambda$$

$$P[x > 2] = 1 - P[x=0] - P[x=1] - P[x=2]$$

$$= 1 - \frac{e^{-\lambda} \cdot \lambda^0}{0!} - \frac{e^{-\lambda} \cdot \lambda^1}{1!} - \frac{e^{-\lambda} \cdot \lambda^2}{2!}$$

$$= 1 - e^{-\frac{10}{3}}\left[1 + \left(\frac{10}{3}\right) + \frac{\left(\frac{10}{3}\right)^2}{2}\right]$$

(b.) $\underbrace{\text{50 products}}_{\text{3 defective}}$ $\underbrace{\text{50 products}}_{\text{2 defective}}$

$$P[\ \ ] = \binom{50}{3}(0.05)^3(0.95)^{47}\binom{50}{2}(0.05)^2(0.95)^{48}$$

$$= \frac{50\times49\times48}{1\times2\times3} \times \frac{50\times49}{1\times2} \times (0.05)^5 (0.95)^{(48+47)}$$