Func^n of 2 RVs → Continuous

$X, Y \rightarrow$ 2 Continuous RVs

$f_{X,Y}(x, y) \rightarrow$ given

$W = g(X, Y) \longrightarrow$ given

pdf of $W \longrightarrow$?

$$F_W(w) \Rightarrow P[W \le w]$$

$$= P[g(X,Y) \le w]$$

$$= P[X, Y \in R_w]$$

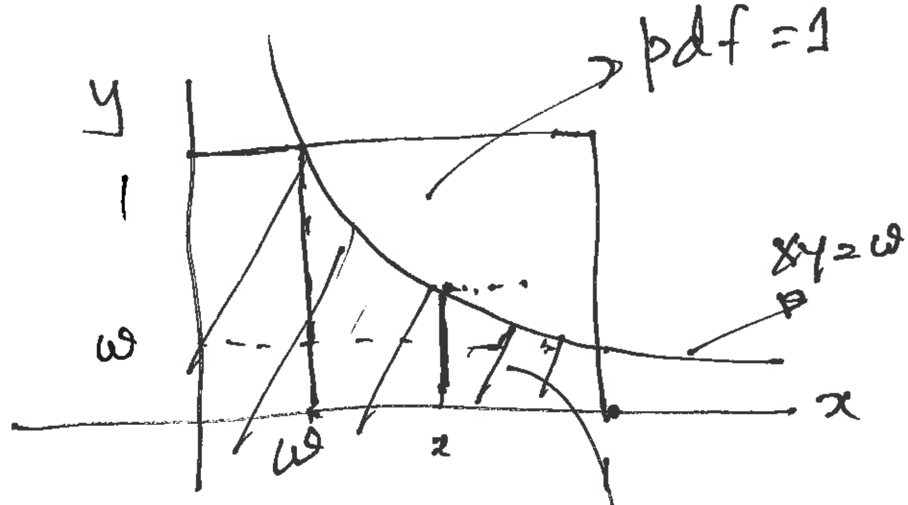
$$= \underbrace{\iint_R f_{X,Y}(x, y)\, dy\, dx}_{\text{func}^n \text{ of } w}$$

$$f_W(w) = \frac{d}{dw} F_W(w)$$

Q 6.4 (Q 4.6)

$$f_{X,Y}(x,y) = \begin{cases} 1, & 0 \le x \le 1 \text{ \& } 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

$W = XY$



W varies from $0 \to 1$

$F_W(\omega) = P[W \le \omega]$

0 to 1

$= P[\cancel{xy}\, XY \le \omega]$

$= \iint_R f_{X,Y}(x,y)\, dy\, dx$

$= \underbrace{\overbrace{1 \times \omega}^{\text{Rectangle}}}_{\text{Area}} \times \underbrace{1}_{\text{pdf}} + \int_{\omega}^{1} \int_{0}^{\omega/x} 1 \, dy\, dx$

5.7 (4.7) Expected value

$E[g(X,Y)]$

Recall:

$$E[g(X)] = \sum_x g(x) \cdot P_X(x)$$

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) \cdot f_X(x)\, dx$$

Discrete case

$$E[g(X,Y)] = \sum_x \sum_y g(x,y)\, P_{X,Y}(x,y)$$

Continuous case

$$E[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\, f_{X,Y}(x,y)\, dy\, dx$$

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$$\mu_X = E[X] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\, f_{X,Y}(x,y)\,dy\,dx$$

$$= \int_{-\infty}^{\infty} x \underbrace{\int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy}_{f_X(x)} \,dx$$

$f_X(x) \rightarrow$ Marginal pdf of X

$$= \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx$$

$Var[X]$, $Var[Y]$, $\mu_Y \longrightarrow$ ✓

* $E[XY] \rightarrow r_{XY}$ is called the correlation of X & Y

* $Cov[X,Y] = E[XY] - \mu_X \mu_Y$ (5)

Covariance of X & Y $= \gamma_{XY} - \mu_X \mu_X$

$E[(X-\mu_X)(Y-\mu_Y)]$

* $$\rho_{X,Y} = \frac{Cov[X,Y]}{\sigma_X \sigma_Y}$$

Correlation Coefficient

Def^{ns}: X & Y are uncorrelated

if $Cov[X,Y] = 0 \rightarrow \rho_{X,Y} = 0$

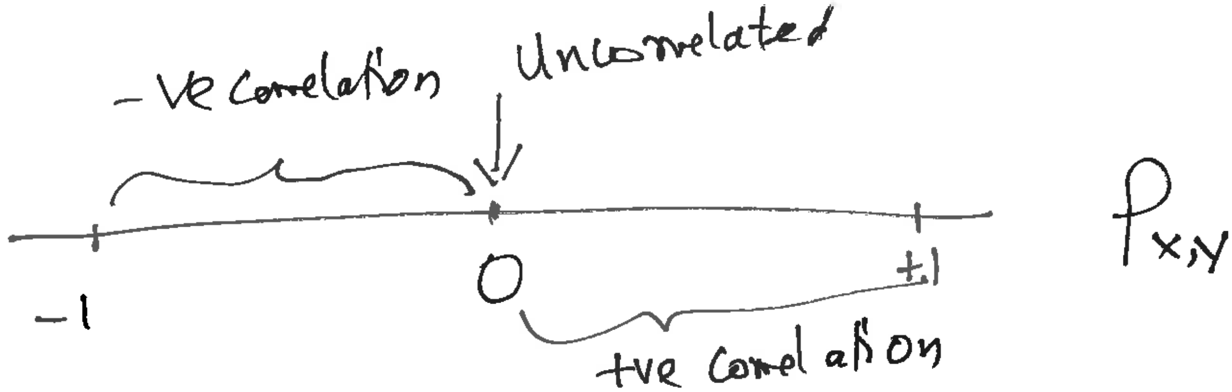
If $\rho_{X,Y} > 0 \rightarrow$ +ve correlation betn X & Y

when X is higher, Y is likely to be higher too

if $\rho_{X,Y} < 0 \rightarrow$ -ve correlation

when X is higher, Y is likely to be lower

*



It can be shown $-1 \leq \rho_{x,y} \leq +1$