Name: **Quiz #4** **10/21/15**

[1] X is a Gaussian random variable with mean 2. If $E[2X^2-3X]=10$, find $P[|X-2|>3]$

$$\mu = 2, \quad E[2x^2-3x] = 2E[x^2] - 3\mu = 10$$

$$2E[x^2] = 10 + 3\mu = 16$$

$$E[x^2] = 8$$

$$Var[x] = 8 - \mu^2 = 4$$

$$\sigma_x = 2$$

$\therefore$ x is $N(2,2)$

$$P[|x-2|>3] = P[x-2>3 \text{ or } x-2<-3]$$

$$= P[x>5] + P[x<-1]$$

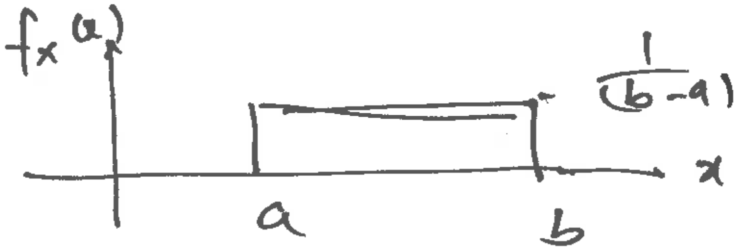
$$P[x>5] = P\left[z > \frac{5-2}{2}\right] = P\left[z > \frac{3}{2}\right] = \Phi\left(\frac{3}{2}\right)$$

$$P[x<-1] = P\left[z < \frac{-1-2}{2}\right] = P\left[z < -\frac{3}{2}\right]$$

$$= \Phi\left(\frac{3}{2}\right)$$

$$\therefore P[|x-2|>3] = 2\,\Phi(3/2)$$

[2] If X is continuous uniform (a,b), prove that Var[X]=(b-a)^2/12.



$$\mu_x = \int_{-\infty}^{\infty} x\, f_x(x)\, dx$$

$$\mu_x = \int_a^b \frac{x}{(b-a)}\, dx = \frac{1}{(b-a)} \left.\frac{x^2}{2}\right|_a^b$$

$$= \frac{1}{2(b-a)} (b^2-a^2) = \frac{a+b}{2}$$

$$E[x^2] = \int_a^b \frac{x^2}{(b-a)}\, dx = \frac{1}{3(b-a)} (b^3-a^3)$$

$$= \frac{1}{3(b-a)} (b-a)(b^2+ab+a^2)$$

$$= \frac{a^2+ab+b^2}{3}$$

$$Var[x] = E[x^2] - \mu_x^2$$

$$= \frac{a^2+ab+b^2}{3} - \frac{(a+b)^2}{4}$$

$$= \frac{4(a^2+ab+b^2) - 3(a^2+2ab+b^2)}{12}$$

$$= \frac{b^2-2ab+a^2}{12} = \frac{(b-a)^2}{12}$$