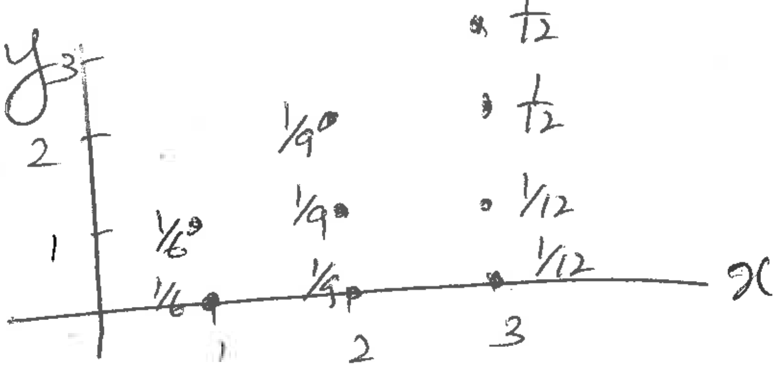
Name: **Quiz #6** **11/18/15**
UTD ID:

[1] The random variable X discrete uniform from 1 to 3. For any given X=x, Y is discrete uniform from 0 to x. Find the covariance of X and Y.



$$E[XY] = \sum_x \sum_y xy\, P_{X,Y}(x,y)$$

$$= (1)(1)\tfrac{1}{6} + (2)(1)\tfrac{1}{9} + (2)(2)\tfrac{1}{9} + (3)(1)(\tfrac{1}{12}) + (3)(2)(\tfrac{1}{12}) + (3)(3)(\tfrac{1}{12}) = a$$

$$\mu_X = 2$$

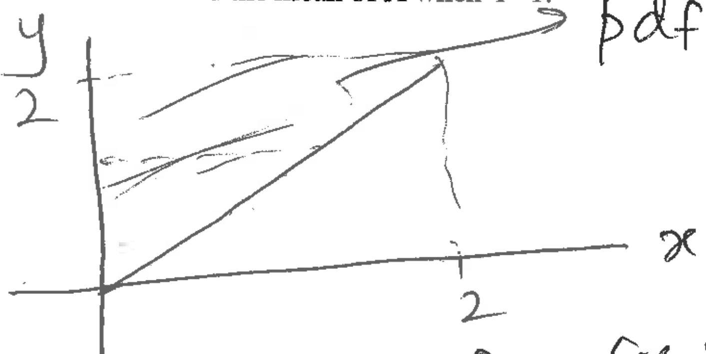
$$\mu_Y = 0 + (1)\left(\tfrac{1}{6} + \tfrac{1}{9} + \tfrac{1}{12}\right) + (2)\left(\tfrac{1}{9} + \tfrac{1}{12}\right) + (3)\left(\tfrac{1}{12}\right) = b$$

$$\mathrm{Cov}[X,Y] = a - 2b$$

[2] The joint probability density function of two continuous random variables X and Y is

$$f_{X,Y}(x,y) = \begin{cases} c, & 0 \le x \le y \le 2 \\ 0; & \text{otherwise} \end{cases}$$

Find the value of c and the mean of X when Y=1.



$pdf = c$

$$\frac{1}{2}(2)(2)c = 1$$
$$c = 1/2$$

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

$$f_Y(y) = \int_0^y c\,dx = \begin{cases} cy, & 0 \le y \le 2 \\ 0, & o/w \end{cases}$$

$$f_{X|Y}(x|1) = \begin{cases} \frac{c}{c(1)}, & 0 \le x \le 1 \\ 0, & o/w \end{cases}$$

$\downarrow$

Continuous uniform (0,1)

$$\mu_{X|Y=1} = 1/2$$