A Predictive Prescription Framework for Unit Commitment Using Boosting Ensemble Learning Algorithms

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Appendix A
Notation and the Mathematical Formulation of the Conditional Stochastic Unit Commitment (CSUC) Problem

In this appendix, we provide the detailed notation and the formulation of the CSUC problem employed in the publication. We begin by providing the nomenclature.

Nomenclature

Sets,Indices

$\mathscr{H}, h$	set, index of hourly periods of a day
$\mathscr{N}, n$	set, index of nodes
$\mathscr{L}, \ell$	set, index of lines
$\mathscr{G}, g$	set, index of dispatchable generators (DGs)
$\mathscr{S}_g, s$	set, index of piecewise cost intervals for DG g

Variables

$u_g[h]$	$\in \{0,1\}$, commitment status of DG g in hour h
$v_g[h]$	$\in \{0,1\}$, startup status of DG g in hour h
$w_g[h]$	$\in \{0,1\}$, shutdown status of DG g in hour h
$p_g[h]$	power generated above minimum by DG g in hour h (MW)
$p_g^s[h]$	power from segment s for DG g in hour h (MW)
$p_c^n[h]$	curtailed load at node n in hour h (MW)

Parameters

Y/y	random net load/its observation
X/x	random auxiliary information/its observation
p_g°	initial power generated above minimum by DG g (MW)
u_g°	$\in \{0,1\}$, initial commitment status of DG g
$T_g^\uparrow (T_g^\downarrow)$	minimum uptime (downtime) of DG g (h)
$T_g^{\uparrow,\circ} (T_g^{\downarrow,\circ})$	number of hours DG g has been online (offline) before the scheduling horizon (h)
$\bar{P}_g^s$	maximum power available from piecewise segment s for DG g (MW)
$\bar{P}_g (\underline{P}_g)$	maximum (minimum) power output of DG g (MW)
α_g^s	cost coefficient for piecewise segment s for DG g (\$/MWh)
α^c	penalty cost for load curtailment (\$/MWh)
α_g^v	startup cost of DG g (\$)
α_g^u	cost of DG g running and operating at $\underline{P}_g$ (\$/h)
$\Delta_g^\uparrow (\Delta_g^\downarrow)$	ramp-up (ramp-down) rate limit of DG g (MW/h)
$\Delta_g^{\uparrow,\circ} (\Delta_g^{\downarrow,\circ})$	startup (shutdown) rate limit of DG g (MW)
$\bar{f}_\ell (\underline{f}_\ell)$	maximum (minimum) real power flow allowed on line ℓ (MW)
Ψ_ℓ^n	injection shift factor of line ℓ with respect to node n

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The first-stage of the CSUC problem is expressed as

$$\underset{\substack{u_g[h], v_g[h],\\ w_g[h]}}{\text{minimize}} \quad \sum_{h \in \mathscr{H}} \sum_{g \in \mathscr{G}} \Big[\alpha_g^u u_g[h] + \alpha_g^v v_g[h] \Big] + \mathbb{E}\big[\mathcal{Q}(z, Y) | X = x \big], \tag{1}$$

subject to

$$u_g[h] - u_g[h-1] = v_g[h] - w_g[h] \quad \forall h \in \mathscr{H} \setminus \{1\}, \tag{2}$$

$$u_g[h] - u_g^\circ = v_g[h] - w_g[h] \quad \forall h \in \{1\}, \tag{3}$$

$$\sum_{h'=h-T_g^\uparrow+1}^{h} v_g[h'] \leq u_g[h] \quad \forall h \in \{T_g^\uparrow, \ldots, 24\}, \tag{4}$$

$$\sum_{h'=h-T_g^\downarrow+1}^{h} w_g[h'] \leq 1 - u_g[h] \quad \forall h \in \{T_g^\downarrow, \ldots, 24\}, \tag{5}$$

$$\sum_{h'=1}^{\min\{u_g^\circ \cdot (T_g^\uparrow - T_g^{\uparrow,\circ}),\, 24\}} w_g[h'] = 0, \tag{6}$$

$$\sum_{h'=1}^{\min\{(1-u_g^\circ) \cdot (T_g^\downarrow - T_g^{\downarrow,\circ}),\, 24\}} v_g[h'] = 0, \tag{7}$$

$$u_g[h], v_g[h], w_g[h] \in \{0,1\} \quad \forall h \in \mathscr{H}, \tag{8}$$

where (2)–(8) hold for all DGs $g \in \mathscr{G}$. The objective (1) of the first stage is to minimize the commitment and startup costs plus the expected dispatch and load curtailment costs. We enforce by (2, 3) the logical constraints that relate the variables $u[h]$, $v[h]$, and $w[h]$, (4, 6) the minimum uptime, and (5, 7) the minimum downtime constraints. For a specific vector of first-stage decision variables z and an observation on net load $Y = \bar{y}$, the value function $\mathcal{Q}(z, \bar{y})$ is computed by solving the following second-stage problem:

$$\underset{\substack{p_g[h], p_g^s[h],\\ p_c^n[h]}}{\text{minimize}} \quad \sum_{h \in \mathscr{H}} \left[\sum_{g \in \mathscr{G}} \sum_{s \in \mathscr{S}_g} \alpha_g^s p_g^s[h] + \sum_{n \in \mathscr{N}} \alpha_c p_c^n[h] \right], \tag{9}$$

subject to

$$p_g[h] \leq (\bar{P}_g - \underline{P}_g) u_g[h] \quad \forall h \in \mathscr{H}, \tag{10}$$

$$p_g[h] \leq p_g^\circ + \Delta_g^\uparrow u_g^\circ + (\Delta_g^{\uparrow,\circ} - \underline{P}_g) v_g[h] \quad \forall h \in \{1\}, \tag{11}$$

$$p_g[h] \geq p_g^\circ - \Delta_g^\downarrow u_g^\circ - (\Delta_g^{\uparrow,\circ} - \Delta_g^\uparrow - \underline{P}_g) w_g[h] \quad \forall h \in \{1\}, \tag{12}$$

$$p_g[h] \leq p_g[h-1] + \Delta_g^{\uparrow} u_g[h-1] \quad \quad$$
$$+ (\Delta_g^{\uparrow,\circ} - \underline{P}_g) v_g[h] \qquad \forall h \in \mathscr{H} \setminus \{1\}, \tag{13}$$
$$p_g[h] \geq p_g[h-1] - \Delta_g^{\downarrow} u_g[h-1]$$
$$- (\Delta_g^{\uparrow,\circ} - \Delta_g^{\uparrow} - \underline{P}_g) w_g[h] \qquad \forall h \in \mathscr{H} \setminus \{1\}, \tag{14}$$
$$p_g[h] = \sum_{s \in \mathscr{S}_g} p_g^s[h] \qquad \forall h \in \mathscr{H}, \tag{15}$$
$$p_g^s[h] \leq \bar{P}_g^s - \bar{P}_g^{s-1} \qquad \forall s \in \mathscr{S}_g, \forall h \in \mathscr{H}, \tag{16}$$
$$p_g[h] \in \mathbb{R}_+ \qquad \forall h \in \mathscr{H}, \tag{17}$$
$$p_g^s[h] \in \mathbb{R}_+ \qquad \forall s \in \mathscr{S}_g, \forall h \in \mathscr{H}, \tag{18}$$
$$p^n[h] = \sum_{g \in \mathscr{G}^n} p_g[h] + p_c^n[h] - \bar{y}_d^n[h] \quad \forall n \in \mathscr{N}, \forall h \in \mathscr{H}, \tag{19}$$
$$\sum_{n \in \mathscr{N}} p^n[h] = 0 \qquad \forall h \in \mathscr{H}, \tag{20}$$
$$\underline{f}_\ell \leq \sum_{n \in \mathscr{N}} \Psi_\ell^n p^n[h] \leq \bar{f}_\ell \qquad \forall \ell \in \mathscr{L}, \forall h \in \mathscr{H}, \tag{21}$$
$$p_c^n[h] \in \mathbb{R}_+ \qquad \forall n \in \mathscr{N}, \forall h \in \mathscr{H}, \tag{22}$$

where (10)–(22) hold for all DGs $g \in \mathscr{G}$. The objective (9) of the second-stage problem is to minimize the dispatch costs of DGs and the penalty cost incurred due to load curtailment. We enforce by (10)–(14) the generation and ramping limits based on the formulation laid out in [1]. The constraints on the power from each linear segment are stated in (15)–(16). We express the net real power injection at each node $n \in \mathscr{N}$ in (19) with the convention that $p^n[h] > 0$ if real power is injected into the system and state the system-wide power balance constraint in (20). We use the DC power flow model to state the transmission constraints and utilize injection shift factors (ISFs) for network representation [2]. In (21), we express the real power flow on each line ℓ in terms of nodal injections and ISFs and constrain it to be within its line flow limits. Finally, a nonnegativity constraint on $p_g[h]$, $p_g^s[h]$, and $p_c^n[h]$ is enforced (17), (18), and (22), respectively.

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