Online Companion to A Data-Driven Methodology for Contextual Unit Commitment Using Regression Residuals

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In this online companion to our article [1], we present four appendices. In Appendix A, we present the nomenclature used in the publication. Appendix B sets forth the detailed mathematical formulation of the unit commitment problem employed in the paper. In Appendices C and D, we report the results of two statistical tests carried out to investigate the validity of some of the assumptions in our article.

Appendix A Nomenclature

We begin by providing the nomenclature used in the publication.

Sets,Indices

$\mathscr{D}/d$	set/index of simulation days
n	sample size, number of simulation days
$\mathscr{D}$	$:= \{d : d = 1, \ldots, n\}$
$\mathscr{H}/h$	set/index of hourly time periods for each day in $\mathscr{D}$
$\mathscr{G}/g$	set/index of dispatchable generators (DGs)
$\mathscr{S}_g, s$	set, index of piecewise cost intervals for DG g
$\mathscr{N}, n$	set, index of nodes
$\mathscr{L}, \ell$	set, index of lines

Variables

$u_g[h]$	binary commitment status of DG g in hour h
$v_g[h]$	binary startup status of DG g in hour h
$w_g[h]$	binary shutdown status of DG g in hour h
$p_g[h]$	power generated above minimum by DG g in hour h (MW)
$p_g^s[h]$	power from segment s for DG g in hour h (MW)
$p_c[h]$	shed load amount in hour h
$p_s[h]$	curtailed renewable generation in hour h
z	vector of first-stage variables comprising $u_g[h]$ and $v_g[h]$
ζ	vector of second-stage variables comprising $p_g[h]$, $p_c[h]$, and $p_s[h]$

Parameters

Y_d/y_d	random net load vector/its observation
X_d/x_d	random covariate vector/its observation
p_g^m/p_g^M	minimum/maximum power output of DG g
$T_g^{\uparrow}/T_g^{\downarrow}$	minimum uptime/downtime of DG g
$\Delta_g^{\uparrow}/\Delta_g^{\downarrow}$	ramp up/down rate limit of of DG g
λ_g^p/λ_g^u	linear/fixed generation cost of DG g
λ_g^v	start-up cost of DG g
λ^c	cost of load curtailment

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Other nomenclature

$f^*(x_d)$	$:= \mathbb{E}[Y_d \mid X_d = x_d]$ underlying conditional expectation function (CEF)
$\hat{f}_n^{OLS}$	ordinary least squares (OLS) estimate of f^*
$\hat{f}_n^{kNN}$	k-nearest neighbors (kNN) estimate of f^*
E_d/e_d	random CEF error/its observation
$\hat{E}_d^{OLS}/\hat{e}_d^{OLS}$	random OLS residual/its observation
$\hat{E}_d^{kNN}/\hat{e}_d^{kNN}$	random kNN residual/its observation
$\lvert\lvert \cdot \rvert\rvert$	Euclidean norm
$(\cdot)^{\mathsf{T}}$	transposition
$det(\cdot)$	determinant

Appendix B Mathematical Formulation of the Conditional Stochastic Unit Commitment (CSUC) Problem

The first-stage of the CSUC problem is expressed as

$$\underset{\substack{u_g[h], v_g[h], \\ w_g[h]}}{\text{minimize}} \quad \sum_{h \in \mathscr{H}} \sum_{g \in \mathscr{G}} \Big[\alpha_g^u u_g[h] + \alpha_g^v v_g[h] \Big] + \mathbb{E}\big[\mathcal{Q}(z, Y) \mid X = x\big], \tag{1}$$

subject to

$$u_g[h] - u_g[h-1] = v_g[h] - w_g[h] \quad \forall h \in \mathscr{H} \setminus \{1\}, \tag{2}$$

$$u_g[h] - u_g^{\circ} = v_g[h] - w_g[h] \quad \forall h \in \{1\}, \tag{3}$$

$$\sum_{h'=h-T_g^{\uparrow}+1}^{h} v_g[h'] \leq u_g[h] \quad \forall h \in \{T_g^{\uparrow}, \ldots, 24\}, \tag{4}$$

$$\sum_{h'=h-T_g^{\downarrow}+1}^{h} w_g[h'] \leq 1 - u_g[h] \quad \forall h \in \{T_g^{\downarrow}, \ldots, 24\}, \tag{5}$$

$$\sum_{h'=1}^{\min\{u_g^{\circ} \cdot (T_g^{\uparrow} - T_g^{\uparrow,\circ}), 24\}} w_g[h'] = 0, \tag{6}$$

$$\sum_{h'=1}^{\min\{(1-u_g^{\circ}) \cdot (T_g^{\downarrow} - T_g^{\downarrow,\circ}), 24\}} v_g[h'] = 0, \tag{7}$$

$$u_g[h], v_g[h], w_g[h] \in \{0, 1\} \quad \forall h \in \mathscr{H}, \tag{8}$$

where (2)–(8) hold for all DGs $g \in \mathscr{G}$. The objective (1) of the first stage is to minimize the commitment and startup costs plus the expected dispatch and load curtailment costs.

We enforce by (2, 3) the logical constraints that relate the variables $u[h]$, $v[h]$, and $w[h]$, (4, 6) the minimum uptime, and (5, 7) the minimum downtime constraints.

For a specific vector of first-stage decision variables z and an observation on net load $Y_{d'} = y_{d'}$, the value function $\mathcal{Q}(z, y_{d'})$ is computed by solving the following second-stage problem:

$$\underset{\substack{p_g[h], p_g^s[h],\\ p_c^n[h]}}{\text{minimize}} \quad \sum_{h \in \mathscr{H}} \left[\sum_{g \in \mathscr{G}} \sum_{s \in \mathscr{S}_g} \alpha_g^s p_g^s[h] + \sum_{n \in \mathscr{N}} \alpha_c p_c^n[h] \right], \quad (9)$$

subject to

$$p_g[h] \leq (\bar{P}_g - \underline{P}_g) u_g[h] \qquad \forall h \in \mathscr{H}, \quad (10)$$

$$p_g[h] \leq p_g^\circ + \Delta_g^\uparrow u_g^\circ + (\Delta_g^{\uparrow,\circ} - \underline{P}_g) v_g[h] \qquad \forall h \in \{1\}, \quad (11)$$

$$p_g[h] \geq p_g^\circ - \Delta_g^\downarrow u_g^\circ - (\Delta_g^{\uparrow,\circ} - \Delta_g^\uparrow - \underline{P}_g) w_g[h] \quad \forall h \in \{1\}, \quad (12)$$

$$p_g[h] \leq p_g[h-1] + \Delta_g^\uparrow u_g[h-1] + (\Delta_g^{\uparrow,\circ} - \underline{P}_g) v_g[h] \qquad \forall h \in \mathscr{H} \setminus \{1\}, \quad (13)$$

$$p_g[h] \geq p_g[h-1] - \Delta_g^\downarrow u_g[h-1] - (\Delta_g^{\uparrow,\circ} - \Delta_g^\uparrow - \underline{P}_g) w_g[h] \quad \forall h \in \mathscr{H} \setminus \{1\}, \quad (14)$$

$$p_g[h] = \sum_{s \in \mathscr{S}_g} p_g^s[h] \qquad \forall h \in \mathscr{H}, \quad (15)$$

$$p_g^s[h] \leq \bar{P}_g^s - \bar{P}_g^{s-1} \qquad \forall s \in \mathscr{S}_g, \forall h \in \mathscr{H}, (16)$$

$$p_g[h] \in \mathbb{R}_+ \qquad \forall h \in \mathscr{H}, \quad (17)$$

$$p_g^s[h] \in \mathbb{R}_+ \qquad \forall s \in \mathscr{S}_g, \forall h \in \mathscr{H}, (18)$$

$$p^n[h] = \sum_{g \in \mathscr{G}^n} p_g[h] + p_c^n[h] - y_{d'}^n[h] \quad \forall n \in \mathscr{N}, \forall h \in \mathscr{H}, (19)$$

$$\sum_{n \in \mathscr{N}} p^n[h] = 0 \qquad \forall h \in \mathscr{H}, \quad (20)$$

$$\underline{f}_\ell \leq \sum_{n \in \mathscr{N}} \Psi_\ell^n p^n[h] \leq \bar{f}_\ell \qquad \forall \ell \in \mathscr{L}, \forall h \in \mathscr{H}, \ (21)$$

$$p_c^n[h] \in \mathbb{R}_+ \qquad \forall n \in \mathscr{N}, \forall h \in \mathscr{H}, (22)$$

where (10)–(22) hold for all DGs $g \in \mathscr{G}$. The objective (9) of the second-stage problem is to minimize the dispatch costs of DGs and the penalty cost incurred due to load curtailment. We enforce by (10)–(14) the generation and ramping limits based on the formulation laid out in [2]. The constraints on the power from each linear segment are stated in (15)–(16). We express the net real power injection at each node $n \in \mathscr{N}$ in (19) with the convention that $p^n[h] > 0$ if real power is injected into the system and state the system-wide power balance constraint in (20). We use the DC power flow model to state the transmission constraints and utilize injection shift factors (ISFs) for network representation [3]. In (21), we express the real power flow on each line ℓ in terms of nodal injections and ISFs and constrain it to be within its line flow limits. Finally, a nonnegativity constraint on $p_g[h]$, $p_g^s[h]$, and $p_c^n[h]$ is enforced (17), (18), and (22), respectively.

Appendix C
One-Sample Two-Tailed Student's t-Test Results for Regression Residuals

We devote this appendix to discussing the results of the one-sample two-tailed Student's t-test [4, Sec. 10.4], which is applied to the OLS and kNN regression residuals that are obtained as spelled out in Sections III and IV of [1]. The null hypothesis of the test for each regression method is that the population mean of the regression residuals is zero, whereas the alternative hypothesis states that the population mean of the regression residuals is not zero, without stipulating the direction of difference. The t-test statistics reported in Table I indicate that the null hypothesis of neither of the tests can be rejected at the 5% level of significance, which supports the CEF model described in Definition 1 and Lemma 1 of [1].

TABLE I
Student's t-Test Results

hour h	Student's t-test statistic		*hour h*	Student's t-test statistic	
	OLS	kNN		OLS	kNN
1	-2.67×10^{-12}	0.67908	13	-4.81×10^{-13}	-0.55183
2	-1.69×10^{-12}	0.97918	14	1.06×10^{-13}	-0.56483
3	-7.03×10^{-13}	1.04591	15	-7.59×10^{-14}	-0.69284
4	-3.33×10^{-13}	0.89226	16	3.05×10^{-13}	-0.59859
5	-1.25×10^{-12}	0.68263	17	-1.90×10^{-13}	-0.39386
6	-5.09×10^{-13}	0.47135	18	-3.50×10^{-13}	0.01416
7	-7.54×10^{-13}	0.61230	19	-1.85×10^{-13}	0.16305
8	-7.29×10^{-13}	0.83388	20	-3.29×10^{-13}	-0.24850
9	-6.46×10^{-13}	0.50639	21	9.23×10^{-14}	-0.56746
10	-2.90×10^{-14}	-0.13002	22	-9.46×10^{-14}	-0.56782
11	-8.35×10^{-13}	-0.34724	23	-6.63×10^{-15}	-0.46928
12	-7.47×10^{-13}	-0.60096	24	2.92×10^{-13}	-0.43174

Appendix D
Modified R/S Test Results for Net Load and Temperature Observations

To corroborate Assumption 1 of [1] that the net load and temperature observations constitute an $\alpha-$mixing sequence, we make use of the modified range over standard deviation (modified R/S) test proposed by Lo [5] in the econometrics literature so as to analyze the short-range dependence of stock returns.

While $\alpha-$mixing cannot be ascertained by statistical tests, the modified R/S statistic utilizes $\alpha-$mixing to define short-range dependence and includes the $\alpha-$mixing condition, in addition to certain moment restrictions, in its null hypothesis. We apply the modified R/S test to the hourly net load and temperature data described in Section IV of [1] for a lag value of 30. The results reported in Table II indicate that we cannot reject the null hypothesis of either of the tests at the 5% level of significance, which corroborates Assumption 1 of [1].

TABLE II
MODIFIED R/S TEST RESULTS

hour h	modified R/S statistic		*hour* h	modified R/S statistic	
	net load	temperature		net load	temperature
1	1.46281	1.15300	13	1.45884	1.19486
2	1.40957	1.14803	14	1.46474	1.23426
3	1.34877	1.15699	15	1.45978	1.24004
4	1.31869	1.16985	16	1.39977	1.25529
5	1.28533	1.16071	17	1.30241	1.23732
6	1.18420	1.16554	18	1.22855	1.21310
7	1.34380	1.18342	19	1.21176	1.17987
8	1.51427	1.21597	20	1.42992	1.17416
9	1.49386	1.19338	21	1.53969	1.17466
10	1.59097	1.18372	22	1.56317	1.16424
11	1.52340	1.17602	23	1.55074	1.15518
12	1.47163	1.18851	24	1.52790	1.15037

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