THREE-EQUATION MELT FORMULATION: COMBINING TO A QUADRATIC EQUATION

JH: (7)

I | $\dot{m} L_i + \dot{m} c_i (T_b - T_i) = c \underbrace{C_d^{1/2} U \Gamma_T}_{\gamma_T} (T - T_b)$

JH: (8)

II | $\dot{m} (S_b - S_i) = \underbrace{C_d^{1/2} U \Gamma_S}_{\gamma_S} (S - S_b)$

JH: (9)

III | $T_b = \lambda_1 S_b + \underbrace{\lambda_2 + \lambda_3 Z_b}_{\beta}$

SOLVE I FOR T_b:

$$T_b = \frac{c \gamma_T T + \dot{m} c_i T_i - \dot{m} L_i}{\dot{m} c_i + c \gamma_T}$$

SOLVE II FOR S_b:

$$S_b = \frac{\dot{m} S_i + \gamma_S S}{\dot{m} + \gamma_S}$$

INSERT T_b AND S_b INTO III:

$$\frac{c \gamma_T T + \dot{m} c_i T_i - \dot{m} L_i}{\dot{m} c_i + c \gamma_T} = \lambda_1 \frac{\dot{m} S_i + \gamma_S S}{\dot{m} + \gamma_S} + \beta$$

MULTIPLY BY $(\dot{m} c_i + c \gamma_T)(\dot{m} + \gamma_S)$

$$(\dot{m} + \gamma_S)(c \gamma_T T + \dot{m} c_i T_i - \dot{m} L_i) = \lambda_1 (\dot{m} S_i + \gamma_S S)(\dot{m} c_i + c \gamma_T) + \beta (\dot{m} c_i + c \gamma_T)(\dot{m} + \gamma_S)$$

EXPAND OUT

$$\dot{m} c \gamma_T T + \dot{m}^2 c_i T_i - \dot{m}^2 L_i + c \gamma_S \gamma_T T + \dot{m} c_i T_i \gamma_S - \dot{m} \gamma_S L$$

$$= \lambda_1 \dot{m}^2 S_i c_i + \lambda_1 \dot{m} S_i c \gamma_T + \lambda \gamma_S S \dot{m} c_i + \lambda_1 \gamma_S S c \gamma_T$$

$$+ \beta \dot{m}^2 c_i + \beta \dot{m} c_i \gamma_S + \beta c \gamma_T \dot{m} + \beta c \gamma_T \gamma_S$$

COLLECT TERMS

$$\dot{m}^2 (c_i T_i - L_i - \lambda_1 S_i c_i - \beta c_i)$$

$$+ \dot{m} (c \gamma_T T + c_i T_i \gamma_S - \gamma_S L - \lambda_1 S_i c \gamma_T - \lambda \gamma_S S c_i - \beta c_i \gamma_S - \beta c \gamma_T)$$

$$+ (c \gamma_S \gamma_T T - \lambda_1 \gamma_S \gamma_T S c - \beta c \gamma_T \gamma_S) = 0$$

SORT

$$\dot{m}^2 (c_i (T_i - (\lambda S_i + \beta)) - L_i)$$

$$+ \dot{m} (c \gamma_T (T - (\lambda_1 S_i + \beta)) + c_i \gamma_S (T_i - (\lambda_1 S + \beta)) - \gamma_S L)$$

$$+ c \gamma_S \gamma_T (T - (\lambda_1 S + \beta)) = 0$$

SE THE FOLLOWING
EFINITIONS

$T_f = \lambda_1 S + \beta$ → Freezing point of (plume) water with salinity S

$T_{fi} = \lambda_1 S_i + \beta$ → Melting point of ice?

$$\dot{m}^2\left(c_i(T_i - T_{fi}) - L_i\right)$$
$$+ \dot{m}\left(c\gamma_T(T - T_{fi}) + c_i\gamma_S(T_i - T_f) - \gamma_S L_i\right)$$
$$+ c\gamma_S\gamma_T(T - T_f) = 0$$

MULTIPLY BY -1, AND NOTE THAT

$$\dot{m} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

IS THE SOLUTION, WITH

$$A = L_i + c_i(T_{fi} - T_i)$$

$$B = \gamma_S\left(c_i(T_f - T_i) + L_i\right) + \gamma_T c(T_{fi} - T)$$

$$C = c\gamma_S\gamma_T(T_f - T)$$

NOTE: JENKINS' MATLAB CODE DOES ANOTHER STEP, DIVIDING BY C.