Mandatory Assignment (opt. B)

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Question 1

Below is a screenshot of the solution to the Sudoku problem. The AMPL files can be found in the “ampl” folder. A small Python script was used to print the solution in a readable way (this script can be found in the “scripts” folder).

```
8 1 2 7 5 3 6 4 9
9 4 3 6 8 2 1 7 5
6 7 5 4 9 1 2 8 3
1 5 4 2 3 7 8 9 6
3 6 9 8 4 5 7 2 1
2 8 7 1 6 9 5 3 4
5 2 1 9 7 4 3 6 8
4 3 8 5 2 6 9 1 7
7 9 6 3 1 8 4 5 2
```

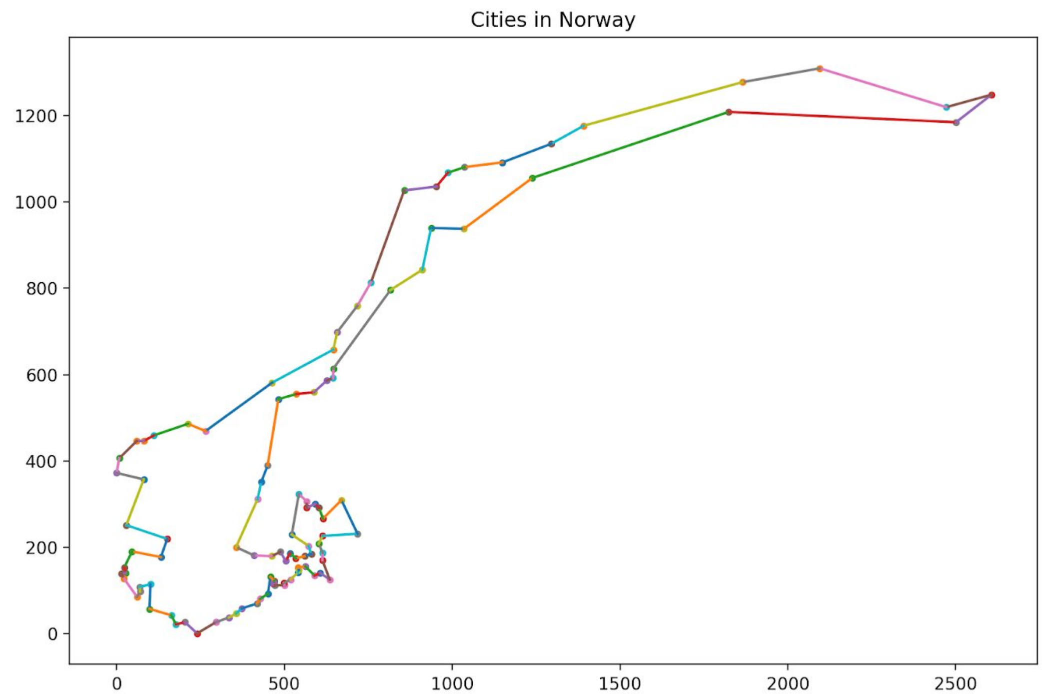
```
ampl: display X;
X [1,*,*]
:   1   2   3   4   5   6   7   8   9    :=
1   0   0   0   0   0   0   0   1   0
2   1   0   0   0   0   0   0   0   0
3   0   1   0   0   0   0   0   0   0
4   0   0   0   0   0   0   1   0   0
5   0   0   0   0   1   0   0   0   0
6   0   0   1   0   0   0   0   0   0
7   0   0   0   0   0   1   0   0   0
8   0   0   0   1   0   0   0   0   0
9   0   0   0   0   0   0   0   0   1
```

The first screenshot above shows the formatted console output from the Python script, while the second screenshot shows the binary matrix for the first line of the solution.

Question 2

Below is a table of my results from question 2 of the mandatory assignment as well as a plot of the optimal solution (Svetska). AMPL files can be found in the “ampl” folder.

Model name	**# of variables**	**# of constraints**	**Objective value**	**Running time**	
MTZ	10609	10712	8077.16	3600 seconds	❌
Gain (Svetska)	21012	10917	8038.69	629 seconds	✔
Steps (Dantzig)	1082120	10712	9221.51	3600 seconds	❌



The plot in the above screenshot was done in Python. The script takes in the list of arcs found by the Svetska model and uses this in combination with the list of x- and y-coordinates for the 103 cities to plot the graph. See the “scripts” folder for the source code (note that the script requires the Matplotlib library to run).

All three models was ran on a MacBook Pro with the specs shown in the above screenshot.

Question 3

Part A

Below is the mathematical model for part A of question 3. The corresponding AMPL files can be found in the "ampl" folder.

$P = \{1, 2, 3, 4, 5\}$
$V = \{1, 2, 3, 4\}$

$C_{ij} = loading\ cost\ per\ barrel\ at\ port\ i\ for\ vessel\ j$
$Cp_i = fixed\ cost\ for\ using\ port\ i$
$Cv_j = fixed\ cost\ for\ using\ vessel\ j$
$Sp_i = supply\ at\ port\ i$
$Uv_j = capacity\ of\ vessel\ j$
$D = 750000$

$x_{ij} = amount\ of\ product\ shipped\ from\ port\ i\ using\ vessel\ j$
$x_{ij} \in \mathbb{R}_+$

$k_{ij} = if\ port\ i\ was\ used\ in\ combination\ with\ vessel\ j,\ otherwise\ 0$
$k_{ij} \in \{0, 1\}$

$y_i = 1\ if\ port\ i\ is\ visited,\ otherwise\ 0$
$y_i \in \{0, 1\}$

$z_i = 1\ if\ vessel\ i\ is\ used,\ otherwise\ 0$
$z_i \in \{0, 1\}$

$$Minimize\ z : \sum_{i=1}^{5} \sum_{j=1}^{4} C_{ij} x_{ij} + \sum_{i=1}^{5} Cp_i y_i + \sum_{j=1}^{4} Cv_j y_j \qquad cost\ of\ loading\ plus\ fixed\ costs$$

subject to

$$\sum_{i=1}^{5} \sum_{j=1}^{4} x_{ij} = D \qquad daily\ demand\ at\ the\ refinery$$

$\sum_{j=1}^{4} x_{ij} \le Sp_i \qquad \forall i \in P$ *supply*

$\sum_{i=1}^{5} x_{ij} \le Uv_j \qquad \forall j \in V$ *vessel capacity*

$\sum_{i=1}^{5} k_{ij} \le 2 \qquad \forall j \in V$ *visit at most two ports*

$x_{ij} - k_{ij}Uv_j \le 0 \quad \forall i \in P, \forall j \in V$ *port-vessel combos used*

$x_{ij} - y_i Sp_i \le 0 \qquad \forall i \in P,\ \forall j \in V$ *fixed cost for port if used*

$x_{ij} - z_j Uv_j \le 0 \qquad \forall i \in P,\ \forall j \in V$ *fixed cost for port if used*

Part B

Most of part B is identical to the model in part A. Below are the updates to the model needed to solve part B. The corresponding AMPL files can be found in the “ampl” folder.

$Q1 = 99999$
$Q2 = 100000$
$Q3 = 179999$
$Q4 = 180000$

$T1_i = section\ one\ tariffs\ for\ port\ i$
$T2_i = section\ two\ tariffs\ for\ port\ i$
$T3_i = section\ three\ tariffs\ for\ port\ i$

$qa_{ij}\ =\ if\ port\ i\ was\ used\ in\ combination\ with\ vessel\ j,\ otherwise\ 0\ (for\ tariff\ in\ section\ one)$
$qa_{ij} \in \{0, 1\}$

$qb_{ij}\ =\ if\ port\ i\ was\ used\ in\ combination\ with\ vessel\ j,\ otherwise\ 0\ (for\ tariff\ in\ section\ two)$
$qb_{ij} \in \{0, 1\}$

$qc_{ij}\ =\ if\ port\ i\ was\ used\ in\ combination\ with\ vessel\ j,\ otherwise\ 0\ (for\ tariff\ in\ section\ three)$
$qc_{ij} \in \{0, 1\}$

$x1_{ij}\ =\ amount\ of\ product\ shipped\ from\ port\ i\ using\ vessel\ j\ for\ the\ tariff\ in\ section\ one$
$x1_{ij} \in \mathbb{R}_+$

$x2_{ij}\ =\ amount\ of\ product\ shipped\ from\ port\ i\ using\ vessel\ j\ for\ the\ tariff\ in\ section\ two$
$x2_{ij} \in \mathbb{R}_+$

$x3_{ij}\ =\ amount\ of\ product\ shipped\ from\ port\ i\ using\ vessel\ j\ for\ the\ tariff\ in\ section\ three$
$x3_{ij} \in \mathbb{R}_+$

$$Minimize\ z : \sum_{i=1}^{5} \sum_{j=1}^{4} T1_i x1_{ij} + T2_i x2_{ij} + T3_i x3_{ij}\ + \sum_{i=1}^{5} Cp_i y_i + \sum_{j=1}^{4} Cv_j y_j$$

$x1_{ij} - Q1Qa_{ij} \leq 0 \qquad \forall i \in P, \forall j \in V \qquad upper\ bound\ on\ intermediate\ x1$

$x2_{ij} - Q2Qb_{ij} \geq 0 \qquad \forall i \in P, \forall j \in V \qquad lower\ bound\ on\ intermediate\ x2$

$x2_{ij} - Q3Qb_{ij} \leq 0$	$\forall i \in P, \forall j \in V$	*upper bound on intermediate x2*
$x3_{ij} - Q4Qc_{ij} \geq 0$	$\forall i \in P, \forall j \in V$	*lower bound on intermediate x3*
$x3_{ij} - Uv_j Qc_{ij} \leq 0$	$\forall i \in P, \forall j \in V$	*upper bound on intermediate x3*
$Qa_{ij} + Qb_{ij} + Qb_{ij} \leq 1$	$\forall i \in P, \forall j \in V$	*make sure at most one tariff is used*
$x1_{ij} + x2_{ij} + x3_{ij} = x_{ij}$	$\forall i \in P, \forall j \in V$	*combine intermediates into main x*