1.

Matoušek and Gartner:

$$c^T x, \quad Ax = b, \quad x \geq 0$$

standard (equational) form.

Vanderbei:

$$c^T x, \quad Ax \leq b, \quad x \geq 0$$

The only diff.

2. Introducing slack variables:

$$x_1 + 2x_2 \leq 10 \Rightarrow x_1 + 2x_2 + x_3 = 10$$

$$\Rightarrow x_3 = 10 - 2x_2 - 3x_3$$

By introducing x_3 we can convert into equality constraint

3., 4. Yes.

5.

6. For matrix A to have a full row rank m, A needs to have at least as many columns as rows.

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$$ $m = 3, n = 2$

This is an overdetermined system, and row 3 can be represented by 2.

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ Here, the row rank is 3, which is a full rank.

7. Def. of basic feasible solution:

a basic feasible solution to the LP:

$\max. \ c^T x, \ \text{s.t.} \ Ax = b, \ x \geq 0$

is a feasible solution $x \in \mathbb{R}^n$ for which there exist an m-element set

$B \subseteq \{0, 1 \ldots n\}$ such that:

- The square matrix of A_B is non-singular, i.e. the columns indexed by B are linearly independent.
- And $x_j = 0 \ \forall j \notin B$.

Note that all non-basic variables are 0 in a basic feasible solution.

E.g. $A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 3 & 4 & 6 \\ 0 & 1 & 3 & 5 & 6 \end{pmatrix}$, $B = \{2, 4\}$

$A_B = \begin{pmatrix} 5 & 4 \\ 1 & 5 \end{pmatrix}$ $A_N = \begin{pmatrix} 1 & 3 & 6 \\ 0 & 3 & 6 \end{pmatrix}$

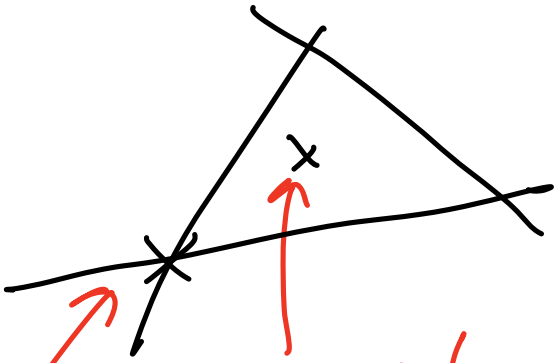
$x = (0, 2, 0, 1, 0)$ ← Basic feasible solution

$$A_B x_B = \begin{pmatrix} 5 & 4 \\ 1 & 5 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \cdot 2 + 4 \cdot 1 \\ 1 \cdot 2 + 5 \cdot 1 \end{pmatrix} = \begin{pmatrix} 14 \\ 7 \end{pmatrix} \quad *$$

$$A_N x_N = \begin{pmatrix} 1 & 3 & 6 \\ 0 & 3 & 6 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$A_B x_B + A_N x_N$ means that adding the matrix of BV's to the matrix of NBV's gives us the solution vector b.

$Ax = b^*$



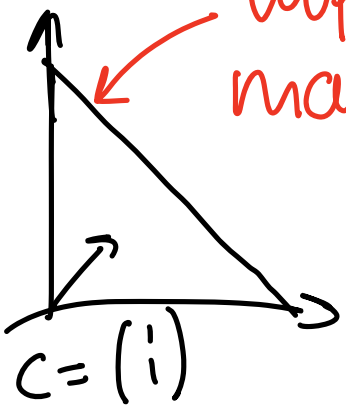
Feasible solution but it's not basic since it is redundant (not lin. ind.).

This one cannot be reconstructed

l. from other points.

10.

b) Yes.



infinite many optimal feasible solutions along this line.

not the case for basic feasible solutions!