# Problem 1.

$$
\begin{array}{llcl}
\max & x_1 + x_2 & & \\
\text{s.t.} & x_1 & \leq & 5 \\
& x_2 & \leq & 5 \\
& x_1 + x_2 & \leq & 8 \\
& x_2 & \geq & 1^{*} \\
& x_1, x_2 & \geq & 0
\end{array}
$$

$$
\begin{array}{rcl}
\zeta & = & x_1 + x_2 \\
\hline
\rightarrow w_1 & = & 5 - x_1 \\
w_2 & = & 5 \qquad - x_2 \\
w_3 & = & 8 - x_1 - x_2
\end{array}
$$

$$x = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad w = \begin{pmatrix} 5 \\ 5 \\ 8 \end{pmatrix}$$

$w_1 = 5 - x_1 \Rightarrow x_1 = 5 - w_1$

$$\zeta = 5 - w_1 + x_2$$

$$x_1 = 5 - w_1$$

$$w_2 = 5 - x_2$$

$$w_3 = 3 + w_1 - x_2$$

$$x = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \quad w = \begin{pmatrix} 0 \\ 5 \\ 3 \end{pmatrix}$$

$$c^T x = 1 \cdot 5 + 1 \cdot 0 = 5$$

$w_3 = 3 + w_1 - x_2 \Rightarrow x_2 = 3 + w_1 - w_3$

$$\zeta = 5 - \cancel{w_1} + 3 + \cancel{w_1} - w_3$$

$$= \boxed{8 - w_3}$$

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$$x_1 = \boxed{5} - w_1$$

$$w_2 = 2 - w_1 + w_3$$

$$x_2 = 3 + w_1 - w_3$$

$$x = \begin{pmatrix} 5 \\ 3 \end{pmatrix}, \quad w = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$$

$$c^T x = 1 \cdot 5 + 1 \cdot 3 = \underline{\underline{8}}$$

## Problem 2.

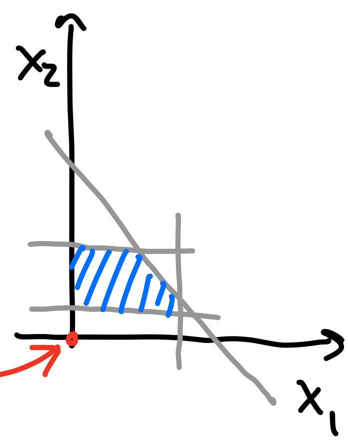
$$
\begin{aligned}
\max \quad & x_1 + x_2 \\
\text{s.t.} \quad & x_1 \le 5 \\
& x_2 \le 5 \\
& x_1 + x_2 \le 8 \\
& x_2 \ge 1^{*} \\
& x_1, x_2 \ge 0
\end{aligned}
$$

$^{*}\; x_2 \ge 1 \Rightarrow -x_2 \le -1$

Simplex tableau:

$$
\begin{array}{rcl}
\zeta & = & \quad\;\; x_1 + x_2 \\
\hline
w_1 & = & 5 - x_1 \\
w_2 & = & 5 \qquad\;\; - x_2 \\
w_3 & = & 8 - x_1 - x_2 \\
w_4 & = & -1 \qquad\; + x_2
\end{array}
$$

Infeasible!




we are here!

Let's solve an aux. problem!

$$
\begin{array}{rcl}
\zeta & = & x_1 + x_2 \\
\hline
w_1 & = & 5 - x_1 \\
w_2 & = & 5 - x_2 \\
w_3 & = & 8 - x_1 - x_2 \\
w_4 & = & -1 + x_2
\end{array}
\qquad (P)
$$

$$
\begin{array}{rcl}
\xi & = & -x_0 \\
\hline
w_1 & = & 5 - x_1 - x_0 \\
w_2 & = & 5 - x_2 - x_0 \\
w_3 & = & 8 - x_1 - x_2 - x_0 \\
w_4 & = & -1 + x_2 - x_0
\end{array}
\qquad (AP)
$$

Initially infeasible

$$
\begin{array}{rcl}
\mathcal{L} & = & - x_0 \\
\hline
w_1 & = & 5 - x_1 - x_0 \\
w_2 & = & 5 - x_2 - x_0 \\
w_3 & = & 8 - x_1 - x_2 - x_0 \\
w_4 & = & -1 + x_2 - x_0
\end{array}
$$

exchange $x_0$ with the most infeasible slack variable.

$$
w_4 = -1 + x_2 - x_0 \Rightarrow x_0 = -1 + x_2 - w_4 \quad \Big| \cdot (-1)
$$
$$
= 1 - x_2 + w_4
$$

$$
\begin{array}{rcl}
\mathcal{L} & = & -1 + x_2 - w_4 \\
\hline
w_1 & = & 4 - x_1 + x_2 - w_4 \\
w_2 & = & 4 - w_4 \\
w_3 & = & 7 - x_1 - w_4 \\
x_0 & = & 1 - x_2 + w_4
\end{array}
$$

Feasible (AP)!

Now apply SM as usual!

$$
\begin{array}{rcl}
\mathcal{L} & = & -1 \qquad\qquad + x_2 - w_4 \\
\hline
w_1 & = & 4 - x_1 + x_2 - w_4 \\
w_2 & = & 4 \qquad\qquad\qquad\quad - w_4 \\
w_3 & = & 7 - x_1 \qquad\qquad - w_4 \\
\rightarrow x_0 & = & 1 \qquad\quad - x_2 + w_4
\end{array}
$$

$$x_0 = 1 - x_2 + w_4 \Rightarrow x_2 = 1 - x_0 + w_4$$

$$
\begin{array}{rcl}
\mathcal{L} & = & \qquad - x_0 \\
\hline
w_1 & = & \boxed{5} - x_1 - x_0 \\
w_2 & = & \boxed{4} \qquad\qquad\qquad - w_4 \\
w_3 & = & \boxed{7} - x_1 \qquad\qquad - w_4 \\
x_2 & = & \boxed{1} \qquad\quad - x_0 + w_4
\end{array}
$$

Optimal

$\xi^{\circ} = 0 \implies$ LP has a feas. point

Now take $\xi$ from LP and drop $x_0$.

original obj. f.

$$
\begin{aligned}
\xi &= x_1 + x_2 \\
&= 1 + x_1 + w_4 \\
\hline
w_1 &= 5 - x_1 - x_0 \\
w_2 &= 4 - w_4 \\
w_3 &= 7 - x_1 - w_4 \\
x_2 &= 1 - x_0 + w_4
\end{aligned}
$$

$$
\begin{array}{llllll}
y & = & 1 & + x_1 & & + w_4 \\
\hline
w_1 & = & 5 & - x_1 & & \\
w_2 & = & 4 & & & - w_4 \\
w_3 & = & 7 & - x_1 & & - w_4 \\
x_2 & = & 1 & & & + w_4
\end{array}
$$

still not opt.

$$w_1 = 5 - x_1 \Rightarrow x_1 = 5 - w_1$$

$$
\begin{array}{llllll}
y & = & 6 & - w_1 & & + w_4 \\
\hline
x_1 & = & 5 & - w_1 & & \\
w_2 & = & 4 & & & - w_4 \\
w_3 & = & 2 & + w_1 & & - w_4 \\
x_2 & = & 1 & & & + w_4
\end{array}
$$

$$w_3 = 2 + w_1 - w_4 \Rightarrow w_4 = 2 + w_1 - w_3$$

$$\begin{array}{rcl} \varphi &=& 8 \qquad\qquad\quad - w_3 \\ \hline x_1 &=& 5 - w_1 \\ w_2 &=& 2 - w_1 + w_3 \\ w_4 &=& 3 + w_1 - w_3 \\ x_2 &=& 3 + w_1 - w_3 \end{array}$$

$$x^0 = \begin{pmatrix} 5 \\ 3 \end{pmatrix}, \quad w^0 = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 3 \end{pmatrix}$$

$$c^T x^0 = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \end{pmatrix} = 1 \cdot 5 + 1 \cdot 3 = \underline{\underline{8}}$$

Optimal solution found!