Lexicographic Method

(to prevent cycling)

Interchange x_1 and w_2.

New dict.:

$$
\begin{array}{lcl}
\zeta & = & \frac{3}{2}\epsilon_2 - \frac{3}{2}w_2 + x_2 \\
\hline
w_1 & = & 0 + \epsilon_1 + \frac{9}{4}\epsilon_2 - \frac{9}{4}w_2 - \frac{1}{2}x_2 \\
x_1 & = & 0 \quad + \frac{1}{3}\epsilon_2 - \frac{1}{4}w_2 - \frac{1}{2}x_2 \\
w_3 & = & 1 \quad + \epsilon_3 \quad - x_2
\end{array}
$$

3 cand. for the pivot row

$$\min\left\{ \frac{\varepsilon_1 + \frac{9}{4}\varepsilon_2}{0{,}5},\ \frac{\frac{1}{4}\varepsilon_2}{0{,}5},\ 1 + \frac{\varepsilon_3}{1} \right\}$$

We only have to compare the first two fractions.

Since $\varepsilon_1 \gg \varepsilon_2 \implies \dfrac{\boxed{\varepsilon_1} + \frac{9}{4}\varepsilon_2}{0{,}5} > \dfrac{\frac{1}{4}\boxed{\varepsilon_2}}{0{,}5}$

$\implies$ Pivot row : $x_1 \implies$ exch. x_1 and x_2.

New dict.:

$$\zeta = 2\varepsilon_2 \overbrace{-2w_2 - 2x_1}^{*}$$

$$\begin{aligned}
w_1 &= \varepsilon_1 + 2\varepsilon_2 - 2w_2 + x_1 \\
x_2 &= \tfrac{1}{2}\varepsilon_2 - \tfrac{1}{2}w_2 - 2x_1 \\
w_3 &= 1 - \tfrac{1}{2}\varepsilon_2 + \varepsilon_3 + \tfrac{1}{2}w_2 + 2x_1
\end{aligned}$$

Note! we are ignoring the ε_i when looking for optimality.

Hence, we are now optimal. *

Optimal vertex can be taken from opt. dict. by ignoring ε-columns.

$$
\begin{aligned}
\zeta &= \cancel{2\varepsilon_2} - 2w_2 - 2x_1 \\
\hline
w_1 &= \cancel{\varepsilon_1} + \cancel{2\varepsilon_2} - 2w_2 + x_1 \\
x_2 &= \cancel{\tfrac{1}{2}\varepsilon_2} - \tfrac{1}{2}w_2 - 2x_1 \\
w_3 &= 1 - \cancel{\tfrac{1}{2}\varepsilon_2} + \cancel{\varepsilon_3} + \tfrac{1}{2}w_2 + 2x_1
\end{aligned}
$$

$$\Rightarrow x^0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \zeta^0 = 0$$

Optimal point (from pplex problem):

$x^{o} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \varphi^{o} = \frac{5}{4}$

2nd technique to prevent (degen. and) cycling:

Bland's rule:

choose the pivot col and the pivot row by selecting the vars w/ the smallest index.

$x_i \longrightarrow x_i, \quad i = 1 \ldots n$

$w_j \longrightarrow w_{n+j}, \quad j = 1 \ldots m$

Try this in practice!!

Fundamental theorem of linear programming:

Any LP problem in std. form is:

1) either infeasible: (feasible set = $\emptyset$)

Phase I doesn't calc. a feasible point.

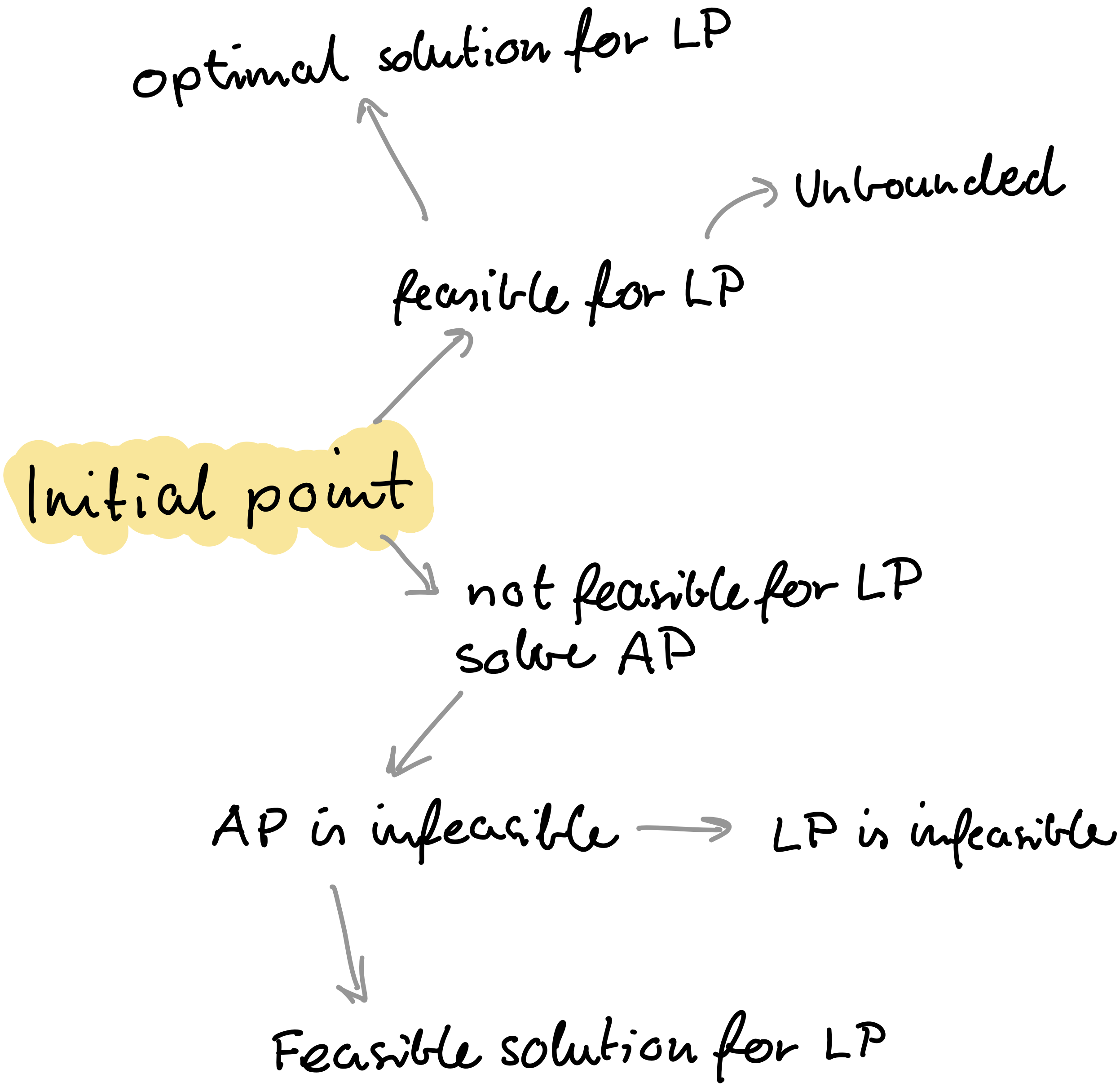
2) or unbounded: (non-empty feasible set. no opt. solution is recognized by a specific pivot col.)

3) or has an opt. solution: (can be calc. in finitely many steps by a cycling avoiding method.)

Proof: If initial basic solution is feasible (f.x. the origin) then the SM (use lexicographic or Bland's rule in the case of cycling) is either unbounded or calc. an optimal solution.

If the initial basic solution is <u>not</u> feasible, we extend the problem with an extra variable x_0. The SM terminates on this aux. problem by:

- detecting infeasibility
- or ending with an initial feasible sol. for the original problem

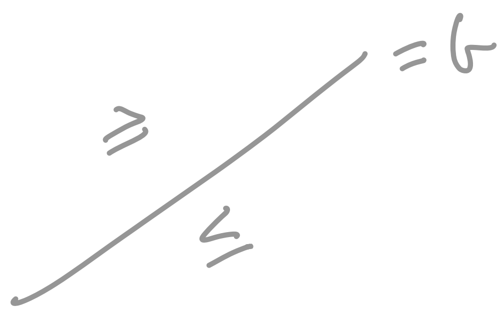

optimal solution for LP
Unbounded
feasible for LP
Initial point
not feasible for LP
solve AP
AP is infeasible
LP is infeasible
Feasible solution for LP

Geometry

$n = 2 \longrightarrow$ graphical method:

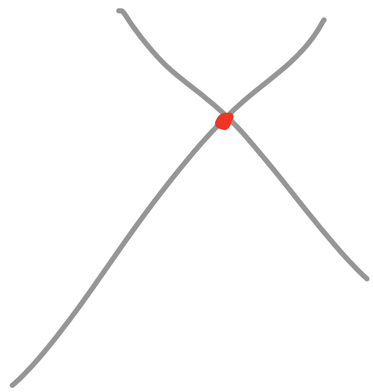
each vertex is described by two or more equations.

$$a_{11}x_1 + a_{12}x_2 = b$$



<u>equations</u>

constr. or non-neg. cond.



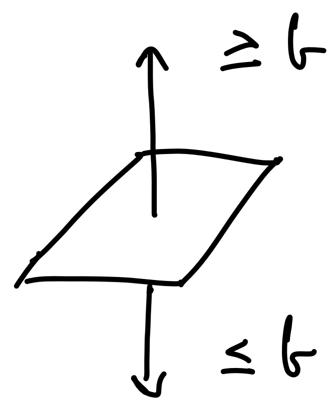
Two NBV's (each of them = 0)

Can be generalized to n dimensions:

Each eq. constraint describes a plane.

$\rightarrow$ Feasible set is the intersection of finitely many half-spaces.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b$$



Facet: intersection of two planes (a system of two eq. constr. describes facet.)

(Intersection of three eq. constr. (3 planes) describes a vertex.

Each vertex has three (n) NBVs (zeros).

$$\begin{pmatrix}0\\0\\0\end{pmatrix} \longrightarrow \begin{pmatrix}0\\0\\1\end{pmatrix} \longrightarrow \begin{pmatrix}1\\0\\1\end{pmatrix} \longrightarrow \begin{pmatrix}3\\2\\0\end{pmatrix}$$

From graphic on the slide)