2,6 (Vanderbei)

$$\begin{aligned}
\max \quad & x_1 + 3x_2 \\
\text{s.t.} \quad & -x_1 - x_2 \le -3 \\
& -x_1 + x_2 \le -1 \\
& x_1 + 2x_2 \le 2 \\
& x_1, x_2 \ge 0
\end{aligned}$$

$$\begin{array}{rcl}
\zeta & = & x_1 + 3x_2 \\
\hline
w_1 & = & -3 + x_1 + x_2 \\
w_2 & = & -1 + x_1 - x_2 \\
w_3 & = & 2 - x_1 - 2x_2
\end{array}$$

(infeasible)

$$
\begin{array}{rcl}
\xi & = & \qquad\qquad\qquad\qquad - x_0 \\
\hline
w_1 & = & -3 + x_1 + x_2 - x_0 \\
w_2 & = & -1 + x_1 - x_2 - x_0 \\
w_3 & = & 2 - x_1 - 2x_2 - x_0
\end{array}
$$

$$w_1 = -3 + x_1 + x_2 - x_0$$

$$\Rightarrow x_0 = 3 - x_1 - x_2 + w_1$$

$$
\begin{array}{rcl}
\xi & = & -3 + x_1 + x_2 - w_1 \\
\hline
x_0 & = & 3 - x_1 - x_2 + w_1 \\
w_2 & = & -4 + 2x_1 \qquad\quad - w_1 \\
w_3 & = & -1 \qquad\quad - x_2 - w_1
\end{array}
$$

Infeasible (AP)!

2,8 (Vanderbei)

$$
\begin{array}{rcl}
\zeta & = & 3x_1 + 2x_2 \\
\hline
w_1 & = & 1 - x_1 + 2x_2 \\
w_2 & = & 2 - x_1 + x_2 \\
w_3 & = & 6 - 2x_1 + x_2 \\
w_4 & = & 5 - x_1 \\
w_5 & = & 16 - 2x_1 - x_2 \\
w_6 & = & 12 - x_1 - x_2 \\
w_7 & = & 21 - x_1 - 2x_2 \\
w_8 & = & 10 - x_2
\end{array}
$$

$w_1 = 1 - x_1 + 2x_2 \Rightarrow x_1 = 1 - w_1 + 2x_2$

$$\zeta = 3(1 - w_1 + 2x_2) + 2x_2$$

$$= 3 - 3w_1 + 6x_2 + 2x_2$$

$$\begin{array}{rcl}
\zeta & = & 3 - 3w_1 + 8x_2 \\
\hline
x_1 & = & 1 - w_1 + 2x_2 \\
\rightarrow\ w_2 & = & 1 + w_1 - x_2 \\
w_3 & = & 4 + 2w_1 - 4x_2 \\
w_4 & = & 4 + w_1 - 2x_2 \\
w_5 & = & 14 + 2w_1 - 5x_2 \\
w_6 & = & 11 + w_1 - 3x_2 \\
w_7 & = & 20 + w_1 - 4x_2 \\
w_8 & = & 10 \qquad\quad\ - x_2
\end{array}$$

$$x^* = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad c^T x^* = 3 \cdot 1 + 2 \cdot 0 = 3$$

$$w_2 = 1 + w_1 - x_2 \Rightarrow x_2 = 1 + w_1 - w_2$$

$$\zeta = 3 - 3w_1 + 8(1 + w_1 - w_2)$$

$$\zeta = 11 + 5w_1 - 8w_2$$

$$\begin{aligned}
x_1 &= 3 + w_1 - 2w_2 \\
x_2 &= 1 + w_1 - w_2 \\
w_3 &= 1 - w_1 + 3w_2 \\
w_4 &= 2 - w_1 + 2w_2 \\
w_5 &= 9 - 3w_1 + 5w_2 \\
w_6 &= 8 - 2w_1 + 3w_2 \\
w_7 &= 16 - 3w_1 + 4w_2 \\
w_8 &= 9 - w_1 + w_2
\end{aligned}$$

$$x^* = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \quad c^T x^* = 3 \cdot 3 + 2 \cdot 1 = 11$$

$$w_3 = 1 - w_1 + 3w_2 \Rightarrow w_1 = 1 - w_3 + 3w_2$$

$$\begin{aligned} y &= 11 + 5(1 - w_3 + 3w_2) - 8w_2 \\ &= 16 - 5w_3 + 7w_2 \end{aligned}$$

(Skipping some vertecies here!)

$$\begin{array}{rcrrr} y & = & 28 & - w_5 & - w_6 \\ \hline x_1 & = & 4 & - w_5 & + w_6 \\ x_2 & = & 8 & - w_5 & + 2w_6 \\ w_1 & = & 13 & + 3w_5 & - 5w_6 \\ w_2 & = & 6 & + 2w_5 & - 3w_6 \\ w_3 & = & 6 & + 3w_5 & - 4w_6 \\ w_4 & = & 1 & + w_5 & - w_6 \\ w_7 & = & 1 & - w_5 & + 3w_6 \\ w_8 & = & 2 & - w_5 & + 2w_6 \end{array}$$

$$x^0 = \begin{pmatrix} 4 \\ 8 \end{pmatrix}, \quad c^T x^0 = 3 \cdot 4 + 2 \cdot 8 = 28$$

Optimal solution!