$$\max \ \zeta = 25x + 30y$$

s.t.

$$\frac{1}{200}x + \frac{1}{140}y \leq 40$$

$$x \leq 6000$$

$$y \leq 400$$

$$x, y \geq 0$$

1. Slack vars

$$w_1 = 40 - \frac{1}{200}x - \frac{1}{140}y$$

$$w_2 = 6000 - x$$

$$w_3 = 4000 - y$$

if $y > 0$ (and $x = 0$):

$$w_1 = 40 - \frac{1}{140}y \Rightarrow 5600 \geq y$$

$$w_3 = 4000 - y \Rightarrow 4000 \geq y$$

$$\min\{5600, 4000\} = 4000$$

exchange y and w_3

new BV

new NBV

$$y = 25x + 30(4000 - w_3)$$
$$= 120000 + 25x - 30w_3$$

$$w_1 = 40 - \frac{1}{200}x - \frac{1}{140}(4000 - w_3)$$
$$= \frac{80}{7} - \frac{1}{200}x + \frac{1}{140}w_3$$

$$w_2 = 6000 - x$$

$$y = 4000 - w_3$$

Updated variables:

$w_1 = \frac{80}{7}$ $\quad x = \underline{0}$

$w_2 = 6000$ $\quad w_3 = 0$

$y = \underline{4000}$

New obj. func. : $25(0) + 30(4000)$

x has a positive coefficient in obj. f. so we're not done!

if $x > 0$ (and $w_3 = 0$):

$$w_1 = \frac{80}{7} - \frac{1}{200}x \implies \frac{16000}{7} \geq x$$

$$w_2 = 6000 - x \implies 6000 \geq x$$

$$\min\left\{\frac{16000}{7}, 6000\right\} = \frac{16000}{7}$$

exchange x and w_1

new BV new NBV

$$\zeta = \frac{1240000}{7} - 5000 w_1 + \frac{40}{7} w_3$$

$$x = \frac{80}{7} - \frac{1}{200} w_1 + \frac{1}{140} w_3 \quad \Big| \cdot 200$$

$$= \frac{16000}{7} - 200 w_1 + \frac{10}{7} w_3$$

$$w_2 = 6000 - x$$

$$= \frac{26000}{7} + 200 w_1 - \frac{10}{7} w_3$$

$$y = 4000 - w_3$$

Updated variables:

$$x = \frac{16000}{7} \qquad w_1 = 0$$

$$w_2 = \frac{26000}{7} \qquad w_3 = 0$$

$$y = 4000$$

w_3 has a positive coeff. still.
Keep going!

if $w_3 > 0$ (and $w_1 = 0$):

$$x = \frac{16000}{7} + \frac{10}{7} w_3 \implies$$

$$w_2 = \frac{26000}{7} - \frac{10}{7} w_3 \implies 2600 \geq w_3$$

$$y = 4000 - w_3 \implies 4000 \geq w_3$$

$$\min\{4000, 2600\} = 2600$$

exchange w_3 and w_2

new BV new NBV

$$\varphi = 192000 - 4200 w_1 - 4 w_2$$

$$x = \frac{16000}{7} - 200 w_1 + \frac{10}{7} w_3$$

$$= \frac{16000}{7} - 200 w_1 + \frac{10}{7}\left(2600 + 140 w_1 - \frac{7}{10} w_2\right)$$

$$= \frac{16000}{7} + \frac{26000}{7} - w_2$$

$$= 6000 - w_2$$

$$w_3 = \frac{26000}{7} + 200 w_1 - \frac{10}{7} w_2 \quad \Big| \begin{array}{l} \cdot 7 \\ \div 10 \end{array}$$

$$= 2600 + 140 w_1 - \frac{7}{10} w_2$$

$$y = 1400 - 140 w_1 + \frac{7}{10} w_2$$

Updated variables:

$x = 6000$ $\quad w_1 = 0$

$w_3 = 2600$ $\quad w_2 = 0$

$y = 1400$

$$c = \begin{pmatrix} 6000 \\ 1400 \end{pmatrix}, \quad X^0 = \begin{array}{c} \begin{array}{ccccc} x^0 & y^0 & w_1^0 & w_2^0 & w_3^0 \end{array} \\ \left(6000, 1400, 0, 0, 2600\right) \end{array}$$

optimal solution!