Problem 1.

$$
\begin{aligned}
\max \quad & x_1 + x_2 \\
\text{s.t.} \quad & x_1 \leq 5 \\
& x_2 \leq 5 \\
& x_1 + x_2 \leq 8 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

$$
\begin{array}{l}
\varsigma = \phantom{5 -{}} x_1 + x_2 \\
\hline
w_1 = 5 - x_1 \\
w_2 = 5 \phantom{{}- x_1} - x_2 \\
w_3 = 8 - x_1 - x_2
\end{array}
$$

$$x = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad w = \begin{pmatrix} 5 \\ 5 \\ 8 \end{pmatrix}$$

$w_1 = 5 - x_1 \Rightarrow x_1 = 5 - w_1$

$$\zeta = 5 - w_1 + x_2$$

$$x_1 = 5 - w_1$$

$$w_2 = 5 \qquad - x_2$$

$$w_3 = 3 + w_1 - x_2$$

$$x = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \quad w = \begin{pmatrix} 0 \\ 5 \\ 3 \end{pmatrix}$$

$$c^T x = 1 \cdot 5 + 1 \cdot 0 = 5$$

$w_3 = 3 + w_1 - x_2 \Rightarrow x_2 = 3 + w_1 - w_3$

$$\varphi = 5 - \cancel{w_1} + 3 + \cancel{w_1} - w_3$$

$$= \boxed{8 \qquad - w_3}$$

$$x_1 = \boxed{5} - w_1$$

$$w_2 = 2 - w_1 + w_3$$

$$x_2 = 3 + w_1 - w_3$$

$$x = \begin{pmatrix} 5 \\ 3 \end{pmatrix}, \quad w = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$$

$$c^T x = 1 \cdot 5 + 1 \cdot 3 = \underline{\underline{8}}$$

Problem 2.

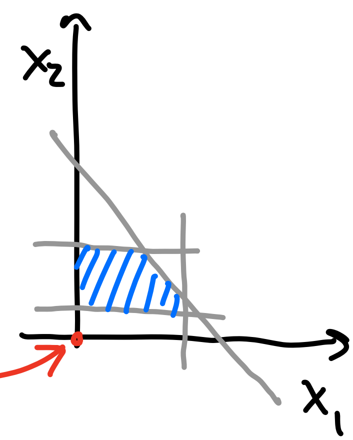
$$\begin{array}{llrl}
\max & x_1 + x_2 & & \\
\text{s.t.} & x_1 & \le & 5 \\
& \quad\quad x_2 & \le & 5 \\
& x_1 + x_2 & \le & 8 \\
& \quad\quad x_2 & \ge & 1^{*} \\
& x_1, x_2 & \ge & 0
\end{array}$$

* $x_2 \ge 1 \Rightarrow -x_2 \le -1$

Simplex tableau:

$$
\begin{array}{rcl}
\zeta & = & x_1 + x_2 \\
\hline
w_1 & = & 5 - x_1 \\
w_2 & = & 5 - x_2 \\
w_3 & = & 8 - x_1 - x_2 \\
w_4 & = & -1 + x_2
\end{array}
$$

Infeasible!



we are here!

Let's solve an aux. problem!

$$
\begin{array}{rcrrr}
\zeta & = & & x_1 & + \; x_2 \\
\hline
w_1 & = & 5 & - \; x_1 & \\
w_2 & = & 5 & & - \; x_2 \\
w_3 & = & 8 & - \; x_1 & - \; x_2 \\
w_4 & = & -1 & & + \; x_2
\end{array}
\qquad (P)
$$

$$
\begin{array}{rcrrrr}
\xi & = & & & & - \; x_0 \\
\hline
w_1 & = & 5 & - \; x_1 & & - \; x_0 \\
w_2 & = & 5 & & - \; x_2 & - \; x_0 \\
w_3 & = & 8 & - \; x_1 & - \; x_2 & - \; x_0 \\
w_4 & = & -1 & & + \; x_2 & - \; x_0
\end{array}
\qquad (AP)
$$

Initially infeasible

$$\begin{array}{rcl}
\mathcal{L} & = & \qquad\qquad\qquad\qquad - x_0 \\
\hline
w_1 & = & 5 - x_1 \qquad\quad - x_0 \\
w_2 & = & 5 \qquad\quad - x_2 - x_0 \\
w_3 & = & 8 - x_1 - x_2 - x_0 \\
w_4 & = & -1 \qquad\quad + x_2 - x_0
\end{array}$$

exchange x_0 with the most infeasible slack variable.

$$w_4 = -1 + x_2 - x_0 \Rightarrow x_0 = -1 + x_2 - w_4 \; \Big| \cdot (-1)$$
$$= 1 - x_2 + w_4$$

$$\begin{array}{rcl}
\mathcal{L} & = & -1 + x_2 - w_4 \\
\hline
w_1 & = & 4 - x_1 + x_2 - w_4 \\
w_2 & = & 4 \qquad\qquad\quad - w_4 \\
w_3 & = & 7 - x_1 \qquad\quad - w_4 \\
x_0 & = & 1 \qquad - x_2 + w_4
\end{array}$$

Feasible (AP)!

Now apply SM as usual!

$$\begin{array}{rcl}
\mathcal{L} & = & -1 \quad + x_2 - w_4 \\
\hline
w_1 & = & 4 - x_1 + x_2 - w_4 \\
w_2 & = & 4 \qquad\qquad\quad - w_4 \\
w_3 & = & 7 - x_1 \qquad\quad - w_4 \\
x_0 & = & 1 \qquad\quad - x_2 + w_4
\end{array}$$

$$x_0 = 1 - x_2 + w_4 \Rightarrow x_2 = 1 - x_0 + w_4$$

$$\begin{array}{rcl}
\mathcal{L} & = & \qquad - x_0 \\
\hline
w_1 & = & 5 - x_1 - x_0 \\
w_2 & = & 4 \qquad\qquad\quad - w_4 \\
w_3 & = & 7 - x_1 \qquad\quad - w_4 \\
x_2 & = & 1 \qquad\quad - x_0 + w_4
\end{array}$$

Optimal

$\xi^\circ = 0 \implies$ LP has a feas. point

Now take ζ from LP and drop x_0.

$$
\begin{array}{rcl}
\zeta & = & x_1 + x_2 \\
 & = & 1 + x_1 \qquad + w_4 \\
\hline
w_1 & = & 5 - x_1 - x_0 \\
w_2 & = & 4 \qquad\qquad - w_4 \\
w_3 & = & 7 - x_1 \qquad - w_4 \\
x_2 & = & 1 \qquad - x_0 + w_4
\end{array}
$$

original obj. f.

$$
\begin{array}{rl}
y = & 1 + \overset{\downarrow}{x_1} + w_4 \\
\hline
\rightarrow w_1 = & 5 - x_1 \\
w_2 = & 4 \qquad\quad - w_4 \\
w_3 = & 7 - x_1 - w_4 \\
x_2 = & 1 \qquad\quad + w_4
\end{array}
$$

still not opt.

$$w_1 = 5 - x_1 \Rightarrow x_1 = 5 - w_1$$

$$
\begin{array}{rl}
y = & 6 - w_1 + \overset{\downarrow}{w_4} \\
\hline
x_1 = & 5 - w_1 \\
w_2 = & 4 \qquad\quad - w_4 \\
\rightarrow w_3 = & 2 + w_1 - w_4 \\
x_2 = & 1 \qquad\quad + w_4
\end{array}
$$

$$w_3 = 2 + w_1 - w_4 \Rightarrow w_4 = 2 + w_1 - w_3$$

$$\begin{array}{l}
\zeta = 8 \qquad\qquad - w_3 \\
\hline
x_1 = 5 - w_1 \\
w_2 = 2 - w_1 + w_3 \\
w_4 = 3 + w_1 - w_3 \\
x_2 = 3 + w_1 - w_3
\end{array}$$

$$x^0 = \begin{pmatrix} 5 \\ 3 \end{pmatrix}, \quad w^0 = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 3 \end{pmatrix}$$

$$c^T x^0 = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \end{pmatrix} = 1 \cdot 5 + 1 \cdot 3 \underline{\underline{= 8}}$$

Optimal solution found!