Mandatory Assignment 1

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Problems

1.

(a)

True. The feasible set equals the solution space of an LP. The set of all solutions to an LP is an intersection of finitely many half-spaces in $\mathbb{R}^n$. Such a set is called a convex polyhedron.

(b)

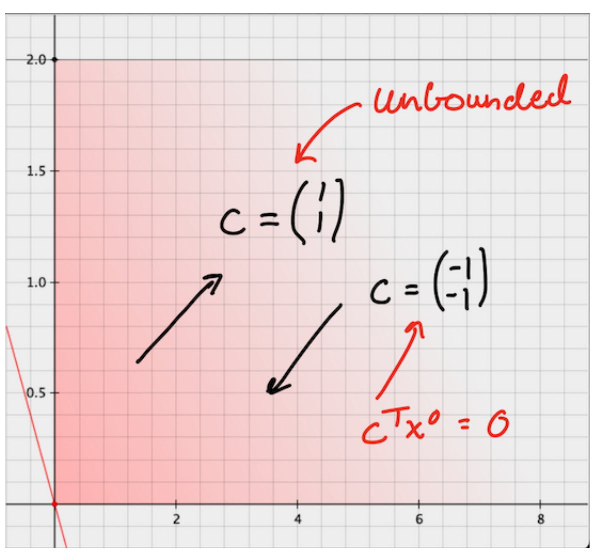
True. The objective function has the general form $c^T x = c_1x_1 + c_2x_2 + \ldots + c_nx_n$ where $\mathbf{c} \in \mathbb{R}^n$ is a given vector.

(c)

False. An optimal solution is always in at least one of the vertices of the feasible set.

(d)

False. Consider the following example:



Let $c = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$, where $c^T x^o = 0$. In this case $-c$ is unbounded.

(e)

False. Consider the following counterexample:

This problem is obviously bounded and since the objective function vector c is perpendicular to the line A, it is clear that there lies infinitely many optimal solutions along this line.

(f)

False. Consider again the example in e). Every basic solution correspond to a vertex in an LP. Since there are infinitely many optimal solutions along the line A, not everyone is on a vertex and hence not every optimal solution is basic.

3.

(a)

$$\begin{aligned} \max \quad & x_1 \\ \text{s.t.} \quad & x_1 + x_2 + w_1 = 5 \\ & x_1 + w_2 = 3 \\ & x_2 + w_3 = 4 \end{aligned}$$

$$x^o = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, c^T x^o = 3$$

(b)

$$\begin{aligned} \max \quad & x_1 + 2x_2 \\ \text{s.t.} \quad & x_1 + x_2 + w_1 = 5 \\ & x_1 + w_2 = 3 \\ & x_2 + w_3 = 4 \end{aligned}$$

$$x^o = \begin{pmatrix} 1 \\ 4 \end{pmatrix}, c^T x^o = 9$$

(c)

$$\begin{aligned} \max \quad & -2x_1 + x_2 \\ \text{s.t.} \quad & x_1 + x_2 + w_1 = 5 \\ & x_1 + w_2 = 3 \\ & x_2 + w_3 = 4 \end{aligned}$$

$$x^o = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, c^T x^o = 4$$

(d)

$$\begin{aligned} \max \quad & x_1 + x_2 \\ \text{s.t.} \quad & x_1 + x_2 + w_1 = 5 \\ & x_1 + w_2 = 3 \\ & x_2 + w_3 = 4 \end{aligned}$$

$$x^o = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, c^T x^o = 5$$

4.

Note the text didn't specify if we should convert the problems into standard form or not, so I left them as minimization problems.

(a)

$$\begin{aligned} \min \quad & 5y_1 + 3y_2 + 4y_3 \\ \text{s.t.} \quad & y_1 + y_2 \geq 1 \\ & y_1 + y_3 \geq 0 \\ & y_1, y_2, y_3 \geq 0 \end{aligned}$$

$$y^o = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, b^T y^o = 5(0) + 3(1) + 4(0) = 3$$

(b)

$$\begin{aligned} \min \quad & 5y_1 + 3y_2 + 4y_3 \\ \text{s.t.} \quad & y_1 + y_2 \geq 1 \\ & y_1 + y_3 \geq 2 \\ & y_1, y_2, y_3 \geq 0 \end{aligned}$$

$$y^o = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, b^T y^o = 5(1) + 3(0) + 4(1) = 9$$

(c)

$$\begin{aligned} \min \quad & 5y_1 + 3y_2 + 4y_3 \\ \text{s.t.} \quad & y_1 + y_2 \geq -2 \\ & y_1 + y_3 \geq 1 \\ & y_1, y_2, y_3 \geq 0 \end{aligned}$$

$$y^o = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, b^T y^o = 5(0) + 3(0) + 4(1) = 4$$

(d)

$$\begin{aligned} \min \quad & 5y_1 + 3y_2 + 4y_3 \\ \text{s.t.} \quad & y_1 + y_2 \geq 1 \\ & y_1 + y_3 \geq 1 \\ & y_1, y_2, y_3 \geq 0 \end{aligned}$$

$$y^o = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, b^T y^o = 5(1) + 3(0) + 4(0) = 5$$