Lecture 2 (26.8)

Remember: $\begin{cases} c^T x \text{ s.t. } Ax = b, \; x \geq 0 \\ c, x \in \mathbb{R}^n, \; A \text{ is an } (m,n) \text{ mat.} \\ \qquad\qquad b \in \mathbb{R}^m \end{cases}$

every opt. problem can be formalized into this.

- what are the decision variables?
  - Answer: x_i
- what is the obj. f.?
  - Answer: $c^T x$
- what are the constraints?
  - Answer: $Ax \leq b$

Example 2: Diet problem (Google it)

Task: find the combination of med. that satisfies the need at min. cost.

x_i = amount of medication $\forall i = 1, 2, 3, 4$

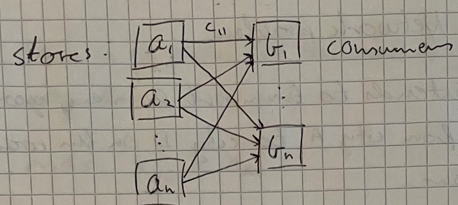
$$c^T x = 1500x_1 + 200x_2 + 1200x_3 + 900x_4$$

s.t.

$$30x_1 + 5x_2 + 20x_3 + 10x_4 \geq 10$$
$$10x_1 + 0x_2 + 10x_3 + 20x_4 \geq 5$$
$$50x_1 + 3x_2 + 50x_3 + 30x_4 \geq 5\tfrac{1}{2}$$
$$x_i \geq 0$$

Example 4 : Transportation problem

A transport company has m stores and wants to deliver a product from these stores to n consumers.
The delivery of one prod. from store i to consumer j costs c_{ij}. Store i has stored A_i items of a product.
Consumer j has a demand of b_j items of a prod.
The comp wants to satisfy the demand of all consumers.
The company wants to minimize delivery cost.



amount to be delivered from $i \to j$

decision variables : x_{ij} ~~[illegible]~~

$i = 1..m \quad j = 1..n$

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \quad \left(\max. \; - \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \right)$$

s.t.

$$\sum_{j=1}^{n} x_{ij} \leq a_i$$ store is fixed here (cap of store)

consumer is fixed here

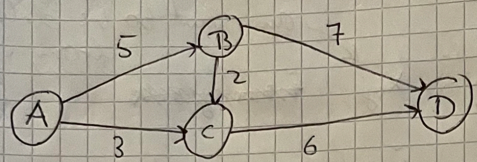
$$\sum_{i=1}^{m} x_{ij} \geq b_j$$ demand of each consumer

$x_{ij} \geq 0$ non-neg.

$i = 1..m \quad j = 1..n$

Example 4 : Network problem

A company intends to transport as many goods as possible from city A to city D on the road network below.



edge numbers denotes capacity

How to model as LP? (later in the course)

Let $V = \{A, B, C, D\}$ denote the nodes in the netw.

Let $E = \{(A,B), (A,C), (B,C), (B,D), (C,D)\}$ denote the edges of the

For each edge (i,j) let x_{ij} denote the amount of goods transported on edge (i,j) and b_{ij} the max. cap. which can be transported on an edge.

$0 \leq x_{ij} \leq b_{ij}$

Assume that no goods are added or lost in intermittent cities. The conservation equation: "out-in =

$X_{BD} + X_{BC} - X_{AB} = 0$ B constraint

$X_{CD} - X_{AC} - X_{BC} = 0$

How to transform an arbitrary LP into standard form? (st. form: $c^T x$ s.t. $Ax \le b$)

I) Inequality constraints:

The constraint $\sum_{j=1}^{n} a_{ij} x_j \geq b$ can be transformed by multiplying by -1.

Then it becomes: $\sum_{j=1}^{n} (\underline{-a_{ij}}) x_j \geq \underline{-b_i}$

II) Equality constraints:

$$\sum_{j=1}^{n} a_{ij} x_j = b_i \quad \rightarrow \quad \sum_{j=1}^{n} a_{ij} x_j \leq b_i, \qquad \sum_{j=1}^{n} a_{ij} x_j \geq b_i$$

split into two constr. to get rid of equality!

now we can use I) for this.

Ex.

$$3x_1 - 4x_2 = 7 \rightarrow 3x_1 - 4x_2 \le 7$$

$$\boxed{3x_1 - 4x_2 \ge 7} \qquad \boxed{-3x_1 + 4x_2 \le -7}$$

III) Free variables:

any real number x_i can be decomposed into $x_i = x_i^+ - x_i^-$

Note that decomp. is not unique.

$$x_i \rightarrow \boxed{x_i = x_i^+ - x_i^-}, \quad x_i^+ \ge 0, \quad x_i^- \ge 0$$

Ex.

$$x_i = 3 \rightarrow x_i = 3 - 0 \; (= 4 - 1 = 5 - 2 = \dots)$$