$$\max \; \zeta = x_1 + x_2$$

s.t.

$$\begin{aligned} -x_1 + x_2 &\leq 1 \\ x_1 \quad\quad &\leq 3 \\ x_2 &\leq 2 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Simplex tableau :

$$\begin{array}{l} \zeta = \overset{\downarrow}{x_1} + x_2 \\ \hline w_1 = 1 + x_1 - x_2 \\ \rightarrow w_2 = 3 - x_1 \\ w_3 = 2 \quad\quad - x_2 \end{array} \qquad (\mathrm{I})$$

Exch. x_1 and w_2

entering/new BV

leaving/new NBV

$$w_2 = 3 - x_1 \Rightarrow x_1 = 3 - w_2$$

$$\begin{aligned}
\varsigma &= 3 - w_2 + x_2 \\
\hline
w_1 &= 4 - w_2 - x_2 \\
x_1 &= 3 - w_2 \\
w_3 &= 2 \qquad\quad - x_2
\end{aligned}$$

(II)

exchange x_2 (NBV) and w_3 (BV)

$$w_3 = 2 - x_2 \implies x_2 = 2 - w_3$$

New simplex tableau:

$$\zeta = 3 - w_2 + 2 - w_3$$
$$= 5 - w_2 - w_3$$

$$w_1 = 4 - w_2 - 2 - w_3$$
$$= 2 - w_2 - w_3$$
$$x_1 = 3 - w_2$$
$$x_2 = 2 - w_3$$

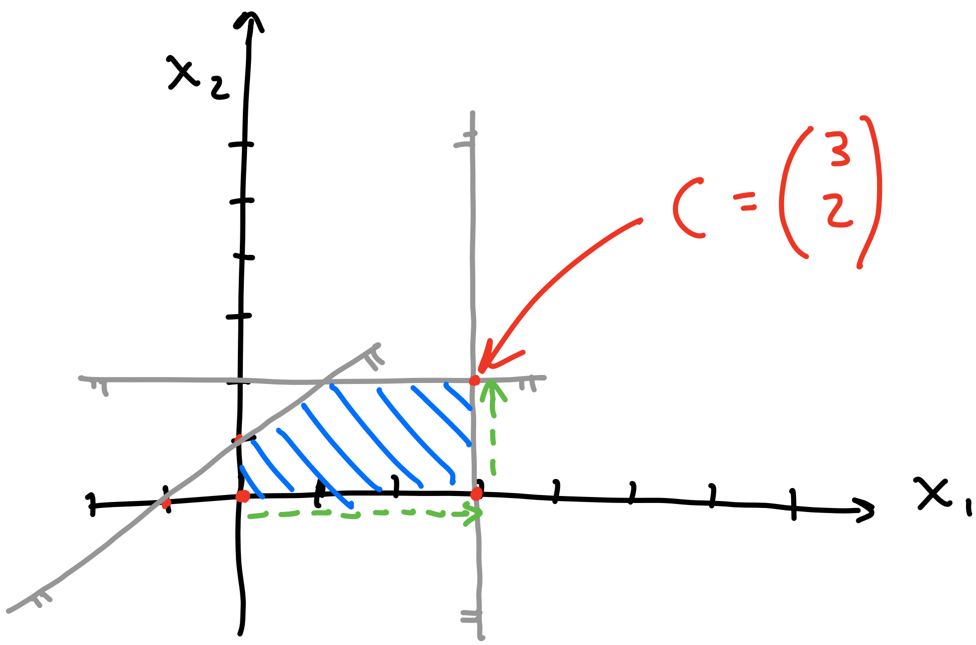
(III)

Not able to increase obj. func. value now, so we are done.

$$c^{*} = \begin{pmatrix}3\\2\end{pmatrix}, \quad x^{0} = \begin{array}{ccccc} x_1 & x_2 & w_1 & w_2 & w_3 \\ \multicolumn{5}{c}{(3,\ 2,\ 2,\ 0,\ 0)} \end{array}$$

$$c^{T}\tilde{x} = \begin{pmatrix}2\\3\end{pmatrix}\begin{pmatrix}3 & 2\end{pmatrix} = 2\cdot 3 + 3\cdot 2 = \underline{\underline{12}}$$

Graphical representation:



$$-x_1 + x_2 \leq 1 \quad (1)$$

$$x_1 \leq 3 \quad (2)$$

$$x_2 \leq 2 \quad (3)$$

1. (p. 57) Yes, because of the added slack variables.

2. (p. 58)

Equational form

$$\max.\ 25x + 30y$$

s.t.

$$\frac{1}{200}x + \frac{1}{140}y + w_1 = 40$$

$$x + w_2 = 6000$$

$$y + w_3 = 4000$$

$$x, y \geq 0$$

Simplex tabl.

$$\zeta = 25x + 30y$$

$$w_1 = 40 - \frac{1}{200}x - \frac{1}{140}y$$

$$w_2 = 6000 - x$$

$$w_3 = 4000 - y$$

3. (p.58)

4. (p.59) Yes, it contains the same information but a new (and impr.) obj. value and a new basic feasible solution.

5. (p.59) Yes.

6. (p.59) A pivot step involves a leaving BV and an entering NBV. The leaving NBV needs to have a positive coeff. in the obj. func for the value to increase. Additionally it needs to be neg. in the pivot row or else it is not restricted and can be arbitrarily large (unbounded).

7. (p. 70) By making pivot steps we are essentially visiting all vertecies of the geometry.

8. (p. 60) Yes. A positive coefficient in the pivot row indicates unboundedness because it means the variable cannot be restricted from the top.

9. (p. 63) A degenerate pivot step might be required to progress with the alg. even though it doesn't increase the obj. v.

10. (p. 63) It sometimes leads to cycling when degenerate pivot steps are repeated since the algorithm looks for a vertex where the obj. value is <u>at least</u> that

of the previous value. There are at most $\binom{m+n}{m}$ steps, so the alg. will repeat after this.

$$\varphi^1 \leq \cdots \leq \varphi^{\binom{m+n}{m}} \leq \varphi^1 \Rightarrow \varphi^1 = \varphi^2 = \cdots = \varphi^{\binom{m+n}{m}}$$