Lecture 3 (1.9)

Remember, we put an LP into std. form because it's better for coding a solution method.

$$\max \ \varphi = c^T x \quad \text{s.t.} \quad Ax \leq b \ , \ x \geq 0$$

Example of converting an LP into std. form:

$$\min \ 1{,}2x_1 + 1{,}8x_2 + x_3$$

s.t.

$x_1 \geq \frac{1}{3}$ $\quad -2x_3 \leq 0$

$x_1 - 2x_3 \leq 0$ $\quad x_2 - x_1 \leq 0$ $\quad x_2, x_3 \geq 0$

$x_3 - 2x_2 \leq 0$ $\quad x_1 + x_2 + x_3 = 1$

Transforming into standard form:

x_1 is a free variable:

$x_1 = x_1^+ - x^-$, $x_1^+, x^- \geq 0$

$x_1 - 2x_3 \leq 0 \Rightarrow x_1^+ - x_1^- - 2x_3 \leq 0$

convert to $\leq$:

$x_1 \geq \frac{1}{3} \Rightarrow -(x_1^+ - x_1^-) \leq \frac{1}{3}$ ← negative?

equality constraint:

$x_1 + x_2 + x_3 = 1 \Rightarrow x_1^+ - x_1^- + x_2 + x_3 \leq 1$

$-x_1^+ + x_1^- - x_2 - x_3 \leq -1$

$$\max\ (-1{,}2\ .\ 1{,}2\ ,\ -1{,}8\ ,\ -1)$$

s.t.

$$\overbrace{\begin{pmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & -2 \\ 1 & -1 & -2 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & 0 & -2 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 \end{pmatrix}}^{A} \leq \overbrace{\begin{pmatrix} 1/3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}}^{b}$$

$$x_1^+\ x_1^-\ x_2\ x_3 \geq 0$$

The Simplex method

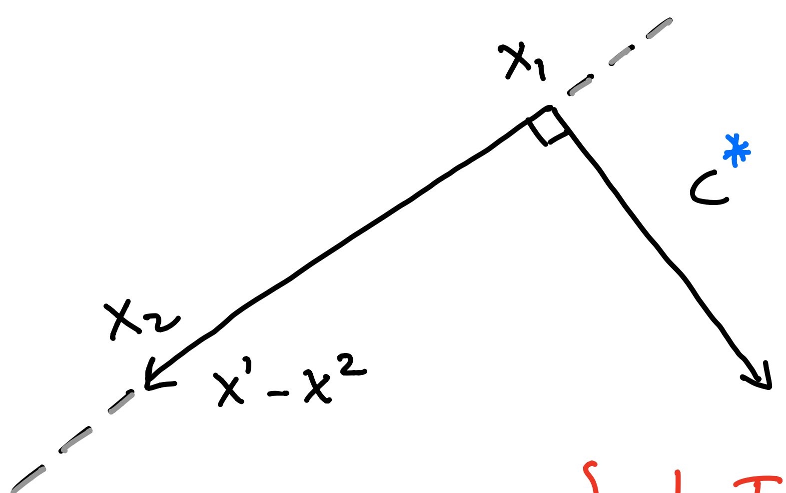
Graphical approach

obj. f. $c^T x = c_1 x_1 + c_2 x_2$

let $x^1, x^2 \in \mathbb{R}^n$: $c^T x^1 = c^T x^2$

$$c^T (x^1 - x^2) = 0$$

$$c \perp (x^1 - x^2)$$



$\{ x \mid c^T x = c^T x^2 \}$

forms a line.

In direction c:

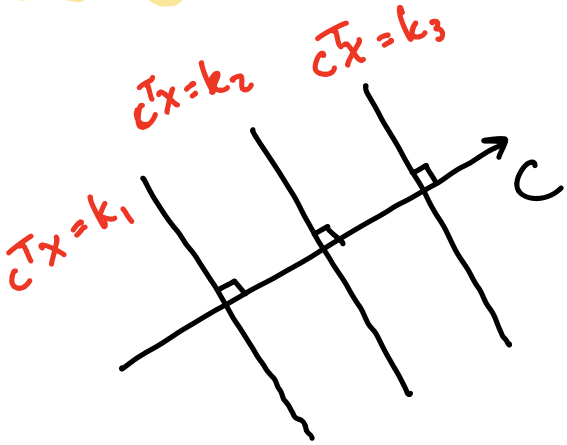
$x^2 + tc \quad , \; t > 0$

What happens to the objective function when we go along the line c?

$$c^T(x^2 + ct) = \boxed{c^Tx^2 + tc^Tc > c^Tx^2}$$

The value of the obj. f. is increasing!

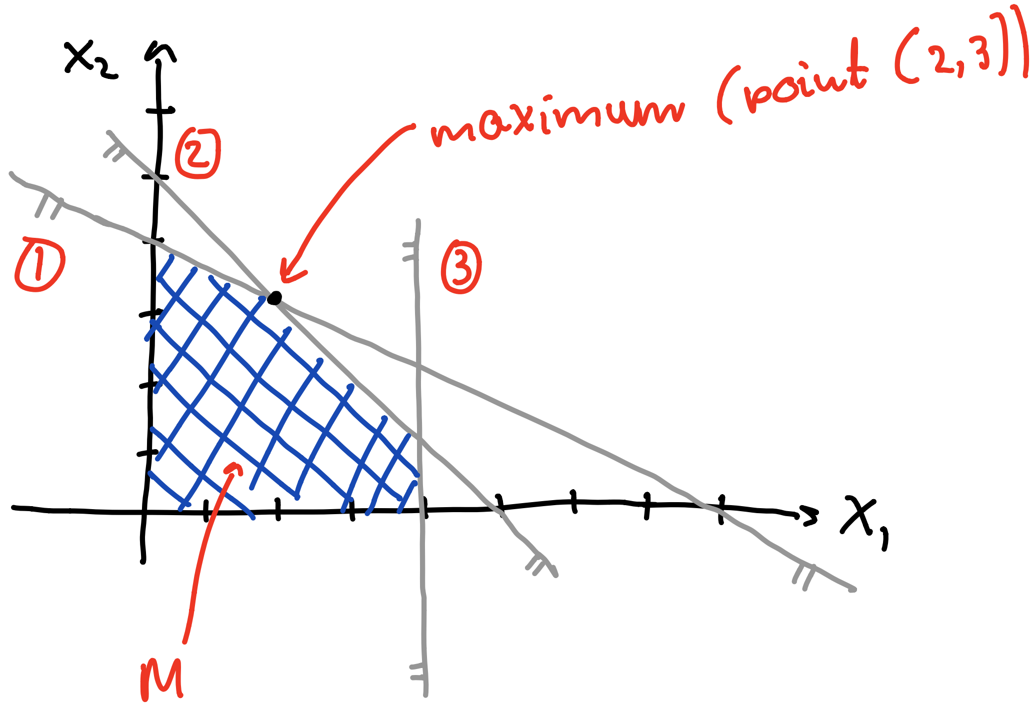
Level lines (explanation)



$k_1 < k_2 < k_3 \ldots$

Example. Properties of a feasible set.

$$M = \left\{ x \in \mathbb{R}^2 \;\middle|\; \begin{array}{l} \overset{①}{x_1 + 2x_2 \le 8} \quad \overset{②}{x_1 + x_2 \le 5} \\ \overset{③}{x_1 \le 4} \quad x_1, x_2 \ge 0 \end{array} \right\}$$



Let $c = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, solution is the point $\overset{\hat{x}}{(2,3)}$.

$$\boxed{\hat{x} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad c^T x = 1 \cdot 2 + 2 \cdot 3 = 8}$$

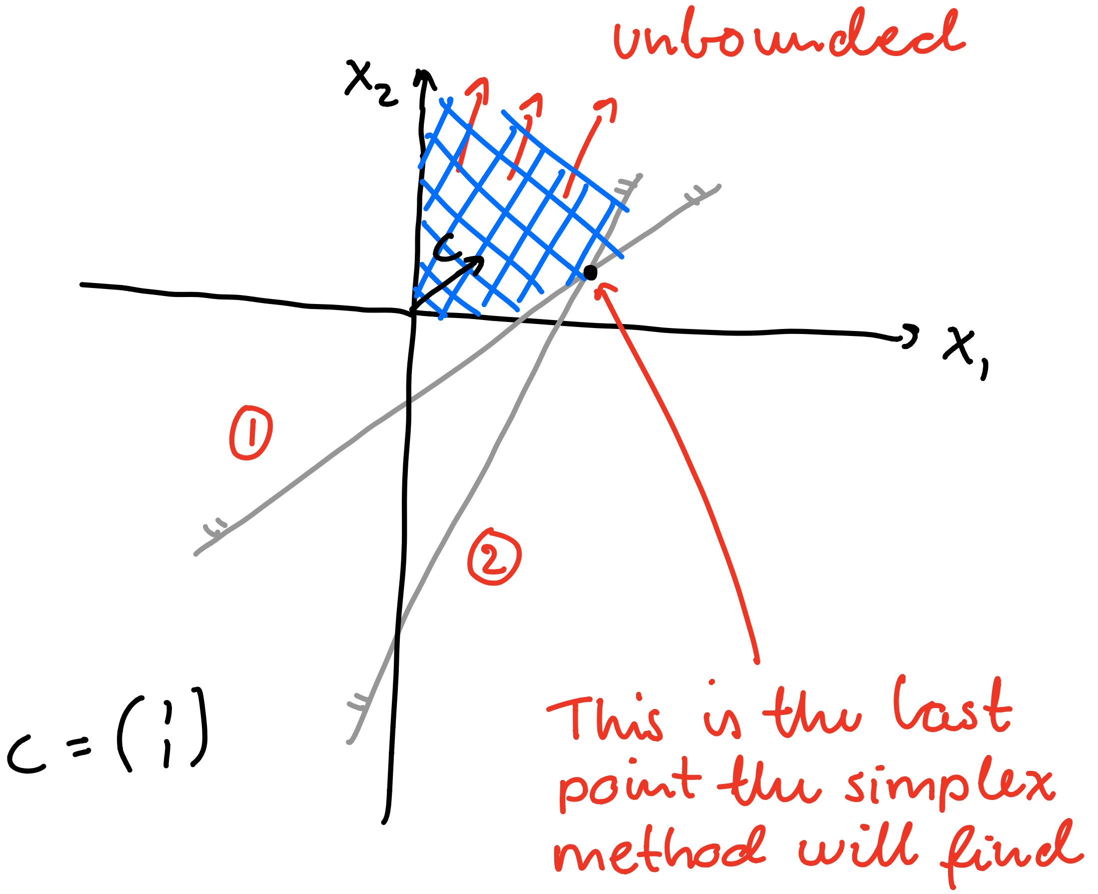
This is the full solution!

Example. No solution

$$\min \ -x_1 - x_2 \longrightarrow \max \ x_1 + x_2$$

s.t.

$$x_1 - x_2 \le 1, \quad 5x_1 - x_2 \le 10 \quad x_1, x_2 \ge 0$$



unbounded
x_2
x_1
c
①
②
$c = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
This is the last point the simplex method will find.