

Lecture 2 (26.8)

Remember: $\left\{ \begin{array}{l} C^T x \text{ s.t. } Ax = b, x \geq 0 \\ C, x \in \mathbb{R}^n, A \text{ is an } (m,n) \text{ mat.} \\ b \in \mathbb{R}^m \end{array} \right.$

every opt. problem can be formalized into this.

- o What are the decision variables?
 - Answer: x_i
- o What is the obj. f.?
 - Answer: $C^T x$
- o What are the constraints?
 - Answer: $Ax \leq b$

Example 2: Diet problem (Google it)

Task: find the combination of med. that satisfies the need at min. cost.

x_i = amount of medication $A_i = 1, 2, 3, 4$

$$c^T x = 1500x_1 + 200x_2 + 1200x_3 + 900x_4$$

s.t.

$$30x_1 + 5x_2 + 20x_3 + 10x_4 \geq 10$$

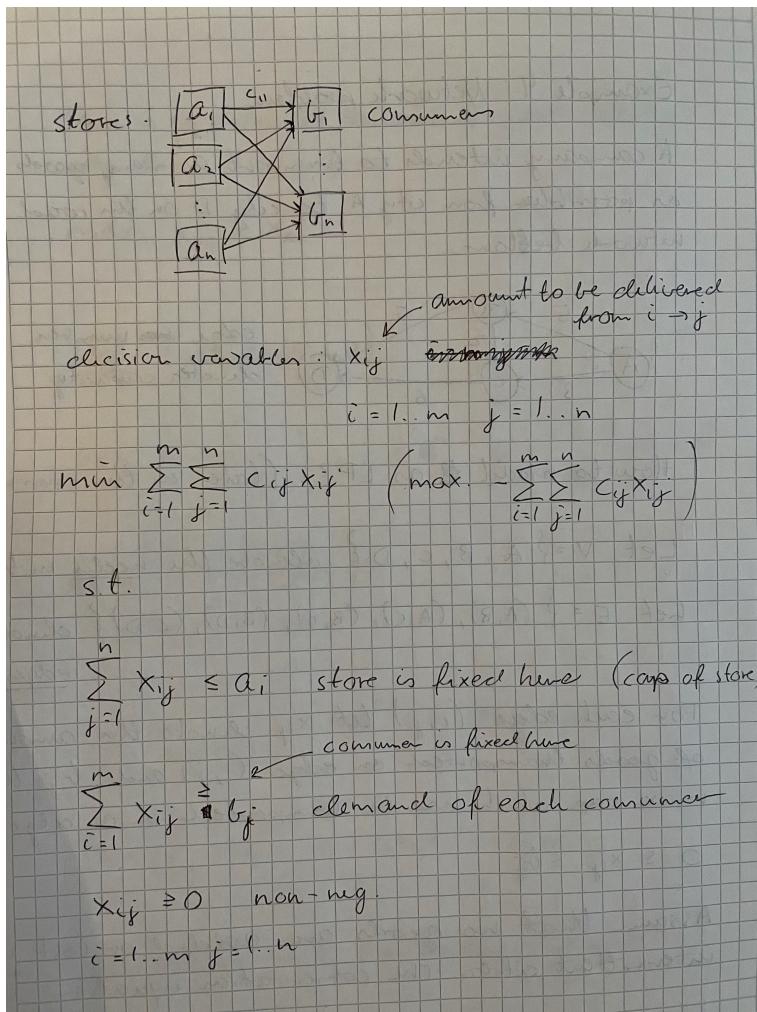
$$10x_1 + 0x_2 + 10x_3 + 20x_4 \geq 5$$

$$50x_1 + 3x_2 + 50x_3 + 30x_4 \geq 5\frac{1}{2}$$

$$x_i \geq 0$$

Example 4 : Transportation problem

A transport company has m stores and wants to deliver a product from these stores to n consumers. The delivery of one prod. from store i to consumer j costs c_{ij} . Store i has stored a_i items of a product. Consumer j has a demand of b_j items of a prod. The comp wants to satisfy the demand of all consumer. The company wants to minimize delivery cost.



Example 4: Network problem

A company intends to transport as many goods as possible from city A to city D on the road network below.



How to model ~~it~~ as LP? (later in the course)

Let $V = \{A, B, C, D\}$ denote the nodes in the netw.

Let $E = \{(A, B), (A, C), (B, C), (B, D), (C, D)\}$ denote the edges of the

For each edge (i, j) let x_{ij} denote the amount of goods transported on edge (i, j) and b_{ij} the max. cap. which can be transported on an edge,

$$0 \leq x_{ij} \leq b_{ij}$$

Assume that no goods are added or lost in intermediate cities. The conservation equation: "out-in =

$$X_{BD} + X_{BC} - X_{AB} = 0 \quad B \text{ constraint}$$

$$X_{CD} - X_{AC} - X_{BC} = 0$$

How to transform an arbitrary LP
into standard form? (st. form:
 $c^T x$ s.t. $Ax \leq b$)

I) Inequality constraints:

The constraint $\sum_{j=1}^n a_{ij} x_j \geq b_i$ can be transformed by multiplying by -1 .

Then it becomes: $\sum_{j=1}^n (-a_{ij}) x_j \leq -b_i$

II) Equality constraints:

$$\sum_{j=1}^n a_{ij} x_j = b_i \rightarrow \sum_{j=1}^n a_{ij} x_j \leq b_i$$

split into two
const. to get
rid of equality!

$$\sum_{j=1}^n a_{ij} x_j \geq b_i$$



now we can use 1)
for this.

Ex.

$$3x_1 - 4x_2 = 7 \rightarrow 3x_1 - 4x_2 \leq 7$$

$$\begin{array}{l} \xrightarrow{\quad} \boxed{3x_1 - 4x_2 \geq 7} \\ \xrightarrow{\quad} \boxed{-3x_1 + 4x_2 \leq -7} \end{array}$$

III) Free variables :

any real number x_i can be
decomposed into $x_i = x_i^+ - x_i^-$

Note that decompos. is not unique.

$$x_i \rightarrow \boxed{x_i = x_i^+ - x_i^-}, \quad x_i^+ \geq 0, \quad x_i^- \geq 0$$

Ex.

$$x_i = 3 \rightarrow x_i = 3 - 0 \quad (= 4 - 1 = 5 - 2 = \dots)$$