5.5 (Vanderbei)

$$\begin{array}{llll}
\text{max.} & 2x_1 + 8x_2 - x_3 - 2x_4 & & \\
\text{s.t.} & 2x_1 + 3x_2 + 6x_4 & \leq & 6 \\
 & -2x_1 + 4x_2 + 3x_3 & \leq & \frac{3}{2} \\
 & 3x_1 + 2x_2 - 2x_3 - 4x_4 & \leq & 4 \\
 & x_1, x_2, x_3, x_4 & \geq & 0
\end{array}$$

$$\begin{array}{l}
\zeta = 2x_1 + 8x_2 - x_3 - 2x_4 \\
\hline
w_1 = 6 - 2x_1 - 3x_2 - 6x_4 \\
w_2 = 1{,}5 + 2x_1 - 4x_2 - 3x_3 \\
w_3 = 4 - 3x_1 - 2x_2 + 2x_3 + 4x_4
\end{array} \quad (P)$$

a)

$$\begin{array}{rcl} \mathcal{Y} & = & -6y_1 - 1{,}5y_2 - 4y_3 \\ \hline z_1 & = & -2 + 2y_1 - 2y_2 + 3y_3 \\ z_2 & = & -8 + 3y_1 + 4y_2 + 2y_3 \\ z_3 & = & 1 + 3y_2 - 2y_3 \\ z_4 & = & 2 + 6y_1 - 4y_3 \end{array} \quad (D)$$

b) Basic (BV) : x_1, w_2, x_3

Non-basic (NBV) : w_1, x_2, w_3, x_4

c) $x = \begin{pmatrix} 3{,}0 \\ 0 \\ 2{,}5 \\ 0 \end{pmatrix}$, $w = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$c^T x = 2 \cdot 3 + 8 \cdot 0 - 1 \cdot 2{,}5 - 2 \cdot 0 = 3{,}5$$

The solution is feasible. x_2 can be exchanged with x_1.

d)

$$
\begin{array}{c} \\ x_1 \\ w_2 \\ x_3 \end{array}
\begin{array}{c}
\begin{array}{ccccc} \phantom{3{,}5} & w_1 & x_2 & w_3 & x_4 \end{array} \\
\begin{pmatrix}
3{,}5 & -0{,}25 & 6{,}25 & -0{,}5 & -1{,}5 \\
3 & -0{,}5 & -1{,}5 & 0 & -3 \\
0 & 1{,}25 & -3{,}25 & -1{,}5 & 13{,}5 \\
2{,}5 & -0{,}75 & -1{,}25 & 0{,}5 & -6{,}5
\end{pmatrix}
\end{array}
$$

$$
\begin{array}{c} \\ y_1 \\ z_2 \\ y_3 \\ z_4 \end{array}
\begin{array}{c}
\begin{array}{cccc} \phantom{-3{,}5} & z_1 & y_2 & z_3 \end{array} \\
\begin{pmatrix}
-3{,}5 & -3 & 0 & -2{,}5 \\
0{,}25 & 0{,}5 & -1{,}25 & 0{,}75 \\
-6{,}25 & 1{,}5 & 3{,}25 & 1{,}25 \\
0{,}5 & 0 & 1{,}5 & -0{,}5 \\
1{,}5 & 3 & -13{,}5 & 6{,}5
\end{pmatrix}
\end{array}
$$

$$\zeta = -3{,}5 - 3{,}0z_1 \qquad - 2{,}5z_3$$

$$y_1 = 0{,}25 - 0{,}5z_1 - 1{,}25y_2 - 0{,}75z_3$$

$$z_2 = -6{,}25 + 1{,}5z_1 + 3{,}25y_2 + 1{,}25z_3$$

$$y_3 = 0{,}5 \qquad + 1{,}5y_2 - 0{,}5z_3$$

$$z_4 = 1{,}5 + 3{,}0z_1 - 13{,}5y_2 + 6{,}5z_3$$

e)

$$y = \begin{pmatrix} 0{,}25 \\ 0 \\ 0{,}5 \end{pmatrix}, \quad z = \begin{pmatrix} 0 \\ -6{,}25 \\ 0 \\ 1{,}5 \end{pmatrix}$$

$$b^T y = -6(0{,}25) - 1{,}5(0) - 4(0{,}5) = -3{,}5$$

The solution is <u>not</u> feas. because of $z_2 = -6{,}25 \ldots$

f) $x = \begin{pmatrix} 3,0 \\ 0 \\ 2,5 \\ 0 \end{pmatrix}, \quad w = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ (P)

$y = \begin{pmatrix} 0,25 \\ 0 \\ 0,5 \end{pmatrix}, \quad z = \begin{pmatrix} 0 \\ -6,25 \\ 0 \\ 1,5 \end{pmatrix}$ (D)

$x_1 z_1 = x_2 z_2 = x_3 z_3 = x_4 z_4 = 0$ ✓

$y_1 w_1 = y_2 w_2 = y_3 w_3 = 0$ ✓

g) No, because there are still variables with positive coeff. in obj. func. $(+6,25 x_2)$.

h) x_2 will enter basis and x_1 will leve. Pivot will <u>not</u> be degenerate.

used for a):

$$\begin{pmatrix} 0 & 2 & 8 & -1 & -2 \\ 6 & -2 & -3 & 0 & -6 \\ 1{,}5 & 2 & -4 & -3 & 0 \\ 4 & -3 & -2 & 2 & 4 \end{pmatrix}$$

$$\begin{array}{cc} & \begin{array}{cccc} \phantom{-1{,}5} & y_1 & y_2 & y_3 \end{array} \\ \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \\ z_4 \end{array} & \begin{pmatrix} 0 & -6 & -1{,}5 & -4 \\ -2 & 2 & -2 & 3 \\ -8 & 3 & 4 & 2 \\ 1 & 0 & 3 & -2 \\ 2 & 6 & 0 & -4 \end{pmatrix} \end{array}$$