Lecture 21

$$y_j - y_i + z_{ij} = c_{ij} \qquad (i,j) \in A$$

g = root node (redundant)

Let $y_g = 0$ ← We fix this value to remove uncert - ainty.

Arc (g,e) : $y_e - y_g + \underbrace{z_{eg}}_{}^{=0} = c_{ge}$

$$y_e - y_g = 19 \Rightarrow \boxed{y_e = 19}$$

From example in the slide.

Arc (b, g) : $y_b - y_g = 33$

$\Rightarrow \boxed{y_b = 33}$

Arc (b, c) : $y_b - y_c = 65$

$\Rightarrow \boxed{y_c = 98}$

Arc (b, f) : $y_b - y_f = 48$

$\Rightarrow \boxed{y_f = -15}$

free variable. can take any value.

Arc (f, a) : $y_a - y_f = 56$

$\Rightarrow \boxed{y_a = 41}$

Arc (a, d) : $y_d - y_a = 28$

$$\Rightarrow \boxed{y_d = 69}$$

(y_i) is part of the dual solution.

Slack variables :

$$z_{ij} \begin{cases} 0 & \text{if } (i,j) \text{ belongs to the spann. tree.} \\ z_{ij} = y_i - y_j + c_{ij} & \end{cases}$$

has to be non-negat.

Now let's check for non-neg.:

$$z_{be} = y_b - y_e + c_{be} = 33 - 19 + 7 = 21$$

$$z_{ac} = y_a - y_c + c_{ac} = 41 - 98 + 48 = \underline{-9}$$

we can stop! Not feasible

Current spanning tree is not optimal since the dual solution is not feasible.

We now need to make a pivot step to get a new spanning tree.

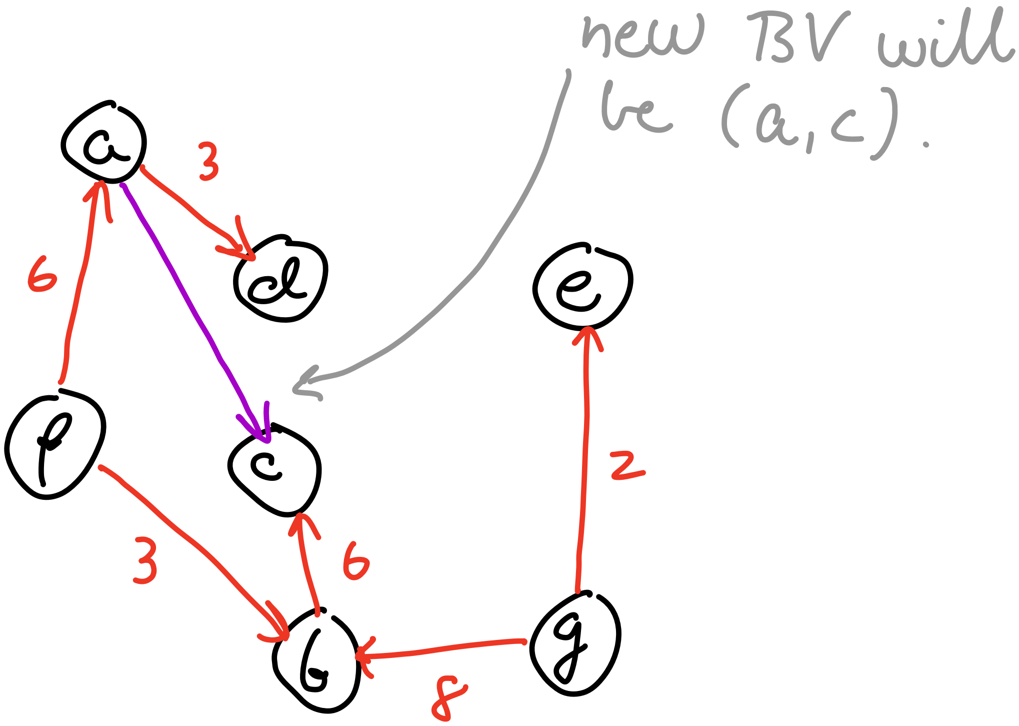
Pivot step in the network SM

Will get this on the exam!!

Choose an infeasible dual slack variable representing an NBV arc which will become a new BV.

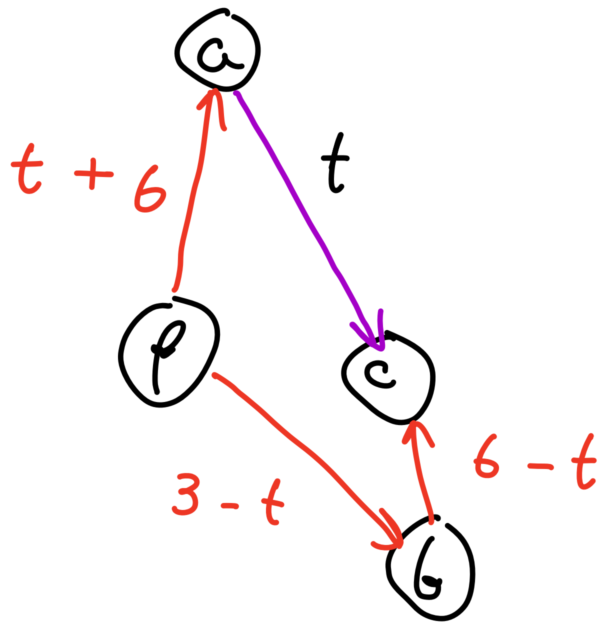
Here, $z_{ac} = -9 \Rightarrow$ arc (a,c) will enter the basis (become new BV).

Which arc should leave the basis?



Remove a red arc from the current solution to preserve the spanning tree.

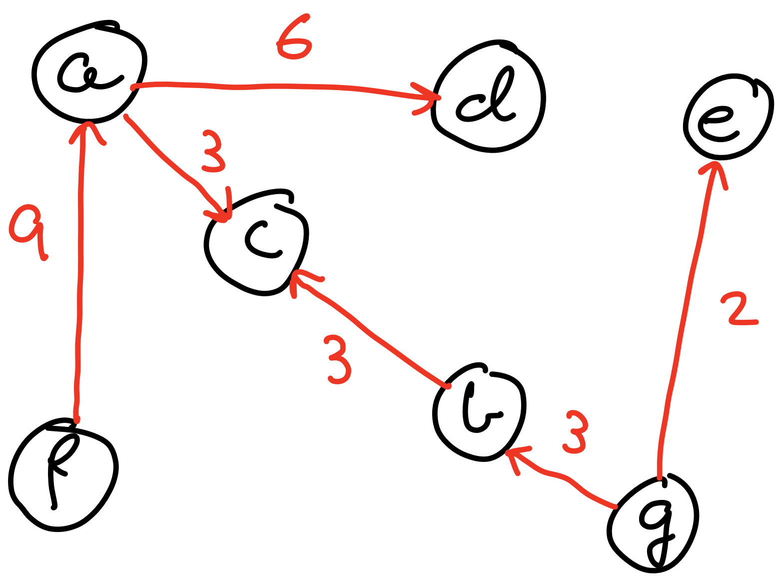
Assign a positive value $t > 0$ on the new BV (a,c) and then the balance constraints have to be fulfilled along the circle (a,c), (c,b), (f,b) and (f,a).



How much can t be increased?

$x_{fb} = 0$ for $t = 3$ $\Rightarrow$ (f,b) leaves the basis. We now have a new spanning tree.

New solution



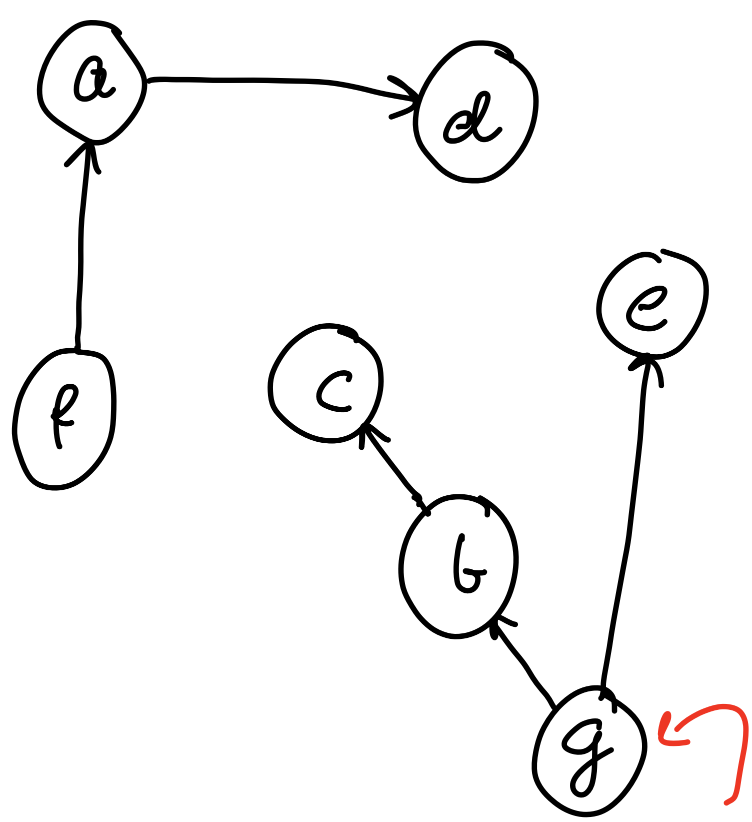
Optimal ?

Check feasibility of the dual solution.

Two connected components because we took out (a,c) and (f,b). (old and new ones).

leaving arc

entering arc



This is still our root/redundant node.

We obtain two sets:

$\{f, a, d\}$ and $\{c, b, d, g\}$

which are not connected.

How to update the dual variables?

Calc. of dual solutions starts with the root node.

$\Rightarrow$ dual solutions in the parts containing the root node will not change, $\{c, b, g, e\}$.

C_{ac}

$(a,c): \; y_c - y_a = 48$

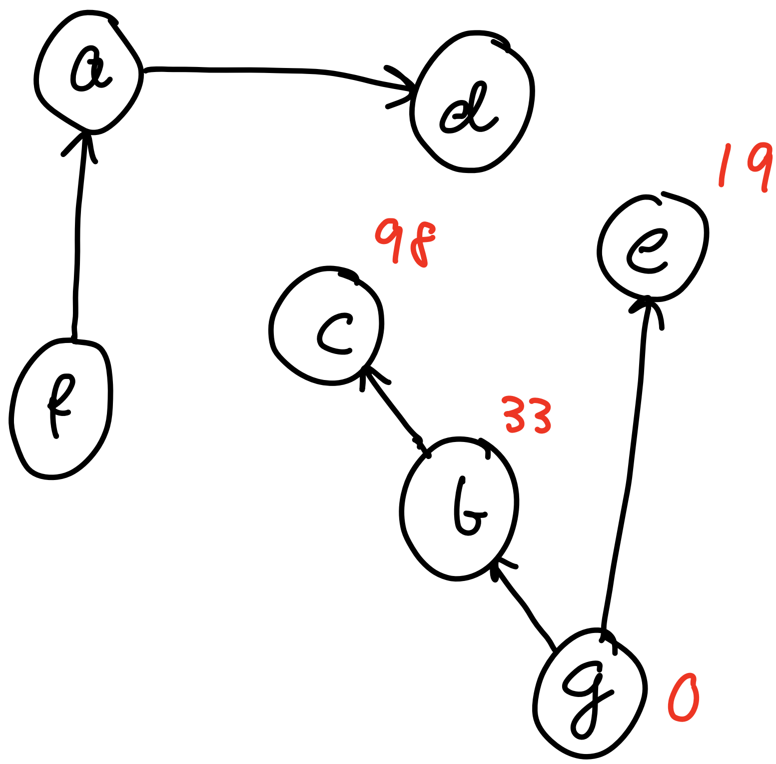
$\Rightarrow \boxed{y_a = 50}$

$(f,a): \; y_a - y_f = 56$

$\Rightarrow \boxed{y_f = -6}$

$(a,d): \; y_d - y_a = 28$

$\Rightarrow \boxed{y_d = 78}$



Previous dual solution:

$$\left.\begin{aligned} y_a &= 41 \\ y_f &= -15 \\ y_d &= 69 \end{aligned}\right\} + 9$$

??

We choose the infeasible dual slack variable $z_{ac} = -9$

$\Longrightarrow$ (a,c) become new BV.

$\Longrightarrow$ new value of z_{ac} after this iteration becomes zero.

After iteration :

$$0 = y_a - y_c + c_{ac}$$

Before iteration :

$$-9 = \tilde{y}_a - \tilde{y}_c + c_{ac}$$

c belongs to root node part

$$\Rightarrow \boxed{y_a = \tilde{y}_a + q}$$

Analogously for all nodes not belonging to the root node part (here, y_f and y_d).

Slack variables (for NBV arcs):

$$z_{ae} - y_a + y_e = c_{ae}$$

unchanged

$\tilde{y} + q$

root node/unchanged

old value

$$\Rightarrow \underset{41}{z_{ae}} = \underset{32}{\tilde{z}_{ae}} + 9$$

Is the solution optimal?

No, since (b, a) is -10. Infeas. dual sol.

Do second / third iteration to become optimal (see example in the book).

If the current solution is primal infeas. and dual feas. then we use the dual network SM.

Example from the book

- Primal infeas. since $x_{db} = -8$
- Dual feas., all slack vars are positive.

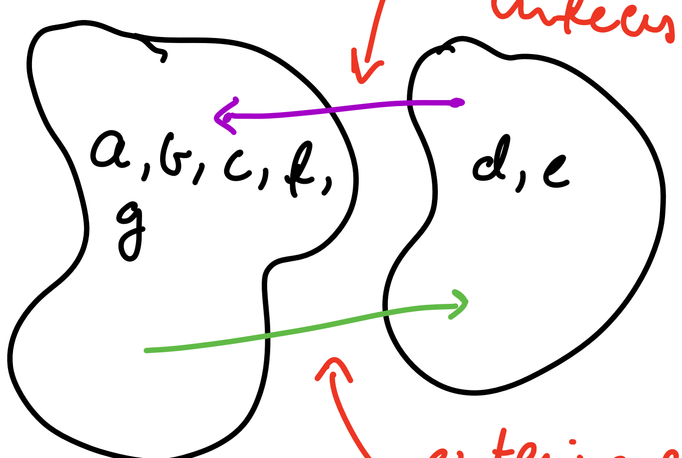
Choose neg. primal variable (here, x_{db}), which represents the leaving arc.

Analogously to the primal sm: after leaving the basis, there remain two disjoint sets:

$\{d, e\}$ and $\{a, b, c, f, g\}$

∘ Observation 1:

we take out the connecting arc with infeas. primal flow $(-\delta)$.



entering arc must have opposite direction. since it has a pos. primal flow it becomes feasible for (P).

Also, the balance constraint must be satisfied.

We have four possibilities for the entering arc according to this criteria.