NTNU

TDT4136 - AI Intro

Exercise 2

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1,3	2,2	3,1	KB	α_2	α_3
				T	
P				T	
W				T	T
PW				T	T
	P				
P	P				
W	P				T
PW	P				T
	W			T	
P	W			T	
	PW				
P	PW				
		P		T	
P		P		T	
W		P	T	T	T
PW		P		T	T
	P	P			
P	P	P			
W	P	P			T
PW	P	P			T
	W	P		T	
P	W	P		T	
	PW	P			
P	PW	P			
		W		T	
P		W		T	
	P	W			
P	P	W			
		PW		T	
P		PW		T	
	P	PW			
P	P	PW			

Table 1: Wumpus

1. We can see that KB $\models \alpha_2$ and KB $\models \alpha_3$ since the single row with KB = True has both α_2 and α_3 True.

2. a. True, False is never true, ergo this statement doesn't violate the definition of entailment.

   b. False. Inverse of above, False is never true, and the statement will therefore always violate the definition.

   c. True. $M(\text{LHS}) \subseteq M(\text{HS})$

   d. False. With both A and B True, the LHS will be true, but the RHS will be false. Thus $M(\text{LHS}) \not\subseteq M(\text{HS})$.

e. True. $M(\text{LHS}) = M(\text{HS})$ for this statement.

f. True. The only case that would make the right hand side False is $A = B = \text{True}, C = False$ for which the LHS is also false.

3. a. $\{T, F\}, \{TF, FT, TT\}, \{T, F\}$. $2 * 3 * 2 = 12$

   b. Only false if all propositions are True, therefore $2^4 - 1 = 15$ models.

   c. 0. $A = T, B = F$ means that $A \implies B$ can never be true.

4. a. Valid. $\text{Smoke} \implies \text{Smoke} \equiv \neg\text{Smoke} \vee \text{Smoke}$

   b. Neither. All models except $\text{Smoke} = \text{True}, \text{Fire} = \text{False}$ satisfies the sentence.

   c. Neither.

$$(\text{Smoke} \implies \text{Fire}) \implies (\neg\text{Smoke} \implies \neg\text{Fire}) \equiv (\neg\text{Smoke} \vee \text{Fire} \implies \neg\text{Fire} \vee \text{Smoke})$$

   False for $\text{Smoke} = \text{False}, \text{Fire} = \text{True}$

5. a. ${}^3/_4 Q$

   b. ${}^5/_8 Q$

   c. ${}^3/_4 Q$

2

1

1. $A \wedge B \wedge C$
2. $A \vee B \vee C$
3. $\neg A \vee B \vee C$
4. $(\neg A \vee C) \wedge (B \vee C)$
5. $\neg A \vee \neg B \vee C \vee D$
6. $(\neg A \vee C) \wedge (\neg A \vee D) \wedge (\neg B \vee C) \wedge (\neg B) \vee D)$
7. $(\neg A \vee \neg B \vee C) \wedge (A \vee \neg C) \wedge (B \vee \neg C)$

2

The items in the KB individually CNF to:

- $(\neg A \vee \neg C) \wedge (B \vee \neg C)$
- $(C \vee \neg E) \wedge D$
- $A \wedge D$

Since these statements are the KB, we can and them together, and simplify:

$$\begin{aligned}(\neg A \vee \neg C) \wedge (B \vee \neg C) \wedge (C \vee \neg E) \wedge D \wedge A \wedge D\\ (\neg A \vee \neg C) \wedge (B \vee \neg C) \wedge (C \vee \neg E) \wedge D \wedge A\\ \neg C \wedge (B \vee \neg C) \wedge (C \vee \neg E) \wedge D \wedge A\\ \neg C \wedge B \wedge \neg E \wedge D \wedge A\end{aligned}$$

And at this point we can see that for the KB to hold true, we must have $\neg E$.

3

1

a. $\text{Occupation}(\text{Emily}, \text{Lawyer}) \vee \text{Occupation}(\text{Emily}, \text{Surgeon})$

b. $\text{Occupation}(\text{Joe}, \text{Actor}) \wedge \exists o\{\text{Occupation}(\text{Joe}, o)\}$

c. $\forall x\{\text{Occupation}(x, \mathit{Doctor}) \wedge \text{Occupation}(x, Surgeon)\}$

2

a. $\forall x\{\text{Even}(x) \Leftrightarrow \exists y\{x = y + y\}\}$

b. $\forall x\{\text{Prime}(x) \Leftrightarrow \forall yz\{x = y \times z \implies z = 1 \vee y = 1\}\}$

c. $\forall x\{\text{Even}(x) \Leftrightarrow \exists yz\{x = y + z \wedge (\text{Prime}(y) \wedge \text{Prime}(z))\}\}$

3

$$\forall k\{\text{Key}(k) \implies \exists t_0\{\text{Before}(\text{Now}, t_0) \wedge \forall t\{\text{Before}(t_0, t) \implies \text{Lost}(k, t)\}\}$$

$$\begin{gathered}\forall xy\{\text{Sock}(x) \wedge \text{Sock}(y) \wedge \text{Pair}(x, y) \implies\\ \exists t_1\{\text{Before}(Now, t_1) \wedge \forall t\{\text{Before}(t_1, t) \wedge \text{Lost}(x, t)\}\} \vee\\ \exists t_2\{\text{Before}(Now, t_2) \wedge \forall t\{\text{Before}(t_2, t) \wedge \text{Lost}(y, t)\}\}\}\end{gathered}$$

4

With the following vocabulary:

$V = \{\text{Person}(x), \text{IsDNAOfPerson}(x, y), \text{IsParentOf}(x, y), \text{IsDerivedFromPersonsDNA}(x, y)\}$

where $\text{Person}(x)$ is true if x is a person, $\text{IsDNAOfPerson}(x, y)$ is true if y is person x's DNA, $\text{IsParentOf}(x, y)$ is true if y is x's parent and $\text{IsDerivedFromPersonsDNA}(x, y)$ is true if DNA x is derived from person x's DNA.

$$\begin{gathered}\forall xy\{\text{Person}(x) \wedge \text{IsDNAOfPerson}(y, x) \implies \neg\exists z\{\text{Person}(z) \wedge \text{IsDNAOfPerson}(y, z)\} \wedge\\ \forall a\{\text{Person}(a) \wedge \text{IsParentOf}(x, a) \implies \text{IsDerivedFromPersonsDNA}(y, a)\}\}\end{gathered}$$

4

1

a. $\Theta = \{x/\text{Rocky}\}$

b. $\Theta = \{x/\text{Leo}, y/\text{Rocky}\}$

c. $\Theta = \{x/\text{Rocky}, y/\text{Leo}\}$

d. Not possible.

e. Not Possible.

f. $\Theta = \{x/\text{Leo}, y/\text{FastestHorse(x)}\}$

g. $\Theta = \{x/\text{Marvin}, y/\text{Leo}\}$

2

Only the last sentence needs to be converted to CNF.

$$\forall x\{\text{Green}(x) \leftrightarrow \text{Bikes}(x) \vee \exists y\{\text{Drives}(x, y) \wedge \text{Hybrid}(y)\}\}$$