Propositional and Predicate Logics

Purpose: Gain experience with propositional and prepositional (first-order) logic by solving many small problems.

For each question with more than 3 parts (e.g. question 5 in section 1), you only need to deliver 3 (any 3) of the parts. As proper preparation for the exam, we strongly advise that you eventually do all the parts. The questions in this homework are very typical of those that appear on exams.

1 Models and Entailment in Propositional Logic

Note: Newer versions of the book appear to have mixed up the order of chapters 6 and 7. The exercises below named "Exercise 7.[...]" refer to exercises in the chapter entitled "Logical Agents", **not** the chapter on "Constraint Satisfaction Problems".

1. Exercise 7.1 ("Suppose the agent has...")

   Build the complete model table and show both entailments using model checking.

2. Exercise 7.4 ("Which of the following are correct?")

   Explain each answer with reference to model-checking, although you need not perform a complete model-check for any case. A verbal description of what the model-check would reveal is sufficient. I.e., you do not need to produce complete model tables unless they make it easier for you to answer the question.

   For example, the following is a sufficient answer to the 3rd question: $A \wedge B \models A \leftrightarrow B$

   > YES. All models in which A and B are both true (i.e. those satisfying the left-hand-side (LHS) of the entailment) are also models in which the right hand side (RHS) is true. The RHS is true whenever A and B have the same truth value, whether true or false.

3. Exercise 7.7 ("Consider a vocabulary with...")

4. Exercise 7.10 ("Decide whether each of the...")

   In this exercise, *neither* means *satisfiable but not valid*. Remember that the words used in this exercise are just symbols. You should ignore their standard interpretations and focus on the structure of the logical sentences. You do **not** need to provide a written proof, though you should understand the basis for each of your answers.

5. Consider a logical knowledge base with 100 variables, $A_1, A_2, \ldots, A_{100}$. This will have $Q = 2^{100}$ possible models. For each logical sentence below, give the number of models that satisfy it. Feel free to express you answer as a fraction of Q (without writing out the whole number $1267650600228229401496703205376 = 2^{100}$) or to use other symbols to represent large numbers.

(a) $A_1 \vee A_{73}$

(b) $A_7 \vee (A_{19} \wedge A_{33})$

(c) $A_{11} \rightarrow A_{22}$

(d) $(A_{11} \rightarrow A_{22}) \vee (A_{55} \rightarrow A_{66})$

(e) $(A_{11} \rightarrow A_{22}) \wedge (A_{55} \rightarrow A_{66})$

(f) $(A_1 \wedge A_2 \wedge \ldots \wedge A_{50}) \wedge (\neg A_{51} \wedge \neg A_{52} \wedge \ldots \wedge \neg A_{100})$

(g) $(A_1 \wedge A_2 \wedge \ldots \wedge A_{50}) \vee (\neg A_{51} \wedge \neg A_{52} \wedge \ldots \wedge \neg A_{100})$

(h) $(A_1 \wedge A_2 \wedge \ldots \wedge A_{50}) \vee (A_{51} \wedge A_{52} \wedge \ldots \wedge A_{100})$

(i) $(A_1 \wedge A_2 \wedge \ldots \wedge A_{50}) \vee [(A_{51} \wedge A_{52} \wedge \ldots \wedge A_{99}) \wedge \neg A_{100}]$

2 Resolution in Propositional Logic

1. Convert each of the following sentences to Conjunctive Normal Form (CNF).

   (a) $A \wedge B \wedge C$

   (b) $A \vee B \vee C$

   (c) $A \rightarrow (B \vee C)$

   (d) $(A \vee \neg C) \rightarrow B$

   (e) $(A \wedge B) \rightarrow (C \vee D)$

   (f) $(A \vee B) \rightarrow (C \wedge D)$

   (g) $(A \wedge B) \leftrightarrow C$

2. Consider the following Knowledge Base (KB):

   - $(A \vee \neg B) \rightarrow \neg C$
   - $D \wedge E \rightarrow C$
   - $A \wedge D$

   Use resolution to show that $KB \models \neg E$.

3 Representations in First-Order Logic

1. Exercise 8.9

2. Exercise 8.21

3. Exercise 8.23 - Feel free to use several logical sentences to express this one natural-language statement.

4. Exercise 8.24

4 Resolution in First-Order Logic

1. Find the unifier (θ) - if possible - for each pair of atomic sentences. Here, Owner, Horse and Rides are predicates, while FastestHorse is a function that maps a person to the name of their fastest horse:

   (a) Horse(x) . . . Horse(Rocky) Answer: $\theta = \{x/Rocky\}$

   (b) Owner(Leo, Rocky) . . . Owner(x, y)

   (c) Owner(Leo, x) . . . Owner(y, Rocky)

   (d) Owner(Leo, x) . . . Rides(Leo, Rocky)

   (e) Owner(x, FastestHorse(x)) . . . Owner(Leo, Rocky)

   (f) Owner(Leo, y) . . . Owner(x, FastestHorse(x))

   (g) Rides(Leo, FastestHorse(x)) . . . Rides(y, FastestHorse(Marvin))

   Do one of the next two resolution proofs as part of your homework. Save the other as preparation for the exam.

2. Use resolution to prove Green(Linn) given the information below. You must first convert each sentence into CNF. Feel free to show only the applications of the resolution rule that lead to the desired conclusion. For each application of the resolution rule, show the unification bindings, θ.

   - Hybrid(Prius)
   - Drives(Linn, Prius)
   - ∀x: Green(x) ↔ Bikes(x) ∨ [∃y: Drives(x, y) ∧ Hybrid(y)]

3. Follow the same procedure as above to prove that the Tigers can win a championship: CWC (Tigers), when given the following facts:

   - ∀x: [PC(x) ∨ [∃y: Player(y, x) ∧ Hero(y)]] → CWC(x)
     If a team is a previous champion (PC) or has a player on it that is a hero, then they Can Win a Championship (CWC).
   - ∀x: Hero(x) ↔ [∃y, z: Player(x, y) ∧ PC(z) ∧ Beat(y, z)]
     A player is a hero if and only if he has played on a team that has beaten a previous champion.
   - PC(Bears)
     The Bears are previous champions.
   - Player(Jack, Tigers)
     Jack plays for the Tigers.
   - Beat(Tigers, Bears)
     The Tigers have beaten the Bears.