

CTRNN Continuous-Time Recurrent Neural Network

The `clj-ctrnn` library simulates CTRNNs using the *forward Euler method* for approximating a series of solutions to the ordinary differential equation that govern neuron membrane potentials. The equation is:

$$\dot{y}_i = \frac{1}{\tau_i}(-y_i + \sum_{j=1}^N w_{ji}\sigma(y_j + \theta_j) + I_i) \quad (1)$$

where:

- τ_i : time-constant of post-synaptic neuron
- y_i : membrane potential of post-synaptic neuron
- w_{ji} : weight of connection between pre-synaptic neuron j and post-synaptic neuron
- $\sigma(x)$: the sigmoid activation function in equation 2
- y_j : membrane potential of pre-synaptic neuron j
- θ_j : bias (input sensitivity) of pre-synaptic neuron j
- I_i : any external input (such as a sensor reading) to post-synaptic node

$$\sigma(x) = \frac{1}{1 + e^{-x}} \quad (2)$$

For each step in time, t , with $t < \tau_i$, `clj-ctrnn` will

1. Approximate the rate of membrane potential change, $\frac{\Delta y}{\Delta t}$, for all neurons
2. Synchronously update all membrane potentials $y_{i+1} = y_i + \frac{\Delta y}{\Delta t} \cdot t$
3. Commit the new network state