Name:

Collaborator(s)[1]:

Math 347, Prof. Hildebrand, Summer 2019

HW 6, DUE IN CLASS MONDAY, 7/8

- **Grading:** 15 points total, broken down as follows:
  - **Presentation/effort:** **3 points**
  - **Graded problems:** **12 points**
- **About his assignment:** The main goal of this assignment is to practice proof-writing skills in a particularly simple context, that of functions and their properties. **Focus on the write-up of your proofs, and aim to produce a proof that is as close to perfect as possible in every respect. Use the class examples as models!** Make sure your proofs are in the correct logical order, have a proper start and end, include any necessary quantifiers (e.g., "for some", "there exists"), and connecting words ("therefore"), and justifications (e.g., "by injectivity of f"). **The quality and care in your write-up of the proofs will be a key factor in the grading of the assignment, so give it your best effort.**

#1 **4.33. Injectivity and surjectivity proofs, I.** Let $f : A \to B$ and $g : B \to C$ be given, and let $h = g \circ f$. Give careful, step-by-step, proofs of the following statements.

(a) If f and g are injective, then h is injective.

(b) If f and g are surjective, then h is surjective.

(c) If f and g are bijective, then h is bijective.

#2 **4.34. Injectivity and surjectivity proofs, II.** Let $f : A \to B$ and $g : B \to C$ be given, and let $h = g \circ f$. Determine which of the following are true. Give careful, step-by-step, proofs for true statements and counterexamples for false statements.

(a) If h is injective, then f is injective.

(b) If h is injective, then g is injective.

(c) If h is surjective, then f is surjective.

(d) If h is surjective, then g is surjective.

#3 **Relations between increasing/decreasing and injective/surjective/bounded functions.** Determine which of the following statements are true. Give careful, step-by-step, proofs for true statements and counterexamples for false statements.

(a) Every decreasing function from $\mathbb{R}$ to $\mathbb{R}$ is surjective.

(b) Every decreasing function from $\mathbb{R}$ to $\mathbb{R}$ is injective.

(c) Every surjective function from $\mathbb{R}$ to $\mathbb{R}$ is unbounded.

(d) Every unbounded function from $\mathbb{R}$ to $\mathbb{R}$ is surjective.

#4 **Surjectivity of specific functions.** Determine which of the following functions are surjective. In each case give a formal proof (by showing that every element in the target occurs as an image of some element in domain), or a disproof (by producing an element in the target, and proving that it is not the image of an element of f).

(a) (4.25(a)) $f(a, b) = a + b$ as a function from $\mathbb{Z} \times \mathbb{Z}$ to $\mathbb{Z}$.

(b) (4.25(b)) $f(a, b) = ab$ as a function from $\mathbb{Z} \times \mathbb{Z}$ to $\mathbb{Z}$.

(c) $f(a, b) = (a + b, a - b)$ as a function from $\mathbb{Z} \times \mathbb{Z}$ to $\mathbb{Z} \times \mathbb{Z}$.

[1] If you worked with another student or in a small group on this assignment, list the names of all students involved.