Worksheet: Some Common Errors in Proof-Writing

Each of the following statements has a logical, or notational, or language, error that makes it nonsensical. Some of the errors are minor issues, others are more serious logical errors. In each case try to find the problem with the given statement, then write down a correct version of the intended statement.

1. **Proof of $A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$:**

   Let $A \times (B \cap C)$. Then ...

2. **Proof of $A - (B - C) \subseteq (A - B) \cup (A - C)$:**

   Since $x \in A - (B - C)$, then $x \in A$ and $x \in (B - C)$, by the definition of a set difference.
   Since $x \in (B - C)$, then $x \in B$ and $x \notin C$, by the definition of a set difference.
   ....

3. Suppose $A - (B \cap C)$. Then ...

4. Since $x \in B \cap C$, then $x \in B \cap x \in C$, by the definition of an intersection.

5. Thus $x \in A$ and $x \notin B$ or $x \in A - C$.

6. Let $x \in A - B - C$. Then ...

7. Therefore $x \in (A \text{ and } C)$.

8. Then $x \in (B \cup C)^c \implies B^c \cap C^c$.

9. Therefore $x \notin B \cap C \implies x \notin B$ or $x \notin C$.

10. **False Proofs:** Find the error in the reasoning.

    (a) **Part of an attempted proof by contradiction:**

    ... Therefore $2x(2x-1) \geq 4x^2 - 1 = (2x-1)(2x+1)$.
    Dividing by $2x - 1$ we get $2x \geq 2x + 1$.
    Subtracting $2x$ on each side, we get $0 \geq 1$.
    Thus we have obtained a contraction. ...

    (b) **Part of another attempted proof by contradiction:**

    ... Therefore $2x(2x-1) \geq 4x^2 + 2x - 1$.
    Subtracting $2x - 1$ on each side, we get $2x(2x-1) - (2x-1) \geq 4x^2$.
    Simplifying, we get $(2x-1)^2 \geq 4x^2$.
    Taking the squareroot on each side gives $2x - 1 \geq 2x$.
    Subtracting $2x$ on each side, we get $-1 \geq 0$.
    Thus we have obtained a contraction. ...

    (c) **"Proof" that $0 = 2$ (thus creating something out of nothing):**

    Obviously, $4x^2 = 4x^2$.
    Rewriting the left and right sides, we get $(-2x)^2 = (2x)^2$.
    Taking the squareroot, we get $-2x = 2x$.
    Adding $x^2 + 1$ to each side gives $-2x + x^2 + 1 = 2x + x^2 + 1$.
    By algebra, this can be written as $(x-1)^2 = (x+1)^2$.
    Taking the squareroot, we get $x - 1 = x + 1$.
    Subtracting $x - 1$ on each side, we get $x - 1 - (x - 1) = x + 1 - (x - 1)$, i.e., $0 = 2$. QED.