Worksheet: Common Errors in Writing up Induction Proofs

Practice Problems Solutions

In each of the following examples, something is wrong with the set-up or write-up of the induction proof. Find the error and try to correct it.

1. **Example 1.**

   - **Base step:** $n = 6$.
   - **Induction step:** Let $k \in \mathbb{N}$ be given and assume $(*)$ is true for $n = k$.

   PROBLEM: *Base step and induction step don't match up. The first k-value for which the induction step is needed is $k = 6$, whereas as stated the step is claimed for all $k \in \mathbb{N}$.*
   FIX: *Add the condition $k \geq 6$ to the induction step. "Let $k \in \mathbb{N}$ with $k \geq 6$ be given ..."*

2. **Example 2.**

   - **Base step:** $n = 1$ and $n = 2$.
   - **Induction step:** Let $k \in \mathbb{N}$ with $k \geq 3$ be given and assume $(*)$ is true for $n = k$ and $n = k - 1$.

   PROBLEM: *Gap between base case and the first case of the induction step: The first case $k = 3$ of the induction step requires the cases 3 and 2, but the base step only gives the cases 1 and 2.*
   FIX: *Start induction step at $k = 2$ rather than $k = 3$: "Let $k \in \mathbb{N}$ with $k \geq 2$ be given . . . "*

3. **Example 3.**

   - **Base step:** $n = 1$ and $n = 2$.
   - **Induction step:** Assume $(*)$ is true for $n = k$ and $n = k - 1$. Then ...

   PROBLEM: *The variable k in the induction step is not quantified.*
   FIX: *Add "Let $k \in \mathbb{N}$ with $k \geq 2$ be given."*

4. **Example 4.**

   - **Base step:** $n = 1$ and $n = 2$.
   - **Induction step:** Let $k \in \mathbb{N}$ be given and assume $(*)$ is true for $n = k$ and $n = k - 1$.

   PROBLEM: *Here the first case induction step is $k = 1$, with the induction hypothesis being the cases $n = k$ and $n = k - 1$. But when $k = 1$, the second of these cases, $n = k - 1 = 0$, is out of range.*
   FIX: *Add the restriction $k \geq 2$ to the induction step: "Let $k \in \mathbb{N}$ with $k \geq 2$ be given."*