# Worksheet: Sum/Product Notations $\boxed{\text{Solutions}}$

## Practice Problems

1. Write the following sums and products explicitly without using sum/product notations, and then evaluate them. (Here $n$ denotes a positive integer and $c$ is a positive constant. (In all cases there is a simple formula for the given sum or product.)

   (a) $\sum_{i=1}^{n} c$ $\boxed{\underbrace{c+c+\dots+c}_{n} = cn}$

   (b) $\prod_{i=1}^{n} c$ $\boxed{\underbrace{c\cdot c\cdots c}_{n} = c^n}$

   (c) $\sum_{i=1}^{n}(n-i)$ $\boxed{(n-1)+(n-2)+\dots+1+0 = \sum_{i=0}^{n-1} i = \frac{n(n-1)}{2}}$

   (d) $\prod_{i=1}^{n}(n-i)$ $\boxed{(n-1)\cdot(n-2)\dots 1\cdot 0 = 0}$

   (e) $\prod_{i=1}^{n} i^n$ $\boxed{1^n\cdot 2^n\cdots n^n = n!^n}$

   (f) $\prod_{i=1}^{n}\left(1+\frac{1}{i}\right)$ $\boxed{\frac{2}{1}\frac{3}{2}\dots\frac{n+1}{n} = \frac{2\cdot 3\dots(n+1)}{1\cdot 2\dots n} = n+1}$

   (g) $\prod_{i=1}^{n}\frac{n+i}{i}$ $\boxed{\frac{(n+1)(n+2)\cdots(2n)}{1\cdot 2\cdots n} = \frac{(2n)!}{n!^2} = \binom{2n}{n}}$

   (h) $\sum_{i=1}^{n}\sum_{j=i}^{n} 1$ $\boxed{\sum_{i=1}^{n}(n-i+1) = \frac{n(n+1)}{2}}$

   (i) $\sum_{i=1}^{n}\sum_{j=1}^{n}(i+j)$ $\boxed{\sum_{i=1}^{n}\sum_{j=1}^{n} i + \sum_{i=1}^{n}\sum_{j=1}^{n} j = \sum_{i=1}^{n}(ni) + \sum_{j=1}^{n}(nj) = 2\cdot n\cdot\frac{n(n+1)}{2} = n^2(n+1)}$

2. Express the following sums in summation notation and evaluate using appropriate summation formulas.

   (a) $3+7+11+\dots+(4n-1)$ $\boxed{\sum_{i=1}^{n}(4i-1) = 4\frac{n(n+1)}{2} - n.}$

   (b) $(1/3)^2-(1/3)^4+(1/3)^6-(1/3)^8+\cdots$ $\boxed{\frac{1}{9}\sum_{i=0}^{\infty}(-\frac{1}{9})^i = \frac{1/9}{1+1/9} = \frac{1}{10}}$

   (c) $y^n + xy^{n-1} + x^2y^{n-2} + \dots + x^{n-1}y + x^n$ $(x, y \neq 0,\ x \neq y)$.
   $\boxed{\sum_{i=0}^{n} x^i y^{n-i} = y^n\sum_{i=0}^{n}(x/y)^i = \frac{y^n(1-(x/y)^{n+1}}{1-(x/y)}.}$