Name:

Collaborator(s)[1]:

Math 347, Prof. Hildebrand, Summer 2019

HW 11, DUE IN CLASS WEDNESDAY, 7/31

- **Grading:** 15 points total, broken down as follows:
  - **Presentation/effort:** **3 points**
  - **Graded problems:** **12 points**

About his assignment: *This is the last homework assignment in this class. It is due on* ***Wednesday*** *rather than Thursday, so that the assignment can be graded and returned by Thursday's class. After this assignment has been graded, the lowest of the homework scores will be dropped, so that ony the 10 highest HW scores are counted towards the grade.*

#1 **More Congruence Magic.** Do the following problems **by hand, using congruences**. (No calculators, wolframalpha, etc. With the right approach, the calculations required are minimal.)

(a) Find the remainder of 1001^{1001} upon division by 9 (i.e., an integer $r \in \{0, 1, \dots, 8\}$ such that $1001^{1001} \equiv r \bmod 9$).

(b) Using congruences, show that 6 divides $n^3 + 5n$ for all $n \in \mathbb{N}$. (Hint: Consider separately the cases when $n \equiv 0, 1, 2, 3, 4, 5 \bmod 6$.)

(c) Using congruences, show that 7 divides $4^{3n+1} + 2^{3n+1} + 1$ for all $n \in \mathbb{N}$.

(d) Let $P(n) = n^2 + n + 17$. Using congruences, find infinitely many integers n such that $P(n)$ is divisible by 19. (Hint: First find one such value n, then use congruences to find infinitely many.)

Proof practice: Properties of Relations: *In each of the following problems, a set S and a relation on S, denoted by $\sim$, are defined. Determine whether the given relation is (a) reflexive, (b) symmetric, (c) transitive.*
For each of the properties (a)–(c), either give a proof, or a ***specific*** *counter-example. The proofs required are short—a line or two is usually enough for symmetric and reflexive properties; a bit more is needed for the reflexive property—but they must be properly written up, in correct logical order, with all quantifiers included.*

#2 $S = \mathbb{N}$. $x \sim y \iff x \mid y$.

#3 $S = \mathbb{R}$. $x \sim y \iff |x - y| \leq 1$.

#4 $S = \mathbb{R}$. $x \sim y \iff x - y \in \mathbb{Z}$.

#5 $S = \mathbb{R}$. $x \sim y \iff$ "There exists $n \in \mathbb{Z}$ such that $x = 2^n y$."

[1] If you worked with another student or in a small group on this assignment, list the names of all students involved.