Name:

Collaborator(s)[1]:

Math 347, Prof. Hildebrand, Summer 2019

HW 2, DUE IN CLASS THURSDAY, 6/20

- **Grading:** 15 points total, broken down as follows:
  - **Presentation/effort:** **3 points**
  - **Graded problems:** **12 points**
- **Use this sheet as cover sheet and staple it to the assignment.** Write legibly and allow a full page for each problem. Group work on these problems is encouraged, provided (i) you write up the solutions yourself, using your own words, and (ii) you write the names of all student(s) in the group on the cover sheet.

HW 2 PROBLEMS

NOTE: Problems with numbered references are taken from the text; for example, 2.4 refers to Exercise 4 in Chapter 2 of the text.

Logical statements. *The first group of problems are all exercises/drills in working with logical statements. Some involve mathematical definitions, others are ordinary English sentence.* ***For problems asking for negations of statements, express the negation as an English sentence (i.e., without using logical symbols), and without using words of negation.*** *See the Logic Worksheet for examples.*

#1 **2.4(a)–(f).** (Negating mathematical statements. **State the negation as an English sentence, without using words of negation.**)

#2 **2.10(a), (c), (d)** (Converting English statements into "if ... then" form and negating such statements. **State your answers using words, not logical symbols.**)

#3 **2.23.** (Negating a complex mathematical definition.)

#4 Consider the statement "If n is a perfect number, then n is even," (where the underlying universe is the set of natural numbers). State (a) the **converse** of this statement; (b) the **contrapositive** of this statement; (c) the **negation** of this statement, avoiding using words of negation; (d) an equivalent statement involving the term "**necessary**".
NOTES: (1) You can use the term "imperfect number" for a number that is not "perfect".
(2) The definition of "perfect number" does not matter at all for this problem, and you do not need to know what "perfect number" means; the logic would remain the exactly same if you replace "perfect number" by **any** other type of number. (If you really want to know, google "odd perfect numbers"...)

Fun problems and puzzles. *These problems don't require any of the formal methods we have developed, just a bit of clever thinking. (See the back of the page for some comments and tips.) None of the problems is particularly difficult, and the point of these problems is not so much on the solution itself, but on the presentation and write-up of the solution.* ***Try to be as clear and as convincing as you can in your write-up. Try to be concise, without skipping any key logical steps.***

#5 **1.22.** (Water/wine puzzle.)

#6 **2.11.** (Penny/dime/dollar puzzle.)

#7 **2.32.** (Liars puzzle.)

***** Turn page for comments, hints, and advice *****

[1]If you worked with another student or in a small group on this assignment, list the names of all students involved.

Homework HW 2 Advice, Hints, and Comment

- **Negations of implications:** One of the most common beginners' mistakes in proofs is to incorrectly negate an implication. As was pointed out in class and on the Logic Handout, an implication $P \Rightarrow Q$ is false if and only if P is true and Q is false, so the negation of $P \Rightarrow Q$ is the statement $P \wedge \neg Q$. This is essentially the only correct way to negate the implication $P \Rightarrow Q$. In particular, and this cannot be emphasized enough:

  A negation of an implication $P \Rightarrow Q$ is never equivalent to another implication involving some combination of P, $\neg P$, Q, $\neg Q$.

- **Implied quantifiers.** Statements like "if n is odd then n is prime", or "$f(x) < f(y)$ whenever $x < y$" contain *implied* (i.e., not explicitly stated) universal quantifiers for the variables n, resp. x and y. In order to come up with the proper negation of these statements, such implied quantifiers must be made explicit (e.g., "$\forall n \in \mathbb{Z}$", resp. "$\forall x \in \mathbb{R})(\forall y \in \mathbb{R})$". After negation, these quantifiers turn into existential quantifiers. and these cannot be omitted. **There is no such thing as an implied existential quantifier.**

- **General advice on puzzles problems.** As mentioned, for these problems the main focus should be on the write-up of your solution, and most of your effort should go towards writing up your solution in as clear and convincing a manner as possible.

  Start out with a draft or outline of your argument on scratch paper, read it over carefully, ask yourself if it makes logical sense and is clear, convincing, and make changes if necessary. **Tips:** Read over your argument again a day or two later, and ask yourself if it still makes sense. Have friends take a look at your solution and provide feedback. If you are working with other students, read and critique each others' write-ups.

- **Water/wine puzzle.** The problem is interesting because it has a surprising answer, and also because it has (at least) two very different solutions, a standard algebraic approach, and a short, slick, and elegant alternate approach that requires no algebra at all! The algebra solution is good enough, but if it piques your interest, try to come up with the slick solution!

- **Logical puzzles.** The first of the two logical puzzles requires a small bit of insight (which may come to you right away after reading the problem, or after a couple of minutes of thinking it over, or the next day after sleeping it over). Don't give up too soon; the problem is not hard, and once you see the solution you wonder why didn't come up with it earlier.

  The second puzzle requires more of a systematic analysis and elimination of cases than a particular insight.