Name:

Collaborator(s)[1]:

Math 347, Prof. Hildebrand, Summer 2019

HW 3, DUE IN CLASS MONDAY, 6/24

- **Grading:** 15 points total, broken down as follows:
  - **Presentation/effort:** **3 points**
  - **Graded problems:** **12 points**
- **Use this sheet as cover sheet and staple it to the assignment.** Write legibly and allow a full page for each problem. Group work on these problems is encouraged, provided (i) you write up the solutions yourself, using your own words, and (ii) you write the names of all student(s) in the group on the cover sheet.

Set-theory and logic problems: *The first group of problems are some additional problems to practice set-theoretic proofs, and the interpretation of logical statements.*

#1 **1.50.** Let f be a function from $\mathbb{R}$ to $\mathbb{R}$.
(a) Prove that, for any sets $C \subseteq \mathbb{R}$ and $D \subseteq \mathbb{R}$, $(*)$ $f(C \cap D) \subseteq f(C) \cap f(D)$.
(b) Give an example (i.e., a specific choice of f and the sets C and D) where equality does not hold in $(*)$. Explain/justify your answer.

(Here, for any subset $A \subseteq \mathbb{R}$, $f(A)$ is defined as the **set** $f(A) = \{y \in \mathbb{R} : \text{ there exists } x \in A \text{ such that } f(x) = y\}$. Thus $(*)$ is a set-theoretic relation of the form $S \subseteq T$, and it should be proved using the standard procedure for proving such relations.)

#2 **2.24.** Let S be a set of real numbers (i.e., $S \subseteq \mathbb{R}$): Consider the following statements:
(A) There is a number M such that, for every x in the set S, $|x| \le M$.
(B) For every x in the set S, there is a number M such that $|x| \le M$.
(a) Which sets satisfy statement (A)? Describe these sets in "simple language" and explain your answer.
(b) Which sets satisfy statement (B)? Describe these sets in "simple language" and explain your answer.
(c) Which of the properties implies the other? Explain your answer, and give an example of a set that satisfies one property, but not the other.

#3 **2.26.** Let $a \in \mathbb{R}$ and f be a function from $\mathbb{R}$ to $\mathbb{R}$. Consider the following statements:
(A) $(\forall \epsilon > 0)(\exists \delta > 0)[(|x - a| < \delta) \Longrightarrow (|f(x) - f(a)| < \epsilon)]$
(B) $(\exists \delta > 0)(\forall \epsilon > 0)[(|x - a| < \delta) \Longrightarrow (|f(x) - f(a)| < \epsilon)]$
(a) Which of the statements is stronger, i.e., implies the other? Give a clear and convincing explanation for your answer.
(b) Find a function that satisfies one statement, but not the other. Justify your answer (i.e., show why this function satisfies one property, but not the other).
(c) **Extra-credit challenge (no collabaration, or help on EC problems!).** Determine, again with a clear, convincing argument, the exact set of functions that satisfy (B).

Sum/product notations. *The following are exercises in properly interpreting and "unwinding" sum/product notations, similar to those on the sum/product notation worksheet. In each case, evaluate the given sum or product for general $n \in \mathbb{N}$, by writing out the sum/product explicitly, or by using basic manipulations of sums and products and summation formulas from the worksheet. (No need to use induction.)* ***All of the given expressions have a simple general formula involving only elementary functions of n and the factorial function $n!$.***

#4 $\sum_{i=1}^{n} n^i$

#5 $\prod_{i=1}^{n} n^i$

#6 $\prod_{i=1}^{n}(2i)$

#7 $\prod_{i=1}^{n}(2i-1)$ (Hint: Write out explicitly and fill in the "holes")

[1] If you worked with another student or in a small group on this assignment, list the names of all students involved.