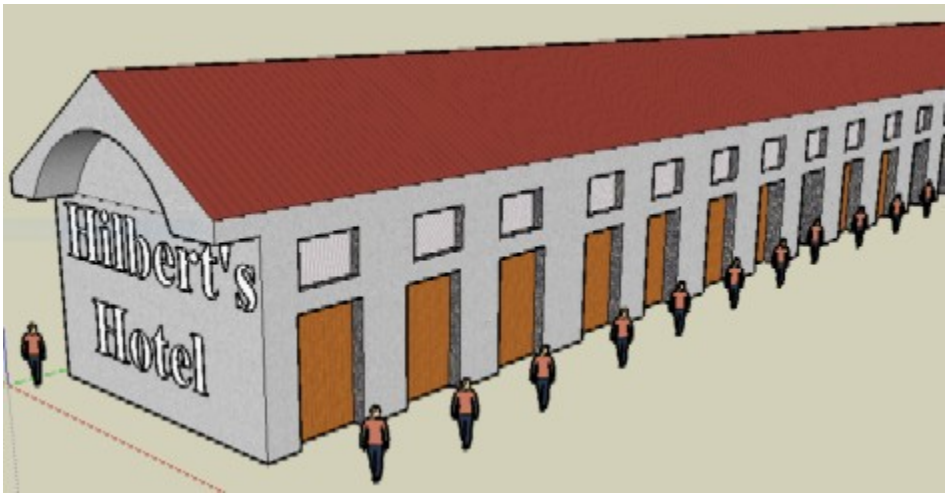
The Hotel Infinity Paradox



Imagine a hotel with an infinite (but countable) number of rooms, all laid out in a single row and numbered consecutively: Room 1, Room 2, Room 3, etc. Suppose every room is occupied by a guest, and the hotel shows a "no vacancy" sign.

1. **One additional guest.** Suppose a guest arrives and pleads with the manager to find him a room. After thinking a bit, the manager lightens up and answers, "No problem." He has come up with a way to reassign his guests to different rooms in such a way that a room becomes available for the new guest while all of his current guests still have a room to themselves.

   Describe, mathematically, how to accomplish this, by an appropriate bijection between the set of rooms and the set of (current and new) guests.

2. **Bus with 30 additional guests.** Suppose a guest arrives and pleads with the manager to find him a room. Now suppose a bus carrying 30 guests arrives at the hotel. Again, describe a bijection that assigns the current and new guests to rooms.

3. **Bus with countably many additional guests.** Do the same assuming the bus carries countably many guests.

4. **Countably many buses with countably many additional guests.** Do the same assuming a countable number of buses arrive, each carrying countably many guests.

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For more information (and some great stories!) about this problem, google "Hotel Infinity" or "Hilbert's Hotel").