

## Worksheet: Common Errors in Writing up Induction Proofs

- **Undeclared/unquantified induction variable:** Starting out an induction step by saying “Suppose  $P(n)$  is true for  $n = k$ ”, without saying what  $k$  is, what restrictions there have to be on  $k$  (e.g.,  $k \geq 2$ ), would make this step incomplete. There are many examples of “false” induction proofs that are due to errors of this type.
- **Not stating, explicitly and precisely, the statement/formula that one seeks to prove.** For example, using language like “suppose it is true for  $n = k$ ”, or “suppose  $P(k)$ ”, without explicitly and precisely stating what “it” or “ $P(k)$ ” means, would be wrong.
- **Induction hypothesis not referenced.** The most crucial part of any induction proof is the place where the induction hypothesis (e.g., the case  $n = k$  of  $P(n)$ , or the cases  $n = k$  and  $n = k - 1$ , etc) is being used. This is the place that can make or break an induction argument, and an error there can have disastrous consequences. **Always clearly state, at the appropriate place in the induction step, where the induction hypothesis is being used.**
- **Insufficient/inappropriate base cases:** The base case(s) must be sufficient to “kick-start” the chain reaction that the induction proof represents. For example, an induction in which the  $k + 1$  case depends on the cases  $k$  and  $k - 1$  requires checking two consecutive  $n$ -values in the base step.
- **Base cases and induction step don’t match up:** The first case of the induction step (i.e., the first  $k$ -value for which the induction step is claimed) must match up with the base case(s), so as not to leave a gap in the chain reaction. There are numerous examples of “false” induction proofs that result from carelessness in this regard.

## Practice Problems

In each of the following examples, something is wrong with the set-up or write-up of the induction proof. Find the error and try to correct it.

### 1. Example 1.

- **Base step:**  $n = 6$ .
- **Induction step:** Let  $k \in \mathbb{N}$  be given and assume  $(*)$  is true for  $n = k$ .

### 2. Example 2.

- **Base step:**  $n = 1$  and  $n = 2$ .
- **Induction step:** Let  $k \in \mathbb{N}$  with  $k \geq 3$  be given and assume  $(*)$  is true for  $n = k$  and  $n = k - 1$ .

### 3. Example 3.

- **Base step:**  $n = 1$  and  $n = 2$ .
- **Induction step:** Assume  $(*)$  is true for  $n = k$  and  $n = k - 1$ . Then ...

### 4. Example 4.

- **Base step:**  $n = 1$  and  $n = 2$ .
- **Induction step:** Let  $k \in \mathbb{N}$  be given and assume  $(*)$  is true for  $n = k$  and  $n = k - 1$ .