Worksheet: Set-theoretic notations and terminology $\boxed{\textbf{Solutions}}$

1. Write the following sets using "set builder" notation:
   (a) The set of *real* solutions to $x^2 - 2x < 3$. $\boxed{\{x \in \mathbb{R} : x^2 - 2x < 3\}}$
   (b) The set of *integer solutions* to $x^2 - 2x < 3$. $\boxed{\{x \in \mathbb{Z} : x^2 - 2x < 3\}}$
   (c) The set of odd integers. $\boxed{\{2k + 1 : k \in \mathbb{Z}\}}$
   (d) The set of odd natural numbers. $\boxed{\{2k - 1 : k \in \mathbb{N}\}.}$
   Note that we need $2k - 1$, not $2k + 1$, so that the number 1 is counted. An alternative would be $\boxed{\{2k + 1 : k \in \mathbb{N} \cup \{0\}\}}$.
   (e) The set $\{1, 4, 7, 10, \dots\}$. $\boxed{\{3k - 2 : k \in \mathbb{N}\}}$
   (f) The set of integers that can be written as sums of two integer squares. (The first few such integers are $0 = 0^2 + 0^2$, $1 = 0^2 + 1^2 = 1$, $2 = 1^2 + 1^2$, $4 = 0^2 + 2^2$.) $\boxed{\{n^2 + m^2 : n, m \in \mathbb{Z}\}}$

2. Given $A = \{1, 2, 3, 4\}$, $B = \{0, 1\}$, find the following sets. *Be sure to use correct set-theoretic notation.*
   (a) $A - B$ $\boxed{\{2, 3, 4\}}$
   (b) $B - A$ $\boxed{\{0\}}$
   (c) $A \times B$ $\boxed{\{(1, 0), (2, 0), (3, 0), (4, 0), (1, 1), (2, 1), (3, 1), (4, 1)\}}$
   (d) $P(B)$ $\boxed{\{\emptyset, \{0\}, \{1\}, \{0, 1\}\}}$
   (e) $B \times \mathbb{N}$ $\boxed{\{(0, 1), (0, 2), (0, 3), \dots, (1, 1), (1, 2), (1, 3), \dots\}}$

3. In the following, E, O denote, respectively, the sets of even integers and odd integers. In each case, write the given set in simpler form.
   (a) $\{p/q : p, q \in E, q \in \mathbb{Z}, q \neq 0\}$ $\boxed{\mathbb{Q}}$
   (b) $\{1/q : q \in \mathbb{Q}, q \neq 0\}$ $\boxed{\mathbb{Q} - \{0\}}$
   (c) $(\mathbb{N} \times \mathbb{Z}) \cap (\mathbb{Z} \times \mathbb{N})$ $\boxed{\mathbb{N} \times \mathbb{N}}$
   (d) $(\mathbb{Z} - E) \cup (\mathbb{Z} - O)$ $\boxed{\mathbb{Z}}$

4. Let $A = \{1, 2\}$ and $B = \{\{1\}, 2\}$. Determine which of the following relations are true:
   (a) $\{1\} \in A$ $\boxed{\textit{FALSE } (\{1\} \textit{ is a subset of } A\textit{, not an element})}$
   (b) $\{1\} \subseteq A$ $\boxed{\textit{TRUE}}$
   (c) $\{1\} \in P(A)$ $\boxed{\textit{TRUE (since the elements of a power set are the subsets of } A)}$
   (d) $\{1\} \subseteq P(A)$ $\boxed{\textit{FALSE}}$
   (e) $\{1\} \in B$ $\boxed{\textit{TRUE}}$
   (f) $\{1\} \subseteq B$ $\boxed{\textit{FALSE}}$
   (g) $\{1\} \in P(B)$ $\boxed{\textit{FALSE (but } \{\{1\}\} \in P(B) \textit{ is TRUE)}}$
   (h) $\{1\} \subseteq P(B)$ $\boxed{\textit{FALSE}}$