Name:

Collaborator(s)[1]:

Math 347, Prof. Hildebrand, Summer 2019

HW 8, DUE IN CLASS THURSDAY, 7/18

- **Grading:** 15 points total, broken down as follows:
  - **Presentation/effort:** **3 points**
  - **Graded problems:** **12 points**

About his assignment: *This assignment and the following one (HW 9) are aimed at gaining experience with writing "epsilonics" type proofs. The first group of problems are "short answer" type problems that ask you to either provide a proof or a disproof (by counterexample) of the given statement. These problems help develop an intuition about limits that will be useful when you are faced with similar questions in an exam. The next two problems are applications of the Monotone Convergence Theorem; here the main task is proving that the two conditions of the theorem (bounded and monotone) are satisfied. The final two problems require you to use directly the appropriate ϵ-definitions (and nothing else!).* ***Focus on the write-up of your proofs, and aim to produce a proof that is as close to perfect as possible in every respect. Use the class and worksheet examples as models.***

#1 **Short proofs and counterexamples.** Determine if the statement is true or false. If it is true, give a proof. You can use any properties and theorems on limits from the class handouts and worksheets, but **you must state clearly which result you are using**. If it is false, give a **specific** counterexample.

(a) If $\{x_n\}$ is unbounded, then $\{x_n\}$ has no limit.

(b) If $\{x_n\}$ is not monotone, then $\{x_n\}$ has no limit.

(c) If $\{x_n\}$ has no limit, then $\{x_n\}$ is unbounded or not monotone.

(d) If $\{x_n^2\}$ diverges, then $\{x_n\}$ diverges.

#2 **Short proofs and counterexamples, II.** As before, determine if the statement is true or false. If it is true, give a proof; if it is false, give a specific counterexample and explain why this example has the desired properties.

(a) If $\{x_n\}$ converges, then for all $\epsilon > 0$ there exists $n \in \mathbb{N}$ such that $|x_{n+1} - x_n| < \epsilon$.

(b) If $\{x_n\}$ converges, then there exists $n \in \mathbb{N}$ such that for all $\epsilon > 0$, $|x_{n+1} - x_n| < \epsilon$.

(c) If $\{x_n\}$ converges, then there exists $\epsilon > 0$ such that for all $n \in \mathbb{N}$, $|x_{n+1} - x_n| < \epsilon$.

#3 **(13.29 in the text).** Let $x_n = \frac{1+n}{1+2n}$. Using the Monotone Convergence Theorem, prove that $\lim_{n\to\infty} x_n$ exists.

#4 **(13.30 in the text).** Let $x_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}$. Using the Monotone Convergence Theorem, prove that $\lim_{n\to\infty} x_n$ exists. (Hint: Note that x_n is a sum of n terms, so x_{n+1} has one more term than x_n. This extra term needs to be taken into account when establishing monotonicity.)

#5 **Cauchy implies bounded.** Prove that any Cauchy sequence is bounded. Your proof should use only the definitions of Cauchy sequences and bounded sequences, and **not** any of the properties and results about such sequences established in class, in the worksheets, or in the book. (In particular, it should not use the fact that a Cauchy sequence is convergent.)

#6 **Convergence implies gaps between consecutive terms go to 0.** Prove that if $\lim_{n\to\infty} a_n$ exists, then $\lim_{n\to\infty} (a_{n+1} - a_n) = 0$. Your proof should use only the ϵ-definition of a limit, and **not** any properties or results of limits established in class, in the worksheets, or the book.

[1] If you worked with another student or in a small group on this assignment, list the names of all students involved.