Review of Set-theoretic Notations and Terminology

- **Sets:** A set is an **unordered** collection of objects, called the **elements** of the set. The standard notation for a set is the brace notation $\{\dots\}$, with the elements of the set listed inside the pair of braces.

- **Set builder notation:** Notation of the form $\{\dots \; : \; \dots\}$, where the colon (:) has the meaning of "such that", the dots to the left of the colon indicate the generic form of the element in the set and the dots to the right of the colon denote any constraints on the element. E.g., $\{x \in \mathbb{R} : x^2 < 4\}$ stands for *"the set of all $x \in \mathbb{R}$ such that $x^2 < 4$"*; this is the same as the interval $(-2, 2)$. Other examples of this notation are $\{2k : k \in \mathbb{Z}\}$ (set of all even integers), $\{x : x \in A \text{ and } x \in B\}$ (intersection of A and B, $A \cap B$).

  Note: Instead of a colon (:), one can also use a vertical bar (|) as a separator; e.g., $\{x \in \mathbb{R} \mid x^2 < 4\}$. Both notations are equally acceptable.

- **Equality of sets:** Two sets are equal if they contain the same elements. E.g., the sets $\{3, 4, 7\}$, $\{4, 3, 7\}$, $\{3, 3, 4, 7\}$ are all equal since they contain the same three elements, namely 3, 4, 7.

- **Some special sets:**

  - **Empty set:** $\emptyset$ (or $\{\}$)
  - **"Double bar" sets:**
    * **Natural numbers:** $\mathbb{N} = \{1, 2, \dots\}$ (note that 0 is **not** an element of $\mathbb{N}$)[1]
    * **Integers:** $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$
    * **Rational numbers:** $\mathbb{Q} = \{p/q : p, q \in \mathbb{Z}, \; q \neq 0\}$
    * **Real numbers:** $\mathbb{R}$
  - **Intervals:**
    * **Open interval:** $(a, b) = \{x \in \mathbb{R} : a < x < b\}$
    * **Closed interval:** $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$
    * **Half-open interval:** $[a, b) = \{x \in \mathbb{R} : a \leq x < b\}$
  - **Universe:** U ("universe", a "superset" of which all sets under consideration are assumed to be a subset). (The meaning of the underlying "universe" is usually clear from the context.)

- **Some set-theoretic operations and relations:**

  - $a \in A$: "a is an element of A". Negation: $a \notin A$
  - $A \subseteq B$: "A is a subset of B". By definition, this means that every element in A is also an element in B. The negation, $A \nsubseteq B$, means that there is an element $x \in A$ with $x \notin B$. Note that the empty set, $\emptyset$, is a subset of any set, and that each set is a subset of itself.
  - $A = B$ (set equality). By definition, this means that every element in A is also an element of B, and that every element of B is also an element of A. Equivalent to "$A \subseteq B$ and $B \subseteq A$".
  - $A \cup B = \{x : x \in A \text{ or } x \in B\}$ (union)
  - $A \cap B = \{x : x \in A \text{ and } x \in B\}$ (intersection)
  - $A - B = \{x : x \in A \text{ and } x \notin B\}$ (set-theoretic difference)[2]
  - $A^c = \{x \in U : x \notin A\}$ (complement)[3]
  - $P(A) = \{B : B \subseteq A\}$ (power set of A)
  - $A \times B = \{(a, b) : a \in A, b \in B\}$ (Cartesian product)

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[1] We follow here the standard convention in the English mathematical literature where 0 is not considered a natural number. In some other fields (e.g., computer science) and cultures (e.g., French) it is common to include 0 in the definition of $\mathbb{N}$.

[2] An alternate notation for the set-theoretic difference $A - B$ is $A \setminus B$ (with a backslash).

[3] Another common notation for the complement is $\overline{A}$ instead of A^c.

- **Venn diagrams:** Venn diagrams can be very helpful to get a "feel" for set operations, relations, and identities such as De Morgan's Law. *However, a Venn diagram does not constitute a proof!* For a refresher on Venn diagrams see the Wikipedia page "Venn diagrams". (Google the term.)

- **Notes on set-theoretic notation:** Normal arithmetic operations like addition, subtraction, multiplication don't make sense in the context of sets. For example, $A + B$ does not make sense, so do not use this notation (you probably meant $A \cup B$). An exception is the minus sign $(-)$, which is used for the set-theoretic difference $A - B$. However, in set-theoretic context the minus sign has a completely different meaning than the ordinary arithmetic minus sign, and properties for the arithmetic minus sign do not, in general, hold for the set-theoretic minus.

- **Further resources:**

  - D'Angelo/West text, pp. 6–10.
  - Wikipedia pages on "Set (mathematics)" and "Venn diagrams". (Google the terms.) Great as a quick refresher on basic set-theoretic concepts.
  - "Discrete Mathematics" by Rosen (on course reserve in the Math Library). This text (which is the text for Math 213 and CS 173) is a great source for additional examples and practice problems.