Name:

Collaborator(s)[1]:

Math 347, Prof. Hildebrand, Summer 2019

HW 9, DUE IN CLASS MONDAY, 7/22

- **Grading:** 15 points total, broken down as follows:
  - **Presentation/effort:** **3 points**
  - **Graded problems:** **12 points**

#1 **Short proofs and counterexamples.** For each of the statements below, determine if it is true or false. If it is true, give a proof. **You can use any properties and theorems about series and limits from the class handouts and worksheets, but you must state clearly which theorem/property you are using**. If it is false, give a **specific** counterexample and explain briefly why this example "works" (e.g., in case of an example of a divergent series say why the series diverges).

(a) If $\sum_{k=1}^{\infty} a_k$ converges, then there exists $k \in \mathbb{N}$ such that $|a_k| \le 1/2^k$.

(b) If $\sum_{k=1}^{\infty} a_k$ converges, then for every $n \in \mathbb{N}$ there exists a $k \in \mathbb{N}$ such that $|a_k| \le 1/2^n$.

(c) If $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ both converge, then $\sum_{k=1}^{\infty} a_k b_k$ converges.

(d) If $\sum_{k=1}^{\infty} |a_k|$ converges and the sequence $\{b_n\}$ converges, then $\sum_{k=1}^{\infty} a_k b_k$ converges.

#2 **Short answers: sup, inf, max, min.** For each of the following sets determine its (i) max, (ii) min, (iii) sup, (iv) inf. If a quantity does not exist, say so (e.g., answer "DNE"). Give a brief (one line) justification. (We will cover these concepts in class on Thursday and Friday; in the text, this is covered at the beginning of Chapter 13.)

(a) $\{\frac{1}{n} : n \in \mathbb{N}\}$

(b) $\{2^n : n \in \mathbb{Z}\}$

(c) $\{x \in \mathbb{R} : x^2 < 2x\}$

(d) $\{(-1)^n + \frac{1}{n} : n \in \mathbb{N}\}$

(e) $\{(-1)^n(1 + \frac{1}{n}) : n \in \mathbb{N}\}$

Proof practice with the ϵ-definition of a limit. *For the following problems, give careful, step-by-step, proofs, using the definitions and results from the class handouts. You can use any properties and theorems about series and limits from the class handouts and worksheets, but you must state clearly which theorem/property you are using (e.g., "by the Comparison Test", "by Cauchy's Criterion for Series").*

As always, pay particular attention to the write-up: Make sure the steps are in the correct logical order, all variables are properly declared/defined, and any necessary quantifiers are included. ***Use the proofs in class, on the worksheets, and the homework solutions as models for your own write-ups.***

#3 **(14.47 in the text) Convergence of "product series".** Prove that if $\sum_{k=1}^{\infty} a_k^2$ and $\sum_{k=1}^{\infty} b_k^2$ both converge, then $\sum_{k=1}^{\infty} a_k b_k$ converges absolutely. (Hint: Use the AGM inequality (i.e., the inequality $2|xy| \le x^2 + y^2$) and the comparison test.)

#4 **(14.58 in the text) Limit comparison test.** Let $\{a_n\}$ and $\{b_n\}$ be sequences of positive numbers, and suppose that $\lim_{n\to\infty} a_n/b_n = L$ for some $L > 0$.

(i) Prove that if $\sum_{k=1}^{\infty} b_k$ converges, then so does $\sum_{k=1}^{\infty} a_k$.

(ii) Prove that if $\sum_{k=1}^{\infty} a_k$ converges, then so does $\sum_{k=1}^{\infty} b_k$.

[1] If you worked with another student or in a small group on this assignment, list the names of all students involved.