Name:

Collaborator(s)[1]:

Math 347, Prof. Hildebrand, Summer 2019

HW 1, DUE IN CLASS MONDAY, 6/17

- **Grading:** 15 points total, broken down as follows:
  - **Presentation/effort:** **3 points**
  - **Graded problems:** **12 points**
- **Use this sheet as cover sheet and staple it to the assignment.** Do the problems in order, and make sure that each problem is clearly labelled. Write legibly and allow plenty of space for each problem (ideally reserve one page for each problem). **Leave some space in the margins or between problems for grading notes, comments, and feedback.**
- **Group work:** I encourage you to work on the problems with another student or two, but each member of the group should write up the solutions yourself, using your own words. **If you collaborate with other students, write the names of the student(s) in the group on the cover sheet.**
- **Questions/help.** I'd be happy to answer questions, look over drafts of your work, and provide feedback. I'm available daily at 11 am in 147 Altgeld, and Wednesdays/Sundays at 4 pm in 141 Altgeld. **SEE BACK OF PAGE FOR COMMENTS, HINTS, AND TIPS ON SPECIFIC PROBLEMS.**

HW 1 PROBLEMS

Even/odd proofs and proof techniques. *For the following problems proceed as in the examples from the even/odd worksheet. For your proofs only use the basic definitions of even, odd, etc., and the basic algebraic properties of integers described on the worksheet.* ***Unless otherwise stated, do not use the results of worksheet problems. If you use a method of proof other than a direct proof, say which method you are using (e.g., contrapositive, proof by contradiction).***

#1 *Let n be an integer. Prove that n is odd if and only if $3n^2 + 1$ is even.*

#2 *Let n be an odd integer. Prove that $n^2 - 1$ is divisible by 8. (See the handout for the definition of divisibility.)*

#3 *(a) Prove that if x is a real number such that x^2 is irrational, then x itself is irrational.*
(b) Show (by a counterexample) that the converse (i.e., the statement "if x is irrational then x^2 is irrational") is not true.

Set-theoretic proofs. *The first three problems are proofs of general set-theoretic relations, of the same type as the examples from the "Settheoretic Proofs" handout, and you should proceed in the same way. The same approach works for the last two problems (from Chapter 1 of the text), which involve relations among* given *sets of real numbers. See Example 1.13 in the text for an illustration.*

#4 *Show that $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.*

#5 *Show that $(A - B) \cup (A - C) = A - (B \cap C)$.*

#6 *Show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.*

#7 ***(1.27.)*** *Let $S = \{x \in \mathbb{R} : |x/(x+1)| \leq 1\}$ and $T = \{x \in \mathbb{R} : x \geq -1/2\}$. Show that $S = T$.*

#8 ***(1.32)*** *Let $S = \{x \in \mathbb{R} : x^2 - 2x - 3 < 0\}$ and $T = \{x \in \mathbb{R} : -1 < x < 3\}$. Show that $S = T$.*

***** Turn page for comments, hints, and advice *****

[1] If you worked with another student or in a small group on this assignment, list the names of all students involved.

HW 1 Advice, Hints, and Comments

- **General comments about writing up proofs**
  - **Use the templates and examples from class and the worksheets as models for your write-up.** For Problems 1–3, use the examples from the "Even/odd Proofs" worksheet as models. For Problems 4–8, the "Set Theory Proofs" worksheet provides a simple template (along with several concrete examples) that is easy to use and virtually foolproof and that can be used for nearly all situations. This is the approach we use in class and which is used and recommended in most books, including our text (except that there the proofs are presented as a single paragraph, rather than an itemized list of steps, but this is only a cosmetic difference).
  - **Use words rather than symbols in formal proofs:** When writing up formal proofs, avoid using symbols such as $\wedge$, $\vee$, $\Rightarrow$, $\forall$, etc., and instead use the equivalent English words: "and", "or", "therefore", "for all", etc. **In particular, never mix words with symbols in the same sentence:**
  - **Proofs should include appropriate connecting words and justification.** Just writing down a list of equations or statements without indicating how these are connected and where these come from does not constitute a proof. For a simple test, string together the bullet items in your proof into a contiguous paragraph and ask yourself: Does this paragraph make logical sense? Does it read well, as a piece of English prose? A good proof should pass both of these tests.

- **Grading:** The homework is worth a total of 15 points, broken down as follows:
  Overall presentation/effort (3 points): This rubric accounts for the overall presentation, appearance, and neatness of your work, the quality and legibility of the writing, and the amount of effort put into writing up the homework.
  Mathematical accuracy (12 points): In each assignment a subset of problems will be chosen for full grading. For full credit, a solution/proof must be mathematically and logically correct, include all necessary steps with appropriate connecting phrases and justifications, use proper terminology and correct mathematical notation.

- **Questions/help.** I'd be happy to answer questions, look over drafts of your work, and provide feedback. I'm available at 11 am each day (in 147 Altgeld), and Wednesdays and Sundays at 4 pm (in 141 Altgeld). You can also email me (ajh@illinois.edu) with a quick question (e.g., "can we use ... for this problem"), but most things are better suited to be discussed/explained in person than via email.

- **Final advice:** Math 347 is a challenging class that requires a substantial investment of time and effort, probably more so than any previous math class you have taken. This is reflected in the homework assignments, and you should take these assignments seriously: Set aside plenty of time to work on the assignments, and give it your best effort. For proofs start with scratch work until you have nailed down the complete logical structure of the argument, then write up a final version of your argument on the homework sheet, paying attention to such things as proper notation, connecting phrases, justifications, etc. *Try your best to produce the perfect proof!* All this takes time (perhaps an hour), but the investment is worth it and will pay many dividends down the road. I am happy to help you along, but I also expect you to do your part and put in the time and effort needed to succeed.