

# Getting Ready for the Exam Paolo Camurati

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# Goal of the Theory exercises in the exam

#### To assess:

- The understanding of fundamental concepts in computer sciences
- The ability to analyze, evaluate, and face conceptual questions

#### What do you need:

- The knowledge of:
  - Complexity analysis
  - Algorithms for online connectivity
  - Sorting algorithms
  - Boolean algebra

# **Topics**

#### Union-find algorithms for online connectivity

- Quick-find
- Quick-union
- Weighted quick-union

#### Quadratic iterative sorting algorithms

- Insertion sort
- Bubble / exchange sort
- Selection sort
- Shell sort

#### Linear iterative sorting algorithms

- Counting sort
- Radix sort

#### Linearithmic iterative sorting algorithms

Bottom-up merge sort

#### Main target:

Analyze and evaluate (step by step) one numerical exercise with the theoretical concepts

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Analyze and evaluate (step by step) one numerical exercise with the theoretical concepts

#### **Example:**

#### Given the following function Foo

```
int Foo (int *collector, int n, int k)
{
    int i, j, acc_l;
    for (i=0; i<n-4; i++)
    {
        acc_l = 0;
        for (j=0; j<4; j++)
            acc_l += collector[i+j];
        if (acc_l == k)
            return i;
    }
    return -1;
}</pre>
```

#### Main target:

Analyze and evaluate (step by step) one numerical exercise with the theoretical concepts

#### **Example:**

```
Given the following function Foo
int Foo (int *collector, int n, int k)
{
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      acc_l = 0;
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        acc_l += collector[i+j];
      if (acc_l == k)
        return i;
   }
   return -1;</pre>
```

Q: Find the worst-case asymptotic complexity (loose bound) of the function T(n) = O(...)

Hint: Observe the loops

Are they nested?

Do they use constant limits?

#### Main target:

Analyze and evaluate (step by step) one numerical exercise with the theoretical concepts

#### **Example:**

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{
    int i, j, acc_l;
    for (i=0; i<n-4; i++)
    {
        acc_l = 0;
        for (j=0; j<4; j++)
            acc_l += collector[i+j];
        if (acc_l == k)
            return i;
     }
    return -1;
}</pre>
```

$$F = (n-4) * (4)$$
  
 $4(n-4) < n$   
 $T(n) = O(n)$ 

#### Main target:

Analyze and evaluate (step by step) one numerical exercise with the theoretical concepts

#### **Quick exercise:**

#### **Given the following function Foo**

```
int Foo (int *collector, int n, int k)
{
   int i, j, acc_l;
   for (i=0; i<n; i++)
   {
      acc_l = 0;
      for (j=0; j<n-3; j++)
        acc_l += collector[i+j];
      if (acc_l == k)
        return i;
   }
   return -1;
}</pre>
```

#### Main target:

Analyze and evaluate (step by step) one numerical exercise with the theoretical concepts

#### **Quick exercise:**

```
Void Bar(int A[][n1], int B[][n1])
{
  int C[m1][m1];
  for(int i = 0; i < n1; i++) {
    for(int j = 0; j < n1; j++) {
        C[i][j] = 0;
        for(int k = 0; k < m1; k++) {
            C[i][j] += A[i][k] * B[k][j];
        }
        printf("%d\t", result[i][j]);
    }
    printf("\n");
    }
}</pre>
```

Let the following sequence of pairs be given:

3-2 3-4 5-1 7-3 5-7 9-1

where relation **i-j** indicates that vertex **i** is adjacent to vertex **j**. Apply an on-line connectivity algorithm with **quick-find**, returning as a result the content of the array at each step. The **vertices** are named with integers between **0** and **9**.

Let the following sequence of pairs be given:

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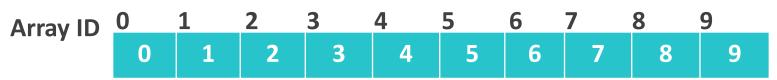
quick-find: updates all array IDs

Let the following sequence of pairs be given:

```
3-2 3-4 5-1 7-3 5-7 9-1
```

- for all (p, q):
  - read (p, q)
  - if (p, q) is connected (id[p] = id[q]),
    - do nothing
  - else
    - Replace id[p] values with id[q] values

Let the following sequence of pairs be given:



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Array ID	0	1	2	3	4	5	6	7	8	9
·	0	1	2	3	4	5	6	7	8	9
3-2	0	1	2	2	4	5	6	7	8	9

Let the following sequence of pairs be given:

Array ID	0	1	2	3	4	5	6	7	8	9
	0	1	2	3	4	5	6	7	8	9
3-2	0	1	2	2	4	5	6	7	8	9
3-4	0	1	4	4	4	5	6	7	8	9

Let the following sequence of pairs be given:

Array ID	0	1	2	3	4	5	6	7	8	9
	0	1	2	3	4	5	6	7	8	9
3-2	0	1	2	2	4	5	6	7	8	9
3-4	0	1	4	4	4	5	6	7	8	9
<b>5-1</b>	0	1	4	4	4	1	6	7	8	9

Let the following sequence of pairs be given:

Array ID	0	1	2	3	4	5	6	7	8	9
	0	1	2	3	4	5	6	7	8	9
3-2	0	1	2	2	4	5	6	7	8	9
3-4	0	1	4	4	4	5	6	7	8	9
5-1	0	1	4	4	4	1	6	7	8	9
7-3	0	1	4	4	4	1	6	4	8	9

Let the following sequence of pairs be given:

Array ID	0	1	2	3	4	5	6	7	8	9
	0	1	2	3	4	5	6	7	8	9
3-2	0	1	2	2	4	5	6	7	8	9
3-4	0	1	4	4	4	5	6	7	8	9
5-1	0	1	4	4	4	1	6	7	8	9
7-3	0	1	4	4	4	1	6	4	8	9
5-7	0	4	4	4	4	4	6	4	8	9

Let the following sequence of pairs be given:

				3-2 3	3-4 5-1	. 7-3 5	-7 9-1			
Array ID	0	1	2	3	4	5	6	7	8	9
7 tirdy 12	0	1	2	3	4	5	6	7	8	9
3-2	0	1	2	2	4	5	6	7	8	9
3-4	0	1	4	4	4	5	6	7	8	9
<b>5-1</b>	0	1	4	4	4	1	6	7	8	9
7-3	0	1	4	4	4	1	6	4	8	9
<b>5-7</b>	0	4	4	4	4	4	6	4	8	9
A.Y. 2023/24	_	0 4	4	4		<b>4</b> XAM	4	6 4	4 8	3 4

**Quick exercise:** 

Let the following sequence of pairs be given:

1-2 3-5 0-1 7-4 5-1 9-0 6-5

where relation **i-j** indicates that vertex **i** is adjacent to vertex **j**. Apply an on-line connectivity algorithm with **quick-find**, returning as a result the content of the array after **5** steps. The **vertices** are named with integers between **0** and **9**.

Let the following sequence of pairs be given:

1-3 3-4 5-1 4-0 7-3 5-7

where relation **i-j** indicates that vertex **i** is adjacent to vertex **j**. Apply an on-line connectivity algorithm with **quick-union**, returning as a result the content of the array at each step. The vertices are named with integers between **0** and **9**.

quick-union: Only updates the evaluated array IDs

Let the following sequence of pairs be given:

```
    for all (p, q):
    read (p, q)
    if(id[p])* = (id[q])*
    do nothing
```

- else
  - id[(id[p])\*] = (id[q])\* (connect the pair).

Let the following sequence of pairs be given:

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Array ID	0	1	2	3	4	5	6	7	8	9
, ,	0	1	2	3	4	5	6	7	8	9
1-3	0	3	2	3	4	5	6	7	8	9

Let the following sequence of pairs be given:

Array ID	0	1	2	3	4	5	6	7	8	9
7 cay 12	0	1	2	3	4	5	6	7	8	9
1-3	0	3	2	3	4	5	6	7	8	9
3-4	0	3	2	4	4	5	6	7	8	9

Let the following sequence of pairs be given:

Array ID	0	1	2	3	4	5	6	7	8	9
	0	1	2	3	4	5	6	7	8	9
1-3	0	3	2	3	4	5	6	7	8	9
3-4	0	3	2	4	4	5	6	7	8	9
5-1	0	3	2	4	4	4	6	7	8	9

Let the following sequence of pairs be given:

Array ID	0	1	2	3	4	5	6	7	8	9
	0	1	2	3	4	5	6	7	8	9
1-3	0	3	2	3	4	5	6	7	8	9
3-4	0	3	2	4	4	5	6	7	8	9
5-1	0	3	2	4	4	4	6	7	8	9
4-0	0	3	2	4	0	4	6	7	8	9

Let the following sequence of pairs be given:

Array ID	0	1	2	3	4	5	6	7	8	9
7 a y 12	0	1	2	3	4	5	6	7	8	9
1-3	0	3	2	3	4	5	6	7	8	9
3-4	0	3	2	4	4	5	6	7	8	9
5-1	0	3	2	4	4	4	6	7	8	9
4-0	0	3	2	4	0	4	6	7	8	9
7-3	0	3	2	4	0	4	6	0	8	9

Let the following sequence of pairs be given:

Array ID	0	1	2	3	4	5	6	7	8	9
, a,	0	1	2	3	4	5	6	7	8	9
1-3	0	3	2	3	4	5	6	7	8	9
3-4	0	3	2	4	4	5	6	7	8	9
<b>5-1</b>	0	3	2	4	4	4	6	7	8	9
4-0	0	3	2	4	0	4	6	7	8	9
7-3	0	3	2	4	0	4	6	0	8	9
A.Y. 2023/24	7	0	3	2	•	O XAM	4	6	0	8 9

**Quick exercise:** 

Let the following sequence of pairs be given:

1-2 3-4 0-1 7-3 5-1 9-0 6-5

where relation **i-j** indicates that vertex **i** is adjacent to vertex **j**. Apply an on-line connectivity algorithm with **quick-union**, returning as a result the content of the array after **4 steps**. The vertices are named with integers between **0** and **9**.

Given the following sequence of pairs, where relation **i-j** indicates that vertex **i** is adjacent to vertex **j**:

Apply an on-line connectivity algorithm with **weighted quick union**. Nodes are named with integers between 0 and 10.

Show the contents of the array at each step and draw the result as a forest.

Given the following sequence of pairs, where relation **i-j** indicates that vertex **i** is adjacent to vertex **j**:

Apply an on-line connectivity algorithm with **weighted quick union**. Nodes are named with integers between 0 and 10.

Show the contents of the array at each step and draw the result as a forest.

**Starting state:** 

SZ	0	1	2	3	4	5	6	7	8	9	10
Id	0	1	2	3	4	5	6	7	8	9	10

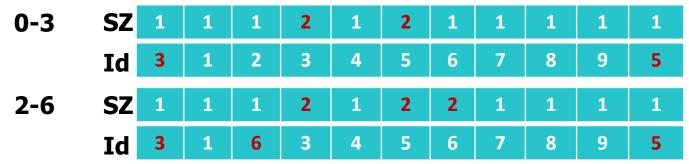
Given the following sequence of pairs, where relation **i-j** indicates that vertex **i** is adjacent to vertex **j**:

<b>Starting state:</b>	SZ	1	1	1	1	1	1	1	1	1	1	1
	Id	0	1	2	3	4	5	6	7	8	9	10
10-5	SZ	1	1	1	1	1	2	1	1	1	1	1
	Id	0	1	2	3	4	5	6	7	8	9	5

Given the following sequence of pairs, where relation **i-j** indicates that vertex **i** is adjacent to vertex **j**:

<b>Starting state:</b>	SZ	1	1	1	1	1	1	1	1	1	1	1
	Id	0	1	2	3	4	5	6	7	8	9	10
10-5	SZ	1	1	1	1	1	2	1	1	1	1	1
	Id	0	1	2	3	4	5	6	7	8	9	5
0-3	SZ	1	1	1	2	1	2	1	1	1	1	1
	Id	3	1	2	3	4	5	6	7	8	9	5

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0-3	SZ	1	1	1	2	1	2	1	1	1	1	1
	Id	3	1	2	3	4	5	6	7	8	9	5
2-6	SZ	1	1	1	2	1	2	2	1	1	1	1
	Id	3	1	6	3	4	5	6	7	8	9	5
5-4	SZ	1	1	1	2	1	3	2	1	1	1	1
	Id	3	1	6	3	5	5	6	7	8	9	5

Given the following sequence of pairs, where relation **i-j** indicates that vertex **i** is adjacent to vertex **j**:

0-3	SZ	1	1	1	2	1	2	1	1	1	1	1
	Id	3	1	2	3	4	5	6	7	8	9	5
2-6	SZ	1	1	1	2	1	2	2	1	1	1	1
	Id	3	1	6	3	4	5	6	7	8	9	5
5-4	SZ	1	1	1	2	1 (	3	2	1	1	1	1
	Id	3	1	6	3	5	5	6	7	8	9	5

Given the following sequence of pairs, where relation **i-j** indicates that vertex **i** is adjacent to vertex **j**:

Apply an on-line connectivity algorithm with **weighted quick union**. Nodes are named with integers between 0 and 10.

5-4

SZ	1	1	1	2	1	3	2	1	1	1	1
Id	3	1	6	3	5	5	6	7	8	9	5

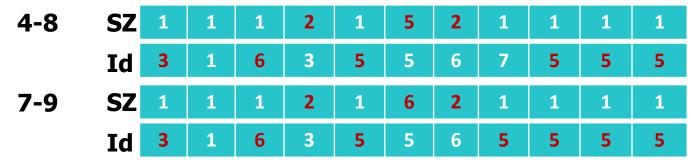
Given the following sequence of pairs, where relation **i-j** indicates that vertex **i** is adjacent to vertex **j**:

5-4	SZ	1	1	1	2	1	3	2	1	1	1	1
	Id	3	1	6	3	5	5	6	7	8	9	5
4-9	SZ	1	1	1	2	1	4	2	1	1	1	1
	Id	3	1	6	3	5	5	6	7	8	5	5

Given the following sequence of pairs, where relation **i-j** indicates that vertex **i** is adjacent to vertex **j**:

5-4	SZ	1	1	1	2	1	3	2	1	1	1	1
	Id	3	1	6	3	5	5	6	7	8	9	5
4-9	SZ	1	1	1	2	1	4	2	1	1	1	1
	Id	3	1	6	3	5	5	6	7	8	5	5
4-8	SZ	1	1	1	2	1	5	2	1	1	1	1
	Id	3	1	6	3	5	5	6	7	5	5	5

Given the following sequence of pairs, where relation **i-j** indicates that vertex **i** is adjacent to vertex **j**:



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10-5, 0-3, 2-6, 5-4, 4-9, 4-8, 7-9, 1-10

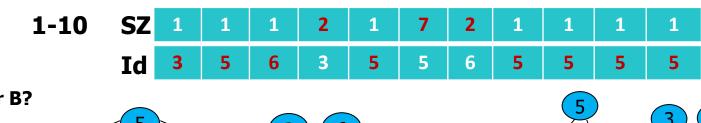
4-8	SZ	1	1	1	2	1	5	2	1	1	1	1
	Id	3	1	6	3	5	5	6	7	5	5	5
7-9	SZ	1	1	1	2	1	6	2	1	1	1	1
	Id	3	1	6	3	5	5	6	5	5	5	5
1-10	SZ	1	1	1	2	1	7	2	1	1	1	1
	Id	3	5	6	3	5	5	6	5	5	5	5

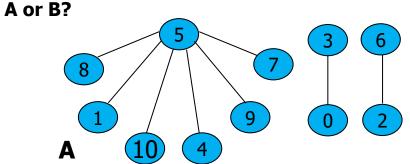
Given the following sequence of pairs, where relation **i-j** indicates that vertex **i** is adjacent to vertex **j**:

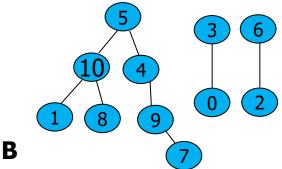
Apply an on-line connectivity algorithm with **weighted quick union**. Nodes are named with integers between 0 and 10.

What about the three diagram?

Given the following sequence of pairs, where relation **i-j** indicates that vertex **i** is adjacent to vertex **j**:







#### **Quick Exercise:**

Given the following sequence of pairs, where relation **i-j** indicates that vertex **i** is adjacent to vertex **j**:

3-9 4-5 2-5 2-4 3-1 5-6 0-10 3-5 8-9 10-3

Apply an on-line connectivity algorithm with **weighted quick union**. Nodes are named with integers between 0 and 10.

Show the contents of the array after **5 steps**.

Sort the following integer array in ascending order by counting sort:

$$12_1 7_1 6_1 1 6_2 8_1 7_2 12_2 6_3 8_2 0 5 4 12_3$$

**Subscripts** identify instances of the same key. Show the contents at each step of the intermediate data structures used.

Sort the following integer array in ascending order by counting sort:

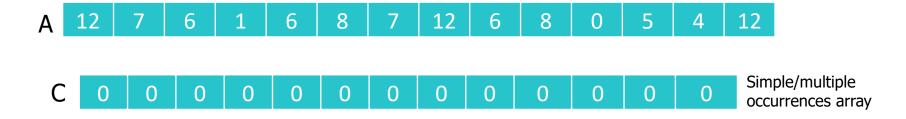
**Subscripts** identify instances of the same key. Show the contents at each step of the intermediate data structures used.

Sort the following integer array in ascending order by counting sort:

$$12_1 \ 7_1 \ 6_1 \ 1 \ 6_2 \ 8_1 \ 7_2 \ 12_2 \ 6_3 \ 8_2 \ 0 \ 5 \ 4 \ 12_3$$

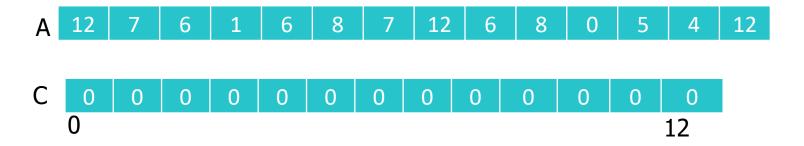
Data are in the range 0.. $\mathbf{k}$ -1. The largest one is 12 =>  $\mathbf{k}$ -1=12,  $\mathbf{k}$ =13

### **Starting state:**

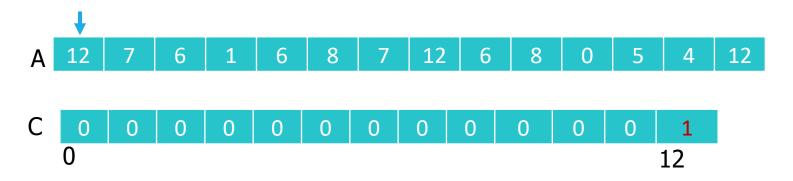


Sort the following integer array in ascending order by counting sort:

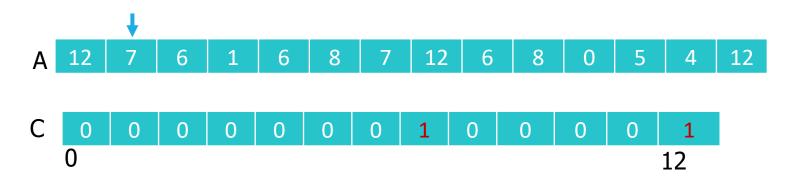
 $12_1 7_1 6_1 1 6_2 8_1 7_2 12_2 6_3 8_2 0 5 4 12_3$ 



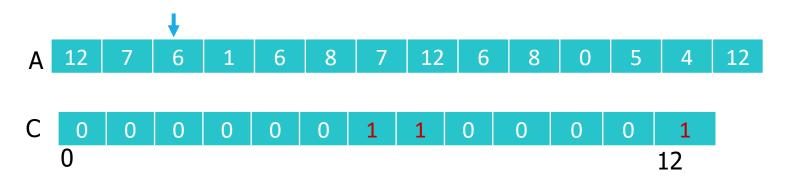
$$12_1 7_1 6_1 1 6_2 8_1 7_2 12_2 6_3 8_2 0 5 4 12_3$$



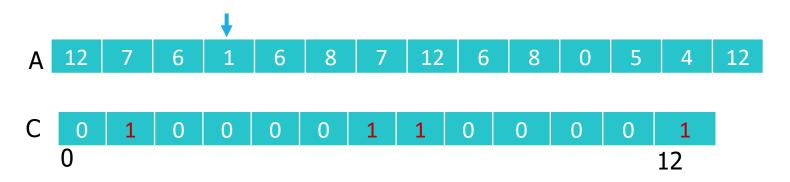
$$12_1 7_1 6_1 1 6_2 8_1 7_2 12_2 6_3 8_2 0 5 4 12_3$$



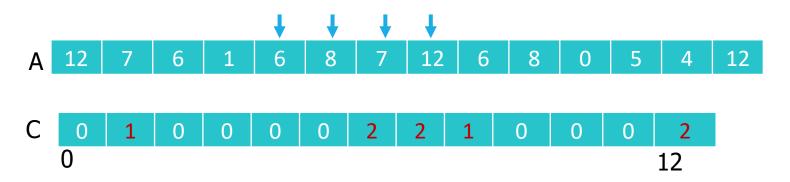
$$12_1 7_1 6_1 1 6_2 8_1 7_2 12_2 6_3 8_2 0 5 4 12_3$$



$$12_1 7_1 6_1 1 6_2 8_1 7_2 12_2 6_3 8_2 0 5 4 12_3$$

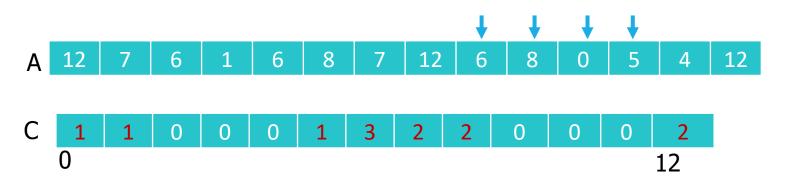


$$12_1 7_1 6_1 1 6_2 8_1 7_2 12_2 6_3 8_2 0 5 4 12_3$$

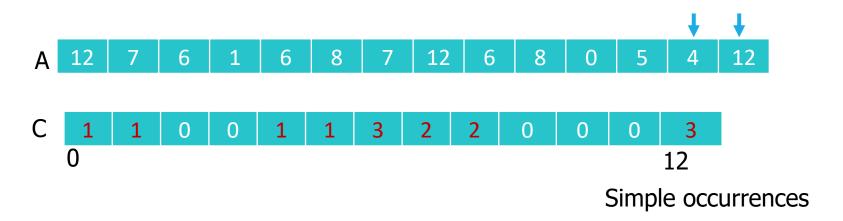


Sort the following integer array in ascending order by counting sort:

 $12_1 7_1 6_1 1 6_2 8_1 7_2 12_2 6_3 8_2 0 5 4 12_3$ 

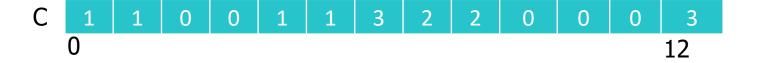


$$12_1 \ 7_1 \ 6_1 \ 1 \ 6_2 \ 8_1 \ 7_2 \ 12_2 \ 6_3 \ 8_2 \ 0 \ 5 \ 4 \ 12_3$$



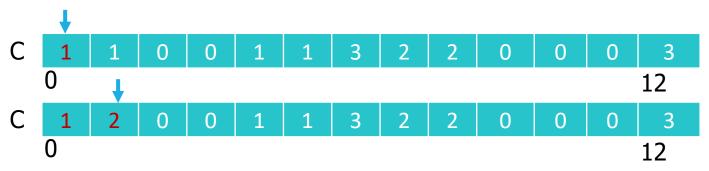
Sort the following integer array in ascending order by counting sort:

$$12_1 7_1 6_1 1 6_2 8_1 7_2 12_2 6_3 8_2 0 5 4 12_3$$



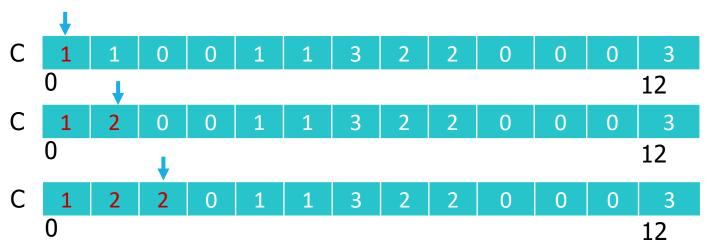
Sort the following integer array in ascending order by counting sort:

$$12_1 \ 7_1 \ 6_1 \ 1 \ 6_2 \ 8_1 \ 7_2 \ 12_2 \ 6_3 \ 8_2 \ 0 \ 5 \ 4 \ 12_3$$



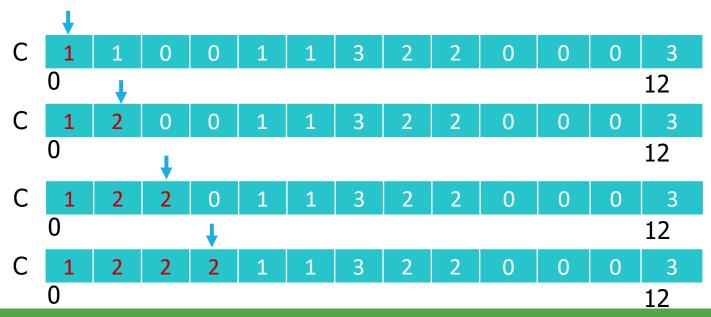
Sort the following integer array in ascending order by counting sort:

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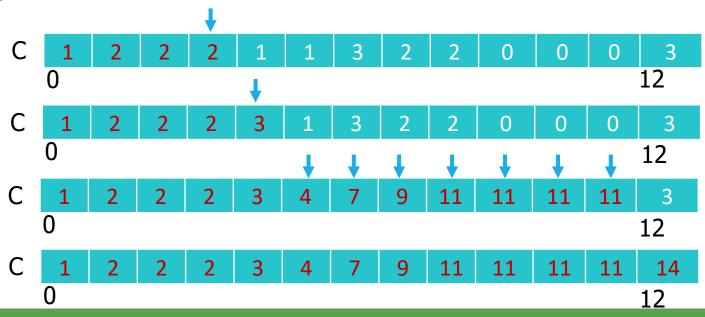
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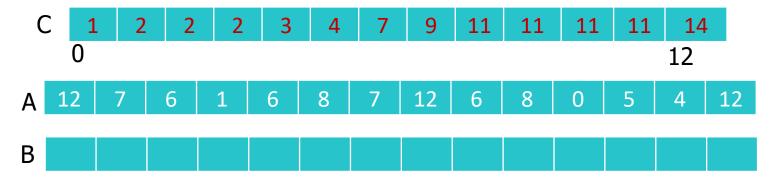
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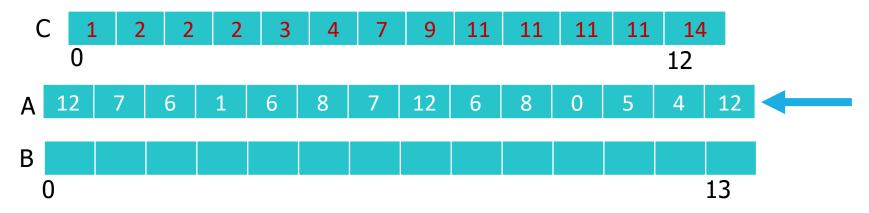
Sort the following integer array in ascending order by counting sort:

$$12_1 \ 7_1 \ 6_1 \ 1 \ 6_2 \ 8_1 \ 7_2 \ 12_2 \ 6_3 \ 8_2 \ 0 \ 5 \ 4 \ 12_3$$



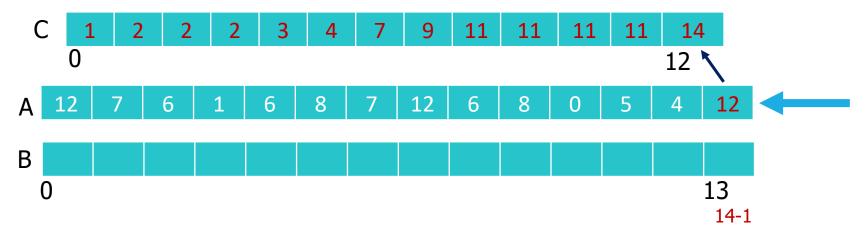
Sort the following integer array in ascending order by counting sort:

$$12_1 7_1 6_1 1 6_2 8_1 7_2 12_2 6_3 8_2 0 5 4 12_3$$



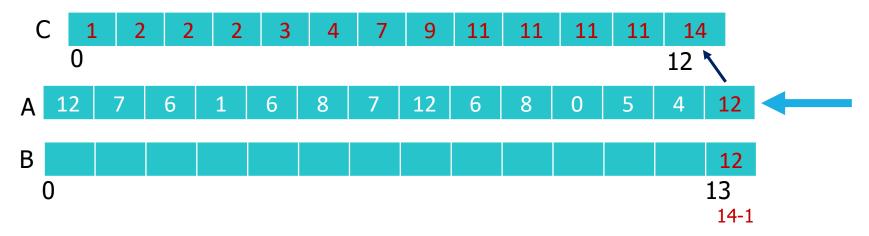
Sort the following integer array in ascending order by counting sort:

$$12_1 7_1 6_1 1 6_2 8_1 7_2 12_2 6_3 8_2 0 5 4 12_3$$



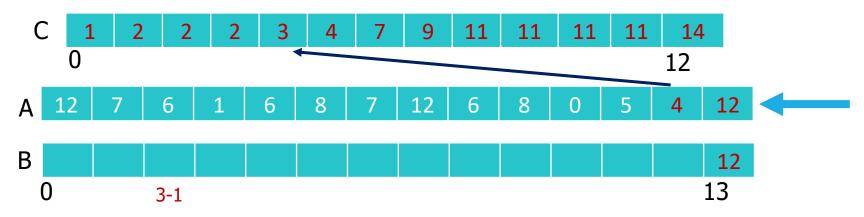
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$$12_1 7_1 6_1 1 6_2 8_1 7_2 12_2 6_3 8_2 0 5 4 12_3$$



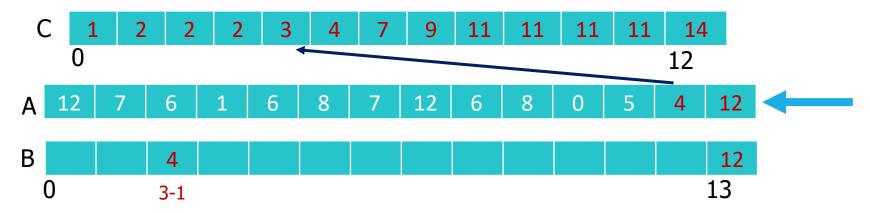
Sort the following integer array in ascending order by counting sort:

$$12_1 7_1 6_1 1 6_2 8_1 7_2 12_2 6_3 8_2 0 5 4 12_3$$



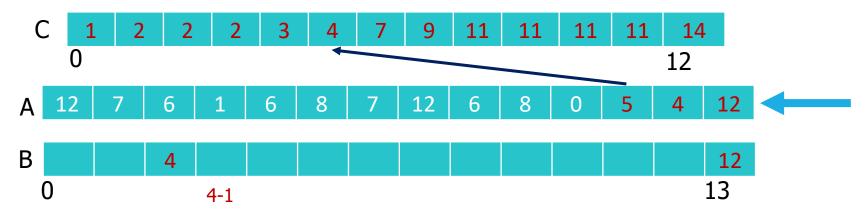
Sort the following integer array in ascending order by counting sort:

$$12_1 7_1 6_1 1 6_2 8_1 7_2 12_2 6_3 8_2 0 5 4 12_3$$



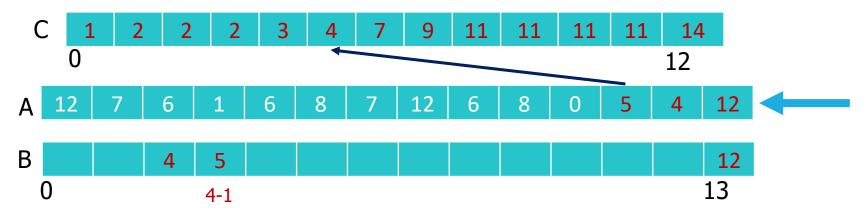
Sort the following integer array in ascending order by counting sort:

$$12_1 7_1 6_1 1 6_2 8_1 7_2 12_2 6_3 8_2 0 5 4 12_3$$



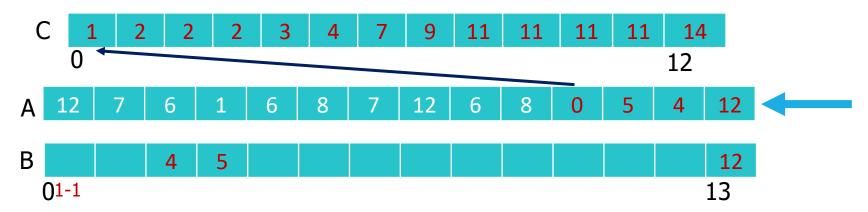
Sort the following integer array in ascending order by counting sort:

$$12_1 7_1 6_1 1 6_2 8_1 7_2 12_2 6_3 8_2 0 5 4 12_3$$



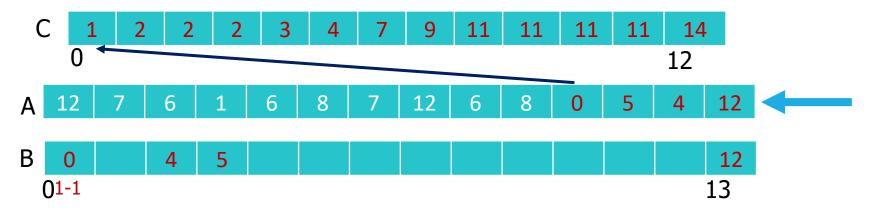
Sort the following integer array in ascending order by counting sort:

$$12_1 \ 7_1 \ 6_1 \ 1 \ 6_2 \ 8_1 \ 7_2 \ 12_2 \ 6_3 \ 8_2 \ 0 \ 5 \ 4 \ 12_3$$



Sort the following integer array in ascending order by counting sort:

$$12_1 7_1 6_1 1 6_2 8_1 7_2 12_2 6_3 8_2 0 5 4 12_3$$



Sort the following integer array in ascending order by counting sort:

$$12_1 7_1 6_1 1 6_2 8_1 7_2 12_2 6_3 8_2 0 5 4 12_3$$

