BLG372E - Analysis of Algorithms
Spring 2012
Homework 2

Total Worth: 5 points
Due: 13.03.2012 17:00

1. [1.5 points] Order the following functions by growing rate. Show all your work.

F1 = n

F2 = $\sqrt{n}$

F3 = $n^{1.5}$

F4 = n^2

F5 = nlogn

F6 = nlog(logn)

F7 = n $\log^2$n

F8 = nlog(n^2)

F9 = 2/n

F10 = 2^n

F11 = $2^{n/2}$

F12 = 37

F13 = n^2logn

F14 = n^3

2. [1 points] Consider the following algorithm for the maximum independent set size problem. Assume that n is a power of 2.

```
0    maxk = 0 ;
1    k=n/2
2    for i=1..log2(n)
3        if there is an independent set of size k
4            maxk = k
5            k = k + k/2
6        else
7            k = k - k/2
```

(Reminder-1: independent set problem: Given a graph with n nodes, what is the maximum size of an independent set. There is an $O(n^2 * 2^n)$ algorithm in the slides.
Reminder-2: You can check if there is an independent set of size k in a graph of n nodes in $O\left(k^2 * \frac{n^k}{k!}\right) = O(n^k)$ time. See the slides.**)**

Since the for loop in line 2 iterates at most log2n times, as in binary search, we can find an independent set in at most $O(log_2 n * n^k)$ time. But this is much faster than the complexity given in (Reminder1) above. What is wrong with this complexity calculation? (Hint: use Stirling's approximation to n!)

3. [1.5points]

```
Algorithm SetDisjointness(S1,…,Sn)
1  for each set Si {
2     for each other set Sj {
3        for each element p of Si {
4           determine whether p also belongs to Sj
5       }
6       if (no element of Si belongs to Sj)
7          report that Si and Sj are disjoint
8     }
9   }
```

Assume that line 4 can be executed in constant (O(1)) time. Assume also that the size of each set is equal to its index, i.e. $|S_i|=i$. What is the worst case running time of this algorithm?

4. [1 points]

a. True or False? Explain your answer.

If $f = \Theta(h)$ and $g = \Theta(h)$ then $f * g = \Theta(h*h)$.

b. Prove that $\log_2(n) = O(n^{0.5})$

Academic Honesty Policy

You may discuss the problem addressed by the homework at an abstract level with your classmates, but you should not share or copy the solution from your classmates or from Internet. You should submit your own, individual homework. Plagiarism and any other forms of cheating will have serious consequences, including failing the course.

Submission Instruction

Please drop the hard copy of your homework into the homework box named 'Analysis of Algorithms' in the department secretary's room. **Please** explain your answers and show all your work clearly and neatly. If a question or statement is not clear, please let the teaching assistant Meryem Uzun-Per know by email (uzunper@itu.edu.tr).