Linear Optimization
HW #10

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Problem 12

Problem: Consider the integer knapsack problem from the chapter. List all allowable choices of the products and the value of the knapsack so filled.

Solution: The only choices of (x_1, x_2, x_3) that satisfy:

$$51x_1 + 50x_2 + 50x_3 \leq 100$$

are the following:

- $(0, 0, 0) \to 0$
- $(1, 0, 0) \to 150$
- $(0, 1, 0) \to 100$
- $(0, 0, 1) \to 99$
- $(0, 2, 0) \to 200$
- $(0, 1, 1) \to 199$
- $(0, 0, 2) \to 198$

We know this is exhaustive since each $x_j \geq 0$ and that increasing any component of any of the above feasible solutions would result in either an unfeasible solution or one already listed.

Problem 13

Problem: Generalize the knapsack problem so that in addition to needing the total weight to be below a critical threshold, there is also a volume constraint. Set this up as a Linear Programming problem.

Solution: If our items have volumes of 10, 30, & 20 respectively, and if our knapsack can only hold a volume of 60, our new problem is:

$$\begin{array}{ll} \text{Maximize} & 150x_2 + 100x_2 + 99x_3 \\ \text{subject to} & 51x_1 + 50x_2 + 50x_3 \leq 100 \quad \text{(weight)} \\ & 10x_1 + 30x_2 + 20x_3 \leq 60 \quad \text{(volume)} \\ \text{and} & \mathbf{x} \geq 0 \end{array}$$