# Intro to Math Reasoning HW 4a

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## Problem 1

**Problem:** Consider the predicate $C(x, y)$ where $x$ and $y$ are real numbers. Let the sets $S_1$ and $S_2$:

$$S_1 = \{x \in \mathbb{R} \mid (\forall y \in \mathbb{R})\ C(x, y)\}$$
$$S_2 = \{x \in \mathbb{R} \mid (\forall y \in \mathbb{R})\ \neg C(x, y)\}$$

Can both $S_1$ and $S_2$ be nonempty?

**Solution:** Yes, and to prove this it suffices to give a single example. Consider the following choice of $C(x, y)$:

$$C(x, y) \equiv (\exists! k \in \mathbb{R})\ kx = y$$

Notice that this is just a more accurate way of saying $\frac{y}{x}$ is defined. We know that for all real numbers $x, y$ the statement is defined and thus the proposition is true *except* when $x = 0$ since $\frac{y}{0}$ is not defined for any $y$.

So in this case $S_1 = \{x \in \mathbb{R} \mid x \neq 0\}$ and $S_2 = \{0\}$ which are both nonempty.

## Problem 2

Consider the predicate $P(A, B, C) \equiv (C \setminus A = C \setminus B) \rightarrow A = B$.

### Part a

**Problem:** Is there an $A, B$ and $C$ such that $P(A, B, C)$ is true?

**Solution:**

### Part b

**Problem:** Is there a unique $(A, B, C)$ such that $P(A, B, C)$ is true?

**Solution:**

### Part c

**Problem:** Is there an $A, B$ and $C$ such that $P(A, B, C)$ is false?

**Solution:**

## Problem 2

### Part a

**Problem:**

**Solution:**

### Part b

**Problem:**

**Solution:**

### Part c

**Problem:**

**Solution:**