

Math Statistics

Semiweekly HW 6

Ozaner Hansha

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Question 1

Problem: Suppose that a population is a normally distributed with unknown mean and variance. If we take a sample of size 6, what is the probability that the population variance is at least 6.25 times as large as the sample variance?

Solution: First note our desired probability is:

$$P(6.25S^2 \leq \sigma^2)$$

Now note that, since our population is normally distributed, we have that the following function of its sample variance S^2 has a chi-distribution:

$$\frac{(n-1)S^2}{\sigma^2} = \frac{5S^2}{\sigma^2} \sim \chi_5^2$$

Now note that:

$$P\left(\frac{5S^2}{\sigma^2} \leq c\right) = P\left(\frac{5S^2}{c} \leq \sigma^2\right)$$

And so setting $\frac{5S^2}{c} = 6.25S^2$ and solving for c nets us:

$$c = 0.8$$

And so we can finally calculate our desired probability:

$$\begin{aligned} P(6.25S^2 \leq \sigma^2) &= P\left(\frac{5S^2}{.8} \leq \sigma^2\right) \\ &= P\left(\frac{5S^2}{\sigma^2} \leq .8\right) \\ &= 1 - P\left(\frac{5S^2}{\sigma^2} \geq .8\right) && \text{(complement)} \\ &= 1 - \chi_{.8,5}^2 \approx 0.02297 && \text{(chi distribution)} \end{aligned}$$