Math Statistics
Semiweekly HW 1

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Question 1

Problem: Show that $E[aX] = aE[X]$, for RV X and constant a.

Solution: Recall the definition of the expectation of a function $g(x)$ of a RV X:

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x)\, dx$$

Letting $g(x) = ax$ for an arbitrary constant a, we find:

$$\begin{aligned}
E[aX] &= \int_{-\infty}^{\infty} ax f_X(x)\, dx && \text{(def. of expectation)}\\
&= a\int_{-\infty}^{\infty} x f_X(x)\, dx && \text{(linearity of integration)}\\
&= aE[X] && \text{(def. of Expectation)}
\end{aligned}$$

Question 2

Problem: Show that $\operatorname{Var}(aX) = a^2 \operatorname{Var}(X)$, for RV X and constant a.

Solution: Recall the definition of the variance of a RV X:

$$\operatorname{Var}(X) = E[(X - E[X])^2]$$

And so the variance of aX for an arbitrary constant a is given by:

$$\begin{aligned}
\operatorname{Var}(aX) &= E[(aX - E[aX])^2] && \text{(def. of variance)}\\
&= E[(aX - aE[X])^2] && \text{(linearity of expectation)}\\
&= E[a^2(X - E[X])^2] && \text{(algebra)}\\
&= a^2E[(X - E[X])^2] && \text{(linearity of expectation)}\\
&= a^2 \operatorname{Var}(X) && \text{(def. of variance)}
\end{aligned}$$

Question 3

Problem: Given two RVs X and Y, is the following true:

$$f_{X+Y}(z) = f_X(z) + f_Y(z)$$

Solution: The pdf of the sum of two RVs is the convolution of their respective pdfs:

$$f_{X+Y}(z) = \int_{-\infty}^{\infty} f(z-y)g(y)\, dy$$

This is clearly not equivalent to the sum of their pdfs in general, and so the statement is false.