Math Statistics
Semiweekly HW 14

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November 7, 2020

Question 1

Problem: Suppose that, without a vaccine, a person in a given population has a $p_{\text{w/o}} = .1$ chance of contracting the flu in a given year. We want to test that, given a particular vaccine, a person has only a $p_{\text{w}} = .06$ chance of contracting the flu.

Suppose we have a sample size of $n = 10$. In this case we have the following null hypothesis H_0, alternative hypothesis H_1, and condition for accepting the null hypothesis $\hat{H}_0$:

$$\begin{aligned}
H_0 &\equiv p_w = .1 && \text{(prob. of infection w/ vaccine is unchanged)} \\
H_1 &\equiv p_w = .06 && \text{(prob. of infection w/ vaccine is .06)} \\
\hat{H}_0 &\equiv X \geq 1 && \text{(At least 1 subject infected within a year)}
\end{aligned}$$

Where X is the number of people infected in our sample. Also let $X_{n,p} \sim B(n,p)$. Note that with a sample size of n and prob p_w of infection we have $X \sim X_{n,p_w}$.

Given these hypotheses and conditions for accepting them, what is the probability of a type I error? What about a type II error?

Solution: For a type I error, i.e. accepting a false alternative hypothesis, we have:

$$\begin{aligned}
P(\text{Type I error}) &= P(\neg\hat{H}_0 \mid H_0) && \text{(def. of type I error)} \\
&= P(X = 0 \mid p_w = .1) && \text{(from def. of hypotheses)} \\
&= P(X_{10,.1} = 0) && (X \sim X_{n,p_w}) \\
&= \binom{10}{0}(.1)^0(1-.1)^{10-0} && \text{(pmf of binomial RV)} \\
&= .9^{10} \approx .348678
\end{aligned}$$

And for a type II error, i.e. accepting a false null hypothesis, we have:

$$\begin{aligned}
P(\text{Type II error}) &= P(\hat{H}_0 \mid H_1) && \text{(def. of type II error)} \\
&= P(X \geq 0 \mid p_w = .06) && \text{(from def. of hypotheses)} \\
&= \sum_{i=1}^{10} P(X_{10,.06} = i) && (X \sim X_{n,p_w}) \\
&= 1 - P(X_{10,.06} = 0) && \text{(complement)} \\
&= 1 - \binom{10}{0}(.06)^0(1-.06)^{10-0} && \text{(pmf of binomial RV)} \\
&= 1 - (.94)^{10} \approx .461385
\end{aligned}$$