

Math Statistics

Semiweekly HW 3

Ozaner Hansha

September 15, 2020

Question 1

Problem: Suppose that in the Calculus II classes at Utgard University, the mean score on the final exam is a 74% with a standard deviation of 10%. In a class of 100 students, assuming that they are randomly chosen, what is the probability that the mean score is at least 76%?

Solution: First note that we expect test scores to have a normal distribution. As such, each of the i.i.d. 100 sampled scores S_i should have the following distribution:

$$S_i \sim \mathcal{N}(.74, .01)$$

**note that since $\sigma = .1$ we have that $\sigma^2 = .01$.*

Now recall that the sum of normal RVs is itself a normal RV whose mean is the sum of the summands' mean and whose variance is the sum of the summands' variance. In this case we have $\mu = 100 * .74$ and $\sigma^2 = 100 * .01$ since all S_i are distributed i.i.d.:

$$\sum_{i=1}^{100} S_i = S \sim \mathcal{N}(74, 1)$$

And so our desired probability is given below:

$$\begin{aligned} P\left(\frac{1}{100}S \geq .76\right) &= P(S \geq 76) \\ &= 1 - P(S \leq 76) && \text{(complement)} \\ &= 1 - F_S(76) && \text{(def. of cdf)} \\ &= 1 - \Phi\left(\frac{76 - 74}{1}\right) && \text{(unit normal RV)} \\ &= 1 - \Phi(2) = \Phi(-2) && \text{(complement of unit RV cdf)} \\ &\approx 0.02275 && \text{(unit normal table lookup)} \end{aligned}$$