

MAT505E NUMERICAL ANALYSIS

HOMEWORK I

- Q.1** Find the roots of the equation $x^2 - 1000.001x + 1 = 0$ by using a calculator with five significant digits. Discuss the results, are the both roots accurate, why? How can you find more accurate results?
- Q.2** Determine absolute and relative truncation errors in computation of the function $\sin(x)$ for each of the values $x = 0.1, 0.2, 0.5, 1$ by taking first five terms of its Taylor's series expansion around $x = 0$.
- Q.3** To compute $e^{0.4}$ within the error 0.001, how many terms are needed in Taylor's series expansion of e^x at $x = 0$.
- Q.4** How can we avoid the cancellation error while computing $f(x) = \sqrt{1+x^2} - 1$ for small $|x|$ values, compute the condition number of the function.
- Q.5** Determine the condition number for the function $f(x) = \sin^{-1} \frac{x}{\sqrt{1+x^2}}$.
- Q.6** Find the forward and backward error values for $f(x) = \pi^x$ for $x = 3$, if the calculated value of the function is $\tilde{f} = 30.2$.
- Q.7** The Babylonians were using the relation $x_{n+1} = \frac{1}{2} \left(x_n + \frac{a}{x_n} \right)$ to compute \sqrt{a} , $a > 0$. Construct a function $f(x)$ as a root finding problem and get the above relation by using Newton method.
- Q.8** How many steps are required to find the fixed point of the function $g(x) = \sqrt{x+2}$ with the error tolerance 10^{-10} with the initial point $x_0 \in (0, 2)$.
- Q.9** Find the root of the function $f(x) = e^{-2x}x + x \cos(x) - 4 \sin(2x)$ on the interval $[1, 2]$ with the error tolerance 10^{-5} , i.e. $(|f(x)| \leq 10^{-5})$ by using
- Bisection method,
 - Regula Falsi method,
 - Newton-Raphson method,
 - Aitken's Δ^2 method with Newton-Raphson's results.
 - Secant method,
 - Fixed Point Iteration method, if possible.

If needed, take the initial values $x_0 = 1$. and $x_1 = 1.1$. Show convergences, iteration numbers and CPU times of all results in a Table and discuss the results.

- Q.10** Find the real roots of the functions

$$f_1(x) = -x^3 + 2x^2 - 2 = 0, \quad x_0 = 1, \quad f_2(x) = x^3 - 2x + 2 = 0, \quad x_0 = 0,$$

$$f_3(x) = (x-1)^3 + 0.512 = 0, \quad x_0 = 5, \quad f_4(x) = \sin x = 0, \quad x_0 = 7.539822$$

with the error tolerance 10^{-3} , i.e. $(|f(x)| \leq 10^{-3})$ and given initial values by using both "Newton-Raphson" and "Fixed Point Iteration" methods, if possible. Then discuss the results.

Due To: 24.10.2019