

MIDDLE EAST TECHNICAL UNIVERSITY
ELECTRICAL AND ELECTRONICS ENGINEERING DEPARTMENT

EE 362 Electromechanical Energy Conversion II

Midterm-II Examination

13 May 2016

Duration: 110 minutes

Attempt all questions

Show all your calculations for full credit.

Please mention it clearly if you continue your solutions on a new page.

Q1 (35 pts).

For a 310 V (1-l rms), $f = 60$ Hz, 3-phase, 6 -poles, Y-connected, slip-ring induction motor the following parameters are given:

$R_1 = 0.2 \Omega$, $X_1 = 0.38 \Omega$, $R_2' = 0.12 \Omega$, $X_2' = 0.38 \Omega$, $X_m = 10 \Omega$. The friction and windage (rotational) and core losses are neglected. The parallel branch is connected at the source terminals. When driving a constant torque load (τ_l) at the rated applied voltage (V_1) and slip, $s_1 = 0.02$ calculate:

Part A:

- (a) The synchronous rpm (n_s) and angular (ω_s) speeds and the rotor rpm (n_r) speed.
- (b) The load torque (τ_l) and the starting torque (τ_{stl}). Can the motor start the load?
- (c) The maximum (τ_{maxl}) torque.
- (d) While the machine is running at the rated speed, the applied voltage is now reduced to a value (V_2), which is the minimum possible voltage value enough to drive the load. (Hint: the motor produces maximum torque at this point and can be operated temporarily). Calculate (V_2) and the corresponding speed (n_{mn}) at this operation mode.

Part B:

While the motor is driving a constant torque load of $\tau_l = 117.6$ Nm, at the rated voltage (V_1), an external resistance (R_{ex}') is connected to the rotor terminals to achieve the maximum starting torque at start and also to control the speed. (For this part neglect the stator winding resistance R_1).

- (e) Calculate the minimum referred external resistance R_{ex}' for the above operating condition.
- (f) Calculate the slip (s_2) when R_{ex}' remains connected in the rotor circuit for the above operating condition.
- (g) Calculate (approximate) the efficiency (η) for the above operating condition. Is this method of speed control suitable? Comment.

Hint: You may use the following equations.

$$\tau = \frac{3}{\omega_s} \cdot \frac{V_1^2}{(R_1 + R_2'/s)^2 + (X_1 + X_2')^2} \times \frac{R_2'}{s}, \quad s_{Tmax} = \frac{R_2'}{\sqrt{R_1^2 + (X_1 + X_2')^2}},$$

Efficiency: $\eta_{ap} \approx 1 - s$

Q1-SOLUTION

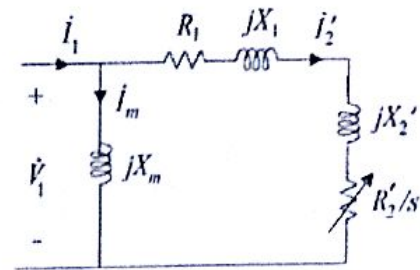
$$(a) V_1 = \frac{310}{\sqrt{3}} = 179 \text{ V}, R_1 = 0.2 \Omega, X_1 = 0.38 \Omega$$

$$R'_2 = 0.12 \Omega, X'_2 = 0.38 \Omega, X_m = 10 \Omega$$

$$n_s = \frac{120f}{P} = \frac{120 \times 60}{6} = 1200 \text{ rpm}$$

$$\omega_s = 2\pi \frac{n_s}{60} = 2\pi \frac{1200}{60} = 125.66 \text{ rad/s}$$

$$s_1 = 0.02, n_1 = n_s(1 - 0.02) = 1200 \times (1 - 0.02) = 1176 \text{ rpm}$$



(Fig.1)

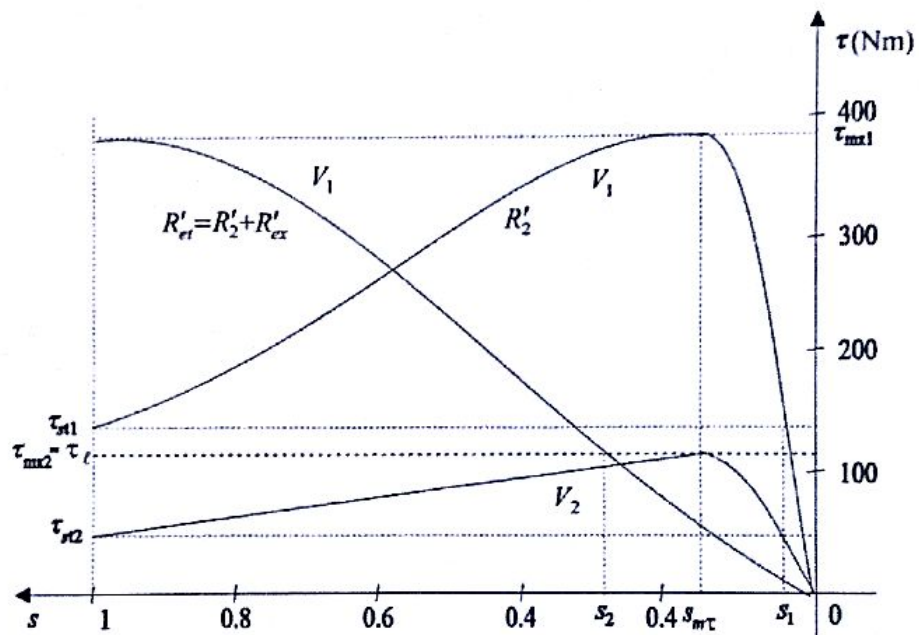


Fig.2

$$(b) \tau_t = \frac{3}{\omega_s} \cdot \frac{V_1^2}{(R_1 + R'_2/s)^2 + (X_1 + X'_2)^2} \times \frac{R'_2}{s}$$

$$\tau_t = \frac{3}{125.66} \times \frac{179^2}{(0.2 + \frac{0.12}{0.02})^2 + (0.76)^2} \times \frac{0.12}{0.02} = 117.6 \text{ Nm}$$

$$\tau_{st1} = \frac{3}{\omega_s} \cdot \frac{V_1^2}{(R_1 + R'_2)^2 + (X_1 + X'_2)^2} \times R'_2 = \frac{3}{125.66} \times \frac{1179^2}{(0.32)^2 + (0.76)^2} \times 0.12 = 135 \text{ Nm}$$

Yes. $\tau_{st1} > \tau_t$, (Fig.1, Fig.2)

$$(c) s_{mT} = \frac{R'_2}{\sqrt{R_1^2 + (X_1 + X'_2)^2}} = \frac{0.12}{\sqrt{0.2^2 + (0.76)^2}} = 0.1527$$

$$\tau_{mx1} = \frac{3}{\omega_s} \cdot \frac{V_1^2}{(R_1 + R'_2/s_{mr})^2 + (X_1 + X')^2} \times \frac{R'_2}{s_{mr}}$$

$$\tau_{mx1} = \frac{3}{125.66} \times \frac{179^2}{(0.2 + \frac{0.12}{0.1527})^2 + (0.76)^2} \times \frac{0.12}{0.1527} = 387.95 \text{ Nm}$$

$$(d) s = s_{mr} \rightarrow \tau_t = \tau_{mx2} \rightarrow n = n_{mn} = n_s(1 - s_{mr}) = 1200 \times (1 - 0.1527) = 1016.76 \text{ rpm}$$

$$\frac{\tau_{mx1}}{\tau_t} = \frac{V_1^2}{V_2^2}, V_2^2 = V_1^2 \frac{\tau_t}{\tau_{mx1}}, V_2 = (179^2 \frac{117.6}{387.95})^{0.5} = 98.6 \text{ V}$$

$$\frac{\tau_{st1}}{\tau_{st2}} = \frac{V_1^2}{V_2^2}, \tau_{st2} = \frac{V_2^2}{V_1^2} \tau_{st1} = (\frac{98.6}{179})^2 \times 135 = 41 \text{ Nm, (Fig.2)}$$

$$(e) s_{mr} = \frac{R'_{2t}}{\sqrt{R_1^2 + (X_1 + X'_2)^2}} = 1, R'_{2t} = \sqrt{0.2^2 + (0.76)^2} = 0.786 \Omega, R'_{ex} = 0.786 - 0.2 = 0.586 \Omega$$

$$R_1 \approx 0, \tau_t = \frac{3}{\omega_s} \cdot \frac{V_1^2}{(R'_{2t}/s_2)^2 + (X_1 + X'_2)^2} \times \frac{R'_{2t}}{s_2}$$

$$\tau_t = \frac{3}{125.66} \cdot \frac{179^2}{(0.786/s_2)^2 + (0.76)^2} \times \frac{0.786}{s_2} = 117.6$$

$$s_2 \times (0.786/s_2)^2 + s_2 \times (0.76)^2 = \frac{3 \times 179^2 \times 0.786}{117.6 \times 125.66} = 5.11$$

$$0.6178/s_2 + 0.5776s_2 - 5.11 = 0, 0.5776 \times s_2^2 - 5.11 \times s_2 + 0.6178 = 0$$

$$s_2 = \frac{5.11 \pm (5.11^2 - 4 \times 0.5776 \times 0.6178)^{0.5}}{2 \times 0.5776} = \frac{5.11 \pm 4.97}{1.1552} = 0.121; 8.725, \text{ Fig.2}$$

$$(h) \eta \approx (1 - s_2) = 1 - 0.121 = 87.9\%$$

$$n_2 = n_s(1 - s_2) = 1200 \times (1 - 0.121) = 1054.8 \text{ rpm}, \omega_m = 2\pi \frac{1054.8}{60} = 110.46 \text{ r/s}$$

$$s_2 = 0.121, \frac{R'_{2t}}{s_2} = \frac{0.786}{0.121} = 6.5 \Omega$$

$$R_1 \approx 0, I_2'^2 = \frac{V_1^2}{(\frac{R'_{et}}{s_2})^2 + X_t^2} = \frac{179^2}{6.5^2 + 0.76^2} = 748.14$$

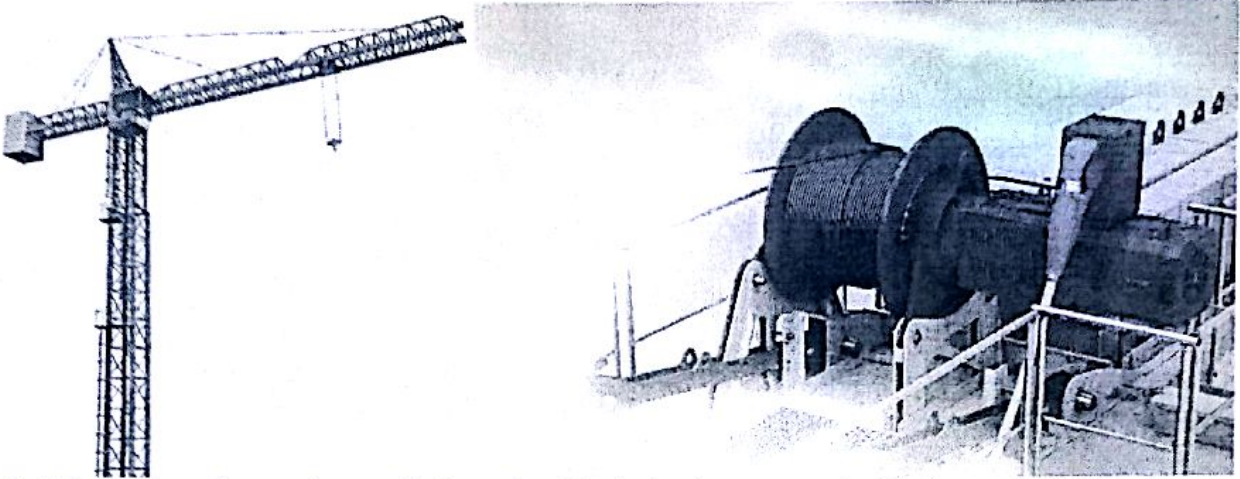
$$P_o = 3P_m = \tau_t \times \omega_m = 3 \times 117.6 \times 110.46 = 38970.3 \text{ W}$$

$$P_{cur} = 3 \times \frac{R'_{et}}{s_2} \times I_2'^2 = 3 \times 6.5 \times 748.14 = 14588.7 \text{ W}$$

$$P_{in} = P_g = P_m + P_{cur} = 38970.3 + 14588.7 = 53559 \text{ W}, \eta = \frac{P_o}{P_{in}} = \frac{38970.3}{53559} = 72.76\%$$

Q2 (35 pts)

Induction motors are commonly used in tower cranes with variable voltage and frequency drives as shown in the figures below.



In this problem, ignore the parallel branch of the induction motor, the friction and windage losses.

a) (4pts) Assume that, the crane operator is lifting (moving up) a mass at constant speed. (Constant-torque load)

i) Please sketch the torque-speed characteristics of a typical induction machine between $s=1$ and $s=-1$. (Label the critical points and clearly show operating mode regions).

ii) In the same graph, draw a load torque line and label the operating point. In what mode does the machine is operating in this case? Describe the direction of the power flow.

b) (6pts) Torque characteristic of induction machines can be approximated using a linear equation ($T_e=ks$), if the rotor speed is close to the synchronous speed, where k is a factor that depends on the machine and supply characteristics.

Starting from the electromechanical torque expression given below, derive the value of k . Please state any assumptions you made, for full credit.

$$\tau = \frac{3}{\omega_s} \cdot \frac{V_1^2}{(R_1 + R_2'/s)^2 + (X_1 + X_2')^2} \times \frac{R_2'}{s}$$

c) (5pts) Assume you have a 400V (l-l) wye-connected, 16kW, 3-phase, 6-pole squirrel-cage induction machine connected to a variable voltage variable frequency drive. The referred rotor resistance (r_2') is 0.5Ω .

The crane operator lifts a mass which exerts 91 Nm of torque on the shaft of the induction machine. Calculate the rotor speed in rpm, if the induction machine is supplied with 50 Hz, 400 V (l-l) voltage.

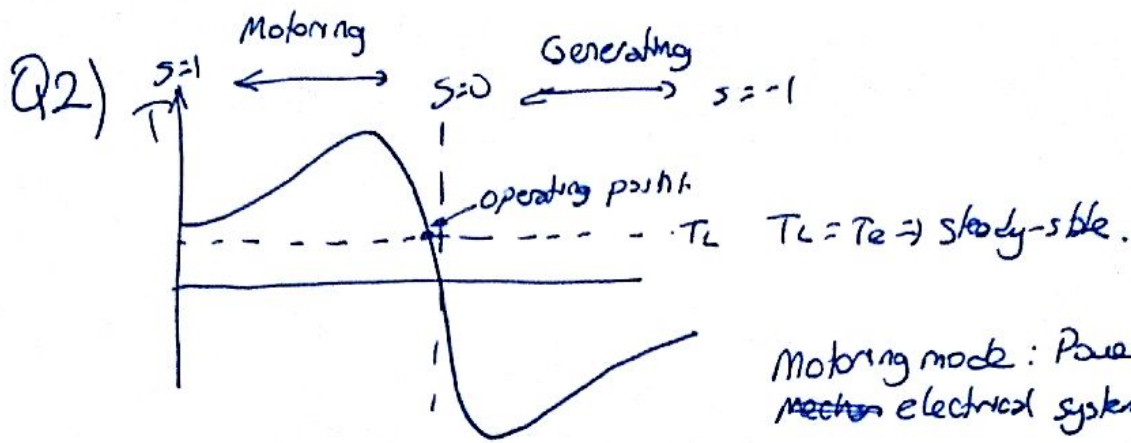
d) (5pts) Now, the operator would like to increase the speed of the load using the motor controller, which responds by suddenly changing the applied frequency to 60 Hz under **constant V/f mode of operation**, calculate the speed of the rotor once the system reaches the steady-state under 60 Hz excitation.

e) (5pts) In a graph, sketch the torque characteristics of the machine (torque vs rotor speed in rpm) in part (c) and label the operating point with label **A**. Then label the steady state operating point of part (d) with label **B**. Describe in detail how the machine moves from the state in point A to the state in point B.

f) (6 pts) For the transient period from point A to point B, sketch the following.

- i) Rotational speed vs time
- ii) Electromagnetic torque vs time
- iii) Gross mechanical power vs time.

g) (4 pts) If the crane is lifting a 1000 kg load at constant speed, and the motor is delivering rated power of 16 kW. What is the linear speed of the load in m/s?



b) $s \approx 0 \Rightarrow \frac{r_2'}{s} \gg r_1, x_1, x_2$ } all three can be neglected

$$T = \frac{3}{\omega_s} \cdot \frac{U_1^2}{(r_2'/s)^2} \cdot \frac{r_2'}{s} = \frac{3 U_1^2}{\omega_s r_2'} \cdot s$$

k

c) $U_{LL} \Rightarrow 400 \Rightarrow U_{ph} = \frac{400}{\sqrt{3}} = 230V$

6 pole $\Rightarrow \omega_s = \frac{100\pi}{3}$

$n_s = 1000 \text{ rpm}$

$$T = \frac{3 U_1^2}{\omega_s \cdot r_2'} \cdot s$$

$$g_1 = \frac{3 \cdot (230)^2}{\frac{100\pi}{3} \cdot 0.5} \cdot s$$

$s = 0.03 \Rightarrow n_r = 970 \text{ rpm}$

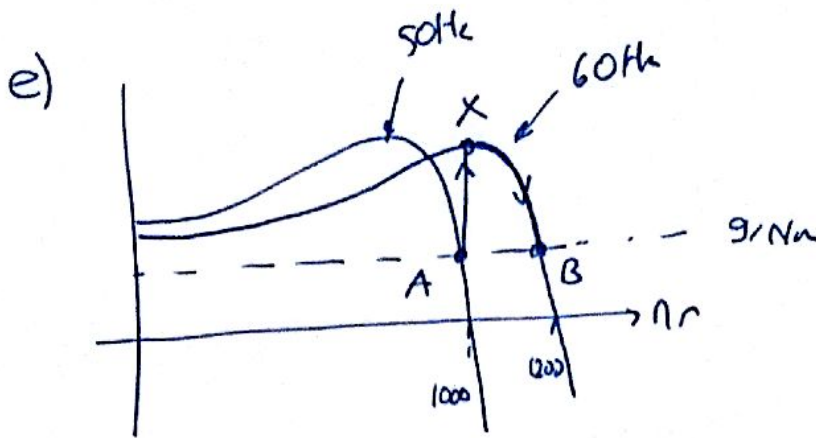
d) Constant $V/f \Rightarrow f_1 \Rightarrow 50 \text{ Hz}, f_2 = 60 \text{ Hz}$

$V_2 = 1.2 V_1$

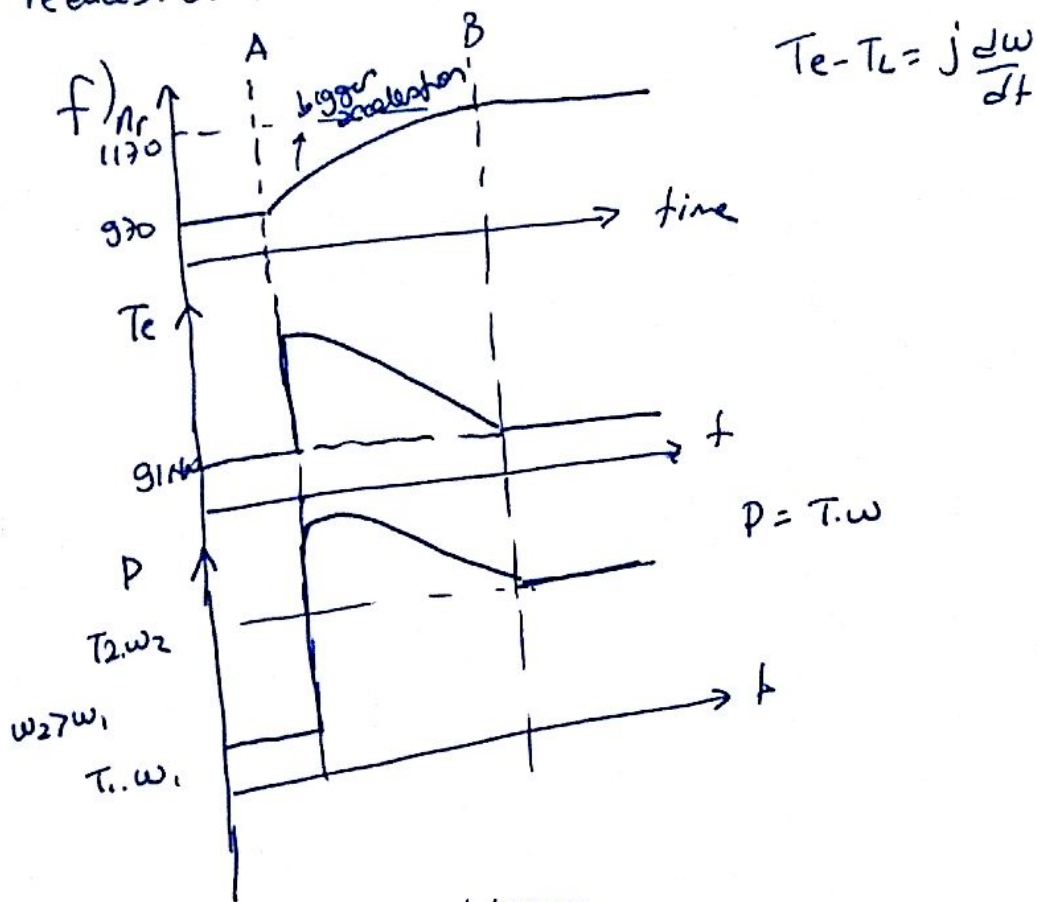
$\omega_{s2} = 1.2 \omega_{s1}$

$$g_1 = \frac{3 \cdot (1.2 \cdot V_1)^2}{1.2 \cdot \omega_{s1} \cdot r_2'} \cdot s_2 \Rightarrow s_2 = \frac{5}{6} \cdot s_1 \Rightarrow s_2 = 0.025 \Rightarrow n_s = 1200 \text{ rpm}$$

$n_r = (1-s) \cdot n_s = 1170 \text{ rpm}$



When the frequency changes, the speed cannot change instantaneously, but the torque characteristics change (point X). That extra torque accelerates the load until the point B is reached. Acceleration is bigger at first then reduces, as it reaches to 1170 rpm.



g)

<u>Rotational</u>	<u>Linear</u>
$P = T\omega$	$P = Fv$
	$16k = m \cdot g \cdot v$
	$16.000 = 1000 \cdot 9.81 \cdot v$
	$v = 1.6 \text{ m/s}$

Q3 (40 pts).

A 3-phase, 400-V, 50-Hz, 2-pole, wye-connected cylindrical-rotor synchronous machine is operating with its armature terminals are connected to a 3-phase, 400-V l-to-l, 50- Hz **infinite bus**. Its parameters are

$$X_s = 1.5 \text{ ohm}, \quad R_a \text{ is negligibly small.}$$

Part A (20 pts). Generating mode of operation.

The above machine is operating in generating mode. The power rating of this machine as given on the nameplate is **69-kVA at 0.8 power factor lagging** for generating mode.

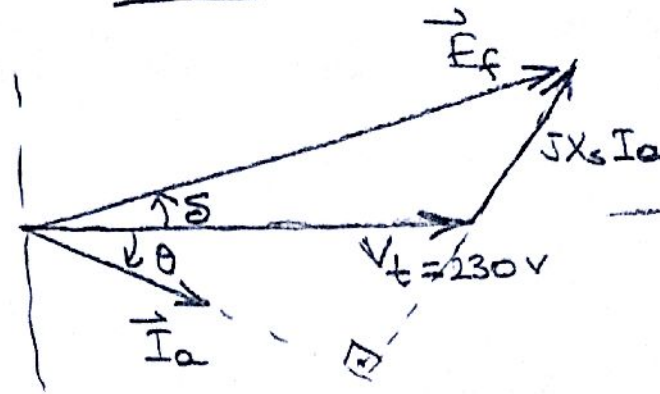
- a. Draw the phasor diagram on complex plane.
- b. Calculate the armature current, I_a by assuming that the synchronous generator is **delivering rated power to the infinite bus at 0.8 power factor lagging**.
Calculate also the active power, P and the reactive power, Q delivered to the infinite bus. State whether the synchronous generator is acting as a **consumer or producer** of the reactive power at this operating point.
- c. Calculate the excitation emf, E_f and load (power) angle, δ .

Part B (20 pts). Motoring mode of operation.

Suppose now that the same machine is operating as a motor to draw 100 A from the infinite-bus at **unity power factor**.

- d. Draw the phasor diagram.
- e. Calculate E_f and δ .
- f. Determine the output electromechanical power, P_e and the corresponding electromechanical torque, T_e .

a)



b) $V_t = 400/\sqrt{3} = 230 \text{ V/ph-}\omega y e$
 $S_o = 3 V_t I_a \rightarrow 69000 = 3 \times 230 \times I_a \Rightarrow$
 $I_a = 100 \text{ A} //$

$\theta = \cos^{-1}(0.8) = 36.87^\circ \rightarrow \sin \theta = 0.6$

c) $P = S \cdot \cos \theta = 69 \times 0.8 = 55.2 \text{ kW} //$
 $Q = S \cdot \sin \theta = 69 \times 0.6 = 41.4 \text{ kVAR} //$
 It is a producer of reactive power.

d) $\vec{E}_f = \vec{V}_t + jX_s \vec{I}_a$
 $E_f (\cos \delta + j \sin \delta) = V_t + jX_s I_a (\cos \theta - j \sin \theta)$
 $E_f (\cos \delta + j \sin \delta) = 230 + j1.5 \times 100 (0.8 - j0.6)$
 $= 230 + j150 (0.8 - j0.6)$
 $E_f \cos \delta + j E_f \sin \delta = 230 + j120 + 90$
 Resolution of real and imaginary components gives

$\cancel{E_f} \sin \delta = \cancel{j} 120$

$E_f \cos \delta = 320$

Dividing both sides

$\frac{\cancel{E_f} \sin \delta}{\cancel{E_f} \cos \delta} = \frac{120}{320} \rightarrow \tan \delta = \frac{12}{32} \rightarrow \delta = \tan^{-1}\left(\frac{12}{32}\right)$

$\delta = 20.6^\circ //$

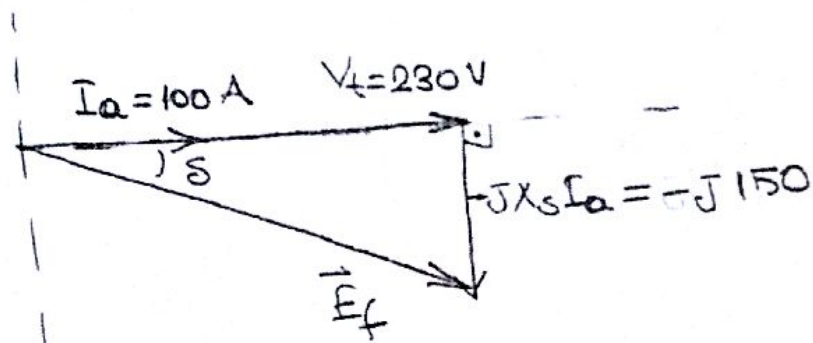
$E_f \sin 20.6 = 120 \Rightarrow$

$E_f = 341.8 \text{ V/ph} //$ and $E_f = 592 \text{ V l-to-l} //$

$\frac{1}{2}$

Part B

e)



$$f) \quad \delta = \tan^{-1}(150/230) \Rightarrow \delta = 33.1^\circ //$$

$$E_f = \sqrt{230^2 + 150^2} = 274.6 \text{ V/ph} //$$

$$\equiv 475.6 \text{ V l-to-l} //$$

g) Since R_a is neglected then $P_e = P_{in} = P$
 Therefore $P_e = 3 \cdot E_f \cdot \cos(\theta + \delta) = P_{in} = 3 \cdot V_t I_a \cos \theta$
 $3 \times 274.6 \times 100 \times \cos(0 + 33.1) = 3 \times 230 \times 100 \times \cos 0$
 $69 \text{ kW} \checkmark = 69 \text{ kW} //$

$$\omega_r = \omega_s = \frac{2\pi f}{(2/2)} = 2\pi 50 = 100\pi \text{ mech rad/s}$$

$$T_e = \frac{P_e}{\omega_r} = \frac{69000}{100\pi} = 219.6 \text{ Nm} //$$

Dr Ermiq