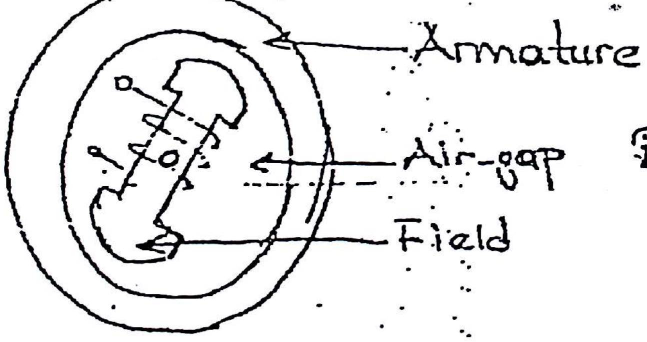
Example

Assume that a 3-phase, 50-Hz, 10-pole Y-connected alternator (synchronous generator) is driven at its synchronous speed by the prime mover.

Armature side:
210 slots, double-layer with a 60° phase spread, 4 turns/coil, Coil pitch is 18 slots.
$D = 1.5$ m, $\ell = 0.5$ m

Field side:
Shaping the air-gap is not very successfull so that the air-gap flux contains some harmonics with significant magnitudes.



$$B(\theta) = \sin\theta + 0.25 \sin 3\theta + 0.15 \sin 5\theta \ \ Wb/m^2$$

Calculate RMS line and phase voltages.

Solution

Synchronous speed, $n = \frac{120f}{P} = \frac{120 \times 50}{10} = 600$ rpm

If this m/c were driven at a speed different than the synchronous speed, say 300 rpm, it would generate $f = np/120 = 300 \times 10/120 = 25$ Hz armature emfs.

Induced emf expression:

$$E_{rms_i} = 4.44\, k_{w_i}\, f_i\, N_{ph}\, \phi_i$$

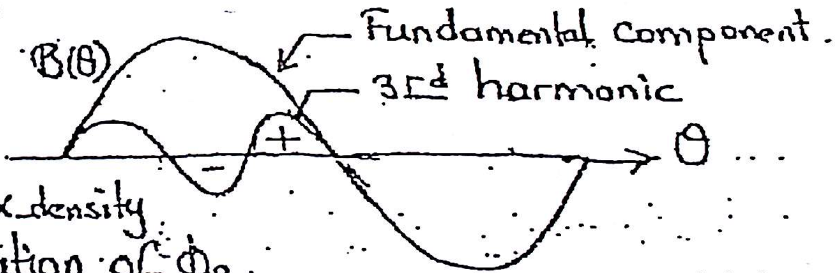
where, i is the no of harmonics ($i = 1, 3, 5$)
and $f_1 = 50$ Hz, $f_3 = 3 \times 50 = 150$ Hz, $f_5 = 5 \times 50 = 250$ Hz

Calculation of Flux components in the air-gap:

$$\phi_i = \left[\frac{2}{\pi} B_{peak_i} \frac{\pi d l}{p}\right] / i$$

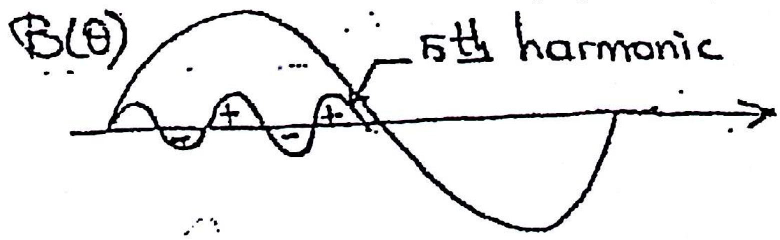
$$\phi_1 = \underbrace{\frac{2}{\pi} \times 1.0}_{B_{average}} \times \underbrace{\frac{\pi \times 1.5 \times 0.5}{10}}_{Area/pole} = 0.15 \text{ Wb}$$

$$\phi_3 = \frac{\frac{2}{\pi} \times 0.25}{3} \times \frac{\pi \times 1.5 \times 0.5}{10} = 0.0125 \text{ Wb}$$



Under each pole, there is three half cycles of the 3rd harmonic flux density component. In the calculation of ϕ_3, the negative half-cycle cancels one of the positive half-cycles. This effectively corresponds to dividing $\frac{2}{\pi} B_{3\,peak}$ by 3.

$$\phi_5 = \frac{\frac{2}{\pi} \times 0.15}{5} \times \frac{\pi \times 1.5 \times 0.5}{10} = 0.0045 \text{ Wb}$$



Calculation of number of series turns/phase :

In double layer winding arranpement the number of coil groups in each phase is equal to the number of poles, i,e:

∴ No of coil groups/phase $= p = 10$

All of them are normally connected in series to form the corresponding armature phase winding, say phase a.

∴ In each coil group/ under each magnetic pole there are $210/10/3 = 7$ coils with 4 turns/coil.

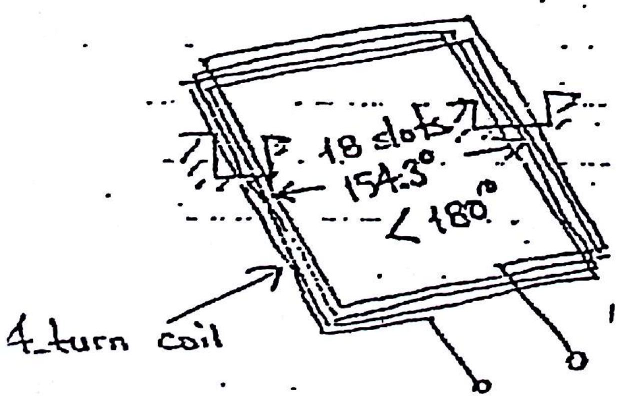
Therefore,

No of series turns/phase, $N_{ph} = 10 \times 7 \times 4 = 280$ turns.

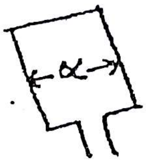
Calculation of winding factors (k_{w_i}) :

No of slots/pole $= 210/10 = 21$ (π radians one pole pitch)
No. of slots/pole/phase $= 21/3 = 7$ (60° phase belt)

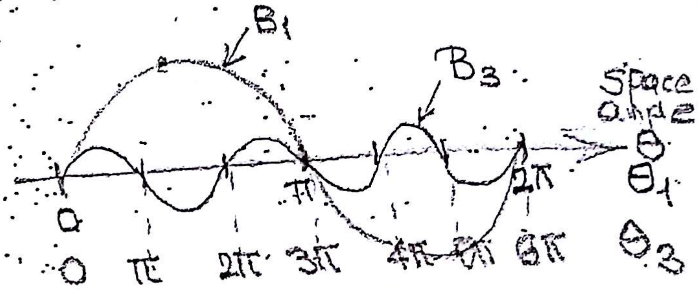
Coil pitch $= 18$ slots $= \frac{18}{21} \times 180^\circ = 154.3^\circ$ electrical which is being an under-pitched armature winding.



$$k_{c_i} = \sin \frac{i\alpha}{2}$$



α is coil span



$$k_{c_1} = \sin \frac{154.3}{2} = 0.975$$

$$k_{c_3} = \sin \frac{3\times 154.3}{2} = -0.78$$

$$k_{c_5} = \sin \frac{5\times 154.3}{2} = 0.433$$

$$k_{s_i} = \frac{\sin i\theta}{q \sin \frac{i\theta}{q}}$$

$2\theta = 60^\circ \Rightarrow \theta = 30^\circ$

$q = 210/10/3 = 7$ slots

$$k_{s_1} = \frac{\sin 30}{7 \sin \frac{30}{7}} = 0.95$$

$\theta_3 = 3 \cdot \theta_1$

$\theta_5 = 5 \cdot \theta_1$

$\theta_i = i \cdot \theta_1$

$$k_{s_3} = \frac{\sin 3\times 30}{7 \sin \frac{3\times 30}{7}} = 0.64$$

$$k_{s_5} = \frac{\sin 5\times 30}{7 \sin \frac{5\times 30}{7}} = 0.196$$

$$k_{w_i} = k_{c_i} \times k_{s_i}$$

$$k_{w_1} = 0.975 \times 0.95 = 0.93$$

$$k_{w_3} = -0.78 \times 0.64 = -0.5$$

$$k_{w_5} = 0.433 \times 0.196 = 0.085$$

$E_{rms_1} = 4.44 \times 0.93 \times 50 \times 280 \times 0.15 = 8671$ Volts/phase

$E_{rms_3} = 4.44 \times 0.5 \times 150 \times 280 \times 0.0125 = 1165.5$ Volts/phase

$E_{rms_5} = 4.44 \times 0.085 \times 250 \times 280 \times 0.0045 = 119$ Volts/phase

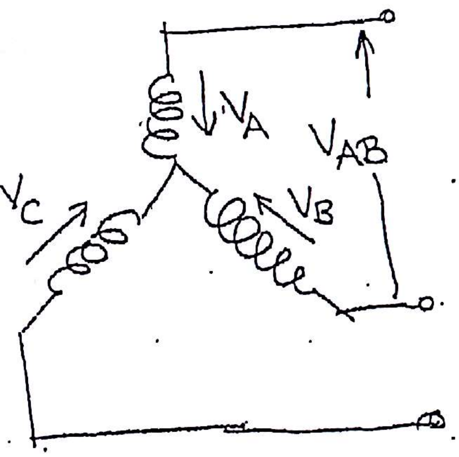
TRUE RMS

↓

$$E_{rms}\,/\text{phase} = \sqrt{\sum E^2_{rms_i}} = \sqrt{E^2_{rms_1} + E^2_{rms_3} + E^2_{rms_5}}$$

$$= \sqrt{8671^2 + 1165.5^2 + 119^2} = 8750 \text{ Volts}$$

3 rd harmonic emfs do not appear in line-to-li[ne] voltages (If the m/c is connected in wye).



$$v_A = V_{A_1} \sin \omega t + V_{A_3} \sin 3\omega t + V_{A_5} \sin 5\omega t$$

$$v_B = V_{B_1} \sin(\omega t - 120^\circ) + V_{B_3} \sin 3(\omega t - 120^\circ) + V_{B_5} \sin 5(\omega t -$$

$$v_C = V_{C_1} \sin(\omega t - 240^\circ) + V_{C_3} \sin 3(\omega t - 240^\circ) + V_{C_5} \sin 5(\omega t$$

$$\begin{aligned} v_{BA} = v_B - v_A &= V_1 [\sin(\omega t - 120^\circ) - \sin \omega t] \\ &+ V_3 [\sin(3\omega t - 360^\circ) - \sin 3\omega t] \\ &+ V_5 [\sin 5(\omega t - 240^\circ) - \sin 5\omega t] \\ &= \sqrt{3}\, V_1 \cos(\omega t - 60^\circ) + \sqrt{3}\, V_5 \cos(5\omega t - 120^\circ) \end{aligned}$$

TRUE RMS

↓

$$E_{rms}\ \ell\text{-to-}\ell = \sqrt{3} \times \sqrt{8671^2 + 119^2} = \sqrt{3} \times 8672$$

$$= 15 \text{ kV}$$

Repeat the calculations by assuming a full-pitch wd[g]

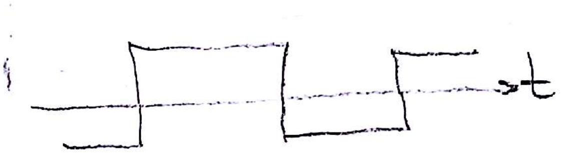
RMS Value , $V_1 = \dfrac{\hat{V}_1}{\sqrt{2}}$

TRUE RMS Value , $V_R = \sqrt{V_1^2 + V_2^2 + V_3^2 + \cdots}$

Total Harmonic Distortion , $THD_F = \dfrac{\sqrt{V_2^2 + V_3^2 + V_4^2 + V_5^2 + \cdots}}{V_1}$

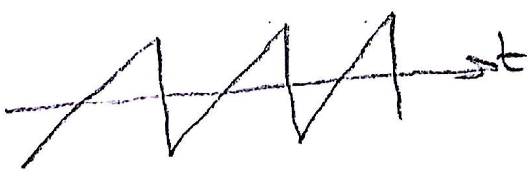
Total Harmonic Distortion , $THD_R = \dfrac{\sqrt{V_2^2 + V_3^2 + V_4^2 + \cdots}}{\underbrace{\sqrt{V_1^2 + V_2^2 + V_3^2 + \cdots}}_{\text{True RMS value}}}$

For a pure square wave



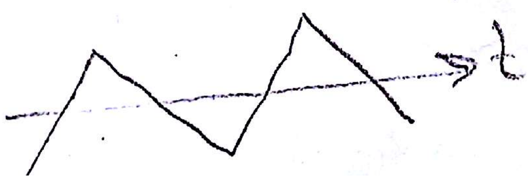
$THD_F = \sqrt{\dfrac{\pi^2}{8} - 1} \cong 0.483 = 48.3\%$

The sawtooth signal



$THD_F = \sqrt{\dfrac{\pi^2}{6} - 1} = 80.3\%$

The pure symmetrical triangular wave



$THD_F = \sqrt{\dfrac{\pi^4}{96} - 1} = 12.1\%$

* Exercise

3-phase 50Hz, 10-pole Y-connected, alternator is driven at its synchronous speed

Armature side

210 slots, double layer winding, with 4 turns/coil

Coil-pitch: 18 slots, coil spread: 60 degrees.

a) Find the synchronous speed: (in rpm) ↳ is always mechanical.

10 pole ⇒

* What should be the mechanical speed in order to get 50 Hz in the stator?

2 pole ⇒ 50Hz ⇒ $50 \times 60 = 3000$ rpm

4 pole ⇒ 1500 rpm

6 pole ⇒ 1000 rpm

$\boxed{10 \text{ pole } \; 600\text{rpm}}$

$$N_{rpm} = \frac{120f}{P} = \frac{60.f}{(P/2)} = \frac{120 \times 50}{10} = 600\text{rpm}$$

b) Slots per pole per phase

$$\frac{210}{10.3} = 7 \text{ slots per pole per phase}$$ → $A_1\ A_2\ A_3\ A_4\ A_5\ A_6\ A_7 \ldots$

c) N_{ph} ⇒ ~~...~~

$$7\,\text{slots} \times 2\,\text{layers} \times \frac{4\,\text{turns}}{\text{coil}} \times \left(\frac{10}{2}\right) = 280 \text{ turns/phase}$$

pole pairs.

↳ repeats itself 5 times

..../......./..............

winding factor.

$$V_{rms}(phase) = \frac{2\pi}{\sqrt{2}} \cdot k_w \cdot f \cdot N_{ph} \cdot \Phi_{pp}$$

Number of turns per phase

flux per pole

~~kw~~?

d) Coil pitch = ? $\frac{210}{10} \Rightarrow 21$ slots per pole $\Rightarrow \pi$ electrical angle (180° electrical)

coil pitch = 18 slots (given)

$\frac{18}{21} \cdot 180 = 154{,}3°$ electrical <u>underpitched</u>

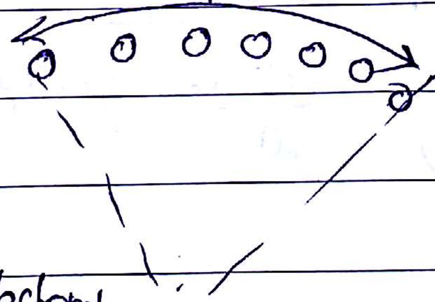
$$k_{p1} = \sin\left(\frac{154{,}3}{2}\right) = 0{,}975$$

$$k_{p3} = \sin\left(\frac{n \cdot \lambda}{2}\right) = \sin\left(\frac{3 \cdot 154{,}3}{2}\right) = -0{,}78$$

$$k_{p5} = \sin\left(5 \cdot \frac{154{,}3}{2}\right) = 0{,}433$$

coil span = 60°

Coil span



coil span $= q\alpha$

number of slots per pole per phase

electrical angle between slots.

$q = 7 \quad q\alpha = 60°$

$$k_d = \frac{\sin\left(\frac{q\alpha}{2}\right)}{q\sin(\alpha/2)} = \frac{\sin(30)}{7 \cdot \sin(30/7)} = 0{,}95$$

$$kd_3 = \frac{\sin(3.30)}{7.\sin\left(\frac{3.30}{7}\right)} = 0,64.$$

$$kd_5 = \frac{\sin(5.30)}{7.\sin\left(\frac{5.30}{7}\right)} = 0,196.$$

$$kw = kp \times kd$$

$$kw_1 = 0,975 \times 0,95 = 0,92625$$

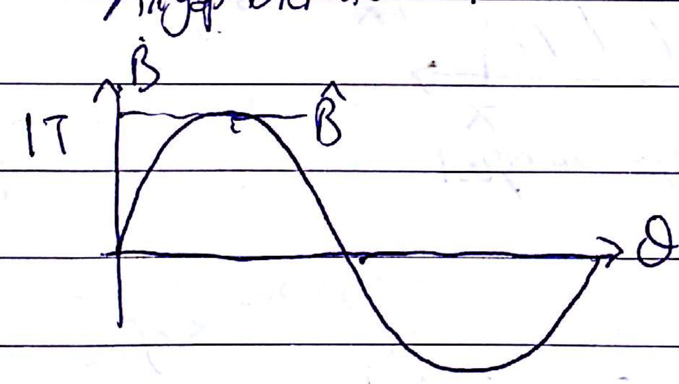
$$kw_3 = -0,75 \times 0,64 = -0,48$$

$$kw_5 = 0,433 \times 0,196 = 0,08487$$

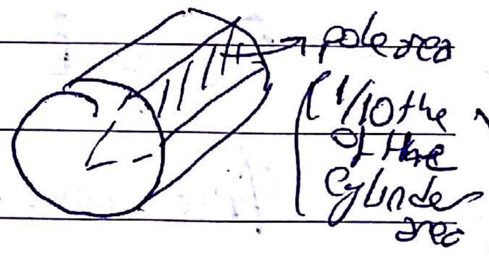
=) Induced Voltage

$$B(\theta) = \sin(\theta) + 0,25 \sin(3\theta) + 0,15 \sin(5\theta)$$

Airgap Diameter = 1,5m   Core length = 0,5m



← 1st component of B.

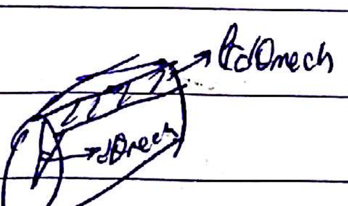


- Formal way

$$\phi_{pole} = \oint B.dA$$

$$\phi_p = \int_{\theta_e=0}^{\theta_e=\pi} \hat{B}.\sin(\theta_e).dA_{mech}$$

$$\phi_p = \int_{\theta_e=0}^{\theta_e=\pi} \hat{B}.\sin(\theta_e).\,l r.\,d\theta_{mech}$$



$$\left(\theta_{mech} = \frac{\theta_e}{P/2}\right)$$

$$\phi_{pole} = \int_0^{\pi} \hat{B} \cdot \sin(\theta_e) \cdot l \cdot r \cdot \frac{d\theta_e}{P/2}$$

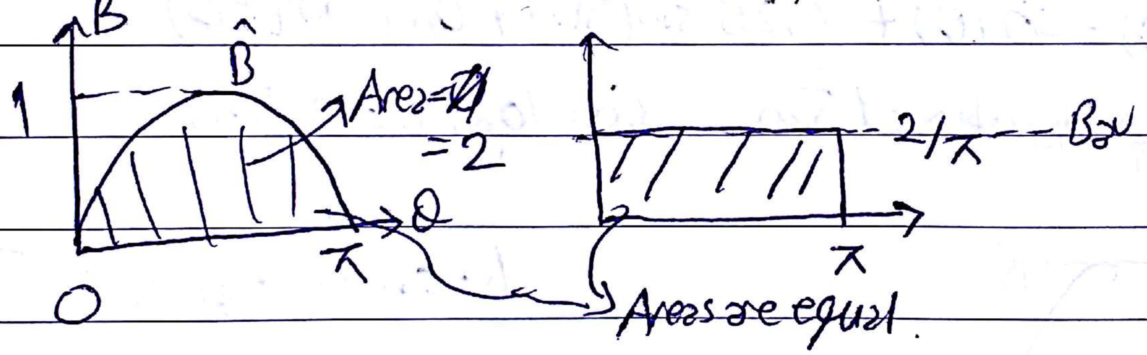
$$= \frac{\hat{B} \cdot l \cdot 2r}{P} \int_{\theta_e=0}^{\theta_e=\pi} \sin(\theta_e) \cdot d\theta_e$$

$$= \frac{\hat{B} \cdot l \cdot 2r}{P} \left(-\cos(\theta_e) \Big|_0^{\pi} \right)$$

$$= \frac{\hat{B} l \cdot 2r}{P} \left(-(-1) - (-1) \right) = \frac{\hat{B} \cdot l \, \overbrace{(2r)}^{\text{diameter}} \cdot 2}{P} = \frac{2 \hat{B} l \cdot D}{P}$$

⇒ Simpler way

⇒ Use the average flux and multiply with pole area



$$\bar{B}_{av} = \frac{2}{\pi} \cdot \hat{B}$$

$$\phi_{pole} = \bar{B}_{av} \cdot A_{pole} = \frac{2}{\pi} \cdot \hat{B} \cdot \frac{\overbrace{2\pi r l}^{\text{total area of rotor surface}}}{P}$$

$$\boxed{\phi_{pole} = \frac{2 \hat{B} l \cdot D}{P}}$$

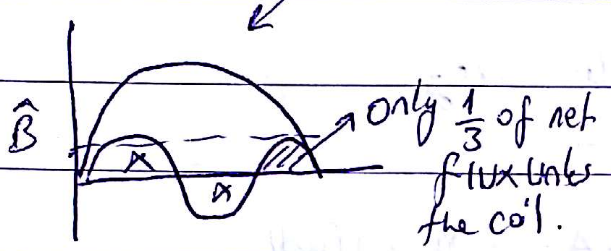
← same equation obtained!

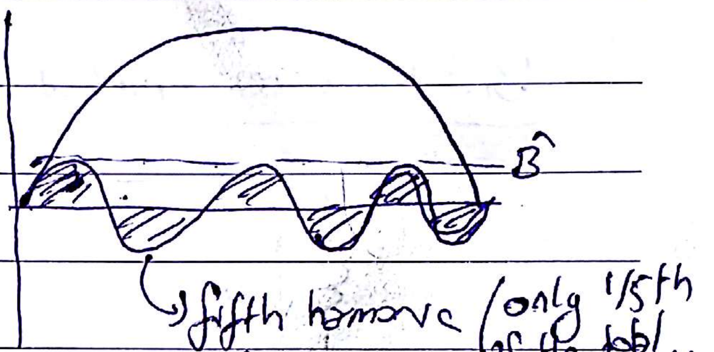
$\phi_1 \Rightarrow \quad B_{av} = \frac{2}{\pi}\hat{B} = \frac{2}{\pi}\ T$

$$\phi_{1\,pole} = \frac{2}{\pi}\cdot\frac{2\pi r l}{P} = \frac{2}{\pi}\cdot\frac{2\pi\cdot 1,5\cdot 0,5}{10}$$

$$= \underline{\underline{0,15\ Wb}}$$

$$\phi_3 = \frac{2}{\pi}\cdot\frac{0,25}{3}\cdot\frac{2\pi\cdot 1,5\cdot 0,5}{10} = \underline{0,0125\ Wb}\ \text{(3rd harmonic)}$$





$$B_{av(3)} = \frac{\frac{2}{\pi}\hat{B}}{3}$$

harmonic order

$$B_{av(5)} = \frac{\frac{2}{\pi}\hat{B}}{5}$$

$$\phi_5 = \frac{\frac{2}{\pi}0.15}{5}\cdot\frac{\pi\cdot 1,5\cdot 0,5}{10} = 0,0045\ Wb\ \text{(5th harmonic)}$$

$$E_{rms1} = \frac{2\pi}{\sqrt{2}}\cdot 0,925\cdot 50\cdot 280\cdot 0,15$$

k_{w1} $\quad$ $\phi_{pole(1)}$

E_{rms1} = ~~8677 V/phase~~ 8642 V/phase

$$E_{rms3} = \frac{2\pi}{\sqrt{2}}\cdot -0,48\ 150\cdot 280\cdot 0,0125$$ (*) be aware of (-)

E_{rms3} = $\underline{\underline{1119}}$ V/phase

$5f_1$

$$E_{rms5} = \frac{2\pi}{\sqrt{2}}\cdot 0,089\,(87)\ 250\cdot 280\cdot 0,0045$$

E_{rms5} = 119 V/phase

$$E_{rms} = \sqrt{\Sigma E_{rms(i)}^2} = \sqrt{E_{rms1}^2 + E_{rms3}^2 + E_{rms5}^2}$$

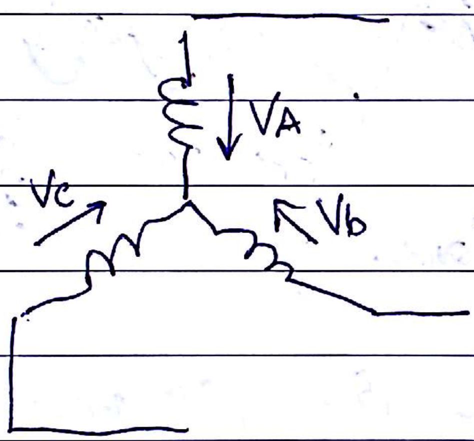
$$= \cancel{\sqrt{8664^2 + 1459^2 + 119^2}}$$

$$= \cancel{8756 \text{ Vrms}}$$

$$= \sqrt{8642^2 + 1119^2 + 119^2}$$

$$= \underline{8715 \text{ Vrms}}$$

<u>3rd harmonics EMF at line to line voltage</u>



$$V_A = V_{A1} \cdot \sin(\omega t)$$
$$V_{A3} \cdot \sin(3\omega t)$$
$$V_{A5} \sin(5\omega t)$$

$$V_B = V_{B1} \cdot \sin(\omega t - 120^\circ)$$
$$V_3 \sin 3(\omega t - 120^\circ)$$
$$V_5 \sin(5(\omega t - 120^\circ))$$

line to line voltage $\Rightarrow V_B - V_A$

$$V_{BA} = V_B - V_A$$
$$= V_1 [\sin(\omega t - 120^\circ) - \sin(\omega t)]$$
$$V_3 [\sin(3(\omega t - 120^\circ)) - \sin(3\omega t)]$$ → cancels eachother
$$V_5 [\sin(5(\omega t - 120^\circ)) - \sin(5\omega t)]$$

$$V_{BA} = \sqrt{3} V_1 \cos(\omega t - 60^\circ)$$
$$+ \sqrt{3} V_5 \cos(5\omega t - 120^\circ)$$

★ No 3rd order voltage harmonics in Y connected line to line voltage

$$E_{rms\,LL} = \sqrt{3}\left(\sqrt{8715^2 + 119^2}\right) \cong \underline{15 \text{kV}}$$ machine