BLG 335E, Analysis of Algoritms 1, Fall 2015, Homework 2

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1. E[X] is the expected number of customers that can get their hats back. Our aim is to find E[X].

X shows how many customers can get their hats back. By using combination, it can be said that the probability of a single person receives his own hat is . In addition, there are n number of people. Thus,

=1

Therefore, the expected number of customers that will get their own hats back is 1.

1. X is the time that takes Alice to reach safety. E[X] is the expected time of Alice to reach safety. There are 3 independent cases and each has possibility. In second and third cases they come back to the mine. Thus;

E[X] = 2. + (3+E[X]). + (5+E[X]).

= + .E[X]

Therefore, E[X] = 10. It means Alice is expected to spend 10 hours before getting back to safety.

|  |  |  |  |
| --- | --- | --- | --- |
| K | 2 | N/e=4 | 8 |
| The Best Applicant Index | 4 | 8 | 10 |
| Applicant Score | 9 | 10 | 5 |
| Running Time | 0.001 | 0.001 | 0.001 |

|  |  |  |  |
| --- | --- | --- | --- |
| K | 2 | N/e=4 | 8 |
| The Best Applicant Index | 10 | 10 | 10 |
| Applicant Score | 6 | 6 | 6 |
| Running Time | 0.001 | 0.001 | 0.001 |

|  |  |  |  |
| --- | --- | --- | --- |
| K | 2 | N/e=4 | 8 |
| The Best Applicant Index | 6 | 6 | 10 |
| Applicant Score | 10 | 10 | 8 |
| Running Time | 0.001 | 0.001 | 0.001 |

3.1)

The chance of hiring the best person does not go higher when we assign a k value closer to n or closer to 1. Because when we choose a k value closer to n, we reject most of the people. And the probability of best person gets rejected is higher. In the other case, we reject less people but since we have to select the first person higher than the best score of the first k person, the probability of hiring the best person decreases.

3.2)

In this algorithm we eliminate all the applicants before key value and we start from k+1 to find the best applicant. For this equation to succeed, it is required that none of the applicant that has a high score which is greater than the best score between (1,k) range.

3.3)

In order to manage Si, both Bi and Oi have to be true. So, it is possible to say that the Pr(Si) is same to the probability of Bi and Oi happening at the same time. Therefore, Si in terms of Bi and Oi will is like below;

3.4)

It is approximated by integrals to bound this summantion from above and below;

By calculating these integrals we find the bounds

Which is a more tight bound for . Since we want to enhance the probability of success, we look at k which maximizes the lower bound on .

We set the equation to 0 to find the lower bound.

k =

Therefore, the value of k is the value that maximize the lower bound of the probability of success is k = n/e.