Business Process Intelligence (BPI) course

ILP-Miner, Inductive Miner

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BPI-Instruction 6







Language

Question 1

Create the inequation system

$$c \cdot \mathbf{1} + A' \cdot x - A \cdot y \ge 0$$

for the following logs:

- a) $L=[\langle a,c,d \rangle, \langle b,c,e \rangle]$
- b) $L=[\langle a,b,d,e \rangle, \langle a,c,d,e \rangle, \langle a,d,b,e \rangle, \langle a,d,c,e \rangle]$





a)
$$L=[\langle a,c,d \rangle, \langle b,c,e \rangle]$$

$$c \cdot \mathbf{1} + A' \cdot x - A \cdot y \ge 0$$

transition occurrences including the last activity

$$A = egin{array}{c} \langle a
angle \ \langle b
angle \ \langle b
angle \ \langle b, c
angle \ \langle b, c, e
angle \end{array} egin{array}{ccccccc} a & b & c & d & e \ 1 & 0 & 0 & 0 & 0 \ 0 & 1 & 0 & 0 & 0 \ 1 & 0 & 1 & 0 & 0 \ 0 & 1 & 1 & 0 & 0 \ 1 & 0 & 1 & 1 & 0 \ 0 & 1 & 1 & 0 & 1 \ \end{array}$$

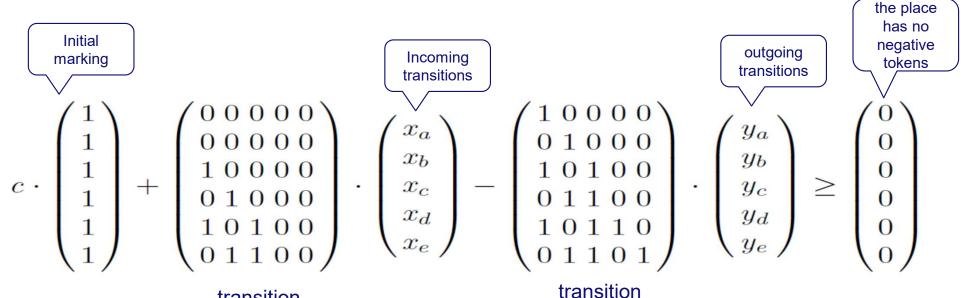
transition occurrences before the last activity





a)
$$L=[\langle a,c,d \rangle, \langle b,c,e \rangle]$$

$$c \cdot \mathbf{1} + A' \cdot x - A \cdot y \ge 0$$



transition occurrences before the last activity

occurrences including the last activity

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a) $L=[\langle a,c,d \rangle, \langle b,c,e \rangle]$

$$c - y_a \ge 0$$

$$c - y_b \ge 0$$

$$c + x_a - y_a - y_c \ge 0$$

$$c + x_b - y_b - y_c \ge 0$$

$$c + x_a + x_c - y_a - y_c - y_d \ge 0$$

$$c + x_b + x_c - y_b - y_c - y_e \ge 0$$

Every valid solution corresponds to a feasible place!





b) $L=[\langle a,b,d,e \rangle, \langle a,c,d,e \rangle, \langle a,d,b,e \rangle, \langle a,d,c,e \rangle]$





b) $L=[\langle a,b,d,e \rangle, \langle a,c,d,e \rangle, \langle a,d,b,e \rangle, \langle a,d,c,e \rangle]$





b) L=[
$$\langle a,b,d,e \rangle$$
, $\langle a,c,d,e \rangle$, $\langle a,d,b,e \rangle$, $\langle a,d,c,e \rangle$] $c-y_a \geq 0$ $c+x_a-y_a-y_b \geq 0$ $c+x_a-y_a-y_c \geq 0$ $c+x_a-y_a-y_d \geq 0$ $c+x_a+x_b-y_a-y_b-y_d \geq 0$ $c+x_a+x_c-y_a-y_c-y_d \geq 0$ $c+x_a+x_c-y_a-y_c-y_d \geq 0$ $c+x_a+x_d-y_a-y_b-y_d \geq 0$ $c+x_a+x_d-y_a-y_c-y_d \geq 0$ $c+x_a+x_d-y_a-y_c-y_d-y_e \geq 0$ $c+x_a+x_c+x_d-y_a-y_c-y_d-y_e \geq 0$ $c+x_a+x_b+x_d-y_a-y_c-y_d-y_e \geq 0$ $c+x_a+x_c+x_d-y_a-y_c-y_d-y_e \geq 0$ $c+x_a+x_c+x_d-y_a-y_c-y_d-y_e \geq 0$





Language-based Regions

Question 2

Consider the following solutions to an inequation system

$$c \cdot \mathbf{1} + A' \cdot x - A \cdot y \ge 0$$

and give the corresponding places:

a)
$$c=1$$
, $x_a=0$, $x_b=0$, $x_c=1$, $y_a=1$, $y_b=0$, $y_c=1$

b) c=2,
$$x_a=0$$
, $x_b=0$, $x_c=0$, $y_a=1$, $y_b=0$, $y_c=1$





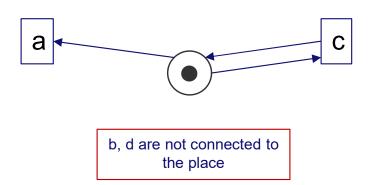
Language-based Regions Solution Q2

Consider the following solutions to an inequation system

$$c \cdot \mathbf{1} + A' \cdot x - A \cdot y \ge 0$$

and give the corresponding places:

a)
$$c=1$$
, $x_a=0$, $x_b=0$, $x_c=1$, $y_a=1$, $y_b=0$, $y_c=1$



Bonus question: Can this place ever be part of a sound workflow net?





Language-based Regions Solution Q2

Consider the following solutions to an inequation system

$$c \cdot \mathbf{1} + A' \cdot x - A \cdot y \ge 0$$

and give the corresponding places:

b)
$$c=2$$
, $x_a=0$, $x_b=0$, $x_c=0$, $y_a=1$, $y_b=0$, $y_c=1$



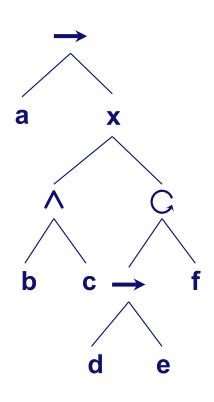
b is not connected to the place







Question 1



Consider the process tree on the left.

a) Is the following trace in accordance with the model? Explain your answer.

$$\sigma = \langle a, c, b, e, d \rangle$$

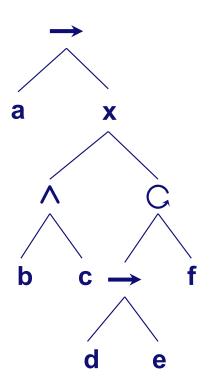
b) Is the following trace in accordance with the model? Explain your answer.

$$\sigma = \langle a, d, e, f, d, e, f \rangle$$

- c) Provide two traces described by the model.
- d) Convert the process tree into a petri net.



Solution Q1 a-c)



- a) No, valid traces must <u>either</u> contain *b* and *c* or *d* and *e*. Also, *d* must be performed <u>before</u> *e*.
- b) No, the loop must always end with the "do" part.
- c) The model describes the following three traces (among others):

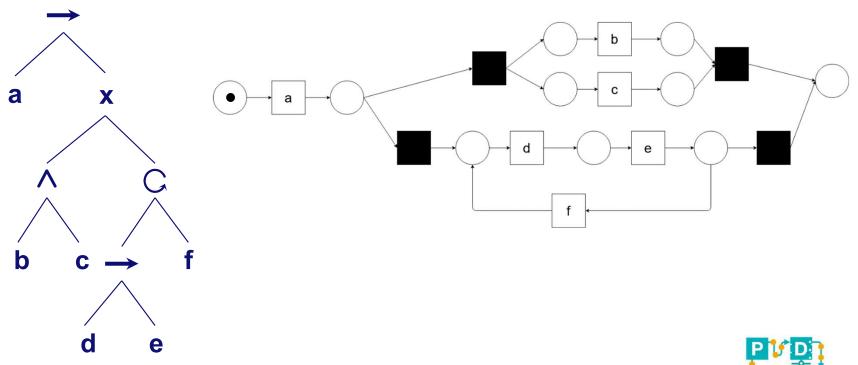
$$\sigma_{1} = \langle a, b, c \rangle$$

$$\sigma_{2} = \langle a, d, e, \rangle$$

$$\sigma_{3} = \langle a, d, e, f, d, e, f, d, e \rangle$$



Convert the process tree to a Petri net.





Question 2

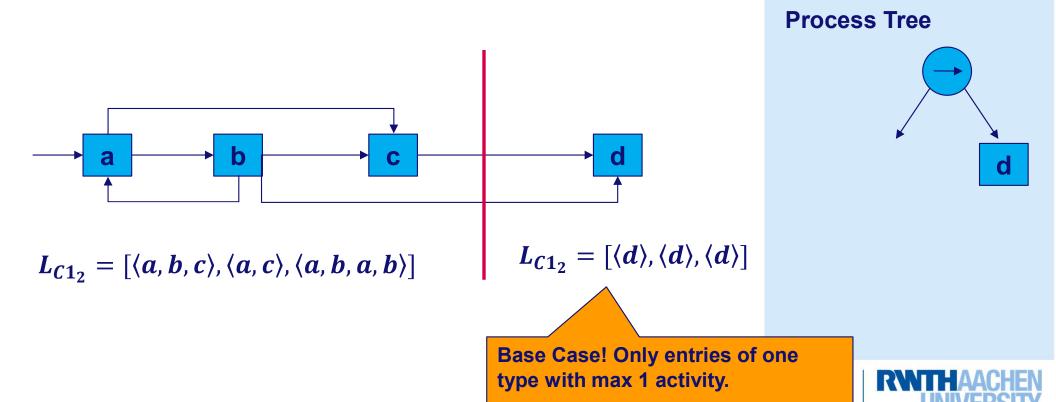
Consider the following event log and perform the inductive miner algorithm on it.

$$L = [\langle a, b, c, d \rangle, \langle a, c, d \rangle, \langle a, b, a, b, d \rangle]$$

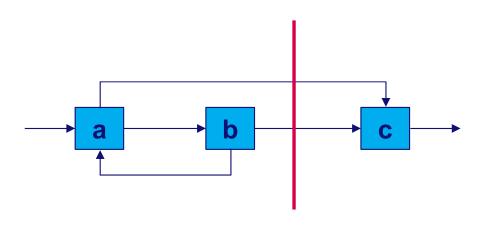




 $L = [\langle a, b, c, d \rangle, \langle a, c, d \rangle, \langle a, b, a, b, d \rangle]$



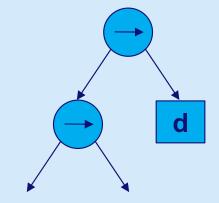
$$L_{C1_2} = [\langle a, b, c \rangle, \langle a, c \rangle, \langle a, b, a, b \rangle]$$



$$L_{C2_1} = [\langle a, b \rangle, \langle a \rangle, \langle a, b, a, b \rangle] \qquad L_{C2_2} = [\langle c \rangle, \langle c \rangle, \langle \rangle]$$

$$L_{C2_2} = [\langle c \rangle, \langle c \rangle, \langle \rangle]$$

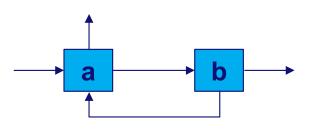
Process Tree







$$L_{C2_1} = [\langle a, b \rangle, \langle a \rangle, \langle a, b, a, b \rangle]$$

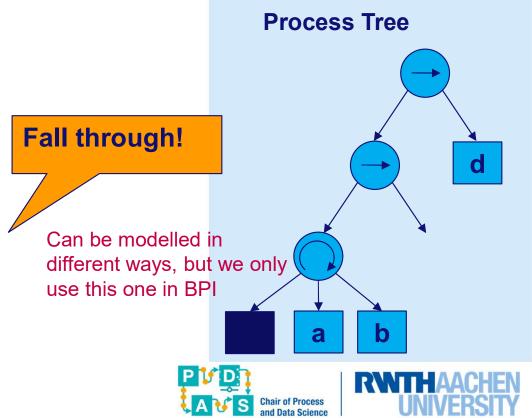


XOR: Arcs between both activities.

Sequence: End activities in both parts.

Parallel: b is not a start activity.

Loop: End activities in both partitions.



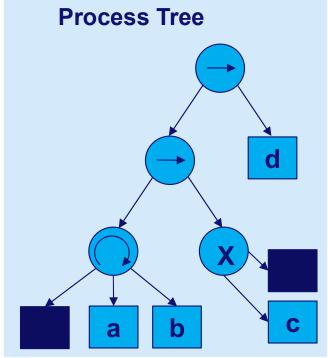
$$L_{C2_1} = [\langle c \rangle, \langle c \rangle, \langle \rangle]$$

Base Case! Only entries of one type with max 1 activity.

$$L_{C3_1} = [\langle c \rangle, \langle c \rangle]$$

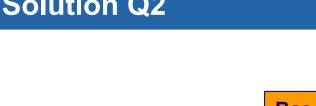
$$L_{C3_2} = [\langle \rangle, \langle \rangle, \langle \rangle]$$

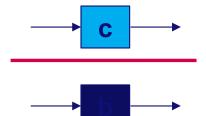
Base Case! Only entries of one type with max 1 activity.



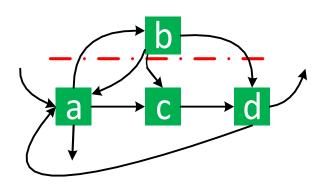








Question 3



Given the following event log and the corresponding directly follows graph:

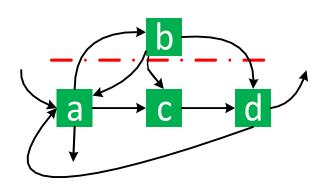
$$L = [\langle a, b, a, c, d \rangle, \langle a, b, c, d \rangle, \langle a, b, d, a \rangle]$$

Name the cut and make the projection on the event log.





$$L = [\langle a, b, a, c, d \rangle, \langle a, b, c, d \rangle, \langle a, b, d, a \rangle]$$



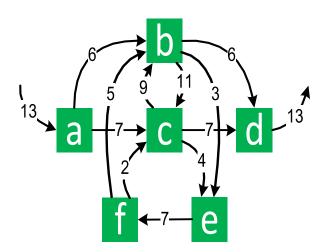
No Cut! → Fall though

(You do not need to know all fallthroughs or how to apply them. You do need to identify sitations where a standard cut is not possible.)





Question 4



Execute the Inductive Miner on the given directly follows graph without log projections (i.e., re-use the partitions in the DFG for recursion without considering the log projections and re-drawing the DFG).

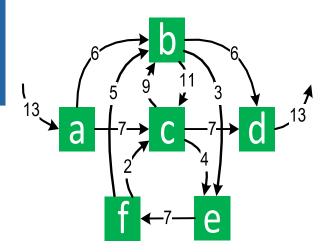
Give the resulting process tree.

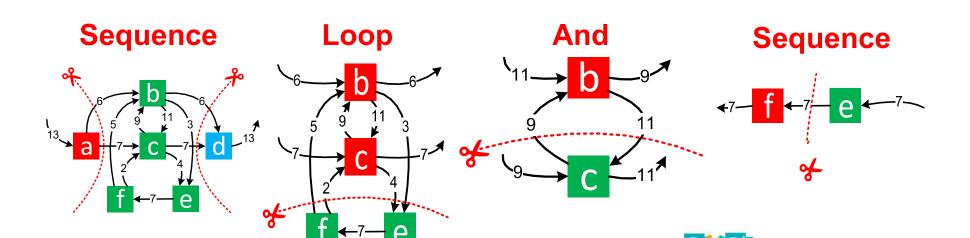


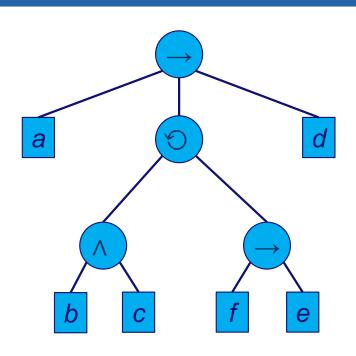


Solution Q4

- **1.** Sequence cut: $\rightarrow (\{a\}, \{b, c, f, e\}, \{d\})$
- **2.** Loop cut: $\rightarrow (a, \bigcirc (\{b,c\}, \{f,e\}), d)$
- **3.** AND cut: $\rightarrow (a, \bigcirc (\land (b, c)), \{f, e\}), d)$
- **4.** Sequence cut: $\rightarrow (a, \bigcirc (\land (b, c)), \rightarrow (f, e), d)$







Sequence cut: \rightarrow ({*a*}, {*b*, *c*, *f*, *e*}, {*d*})

Loop cut: \rightarrow (a, \heartsuit ($\{b,c\}$, $\{f,e\}$), d)

AND cut: \rightarrow (a, \bigcirc (\land (b, c), {f, e}), d)

Sequence cut: \rightarrow (a, \bigcirc (\land (b, c), \rightarrow (f, e)), d)

$$\rightarrow$$
 $(a, \circlearrowleft (b, c), \rightarrow (f, e)), d)$





Question 5

Discover a process tree using the Inductive Miner for event logs given below, and give a trace that is possible according to the tree but has not been seen in the event log (if possible).

- a) Without log projections.
- b) With log projections.
 - i) $L = \{(a, b, c, d, e, f, b, d, c, e, g), (a, b, d, c, e, g), (a, b, c, d, e, f, b, c, d, e, f, b, d, c, e, g)\}$
 - ii) $L = \{(a, d, e, f, h), (a, e, d, b, f, h), (g, h), (a, b, c, d, f, h), (a, c, b, d, f, h), (a, b, d, c, e, f, h), (a, e, b, e, c, f)\}$
 - iii) $L = \{\langle a, c, d, e \rangle, \langle a, d, c, e \rangle, \langle a, d, e, c, f, d, e \rangle, \langle b, d, e, c \rangle, \langle b, c, d, e, f, d, e \rangle, \langle b, d, e, f, c, d, e \rangle\}$
 - iv) $L = \{\langle a, b, c, d, f \rangle, \langle a, c, b, d, f \rangle, \langle a, b, d, c, f \rangle, \langle a, c, d, b, f \rangle, \langle a, d, e, f \rangle, \langle a, e, d, f \rangle\}$

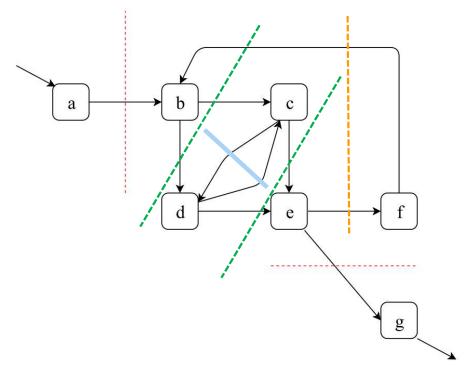




Solution Q5 a i)

Solution for a) (without log projection):

i. $L = \{(a, b, c, d, e, f, b, d, c, e, g), (a, b, d, c, e, g), (a, b, c, d, e, f, b, c, d, e, f, b, d, c, e, g)\}$



$$\rightarrow$$
({a}, {b, c, d, e, f}, {g})
 \rightarrow (a, \heartsuit ({b, c, d, e}, {f}), g)
 \rightarrow (a, \heartsuit (\rightarrow ({b}, {c, d}, {e}), f), g)

$$\rightarrow$$
 $(a, \circlearrowleft) (\rightarrow (b, \land (\{c\}, \{d\}), e), f), g)$

$$\rightarrow$$
 $(a, \bigcirc (\rightarrow (b, \land (c,d), e), f), g)$

Trace in tree but not in log: (a, b, c, d, e, g)

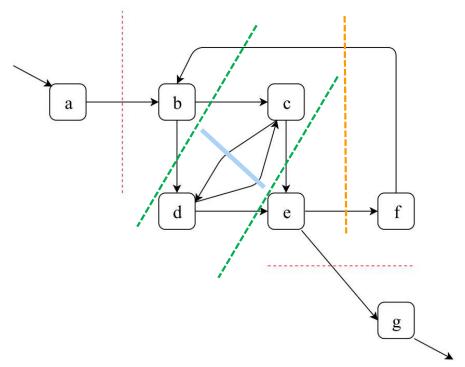




Solution Q5 b i)

Solution for b) (with log projection) same as for a):

i. $L = \{(a, b, c, d, e, f, b, d, c, e, g), (a, b, d, c, e, g), (a, b, c, d, e, f, b, c, d, e, f, b, d, c, e, g)\}$



$$\rightarrow (\{a\}, \{b, c, d, e, f\}, \{g\})$$

$$L_a = [\langle a \rangle], L_g = [\langle g \rangle]$$

$$L_1 = [\langle b, c, d, e, f, b, d, c, e \rangle, \langle b, d, c, e \rangle, \langle b, c, d, e, f, b, c, d, e, f, b, d, c, e \rangle]$$

$$\rightarrow (a, \bigcirc (\{b, c, d, e\}, \{f\}), g)$$

$$L_f = [\langle f \rangle]$$

$$L_2 = [\langle b, c, d, e \rangle, \langle b, d, c, e \rangle]$$

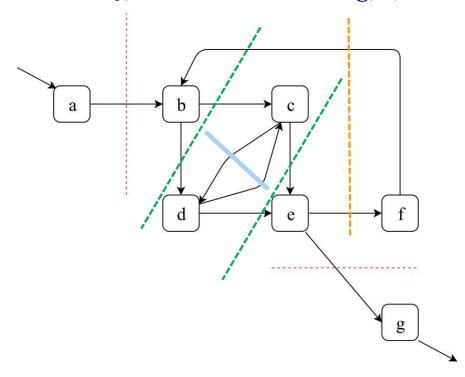




Solution Q5 b i)

Solution for b) (with log projection) same as for a):

i. $L = \{(a, b, c, d, e, f, b, d, c, e, g), (a, b, d, c, e, g), (a, b, c, d, e, f, b, c, d, e, f, b, d, c, e, g)\}$



$$\rightarrow (a, \mathfrak{O}) (\{b, c, d, e\}, \{f\}), g)$$

$$L_2 = [\langle b, c, d, e \rangle, \langle b, d, c, e \rangle]$$

$$\rightarrow (a, \mathfrak{O}) (\rightarrow (\{b\}, \{c, d\}, \{e\}), f), g)$$

$$L_b = [\langle b \rangle], L_e = [\langle e \rangle], L_3 = [\langle c, d \rangle, \langle d, c \rangle]$$

$$\rightarrow (a, \mathfrak{O}) (\rightarrow (b, \Lambda) (\{c\}, \{d\}), e), f), g)$$

$$L_c = [\langle c \rangle], L_d = [\langle d \rangle]$$







Solution Q5 a ii)

Solution for a) (without log projection):

ii. $L = \{\langle a, d, e, f, h \rangle, \langle a, e, d, b, f, h \rangle, \langle g, h \rangle, \langle a, b, c, d, f, h \rangle, \langle a, c, b, d, f, h \rangle, \langle a, e, f, h \rangle, \langle a, e, b, e, c, f \rangle\}$

 \rightarrow ({a, b, c, d, e, f, g}, {h})

 \rightarrow (x({a,b,c,d,e,f},{g}),h)

 \rightarrow (x(\rightarrow ({a},{b,c,d,e},{f}),g),h)

 $\rightarrow (x(\rightarrow (a, \land (\{b\}, \{c\}, \{d\}, \{e\}), f), g), h)$

 \rightarrow (X(\rightarrow (a, \land (b, c, d, e), f), g), h)

Trace in tree but not in log: (a, b, c, d, e, f, h)

b C а d e h g

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Solution Q5 b ii)

Solution for b) (with log projection):

ii. $L = \{\langle a, d, e, f, h \rangle, \langle a, e, d, b, f, h \rangle, \langle g, h \rangle, \langle a, b, c, d, f, h \rangle, \langle a, c, b, d, f, h \rangle, \langle a, e, f, h \rangle, \langle a, e, b, e, c, f \rangle\}$

 \rightarrow ({a, b, c, d, e, f, g}, {h})

 $L_1 = \{\langle a, d, e, f \rangle, \langle a, e, d, b, f \rangle, \langle g \rangle, \langle a, b, c, d, f \rangle, \langle a, c, b, d, f \rangle, \langle a, b, d, c, e, f \rangle, \langle a, e, b, e, c, f \rangle\}$

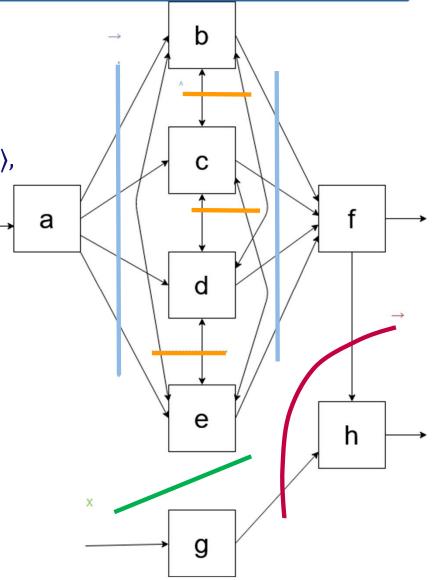
 $L_2 = \{\langle h \rangle, \langle \rangle \}$

 $\rightarrow (x(\{a,b,c,d,e,f\},\{g\}),x(h,\tau))$

 $L_3 = \{\langle a, d, e, f \rangle, \langle a, e, d, b, f \rangle, \langle a, b, c, d, f \rangle,$

 $\langle a, c, b, d, f \rangle$, $\langle a, b, d, c, e, f \rangle$, $\langle a, e, b, e, c, f \rangle$

 $L_4 = \{\langle g \rangle\}$



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Solution Q5 b ii)

Solution for b) (with log projection):

ii. $L = \{\langle a, d, e, f, h \rangle, \langle a, e, d, b, f, h \rangle, \langle g, h \rangle, \langle a, b, c, d, f, h \rangle, \langle a, c, b, d, f, h \rangle, \langle a, e, f, h \rangle, \langle a, e, b, e, c, f \rangle\}$

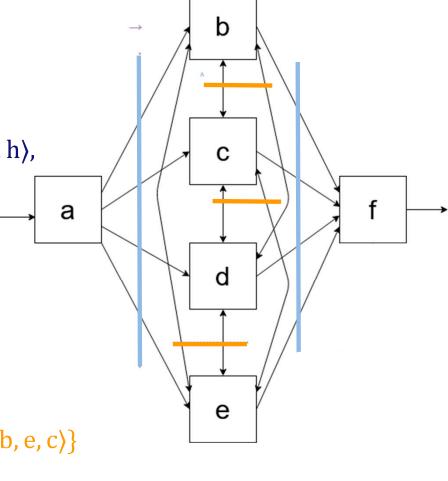
 $\rightarrow (x(\{a,b,c,d,e,f\},g),x(h,\tau))$ $L_3 = \{\langle a,d,e,f\rangle,\langle a,e,d,b,f\rangle,\langle a,b,c,d,f\rangle,\langle a,c,b,d,f\rangle,\langle a,b,d,c,e,f\rangle,\langle a,e,b,e,c,f\rangle\}$

$$\rightarrow (\mathsf{x}(\rightarrow(\{a\},\{b,c,d,e\},\{f\}),g),\mathsf{x}(h,\tau))$$

 $L_5 = \{\langle a \rangle\}$

 $L_6 = \{\langle d, e \rangle, \langle e, d, b \rangle, \langle b, c, d \rangle, \langle c, b, d \rangle, \langle b, d, c, e \rangle, \langle e, b, e, c \rangle\}$

 $L_7 = \{\langle f \rangle\}$



Solution Q5 b ii)

Solution for b) (with log projection):

ii. $L = \{\langle a, d, e, f, h \rangle, \langle a, e, d, b, f, h \rangle, \langle g, h \rangle, \langle a, b, c, c \rangle, \langle a, c, b, d, f, h \rangle, \langle a, b, d, c, e, f, h \rangle, \langle a, e, b, e, c, f \rangle$

$$\rightarrow (\mathbf{x}(\rightarrow (a, \{b, c, d, e\}, f), g), \mathbf{x}(h, \tau))$$

 $L_6 = \{\langle d, e \rangle, \langle e, d, b \rangle, \langle b, c, d \rangle, \langle c, b, d \rangle, \langle b, d, c, e \rangle, \langle e, b, e, c \rangle\}$

$$\rightarrow (x(\rightarrow (a, \land (\{b\}, \{c\}, \{d\}, \{e\}), f), g), x(h, \tau))$$

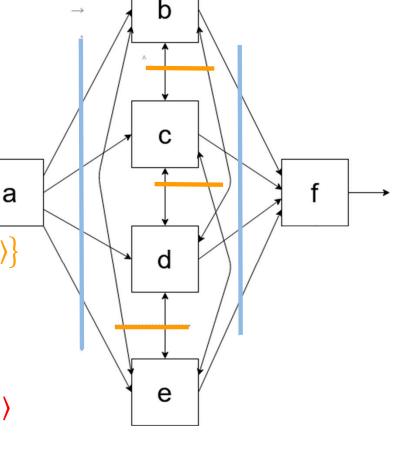
$$L_b = \{\langle \rangle, \langle b \rangle\}$$

$$L_c = \{\langle \rangle, \langle c \rangle\}$$

$$L_d = \{\langle d \rangle, \langle \rangle \}$$

$$L_e = \{\langle e \rangle, \langle \rangle, \langle e, e \rangle\}$$

Trace in tree but not in log: $\langle g \rangle$



 $\rightarrow (\mathsf{X}(\rightarrow (a, \land (\mathsf{X}(b,\tau), \mathsf{X}(c,\tau), \mathsf{X}(d,\tau), \mathsf{X}(\circlearrowleft (e,\tau),\tau)), f), g), \mathsf{X}(h,\tau))$

Solution Q5 a/b iii)

Solution Part 1 for a) and b) (with and without log projection):

iii. $L = \{\langle a, c, d, e \rangle, \langle a, d, c, e \rangle, \langle a, d, e, c, f, d, e \rangle, \langle b, d, e, c \rangle, \langle b, c, d, e, f, d, e \rangle, \langle b, d, e, f, c, d, e \rangle\}$

$$\rightarrow (\{a,b\}, \{c,d,e,f\})$$

$$L_1 = \{\langle a \rangle, \langle b \rangle\}$$

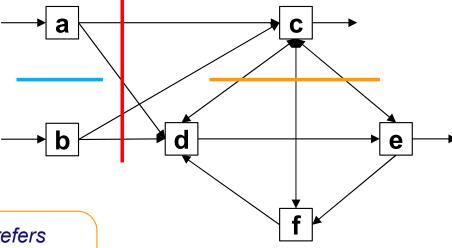
$$L_2 = \{\langle c,d,e \rangle, \langle d,c,e \rangle, \langle d,e,c,f,d,e \rangle, \langle d,e,c \rangle, \langle d,e,f,c,d,e \rangle\}$$

$$\rightarrow (\times (\{a\}, \{b\}), \land (\{c\}, \{d, e, f\}))$$

$$L_a = \{\langle a \rangle\}, L_b = \{\langle b \rangle\}, L_c = \{\langle c \rangle\}$$

$$L_3 = \{\langle d, e \rangle, \langle d, e, f, d, e \rangle\}$$

IM prefers parallel cut over loop cut (both possible on L₂)







Solution Q5 a iii)

Solution Part 2 for a) (with log projection):

```
iii. L = \{\langle a, c, d, e \rangle, \langle a, d, c, e \rangle, \langle a, d, e, c, f, d, e \rangle, \langle b, d, e, c \rangle, \langle b, c, d, e, f, d, e \rangle, \langle b, d, e, f, c, d, e \rangle\}
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```
\rightarrow (\times (\{a\}, \{b\}), \land (\{c\}, \{d, e, f\}))

L_a = \{\langle a \rangle\}, L_b = \{\langle b \rangle\}, L_c = \{\langle c \rangle\}

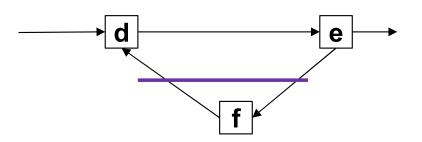
L_3 = \{\langle d, e \rangle, \langle d, e, f, d, e \rangle, \}
```

$$\rightarrow (\times (a, b), \land (c, \circ) (\{d, e\}, \{f\})))$$

$$L_4 = \{\langle d, e \rangle\}$$

$$L_f = \{\langle f \rangle\}$$

DFG based on projected event log L₃ allows for **loop cut**:







Solution Q5 a iii)

Solution Part 2 for a) (with log projection):

iii. $L = \{\langle a, c, d, e \rangle, \langle a, d, c, e \rangle, \langle a, d, e, c, f, d, e \rangle, \langle b, d, e, c \rangle, \langle b, c, d, e, f, d, e \rangle, \langle b, d, e, f, c, d, e \rangle\}$

$$\rightarrow (\times (a, b), \land (c, \circlearrowleft (\{d, e\}, \{f\})))$$

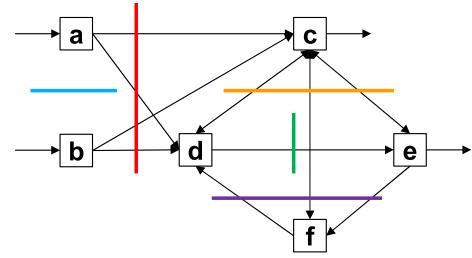
$$L_4 = \{\langle d, e \rangle\}$$

$$L_f = \{\langle f \rangle\}$$

$$\rightarrow (\times (a,b), \land (c, \circlearrowleft (\rightarrow (\{d\}, \{e\}), f)))$$

$$L_d = \{\langle d \rangle\}, L_e = \{\langle e \rangle\}$$

$$\rightarrow$$
 (× (a, b), \land (c, \circlearrowleft (\rightarrow (d, e), f))







Solution Q5 b iii)

Solution Part 2 for b) (without log projection):

iii. $L = \{\langle a, c, d, e \rangle, \langle a, d, c, e \rangle, \langle a, d, e, c, f, d, e \rangle, \langle b, d, e, c \rangle, \langle b, c, d, e, f, d, e \rangle, \langle b, d, e, f, c, d, e \rangle\}$

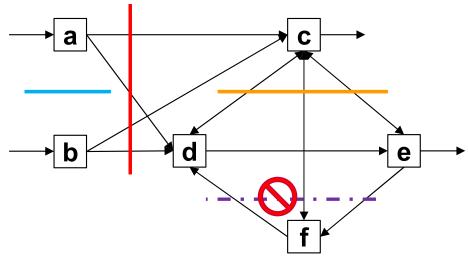
$$\rightarrow (\times (\{a\}, \{b\}), \land (\{c\}, \{d, e, f\})))$$

$$L_a = \{\langle a \rangle\}, L_b = \{\langle b \rangle\}, L_c = \{\langle c \rangle\}$$

$$L_3 = \{\langle d, e \rangle, \langle d, e, f, d, e \rangle, \}$$

With the basic techniques introduced in the lecture, the loop cut cannot be discovered without redrawing the DFG (because f is a start activity).

Note, that the 'real' Inductive Miner includes strategies to discover this loop cut (not relevant for the exam).







Solution Q5 a/b iv)

Solution for a) and b) (with and without log projection):

iv. $L = \{\langle a, b, c, d, f \rangle, \langle a, c, b, d, f \rangle, \langle a, b, d, c, f \rangle, \langle a, c, d, b, f \rangle, \langle a, d, e, f \rangle, \langle a, e, d, f \rangle\}$

