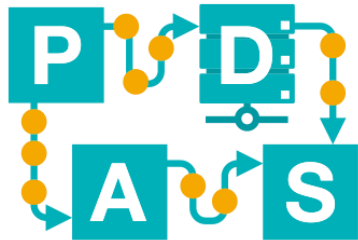


Conformance Checking Footprint & Token-Based Replay

BPI-I 8

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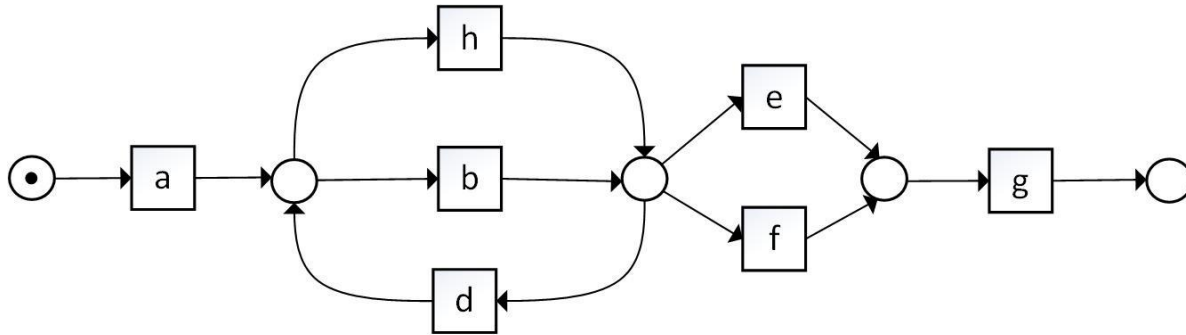


Chair of Process
and Data Science

RWTHAACHEN
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Exercise 1

- Compute the footprint-based conformance for the event log L and the presented model.
- $L = [\langle a, b, f, e, g \rangle^{45}, \langle a, b, d, b, e, f, g \rangle^5, \langle a, h, e, f, g \rangle^{25}, \langle a, h, f, e, g \rangle^{25}]$

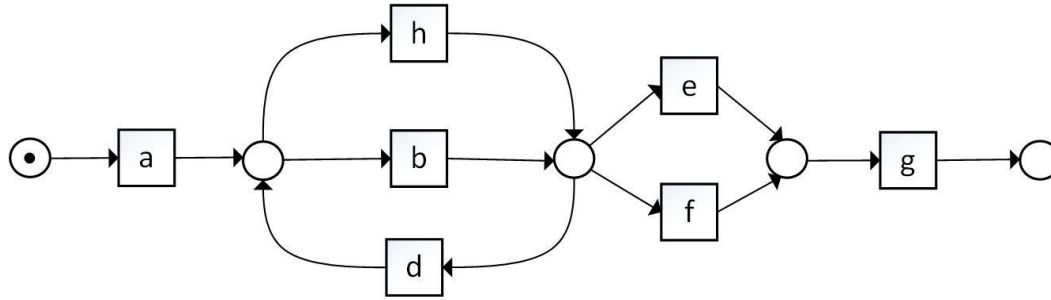


Exercise 1 Solution

- $L = [\langle a, b, f, e, g \rangle^{45}, \langle a, b, d, b, e, f, g \rangle^5, \langle a, h, e, f, g \rangle^{25}, \langle a, h, f, e, g \rangle^{25}]$

	a	b	d	e	f	g	h
a	#	→	#	#	#	#	→
b	←	#		→	→	#	#
d	#		#	#	#	#	#
e	#	←	#	#		→	←
f	#	←	#		#	→	←
g	#	#	#	←	←	#	#
h	←	#	#	→	→	#	#

Exercise 1 Solution



	a	b	d	e	f	g	h
a	#	→	#	#	#	#	→
b	←	#		→	→	#	#
d	#		#	#	#	#	
e	#	←	#	#	#	→	←
f	#	←	#	#	#	→	←
g	#	#	#	←	←	#	#
h	←	#		→	→	#	#

Exercise 1 Solution

Event log

	a	b	d	e	f	g	h
a	#	→	#	#	#	#	→
b	←	#		→	→	#	#
d	#		#	#	#	#	#
e	#	←	#	#		→	←
f	#	←	#		#	→	←
g	#	#	#	←	←	#	#
h	←	#	#	→	→	#	#

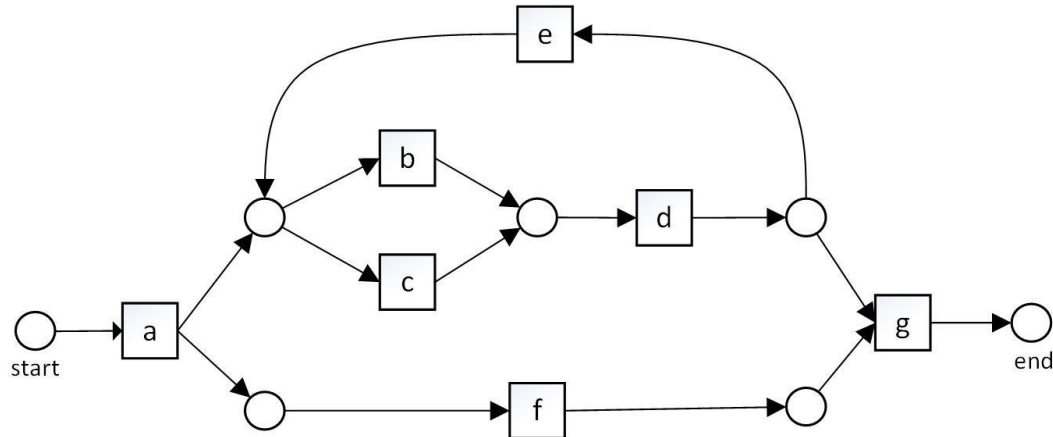
Process Model

	a	b	d	e	f	g	h
a	#	→	#	#	#	#	→
b	←	#		→	→	#	#
d	#		#	#	#	#	
e	#	←	#	#	#	→	←
f	#	←	#	#	#	→	←
g	#	#	#	←	←	#	#
h	←	#		→	→	#	#

Footprint-based conformance: $1 - \frac{4}{49} = 0.918$

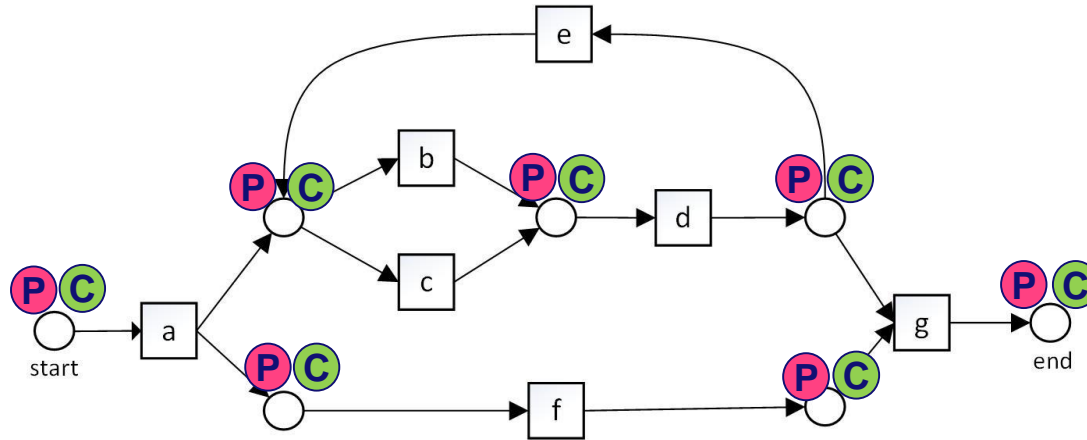
Exercise 2

- Compute the token-based replay fitness for the event log L and the presented model.
- $L = [\langle a, b, d, f, g \rangle^8, \langle a, c, d, e, b, f, g \rangle^4, \langle a, b, c, d, f, g \rangle^5, \langle a, b, f, g \rangle^1, \langle b, d, f, g \rangle^2]$



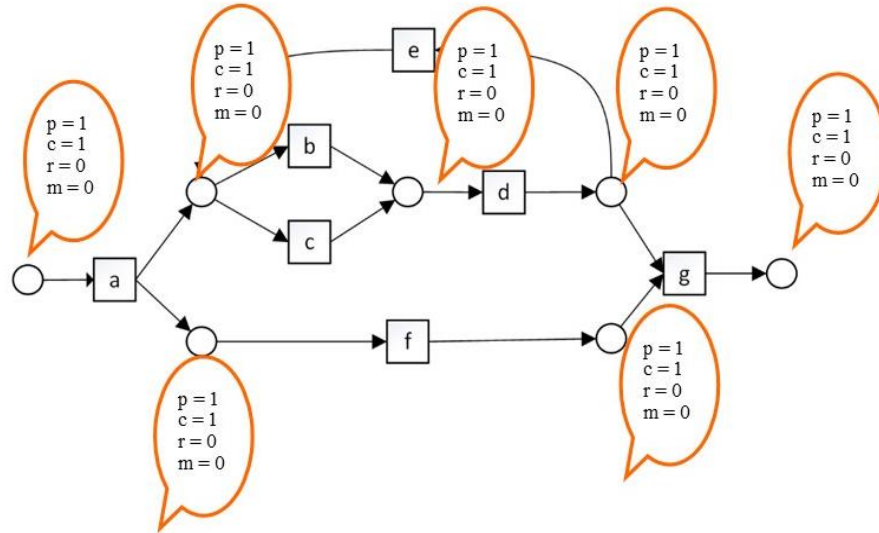
Exercise 2 Solution

$\langle a, b, d, f, g \rangle$



Exercise 2 Solution

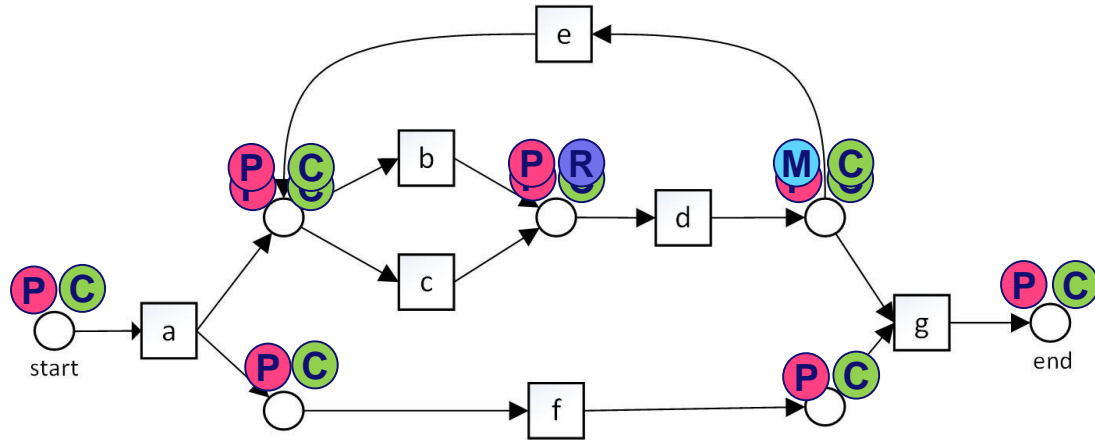
$\langle a, b, d, f, g \rangle$



$p = 7; c = 7; r = 0; m = 0$

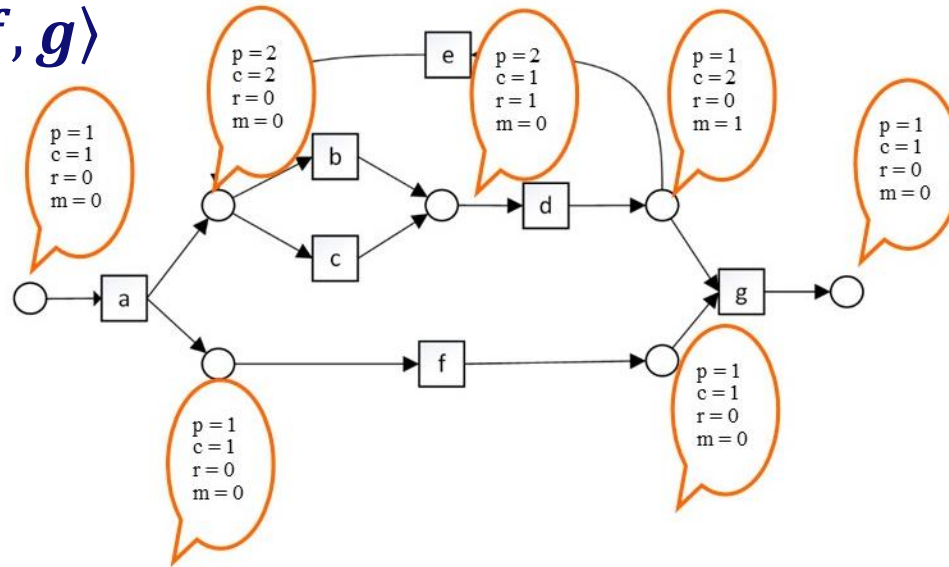
Exercise 2 Solution

$\langle a, c, d, e, b, f, g \rangle$



Exercise 2 Solution

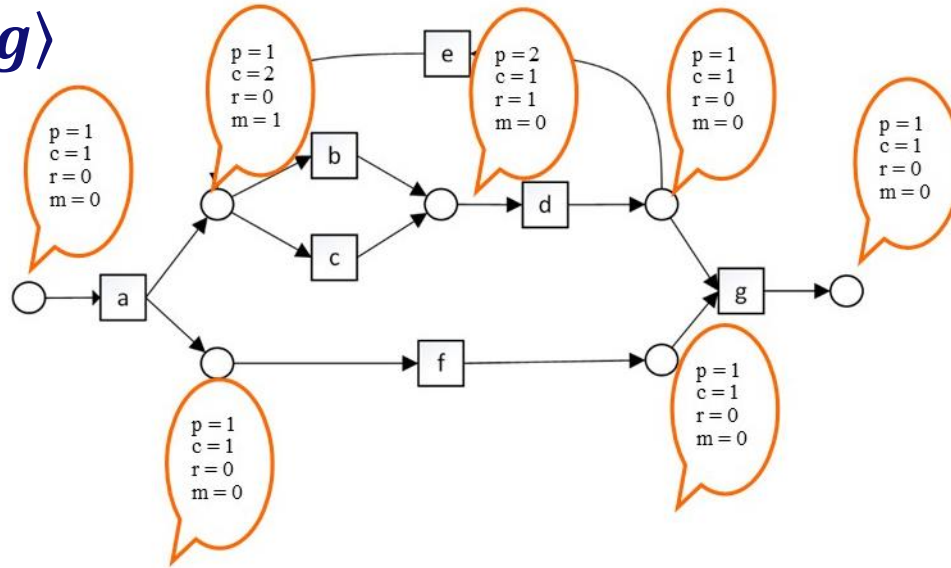
$\langle a, c, d, e, b, f, g \rangle$



$$p = 9; c = 9; r = 1; m = 1$$

Exercise 2 Solution

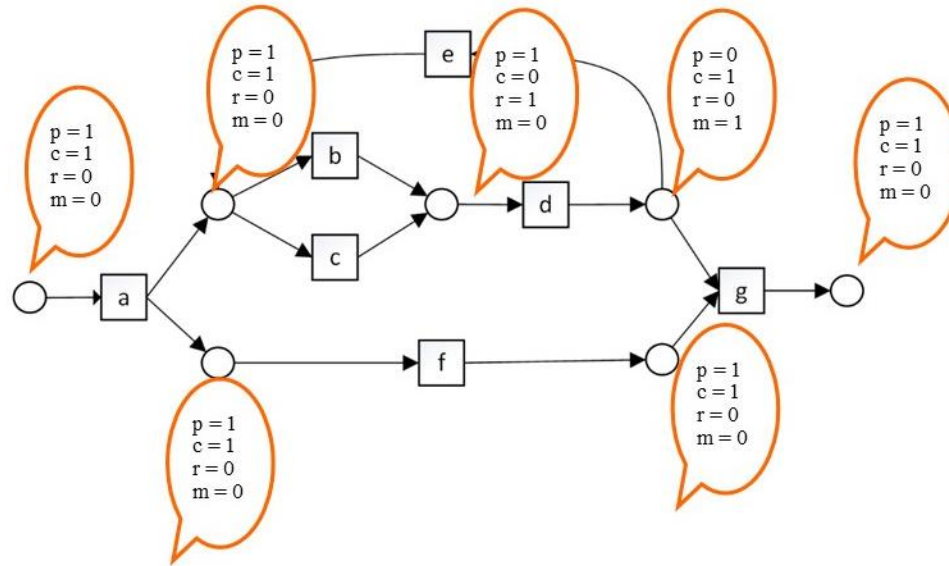
$\langle a, b, c, d, f, g \rangle$



$$p = 8; c = 8; r = 1; m = 1$$

Exercise 2 Solution

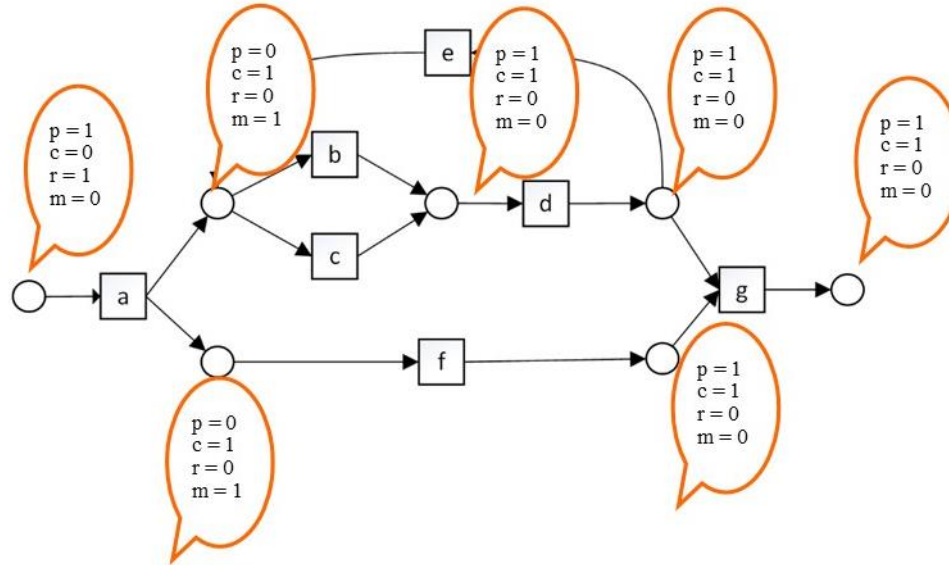
$\langle a, b, f, g \rangle$



$$p = 6; c = 6; r = 1; m = 1$$

Exercise 2 Solution

$\langle b, d, f, g \rangle$



$$p = 5; c = 6; r = 1; m = 2$$

Exercise 2 Solution

- **Token-Based Fitness:**

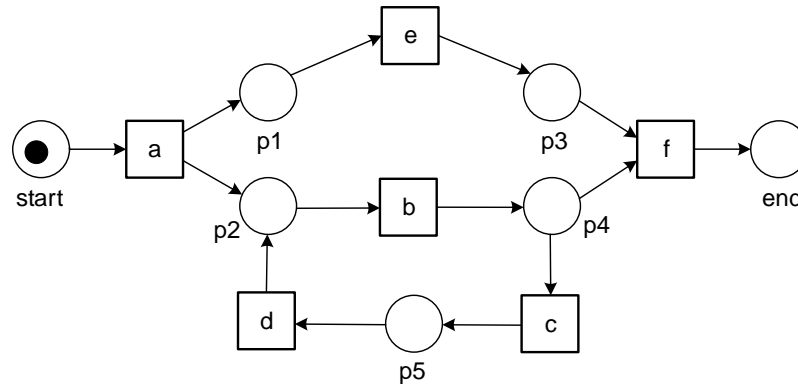
$$\text{fitness}(L, N) = \frac{1}{2} \left(1 - \frac{\sum_{\sigma \in L} L(\sigma) \cdot m_{N,\sigma}}{\sum_{\sigma \in L} L(\sigma) \cdot c_{N,\sigma}} \right) + \frac{1}{2} \left(1 - \frac{\sum_{\sigma \in L} L(\sigma) \cdot r_{N,\sigma}}{\sum_{\sigma \in L} L(\sigma) \cdot p_{N,\sigma}} \right)$$

- $\sum_{\sigma \in L} L(\sigma) \cdot p_{N,\sigma} = (8 \cdot 7) + (1 \cdot 6) + (5 \cdot 8) + (2 \cdot 5) + (4 \cdot 9) = 148$
- $\sum_{\sigma \in L} L(\sigma) \cdot c_{N,\sigma} = (8 \cdot 7) + (1 \cdot 6) + (5 \cdot 8) + (2 \cdot 6) + (4 \cdot 9) = 150$
- $\sum_{\sigma \in L} L(\sigma) \cdot r_{N,\sigma} = (8 \cdot 0) + (1 \cdot 1) + (5 \cdot 1) + (2 \cdot 1) + (4 \cdot 1) = 12$
- $\sum_{\sigma \in L} L(\sigma) \cdot m_{N,\sigma} = (8 \cdot 0) + (1 \cdot 1) + (5 \cdot 1) + (2 \cdot 2) + (4 \cdot 1) = 14$

$$\frac{1}{2} \left(1 - \frac{14}{150} \right) + \frac{1}{2} \left(1 - \frac{12}{148} \right) = 0.912$$

Exercise 3

- Compute the footprint and token-based replay fitness for the event log L and the presented model.
 - $L = [\langle a, b, e, f \rangle^2, \langle a, c, d, f \rangle^3]$



Exercise 3 Solution

Footprint

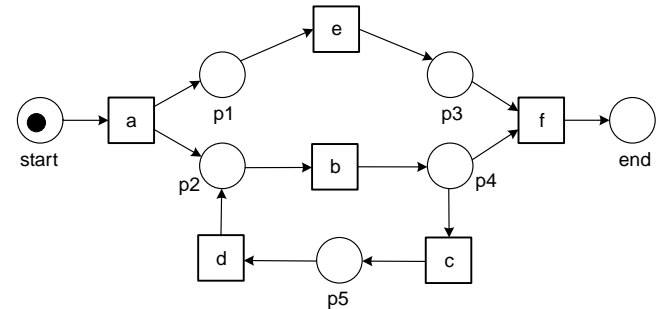
From the log

L	a	b	c	d	e	f
a	#	→	→	#	#	#
b	←	#	#	#	→	#
c	←	#	#	→	#	#
d	#	#	←	#	#	→
e	#	←	#	#	#	→
f	#	#	#	←	←	#

From the model

M	a	b	c	d	e	f
a	#	→	#	#	→	#
b	←	#	→	←		→
c	#	←	#	→		#
d	#	→	←	#		#
e	←				#	→
f	#	←	#	#	←	#

$$L = [\langle a, b, e, f \rangle^2, \langle a, c, d, f \rangle^3]$$



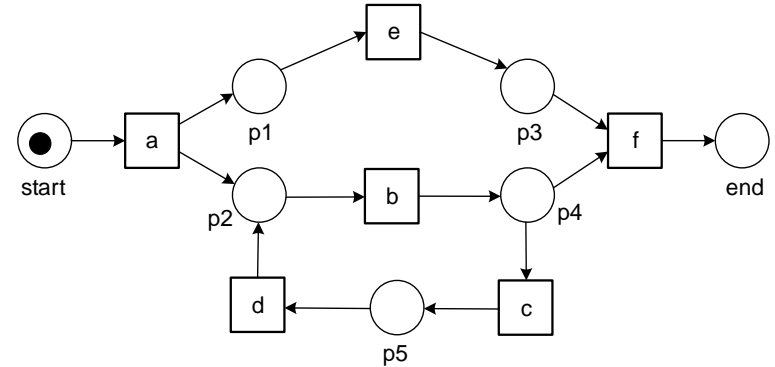
$$1 - \frac{18}{36} = 0.5$$

Exercise 3 Solution

Token-based Replay

- $L = [\langle a, b, e, f \rangle^2, \langle a, c, d, f \rangle^3]$

p = ?
c = ?
m = ?
r = ?



First trace?

Trace	Frequency	P	R	C	M	P <all>	R <all>	C <all>	M <all>
<a, b, e, f>	2	6	0	6	0	12	0	12	0
<a, c, d, f>	3								
total									

Why?

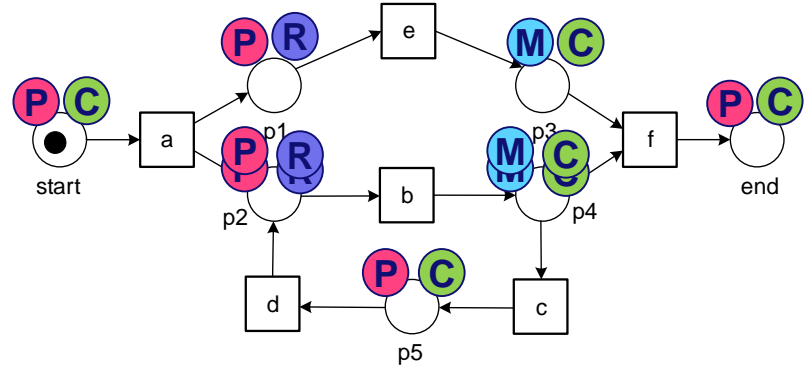
Exercise 3 Solution

Token-based Replay

$$L = [\langle a, b, e, f \rangle^2, \langle a, c, d, f \rangle^3]$$

Second trace?

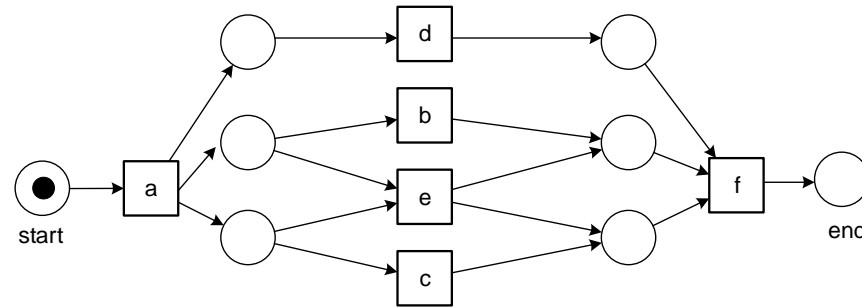
$$\mathbf{fitness} = \frac{1}{2} \left(1 - \frac{9}{30} \right) + \frac{1}{2} \left(1 - \frac{9}{30} \right) = 0.7$$



Trace	Frequency	P	R	C	M	P <all>	R <all>	C <all>	M <all>
<a, b, e, f>	2	6	0	6	0	12	0	12	0
<a, c, d, f>	3	6	3	6	3	18	9	18	9
total						30	9	30	9

Exercise 4

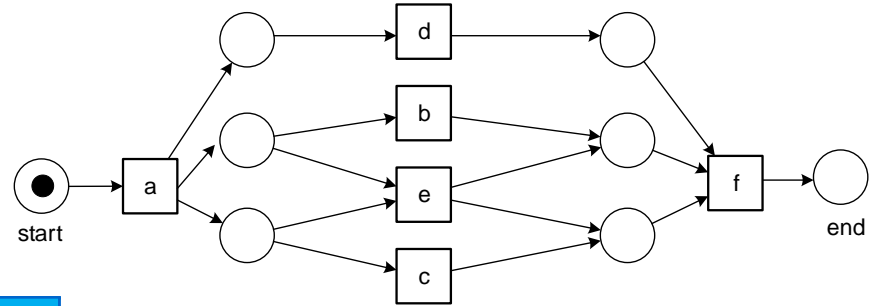
- Compute the footprint and token-based replay fitness for the event log L and the presented model.
 - $L = [\langle a, c, d, f \rangle, \langle a, c, b, d, f \rangle^2, \langle a, b, c, f \rangle, \langle a, e, f \rangle^2]$



Exercise 4 Solution

Footprint

- $L = [\langle a, c, d, f \rangle, \langle a, c, b, d, f \rangle^2, \langle a, b, c, f \rangle, \langle a, e, f \rangle^2]$



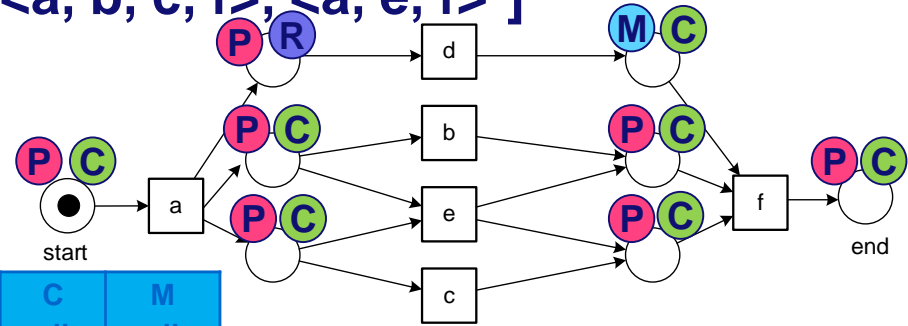
M : L	a	b	c	d	e	f
a	#	→	→	→ : #	→	#
b	←	#		: →	#	→ : #
c	←		#	: →	#	→
d	← : #	: ←	: ←	#	: #	→
e	←	#	#	: #	#	→
f	#	← : #	←	←	←	#

$$1 - \frac{10}{36} = 0.72$$

Exercise 4 Solution

Token-based Replay

- $L = [<a, c, d, f>, <a, c, b, d, f>^2, <a, b, c, f>, <a, e, f>^2]$

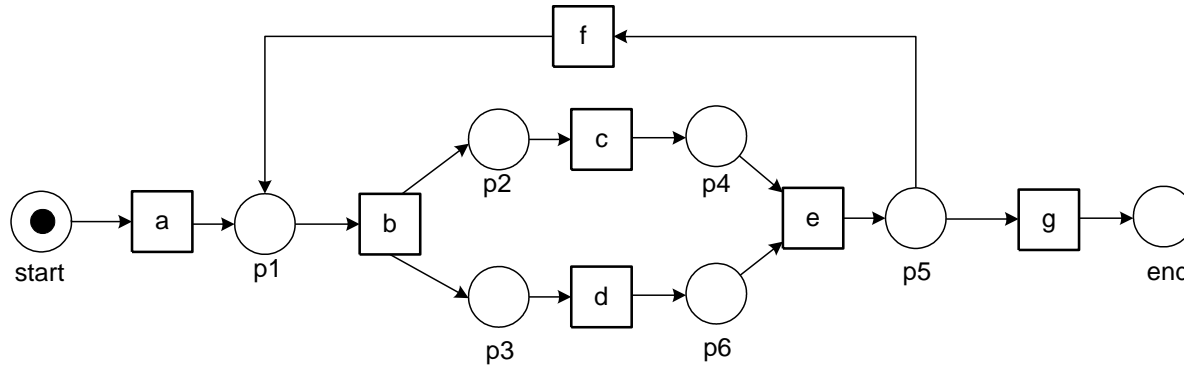


Trace	Frequency	P	R	C	M	P <all>	R <all>	C <all>	M <all>
<a, c, d, f>	1	7	1	7	1	7	1	7	1
<a, c, b, d, f>	2	8	0	8	0	16	0	16	0
<a, b, c, f>	1	7	1	7	1	7	1	7	1
<a, e, f>	2	7	1	7	1	14	2	14	2
total						44	4	44	4

$$\text{fitness} = \frac{1}{2} \left(1 - \frac{4}{44} \right) + \frac{1}{2} \left(1 - \frac{4}{44} \right) = 0.9$$

Exercise 5

- Compute the footprint and token-based replay fitness for the event log L and the presented model.
- $L = [\langle a, b, c, d, e, f, b, d, c, g \rangle, \langle a, b, d, c, e, g \rangle, \langle a, b, c, d, e, f, c, d, e, f, b, d, e, g \rangle]$



Exercise 5 Solution

Footprint

L	a	b	c	d	e	f	g
a	#	→	#	#	#	#	#
b	←	#	→	→	#	←	#
c	#	←	#		→	←	→
d	#	←		#	→	#	#
e	#	#	←	←	#	→	→
f	#	→	→	#	←	#	#
G	#	#	←	#	←	#	#

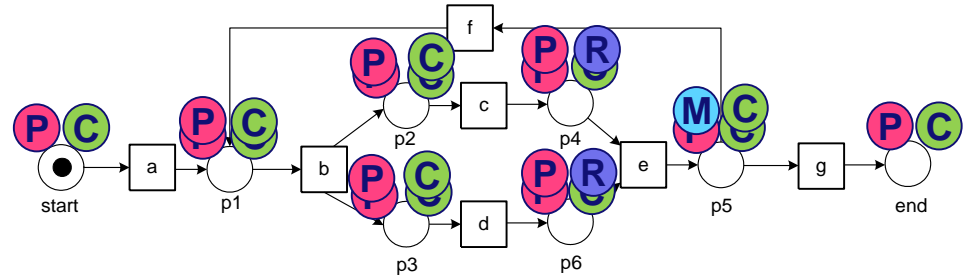
$$1 - {}^4/_{49} = 0.9184$$

M	a	b	c	d	E	f	g
a	#	→	#	#	#	#	#
b	←	#	→	→	#	←	#
c	#	←	#		→	#	#
d	#	←		#	→	#	#
e	#	#	←	←	#	→	→
f	#	→	#	#	←	#	#
g	#	#	#	#	←	#	#

Exercise 5 Solution

Token-based Replay

- $L = [<a, b, c, d, e, f, b, d, c, g>, <a, b, d, c, e, g>, <a, b, c, d, e, f, c, d, e, f, b, d, e, g>]$

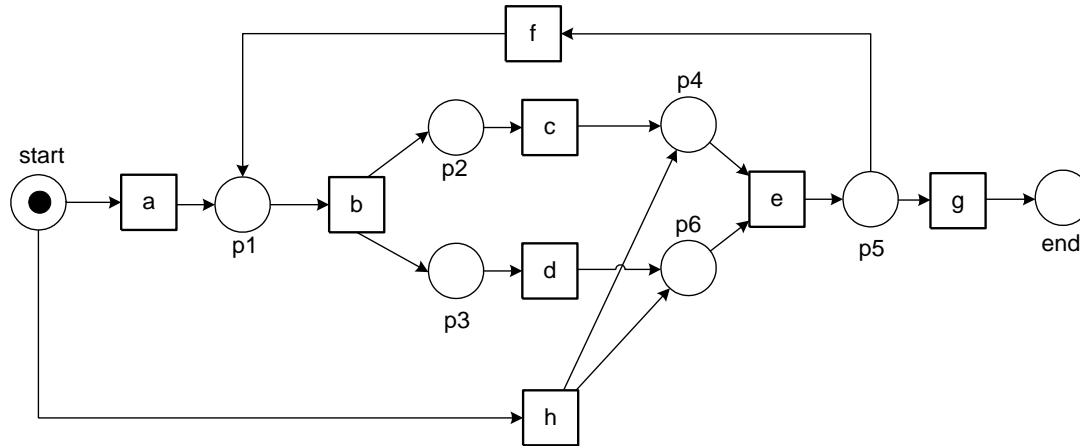


Trace	Frequency	P	R	C	M	P <all>	R <all>	C <all>	M <all>
<a, b, c, d, e, f, b, d, c, g>	1	13	2	12	1	13	2	12	1
<a, b, d, c, e, g>	1	8	0	8	0	8	0	8	0
<a, b, c, d, e, f, c, d, e, f, b, d, e, g>	1	17	2	18	3	17	2	18	3
total						38	4	38	4

$$\text{fitness} = \frac{1}{2} \left(1 - \frac{4}{38} \right) + \frac{1}{2} \left(1 - \frac{4}{38} \right) = 0.8947$$

Exercise 6

- Compute the footprint and token-based replay fitness for the event log L and the presented model.
- $L = [\langle a, b, c, e, f, b, d, c, e, g \rangle, \langle a, b, e, g \rangle, \langle h, e, g \rangle]$

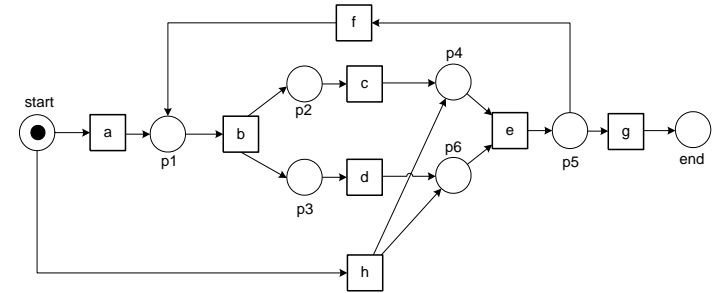


Exercise 6 Solution

Footprint

$$L = [\langle a, b, c, e, f, b, d, c, e, g \rangle, \langle a, b, e, g \rangle, \langle h, e, g \rangle]$$

M : L	a	b	c	d	e	f	g	h
a	#	→	#	#	#	#	#	#
b	←	#	→	→	# : →	←	#	#
c	#	←	#	: ←	→	#	#	#
d	#	←	: →	#	→ : #	#	#	#
e	#	# : ←	←	← : #	#	→	→	←
f	#	→	#	#	←	#	#	#
g	#	#	#	#	←	#	#	#
h	#	#	#	#	→	#	#	#



Footprint-based Conformance:

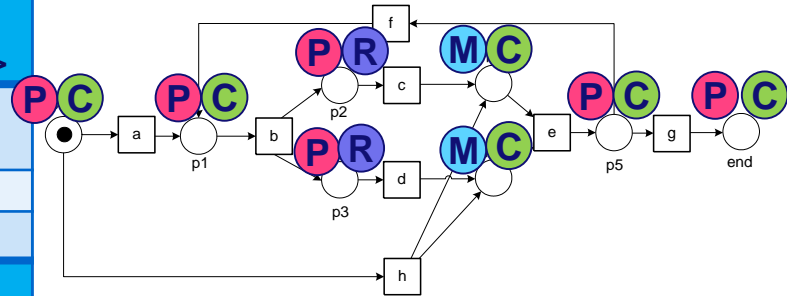
$$1 - \frac{6}{64} = 0.906$$

Exercise 6 Solution

Token-based Replay

$L = [<a, b, c, e, f, b, d, c, e, g>, <a, b, e, g>, <h, e, g>]$

Trace	Frequency	P	R	C	M	P <all>	R <all>	C <all>	M <all>
<a, b, c, e, f, b, d, c, e, g>	1	13	1	13	1	13	1	13	1
<a, b, e, g>	1	6	2	6	2	6	2	6	2
<h, e, g>	1	5	0	5	0	5	0	5	0
total						24	3	24	3



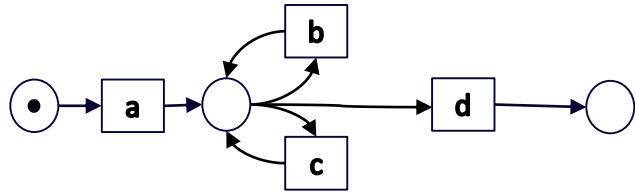
$$\text{fitness} = \frac{1}{2} \left(1 - \frac{3}{24} \right) + \frac{1}{2} \left(1 - \frac{3}{24} \right) = 0.875$$

Exercise 7

- Design a sound workflow net using **all** the activities $\{a, b, c, d\}$ and **only** those activities such that it can replay the following trace but the fitness score using footprint table is below 0.7.
 - $\langle a, b, c, d \rangle$
 - What will be the token-based replay fitness, higher or less than footprint? Why?

Exercise 7 Solution

$\langle a, b, c, d \rangle$



M : L	a	b	c	d
a	#	→	→:#	→:#
b	←	:#	:→	→:#
c	←:#	:←	:#	→
d	←:#	←:#	←	#

$$p = 5$$

$$c = 5$$

$$m = 0$$

$$r = 0$$

Footprint-Based Conformance: $1 - \frac{10}{16} = 0.375$

Token-Based Replay Fitness: $\frac{1}{2} \left(1 - \frac{0}{5} \right) + \frac{1}{2} \left(1 - \frac{0}{5} \right) = 1.00$