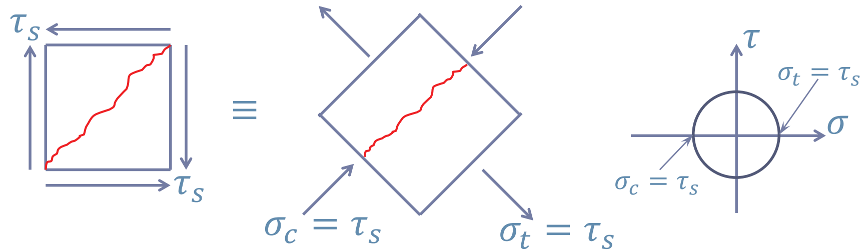
CE 382 Reinforced Concrete Fundamentals

Shear – Diagonal Tension

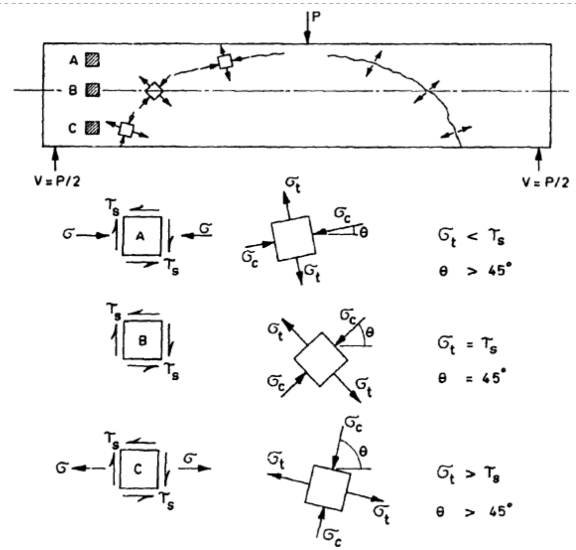
Introduction

- Shear by itself does not create any serious problems in RC members
- Principal tensile stresses caused by shear cause problems
- Tensile strength of concrete < shear strength of concrete

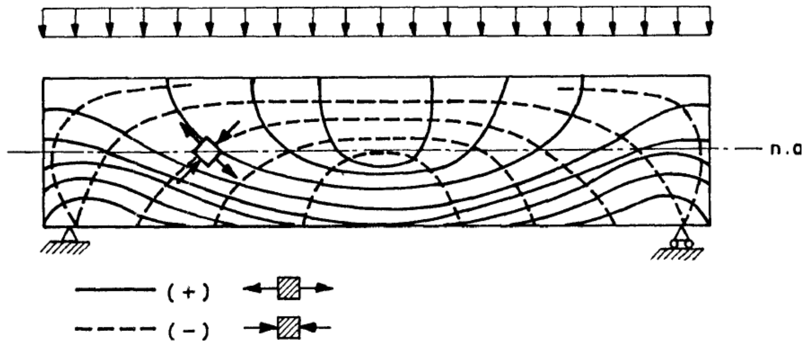


- Failure in tension is very sudden & brittle

Cracking due to shear



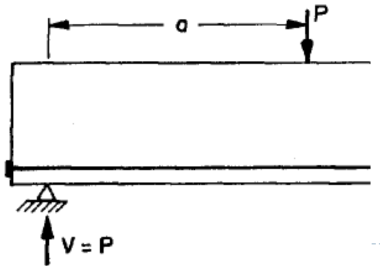
Direction of principal stresses



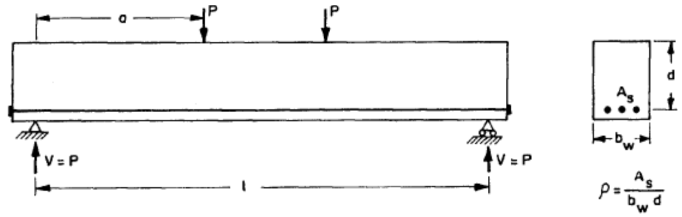
- *Prof. Leonhardt*: shear stresses do not exist; shear stress means that the principal stresses are not parallel to the x & y axis

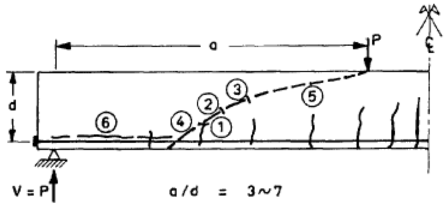
Behavior & strength of beams without shear reinforcement

- Shear span « a » : the distance from the load to the reaction.
- Dimensionless: a/d
- $a/d\ > 7.0\ \ or\ \ 8.0$: Flexural failure (for normal steel ratios). Yielding of longitudinal tension reinforcement and then crushing of concrete



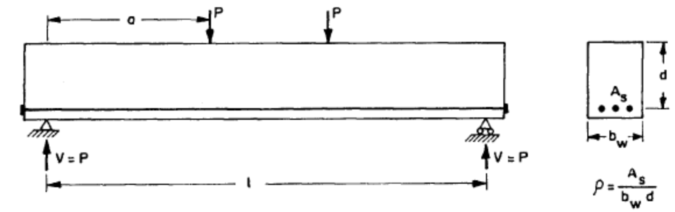
Diagonal Tension Failure

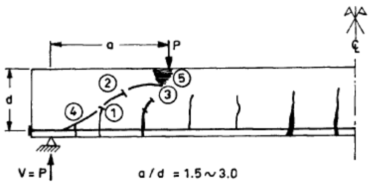




- Failure load lower than flexural capacity
- Flexural cracks at maximum moment region
- ① vertical cracks at shear span
- ②, ③ cracks inclined towards the load
- ④ inclined crack progresses downward
- ④, ②, ③ «true diagonal crack»
- Redistribution → steel stress⇗
- ⑤, ⑥ sudden, extremely brittle and destructive failure

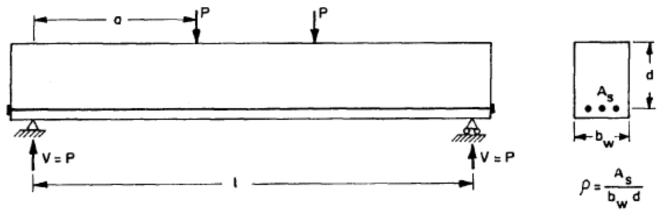
Shear-Compression Failure

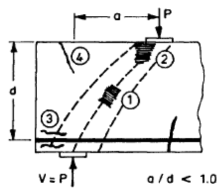




- Failure load lower than flexural capacity
- Flexural cracks at maximum moment region
- ① vertical cracks at shear span
- ② cracks inclined towards the load
- ③ inclined crack extends toward the load
- ④ inclined crack reaches the level of longitudinal steel
- ④, ②, ③ fully developed diagonal crack
- Continue to carry the increasing load
- ⑤ brittle failure by crushing of concrete in the compression zone

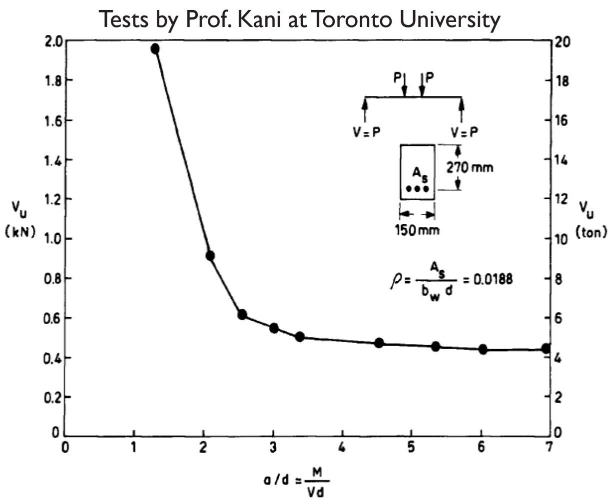
Tied Arch





- Failure due to principal tensile stresses is not possible
- Load is directly transferred to the support through a compression strut
- Failure by crushing of concrete in the compression zone, crushing of the web or by anchorage failure near the support

The effect of $\frac{a}{d}$ ratio on shear capacity



Generalized for distributed load

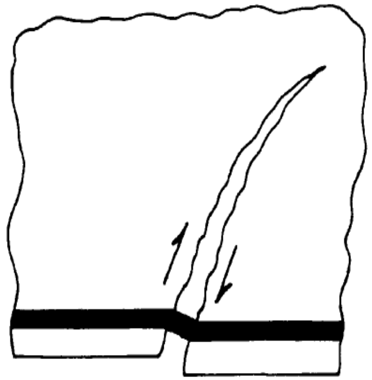
$$\frac{a}{d} = \frac{M}{Vd}$$

Variables affecting the behavior and strength of beams without web reinforcement

$$\rho = \frac{A_s}{b_w d}$$

$$f_{ct}$$

Dowel Action



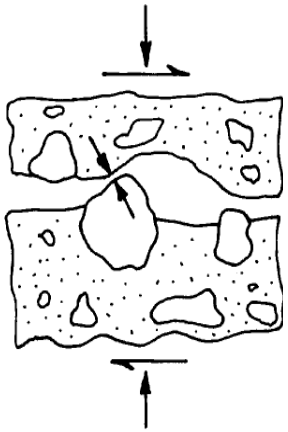
The longitudinal reinforcement tries to resist shear displacement.

Dowel capacity depends mainly on:

- Tensile strength of concrete
- The width of the crack
- Concrete cover
- Diameter of the tension reinforcement

Splitting cracks decrease effectiveness of the dowel action.

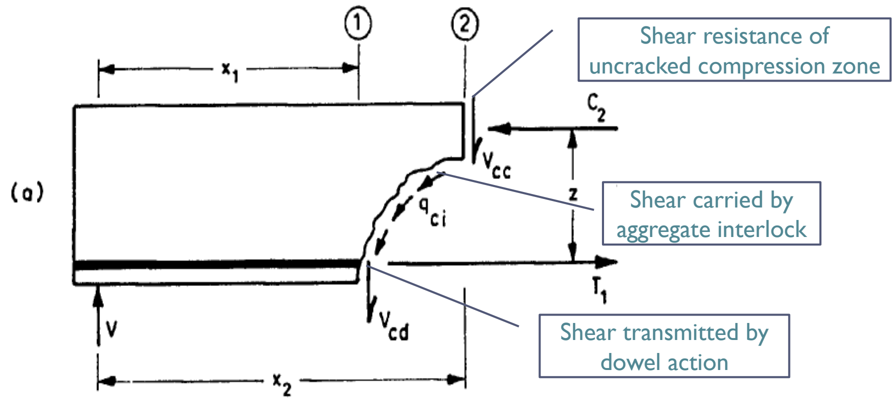
Aggregate Interlock



Roughness of coarse aggregate particles prevents shear displacement between the two faces of a crack of moderate width.

Considerable shear can be transmitted along the crack.

Shear resistance of the uncracked compression zone

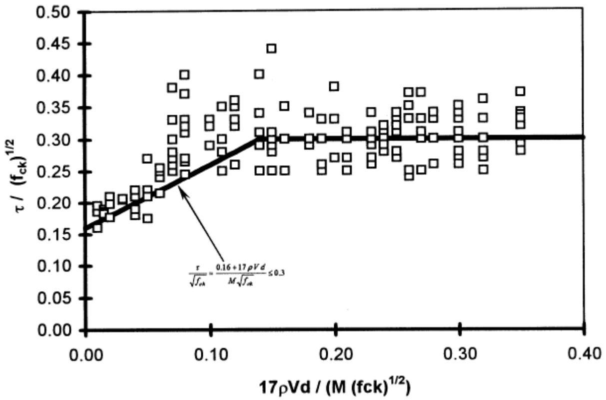


Shear carried by the three mechanisms are not necessarily additive at the failure stage.

Shear transferred by aggregate interlock and dowel action decrease significantly near the ultimate stage due to widening of cracks.

Diagonal Cracking Strength

- By Viest:
- $V_{cr} = \left(0.16\sqrt{f_{ck}} + 17\rho\frac{Vd}{M}\right) b_w d \leq 0.3\sqrt{f_{ck}} b_w d$

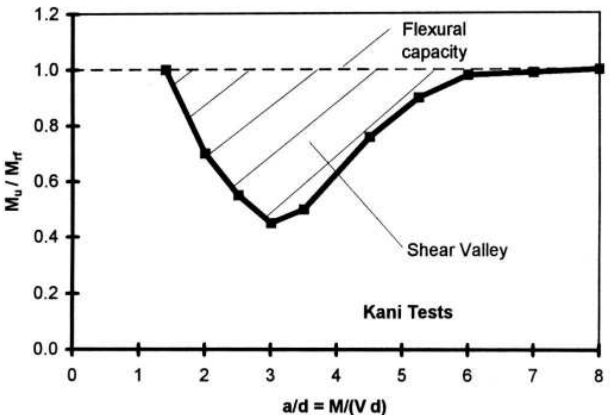


Diagonal Cracking Strength

- TS 500-2000:
- $V_{cr} = 0.65 f_{ctd} b_w d \left(1 + \gamma \frac{N_d}{A_c}\right)$
- Axial compression: $\gamma = 0.07$
- Axial tension: $\gamma = -0.3$

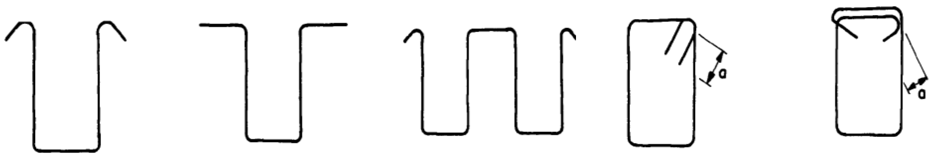
Function of Shear Reinforcement

- Prevent diagonal tension failure
- Insure flexural failure, $\frac{M_u}{M_{rf}} \geq 1.0$



Function of Shear Reinforcement

- Shear reinforcement:
  - Stirrups (ties)
  - Bent bars
- Turkish Seismic Code
  - Only stirrups are considered as shear reinforcement
- $\frac{A_{sw}}{s} = \frac{V_w}{f_{ywd} d} \qquad A_{sw} = nA_0$



Design for Shear

- $V_r \geq V_d$
  - V_d: maximum design shear
  - V_r: shear strength
- $V_r = V_w + V_c$
  - V_c: resistance of concrete
  - V_w: resistance of shear reinforcement
- $V_c = 0.8V_{cr}$
- $V_{cr} = 0.65 f_{ctd} b_w d\, \psi$ $\qquad \psi = \left(1 + \gamma \frac{N_d}{A_c}\right)$
- $V_r = \frac{A_{sw}}{s} f_{ywd} d + 0.52 f_{ctd} b_w d$
- $\frac{A_{sw}}{s} = \frac{V_d - V_c}{f_{ywd} d}$

Design for Shear

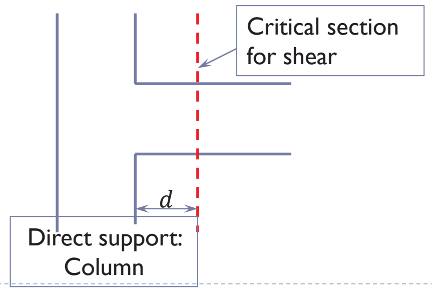
- If $V_E > 0.5V_{G+Q+E} \rightarrow V_c = 0$ (TEC2007)
- Contribution of bent bars to shear is not taken into account (TS500 & TEC2007)
- Minimum shear reinforcement: $\frac{A_{sw}}{s} \geq 0.3 \frac{f_{ctd}}{f_{ywd}} b_w$
  - To prevent brittle shear failure due to principal tensile stresses.
- Maximum shear force: $V_d \leq V_{max} = 0.22 f_{cd} b_w d$
  - Brittle failure due to crushing of web concrete due to principal compressive stresses.

Design for Shear

- $s \leq \frac{d}{2}$
- $s \leq \frac{d}{4}$ if $V_d > 3V_{cr}$
- Both ends of the beams should be confined by closely spaced ties.
  - $confined\ zone = 2h$
  - $s \leq \frac{d}{4}$
  - $s \leq 8\phi_\ell$
  - $s \leq 150\ mm$
- Additional detailing provisions in TEC2007 for beams and columns separately.

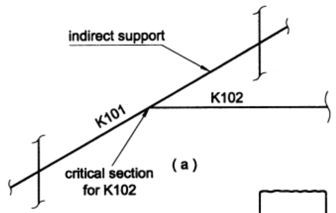
Critical Section for Design Shear

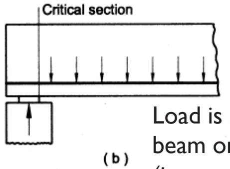
- The design shear is at the face of the support (not at the center of column)
- Due to local compressive forces from the support, diagonal tension failure can not occur very near to the face of the support $\rightarrow$ critical section = «d» away from the support face (for direct support)



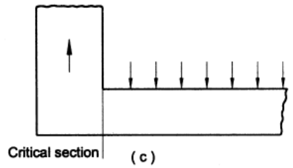
Critical Section for Design Shear

- Indirect support: which does not exert local pressure(compression) to the beam near by the support face.



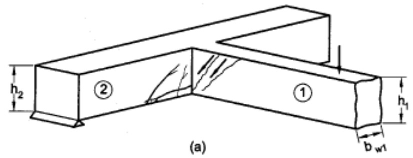


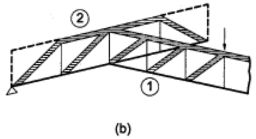
Load is applied to the beam on the tensile face (inverted beam)

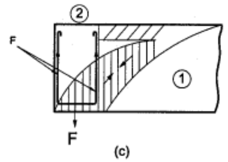


The beam is hanging to another structural member causing tension in that member

Indirect Support







- Beam ① is supported by beam ② → indirect support
- Beam ① applies the reaction to the bottom (tension) face of beam ②
- Diagonal compression strut pushes the bottom longitudinal bar of beam ② downward → diagonal shear cracks and horizontal bond cracks
- Inclined compression from beam ① should be transferred to the compression zone of beam ② → use along $b_{w1} + h_2$ length stirrups (hangers)

Example 1

- Simply supported flanged beam
- $b = 750\ mm \qquad b_w = 250\ mm \qquad \ell = 10\ m$
- C25 ($f_{cd} = 17\ MPa$ & $f_{ctd} = 1.2\ MPa$)
- S420 ($f_{yd} = f_{ywd} = 365\ MPa$)
- Preliminary design:
  - Estimate $g = 20\ kN/m$ & $q = 10\ kN/m$
  - $p_d = 1.4 \times 20 + 1.6 \times 10 = 44\ kN/m$
  - $V_d = p_d \frac{\ell}{2} = 220\ kN$
  - $\boxed{b_w d = \frac{0.9V_d}{f_{ctd}}} = 165000\ mm^2 \rightarrow d = 660\ mm$
  - $\rightarrow h = 700\ mm$

Example 1

- Final Design
  - $V_d = 173.6\ kN$ (d away from support)
  - $V_{cr} = 0.65 f_{ctd} b_w d = 0.65 \times 1.2 \times 250 \times 660 = 128.7\ kN$
  - $V_{max} = 0.22 f_{cd} b_w d = 0.22 \times 17 \times 250 \times 660 = 617.1\ kN$
  - $V_{cr} < V_d < V_{max}$
  - $V_c = 0.8 V_{cr} = 103\ kN$
  - $\frac{A_{sw}}{s} = \frac{V_d - V_c}{f_{ywd} d} = \frac{173600-103000}{365 \times 660} = 0.29\ mm$

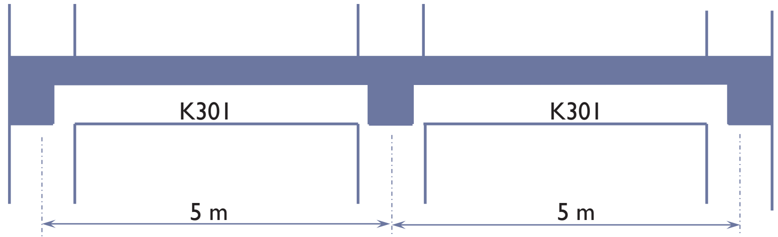
Example 1

- Final Design
  - $\min \frac{A_{sw}}{s} = 0.3 \frac{f_{ctd}}{f_{ywd}} b_w = 0.3 \frac{1.2}{365} 250 = 0.246\ mm$
  - Two leg ϕ8: $A_{sw} = nA_0 = 2 \times 50 = 100\ mm^2$
  - $\rightarrow s = \frac{100}{0.29} = 345\ mm$
  - But $s \leq \frac{d}{2} = 330\ mm$
  - → use ϕ8/330 mm

Study Example 7.2
Study Example 7.3

Example 2

- Given: two span beam, C20, S420, columns 400×400 mm



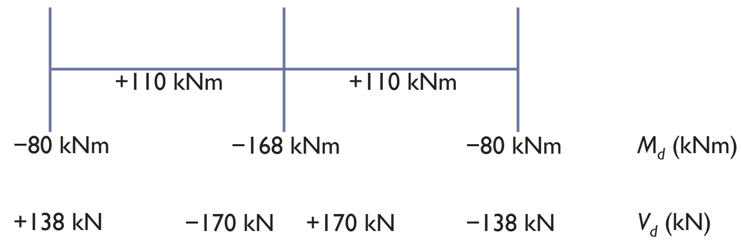
- Preliminary Design:
  - Estimate loads: $g = 30\ kN/m \quad q = 15\ kN/m$
  - $p_d = 1.4 \times 30 + 1.6 \times 15 = 66\ kN/m$
  - max. estimated moment $\cong \frac{1}{9} p_d \ell^2 = \frac{1}{9} 66 \times 5^2 = 183\ kNm$
  - Estimated shear $V \cong \frac{p\ell}{2} = \frac{66 \times 5}{2} = 165\ kN$

Example 2

- $K_\ell = 380 \frac{mm^2}{kN}$
- $b_w d^2 = M_d K_\ell = 183000 \times 380 = 540000\ mm^3$
- if $b_w = 250\ mm \rightarrow d = 527\ mm$
- if $b_w = 300\ mm \rightarrow d = 481\ mm$
- $b_w d = \frac{0.9 V_d}{f_{ctd}} = 0.9 \frac{165000}{1.0} = 148500\ mm^2$
- if $b_w = 250\ mm \rightarrow d = 594\ mm$
- if $b_w = 300\ mm \rightarrow d = 495\ mm$
- → use 300×500 mm

Example 2

- Loads $g = 28\ kN/m, q = 14\ kN/m \rightarrow p_d = 61.6\ kN/m$
- from analysis (centerline values)



- $\Delta M = \frac{V_d a}{3} = \frac{170 \times 0.4}{3} = 22\ kNm$ & $\Delta M = \frac{138 \times 0.4}{3} = 18$
- External support: $M_d = 80 - 18 = 62\ kNm$
- Internal support: $M_d = 168 - 22 = 146\ kNm$

Example 2

- Span:
  - K301, 300×500, $+M_d = 110\ kNm$, T-beam, $d = 460\ mm$
  - $jd = 0.9d = 414\ mm$
  - $+A_s = \frac{110000000}{365 \times 414} = 728\ mm^2$
  - $\min A_s = 0.8 \frac{f_{ctd}}{f_{yd}} b_w d = 302\ mm^2$
  - 2ϕ16 straight + 2ϕ16 bent = 800 mm²
- Supports:
  - External support (rectangular beam):
  - $M_d = 62\ kNm \qquad K = \frac{b_w d^2}{M_d} = 1023 > K_\ell$ OK ✓ single reinf.
  - $A_s = \frac{62 \times 10^6}{365 \times 0.86 \times 460} = 429\ mm^2$
  - with correct $j = 0.96 \rightarrow 385\ mm^2$

Example 2

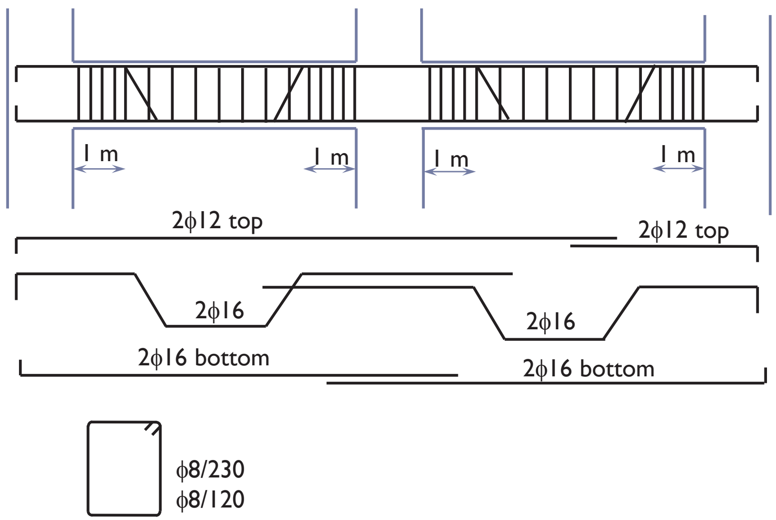
- Available: 2ϕ12 hanger + 2ϕ16 bent = 626 mm² > 429 OK✓
- Internal support
- $M_d = 146\ kNm \quad \rightarrow \quad K = 435 > K_\ell$ OK ✓ single reinf.
- $A_s = \frac{146 \times 10^6}{365 \times 0.86 \times 460} = 1011\ mm^2$
- with correct $j = 0.88 \rightarrow 988\ mm^2$
- Available: 2ϕ12 hanger + 4ϕ16 bent = 1026 mm² > 988 OK✓

- Shear Design:
  - Critical shear $V_d = 170\ kN$ (℄ of support)
  - $V_d' = V_d - p_d \left(\frac{a}{2} + d\right) = 170 - 61.6(0.2 + 0.46) = 130\ kN$
  - $V_{cr} = 0.65 \times 1.0 \times 300 \times 460 = 89.7\ kN$

Example 2

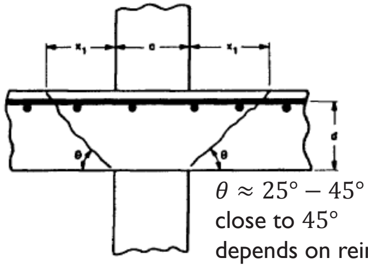
- $V_c = 0.8V_{cr} = 72\ kN$
- $V_{max} = 0.22 \times 13 \times 300 \times 460 = 394\ kN$
- $V_{cr} < V_d' < V_{max}$
- $\min \frac{A_{sw}}{s} = 0.3 \frac{1.0}{365} 300 = 0.25\ mm$
- $\frac{A_{sw}}{s} = \frac{V_d - V_c}{f_{ywd} d} = \frac{130000 - 72000}{365 \times 460} = 0.35 > \min \frac{A_{sw}}{s}$
- if ϕ8 is used $A_{sw} = 2 \times 50 = 100\ mm^2 \quad \rightarrow \quad s = 289\ mm$
- $\max s = \frac{d}{2} = 230\ mm \quad \rightarrow$ use ϕ8/230

Example 2

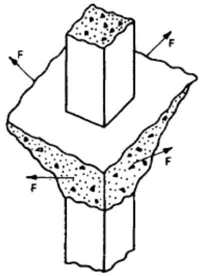


Punching Shear

- Slabs without beams → flat plate or flat slab or footings



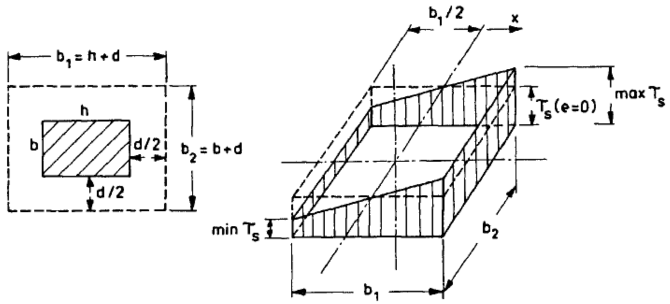
$\theta \approx 25° - 45°$
close to 45°
depends on reinforcement



  - Flat slab supported directly by column
  - Normal & shear stresses → very high principal tensile stresses
  - Failure → very sudden & brittle
  - for $\theta = 45° \rightarrow x_1 = d$

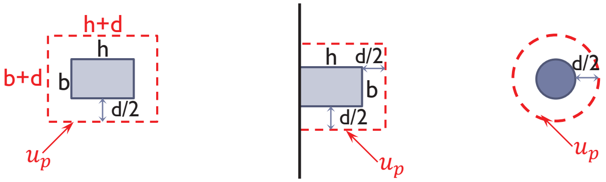
Punching Shear

- If moment exists in column, it has to be transferred to slab by flexure and torsion.
- The shear stresses caused by torsion increase the shear stresses caused by the axial loads on one face and decrease them on the other face.

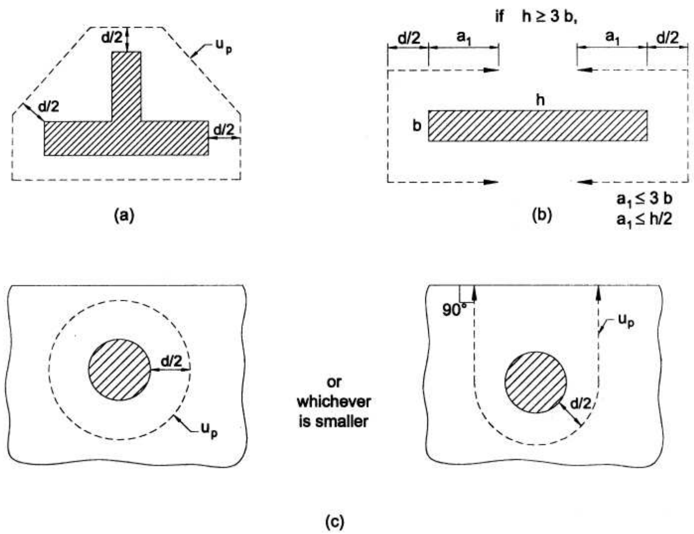


Punching Strength

- $V_{pc} = \gamma f_{ctk} u_p d$
- γ: a coefficient depending on moment transfer
- f_{ctk}: characteristic tensile strength of concrete
- u_p: critical perimeter
- d: effective depth

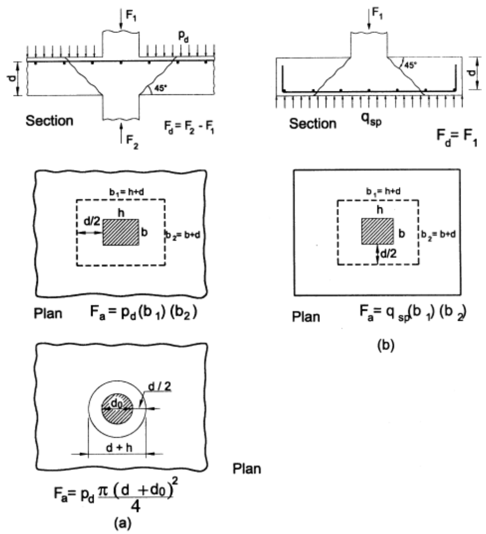


Critical Perimeter



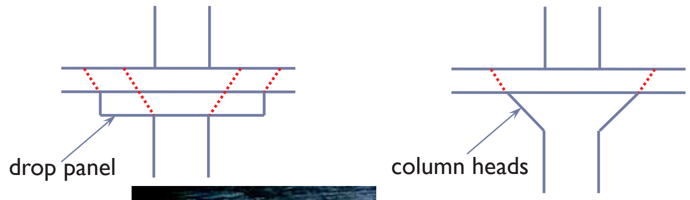
Design requirements for punching shear

- $V_{pc} > V_d$
- $V_d = F_d - F_a$
- $F_d = F_2 - F_1$
- F_d: design shear
- F_a: portion of load remaining within the critical perimeter

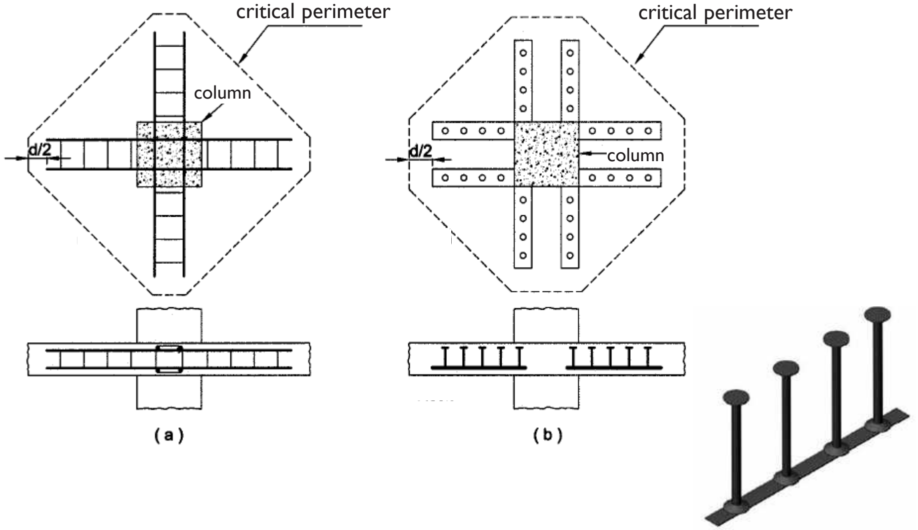


Increasing the Punching Shear Strength

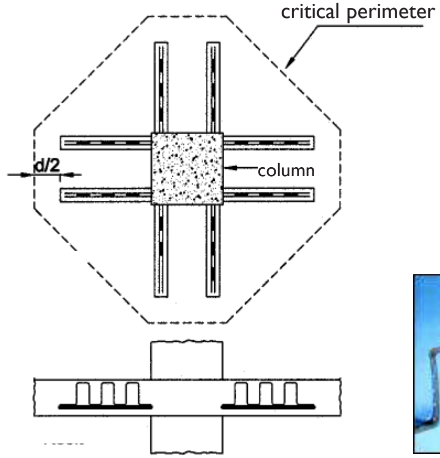
- Increase the concrete strength $V_{pc} = \gamma \boldsymbol{f_{ctk}} u_p d$
- Increase column dimensions $V_{pc} = \gamma f_{ctk} \boldsymbol{u_p} d$
- Increase the slab thickness $V_{pc} = \gamma f_{ctk} \boldsymbol{u_p d}$
- Column heads & drop panel $V_{pc} = \gamma f_{ctk} \boldsymbol{u_p d}$



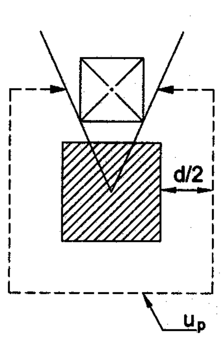
Punching Reinforcement

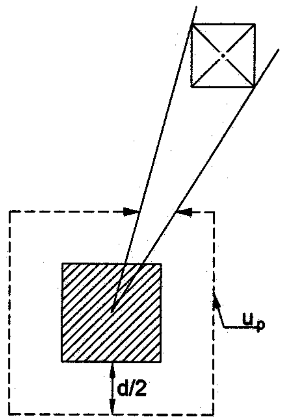


Punching Reinforcement



Openings in Slabs





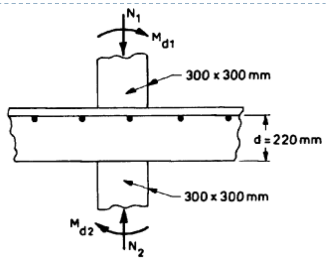
- Due to heating and plumping there are openings in the slab
- TS500 → reduce u_p if the opening in the slab is closer than five times the thickness of the slab (to the column face)

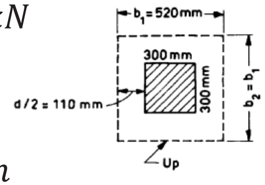
Cruces of Punching Shear Design

- Tensile strength of concrete is directly proportional to punching strength → concrete quality is extremely important
- Punching shear reinforcement may remain inside the punch pyramid → ineffective
- Openings especially close to columns are very important
- Self weight of slab is generally higher than live load → during casting of concrete, the floor below should be properly reshored.
- Corner and edge columns are trouble spots for punching → provide spandrel beam or extend the slab beyond the column

Example 3

- Column 300×300 mm
- C20
  - $f_{cd} = 13\ MPa$ & $f_{ctd} = 1.0\ MPa$
- S420 ($f_{yd} = 365\ MPa$)
- $t = 250\ mm$ & $d = 220\ mm$
- $p_d = 10\ kN/m^2$
- $N_{d1} = 880\ kN$ & $N_{d2} = 1300\ kN$
- $M_{d1} = M_{d2} = 0$
- $u_p = (300 + 220)4 = 2080\ mm$





Example 3

- $F_d = N_{d2} - N_{d1} = 1300 - 880 = 420\ kN$
- $F_a = b_1 b_2 p_d = 0.52 \times 0.52 \times 10 = 2.7\ kN$
- $V_d = F_d - F_a = 420 - 2.7 = 417.3\ kN$
- in flat plate F_a is small, but in footings its value is high
- $V_{pc} = \gamma f_{ctd} u_p d = 1.0 \times 1.0 \times 2080 \times 220 = 457.6\ kN$
- $V_{pc} > V_d$ OK ✓ slab is safe in punching