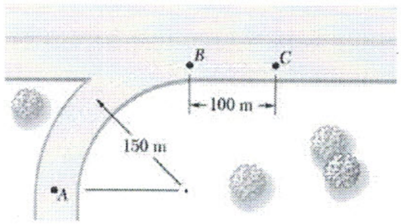
R1

PROBLEM 11.140



A motorist starts from rest at Point A on a circular entrance ramp when $t = 0$, increases the speed of her automobile at a constant rate and enters the highway at Point B. Knowing that her speed continues to increase at the same rate until it reaches 100 km/h at Point C, determine (a) the speed at Point B, (b) the magnitude of the total acceleration when $t = 20$ s.

SOLUTION

Speeds: $v_0 = 0 \qquad v_1 = 100 \text{ km/h} = 27.78 \text{ m/s}$

Distance: $s = \dfrac{\pi}{2}(150) + 100 = 335.6 \text{ m}$

Tangential component of acceleration: $v_1^2 = v_0^2 + 2a_t s$

$$a_t = \frac{v_1^2 - v_0^2}{2s} = \frac{(27.78)^2 - 0}{(2)(335.6)} = 1.1495 \text{ m/s}^2$$

At Point B, $\quad v_B^2 = v_0^2 + 2a_t s_B \quad$ where $\quad s_B = \dfrac{\pi}{2}(150) = 235.6 \text{ m}$

$$v_B^2 = 0 + (2)(1.1495)(235.6) = 541.69 \text{ m}^2/\text{s}^2$$

$$v_B = 23.27 \text{ m/s}$$

$v_B = 83.8 \text{ km/h}$ ◀

(a) At $t = 20$ s, $\quad v = v_0 + a_t t = 0 + (1.1495)(20) = 22.99 \text{ m/s}$

Since $v < v_B$, the car is still on the curve. $\quad \rho = 150 \text{ m}$

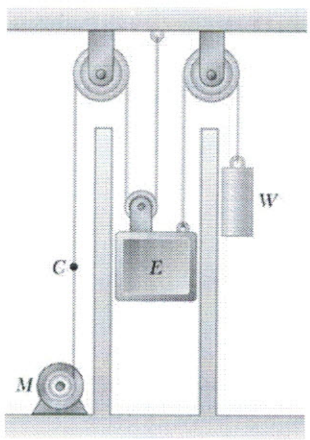
Normal component of acceleration: $\quad a_n = \dfrac{v^2}{\rho} = \dfrac{(22.99)^2}{150} = 3.524 \text{ m/s}^2$

(b) Magnitude of total acceleration: $\quad |a| = \sqrt{a_t^2 + a_n^2} = \sqrt{(1.1495)^2 + (3.524)^2}$

$|a| = 3.71 \text{ m/s}^2$ ◀

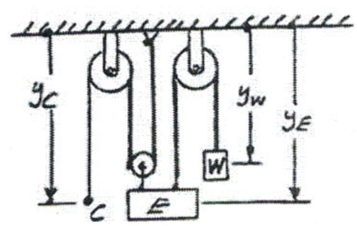
R1

PROBLEM 11.47



The elevator shown in the figure moves downward with a constant velocity of 4 m/s. Determine (*a*) the velocity of the cable *C*, (*b*) the velocity of the counterweight *W*, (*c*) the relative velocity of the cable *C* with respect to the elevator, (*d*) the relative velocity of the counterweight *W* with respect to the elevator.

SOLUTION



Choose the positive direction downward.

(*a*) Velocity of cable *C*.

$$y_C + 2y_E = \text{constant} = L_{T1}$$

$$v_C + 2v_E = 0$$

But, $v_E = 4 \text{ m/s}$

or $v_C = -2v_E = -8 \text{ m/s}$

$\mathbf{v}_C = 8.00 \text{ m/s} \uparrow$ ◀

(*b*) Velocity of counterweight *W*.

$$y_W + y_E = \text{constant} = L_{T2}$$

$$v_W + v_E = 0 \quad v_W = -v_E = -4 \text{ m/s}$$

$\mathbf{v}_W = 4.00 \text{ m/s} \uparrow$ ◀

(*c*) Relative velocity of *C* with respect to *E*.

$$v_{C/E} = v_C - v_E = (-8 \text{ m/s}) - (+4 \text{ m/s}) = -12 \text{ m/s}$$

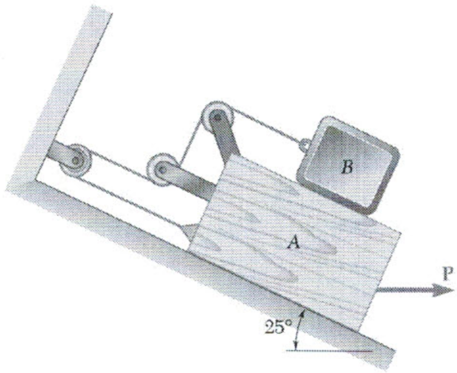
$\mathbf{v}_{C/E} = 12.00 \text{ m/s} \uparrow$ ◀

(*d*) Relative velocity of *W* with respect to *E*.

$$v_{W/E} = v_W - v_E = (-4 \text{ m/s}) - (4 \text{ m/s}) = -8 \text{ m/s}$$

$\mathbf{v}_{W/E} = 8.00 \text{ m/s} \uparrow$ ◀

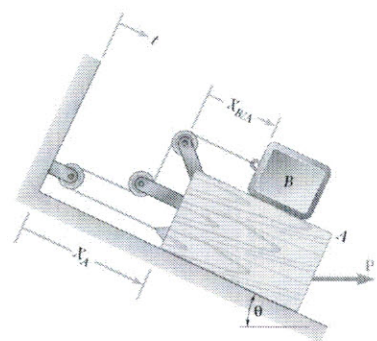
R2



PROBLEM 12.19

Block A has a mass of 40 kg, and block B has a mass of 8 kg. The coefficients of friction between all surfaces of contact are $\mu_s = 0.20$ and $\mu_k = 0.15$. If $P = 40\text{ N} \longrightarrow$, determine (*a*) the acceleration of block B, (*b*) the tension in the cord.

SOLUTION



From the constraint of the cord.

$$2x_A + x_{B/A} = \text{constant}$$

Then $$2v_A + v_{B/A} = 0$$

and $$2a_A + a_{B/A} = 0$$

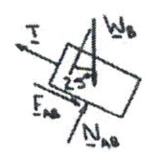
Now $$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

Then $$a_B = a_A + (-2a_A)$$

or $$a_B = -a_A \qquad (1)$$

First we determine if the blocks will move for the given value of P. Thus, we seek the value of P for which the blocks are in impending motion, with the impending motion of a down the incline.

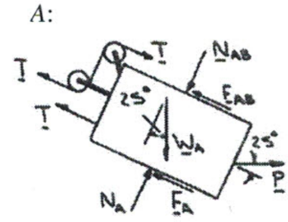
B: $\quad +\!\nearrow \Sigma F_y = 0: \quad N_{AB} - W_B \cos 25° = 0$



or $$N_{AB} = m_B g \cos 25°$$

Now $$F_{AB} = \mu_s N_{AB}$$
$$= 0.2\, m_B g \cos 25°$$

$$\overset{+}{\searrow} \Sigma F_x = 0: \quad -T + F_{AB} + W_B \sin 25° = 0$$



or $$T = 0.2\, m_B g \cos 25° + m_B g \sin 25°$$
$$= (8\text{ kg})(9.81\text{ m/s}^2)(0.2\cos 25° + \sin 25°)$$
$$= 47.39249\text{ N}$$

A: $\quad +\!\nearrow \Sigma F_y = 0: \quad N_A - N_{AB} - W_A \cos 25° + P \sin 25° = 0$

or $$N_A = (m_A + m_B) g \cos 25° - P \sin 25°$$

PROBLEM 12.19 (Continued)

Now $$F_A = \mu_s N_A$$

or $$F_A = 0.2[(m_A + m_B)g \cos 25° - P \sin 25°]$$

$$\overset{+}{\searrow} \Sigma F_x = 0: \quad -T - F_A - F_{AB} + W_A \sin 25° + P \cos 25° = 0$$

or $$-T - 0.2[(m_A + m_B)g \cos 25° - P \sin 25°] - 0.2 m_B g \cos 25° + m_A g \sin 25° + P \cos 25° = 0$$

or $$P(0.2 \sin 25° + \cos 25°) = T + 0.2[(m_A + 2m_B)g \cos 25°] - m_A g \sin 25°$$

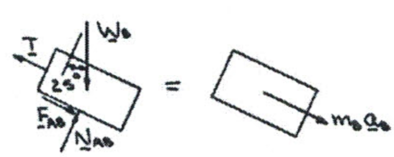
Then $$P(0.2 \sin 25° + \cos 25°) = 47.39249 \text{ N} + 9.81 \text{ m/s}^2\{0.2[(40 + 2 \times 8)\cos 25° - 40 \sin 25°] \text{ kg}\}$$

or $$P = -19.04 \text{ N} \text{ for impending motion.}$$

Since $P, < 40$ N, the blocks will move. Now consider the motion of the blocks.

(*a*) $\overset{+}{\nearrow} \Sigma F_y = 0: \quad N_{AB} - W_B \cos 25° = 0$

B:



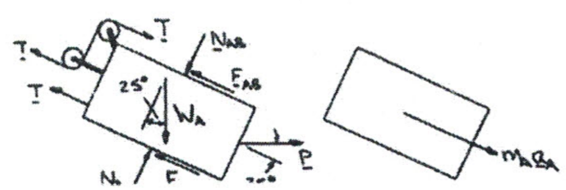
or $$N_{AB} = m_B g \cos 25°$$

Sliding: $$F_{AB} = \mu_k N_{AB}$$
$$= 0.15\, m_B g \cos 25°$$

$$\overset{+}{\searrow} \Sigma F_x = m_B a_B: \quad -T + F_{AB} + W_B \sin 25° = m_B a_B$$

or $$T = m_B[g(0.15 \cos 25° + \sin 25°) - a_B]$$
$$= 8[9.81(0.15 \cos 25° + \sin 25°) - a_B]$$
$$= 8(5.47952 - a_B) \qquad \text{(N)}$$

A:



$$\overset{+}{\nearrow} \Sigma F_y = 0: \quad N_A - N_{AB} - W_A \cos 25° + P \sin 25° = 0$$

or $$N_A = (m_A + m_B)g \cos 25° - P \sin 25°$$

Sliding: $$F_A = \mu_k N_A$$
$$= 0.15[(m_A + m_B)g \cos 25° - P \sin 25°]$$

$$\overset{+}{\searrow} \Sigma F_x = m_A a_A: \quad -T - F_A - F_{AB} + W_A \sin 25° + P \cos 25° = m_A a_A$$

Substituting and using Eq. (1)

$$\begin{aligned}T &= m_A g \sin 25° - 0.15[(m_A + m_B)g \cos 25° - P \sin 25°] \\ &\quad - 0.15\, m_B g \cos 25° + P \cos 25° - m_A(-a_B) \\ &= g[m_A \sin 25° - 0.15(m_A + 2m_B)\cos 25°] \\ &\quad + P(0.15 \sin 25° + \cos 25°) + m_A a_B \\ &= 9.81[40 \sin 25° - 0.15(40 + 2\times 8)\cos 25°] \\ &\quad + 40(0.15 \sin 25° + \cos 25°) + 40a_B \\ &= 129.94004 + 40a_B\end{aligned}$$

Equating the two expressions for T

$$8(5.47952 - a_B) = 129.94004 + 40a_B$$

or $$a_B = -1.79383 \text{ m/s}^2$$

$\mathbf{a}_B = 1.794 \text{ m/s}^2 \searrow 25°$ ◀

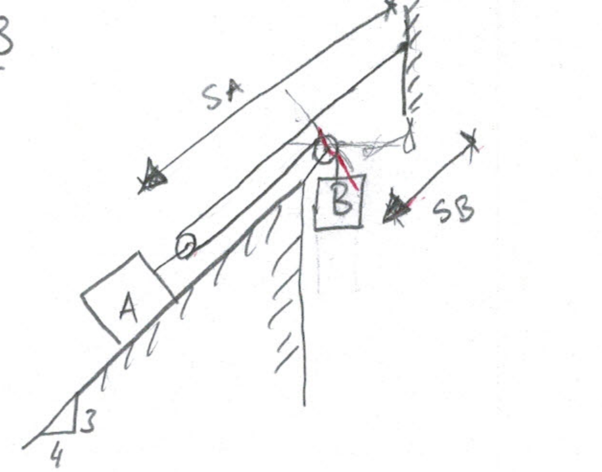
(*b*) We have $$T = 8[5.47952 - (-1.79383)]$$

or $T = 58.2$ N ◀

14-22

R3

The two blocks A and B have weights W_A 60 N and $W_b = 5$ N. If the kinetic coefficient of friction between the incline and block A is $\mu_k = 0,25$, determine the speed of A after it moves 2 m down the plane starting from rest. Neglect the mass of the cord and pulleys.



Solution

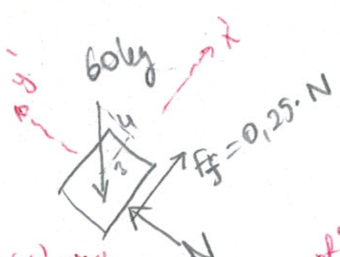
Position coordinate equation →

$s_A + (s_A - s_B) = l$ $\qquad \boxed{2s_A - s_B = l}$

$2\Delta s_A - \Delta s_B = 0 \qquad \Delta s_B = 2\Delta s_A = 2\cdot(2^m) = 4^m$

$2V_A - V_B = 0$

$\Sigma F_{y'} = m \cdot a_{y'}$; $\quad N - 60\left(\frac{4}{5}\right) = \frac{60}{9,81}\cdot(0)$; $N = 48$ ~~kg~~ N



$\mu_k \cdot N$

$F_f = 0{,}25 \cdot 48 = 12$ ~~kg~~ N

T (the cable force) do no work since these forces represents the reactions at the support, so they do not move while the block are displaced.

Principle work and Energy

$W_A \to (+)$ work acts in direction of displacement

$F_f \to (-)$ work in the opposite direction of disp.

→ kinetic energy → potential energy

$$\left[T_1 + \Sigma U_{1-2} = T_2\right]$$

$\hookrightarrow T_1 = 0$ (at rest) initially

work $= f \cdot d$

$$\overbrace{0 + W_A\left(\tfrac{3}{5}\Delta s_A\right)}^{PE} - \overbrace{F_f \cdot \Delta s_A}^{work} - \overbrace{W_B \Delta s_B}^{PE}$$

$$= \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2$$

$$60\cdot\left(\frac{3}{5}\cdot 2\right) - 12\cdot 2 - 5\cdot 4 = \frac{1}{2}\left(\frac{60}{9,81}\right)\cdot V_A^2 + \frac{1}{2}\cdot\frac{5}{9,81}\cdot V_B^2$$

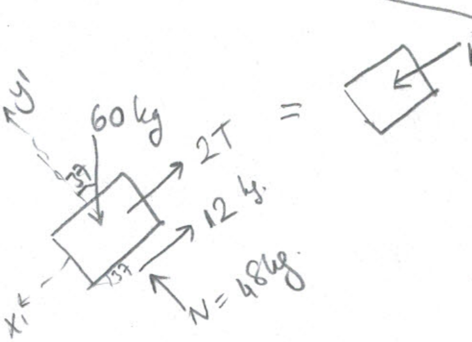
olack 5

$\Rightarrow 3V_A^2 + V_B^2 - 27,5 = 0$

$2V_A = V_B$ $\qquad 4{,}078\, V_A^2 = 27{,}5$

$V_A = 2{,}62$ m/s

$V_B = 5{,}24$ m/s



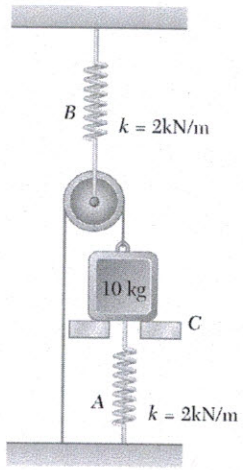
$= \quad m_A \cdot a_A$

$V_A^2 = 0^2 + 2a_A \cdot 2$

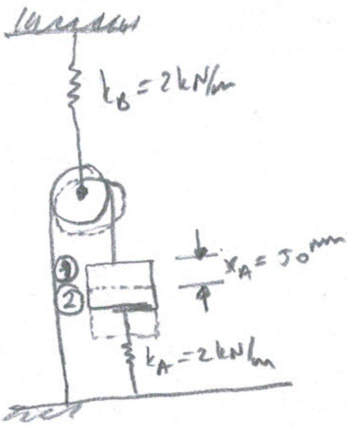
$\Sigma F_y' = 0 \quad N = 48$ kg

$\Sigma F_x' = m a_x' \Rightarrow 2T - 60 \sin 37 + 12 = \frac{60}{\ldots} \cdot a_A$

Chapter 13, Problem 30.



A 10-kg block is attached to spring *A* and connected to spring *B* by a cord and pulley. The block is held in the position shown with both springs unstretched when the support is removed and the block is released with no initial velocity. Knowing that the constant of each spring is 2 kN/m, determine (*a*) the velocity of the block after it has moved down 50 mm, (*b*) the maximum velocity achieved by the block.



a) $W = 10 * 9,81 = 98,1\ N$

$x_B = \frac{1}{2} x_A$

Principle of Work and Energy

$$\boxed{\cancel{T_1}^{\,0} + \Sigma U_{1-2} = T_2}$$

$$\Sigma U_{1-2} = \underbrace{W(x_A)}_{\text{Gravity}} - \underbrace{\frac{1}{2} k_A (x_A)^2}_{\text{Spring A}} - \underbrace{\frac{1}{2} k_B (x_B)^2}_{\text{Spring B}}$$

$$= (98,1^{N})(0,05^{mm}) - \frac{1}{2}(2000\ N/m)(0,05^{m})^2 - \frac{1}{2}(2000\ N/m)\left(\frac{0,05^{m}}{2}\right)^2$$

$=$

$$\Sigma U_{1-2} = T_2 = \frac{1}{2} m v^2 = \frac{1}{2}(10\ kg)\, v^2$$

$$4,905 - 2,5 - 0,625 = \frac{1}{2}\, 10\, v^2 \longrightarrow v = 0,597\ m/s$$

b) Let x = Distance moved down by the 10 kg block

$$U_{1-2} = W(x) - \frac{1}{2} k_A x^2 - \frac{1}{2} k_B \left(\frac{x}{2}\right)^2 = \frac{1}{2} m v^2$$

For $x = 0,0436$

$U = 4,2772 - 1,9010 - 0,4752 = \frac{1}{2}(10) \cdot v^2$

$\boxed{V_{max} = 0,6166\ m/s}$

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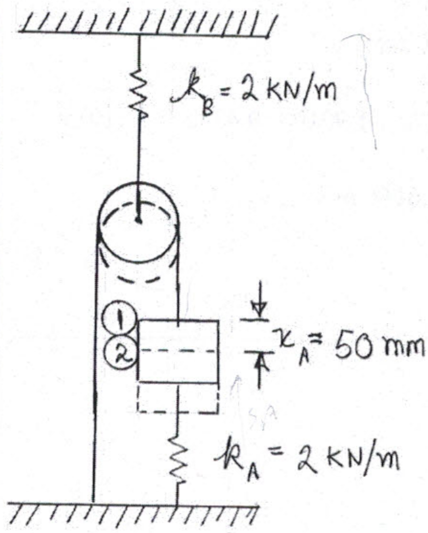
$\frac{1}{2} k_A \ldots \quad k_B\ (2x) = 0$

Chapter 13, Solution 30.

$W = \vec{F} \cdot \vec{d} \rightarrow dW = F\,dx$

$\int dW = k \int x\,dx$



(*a*) W = Weight of the block $= 10\,(9.81) = 98.1$ N

work done

$$x_B = \frac{1}{2} x_A \qquad T_1 + \Sigma U_{1-2} = T_2$$

kinetic E.

mgh

$$U_{1-2} = W(x_A) - \frac{1}{2}k_A(x_A)^2 - \frac{1}{2}k_B(x_B)^2$$

PE

(Gravity) (Spring *A*) (Spring *B*)

$$U_{1-2} = (98.1\text{ N})(0.05\text{ m}) - \frac{1}{2}(2000\text{ N/m})(0.05\text{ m})^2$$

$$-\frac{1}{2}(2000\text{ N/m})(0.025\text{ m})^2$$

$$U_{1-2} = \frac{1}{2}(m)\,v^2 = \frac{1}{2}(10\text{ kg})\,v^2$$

$$4.905 - 2.5 - 0.625 = \frac{1}{2}(10)\,v^2$$

$v = 0.597$ m/s ◀

(*b*) Let x = Distance moved down by the 10 kg block

$$U_{1-2} = W(x) - \frac{1}{2}k_A(x)^2 - \frac{1}{2}k_B\left(\frac{x}{2}\right)^2 = \frac{1}{2}(m)\,v^2$$

$$\frac{d}{dx}\left[\frac{1}{2}(m)\,v^2\right] = 0 = W - k_A(x) - \frac{k_B}{8}(2x)$$

$S_B + S_B - S_A = 0$

$\boxed{2S_B = S_A}$

$S_B = S_A/2$

$KE = W - PE$

$W + PE_0 + KE_0 = PE_f + KE_f +$ heat lost

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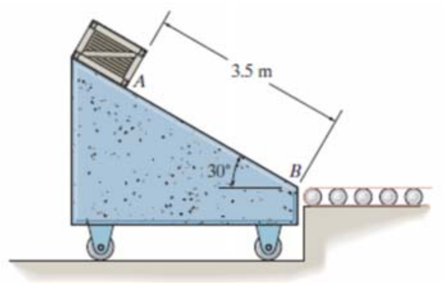
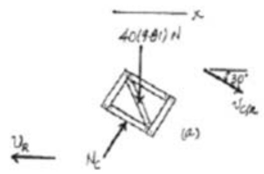
1)

The free-rolling ramp has a mass of 40 kg. A 10-kg crate is released from rest at *A* and slides down 3.5 m to point *B*. If the surface of the ramp is smooth, determine the ramp's speed when the crate reaches *B*. Also, what is the velocity of the crate?



Conservation of Energy: The datum is set at lowest point B. When the crate is at point A, it is $3.5 \sin 30^\circ = 1.75$ m *above* the datum. Its gravitational potential energy is $10(9.81)(1.75) = 171.675$ N · m. Applying Eq. 14–21, we have

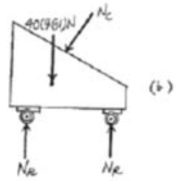


$$T_1 + V_1 = T_2 + V_2$$

$$0 + 171.675 = \frac{1}{2}(10)v_C^2 + \frac{1}{2}(40)v_R^2$$

$$171.675 = 5v_C^2 + 20v_R^2 \qquad \textbf{(1)}$$

Relative Velocity: The velocity of the crate is given by



$$\mathbf{v}_C = \mathbf{v}_R + \mathbf{v}_{C/R}$$

$$= -v_R\mathbf{i} + (v_{C/R}\cos 30^\circ\mathbf{i} - v_{C/R}\sin 30^\circ\mathbf{j})$$

$$= (0.8660v_{C/R} - v_R)\mathbf{i} - 0.5v_{C/R}\mathbf{j} \qquad \textbf{(2)}$$

The magnitude of v_C is

$$v_C = \sqrt{(0.8660\,v_{C/R} - v_R)^2 + (-0.5v_{C/R})^2}$$

$$= \sqrt{v_{C/R}^2 + v_R^2 - 1.732v_R v_{C/R}} \qquad \textbf{(3)}$$

Conservation of Linear Momentum: If we consider the crate and the ramp as a system, from the FBD, one realizes that the normal reaction N_C (*impulsive force*) is *internal* to the system and will cancel each other. As the result, the linear momentum is conserved along the x axis.

$$0 = m_C(v_C)_x + m_R v_R$$

$$(\overset{+}{\leftarrow}) \qquad 0 = 10(0.8660v_{C/R} - v_R) + 40(-v_R)$$

$$0 = 8.660v_{C/R} - 50v_R \qquad \textbf{(4)}$$

Solving Eqs. (1), (3), and (4) yields

$$v_R = 1.101 \text{ m/s} = 1.10 \text{ m/s} \quad v_C = 5.43 \text{ m/s} \qquad \textbf{Ans.}$$

$$v_{C/R} = 6.356 \text{ m/s}$$

From Eq. (2)

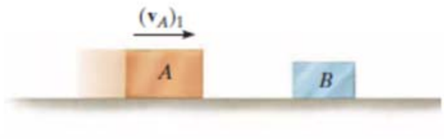
$$\mathbf{v}_C = [0.8660(6.356) - 1.101]\mathbf{i} - 0.5(6.356)\mathbf{j} = \{4.403\mathbf{i} - 3.178\mathbf{j}\} \text{ m/s}$$

Thus, the directional angle ϕ of v_C is

$$\phi = \tan^{-1}\frac{3.178}{4.403} = 35.8^\circ \quad \triangleleft\!\!\phi \qquad \textbf{Ans.}$$

2)

Block A has a mass of 3 kg and is sliding on a rough horizontal surface with a velocity $(V_A)_1$=2 m/s when it makes a direct collision with block B, which has a mass of 2 kg and is originally at rest. If the collision is perfectly elastic (e=1) determine the velocity of each block just after collision and the distance between the blocks when they stop sliding. The coefficient of kinetic friction between the blocks and the plane is $\mu_k = 0.3$.

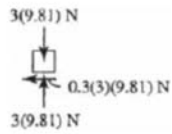


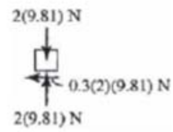
$(\overset{+}{\rightarrow})\qquad \sum mv_1 = \sum mv_2$

$$3(2) + 0 = 3(v_A)_2 + 2(v_B)_2$$

$(\overset{+}{\rightarrow})\qquad e = \dfrac{(v_B)_2 - (v_A)_2)}{(v_A)_1 - (v_B)_1}$

$$1 = \frac{(v_B)_2 - (v_A)_2}{2 - 0}$$





Solving

$(v_A)_2 = 0.400$ m/s → **Ans.**

$(v_B)_2 = 2.40$ m/s → **Ans.**

Block A:

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(3)(0.400)^2 - 3(9.81)(0.3)d_A = 0$$

$$d_A = 0.0272 \text{ m}$$

Block B:

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(2)(2.40)^2 - 2(9.81)(0.3)d_B = 0$$

$$d_B = 0.9786 \text{ m}$$

$d = d_B - d_A = 0.951$ m **Ans.**