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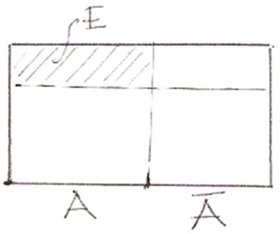
Deformations that may develop over concrete bridge girders such as due to creep will cause a vertical harmonic excitation in a vehicle travelling over the bridge with constant speed. When the bridge deflection exceeds 1.6 cm there may occur vertical motion on the vehicle depending on the speed of the vehicle. If the speed is less than 40 km/hr there occurs no vertical excitation, otherwise, there occurs vertical harmonic excitation (this is important for the design of shock absorbers to provide vibration-isolation system). The probability that the bridge deflection exceeds 1.6 cm is 3% and if the deflection exceeds 1.6 cm the probability that the speed of the vehicle will be exceeding 40 km/hr is 45%.

a) Define all the events related with your problem, give a Venn Diagram.
b) Find the probability that the vehicle over the bridge will be excited by vertical motion.
c) If it is known that no vertical excitation has occurred on the vehicle find the probability that the deflection has not exceeded 1.6 cm.

Let A be the event that velocity, $V > 40$ km/hr
B " " " " deflection, $\delta > 1{,}6$ cm.
E " " " " there is vertical vibration

a)



Note: $E = AB$

b) Given: $P(A/B) = 0{,}45$ & $P(B) = 0{,}03$

$$P(E) = P(AB) = P(A/B) \cdot P(B) = 0{,}45 \times 0{,}03 = 0{,}0135$$

c) $$P(\bar{B}/\bar{E}) = \frac{P(\bar{B}\bar{E})}{P(\bar{E})} = \frac{1-0{,}03}{1-0{,}0135} = \frac{0{,}97}{0{,}9865} = 0.9833$$

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3. Let E_1, E_2, and E_3 denote the events of excessive snowfall in the first, second, and third winters, respectively, from this fall. Statistical records of snowfalls indicate that during any winter, the probability of excessive snow is 0.10. However, if excessive snowfall occurred in the previous winter, the probability of excessive snowfall in the following winter is increased to 0.40, whereas if the preceding two winters are both subjected to excessive snowfalls, the probability of excessive snow in the following winter will be 0.20.

a) From the information given above, determine the following:
$P(E_1)$, $P(E_2)$, $P(E_2|E_1)$, $P(E_3|E_1 E_2)$, $P(E_3|E_2)$

b) What is the probability that excessive snowfall will occur in at least one of the next two winters?

c) What is the probability that excessive snowfall will occur in each of the next three winters?

d) If the preceding winter did not experience excessive snowfall, what is the probability that the subsequent winter will not suffer excessive snowfall? In other words, determine $P(\overline{E}_2|\overline{E}_1)$.

Hint: Start out with the following relationship:

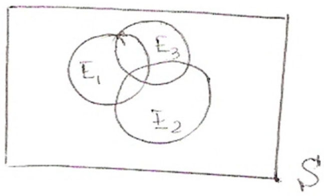
$$P(\overline{E}_1 \cup \overline{E}_2) = 1 - P(\overline{\overline{E}_1 \cup \overline{E}_2}) = 1 - P(E_1 E_2)$$

Note that notations for complement and intersection of events are as given in your text book. That is, $\overline{A} = A'$ and $AB = A \cap B$

a) $P(E_1) = 0.1$, $P(E_2) = 0.1$

$P(E_2/E_1) = 0.40$ & $P(E_3/E_1E_2) = 0.2$

$P(E_3/E_2) = 0.40$



b) $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1E_2)$

$= 0.1 + 0.1 - P(E_2/E_1)P(E_1) = 0.2 - 0.4 \times 0.1 = 0.16$

c) $P(E_1E_2E_3) = P(E_3/E_1E_2)P(E_1E_2) = 0.2 \times P(E_2/E_1)P(E_1)$

$= 0.2 \times 0.4 \times 0.1 = 0.008$.

d) $P(\overline{E}_2/\overline{E}_1) \Rightarrow$ $P(\overline{E}_1 \cup \overline{E}_2) = P(\overline{E}_1) + P(\overline{E}_2) - P(\overline{E}_1\overline{E}_2)$

$= 0.9 + 0.9 - P(\overline{E}_2/\overline{E}_1)P(\overline{E}_1)$

$P(\overline{E}_1 \cup \overline{E}_2) = 1 - P(E_1E_2) = 1 - P(E_2/E_1)P(E_1) = 1 - 0.1 \times 0.4 = 0.$

Thus, $0.96 = 1.8 - P(\overline{E}_2/\overline{E}_1)(1 - 0.1)$

$P(\overline{E}_2/\overline{E}_1) = \dfrac{1.8 - 0.96}{0.9} = 0.9333$

4. Before the design of a tunnel through a rocky region, geological exploration was conducted to investigate the joints and potential slip surfaces that exist in the rock strata. For economic reasons, only portions of the strata are explored. In addition the measurements recorded by the instruments are not perfectly reliable. Thus the geologist can only conclude that the condition of the rock may be either highly fissured (H), medium fissured (M), or slightly fissured (L) with relative likelihoods of 1 : 2 : 7. Based on this information, the engineer designs the tunnel and estimates that if the rock condition is L, the reliability of the proposed design is 95 %. However, if it turns out that the rock condition is M, the probability of failure will be doubled; similarly, if the rock condition is H, the probability of failure is 8 times that for condition L.

a) What is the expected reliability of the proposed tunnel design ?

b) A more reliable device is subsequently used to improve the prediction of rock condition. Its results indicate that a highly fissured condition for the rock around the tunnel is practically impossible, but it cannot give better information on the relative likelihood between rock conditions M and L. In light of this information, what would be the revised reliability of the proposed tunnel design?

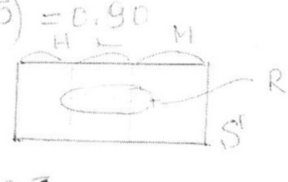
c) If the tunnel collapsed, what should be the updated probabilities of M and L?

Let R be the event that design is reliable.

Given: $P(H) = 1/10$, $P(M) = 2/10$, $P(L) = 7/10$

$$P(R/L) = 0.95, \quad P(R/M) = 1 - 2(1-0.95) = 0.90$$

$$P(R/H) = 1 - 0.05 \times 8 = 0.60$$



a) $$P(R) = 0.95 \times 0.7 + 0.90 \times 0.2 + 0.60 \times 0.1$$
$$= 0.905$$

b) $\overset{0}{\cancel{1}} : 2 : 7 \Rightarrow P(M) = 2/9, \quad P(L) = 7/9$

$$P(R) = 0.90 \times 2/9 + 0.95 \times 7/9 = 0.9389$$

c) $$P(M/\bar{R}) = \frac{P(\bar{R}/M)\,P(M)}{P(\bar{R})} = \frac{(1-0.90) \times 2/9}{1-0.9389} = 0.3636$$

$$P(L/\bar{R}) = \frac{P(\bar{R}/L)\,P(L)}{P(\bar{R})} = \frac{(1-0.95) \times 7/9}{1-0.9389} = 0.6364$$