CE382 RECITATION FOR MIDTERM 1

Q1) A normal density concrete has a compressive strength of 50 MPa. A compressive stress is applied to the concrete. This stress is increased in value from zero to 40 MPa and is then reduced to 15 MPa.

(i) Calculate the compressive strain in concrete at this point in time.

(ii) The stress is then increased to 45 MPa. Calculate the compressive strain at this point in time.

(iii) Finally the stress is reduced to zero. Calculate the residual compressive strain that remains in the concrete after the compressive stress has been removed.

Use Hognestad stress-strain model in your calculations and assume linear elastic unloading-reloading rules with initial elastic stiffness before peak strength.

$f_c' = 50\,MPa$ Assume $\epsilon_0 = 0.002$ $E_c = \dfrac{2 f_c'}{\epsilon_0} = 50\,000\ MPa$

Hognestad Parabola : $\sigma = f_c' \left\{ \dfrac{2\epsilon}{\epsilon_0} - \left(\dfrac{\epsilon}{\epsilon_0}\right)^2 \right\} \Rightarrow$ Solve for ϵ $\epsilon = \epsilon_0 \left(1 - \sqrt{1 - \dfrac{\sigma}{f_c'}}\right)$

i) $0 \rightarrow 40MPa \rightarrow 15MPa$

$\epsilon_{40} = 0.002 * \left(1 - \sqrt{1 - \dfrac{40}{50}}\right) = 0.0011$

Elastic unloading between 40 and 15MPa.

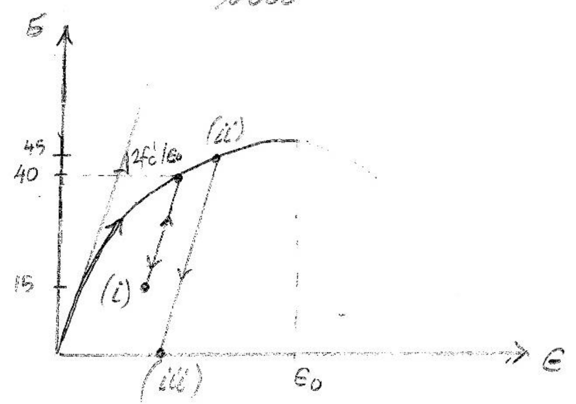
$\epsilon_{15} = 0.0011 - \dfrac{40-15}{50\,000} =$ 0.0006

ii) $15 \rightarrow 45\,MPa$ (Back on Hognestad)

$\epsilon_{45} = 0.002 * \left(1 - \sqrt{1 - \dfrac{45}{50}}\right) = 0.0014$

iii) $45MPa \rightarrow 0\ MPa$ (Residual strain)

$\epsilon_r = 0.0014 - \dfrac{45}{50000} = 0.0005$



Q2) Determine the axial load capacity of the given double spiral rectangular column section

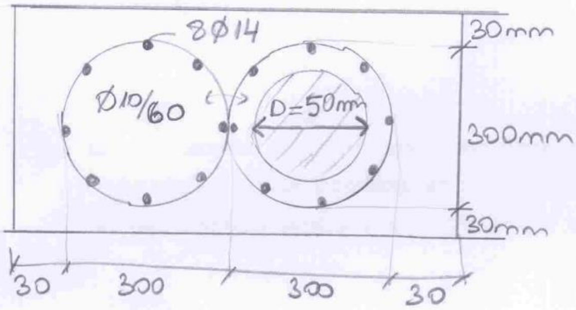
a) For the first peak, N_{01}

b) For the second peak, N_{02}

Consider, $f_{cc} = 0.85 f_c + 5\sigma_2$

Remember, $\sigma = pr/t$ for thin walled cylinders

Given, $f_c = 25$ MPa, $f_y = 420$ MPa, $f_{yw} = 420$ MPa



$* \; A_c = 660\times360 - \frac{\pi\times\cancel{80}^2}{4}$ =660*360-π*50^2/4

$\boxed{A_c = \cancel{232576}\ mm^2}$ =235637.5 mm^2

$* \; A_{ck} = 2*\pi*\frac{300^2}{4} - \pi*\frac{50^2}{4}$

$\boxed{A_{ck} = 139337.5\ mm^2}$

$* \; A_{st} = 16*\pi\times\frac{14^2}{4} \Rightarrow \boxed{A_{st} = 2461.76\ mm^2}$

a) First Peak;

$N_{01} = 0.85 f_{ck} A_c + A_{st} f_{yk}$

$N_{01} = (0.85\times25\times\cancel{232576} + 420\times2461.76)*10^{-3}$

$N_{01} = \cancel{5976.2}$ kN =6041.2 kN

b) Second Peak;

$N_{02} = A_{ck}\cdot f_{cc} + A_{st}*f_{yk}$; $f_{cc} = 0.85 f_{ck} + 5\sigma_2$; $A_s = 78.5\ mm^2$

$* \; f_{cc} = 0.85*25 + 5\times\underbrace{\frac{2*78.5*420}{300*60}}_{3.66\ MPa} \Rightarrow f_{cc} = 39.57\ MPa$

$N_{02} = (39.57*139337.5 + 420*2461.76)*10^{-3}$

$* \; \sigma_2 = \frac{2A_s f_{ywk}}{D*s} = \frac{2\times78.5\times420}{300\times60} = 3.66\ MPa$

$N_{02} = 6547$ kN

$* \; N_{01} = 6083$ kN

$* \; N_{02} = 6625$ kN

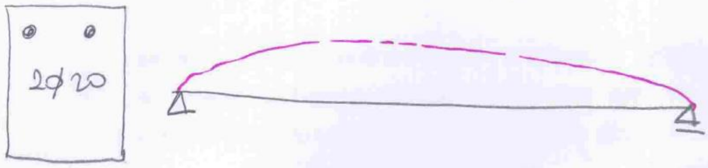
"No hole case"

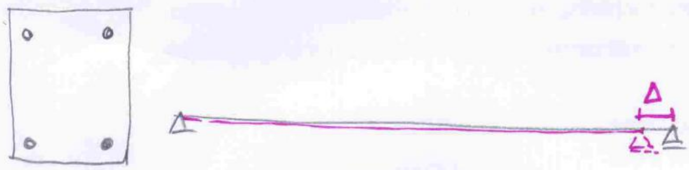
$\frac{N_{02}}{N_{01}} = \frac{6625}{6083} = 1.089$ $\boxed{8.9\%}$

$\frac{N_{02}}{N_{01}} = \frac{6547}{5976.2} = 1.096$ $\boxed{9.6\%}$

"With hole case" D=50mm

Q3) For four simple beams given below, sketch the deflected shapes caused by shrinkage alone (no other loads)



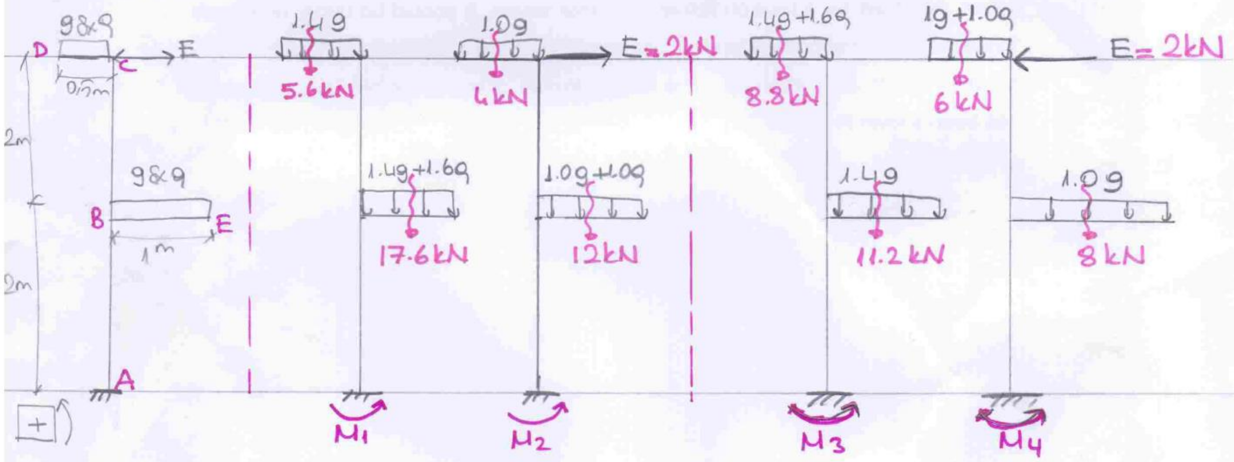


No deflection, only shortening!

$\Delta' > \Delta$

No deflection, only shortening!

Q4) Free standing cantilever column AC has two extensions BE and CD which carry uniformly distributed dead and live loads $g = 8$ kN/m and $q = 4$ kN/m, respectively. The earthquake load $E = 2.0$ kN is a reversed (alternating sense) cyclic load. Determine the maximum moment M_{Ad} (at section A) to be used in the design of this column which is known to be short column. Consider all necessary live load arrangements and ($F_{d1} = 1.4G + 1.6Q$) and ($F_{d2} = 1.0G + 1.0Q + 1.0E$) load combinations



$\boxed{+}$

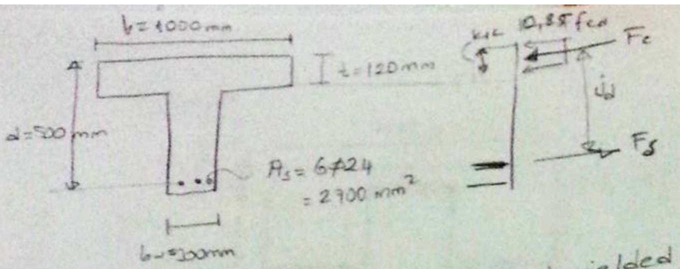
* $M_1 - 17.6 * 0.5 + 5.6 * 0.25 = 0 \Rightarrow M_1 = 7.4$ kN.m

* $\boxed{M_2 - 12 \times 0.5 - 2 \times 4 + 4 \times 0.25 = 0 \Rightarrow M_2 = 13 \text{ kN.m}}$ ✓

* $11.2 \times 0.5 - M_3 - 8.8 * 0.25 = 0 \Rightarrow M_3 = +3.4$ kN.m

* $8 * 0.5 - M_4 - 2 \times 4 - 6 * 0.25 = 0 \Rightarrow M_4 = -5.5$ kN.m

$M_{Ad} = 13$ kN.m



C20 → fcd = 13MPa
S420 → fyd = 365 MPa
→ Mr = ?

Assume $k_1c = t = 120mm$ & steel yielded

$F_c = 0{,}85\, f_{cd} \cdot b \cdot k_1 \cdot c = 0{,}85 \times 13 \times 1000 \times 120 = 1326\,kN$

$F_s = A_s \cdot f_{yd} = 2700 \times 365 = 985\,kN$

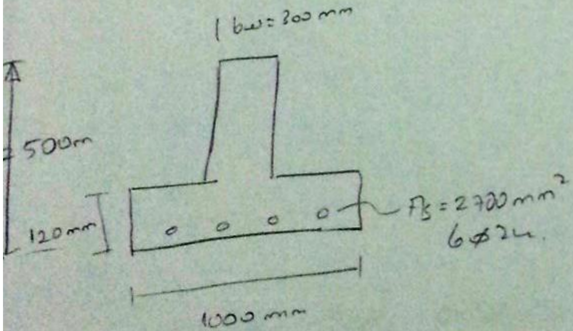
$F_c > F_s \Rightarrow k_1c < t$

$\Sigma N = 0; \quad -F_c + F_s = 0 \qquad 0{,}85 \cdot f_{cd} \cdot b \cdot k_1 c = A_s \cdot f_{yd}$

$k_1c = \dfrac{A_s \cdot f_{yd}}{0{,}85\, f_{cd} \cdot b \cdot k_1} = 89\,mm$ // $\qquad c = \dfrac{89}{0{,}85}$

$\varepsilon_s = 0{,}003 \cdot \dfrac{d-c}{c} = 0{,}01133 > \varepsilon_{yd} = 0{,}001825$

$M_r = A_s \cdot f_{yd} \cdot jd = 2700 \times 365 \times \left(500 - \dfrac{89}{2}\right) = 449\,kNm$ //



C20 → fcd = 13MPa
S420 → fyd = 365 MPa
Mr = ?

Assume ε_s yielded; $\quad \varepsilon_s = 0{,}003 \cdot \left(\dfrac{d-c}{c}\right)$

$F_c = 0{,}85 \cdot f_{cd} \cdot b_w \cdot k_1 c$

$F_s = A_s \cdot f_{yd} = 2700 \times 365$

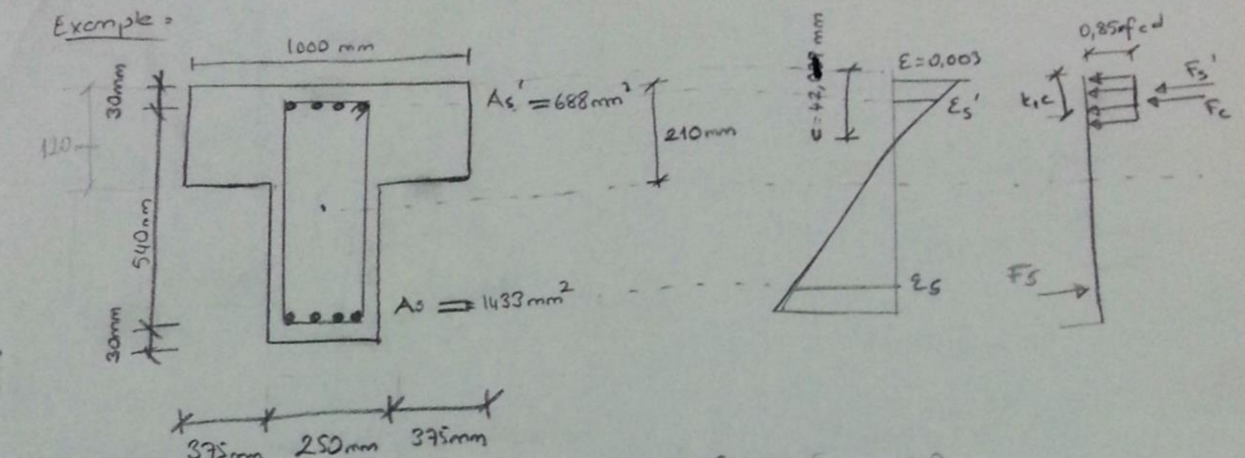
$\Sigma N = 0; \quad -F_c + F_s = 0$

$k_1c = \dfrac{A_s \cdot f_{yd}}{0{,}85 \times f_{cd} \times b_i} = \dfrac{2700 \times 365}{0{,}85 \times 13 \times 300}$

$k_1c = 297{,}29\,mm \Rightarrow c = 349{,}75\,mm$

$\varepsilon_s = 0{,}003 \cdot \dfrac{500 - 349{,}75}{349{,}75} = 0{,}0013$

Example :



$M_r = ?$

$E_s = 200 GPa$

$f_{ck} = 20\ N/mm^2 \qquad f_{yk} = 420\ N/mm^2$

$\varepsilon_{sy} = \frac{420}{200000} = 0,0021$

$$\varepsilon_s' = \varepsilon_{co} \cdot \left(\frac{c-30}{c}\right) = 0,003 \cdot \left(\frac{c-30}{c}\right)$$

$$\varepsilon_s = \varepsilon_{co} \cdot \left(\frac{d-c}{c}\right) = 0,003 \cdot \left(\frac{570-c}{c}\right)$$

First assume that: $c = 120$ mm

$\varepsilon_s' = 0,003\left(\frac{120-30}{120}\right) = 0,0022 > \varepsilon_{sy} \rightarrow F_s' = A_s' \cdot f_{yd} = 688 \cdot \frac{420}{1,15} = 251269,6\ N$

$\varepsilon_s = 0,003\left(\frac{570-120}{120}\right) = 0,0112 \rightarrow F_s = A_s \cdot f_{yd} = 1433 \cdot \frac{420}{1,15} = 523356,52\ N$

$$F_c = 0,85 f_{cd} \cdot k_1 c \cdot b_w = 0,85 \cdot \frac{20}{1,5} \cdot 0,85 \cdot 120 \cdot 1000$$

$$F_c = 1156000$$

$\Sigma N = 0; \quad F_s - F_s' - F_c = 0$

$\qquad F_s < F_c + F_s'$

so ; $\underline{c < 120}$

ε_s

$F_s' = A_s' \cdot f_{yd} = 251269{,}6\ N$

$F_s = A_s \cdot f_{yd} = 523356{,}52\ kN$

$F_c = 0{,}85 \times 13{,}3 \times 1000 \times k_1 c$

$\Sigma N = 0;$

$k_1 c = \dfrac{F_s - F_s'}{0{,}85 \times 13{,}3 \times 1000}$

$k_1 c = \dfrac{272086{,}92}{11305} = 24{,}0678$

$c = 28{,}32$ < smaller cover

$\varepsilon_s' < \varepsilon_{syd}$

$\varepsilon_s' = 0{,}003 \cdot \left(\dfrac{c-30}{c}\right)$

So; Assume that $\varepsilon_s' < \varepsilon_{syd}$

$F_s' = A_s' \cdot (\varepsilon_s \cdot E_s) = 688 \cdot \left(0{,}003 \left(\dfrac{c-30}{c}\right)\right) \cdot 200000 = 412800 \left(\dfrac{c-30}{c}\right)$

$F_s = A_s \cdot f_{yd} = 1433 \times \dfrac{420}{1{,}15} = 523356{,}52\ N$

$F_c = 0{,}85 \times 13{,}3 \times 1000 \times k_1 C = 11305\ k_1 c$

$11305\ k_1 c + 412800 \left(\dfrac{c-30}{c}\right) = 523356{,}52$

$c = 42{,}11\ mm$

Check $\varepsilon_s' = \left(\dfrac{42{,}11 - 30}{42{,}11}\right) \cdot 0{,}003 = 8{,}63e{-4} < \varepsilon_{syd}$. Our assumption is true.

$F_s' = A_s' (\varepsilon_s' \cdot E_s) = 8{,}63e{-4} \times 688 \cdot 200000 = 118713{,}1\ N$

$F_s = 523356{,}52\ N$

$F_c = 0{,}85 \times 13{,}3 \times 1000 \times 0{,}85 \times 42{,}11 = 404645{,}52\ N$

$M_r = F_c \cdot \left(210 - \dfrac{k_1 c}{2}\right) + F_s' \cdot (210 - 30) + F_s \cdot (570 - 210)$

$M_r = 77733719{,}01 + 21368358 + 188408347$

$M_r = 287{,}51\ kNm$ //