SYSTEM OF LINEAR EQUATIONS

Direct Methods

Naive Gauss Elimination M.

- Using elementary row operations convert $[A|b]$ into $[\,U\,|\,b^*]$
- U is an upper triangular matrix
- Solve

Gauss Elimination M.

- Partial pivoting
- Using elementary row operations convert $[A|b]$ into $[\,U\,|\,b^*]$
- U is an upper triangular matrix
- Solve

Simpler methods

- Graphical $(n=2)$
- Cramer's $(n=3)$
- Elimination of unknowns
- Matrix inversion
  $\underline{X} = A^{-1}.\underline{b}$

Gauss-Jordan Elimination M.

- Partial pivoting
- Using elementary row operations convert $[A|b]$ into $[\,I\,|\,b^*]$
- U is an upper triangular matrix whose diagonal elements are all 1
- Solve

LU Decomposition M.

- Decompose A into LU where L and U are lower and upper triangular matrices, respectively.
- Doolittle's Algorithm for decomposition: diagonal elements of L are all 1
- $A\underline{X} = \underline{b}$
  $A = LU$
  $LU\underline{X} = \underline{b}$
  $U\underline{X} = \underline{Y}$
  $L\underline{Y} = \underline{b}$

Indirect Methods

Gauss Jacobi M.

- Apply elementary row operations to obtain all diagonal elements of A as 1, call that A*
- $A\underline{X} = \underline{b}$
  $A^* = I + B$
  $(I+B)\underline{X} = \underline{b}^*$
  $I\underline{X} + \underline{BX} = \underline{b}^*$
  $I\underline{X} = -\underline{BX} + \underline{b}^*$
  $\underline{X} = -\underline{BX} + \underline{b}^*$
  $\underline{X}^{i+1} = -\underline{BX}^{i} + \underline{b}^*$
- or

$$x_i^{k+1} = \frac{b_i - \sum_{\substack{j=1 \\ i \neq j}}^{n} a_{ij} x_j^k}{a_{ii}}$$

$i = 1,2,\ldots,n$

Gauss Seidel M.

- Similar to Gauss Jacobi but use updated values of currunt iteration.

$$x_i^{k+1} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{k+1} - \sum_{j=i+1}^{n} a_{ij} x_j^k}{a_{ii}}$$

$i = 1,2,\ldots,n$