MIDDLE EAST TECHNICAL UNIVERSITY
FACULTY OF ENGINEERING
CE 204 UNCERTAINTY and DATA ANALYSIS
Spring Semester 2014-15

Homework 2- Date Due: April 27th, 2015 Monday till 12.00pm (no late submissions will be accepted).

IMPORTANT NOTICE:

- ***Please submit the homework to K2-207 room at the structural engineering lab. building.***
- ***You are allowed to collaborate with other students (or ask questions to your assistants/ instructors) on homework provided that you stay away from plagiarizing (according to dictionaries "to plagiarize" means to steal and pass off ideas and/or words/ solutions of another as one's own without citing the source). That is, collaboration is accepted if you write and give your own solutions. If you are caught on plagiarizing or cheating by handing in "too similar" homework, you will be graded by zero on this homework.***
- ***You are to hand in answers to all questions but you will be graded on only one randomly chosen answer.***

Q1) The recurrence time (T) of category 5 hurricanes over a region is modeled using log-normal distribution with mean 15 years and standard deviation of 8 years.
a) Find parameters of log-normally distributed recurrence time.
b) What is the probability that a category 5 hurricane will occur within 5 years?
c) Assume the last sever earthquake in the region took place 100 years ago. What is the probability that a category 5 hurricane will hit this region during next year?

Q2) The number of people shopping at mall Cepa on weekends is modeled by random variable X which has normal distribution with mean 15,000 and standard deviation 3000. To satisfy the parking lot construction design criteria's, it is assumed that one in two people come to the mall with their own car (i.e. number of cars visiting Cepa, Y=X/2). Given there are only 9000 parking lots available
a) What are the chances that people will wait for parking lots any given weekend (i.e. Y > 9000)?
b) Plot the probability distribution function (PDF) and cumulative distribution function (CDF) of Y.
c) What is the difference between mean, median, and mode for random variable Y? Give their definitions.
d) Show the area over PDF that represents probability that 70% of the time parking lots will not be sufficient for the incoming customers.

Q3) Three loads, X, Y, Z are applied to a cantilever beam, A-B, as shown in the figure below. The following information is given with respect to the loads.
Load X: Mean value: 2 kN; and coefficient of variation, $\delta_X = 0.25$.
Load Y: Mean value: 3 kN; and coefficient of variation, $\delta_Y = 0.20$.
Load Z: Mean value: 4 kN; and coefficient of variation, $\delta_X = 0.15$.
All three loads are correlated with each other with a correlation coefficient of $\rho = 0.50$ (i.e. $\rho_{xy} = \rho_{xz} = \rho_{yz} = 0.50$)
a) Compute the expected value and the variance of the bending moment (M_A) at the fixed end (i.e. at A: $M_A = 2X+4Y+6Z$).
b) Assume that the bending moment created at the fixed end by these three loads is lognormally distributed with the mean value and variance as computed in part (a). If the bending moment capacity of this beam is either 50 kN-m or 45 kN-m, with probabilities of 0.3 and 0.7 respectively, compute the failure probability of this beam in the flexural failure mode.

Q4) Drinking water samples offered in a city should meet the criteria for PH range of water to be between 6.5 and 8.0. Past experience tells that the PH of water samples originating from reservoir A has log-normal distribution with median 7.2 and mean 7.211. Based on these statistics
a) Find parameters of this log-normal distribution.

b) What are the chances that any water sample will meet the drinking water criteria?
c) The city regulations state that in order a reservoir to be considered safe for drinking water supply 10 samples need to be tested and not more than one sample should fail the test. What are the chances that this reservoir will satisfy the regulations?
d) What are the differences between Binominal and Poisson distributions? Particularly the difference in the type of events that they are used.
e) What is the difference between Poisson and Exponential distributions?

Q5) The joint probability density function of the random variables X and Y is modelled as

$$f_{XY}(x,y) = ax^2 , \quad 0 \le x \le 1 \text{ and } 0 \le y \le x$$

$$f_{XY}(x,y) = 0 , \quad \text{elsewhere.}$$

a) Find the constant "a" so that $f_{XY}(x,y)$ is a proper joint probability density function.
b) Find the marginal probability density function of the random variable X.
c) Find the expected value and standard deviation of X.
d) If a random variable Z is given as $Z=X^2$, find the probability density function of Z.
e) Find the expected value and standard deviation of the random variable Z.
f) If the expected values of X and Z are 0.800 , 0.667 with standart deviations 0.163 and 0.333, respectively, determine the first order coefficient of variation of the random variable W defined as $W=Xe^Z$, assuming X and Z as statistically indepemdent variables.

Q6) Arrival of a bus that run between Kizilay and Cayyolu is modeled using poisson distribution. There are total 10 busses running over this route and each make 10 complete circles every day between 06.00am and 23.00pm (i.e. 100 rounds within 17 hours). Based on this:
a) What is the return period of a bus coming to a particular bus station.
b) Find the probability that someone will wait more than 10 minutes at a bus stop for the next bus.
c) What are the chances that no bus will arrive a particular bus stop for 30 minutes.

Q7) The size (in millimeter) of a crack on a structural weld is described by a random variable X with the following PDF:

$$f_x(x) = \begin{cases} x/8 & 0 < x \le 2 \\ 1/4 & 2 < x \le 5 \\ 0 & \text{elsewhere} \end{cases}$$

a) Sketch the PDF and CDF on a piece of graph paper.
b) Determine the mean crack size.
c) What is the probability that a crack will be smaller than 4 mm?
d) Determine the median crack size.
e) Suppose there are four cracks in the weld. What is the probability that only one of these four cracks larger than 4 mm?

Q8) The daily water levels (normalized to the respective full condition) of two reservoirs A and B are denoted by two random variables X and Y having the following joint PDF:

$$f(x,y) = (6/5)(x+y^2), \qquad 0 < x < 1 \text{ and } 0 < y < 1$$

a) Determine the marginal density function of the daily water level for reservoir A.
b) If reservoir A is half full on a given day, what is the probability that the water level will be more than half full?
c) Is there any statistical correlation between the water levels in the two reservoirs?