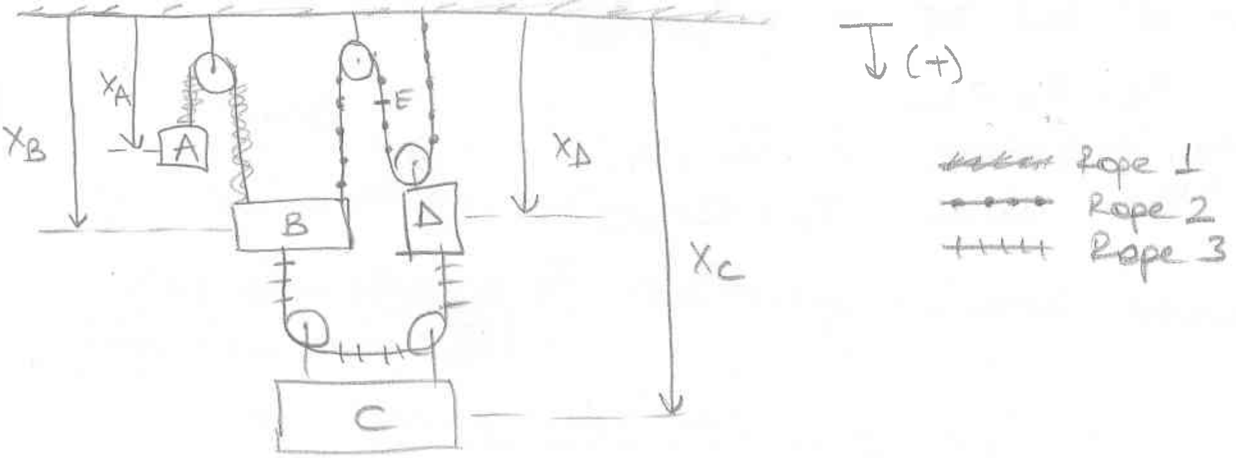
— 2013-2014 SPRING SEMESTER —
— AE 262 HW #1 SOLUTION —

Q.1) FBD of the system:



ⓐ – Wires are rods and inextensible. Rope 1,2,3 have constant lengths.

Length of Rope 1,

$$\vec{X}_A + \vec{X}_B = C_1$$

Taking derivative,

$$\vec{V}_A + \vec{V}_B = 0$$

Once more,

$$\vec{a}_A + \vec{a}_B = 0$$

$$\boxed{\vec{a}_A = -\vec{a}_B}$$

Length of Rope 2,

$$\vec{X}_B + 2\vec{X}_\Delta = C_2.$$

Taking derivative,

$$\vec{V}_B + 2\vec{V}_\Delta = 0.$$

Once more,

$$\vec{a}_B + 2\vec{a}_\Delta = 0.$$

$$\boxed{\vec{a}_\Delta = -\vec{a}_B/2}$$

Length of Rope 3,

$$(\vec{X}_C - \vec{X}_B) + (\vec{X}_C - \vec{X}_\Delta) = C_3$$

$$2\vec{X}_C - \vec{X}_B - \vec{X}_\Delta = C_3$$

Taking derivative,

$$2\vec{V}_C - \vec{V}_B - \vec{V}_\Delta = 0.$$

Once more,

$$2\vec{a}_C - \vec{a}_B - \vec{a}_\Delta = 0.$$

$$\boxed{\vec{a}_C = \vec{a}_B/4}$$

– Given that at $t = 5\,s$, $V_{A/\Delta} = 2.4\ m/s$

$$|\vec{V}_{A/\Delta}| = |\vec{a}_{A/\Delta}| \cdot t$$

$$|\vec{a}_{A/\Delta}| = \frac{|\vec{V}_{A/\Delta}|}{t} = \frac{2.4\,m/s}{5\,s} = 0.48\ m/s^2$$

Noting that, $\vec{V}_{A/\Delta} = \vec{V}_A - \vec{V}_\Delta$

$$\vec{a}_{A/\Delta} = \vec{a}_A - \vec{a}_\Delta = -\vec{a}_B - \left(-\frac{\vec{a}_B}{2}\right) = -\frac{\vec{a}_B}{2}$$

$$|\vec{a}_{A/\Delta}| = \left|-\frac{\vec{a}_B}{2}\right| = 0.48\ m/s^2 \longrightarrow |\vec{a}_B| = 0.96\ m/s^2$$

Therefore, $\vec{a}_C = \frac{\vec{a}_B}{4} = \frac{0.96\,m/s^2}{4} = 0.24\ m/s^2\ (\downarrow)$

$$\boxed{\vec{a}_C = 0.24\ m/s^2\ (\downarrow)}$$

Moreover,

$\vec{a_B} = 0.96\, m/s^2\ (\downarrow)$, $\vec{a_B} = 0.48\, m/s^2\ (\uparrow)$, $\vec{a_A} = 0.96\, m/s^2\ (\uparrow)$

(b) To obtain $\vec{a_E}$,

Length of Rod BE is determined.

$$\vec{X_B} + \vec{X_E} = C_4$$

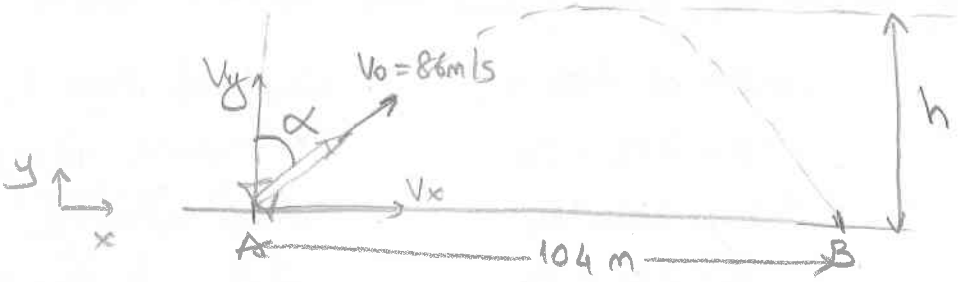
Taking derivative, $\vec{V_B} + \vec{V_E} = 0$

once more, $\vec{a_B} + \vec{a_E} = 0$

$$\vec{a_E} = -\vec{a_B} = 0.96\, m/s^2\ (\uparrow)$$

$$\boxed{a_E = 0.96\, m/s^2\ (\uparrow)}$$

Q.2) Schematic of the problem



(a)

- Noticing that, $V_x = V_0 \sin\alpha$ and $V_y = V_0 \cos\alpha - gt$

Horizontal motion: $X = (V_0 \sin\alpha)t$

Vertical motion: $Y = (V_0 \cos\alpha)t - \frac{gt^2}{2}$

- at point B, $Y_B = 0$, and $X_B = 104\, m$.

$X_B = V_0 \sin\alpha\, t_B \longrightarrow t_B = \frac{X_B}{V_0 \sin\alpha}$

$Y_B = V_0 \cos\alpha\, t_B - \frac{g t_B^2}{2}$, Substituting t_B obtained above into Y_B formula

$$Y_B = V_0 \cos\alpha \left(\frac{X_B}{V_0 \sin\alpha}\right) - \frac{g}{2}\left(\frac{X_B}{V_0 \sin\alpha}\right)^2 = 0$$

$$\frac{\cos\alpha}{\sin\alpha} X_B = \frac{g}{2} \frac{X_B^2}{V_0^2 \sin^2\alpha} \longrightarrow 2\cos\alpha \sin\alpha = \frac{g X_B}{V_0^2}$$

Recalling that, $2\sin\alpha\cos\alpha = \sin 2\alpha$

Therefore, $\sin 2\alpha = \dfrac{g\, x_B}{V_0^2} = \dfrac{(9.81\ m/s^2)(104\ m)}{(86\ m/s)^2} = 0.138$

$\sin 2\alpha = 0.138$

$\boxed{\alpha \cong 3.966^\circ}$

(b) at h, $V_y = 0$.

$V_y = V_0 \cos\alpha - g t_1 = 0$ at h.

$t_1 = \dfrac{V_0 \cos\alpha}{g}$, Substituting numerical values,

$t_1 = \dfrac{(86\ m/s)\cos(3.966^\circ)}{9.81\ m/s^2} = 8.745\ s$

Then, $h = V_0 \cos\alpha\, t_1 - \dfrac{g t_1^2}{2}$

$= (86\ m/s)\cos(3.966^\circ)(8.745\ s) - \dfrac{(9.81\ m/s^2)(8.745\ s)^2}{2}$

$\boxed{h = 375.2\ m}$

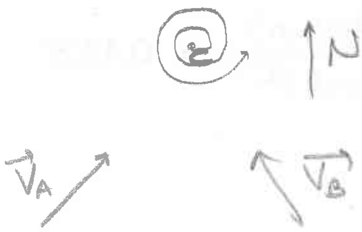
(c) Duration of flight can be obtained from,

$x_B = V_0 \sin\alpha\, t_B$

$t_B = \dfrac{x_B}{V_0 \sin\alpha} = \dfrac{104\ m}{(86\ m/s)\sin(3.964)} = 17.48\ s$

duration of flight $= \boxed{t_B = 17.48\ s}$

Q.3) Schematic of the problem



Given, $\vec{V}_{C/A} = 470$ km/h 

$\vec{V}_{C/B} = 520$ km/h 

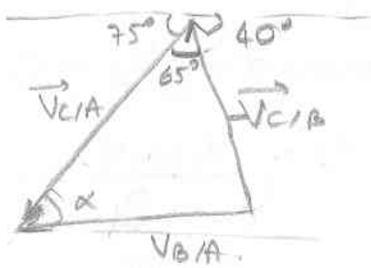
(a)

$$\vec{V}_{C/A} = \vec{V}_C - \vec{V}_A$$

$$\vec{V}_{C/B} = \vec{V}_C - \vec{V}_B$$

$$\vec{V}_{B/A} = \vec{V}_B - \vec{V}_A$$

$$\boxed{\vec{V}_{B/A} = -\vec{V}_{C/B} + \vec{V}_{C/A}}$$



Using Law of Cosines,

$$|\vec{V}_{B/A}|^2 = |\vec{V}_{C/A}|^2 + |-\vec{V}_{C/B}|^2 - 2\,|\vec{V}_{C/A}|\,|-\vec{V}_{C/B}| \cos 65°$$

$$= (470^2 + 520^2 - 2 \cdot 470 \cdot 520 \cdot \cos 65°)\ (\text{km/h})^2$$

$$= 284.72 \times 10^3\ (\text{km/h})^2$$

$$|\vec{V}_{B/A}| = 533.6 \text{ km/h}.$$

Using Law of Sines,

$$\frac{|\vec{V}_{B/A}|}{\sin 65°} = \frac{|-\vec{V}_{C/B}|}{\sin \alpha} \longrightarrow \sin \alpha = 0.883$$

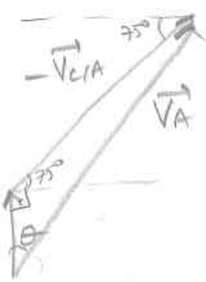
$$\alpha \cong 62.01°$$

$\beta = 75° - 62.01° = 12.99°$

$\Rightarrow$ $\boxed{\vec{V}_{B/A} = 533.6 \text{ km/h}}$ 

(b)

$$\vec{V}_A = \vec{V}_C - \vec{V}_{C/A}$$



Using Law of Cosines,

$$|\vec{V}_A|^2 = |-\vec{V}_{C/A}|^2 + |\vec{V}_C|^2 - 2\,|-\vec{V}_{C/A}|\,|\vec{V}_C| \cos 165°$$

$$|\vec{V}_A|^2 = (470^2 + 48^2 - 2 \cdot 470 \cdot 48 \cdot \cos 165°)\ (\text{km/h})^2$$

$$|\vec{V}_A| = 516.5 \text{ km/h}.$$

Using Law of Sines,

$$\frac{|\vec{V}_A|}{\sin 165^\circ} = \frac{|-\vec{V}_{C/A}|}{\sin \theta} \qquad \sin\theta = 0.235$$
$$\theta = 13.59^\circ$$

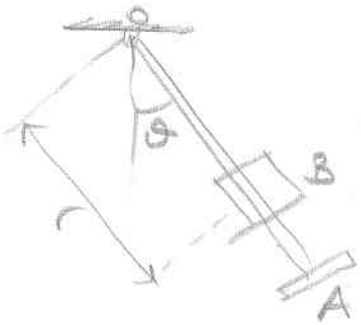
$$\boxed{\vec{V}_A = 516.5 \text{ km/h} \quad 13.59^\circ}$$

ⓒ $\vec{X}_{C/B} = \vec{V}_{C/B} \cdot t$

$$= 520 \text{ km/h} \cdot (15/60) \text{h} = 130 \text{ km/h} \quad 40^\circ$$

$$\boxed{\vec{X}_{C/B} = 130 \text{ km/h} \quad 40^\circ}$$

Q.4) Schematic of the problem



Given,

$$\theta = 0.5\, e^{-0.8t} \sin(3\pi t)$$
$$r = 1 + 2t - 6t^2 + 8t^3$$

ⓐ $\vec{V}_{collar} = \dot{r}\,\vec{e_r} + r\dot{\theta}\,\vec{e_\theta}$ (1)

$\dot{r} = 2 - 12t + 24t^2$ when $t = 0.5s$, $\dot{r} = 2$ m/s

$r = 1 + 2t - 6t^2 + 8t^3$ when $t = 0.5s$, $r = 1.5$ m

$\dot{\theta} = -0.4\, e^{-0.8t} \sin(3\pi t) + 1.5\pi\, e^{-0.8t} \cos(3\pi t)$ when $t = 0.5s$, $\dot{\theta} = 0.268$ rad/s

Substituting these into (1),

$$\vec{V}_{collar} = (2 \text{m/s})\,\vec{e_r} + (1.5)(0.268) \text{ m/s}\,\vec{e_\theta}$$

$$\boxed{\vec{V}_{collar} = (2 \text{m/s})\,\vec{e_r} + (0.402 \text{ m/s})\,\vec{e_\theta}}$$

ⓑ $\vec{a}_{collar} = (\ddot{r} - r\dot{\theta}^2)\,\vec{e_r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\vec{e_\theta}$ (2)

$\dot{r} = 2$m/s, $r = 1.5$m, $\dot{\theta} = 0.268$ rad/s as obtained in part a.

$\ddot{r} = -12 + 48t$ when $t = 0.5s$, $\ddot{r} = 12$ m/s^2

$$\ddot{\theta} = \left[0.32e^{-0.8t}\sin(3\pi t) - 1.2\pi e^{-0.8t}\cos(3\pi t)\right] + \left[-1.2\pi e^{-0.8t}\cos(3\pi t) + \left(-4.5\pi^2 e^{-0.8t}\right)\sin(3\pi t)\right]$$

When $t = 0.5s$, $\ddot{\theta} = 29.54$ rad/s.

Substituting the numerical values of $\ddot{r}, \dot{r}, r, \ddot{\theta}, \dot{\theta}$ into (2),

$$\vec{a}_{collar} = (12 - 1.5\times(0.268)^2)\vec{e_r} + (1.5\times 29.54 + 2\times 2\times 0.268)\vec{e_\theta}$$

$$\boxed{\vec{a}_{collar} = (11.892\ m/s^2)\vec{e_r} + (45.382\ m/s^2)\vec{e_\theta}}$$

(c) $\vec{a}_{collar/rod} = \ddot{r}\,\vec{e_r}$

$\ddot{r} = 12\ m/s^2$ as obtained in part b.

$$\boxed{\vec{a}_{collar/rod} = (12\ m/s^2)\vec{e_r}}$$