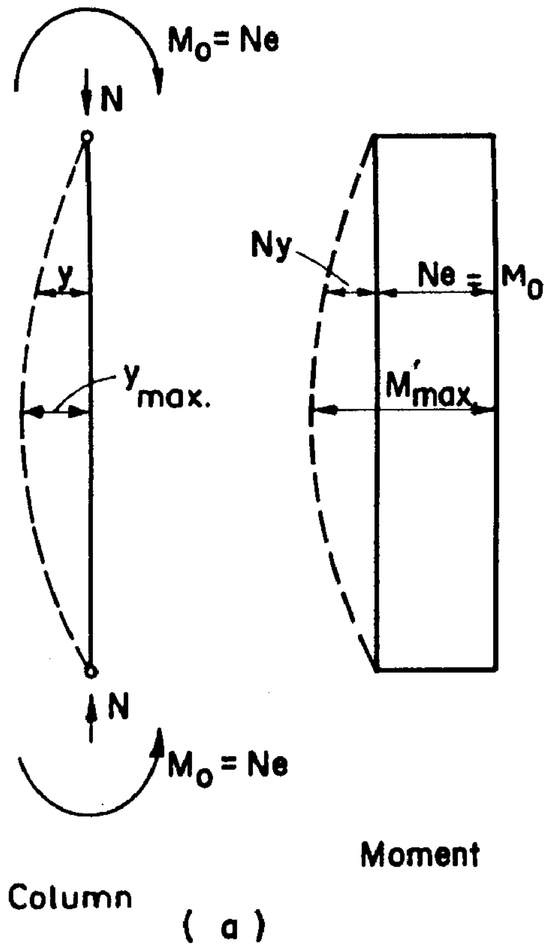
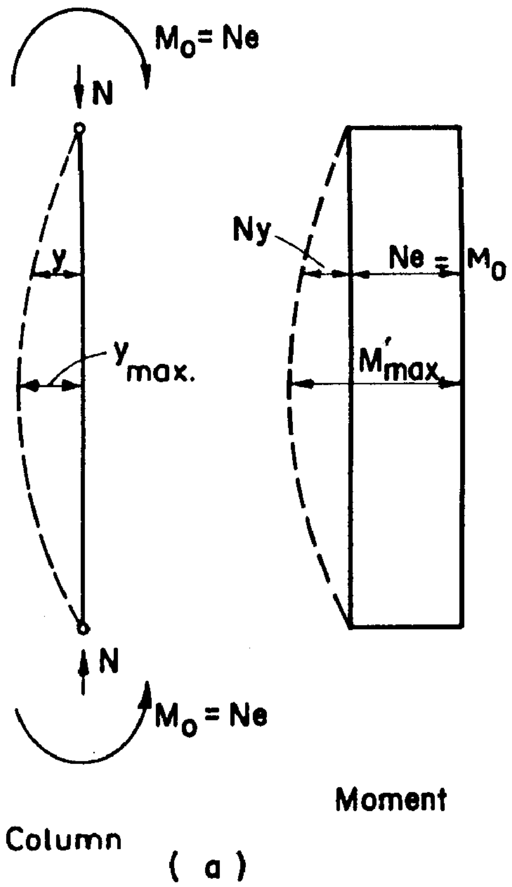
Deflected Shape of the Column



Single Curvature –

If there is no point of inflection between the ends of the column, the column is said to be in single curvature.

Deflected Shape of the Column

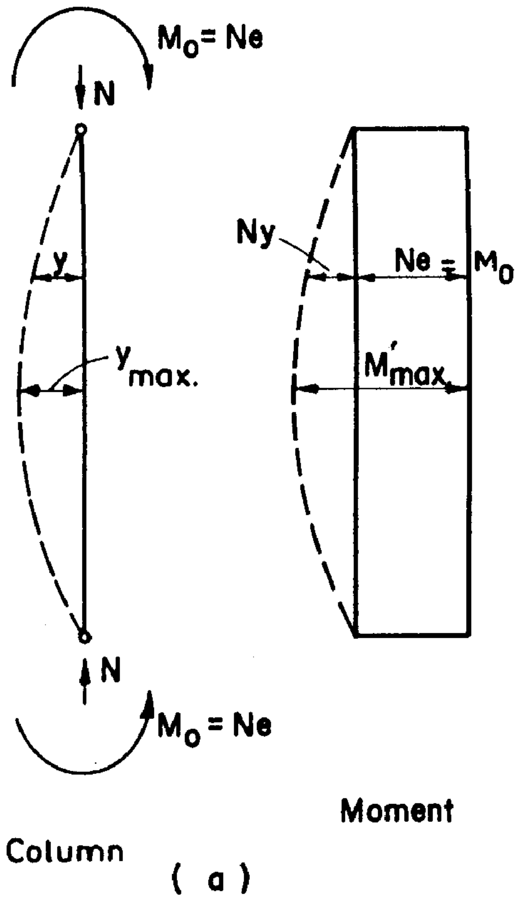


Single Curvature –

$$M'_d = M_0 + N \times y = N \times e + N \times y = N(e + y)$$

$$\text{maximum} \quad M'_d = N(e + y_{max})$$

Deflected Shape of the Column



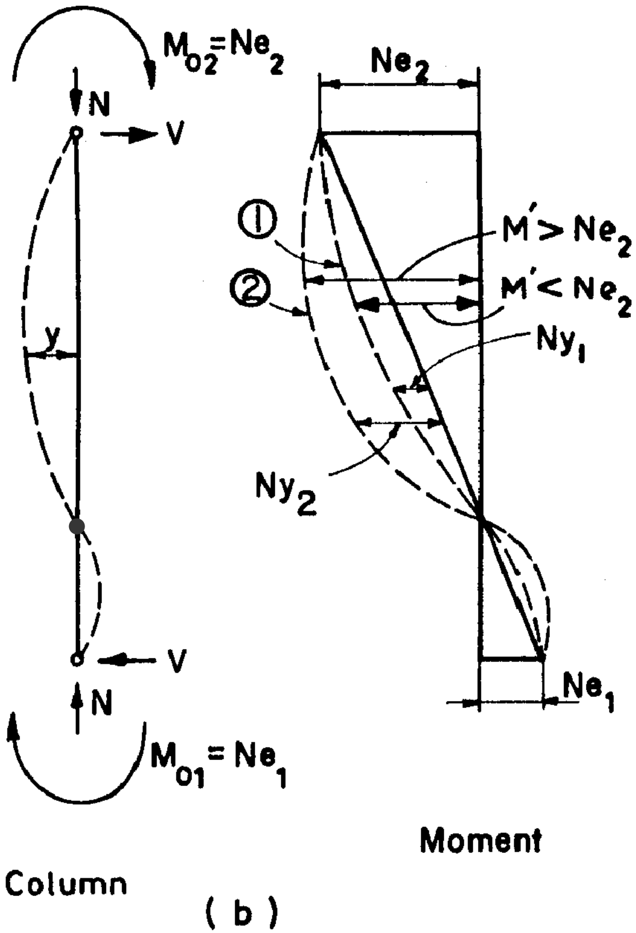
Single Curvature –

"y_{max}" depends on "l_k/i", i.e. the slenderness of the column.

As "l_k/i" ↓ "y_{max}" ↓

and column tends to act as a "SHORT COLUMN".

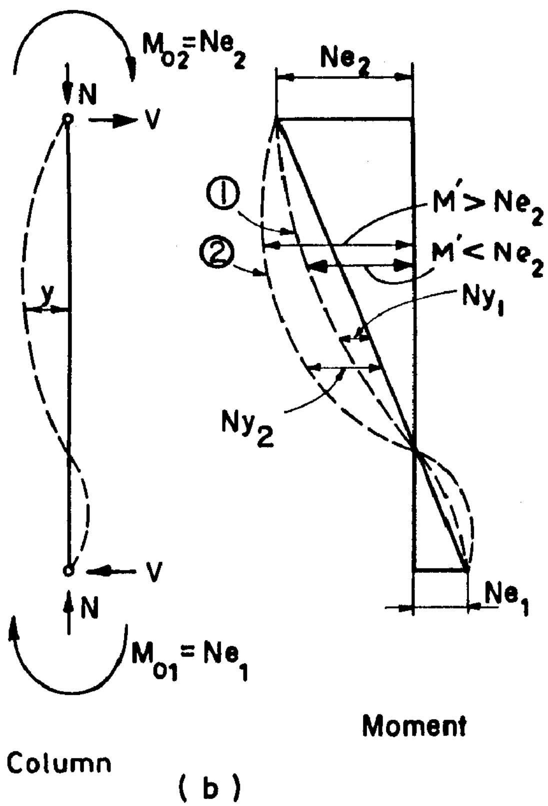
Deflected Shape of the Column



Double Curvature **–**

If there is a point of inflection between the ends of the column, the column is in <u>double curvature</u>.

Deflected Shape of the Column



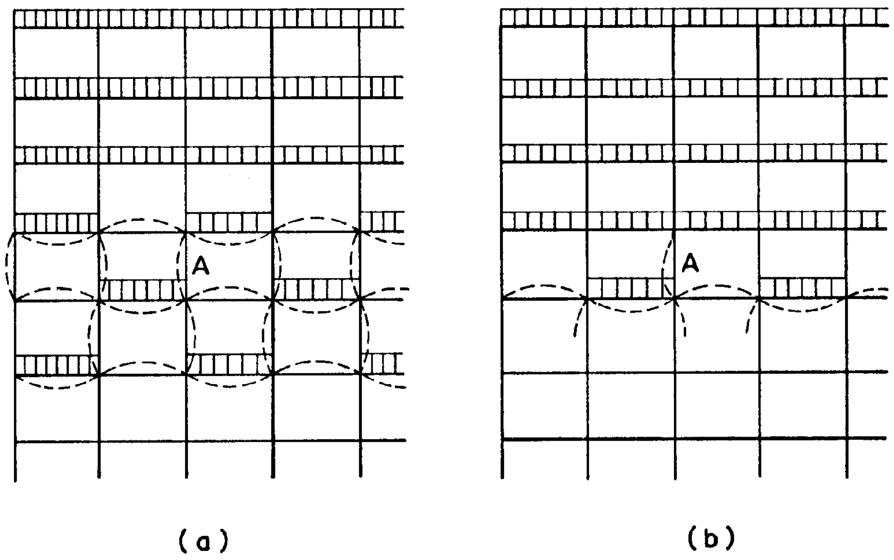
Double Curvature –

In this case, the maximum first order moments (N×e) are at the ends of the column.

- For columns with medium slenderness => $M' < N \times e_2$
- For very slender columns => $M' > N \times e_2$

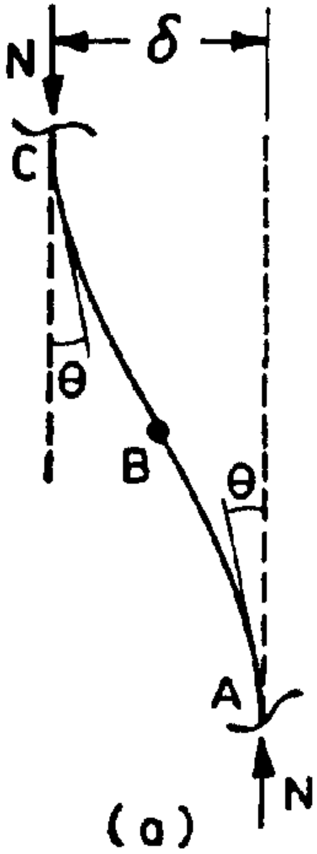
Deflected Shape of the Column

NON-SWAY FRAMES –



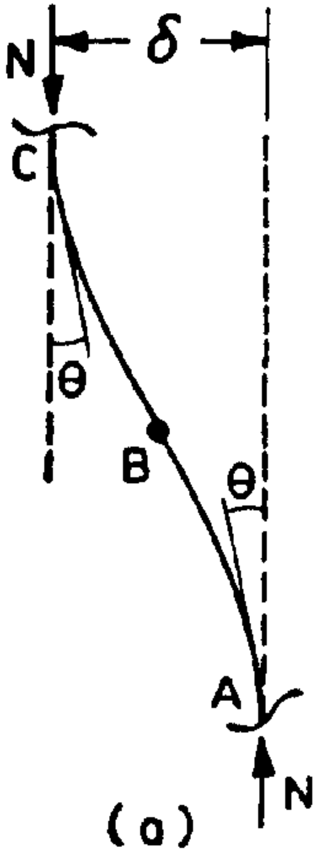
The deflected shape of the column depends on boundary conditions and arrangement of live loads. The same column can either be in single or double curvature.

Deflected Shape of the Column



*In case of **<u>sway frames</u>**, two ends of the column are displaced with respect to each other. All columns in such sway frames are in double curvature.*

Deflected Shape of the Column

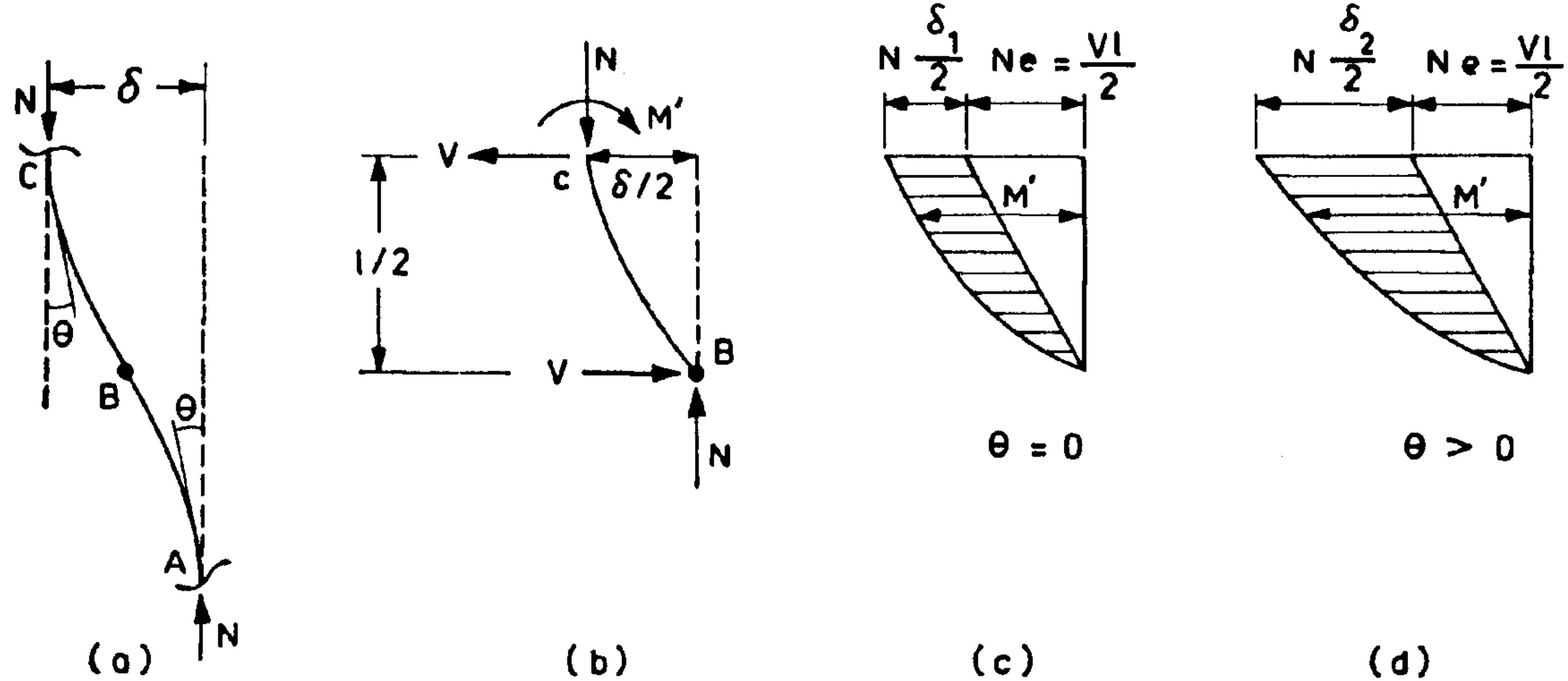


*In case of **sway frames**, depending on the relative stiffness of columns with respect to beams column ends may rotate more.*

$$\alpha = \frac{\sum \frac{I}{\ell}\,(columns)}{\sum \frac{I}{\ell}\,(\,floor members)}$$

As "α" $\downarrow$ "θ" $\downarrow$

Deflected Shape of the Column



More rotation => more displacement, δ

more δ *=> more* $\Delta M = N \times \delta / 2$

Deflected Shape of the Column

SWAY FRAMES –

1. *In flat plate or flat slab floor systems , floor member is the slab which is very flexible, or*
2. *In block joist floors with shallow, wide beams behave very similar to flat plates.*

very large second order moments will develop due to the rotation of the flexible floor members . These moments can lead to severe damage or even collapse of the building.

=> Provide reinforced concrete structural walls in both directions to take care of the lateral loads.

Deflected Shape of the Column

SWAY FRAMES –

In beam-column type of structural systems, beams in the frame direction are floor members. Such elements having higher stiffnesses than the flat plates or block joists provide superior control of the column end rotations.

Deflected Shape of the Column

SWAY FRAMES –

Sidesway of a frame can be prevented by introducing bracing elements. The most common types of bracings used in our structures are structural walls, cross-bracings and trusses.

Deflected Shape of the Column

SWAY FRAMES –

In sidesway permitted frames second order moments will increase the design moments no matter how small the slenderness ratio is. Since all the columns supporting a given floor will provide lateral resistance sidesway becomes a problem of the entire story rather than the individual column. Therefore, codes require the computation of second order moments by considering all the columns in that story.

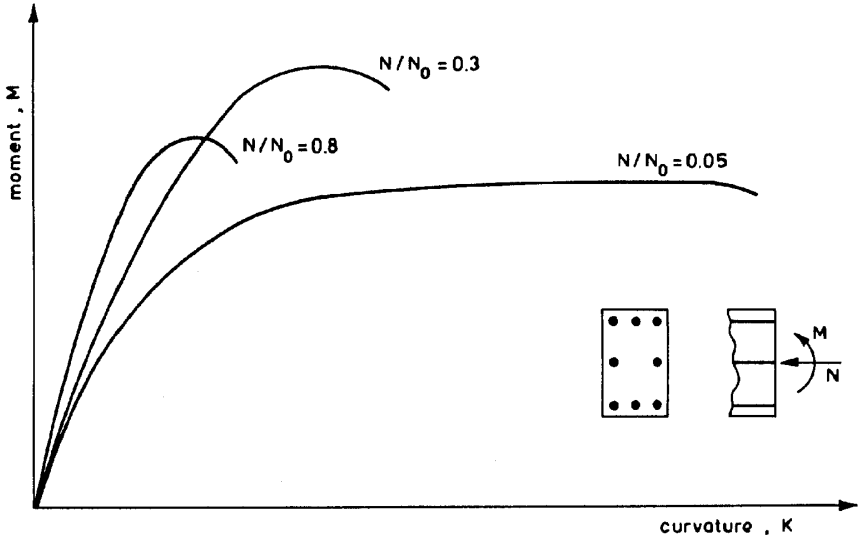
Deflected Shape of the Column

The displacement or deflection of a column which creates the second order moments depends mainly on the ***effective length*** *of the column and* ***flexural rigidity****.*

Flexural Rigidity

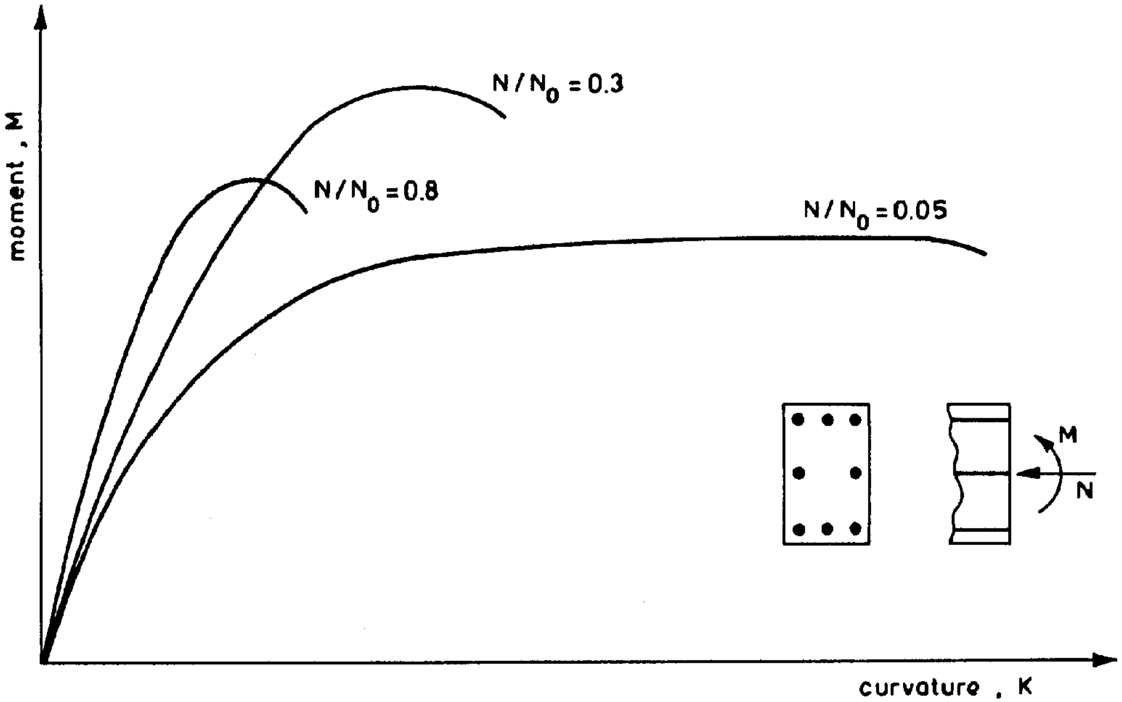
Flexural rigidity of a structural element can be defined as the slope of moment-curvature diagram.

Flexural Rigidity



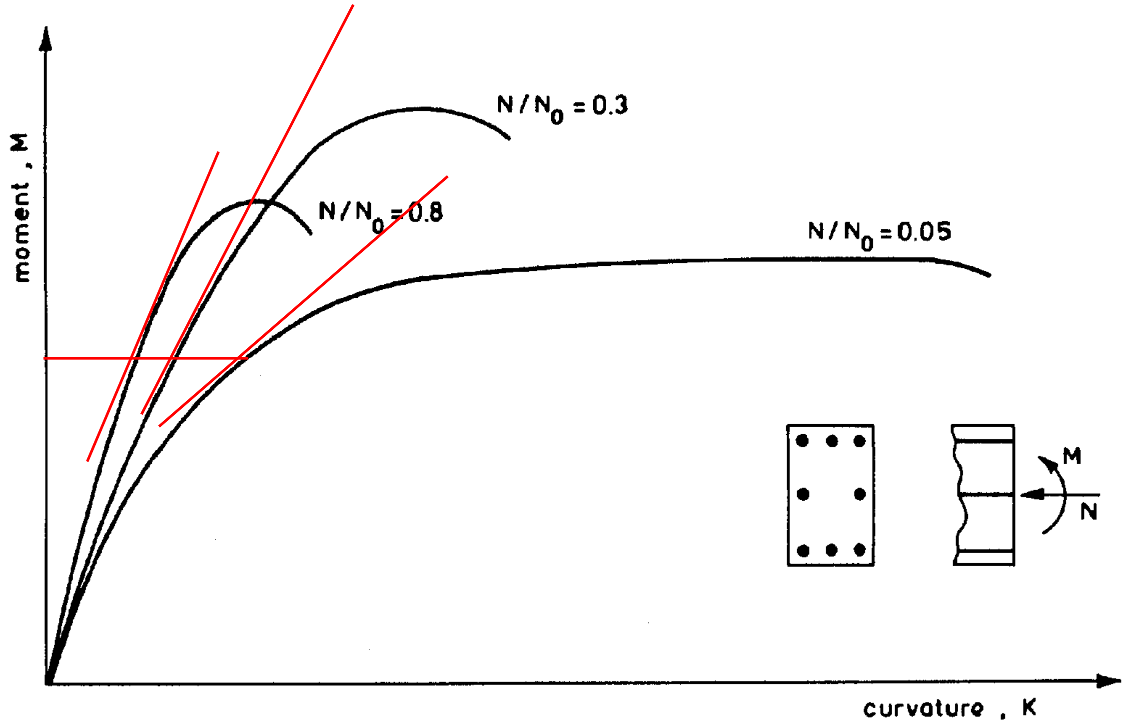
Three moment-curvature diagrams, each corresponding to a different level of axial load are shown in the figure. The following observations can be:

Flexural Rigidity



The moment-curvature relationships are not linear

Flexural Rigidity



Shape of M-K depends on the level of axial load. Hence, there is no unique M-K curve for a given cross-section.

Flexural Rigidity

These observations reveal that the flexural rigidity, which can be called "EI" for the purpose of this discussion, is not constant and depends on the level of the axial load.

Flexural Rigidity

In members under high axial loads such as columns, the modulus of elasticity of concrete reduces down to one third of its initial value as a result of creep. This also changes the flexural stiffness of the column considerably.

As "E" ↓ *"the flexural rigidity, EI"* ↓

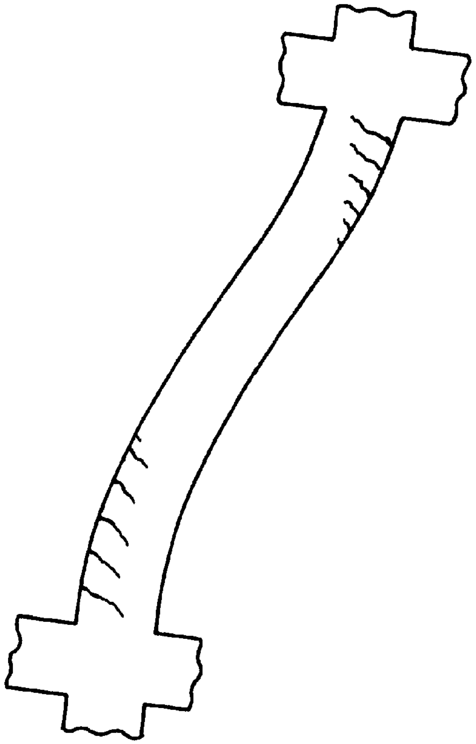
Flexural Rigidity

Concrete has a very low tensile strength.

Flexural moments + axial loads =>

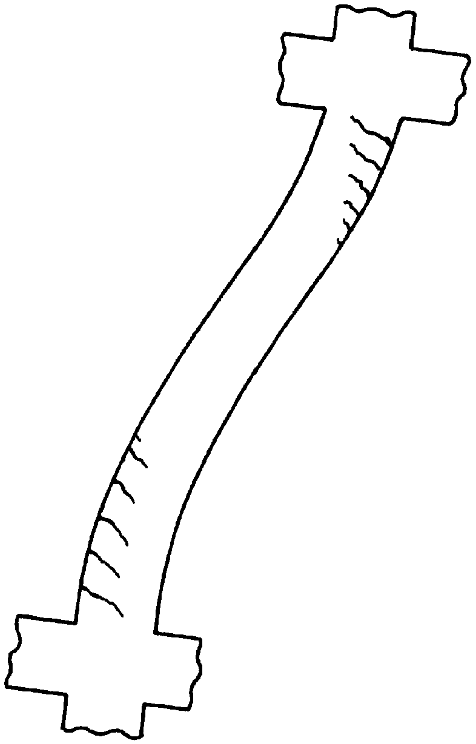
cracking of column under service loads.

Flexural Rigidity



In many cases, moment is not constant along the length of the column. Therefore, it is possible that some portions of the column will be cracked and other portions will not be crack. Also, the degree of cracking will vary from point to point as shown in figure below.

Flexural Rigidity



Therefore the moment of inertia is not constant along the length of the column and its variation depends on the degree of cracking. It should not be forgotten that shrinkage strains can cause micro cracking and cracking does have an influence the moment of inertia.

Cracking decreases "*I*".

As "I" $\downarrow$ *"flexural rigidity, EI"* $\downarrow$

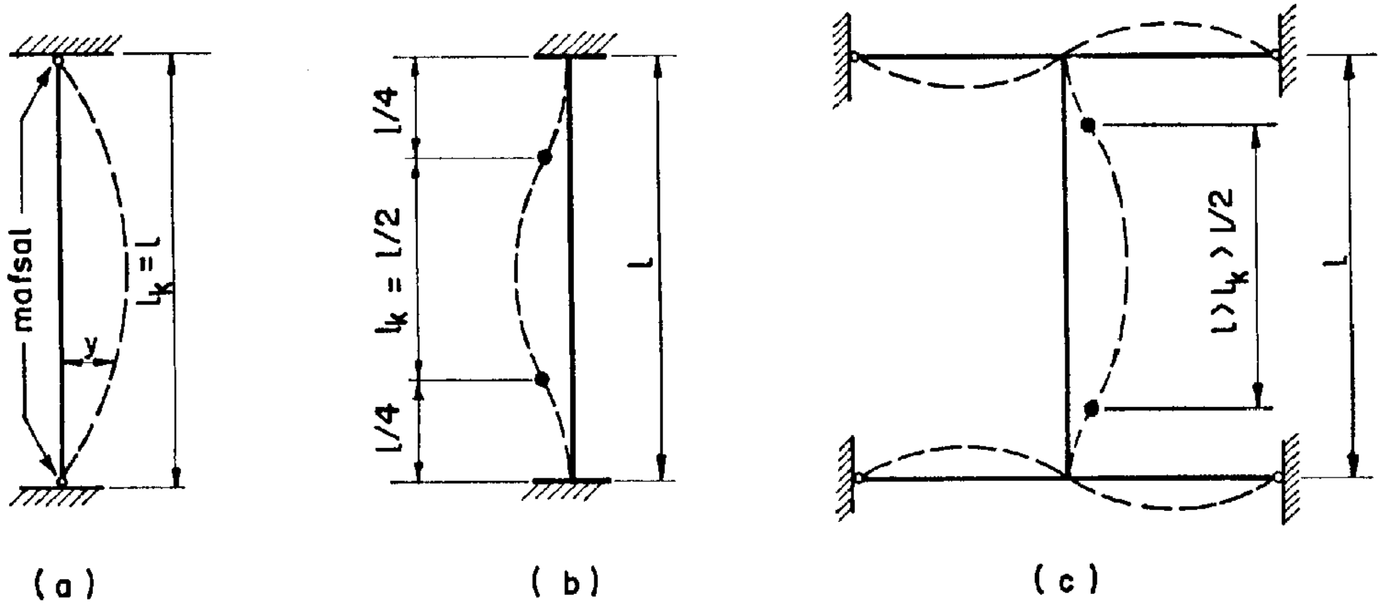
Effective Length of Columns

The deflection of an elastic beam-column depends on the ratio of the axial load to the buckling load of that member.

$$\frac{N}{N_{cr}} = \frac{N}{\frac{\pi^2 EI}{\ell_k^2}}$$

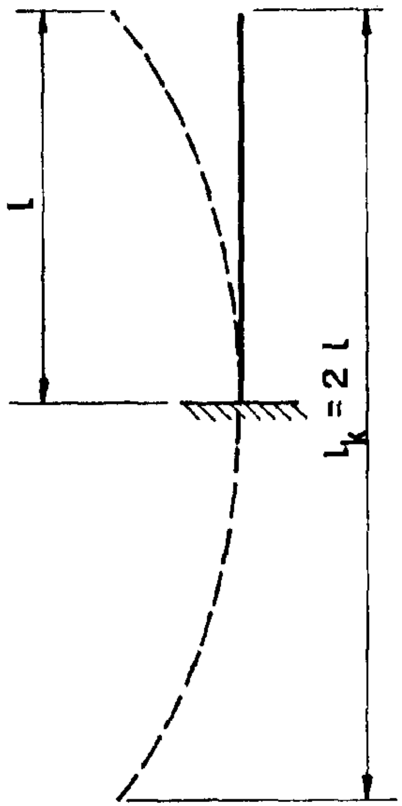
In this equation, ℓ_k is the effective length (distance between the point of inflections) and depends mainly on boundary conditions.

Effective Length of Columns



For braced columns, i.e. non-sway frames, depending on the rigidity of the floor members, effective length of the column will vary between $\ell/2$ and ℓ .

Effective Length of Columns

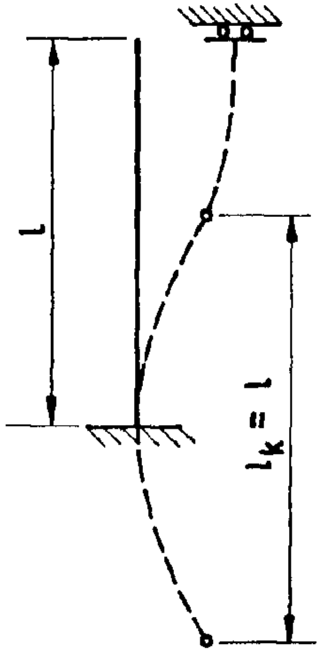


(a)

Sway Frames –

For the free cantilever (flag pole), the effective length is twice the actual length, i.e. 2ℓ .

Effective Length of Columns

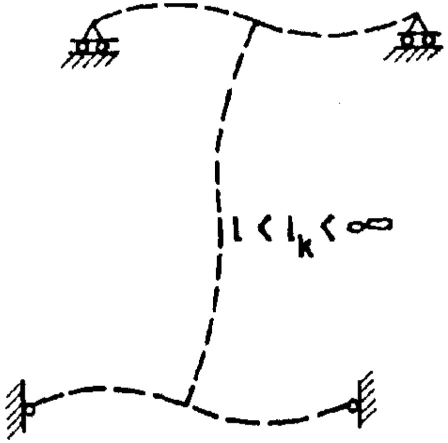


(b)

Sway Frames –

When the rotation at the free end is prevented, the effective length becomes equal to ℓ.

Effective Length of Columns



Sway Frames –

Deformations of columns in real structures depend on the relative stiffnesses of members at the joint as illustrated in the figure.

Effective Length of Columns

Therefore for *"Sway Frames"*, *the* effective length of the column will vary between ℓ and ∞.