CE 231 – ENGINEERING ECONOMY

PROBLEM SET 2

PROBLEM 1

a) What is the effective interest rate per semiannual period if the effective interest rate is 2% per month?
b) Calculate the effective interest rate per semiannual period if the effective interest rate is 5% per quarter.
c) Calculate the effective annual rate if the effective interest rate is 5% per quarter.

SOLUTION 1

a) i = $(1+0{,}02)^6 - 1 =$ **12,62 %**

b) i = $(1+0{,}05)^2 - 1 =$ **10,25 %**

c) i = $(1+0{,}05)^4 - 1 =$ **21,55 %**

PROBLEM 2

What yearly deposits would be equivalent to a deposit of 600 TL every 6 months for 2 years if the interest rate is 24% per year compounded quarterly?

SOLUTION 2

$$i_q = \frac{24}{4} = 6\% \qquad i_{yearly} = \left(1+\frac{0{,}24}{4}\right)^4 - 1 = 26{,}25\% \qquad i_{semiannual} = \left(1+\frac{0{,}12}{2}\right)^2 - 1 = 12{,}36\%$$

600 × (F/A, 12,36%, 4) = X (F/A, 26,25%, 2)

600 × 4,806 = 2,2625 X

X = 1274,15 TL

PROBLEM 3

A woman deposited 10.000 TL into her bank account. The money was left on deposit for 10 years. During the first 5 years the woman earned a yearly nominal interest rate of 15 % compounded monthly. The bank then changed its interest policy so that in the second five years, the woman earned a yearly nominal interest rate of 18% compounded quarterly. Calculate the amount that accumulated at the end of 10 years.

SOLUTION 3

$$i = \left(1 + \frac{0,15}{12}\right)^{12} - 1 = 16,08\% \longrightarrow \text{for the first 5 years}$$

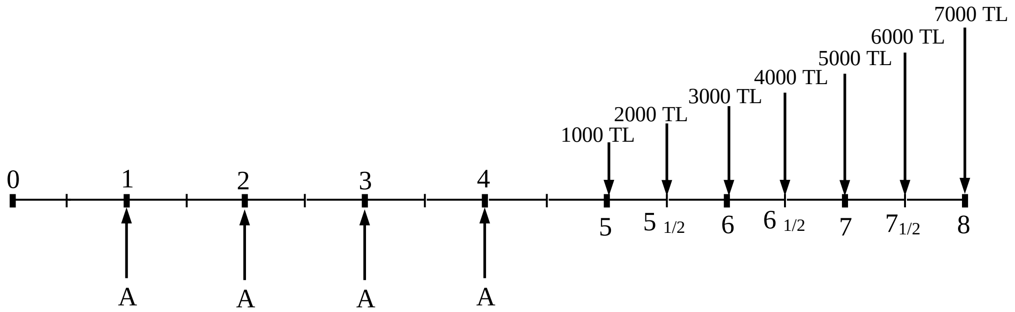
$F_{\text{at the end of first 5 years}} = 10.000 \times (1 + 0,1608)^5 =$ **21.071,81 TL**

in the next 5 years $\longrightarrow$ $i = \left(1 + \frac{0,18}{4}\right)^{4} - 1 = 19,25\%$

$F_{\text{at the end of second 5 years}} = 21.071,81 \times (1 + 0,1925)^5 =$ **<u>50.819,18 TL</u>**

PROBLEM 4

What must be the equal amount deposits, A, through years 1 to 4 so that the following semi annual withdrawals can be made through years 5 to 8 as shown in the cash flow below. Interest rate per year is 8 % compounded semiannually.



SOLUTION 4

$i_{semiannual} = 4\ \%$ $\qquad$ $i = (1+0,04)^2 – 1 = 8,16\ \%$ per year

F_A at the end of 4^{th} year = A (F/A, 8,16%, 4)

$$(F/A, 8,16\%, 4) = \frac{(1,0816)^4 - 1}{0,0816} = 4,5168$$

P_G at the end of 4^{th} year = **[**(1.000+1.000(A/G, 4%, 7)**]** (P/A, 4%, 7) (P/F, 4%, 1)

$= (1.000 + 1.000 \times 2{,}8433) \times (6{,}0021) \times (0{,}9615)$

$= 22.179{,}76$ TL

$F_A = P_G \longrightarrow \boxed{A = \dfrac{22.179{,}76}{4{,}5168} = 4910{,}50 TL}$

PROBLEM 5

A company borrows 8.000 TL at a nominal interest rate of 12% per year compounded monthly. Company wants to repay the loan in 14 equal monthly payments, with the first payment starting 1 month from now.

- **a)** What should be the size of each payment?
- **b)** After making 8 payments, if company decides to pay off the remaining loan in the 9^{th} month, how much must the company pay?

SOLUTION 5

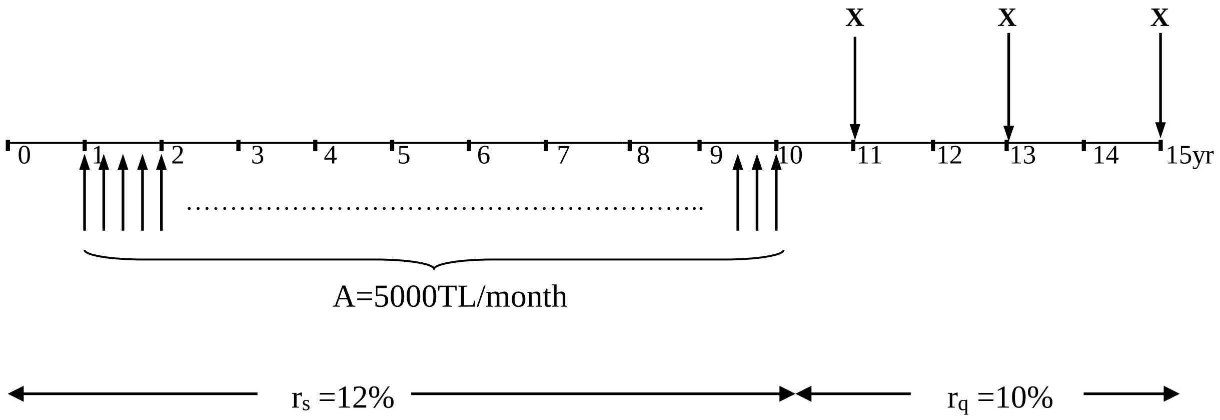
$\dfrac{0{,}12}{12} = 1\%$ monthly

a) A = 8000 (A/P, 1%, 14)
$\cong$ **615,21 TL**

b) P_f = A + A(P/A, 1%, 5)
= 615,21 **[** 1 + (P/A, 1%, 5)
P_f = 3.601 TL (payment at the end of 9^{th} month)

PROBLEM 6

The savings of a family are deposited each month during a 10 year-period. The interest rate is 12% compounded semi-annually. The family wishes to get back the accumulated money at 3 equal payments; 1, 3 and 5 years after the last deposit. For this 5year-period, the interest rate is 10% compounded quarterly. Find the value of these 3 payments. The cash flow diagram is given below:



SOLUTION 6

For the first 10 years

$i_s = r_s/2 = 12\% / 2 = 6$
$i = (1+ i_s)^2 - 1 = (1+0{,}06)^2 - 1 = 0{,}1236 = 12{,}36\%$
$0{,}1236 = (1+i_m)^{12} - 1 \Rightarrow i_m \cong 0{,}01 = 1\%$

For the next 5 years

$i = (1+ i_q)^4 - 1 = (1+0{,}1/4)^4 - 1 = 0{,}104 = 10{,}4\ \%$

$F = 5.000(F/A,1\%, 120) = 5.000([1+0{,}01]^{120} - 1)/0{,}01 = 1.150.193$ TL

$= X\underbrace{(P/F,10{,}4\%, 1)}_{\mathbf{0{,}9059}} + X\underbrace{(P/F,10{,}4\%, 3)}_{\mathbf{0{,}7434}} + X\underbrace{(P/F,10{,}4\%, 5)}_{\mathbf{0{,}6102}}$

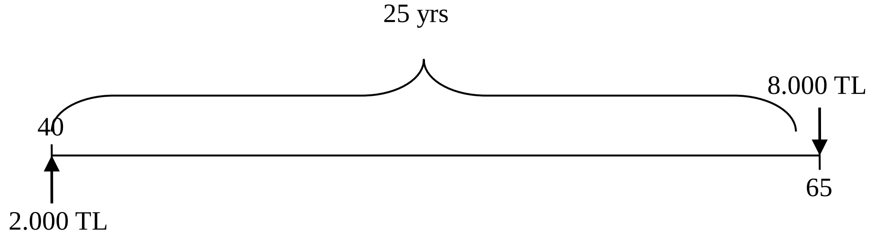
1.150.193= 2,2595X

X=509.047,58 TL

PROBLEM 7

A businessman now 40 years old plans to retire at age 65 and would like to have 8,000 TL at that time to purchase a beach cottage in Kuşadası. If he now has 2,000 TL to deposit in bank, where it will draw interest at the rate of 6 % compounded semiannually, will this achieve his goal?

SOLUTION 7



r = 6% compounded semiannually

$i_s = \dfrac{0,06}{2} = 0,03$

n = 25x2 = 50

F = 2.000 (F/P, i_s, n)

F = 2.000 (F/P, 3, 50)

= 2.000 x 4,3839

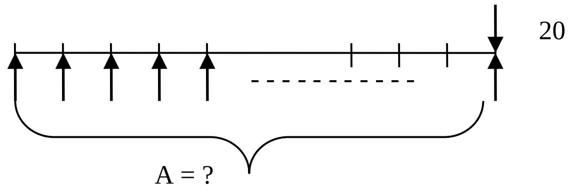
= 8.767,8 > 8.000 TL YES

PROBLEM 8

What semiannual deposit must be made into sinking fund to amount to 8.000 TL in 10 years with interest?

a) at 6 % compounded semi annually?
b) at 6% compounded yearly?
c) at 6% compounded quarterly?

SOLUTION 8



a) n_s = 20 semiannual periods

r_s = 6%

A = ? TL/half year

$i_s = \dfrac{r_s}{2} = \dfrac{6}{2} = 3\%$

A = F(A/F, i_s, n_s)

= 8.000(A/F, 3, 20)

= 8.000 x 0,03722

= 297,76 TL / half year

b) n_s = 20 semiannual periods

r = i = 6 %

A = ? TL/half year

$$i = \left(1 + \frac{r_s}{2}\right)^2 - 1$$

$$0{,}06 = \left(1 + \frac{r_s}{2}\right)^2 - 1$$

$$2Log\left(1 + \frac{r_s}{2}\right) = Log1{,}06$$

$$Log\left(1 + \frac{r_s}{2}\right) = \frac{Log1{,}06}{2}$$

$$Log\left(1 + \frac{r_s}{2}\right) = \frac{0{,}0253}{2} = 0{,}012653$$

$$\frac{r_s}{2} = i_s = 0{,}02956 \text{ compounded every half year}$$

$$A = F\left[\frac{i_s}{(1+i_s)^{n_s} - 1}\right] = 8.000\left[\frac{0{,}02956}{(1+0{,}02956)^{20} - 1}\right]$$

= 299,06 TL / half year

c) n_s = 20 semiannual periods r_Q = 6 % A = ? TL/half year

$$i = \left(1 + \frac{r_q}{4}\right)^4 - 1$$

$$= \left(1 + \frac{0{,}06}{4}\right)^4 - 1$$

= 0,06136

$$i = \left(1 + \frac{r_s}{2}\right)^2 - 1$$

$$0{,}06136 = \left(1 + \frac{r_s}{2}\right)^2 - 1$$

This gives : $\frac{r_s}{2} = i_s = 0{,}03022$

$$A = F\left[\frac{i_s}{(1+i_s)^{n_s} - 1}\right] = 8.000\left[\frac{0{,}03022}{(1+0{,}03022)^{20} - 1}\right]$$

= 297,06 TL / half year

PROBLEM 9

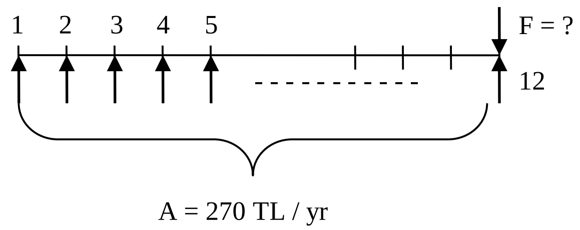
What is the compound amount of 270 TL paid yearly for 12 years with interest at 4 % compounded quarterly?

SOLUTION 9

A = 270 TL / yr

n = 12 yrs

r_Q = 4% F = ?



$$i = \left(1 + \frac{r_q}{4}\right)^4 - 1 = \left(1 + \frac{0,04}{4}\right)^4 - 1 = 0,00406$$

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 270\left[\frac{(1+0,0406)^{12} - 1}{0,0406}\right]$$

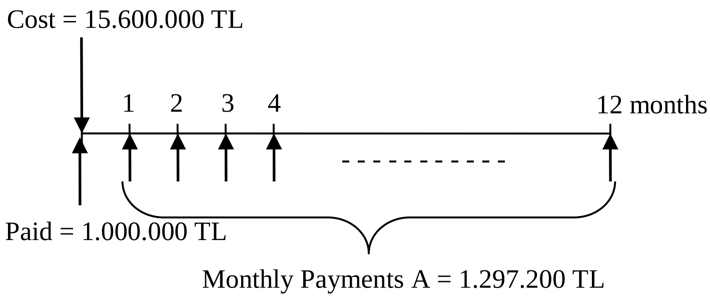
= 4.070,95 TL

PROBLEM 10

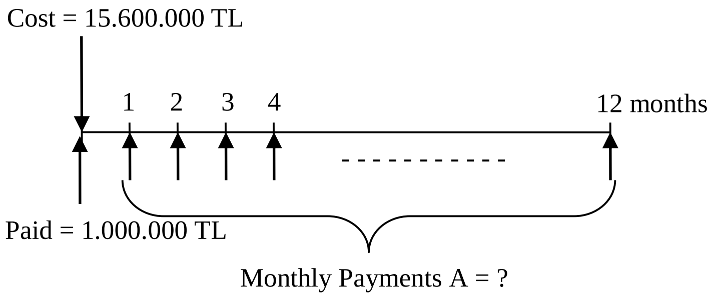
A company purchased a fleet of trucks for 15.600.000 TL. Payment was made by an immediate cash payment of 1.000.000 TL and 12 month-end payments of 1.297.200 TL each. Another dealer offered to finance the same purchase with an initial payment of 1.000.000 TL and 12 monthly payments for the unpaid balance at an interest rate of 1% per month. Which offer should the company have accepted?

SOLUTION 10

First alternative;



Second alternative;



P = 14.600.000 TL

A = 1.297.200 TL

i_m = 1 %

n_m = 12 months

A = P(A/P, i_m , n_m)

= 14.600.000 (A/P, 1%, 12)

= 14.600,000 x 0,08885

= 1.297.210 > 1.297.200

∴ Prefer the first offer

PROBLEM 11

A man has borrowed 10.000 TL which he will repay in 60 equal monthly installments. At the time of his twenty-fifth payment he desires to pay the remainder of the loan as an additional single payment. At 24 % interest compounded monthly what is the amount of the additional single payment?

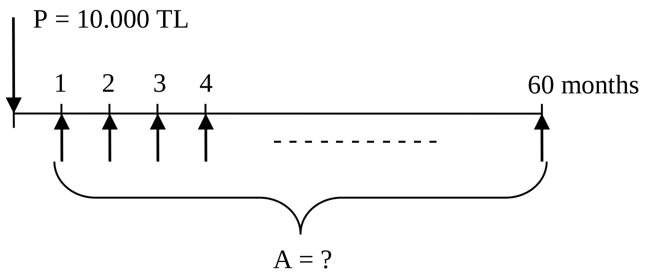
SOLUTION 11

P = 10.000 TL

n_m = 60 months

A = ?

i_m = 24 %



$$i_m = \frac{n_m}{12} = \frac{24}{12} = 2\,\%$$

A = P (A/P, i_m , n_m)

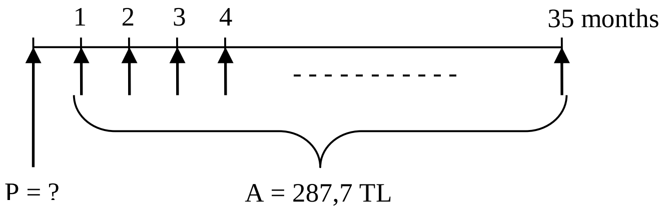
= 10.000 (A/P, 2%, 60)

= 10.000 x 0,02877

= 287,7 TL / month

n_m = 60 – 25 = 35 months

P = ?



P = A (P/A, i_m ,n_m)

= 287,7 x (P/A, 2 , 35)

= 287,7 x 24,999

= 7.192,21 TL → Additional Single Payment

Total Payment at 25th month = 287,7+7.192,21 = 7.479,91 TL

PROBLEM 12

What is the present worth of 480 TL due 14 years hence with interest at 2 % compounded semiannually?

SOLUTION 12

F = 480 TL
r_s = 2%
n = 14 yrs
P = ?

$$i = \left(1 + \frac{r_s}{2}\right)^2 - 1 \quad = \left(1 + \frac{0,02}{2}\right)^2 - 1 \quad = 2,01\,\%$$

$$P = F\left[\frac{1}{(1+i)^n}\right] = 480\left[\frac{1}{(1+0,0201)^{14}}\right] = 363,28\,\text{TL}$$

OR

$$i_s = \frac{r_s}{2} = \frac{2}{2} = 1\,\%$$

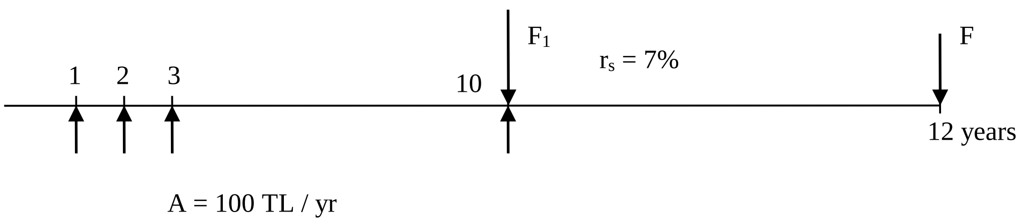
n_s = 2 x 14 = 28 half years

$$P = F\left[\frac{1}{(1+i_s)^{n_s}}\right] = 480\left[\frac{1}{(1+0,01)^{28}}\right] = 363,28\ \text{TL}$$

PROBLEM 13

Monthly deposits 100 TL are made into an account paying 4 % compounded quarterly. Ten monthly deposits are made. Determine how much will be accumulated in the account 2 years after the last deposit if the interest rate for this period is 7 % compounded semiannually.

SOLUTION 13



r_q = 4% ; i_q = 1%

$$i = \left(1 + \frac{0{,}04}{4}\right)^4 - 1 = (1{,}01)^4 - 1 = 0{,}0406$$

$$0{,}0406 = (1 + i_m)^{12} - 1$$

i_m = 0,0033 = 0,33 %

F = 100(F/A, 0,33, 10) (F/P, $i_{semiannually}$, 4)

OR

(F/P, $i_{annually}$, 2)

r_s = 7 % ; i_s = 3,5

i = $(1 + 0{,}035)^2 - 1$ = 7,12 %

$$F = 100\left[\frac{(1+i)^n - 1}{i}\right](1+i)^n$$

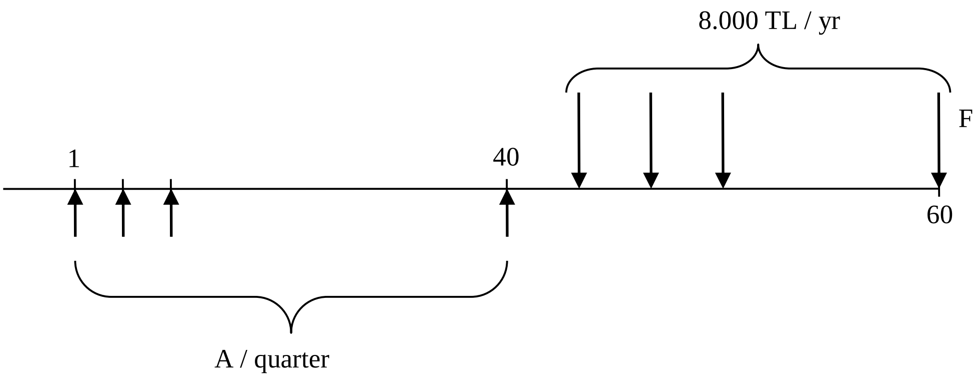
$$F = 100\left[\frac{(1+0{,}0033)^{10} - 1}{0{,}0033}\right](1+0{,}035)^4$$

= 100 x (10,1498)(1,1475) = 1.164,69 TL

PROBLEM 14

A man is planning to retire in 40 years. He wishes to deposit a regular amount every 3 months until he retires, so that beginning one year following his retirement he will receive annual payment of 8.000 TL for the next 20 years. How much must he deposit if the interest rate is 8% compounded quarterly?

SOLUTION 14



r_q = 8 %

i_q = 0,08/4 = 0,02

i = $(1 + 0{,}02)^4 - 1$ = 0,0824

$$i = \left[1 + \frac{r}{c}\right]^c - 1$$

i = 8,24%

n = 20, i = 8 P/A = 9,818

A (F/A, 2%, 160) = 8.000(P/A, 8.24, 20)

n = 20, i = 9 P/A = 9,129

1 0,689

0,24 x

x = 0,516247

$$A = \frac{8.000\left[\dfrac{(1+i)^n - 1}{i(1+i)^n}\right]}{\dfrac{(1+i)^n - 1}{i}}$$

$$A = \frac{8.000\left[\dfrac{(1{,}0824)^{20} - 1}{0{,}0824(1{,}0824)^{20}}\right]}{\dfrac{(1{,}02)^{160} - 1}{0{,}02}}$$

$$A = 8.000\frac{9{,}645247}{1138{,}5} = 67{,}81 \text{ TL / quarter}$$

PROBLEM 15

- **a)** How long does it take a given amount of money to double if it is invested at an nominal interest rate of 12 % compounded monthly?
- **b)** What equal annual payments are necessary to repay 50.000 TL in 5 years with a nominal interest rate 10 % compounded semi-annually?
- **c)** What is the accumulated value of 1.000 TL deposited at the end of each quarter for 10 years with a nominal interest rate of 12 % compounded monthly?

SOLUTION 15

a)

r_m = 12 % compounded monthly

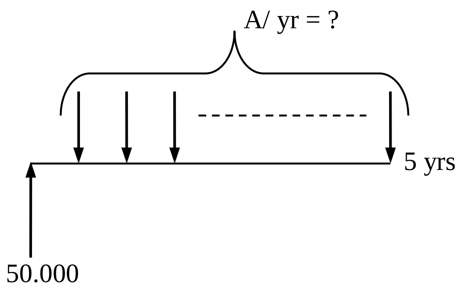
$$i_m = \frac{0{,}12}{12} = 0{,}01$$

$2P = P\,(1 + 0{,}01)^n$

$\log 2 = n \log 1{,}01$

$n = \dfrac{\log 2}{\log 1{,}01} = \dfrac{0{,}3010}{0{,}0043} = 69{,}95$ months $\longrightarrow$ $n = 69{,}95$ months

b)



$r_s = 10\,\%$ compounded semiannually

$i = (1 + \dfrac{0{,}10}{2})^2 - 1 = (1 + 0{,}05)^2 - 1$

$i = 10{,}25\,\%$

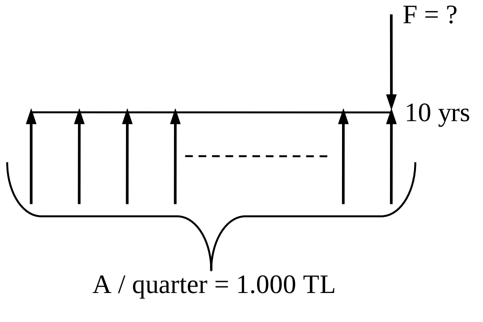
$A = P(A/P,\ 10{,}25,\ 5)$

$A = 50.000\ \dfrac{i(1+i)^n}{(1+i)^n - 1}$

$A = 50.000\ \dfrac{0{,}1025(1{,}1025)^5}{(1{,}1025)^5 - 1} = 50.000 * \dfrac{0{,}1670}{0{,}6289}$

A = 50.000 * 0,2655 = **13.275 TL/yr**

c)



$r_m = 12\,\%$ compounded monthly

$i = (1 + \dfrac{0{,}12}{12})^{12} - 1 = (1 + 0{,}01)^{12} - 1$

$i = 0{,}1268 = 12{,}68\,\%$

$0{,}1268 = (1 + \dfrac{r}{4})^4 - 1$

$4 \log (1 + i_q) = \log 1{,}1268$

$\log (1 + i_q) = \dfrac{0{,}0518}{4} = 0{,}0130$

$i_q = 3{,}04\,\%$

$$F = 1.000\left[\frac{(1+i)^n - 1}{i}\right] = 1.000\left[\frac{(1{,}0304)^{40} - 1}{0{,}0304}\right]$$

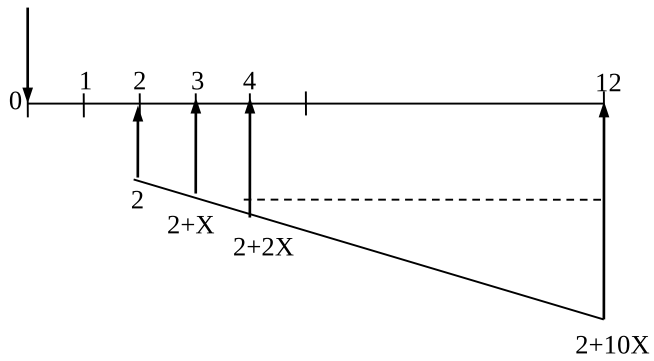
$$A = \frac{1000 * 2{,}313}{0{,}0304}$$

A = **76.085,52 TL**

PROBLEM 16

You borrow 50 TL on January 1st with the condition that you must pay back in 1 year, your monthly payments starting at the end of February. Your first payment will be 2 TL, second one (2+X) TL, third one (2+2X) TL,………….., last one (2+10X) etc. If the nominal interest rate is 36 % compounding monthly, calculate the value of X.

SOLUTION 16



$$i = \frac{r}{c} = \frac{36}{12} = 3\ \%$$

$$[2+X\underset{4{,}7049}{(A/G,3\%,11)}]\ \underset{9{,}2526}{(P/A,3\%,11)}\ \underset{0{,}9709}{(P/F,3\%,1)} = 50$$

(2 + 4,7049X) (8,98335) = 50

17,967 + 42,266X = 50

X = 0,758 TL

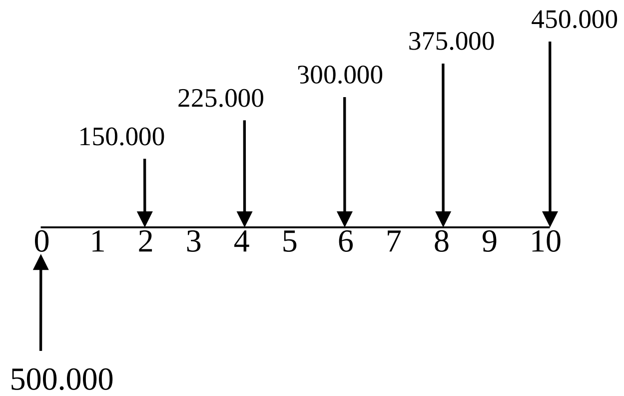
PROBLEM 17

A company plans to give an offer for a construction job. According to the company, they have to purchase several equipments right now, whose total cost is 500.000 TL and they expect an income of 150.000 TL at the end of second year which will increase by 75.000 TL every two years.

Assuming that the job continues 10 years with an interest rate of 20% per year, determine whether this is a good offer or not by calculating the present worth of all incomes and payments.

USE UNIFORM GRADIENT SERIES IN YOUR CALCULATIONS.

SOLUTION 17



Interest rate per two years: $(1+i)^2 - 1 = (1+0.20)^2 - 1 = 1.44 - 1 = 0.44 = 44\%$

$$P = -500000 + \left(150000 + 75000\left(1/0.44 - 5/\left((1+0.44)^5 - 1\right)\right)\right) \times (P/A,44\%,5)$$

$$(P/A,44\%,5) = \frac{(1+0.44)^5 - 1}{0.44(1+0.44)^5} = \frac{5.1917}{2.7243} = 1.9057$$

$$P = -500000 + 473052 = -26948 \text{ TL}$$

- It is not a good offer.

PROBLEM 18

If the nominal interest per year is 3,64% and effective interest rate per year is 3,70%, find the interest compounding period.

SOLUTION 18

$$i = \left(1 + \frac{r}{c}\right)^{c} - 1$$

$$0{,}0370 = \left(1 + \frac{0{,}0364}{c}\right)^{c} - 1$$

As compounding periods are generally expressed in terms of; daily, weekly, monthly, quarterly, semi-annually and yearly periods; try only these compounding periods.

By trial and error method finally; c = 365 is found