CHAPTER 5

PILE FOUNDATIONS

5.2. CAPACITY OF SINGLE PILE UNDER VERTICAL LOADS

5.2.1. STATIC FORMULA

Ultimate capacity of a pile; $Q_{ult} = Q_p + Q_s$

where,

Q_p = End (point) bearing capacity of the pile at the base

Q_f or Q_s = (Skin) friction capacity along pile perimeter

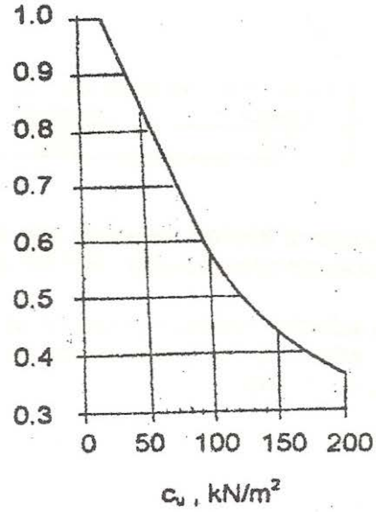


Figure.1

1. COHESIVE SOILS ($\phi_u = 0$) :

1A. (Skin) friction capacity (kN) :

$Q_s = q_s \cdot A_s = \alpha \cdot c_u \cdot A_s$

q_s (or f_s) = unit skin friction (or adhesion in this case) between pile and clay (kN/m^2)

α = adhesion factor (See Fig.1)

c_u = undrained shear strength

A_s = shaft (friction) area

If there are n different layers;

$$Q_s = \sum_{i=1}^{n} \alpha_i \cdot c_{ui} \cdot A_{si}$$

Since A_s = (perimeter, p) . ΔL , $\quad Q_s = p \cdot \sum_{i=1}^{n} \alpha_i \cdot c_{ui} \cdot \Delta L_i$

1B. End bearing capacity (kN) :

General expression, $\quad Q_p = A_p\ (c\, N_c + 1/2\, \gamma\, B\, N_\gamma + \gamma\, D_f\, N_q)$

where N_c, N_γ, N_q : bearing capacity factors for deep foundations

$Q_p = q_p \cdot A_p = N_c \cdot c_u \cdot A_p \quad$ (N_c = 9 is commonly used)

$Q_p = 9 \cdot c_u \cdot A_p$

where, c_u is undrained shear strength at pile point level

N_c is bearing capacity factor for cohesion for deep foundations

A_p is base (point) area of pile

2. COHESIONLESS SOILS :

2A. (Skin) friction capacity (kN) :

$Q_s = q_s$ (or q_f) . $A_s = K_s . \sigma'_{vo}$. $\tan\delta$. A_s

where, q_s or q_f is unit skin friction (kPa), ($q_s = \sigma'_h . \tan\delta$)
K_s is a coefficient of earth pressure depending on relative density, volume displacement of pile, material and shape of pile (See Table.1)

Table.1 K_s values

Driven Pile (Displacement Pile)		Small Displacement Pile		Bored Pile
Low D_r	High D_r	Low D_r	High D_r	
1.0	2.0	0.5	1.0	0.5

δ is angle of friction between pile and soil
(recommended values : 20° for steel piles, 3/4 ϕ for concrete piles)

σ'_{vo} is effective vertical pressure at the level considered
σ'_h is effective horizontal pressure at the level considered, $\sigma'_h = K_s . \sigma'_{vo}$
A_s is shaft area

If there are different cohesionless layers;

$Q_s = \sum_{i=1}^{n} K_{si} . \sigma'_{vo} . \tan\delta . A_s$ since $A_s = p . \Delta L$

$Q_s = p . \tan\delta . \sum_{i=1}^{n} K_{si} . \sigma'_{vo} . \Delta L_i$ Note that σ'_{vo} is average for each layer considered (i.e. taken at mid-height)

K_s is usually constant but if D_r's of the layers are significantly different, use Table.1

2B. End bearing capacity (kN) :

$Q_p = q_p . A_p = N_q . \sigma'_{vo} . A_p$ (Note that the term with N_γ in the general eqn. is comparatively small)

where, q_p is unit base resistance (kPa), A_p is base area of pile
N_q is bearing capacity factor for deep foundations, see Table.2
σ'_{vo} is vertical effective stress at base level of pile, constant after 20D (or B) which is called "critical depth".

Table.2 N_q values

ϕ'	20	25	28	30	32	34	36	38	40	42	45
N_q (Driven)	8	12	20	25	35	45	60	80	120	160	230
N_q (Bored)	4	5	8	12	17	22	30	40	60	80	115

5.2.3 EMPRICAL METHODS BASED ON FIELD TESTS

5.2.3.1 FROM SPT

1 **Schmertman (1975):** For conservative ultimate values for unit side friction &end bearing straight sided driven precast piles.

Type of soil	Soil Gr.	q_c/N	$R_f(\%)$	Side Friction(tsf)	EndBearing(tsf)
1. Clean Sand (above&below GWT)	GW,GP,GM SW,SP,SM	3.5	0.6	0.019N	3.2N
2.Clay-Silt-Sand mix, silty sand silts,marls	GC,SC ML,CL	2.0	2.0	0.04N	1.6N
3. Plastic Clay	CH,OH	1.0	5.0	0.05N	0.7N
4. Soft Limestone Lime Rocks	–	4.0	0.25	0.01N	3.6N

NOTE: Loose Sand ∵ Driving generates (+) p.p.,σ_v' decrease, low N value
NC. Clay

Dense Sand : Driving generates (-) p.p., σ_v' increase, high N value
OC. Clays

$q_p = 40 N_{60} \frac{L}{D} \leq 300 N_{60}$ for silts

2 **Meyerhof (1976)(Das) :** q_p (kN/m^2)=$40N_{60}L/D \leq 400N$, where (sand & gravel)

q_p : unit tip resistance

N :average standard penetration number near the pile point(10D above,4D below)

Driven high displacement piles : $(f_s)_{av}$ (kN/m^2)=$2\bar{N}$

Driven low displacement piles : $(f_s)_{av}$ (kN/m^2)=$\bar{N}$, where

f_s: unit shaft resistance

$\bar{N}$: average over pile shaft (with correction)

Estimate ∅ from N, Estimate D_r from N → Vesic:
(theoritical approach)

$f_s = x_v(10)^{1.54D}r^4$ (kN/m^2)

x_v=8 for large volume displacement

=2.5 for bored, open end, H piles

3 Biraud 1985

$q_p = 19.7\, \sigma_r (N_{60})^{0.36}$

$q_s = 0.224\, \sigma_r (N_{60})^{0.29}$

$\sigma_r = 100$ kPa

NOTE:

$q_p = 40\ N\ (L/D) < 400\ N$ kPa, N : average at pile tip, 10 D above, 4 D below the pile tip

$f_s = 2\ N$ kPa driven piles

$f_s = N$ kPa bored piles

N : average SPT value along the pile shaft

5.2.3.2 FROM CPT

$q_p = k_c\ q_{cav}$ k_c : bearing capacity factor;
q_{cav} : equivalent q_c at pile tip (1.5D above & 1.5D below the pile tip)

Nature of Soil		q_c(MPa)	k_c(bored pile)	k_c(driven pile)
CLAY	Soft	<1	0.40	0.50
	Medium	1-5	0.35	0.45
	Stiff	>5	0.45	0.55
SAND	Loose	<5	0.40	0.50
	Medium	5-12	0.40	0.50
	Dense	>12	0.30	0.40

$f_s = 1/\alpha(q_c)$ α=friction coefficient q_c=average along the shaft

			BORED		DRIVEN			
Nature of Soil		q_c (MPa)	α	f_{smax} (MPa)	R/C		STEEL	
					α	f_{smax}(MPa)	α	f_{smax}(MPa)
CLAY	Soft	<1	30	0.015	90	0.015	30	0.015
	Medium	1-5	40	0.035	40	0.035	80	0.035
	Stiff	>5	60	0.035	60	0.035	120	0.035
SAND	Loose	<5	60	0.035	60	0.035	120	0.035
	Medium	5-12	100	0.080	100	0.080	200	0.080
	Dense	>12	150	0.120	150	0.120	200	0.120

↑ upper limit ↑ upper limit ↑ upper limit

5.2.3.3 FROM PMT

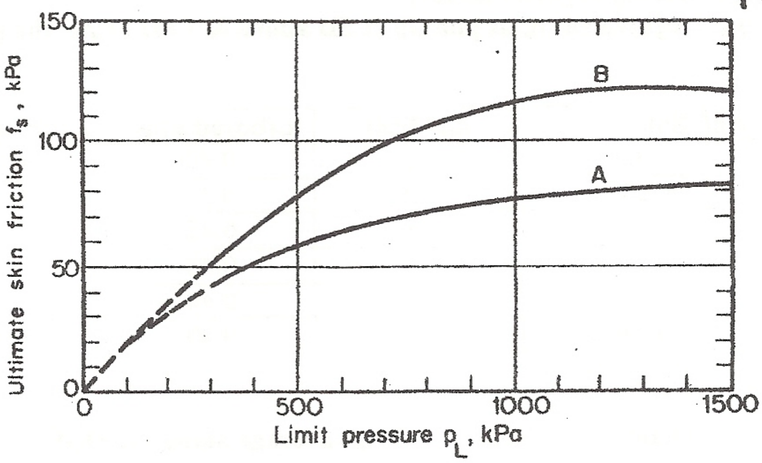
TABLE 7.6
PRESSUREMETER BEARING CAPACITY FACTOR k_q FOR DEEP FOUNDATIONS*

Material	Class	P_{Ln},† tsf	Bearing capacity factor k_q,‡ Driven piles	Bearing capacity factor k_q,‡ Bored piles
Clays: soft to firm	1	0–12	2.0	1.8
Silts: loose		0–7		
Sands: loose§	2	4–8	3.6	3.2
Silts: dense		12–30		
Clays: stiff		18–40		
Rock: soft, decomposed, friable		10–30		
Sands and gravels: medium dense	3	10–20	5.8	5.2
Rock: moderately soft, decomposed		40–100		
Sands and gravels: very dense	4	30–80	9.0	7.1
Rock: medium to high strength		80–100+		

*After CGS (1978),[8] Menard (1975).[28]
†Net limiting pressure, $P_{Ln} = P_L - P_{oh}$.
‡For penetrations of z_{tip}/B = 10 or more; k_q decreases for shallower depths for a given B.
§Calcareous sands: <2 [From Tirant (1979)].

k_g : bearing capacity factor

$q_p = k_g P_{Ln}$ $\quad$ P_{Ln} : net limit pressure at pile tip



SAND: Curve A for nondisplacement concrete piles and displacement steel piles. None displacement steel piles apply 50% reduction

Curve B for displacement concrete piles.

CLAY: Curve A for concrete piles; apply 25% reduction for steel piles

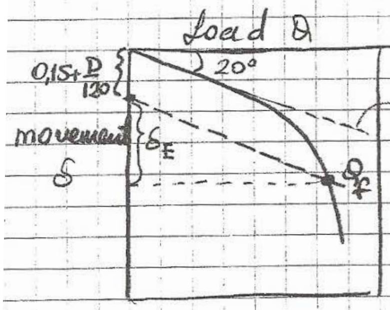
UPPER LIMITS: Concrete displacement: SAND f_s = 125 kPa; CLAY f_s = 85 kPa.

Nondisplacement piles f_s = 40 kN/m²

Steel displacement: CLAY f_s = 65 kPa; SAND f_s = 85 kPa

5.2.4. INTERPRETATION OF PILE LOAD TEST RESULTS

1. DAVISSON's method:



scale: elastic line ≅ 20°

→ Elastic compression of the pile:

$$\delta_E = \frac{QL}{AE}$$

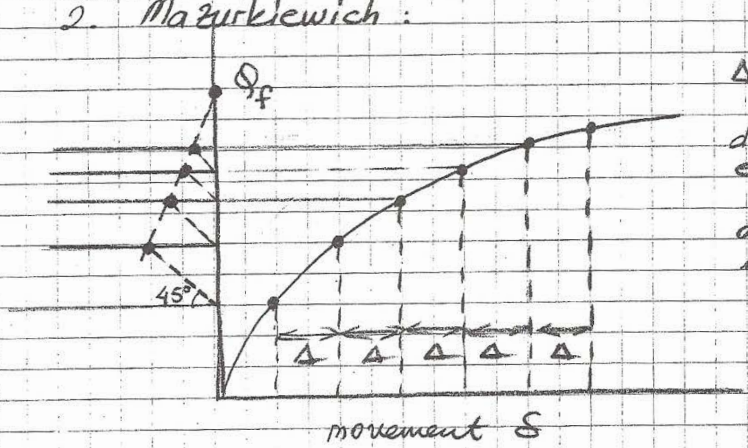
Failure Load: the load that cause a settlement of:

$$\delta = \delta_E + \left(0.15 + \frac{D''}{120}\right) \text{ (inches)}$$

D: in pile diam in inch.
0,15 inch. (3.81mm)

$$\delta = \delta_E + 3.81 + 0.008 \times D \text{ (mm)}$$

2. Mazurkiewich:

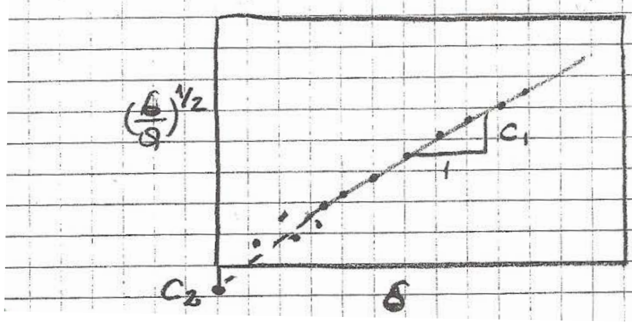


Δ: equal settlement

draw horizontal from each point.

draw 45°, mark the intersection with horizontal above.

3. Brinch-Hansen 80% criteria



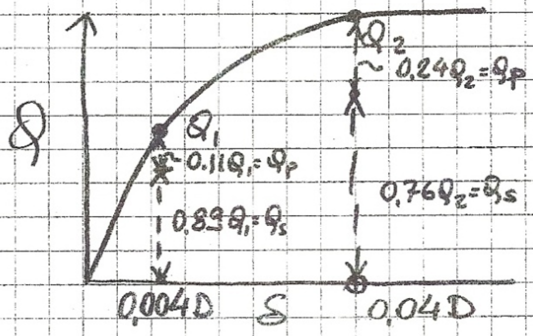
$$Q_u = \frac{1}{2\sqrt{C_1 C_2}}$$

$\delta_u = C_2/C_1$ ultimate settlement

C_1: SLOPE

C_2: INTERCEPT

4. Kulhawy & Hirany :



Q_1 = Elastic Limit.

Q_2 = ultimate load capacity

Q_2 : occurs at 4% Diameter

Data indicates that: $Q_1 = Q_2/2$

Q_1 occurs at 0,004D

at Q_1 : $Q_P = 0,11\,Q_1$

$Q_S = 0,89\,Q_1$

at Q_2 : $Q_P = 0,24\,Q_2$

$Q_S = 0,76\,Q_2$

Convenient method for underloaded piles.

MANY OTHER METHODS ARE AVAILABLE.

(Pages 8-13 are adopted from Poulos 1980, Pile Foundation Analysis and Design)

SETTLEMENT OF SINGLE PILE

5.3.2. Load Transferred to Pile Tip

lified presentation given by Poulos (1972*d*), the proportion of load transferred to the pile tip, β, is expressed in terms of the value β_0 for an incompressible floating pile in a semi-finite mass, multiplied by correction factors to take account of pile compressibility and the relative stiffness of the bearing stratum.

a) Floating Pile

$$\beta = \beta_0 C_K C_\nu \qquad (5.31)$$

where

$\beta = P_b/P$ = proportion of applied load transferred to pile tip

β_0 = tip-load proportion for incompressible pile in uniform half-space (Poisson's ratio = 0.5)

C_K = correction factor for pile compressibility

C_ν = correction factor for Poisson's ratio of soil

Values of β_0, C_K, and C_ν are plotted in Figs. 5.11, 5.12, and 5.13 for a wide range of parameters. The effect of pile compressibility is to decrease the amount of load transferred to the tip—that is, C_K is less than 1. The presence of an enlarged base increases β significantly. β is not significantly affected if the pile is situated in a finite layer rather than a half-space, provided the hard base of the layer is more than $0.2L$ below the bottom of the pile.

b) End-Bearing Pile on Stiffer Stratum

$$\beta = \beta_0 C_K C_b C_\nu \qquad (5.32)$$

where

β, β_0 C_K, and C_ν are defined as above and to sufficient accuracy, may be assumed to take their previous values

C_b = correction factor for stiffness of bearing stratum

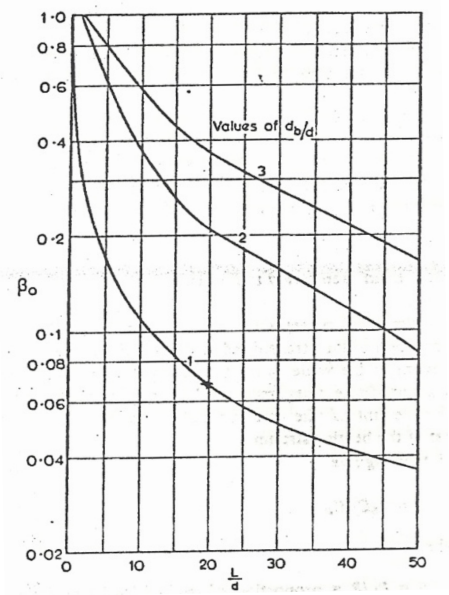


Figure 5.11 Proportion of base load, β_o

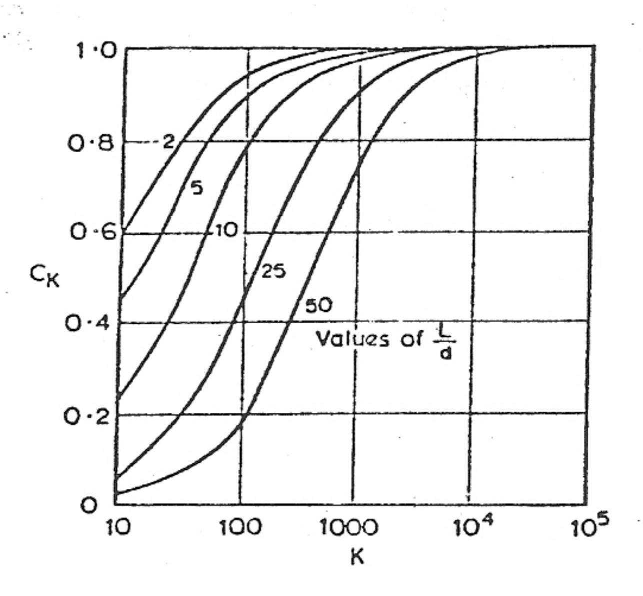


Figure 5.12 Compressibility correction factor for base load, C_k

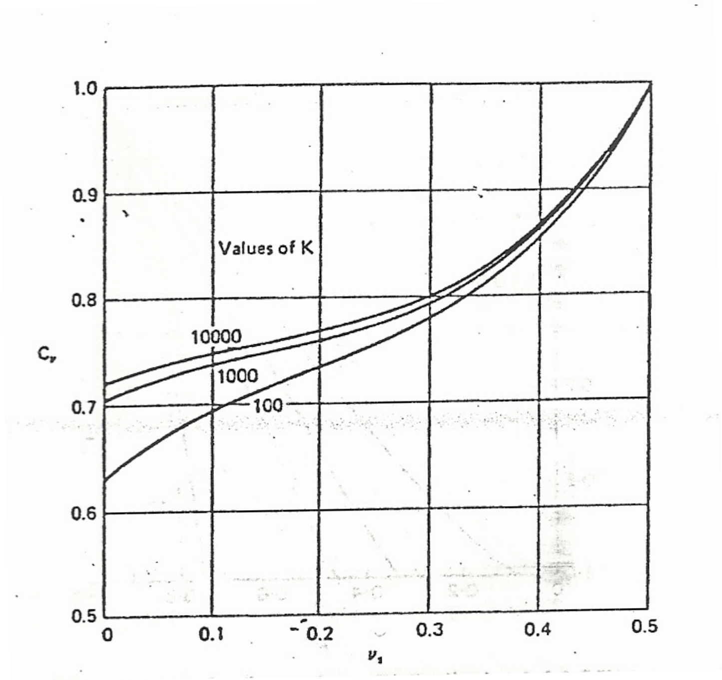


Figure 5.13 Poisson 's ratio correction factor for base load, C_v

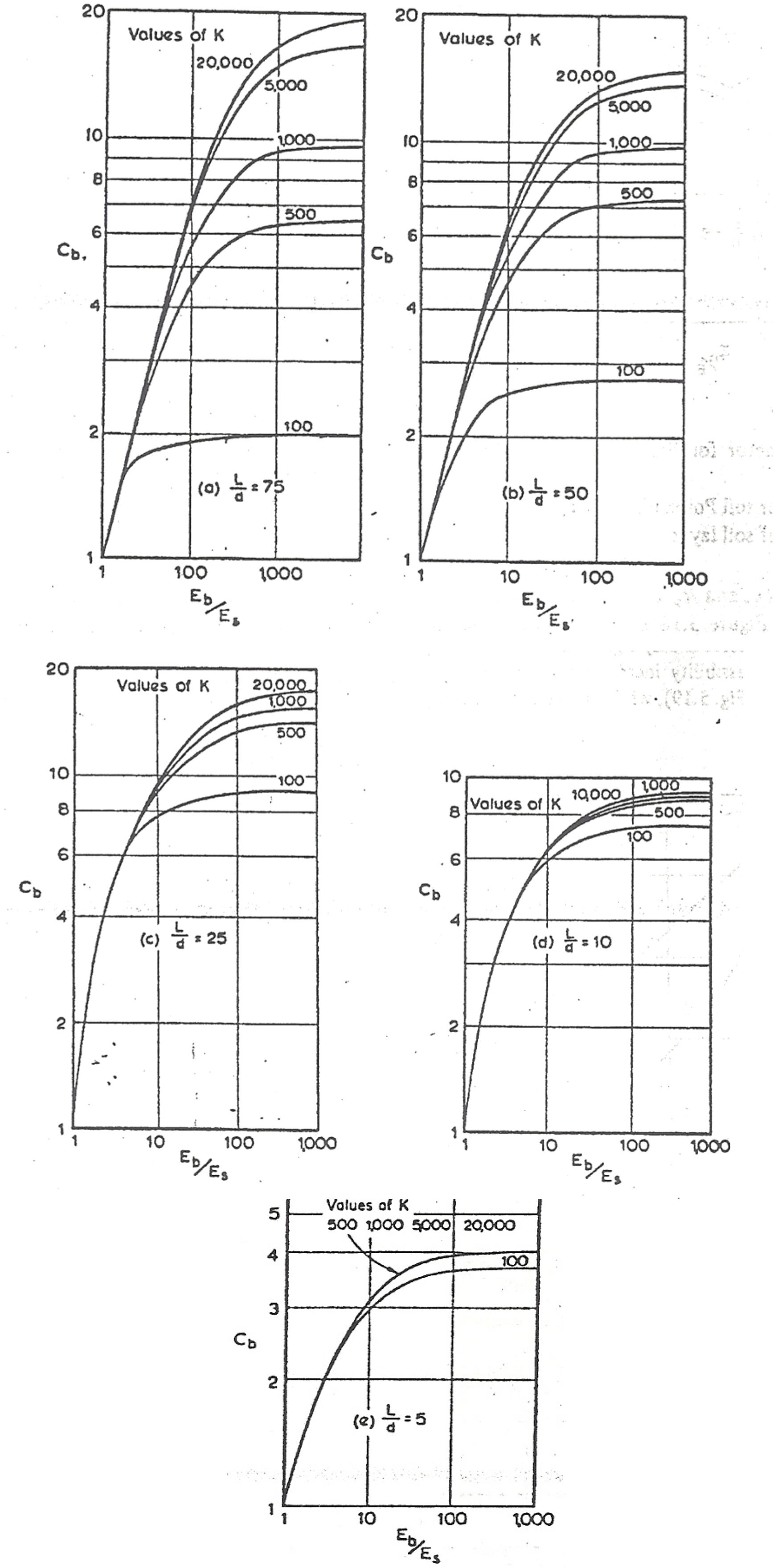


Figure 5.14 Base modulus correction factor for base load, C_b

Settlement of Pile

As with the tip load on a pile, the settlement of the top of the pile may be expressed, to sufficient accuracy, in terms of the settlement of an incompressible pile in a half-space, with correction factors for the effects of pile compressibility, and so on. It is again convenient to consider two cases for a homogeneous soil mass having constant Young's modulus E_s and Poisson's ratio ν_s:

a) Floating Pile

$$\rho = \frac{PI}{E_s d} \tag{5.33}$$

where

$I = I_0 R_K R_h R_\nu$ (5.33a)
ρ = settlement of pile head
P = applied axial load
I_0 = settlement-influence factor for incompressible pile in semi-infinite mass, for $\nu_s = 0.5$.
R_K = correction factor for pile compressibility
R_h = correction factor for finite depth of layer on a rigid base
R_ν = correction for soil Poisson's ratio ν_s
h = total depth of soil layer

b) End-Bearing Pile on Stiffer Stratum

$$\rho = \frac{PI}{E_s d}$$

$I = I_0 R_K R_b R_\nu$ (5.34a)
(I_0, R_K, R_ν are defined as for Eq. 5.33 and take the same values to sufficient accuracy)
R_b = correction factor for stiffness of bearing stratum

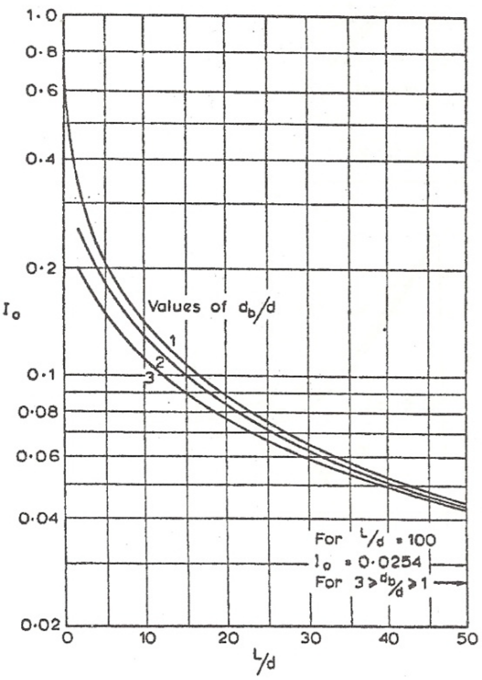


Figure 5.18 Settlement-influence factor, I_o

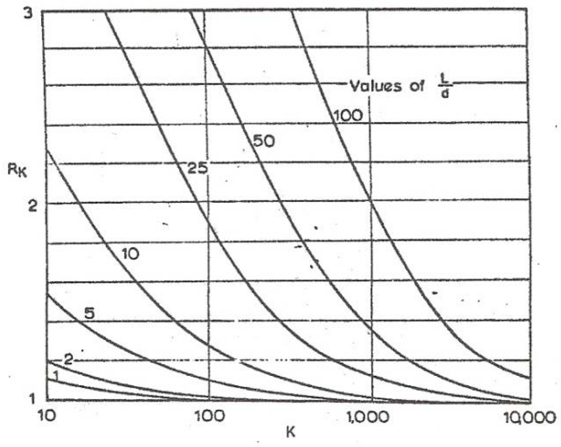


Figure 5.19 Compressibility correction factor for settlement, R_K

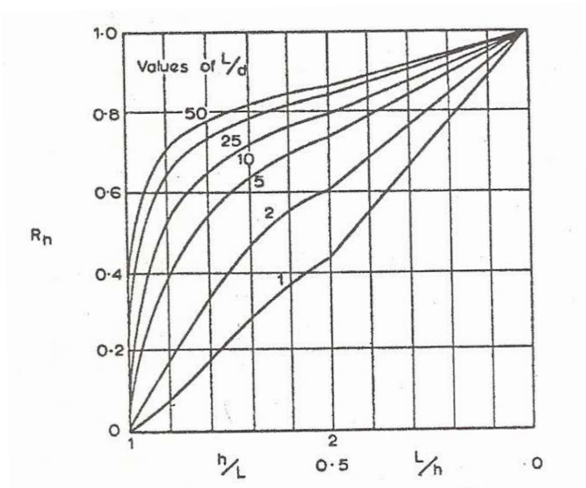


Figure 5.20 Depth correction factor for settlement, R_h

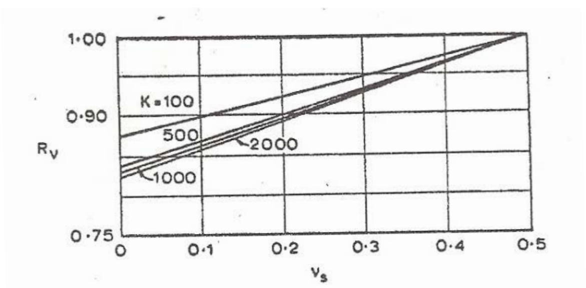


Figure 5.21 Poisson 's ratio correction factor for settlement, R_v

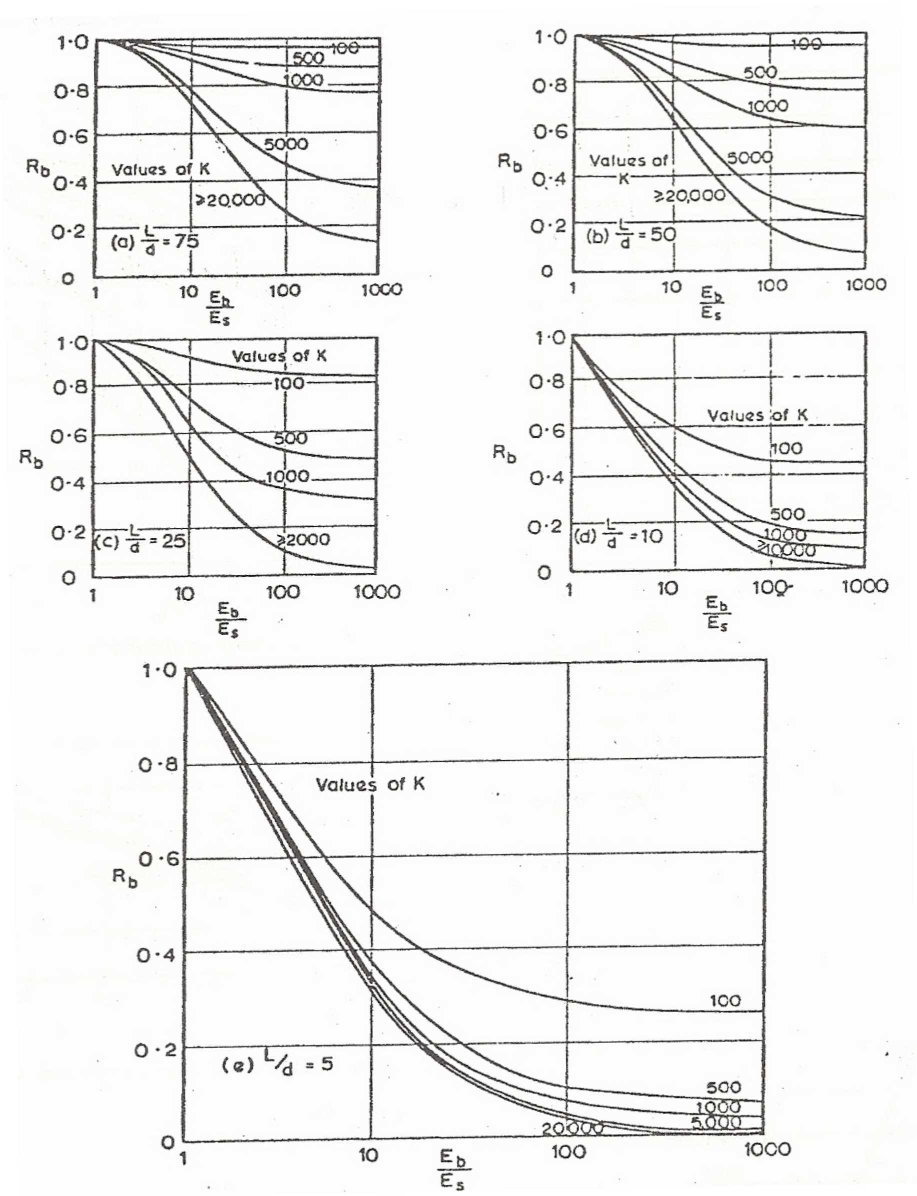


Figure 5.22 Base modulus correction factor for settlement, R_b

SETTLEMENT OF PILE GROUPS

Elastic Solution

TABLE 6.2 THEORETICAL VALUES OF SETTLEMENT RATIO R_s FRICTION PILE GROUPS, WITH RIGID CAP, IN DEEP UNIFORM SOIL MASS

No of Piles in Group		4				9				16				25			
L/d	s/d K	10	100	1000	∞	10	100	1000	∞	10	100	1000	∞	10	100	1000	∞
10	2	1.83	2.25	2.54	2.62	2.78	3.80	4.42	4.48	3.76	5.49	6.40	6.53	4.75	7.20	8.48	8.68
	5	1.40	1.73	1.88	1.90	1.83	2.49	2.82	2.85	2.26	3.25	3.74	3.82	2.68	3.98	4.70	4.75
	10	1.21	1.39	1.48	1.50	1.42	1.76	1.97	1.99	1.63	2.14	2.46	2.46	1.85	2.53	2.95	2.95
25	2	1.99	2.14	2.65	2.87	3.01	3.64	4.84	5.29	4.22	5.38	7.44	8.10	5.40	7.25	9.28	11.25
	5	1.47	1.74	2.09	2.19	1.98	2.61	3.48	3.74	2.46	3.54	4.96	5.34	2.95	4.48	6.50	7.03
	10	1.25	1.46	1.74	1.78	1.49	1.95	2.57	2.73	1.74	2.46	3.42	3.63	1.98	2.98	4.28	4.50
50	2	2.43	2.31	2.56	3.01	3.91	3.79	4.52	5.66	5.58	5.65	7.05	8.94	7.26	7.65	9.91	12.66
	5	1.73	1.81	2.10	2.44	2.46	2.75	3.51	4.29	3.16	3.72	5.11	6.37	3.88	4.74	6.64	8.67
	10	1.38	1.50	1.78	2.04	1.74	2.04	2.72	3.29	2.08	2.59	3.73	4.65	2.49	3.16	4.76	6.04
100	2	2.56	2.31	2.26	3.16	4.43	4.05	4.11	6.15	6.42	6.14	6.50	9.92	8.48	8.40	10.25	14.35
	5	1.88	1.88	2.01	2.64	2.80	2.94	3.38	4.87	3.74	4.05	4.98	7.54	4.68	5.18	6.75	10.55
	10	1.47	1.56	1.76	2.28	1.95	2.17	2.73	3.93	2.45	2.80	3.81	5.82	2.95	3.48	5.00	7.88

$\rho_{gr} = \rho_{single} R_s$ $\quad \rho_{single} = \dfrac{PI}{E_s d}$ (elastic solutions)

n: no of piles in a group

upto n = 25 R_s is taken from Tables 6.2 & 6.3 (use interpolations)

n > 25 $\quad R_s = (R_{25} - R_{16})\{\sqrt{n} - 5\} + R_{25}$

Strain Integration Method for Equivalent Raft

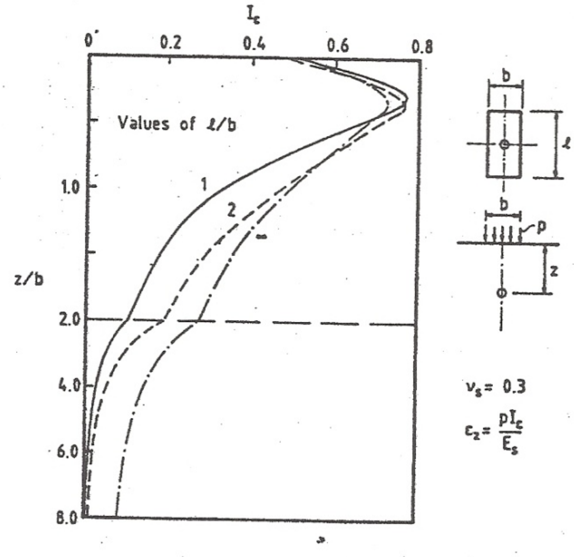


FIG.2 VERTICAL STRAIN FACTORS BELOW CENTRE OF RECTANGULAR AREA

Poulos 1993, BAP II
Balkema

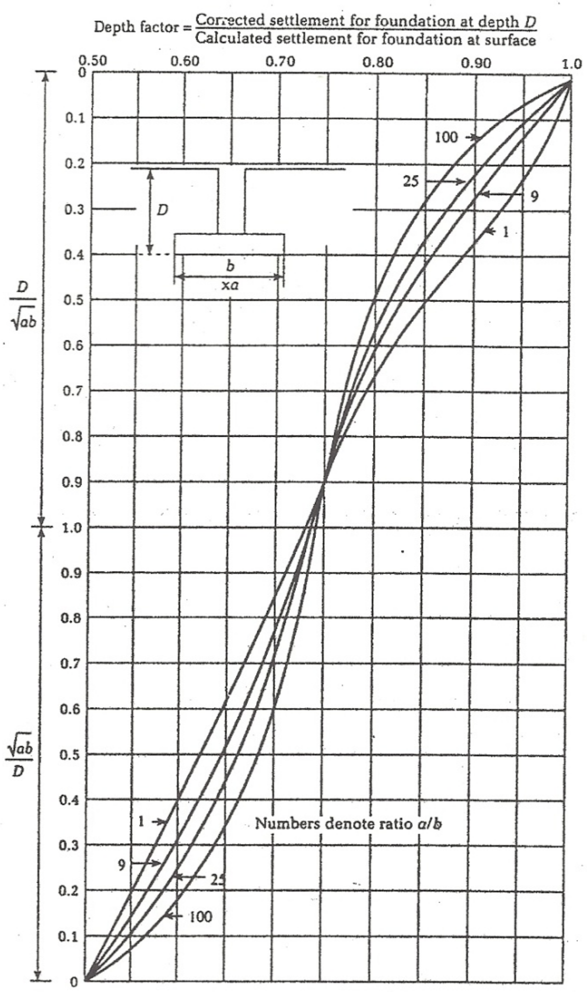


Figure 2.39 Fox's correction curves for elastic settlement of flexible rectangular foundations at depth.

Tomlinson 2001 Found. Design
and Construction

PILED-RAFT SYSTEMS

DATE

PILE-RAFT SYSTEMS

Total load applied to pile cap is shared by piles and pile cap. In this case piles work as settlement reducing members.

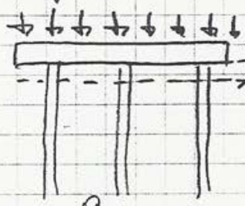
It is a complicated soil-structure interaction problem.

Randolph & Clancy (1993) gives simplified solution:

The overall stiffness of the piled raft system is given by the following expression:

$$k_{pr} = \frac{k_p + (1 - 2\alpha_{rp})\, k_r}{1 - \alpha_{rp}^2 (k_r / k_p)}$$

here it is assumed that average settlements of the piles and the raft are identical in a piled raft.



$$k_{pr} = \frac{q}{s} \qquad s = q / k_{pr}$$

Proportion of the load carried by the raft:

$$\frac{P_r}{\underbrace{P_r + P_p}_{\text{total load}}} = \frac{(1 - \alpha_{rp})\, k_r}{k_p + (1 - 2\alpha_{rp})\, k_r}$$

k_r : relative stiffness of raft

k_p : pile group stiffness.

α_{rp} : interaction factor

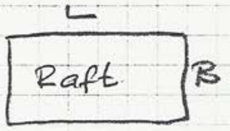
$\alpha_{rp} \cong 0.8$ for most cases.

DATE

ELASTIC SOLUTIONS

$$k_r = \frac{\beta \sqrt{BL}}{1-\nu^2} E_s$$

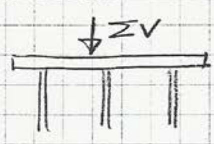
$\frac{L}{B}$	β
1.0	1.10
2.0	1.20
4.0	1.45
6.0	1.65



k_p : consider the piles only.
compute settlement of single pile
compute the settlement of the group

$$k_p = \frac{\text{Load on the group}}{\text{settlement on the group.}}$$

Single pile settlement:



n: number of piles.

P: load on each pile.

$$P = \frac{\Sigma V}{n}$$

Settlement of single pile:

$$\rho_s = \frac{PI}{E_s d}$$

$I = I_o R_v R_k R_b$ previously discussed.

Pile group settlement:

$$\rho_{gr} = \rho_s \times R_s \qquad R_s = \text{settlement ratio}$$

p.122 Table 6.2 & 6.3 Poulos & Davis (1980)
Floating. End bearing

$$k_p = \frac{\Sigma V}{\rho_{gr}} \Rightarrow k_p = \frac{\Sigma V}{\rho_s \cdot R_s}$$

$$R_s = (R_{25} - R_{16})(\sqrt{n} - 5) + R_{25}$$

R_{25}: R_s for 25 piles

R_{16}: R_s for 16 piles

n : no of piles in group

DODİ

DATE

Consider the foundation Lay out in the figure
A 36m square raft is supported by
9x9 group of piles at 4m spacing. Total load acting on the pile group = 97.2 MN. Diameter 0.8m

Soil data	Pile data	Raft data
$E_s = 280\ MPa$	$E_p = 35000\ MPa$	$L = 36$
$\nu_s = 0.4$	$L = 20\ m$	$B = 36$
	$d = 0.8\ m$	

$$k_r = \frac{\beta\sqrt{BL}}{1-\nu^2} E_s$$ (relative stiffness of raft from elastic theory.

$\beta = 1.1$ for $L/B = 1.0$ (square)

$$k_r = \frac{1.1\sqrt{36\times36}\cdot 280\ MPa}{1-0.4^2} = 13200\ MN/m = 13.2\ MN/mm.$$

Pile group stiffness:

Single pile load: $P = $ Total load/no of pile

$P = 97.2/81 = 1.2\ MN$ ($\sim 120t$)

$K = E_p/E_s = 35000/280 = 125$

$L/d = 20/0.8 = 25$

Elastic settlement of single pile

$$\delta = \frac{P\,I}{E_s\,d} \qquad I = I_o R_k R_\nu R_b$$

$I_o = 0.09 \quad R_\nu = 0.95 \quad R_k = 1.9 \quad R_b = 1.0$ (floating pile)

$I = 0.09\times0.95\times1.9\times1.0 = 0.162$

$$\delta = \frac{1.2\times0.162}{280\times0.8} = 8.7\times10^{-4}\,m = 0.87\,mm.$$

DODI

DATE

$S/d = 4/0.8 = 5$ (spacing / diameter)

$L/d = 20 \sim 25$

$K = 125 \sim 100$

Go to table 6.2 of Poulos & Davis

$R_{16} = 3.54 \quad R_{25} = 4.48$

for 81 piles

$R_S = (R_{25} - R_{16})(\sqrt{n} - 5) + R_{25}$

$R_S = (4.48 - 3.54)(\sqrt{81} - 5) + 4.48$

$= 8.24$

settlement of pile group:

$\rho_{gr} = \rho_{single} \times R_S = 0.87 \times 8.24 = 7.17$ mm.

Stiffness of the pile group:

$k_p =$ Total load on group / settlement of group

$k_p = 97.2$ MN / 7.17 mm $= 13.6$ MN/mm.

Then overall stiffness of piled-raft system:

$$K_{pr} = \frac{k_p + (1 - 2\alpha_{rp})\, k_r}{1 - \alpha_{rp}^2 (k_r / k_p)}$$

$k_r = 13.2$ MN/mm

$k_p = 13.6$ MN/mm

$\alpha_{rp} \cong 0.8$

$$= \frac{13.6 + (1 - 2 \times 0.8)\, 13.2}{1 - \overline{0.8}^2 (13.2/13.6)}$$

$= 5.68 / 0.38$

$K_{pr} = 14.95$ MN/mm.

DATE

Load Sharing:

$$\frac{P_r}{P_r + P_p} = \frac{(1-\alpha_{rp})k_r}{k_p + (1-2\alpha_{rp})k_r}$$

P_r = load on raft

P_p = load on piles.

$P_r + P_p = P_t$ = total load.

$$\frac{P_r}{P_r + P_p} = \frac{(1-0.8)(13.2)}{13.6 + (1-2\times0.8)\,13.2} = \frac{2.64}{5.68} = 0.46$$

Conclusion 46% of the load is carried by RAFT

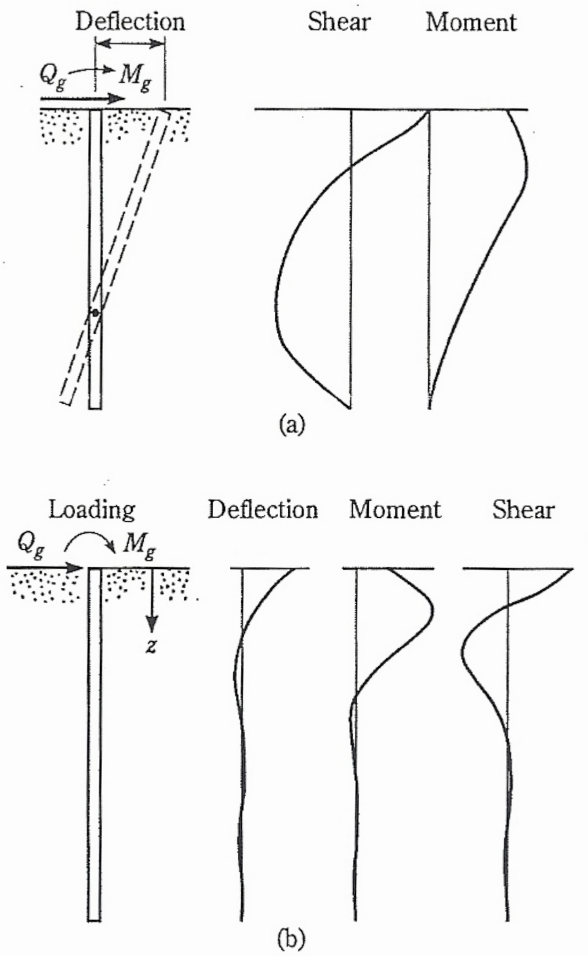
%54 of the load is carried by PILES.

(Pages 21-30 are adopted from Das 1999, Principles of Foundation Engineering)

ANALYSIS OF PILE FOUNDATIONS UNDER LATERAL LOADS

Laterally Loaded Piles

A vertical pile resists lateral load by mobilizing passive pressure in the soil surrounding it (Figure 9.1c). The degree of distribution of the soil reaction depends on (a) the stiffness of the pile, (b) the stiffness of the soil, and (c) the fixity of the ends of the pile. In general, laterally loaded piles can be divided into two major categories: (1) short or rigid piles and (2) long or elastic piles. Figure 9.37a and



▼ FIGURE 9.37 Nature of variation of pile deflection, moment, and shear force for (a) rigid pile, (b) elastic pile

9.37b show the nature of variation of the pile deflection and the moment and shear force distribution along the pile length when subjected to lateral loading. Following is a summary of the solutions presently available for laterally loaded piles.

Elastic Solution

A general method for determining moments and displacements of a vertical pile embedded in a *granular soil* and subjected to lateral load and moment at the ground surface was given by Matlock and Reese (1960). Consider a pile of length L subjected to a lateral force Q_g and a moment M_g at the ground surface ($z = 0$), as shown in Figure 9.38a. Figure 9.38b shows the general deflected shape of the pile and the soil resistance caused by the applied load and the moment.

According to a simpler Winkler's model, an elastic medium (soil in this case) can be replaced by a series of infinitely close independent elastic springs. Based on this assumption,

$$k = \frac{p' \text{ (kN/m or lb/ft)}}{x \text{ (m or ft)}} \tag{9.81}$$

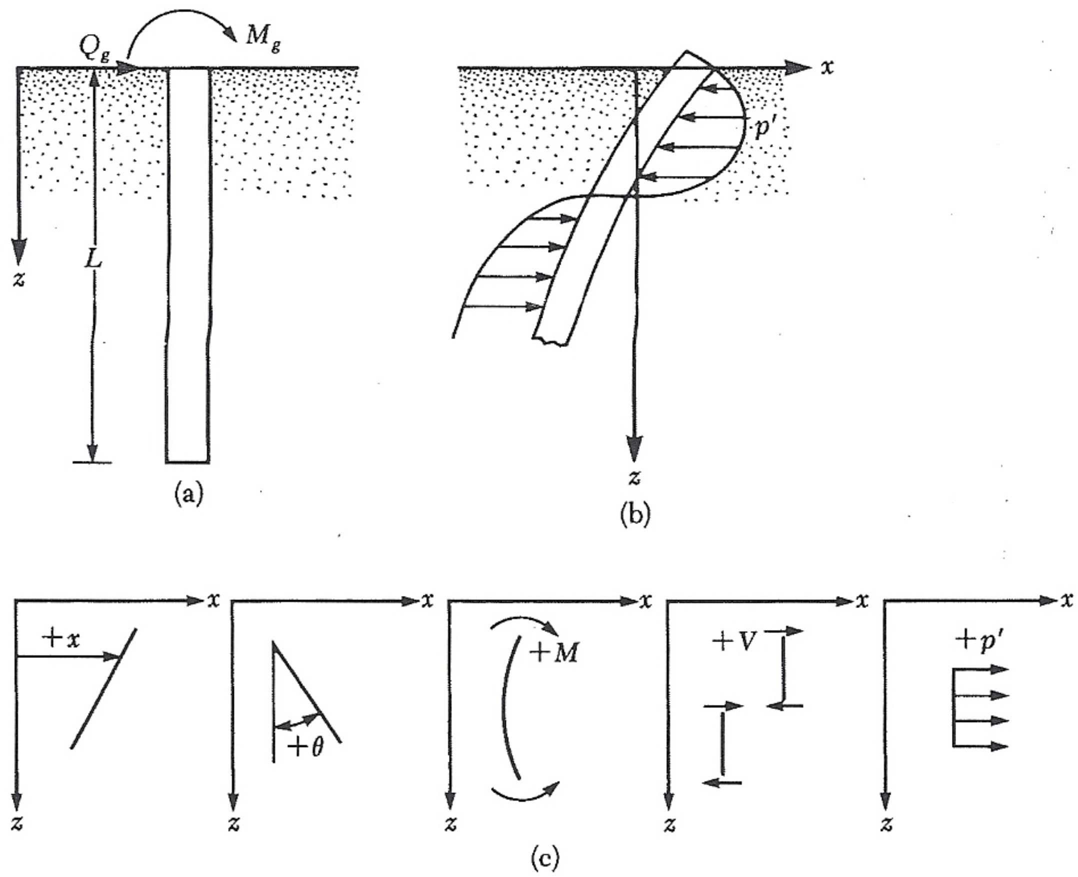


FIGURE 9.38 (a) Laterally loaded pile; (b) soil resistance on pile caused by lateral load; (c) sign conventions for displacement, slope, moment, shear, and soil reaction

where k = modulus of subgrade reaction
p' = pressure on soil
x = deflection

The subgrade modulus for *granular soils* at a depth z is defined as

$$k_z = n_h z \tag{9.82}$$

where n_h = constant of modulus of horizontal subgrade reaction

Referring to Figure 9.38b and using the theory of beams on an elastic foundation, we can write

$$E_p I_p \frac{d^4x}{dz^4} = p' \tag{9.83}$$

where E_p = modulus of elasticity in the pile material
I_p = moment of inertia of the pile section

Based on Winkler's model

$$p' = -kx \tag{9.84}$$

The sign in Eq. (9.84) is negative because the soil reaction is in the direction opposite to the pile deflection.

Combining Eqs. (9.83) and (9.84) gives

$$E_p I_p \frac{d^4x}{dz^4} + kx = 0 \tag{9.85}$$

The solution of Eq. (9.85) results in the following expressions:

Pile Deflection at Any Depth [$x_z(z)$]

$$\boxed{x_z(z) = A_x \frac{Q_g T^3}{E_p I_p} + B_x \frac{M_g T^2}{E_p I_p}} \tag{9.86}$$

Slope of Pile at Any Depth [$\theta_z(z)$]

$$\boxed{\theta_z(z) = A_\theta \frac{Q_g T^2}{E_p I_p} + B_\theta \frac{M_g T}{E_p I_p}} \tag{9.87}$$

Moment of Pile at Any Depth [$M_z(z)$]

$$\boxed{M_z(z) = A_m Q_g T + B_m M_g} \tag{9.88}$$

Shear Force on Pile at Any Depth [$V_z(z)$]

$$\boxed{V_z(z) = A_v Q_g + B_v \frac{M_g}{T}} \tag{9.89}$$

Soil Reaction at Any Depth [$p'_z(z)$]

$$p'_z(z) = A_{p'} \frac{Q_g}{T} + B_{p'} \frac{M_g}{T^2} \tag{9.90}$$

where A_x, B_x, A_θ, B_θ, A_m, B_m, A_v, B_v, $A_{p'}$, and $B_{p'}$ are coefficients
T = characteristic length of the soil–pile system

$$= \sqrt[5]{\frac{E_p I_p}{n_h}} \tag{9.91}$$

n_h has been defined in Eq. (9.82)

When $L \geq 5T$, the pile is considered to be a *long pile*. For $L \leq 2T$, the pile is considered to be a *rigid pile*. Table 9.9 gives the values of the coefficients for long piles ($L/T \geq 5$) in Eqs. (9.86) to (9.90). Note that, in the first column of Table 9.9,

▼ **TABLE 9.9** Coefficients for Long Piles, $k_z = n_h z$

Z	A_x	A_θ	A_m	A_v	A'_p	B_x	B_θ	B_m	B_v	B'_p
0.0	2.435	−1.623	0.000	1.000	0.000	1.623	−1.750	1.000	0.000	0.000
0.1	2.273	−1.618	0.100	0.989	−0.227	1.453	−1.650	1.000	−0.007	−0.145
0.2	2.112	−1.603	0.198	0.956	−0.422	1.293	−1.550	0.999	−0.028	−0.259
0.3	1.952	−1.578	0.291	0.906	−0.586	1.143	−1.450	0.994	−0.058	−0.343
0.4	1.796	−1.545	0.379	0.840	−0.718	1.003	−1.351	0.987	−0.095	−0.401
0.5	1.644	−1.503	0.459	0.764	−0.822	0.873	−1.253	0.976	−0.137	−0.436
0.6	1.496	−1.454	0.532	0.677	−0.897	0.752	−1.156	0.960	−0.181	−0.451
0.7	1.353	−1.397	0.595	0.585	−0.947	0.642	−1.061	0.939	−0.226	−0.449
0.8	1.216	−1.335	0.649	0.489	−0.973	0.540	−0.968	0.914	−0.270	−0.432
0.9	1.086	−1.268	0.693	0.392	−0.977	0.448	−0.878	0.885	−0.312	−0.403
1.0	0.962	−1.197	0.727	0.295	−0.962	0.364	−0.792	0.852	−0.350	−0.364
1.2	0.738	−1.047	0.767	0.109	−0.885	0.223	−0.629	0.775	−0.414	−0.268
1.4	0.544	−0.893	0.772	−0.056	−0.761	0.112	−0.482	0.688	−0.456	−0.157
1.6	0.381	−0.741	0.746	−0.193	−0.609	0.029	−0.354	0.594	−0.477	−0.047
1.8	0.247	−0.596	0.696	−0.298	−0.445	−0.030	−0.245	0.498	−0.476	0.054
2.0	0.142	−0.464	0.628	−0.371	−0.283	−0.070	−0.155	0.404	−0.456	0.140
3.0	−0.075	−0.040	0.225	−0.349	0.226	−0.089	0.057	0.059	−0.213	0.268
4.0	−0.050	0.052	0.000	−0.106	0.201	−0.028	0.049	−0.042	0.017	0.112
5.0	−0.009	0.025	−0.033	0.015	0.046	0.000	−0.011	−0.026	0.029	−0.002

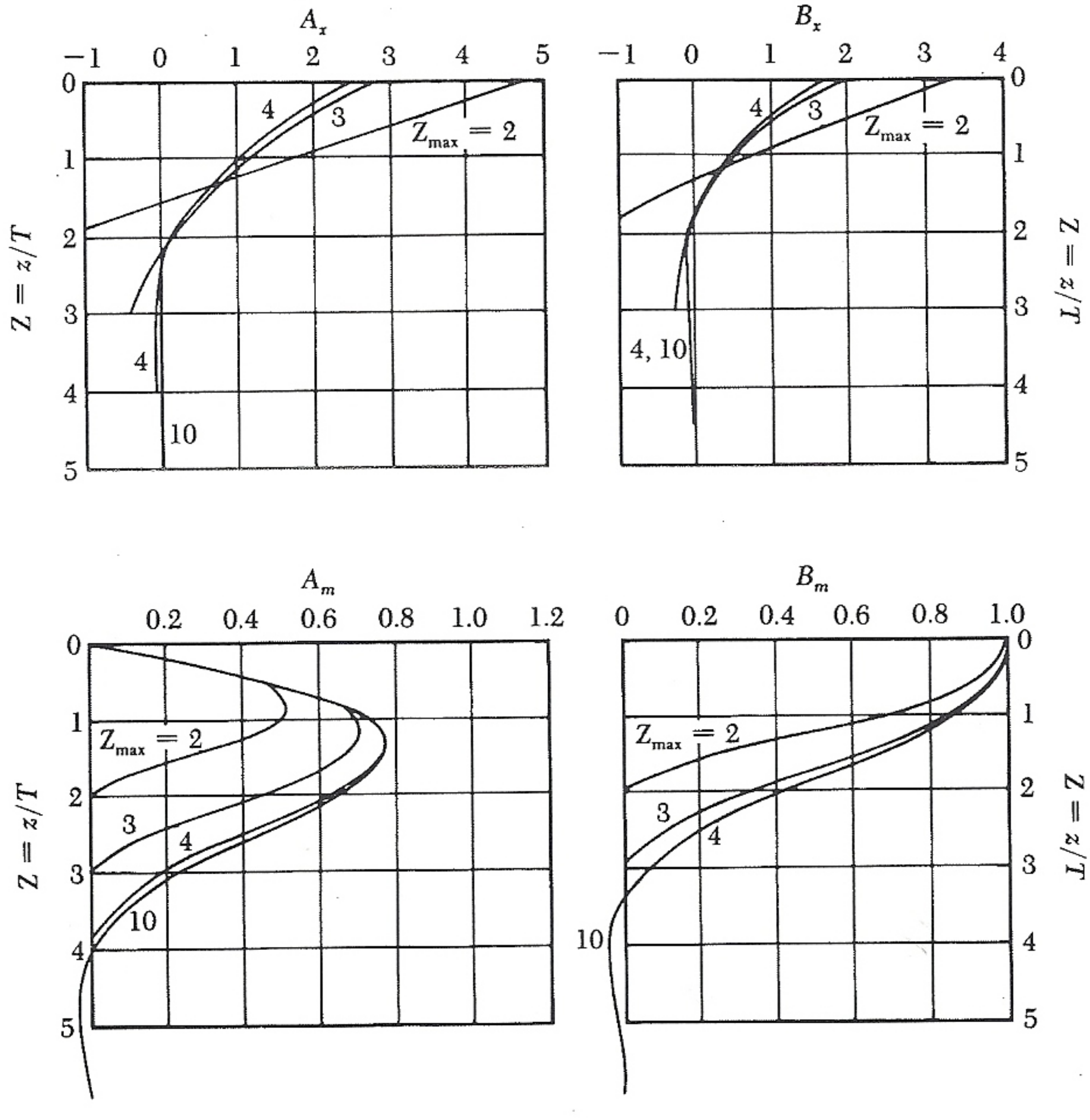
Z is the nondimensional depth, or

$$Z = \frac{z}{T} \tag{9.92}$$

The positive sign conventions for $x_z(z)$, $\theta_z(z)$, $M_z(z)$, $V_z(z)$, and $p'_z(z)$ assumed in the derivations in Table 9.9 are shown in Figure 9.38c. Also, Figure 9.39 shows the variation of A_x, B_x, A_m, and B_m for various values of $L/T = Z_{\text{max}}$. It indicates that, when L/T is greater than about 5, the coefficients do not change, which is true of long piles only.

Calculating the characteristic length T for the pile requires assuming a proper value of n_h. Table 9.10 gives some representative values of n_h.

Elastic solutions similar to those given in Eqs. (9.86)–(9.90) for piles embedded in *cohesive soil* were developed by Davisson and Gill (1963). These relationships are given in Eqs. 9.93–9.97.



▼ **FIGURE 9.39** Variation of A_x, B_x, A_m, and B_m with Z (after Matlock and Reese, 1960)

▼ **TABLE 9.10** Representative Values of n_h

Soil	n_h lb/in³	n_h kN/m³
Dry or moist sand		
Loose	6.5–8.0	1800–2200
Medium	20–25	5500–7000
Dense	55–65	15,000–18,000
Submerged sand		
Loose	3.5–5.0	1000–1400
Medium	12–18	3500–4500
Dense	32–45	9000–12,000

$$\boxed{x_z(z) = A'_x \frac{Q_g R^3}{E_p I_p} + B'_x \frac{M_g R^2}{E_p I_p}} \tag{9.93}$$

and

$$\boxed{M_z(z) = A'_m Q_g R + B'_m M_g} \tag{9.94}$$

where A'_x, B'_x, A'_m, and B'_m are coefficients

$$R = \sqrt[4]{\frac{E_p I_p}{k}} \tag{9.95}$$

The values of the A' and B' coefficients are given in Figure 9.40. Note that

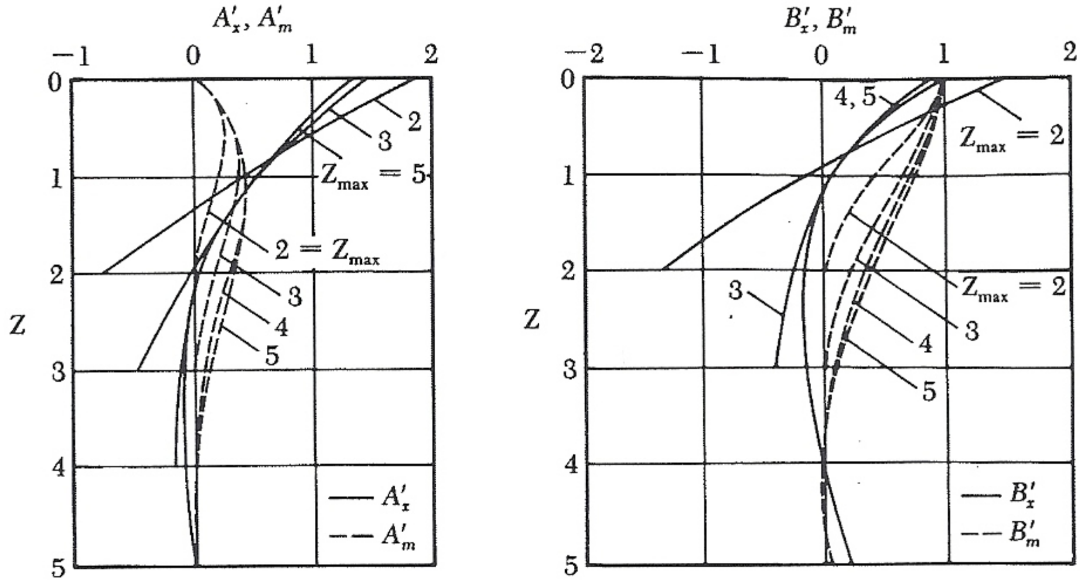
$$Z = \frac{z}{R} \tag{9.96}$$

and

$$Z_{\max} = \frac{L}{R} \tag{9.97}$$

The use of Eqs. (9.93) and (9.94) requires knowing the magnitude of the characteristic length, R. It can be calculated from Eq. (9.95), provided the coefficient of the subgrade reaction is known. For sands, the coefficient of subgrade reaction was given by Eq. (9.82), which showed a linear variation with depth. However, in cohesive soils, the subgrade reaction may be assumed to be approximately constant with depth. Vesic (1961) proposed the following equation to estimate the value of k:

$$k = 0.65 \sqrt[12]{\frac{E_s D^4}{E_p I_p}} \frac{E_s}{1 - \mu_s^2} \tag{9.98}$$



▼ **FIGURE 9.40** Variation of A'_x, B'_x, A'_m, and B'_m with Z (after Davisson and Gill, 1963)

where E_s = modulus of elasticity of soil
D = pile width (or diameter)
μ_s = Poisson's ratio of the soil

Ultimate Load Analysis—Broms' Method

Broms (1965) developed a simplified solution for laterally loaded piles based on the assumptions of (a) shear failure in soil, which is the case for short piles, and (b) bending of the pile governed by plastic yield resistance of the pile section, which is applicable for long piles. Broms' solution for calculating the ultimate load resistance, $Q_{u(g)}$, for *short piles* is given in Figure 9.41a. A similar solution for piles embedded in cohesive soil is shown in Figure 9.41b. In using Figure 9.41a, note that

$$K_p = \text{Rankine passive earth pressure coefficient} = \tan^2\left(45 + \frac{\phi}{2}\right) \tag{9.99}$$

Similarly, in Figure 9.41b,

$$c_u = \text{undrained cohesion} \approx \frac{0.75q_u}{FS} = \frac{0.75q_u}{2} = 0.375q_u \tag{9.100}$$

where FS = factor of safety (=2)
q_u = unconfined compression strength

Figure 9.42 shows Broms' analysis of long piles. In this figure, M_y is the yield moment for the pile, or

$$M_y = SF_Y \tag{9.101}$$

where S = section modulus of the pile section

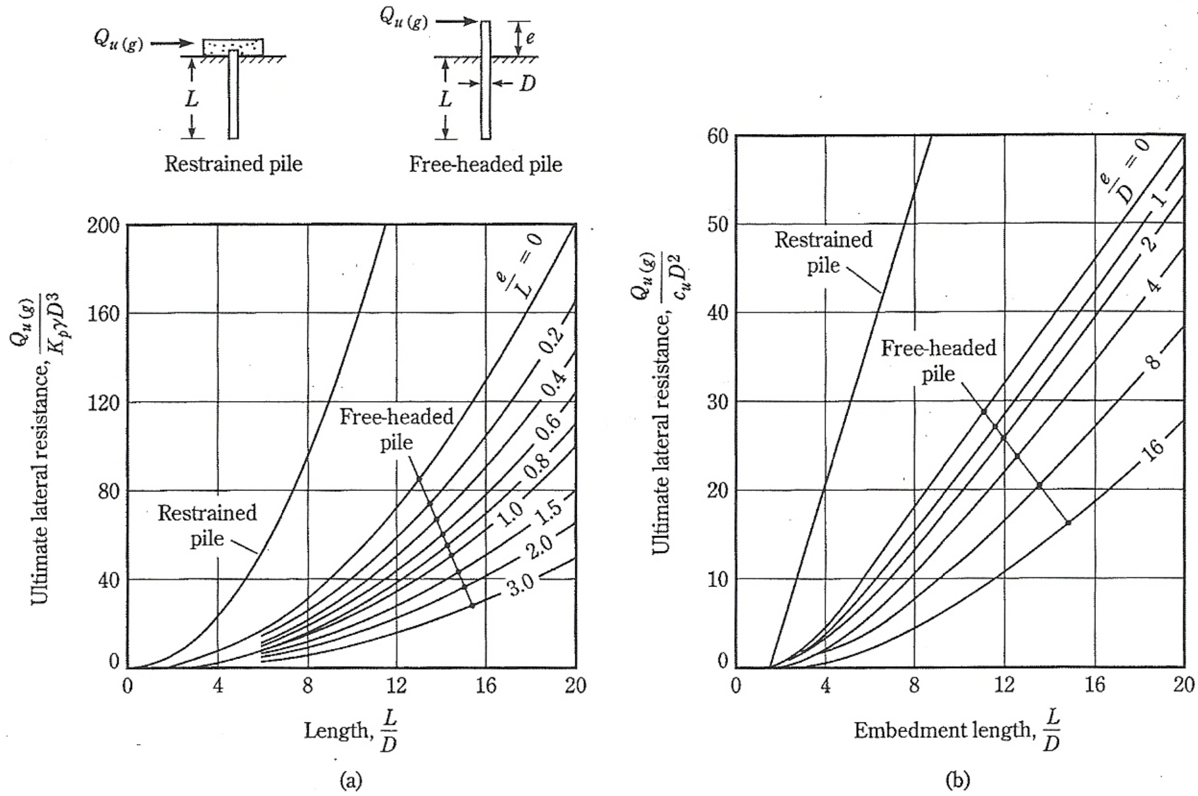


FIGURE 9.41 Broms' solution for ultimate lateral resistance of short piles (a) in sand, (b) in clay

F_Y = yield stress of the pile material

In solving a given problem, both cases (that is, Figure 9.41 and Figure 9.42) should be checked.

The deflection of the pile head, x_o, under working load conditions can be estimated from Figure 9.43. In Figure 9.43a, the term η can be expressed as

$$\eta = \sqrt[5]{\frac{n_h}{E_p I_p}} \tag{9.102}$$

The range of n_h for granular soil is given in Table 9.10. Similarly in Figure 9.43b, which is for clay, the term K is the horizontal soil modulus and can be defined as

$$K = \frac{\text{pressure (lb/in}^2 \text{ or kN/m}^2)}{\text{displacement (in. or m)}} \tag{9.103}$$

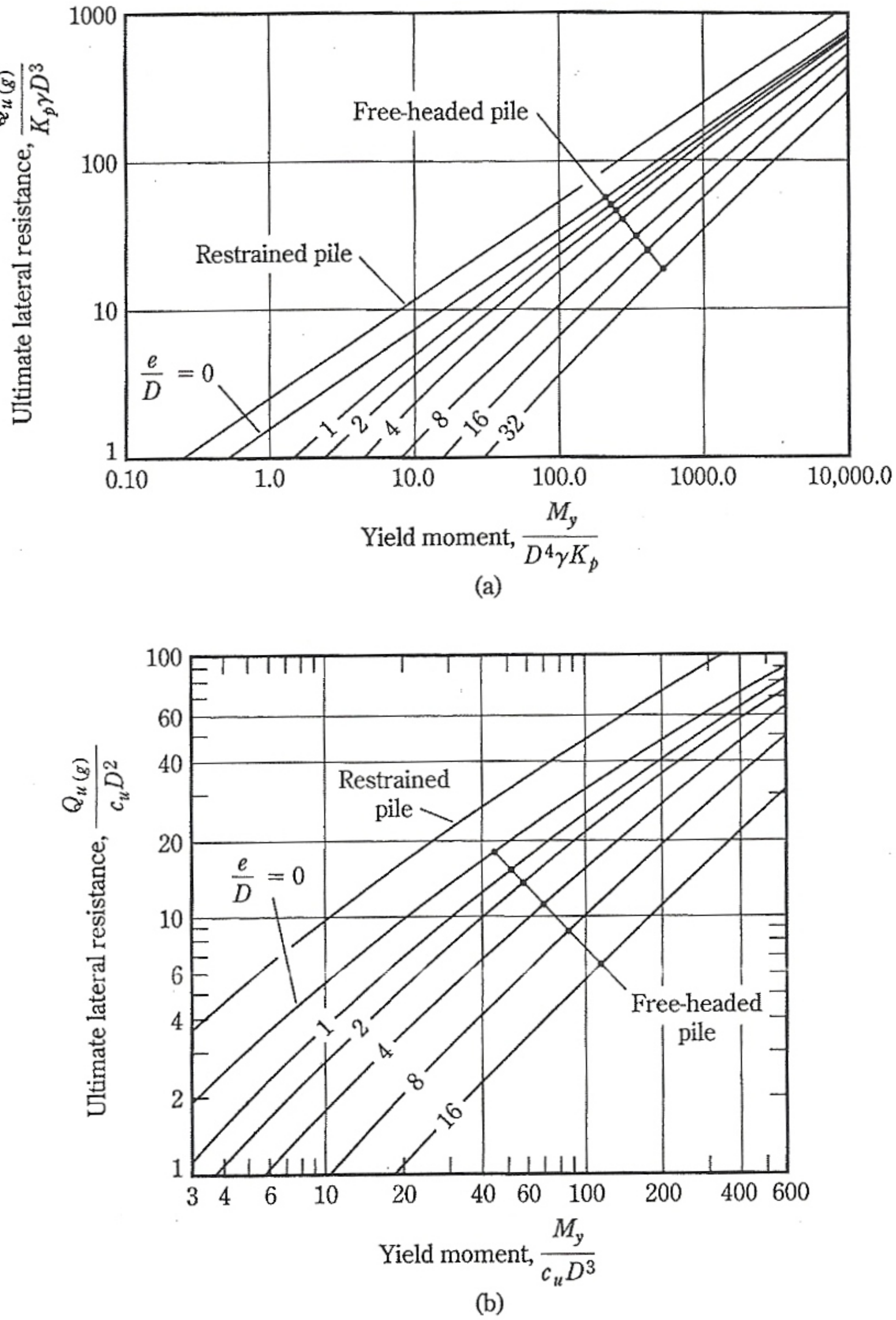


FIGURE 9.42 Broms' solution for ultimate lateral resistance of long piles (a) in sand, (b) in clay

Also, the term β can be defined as

$$\beta = \sqrt[4]{\frac{KD}{4E_p I_p}} \tag{9.104}$$

Note that, in Figure 9.43, Q_g is the working load.

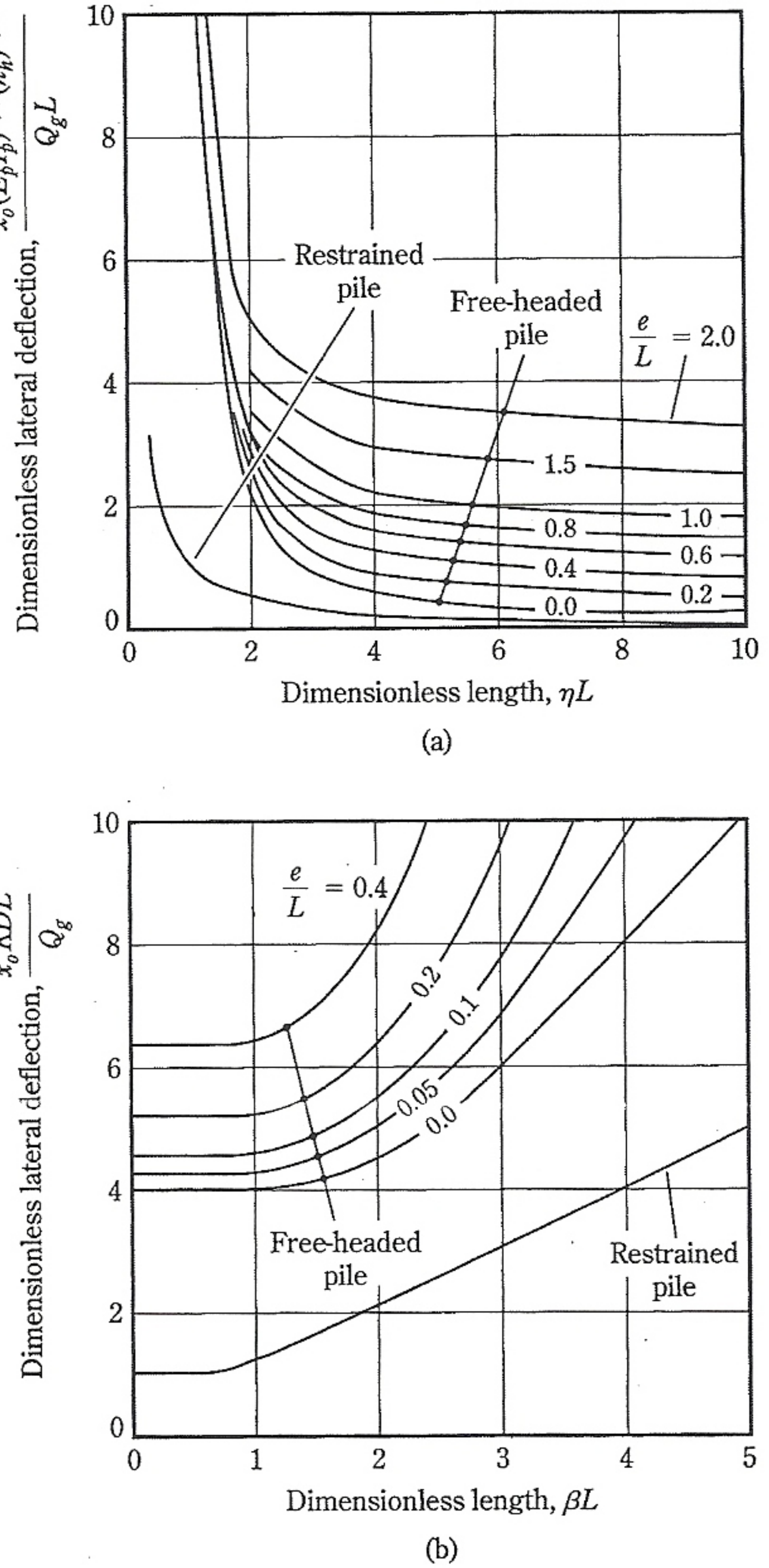


FIGURE 9.43 Broms' solution for estimating deflection of pile head (a) in sand, and (b) in clay

(page 31& 32 is adopted from Prakash and Sharma 1990, Pile Foundations in Engineering Practice)

ULTIMATE LATERAL LOAD RESISTANCE OF PILE GROUPS IN COHESIONLESS SOIL

TABLE 6.2 Group Efficiency G_e for Cohesionless Soils[a]

S/B[b]	G_e
3	0.50
4	0.60
5	0.68
6	0.70

[a]These are interpolated values from graphs provided by Oteo (1972).
[b]S = center-to-center pile spacing.
B = pile diameter or width.

LATERAL DEFLECTION OF PILE GROUPS IN COHESIONLESS SOIL

TABLE 6.6 Group Reduction Factor for the Coefficient of Subgrade Reaction (Davisson 1970)[a]

Pile Spacing in the Direction of Loading	Group Reduction Factor for n_h or k[b]
$3B$	0.25
$4B$	0.40
$6B$	0.70
$8B$	1.00

[a]Also adopted in *Canadian Foundation Engineering Manual*, 1985. *Foundation and Earth Structures, Design Manual 7.2*, NAVFAC, DM 7.2 (1982) also recommends these values.
[b]n_h is applicable for soil modulus linearly increasing with depth, and k is applicable for soil modulus constant with depth.

ULTIMATE LATERAL LOAD RESISTANCE OF PILE GROUPS IN COHESIVE SOIL

Group Efficiency G_e for Cohesive Soils

S/B	**G_e**
3	0.40
4	0.50
5	0.55
6	0.65