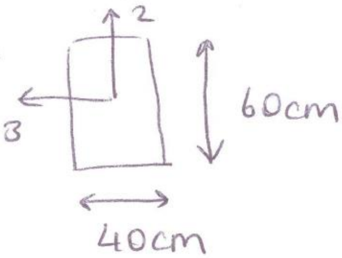
CE 425

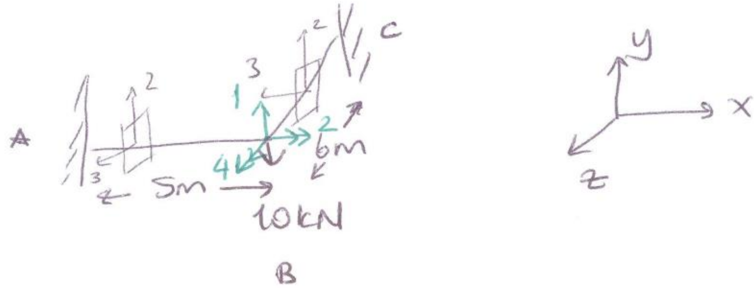
HOMEWORK - 3

SOLUTIONS

Q1) Draw the torsion and moment diagrams for the given structure. $E = 20.000$ MPa. Assume axial rigidity.

Cross Section :





Member AB | Member BC

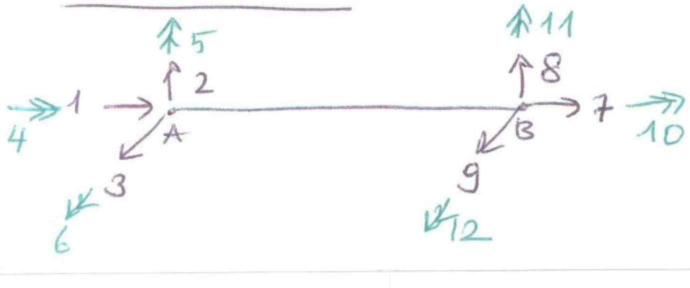
$I_z = \frac{1}{12}(0.4).(0.6)^3 = 7{,}2 \times 10^{-3}\, m^4 = I_x$

$J = 7{,}512 \times 10^{-3}\, m^4$ (from the paper given, see website)

$\nu = 0{,}5$ (assumed) (although it is not a realistic situation, as most of you considered that way chosen to be used)

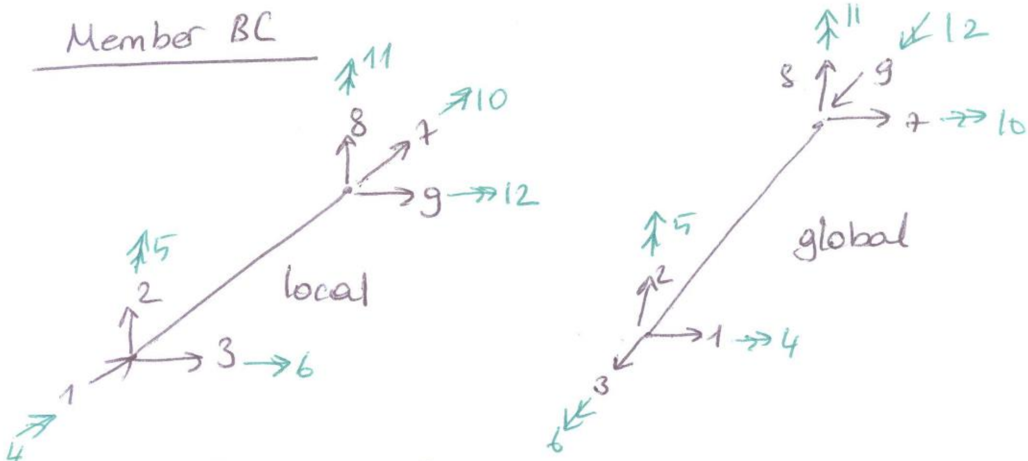
$G = \frac{E}{2(1+\nu)} = 20.000/3 = 6667$ MPa

Member AB



(local and global coordinates are same)

Member BC



$$R = \begin{bmatrix} r & & & 0 \\ & r & & \\ & & r & \\ 0 & & & r \end{bmatrix}$$, will be used for the 12x12 translation.

$$r = \begin{bmatrix} \cos\theta x'x & \cos\theta x'y & \cos\theta x'z \\ \cos\theta y'x & \cos\theta y'y & \cos\theta y'z \\ \cos\theta z'x & \cos\theta z'y & \cos\theta z'z \end{bmatrix}$$

$\theta x'x = 90°$
$= \theta y'x = \theta x'y$
$= \theta y'z = \theta z'y$
$= \theta z'z$
$\theta y'y = \theta z'x = 0$
$\theta x'z = 180°$

$$r = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$k_{BC} = R^{-1} . k_{BC}' . R$$

(However, in this solution local coordinates will be used directly to save time)

$$
k = \begin{bmatrix}
\frac{AE}{L} \\
0 & \frac{12EI_z}{L^3} \\
0 & 0 & \frac{12EI_y}{L^3} \\
0 & 0 & 0 & \frac{GJ}{L} \\
0 & 0 & -\frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} \\
0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L} \\
-\frac{AE}{L} & 0 & 0 & 0 & 0 & 0 & \frac{AE}{L} \\
0 & -\frac{12EI_z}{L^3} & 0 & 0 & 0 & -\frac{6EI_z}{L^2} & 0 & \frac{12EI_z}{L^3} \\
0 & 0 & -\frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{12EI_y}{L^3} \\
0 & 0 & 0 & -\frac{GJ}{L} & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{L} \\
0 & 0 & -\frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0 & 0 & 0 & \frac{6I_y}{L^2} & 0 & \frac{4EI_y}{L} \\
0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} & 0 & -\frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L}
\end{bmatrix}^T
$$

$$
k_{AB} = \begin{array}{cccccccccccc|cc}
 & & & & & & & 1 & & 2 & & 3 & & \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & & \\
\hline
 & & & & & & & & & & & & 1 & \\
 & & & & & & & & & & & & 2 & \\
 & & & & & & & & & & & & 3 & \\
 & & & & & & & & & & & & 4 & \\
 & & & & & & & & & & & & 5 & \\
 & & & & & & & & & & & & 6 & \\
 & & & & & & & & & & & & 7 & \\
 & & & & & & & \times & & \times & & \times & 8 & 1 \\
 & & & & & & & & & & & & 9 & \\
 & & & & & & & \times & & \times & & \times & 10 & 2 \\
 & & & & & & & & & & & & 11 & \\
 & & & & & & & \times & & \times & & \times & 12 & 3
\end{array}
$$

$$k'_{BC} = \begin{array}{c} \begin{array}{cccccccccccc} & 1 & & 2 & & 3 & & & & & & \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \end{array} \\ \left[\begin{array}{cccccccccccc} & & & & & & & & & & & \\ & \times & & \times & & \times & & & & & & \\ & & & & & & & & & & & \\ & \times & & \times & & \times & & & & & & \\ & & & & & & & & & & & \\ & \times & & \times & & \times & & & & & & \\ & & & & & & & & & & & \\ & & & & & & & & & & & \\ & & & & & & & & & & & \\ & & & & & & & & & & & \\ & & & & & & & & & & & \\ & & & & & & & & & & & \end{array}\right] \end{array} \begin{array}{ll} 1 & \\ 2 & 1 \\ 3 & \\ 4 & 2 \\ 5 & \\ 6 & 3 \\ 7 & \\ 8 & \\ 9 & \\ 10 & \\ 11 & \\ 12 & \end{array}$$

$$K_T = \begin{bmatrix} \frac{12EI_z}{L_1^3} + \frac{12EI_z}{L_2^3} & & \text{sym.} \\ \frac{6EI_z}{L_2^2} & \frac{GJ}{L_1} + \frac{4EI_z}{L_2} & \\ -\frac{6EI_z}{L_1^2} & 0 & \frac{4EI_z}{L_1} + \frac{GJ}{L_2} \end{bmatrix}$$

$$K_T = \begin{bmatrix} 21824 & & \text{sym.} \\ 24000 & 106016 & \\ -34560 & 0 & 123547 \end{bmatrix} \qquad F = \begin{bmatrix} -10 \\ 0 \\ 0 \end{bmatrix}$$

$$D = \begin{bmatrix} -0.0015 \\ 0.00034 \\ -0.00042 \end{bmatrix} \begin{array}{l} m \\ rad \\ rad \end{array}$$

$$M_{AB} = \frac{2EI_z}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$= \frac{2.20.000.\,7{,}2}{5}\left(-0.00042 + \left(\frac{3.0.0015}{5}\right)\right)$$

$$= 27{,}65\ kN.m$$

$$M_{BA} = \frac{2.20.000.\,7{,}2}{5}\left(2.(-0.00042) + \frac{3\times0.0015}{5}\right)$$

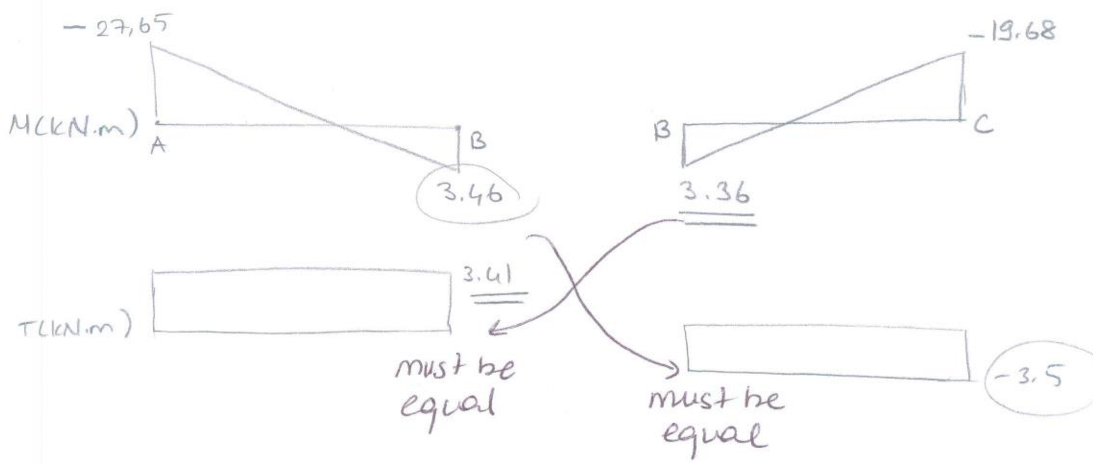
$$= 3.46\ kN.m$$

$$T = \frac{GJ}{L}\cdot\alpha = \frac{6667\times1000\times7{,}512\times10^{-3}}{5}\cdot 0{,}00034 = 3{,}41\ kN.m$$

$$M_{BC} = \frac{2.20.000.\,7{,}2}{6}\left(2.\,0{,}00034 - \frac{3\times0.0015}{6}\right) = -3.36\ kN.m$$

$$M_{CB} = \frac{2.20.000.\,7{,}2}{6}\left(0{,}00034 - \frac{3\times0.0015}{6}\right) = -19.68\ kNm$$

$$T = \frac{6667\times7{,}512}{6}\times -0.00042 = -3.5\ kN.m$$



(since they are nearly same, acceptable ✓)