1)

a) $\lambda = lnx_{median} = 8.163$

$\xi = \sqrt{\ln(1+\delta^2)} = 0.34$

b) $P(C > a) = 0.10$

$1 - P(C \leq a) = 0.10$

$P(C \leq a) = 0.90$

$$P\left(\frac{lnC - \lambda}{\xi} < \frac{lna - 8.163}{0.34}\right) = 0.90 \;\Rightarrow\; \frac{lna - 8.163}{0.34} = 1.282$$

$$a = 5425.6\, mm$$

c) $P(C > 800) = P\left(\frac{lnC - \lambda}{\xi} < \frac{ln\,800 - 8.163}{0.34}\right) = P(z > -4.35) \cong 1$

Therefore, it is almost sure that rainfall intensity exceeding 800 mm in a given year can be observed.

d) $\left.\begin{array}{l} \xi = \sqrt{\ln(1+\delta^2)} = 0.34 \\ \lambda = ln\mu - \frac{\xi^2}{2} = 8.163 \end{array}\right\}$ Therefore, the parts a, b and c have the same answers.

2)

a) $\lambda = 1\; for\; 8\; years \;\Rightarrow\; \lambda = 1.25\; for\; 10\; years$

$P(No\;Flood) = P(F = 0) = \frac{e^{-\lambda}\lambda^n}{n!} = \frac{e^{-1.25}1.25^0}{0!} = 0.287$

$P(F = 1) = \frac{e^{-\lambda}\lambda^n}{n!} = \frac{e^{-1.25}1.25^1}{1!} = 0.358$

$P(F > 3) = 1 - P(F = 0) - P(F = 1) - P(F = 2) - P(F = 3)$

$= 1 - \frac{e^{-1.25}1.25^0}{0!} - \frac{e^{-1.25}1.25^1}{1!} - \frac{e^{-1.25}1.25^2}{2!} - \frac{e^{-1.25}1.25^3}{3!}$

$= 1 - 0.287 - 0.358 - 0.224 - 0.093$

$= 0.038$

b) $P(S \cap F = 0) = P(F = 0) * P(S/F = 0) = 0.287 * 1 = 0.287$

$P(S \cap F = 1) = P(F = 1) * P(S/F = 1) = 0.358 * 0.95 = 0.34$

$P(S \cap F = n) = P(F = n) * P(S/F = n) = \frac{e^{-\lambda}\lambda^n}{n!} * 0.95^n = \frac{e^{-1.25}1.25^n}{n!} * 0.95^n = \frac{0.287 * 1.188^n}{n!}$

3)

a) $P(Acceptable) = P(x = 4) + P(x = 5)$

$= \binom{5}{4} * 0.80^4 * 0.20^1 + \binom{5}{5} * 0.80^5 * 0.20^0$

$= 0.4096 + 0.328$

$= 0.738$

b) $P(Acceptable) = P(x = 4) + P(x = 5)$

$= \binom{5}{4} * p^4 * (1-p)^1 + \binom{5}{5} * p^5 * (1-p)^0$

$= 5 * (p^4 - p^5) + p^5 = -4p^5 + 5p^4$

$$P(Acceptable) = -4p^5 + 5p^4 = 0.80$$

$$p_1 = -0.575 < 0$$

$$p_2 = 0.831$$

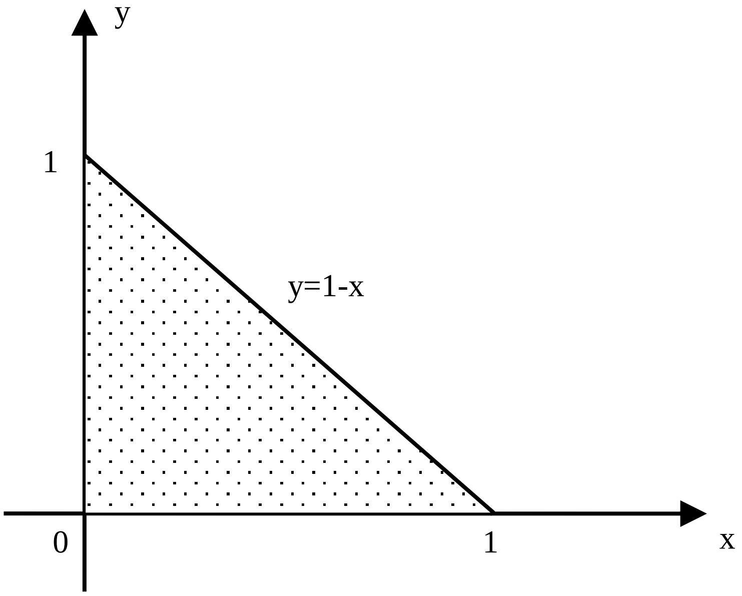
$$p_3 = -0.065 + 0.606i\ (Not\ real)$$

$$p_4 = -0.065 - 0.606i\ (Not\ Real)$$

$$p_5 = 1.125 > 1$$

Therefore, the probability of each sample must be 0.831.

4) $f_{xy}(x,y) = \begin{cases} c & for\ 0 \le y \le 1-x \le 1 \\ 0 & otherwise \end{cases}$



a) $F_{xy} = \int_0^1 \int_0^{1-x} c * dy * dx = \int_0^1 c * y\Big|_0^{1-x} dx = \int_0^1 c * (1-x) * dx = \frac{-c*x^2}{2} + cx\Big|_0^1 = \frac{-c}{2} + c = 1 \Rightarrow c = 2$

b) $f_x(x) = \int_0^{1-x} f_{xy} * dy = \int_0^{1-x} 2 * dy = 2 - 2x \quad for\ 0 < x < 1$

$f_y(y) = \int_0^{1-y} f_{xy} * dx = \int_0^{1-y} 2 * dy = 2 - 2y \quad for\ 0 < y < 1$

c) $f_{xy}(x,y) = 2 \neq f_x(x) * f_y(y) = (2-2x) * (2-2y) = 4xy - 4x - 4y + 4$

Therefore, x and y are not statistically independent.

d) $E(x) = \int_0^1 x * f_x(x) * dx = \int_0^1 x * (2-2x) * dx = 2 * \left(\frac{-x^3}{3} + \frac{x^2}{2}\right)\Big|_0^1 = 0.333$

$E(y) = \int_0^1 y * f_y(y) * dy = \int_0^1 y * (2-2y) * dy = 2 * \left(\frac{-y^3}{3} + \frac{y}{2}\right)\Big|_0^1 = 0.333$

$E(xy) = \int_0^1 \int_0^{1-x} x * y * f_{xy}(x,y) * dy * dx = \int_0^1 \int_0^{1-x} x * y * 2 * dy * dx = \int_0^1 2 * x * \frac{y^2}{2}\Big|_0^{1-x} * dx$

$= 2 * \int_0^1 \frac{x*(1-x)^2}{2} * dx$

$= \left(\frac{x^2}{2} - \frac{2x^3}{3} + \frac{x^4}{4}\right)\Big|_0^1$

$= 0.083$

$cov(xy) = E(xy) - E(x) * E(y) = 0.083 - 0.333 * 0.333 = -0.0278$

Hint:

$$cov(xy) = \int_0^1 \int_0^{1-x} (x-\mu_x) * (y-\mu_y) * f_{xy}(x,y) * dy * dx = \int_0^1 \int_0^{1-x} (x-0.333) * (y-0.333) * 2 * dy * dx$$
$$= \int_0^1 (x-0.5) * (y^2 - 0.666y)\Big|_0^{1-x} * dx$$
$$= \int_0^1 (x-0.5) * [(1-x)^2 - 0.666 * (1-x)] * dx$$
$$= \frac{x}{6000} * (1500x^3 - 3668x^2 + 3003x - 1002)\Big|_0^1$$
= -0.0278

$$E(x^2) = \int_0^1 x^2 * f_x(x) * dx = \int_0^1 x^2 * (2-2x) * dx = 2 * \left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1 = 0.167$$
$$E(y^2) = \int_0^1 y^2 * f_y(y) * dy = \int_0^1 y^2 * (2-2y) * dy = 2 * \left(\frac{y^3}{3} - \frac{y^4}{4}\right)\Big|_0^1 = 0.167$$
$$V(x) = E(x^2) - (E(x))^2 = 0.167 - 0.333^2 = 0.056$$
$$V(y) = E(y^2) - (E(y))^2 = 0.167 - 0.333^2 = 0.056$$
$$\sigma(x) = \sqrt{V(x)} = \sqrt{0.056} = 0.237$$
$$\sigma(y) = \sqrt{V(y)} = \sqrt{0.056} = 0.237$$

$$\rho = \frac{cov(xy)}{\sigma(x) * \sigma(y)} = \frac{-0.0278}{0.237 * 0.237} = -0.49$$

Therefore, there is no strong linear correlation between x and y. Moreover, it can be inferred that y is inversely proportional to x as the correlation coefficient has a negative sign.

e) $f_{x/y}(x/y) = \frac{f_{xy}(x,y)}{f_y(y)} = \frac{2}{2-2y} = \frac{1}{1-y} \quad for\ 0 < x < 1\ and\ 0 < y < 1-x$

f) $E(x/y) = \int_0^{1-y} x * f_{x/y}(x/y) * dx = \int_0^x x * \frac{1}{1-y} * dx = \frac{x^2}{2*(1-y)}\Big|_0^{1-y} = \frac{1-y}{2}\ for\ 0 < y < 1$

5)

a)

y \ x	0	1	2	$P_y(y)$
0	1/3	0	1/3	2/3
1	0	1/3	0	1/3
$P_x(x)$	1/3	1/3	1/3	1

$$E(x) = \sum x * P_x(x) = 0 * \frac{1}{3} + 1 * \frac{1}{3} + 2 * \frac{1}{3} = 1$$
$$E(y) = \sum y * P_y(y) = 0 * \frac{2}{3} + 1 * \frac{1}{3} = 0.333$$
$$E(xy) = \sum x * y * P_{xy}(x,y) = 0 * 0 * \frac{1}{3} + 0 * 1 * 0 + 0 * 2 * \frac{1}{3} + 1 * 0 * 0 + 1 * 1 * \frac{1}{3} + 1 * 2 * 0 = 0.333$$
$$cov(xy) = E(xy) - E(x) * E(y) = 0.333 - 1 * 0.333 = 0$$
$$E(x^2) = \sum x^2 * P_x(x) = 0^2 * \frac{1}{3} + 1^2 * \frac{1}{3} + 2^2 * \frac{1}{3} = 1.667$$
$$E(y^2) = \sum y^2 * P_y(y) = 0^2 * \frac{2}{3} + 1^2 * \frac{1}{3} = 0.333$$

$V(x) = E(x^2) - (E(x))^2 = 1.667 - 1^2 = 0.667$
$V(y) = E(y^2) - (E(y))^2 = 0.333 - 0.333^2 = 0.222$
$\sigma(x) = \sqrt{V(x)} = \sqrt{0.667} = 0.817$
$\sigma(y) = \sqrt{V(y)} = \sqrt{0.222} = 0.471$

$$\rho = \frac{cov(xy)}{\sigma(x) * \sigma(y)} = \frac{0}{0.817 * 0.471} = 0$$

Therefore, there is no correlation between x and y.

b) For x=0 and y=0,

$$P_x(x = 0) * P_y(y = 0) = \frac{1}{3} * \frac{2}{3} = \frac{2}{9} \neq P_{xy}(0,0) = \frac{1}{3}$$

Therefore, x and y are not statistically independent.

c) $P_{y/x}(y/x) = \frac{P_{xy}(x,y)}{P_x(x)}$

y \ x	0	1	2
0	1	0	1
1	0	1	0

The conditional probability values of $P_{y/x}(y/x)$

For x=0,

$$E(y/x) = \sum y * P_{y/x}(y/x) = 0 * 1 + 1 * 0 = 0$$

For x=1,

$$E(y/x) = \sum y * P_{y/x}(y/x) = 0 * 0 + 1 * 1 = 1$$

For x=2,

$$E(y/x) = \sum y * P_{y/x}(y/x) = 0 * 1 + 1 * 0 = 0$$

6)

a) $F_x(x) = \int_0^1 (cx + 0.5) * dx = \left(\frac{cx^2}{2} + 0.5x\right)\Big|_0^1 = \frac{c}{2} + 0.5 = 1 \Rightarrow c = 1$

b) $F_x(x) = \int_0^x (x + 0.5) * dx = \left(\frac{x^2}{2} + 0.5x\right)\Big|_0^x = \frac{x^2}{2} + 0.5x \; for \; 0 < x < 1$

$y = x^2 \Rightarrow x = \sqrt{y}$

$F_y(y) = \frac{(\sqrt{y})^2}{2} + 0.5\sqrt{y} = 0.5y + 0.5\sqrt{y} \quad for \; 0 < y < 1$

$f_y(y) = \frac{dF_y(y)}{dy} = \frac{0.25}{\sqrt{y}} + 0.5 \quad for \; 0 < y < 1$

$E(y) = \int_0^1 y * f_y(y) * dy = \int_0^1 y * \left(\frac{0.25}{\sqrt{y}} + 0.5\right) * dy = \left(\frac{y^{1.5}}{6} + \frac{y^2}{4}\right)\Big|_0^1 = 0.417$

7)

a) $M_A = 2S + P$

$E(M_A) = 2E(S) + E(P) = 2 * 10 + 5 = 25\ kN$

$V(M_A) = 4 * V(S) + V(P) + 2 * 2 * 1 * \sigma_S * \sigma_P * \rho_{PS} = 4 * 1^2 + 1^2 + 2 * 2 * 1 * 1 * 1 * 0.4 = 6.6\ kN^2$

b) $N_R(30\ kN, 3\ kN)$

$$P(Failure) = P(R < M_A) = P(R - M_A < 0) = P(F < 0)$$

$E(F) = E(R) - E(M_A) = 30 - 25 = 5\ kN$

$V(F) = V(R) + V(M_A) = 3^2 + 6.6 = 15.6\ kN^2$

$$P(F < 0) = P\left(\frac{F - \mu_F}{\sigma_F} < \frac{0 - 5}{\sqrt{15.6}}\right) = P(z < -1.266) = 1 - P(z < 1.266) = 1 - 0.8973 = 0.1027$$

c) $P(Failure) = P(R < 2 * S) = 0.01$

$P\left(\frac{R - \mu_R}{\sigma_R} < \underbrace{\frac{2 * S - 30}{3}}_{a}\right) = 0.01$

From normal distribution table, a=-2.326

$$\frac{2 * S - 30}{3} = -2.326$$

$$S = 11.51\ kN$$

8)

a) $\theta = \frac{1}{\mu} = \frac{1}{20} = 0.05$

$P(x < 5) = \int_0^5 \theta e^{-\theta x}\, dx = -e^{-\theta x} \Big|_0^5 = -e^{-0.05*5} + e^{-0.05*0} = 0.221$

b) $Probability\ of\ at\ most\ one\ repair = P(x > 5)^3 + 3 * P(x < 5) * P(x > 5) * P(x > 5)$

$= (1 - 0.221)^3 + 3 * 0.221 * (1 - 0.221) * (1 - 0.221)$

$= 0.473 + 0.402$

$= 0.875$