MIDDLE EAST TECHNICAL UNIVERSITY
FACULTY OF ENGINEERING

UNCERTAINTY and DATA ANALYSIS
Spring Semester 2009-10

Homework No: 2- Date Due: 1.04.2010

Please, solve only six of the following questions.

1) Strong wind at a particular site may come from any direction between due east ($\theta = 0^{o}$) and due north ($\theta = 90^{o}$). All values of wind speed V are possible.

a) Sketch the sample space for wind speed and direction.

b) Let $A = \{V > 30\ \text{km/hr}\}$; $B = \{28 < V \leq 50\ \text{km/hr}\}$; $C = \{\theta \leq 30^{o}\}$

Identify the events A, B, C and $\overline{A}$ in the sample space sketched in part (a).

b) Use new sketches to identify the following events:

$$D = A \cap C = AC;\ E = A \cup B;\ F = A \cap B \cap C$$

c) Are the events D and E are mutually exclusive? How about events A and C?

2) Ministry of Public Works evaluates the feasibility of preliminary construction projects. In the past, 95 % of highly successful projects were evaluated as feasible, 60 % of the moderately successful projects and 10% of the very poor projects were also evaluated as feasible.
According to previous records of Chamber of Civil Engineers only 40% of the preliminary construction projects were highly successful, 35% were moderately successful and 25% have been poor projects.

a) What is the probability that a project will be evaluated as feasible?

b) If a project is not evaluated as feasible what is the probability that it will be a highly successful project?

3) You are supervising the erection of a precast concrete water tower. The lifting of the heavy concrete segments into position will require the use of a crane. The erection is planned to be started and completed on Tuesday. Weather conditions affect the speed at which the segments can be lifted and put in position. The erection can't be completed in one single day in bad weather. The chance that the erection of the tower can be completed in one day in good weather is 70%. The chance that the weather will be bad on Tuesday is 20%.
The crane can be rented from the local company "VINC" for daily use. Since there is only one crane rental company in town and three construction companies that are equally likely to rent the crane on any given day, it is not available everyday. Also, for safety, the crane rental company only rents it when the weather is good.

a) What is the probability that your company will not be able to use the crane on Tuesday?

b) Determine the probability that the tower erection can be started and completed, on Tuesday?

c) What is the probability that tower erection will last longer than one day?

4) In a typical month, the demand for cement material in a city may be low (L), average (A), or high (H) with respective likelihoods of 6: 3: 1. The various suppliers of cement material in the city can definitely handle a low demand (L), but if the is average (A) or high (H), the supply may be inadequate with probabilities of 0.10 and 0.50, respectively.

a) What is the probability of shortage of cement material in a given month?

b) If shortage occurred in a month, what is the probability that the demand had been average?

c) What is the probability of shortage of cement in at least one month over two month period? Assume that the demand and supply of cement material are statistically independent between consecutive months.

5) A system contains two components A and B, connected in parallel as shown in the following diagram. Assume components A and B function independently. For the system to function either A and B must function.

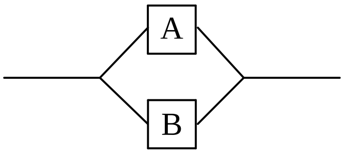


Diagram for part (a), (b) and (c)

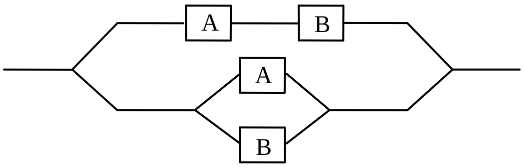


Diagram for part (b)

a) If the probability A fails is 0.08 and the probability that B fails is 0.12, find the probability that the system functions.
b) If both A and B have probability p of failing, what must be the value of p so that the probability that the system functions is 0.99?
c) If components function independently and each component has a probability 0.5 of failing, what is the minimum number of components that must be connected in parallel so that the probability that the system function is at least 0.99?
d) Consider now a system with four components as shown in the above diagram. Assume the same failure probabilities for components A and B as before and assume that all of the components function independently. What is the probability that the system functions?

6) The total load, X (in kN), on roof of a building has the following probability density function (pdf) :

$$f_X(x) = \begin{cases} \dfrac{c}{x^2} & 3 \le x \le 6 \\ 0 & \text{elsewhere} \end{cases}$$

a) Determine the constant "c" so that $f_X(x)$ is a proper pdf.
b) Determine and plot cumulative distribution function (cdf) of X, i.e., $F_X(x)$.
c) What is the expected load?
d) Find the median and mode of the load.
e) Determine the coefficient of variation of the load?
e) Suppose that the roof can carry only 5.5 kN before collapse. What is the probability that the roof will collapse?

7) The maximum load S (in kN) on a structure is modelled by a continuous random variable S whose cumulative probability distribution function is as given below:

$$F_S(s) = \begin{cases} 0 & \text{for } s \le 0 \\ -\dfrac{s^3}{864} + \dfrac{s^2}{48} & \text{for } 0 \le s \le 12 \\ 1 & \text{for } s > 12 \end{cases}$$

a) Determine the mode and mean value of S.
b) The strength R of a structure can be modelled by a discrete random variable with the following probability mass function (pmf):

$$P_R(r) = \begin{cases} 0.75 & \text{for } r = 10\,\text{kN} \\ c & \text{for } r = 13\,\text{kN} \\ 0 & \text{otherwise.} \end{cases}$$

Determine the constant c so that $P_R(r)$ is a proper pmf and then find the probability of failure, i.e., the probability that load S is greater than the strength R.

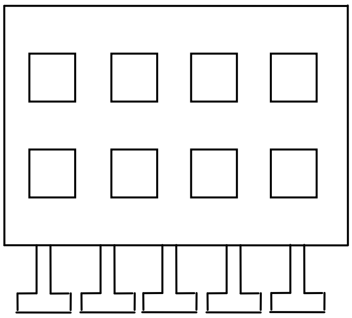
8) A study assumes that the time spent (T) on commercials per hour on a TV channel is a random variable normally distributed with mean μ = 12 minutes and coefficient of variation δ = 0.2 during a randomly selected hour

a) What is the probability that T is between 10 and 15 minutes?
b) What are the probabilities that i) T will differ from its mean value by less than 3 (i.e. $P\ (|T-\mu|\leq 3)$) and ii) T will differ from its mean value by more than 2 (i.e. $P\ (|T-\mu|\geq 2)$).
c) Find $P(T > 12 \mid 8 < T < 16)$.
d) Find t so that $P(T < t) = 0.15$.
e) Is assuming time spent on commercials as a normal random variable reasonable?
Why yes and why not? What types of uncertainties are involved in the variable T?

9) A temporary flood protection structure has been built to protect a township (a subdivision of a country) during relocation of that township. The relocation will take five years. The structure was built according to a 25 - year design flood magnitude.

a) What is the probability that the structure will be overtopped for the first time during the five-year period of relocation of the township?
b) What is the probability that the structure will be overtopped exactly twice during the relocation of the township?
c) What is the probability that the structure will be overtopped for the first time in the fifth year? What is the probability that no overtopping will take place during the first four years? Why are the two answers different?

10) A hospital building is supported by **five** identical individual (and **statistically independent**) footings as shown in the figure. The capacity of each footing to resist the lateral forces is a **lognormal** distribution with parameters λ and ξ as **4.60 kN** and **2.38 kN**, respectively. The maximum lateral capacity of the footing is **150 kN**. During the lifetime of the building, if one of the footings fail, the building will get a "**Yellow Flag**" which requires the hospital to be closed for 60 days for maintenance. The probability that the hospital will be used after maintenance is **0.5**. During the lifetime of the building, if two or more footings fail, the building will get a "**Red Flag**" and will be closed.



a) Calculate the probability of failure for a footing.
b) Calculate the probability that the building will get "Yellow Flag".
c) Calculate the probability that the building will get "Red Flag".
d) Calculate the probability that the building will be out of service.