Additional Example on Solving Nonlinear System of Equations

Solve the following set of nonlinear equations using;

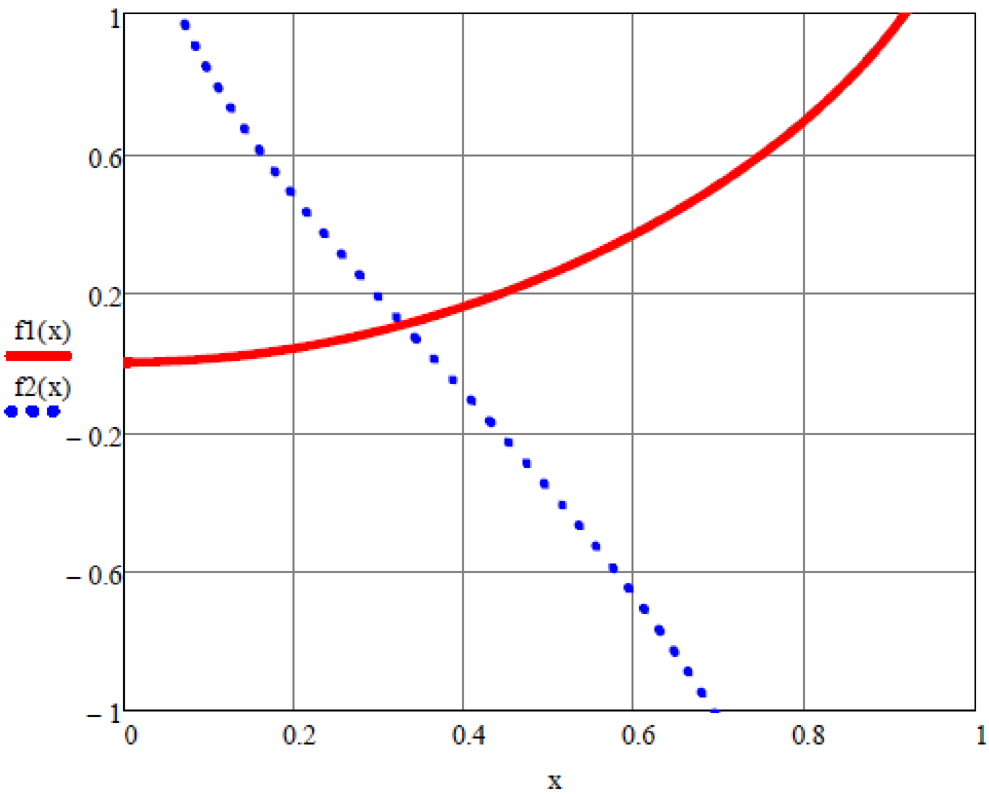
(i) Graphical Method,
(ii) Fixed Point Iteration, and
(iii) Newton Jacobi Method,

given the tolerance = 0.005 and (x0,y0) = (0.5, 0.5)

$$x^2 = \sin(y)$$
$$\ln\left(\frac{1}{x}\right) = e^y$$

(i) Graphical Method:

$f1(x) := \operatorname{asin}(x^2)$ $\qquad$ $f2(x) := \ln\left(\ln\left(\frac{1}{x}\right)\right)$



(ii) Fixed Point Iteration

$$x = \sqrt{\sin(y)}$$

$$y = \ln(\ln(1/x))$$

$x := 0.5$ $f1(x) = 0.6924$

$x := -0.3665$ $f1(x) = 0.5986i$

Complex Number

$y := 0.5$ $f2(y) = -0.3665$

$y := 0.6924$ $f2(y) = -1.0008$

$y := -1.0008$ $f2(y) = 1.1447 + 1.5711i$

Complex Number

The solution is **divergent** when the equations are expressed using the formulas above, rewriting the same equation given below will produce:

$$x = \frac{1}{e^{e^y}}$$

$$y = \arcsin(x^2)$$

$x0 := 0.5$	$f1(x0) = 0.1923$	$y0 := 0.5$	$f2(y0) = 0.2527$
$x1 := 0.2527$	$f1(x1) = 0.276$	$y1 := 0.1923$	$f2(y1) = 0.037$
$x2 := 0.0361$	$f1(x2) = 0.3546$	$y2 := 0.276$	$f2(y2) = 0.0762$
$x3 := 0.0762$	$f1(x3) = 0.3399$	$y3 := 0.3546$	$f2(y3) = 0.1261$
$x4 := 0.1261$	$f1(x4) = 0.3216$	$y4 := 0.3399$	$f2(y4) = 0.1158$
$x5 := 0.1158$	$f1(x5) = 0.3254$	$y5 := 0.3216$	$f2(y5) = 0.1036$
$x6 := 0.1036$	$f1(x6) = 0.3298$	$y6 := 0.3254$	$f2(y6) = 0.1061$
$x7 := 0.1061$	$f1(x7) = 0.3289$	$y7 := 0.3298$	$f2(y6) = 0.1061$

$\varepsilon\_AA\_x\_1 := \lvert x1 - x0 \rvert$	$\varepsilon\_AA\_y\_1 := \lvert y1 - y0 \rvert$	$\max(\varepsilon\_AA\_x\_1, \varepsilon\_AA\_y\_1) = 0.3077$
$\varepsilon\_AA\_x\_2 := \lvert x2 - x1 \rvert$	$\varepsilon\_AA\_y\_2 := \lvert y2 - y1 \rvert$	$\max(\varepsilon\_AA\_x\_2, \varepsilon\_AA\_y\_2) = 0.2166$
$\varepsilon\_AA\_x\_3 := \lvert x3 - x2 \rvert$	$\varepsilon\_AA\_y\_3 := \lvert y3 - y2 \rvert$	$\max(\varepsilon\_AA\_x\_3, \varepsilon\_AA\_y\_3) = 0.0786$
$\varepsilon\_AA\_x\_4 := \lvert x4 - x3 \rvert$	$\varepsilon\_AA\_y\_4 := \lvert y4 - y3 \rvert$	$\max(\varepsilon\_AA\_x\_4, \varepsilon\_AA\_y\_4) = 0.0499$
$\varepsilon\_AA\_x\_5 := \lvert x5 - x4 \rvert$	$\varepsilon\_AA\_y\_5 := \lvert y5 - y4 \rvert$	$\max(\varepsilon\_AA\_x\_5, \varepsilon\_AA\_y\_5) = 0.0183$
$\varepsilon\_AA\_x\_6 := \lvert x6 - x5 \rvert$	$\varepsilon\_AA\_y\_6 := \lvert y6 - y5 \rvert$	$\max(\varepsilon\_AA\_x\_6, \varepsilon\_AA\_y\_6) = 0.0122$
$\varepsilon\_AA\_x\_7 := \lvert x7 - x6 \rvert$	$\varepsilon\_AA\_y\_7 := \lvert y7 - y6 \rvert$	$\max(\varepsilon\_AA\_x\_7, \varepsilon\_AA\_y\_7) = 4.4 \times 10^{-3}$

Please note that the equations are calculated on the basis of updated guesses from the previous iteration, i.e., the updated guesses are not used until all the equations in the system are calculated. The absolute approximate error is calculated using the infinity ∞ norm.

(iii) Newton-Jacobi Method

$$F(x1,x2) := \begin{pmatrix} x1^2 - \sin(x2) \\ \ln\left(\frac{1}{x1}\right) - e^{x2} \end{pmatrix}$$

$$J(x1,x2) := \begin{pmatrix} \frac{d}{dx1}F(x1,x2)_1 & \frac{d}{dx2}F(x1,x2)_1 \\ \frac{d}{dx1}F(x1,x2)_2 & \frac{d}{dx2}F(x1,x2)_2 \end{pmatrix} \qquad J(x1,x2) := \begin{pmatrix} 2\cdot x1 & -\cos(x2) \\ -\frac{1}{x1} & -e^{x2} \end{pmatrix}$$

X(x1,x2) F(x1,x2) J(x1,x2)

$$X\_0 := \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} \qquad F(0.5,0.5) = \begin{pmatrix} -0.2294 \\ -0.9556 \end{pmatrix} \qquad J(0.5,0.5) = \begin{pmatrix} 1 & -0.8776 \\ -2 & -1.6487 \end{pmatrix}$$

$$J(0.5,0.5)^{-1} = \begin{pmatrix} 0.4844 & -0.2578 \\ -0.5876 & -0.2938 \end{pmatrix} \qquad X\_1 := X\_0 - J(0.5,0.5)^{-1}\cdot F(0.5,0.5)$$

$$X\_1 = \begin{pmatrix} 0.3648 \\ 0.0845 \end{pmatrix}$$

$$X\_1 := \begin{pmatrix} 0.3648 \\ 0.0845 \end{pmatrix} \qquad F(0.3648,0.0845) = \begin{pmatrix} 0.0487 \\ -0.0798 \end{pmatrix} \qquad J(0.3648,0.0845) = \begin{pmatrix} 0.7296 & -0.9964 \\ -2.7412 & -1.0882 \end{pmatrix}$$

$$J(0.3648,0.0845)^{-1} = \begin{pmatrix} 0.3087 & -0.2826 \\ -0.7776 & -0.207 \end{pmatrix} \qquad X\_2 := X\_1 - J(0.3648,0.0845)^{-1}\cdot F(0.3648,0.0845)$$

$$X\_2 = \begin{pmatrix} 0.3272 \\ 0.1058 \end{pmatrix}$$

$$X\_2 := \begin{pmatrix} 0.3272 \\ 0.1058 \end{pmatrix} \qquad F(0.3272,0.1058) = \begin{pmatrix} 1.4571 \times 10^{-3} \\ 5.5841 \times 10^{-3} \end{pmatrix} \qquad J(0.3272,0.1058) = \begin{pmatrix} 0.6544 & -0.9944 \\ -3.0562 & -1.1116 \end{pmatrix}$$

$$J(0.3272,0.1058)^{-1} = \begin{pmatrix} 0.2951 & -0.264 \\ -0.8114 & -0.1737 \end{pmatrix} \qquad X\_3 := X\_2 - J(0.3272,0.1058)^{-1}\cdot F(0.3272,0.1058)$$

$$X\_3 = \begin{pmatrix} 0.3282 \\ 0.108 \end{pmatrix}$$