M E T U
Department of Mathematics

	Basic Linear Algebra Final	
Code: *Math 260* Acad.Year: *2006-2007* Semester: *Spring* Date: *29.5.2007* Time: *16:40* Duration: *120 minutes*	Last Name: Name: Student No.: Department: Section: Signature:	
	4 QUESTIONS ON 4 PAGES TOTAL 80 POINTS	

1	2	3	4		

Please carefully write the logical steps leading to your answers. Correct answers without any correct reasoning will not get any points.

1. (20 pts) Let V be the vector space of polynomials in x of degree less than or equal to 3. Let

$$(f|g) = \int_0^1 x^2 f(x)g(x)dx$$

(a) Show that $(\ | \ )$ is an inner product.

(b) Find an orthogonal basis of the subspace S of V spanned by $1, x^2$ and x^3.

(c) Find the orthogonal projection of x to the subspace S.

2. (20 pts) Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation such that $T((1,1,0,0)) = (1,0,0,1)$, $T((1,-1,0,0)) = (0,1,1,0)$, $T((0,0,1,1)) = (1,1,1,1)$, $T((0,0,1,-1)) = (1,1,0,0)$.

(a) Find $T((x_1, x_2, x_3, x_4))$.

(b) Find a basis for $Ker(T)$.

(c) Find a basis for $Range(T)$.

(d) Find the matrix of T with respect to the standard bases $\{(1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1)\}$ in the domain and range.

3. (20 pts) A 3×3 matrix A has eigenvalues $1, -1$ and 2 with corresponding eigenvectors $v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

(a) Is A invertible?

(b) Is A diagonalizable? If it is, determine an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

(c) Find A.

(d) Find $Tr(A)$ and $det(A)$.

4. (20 pts) (a) Suppose that $A^2 = I$. Show that every eigenvalue of A is equal to 1 or -1.

(b) Let $V = \mathbb{R}^3$ and W be the $z-$axis in $\mathbb{R}^3$. Let $T : V \to W$ be orthogonal projection. Choose bases for V, W, and find the matrix of T with respect to these bases.

(c) Show that for any real numbers x_1, x_2, y_1, y_2,

$$2x_1y_1 + 2x_1y_2 + 3x_2y_2 \leq \sqrt{2x_1^2 + 2x_1x_2 + 3x_2^2}\sqrt{2y_1^2 + 2y_1y_2 + 3y_2^2}$$

(Hint: first define an appropriate inner product on $\mathbb{R}^2$)