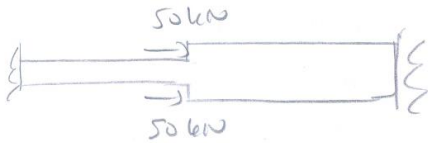
CE 425

FALL 2014

HOMEWORK #6 SOLUTIONS

(1)



(1)

Virtual work

$$\int_0^4 EA \frac{du}{dx}\frac{dv}{dx}\,dx = 100\times10^3\, v(2)$$

$$u = C_1 x(4-x) + C_2 x^2(4-x) = \sum C_i N_i(x)$$

$$\frac{du}{dx} = C_1[4-2x] + C_2[8x-3x^2]$$

$$v_1 = x(4-x) \qquad v_2 = x^2(4-x)$$

$$\frac{dv_1}{dx} = [4-2x] \qquad \frac{dv_2}{dx} = 8x-3x^2$$

$$\sum C_i \int_0^4 EA \frac{dN_i}{dx}\frac{dN_j}{dx}\,dx = 100\times10^3\, N_j(2)$$

or (Potential energy)

$$\Pi = \int_0^4 \frac{EA}{2}\left(\frac{du}{dx}\right)^2 dx - 100\times10^3\, u(2)$$

Substitute $u = \sum C_i N_i$ and minimize for C_i.

$$10^{10}\begin{bmatrix} 1.7067 & 4.0533 \\ 4.0533 & 1.3483 \end{bmatrix}\begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 400\,000 \\ 800\,000 \end{Bmatrix}$$

$$C_1 = 0.3267\times10^{-4}$$

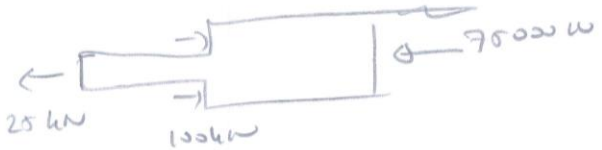
$$C_2 = -0.0389\times10^{-4}$$

(1) Strength of Mat. solution.

(2)

$$\frac{100\times10^3 \times 2}{4\times10^8} = R\left[\frac{2}{4\times10^8} + \frac{2}{12\times10^8}\right]$$

$$R = 75000\ N$$



$$\frac{25\times10^3\times2}{4\times10^8} = 1.25\times10^{-4}$$ (disp at the mid point)

(2) (3)

$$\Pi = \int_0^2 \frac{GJ}{2}\left(\frac{d\phi}{dx}\right)^2 dx - 1000\times\phi(2)$$

virtual work

$$\int_0^2 GJ\,\frac{d\phi}{dx}\,\frac{dv}{dx}\,dx = 1000\,v(2)$$

One parameter solution

$\phi = c_1 x$

$v = x$

$$\int_0^2 G(x)J\,c_1\,dx = 1000\times 2$$

$1.5\times10^{11}\,J\,c_1 = 2000$

$c_1 = \dfrac{2000}{1.5\times10^{11}\,J}$

$c_1 = 1.358\times10^{-3}$

$\phi(2) = 2.72\times10^{-3}$ rad

two parameter solution

$\phi = c_1 x + c_2 x^2 = \sum c_i N_i$

$v_1 = x \quad v_2 = x^2$

$$\sum c_i \int GJ\,\frac{dN_i}{dx}\,\frac{dN_j}{dx}\,dx = 1000\,N_j(2)$$

$$10^6\begin{bmatrix} 1.4726 & 2.62 \\ \text{Sym} & 6.545 \end{bmatrix}\begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix} = \begin{bmatrix} 2000 \\ 4000 \end{bmatrix}$$

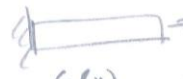
$c_1 = 0.941\times10^{-3}$

$c_2 = 0.2343\times10^{-3}$

$\phi(L) = 2.82\times10^{-3}$ rad

---

exact solution



$$\phi(L) - \phi(0) = \int_0^2 \frac{T}{GJ}\,dx = 2.8242\times10^{-6}$$

$\phi(2) = 2.824\times10^{-3}$ radians

or

$$3.54\times10^5\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} \phi(0) \\ \phi(2) \end{Bmatrix} = \begin{bmatrix} T_R \\ 1000 \end{bmatrix}$$

(4)

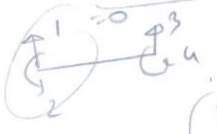
(3)

$$\Pi = \int_0^4 \frac{EI}{2}\left(\frac{d^2y}{dx^2}\right)^2 dx + \int_0^L wy\,dx + \frac{k\,y(L)^2}{2}$$

or

virtual work

$$\int_0^4 EI \frac{d^2y}{dx^2}\frac{d^2v}{dx^2}\,dx + k\,y(L)\,v(L) = -\int_0^L wv\,dx$$



$\frac{wL^2}{12}$ $\frac{wL}{2}$

$$\begin{bmatrix} \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -wL/2 - ky_2 \\ -wL^2/12 \end{bmatrix}$$

$$\begin{bmatrix} \frac{12EI}{L^3}+k & -\frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -wL/2 \\ +wL^2/12 \end{bmatrix}$$

$$y_2 = \frac{-32^4 w}{8kL^3 + 24EI}$$

$k \to \infty \quad y_2 = 0$

$k \to 0 \quad y_2 = \frac{L^4 w}{8EI}$

$$y_2 = -\frac{3 \times 4^4 \times 2000}{8 \times 10^6 \times 4^3 + 24 \times 200 \times 10^9 \times \frac{0.05^4}{12}} = -2.985 \times 10^{-3}\ m$$

(5)

$F_5 = 2985\ N$