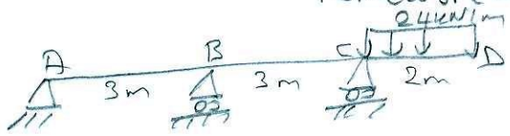
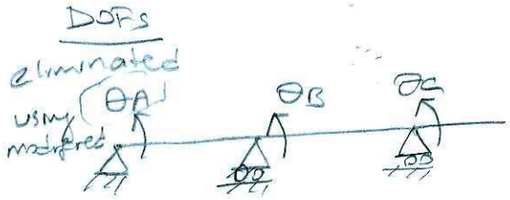
Homework – 3 Solutions

1 –



DOFs



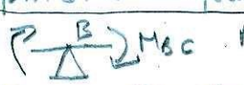
FEMs

No fixed end moments for any member

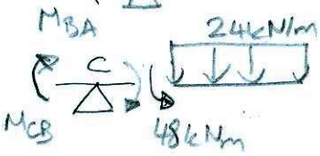
SD Equations

$M_{AB}=0 \quad M_{BA}=\frac{3EI}{3}\theta_B \quad M_{BC}=\frac{2EI}{3}(2\theta_B+\theta_C) \quad M_{CB}=\frac{2EI}{3}(2\theta_C+\theta_B)$

Equilibrium Equations



$M_{BA}+M_{BC}=0 \quad \frac{EI}{3}(7\theta_B+2\theta_C)=0 \quad 7\theta_B+2\theta_C=0 \ldots (1)$

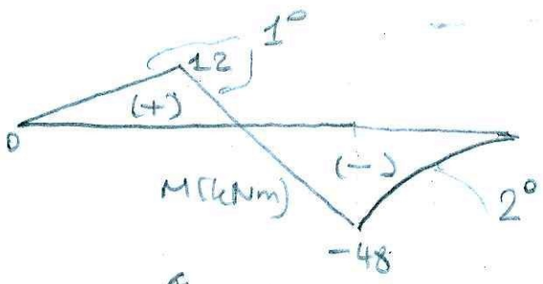


$M_{CB}+48=0 \quad \frac{2EI}{3}(2\theta_C+\theta_B)=-48$

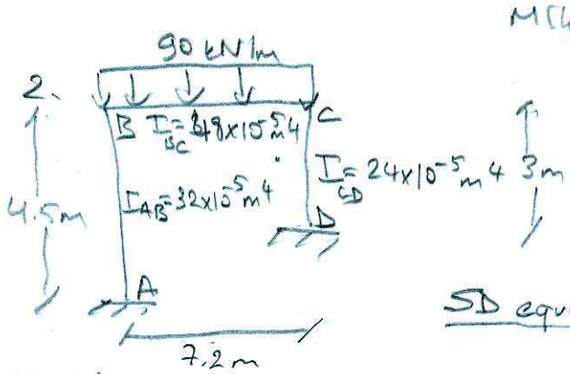
$2\theta_C+\theta_B=\frac{-72}{EI} \ldots (2)$

Solving (1) and (2) $\theta_C=-\frac{42}{EI} \quad \theta_B=\frac{12}{EI}$

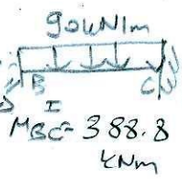
Backsubstition: $M_{BA}=12$ kNm $M_{BC}=-12$ kNm $M_{CB}=-48$ kNm



2.



FEMs:

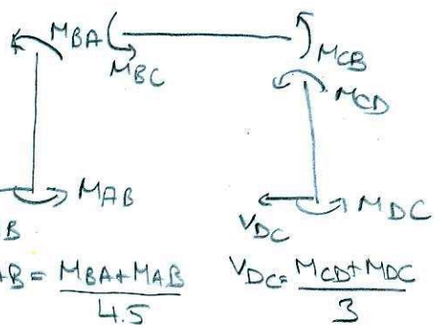


SD equations: $M_{AB}=\frac{2EI_{AB}}{4.5}\left(\theta_B+\frac{3\Delta}{4.5}\right) \quad M_{BA}=\frac{2EI}{4.5}\left(2\theta_B+\frac{3\Delta}{4.5}\right)$

$M_{BC}=\frac{2EI_{BC}}{7.2}(2\theta_B+\theta_C)+388,8 \quad M_{CB}=\frac{2EI_{BC}}{7.2}(2\theta_C+\theta_B)-388,8$

$M_{CD}=\frac{2EI_{CD}}{3}\left(2\theta_C+\frac{3\Delta}{3}\right) \quad M_{DC}=\frac{2EI_{CD}}{3}\left(\theta_C+\frac{3\Delta}{3}\right)$

Equilibrium Equations:



$V_{AB}=\frac{M_{BA}+M_{AB}}{4.5} \quad V_{DC}=\frac{M_{CD}+M_{DC}}{3}$

$M_{BA}+M_{BC}=0$ (1) $110222,22\theta_B+26666,67\theta_C+18962,96\Delta=-388,8$

$M_{CB}+M_{CD}=0$ (2) $26666,67\theta_B+117333,33\theta_C+32000\Delta=388,8$

$V_{AB}+V_{DC}=0$ (3) $18962,96\theta_B+32000\theta_C+29761,32\Delta=0$

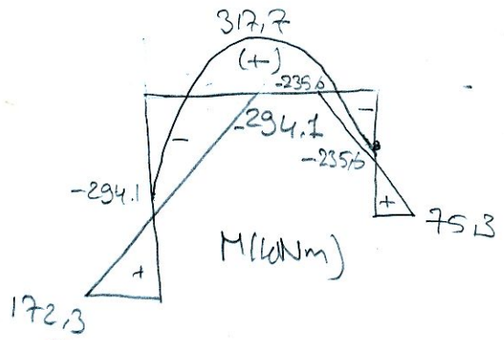
Solving (1) (2) (3): $\theta_B=-4.279\times10^{-3}$ rad

$\theta_C=5,009\times10^{-3}$ rad

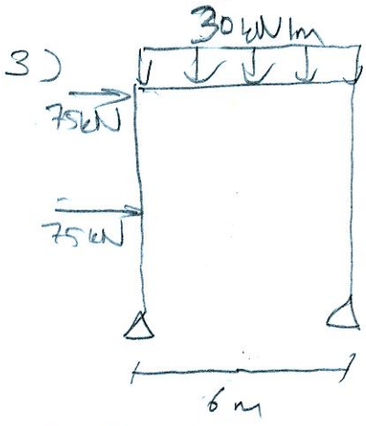
$\Delta=-2,659\times10^{-3}$ m

Backsubstitution: $M_{AB} = -172,34\,kNm$ $M_{BC} = 294,1\,kNm$ $M_{CD} = 235,6\,kNm$

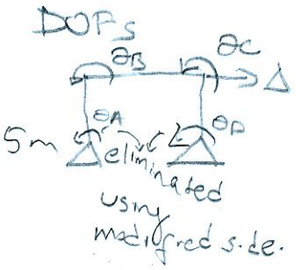
$M_{BA} = -294.1\,kNm$ $M_{CB} = -235,6\,kNm$ $M_{DC} = 75,3\,kNm$



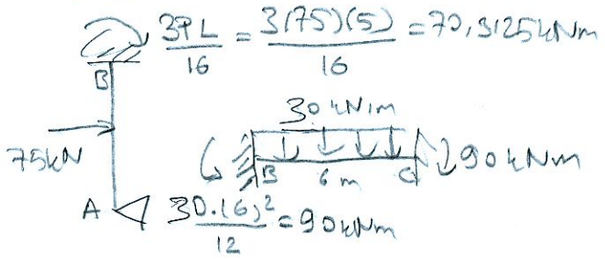
3)



DOFs



FEMs

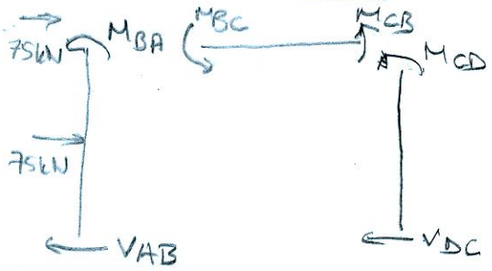


SD Equations:

$$M_{BA} = \frac{3EI}{5}\left(\theta_B + \frac{\Delta}{5}\right) - 70,3125 \qquad M_{CB} = \frac{2EI}{6}(2\theta_C + \theta_B) - 90$$

$$M_{BC} = \frac{2EI}{6}(2\theta_B + \theta_C) + 90 \qquad M_{CD} = \frac{3EI}{5}\left(\theta_C + \frac{\Delta}{5}\right)$$

Equilibrium Equations:



$$V_{AB} = \frac{M_{BA} + 75(2.5)}{5} \qquad V_{DC} = \frac{M_{CD}}{5}$$

$M_{BA} + M_{BC} = 0$ ..... (1)

$M_{CB} + M_{CD} = 0$ ..... (2)

$V_{AB} + V_{DC} = 150 \Rightarrow M_{BA} + M_{CD} = 562,5$ ..(3)

From (1): $1,267\,\theta_B + 0,333\,\theta_C + 0,12\,\Delta = \frac{-19,69}{EI}$

From (2): $0,333\,\theta_B + 1,267\,\theta_C + 0,12\,\Delta = \frac{90}{EI}$

From (3) $0,6\,\theta_B + 0,6\,\theta_C + 0,24\,\Delta = \frac{652,81}{EI}$

Solving (1) (2) and (3).

$$\theta_B = \frac{-350}{EI} \qquad \theta_C = \frac{-232,5}{EI} \qquad \Delta = \frac{4176,3}{EI}$$

Backsubstitution: $M_{BA} = 211,24\,kNm$ $M_{CB} = -351,26\,kNm$

$M_{BC} = -211,24\,kNm$ $M_{CD} = 351,26\,kNm$