Groundwater Hydrology

Introduction

- Groundwater is the largest available freshwater source in the hydrologic cycle.
- Relatively free from pollution.
- Especially imp. for irrigation & domestic use for small towns.
- Extensively used in arid regions.
- Already existing storage for the continued availability.
- 30 % of streamflow is from groundwater in the world

  40 % in Turkey.

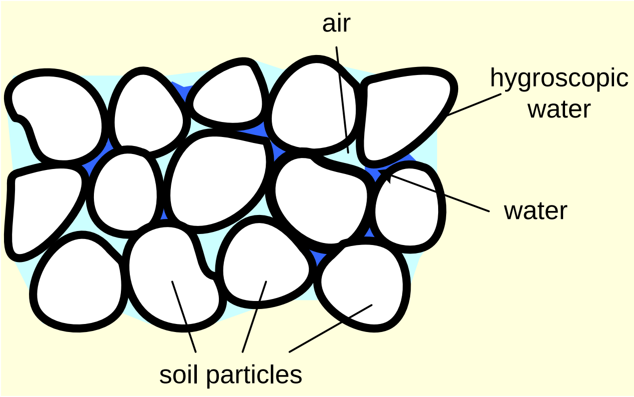
- Subsurface formations containing water are composed of two horizontal zones
  - Zone of Aeration (Unsaturated Zone above GWT)
  - Zone of Saturation (Below GWT)

- Uncontrolled pumping from the wells used for irrigation in Konya Closed Basin caused decreasing of the GWT around 22-40 m in the last 20 years.
- 92 000 wells in the basin, 66 000 of them are uncontrolled
- Water used from groundwater affects the surface water in long term.

Konya basin

Çavuşçu lake, 20/July/2008

Occurence of Groundwater



Gravity water

Capillary water

Hygroscopic water

Water vapor

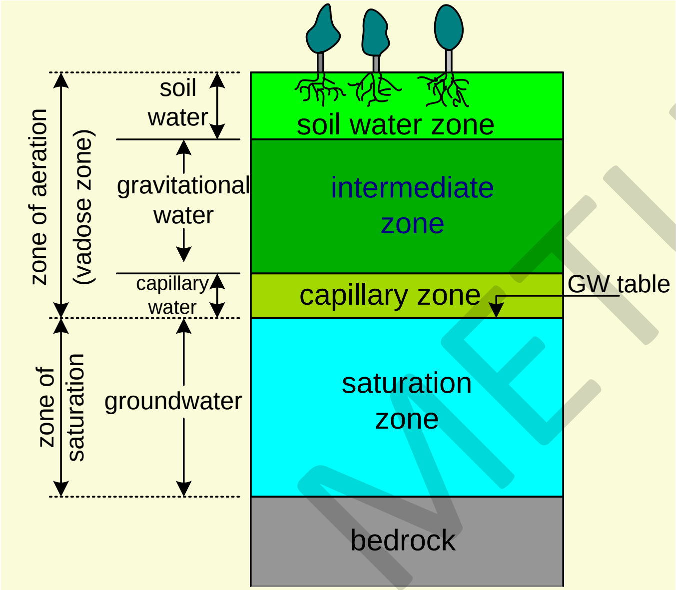


Figure 12.1 Types of subsurface waters and soil zones

- Soil water zone depth = f (Soil type, vegetation)
- Capillary zone depth = f (Pore size)
  - Fine gravel → 2.5 cm rise (5 – 2 mm grain size)
  - Silt → 100 cm rise (0.1 – 0.05 mm grain size)
- Intermediate zone depth 0 - 100 m

Aquifers

- Aquifers are formations that are characterized by their ability to
  - store &
  - transmit water
- There are two types of aquifers;
  - Unconfined Aquifer (Water Table Aquifer)
  - Confined Aquifer (Pressure Aquifer)

Occurence of Groundwater

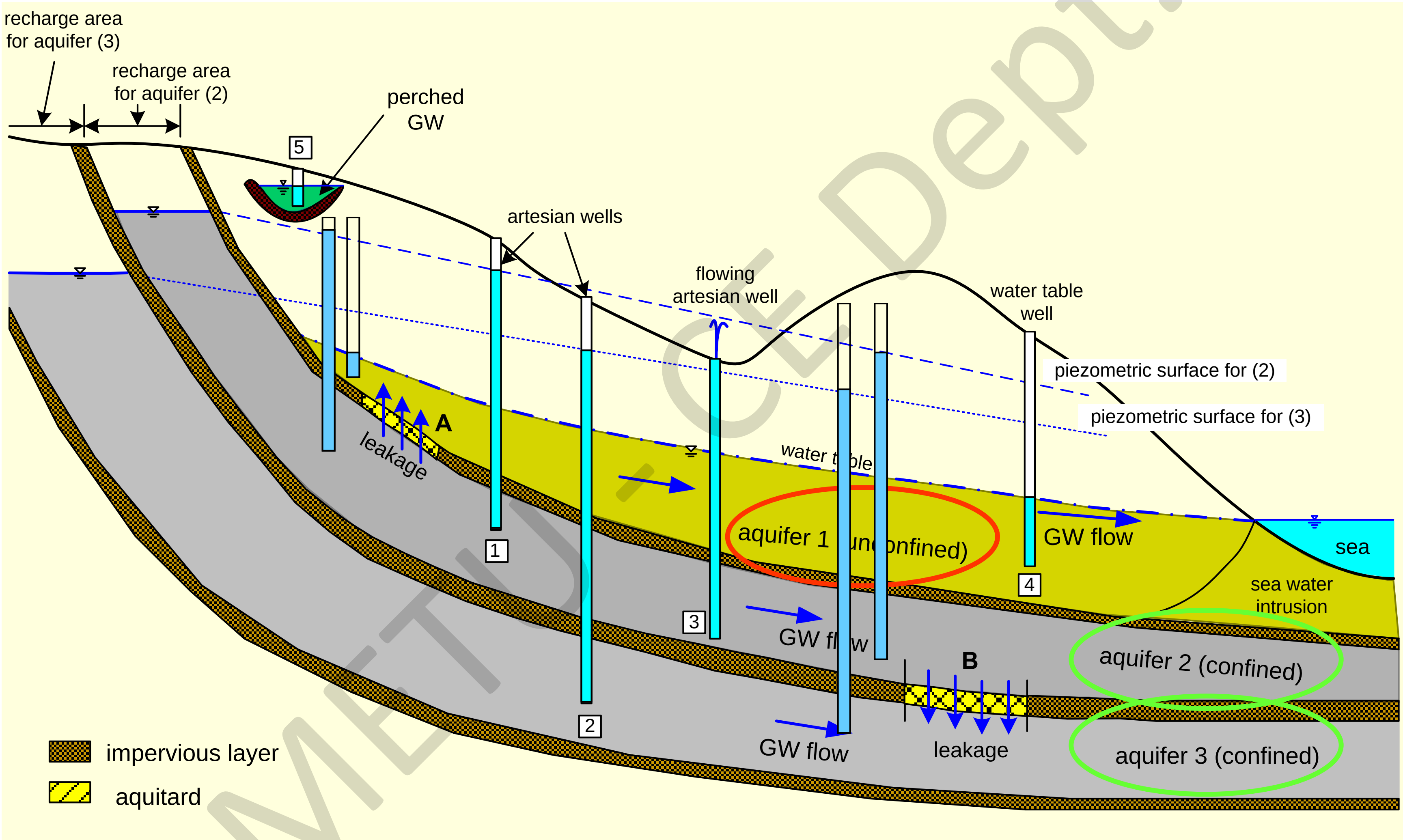


Figure 12.3 Types of aquifer

Storage characteristics of confined & unconfined aquifers

$$Porosity = \frac{\text{Volume of pores}}{\text{Total volume}}$$

Material	Porosity
Gravel (coarse)	28
Sand (coarse)	39
Clay	42
Basalt	17

- Amount of water that can be stored in an aquifer is a function of porosity.
- However, productivity of an aquifer is not only related to the porosity.
- Clay formations have high porosity but do not yield water.
- Sand & gravel have lower porosity but yield more water (good aquifer materials).

Storativity or Storage Coefficient, S

- Storage Coefficient or Storativity → represent aquifer's total water storage and ability to transmit it.

- S: volume of water released from (or added to) storage in the aquifer per unit horizontal area of the aquifer and per unit decline (or rise) of the average piezometric head in the aquifer.

$$S = \frac{volume\ of\ water\ released\ (added) / unit\ horizontal\ area}{unit\ decline\ in\ head}$$

Storativity

FOR CONFINED AQUIFERS

Storativity of a saturated confined aquifer of thickness b = the volume of water that an aquifer releases from storage per unit surface area of aquifer per unit decline in the component of hydraulic head normal to that surface.

FOR UNCONFINED AQUIFERS

The storage term for unconfined aquifers is known as the specific yield, S_y.

Specific yield = the volume of water that an unconfined aquifer releases from storage per unit surface area of aquifer per unit decline in the water table.

Specific yields of unconfined aquifers >> Storativity of confined aquifers

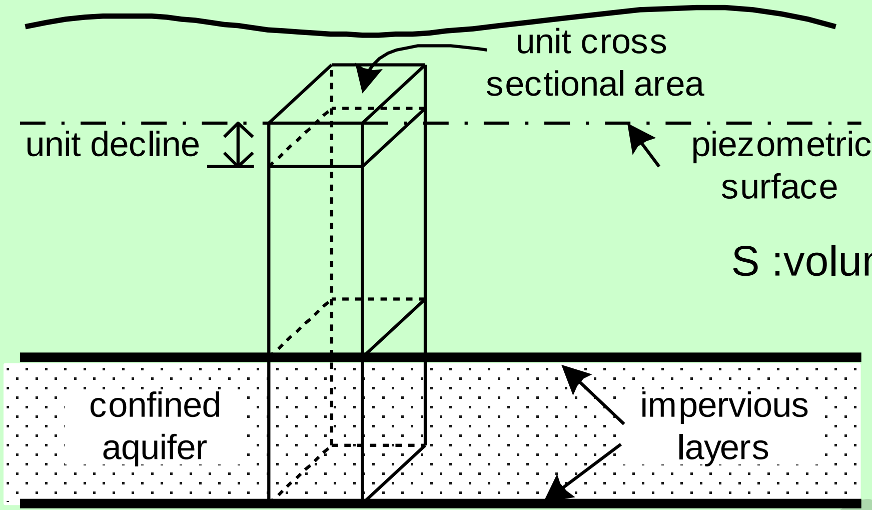


Figure 12.4 Storage coefficient for confined aquifer

S :volume / unit horizontal area per unit decline in head

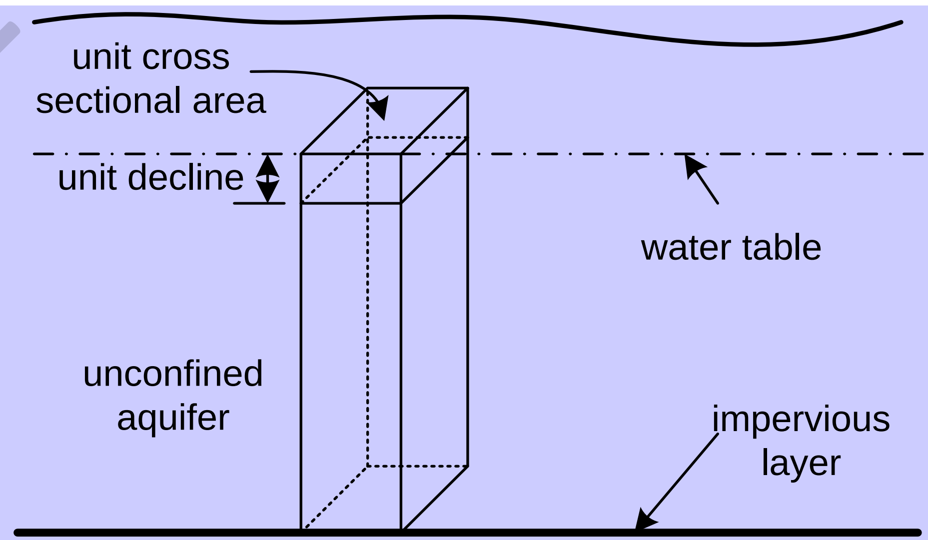


Figure 12.5 Storage coefficient for unconfined aquifer

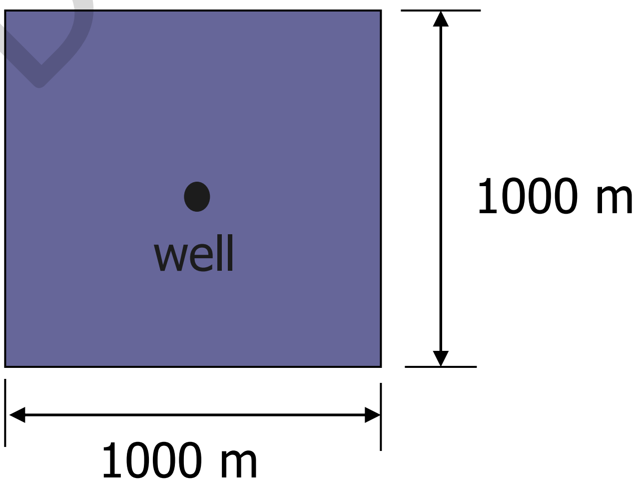
- FOR UNCONFINED AQUIFERS

- Storativity of unconfined aquifers is less than the porosity

$$\text{Porosity} = \underbrace{\text{Retention}}_{\text{Capillary \& hygroscopic water}} + \underbrace{\text{Specific yield}}_{\text{Gravity water}}$$

Material	Porosity	S_y (%)	% retention
Gravel (coarse)	28	23	5
Sand (coarse)	39	27	12
Clay	42	3	39

Example 16 A pumping well fully penetrates an aquifer. It works at a rate of Q= 2.5 m^3/hr for 10 days, then it is shut off. If piezometric head decreases 60 cm, determine the storage coefficient, S.



Solution:

ΔV = Total Volume of pumped water

$$\Delta V = 2.5 * 24 * 10 = 600 \text{ m}^3$$

$$S = \frac{\Delta V}{A.\ \Delta h} = \frac{600}{1000 * 1000 * 0.60} = 0.001$$

Darcy's law (1856)

$$Q = -KA\frac{h_2 - h_1}{L}$$

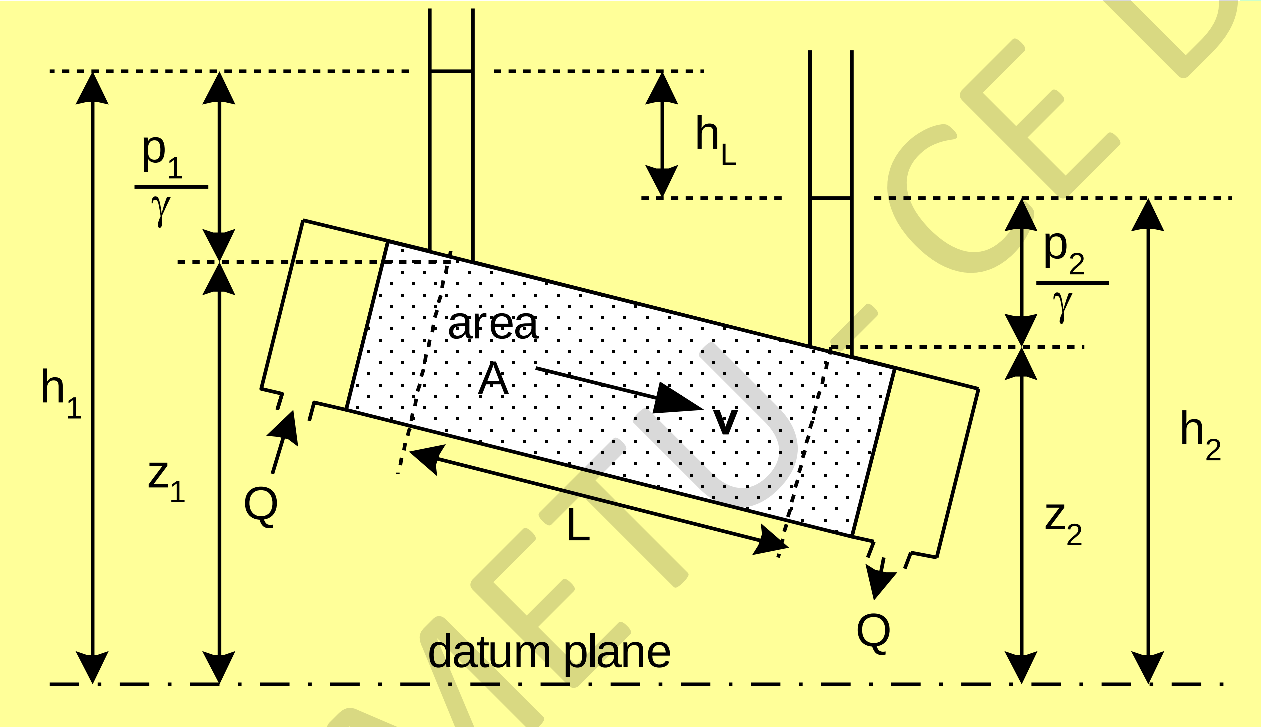
$$Q = -KA\frac{dh}{dl}$$

$$\frac{Q}{A} = v = -K\frac{dh}{dl}$$

$K = \text{hydraulic conductivity}$

$\frac{dh}{dl} = \text{hydraulic gradient}$

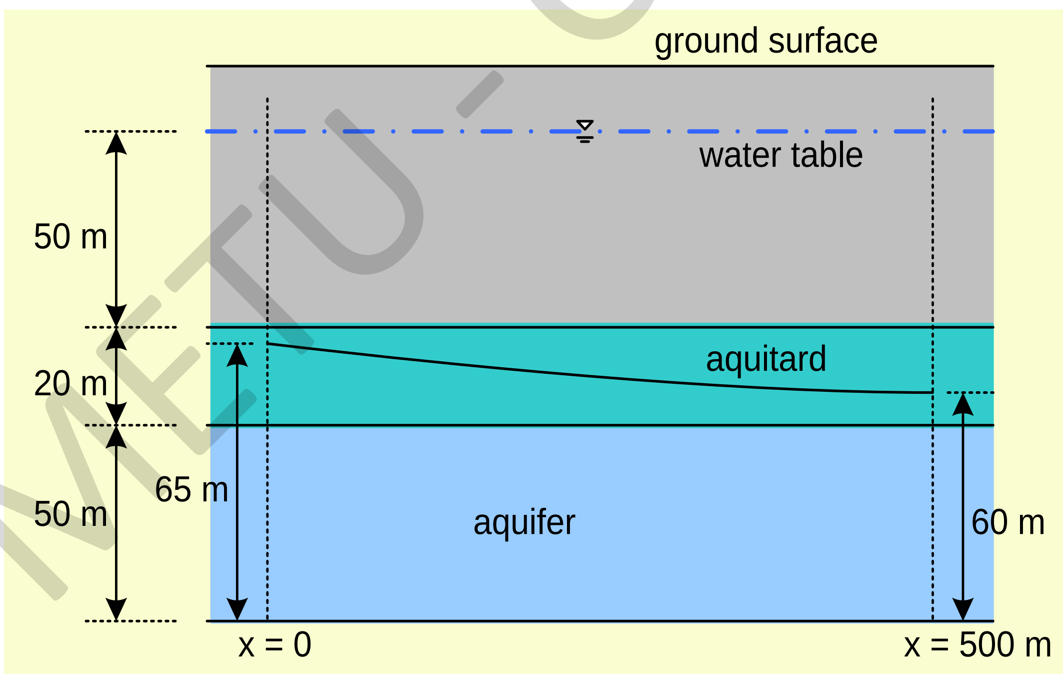
$v = \text{specific discharge}$

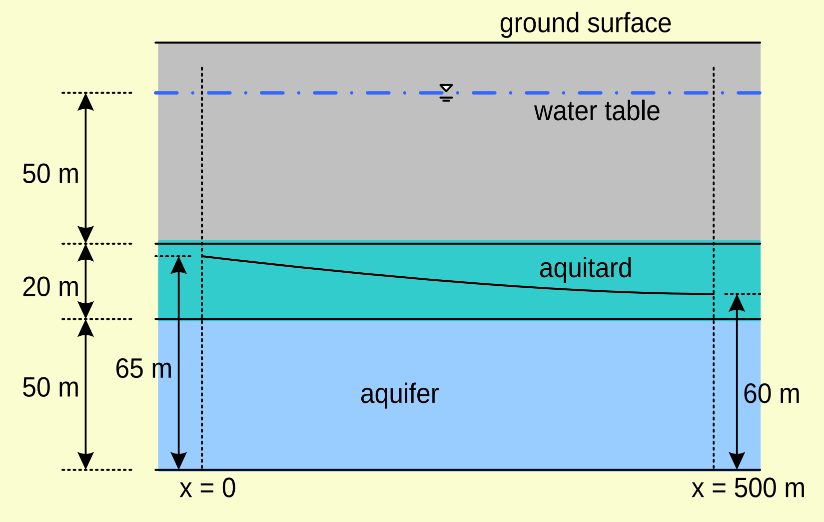


K = f(fluid & porous medium)

Material	Hydraulic Conductivity (m/day)
Gravel (coarse)	150
Sand (coarse)	45
Clay	0.0002

Example 17 Consider a leaky confined aquifer as shown in figure below. The water table in the source bed is horizontal at an elevation 50 m above the top of the aquitard. The piezometric surface for the confined aquifer slopes down linearly in the flow direction as shown. Vertical hydraulic conductivity of the aquitard is 4.0×10^{-6} m/s and its thickness is 20 m. Compute the total rate of leakage per unit width into the aquifer between the flow section x = 0 and x = 500 m.





$$q_v = -K' \frac{\Delta h}{b'} \quad \text{(Darcy's Law)}$$

K' = 4*10^{-6} m/s

b'= 20 m

Δh varies along x

$$\Delta h = 120 - 65 + \frac{5}{500} x = 55 + 0.01x$$

$$Q_v = q_v A \qquad \text{and} \qquad dQ_v = q_v dA$$

$$dQ_v = \underbrace{\frac{K'}{b'}(55 + 0.01x)}_{q_v}(\underbrace{dx * 1}_{\text{area}})$$

$$Q_v = \frac{K'}{b'} \int_0^{500} (55 + 0.01x)\, dx$$

$$= \frac{K'}{b'} \left(55x + 0.01\frac{x^2}{2}\right)\Bigg|_0^{500}$$

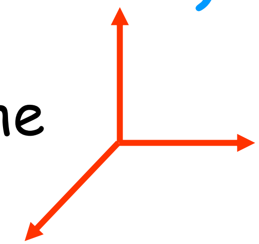
$$= \frac{4*10^{-6}}{20}\left(55*500 + \frac{0.01}{2}*500^2\right)$$

Q_v = 5.75*10^{-3} m^3/s/m

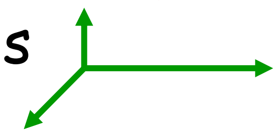
Q_v = 5.75 lt/s/m

Heterogeneity (location) & Anisotropy (direction)

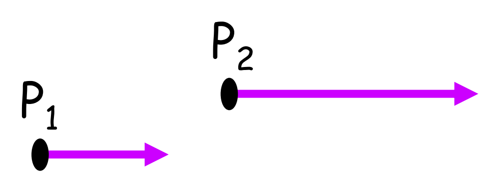
- If K at a point is the same for all directions, then the medium is isotropic.



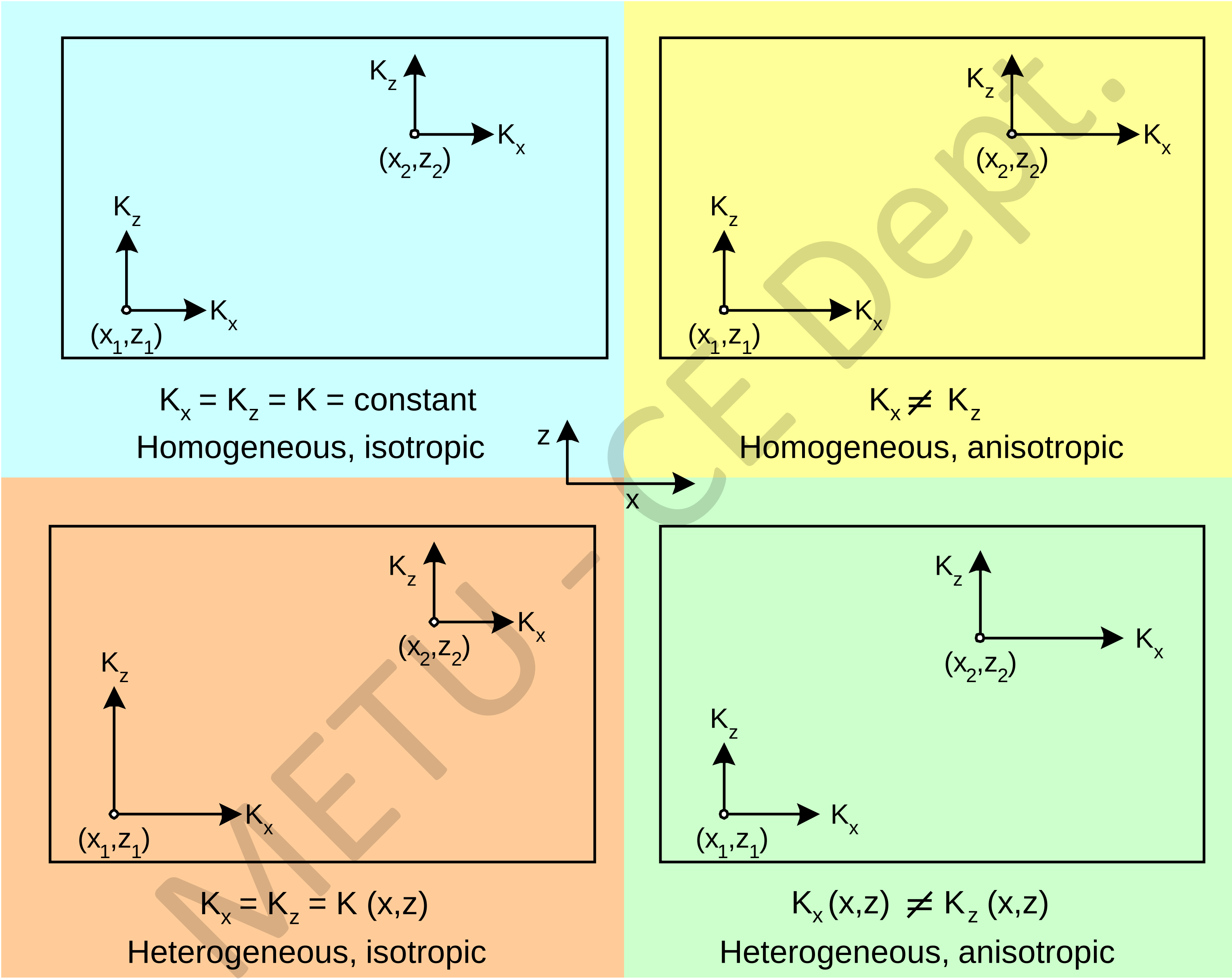
- If K varies with direction of flow, the medium is anisotropic.



- The directions corresponding to the angle at which K attains its maximum & its minimum are called principle directions of anisotropy.
- Principle directions are always perpendicular to one another.
- Cause of anisotropy:
  1. Particle orientation
  2. Layering of materials with different K



- Heterogeneity implies variation of hydraulic conductivity from one point to another point.


K_z
K_x
(x_2,z_2)
K_z
K_x
(x_1,z_1)
$K_x = K_z = K =$ constant
Homogeneous, isotropic
K_z
K_x
(x_2,z_2)
K_z
K_x
(x_1,z_1)
$K_x \neq K_z$
Homogeneous, anisotropic
z
x
K_z
K_x
(x_2,z_2)
K_z
K_x
(x_1,z_1)
$K_x = K_z = K\,(x,z)$
Heterogeneous, isotropic
K_z
K_x
(x_2,z_2)
K_z
K_x
(x_1,z_1)
$K_x\,(x,z) \neq K_z\,(x,z)$
Heterogeneous, anisotropic

Groundwater Flow Equations

Differential Equation of GW Flow

$$\frac{\partial}{\partial x}\left(T_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_y \frac{\partial h}{\partial y}\right) + q_v = S\frac{\partial h}{\partial t}$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \,!\right)$$

(T_x, T_y !)

(Heterogeneous + Anisotropic)

$$T_x \frac{\partial^2 h}{\partial x^2} + T_y \frac{\partial^2 h}{\partial y^2} + q_v = S\frac{\partial h}{\partial t}$$

(T_x, T_y !)

(Homogeneous + Anisotropic)

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{q_v}{T} = \frac{S}{T}\frac{\partial h}{\partial t}$$

($T_x = T_y = T$)

(Homogeneous + Isotropic)

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T}\frac{\partial h}{\partial t}$$

($q_v = 0$)

(Homogeneous + Isotropic + Non Leaky)

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

(Homogeneous + Isotropic + Non Leaky + Steady)

Laplace Equation

$$\text{no } \frac{\partial h}{\partial t}$$

Unsteady radial flow (well hydraulics)

- When piezometric head changes with time, groundwater flow becomes unsteady.
- When this flow occurs towards a well then it will also be radial.
- We will investigate different cases
  - Fully penetrating well in a confined aquifer
  - Fully penetrating well in a leaky confined aquifer

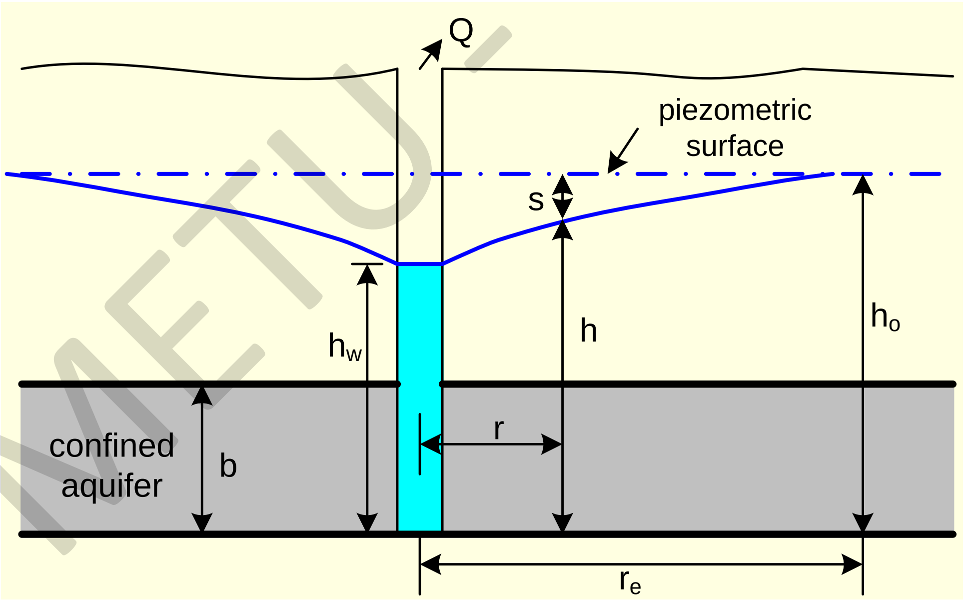
Assumptions:

1) Aquifer is homogeneous, isotropic, and areally entensive

2) Aquifer has a constant thickness and negligible slope

3) Well diameter is infinitesimal & pumping is continuous at a constant rate

4) Initial piezometric surface is horizontal



fully penetrating well in a confined aquifer

Fully Penetrating Well in a Confined Aquifer

$$s = \frac{Q}{4\pi T} W(u)$$

$$u = \frac{r^2 S}{4Tt}$$

Theis Solution

s	= drawdown (m)
Q	= discharge (m^3/s)
r	= radial distance (m)
S	= storage coefficient
T	= transmissivity (m^2/s)
t	= time from the start of pumping (s)
W(u)	= well function

$$W(u) = -0.5772 - \ln u + u - \frac{u^2}{2 \times 2!} + \frac{u^3}{3 \times 3!} \pm \ldots$$

Fully Penetrating Well in a Confined Aquifer

Well function

u	10^{-15}	10^{-14}	10^{-13}	10^{-12}	10^{-11}	10^{-10}	10^{-9}	10^{-8}	10^{-7}	10^{-6}
1	33.96	31.66	29.36	27.05	24.75	22.45	20.15	17.84	15.54	13.24
1.5	33.56	31.25	28.95	26.65	24.35	22.04	19.74	17.44	15.14	12.83
2	33.27	30.97	28.66	26.36	24.06	21.76	19.45	17.15	14.85	12.55
2.5	33.05	30.74	28.44	26.14	23.84	21.53	19.23	16.93	14.62	12.32
3	32.86	30.56	28.26	25.96	23.65	21.35	19.05	16.74	14.44	12.14
3.5	32.71	30.41	28.10	25.80	23.50	21.20	18.89	16.59	14.29	11.99
4	32.58	30.27	27.97	25.67	23.36	21.06	18.76	16.46	14.15	11.85
4.5	32.46	30.15	27.85	25.55	23.25	20.94	18.64	16.34	14.04	11.73

Fully Penetrating Well in a Confined Aquifer

- The solution of groundwater flow equation, either by Theis method has some practical applications:

    1) Computation of s when r, t , Q, S and T are known
    2) Computation of Q when r, s, t and S & T are known
    3) Computation of S and T (Aquifer characteristics) by performing pumping tests with observations of Q, s and t

Example 18

- Calculate the drawdown in a confined aquifer at r=0.3 m after 7 hrs of pumping with constant $Q=0.0315 m^3/sec$. (Assume $S=0.001$, $T=0.0094\ m^2/sec$.)

Theis Solution

$$s = \frac{Q}{4\pi T} W(u)$$

$$s = \frac{0.0315}{4\pi \times 0.0094} \times 15.592$$

$$s = 4.15\ m$$

$$u = \frac{r^2 S}{4Tt} = \frac{0.3^2 \times 0.001}{4 \times 0.0094 \times 7 \times 3600}$$

$$u = 9.499 \times 10^{-8} < 0.01$$

$$u = 9.499 \times 10^{-8}$$

$$\rightarrow W(u) = 15.592$$

Determination of aquifer characteristics: T and S

Theis, Graphical Method

$$s = \frac{Q}{4\pi T} W(u)$$

Take logarithms of both sides

$$\log s = \log\left(\frac{Q}{4\pi T}\right) + \log[W(u)]$$

Take logarithm of Boltzman variable after rearranging

$$u = \frac{r^2 S}{4Tt} \rightarrow \frac{r^2}{t} = \frac{4T}{S} u$$

$$\log\left(\frac{r^2}{t}\right) = \log\left(\frac{4T}{S}\right) + \log u$$

Assume that a pumping test is conducted

Q is known (constant rate)

r is known (fixed)

s values are recorded as a funcion of t

Determination of aquifer characteristics

$$\log s = \log \frac{Q}{4\pi T} + \log[W(u)]$$

$$\log\left(\frac{r^2}{t}\right) = \log\frac{4T}{S} + \log u$$

Log s = Constant$_1$ + log [W(u)]

Log (r^2/t) = Constant$_2$ + log u

Procedure

1) Prepare a plot of [W(u) vs u] on Log-Log paper (TYPE CURVE)
2) Prepare a plot of [s vs r^2/t] on Log-Log paper (transparent paper)
3) Superimpose Plot 2 and Plot 1 keeping appropriate axes parallel to each other.

   Adjust so that most of the data points fall on TYPE CURVE
4) Select an arbitrary point (not necessarily on the curves) and record

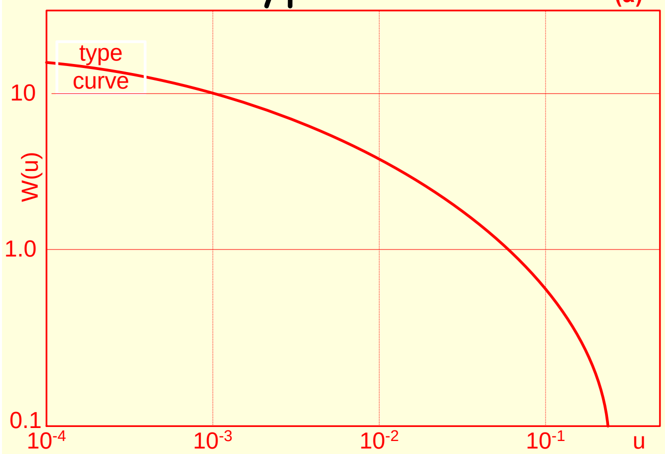
   W(u)*, u*, s*, (r^2/t)*

$$T = Q\frac{W(u)^*}{4\pi s^*} \quad \boxed{\text{from} \quad s = \frac{Q}{4\pi T}W(u)}$$

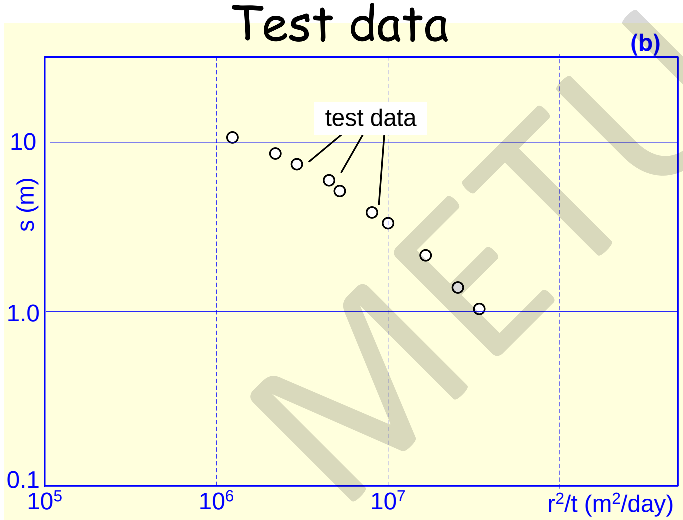
$$S = 4T\left(\frac{t}{r^2}\right)^* u^* \quad \boxed{\text{from } u = \frac{r^2}{4T}\frac{S}{t}}$$

Determination of aquifer characteristics

Type curve



Test data



Superimposition

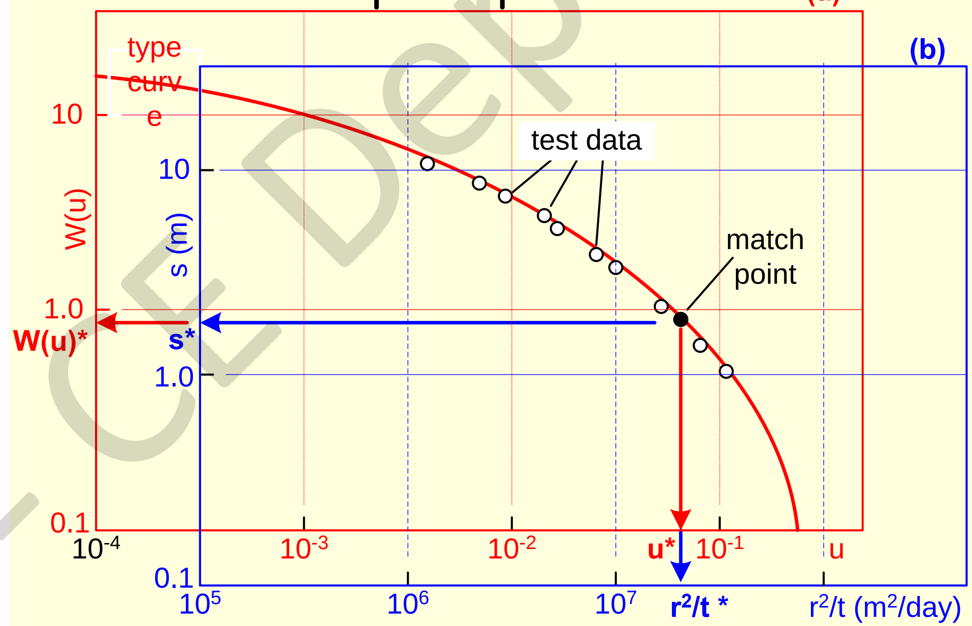


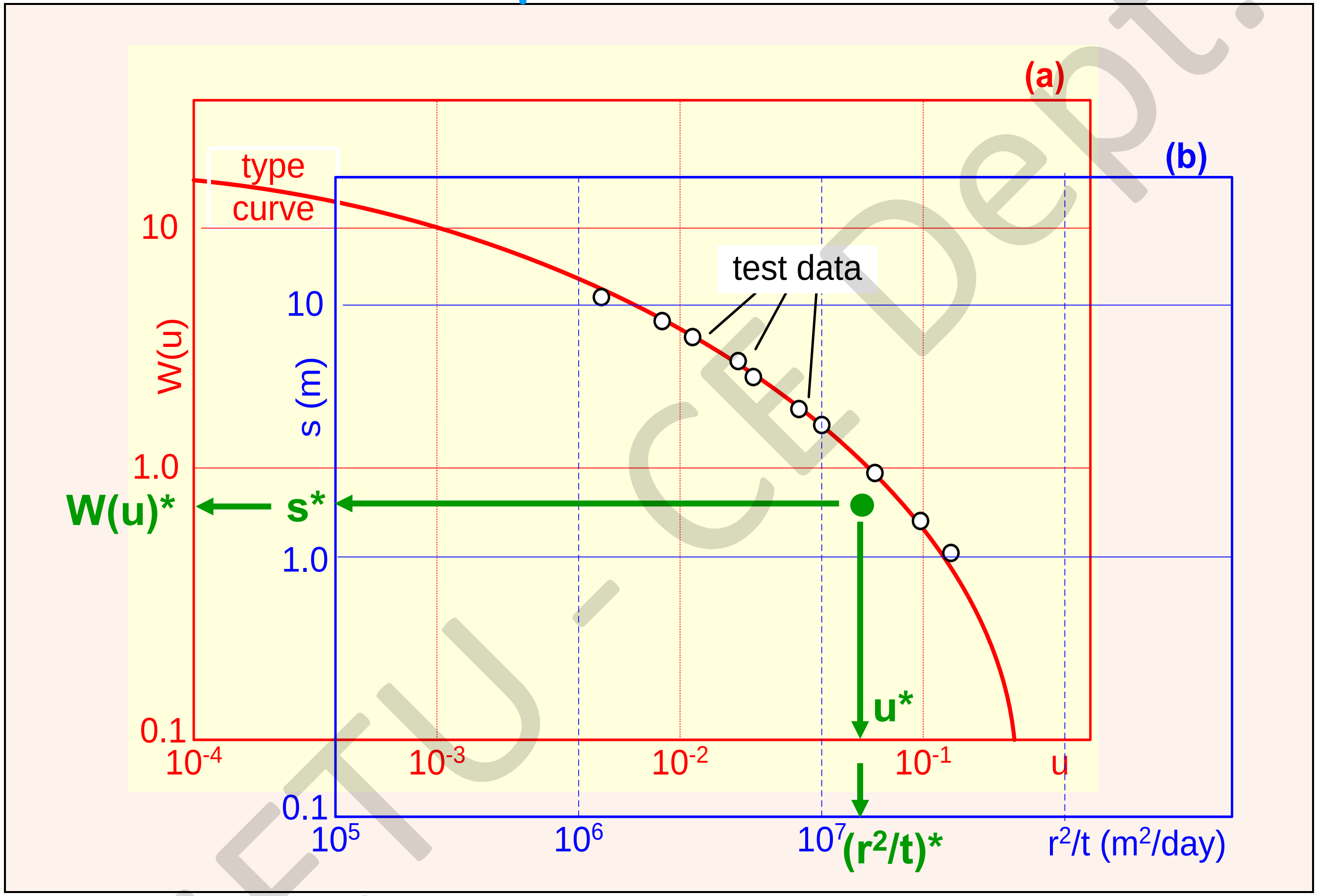
Figure 12.18 Theis graphical method

Read values $w(u)^*$, u^*, s^*, $(r^2/t)^*$

$$T = \frac{Q \times W(u)^*}{4\pi s^*} \qquad \boxed{\text{from} \quad s = \frac{Q}{4\pi T} W(u)}$$

$$S = 4T \left(\frac{t}{r^2}\right)^* u^* \qquad \boxed{\text{from} \quad u = \frac{r^2 S}{4Tt}}$$

Determination of aquifer characteristics



W(u)*, s*

$$T = \frac{Q \times W(u)^*}{4\pi s^*} \quad \text{from} \quad \boxed{s = \frac{Q}{4\pi T} W(u)}$$

(t/r²)*, u*

$$S = 4T\left(\frac{t}{r^2}\right)^* u^* \quad \boxed{\text{from} \quad u = \frac{r^2 S}{4Tt}}$$

Determination of aquifer characteristics

$$T = \frac{Q \times W(u)^*}{4\,\pi\, s^*} \qquad \boxed{\text{from } s = \frac{Q}{4\pi T} W(u)}$$

$$S = 4\,T \left(\frac{t}{r^2}\right)^* u^* \qquad \boxed{\text{from } u = \frac{r^2 S}{4Tt}}$$

Q: m^3/ day, m^3/ hr, m^3/ min	s: m
T: m^2/ day, m^2/ hr, m^2/ min	r: m
S: dimensionless	t: day, hr, min

Fully penetrating well in a leaky confined aquifer

Initially

Piezometric head of the unconfined aquifer and the aquitard (unconfined aquifer) coincides

After pumping starts

Does not change

Does change

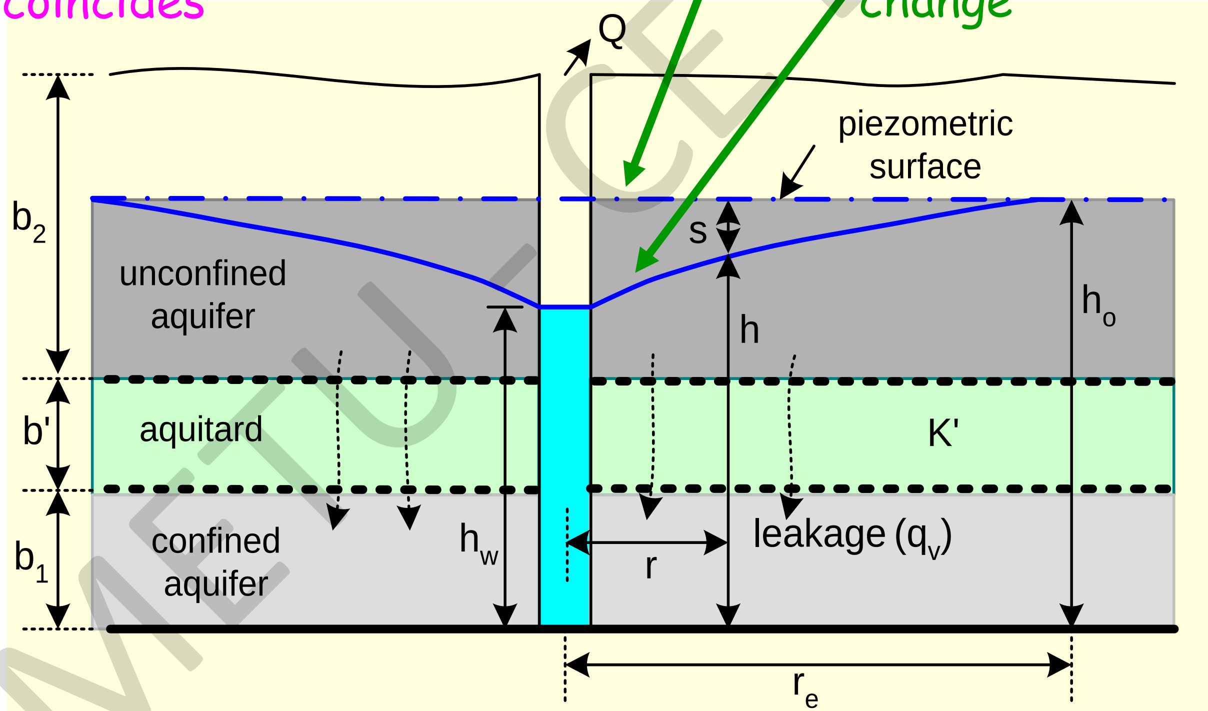


Figure 12.20 Fully penetrating well in a leaky confined aquifer

Fully penetrating well in a leaky confined aquifer

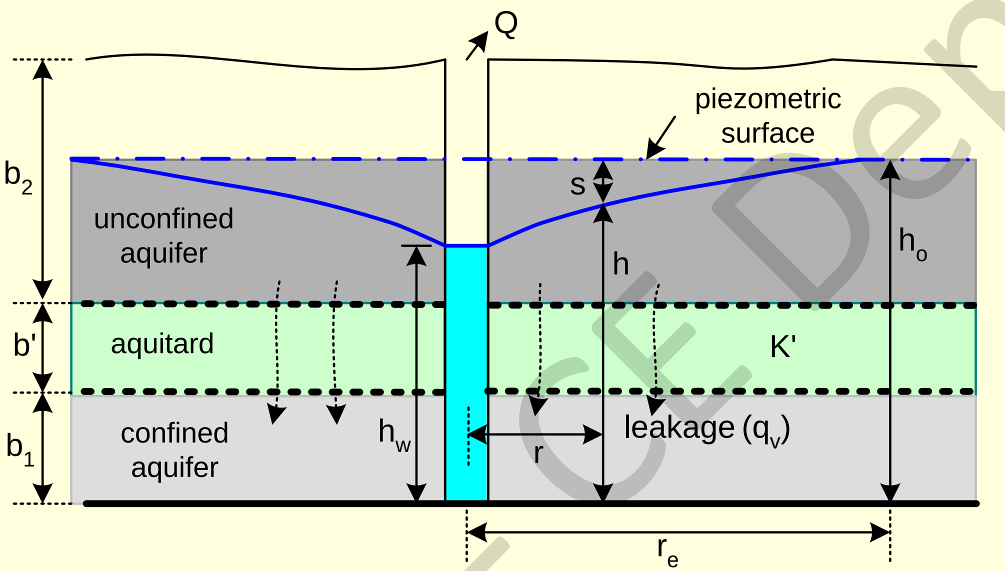


Figure 12.20 Fully penetrating well in a leaky confined aquifer

$$q_v = -K'\frac{h_0 - h}{b'} = -K'\frac{s}{b'}$$

q_v = rate of leakage per unit area

K' = vertical hydraulic conductivity of aquitard

b' = thickness of the aquitard

h_0 = initial head

s = drawdown

Fully penetrating well in a leaky confined aquifer

$$q_v = -K' \frac{h_0 - h}{b'} = -K' \frac{s}{b'}$$

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} + \frac{q_v}{T} = \frac{S}{T}\frac{\partial s}{\partial t}$$

Define a new variable

$$B^2 = \frac{T\,b'}{K'}$$

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} - \frac{s}{B^2} = \frac{S}{T}\frac{\partial s}{\partial t}$$

Solution of this equation will have the same boundary conditions as in the "fully penetrating well in confined aquifer " case.

Then the solution: $s = \frac{Q}{4\pi T} \underbrace{W\left(u, \frac{r}{B}\right)}_{\text{Well function for leaky aquifers}}$ where $u = \frac{r^2 S}{4Tt}$

Fully penetrating well in a leaky confined aquifer

Well function for leaky aquifer

W (u, r/b)										
u r/b	0.002	0.004	0.005	0.007	0.01	0.02	0.04	0.06	0.08	0.10
0	12.661	11.275	10.829	10.156	9.443	8.057	6.673	5.866	5.295	4.854
1×10^{-6}	12.442	11.271	10.828							
2×10^{-6}	12.101	11.226	10.817	10.155						
5×10^{-6}	11.438	10.964	10.682	10.129	9.443					
8×10^{-6}	11.038	10.715	10.503	10.060	9.431					
1×10^{-5}	10.838	10.573	10.396	10.003	9.418	8.057				
2×10^{-5}	10.193	10.052	9.953	9.713	9.296	8.056				
5×10^{-5}	9.306	9.248	9.205	9.096	8.883	8.008	6.673			
7×10^{-5}	8.976	8.934	8.903	8.822	8.663	7.946	6.673			
1×10^{-4}	8.623	8.594	8.572	8.515	8.398	7.838	6.669	5.866		
2×10^{-4}	7.935	7.920	7.896	7.880	7.819	7.497	6.624	5.864	5.295	
5×10^{-4}	7.022	7.016	7.012	7.000	6.975	6.835	6.363	5.801	5.285	4.853

Fully penetrating well in a leaky confined aquifer

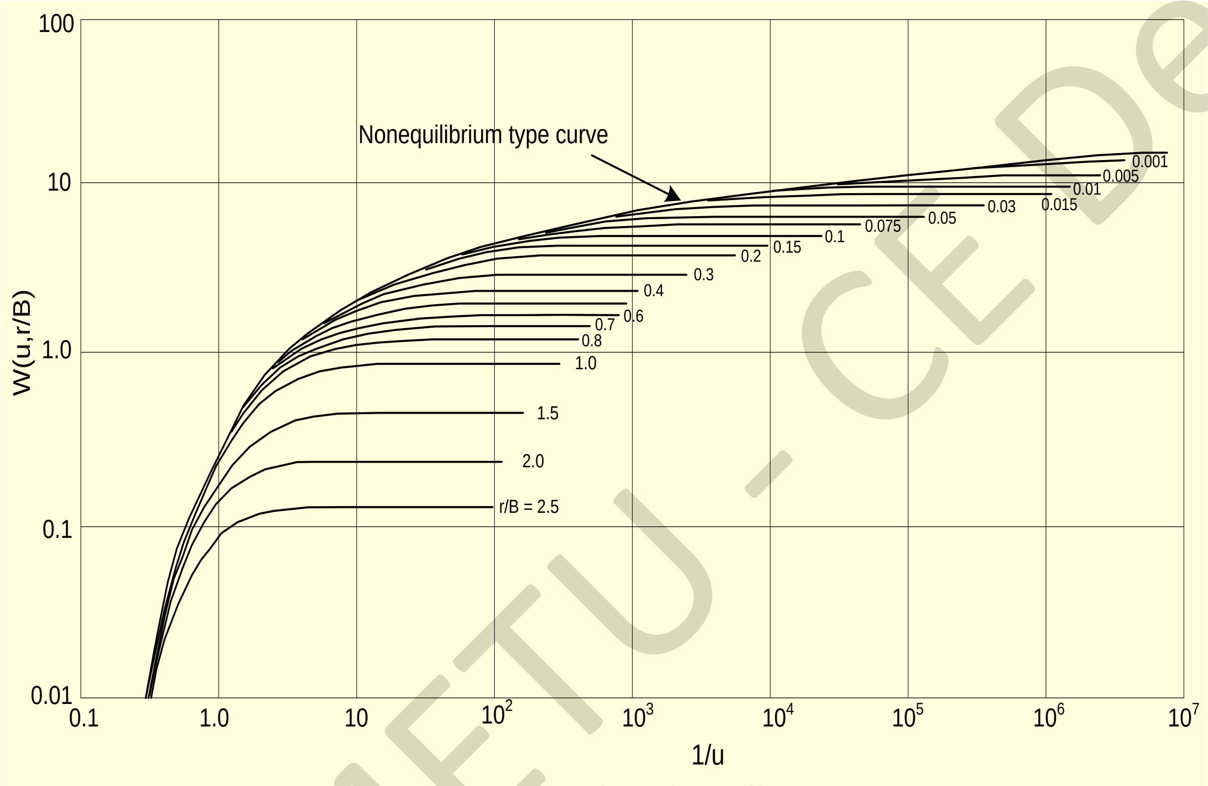
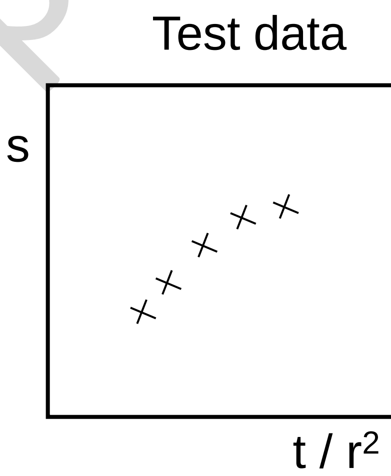


Figure 12.21 Type curves for leaky aquifers



$$T = \frac{Q \times W(u, r/B)^*}{4\pi s^*}$$

$$S = 4Tu^* \left(\frac{t}{r^2}\right)^*$$

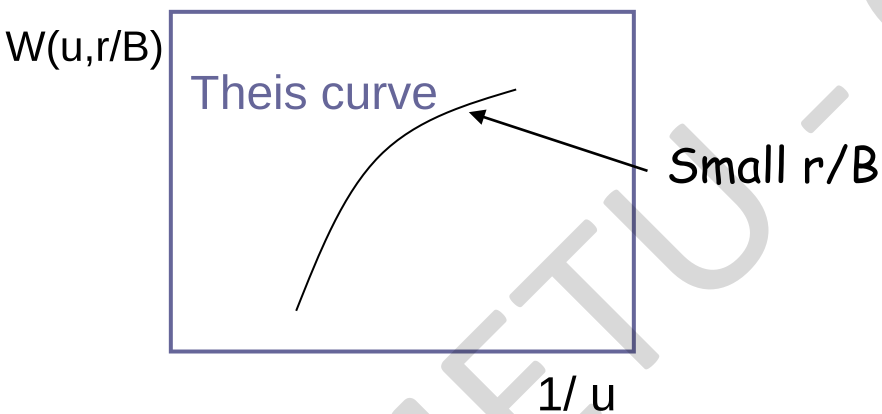
Type curves of W (u, r/B) vs 1/u for a leaky aquifer

Fully penetrating well in a leaky confined aquifer

$\frac{r}{B}$ Very small → Theis curve

B → Very big

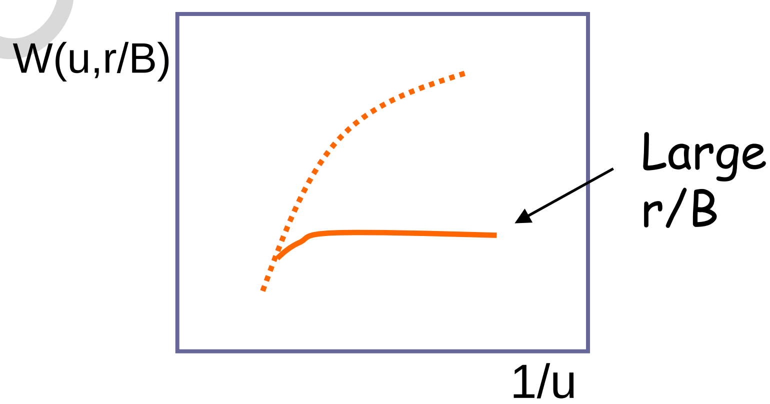
$B^2 = \frac{T\,b'}{K'}$ → **Very small K'** **Very big b'** **Very big T**



$\frac{r}{B}$ Very big

B → Very small

$B^2 = \frac{T\,b'}{K'}$ → **Very big K'** **Very small b'** **Very small T**



How about drawdowns? $s = \frac{Q}{4\pi T} W(u, \frac{r}{B})$

Less drawdown (Because aquifer is fed by the aquitard considerably)

Generalization of solutions by superposition

- All analytical solutions presented previously involve the following assumptions
  - Single well
  - Constant and continuous pumping rate
  - Infinitely large aquifer
- In order to handle, Multiple well systems, Finite aquifers, and Variable pumping rate
  - linearity of the governing equation is accepted
  - method of superposition is used

Generalization of solutions by superposition

Multiple Well Systems

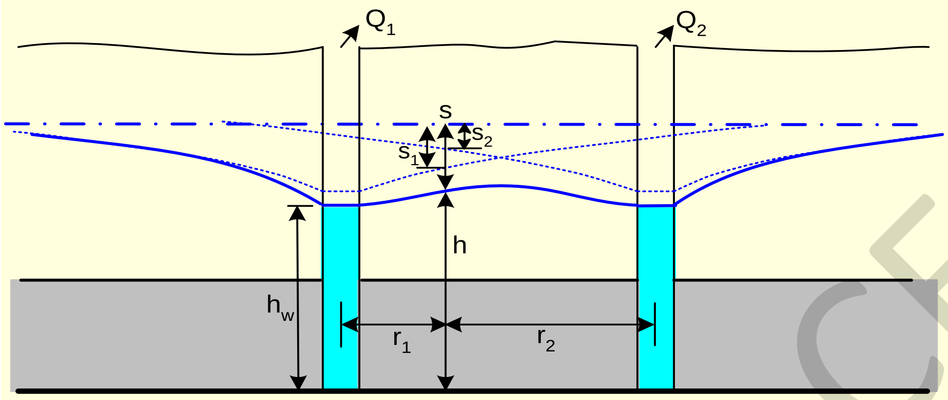


Figure 12.22 Drawdown in a confined aquifer with two pumping wells

$$s = s_1 + s_2 = \frac{Q_1}{4\pi T} W(u_1) + \frac{Q_2}{4\pi T} W(u_2)$$

$$u_1 = \frac{r_1^2 S}{4Tt} \quad \text{and} \quad u_2 = \frac{r_2^2 S}{4Tt}$$

Both wells start to pump at time, t=0

General Formulation

$$s = \sum_{i=1}^{n} \frac{Q_i}{4\pi T} W(u_i)$$

s = drawdown

Q_i = discharge of i^{th} well

u_i = $r_i^2 S/(4Tt_i)$ Boltzman variable of i^{th} well

r_i = distance of i^{th} well to the observation point

t_i = time from the start of pumping in i^{th} well

n = number of wells

Generalization of solutions by superposition

Variable pumping rate case

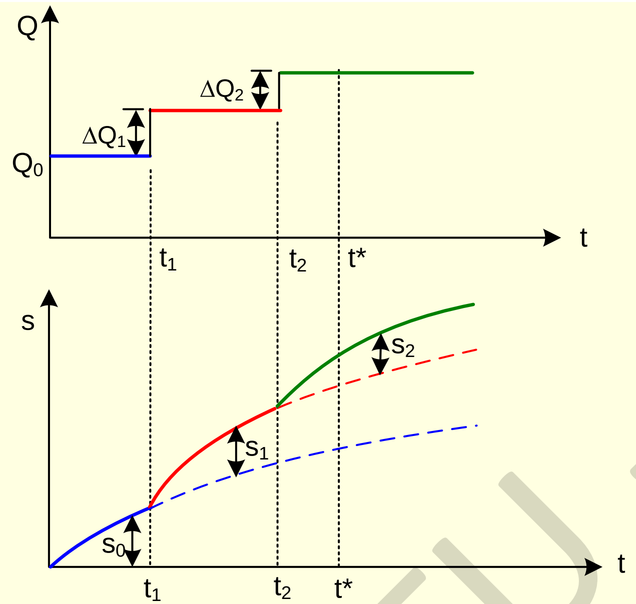


Figure 12.24 Superposition of drawdowns for stepwise pumping

$$s = s_0 + s_1 + s_2$$

$$s_0 = \frac{Q_0}{4\pi T} W(u_0) \quad \text{where} \quad u_0 = \frac{r^2 S}{4Tt^*}$$

$$s_1 = \frac{\Delta Q_1}{4\pi T} W(u_1) \quad \text{where} \quad u_1 = \frac{r^2 S}{4T(t^* - t_1)}$$

$$s_2 = \frac{\Delta Q_2}{4\pi T} W(u_2) \quad \text{where} \quad u_2 = \frac{r^2 S}{4T(t^* - t_2)}$$

In general

$$s = \frac{Q_0}{4\pi T} W(u_0) + \frac{1}{4\pi T} \sum_{i=1}^{n} \Delta Q_i W(u_i)$$

$$u_0 = \frac{r^2 S}{4Tt^*} \qquad u_i = \frac{r^2 S}{4T(t^* - t_i)}$$

t* = the time at which drawdown is required

t_i = the time at which ΔQ_i increment occurs

Generalization of solutions by superposition

Recovery of a well

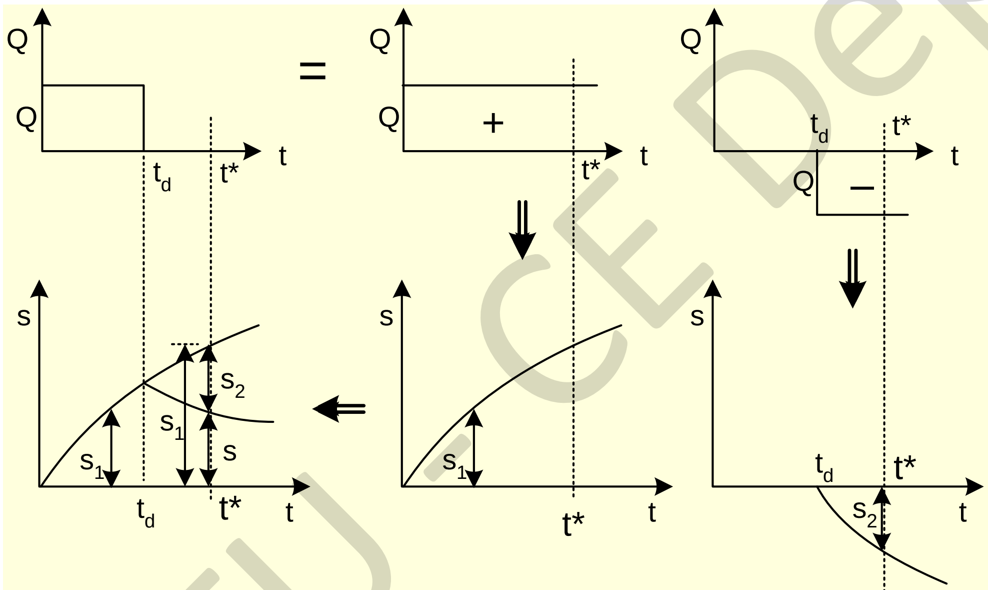


Figure 12.25 Drawdown for recovery after pumping is stopped

$$s = \frac{Q}{4\pi T} W(u_1)$$

for $0 < t < t_d$

$$s = \frac{Q}{4\pi T}\left[W(u_1) - W(u_2)\right]$$

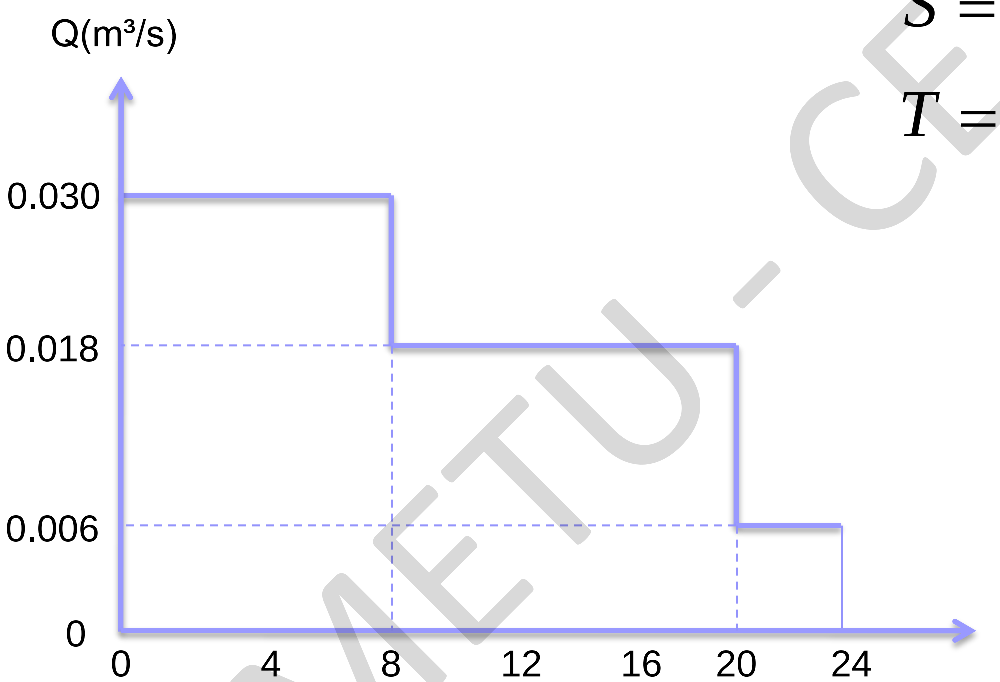
for $t > t_d$

$$u_1 = \frac{r^2 S}{4Tt^*}$$

$$u_2 = \frac{r^2 S}{4T(t^* - t_d)}$$

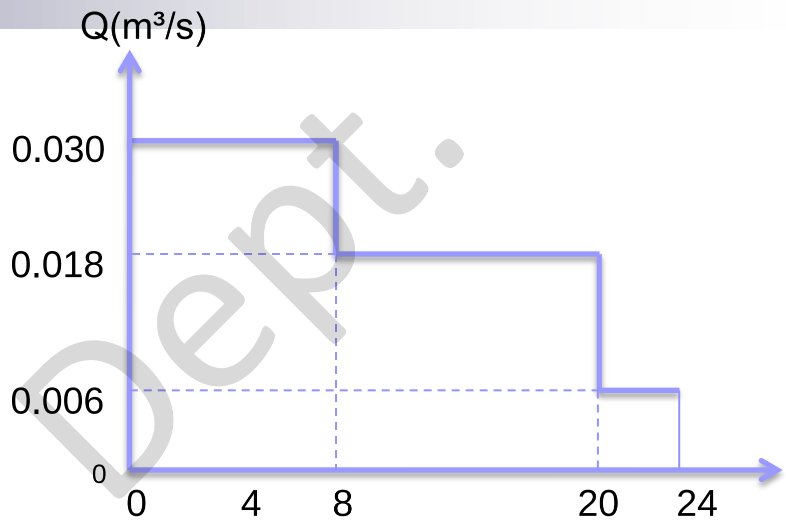
Example 19

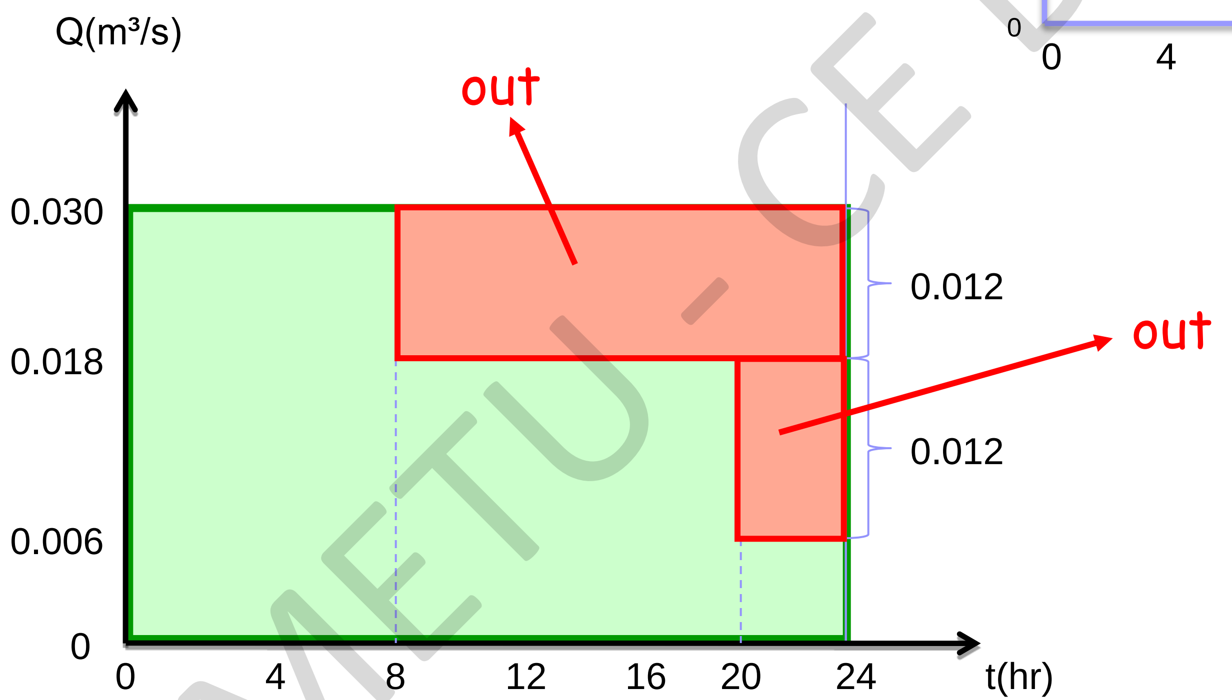
For the given pumping schedule, determine the observed drawdown at r=30 m, t=24 hr.

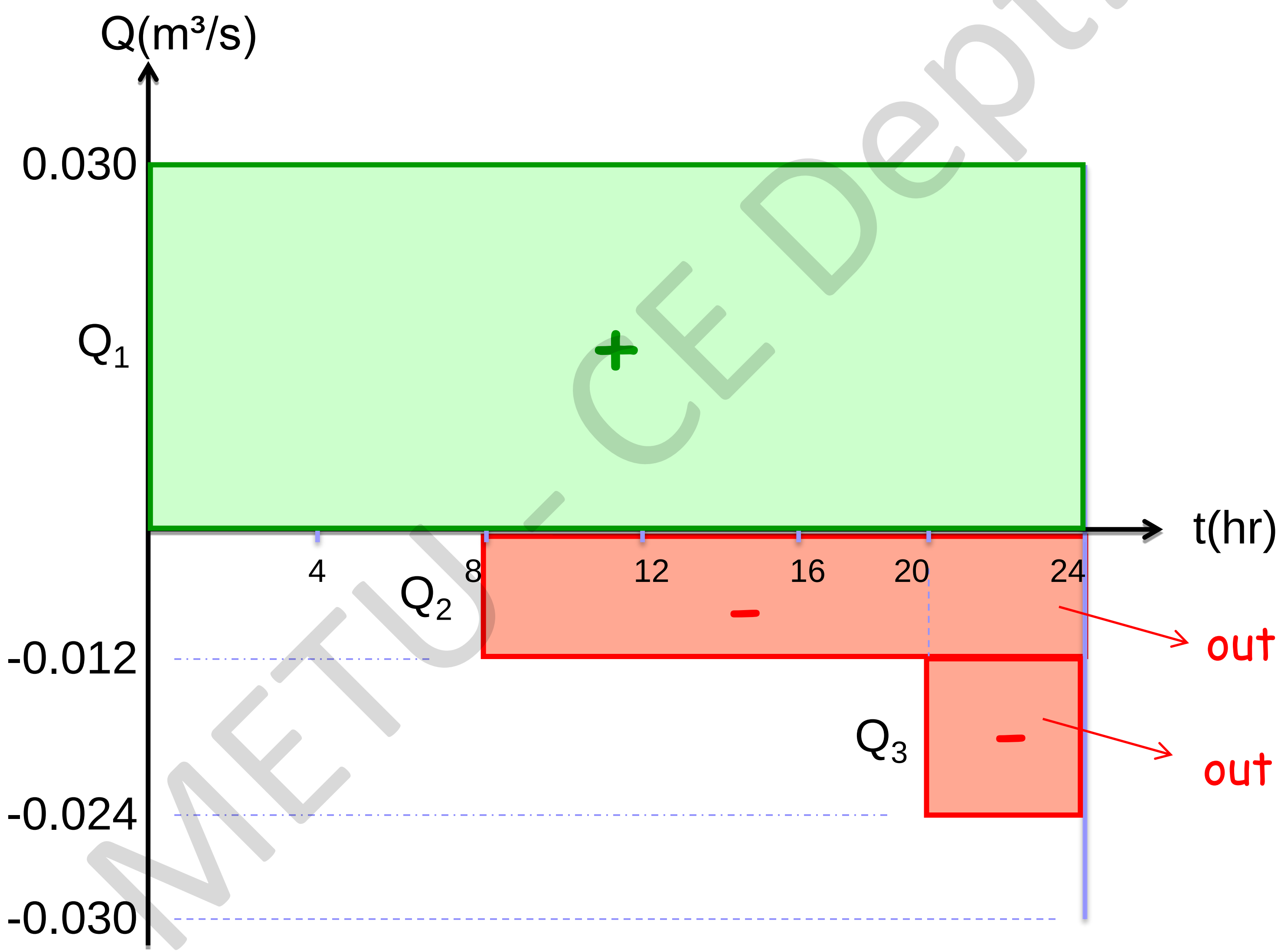


$$S = 1 \times 10^{-4}$$

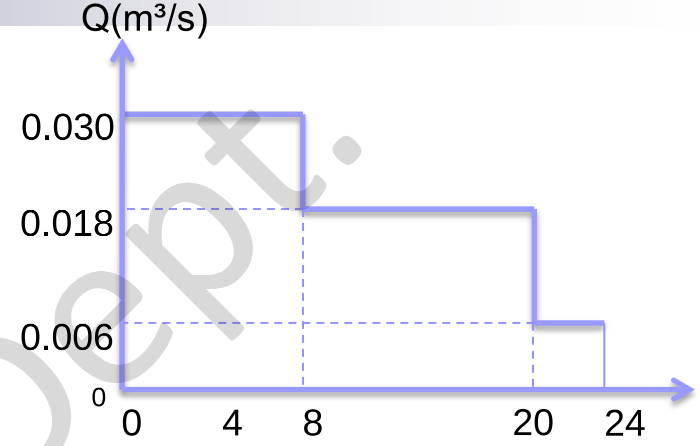
$$T = 0.0012\ m^2 / s$$


Q(m³/s)
0.030
0.018
0.006
0
0
4
8
20
24


Q(m³/s)
out
0.030
0.018
0.006
0
0.012
0.012
out
0
4
8
12
16
20
24
t(hr)



Q(m³/s)
0.030
Q1
+
t(hr)
4
8
12
16
20
24
Q2
-
out
-0.012
Q3
-
out
-0.024
-0.030



$$s = \frac{Q_1}{4\pi T}W(u_1) - \frac{Q_2}{4\pi T}W(u_2) - \frac{Q_3}{4\pi T}W(u_3)$$

$$s = \frac{1}{4\pi T}\left[Q_1 W(u_1) - Q_2 W(u_2) - Q_3 W(u_3)\right]$$

$$u_1 = \frac{r^2 S}{4Tt^*} = \frac{30^2 \times 0.0001}{4 \times 0.0012 \times 24 \times 3600} = 0.000217 \rightarrow w(u_1) = 7.85$$

$$u_2 = \frac{r^2 S}{4T(t^* - t_1)} = \frac{30^2 \times 0.0001}{4 \times 0.0012 \times (24-8) \times 3600} = 0.000325 \rightarrow w(u_2) = 7.45$$

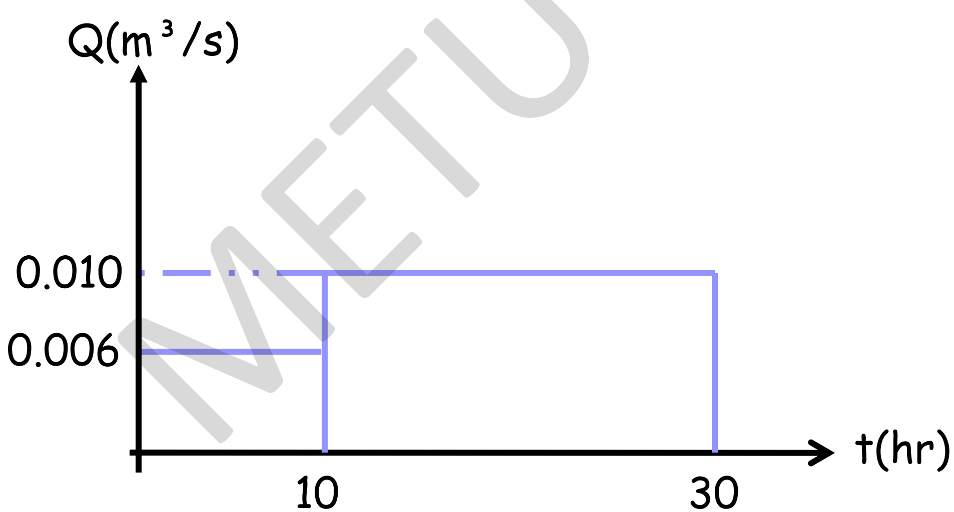
$$u_3 = \frac{r^2 S}{4T(t^* - t_2)} = \frac{30^2 \times 0.0001}{4 \times 0.0012 \times (24-20) \times 3600} = 0.0013 \rightarrow w(u_2) = 6.08$$

$$s = \frac{1}{4\pi \times 0.0012}(0.030 \times 7.85 - 0.012 \times 7.45 - 0.012 \times 6.08)$$

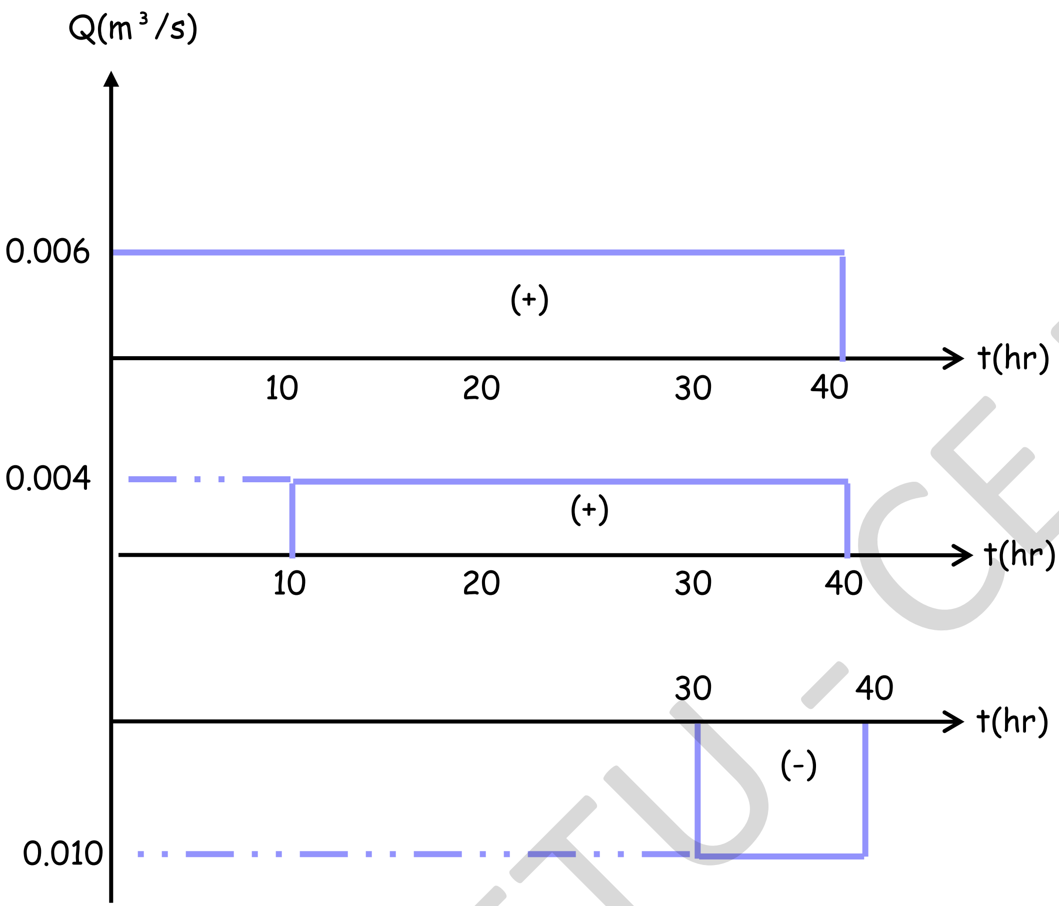
$$s = 4.85m$$

Example 20

A discharge well in a confined aquifer is pumped according to the schedule shown below. The aquifer has the characteristics of T=0.006 m^2/s and S=0.0001. Initially the piezometric surface is horizontal. Determine the drawdown at a point 500 m from the well 40 hours after pumping starts.



$T = 6*10^{-3}\ m^2/s$
$S = 0.0001$
$r = 500\ m$
$t = 40\ hr$



$$u_1 = \frac{500^2 \times 0.0001}{4 \times 0.006 \times 40 \times 3600} = 7.23 \times 10^{-3}$$
$$W(u_1) = 4.36$$

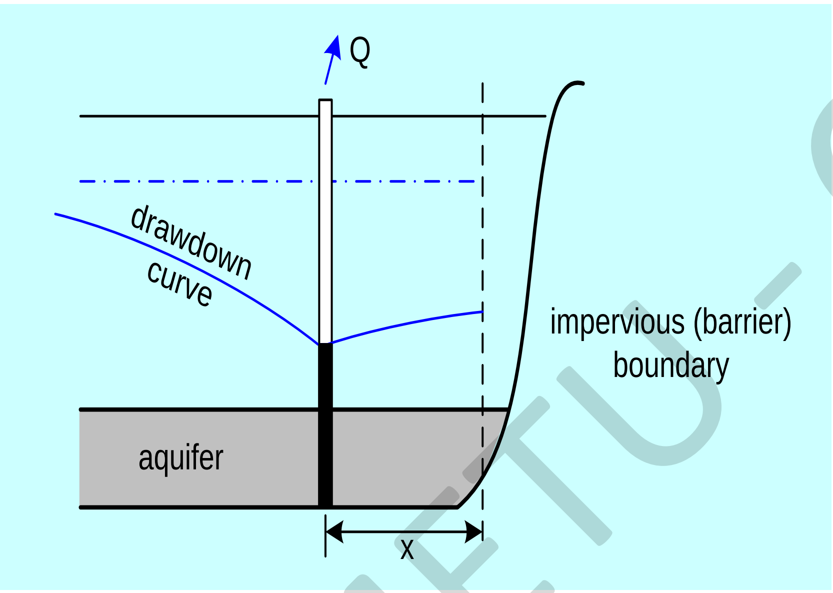
$$u_2 = \frac{500^2 \times 0.0001}{4 \times 0.006 \times 30 \times 3600} = 9.65 \times 10^{-3}$$
$$W(u_2) = 4.07$$

$$u_3 = \frac{500^2 \times 0.0001}{4 \times 0.006 \times 10 \times 3600} = 2.89 \times 10^{-2}$$
$$W(u_3) = 3.00$$

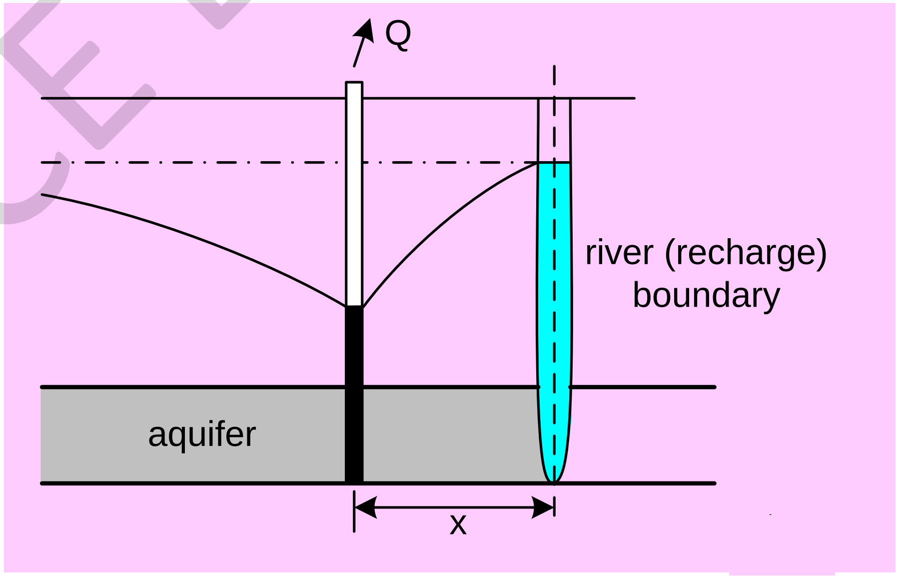
$$s = s_1 + s_2 + s_3$$
$$s = \frac{1}{4\pi 0.006}(0.006 \times 4.36 + 0.004 \times 4.07 - 0.010 \times 3.00)$$
$$s = 0.165\ m$$

FINITE AQUIFERS

- Image well concept



impervious
boundary



recharge
boundary

Generalization of solutions by superposition

Impervious boundary → an image well with same Q is placed at a point symmetrical to the real well

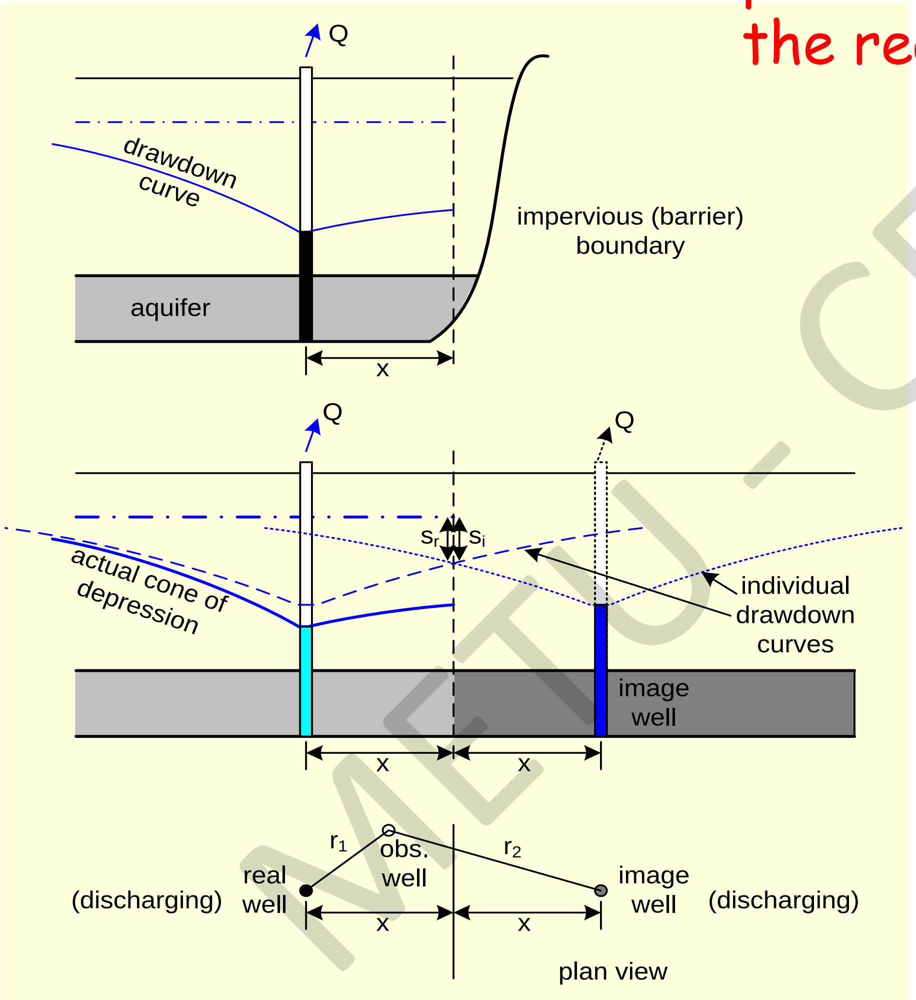


Figure 12.26 Effect of impervious boundary

$$s = \frac{Q}{4\pi T}\left[W(u_r) + W(u_i)\right]$$

$$u_r = \frac{r_r^2 S}{4Tt} \qquad u_i = \frac{r_i^2 S}{4Tt}$$

Generalization of solutions by superposition

Recharge boundary → an image well with -Q is placed at a point symmetrical to the real well

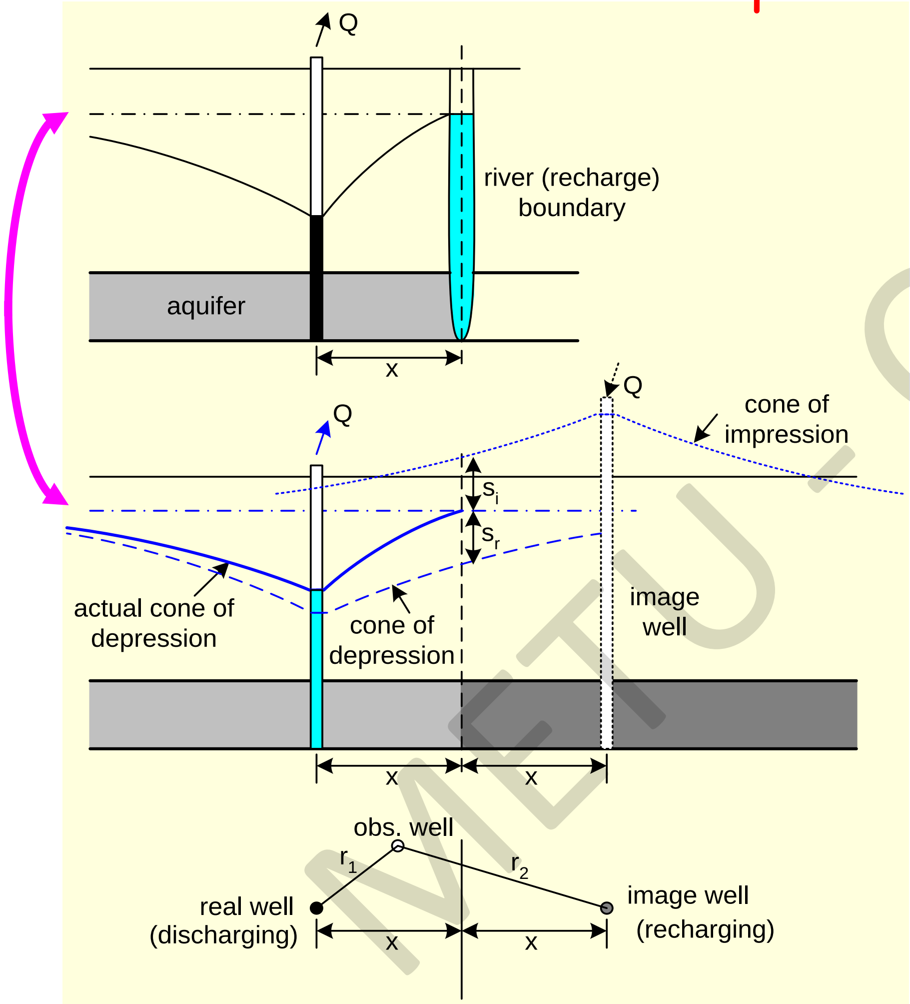


Figure 12.27 Effect of recharge boundary

$$s = \frac{Q}{4\pi T}\left[W(u_r) - W(u_i)\right]$$

$$u_r = \frac{r_r^2 S}{4Tt} \qquad u_i = \frac{r_i^2 S}{4Tt}$$

Generalization of solutions by superposition

Parallel boundaries (both impervious)

A pumping well in a confined alluvial aquifer in a more or less straight valley

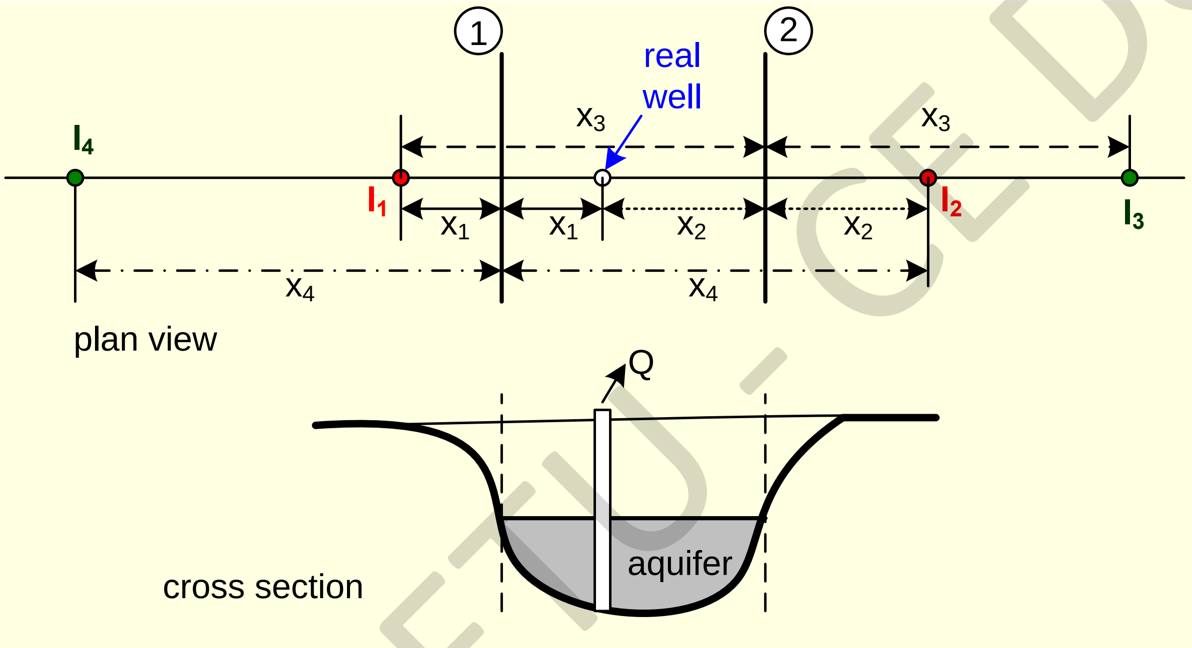


Figure 12.28 Image wells in a confined valley aquifer

- all wells are discharge wells
- infinitely may wells are req'd
- when effect on s gets small, stop adding wells

$$s = \frac{Q}{4\pi T}\left[W(u_r) + \boxed{W(u_1) + W(u_2)} + \boxed{W(u_3) + W(u_4)} + \ldots\right]$$

$$u_r = \frac{r_r^2 S}{4Tt}$$

$$u_i = \frac{r_i^2 S}{4Tt}$$

Generalization of solutions by superposition

Parallel boundaries (one impervious the other recharge)

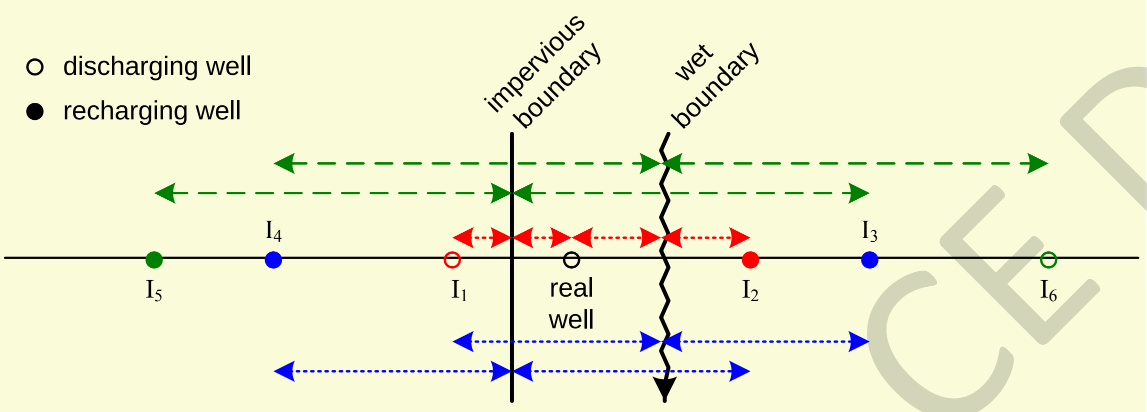


Figure 12.29 Image well system for parallel boundaries

I_1, I_2	Primary
I_3, I_4	Secondary
I_5, I_6	Tertiary

$$S_T = S_r + \boxed{S_1 + S_2} + \boxed{S_3 + S_4} + \boxed{S_5 + S_6}$$

It is possible to use image well approach to provide predictions of drawdown in systems with more than one boundary

Real pumping well I_r

Image well I_1 →	Image well I_3 →	Image well I_5
Image well I_2 →	Image well I_4 →	Image well I_6

When to stop putting image wells?

Generalization of solutions by superposition

Application of the method of images to:

1) Semi Infinite strip aquifers

2) Rectangular aquifers

3) Wedge-shaped aquifers

For example: Semi infinite strip aquifer

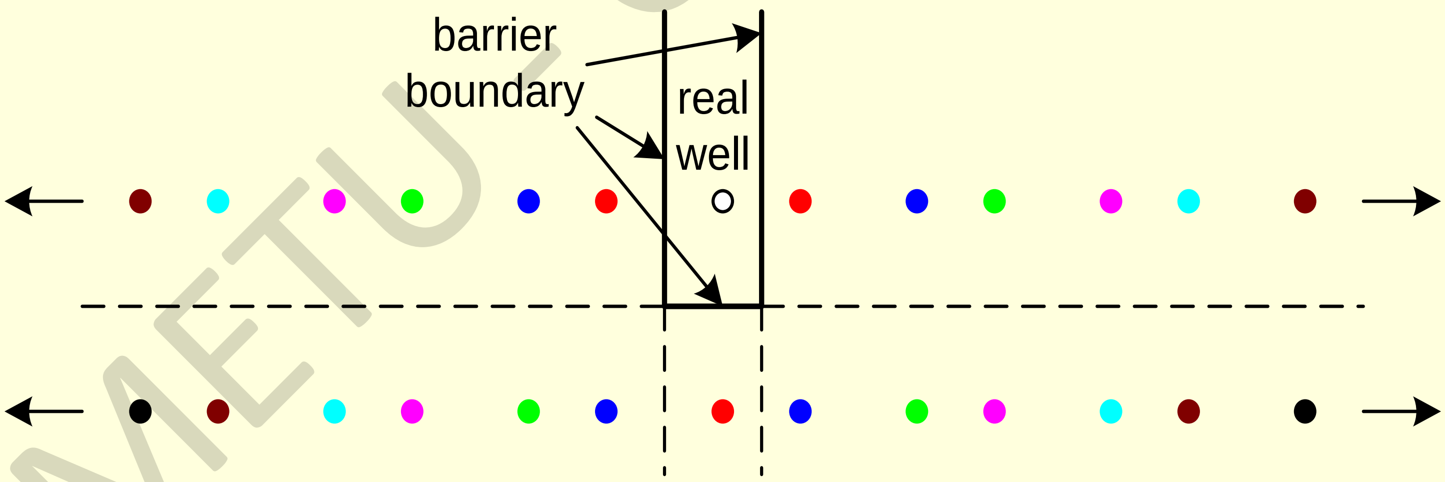
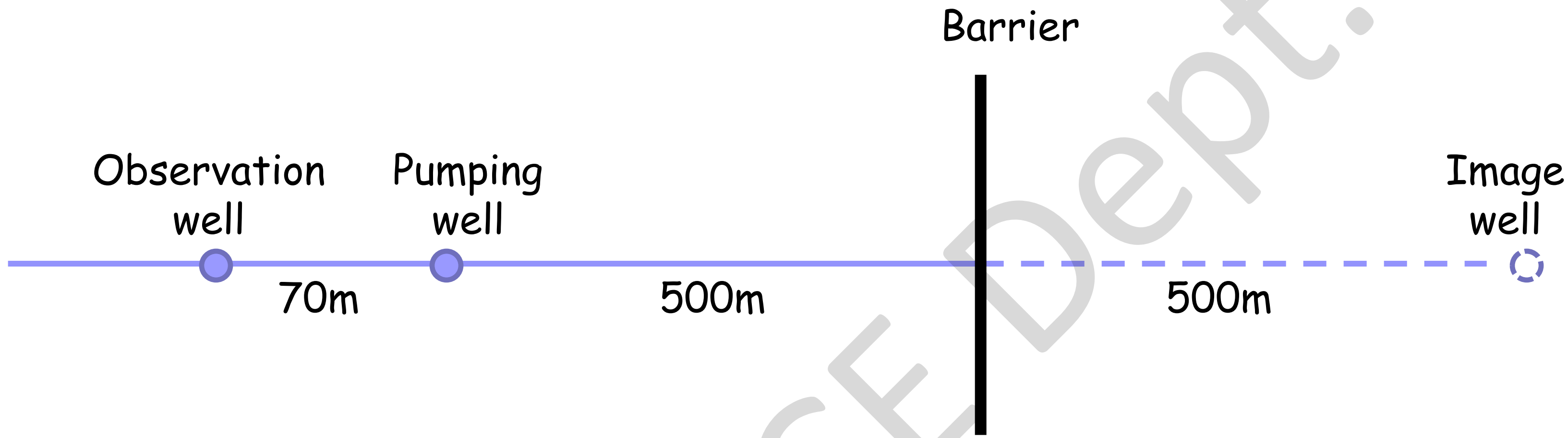


Figure 12.30 Image well system for semi infinite aquifer

Example 21

A pumping test with a constant discharge of Q=0.072 m^3/s conducted in a confined aquifer. There is a barrier boundary at 500 m away from the pumping well. The drawdowns are measured in an observation well located 70 m away from the pumping well. At what time after the pumping started, the boundary has started to influence the drawdown in the observation well?

- S=0.10, T=0.015 m^2/s
- Assume a smallest drawdown of 0.005 m was detectable in the field.



$$W(u) = \frac{4\pi T}{Q} \times s = \frac{4\pi \times 0.015}{0.072} \times 0.005$$

$W(u) = 0.01309 \longrightarrow$ from table u=3

$$u = \frac{r^2 S}{4Tt}, \qquad t = \frac{r^2 S}{4Tu} = \frac{1070^2 \times 0.10}{4 \times 0.015 \times 3} = 636055 \text{ sec}$$

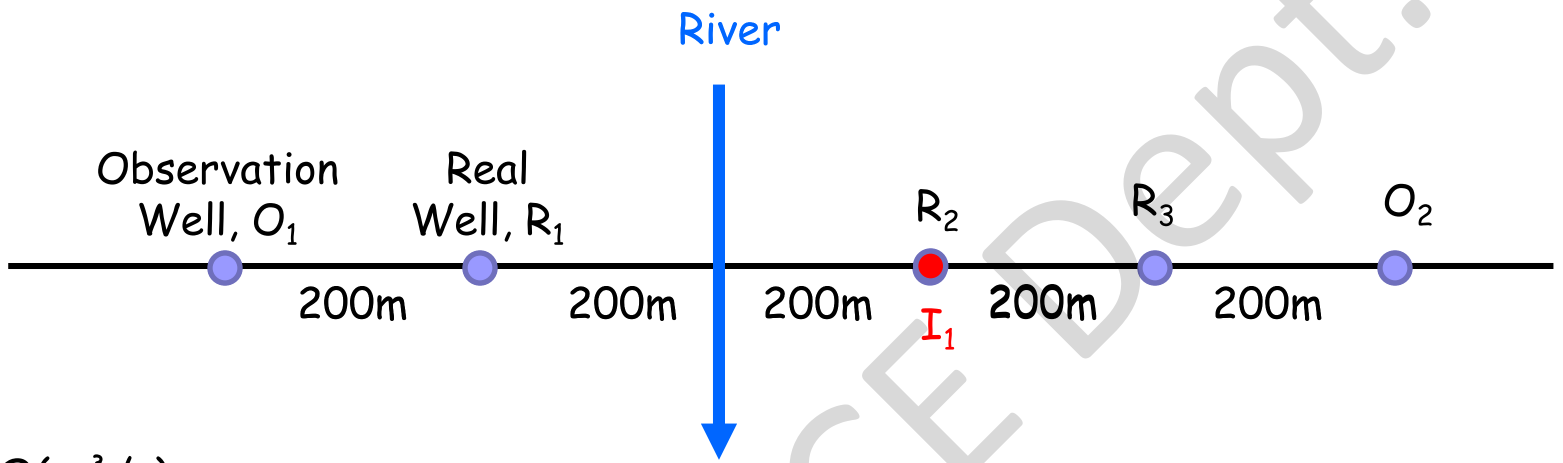
$$t = 176.68\ hr$$

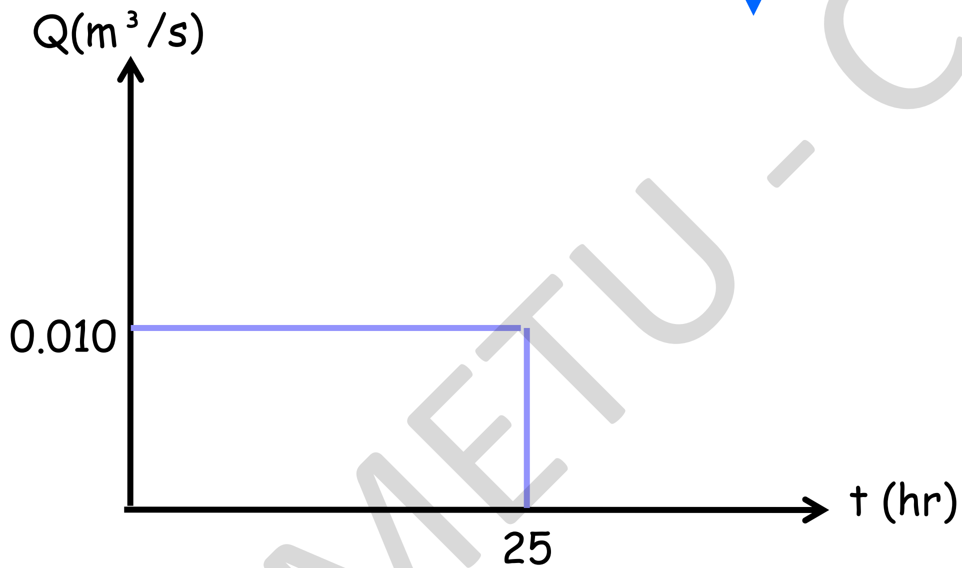
Example 22

The discharge wells are fully penetrated into a confined aquifer on both sides of a river boundary with two observation wells as shown in the figure. If $S=0.0001$, $T=0.006\ m^2/s$ and $r=1.0$ m determine;

a) The drawdown in the observation wells at 20 hours after pumping starts if all the wells are pumping according to the schedule as shown on figure.

b) The drawdown in the discharge well (R_1) and observation well (O_2) at 30 hours after pumping starts if only discharge well (R_1) pumps according to the schedule as shown on figure.





Pumping Schedule

$$S = 0.0001$$

$$T = 0.006\ m^2 / s$$

$$r_w = 1\ m$$

a) Drawdown in O_1 and O_2 at t=20 hr (All wells are pumping)

observation well 1, O_1

	r	Q	t	$u=\frac{r^2 S}{4Tt}$	W(u)	$s=\frac{Q}{4\pi T}w(u)$	Sign
R_1	200	0.01	72000	0.0023	5.50	s_1=0.7295	(+)
Image of R_1, I_1	600	0.01	72000	0.021	3.30	s_1'=0.4377	(-)

$$s = s_1 - s_1'$$

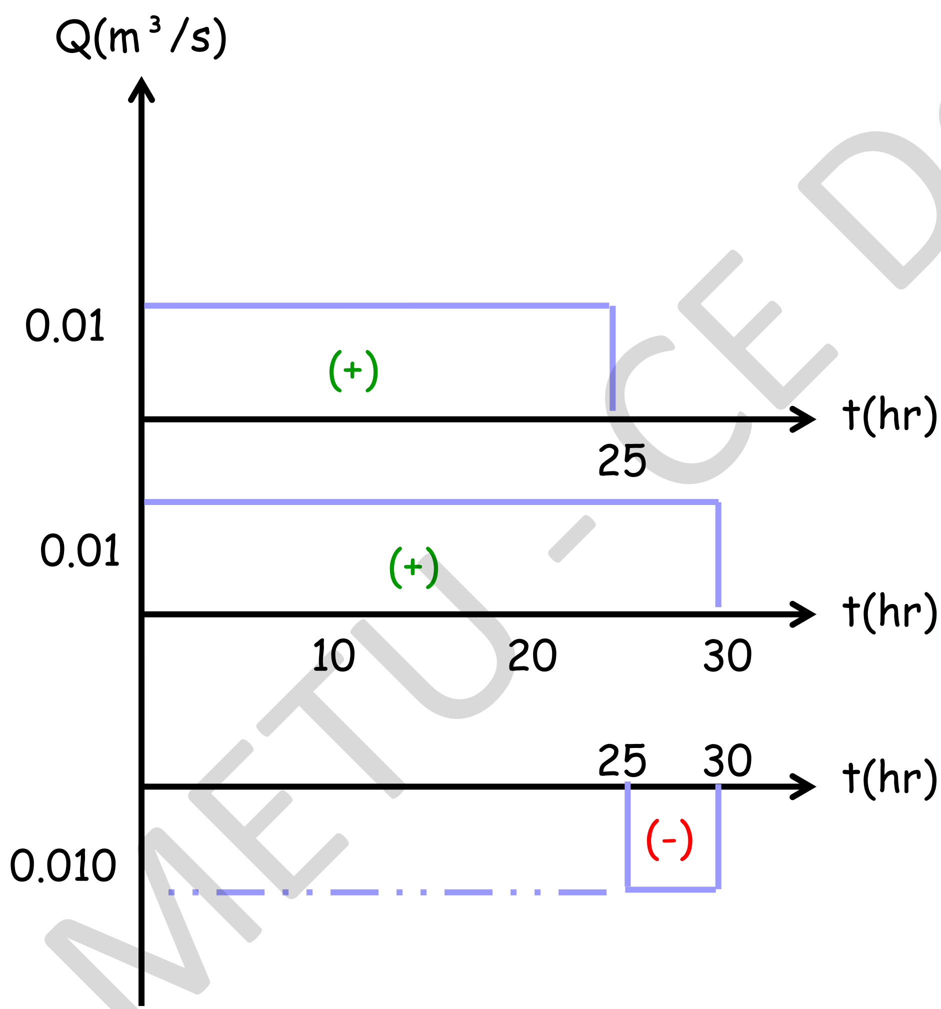
$$s = 0.292m$$

observation well 2, O_2

	r	Q	t	$u=\frac{r^2 S}{4Tt}$	W(u)	$s=\frac{Q}{4\pi T}w(u)$	Sign
R_2	400	0.01	72000	0.0093	4.12	s_2=0.5463	(+)
R_3	200	0.01	72000	0.0023	5.5	s_3=0.7295	(+)
Image of R_2, I_2	800	0.01	72000	0.037	2.76	s_2'=0.3660	(-)
Image of R_3, I_3	1000	0.01	72000	0.058	2.34	s_3'=0.3103	(-)

$$s = 0.5995m$$

b) Drawdown in R_1 and O_2 at t=30 hr (only R_1 is pumping)



a) Drawdown at R_1

Drawdown at R_1 due to R_1

$$u_1 = \frac{1^2 \times 0.0001}{4 \times 0.006 \times 30 \times 3600} = 3.85 \times 10^{-8} \qquad W(u_1) = 16.59$$

$$u_2 = \frac{1^2 \times 0.0001}{4 \times 0.006 \times 5 \times 3600} = 2.32 \times 10^{-2} \qquad W(u_2) = 14.75$$

$$s_1 = \frac{Q}{4\pi T}\left[W(u_1) - w(u_2)\right] = \frac{0.01}{4\pi \times 0.006}(16.59 - 14.75) = 0.244\ m$$

Drawdown at R_1 due to image of R_1 (recharging)

$$u_1' = \frac{400^2 \times 0.0001}{4 \times 0.006 \times 30 \times 3600} = 6.2 \times 10^{-3} \qquad W(u_1') = 4.5$$

$$u_2' = \frac{400^2 \times 0.0001}{4 \times 0.006 \times 5 \times 3600} = 3.7 \times 10^{-2} \qquad W(u_2') = 2.76$$

$$s_1' = \frac{0.01}{4\pi \times 0.006}(-4.5 + 2.76) = -0.231\ m$$

Total $s = s_1 + s_1' = 0.244 - 0.231 = 0.0132$ m

$$s\big|_{O_2} = 0$$

b) Drawdown at O_2 – Since R_1 does not effect the other side of the river