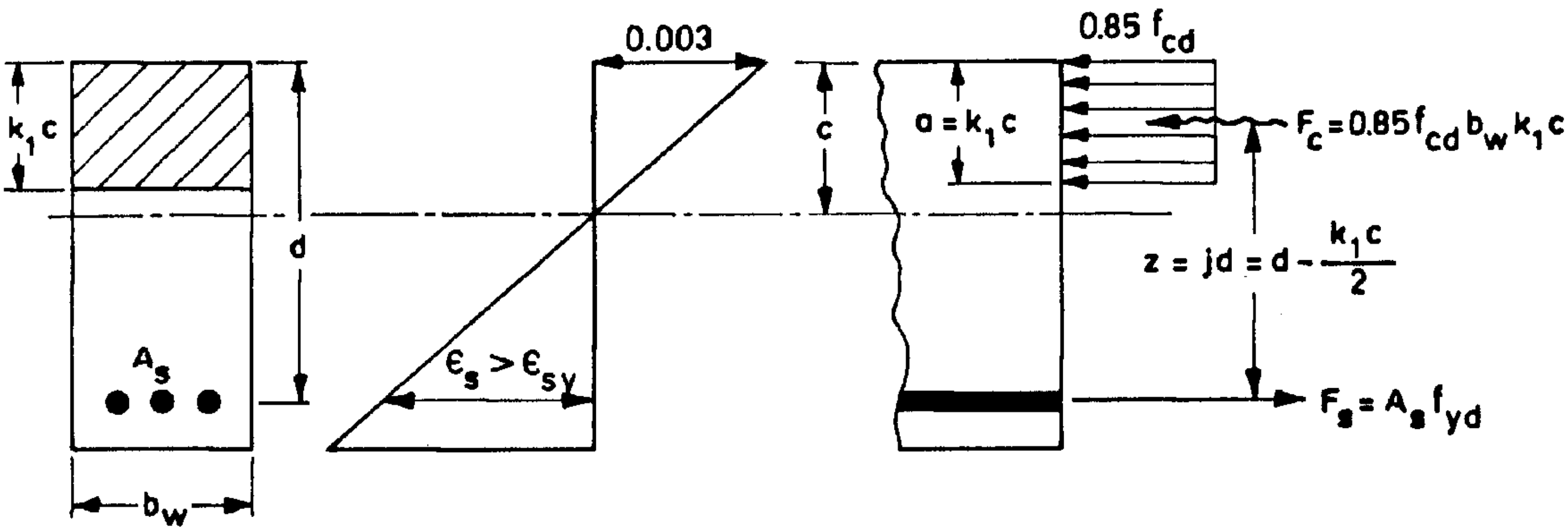
UNDERREINFORCED BEAMS ($A_s<A_{sb}$, $\varepsilon_s>\varepsilon_{sy}$)

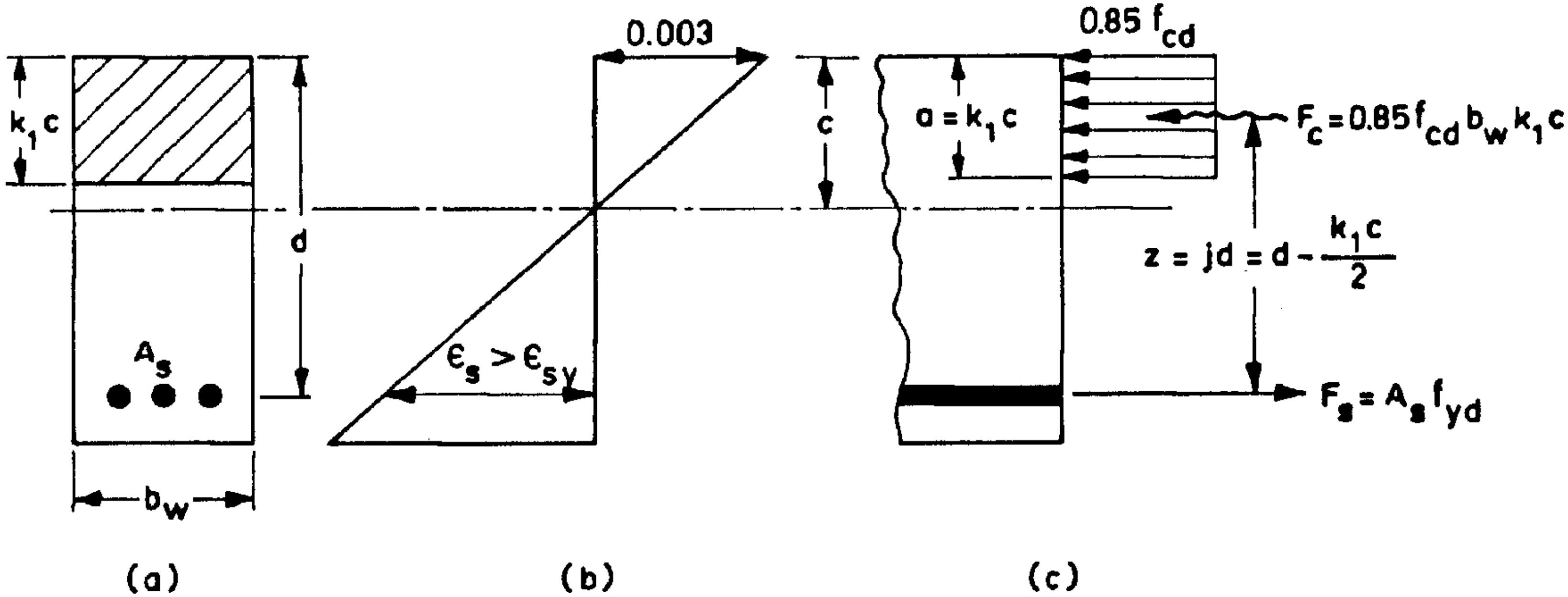


Equilibrium:

$$\Sigma F = 0; \quad F_s - F_c = 0; \quad A_s f_{yd} - 0.85 f_{cd} b_w k_1 c = 0$$

$$\frac{c}{d} = \rho \frac{1.0}{0.85 k_1} \times \frac{f_{yd}}{f_{cd}} \qquad \text{where} \qquad \rho = \frac{A_s}{b_w d}$$

UNDERREINFORCED BEAMS



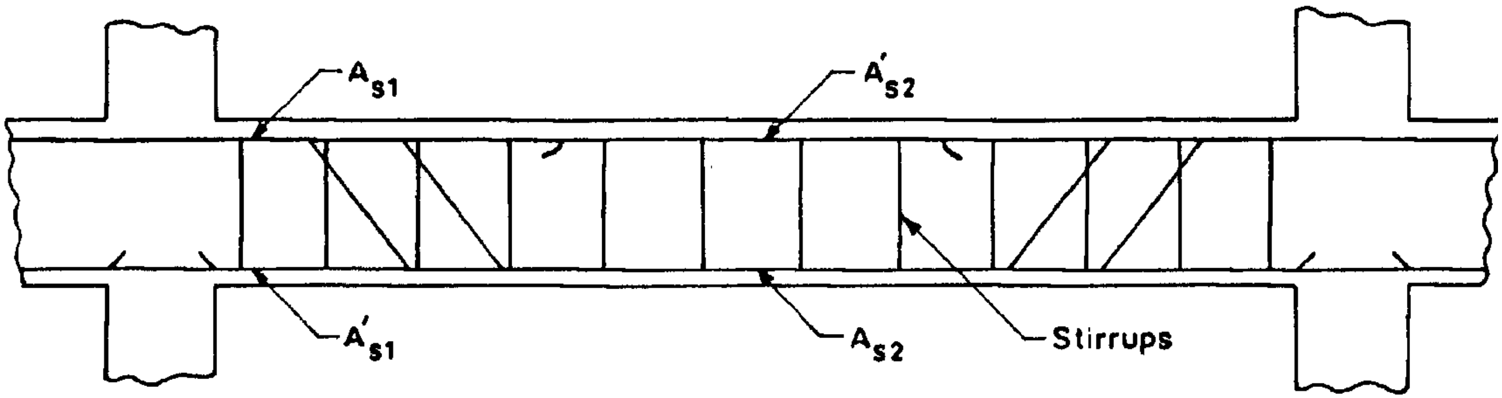
Equilibrium:

$$\Sigma M = 0 \qquad M_r = A_s f_{yd}\left(d - \frac{k_1 c}{2}\right) = A_s f_{yd}(j)d$$

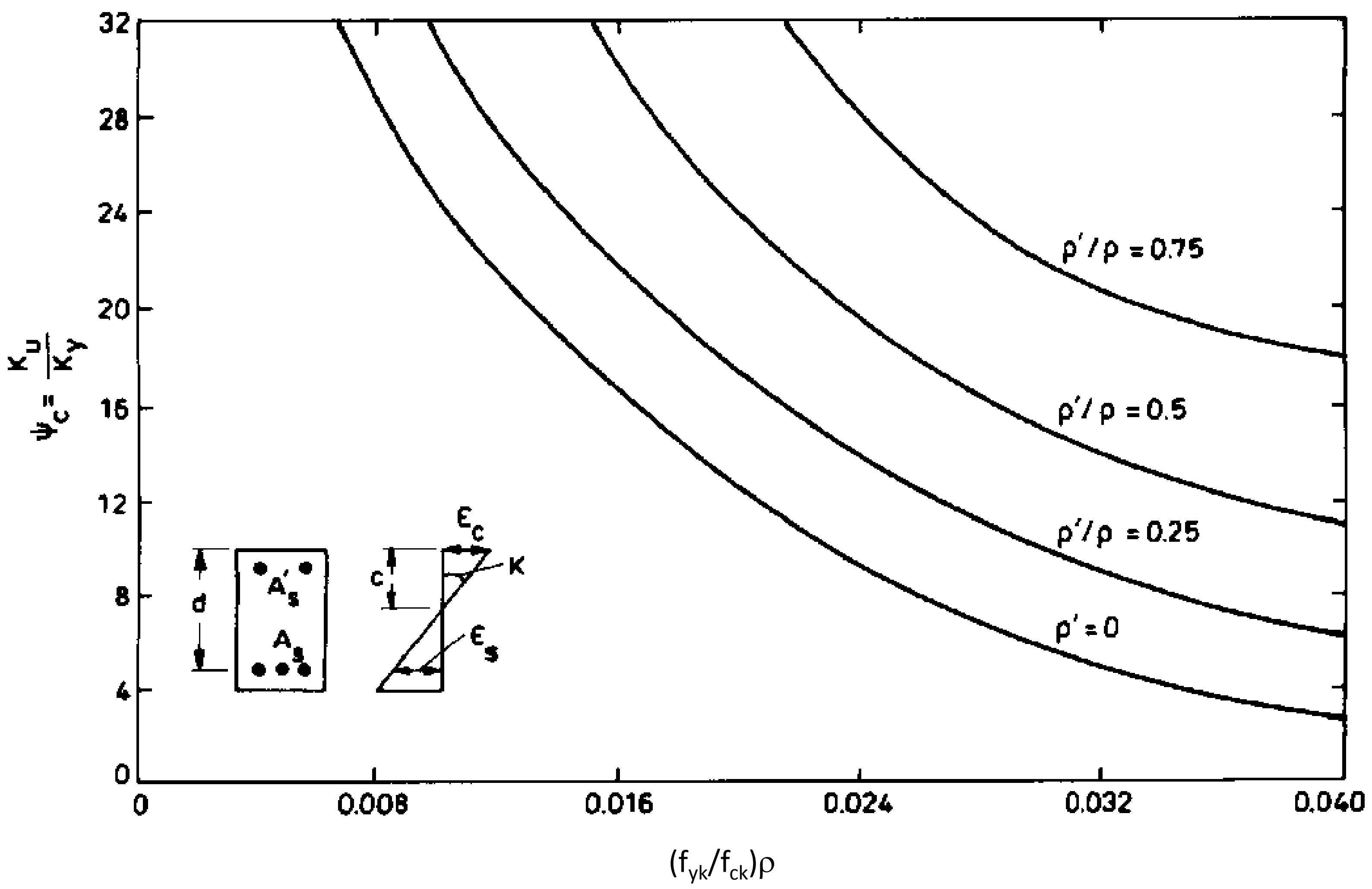
Double Reinforced Beams

In many cases, additional bars are placed in the compression zone of beams to act as compression steel. Compression steel is provided to:

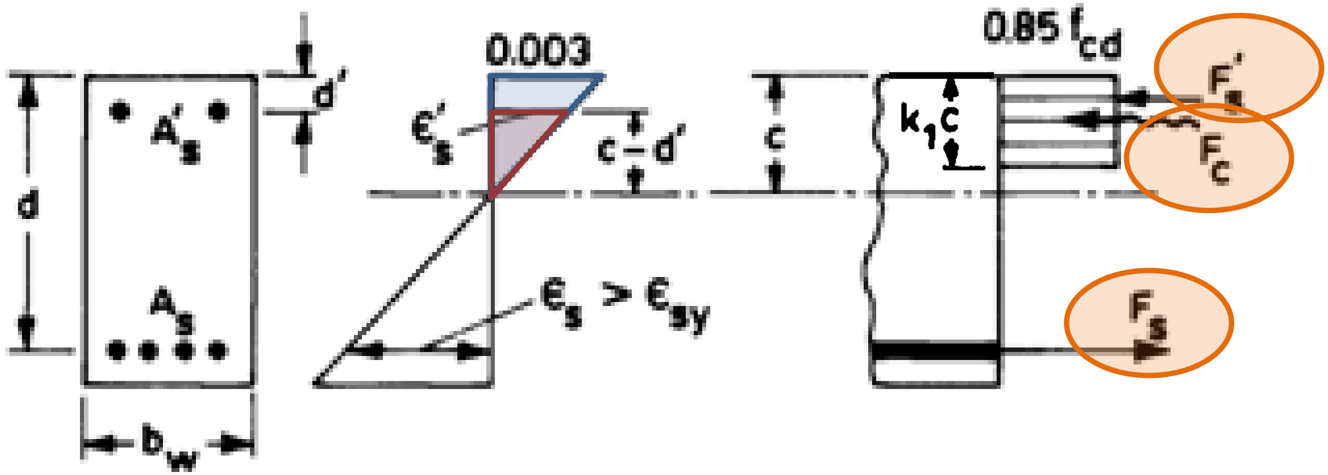
1. decrease the time dependent deformations (deflection)
2. provide support for the stirrups and
3. increase the strength and ductility of the beam under consideration.



Double Reinforced Beams



DOUBLE REINFORCED BEAMS



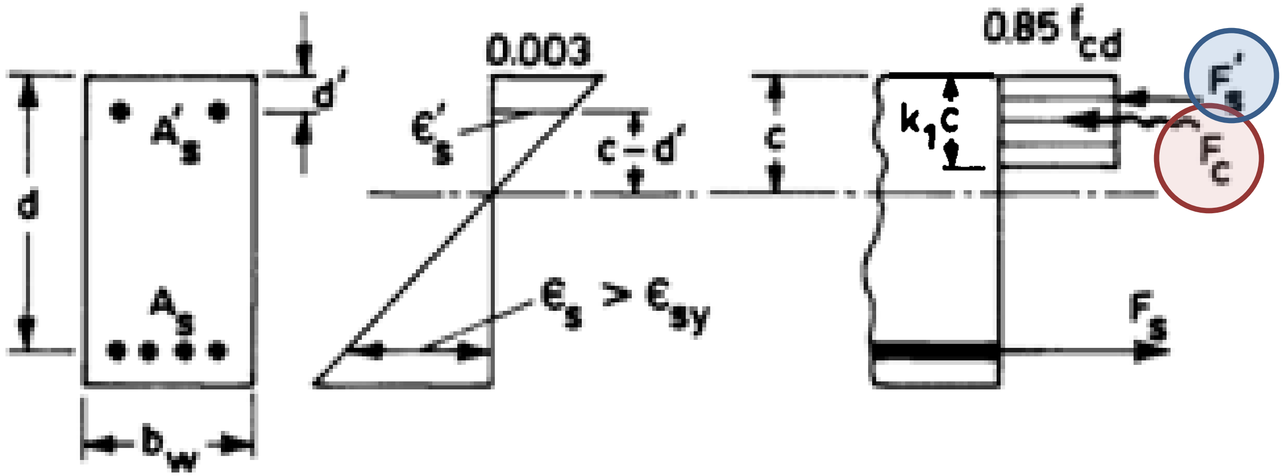
Equilibrium:

$$\Sigma F = 0 \quad A_s f_{yd} - 0.85 f_{cd}\, b_w\, k_1 c - A'_s \sigma'_s = 0$$

Compatibility: $$\varepsilon'_s = 0.003 \frac{c - d'}{c}$$

Stress-Strain: $$\sigma'_s = \varepsilon'_s E_s \le f_{yd}$$

DOUBLE REINFORCED BEAMS

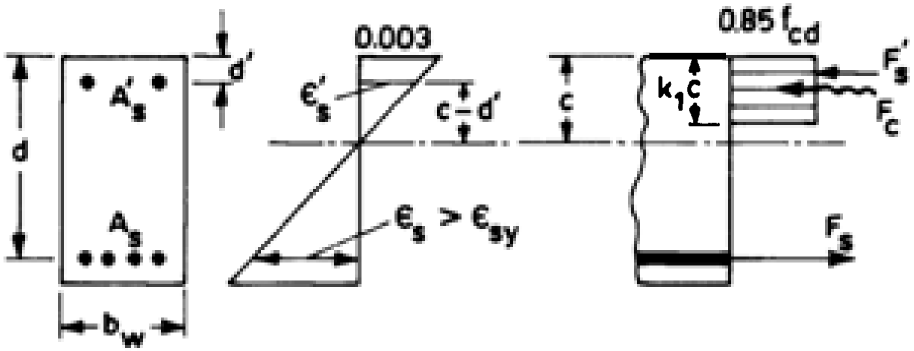


Equilibrium:

$\Sigma M = 0$, taking moment about tension steel, we have;

$$M_r = 0.85 f_{cd}\, b_w\, k_1 c \left(d - \frac{1}{2} k_1 c \right) + A'_s \sigma'_s \left(d - d' \right)$$

Double Reinforced Beams



In an analysis problem, A_s, A'_s, b_w, d, d′, f_{cd} and f_{yd} will be given, the only unknowns are c, σ'_s, ε'_s and M_r . The four equations given below are adequate for the solution.

Equilibrium:
$\Sigma F=0$

$$A_s f_{yd} - 0.85 f_{cd} b_w k_1 c - A'_s \sigma'_s = 0$$

$\Sigma M=0$, about tension steel

$$M_r = 0.85 f_{cd} b_w k_1 c\left(d - \frac{1}{2} k_1 c\right) + A'_s \sigma'_s (d - d')$$

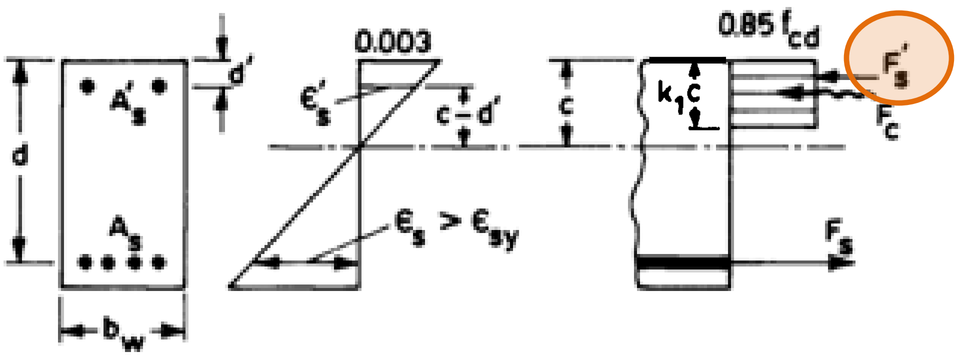
Compatibility:

$$\varepsilon'_s = 0.003 \frac{c - d'}{c}$$

Force Deformation:

$$\sigma'_s = \varepsilon'_s E_s \le f_{yd}$$

DOUBLE REINFORCED BEAMS



Equations simplify considerably if the compression steel also yields. You may, therefore, assume that $\sigma'_s = f_{yd}$ and solve the equilibrium equation fo "c." This assuption, however, requires verification at the end of the analysis.

Equilibrium: $\Sigma F=0$

$$A_s f_{yd} - 0.85 f_{cd} b_w k_1 c - A'_s \sigma'_s = 0$$

$$A_s f_{yd} - 0.85 f_{cd} b_w k_1 c - A'_s(\sigma'_s = f_{yd}) = 0$$

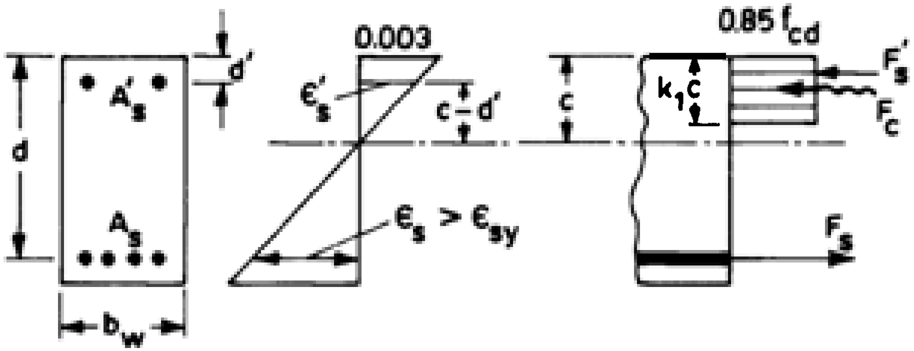
Rearranging the terms

$$f_{yd}(A_s - A'_s) = 0.85 f_{cd} b_w k_1 c$$

Solving for "c" we have;

$$c = \frac{f_{yd}(A_s - A'_s)}{0.85 f_{cd} b_w k_1}$$

DOUBLE REINFORCED BEAMS



Analysis may be completed by verifying the assumption made in the previous slide. If this assumption holds true you may finally take moment about the tension steel to find the capacity of the double reinforced beam.

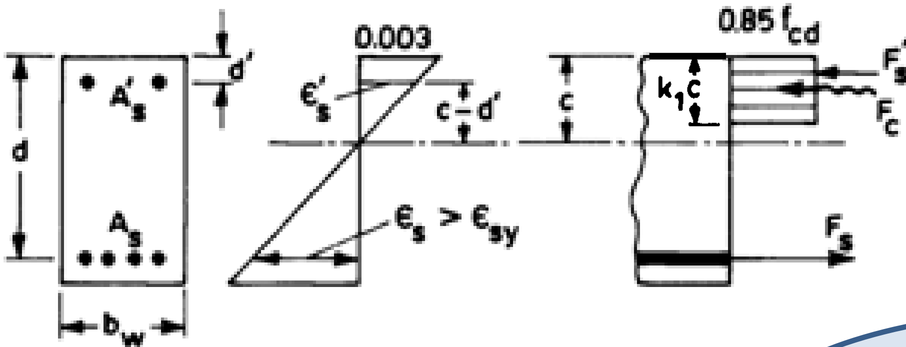
Compatibility: $$\varepsilon_s' = 0.003\,\frac{c-d'}{c} \overset{?}{\geq} \varepsilon_{sy}$$ if "YES" then;

ΣM=0, about tension steel $$M_r = 0.85 f_{cd} b_w k_1 c\left(d - \frac{1}{2}k_1 c\right) + A_s' \sigma_s' (d - d')$$

Reconsider the equilibrium equation in the previous slide. $$f_{yd}(A_s - A_s') = 0.85 f_{cd} b_w k_1 c$$

Dividing both sides by $b_w d$, the critical percentage is obtained. $$(\rho - \rho') = 0.85 k_1 \frac{c}{d} \frac{f_{cd}}{f_{yd}}$$

Double Reinforced Beams



Balanced reinforcement demand of a companion beam with same , b_w, d, f_{cd} and f_{yd}.

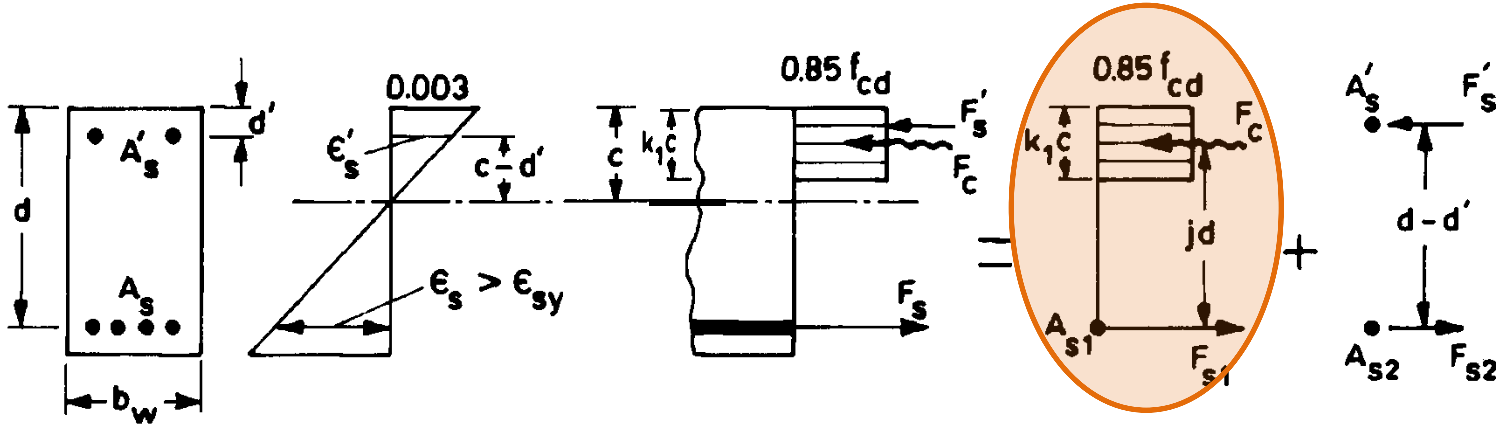
$$(\rho-\rho')=0.85k_1\frac{c}{d}\frac{f_{cd}}{f_{yd}} \leq \rho_b=\frac{0.85f_{cd}}{f_{yd}}k_1\frac{c_b}{d}$$

If the above expression holds true, then the double reinforced beam will be an underreinforced beam. If the compression steel does not yield the above expression changes slightly to:

$$\left(\rho-\rho'\frac{\sigma'_s}{f_{yd}}\right)\leq\rho_b$$

In practice, $\rho-\rho' \leq \rho_b$ can be taken as the basis regardless of the yielding of the compression steel. Usually this does not introduce any serious error.

DOUBLE REINFORCED BEAMS



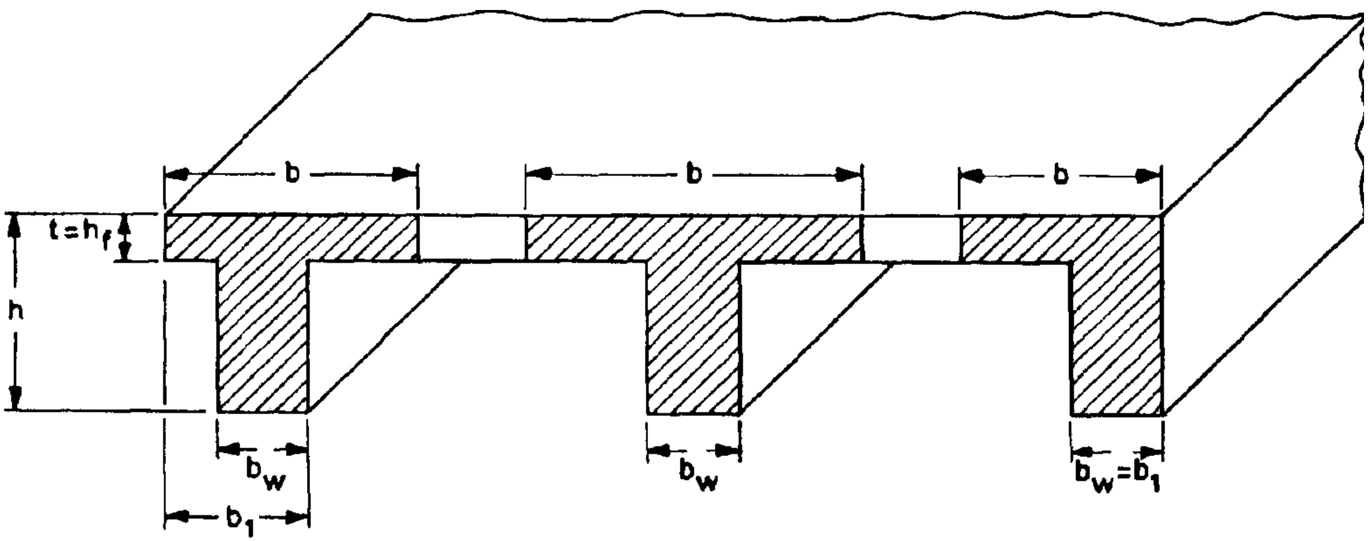
Note that, if the compression steel yields, it is possible to decompose the force system into two components as shown in figure above.

The first component involves single reinforced concrete beam, therefore, it is the component depicting the type of failure.

Hence, $\rho\text{-}\rho' \leq \rho_b$ can be taken as a measure in limiting the amount of tension steel to ensure tension failure in double reinforced beams.

FLANGED SECTIONS

When reinforced concrete beams are cast monolithically with the floor slab they support, the beam rib only acts in bending by bringing the slab into play as a compression flange. This interaction between the rib and slab give rise to beam sections of T and L shapes as indicated in figure below.



FLANGED SECTIONS

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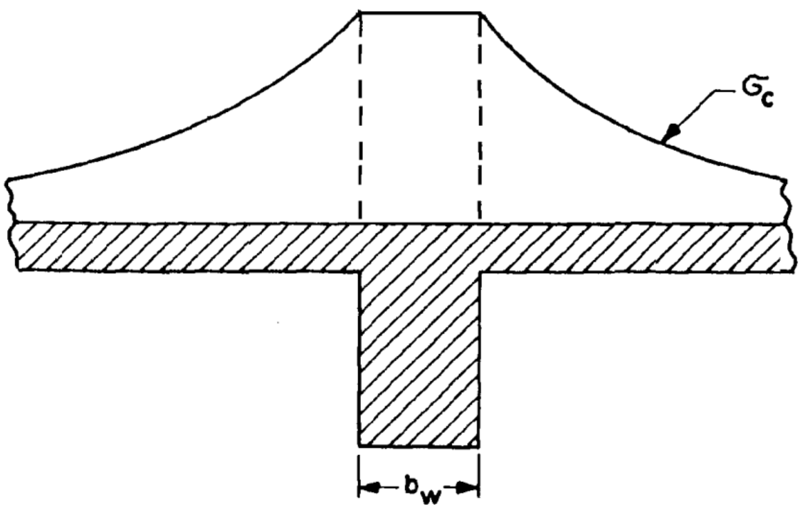
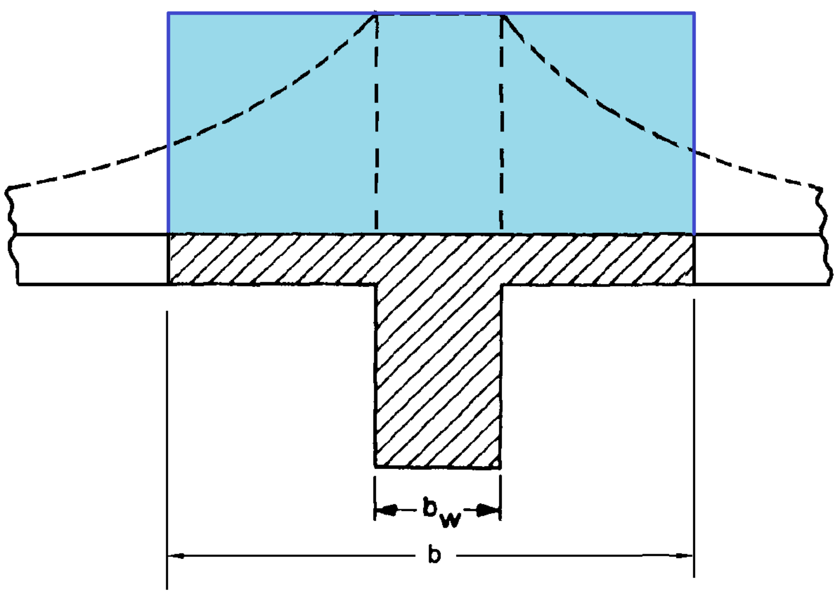
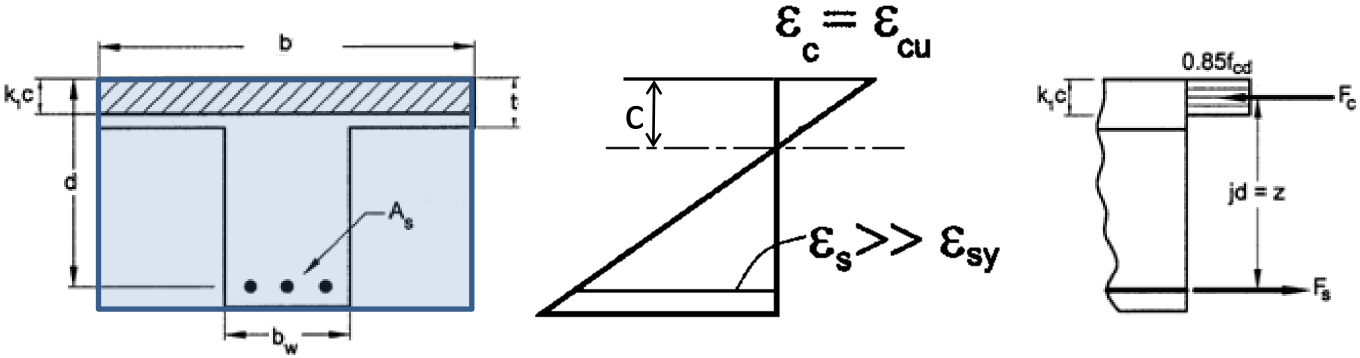


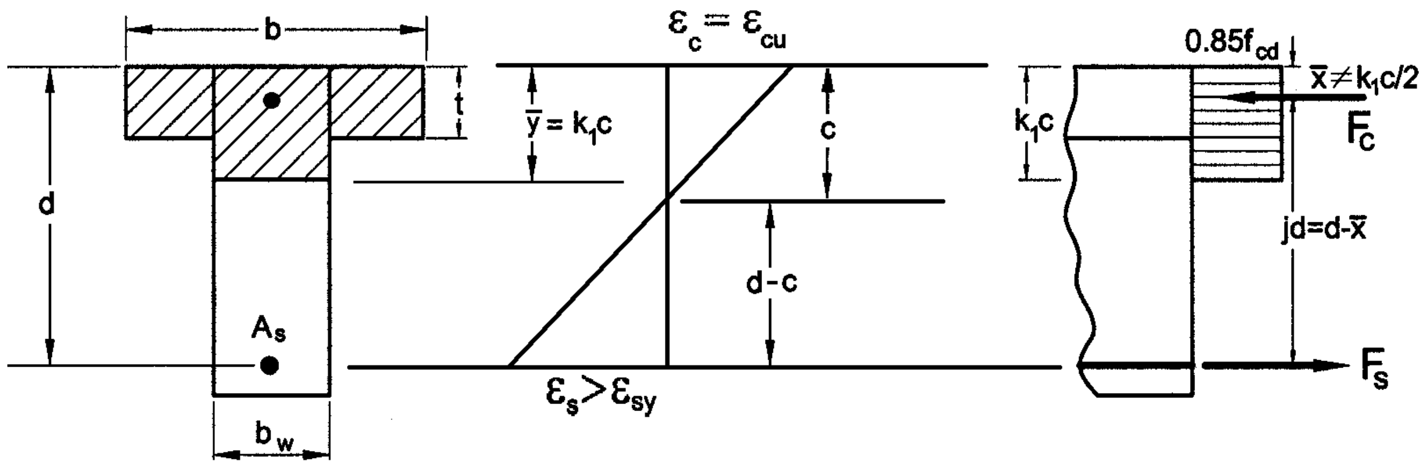


FLANGED SECTIONS

When $k_1 c \leq t$

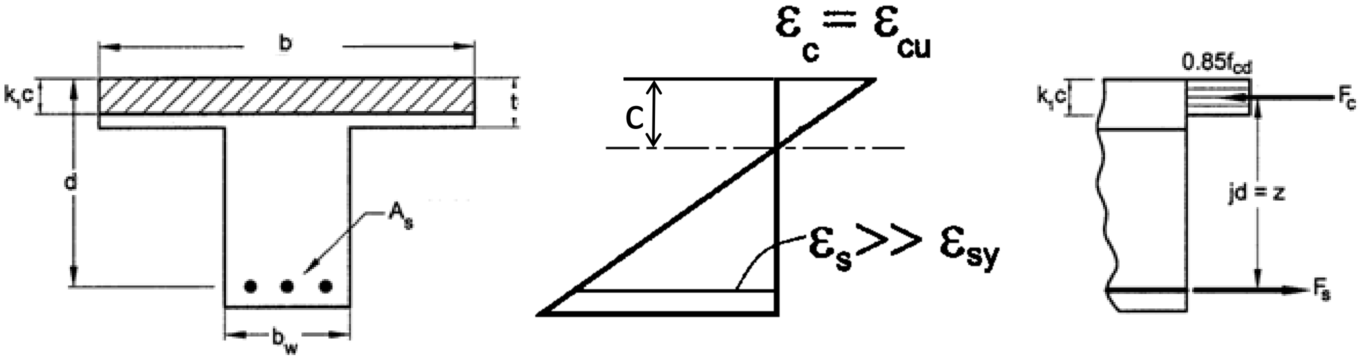


When $k_1 c > t$



Flanged Sections

When $k_1 c \leq t$



Equilibrium:

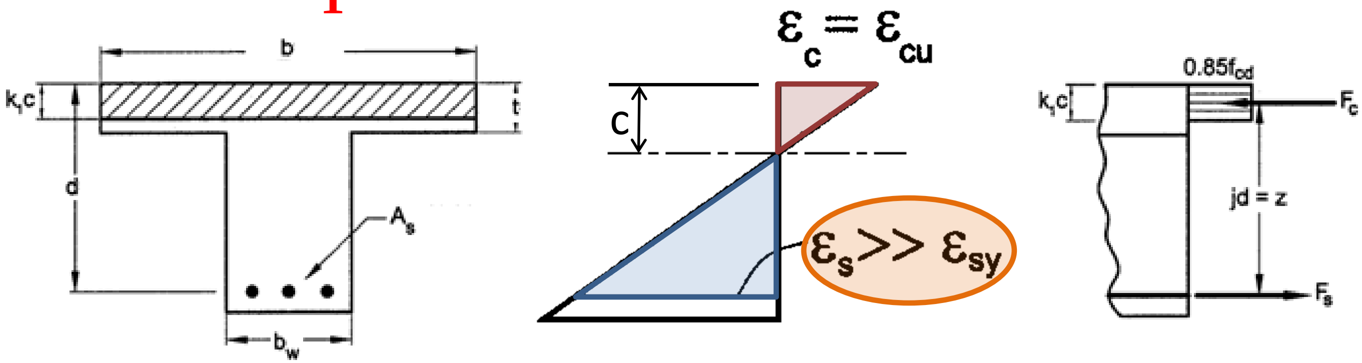
$$\Sigma F = 0 \qquad A_s f_{yd} - 0.85 f_{cd} b k_1 c = 0$$

Solving for "c," we have; $$c = \frac{A_s f_{yd}}{0.85 f_{cd} b\, k_1}$$

Taking moment about tension steel, we have; $$M_r = A_s f_{yd}\left(d - \frac{k_1 c}{2}\right) = A_s f_{yd}(j)d$$

FLANGED SECTIONS

When $k_1 c \leq t$



At this point, it is useful to verify the yielding tension steel assumption.

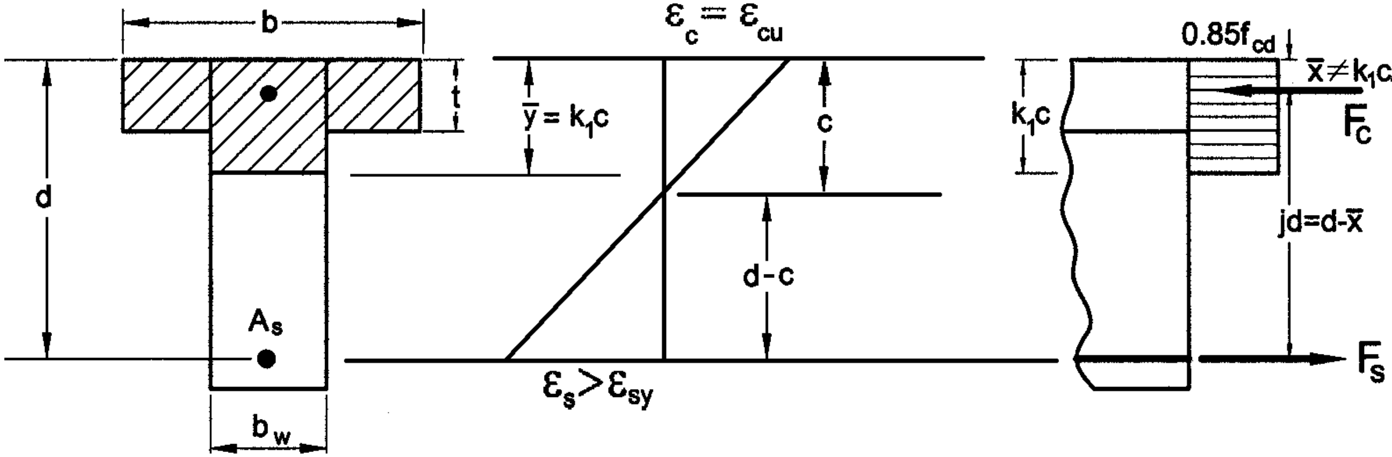
From similar triangles, noting that $\varepsilon_{cu} = 0.003$, we have;

$$\varepsilon_s = 0.003 \frac{c}{d-c}$$

If $\varepsilon_s \geq \varepsilon_{sy}$ then calculations are correct, otherwise you have to repeat the analysis with non yielding steel assumption.

FLANGED SECTIONS

When $k_1 c > t$



Equilibrium:

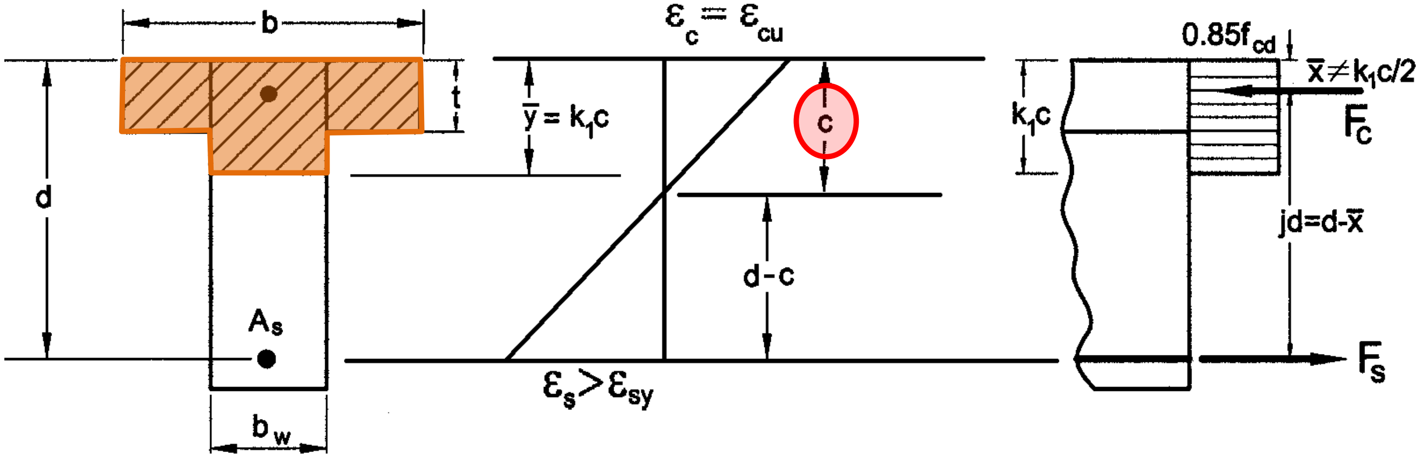
$$\Sigma F = 0 \qquad F_c = F_s \qquad 0.85\, f_{cd}\, A_{cc} = A_s\, f_{yd}$$

where, $A_{cc} = (b - b_w)t + b_w \times k_1 c$

Above expression has only one unkown, i.e. “c.” This equation, therefore, can be used to determine the depth of neutral axis, “c.”

FLANGED SECTIONS

When $k_1 c > t$



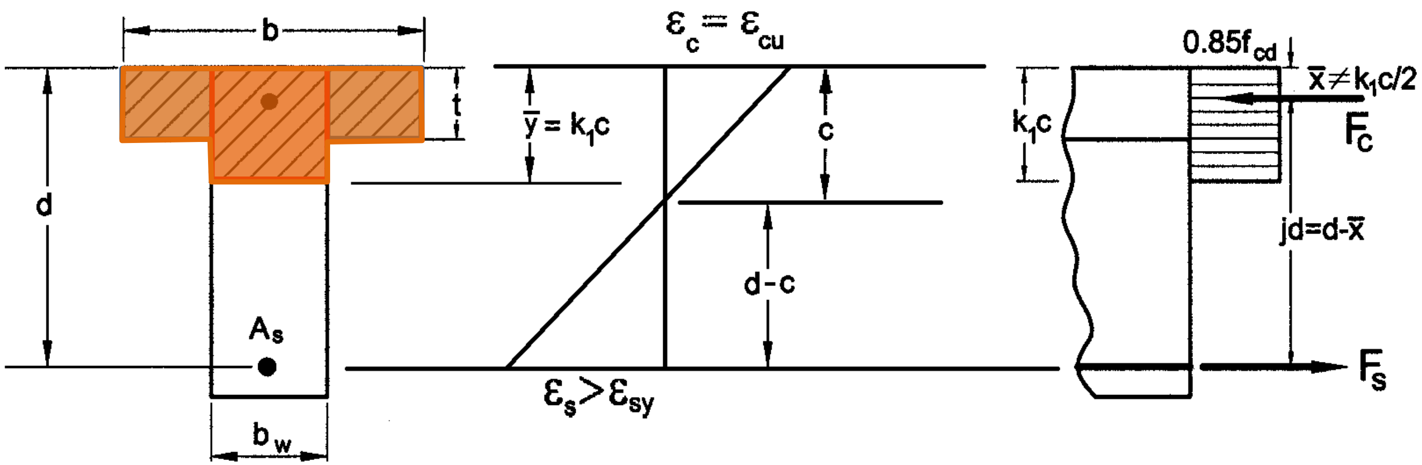
Equilibrium:

$$0.85\, f_{cd} \boxed{[(b - b_w)t + b_w \times k_1 c]} = A_s\, f_{yd}$$

$$c = \frac{\left[\dfrac{A_s\, f_{yd}}{0.85\, f_{cd}} - (b - b_w)t \right]}{b_w \times k_1}$$

FLANGED SECTIONS

When $k_1 c > t$

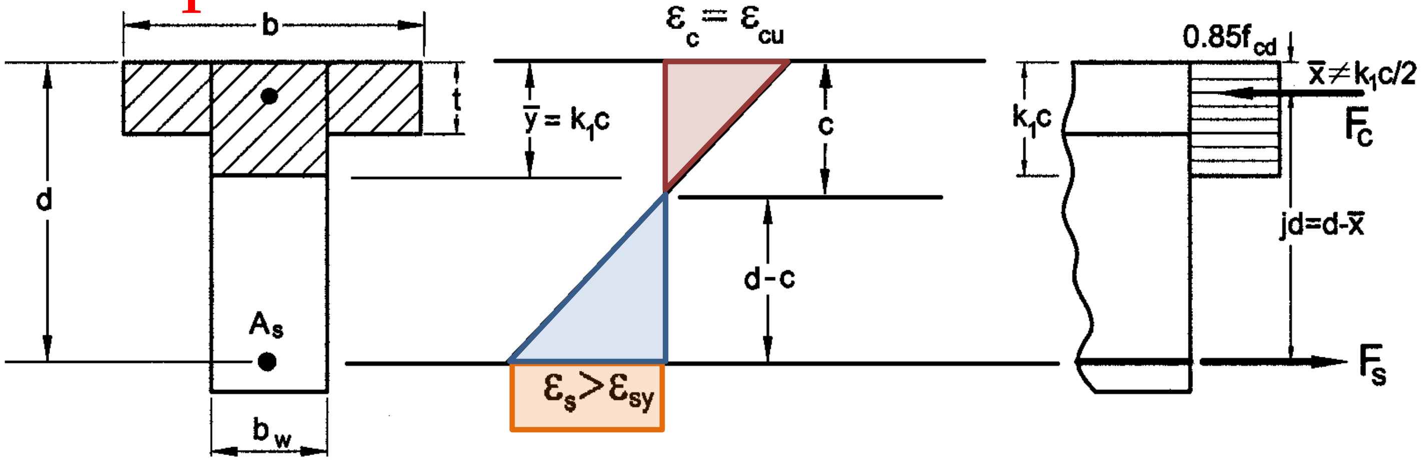


The centroid of the hatched zone can be found by taking its first area moment w.r.t. outermost compression fiber as;

$$\bar{x} = \frac{(b-b_w)\times t\times\frac{t}{2} + b_w\times k_1c\times\frac{k_1c}{2}}{(b-b_w)\times t + b_w\times k_1c} \leq k_1c/2$$

FLANGED SECTIONS

When $k_1 c > t$



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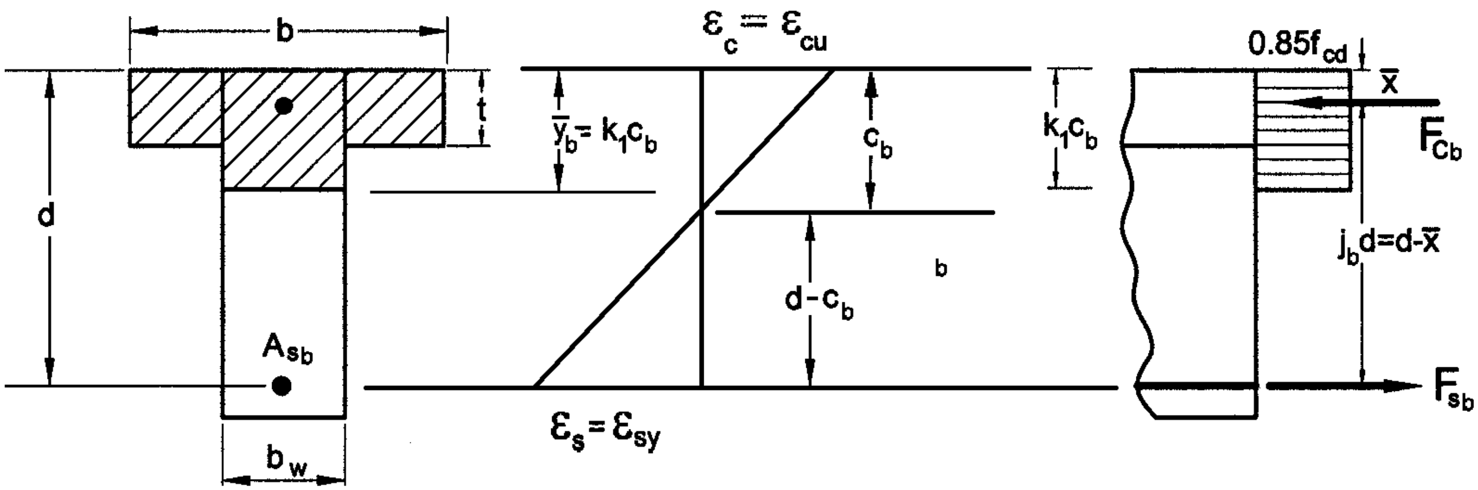
$$\varepsilon_s = 0.003 \frac{c}{d-c}$$

If $\varepsilon_s \geq \varepsilon_{sy}$ then calculations are correct, otherwise you have to repeat the analysis with non yielding steel assumption.

FLANGED SECTIONS

In T- or flanged sections, the percentage of tension steel for a balanced case is very high due to the large compression area provided. Therefore, usually flanged sections can be classified as underreinforced sections without making a check. However, the balanced percentage of steel can be derived by referring to the figure below easily.



Obviously for this case $k_1 c > t$