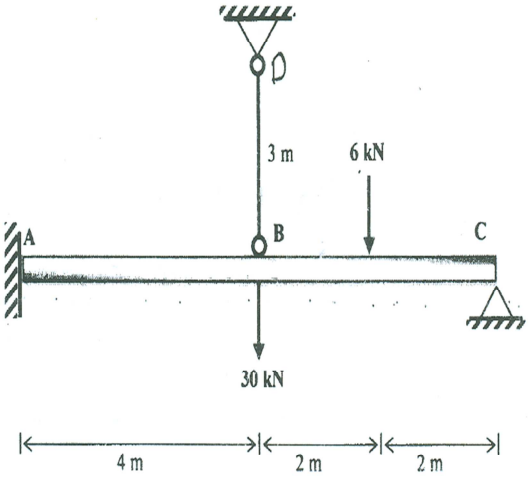
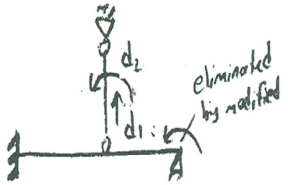
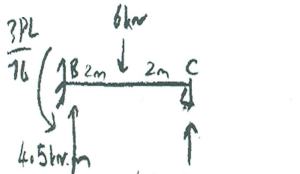
The cantilever beam shown is connected to a bar at joint B by a pin. Compute all reactions using **general stiffness method**. EI= 2.13×10^3 kN.m^2 for beam and EA = 6×10^3 kN for truss.





FEM's



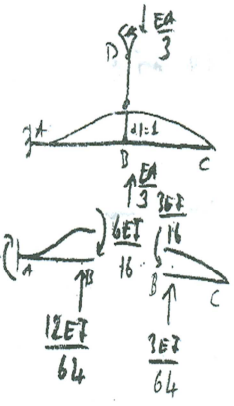
$F = Q^A - Q^F$

$$= \begin{Bmatrix} -30 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 4.125 \\ 4.5 \end{Bmatrix}$$

$$F = \begin{Bmatrix} -34.125 \\ -4.5 \end{Bmatrix}$$

k_{j1}

$d_1 = 1,\ d_2 = 0$

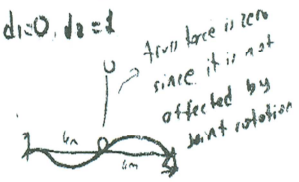


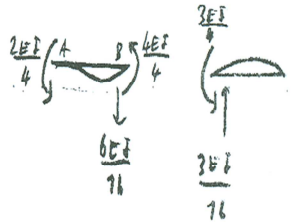
$k_{11} = \frac{12EI}{64} + \frac{3EI}{64} + \frac{EA}{3} = 2499.2$

$k_{21} = \frac{3EI}{16} - \frac{6EI}{16} = -399.4$

k_{j2}

$d_1 = 0,\ d_2 = 1$





$k_{12} = \frac{3EI}{16} - \frac{6EI}{16} = -399.4$

$k_{22} = \frac{4EI}{4} + \frac{3EI}{4} = 3727.5$

$$k = \begin{bmatrix} 2499.2 & -399.4 \\ -399.4 & 3727.5 \end{bmatrix}$$

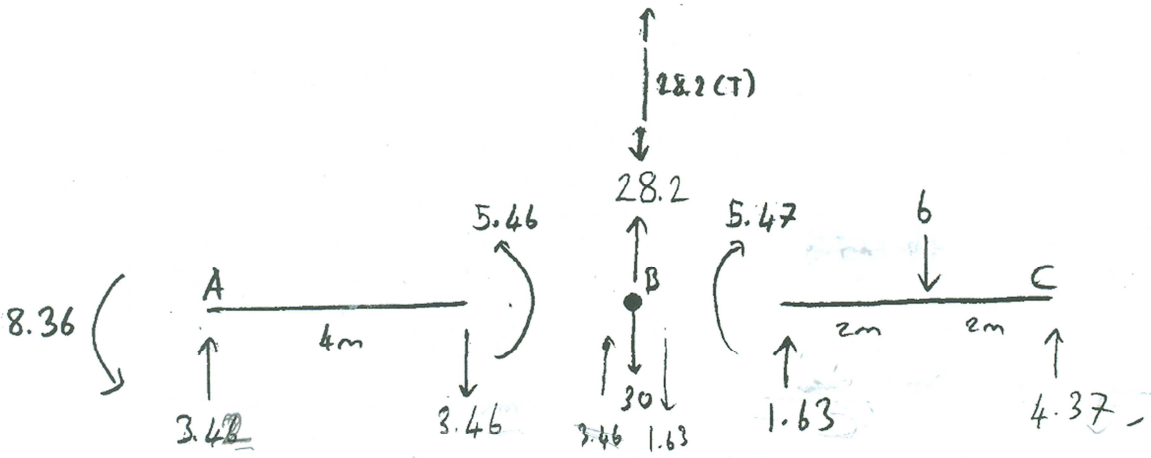
$$u = k^{-1} * F = \begin{Bmatrix} -0.01441 \\ -0.00272 \end{Bmatrix} \begin{matrix} m \\ rad \end{matrix}$$

$$M_{AB} = \frac{2EI}{4}\left(d_3 - 3\frac{d_2}{4}\right) = 8.36\ kN.m$$

$$M_{BA} = \frac{2EI}{4}\left(2d_3 - 3\frac{d_2}{4}\right) = 5.46\ kN.m$$

$$M_{BC} = \frac{3EI}{4}\left(d_3 + \frac{d_2}{4}\right) + 4.5 = -5.47\ kN.m$$

$$F_{axial} = -\frac{EA}{2} * d_2 = 28.2\ kN(T)$$

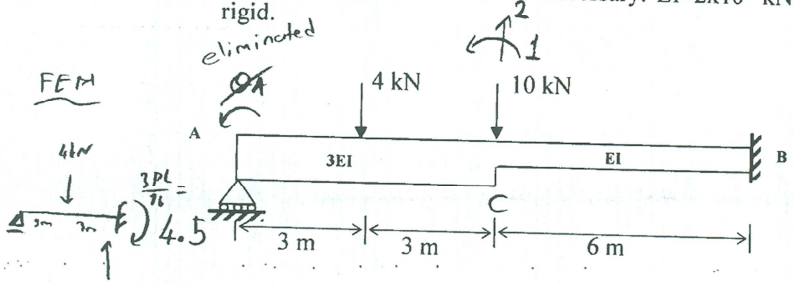


check Joint B

$\curvearrowright + \sum M = 5.47 - 5.46 \simeq 0$ ✓

$\uparrow + \sum F_y = -30 + 3.46 + 1.63 + 28.2 \simeq 0$ ✓

Use the **general stiffness method** to determine all support reactions. Use the minimum degrees of freedom necessary. EI=2x10^4 kN.m^2. The beam is axially rigid.



2 DOF (Δ_B, θ_B)

FEM

4kN, $\frac{3PL}{16}$ = 4.5

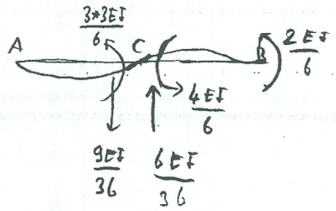
$\frac{4.5+6\times3}{6} = 2.75$

$F = \varphi^A - \varphi^F$

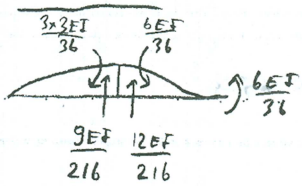
$= \begin{Bmatrix} 0 \\ -10 \end{Bmatrix} - \begin{Bmatrix} -4.5 \\ 2.75 \end{Bmatrix}$

$F = \begin{Bmatrix} 4.5 \\ -12.75 \end{Bmatrix}$

$D_1 = 1, D_2 = 0$



$D_1 = 0, D_2 = 1$



$k_{11} = \frac{9EI}{6} + \frac{4EI}{6} = \frac{13EI}{6}$ $\quad k_{12} = \frac{6EI}{36} - \frac{9EI}{36} = -\frac{EI}{12}$

$k_{21} = \frac{6EI}{36} - \frac{9EI}{36} = -\frac{EI}{12}$ $\quad k_{22} = \frac{9EI}{216} + \frac{12EI}{216} = \frac{7EI}{72}$

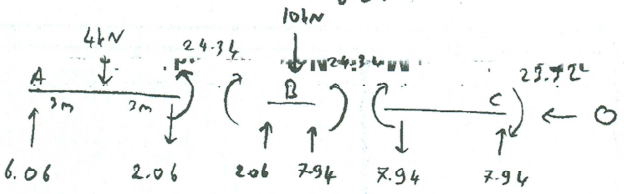
$K = EI \begin{bmatrix} 13/6 & -1/12 \\ -1/12 & 7/72 \end{bmatrix}$

$M_{CA} = \frac{3\times3EI}{6}\left(\frac{-3.068}{EI} + \frac{133.77}{6EI}\right) - 45 = 24.34\,kN.m$

$M_{CB} = \frac{2EI}{6}\left(2\times\frac{-3.068}{EI} - 3\times\frac{133.77}{6EI}\right) = -24.34\,kN.m$

$M_{BC} = \frac{2EI}{6}\left(\frac{-3.068}{EI} - 3\times\frac{133.77}{6EI}\right) = -23.32\,kN.m$

$\Delta = K^{-1}\times F = \begin{Bmatrix} \frac{-3.068}{EI} \\ \frac{-133.77}{EI} \end{Bmatrix}$



$A_y = 6.06\,kN\ (\uparrow)$

$B_x = 0\,kN$

$B_y = 7.94\,kN\ (\uparrow)$

$M_B = 23.32\,kN.m\ (CCW)$