Integration Formulas

a) Trapezoidal Rule:

$$I \approx \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n) \right]$$

or

$$I \approx \frac{(b-a)}{2n}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n) \right];$$

$$E = -\frac{(b-a)^3}{12n^3}\sum_{i=1}^{n} f^{(2)}(\varepsilon_i) \quad \text{or} \quad E = -\frac{(b-a)^3}{12n^2}\bar{f}^{n}$$ for n segments (or intervals).

b) Simpson's 1/3 Rule:

$$I \approx \frac{h}{3}\left[f(x_0) + 4\sum_{i=1,3,5,\ldots}^{n-1} f(x_i) + 2\sum_{i=2,4,6,\ldots}^{n-2} f(x_i) + f(x_n) \right]$$

or

$$I \approx \frac{b-a}{3n}\left[f(x_0) + 4\sum_{i=1,3,5,\ldots}^{n-1} f(x_i) + 2\sum_{i=2,4,6,\ldots}^{n-2} f(x_i) + f(x_n) \right]$$

$$E = -\frac{(b-a)^5}{90n^5}\sum_{i=1}^{n/2} f^{(4)}(\varepsilon_{2i}) = -\frac{(b-a)^5}{180n^4}$$ for n segments (or intervals).

c) Gauss-Legendre Formulas:

Let your original integral be of the form:

$$\int_a^b g(t)dt \tag{1}$$

Gauss Legendre Formulation has the variable x for $\int_{-1}^{1} f(x)dx$

Now, change (transform) t to x by;

$$t = \frac{b+a}{2} + \frac{b-a}{2}x \tag{2}$$

and dt to dx by;

$$dt = \frac{b-a}{2}dx \qquad (3)$$

Now substitute equations (2) and (3) into equation (1) and get ready to use Gauss-Legendre formula $\int_{-1}^{1} f(x)dx \approx \sum_{i=0}^{n} w_i f_i$ with

N	Abscissas, $x_{N,k}$ (x_i)	Weights, $w_{N,k}$ (w_i)	Truncation Error
2	-0.5773502692 0.5773502692	1.0000000000 1.0000000000	$\frac{f^{(4)}(\varepsilon)}{135}$
3	±0.7745966692 0.0000000000	0.5555555556 0.8888888888	$\frac{f^{(6)}(\varepsilon)}{15,750}$
4	±0.8611363116 ±0.3399810436	0.3478548451 0.6521451549	$\frac{f^{(8)}(\varepsilon)}{3,472,875}$
5	±0.9061798459 ±0.5384693101 0.0000000000	0.2369268851 0.4786286705 0.5688888888	$\frac{f^{(10)}(\varepsilon)}{1,237,732,650}$