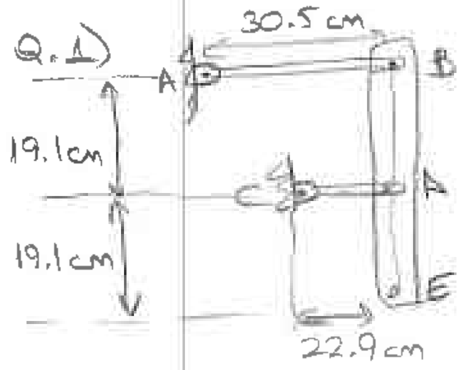
— 2013–2014 SPRING SEMESTER —
— AE 262 HW#3 SOLUTION —

Q.1)



$\omega_{AB} = 3\,rad/s\ (\circlearrowright)$, $\omega_{CD} = \omega_{CD}\ (\circlearrowright)$, $\omega_{BDE} = \omega_{BD}\ (\circlearrowright)$

ⓐ $\vec{a}_D = \vec{a}_B + (\vec{a}_{D/B})_t + (\vec{a}_{D/B})_n$ where $\vec{a}_B = |AB|\cancel{\alpha_{AB}}^{0} + \omega_{AB}^2|AB|\ (\leftarrow)$

$= (3^2 \times 30.5)\ cm/s^2\ (\leftarrow)$

$= 274.5\ cm/s^2\ (\leftarrow)$

velocity analysis / $\vec{\omega}_{AB} = 3\,rad/s\ (\circlearrowright)$, $\vec{\omega}_{CD} = \omega_{CD}\ (\circlearrowright)$, $\vec{\omega}_{BDE} = \omega_{BD}\ (\circlearrowright)$.

$\vec{V}_B = \omega_{AB} \times |AB| = 91.5\ cm/s\ (\downarrow)$, $\vec{V}_D = \omega_{CD} \times |CD| = 22.9\,\omega_{CD}\ (\downarrow)$.

$\vec{V}_{D/B} = \vec{V}_D - \vec{V}_B = 22.9\,\omega_{CD}\ (\downarrow) - 91.5\ cm/s\ (\downarrow)$. and $\vec{V}_{D/B} = \omega_{BD} \times |AB|$
$= 19.1\,\omega_{DB}\ (\leftarrow)$

$19.1\,\omega_{DB}\ (\leftarrow) = 22.9\,\omega_{CD}\ (\downarrow) - 91.5\ cm/s\ (\downarrow)$. $\rightarrow \omega_{DB} = 0$, $\boxed{\omega_{CD} = 4\,rad/s\ (\circlearrowright)}$

Now, $(\vec{a}_{D/B})_t = \alpha_{BD} \times |BD| = 19.1\,\alpha_{BD}\ (\leftarrow)$ / $\alpha_{BCD} = \alpha_{BD}\ (\circlearrowright)$

$(\vec{a}_{D/B})_n = |BD|\,\omega_{BD}^2\ (\uparrow) = 0$

Moreover, $\vec{a}_D = \cancel{\vec{a}_C}^{0} + (\vec{a}_{C/D})_t + (\vec{a}_{C/D})_n$, $\alpha_{CD} = \alpha_{CD}(\circlearrowright)$, $\alpha_{AB} = 0$.

$(\vec{a}_{C/D})_t = \alpha_{CD} \times |CD| = 22.9\,\alpha_{CD}\ (\downarrow) = 0$

$(\vec{a}_{C/D})_n = |CD|\,\omega_{CD}^2 = 22.9\,\omega_{CD}^2\ (\leftarrow) = 366.4\ cm/s^2\ (\leftarrow)$

Solving two $\vec{a}_D$ formulas together,

$274.5\ cm/s^2\ (\leftarrow) + 19.1\,\alpha_{BD}\ (\leftarrow) + 0 = 22.9\,\alpha_{CD}\ (\downarrow) + 366.4\ cm/s^2\ (\leftarrow)$.

in x direction, $274.5 + 19.1\,\alpha_{BD} = 366.4 \rightarrow \alpha_{BD} = 4.8\ rad/s\ (\circlearrowright)$

in y direction, $0 = 22.9\,\alpha_{CD} \rightarrow \alpha_{CD} = 0$.

$\vec{a}_D = 22.9\,\cancel{\alpha_{CD}}^{0} + 366.4\ cm/s^2\ (\leftarrow) = 366.4\ cm/s^2\ (\leftarrow)$.

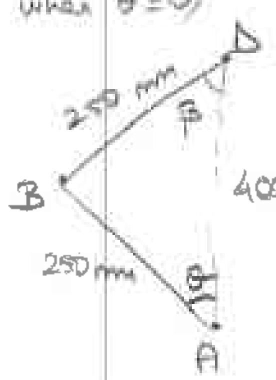
ⓑ $\vec{a}_E = \vec{a}_D + (\vec{a}_{E/D})_t + (\vec{a}_{E/D})_n$

$= 366.4\ cm/s^2\ (\leftarrow) + \alpha_{BD}|DE|\ (\leftarrow) + |DE|\,\cancel{\omega_{BD}^2}^{0}\ (\uparrow)$

$= 366.4\ cm/s^2\ (\leftarrow) + (4.8 \times 19.1)\ cm/s^2\ (\leftarrow) = 458.1\ cm/s^2\ (\leftarrow)$

Q.2) when $\theta = 0$,

ⓐ



$\vec{V_D} = 0.6\,m/s\ (\rightarrow)$

$\vec{V_D} = \vec{V_B} + \vec{V_{D/B}}$

$0.6\vec{i} = \omega_{AB} \times r_B + \omega_{BD} \times r_{D/B}$

$= \omega_{AB}\vec{k} \times (0.25\cos\theta\vec{j} - 0.25\sin\theta\vec{i}) + \omega_{BD}\vec{k} \times (0.25\sin\beta\vec{i} - 0.25\cos\beta\vec{j})$

in $\vec{i}$ direction: $0.25\cos\theta\,\omega_{AB} + 0.25\cos\beta\,\omega_{B} = 0.6$ (1)

in $\vec{j}$ direction: $0.25\sin\theta\,\omega_{AB} - 0.25\sin\beta\,\omega_{BD} = 0$. (2)

Using cosine theorem,

$250^2 = 250^2 + 400^2 - 2 \times 250 \times 400 \times \cos\theta \rightarrow \theta = 36.87°,\ \theta = \beta = 36.87$

Substituting $\theta = \beta = 36.87°$ into (1) and (2),

from (2), $\omega_{AB} = \omega_{BD}$ and from (1) $\vec{\omega_{AB}} = \frac{0.6}{2 \times 0.25 \times \cos\theta} = 1.5\ rad/s$ (↻)

Therefore, $\boxed{\vec{\omega_{BD}} = 1.5\ rad/s\ (↺)}$

ⓑ $\vec{a_B} = \cancel{\vec{a_A}}^{0} + (\vec{a_{B/A}})_t + (\vec{a_{B/A}})_n$

$= \alpha_{B/A}\vec{k} \times (-0.25\cos\theta\vec{i} + 0.25\sin\theta\vec{j}) + \omega_{BA}^2(-0.25\cos\theta\vec{i} + 0.25\sin\theta\vec{j})$ (3)

and $\cancel{\vec{a_D}}^{0} = \vec{a_B} + (\vec{a_{D/B}})_t + (\vec{a_{D/B}})_n$

$= \vec{a_B} + \alpha_{DB}\vec{k} \times (0.25\sin\beta\vec{i} + 0.25\cos\beta\vec{j}) + \omega_{DB}^2(0.25\sin\beta\vec{i} + 0.25\cos\beta\vec{j})$ (4)

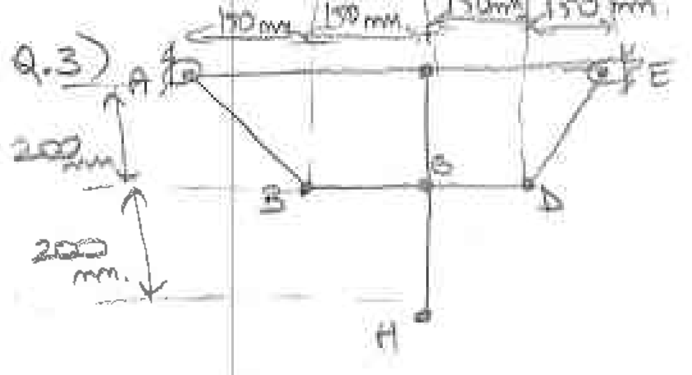
Solving (3) and (4) together,

$0 = -0.25\cos\theta\,\alpha_{B/A}\vec{j} - 0.25\sin\theta\,\alpha_{BA}\vec{i} - 0.25\cos\theta(1.5)^2\vec{i} + 0.25\cos\beta(1.5)^2\vec{j} +$
$0.25\sin\beta\,\alpha_{DB}\vec{j} - 0.25\cos\beta\,\alpha_{DB}\vec{i} + 0.25\sin\beta(1.5)^2\vec{i} + 0.25\cos\beta(1.5)^2\vec{j}$

$\Rightarrow$ $\alpha_{AB} = 3\ rad/s^2$ (↻)

$\alpha_{BD} = 3\ rad/s^2$ (↺)

Q.3)



$\omega_{AB} = 4\ rad/s\ (\circlearrowright)$
$= 4\ rad/s\ (-\vec{k})$

ⓐ for link AB, $\vec{V}_B = \cancel{\vec{V}_A}^{\,0} + \vec{V}_{B/A} = \vec{\omega}_{AB} \times \vec{r}_{BA}$
$= -4\vec{k} \times (0.15\vec{i} - 0.2\vec{j}) = -0.8\vec{i} - 0.6\vec{j}$

for cross BD, $\vec{V}_D = \vec{V}_B + \vec{V}_{D/B} = (-0.8\vec{i} - 0.6\vec{j}) + \vec{\omega}_{DB} \times \vec{r}_{DB}$
$= -0.8\vec{i} - 0.6\vec{j} + (\omega_{DB}\vec{k} \times 0.3\vec{i})$
$= -0.8\vec{i} + (3\omega_{DB} - 0.6)\vec{j}$

for link DE, $\cancel{\vec{V}_E}^{\,0} = \vec{V}_D + \vec{V}_{E/D} = -0.8\vec{i} + (3\omega_{DB} - 0.6)\vec{j} + \vec{\omega}_{ED} \times \vec{r}_{ED}$
$0 = -0.8\vec{i} + (3\omega_{DB} - 0.6)\vec{j} + \omega_{ED}\vec{k} \times (0.15\vec{i} + 0.2\vec{j})$
$0 = (-0.8 - 0.2\omega_{ED})\vec{i} + (3\omega_{DB} - 0.6 + 0.15\omega_{ED})\vec{j}$

in $\vec{i}$ direction, $-0.8 - 0.2\,\omega_{ED} = 0 \rightarrow \omega_{ED} = -4^{rad/s} = 4\ rad/s\ (\circlearrowright)$

in $\vec{j}$ direction, $3\omega_{DB} - 0.6 + 0.15 \times (-4) = 0 \rightarrow \omega_{DB} = 4\ rad/s\ (\circlearrowleft)$

ⓑ for link AB, $\vec{a}_B = \cancel{\vec{a}_A}^{\,0} + (\vec{a}_{B/A})_t + (\vec{a}_{B/A})_n = \cancel{\alpha_{BA}}^{\,0} \times \vec{r}_{BA} + \omega_{BA}\vec{k} \times (\omega_{BA}\vec{k} \times \vec{r}_{BA})$
$= -2.4\ m/s^2\,\vec{i} + 3.2\ m/s^2\,\vec{j}$

for cross BD, $\vec{a}_{D/B} = (\vec{a}_{D/B})_t + (\vec{a}_{D/B})_n = \alpha_{D/B}\vec{k} \times \vec{r}_{D/B} + \omega_{BD}\vec{k} \times (\omega_{BD}\vec{k} \times \vec{r}_{D/B})$
$= 0.3\,\alpha_{D/B}\,\vec{j} - 4.8\vec{i}$

therefore, $\vec{a}_D = \vec{a}_B + \vec{a}_{D/B} = -7.2\vec{i} + (3.2 + 0.3\,\alpha_{D/B})\vec{j}$

for link DE, $\vec{a}_{E/D} = (\vec{a}_{E/D})_t + (\vec{a}_{E/D})_n = \alpha_{DE}\vec{k} \times \vec{r}_{E/D} + \omega_{DE}\vec{k} \times (\omega_{DE}\vec{k} \times \vec{r}_{D/E})$
$= \alpha_{DE}\vec{k} \times (0.15\vec{i} + 0.2\vec{j}) + 4\vec{k} \times (4\vec{k} \times (0.15\vec{i} + 0.2\vec{j}))$
$= 0.15\alpha_{DE}\vec{j} - 0.2\alpha_{DE}\vec{i} - 2.4\vec{i} - 3.2\vec{j}$
$= (-2.4 - 0.2\alpha_{DE})\vec{i} + (-3.2 + 0.15\,\alpha_{DE})\vec{j}$

$\vec{a}_E = \vec{a}_D + \vec{a}_{E/D} = 0$ since point E is fixed.
$0 = -7.2\vec{i} + (3.2 + 0.3\,\alpha_{DB})\vec{j} + (-2.4 - 0.2\alpha_{DE})\vec{i} + (-3.2 + 0.15\alpha_{DE})\vec{j}$

in $\vec{i}$ direction $-7.2 - 2.4 - 0.2\,\alpha_{DE} = 0 \longrightarrow \alpha_{DE} = -48 = 48\ rad/s^2\ (\circlearrowright)$

in $\vec{j}$ direction $3.2 + 0.3\alpha_{DB} - 3.2 + 0.15\,\alpha_{DE} = 0 \longrightarrow \alpha_{DB} = 24 = 24\ rad/s^2\ (\circlearrowleft)$

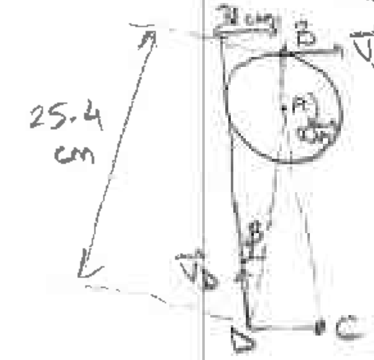
(c) $\vec a_H = \vec a_B + (\vec a_{H/B})_t + (\vec a_{H/B})_n = a_B + \alpha_{B/D}\times r_{H/B} + \vec\omega_{BD}\times(\vec\omega_{BD}\times\vec r_{HB})$

$= [-2.4\vec i + 3.2\vec j + 24\vec k\times(0.15\vec i - 0.2\vec j) + 4\vec k\times(4\vec k\times(0.15\vec i - 0.2\vec j))]$ m/s²

$= (\cancel{-2.4\vec i} + 3.2\vec j + 3.6\vec j + \cancel{4.8\vec i} - \cancel{2.4\vec i} + 3.2\vec j)$ m/s²

$= 10$ m/s² $\vec j$

Q.4) (a) at $\theta = 0°$



$\vec V_B = |AB|\,\omega_A = (7.1\times 15)$ cm/s $= 1.07$ m/s (→)

$\vec a_B = |AB|\,\omega_A^2 = (7.1\times 15^2)$ cm/s² $= 16$ m/s² (↓)

$\omega_{BD} = V_B/|BC| = 1.07/(25.4\cos\beta)$ where

$\beta = \sin^{-1}(7.1/25.4) = 16.23°$

$\omega_{BD} = 4.36$ rad/s (↻)

$\Rightarrow \vec a_{D/B} = \alpha_{D/B}(↻)\times\vec r_{DB} + \omega_{D/B}(↻)\times(\omega_{DB}(↻)\times\vec r_{DB})$

$= \alpha_{DB}\vec k\times(-7.1\vec i - 24.38\vec j) + 4.36\vec k\times(4.36\vec k\times(-7.1\vec i - 24.38\vec j))$

$= -7.1\alpha_{DB}\vec j + 24.38\alpha_{DB}\vec i + 134.97\vec i + 463.45\vec j$

$= (-7.1\alpha_{DB} + 463.45)\vec j + (24.38\alpha_{DB} + 134.97)\vec i$

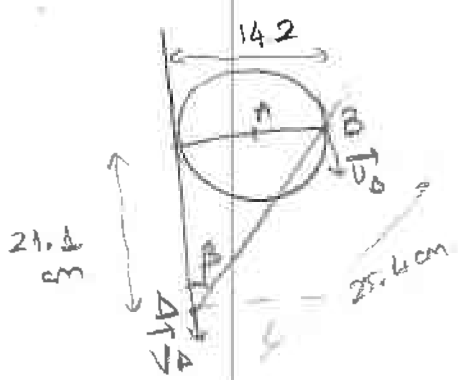
Moreover, $\vec a_D = \vec a_B + \vec a_{D/B}$, $\vec a_D = a_D(↓)$.

$\vec a_D = (-1597.5 - 7.1\alpha_{DB} + 463.45)\vec j + (24.38\alpha_{DB} + 134.97)\vec i$.

Therefore, $24.38\alpha_{DB} + 134.97 = 0 \rightarrow \alpha_{DB} = -5.54$ rad/s² $= 5.54$ rad/s² (↻).

$-1597.5 - 7.1\times -5.54 + 463.45 = -1094.7$ cm/s² $\longrightarrow \vec a_D = 1094.7$ cm/s² (↓)

(b) at $\theta = 90°$



$\vec V_B = |AB|\,\omega_A = (7.1\times 15)$ cm/s $= 106.5$ cm/s (↓)

$\vec V_D = v_D(↓)$, $\omega_{BD} = 0$ (inst. center of BD @ ∞)

$\vec a_B = |AB|\,\omega_A^2 = 1597.5$ cm/s² (←)

$\vec a_D = a_D(↓)$, $\vec\alpha_{DB} = \alpha_{DB}(↺)$

$\vec a_{D/B} = \vec\alpha_{DB}\times\vec r_{DB} + \vec\omega_{DB}\times(\vec\omega_{DB}\times\vec r_{DB})$

$= \alpha_{DB}\vec k\times(-14.2\vec i - 21.1\vec j) + 0$

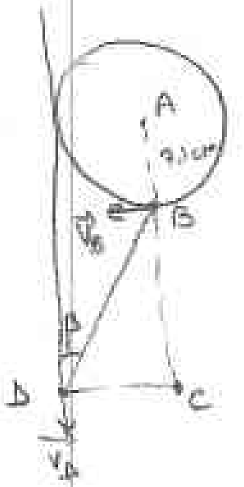
$= -14.2\alpha_{DB}\vec j + 21.1\alpha_{DB}\vec i$

$\vec a_D = \vec a_B + \vec a_{D/B} = -1597.5\vec i + 21.1\alpha_{DB}\vec i - 14.2\alpha_{DB}\vec j$

- $-1597.5 + 21.1\alpha_{DB} = 0 \longrightarrow \boxed{\alpha_{DB} = 75.7 \text{ rad/s}^2 (↺)}$
- $\vec a_D = -14.2\times 75.7 = -1075.1$ cm/s²

$\boxed{\vec a_D = 1075.1 \text{ cm/s}^2 (↓)}$

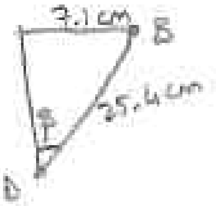
(c) at $\theta = 180°$



$\vec{V}_B = |AB|\,\omega_A = (7.1\times15)\ \text{cm/s} = 106.5\ \text{cm/s}\ (\leftarrow)$

$\vec{V}_D = V_D\ (\downarrow)$

$\omega_{BD} = V_B/(|BD|\cos\beta)$ where,



$\beta = \sin^{-1}(7.1/25.4) = 16.23°$

Hence, $\omega_{BD} = 106.5/[25.4\times\cos(16.23°)] = 4.44\ \text{rad/s}$

$\vec{\omega}_{BD} = 4.44\ \text{rad/s}\ (\circlearrowright)$

Moreover, $\vec{a}_B = |AB|\,\omega_A^2\,(\uparrow) = (7.1\times15^2)\ \text{cm/s}^2\,(\uparrow) = 1597.5\ \text{cm/s}^2\,(\uparrow)$

$\vec{a}_{D/B} = \alpha_{DB}\vec{k}\times\vec{r}_{DB} + \vec{\omega}_{DB}\vec{k}\times(\omega_{DB}\vec{k}\times\vec{r}_{DB})$

$= \alpha_{DB}\vec{k}\times(-7.1\vec{i} - 24.4\vec{j}) + 4.44\vec{k}\times(4.44\vec{k}\times(-7.1\vec{i} - 24.4\vec{j}))$

$= -7.1\,\alpha_{DB}\vec{j} + 24.4\,\alpha_{DB}\vec{i} + 140\,\vec{i} + 481\,\vec{j}$

$\vec{a}_D = \vec{a}_B + \vec{a}_{D/B}$, $\vec{a}_D = a_D\,(\uparrow)$

$24.4\,\alpha_{DB} + 140 = 0. \rightarrow \alpha_{DB} = -5.74\ \text{rad/s}^2 = 5.74\ \text{rad/s}^2\ (\circlearrowright)$

$\vec{a}_{D/B} = (-7.1\,\alpha_{DB} + 481 + 1597.5)\ \text{cm/s}^2 = 2119.3\ \text{cm/s}^2\ (\uparrow)$