MIDDLE EAST TECHNICAL UNIVERSITY
FACULTY OF ENGINEERING

UNCERTAINTY and DATA ANALYSIS
Spring Semester 2009-2010

Homework No: 3 – Date Due: 30.04.2010 till 17:00

1) A rain gage (an instrument to measure intensity of rainfall) is being designed for the Deriner Dam site. The intensity of rainfall in the Çoruh Basin is designed by a lognormal distribution. The median of the annual (one year period) rainfall intensity is 3510 mm and the coefficient of variation of its distribution is 0.35.

a) Determine the parameters of the lognormal distribution.

b) What must be the minimum capacity of the rain gage (in mm) to ensure a 90% probability that it won't be overloaded in one year period?

c) According to this annual rainfall intensity distribution, is it possible to observe rainfall intensity exceeding 800 mm in a given year? Verify your answer with a probability estimation.

d) Solve parts (a), (b) and (c) if it is known that the mean is 3719 mm and the coefficient of variation is 0.35.

2) The occurrences of flood may be modeled by a Poisson process. If the mean occurrence rate of floods for a certain region A is once every 8 years, determine

a) The probabilities of **no floods**; of **1 flood** and of **more than 3 floods in a 10-year period.**

b) A structure is located in the region A. The probability that it will be inundated, when a flood occurs, is 0.05. Compute the probability that the structure will survive if there are **no floods**; if there is **one flood**; if there are **n floods in a 10-year period**. Assume statistical independence between floods.

3) A compacted subgrade is required to have a specified density of 110 kg/cm^3. It will be acceptable if 4 out of 5 cored samples have at least the specified density.

a) Assuming each sample has a probability of 0.80 of meeting the required density, what is the probability that the subgrade will be acceptable?

b) What should the probability of each sample be in order to achieve a 80% probability of an acceptable subgrade?

4) A point P has random location in the x-y plane with its random coordinates as X and Y. The joint probability density function of X and Y is in the following form.

$$f_{XY}(x,y) = c \qquad 0 \le y \le 1-x \le 1$$
$$f_{XY}(x,y) = 0 \qquad \text{otherwise.}$$

a) Find the constant "c" so that $f_{XY}(x,y)$ is a proper joint density function.

b) Find the marginal probability density functions of X and Y.

c) Are X and Y statistically independent? Why yes/why no?

d) Find the correlation coefficient ρ_{XY} and give your comments on its value.

e) Find the conditional probability density function of X given Y (i.e. $f_{X/Y}(x/y)$).

f) What is the conditional expectation of X given Y. (i.e. $E(x/y) = \int x\, f_{X/Y}(x/y)\, dx$)

5) The joint probability mass function of the random variables X and Y are as given in the following table.

Y \ X	**0**	**1**	**2**
0	$\frac{1}{3}$	*0*	$\frac{1}{3}$
1	*0*	$\frac{1}{3}$	*0*

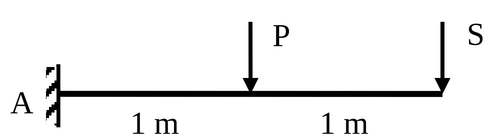
a) Find the correlation coefficient ρ_{XY} and give your comments on its value?
b) Are X and Y statistically independent? Why yes/why no?
c) Find the conditional expectation of Y given X (i.e. $\sum y\, P_{Y/X}(x/y)$).

6) The random variable X has probability density function as

$$f_X(x) = c\,x + 0.5 \qquad 0 \le x \le 1$$
$$f_X(x) = 0 \qquad \text{elsewhere}$$

a) Find the constant c so that $f_X(x)$ is a proper probability density function of the random variable X.
b) If $y = x^2$, find the probability density function of the random variable y and its expected value.

7) The cantilever beam shown below is subjected to two random concentrated loads P and S, as shown below. P and S have the following statistical parameters: **P: Normal; N(5 kN, 1 kN)** and **S: Normal; N(10 kN, 1 kN).** The two loads are correlated with a correlation coefficient of 0.4.
a) Compute the expected value and the variance of the bending moment at the fixed end of the beam. (i.e. at A)
b) If the resisting moment of the beam is statistically independent from loads and is also normal with a mean value of 30 kN.m and has a coefficient of variation of 0.1, compute the probability of failure of the beam. (Hint: Assume the bending moment created at the fixed end to be normally distributed with the mean value and variance as computed part (a).)
c) Assume that the beam has the same moment resisting capacity as described in part (b) and subjected only to load S. Assume S to be a deterministic quantity. If survival probability of 0.99 is desired, what will be the maximum allowable value of S?



8) The time of operation of a construction equipment until breakdown follows an exponential distribution with a mean of 20 months and the present inspection program is scheduled at every 5 months.
a) What is the probability that equipment will need repair at the first scheduled inspection date?
b) The company owns 3 pieces of a certain type of equipment; assuming that the service lives of equipment is statistically independent, determine the probability that at most one piece of equipment will need repair at the scheduled inspection date.