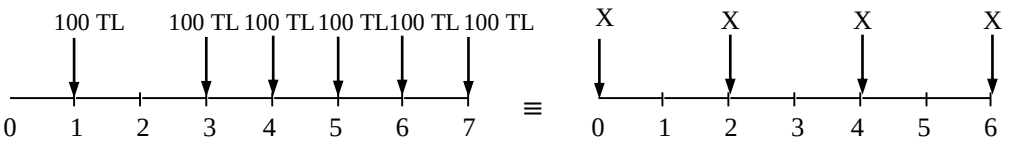
CE 231 – ENGINEERING ECONOMY

PROBLEM SET 1

PROBLEM 1

The following two cash-flow operations are said to be equivalent at 10 % interest rate compounded annually. Find X that satisfies the equivalence.



SOLUTION 1

P = 100 (P/A, 10%, 7) – 100 (P/F, 10%, 2) = 404,19 TL
(4,8684) (0,8265)

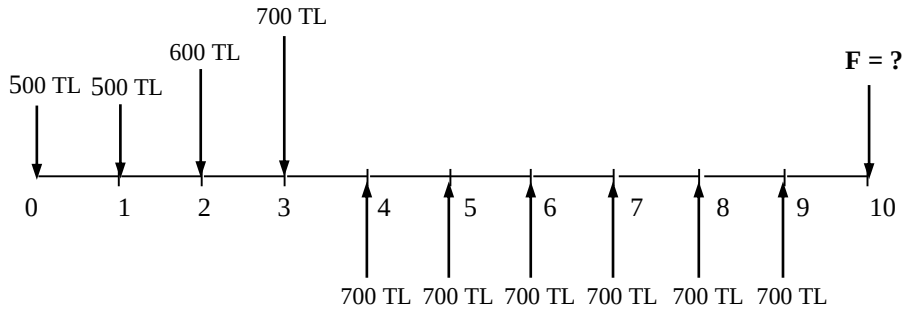
404,19 = X + X (P/F, 10%, 2) + X (P/F, 10%, 4) + X (P/F, 10%, 6)

404,19 = 3,074X

X ≅ 131,49 TL

PROBLEM 2

What must be the value of F at the end of the 10^{th} year so that, future worth of the cash flow equals to zero at the end of 10 years. Interest rate is 10% per year.



SOLUTION 2

F + 500 (F/P, 10%, 10) + **[**500 + 100 (A/G, 10%, 3)**]** (F/A, 10%, 3) (F/P, 10%, 7)
- 700 (F/A, 10%, 6) (F/P, 10%, 1) = 0

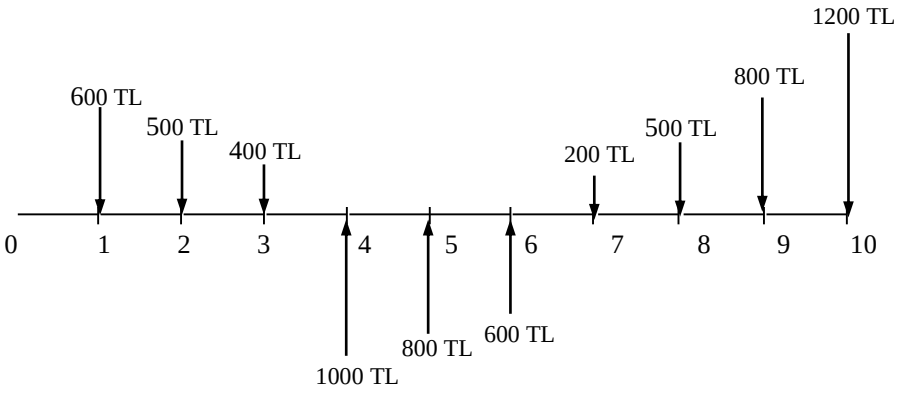
F + 500 × 2,594 + (500 + 100 × 0,9366) × (3,310) × (1,949) – 700 × 7,716 × 1,1 = 0

F + 1.297 + 3.829,81 – 5.941,32 = 0

F = 814,51 TL

PROBLEM 3

Find the Present Value (at year zero) of the cash-flow given below. Interest rate is 10% per year for the first 6 years and 12% per year for the next 4 years (Use gradient series approach).



SOLUTION 3

P = **[** 600 - 100(A/G, 10%, 3)**]** × (P/A, 10%, 3) – **[** 1.000 - 200(A/G, 10%, 3)**]** × (P/A, 10%, 3) (P/F, 10%, 3) + **[** 200 + 300(A/G, 12%, 4)**]** × (P/A, 12%, 4) × (P/F, 10%, 6) + 100 (P/F, 12%, 4) (P/F, 10%, 6)

P = (600-100×0,9366)×(2,4869) – (1000-200×0,9366)×(2,4869)×(0,7513) + (200+300×1,3589)×(3,0373)×(0,5645) + 100(0,6355)×(0,5645)

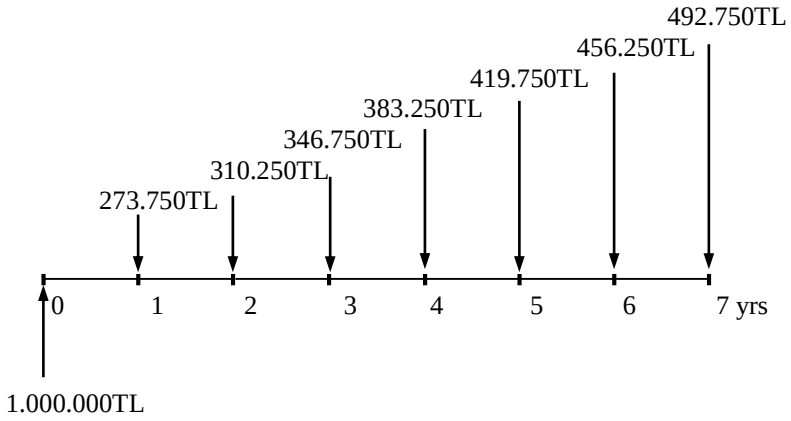
= 1259,217 – 1518,418 + 1041,884 + 35,874

P = 818,56 TL

PROBLEM 4

As a civil engineer, you are asked to check the feasibility of financing a new traffic signal system on a busy intersection. The system is planned to serve 15.000 cars/day with an increasing rate of 2.000 cars/day each year. Installation process is estimated to cost 1.000.000 TL (cash outflow) and the system will provide a saving of 0,05 TL/car (cash inflow) from time, fuel and wear expenses point of view. With an interest rate of 5% and for a period of 7 years, state whether this investment is feasible or not by computing the future worth of the project at the end of 7 years. Assume that it is feasible if the future worth at the end of 7 years is greater than 0 (1 year = 365 days).

SOLUTION 4



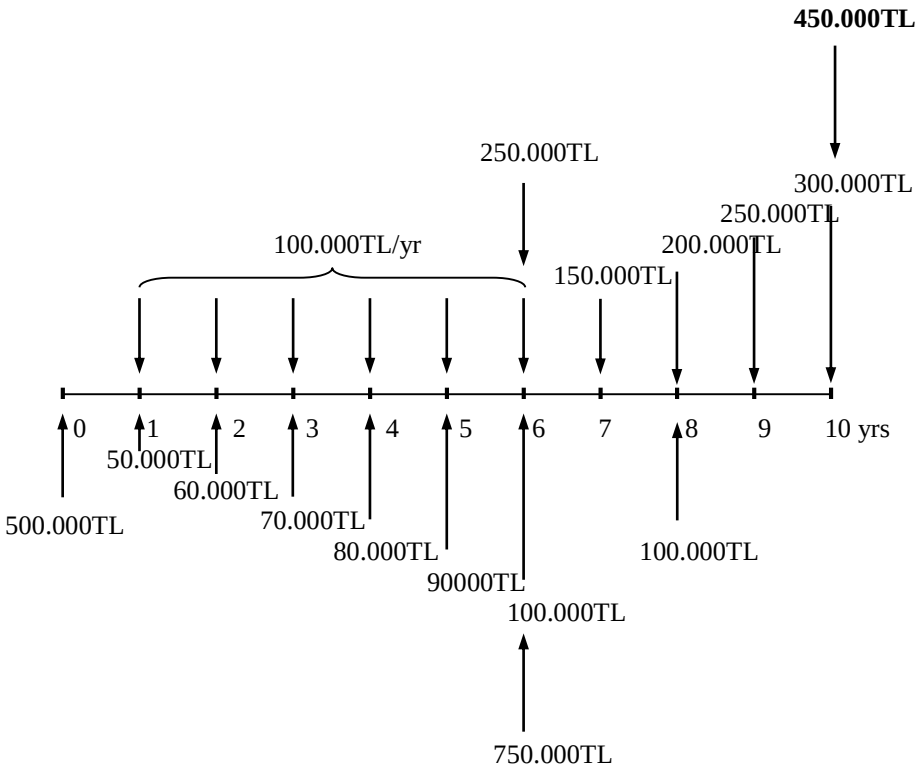
15.000 cars/day * 0,05 TL/car * 365 days =273.750 TL/yr
Gradient = 2.000 cars/day * 0,05 TL/car * 365 =36.500 TL/yr

F = -1.000.000(F/P, 5%, 7)+[273.750+36.500(A/G, 5%, 7)] (F/A, 5%, 7)
1,407 **2,8052** **8,142**

F =1.655.530 TL ⟶ feasible!

PROBLEM 5

A construction company is planning to complete a soil compaction process within 10 years. Only one machine will work during this period. The company decides on purchasing a new machine costing 500.000TL and having a useful life of 6 years. Maintenance and operating costs are 50.000TL/yr with an increasing rate of 10.000TL/yr. This machine will provide an annual income of 100.000TL during its lifetime and a salvage value of 250.000TL at the end of 6 years. At that time another machine is planned to be bought at a cost of 750.000TL having a life time of 4 years and a salvage value of 450.000TL. Maintenance and operation cost is estimated to be 100.000TL at the end of the 2nd year of its useful life. Annual incomes are 150.000TL/yr with an increasing rate of 50.000TL/yr till the end of its lifetime. Considering that the interest rate is 7% for the first 6 years and 9% for the next 4 years, find the present worth of these investments.



SOLUTION 5

P= -500.000 TL-[50.000+10.000(A/G, 7%, 6)](P/A, 7%, 6)-750.000(P/F, 7%, 6)
2,3032 **4,7665** **0,6663**

+100.000(P/A, 7%, 6)+250.000(P/F, 7%, 6)-100.000(P/F, 9%, 2)(P/F, 7%, 6)
4,7665 **0,6663** **0,8417** **0,6663**

+[150.000+50.000(A/G, 9%, 4)](P/A, 9%, 4)(P/F, 7%, 6)+450.000(P/F, 9%,4)(P/F, 7%, 6)
1,3925 **3,2397** **0,6663** **0,7084** **0,6663**

P= -76.549,21 TL⇒ unacceptable!

PROBLEM 6

A person deposited 100 TL in a bank in 1850, at an interest of 7 % compounded annually, with instructions that it shall be withdrawn and donated to a cancer research institute when it amounts to 100.000 TL. At the end of what year should the money be withdrawn?

SOLUTION 6

P = 100 TL
i = 7 %
F = 100.000 TL
n = ?

$F = P(1 + i)^n$

$$(1+i)^n = \frac{F}{P}$$

$$(1+0{,}07)^n = \frac{100.000}{100}$$

$$(1+0{,}07)^n = 1000$$

$$nLog(1{,}07) = Log(1000)$$

$$n = \frac{Log1000}{Log1{,}07}$$

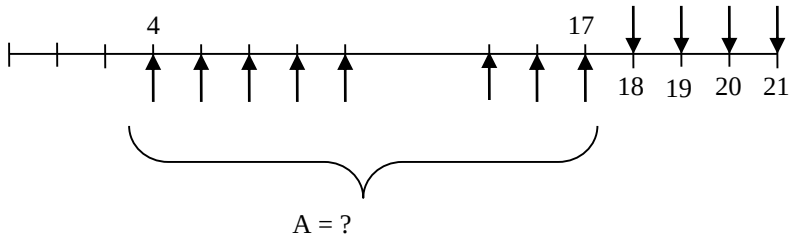
$$= \frac{3}{0.0294} = 102{,}04 \text{ yrs}$$

Withdrawal year:
1.850 + 102 = 1.952

PROBLEM 7

A young couple has decided to make advance plans for financing their 3-year-old son's college education. Money can be deposited at 7 % compounded annually. What annual deposit on each birthday from 4th to the 17th inclusive must be made to provide 3.000 TL on each birthday from the 18th to the 21st inclusive?

SOLUTION 7



A_1 = 3.000 TL
n_1 = 4 yrs
n = 14 yrs
i = 7 %
A = ?

$A = A_1(P/A, i, n_1)(A/F, i, n) \rightarrow (A/F, i, n) = 1 / (F/A, i, n)$

$= 3.000(P/A, 7, 4)(A/F, 7, 14)$

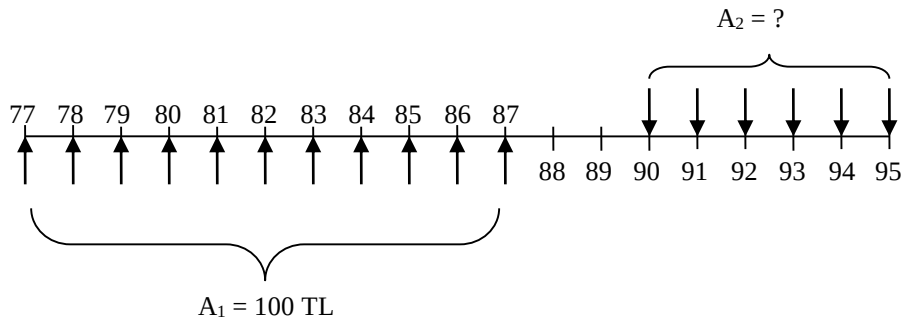
$= 3.000(3,387)(0,04434)$

$= 450,54$ TL/yr

PROBLEM 8

On 1st January 1977, 100 TL is deposited in a fund drawing 3 % interest compounded annually. 100 TL is to be deposited on each 1st January up to and including 1st January 1987. The purpose of the fund is to provide a series of uniform annual withdraws starting 1st January 1990. The final withdrawal on 1st January 1995 will exhaust the fund. How much can be withdrawn each year during the period 1990-1995?

SOLUTION 8



$A_1 = 100$ TL/yr

$n_1 = 11$ yrs

$n_2 = 6$ yrs

$i = 3$ %

$A_2 = ?$

$A_2 = A_1(F/A, i, n_1)(F/P, i, 2)(A/P, i, n_2)$

$= 100(F/A, 3, 11)\ (F/P, 3, 2)\ (A/P, 3, 6)$

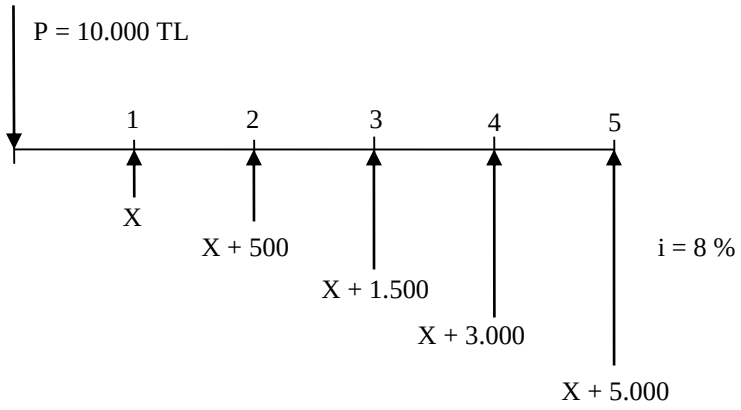
$= 100(12,808)\ (1,0609)\ (0,18460)$

$= 250,84$ TL/yr

PROBLEM 9

A person borrows 10.000 TL at 8% interest rate compounded annually and wishes to pay the loan back over a five year period with annual payments. However, the second payment is to be 500 TL greater than the first payment, the third payment must be 1.000 TL greater than the second payment; the forth payment must be 1.500 TL greater than the third payment; and the fifth payment must be 2.000 TL greater than the fourth payment. Determine the size of the first payment and use Annual Equivalent approach in your solution.

SOLUTION 9



10.000(A/P, 8, 5) = X + 500(P/F, 8, 2) (A/P, 8 , 5) + 1.500(P/F, 8, 3) (A/P, 8, 5)
+ 3.000(P/F, 8, 4) (A/P, 8, 5) + (A/F, 8, 5)

10.000 x 0,25046 = X + 500 x 0,8573 x 0,25046 + 1.500 x 0,7938 x 0,25046
+ 3.000 x 0,7350 x 0,25046 + 5.000 x 0,17046

2.504,60 = X + 107,36 + 298,22 + 552,26 + 852,30

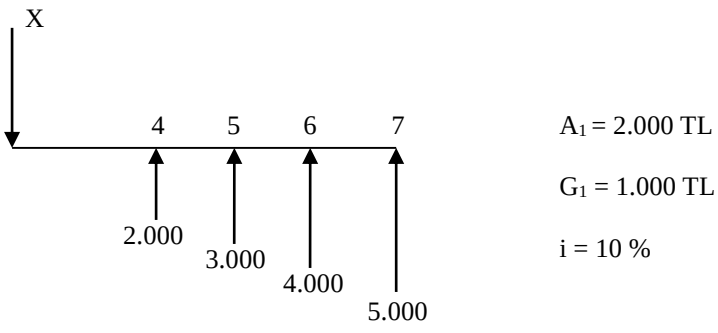
X = 2.504,60 – 1.810,14

= 694,46 TL

PROBLEM 10

X TL is borrowed from a bank. It is agreed that payments of 2.000 TL, 3.000 TL, 4.000 TL, and 5.000 TL made at the end of the 4^{th}, 5^{th}, 6^{th}, and 7^{th} years respectively will satisfy the borrowed money if an interest rate of 10 % compounded annually is the appropriate interest rate. By use of the uniform gradient series factor and any other relevant interest factor(s), determine the amount of **X** TL.

SOLUTION 10



X = [2.000 + 1.000(A/G,10,4)](F/A,10,4)(P/F,10,7)

= [2.000 + 1.000*1,38]* 4,641* 0,5132

= [2.000 + 1.380]* 4,641* 0,5132

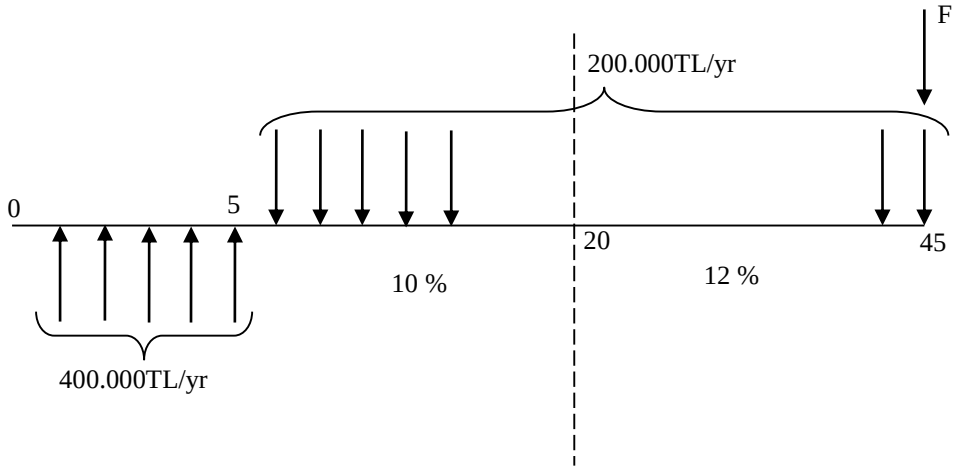
= 3.380* 4,641* 0,5132

= 8.050,35 TL

PROBLEM 11

A building will be constructed in 5 years. The yearly expenditures in these five years will be 400.000 TL/yr. The building's life is 40 years after the end of the construction. The yearly net rental income during these 40 years will be 200.000 TL/yr. Assuming an interest rate of 10 % in the first 20 years (including 5 years of construction time and 15 years of the building's life) and an interest rate of 12 % in the remaining 25 years of the building's life, calculate the value of the building at the end of its life.

SOLUTION 11



F_1= 400.000(F/A,10,5)(F/P,10,15)(F/P,12,25)

F_1= 400.000 * 6,105 * 4,177 * 17,00

F_1= 173.403.978

F_2= 200.000(F/A,10,15)(F/P,12,25)

F_2= 200.000 * 31,772 * 17,00

F_2= 108.024.800

F_3= 200.000(F/A,12,25)

F_3= 200.000 * 133,334

F_3= 26.668.800

F_4= F_2 + F_3 = 108.024.800 + 26.668.800

F_4= 134.691.600

F= F_4-F_1 = 134.691.600 – 173.403.978

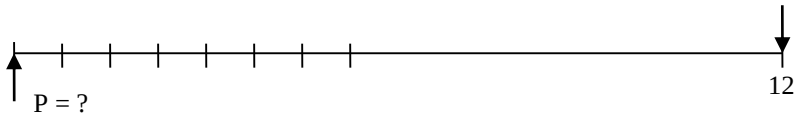
F= -38.712.378 TL

PROBLEM 12

To get 8.000 TL 12 hence, how much must be invested now? Take the interest rate as 8 % compounded annually.

SOLUTION 12

F = 8.000 TL
n = 12 years
i = 8 %
P = ?



P = F(P/F, i, n)
= 8.000(P/F, 8%, 12)
= 8.000(0,3971)
= 3.176,80 TL

OR $P = F\left[\frac{1}{(1+i)^n}\right]$

$= 8.000\left[\frac{1}{(1+0{,}08)^{12}}\right]$

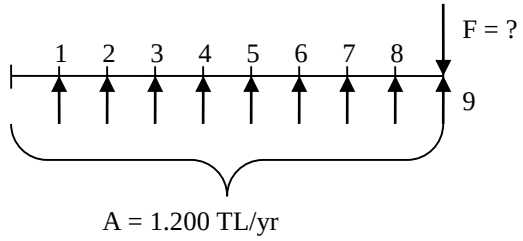
= 3.176,91 TL

PROBLEM 13

A man who is planning to retire in 9 years' time, deposits 1.200 TL at the end of each year. If the interest rate is 12 % compounded annually, what will be the amount he will receive when he retires?

SOLUTION 13

A = 1.200 TL/yr
i=12%
n = 9 yrs
F = ?



F = A(F/A, i, n)
= 1.200(F/A, 12, 9)
= 1.200 x 14,776
= 17.731,20 TL

OR $F = A\left[\frac{(1+i)^n - 1}{i}\right]$

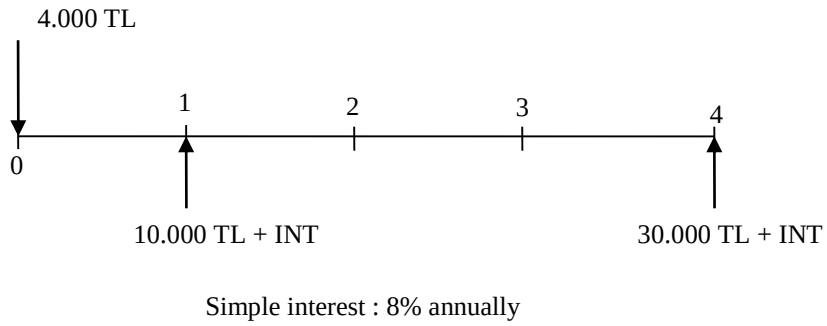
$= 1.200\left[\frac{(1+0{,}12)^9 - 1}{0{,}12}\right]$

= 17.730,78 TL

PROBLEM 14

A contractor borrows 40.000 TL from the bank. According to the arrangement between them, the contractor agrees to pay 10.000 TL plus accrued interest at the end of the first year, and 30.000 TL plus the accrued interest at the end of the forth year. What are the amounts for the two payments if 8 % annual simple interest applies?

SOLUTION 14



Simple interest : 8% annually

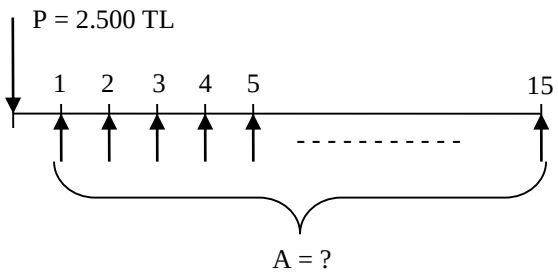
First payment at the end of first year:
= 10.000 + Pni
= 10.000 + (40.000x1x0,08)
= 10.000 + 3.200
= 13.200 TL

Second payment at the end of forth year:
= 30.000 + Pni
= 30.000 + (30.000x3x0,08)
= 30.000 + 7.200
= 37.200 TL

PROBLEM 15

If 2.500 TL is deposited now, what uniform amount could be withdrawn at the end of each year for 15 years and have nothing left at the end of the 15th year? The interest rate is accepted to be 4 % compounded annually.

SOLUTION 15



A = (A/P, i, n)
= 2.500 (A/P, 4, 15)
= 2.500 x 0,08994
= 224,85 TL / year

OR

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

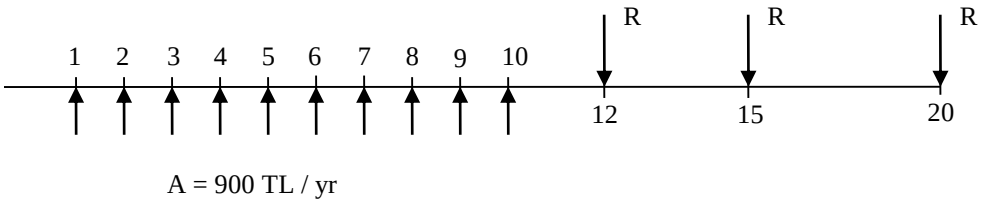
$$= 2.500\left[\frac{0{,}04(1+0{,}04)^{15}}{(1+0{,}04)^{15} - 1}\right]$$

= 224,85 TL / yr

PROBLEM 16

A series of 10 annual payments of 900 TL is equivalent to three equal payments at the end of years 12, 15 and 20 at 9% interest compounded annually. What is the amount of these three payments?

SOLUTION 16



i = 9 %

A (P/A, 9, 10) = R[(P/F, 9, 12) + (P/F, 9, 15) + (P/F, 9, 20)]

$$R = \frac{A(P/A, 9, 10)}{(P/F, 9, 12) + (P/F, 9, 15) + (P/F, 9, 20)}$$

$$= \frac{900x6{,}418}{0{,}3555 + 0{,}2745 + 0{,}1784} = \frac{5776{,}20}{0{,}8084}$$

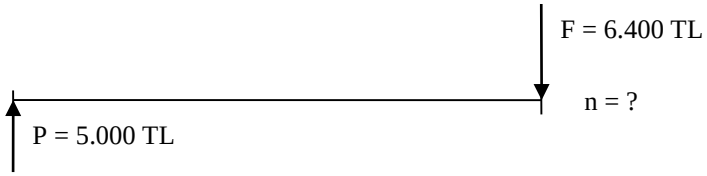
= 7146,03 TL

PROBLEM 17

For what period of time will 5.000 TL have to be invested to amount to 6.400 TL if it earns 8% simple interest per annum?

SOLUTION 17

P = 5.000 TL
F = 6.400 TL
i = 8 % simple interest
n = ?



F = P + I
Where I = Pni
Then:
F = P + Pni

$$\therefore \; n = \frac{F - P}{Pi} = \frac{6.400 - 5.000}{5.000x0{,}08} = 3{,}50 \text{ years}$$