SM'14 Homework 7 Solution

Note that the numbers in this homework rely on measurements by ruler and protractor. Your numbers may have 0.05 m and 0.5° from those in this solution.

a) For cohesionless soils, infinite slope analysis gives

$$FS = \frac{\tan\phi'}{\tan\alpha} = \frac{\tan 39°}{8.1/12.7} = \mathbf{1.27}$$

(8.1 and 12.7 are height and width of the slope, measured from the figure)

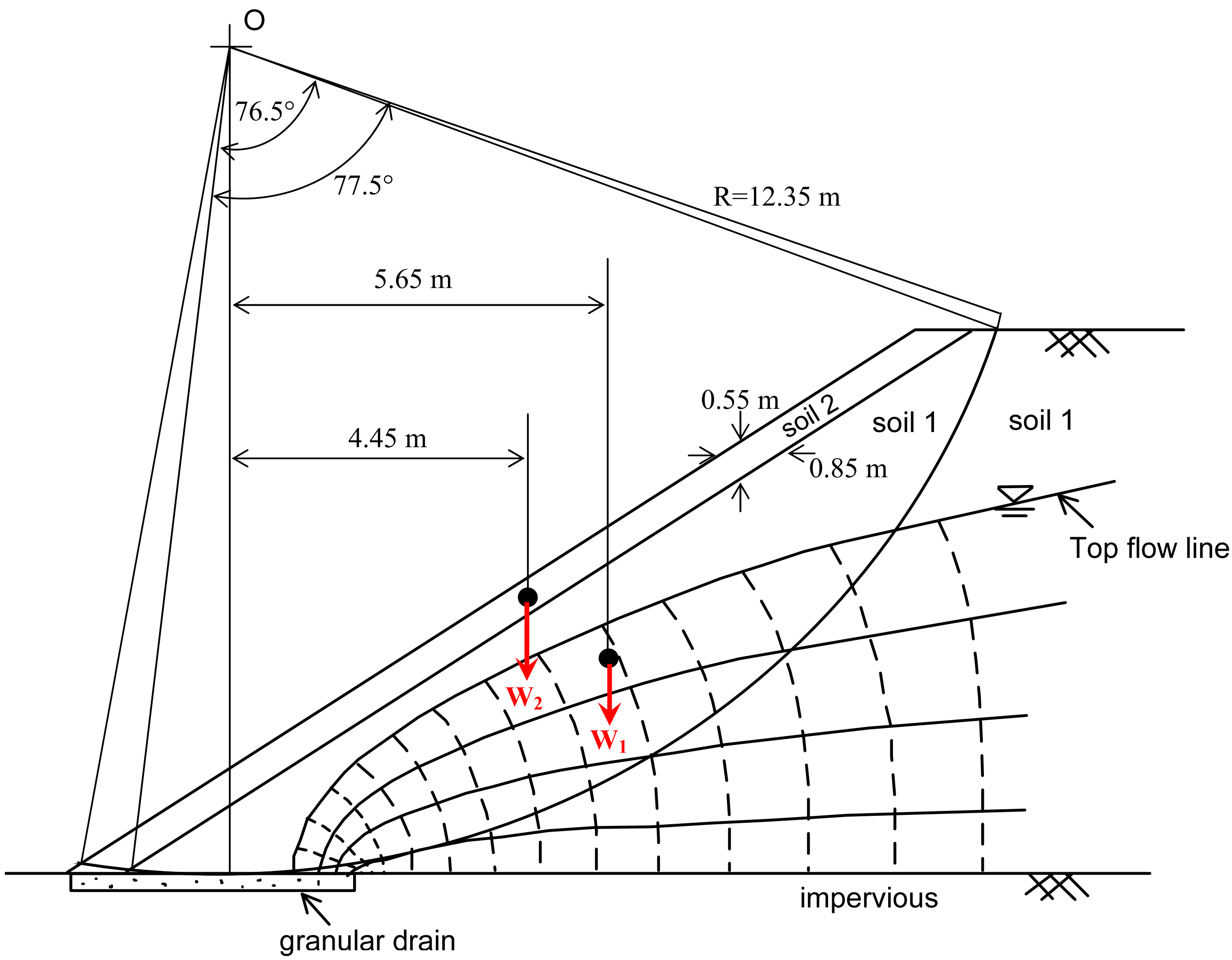
b) All calculations are per unit length into the page. Let's calculate the areas (and weights) of two soils separately. The centroid locations in the figure are approximate, determined by eye.

$$A_1 = \pi R^2 \frac{77.5°}{360°} - \left(R\cos\frac{77.5°}{2}\right)\left(R\sin\frac{77.5°}{2}\right) = 12.35^2 \cdot \left(\pi\frac{77.5°}{360°} - \frac{\sin 77.5°}{2}\right) = 28.7m^2$$

The areas of soil 1 above and below groundwater are about equal.

$W_1 = A_1 \times (\gamma_{dry}+\gamma_{sat})/2 = 28.7 \times 19 = 545.3$ kN

$A_2 = 0.85 \times 8.1 = 6.88$ m^2 $\Rightarrow$ $W_2 = 6.88 \times 16 = 110$ kN



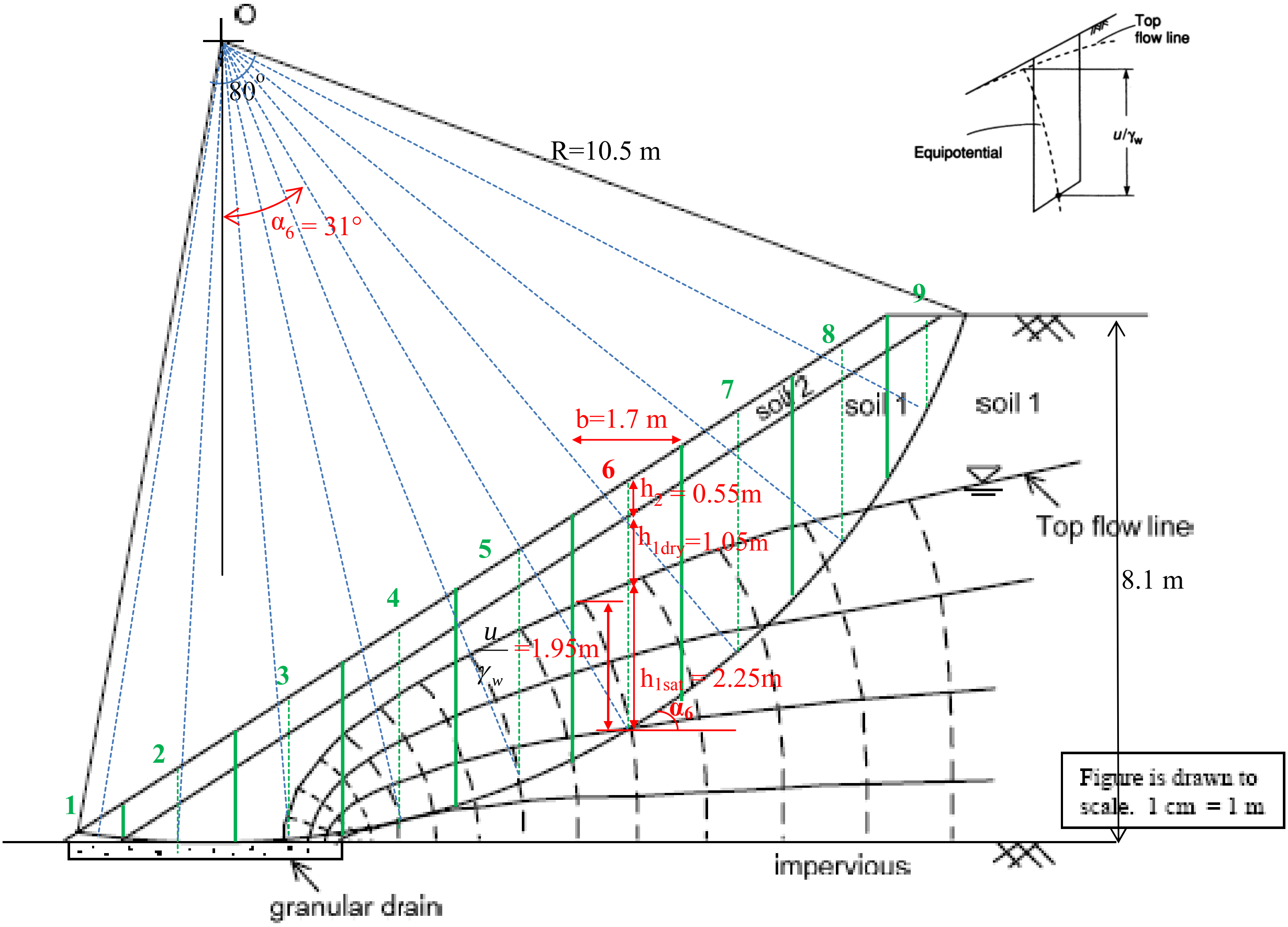
Moment of weights around O = 545.3x5.65 + 110x4.45 = 3570 kNm

$$F.S = \frac{M_{resisting}}{M_{driving}} = \frac{c_u \cdot L \cdot R}{\sum (W \cdot x)} = \frac{50 \cdot \left(\pi \cdot \frac{76.5°}{180°} \cdot 12.35 \right) \cdot 12.35}{3570} = \boldsymbol{2.85}$$

The question said "assume that all of the length of the circular arc passes through soil 1". The strength of Soil 2 is proportional to stresses in it. As it is a very thin layer, it has very small stress and hence shear strength (average strength in it is 0.55/2 x 16 x tan39° = 3.6 kPa, which is much smaller than 50 kPa strength in Soil 1). So the length of the arc in Soil 2 is ignored here. If you obeyed the question and assumed that portion of the arc also has Soil 1, the only change to the calculations would be replacing 76.5° with 80° in the arc length calculation. Then FS increases to 2.98 as the assumption introduces an error that stabilizes the slope.

If desired, the contribution of Soil 2 (3.6x0.75x12.35) can be added into the $M_{resisting}$. This increases the FS to 2.86.

c) Applying Swedish method of slices with 9 slices (width of slices 1 and 9 are fixed because there are geometry and material discontinuities). All red dimensions belong to slice 6.



	measured from figure									given in question			
slice	b	h_{1dry}	h_{1sat}	h_2	u/γ_w	α (^{o})	$L=b/\cos\alpha$	$W=(h_{1dry}\cdot\gamma_{dry}+h_{1sat}\cdot\gamma_{sat}+h_2\cdot\gamma_2).b$	u (kPa)	ϕ' (^{o})	c' (kPa)	$c'L+(W\cos\alpha-uL)\tan\phi'$	$W\sin\alpha$
1	0.7	0	0	0.28	0	-8	0.71	3.08	0	39	0	2.47	-0.43
2	1.75	0.55	0	0.55	0	-3	1.75	32.73	0	30	8	32.89	-1.71
3	1.65	1.1	0.5	0.55	0.1	12	1.69	63.69	1	30	8	48.49	13.24
4	1.75	0.5	1.95	0.55	1.35	15	1.81	99.4	13.5	30	8	55.81	25.73
5	1.8	0.7	2.35	0.55	1.95	22	1.94	123.12	19.5	30	8	59.58	46.12
6	1.7	1.05	2.25	0.55	1.95	31	1.98	123.59	19.5	30	8	54.70	63.65
7	1.7	1.5	1.65	0.55	1.4	40	2.22	116.96	14	30	8	51.54	75.18
8	1.45	2.1	0.35	0.55	0.3	55	2.53	77.72	3	30	8	41.58	63.66
9	1.2	1.3	0	0.15	0	60	2.40	30.96	0	30	8	28.14	26.81
											$\Sigma =$	375.20	312.26

Notes: Red numbers are illustrated on the figure on the previous page.

Blue numbers involve some sort of averaging, because the measurement at the middle of the slice is not representative of the entire slice width. For example, on the mid-line of slice 3, h_{1sat} is close to 0, but we can see that about a third of the slice is saturated. So I divided the height of 1.6m in soil 1 as 0.5 and 1.1m. In a better analysis with more slices, you could have slices end at any discontinuity (as I did for slices 1 and 9), including groundwater surface, so you wouldn't have such issues.

$$F.S = \frac{\Sigma\,[c'L+(W\cos\alpha-uL)\tan\phi']}{\Sigma\,W\sin\alpha} = \frac{375.20}{312.26} = \boldsymbol{1.20}$$