RULES

1. The answers of the exercises will be posted on our website at 17:00 PM on 10.11.2012: www2.ce.metu.edu.tr/~ce305 .

2. This is the **version 1.0**. In case there are any corrections for this exercise, we will post an updated version on our website. You can follow the changes in the exercises by the **Version History** section below.

Version History

v1.0 Exercise is released.

1. Find the root of the function $f(x) = e^x - 3$ in the interval [0, 2] using

a) Bisection method

b) Regula-Falsi (False position) method

c) Newton-Raphson method (use an initial guess of x_0=0.3)

d) Compare your results of these three methods in terms of number of iterations and speed of convergence. Comment briefly.

For all methods, use a percent relative approximate error tolerance of $5x10^{-3}$. Carry out your iterations with 6 decimal places.

2.

a) Solve the same problem given in Question 1 by MatLAB, using Secant method with x_0=0.3, x_1=0.4 and much lower percent relative error tolerance of $5x10^{-7}$. Define an additional termination criteria in the form of maximum number of iterations N=20.

b) Plot the function in the given interval using MatLAB.

c) If you were to change the initial guesses to be x_0=0.4 and x_1=0.3 how would your results vary? Comment briefly.

d) If you select the initial guesses to bracket the root, how would your results of Secant method vary? Comment briefly.

3. You are given the function: $f(x) = x - \cos(x)$. Assume you are to find the root of this function in [-2, 2].

a) State 1 initial guess x_0 within the interval [-2, 2] where the Newton-Raphson method will not work if the algorithm starts with this guess. Also explain why the method will not converge using this initial guess. (Do NOT perform any Newton Raphson iterations, just comment very briefly.)

b) Find the root of the given function in [-2, 2] with Secant method starting with $x_0 = 0$ and $x_1 = 0.25$. Perform 2 iterations only. Use 4 decimal places in your calculations. Calculate your relative percent error in each step.

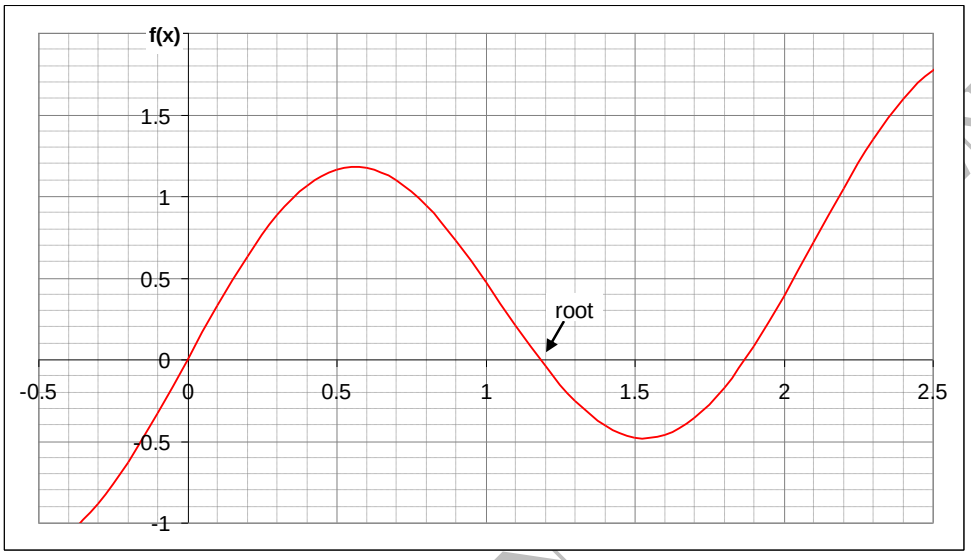
c) When would you prefer Secant method over Newton Raphson method? Explain very briefly.

4. Given the function $f(x) = 2x - 2 - \sin x$.

a) Plot the function f(x) in the interval [0, π/2] using 6 points within the interval.

b) Find the root of f(x) in the interval [0, π/2] using Secant method. Start with initial estimates of $x_0 = 0.75$ and $x_1 = 1.0$. Perform only 4 iterations, and use an error tolerance corresponding to 3 decimal place accuracy.

5. $f(x)=\sin(3x)+\frac{x}{3}$ is plotted below. Find the root between 1 and 1.5 using Newton-Raphson method. Use 4 significant figures.



a) **Initial guess:** Since you know that the root is near $x=1$ and $x=1.5$, you will be using either one of these two values as your initial guess (x_0). Only one of these two x_0 values converges to the correct root.

   i. Explain which x_0 you must choose ($x_0=1$ or $x_0=1.5$) in order to obtain the root, by showing the first iteration of Newton-Raphson method graphically for the ***WRONG*** x_0. You may use the plot given above.

   ii. Based on the graphical explanation in part (i), graphically estimate what result you will get after many iterations, if you start the Newton-Raphson method from the wrong x_0. Do not do any calculation.

b) **Iteration:** Perform 2 iterations. Start from the correct x_0 you chose in part (a). Fill the resulting numbers into the table below. You may use an error tolerance of 10^{-5}.

i	x_{i-1}	$f(x_{i-1})$	$f'(x_{i-1})$	x_i	Error
1					
2					