CE305 Numerical Methods for Civil Engineers — 2014 Fall

Homework I

Question 1. Given $f(x) = 2x - 2 - \sin x$.

a) Plot the function f(x) in the interval [0, π/2] using 6 points within the interval.

b) Find the root of f(x) in the interval [0, π/2] using Secant method. Start with initial estimates of $x_0 = 0.75$ and $x_1 = 1.0$. Perform **only 4** iterations. Calculate the rel. approximate (percent) error at each iteration.

c) Repeat part (b) with Newton method. Use $x_0 = 0.75$.

Question 2. Same function given in Question 1. Solve the problem with Fixed Point Iteration method with $x_0 = 0.75$. Find two alternative x=g(x) forms. Do your solutions converge? Why/ why not?

Question 3. Given $f(x) = e^x - 2$

i. Plot the function in the interval [0, 2] using 6 points (i.e., x = 0, 0.4, 0.8, 1.2, 1.6, 2).

ii. Find the root of the function located in the interval [0, 2] using

a) Bisection method

b) Regula-Falsi (False position) method

c) Newton-Raphson method (use an initial guess of x_0=0.3)

iii. Compare the results of these three methods in terms of number of iterations and speed of convergence.

Question4. For $f(x) = (x/2)^4 + 3x^2 - 5$, use fixed point iteration to find the root of the function in the following sections. Your calculations should be well enough to show 3 significant digit accuracy. (3 points)

a) Determine three fixed point functions, g(x),(i.e. f(x)=0 can be written as f(x)=0=x-g(x)) to do fixed point iterations.

b) Choose one of the fixed point functions and do three iterations to get the root of the function, start at x_0=1.0. Did you converge?

c) How can you make sure that you converge to a root? Apply this criteria to the first fixed point function you chose.

Question 5. Define the rate of convergence for a root-finding algorithm. Explain what we mean when we state that "a method converges quadratically"? Linearly? Which one converges faster? What is the physical meaning of rate of convergence?

Question 6. Find a physical problem from any field in civil engineering (solid mechanics, fluid mechanics, structures. etc) that can be placed in the form of a "nonlinear equation of a single variable". Design your problem similar to the ones given in Question 1 or 3. Choose an initial guess and solve it with a preferred root-finding method. Make sure it converges to a root. Show your iterations. (You may use Matlab or Excel to check your manual calculations).

Question 7. Estimate the value of ln(3) using a 4th order Taylor series around x=2.

Question 8. By using Newton-Raphson iteration formula as a fixed point scheme, show that Newton-Raphson formula has quadratic convergence property. (Hint: You will need to use Taylor series expansion for the fixed point function near the root, p.) The definition of quadratic convergence, as given in class, is as follows. (2 points)

$$\frac{|p_{n+1} - p|}{|p_n - p|^2} < \lambda$$ *where p is the real root, p_n and p_{n+1} are n^{th} and the $n{+}1^{th}$ estimations*