**Examination Date**: 14.12.2011, Wednesday
**Examination Duration**: 120 minutes

| | |
|---|---|
| **Surname, Name** | |
| **Student ID** | |
| **Signature** | |

**For the attention of students:**

- THIS EXAM PAPER CONSISTS OF 7 PAGES. PLEASE CHECK THAT THERE ARE NO BLANK PAGES. IN ADDITION, FORMULA SHEET WILL BE PROVIDED SEPERATELY.
- STUDENTS CANNOT LEAVE THE CLASS IN **THE FIRST 30** AND **LAST 10 MINUTES EVEN FOR THE RESTROOM**.
- MOBILE PHONES / COMPUTERS MUST BE **OFF** DURING THE EXAM.
- PLEASE SHOW EVERY DETAIL OF YOUR SOLUTION (**GIVE DEFINITION OF EVENTS, STEPS TO BE TAKEN, MATHEMATICAL OPERATIONS, ETC. FEEL FREE TO MAKE ANY ASSUMPTIONS YOU NEED**). PLEASE DO NOT DETACH THE STAPLED SHEETS.
- THE USE OF PROGRAMS AND STORED DATA ON PROGRAMMABLE CALCULATORS IS STRICTLY FORBIDDEN.

**GOOD LUCK IN THE EXAM!**

| Question | Grade |
|---|---|
| 1 (25%) | |
| 2 (25%) | |
| 3 (13%) | |
| 4 (12%) | |
| 5 (25%) | |
| **Total (100%)** | |

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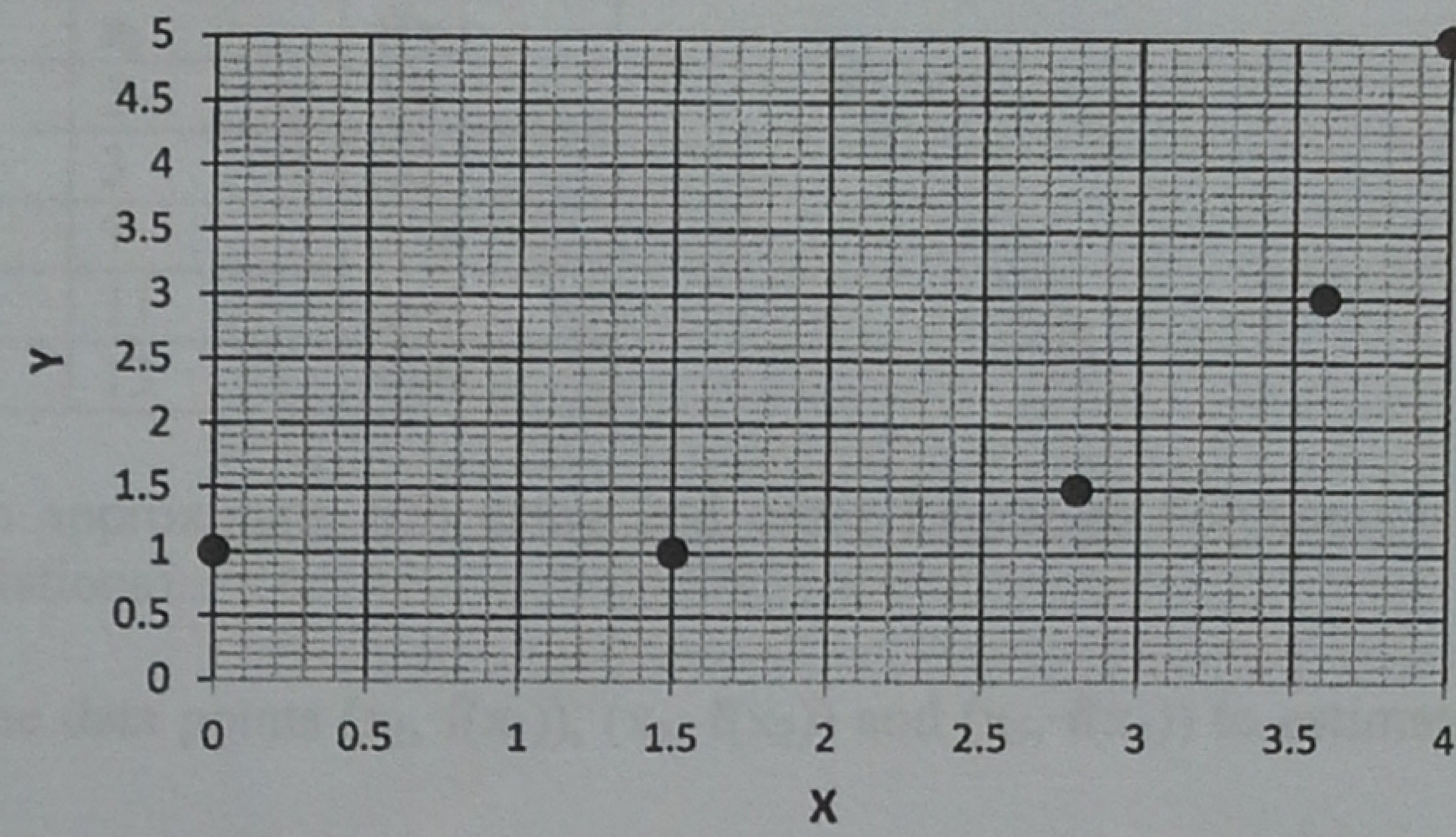
| 5 (25%) | |
| **Total (100%)** | |

**Surname, Name:**
**Student ID:**

**(25%) Question1:**

You have the following data:

| x | y |
|---|---|
| 0 | 1 |
| 1.5 | 1 |
| 2.8 | 1.5 |
| 3.6 | 3 |
| 4 | 3.5 |




**a)** The line $y-0.7x=1$ is suggested as a rough approximation to the data. Calculate the Root Mean Square Error of this approximation.

**b)** The data are obviously non-linear. Perhaps you can fit a parabola ($y=ax^2+b$) through the points. Write a system of 2 equations with 2 unknowns (a,b). Determine a and b using "Least Squares Regression".

$y = 1+0,7x = f(x)$

| x | y | f(x) | e |
|---|---|---|---|
| 0 | 1 | 1 | 0 |
| 1,5 | 1 | 2,05 | 1,05 |
| 2,8 | 1,5 | 2,96 | 1,46 |
| 3,6 | 3 | 3,52 | 0,52 |
| 4 | 3,5 | 3,8 | 0,3 |

$$E = \sqrt{\frac{0^2+1{,}05^2+1{,}46^2+0{,}52^2+0{,}3^2}{n}} = 0{,}9692$$

b) $(ax^2+b-y_i)^2 = f(x)$

$\frac{d}{da}(ax^2+b-y_i)^2 = 2x^2(ax^2+b-y_i) = 0 \quad \Sigma ax^4 + \Sigma bx^2 = \Sigma y_i x^2$

$\frac{d}{db}(ax^2+b-y_i)^2 = 2(ax^2+b-y_i) = 0. \quad \Sigma ax^2 + Nb = \Sigma y_i$

$N=5 \quad \sum_{i=0}^{5} x^4 = 490{,}49 \quad \sum_{i=0}^{5} x^2 = 39{,}05$

$140a + 39{,}05b = 108{,}9$
$39{,}05a + 5b = 10.$

$a = -0{,}18669$
$b = 3{,}45804$

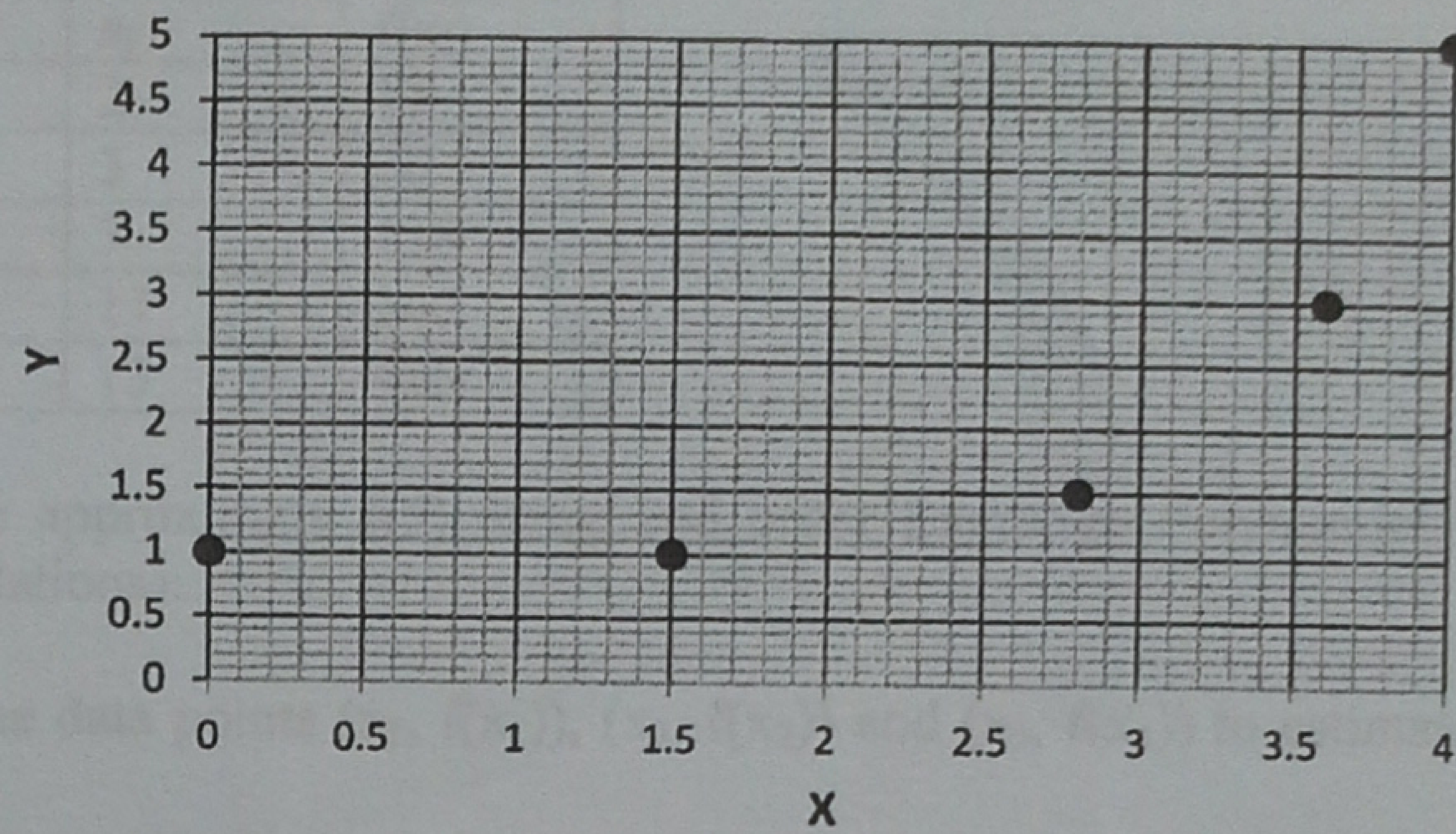
$y = -0{,}18669x^2 + 3{,}45804$

Surname, Name:
Student ID:

**(25%) Question1:**

You have the following data:

| x | y |
|---|---|
| 0 | 1 |
| 1.5 | 1 |
| 2.8 | 1.5 |
| 3.6 | 3 |
| 4 | 3.5 |




**a)** The line $y-0.7x=1$ is suggested as a rough approximation to the data. Calculate the Root Mean Square Error of this approximation.

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$y = 1 + 0,7x = f(x)$

| x | y | f(x) | e |
|---|---|---|---|
| 0 | 1 | 1 | 0 |
| 1,5 | 1 | 2,05 | 1,05 |
| 2,8 | 1,5 | 2,96 | 1,46 |
| 3,6 | 3 | 3,52 | 0,52 |
| 4 | 3,5 | 3,8 | 0,3. |

$\varepsilon = \sqrt{\dfrac{0^2 + 1,05^2 + 1,46^2 + 0,52^2 + 0,3^2}{n}} = 0,9692$

b) $(ax^2+b-y_i)^2 = f(x)$

$\frac{d}{da}(ax^2+b-y_i)^2 = 2x^2(ax^2+b-y_i) = 0 \quad \Sigma a x^4 + \Sigma b x^2 = \Sigma y_i x^2$

$\frac{d}{db}(ax^2+b-y_i)^2 = 2(ax^2+b-y_i) = 0. \quad \Sigma a x^2 + Nb = \Sigma y_i$

$N=5 \quad \sum_{i=0}^{5} x^4 \; 49928 \quad \sum_{i=0}^{5} x^2 = 39,05$

$140a + 39,05b = 108,9$

$39,05a + 5b = 10.$

$a = -0,18669$

$b = 3,45804$

$y = -0,18669x^2 + 3,45804$

**Surname, Name:**
**Student ID:**

**(25%) Question 2:**

For the data points given below:

| i | $x_i$ | $f(x_i)$ |
|---|---|---|
| 0 | 2 | 13 |
| 1 | 3 | 26 |
| 2 | 7 | 138 |
| 3 | 11 | 346 |
| 4 | 13 | 486 |

**a)** (4%) Which points would you use to approximate f(5) using 2nd order Lagrange Polynomials? Briefly explain (Do not perform any calculations).

**b)** (13%) Fit a Lagrange Polynomial to the data points ($x_1$, $f(x_1)$), ($x_2$, $f(x_2)$) and ($x_3$, $f(x_3)$) to estimate the value of f(6).

**c)** (4%) What can you say about the accuracy of the approximation made in part (b) briefly explain:
i) If you know that f(x) is a 2nd order polynomial.
ii) If you know that f(x) is a 3rd order polynomial.

**d)** (4%) When and why one should avoid using Lagrange polynomials for approximation?

a) $x = 2, 3, 7$ or $x = 3, 7, 11$

b) $L_0 = \frac{x - x_1}{x_0 - x_1} \cdot \frac{x - x_2}{x_0 - x_2} = -0{,}6$

$L_1 = \frac{x - x_0}{x_1 - x_0} \cdot \frac{x - x_2}{x_1 - x_2} = 1$

$L_2 = \frac{x - x_0}{x_2 - x_0} \cdot \frac{x - x_1}{x_2 - x_1} = 0{,}6$

$f(x) = f(x_0) \cdot L_0 + f(x_1) \cdot L_1 + f(x_2) \cdot L_2$

$f(x) = -0{,}6 \cdot 13 + 1 \times 26 + 138 \times 0{,}6$

$f(x) = 101$

c
i) exact integral but includes error due to function
ii) includes error from integral also

Surname, Name:
Student ID:

**(25%) Question 2:**

For the data points given below:

| i | $x_i$ | $f(x_i)$ |
|---|---|---|
| 0 | 2 | 13 |
| 1 | 3 | 26 |
| 2 | 7 | 138 |
| 3 | 11 | 346 |
| 4 | 13 | 486 |

**a)** (4%) Which points would you use to approximate f(5) using 2nd order Lagrange Polynomials? Briefly explain (Do not perform any calculations).

**b)** (13%) Fit a Lagrange Polynomial to the data points ($x_1$, $f(x_1)$), ($x_2$, $f(x_2)$) and ($x_3$, $f(x_3)$) to estimate the value of f(6).

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$f(x) = f(x_0) \cdot L_0 + f(x_1) \cdot L_1 + f(x_2) \cdot L_2$

$L_1 = \frac{x - x_0}{x_1 - x_0} \cdot \frac{x - x_2}{x_1 - x_2} = 1$

$f(x) = -0{,}6 \cdot 13 + 1 \times 26 + 138 \times 0{,}6$

$f(x) = 101$

$L_2 = \frac{x - x_0}{x_2 - x_0} \cdot \frac{x - x_1}{x_2 - x_1} = 0{,}6$

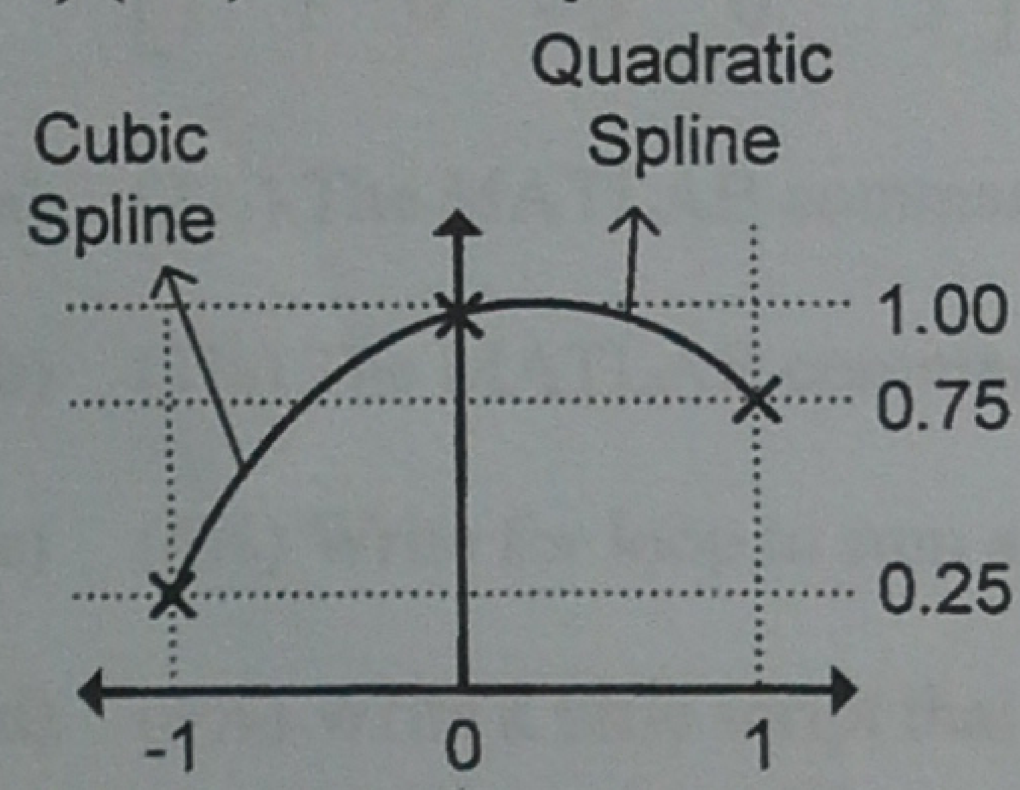
c
i) exact integral but includes error due to function
ii) includes error from integral also

**Surname, Name:**
**Student ID:**

**(13%) Question 3:**

**a)** (12%) For the spline functions providing a fit to the following three points, determine the matrix equation (Ax=b) with which the spline functions can be determined (i.e. their coefficients estimated).

**b)** (1%) How can you solve this equation?




Cubic

$f(-1) = 0,25$

$f(0) = 1.00$

Quadratic

$f(1) = 0,75$

$f(0) = 1,00.$

$y_c = c_1 \cdot f(-1) + c_2 \cdot f(0) + c_3 \cdot f(1).$

$y_a = q_1 \cdot f(-1) + q_2 f(0) + q_3 f(1)$

$y_c(0) = y_q(0)$

**Surname, Name:**
**Student ID:**

**(13%) Question 3:**

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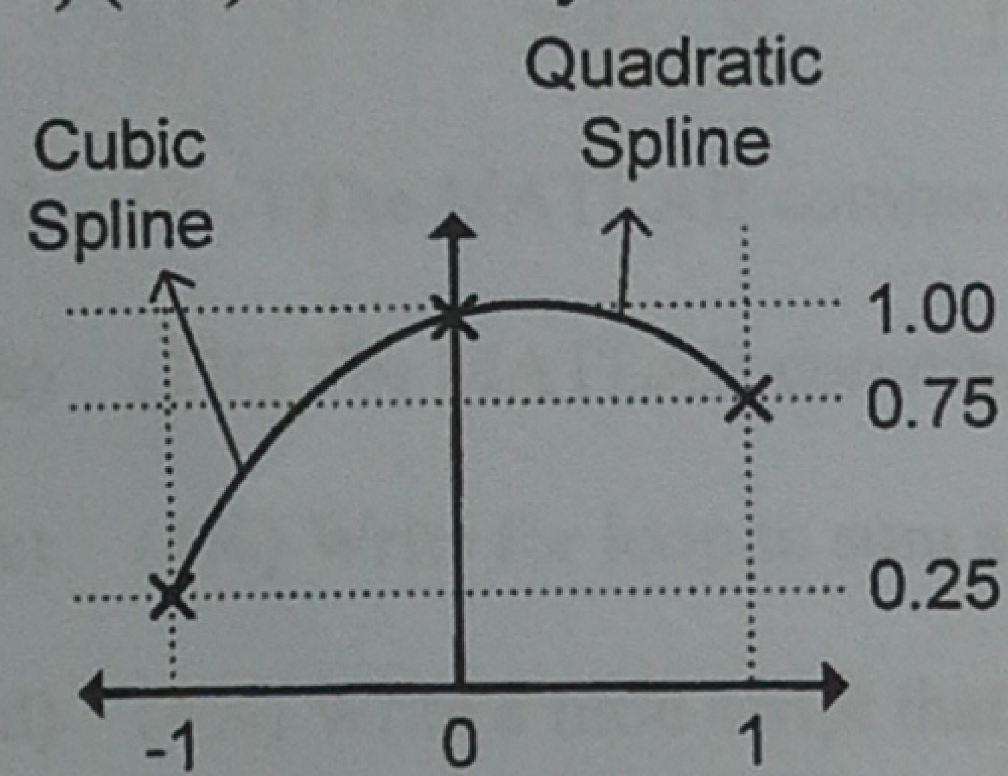



Cubic

$f(-1) = 0.25$

$f(0) = 1.00$

$y_c = c_1 \cdot f(-1) + c_2 \cdot f(0) + c_3 \cdot f(1).$

Quadratic

$f(1) = 0.75$

$f(0) = 1.00.$

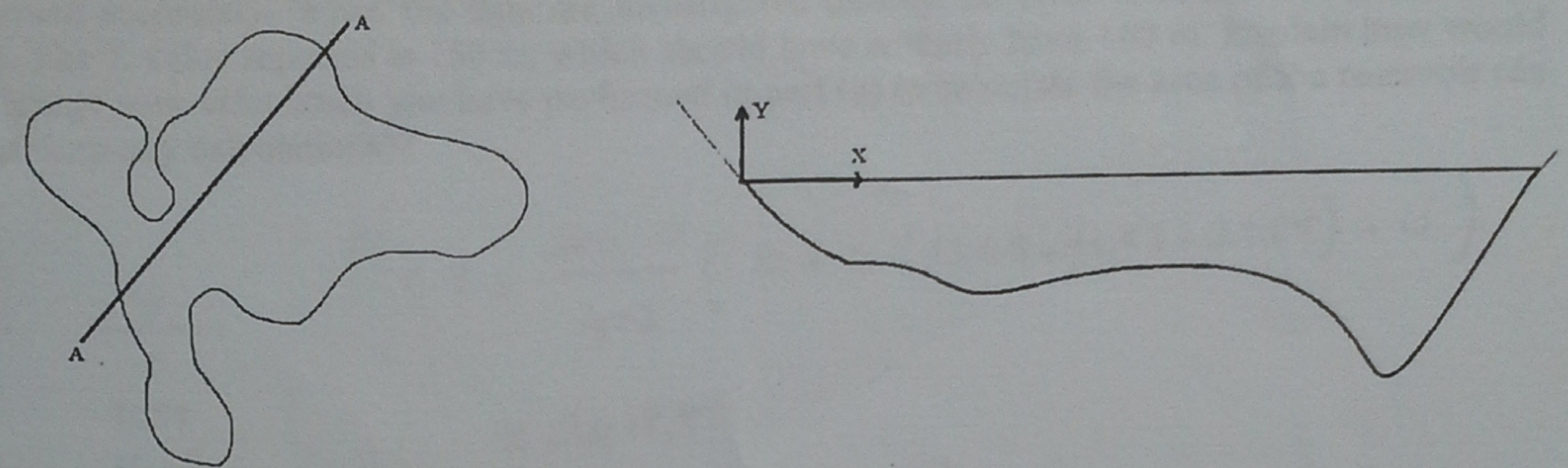
$y_a = q_1 \cdot f(-1) + q_2 \cdot f(0) + q_3 f(1)$

$y_c(0) = y_q(0)$

**Surname, Name:**
**Student ID:**

**(25%) Question 5:**

In the scope of Laboratory #6, CE 305 Ltd. consulted you on Lake Newton project to interpolate the depth of the Lake Newton to accurately calculate the volume of the reservoir. There you were given section A-A where the vertical profile changed rapidly. Since the volume of a reservoir mainly depends on the depth and the area of the reservoir, depth measurements play a significant role in volume calculations. This time, you are required to calculate the area of the profile along A-A (Fig 1.a).




(a) Plan View (b) Profile of Lake Newton along A-A

Figure 1: Lake Newton Project

It is planned that the depth of the reservoir needs to be measured in every 50 meters (0, 50, 100, 150 and 200 m in x direction) along A-A (Fig 1.b). The measurements are provided in Table 1 and sketched in Figure 2. Calculate the area of the water in the reservoir for section A-A using;

Table 1: Depth and Length Measurements of Lake Newton along Section A-A

| Measurement No. | x, Length (m) | y, Depth(m) |
|---|---|---|
| 1 | 0 | 0 |
| 2 | 50.00 | -12.99 |
| 3 | 100.00 | -11.53 |
| 4 | 150.00 | -23.85 |
| 5 | 200.00 | 0 |

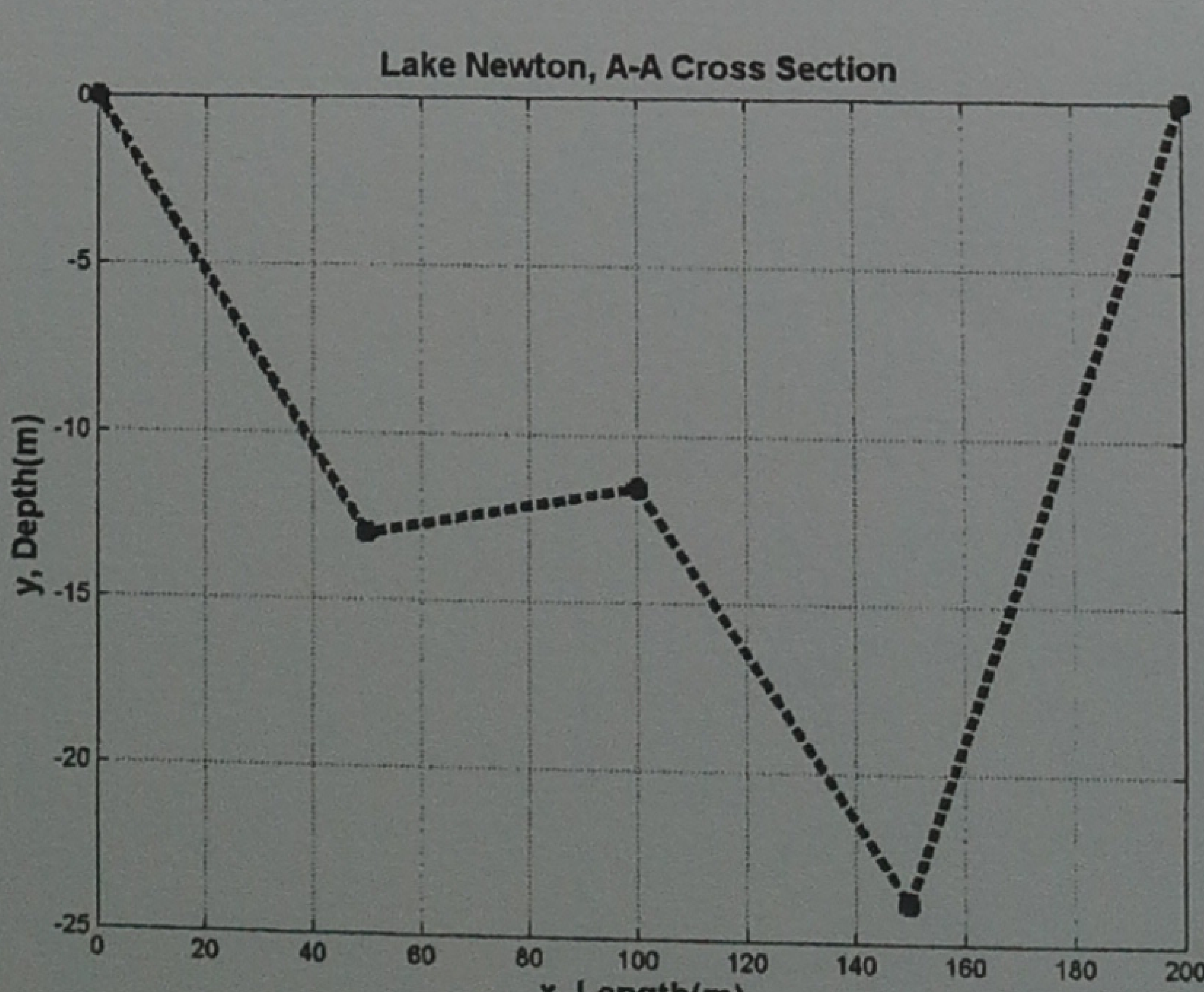



Figure 2: Sketch of Depth Measurements Lake Newton Project

**Surname, Name:**
**Student ID:**

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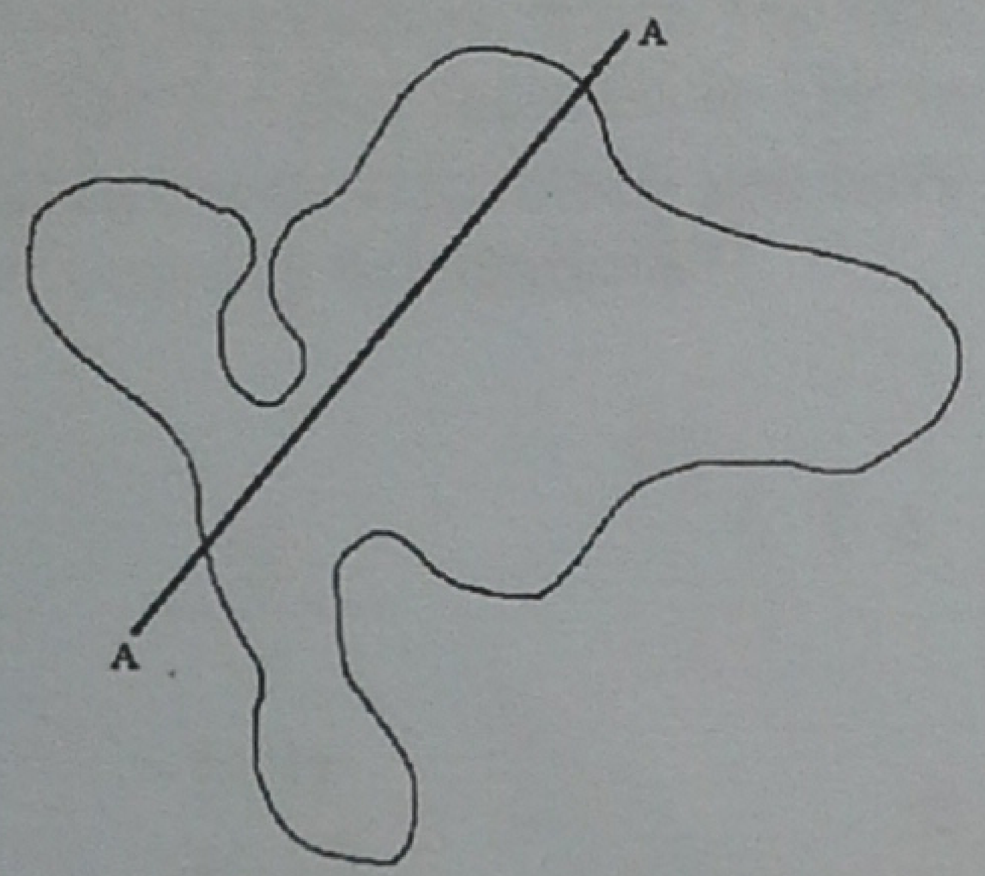


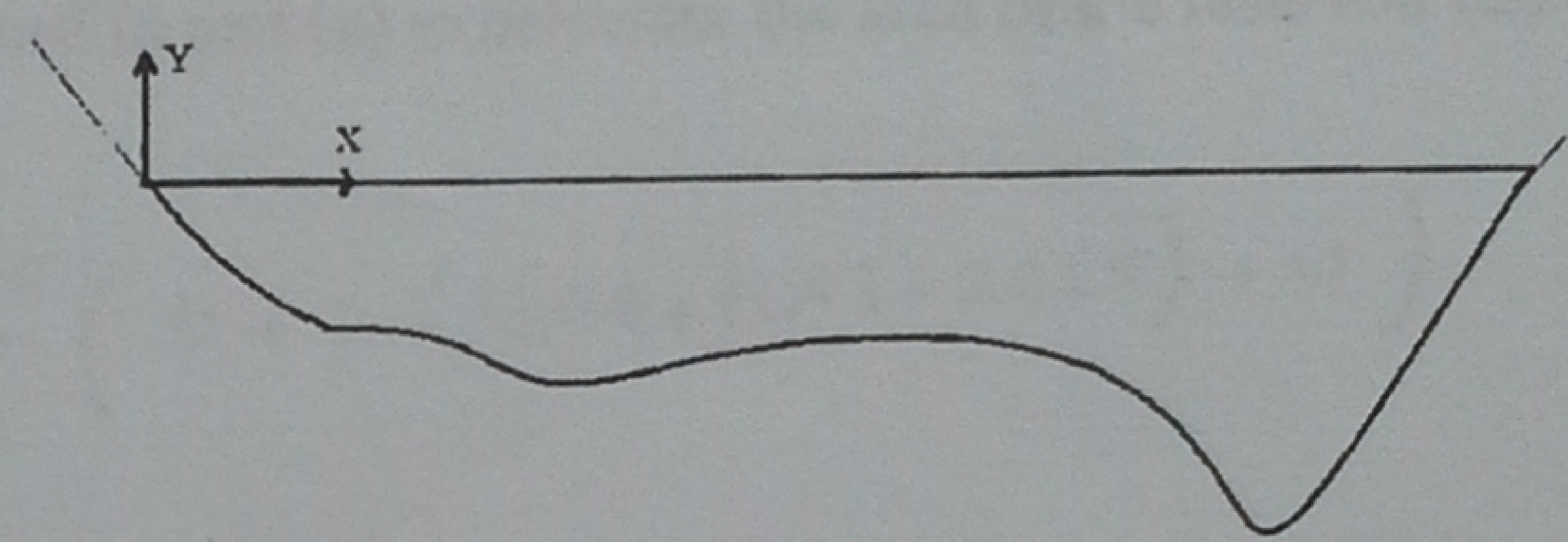


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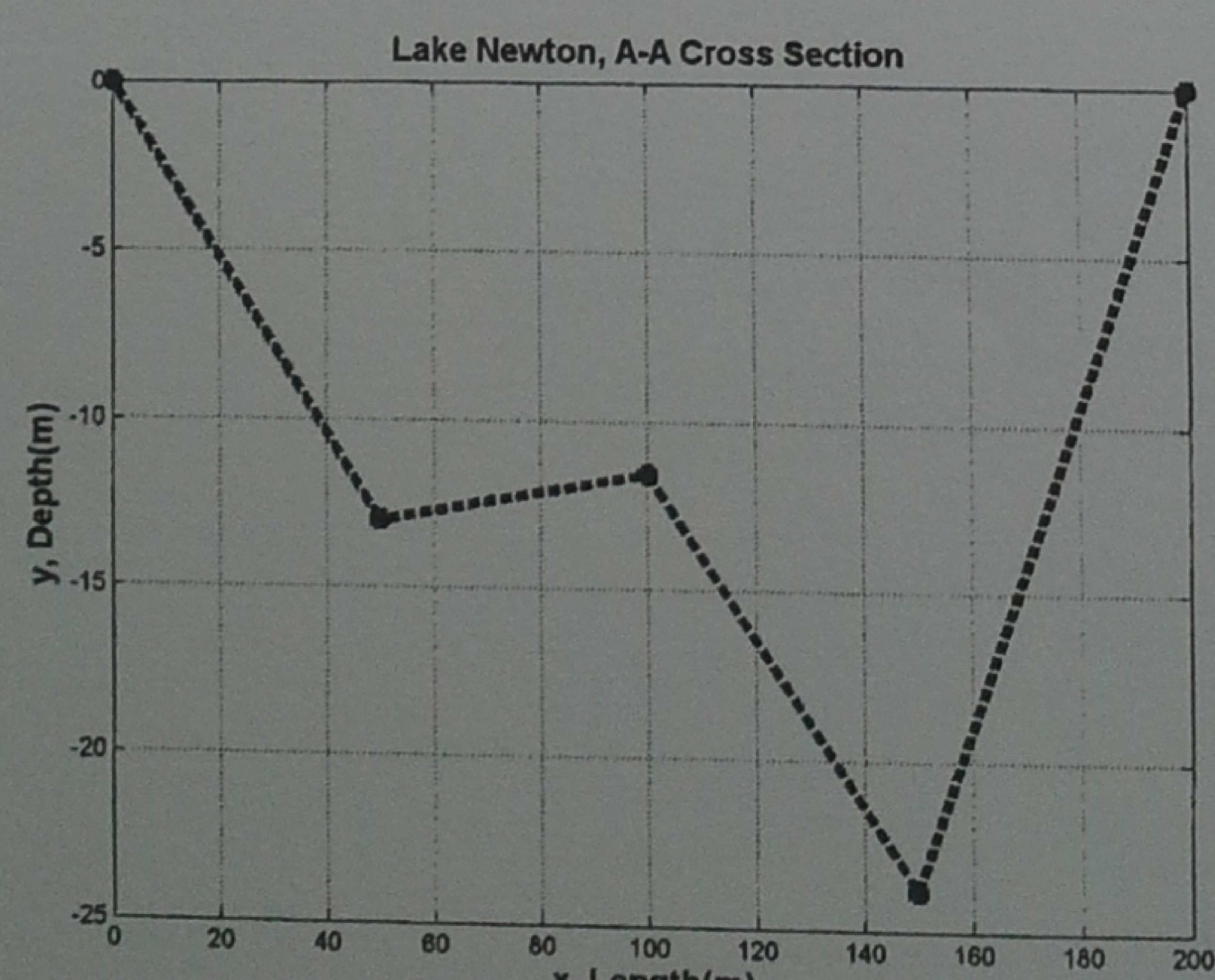


Figure 2: Sketch of Depth Measurements Lake Newton Project

**Surname, Name:**
**Student ID:**

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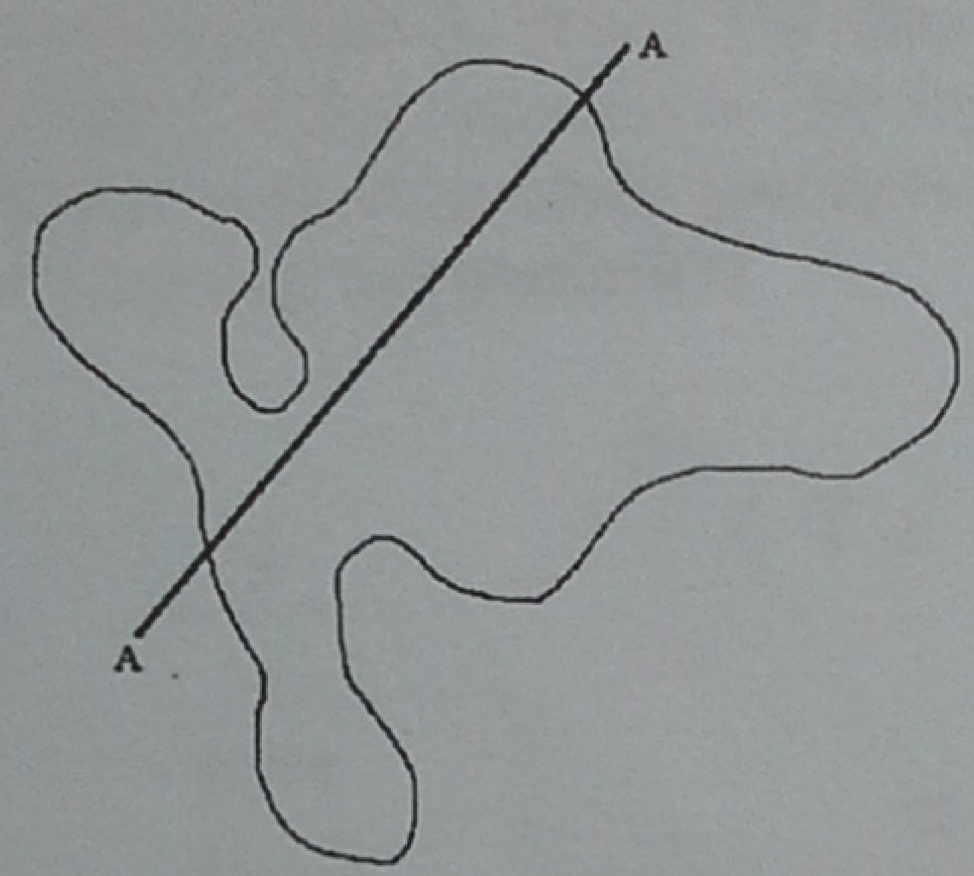
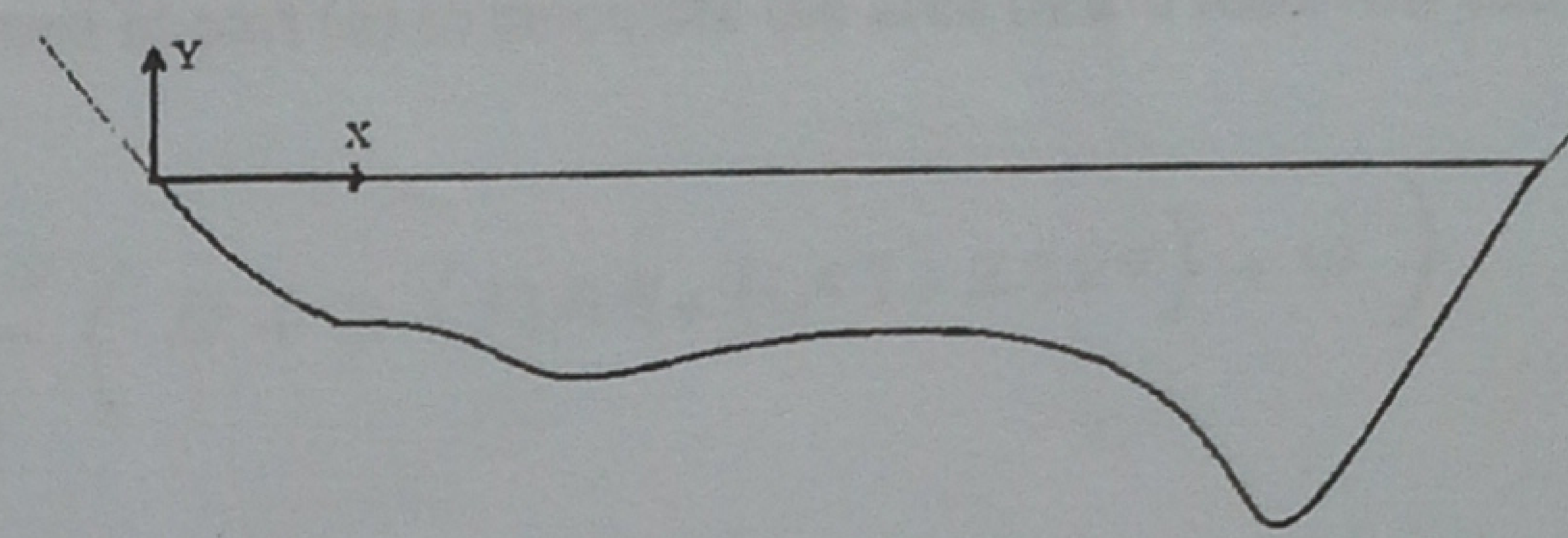



(a) Plan View




(b) Profile of Lake Newton along A-A

Figure 1: Lake Newton Project

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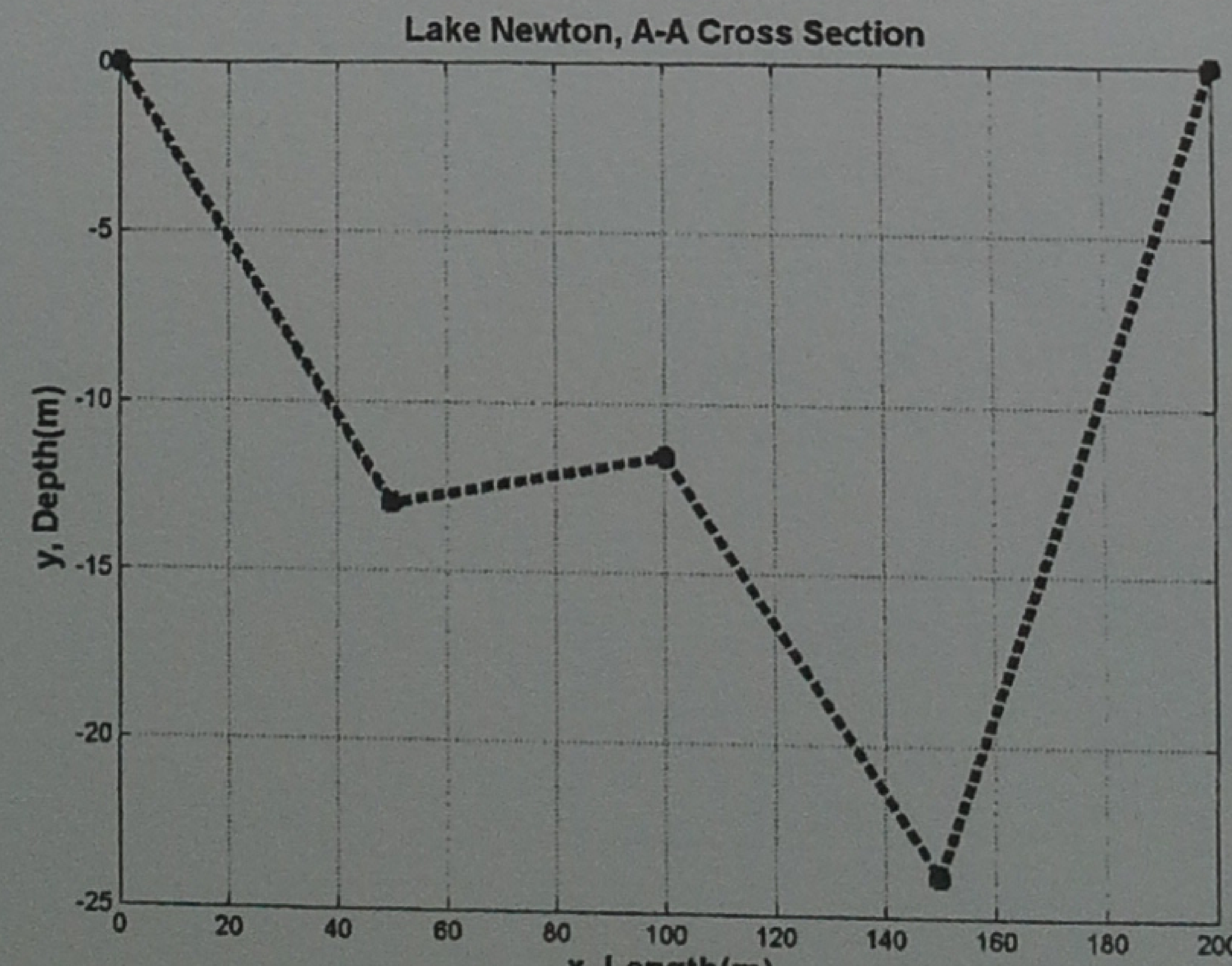


Figure 2: Sketch of Depth Measurements Lake Newton Project

**Surname, Name:**
**Student ID:**

**a) (10%)** Composite Trapezoidal rule given the above intervals. (Show all your calculations explicitly.)

**b) (10%)** Composite Simpson's 1/3 rule using 2 intervals. (Show your calculations explicitly.)

**c) (5%)** Since the equipment of the company is too old, it is known some measurements could not be performed accurately. When the data are investigated closely, an error is found on Measurement No. 4. The X value reported is 150 m, which should have actually been 160 m. Explain how would you change your calculation you have performed in part (a) to calculate the area of the reservoir (do not perform any calculations)?

| x | y |
|---|---|
| 0 | 0 |
| 50 | -12,99 |
| 100 | -11,53 |
| 150 | -23,85 |
| 200 | 0 |

a) $CT = \frac{200-0}{4 \cdot 2}\left(0 + 2\cdot(12{,}99 + 11{,}53 + 23{,}85) + 0\right)$

$= 2418{,}5$

b) $CS = \frac{200-0}{2 \cdot 3}\left(0 + 11{,}53 \cdot 4 + 0\right) = 1537{,}33$

c)

**Surname, Name:**
**Student ID:**

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|---|---|
| 0 | 0 |
| 50 | -12,99 |
| 100 | -11,53 |
| 150 | -23,85 |
| 200 | 0 |

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$= 2418{,}5$

b) $CS = \frac{200-0}{2 \cdot 3}\left(0 + 11.53 \cdot 4 + 0\right) = 1537{,}33$

c)

**Least Squares Fit**

For fitting an n[th] degree polynomial of the form $y = C_0x^0 + C_1x^1 + C_2x^2 + \ldots + C_nx^n$,

$$\begin{bmatrix} \sum x^0 & \sum x^1 & \cdots & \sum x^n \\ \sum x^1 & \sum x^2 & \cdots & \sum x^{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ \sum x^n & \sum x^{n+1} & \cdots & \sum x^{2n} \end{bmatrix} \cdot \begin{bmatrix} C_0 \\ C_1 \\ \vdots \\ C_n \end{bmatrix} = \begin{bmatrix} \sum x^0 y \\ \sum x^1 y \\ \vdots \\ \sum x^n y \end{bmatrix}$$

**Numerical Integration**

***Trapezoidal Rule:*** $I = \int_a^b f(x)dx = (b-a)\dfrac{f(a)+f(b)}{2}$

***Simpson's Rule:*** $I = \int_a^b f(x)dx = (b-a)\dfrac{f(x_0)+4f(x_1)+f(x_2)}{6}$

***Composite Trapezoidal Rule:*** $I = \dfrac{h}{2}\left[ f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n) \right]$

*Truncation Error:*

$E_t = -\dfrac{(b-a)^3}{12n^3}\sum_{i=1}^{n} f''(\xi_i)$ where $\xi_i \in (x_{i-1}, x_i)$, i.e. $\xi_i$ is any value within the segment.

***Composite Simpson's Rule:*** $I = \dfrac{h}{3}\left[ f(x_0) + 4\sum_{i=1,3,\cdots}^{n-1} f(x_i) + 2\sum_{j=2,4,\cdots}^{n-2} f(x_j) + f(x_n) \right]$

*Truncation Error:*

$E_t = -\dfrac{(b-a)^5}{180n^5}\sum_{i=1}^{n} f^{(4)}(\xi_i)$ where $\xi_i \in (x_{i-1}, x_i)$, i.e. $\xi_i$ is any value within the segment.

***Gauss Quadrature:***

$$\int_a^b g(t)dt = \int_{-1}^{1} f(x)dx$$

$$t = \frac{b+a}{2} + \frac{b-a}{2}x$$

$$dt = \frac{b-a}{2}dx$$

$$\int_{-1}^{1} f(x)dx \approx \sum_{i=0}^{n} w_i f_i$$

| N | Abscissas, $x_{N,k}$ ($X_i$) | Weights, $w_{N,k}$ ($W_i$) | Truncation Error |
|---|---|---|---|
| 2 | -0.5773502692<br>0.5773502692 | 1.0000000000<br>1.0000000000 | $\dfrac{f^{(4)}(\varepsilon)}{135}$ |
| 3 | ±0.7745966692<br>0.0000000000 | 0.5555555556<br>0.8888888888 | $\dfrac{f^{(6)}(\varepsilon)}{15,750}$ |
| 4 | ±0.8611363116<br>±0.3399810436 | 0.3478548451<br>0.6521451549 | $\dfrac{f^{(8)}(\varepsilon)}{3,472,875}$ |
| 5 | ±0.9061798459<br>±0.5384693101<br>0.0000000000 | 0.2369268851<br>0.4786286705<br>0.5688888888 | $\dfrac{f^{(10)}(\varepsilon)}{1,237,732,650}$ |

## Least Squares Fit

For fitting an n[th] degree polynomial of the form $y = C_0x^0 + C_1x^1 + C_2x^2 + \ldots + C_nx^n$,

$$\begin{bmatrix} \sum x^0 & \sum x^1 & \cdots & \sum x^n \\ \sum x^1 & \sum x^2 & \cdots & \sum x^{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ \sum x^n & \sum x^{n+1} & \cdots & \sum x^{2n} \end{bmatrix} \cdot \begin{bmatrix} C_0 \\ C_1 \\ \vdots \\ C_n \end{bmatrix} = \begin{bmatrix} \sum x^0 y \\ \sum x^1 y \\ \vdots \\ \sum x^n y \end{bmatrix}$$

## Numerical Integration

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***Simpson's Rule:*** $I = \int_a^b f(x)dx = (b-a)\frac{f(x_0)+4f(x_1)+f(x_2)}{6}$

***Composite Trapezoidal Rule:*** $I = \frac{h}{2}\left[ f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n) \right]$

*Truncation Error:*

$E_t = -\frac{(b-a)^3}{12n^3}\sum_{i=1}^{n} f''(\xi_i)$ where $\xi_i \in (x_{i-1}, x_i)$, i.e. $\xi_i$ is any value within the segment.

***Composite Simpson's Rule:*** $I = \frac{h}{3}\left[ f(x_0) + 4\sum_{i=1,3,\cdots}^{n-1} f(x_i) + 2\sum_{j=2,4,\cdots}^{n-2} f(x_j) + f(x_n) \right]$

*Truncation Error:*

$E_t = -\frac{(b-a)^5}{180n^5}\sum_{i=1}^{n} f^{(4)}(\xi_i)$ where $\xi_i \in (x_{i-1}, x_i)$, i.e. $\xi_i$ is any value within the segment.

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$$\int_a^b g(t)dt = \int_{-1}^{1} f(x)dx$$

$$t = \frac{b+a}{2} + \frac{b-a}{2}x$$

$$dt = \frac{b-a}{2}dx$$

$$\int_{-1}^{1} f(x)dx \approx \sum_{i=0}^{n} w_i f_i$$

| N | Abscissas, $x_{N,k}$ ($X_i$) | Weights, $w_{N,k}$ ($W_i$) | Truncation Error |
|---|---|---|---|
| 2 | -0.5773502692<br>0.5773502692 | 1.0000000000<br>1.0000000000 | $\frac{f^{(4)}(\varepsilon)}{135}$ |
| 3 | ±0.7745966692<br>0.0000000000 | 0.5555555556<br>0.8888888888 | $\frac{f^{(6)}(\varepsilon)}{15,750}$ |
| 4 | ±0.8611363116<br>±0.3399810436 | 0.3478548451<br>0.6521451549 | $\frac{f^{(8)}(\varepsilon)}{3,472,875}$ |
| 5 | ±0.9061798459<br>±0.5384693101<br>0.0000000000 | 0.2369268851<br>0.4786286705<br>0.5688888888 | $\frac{f^{(10)}(\varepsilon)}{1,237,732,650}$ |

**Some Basic Formulae:**

***Derivatives:*** $\frac{d}{dx}e^{kx} = k \cdot e^{kx}$, $\frac{d}{dx}\arctan(kx) = \frac{k}{1+(kx)^2}$

***Inverse of a 2x2 matrix:*** $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-cb} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

**Error Formulae:**

$\varepsilon_{RA} = \left| \frac{x_{k+1} - x_k}{x_{k+1}} \right|$ $\varepsilon_{RA\%} = \left| \frac{x_{k+1} - x_k}{x_{k+1}} \right| \times 100$ $\varepsilon_{AA} = |x_{k+1} - x_k|$ $RMS = \sum_{i=1}^{n} \sqrt{\frac{1}{n}(f(x_i) - y_i)^2}$

**Taylor Series:**

$$f(x+h) = f(x) + hf'(x) + \frac{h^2 f''(x)}{2!} + \frac{h^3 f^{(3)}(x)}{3!} + .... \frac{h^n f^{(n)}(x)}{n!} + R_n$$

where $R_n = h^{n+1} \frac{f^{(n+1)}(\xi)}{(n+1)!}$, $\xi$ lies in the interval $[x, x+h]$.

**Newton-Raphson Formula:** $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

**Secant Formula:** $x_{k+1} = x_k - \frac{f(x_k)(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})}$

**Fixed Point Iteration:** $x_{i+1} = g(x_i)\, such \;\; that \; f(x) = 0$

**Newton-Jacobi Method:**

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad F = \begin{bmatrix} f_1(x_1, x_2, \cdots, x_n) \\ f_2(x_1, x_2, \cdots, x_n) \\ \vdots \\ f_n(x_1, x_2, \cdots, x_n) \end{bmatrix} \quad J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

$$X_{k+1} = X_k - J_k^{-1} \cdot F_k$$

**Lagrange's Polynomial:**

$f_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$ where $L_i(x) = \prod_{j=0; j \neq j}^{n} \frac{(x - x_j)}{(x_i - x_j)}$

$$E_n(x) = \prod_{i=0}^{n} (x - x_i) \frac{f^{(n+1)}(\xi)}{(n+1)!};\; x_0 < \xi < x_n$$

**Some Basic Formulae:**

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