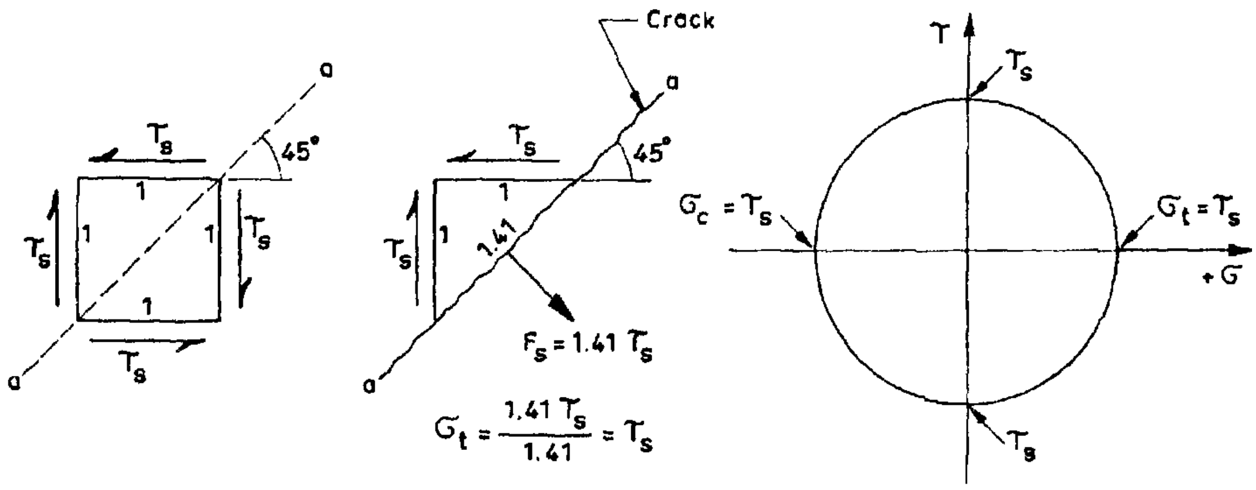
Shear and Diagonal Tension

INTRODUCTION

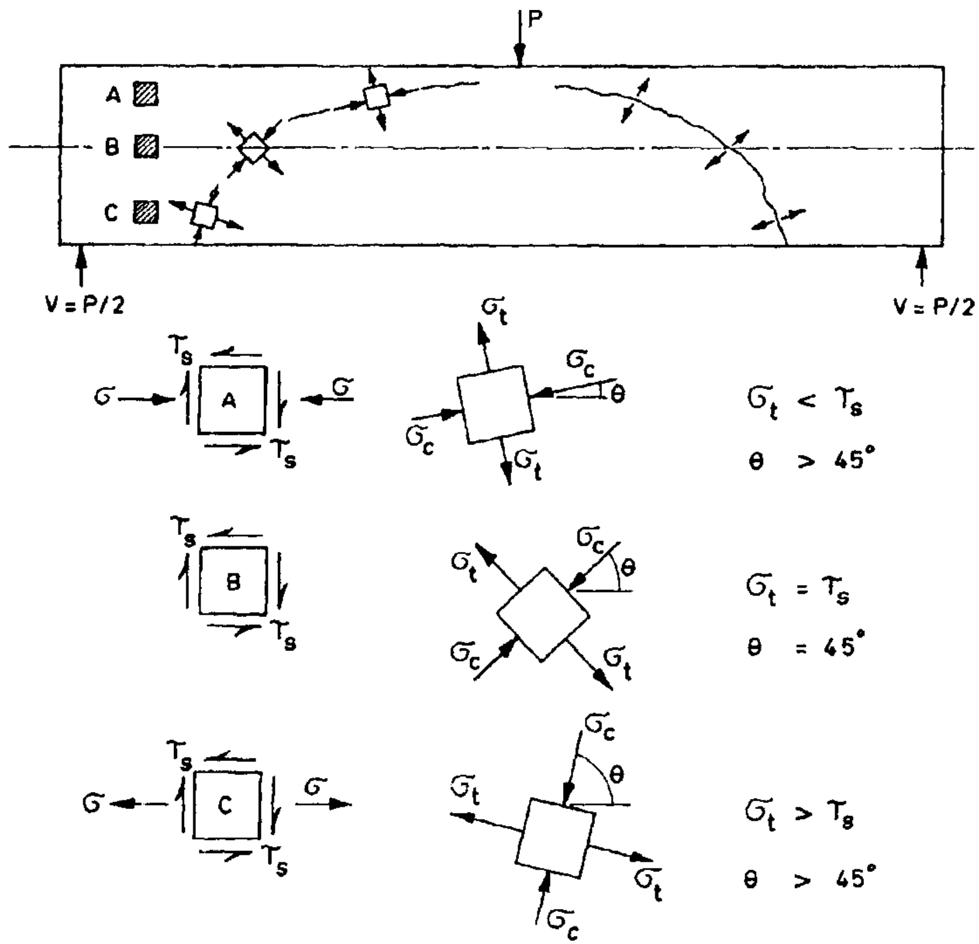
Members of reinforced concrete structures are usually subjected to shear in addition to flexure and axial loads.

Concrete is quite strong in shear. Therefore, shear by itself does not create any serious problems in reinforced concrete members. However, principal tensile stresses caused by shear combined with other stresses create a serious problem for reinforced concrete members, since the tensile strength of concrete is very low.

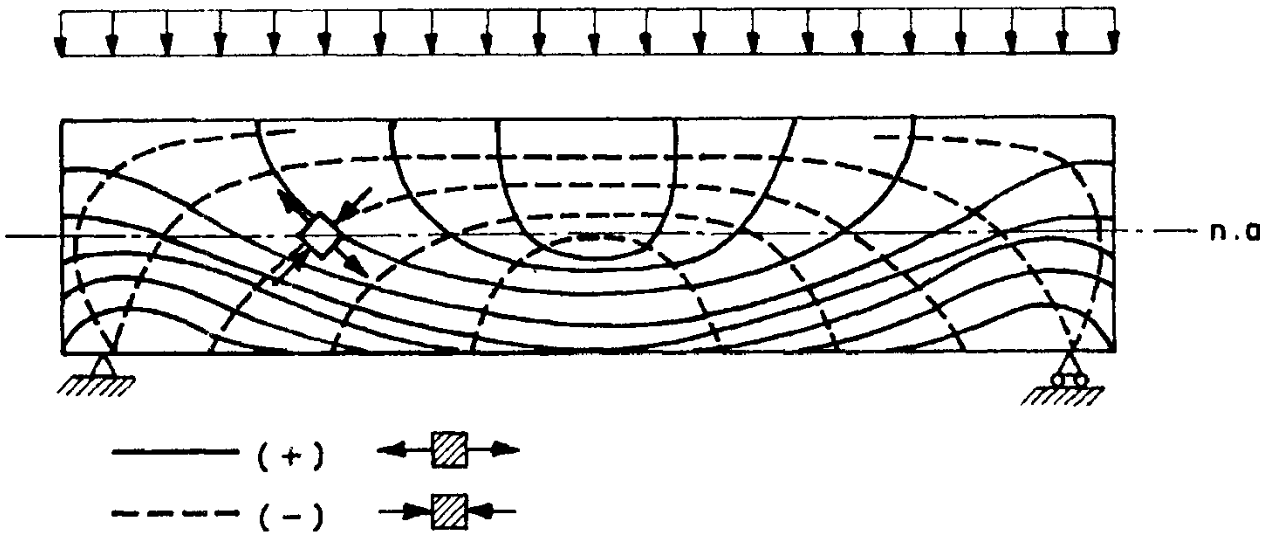


Diagonal tension (principal tensile stresses) can cause severe diagonal cracking which can lead to failure of the member if necessary precautions are not taken by properly designed web reinforcement. Diagonal tension failures are usually very sudden and brittle and therefore such failures should be prevented if a ductile behavior is desired.

The shape or the inclination of the diagonal tension cracks is determined by the orientation of the principal tensile stresses.



The shape or the inclination of the diagonal tension cracks is determined by the orientation of the principal tensile stresses. The trajectories shown in the figure below are for an uncracked beam subjected to uniform loading.



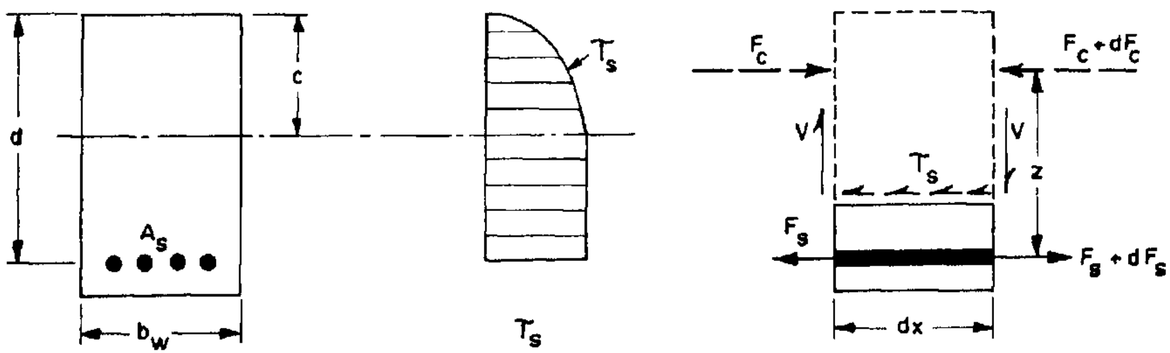
Early designers of reinforced concrete structures (late in 19 and early in 20^{th} century) treated shear stresses in a similar way to what they did for timber members. These designers considered the horizontal shear as the main cause for shear failures. Stirrups (transverse steel placed perpendicular to the longitudinal steel) were designed as bolts in timber beams, which were assumed to carry the horizontal shear.

Ritter: The problem was not the shear stresses but principal tensile stresses caused by these shear stresses (diagonal tension).

Mörsch (1916) after a set of beam tests had convinced the engineers that the shear cracks (diagonal tension cracks) were caused by principal tensile stresses. Mörsch proposed to use shear stresses <u>as a measure</u> of the magnitude of diagonal tensile stresses (principal stresses).

He derived a formula for the shear stress, which is based on a parabolic shear stress distribution in the compression zone, and a uniform distribution in the tension zone.

From the free body shown in figure below:



$$\tau_s \cdot b_w \cdot dx = dF_s = \frac{dM}{z}$$

$$\frac{dM}{dx} = V \quad , \quad \tau_s = \frac{V}{b_w\, z}$$

Mörsch believed that concrete could not contribute to the shear strength of the member once a certain shear stress level is exceeded (stress, which causes diagonal cracking). In such cases, Mörsch recommended that the total shear stress (or total principal tension) should be taken care of by properly designed transverse reinforcement. This approach of Mörsch, which neglects concrete contribution in shear design, was consistent with the flexural design, in which tensile strength of concrete is neglected.

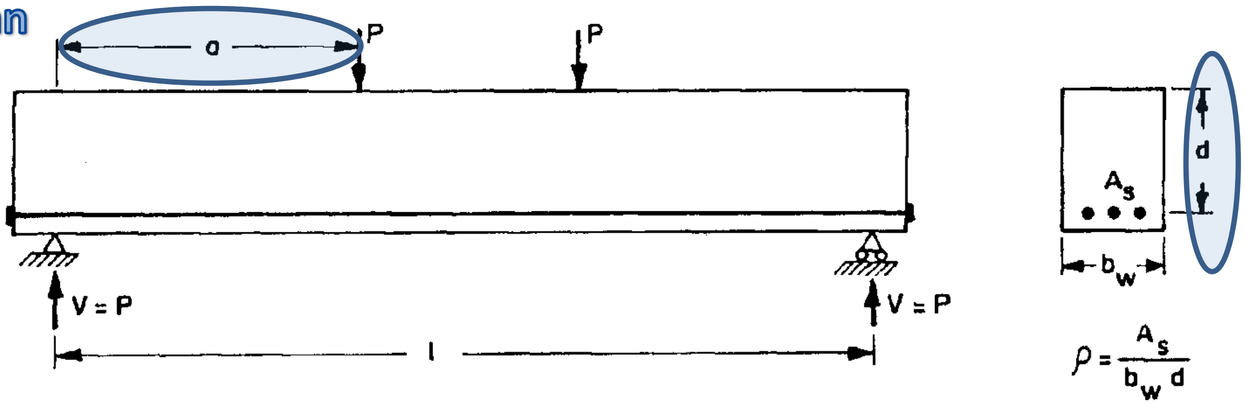
Mörsch developed analogy, which is known as "Mörsch-Ritter Truss Analogy.“ In this analogy, the longitudinal and transverse reinforcement and concrete in compression were considered as members of a truss. This analogy will be discussed later in this chapter.

The shear theory established by Mörsch found a universal acceptance. Extensive experimental research on shear, which took place after 1950, brought new dimensions to the problem. Some new variables were introduced, and it was found out that Mörsch’s theory did not apply to beams with small shear span. As a result of the extensive research, after 1950 new design methods and recommendations have been developed.

Behavior and Strength of Beams without Web Reinforcement

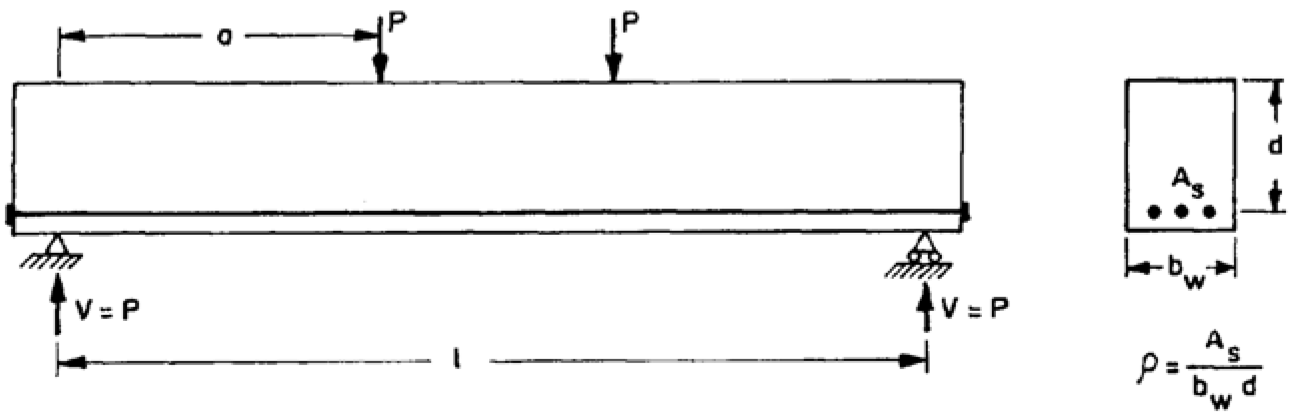
Most design codes prohibit beams without transverse reinforcement (web reinforcement). Such codes require uniformly spaced stirrups (transverse steel) along the entire span no matter how small the shear stresses are (shear stress as a measure of diagonal tension). Although this is the case, behavior of beams without web reinforcement will be discussed here since the behavior of beams with web reinforcement cannot be clearly understood without knowing the behavior of beams without web reinforcement.

Shear Span

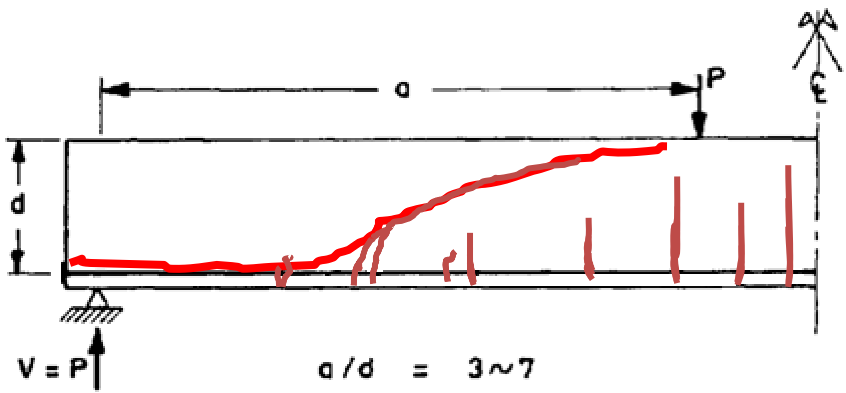


Researchers have observed that the dimensionless shear span or a/d influenced the behavior and strength of reinforced concrete beams significantly. Therefore, the beam behavior will be discussed in relation to a/d.

When a/d is large, i.e. greater than 7.0 or 8.0, then the behavior will be dominated by flexure. In such beams although the flexural cracks in the shear span are slightly inclined, no diagonal cracks form.

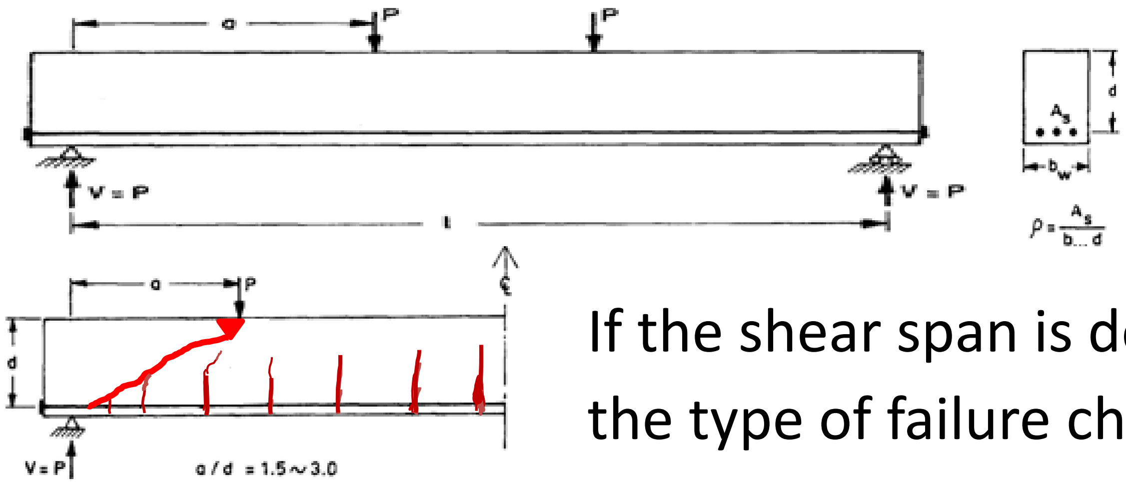


(a)



(b)

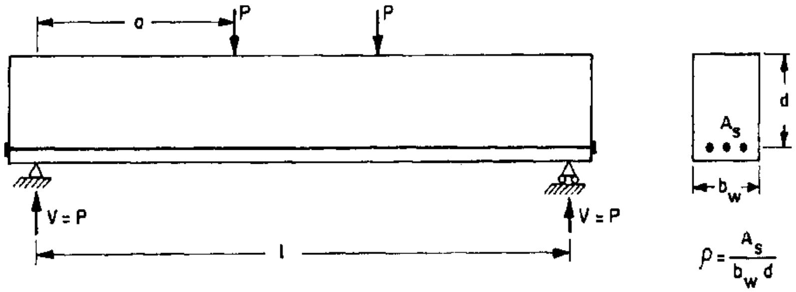
Beams with 7 > a/d > 3 fail by the formation of a diagonal crack at **loads lower than the ones corresponding to flexural capacity**. Typical crack growth for such a beam is shown in Fig. (b).



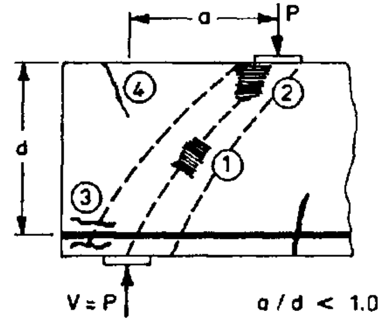
(c)

If the shear span is decreased (1.5 < a/d < 3.0), the type of failure changes.

In beams with shorter shear spans, 1.5 < a/d <3, failure load is greater than the diagonal cracking load. Therefore, although the failure is still brittle, it is not as sudden and brittle as the "diagonal tension failure.”



(a)



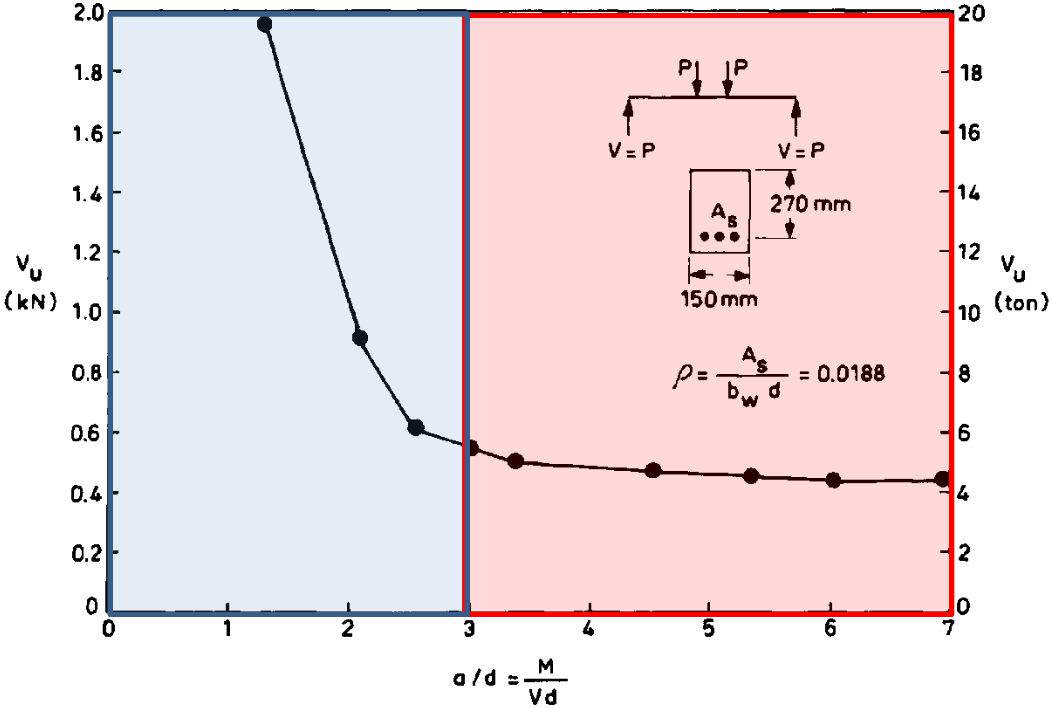
(d)

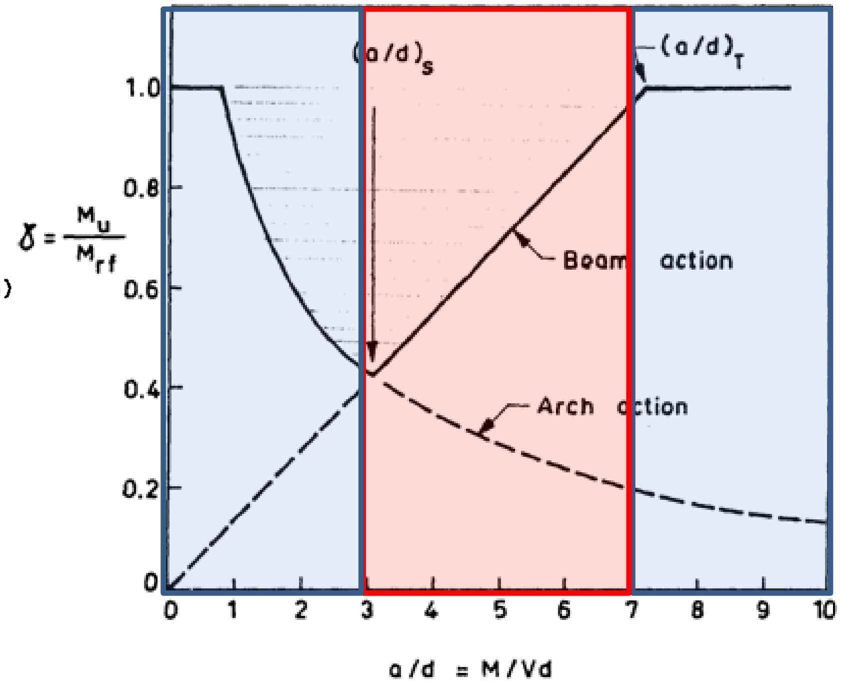
In beams with very short shear spans, a/d < 1.0, failure due to principal tensile stresses is not possible.

in such cases the load is directly transferred to the support with a compression strut forming in the web. In this case, the beam reaches **its flexural capacity**.

The final failure can take place by crushing of concrete in the compression zone, crushing of the web (if web width is small) or by anchorage failure near the support.

Members with such a short shear span behave like a "tied arch" rather than a beam. Therefore, the longitudinal steel acting as a tie rod creates severe anchorage problems in the vicinity of the reaction.

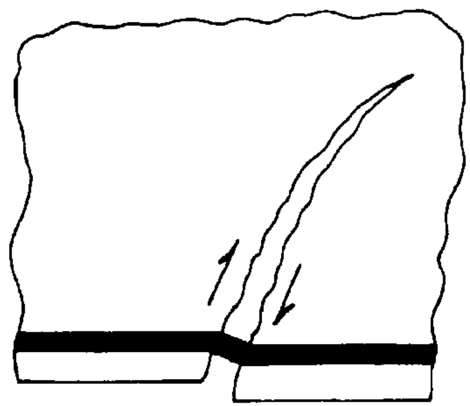




This ratio can be generalized as M/(Vd) so that it can also be applied to beams subjected to distributed loads.

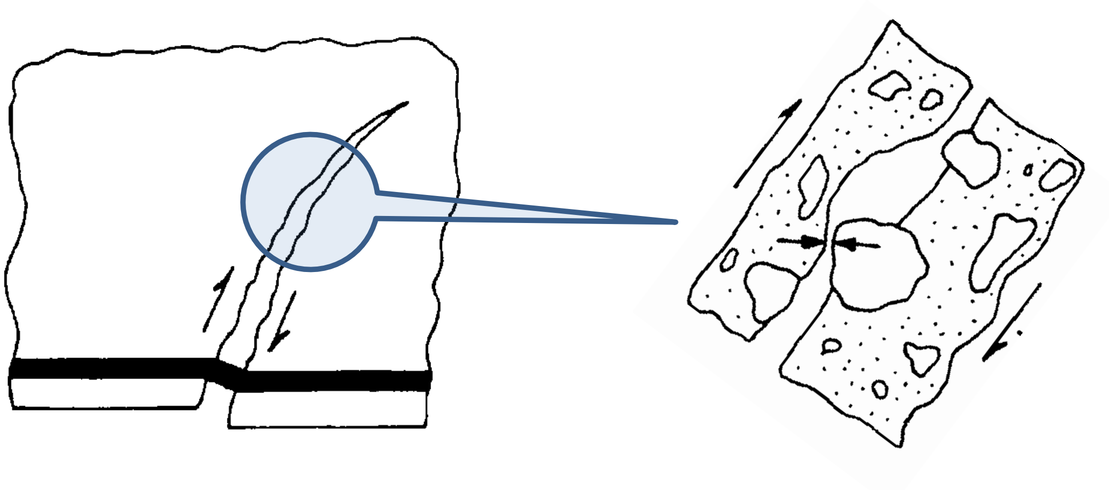
This discussion reveals the importance of a/d ratio on the behavior of beams without web reinforcement.

Thousands of tests made at different laboratories all over the world revealed that two other main variables in addition to M/(Vd) ratio affected the behavior and strength of beams without web reinforcement. These were the ratio of tensile reinforcement, $\rho = A_s/(b_w d)$ and the tensile strength of concrete.



Once the splitting cracks occur, the dowel action will loose its effectiveness. The shear carried by dowel action is about 20% of the total shear in beams without web reinforcement. Effectiveness of dowel action depends on the tensile strength of concrete, on the effective area between bars and the stiffness of the reinforcement (crack width and bar diameter).

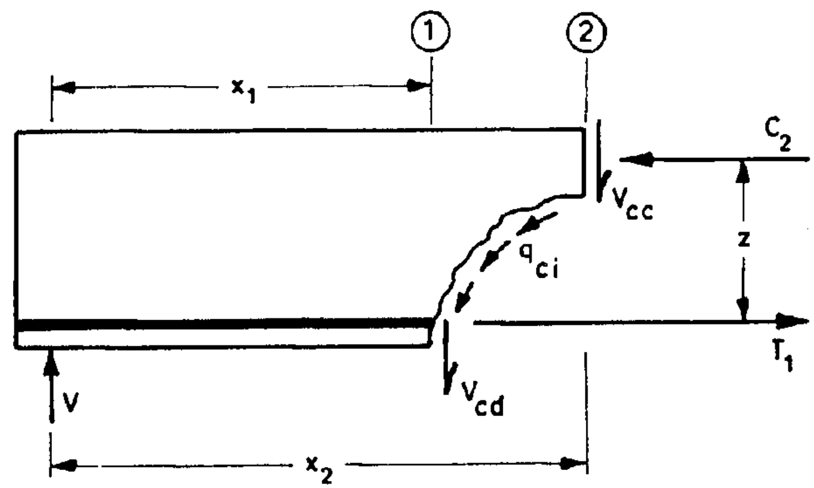
The interface or surface of cracks are not smooth, the coarse aggregate particles projecting across the crack produce roughness. When the two faces of a crack of moderate width are given a shear displacement, these projecting aggregate particles try to prevent this displacement and thus considerable shear can be transmitted along the crack. The resistance mechanism can be seen in figure below. Measurements on test beams without web reinforcement indicated that 50 to 70% of the total shear was resisted by the aggregate interlock mechanism.



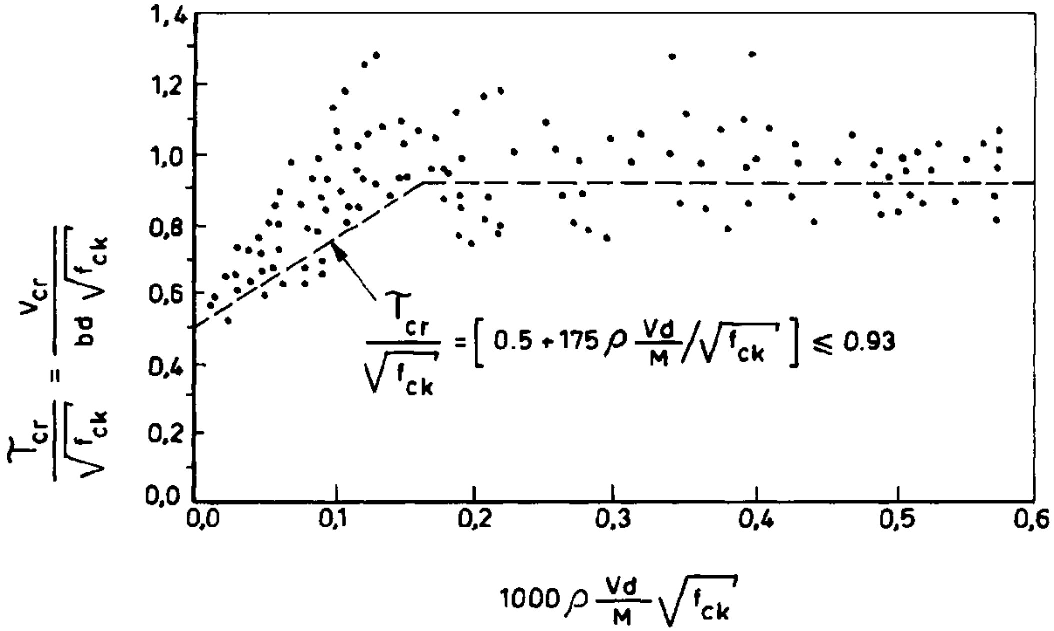
It is important to note that the maximum capacities of the mechanisms (compression zone, dowel action and aggregate interlock) are not necessarily additive at the failure stage.

It is natural to assume that some shear will be carried across the uncracked compression zone. Thus; equilibrium after diagonal cracking can be shown in the following slide.

BEAMS WITHOUT WEB REINFORCEMENT: EQUILIBRIUM AFTER CRACKING



DIAGONAL CRACKING STRENGTH



In S.I. units (kN):

$$V_{cr} = \left[\left(0.16\sqrt{f_{ck}} + 17\rho\frac{Vd}{M}\right)b_w d\right] \leq 0.3\sqrt{f_{ck}}\, b_w d$$

In metric units (tons):

$$V_{cr} = \left[\left(0.53\sqrt{f_{ck}} + 175\rho\frac{Vd}{M}\right)b_w d\right] \leq 0.93\sqrt{f_{ck}}\, b_w d$$

In the Turkish Code TS-500, V_{cr} is simply expressed in terms of design tensile strength of concrete:

$$V_{cr} = 0.65\, f_{ctd}\, b_w\, d \qquad (7.27)$$

In case of axial compression, V_{cr} given above should be multiplied by the following factor:

$$\beta = \left(1 + 0.07\frac{N_d}{A_c}\right) \quad \text{S.I. units}$$

$$\beta = \left(1 + 0.007\frac{N_d}{A_c}\right) \quad \text{Metric}$$

When axial tensile stress is smaller than 0.5 MPa or 5 kg/cm² it is recommended to compute the cracking shear using Eq. (7.27) without any reduction. When the axial tensile stress is greater than this value then the cracking strength should be computed by:

$$V_{cr} = 0.65 f_{ctd}\, b_w\, d\left(1 - 0.30\frac{N_d}{A_c}\right) \quad \text{S.I. units}$$

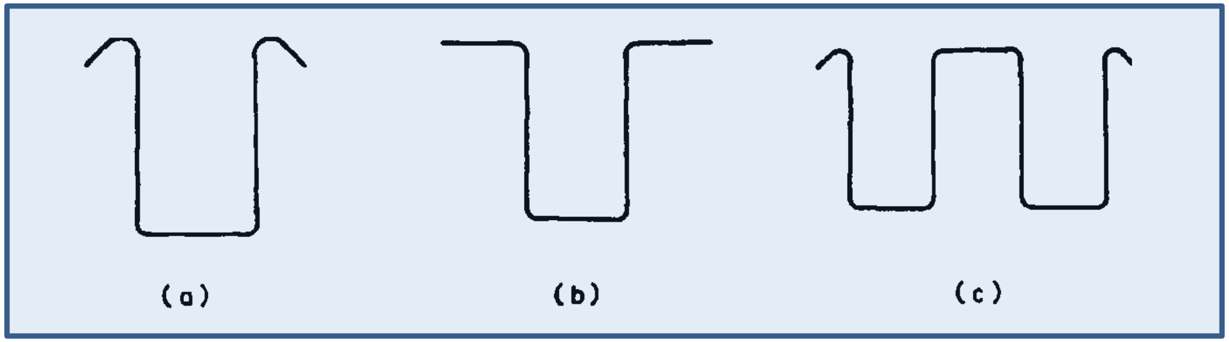
$$V_{cr} = 0.65 f_{ctd}\, b_w\, d\left(1 - 0.03\frac{N_d}{A_c}\right) \quad \text{Metric}$$

Behavior and Strength of Beams with Web Reinforcement

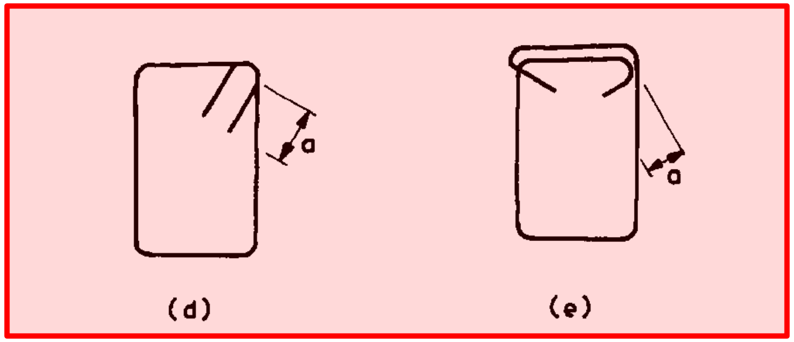
Failure of beams without web reinforcement by diagonal tension is a brittle and undesirable type of failure. In such beams, the ultimate moment corresponding to failure is less than the flexural capacity.

In designing reinforced concrete members, web reinforcement (transverse reinforcement) is provided to prevent diagonal tension failures, to insure failure to occur in flexure, $M_u/M_{rf} \geq 1.0$.

The most commonly used stirrup types are shown in Fig. 7.18. Types shown in (d) and (e) are usually called hoops rather than stirrups. However, in this chapter "stirrup" is used as a general term, which also covers hoops.



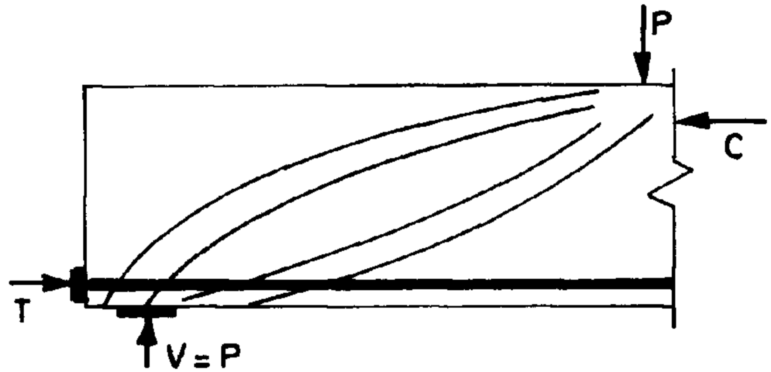
Good for gravity design. Not recommended in seismic regions.



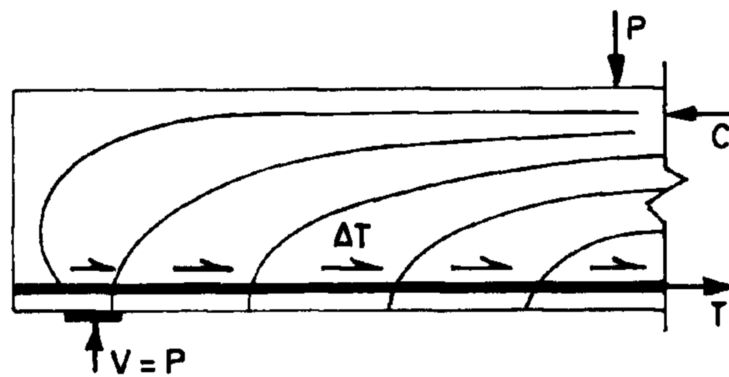
Good for seismic design. Provides better confinement

Web reinforcement cannot prevent diagonal cracking. However properly designed web reinforcement can keep cracks from opening up widely; thus induce the formation of minute cracks instead of a few widely opened cracks.

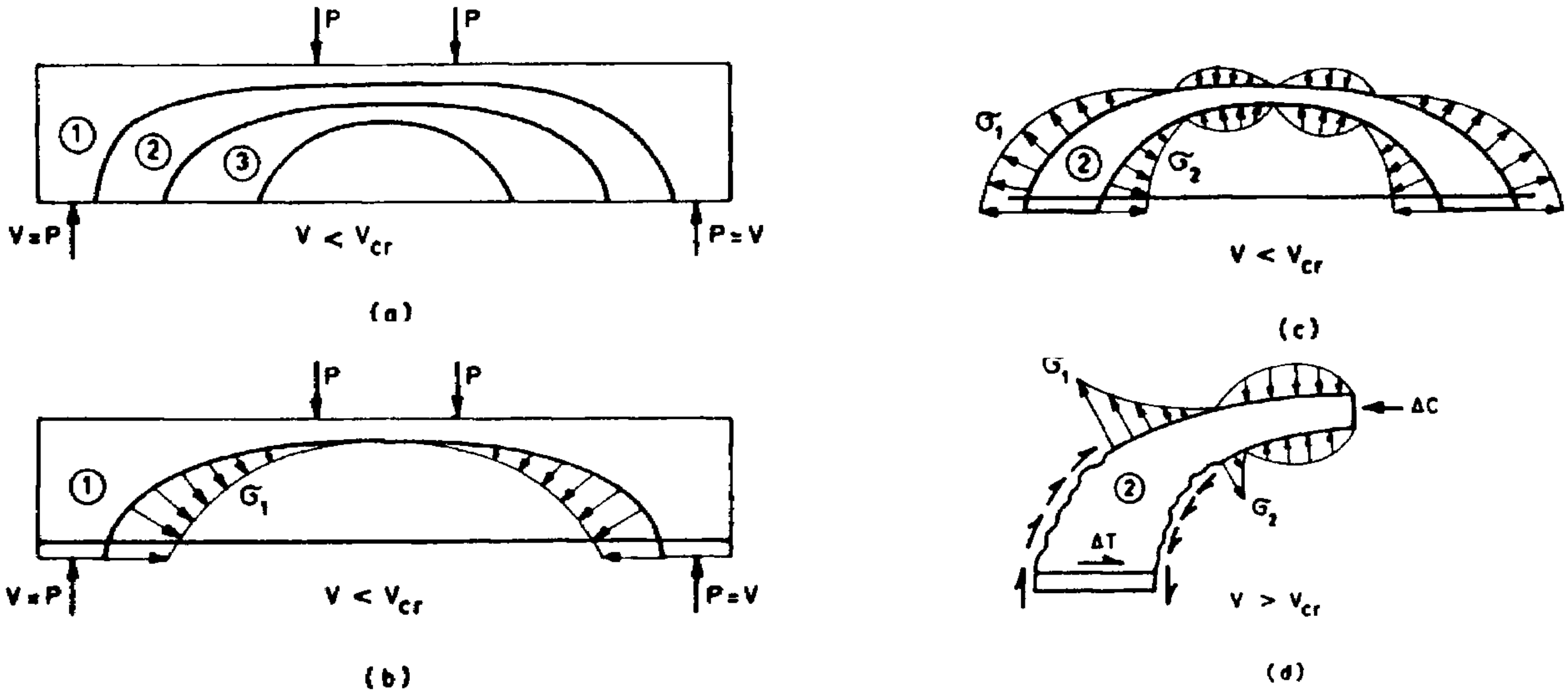
Since the main purpose for providing web reinforcement is to prevent the brittle type of diagonal tension failure, enabling the beam to reach its full flexural capacity and to keep the crack width minimum, the question is what kind of web reinforcement is to be used and where to place the web reinforcement to fulfill these functions. This can be best understood if the internal mechanism is known.



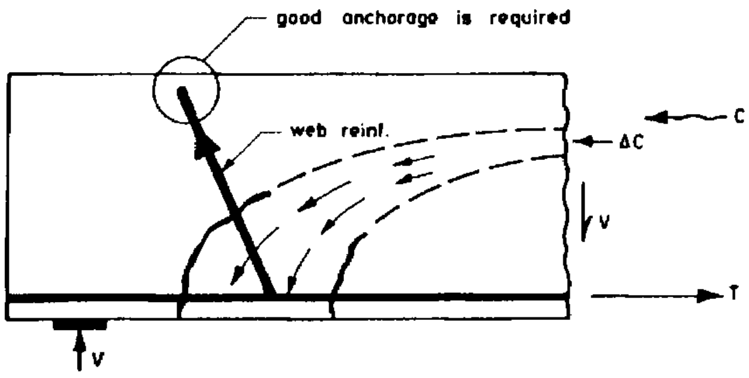
(a) No Bond

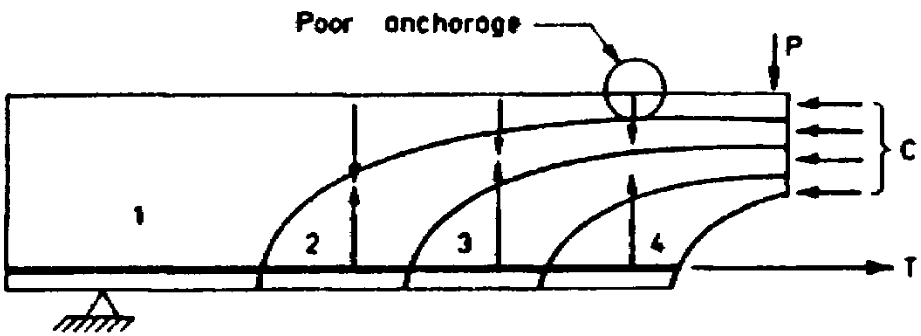


(b) With Bond



Since the main reason for the reduction of resistance of intermediate arches is the gradual loss of hanging supports, it appears logical that creating some replacement supports for the intermediate arches would increase their load carrying capacity. Such arch supports can be provided by the stirrups inclined at an angle of b with the beam axis.





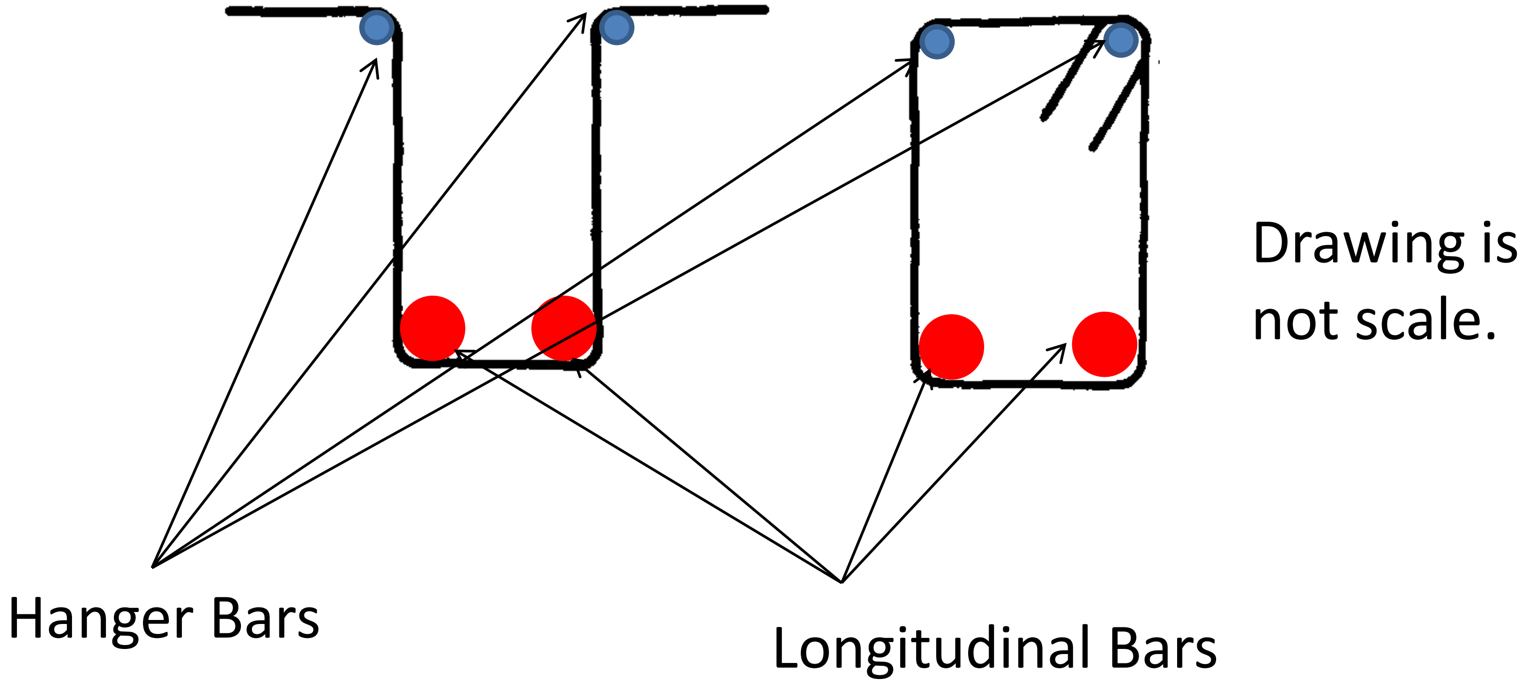
In order to provide firm supports to the intermediate arches, the web reinforcement have to be anchored well in the compression zone of the beam.

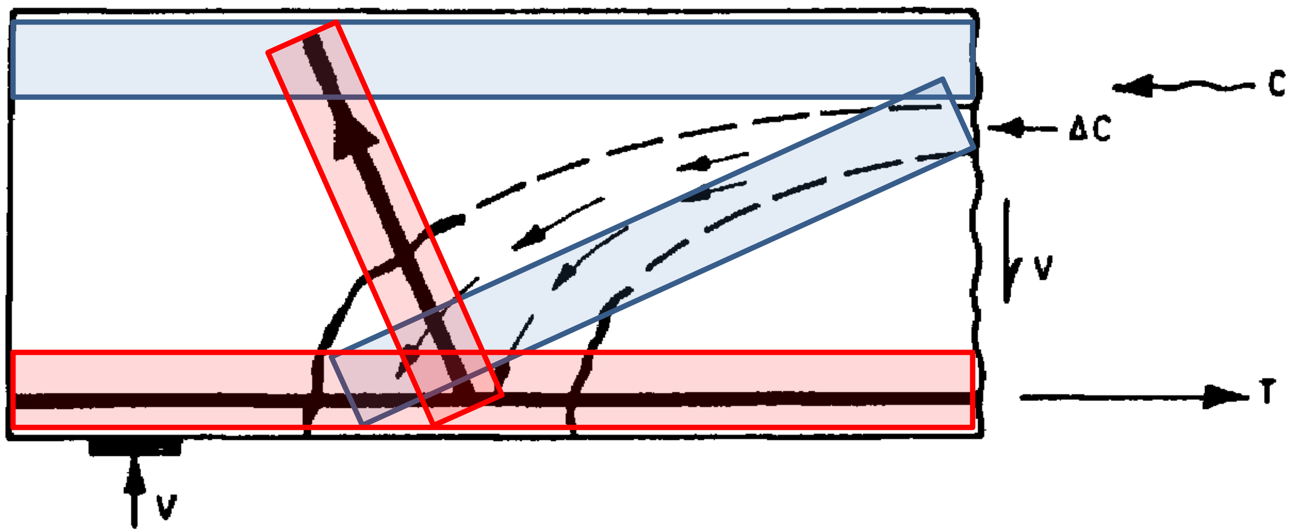
It should be noticed that no web reinforcement is required near the reaction since Arch-1 needs no such support.

It should also be noticed that the stirrup close to the load point cannot be satisfactorily anchored, since the depth of the rib of the main arch at this point is quite small.

Hanger Bars

Hanger Bars

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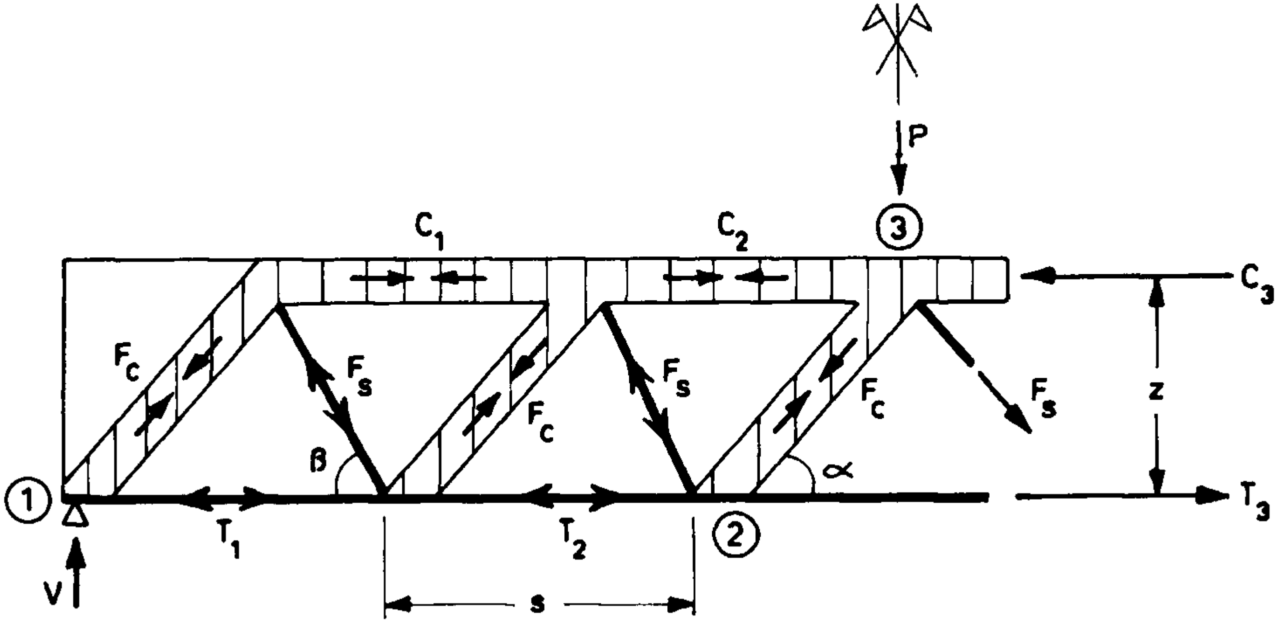
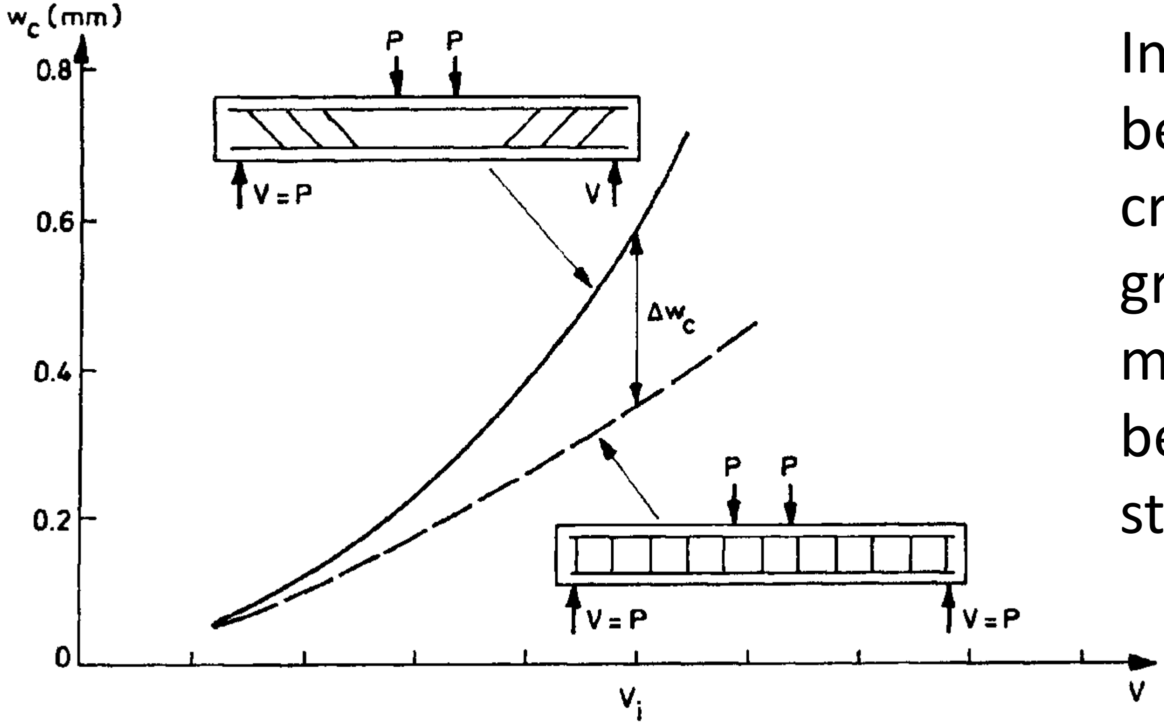


$$A_{sw} = \frac{V(s)}{(z)\sigma_{sw}} \qquad \text{for } \beta = 90°$$

TRUSS ANALOGY

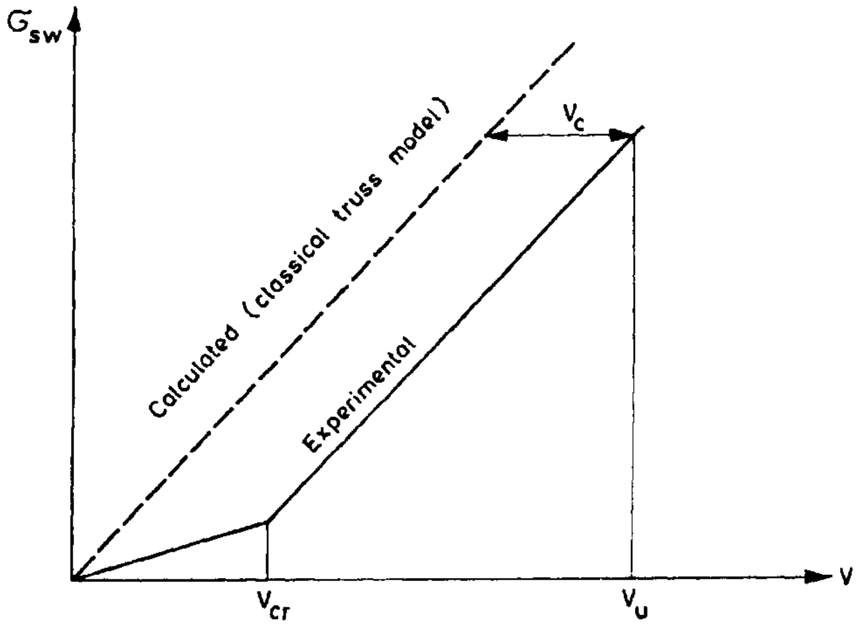
$$A_{sb} = \frac{V(s)}{\sqrt{2}(z)\sigma_s} \qquad \text{for } \beta = 45°$$

Stuttgart Tests



In beams reinforced with bent bars, measured crack width are much greater than the ones measured on identical beams having vertical stirrups.

Stuttgart Tests



Strain measurements taken on web reinforcement do not agree with the results obtained from classical truss analogy. The results indicate that the stresses in web reinforcement are always below the values predicted by the truss model.

Stuttgart Tests

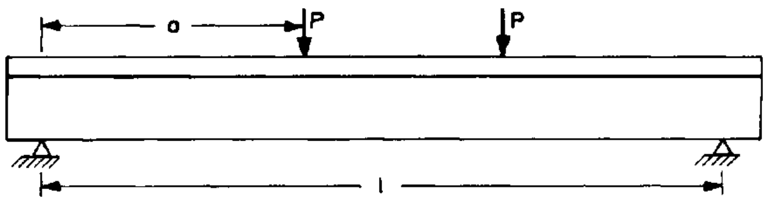
In Stuttgart tests, beams which had only 47% of the web reinforcement required by the classical truss model behaved well and reached the flexural capacity without premature shear failure.

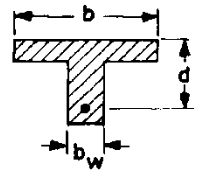
There is experimental evidence showing that web reinforcement becomes more effective and carries greater portion of the shear as the ratio of flange width to web thickness increases, b/b_w.

Stuttgart Tests

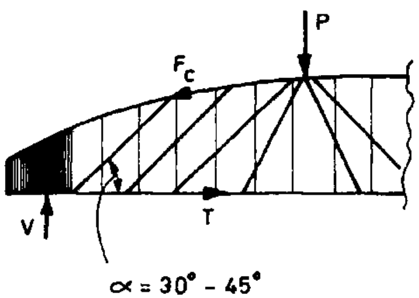
In Stuttgart tests, strains measured on the top surface of T-beams, showed considerable tension near the end of the beam reaching as far as 1.6d from the support.

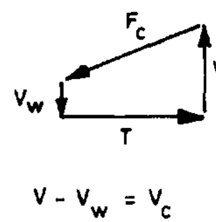
These experimental findings have led researchers to develop new analytical models. Prof. Leonhardt proposed a revised truss model. Leonhardt recommended to use the formula derived from classical truss analogy, by replacing V with V-V_c.





$$\frac{A_{sw}}{s} = \frac{V - V_c}{(z)\sigma_{sw}}$$





For ultimate strength design;

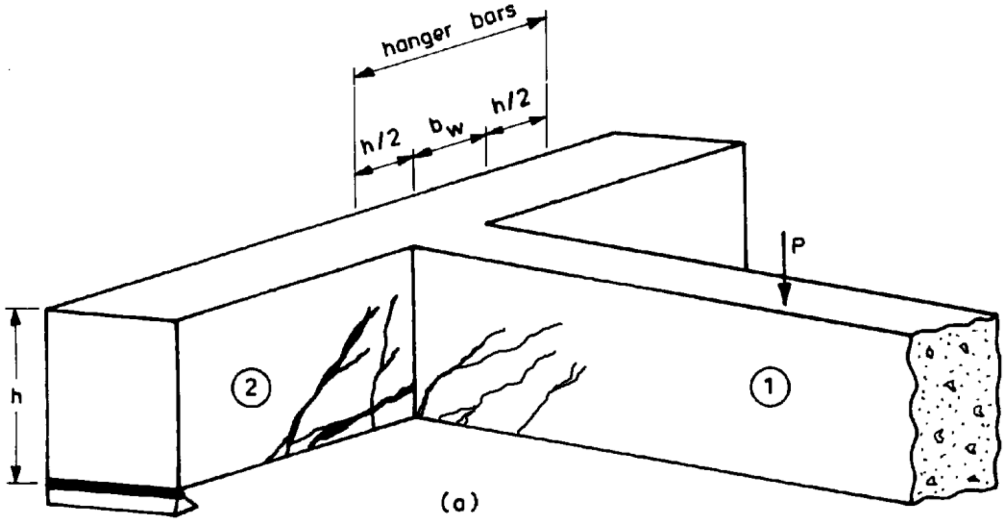
$$\frac{A_{sw}}{s} = \frac{V_d - V_c}{f_{ywd}(z)} = \frac{V_W}{f_{ywd}(z)}$$

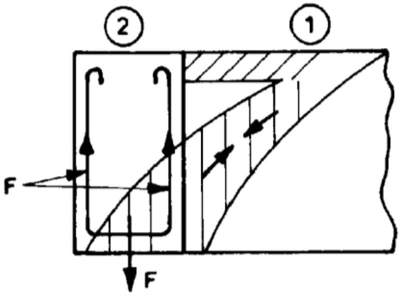
TS-500 : z = d and $V_c = 0.8V_{cr} = 0.5f_{ctd}b_w d$

BENT BARS

In Turkey and in many of the European countries usually beams are detailed to have some bent bars. Until recent times, these bent bars are considered as web reinforcement. However, they lead to very large diagonal cracks and, more importantly, in case of earthquake when shear force change direction they become ineffective in resisting the applied load. Therefore do not rely on bent bars as effective shear reinforcement.

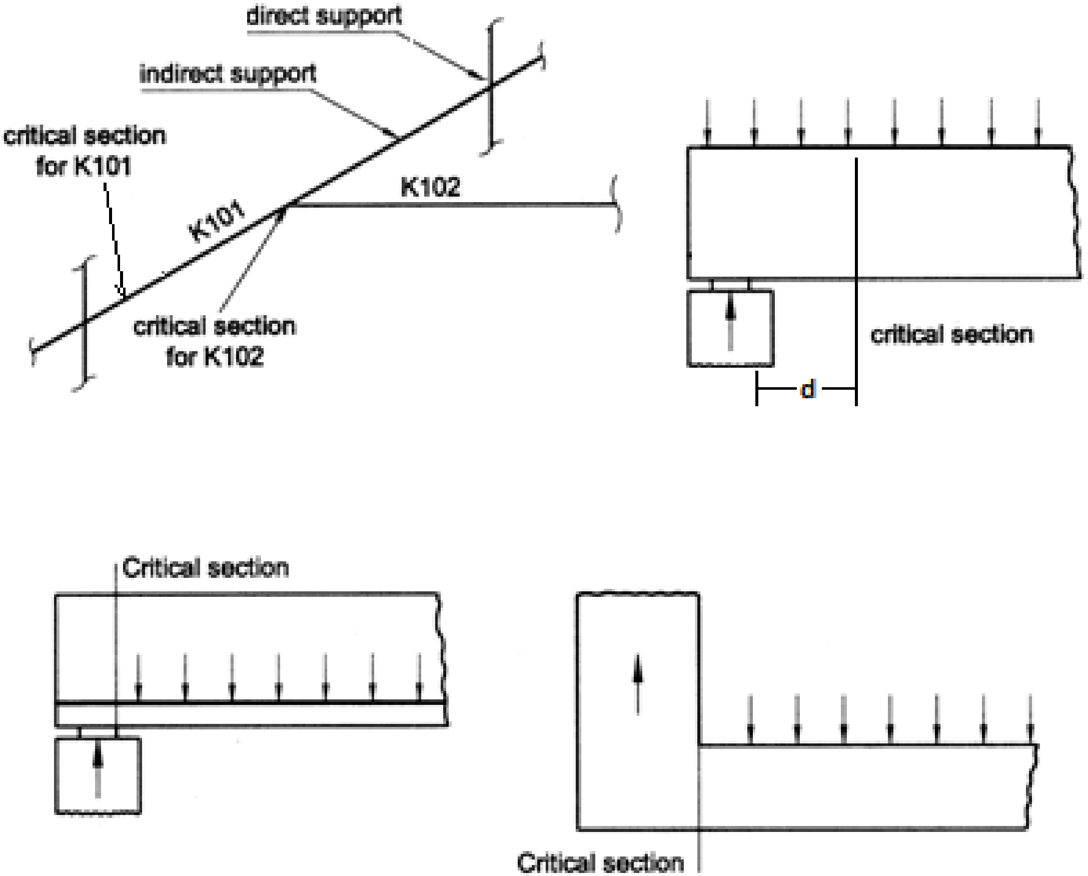
INDIRECT SUPPORTS





CRITICAL SECTION FOR DESIGN SHEAR

Due to local compressive forces from the support, diagonal tension failure cannot occur very near to the face of the support. For this reason, in TS500-2000 the critical section for the design shear is defined at a distance "d" from the support face. "d", is the effective depth of the beam. However if the support is not a direct support (column, wall etc.), then the critical section should be taken at the face of the support.



DESIGN RECOMMENDATIONS

$$V_r \geq V_d$$

$$V_d \leq V_{max}$$

$$V_r = V_w + V_c$$

In the design, the aim is to calculate the shear reinforcement necessary which will make the member reach the ultimate limit state by flexure. Once the dimension of the cross-section and material strengths are known and V_d has been calculated from analysis.

The design shear V_d needs not to be calculated at the face of the support but generally at a distance "d" from the face. However, in case of indirect supports, the design shear should be calculated at the face of the support.

In determining the amount of reinfocement, V_r is replaced by V_d and the equation can be written in the following format.

DESIGN RECOMMENDATIONS

$$\frac{A_{sw}}{s} = \frac{V_d - V_c}{f_{ywd} \times d}$$

$$V_c = 0.52 f_{ctd} b_w d \psi$$

According to the Turkish Seismic Code, if certain conditions exist, V_c at the end regions of beams and columns should be taken as zero (V_c=0).

Preventing Brittle Failure

$$\frac{A_{sw}}{s} = \frac{V_d - V_c}{f_{ywd}(z)} \qquad => V_r = V_d = \frac{A_{sw}}{s} f_{ywd}(z) + V_c$$

$V_c = 0.4 f_{ctk} b_w d$ *(which is appr'x =75% - 80% of* V_{cr}*)*

$z = d$

instead of f_{ywd} *use* f_{ywk}

$$V_r = \frac{A_{sw}}{s} f_{ywk}\, d + 0.40\, f_{ctk}\, b_w\, d$$

$$V_r = V_{cr} = 0.65\, f_{ctk}\, b_w\, d$$

$$\min\ \rho_w = \frac{A_{sw}}{b_w\, s} = 0.25 \frac{f_{ctk}}{f_{ywk}} => 0.3 \frac{f_{ctd}}{f_{ywd}}$$

In cases where shear is very low, $V_d \leq V_{cr}$ the minimum reinforcement should be provided along entire span.

Preventing Brittle Failure

Stirrups or ties are not effective unless crossed by a potential diagonal crack. Therefore even when $\rho_w > \min \rho_w$, if the spacing of ties is greater than d/2, it is possible to have brittle failure since a diagonal crack can develop in between the two ties.

In TS500-2000 the following requirements are imposed on tie spacing.

$$s \leq d/2$$

if $V_d \geq V_{max}$ then $s \leq d/4$

In seismic resistant frames, both ends of the beams should be confined by closely spaced ties over a length 2h. The tie spacing in this region should not exceed d/4.

Preventing Brittle Failure

It can be concluded the strength of the beam can be increased by increasing the web reinforcement. This of course is true but there is a limit. More reinforcement will lead to high high shear forces. However, as the shear forces get higher, principal compressive stresses in the web can reach the crushing strength of concrete (strength under combined stresses) and can cause brittle failure due to web crushing.

This type of a failure can be prevented by limiting the shear force.

$$V_{\max} = 0.22\ f_{cd}\, b_w\, d$$

Preliminary Design

In Chapter 5, it was recommended to establish the size of beam cross-sections using the relationship, $b_w d^2 = M_d K_\ell$. However it was pointed out that in establishing the size of beam section, shear should also be taken into account.

Shear strength of a beam is the sum of the resistances provided by the shear reinforcement and concrete.

$$V_r = V_w + V_c = \rho_w f_{ywd} b_w d + 0.52 f_{ctd} b_w d$$

If we provide $\rho_w = 2 \times \min \rho_w$

$$\rho_w = 2 \times 0.3 \times \left(f_{ctd} / f_{ywd} \right) = 0.6 \left(f_{ctd} / f_{ywd} \right)$$

Preliminary Design

$$V_d = 0.6\left({f_{ctd}}/{f_{ywd}} \right) \times f_{ywd} \times b_w d + 0.52 f_{ctd} \times b_w d$$

Solving for $b_w d$

$$b_w d = \frac{0.9V_d}{f_{ctd}}$$

To summarize, the cross-sectional dimensions of the beam can be estimated considering the flexure and shear.

$$b_w d^2 = K_\ell M_d \quad \text{and} \quad b_w d = 0.9 V_d / f_{ctd}$$

whichever governs.