Ce 382

HW #1

Part A

$f_c = 50 \Rightarrow \varepsilon_{co} = \frac{2 \times 50}{E_c} \Rightarrow \frac{100}{35680} = 2,80 \times 10^{-3}$

(i)

?

$E_c = 12680 + 460(50) = 35680$ MPa

Firstly, we apply a 40 MPa compressive force.
we will find it from Hognestad equation;

$$40 = 50 \left[\frac{2 \varepsilon_c}{2,8 \times 10^{-3}} - \left(\frac{\varepsilon_c}{2,8 \times 10^{-3}} \right)^2 \right]$$

$\boxed{\varepsilon_{c_1} = 1,55 \times 10^{-3}}$ or $\varepsilon_{c_2} = \cancel{4,05 \times 10^{-3}}$

reject this, bigger than ε_{co}

In question, the stress is reduced to 15 MPa from 40 MPa. I should determine the change in ε by Hooke's law,

$\sigma = E \cdot \varepsilon \Rightarrow (40-15) = 35680 \cdot \varepsilon$

$\Rightarrow \varepsilon = 7 \times 10^{-4}$

so strain @ this point (i.e $\sigma = 15$ MPa) = 1,5

$1,55 \times 10^{-3} - 7 \times 10^{-4} = \boxed{0,85 \times 10^{-3}}$

(ii) This time, we are wanted to calculate ε @ 45 MPa

$$45 = 50 \left[\frac{2 \times \varepsilon_c}{2,80 \times 10^{-3}} - \left(\frac{\varepsilon_c}{2,8 \times 10^{-3}} \right)^2 \right] \Rightarrow \varepsilon_{c_1} = \cancel{3,68 \times 10^{-3}}$$

$\varepsilon_{c_2} = 1,91 \times 10^{-3}$

$\varepsilon_{c_2} = 1,91 \times 10^{-3}$ @ $\sigma = 45$ MPa

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(iii) To find the residual compressive strain, I should also determine the strain value from Hooke's law

$\sigma = E \cdot \epsilon$

$45 = 35680 \times \epsilon \Rightarrow 1{,}26 \times 10^{-3}$

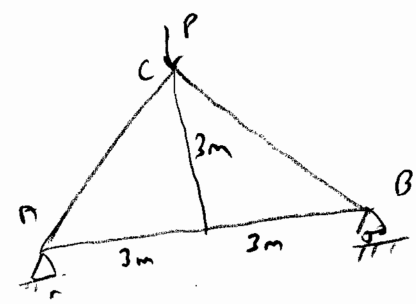
residual compressive strain =

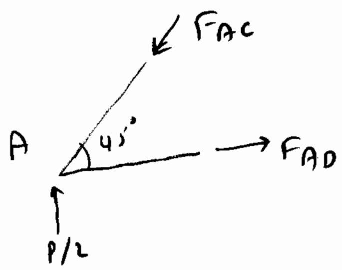
$1{,}91 \times 10^{-3} - 1{,}26 \times 10^{-3} = \underline{0{,}65 \times 10^{-3}}$

(2) 1,2-9

$f_c = 20$ MPa.
Find max P carried by the structure below

a) $A_C = 150 \times 150 = 22500$ mm^2





$F_{AC} \sin 45 = P/2 \Rightarrow F_{AC} = \frac{\sqrt{2}}{2} P$ (C)

$F_{AC} \cos 45 = F_{AD} \Rightarrow F_{AD} = \frac{P}{2}$ (T)

$F_{AD} = F_{BD} = \frac{P}{2}$ (due to symmetry)

$F_{CD} = 0$

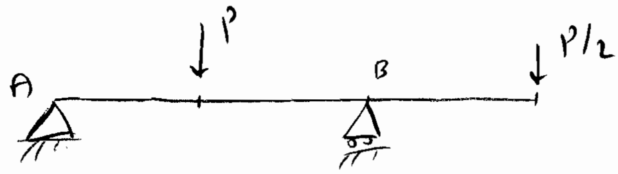
Direct tension $\rightarrow f_{ct} = 0{,}35\sqrt{f_c}$

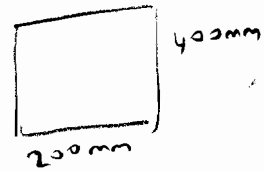
$F_{AD} = \frac{P}{2} = f_{ct} \cdot A_c$

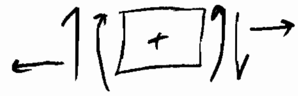
$\frac{P}{2} = 0{,}35\sqrt{20} \times 22500 \Rightarrow \underline{P = 70{,}4 \text{ kN}}$

(2)

(b)



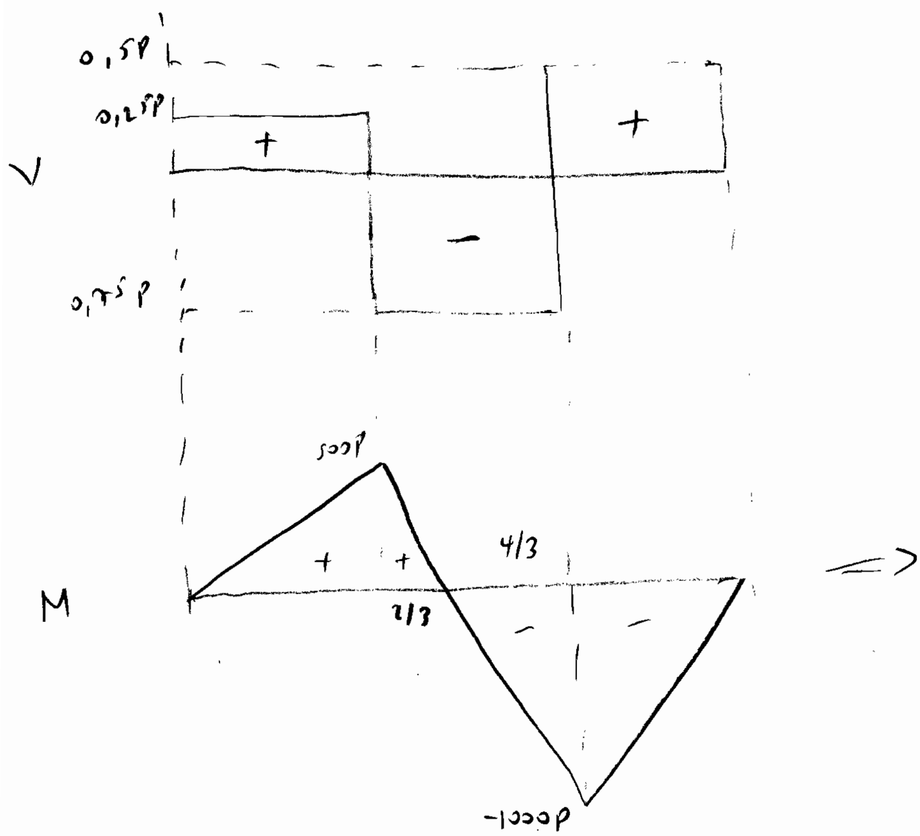




$\Sigma M_A = 0$

$4R_B - 2P - 3P = 0$, $R_b = \underline{\underline{1{,}25 P}} \uparrow$

$\uparrow \Sigma F_y = 0 \qquad R_A + R_B - P - \frac{P}{2} = 0 \implies \underline{\underline{R_A = 0{,}25 P}} \uparrow$

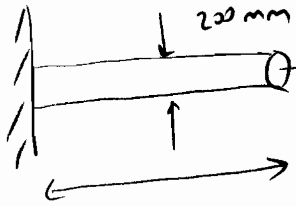


$\Rightarrow$

$$\underbrace{\sigma_{ctf}}_{\text{due to flexure}} = \frac{My}{I} = \frac{1000P \times 400/2}{\frac{1}{12}\, 200 \cdot 400^3} = f_{ctf} = 0{,}7\sqrt{f_c}$$

$\implies \underline{\underline{P = 16{,}7 \text{ kN}}}$

(c)



$f_{c+s} = 0{,}5\sqrt{f_c} = \sigma_{c+s}$

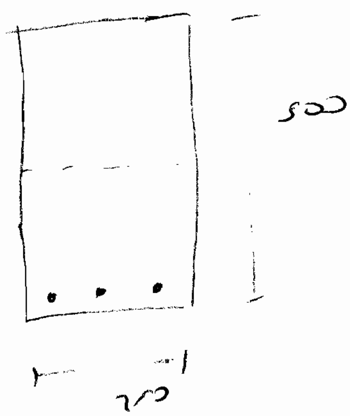
due to torsion $\longrightarrow$ $\sigma_{c+s} = M.r / J$

or shear stress $\longrightarrow$ $\tau_{c+s} = VQ/It$

$$\sigma_{c+s} = \frac{M.r}{J} \quad r \to d/2, \quad J \to \frac{\pi d^4}{32} \quad = \frac{10^3 P\,(N.mm) \times 200/2}{\frac{\pi (200)^4}{32}}$$

$\underline{\underline{P = 3{,}5 \text{ kN}}}$

1.4 shear capacity $= \tau = \dfrac{VQ}{It}$



$Q = A \times d$

$= 250 \times 250 \times 250/2$

$= 7812500 \text{ mm}^3$

$I = \frac{1}{12} b h^3 = \frac{1}{12}(250)(500)^3$

$= 2604166667 \text{ mm}^4$

$t = 250 \text{ mm}$

$V_{max} = \dfrac{P}{2}$

$$\tau = \frac{VQ}{It} = \frac{P/2 * 7812500}{2604166667 * 250} = P \times 6 \times 10^{-6}$$

$f_{c+t} = 0{,}7\sqrt{f_c} = 3{,}14 \text{ MPa}$

$3{,}14 \text{ MPa} = 6 \times 10^{-6} \times P \Rightarrow \boxed{P_{max} = 521{,}7 \text{ kN}}$

this is arrived when member has a 45° about x-axis / tension crack angle

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1.6

$P = 1750$ kN
$\sigma_c = 8$ MPa
$\sigma_{steel} = 120$ MPa
$\phi_{CE} = 2.0$
$E_{c_{28}} = 20{,}000$ MPa

given
compute σ_c and σ_{steel} at the end of 2 years.

$$\varepsilon_{ce} = \frac{\sigma_{co}\,\phi_{cE}}{E_c} = \frac{8 \times 2}{20000} = 8 \times 10^{-4}$$

(1) Equilibrium

$$F_s - F_c = 0$$
$$\sigma_s A_s - \sigma_c A_c = 0$$
$$\varepsilon_s E_s A_s - \varepsilon_c E_c A_c = 0$$
$$\varepsilon_s (200000)\left(8 \times \left(\frac{26^2 \times \pi}{4}\right)\right) - \varepsilon_c \left(20000 \times (400 \times 400)\right) = 0$$
$$\varepsilon_s (8{,}495 \times 10^8) - \varepsilon_c (32 \times 10^8) = 0$$
$$\Rightarrow \boxed{\varepsilon_s = 3{,}76\ \varepsilon_c} \ \ldots\ (1)$$

(2) compatibility

$$\varepsilon_{steel} + \varepsilon_{concrete} = \varepsilon_{creep}$$
$$\boxed{\varepsilon_{steel} + \varepsilon_{concret} = 8 \times 10^{-4}} \ \text{--}\ (2)$$

by (1) and (2) $\quad \varepsilon_s = 6{,}32 \times 10^{-4} \quad \varepsilon_c = 1{,}68 \times 10^{-4}$

N $\sigma_s = E_s \varepsilon_s = 200000 \times 6{,}32 \times 10^{-4} = 126{,}4$ MPa (compression)
N $\sigma_c = E_c \varepsilon_c = 20000 \times 1{,}68 \times 10^{-4} = 3{,}36$ MPa (tension)

New Loads after creep

* $\sigma_c = 8$ MPa (compression) $+ 3{,}36$ MPa (tension) $=$ $4{,}64$ MPa (comp)
* $\sigma_s = 120$ MPa (comp) $+ 126{,}4$ MPa (comp) $=$ $246{,}4$ MPa (comp)

1.7 $\quad l_e = \frac{2 \cdot A_c}{u} = \frac{2(250 \times 700)}{2(250+700)} = 184 \text{ mm}$

$\varepsilon_{cs} = 0{,}00059$ (interpolation from table 1.3, p. 48)
assuming a dry climate

(1) Equilibrium

$$-F_c + F_s = 0$$
$$-\sigma_c A_c + \sigma_s A_s = 0$$
$$-\varepsilon_c E_c A_c + \varepsilon_s E_s A_s = 0 \qquad \left(E_c = 28500 \text{ from book}\right)$$
$$-\varepsilon_c \times 28500 \, (250 \times 700) + \varepsilon_s (200000) \, 10\left(\frac{\pi \times 16^2}{4}\right) = 0$$

gross area

$$-4{,}98 \times 10^9 \, \varepsilon_c + 0{,}402 \times 10^9 \, \varepsilon_s = 0$$
$$\boxed{\varepsilon_s = 12{,}38 \, \varepsilon_c} \quad --- (1)$$

(2) Compatibility

$$\boxed{\varepsilon_c + \varepsilon_s = 0{,}00059} \quad --- (2)$$

by (1) and (2) $\quad * \ \varepsilon_c = 4{,}41 \times 10^{-5}$
$\quad * \ \varepsilon_s = 5{,}45 \times 10^{-4}$

$\sigma_s = E_s \varepsilon_s = 200000 \times 5{,}45 \times 10^{-4} = \underline{109 \text{ MPa}}$ (comp)

$\sigma_c = E_c \varepsilon_c = 28500 \times 4{,}41 \times 10^{-5} = \underline{1{,}25 \text{ MPa}}$ tension

Clearly, then, the city is not a CONCRETE jungle;
it is a HUMAN zoo...
William Hamilton