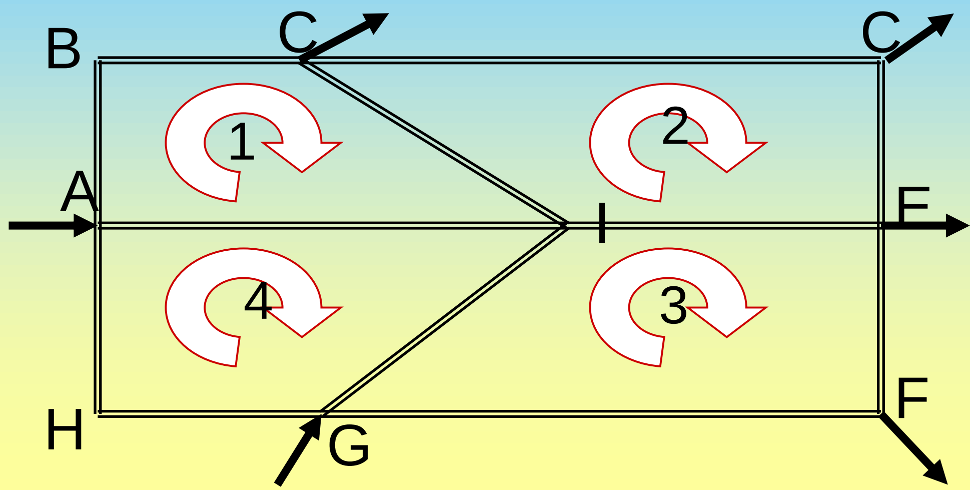
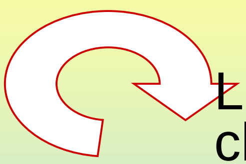
NETWORK OF PIPES



In general we know the discharges coming to a loop. However we don't know the discharge in each pipe. Therefore, it is necessary to compute them.

We have two equations:



LOOP "+" clockwise

A,...,I:JUNCTIONS/ NODES

EF:LINK/BRANCH

- **Conservation of Mass**: Flow into each junction must be equal flow out of the junction
- **Energy Conservation**: Algebraic sum of head losses around each and every loop must be zero.

Darcy-Weisbach Equation for head loss

$$h_L = f\frac{L}{D^5}\frac{8Q^2}{\pi^2 g} + K_m\frac{}{D^4}\frac{8Q^2}{\pi^2 g} = K_f Q^2 + K_m Q^2 = K_L Q^2$$

$$h_l = K_L Q|Q|$$

Where

$$K_f = 8f\frac{L}{D^5 g\pi^2}$$

K_m = Local Loss coefficient,

which can be wrítten in terms of equivalent length, and

K_L = combined loss coefícient

Conservation of Energy around any loop

$$\sum_{1}^{N} h_{l\,loop} = \sum_{1}^{N} K_i Q_i^{\,n} = \sum_{1}^{N} K_i (Q_o + \Delta Q)^n$$

$$= \sum_{1}^{N} KQ_o^n + \sum_{1}^{N} nK\Delta Q Q_o^{n\ \ 1} + \ldots\ldots$$

$$where\ \ \text{N} = \text{number of links}$$

$$h_{loop} = \sum_{1}^{N} KQ_o^n + \sum_{1}^{N} nK\Delta Q Q_o^{n\ \ 1} + \ldots\ldots + \ldots = o$$

$$If \quad n = 2\ and$$

$$terms\ \text{containing powers of } \Delta\text{Q neglected}:$$

$$\Delta Q = -\frac{\sum_{1}^{N} K_i Q_{oi} \left| Q_{oi} \right|}{\sum_{1}^{N} 2K_i \left| Q_{oi} \right|}$$

Solution Procedure

1)Assume the best distribution of flow that satisfies the continuity equation at each junction.

Let Q_0 be the assumed discharge in a pipe, and Q be the actual discharge.

2) Then $Q=Q_0+\Delta Q$ for each pipe, where ΔQ is the error in estimation. Therefore, it is necessary to calculate the error, ΔQ, for each loop.

3) Calculaion of ΔQ:

The head loss can be written as: hf=KQn. For Darcy-Weisbach equation n=2, and

$$K = 8f\frac{L}{D^5 g\pi^2}$$

4) Around any loop, algebraic summation od head loss must be zero:

$$\sum h_f = \sum_1^N K_i Q_i^2 = \sum_1^N K_i \left(Q_{oi}^2 + \Delta Q\right)^2 = \sum_1^N K_i \left(Q_{oi}^2 + 2\Delta Q \bullet Q_{0i} + \Delta Q^2\right)$$

Assuming that ΔQ is small, ΔQ^2 can be neglected, and

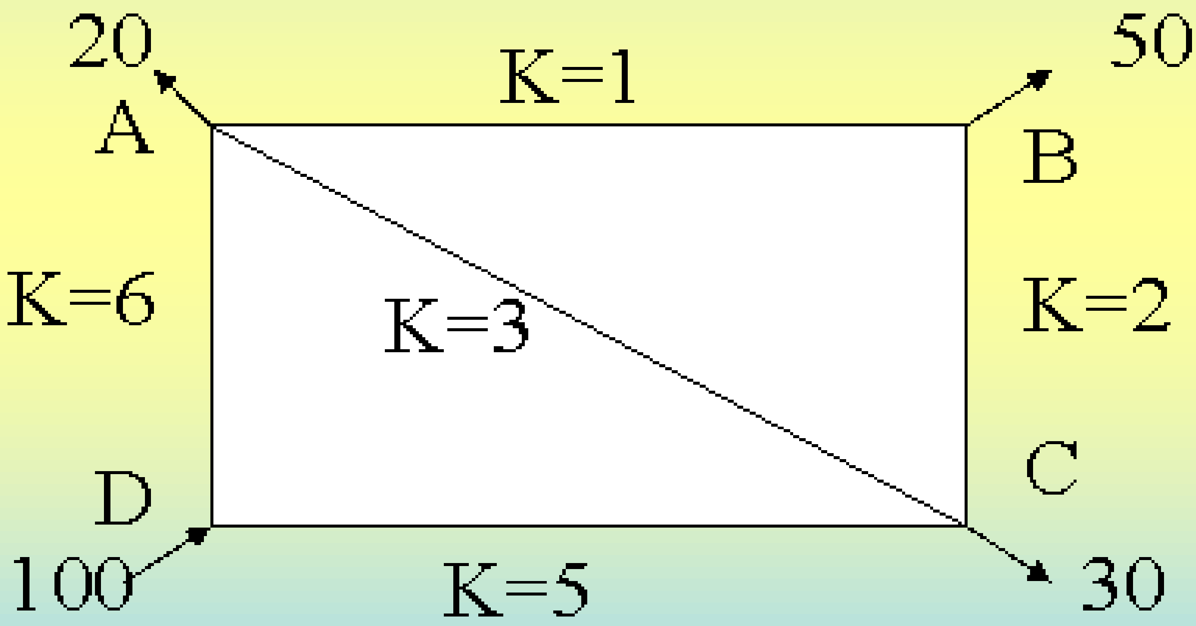
the above equation can be solved for ΔQ as :

$$\Delta Q = -\frac{\sum_1^N K_i Q_{0i}\left|Q_{0i}\right|}{\sum_1^N 2K_i\left|Q_{0i}\right|}$$

- ΔQ for each and every loop must be smaller than a tolerable magnitude. Otherwise, it is necessary to iterate the solution until the error term ΔQ becomes acceptably small. The details can be explained best by an example:

Example 2.8

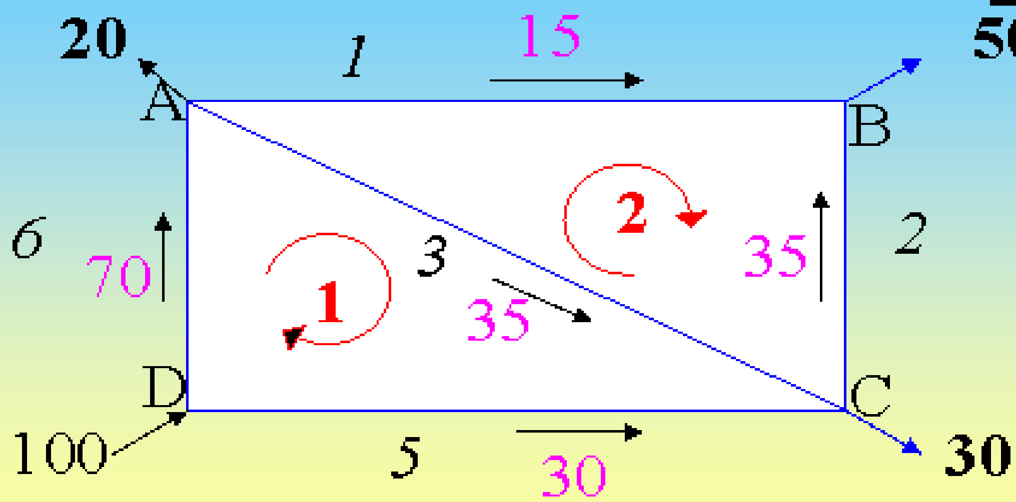
Given is the network shown in figure below. Find the discharges in each and every link.



Q's in m^3/sec

K's in "s^2/m^5"

Initial Guess



First Iteration

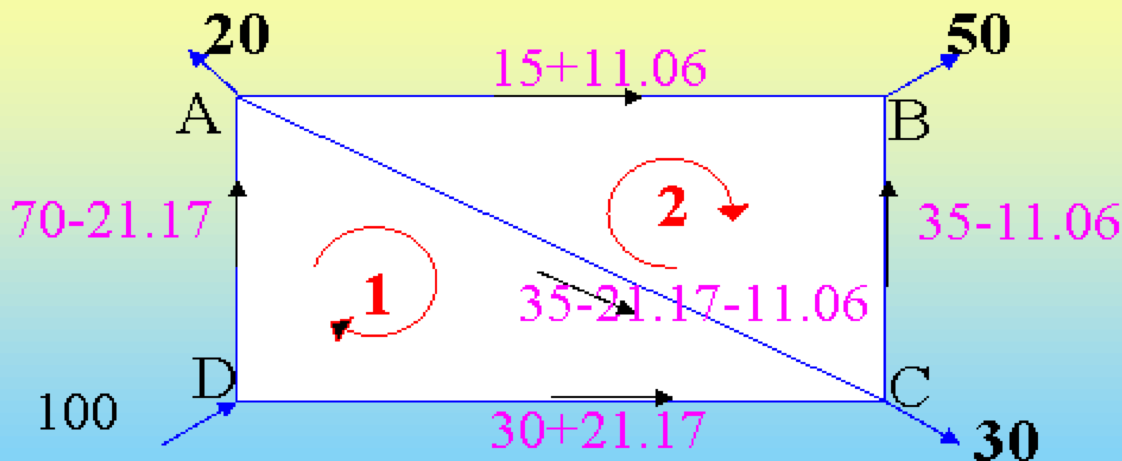
LOOP 1

Pipe	K Q |Q|	2 K |Q|
AD	6*70*70=29400	2*6*70=840
AC	3*35*35 =3675	2*3*35=210
DC	*-5*30*30 =-4500	2*5*30=300
	Σ=28575	Σ=1350

$$\Delta Q_1 = -\frac{28575}{1350} = -21.17$$

AC = 35-21.17

Flow Distribution After First Iteratıon

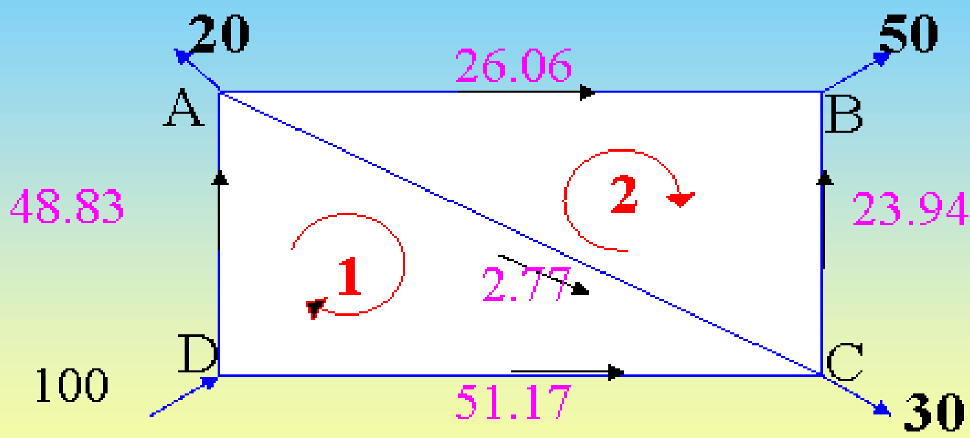


LOOP 2

Pipe	K Q |Q|	2 K |Q|
AB	1*15*15 =225	2*1*15=300
BC	2*-35*35=-2450	2*2*35=140
AC	3*-*13.83**13.83 =-574	2*3*13.83=83
	Σ=–2799	Σ=253

$$\Delta Q_2 = \frac{2799}{253} = 11.06$$

Second iteration



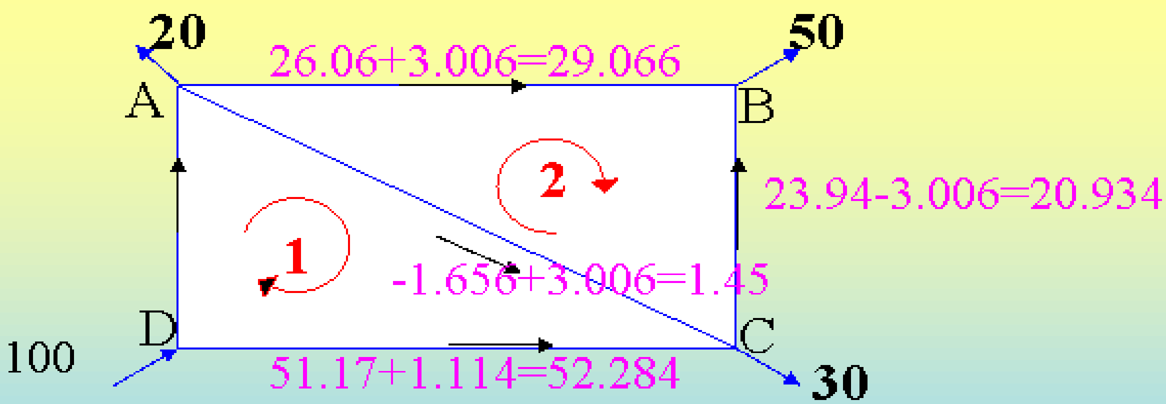
LOOP 1

Pipe	K Q |Q|	2 K |Q|
AD	6*48.83*48.83 =14308	2*6*48.83 =586
AC	3*2.77*2.77 =23	2*3*2.77 =17
CD	5*-51.17*51.17 =-13090	2*5*51.17 =511
	Σ=1241	Σ=1114

$$\Delta Q_1 = -\frac{1241}{1114} = -1.114$$

AC 2.77-1.114

Final Distribution of Second Iteration



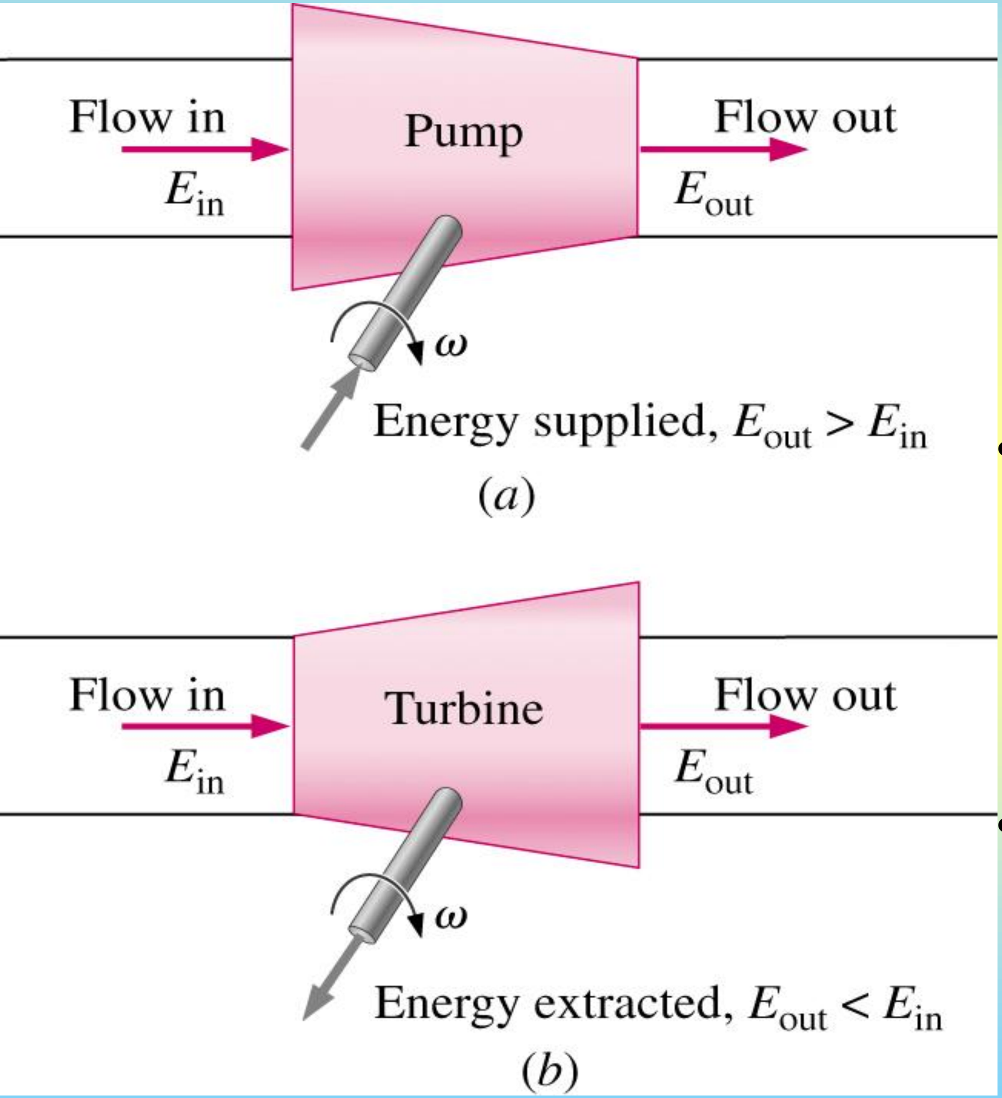
LOOP 2

Pipe	K Q |Q|	2 K |Q|
AB	1*26.06*26.06 = 679	2*1*26.06=52
BC	2*-23.94*23.94= -1146	2*2*23.94=96
AC	3*-*1.656**1.656= -8	2*3*1.656=10
	Σ=−475	Σ=158

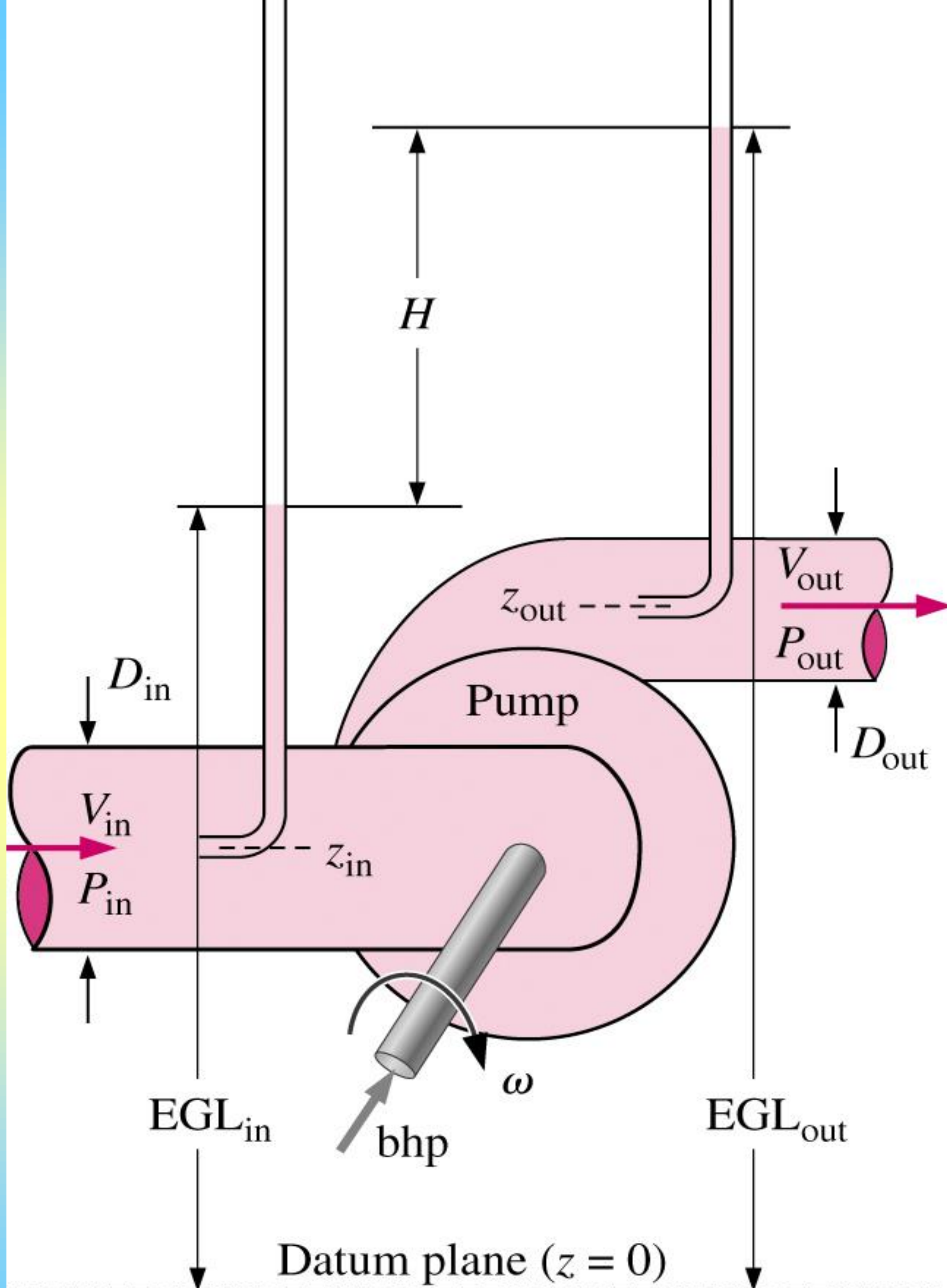
Attention: One more iteration is needed

$$\Delta Q_2 = \frac{475}{158} = 3.006$$

HYDRAULICS AND OPERATION OF PUMPED DISCHARGE LINES



- A Pump is a mechanical device which adds energy to a fluid, as result of dynamic interaction between the device and fluid.
- Energy is supplied to the rotating shaft (by a motor) and transferred to the fluid by the blades.
- Turbine is a mechanical device which extracts energy from the fluid.



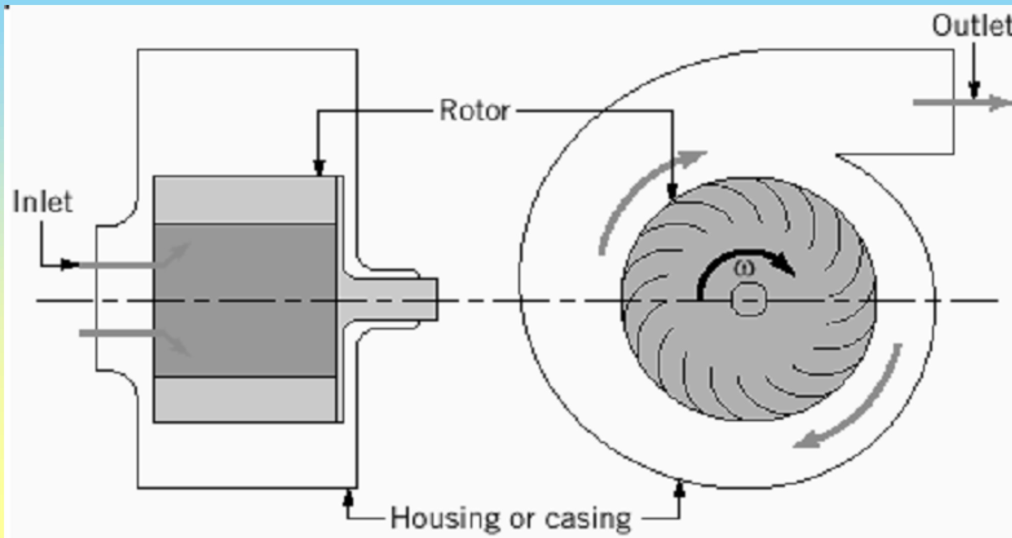
H
V_{out}
z_{out}
P_{out}
D_{in}
Pump
D_{out}
V_{in}
z_{in}
P_{in}
ω
EGL_{in}
bhp
EGL_{out}
Datum plane ($z = 0$)

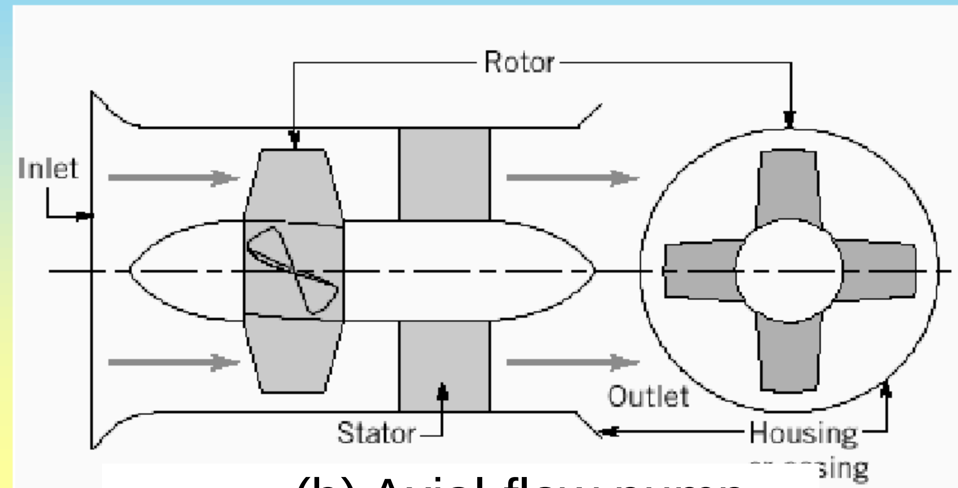
- Many Turbomachines contain some type of housing or casing that surrounds the rotating blades or rotor, thus forming an internal flow passageway through which the fluid flows.
- Turbomachines are classified depending on the predominant direction of the fluid motion relative to the rotor's axis as the flow passes the blades:

1. Axial – flow machine
2. Mixed – flow machine
3. Radial – flow machine

For an axial flow machine the fluid maintains a significant axial flow direction component from the inlet to the outlet of rotor.

Types of Pumps

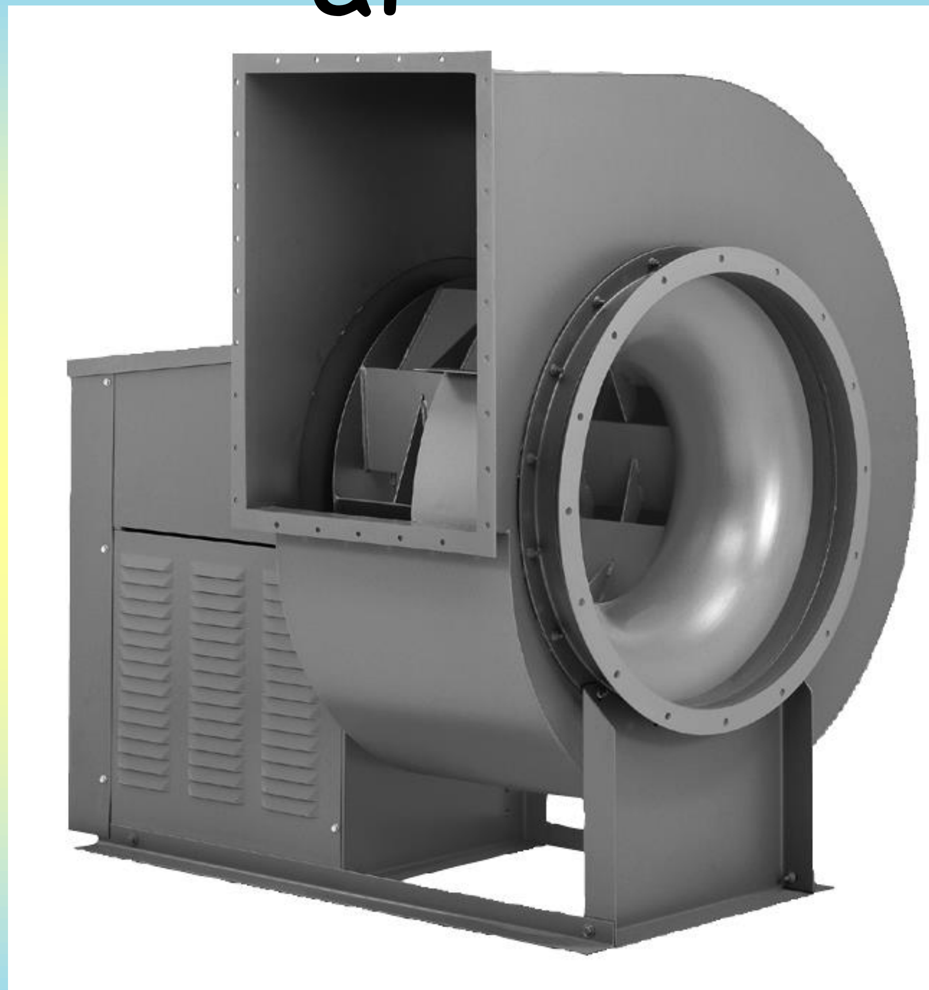




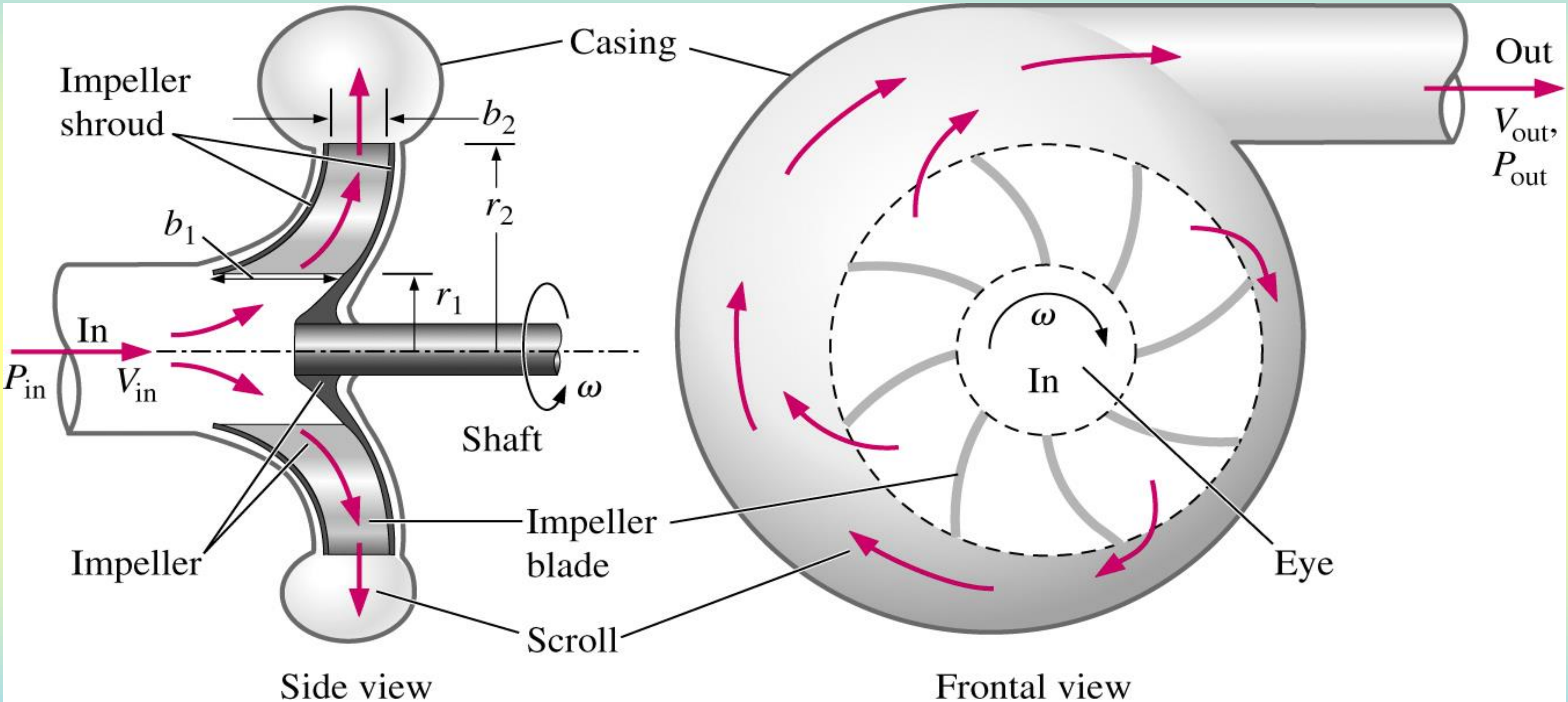
(b) Axial-flow pump

Axial

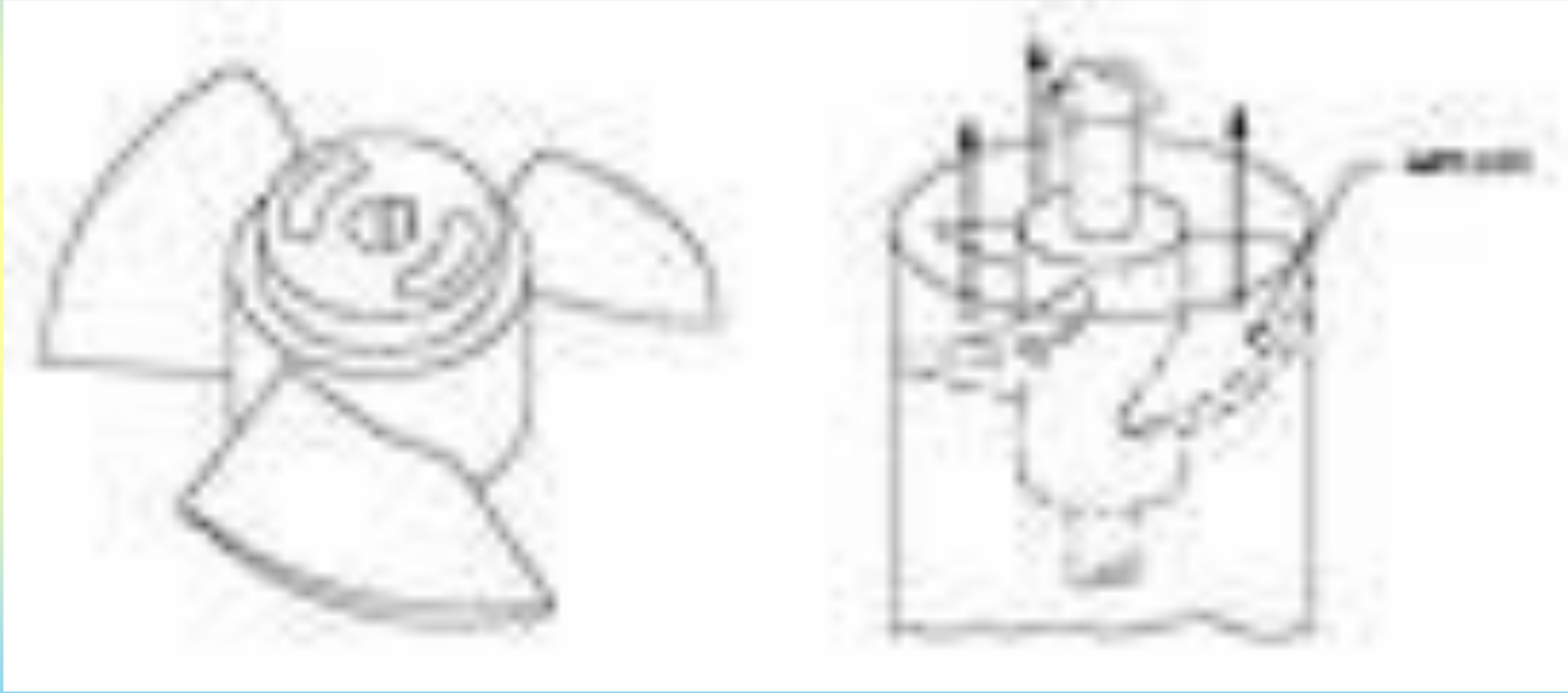
Centrifug
al



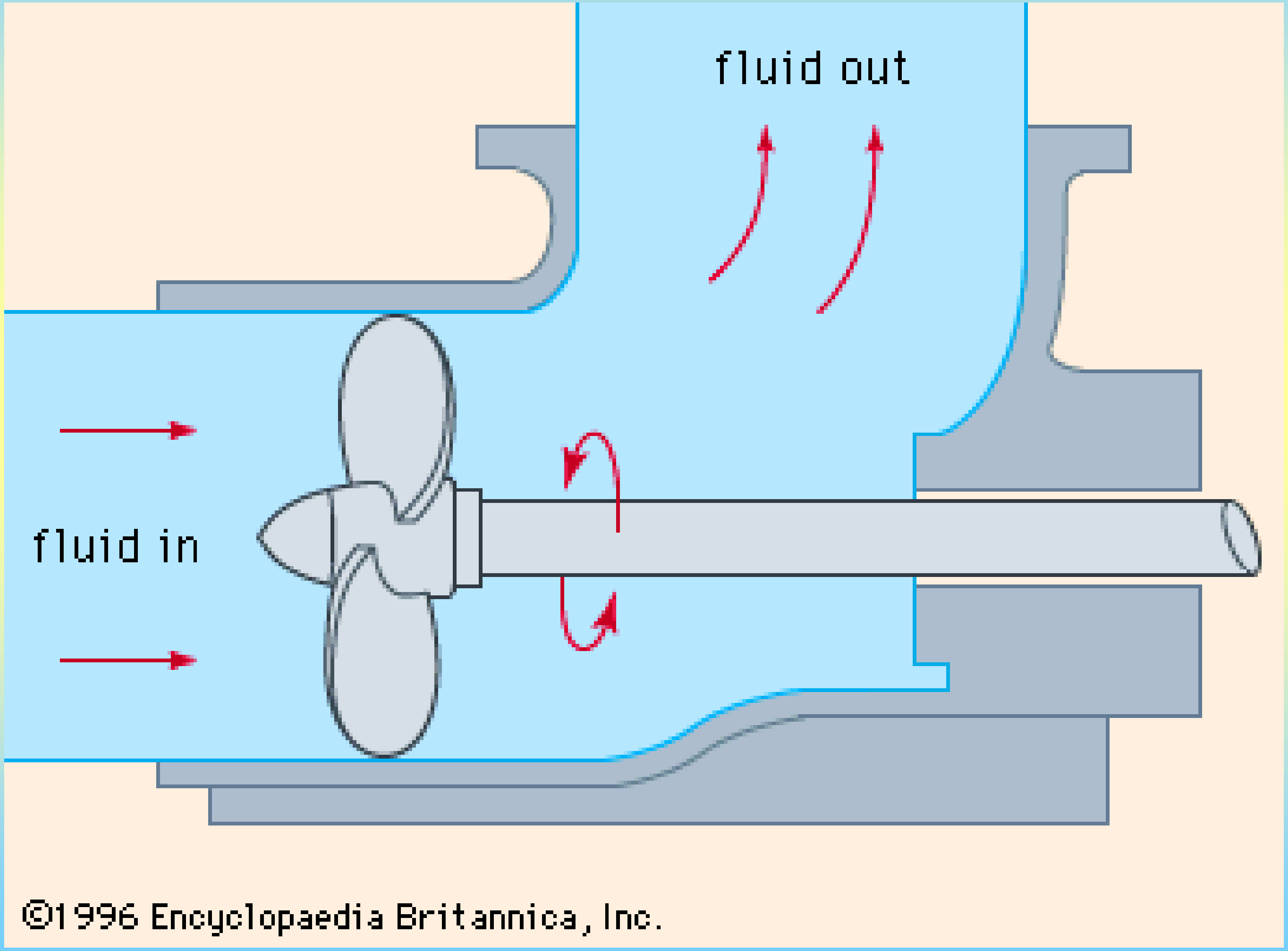
Centrifugal Pumps



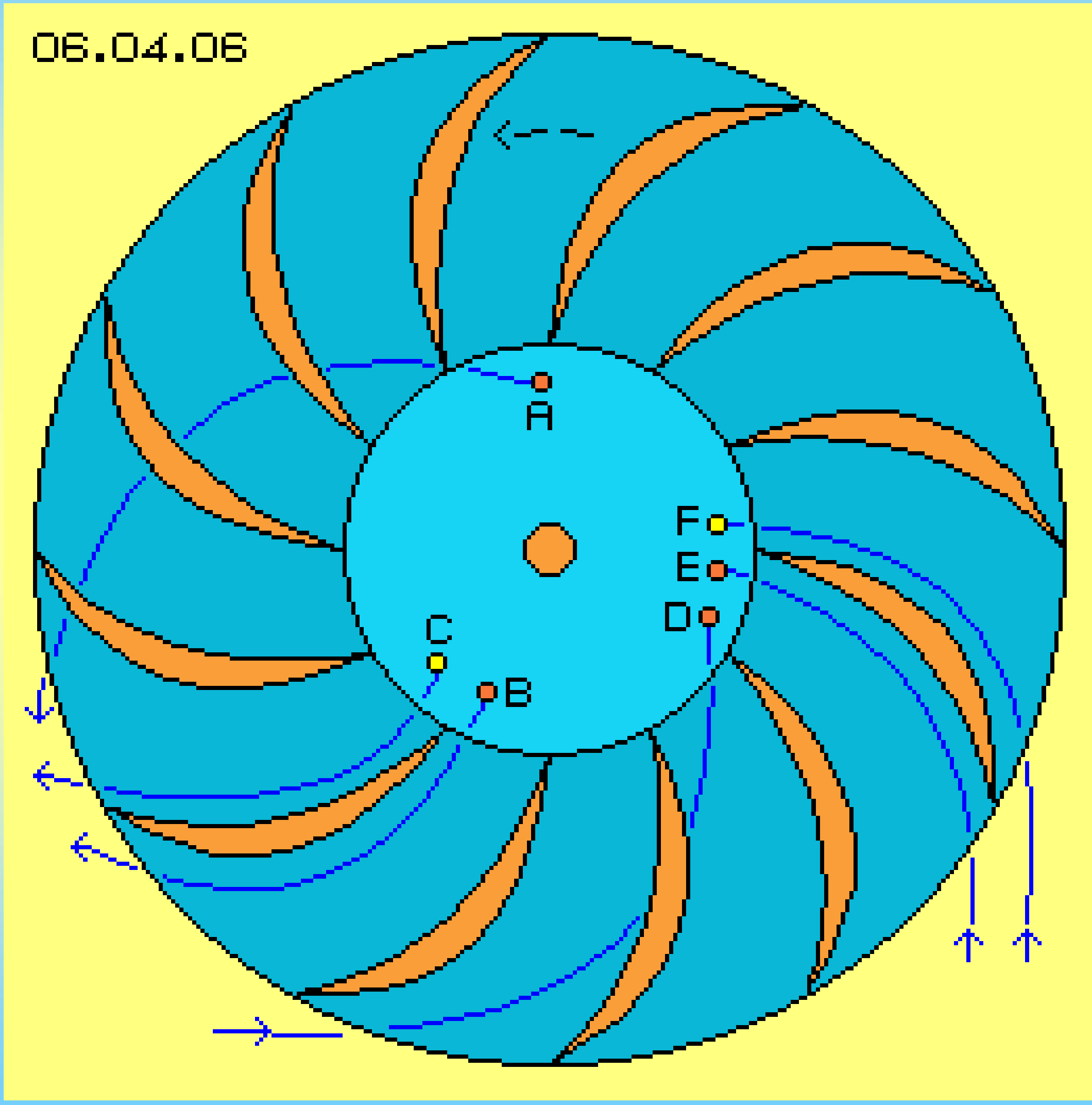
- Axial-flow pump



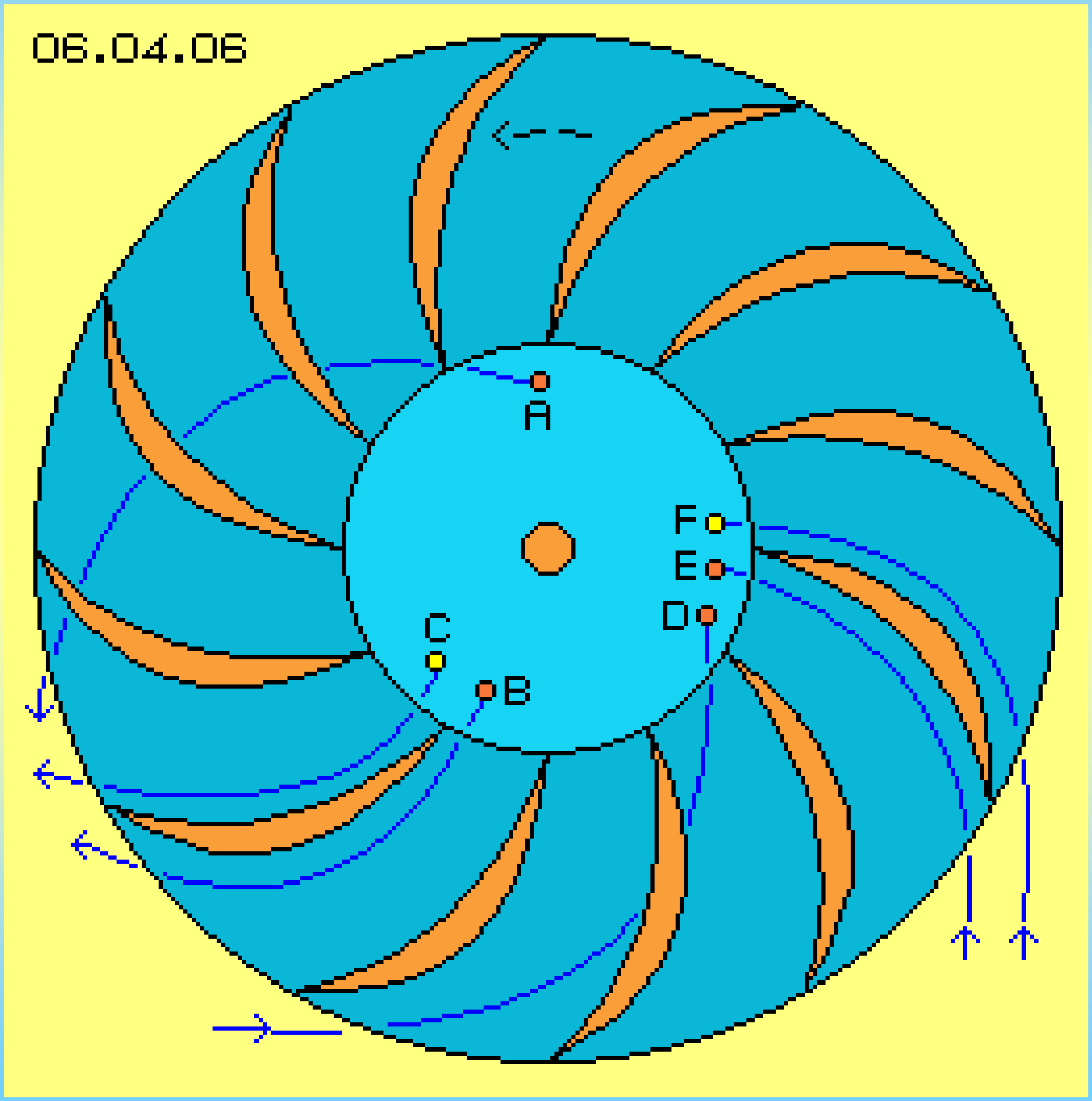
axial flow centrifugal pump



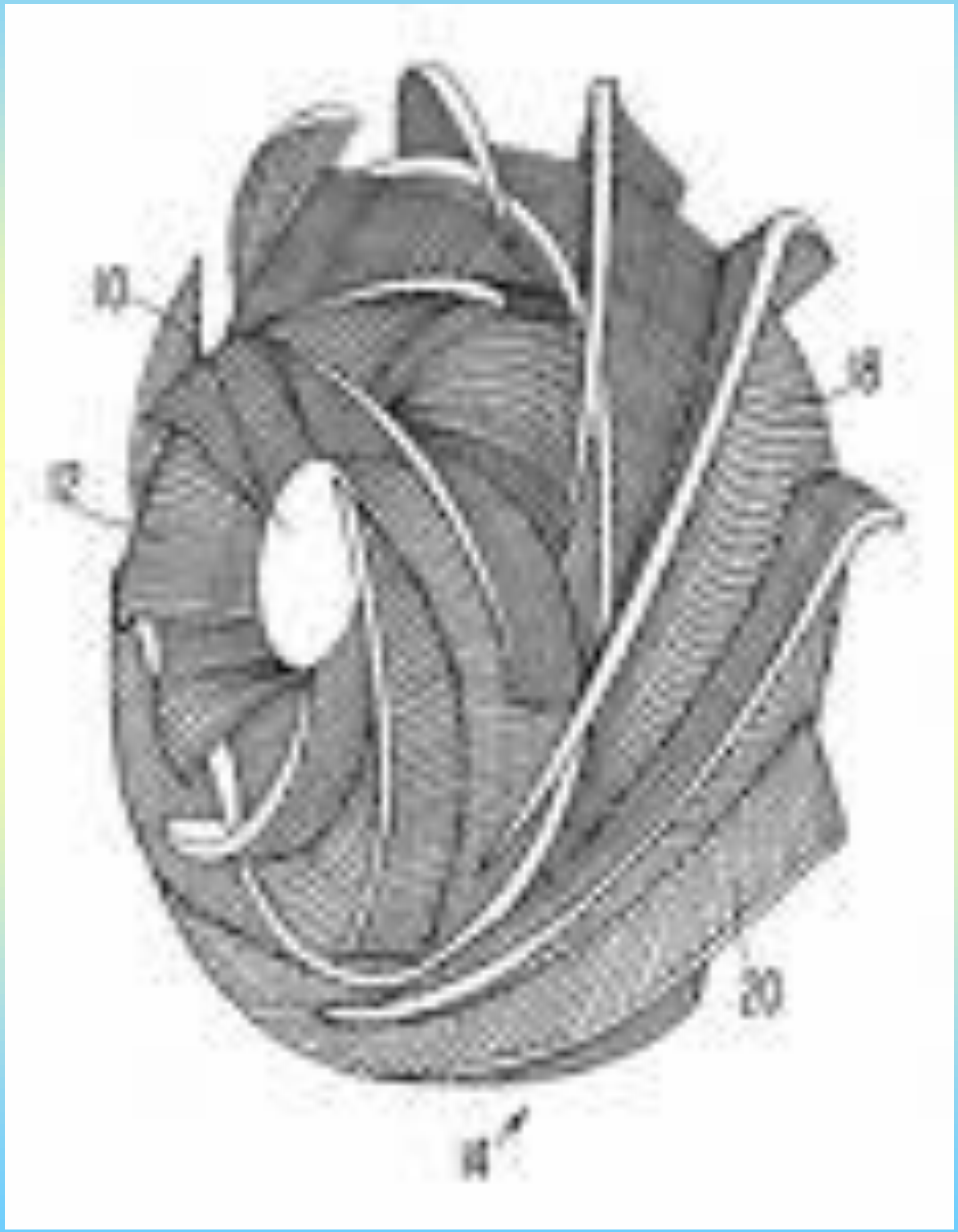
- For a radial-flow machine, the flow across the blades involves a substantial radial flow component at rotor inlet, exit, or both.



view of normal radial pump



Mixed flow pump.



In mixed-flow machines, there may be significant radial- and axial-flow velocity components for the flow through the rotor row

- Each type of machine has advantages and disadvantages for different applications and in terms of fluid mechanical performance.
- The power P_f, gained by the fluid is given by:
- $P_f=\gamma Q H_P$
- Where
- γ = the specific weight of fluid in N/m^3
- Q = discharge in m^3/s
- H_P= Head supplied by the pump in m.
- P_f= Power gained by the fluid in Watts

- This quantity expressed in terms of horsepower is tradionally called the water horsepower.

- 1 hp=745.7 Watts $\rightarrow P_f = \dfrac{\gamma Q H_P}{745.7}$ **P_f is in terms of hp**.

- The power required to drive the pump is the brake horsepower:
- $bhp = wT = P_P$, where
- w= shaft angular velocity
- T= shaft torque
- If there were no losses P_f and brake horsepower would be equal. However there are always losses such as:

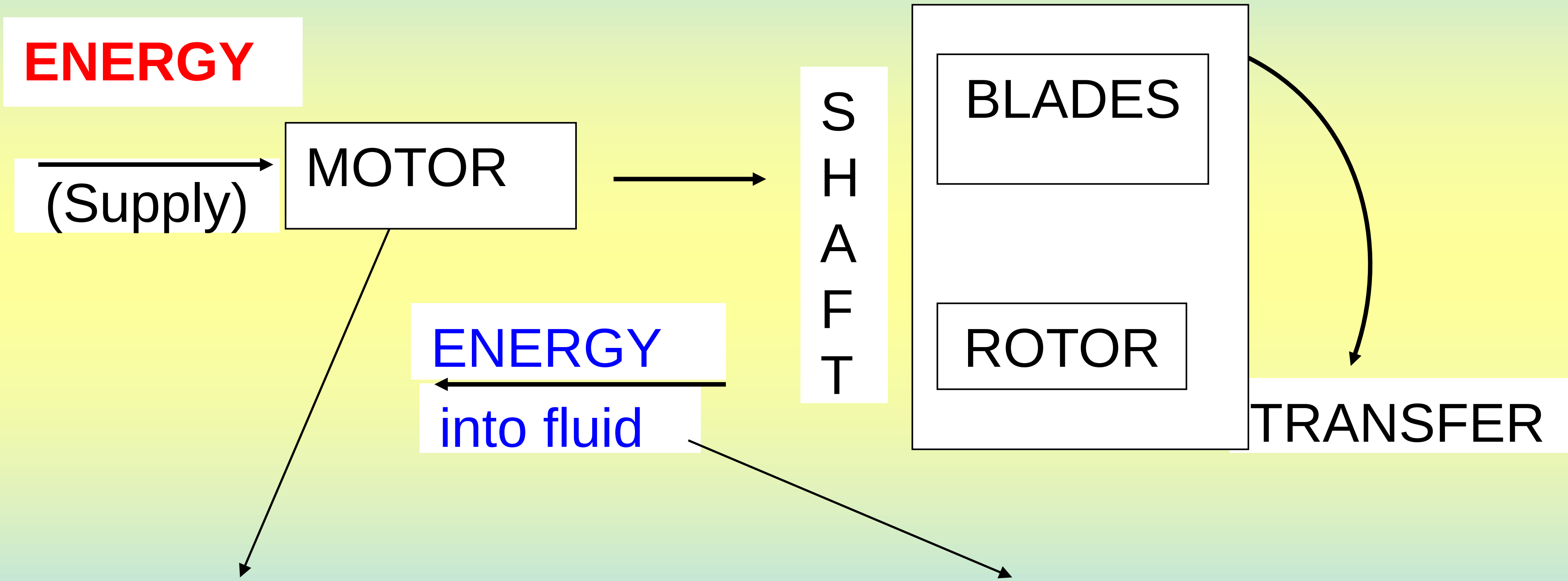
1. Mechanical losses in the bearing and seals
2. There may be power loss due to leakage of fluid between the back surface of the impeller hub plate and the casing, or through other components of the pump
3. There will be frictional ana local losses

Therefore the overall efficiency of the pump reduces. Therefore the water horsepower is actually less than the bhp, and the efficiency of the pump is defined as:

$$\eta = \frac{P_f}{bhp} = \frac{\gamma Q H_P}{wT} \quad \text{and hence} \quad P_P = \frac{\gamma Q H_P}{\eta}$$

P_P= Power of the pump required to supply a power of $\gamma Q H_P$ to the fluid.

The power required to derive the motor is bhp=ωT

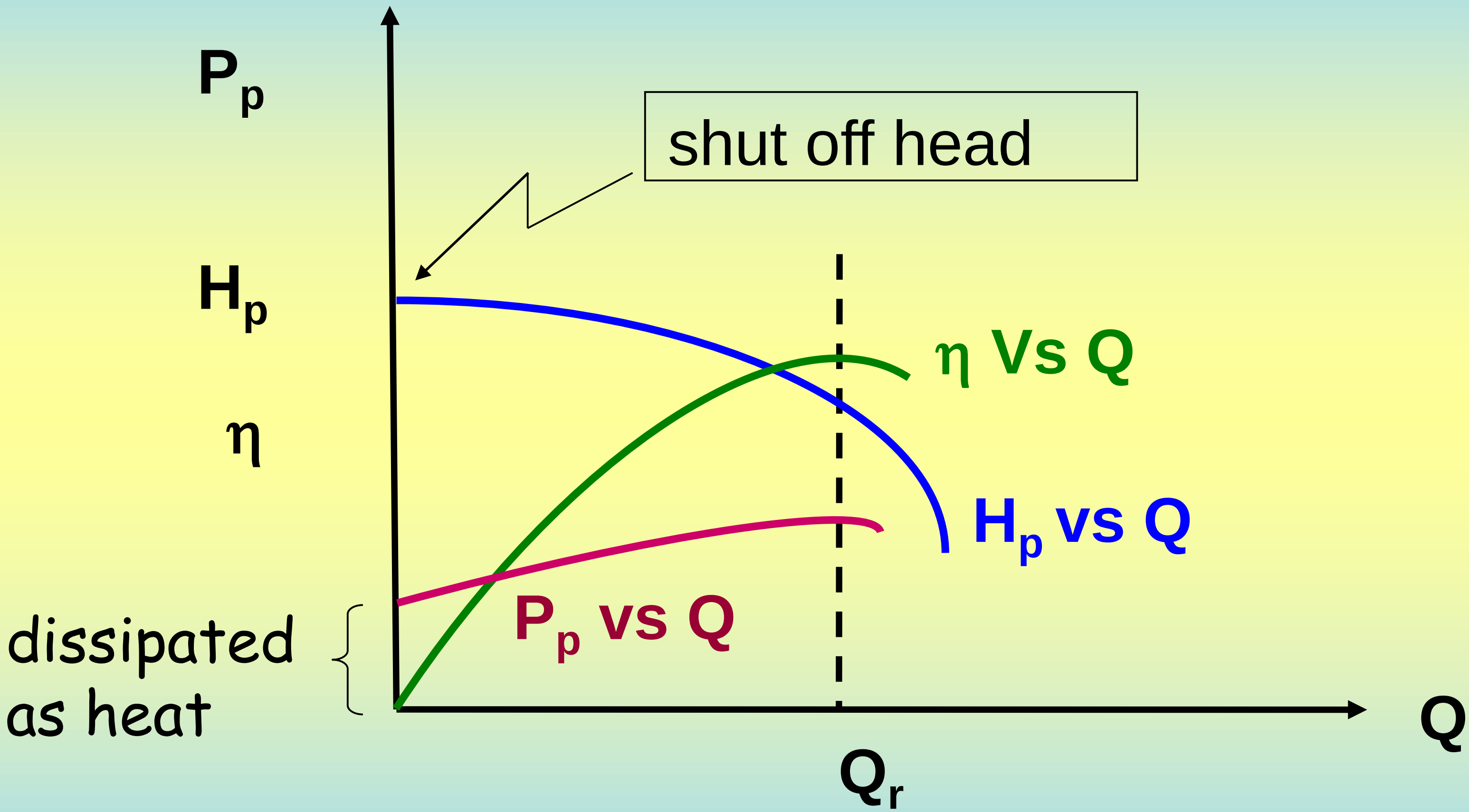


The power required to drive the motor is known as break horsepower (input power P_P

The power gained $P_f = \gamma\ Q\ H_p$

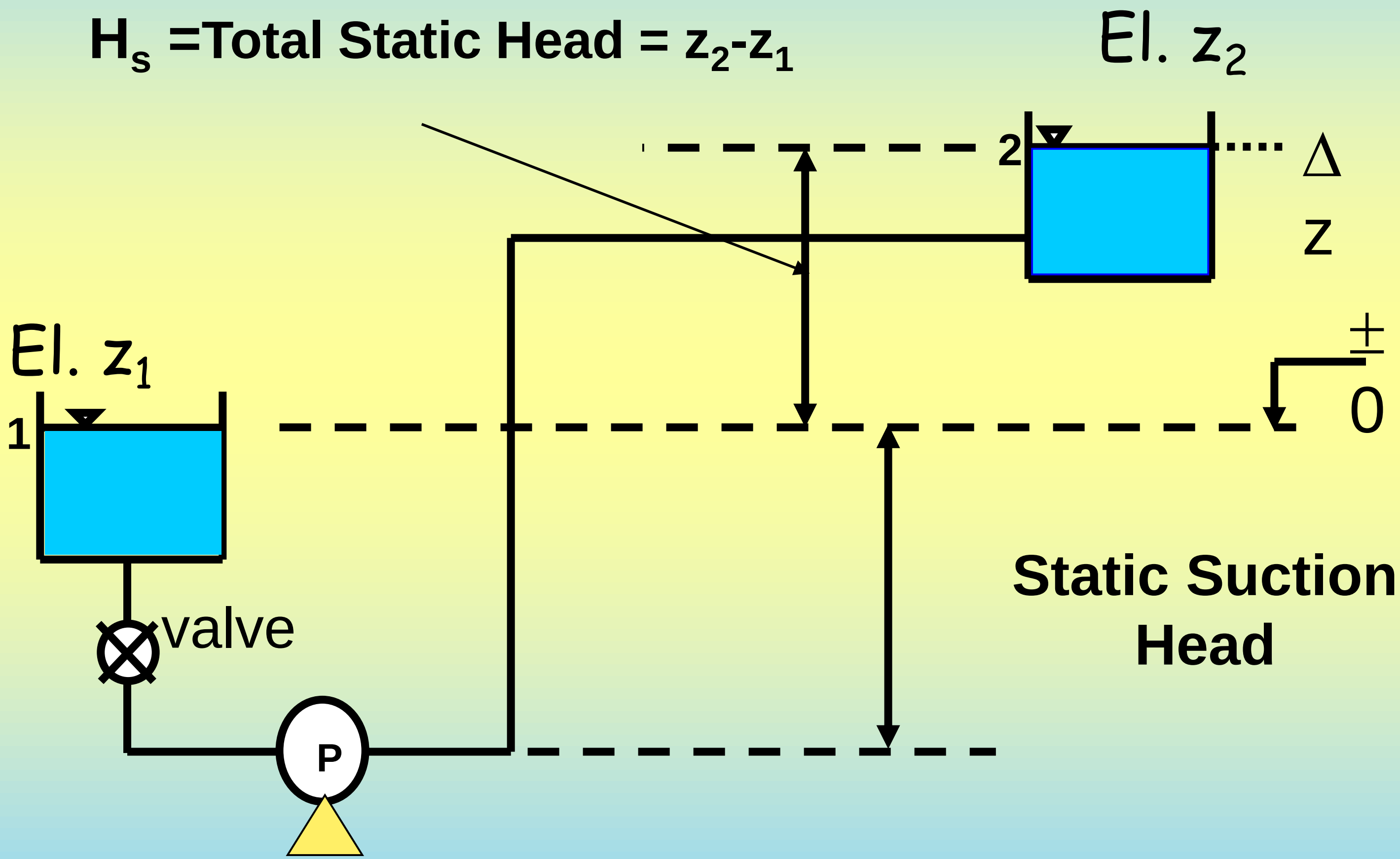
- In other words, to supply a power of $P_f=\gamma Q H_P$ to the fluid, we need a pump which would supply a larger power if the pump is not working efficiently.
- The chief aim of the pump designer is to make the efficiency η as high as possible over a broad range of discharge Q.
- Performance characteristics of a given pump geometry and operating speed are usually given in the form of plots of H_P, η, and bhp versus Q (commonly referred to as capacity) as shown:

Pump Performance Curves



Q_r= rated capacity
(normal or design flowrate)

- The head developed by the pump at zero discharge is called the shut off head, and it represents the rise in pressure head across the pump with the discahrge valve closed. Since there is no flow with the valve closed, the related efficiency is zero, and the power supplied by the pump (bhp at Q=0) is simply dissipated as heat. Although centrifugal pumps can be operated for short periods of time with the discharge valve closed, damage will occur due to overheating and large mechanical stress with any extended operation with valve closed.
- The efficiency h is a function of the flowrate, and reaches a maximum value at some particular value of flowrate, commonly referred as rated capacity, or normal or design discharge.
- The points on the various curves corresponding to the maximum efficiency are denoted as the BEST EFFICIENCY POINTS (BEP)
- It is apparent that when selecting a pump for a particular application, it is usually desirable to have the pump operate near its maximum efficiency.
- These performance curves are very important to the engineer responsible for the selection of pumps for a particular flow system.
- Consider a pipeline system, where water is pumped from a lower reservoir to a higher one:



H_s =Total Static Head = z_2-z_1
El. z_2
2
Δ
z
±
0
El. z_1
1
valve
P
Static Suction
Head

- To select a pump for a particular application, it is necessary to plot both
- $(H_P \text{ vs } Q)_{pump}$ and
- $(H_P \text{ vs } Q)_{system}$
- A pump characteristic curve is normally given by the manufacturer for a certain operating speed. For example:

Q(lt/s)	0	10	20	30	40	50
H_P (m)	30	28	25	20	13	5

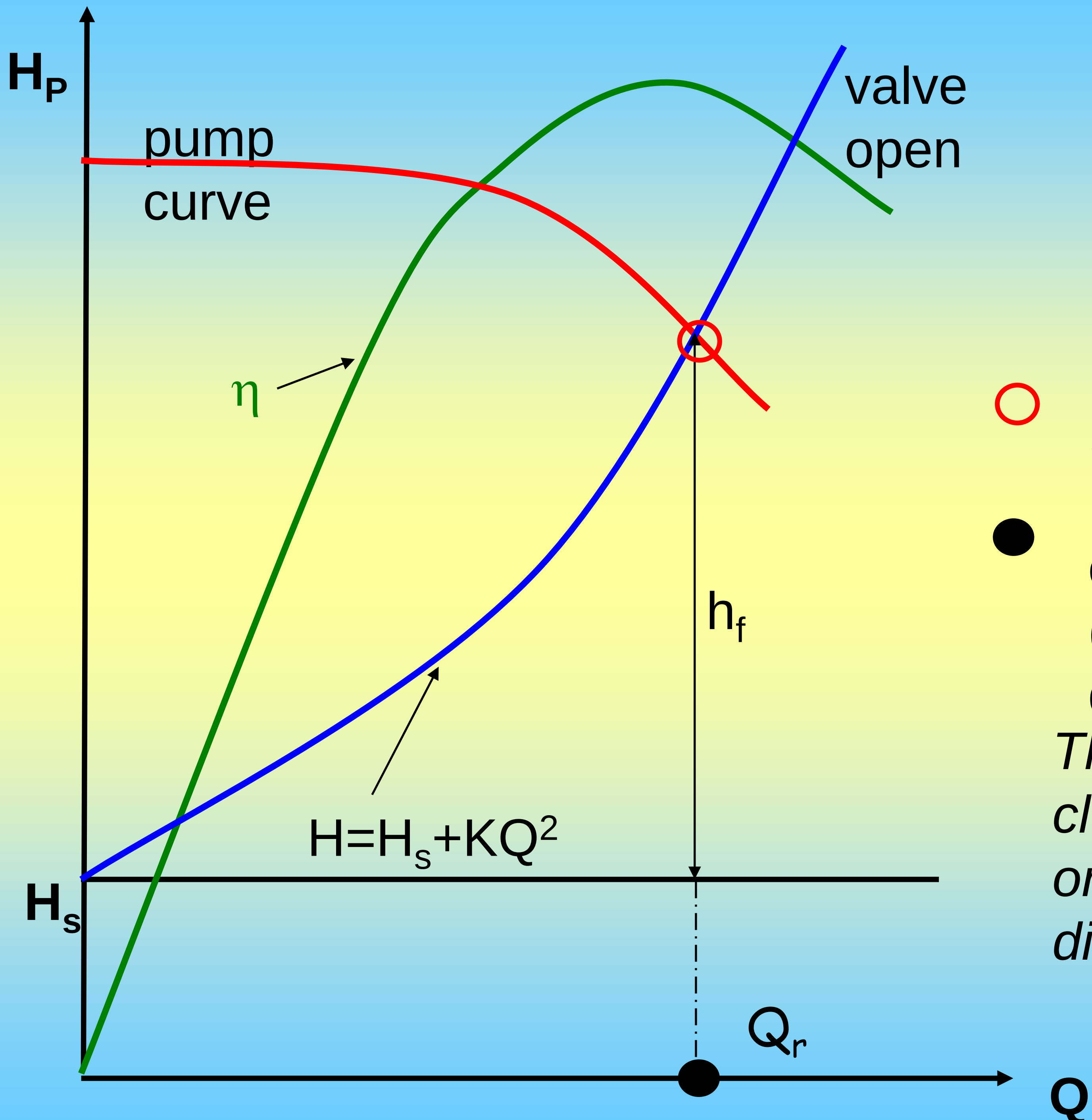
- The system equation can be written as the energy equation between two reservoirs: $H_1 = H_2 - H_P + h_L$

$$H_1 = z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = z_1$$

$$H_2 = z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} = z_2$$

$$h_L = h_f = 8f \frac{L}{D^5} \frac{Q^2}{g\pi^2} = KQ^2 \qquad \text{K is known for a given pipesystem}$$

- $z_1 = z_2 - H_P + KQ^2$ solving for H_P
- $H_P = (z_2 - z_1) + KQ^2 = H_P = H_s + KQ^2 \rightarrow (H_P \text{ vs } Q)_{system}$
- $H_s = (z_2 - z_1)$
- Therefore a plot of $(H_P \text{ vs } Q)_{system}$ and $(H_P \text{ vs } Q)_{pump}$ can be drawn.
- The intersection of the pump curve with the system curve gives **the operating point** of the given pump for a given pipeline system, which means that the pump will deliver the maximum discharge corresponding to the operating point, provided that the valve on the discharge line is fully open.

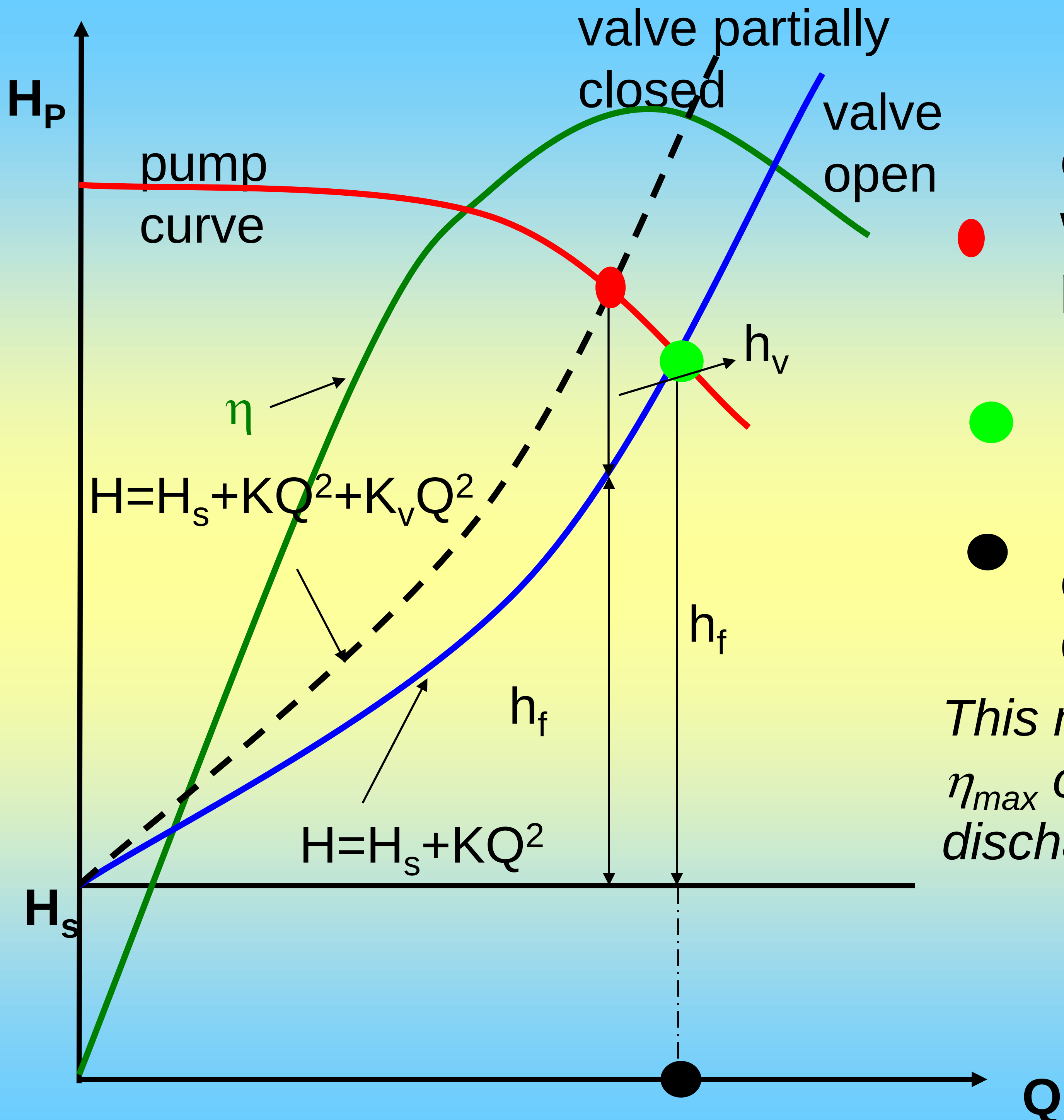


operating point when valve is open

operating (design) discharge

This must be closer to the η_{max} or pump's design discharge

- If the discharge valve is partly closed, i,e; regulated, then a reduced discharge is obtained. For a partly closed valve, LOCAL LOSS must be included:
- $H_1=H_2-H_P+h_f+h_V$, hence system curve becomes:
- $H_P=H_s+KQ^2+K_VQ^2$
- For a given pump, it is clear that as the system equation changes, the operating point will shift.



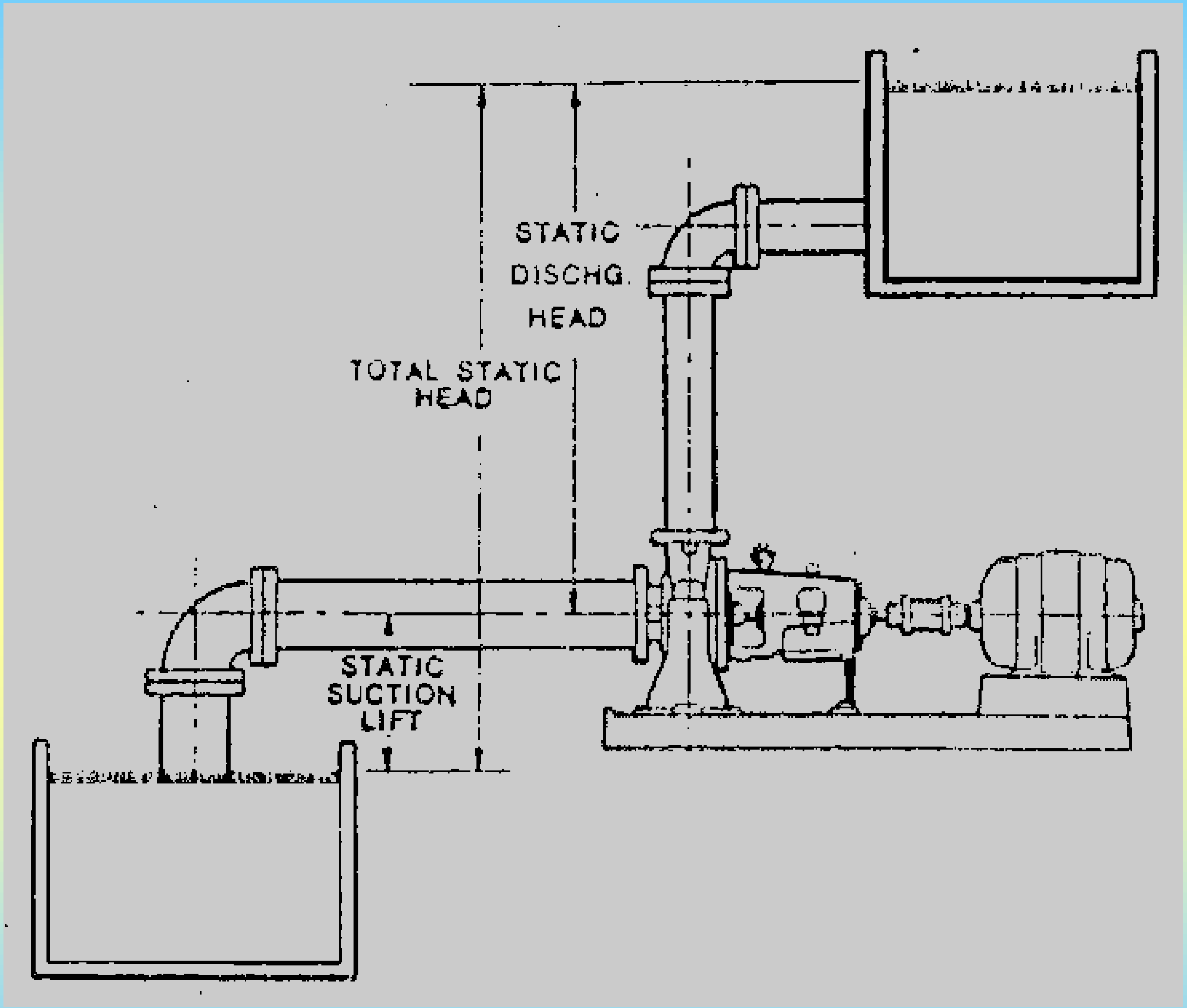
operating point when valve is partially closed

operating point when valve is open

operating (design) discharge

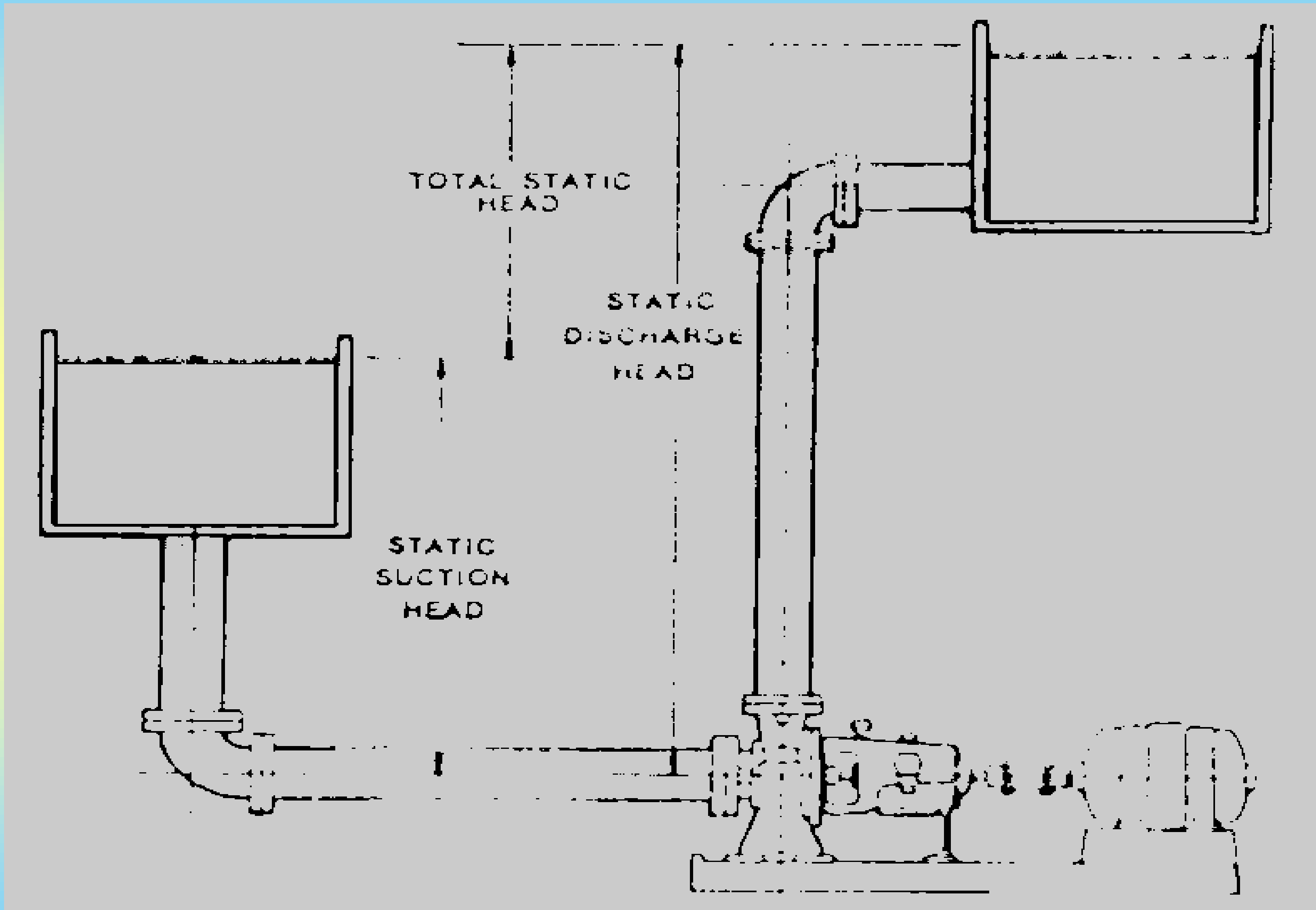
This must be closer to the η_{max} or pump's design discharge

Suction Lift



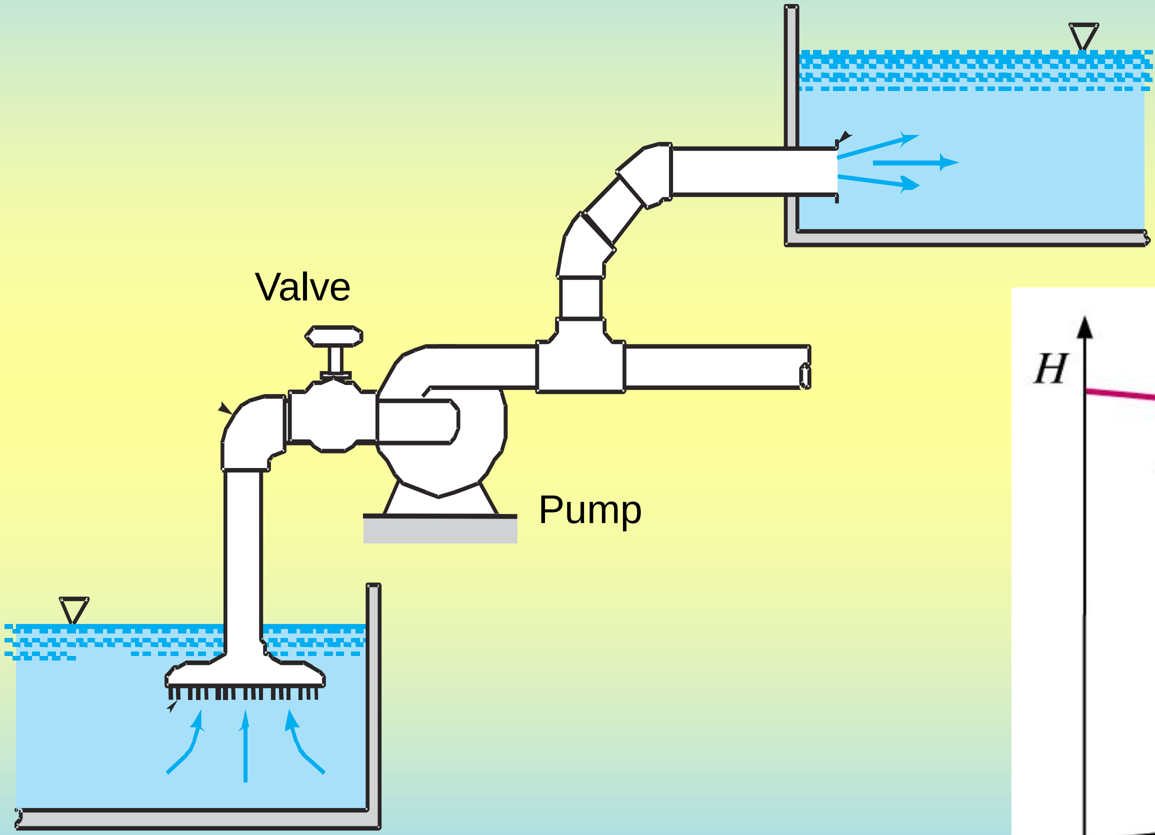
Static head when the pump is located above the suction tank (Static Suction Lift)

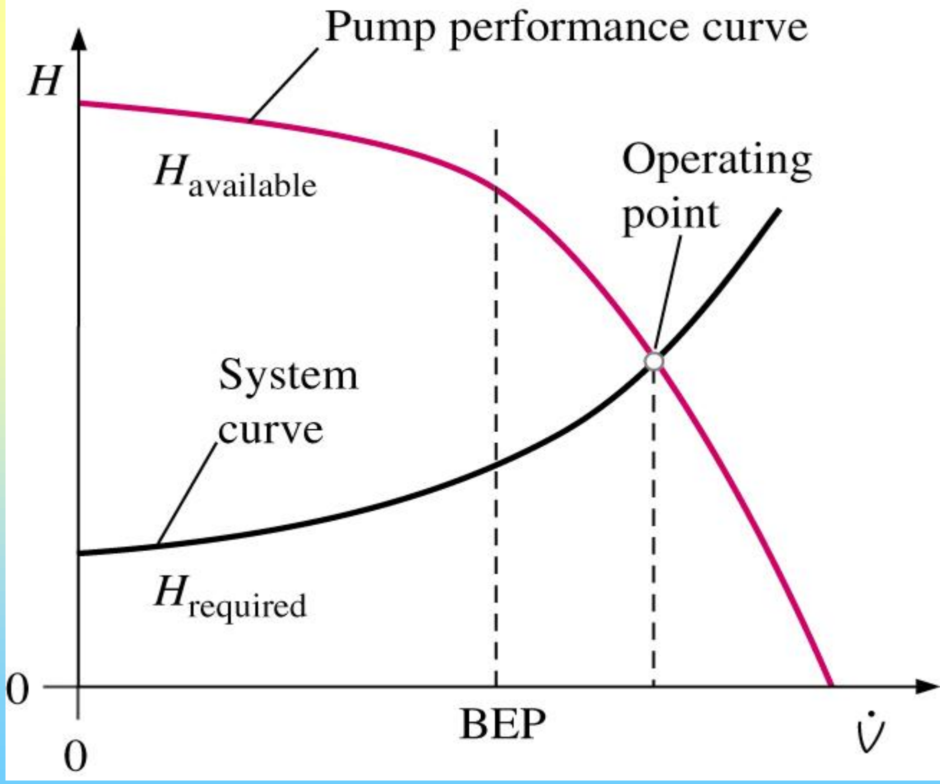
Suction Head



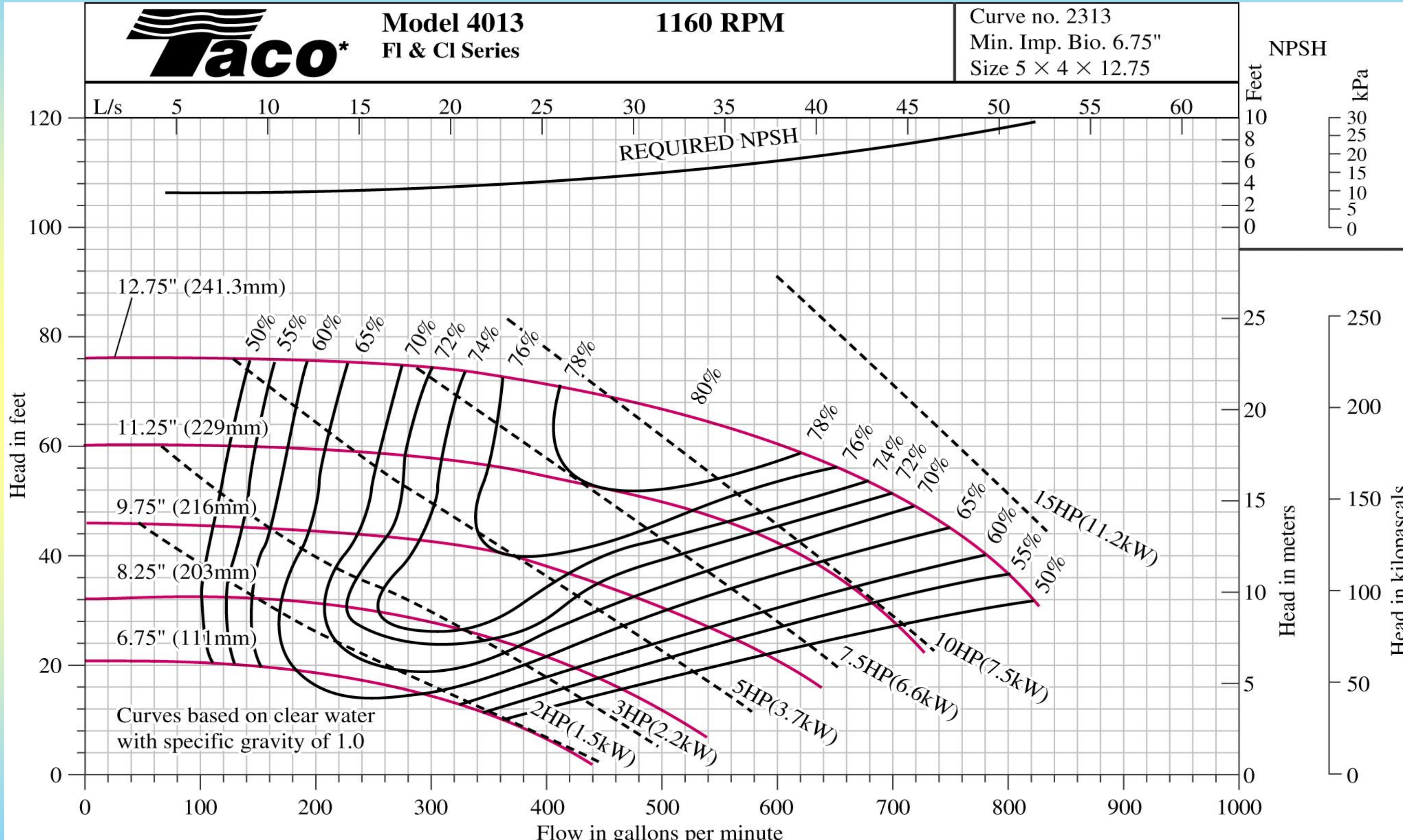
Static head when the pump is located below the suction tank (Static Suction Head)

SELECTION OF A PUMP

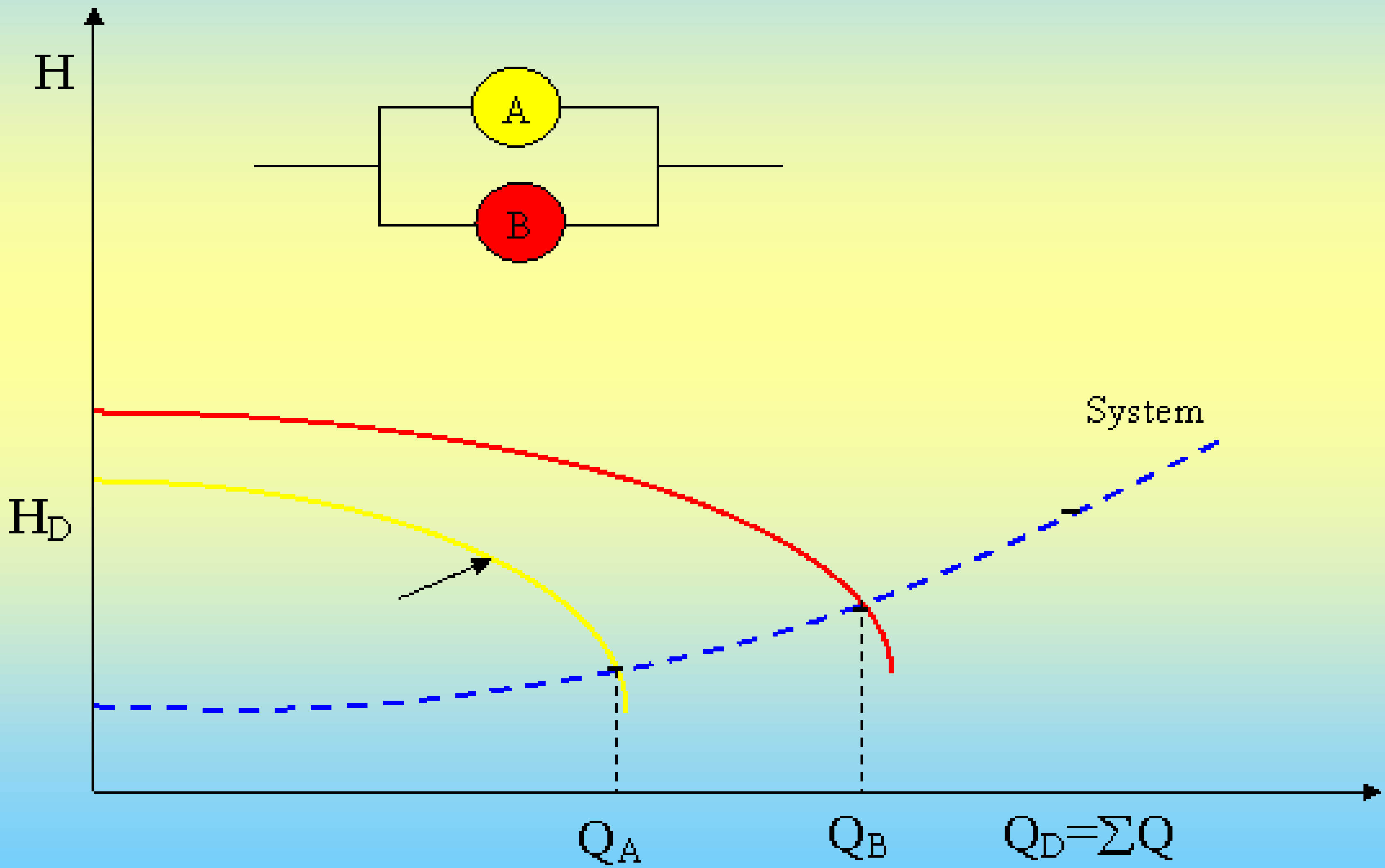




Manufacturer Performance Plot

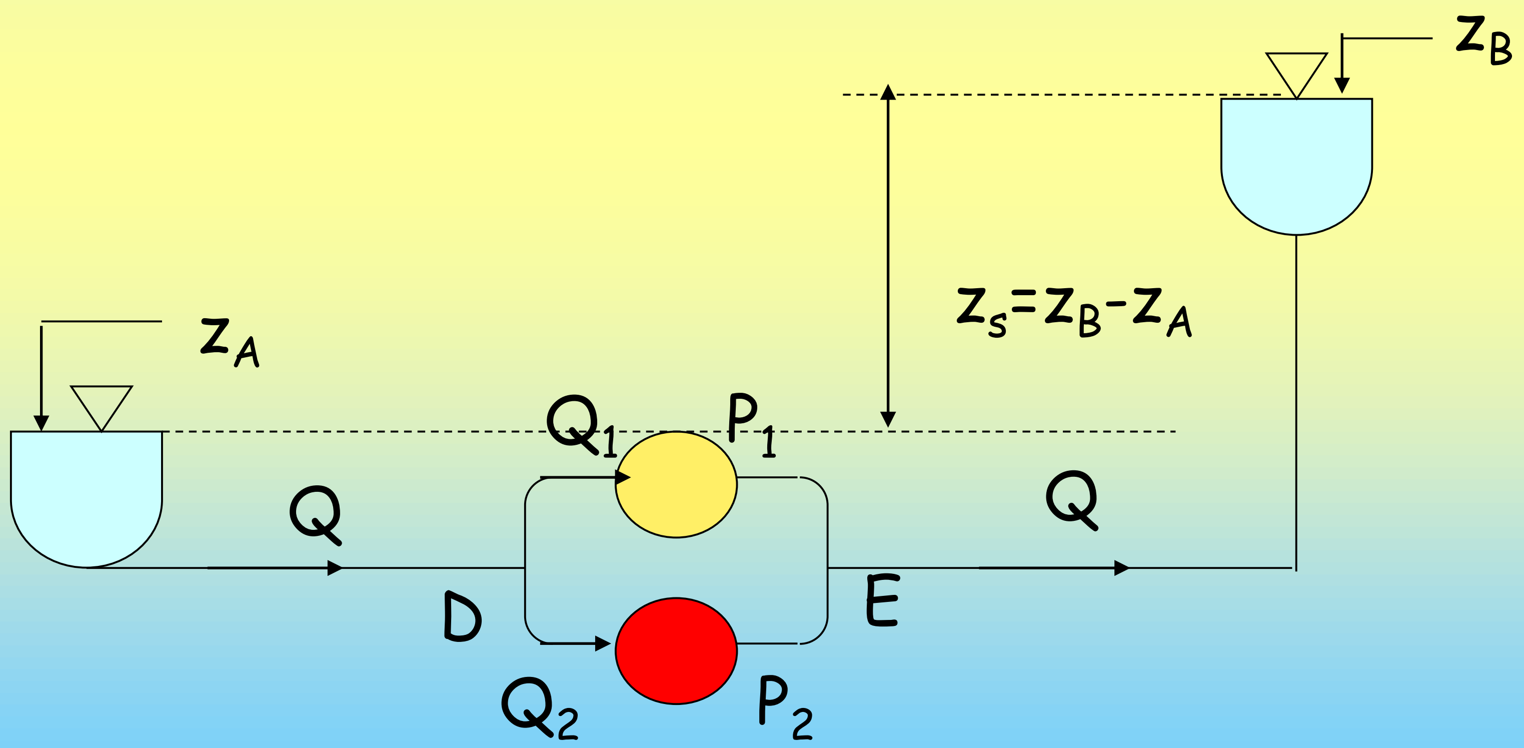


Pumps in Parallel

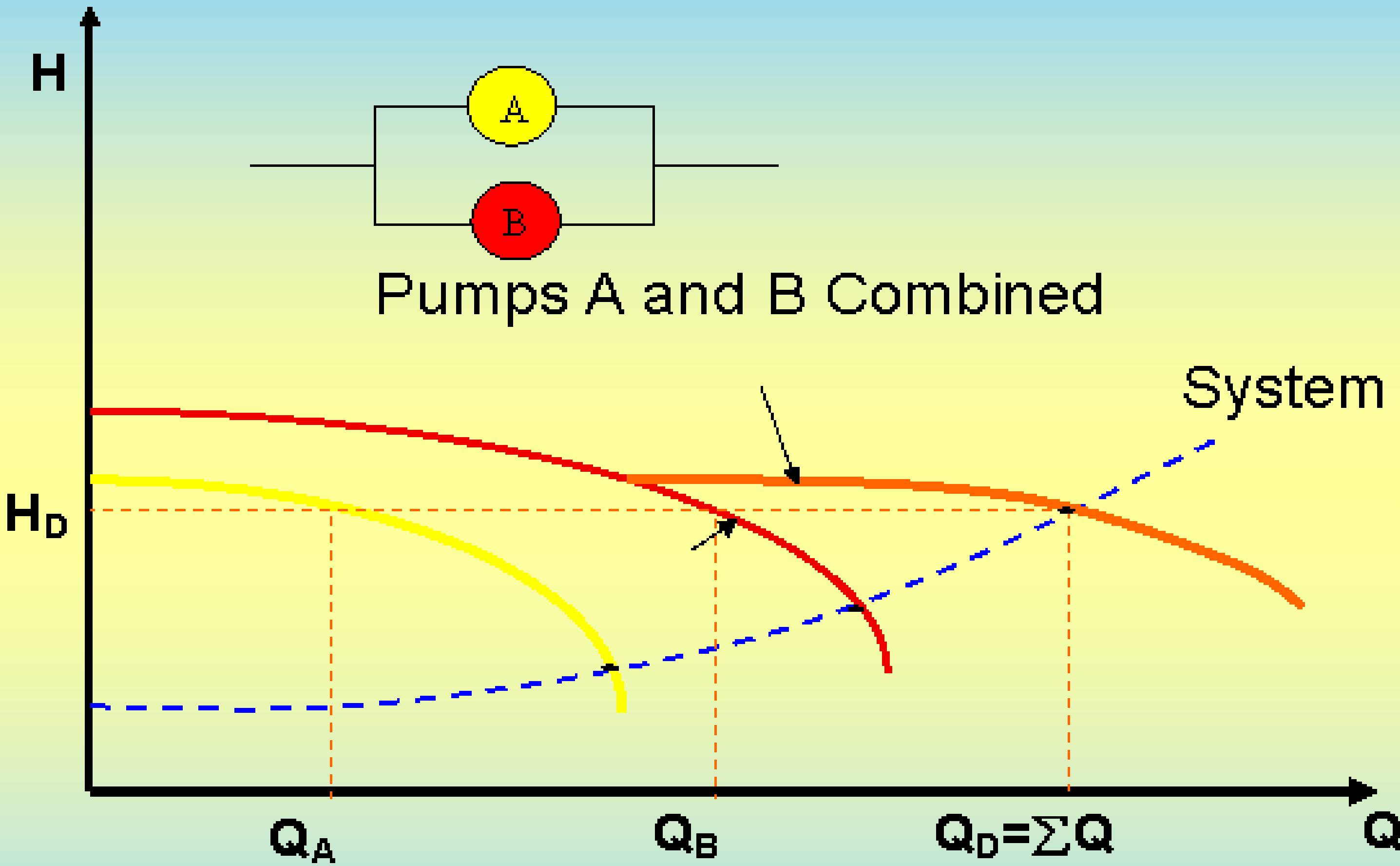


Pumps in Parallel

- Where a large variation in flow demand is required, two or more pumps are placed in a parallel configuration.
- Pumps are turned on individually to meet the required flow demand. In this way, operation at a higher efficiency can be attained.
- Consider two pumps, A and B, which are connected in parallel



Pumps in Parallel



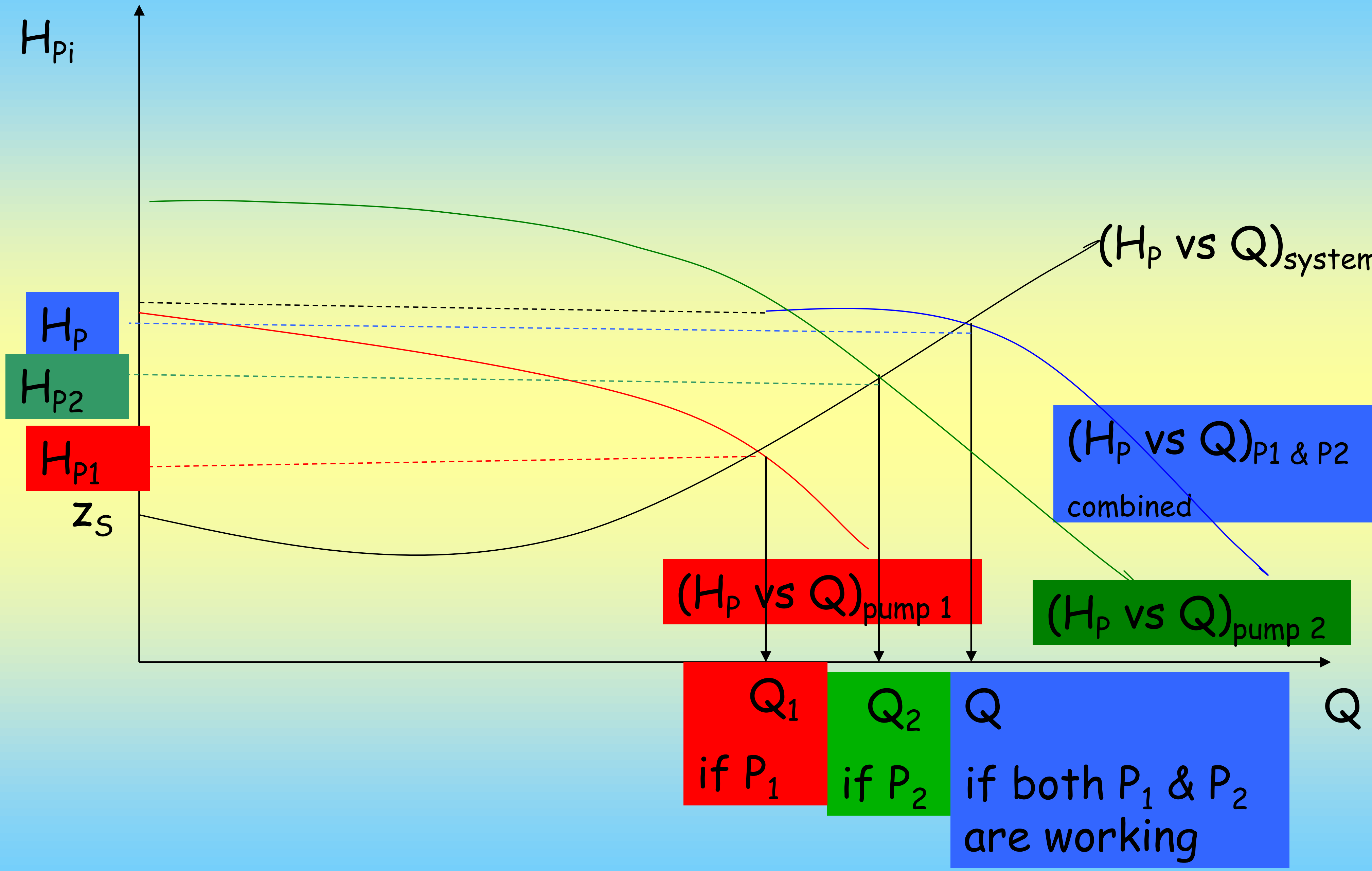
It is not necassary to have identical pumps. Basic principles:

1. Head across each pipe is identical,
2. Total discharge, $Q=\sum Q_i$, $i=1,\ldots,n$, if n pumps are joined in parallel.

Energy equation between reservoirs A and B, if only pump (i) is working:

$z_A=z_B+\sum h_f-H_{Pi}$, solving for H_{Pi}:

$$H_{Pi}=z_B-z_A+\sum h_f = z_S+\sum h_f, \quad i=1,2$$



H_{Pi}
H_P
H_{P2}
H_{P1}
z_S
$(H_P \text{ vs } Q)_{system}$
$(H_P \text{ vs } Q)_{P1 \,\&\, P2}$
combined
$(H_P \text{ vs } Q)_{pump\ 1}$
$(H_P \text{ vs } Q)_{pump\ 2}$
Q_1
if P_1
Q_2
if P_2
Q
if both P_1 & P_2 are working
Q

- If both pumps are working, then
- $H_D = H_E - H_{P1}$, also
- $H_D = H_E - H_{P2}$, then $H_{P1} = H_{P2} = H_P$ for the system

$$P_i = \frac{\gamma Q_i H_P}{\eta_i} \qquad \text{Therefore} \quad Q_i = \frac{P_í \eta_i}{\gamma H_P} \quad í = 1,2$$

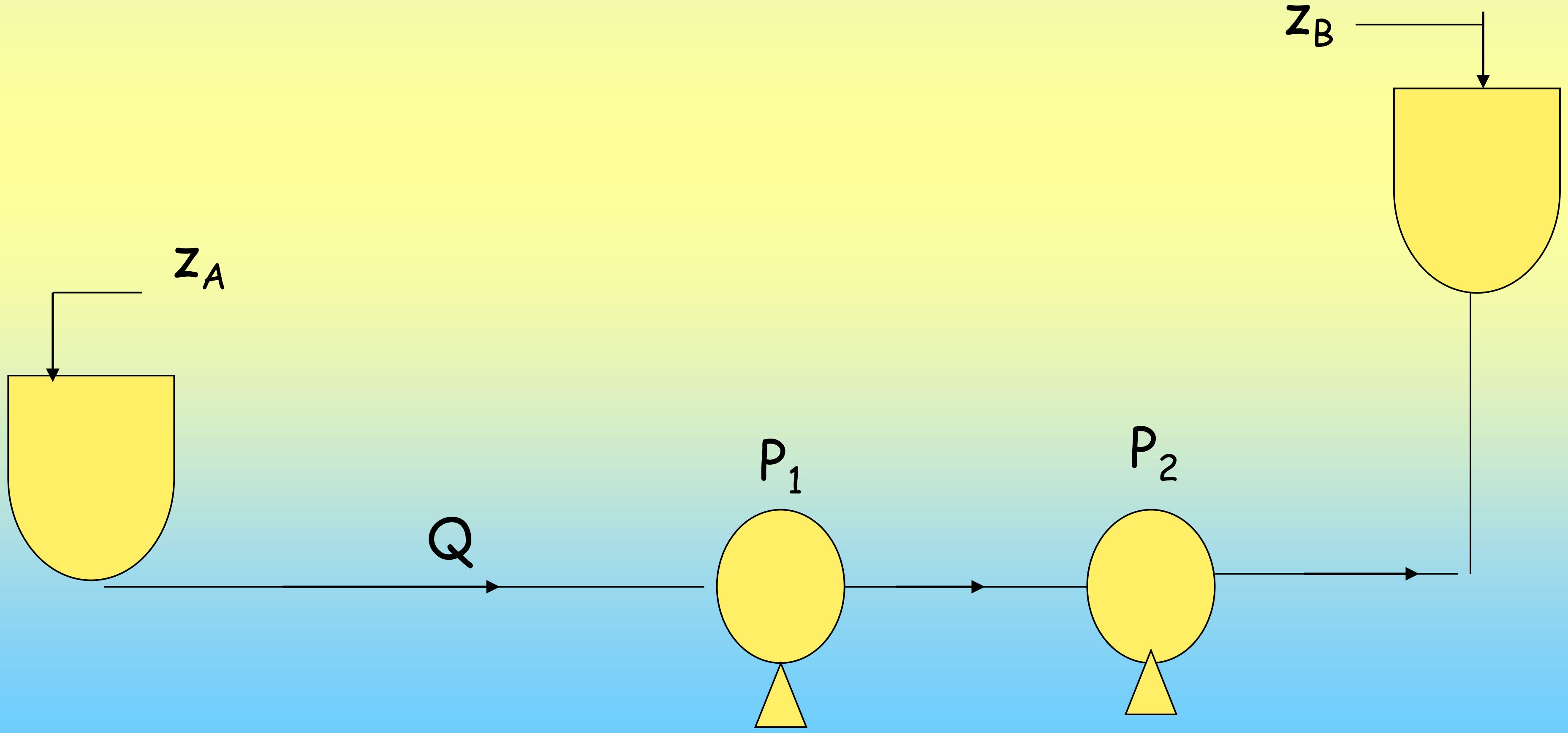
Also, $Q = Q_1 + Q_2 = (\sum P_i \eta_i)/\gamma H_P$, therefore the overall efficiency, η_o, of the system is:

$\eta_o \sum P_i = \sum \eta_i P_i$, therefore $\eta_o = \sum \eta_i P_i / \sum P_i$, but $\sum \eta_i P_i = \gamma Q H_P = \gamma H_P \sum Q_i$

$$\eta_p = \frac{\gamma H_D \sum Q}{\sum P_P}$$

Pump in Series

- Where high head demand is required in the flow, two or more pumps are placed in series configuration.



It is not necassary to have identical pumps. Basic principles:

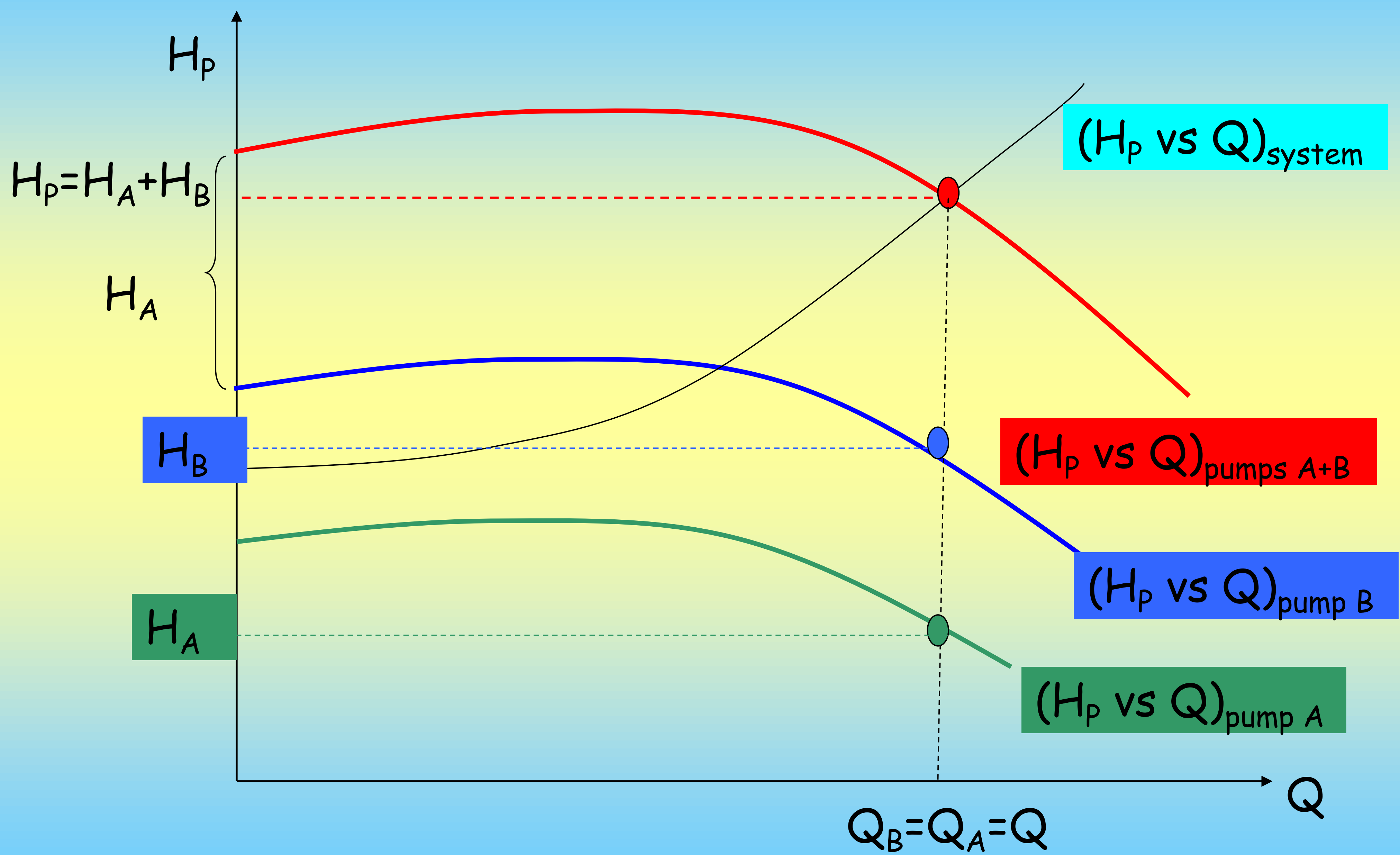
1. The discharge through each pipe is identical, i.e: $Q=Q_1+Q_2$
2. Total head developed by the pumps, $H_P=\sum H_{Pi}$, i=1,...,n, if n pumps are joined in series.

   The characteristic curve for the pumps is found by summing the head across each pump.

   The overall efficiency is:

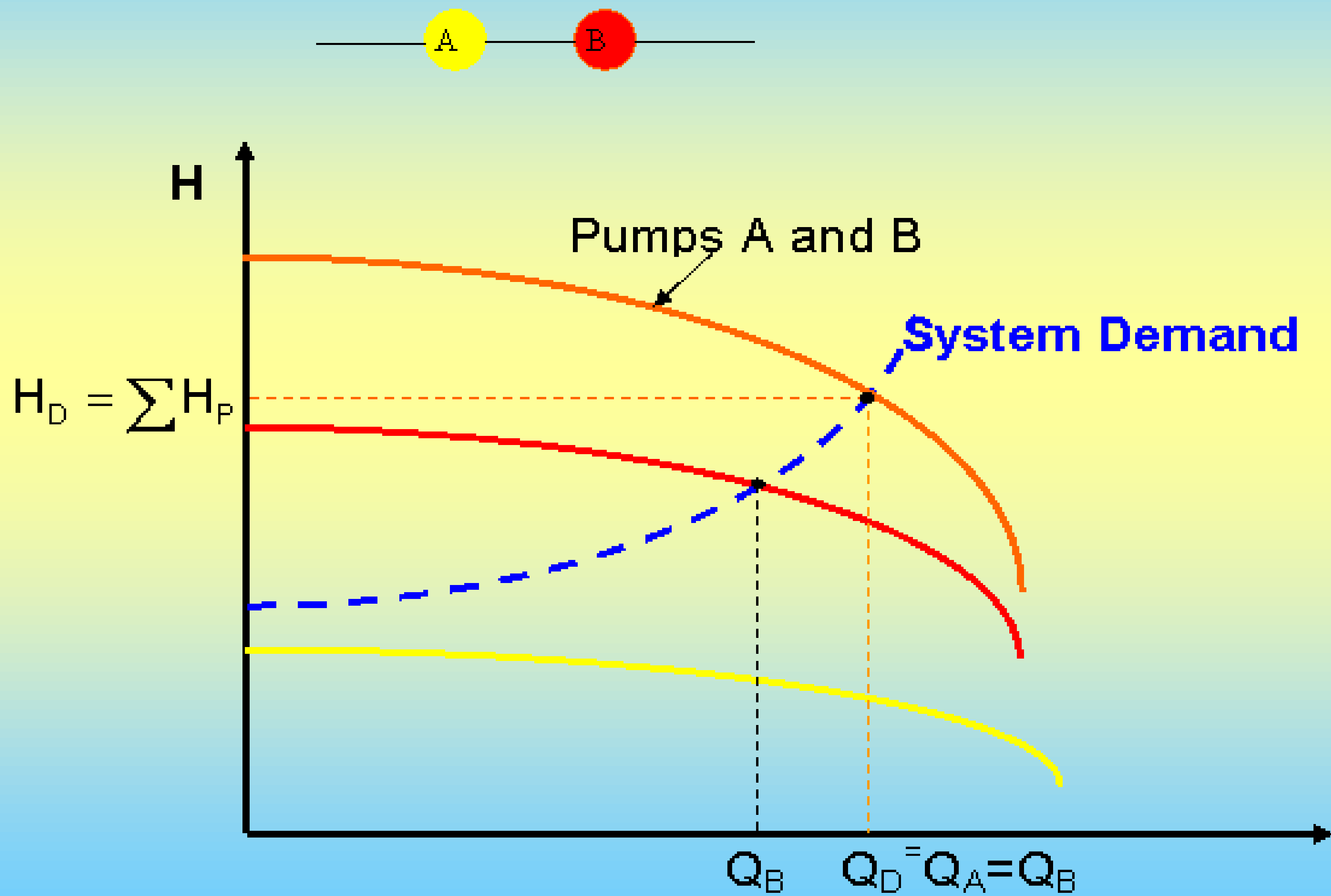
$$\eta_P = \frac{\gamma\left(\sum H_P\right)Q_D}{\sum P_P}$$

- $\sum H_{Pi}$= sum of the individual heads across each pump.



H_P
$H_P=H_A+H_B$
H_A
H_B
H_A
$(H_P \text{ vs } Q)_{system}$
$(H_P \text{ vs } Q)_{pumps\ A+B}$
$(H_P \text{ vs } Q)_{pump\ B}$
$(H_P \text{ vs } Q)_{pump\ A}$
Q
$Q_B=Q_A=Q$

Pump in Series



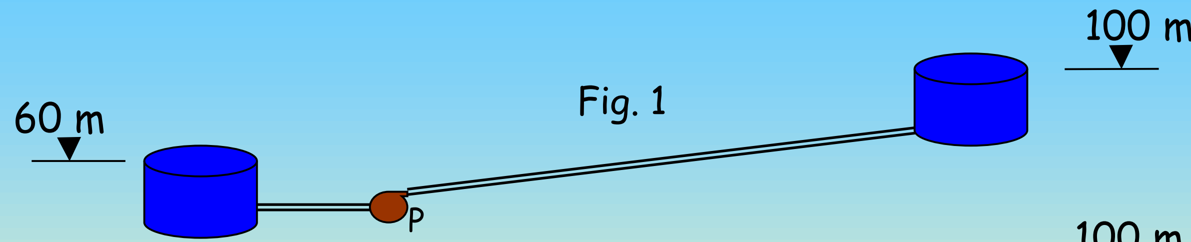
EXAMPLE 2.9:

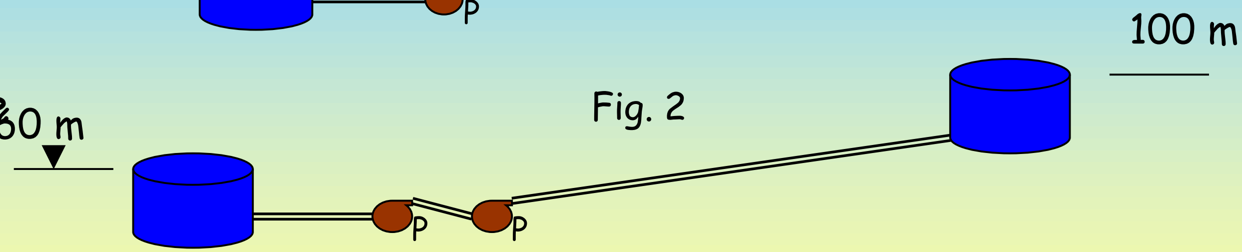
The following pumped discharge pipelines are given;

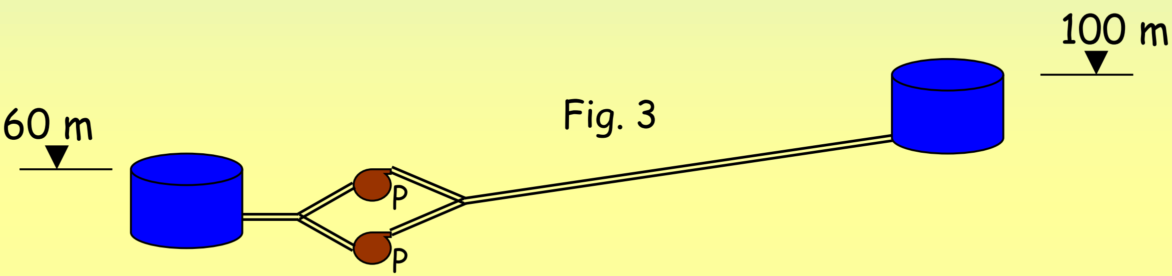
a) Determine the discharge when only one pump is operating (as shown in figure 1.) and compute the power consumption.

b) Determine the discharge when both pumps are operating in series (as shown in figure 2.) and compute the power consumption.

c) Determine the discharge when both pumps are operating in parallel (as shown in figure 3.) and compute the power consumption.







Pipeline Characteristics

L=2000 m; D=0.2 m; ε=0.0002m; ν=1* 10^{-6} m^2/s

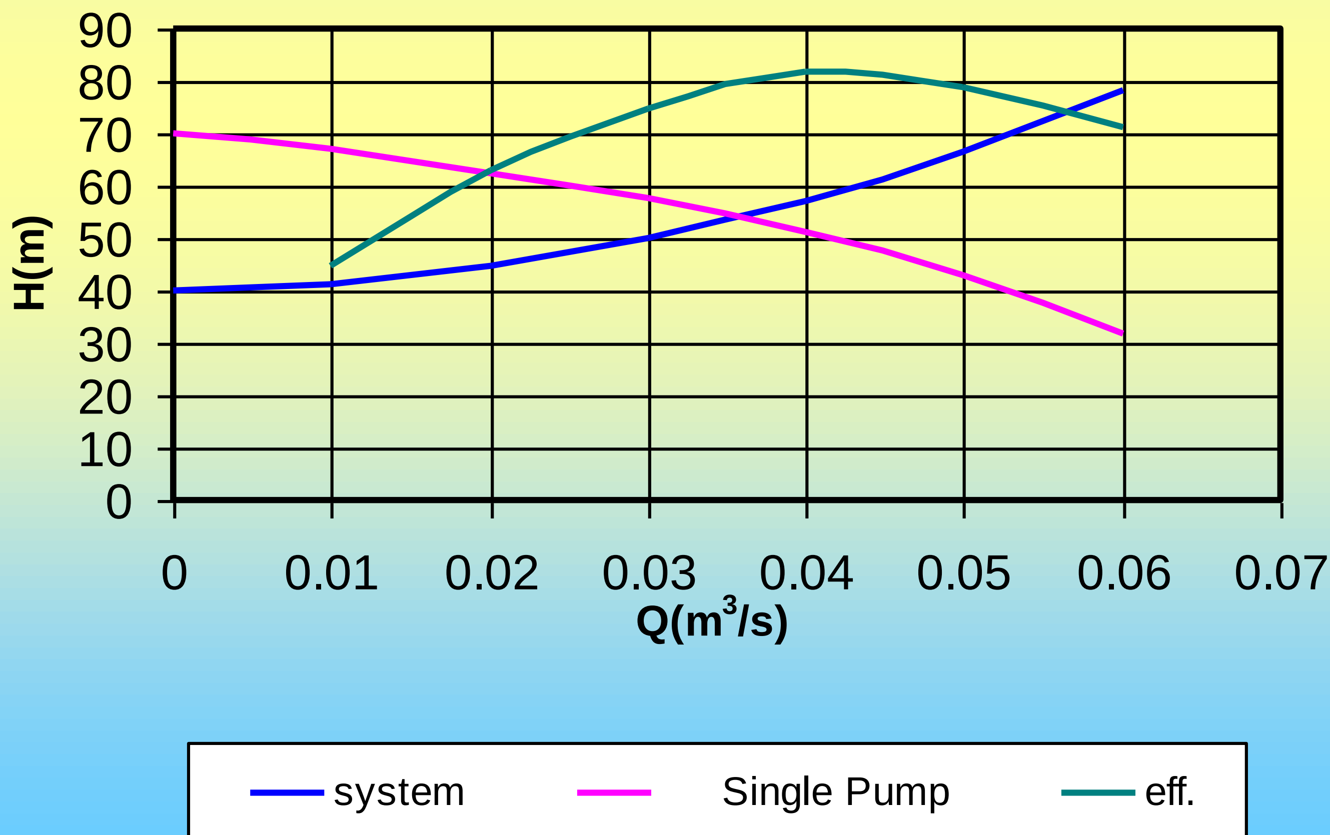
Pump Charecteristics

Q (lt/s)	0	10	20	30	40	50	60
H (m)	70	67	62.5	57.5	51	43	32
η (%)		45	63	75	82	79	71

$H_{!}=H_2+h_f-H_P$

$$H_P = 40 + h_f = 40 + \underbrace{\left(8f \frac{L}{g\pi^2 D^5}\right)}_{K} Q^2 = 40 + KQ^2$$

Case 1: Single Pump



Single Pump

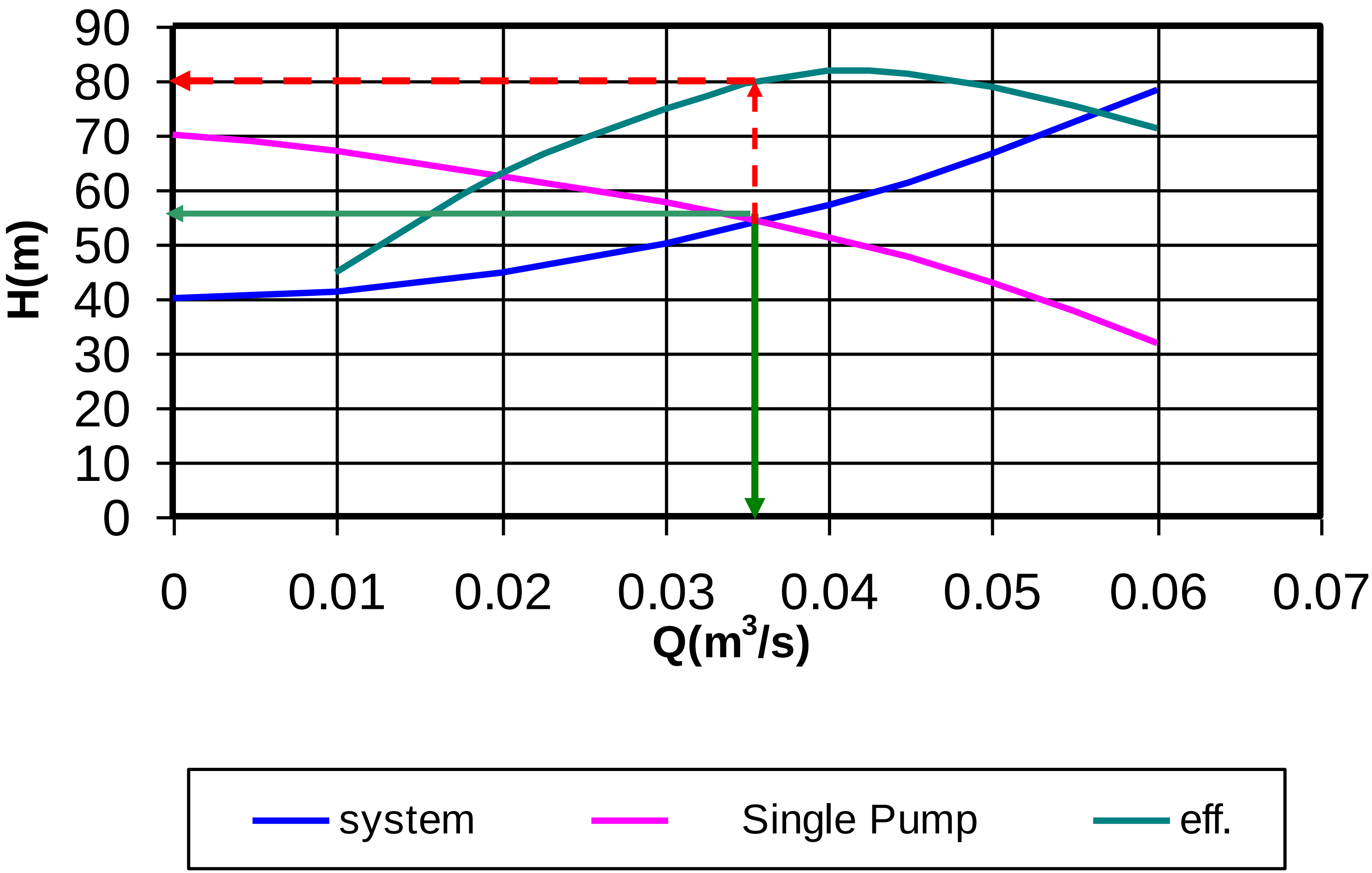
L (m)	2000
D (m)	0.2
ε (m)	0.0002
υ (m²/s)	0.000001

Q	**Re**	**f**	**h_l**	**$(H_P)_{system}$**
0	0	0	0.00	40.00
0.01	63662	0.0234	1.21	41.21
0.02	127324	0.0219	4.52	44.52
0.03	190986	0.0212	9.88	49.88
0.04	254648	0.0209	17.28	57.28
0.05	318310	0.0207	26.72	66.72
0.06	381972	0.0205	38.19	78.19

Single Pump Characteristic Curve

H	**Q**
70	0
67	0.01
62.5	0.02
57.5	0.03
51	0.04
43	0.05
32	0.06

Case 1: Single Pump



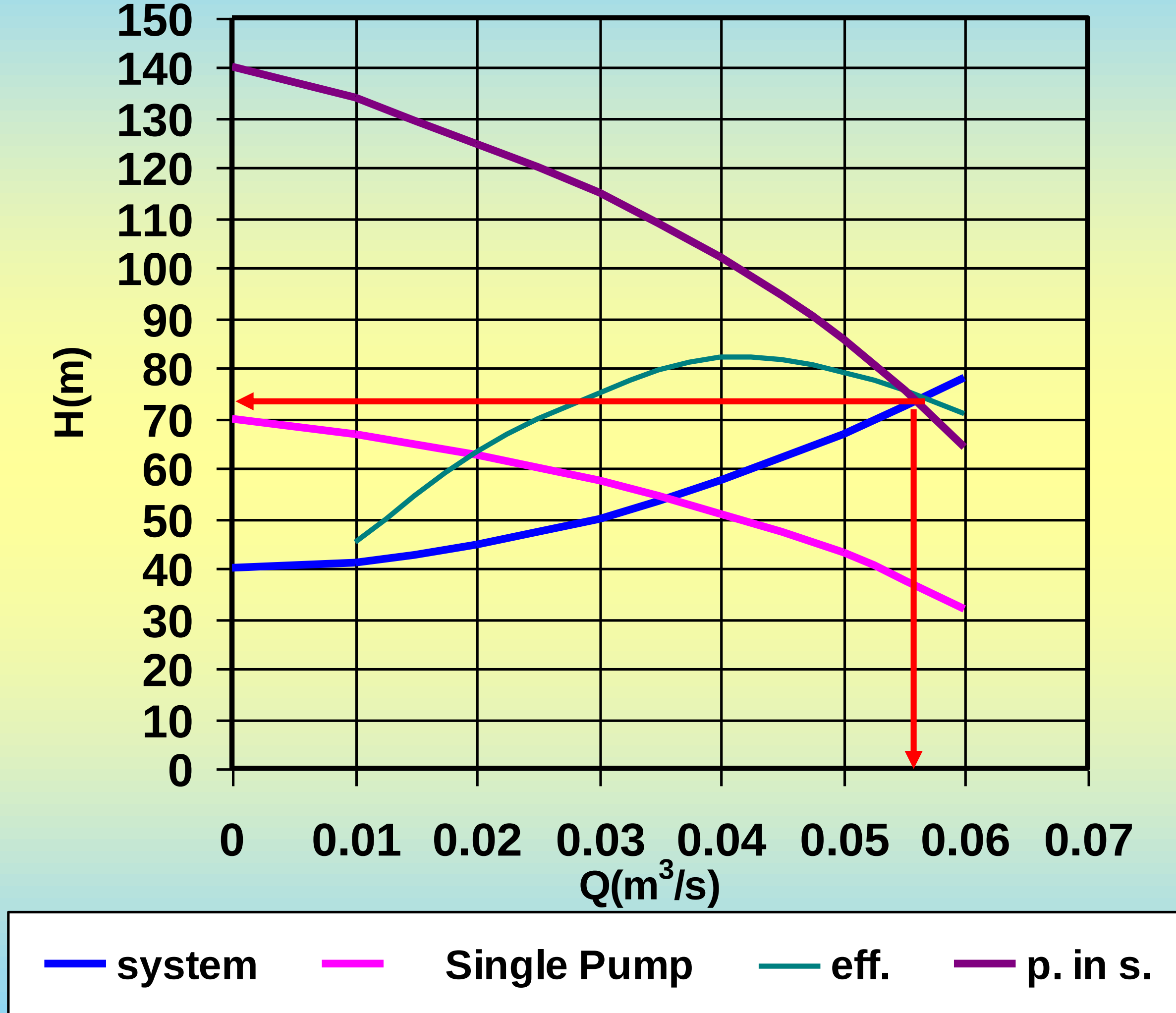
SOLUTION
Q=35 *lt*/s
η=80%
(for Q=35 ℓ/s)
H=54m
P=23.2kW

Pumps in Series

H(m)	Q(m³/s)
140	0
134	0.01
125	0.02
115	0.03
102	0.04
86	0.05
64	0.06

$$H=\sum H_{P_i}=2H_P$$

Case 2: Pump in Series



SOLUTION

Q=56 ℓ/s

η=75%

(for Q=56 ℓ/s)

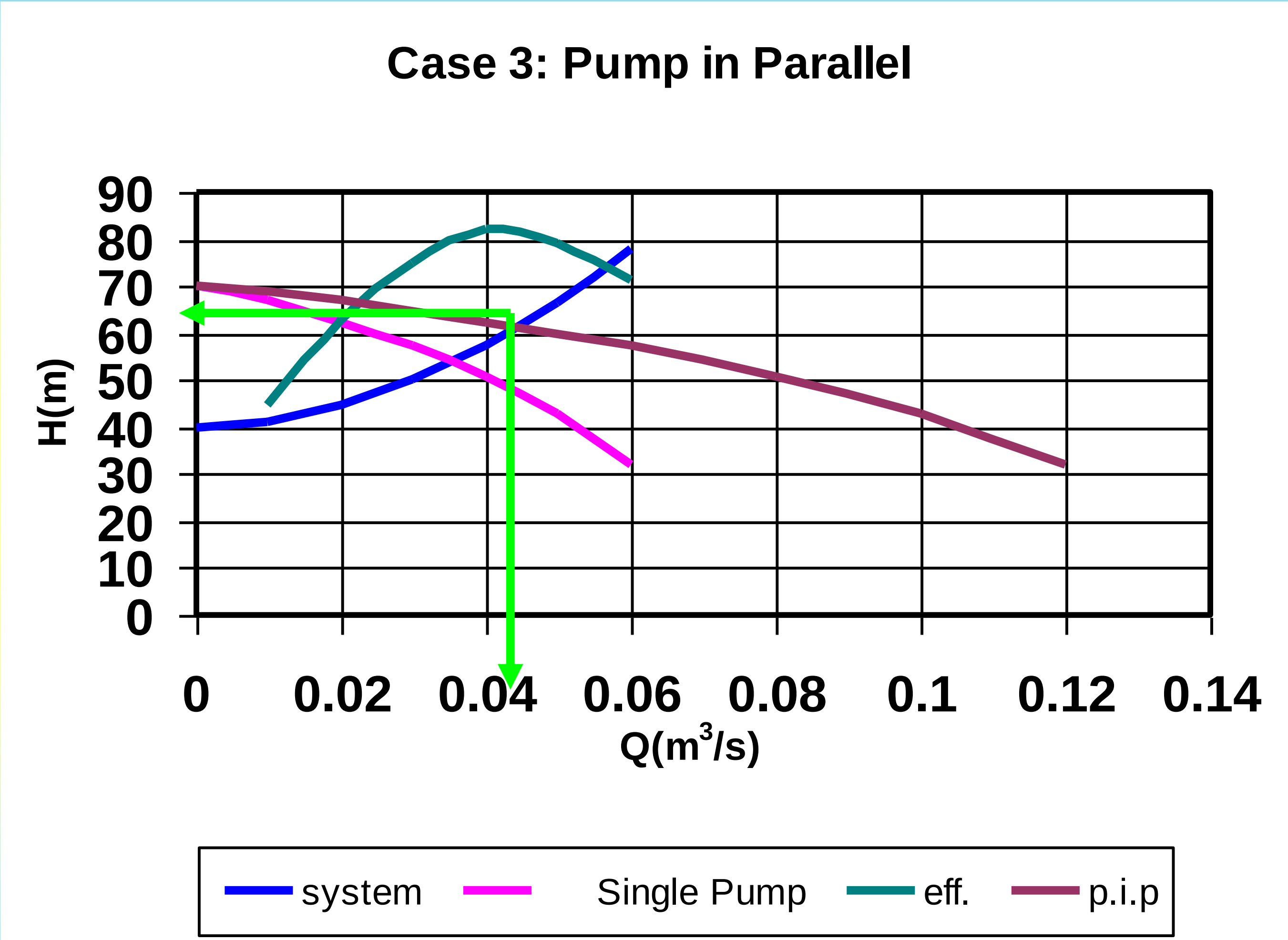
∑H=72m

∑P=52.8kW

Parallel Pumps

H (m)	Q (m^3/s)
70	0
67	0.02
62.5	0.04
57.5	0.06
51	0.08
43	0.1
32	0.12

$$Q = \sum Q_i = 2Q$$



SOLUTION

Q=46 ℓ/s

η=68%

(for Q=23ℓ/s)

$\sum$H=61m

$\sum$P=40.5kW

summary

	single	series	paralel	
Q (lt/s)	35	56	46	(2*23)
η (%)	80	75	68	
H (m)	54	72	61	
P (kW)	23.2	52.7	40.5	(2*20.24)
P/Q (kW.s/lt)	0.66	0.94	0.88	

EXAMPLE 2.10

Water is pumped from reservoir A to reservoirs C and D. The system and pump characteristics are given below. Determine

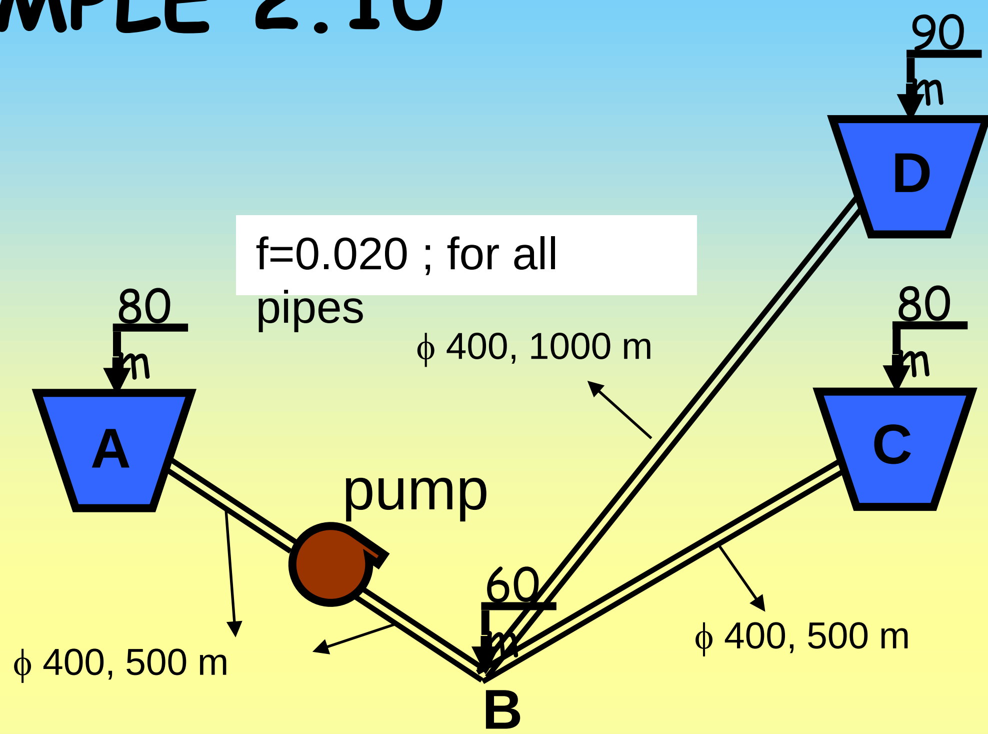
a)The discharge in each pipe,

b)The head added to the system by pump,

c)The head loss in each pipe for the computed discharges,

d)Complete the HGL,

e) Calculate the power of the pump for η=0.80.



Pump Characteristics

Q (lt/s)	0	100	200	300	400	500
H_p (m)	50	48	45	38	30	18

SYSTEM CURVES

$$h_f = \frac{8fL}{g\pi^2 D^5} Q^2$$

Branch AB:

HA = HB-Hp+hfAB

80 = HB+hfAB-Hp

HB = 80+Hp-hfAB ....Eq.(1)

$$h_{fAB} = h_{fBC} = 80.69Q^2$$

Branch BC:

HB = HC +hfBC

HB = 80+hfBC .... Eq.(2)

Branch BD:

$$h_{fBD} = 161.38Q^2$$

HB = HD +hfBD

HB = 90+hfBD .... Eq.(3)

To find the operating point, i.e: to find thw solution, Equation of continuity at junction must be satisfied.

$Q_{AB} = Q_{BC} + Q_{BD}$ .... Eq.(4)

1. In other words, the head value, H_B, obtained by using the system curves of each pipe must be the same. Therefore, it is necessary to construct the system curves for AB, BC and BD.

2. The variations of head loss with discharge for each pipe should be obtained. Therefore from The Darcy-Weisbach equation:

$$h_{fAB} = h_{fBC} = 80.69Q^2$$

$$h_{fBD} = 161.38Q^2$$

Variation of head loss for each pipe

Q (lt/s)	h_{fAB} (m)	h_{fBD} (m)	h_{fBC} (m)
0	0	0	0
100	0.8	1.6	0.8
200	3.2	6.4	3.2
300	7.2	14.5	7.2
400	12.9	25.8	12.9
500	20.2	40.3	20.2

Then using these values, variation of H_B with discharge for each pipe can be obtained

Head of Junction, H_B

Q (lt/s)	H_p (m)	from AB H_B (m) Eq.(1)	from BC H_B (m) Eq.(2)	from BD H_B (m) Eq.(3)
0	50	130	80	90
100	48	127.2	80.8	91.6
200	45	121.8	83.2	96.5
300	38	110.8	87.2	104.5
400	30	97.1	92.9	115.8

Attention:The minimum HB value is 90 m.; above which flow in branch BC and BD will be in the assumed direction!

$Q_{AB} = Q_{BC} + Q_{BD}$

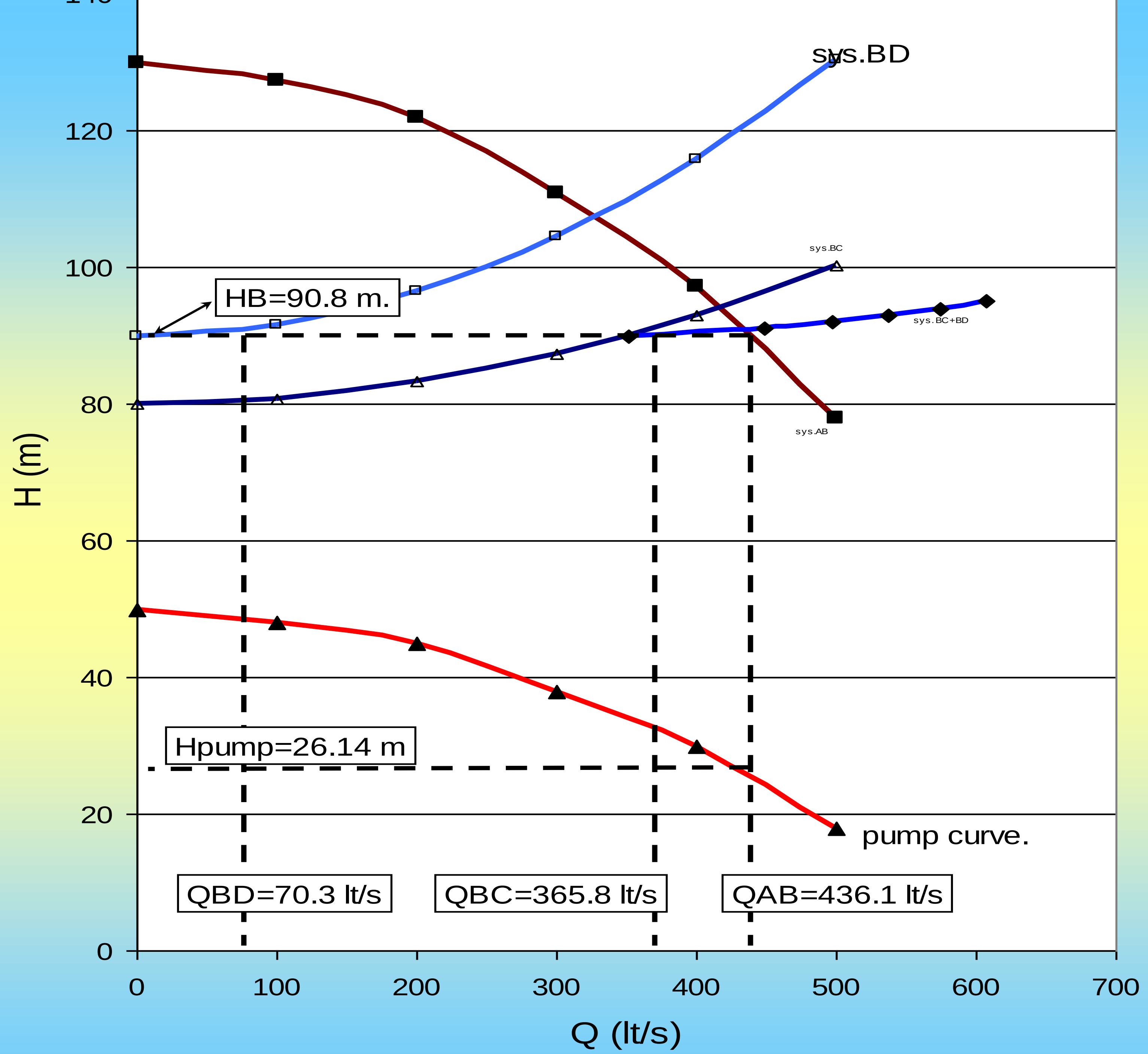
Rearranging Eqs. (2) and (3):

$$Q_{BC} = 1000.\left(\frac{H_B - 80}{80.69}\right)^{1/2}$$

$$Q_{BD} = 1000.\left(\frac{H_B - 90}{161.38}\right)^{1/2}$$

H_B (m)	Q_{BC} (lt/s)	Q_{BD} (lt/s)	$Q_{BC}+Q_{BD}$ (lt/s)
90	352	0	352
91	369.2	78.7	447.9
92	385.6	111.3	496.9
93	401.4	136.3	537.7
94	416.5	157.4	573.9
94	431.2	176	607.2

Plotting H_B vs Q_{AB}, H_B vs Q_{BD}, H_B vs Q_{BC} and H_B vs $Q_{(BC+BD)}$, the discharge in the system and discharges in each pipe can be obtained.



sys.BD
sys.BC
sys.BC+BD
sys.AB
pump curve.
HB=90.8 m.
Hpump=26.14 m
QBD=70.3 lt/s
QBC=365.8 lt/s
QAB=436.1 lt/s
H (m)
Q (lt/s)
0
20
40
60
80
100
120
0
100
200
300
400
500
600
700

SUMMARY OF RESULTS:

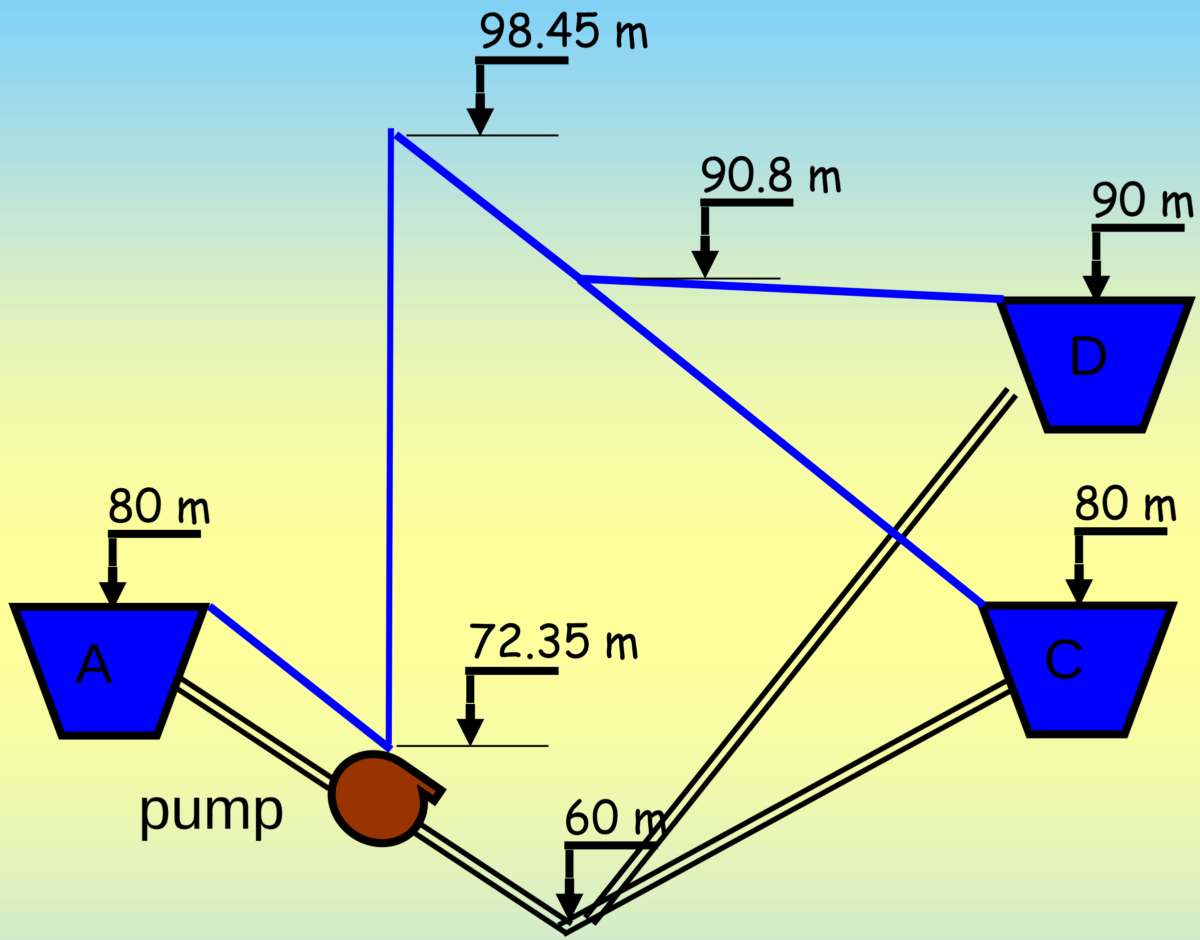
- From the graph, the intersection of HB vs Q_{AB} and H_B vs $Q_{BC}+Q_{BD}$ is read to be

  at **H_B=90.8m** and **QAB=436.1 lt/s.**

  Furthermore, these values correspond to **Q_{BC}=365.8 lt/s** and **Q_{BD}=70.3 lt/s.**

- The pump head **H_{pump}=26.1 m**
  h_{fAB}=15.34 m
  h_{fBC}=10.8 m.
- **h_{fBD}=0.8 m.**

d)Hydraulic Grade Line



e) $P_{pump} = \gamma g Q H_{pump}/h = 139.6\ kW$

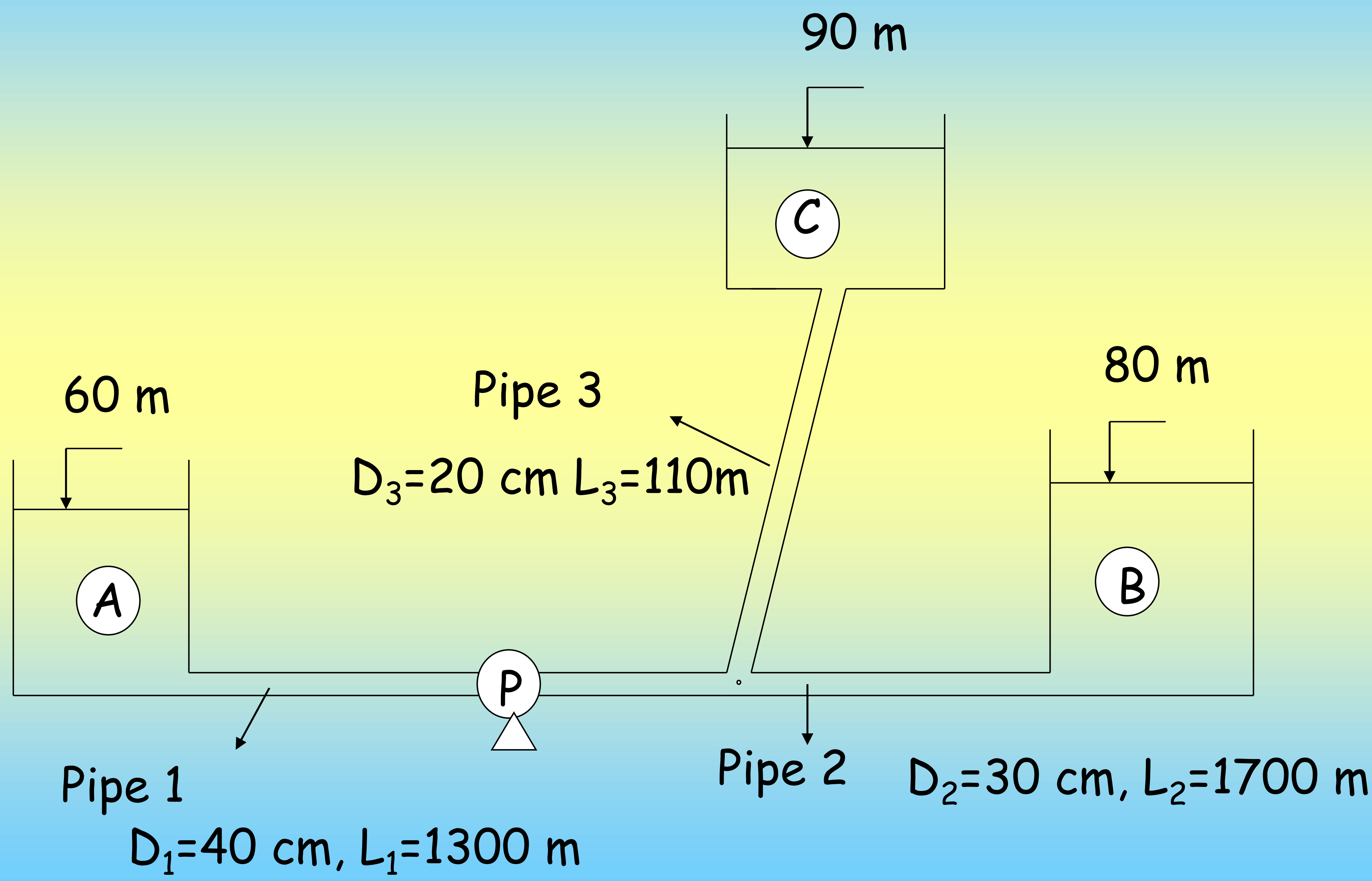
Example 2.11

A branching pipe system with three reservoirs and a pump located on Pipe 1 is given. The pump head-discharge relationship is given as,

$$H_p = 50 - 128Q^2$$

Water is to be transported from reservoir A to the junction, then to reservoirs B and C. Determine:

a) the discharge in each pipe,
b) the head added to the system by the pump,
c) the head loss in each pipe for the computed discharges,
d) the power required by the pump if efficiency is $\eta = 0.8$ and
e) draw the piezometric line for the $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.015$ cm



90 m
C
80 m
60 m
Pipe 3
D3=20 cm L3=110m
B
A
P
Pipe 1
D1=40 cm, L1=1300 m
Pipe 2
D2=30 cm, L2=1700 m

Solution:

a) Energy equation between A and J;

$$z_A + H_p - h_{f1} = h_j \Longrightarrow h_j = 60 + H_p - h_{f1} \quad \text{................ (Pipe 1)}$$

Energy equation between J and B;

$$h_j - h_{f2} = z_B \Rightarrow h_j = 80 + h_{f2} \quad \text{................ (Pipe 2)}$$

Energy equation between J and C;

$$h_j - h_{f3} = z_C \Rightarrow h_j = 90 + h_{f3} \quad \text{................(Pipe 3)}$$

- So from the operational point obtained from graph 1 the discharges are found as ;
- Q_1 = 222 lt/s
- Q_2 =124 lt/s
- Q_3 = 98 lt/s
-

b) By using the equation of pump head-discharge relationship, the Hp is obtained as;

$$H_p = 50 - 128Q^2 = 50 - 128(0.222)^2 = 43.7m$$

Also the pump head can be found from graph 2 where we have the system curve and pump curve:

$H_p = h_j + h_{f1} - 60 = 95.2 + h_{f1} - 60$

Hp = 35.2 + hf1 ...System Equation

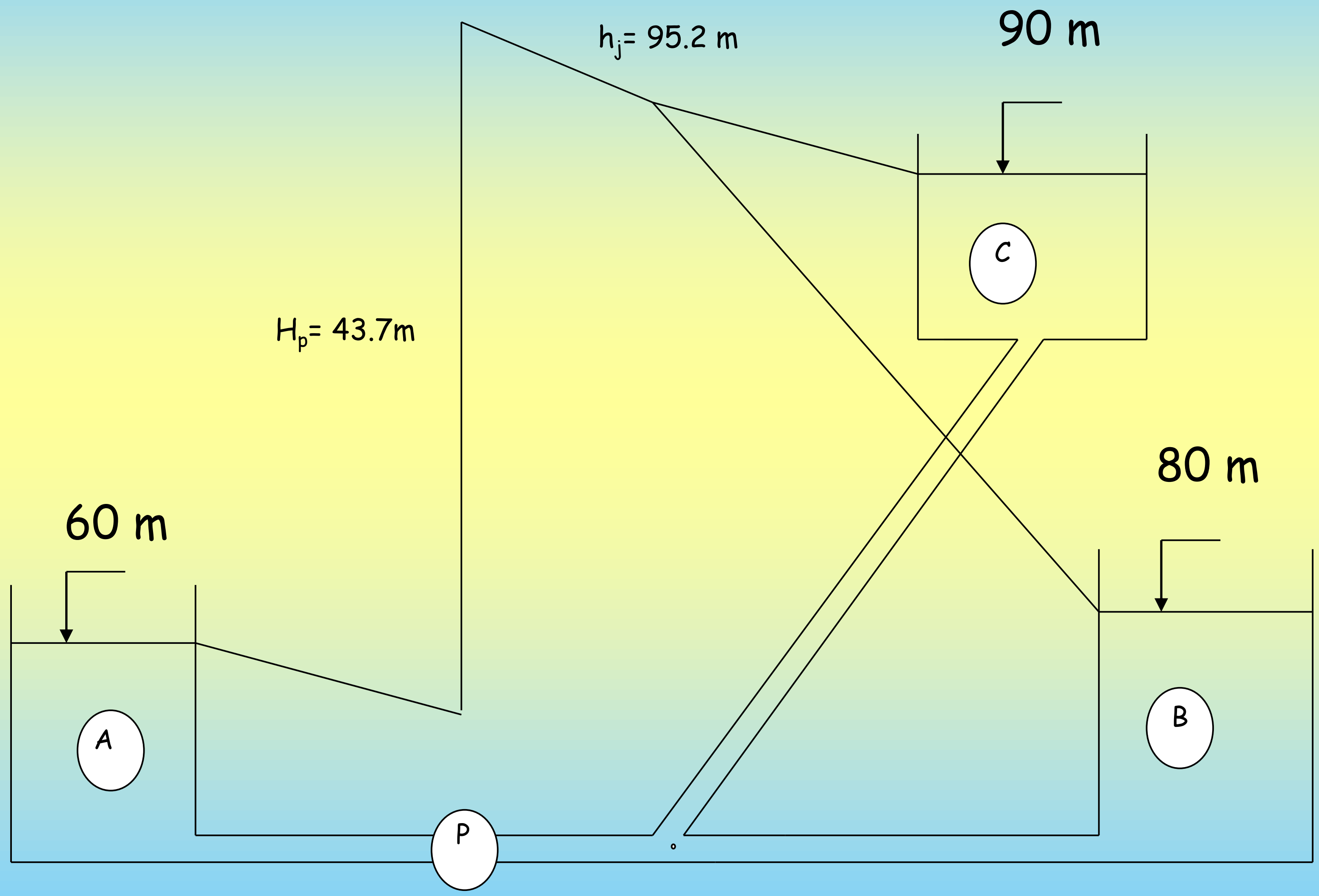
Hp = 50 – 128Q^2...Pump curve

- c) From Darcy-Weisbach equation, the head loss in each pipe is found as;
  - $h_{f1} = 8.5$ m
  - $h_{f2} = 15.75$ m
  - $h_{f3} = 5.2$ m

d) For $\eta = 0.8$ the required power is;

$$P_p = \frac{\gamma Q H_p}{\eta} = \frac{(9.81)(0.222)(43.7)}{0.8} = 118.96 \text{kW}$$

e) the piezometric line



Example 2.11

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$$H_p = 50 - 128Q^2$$

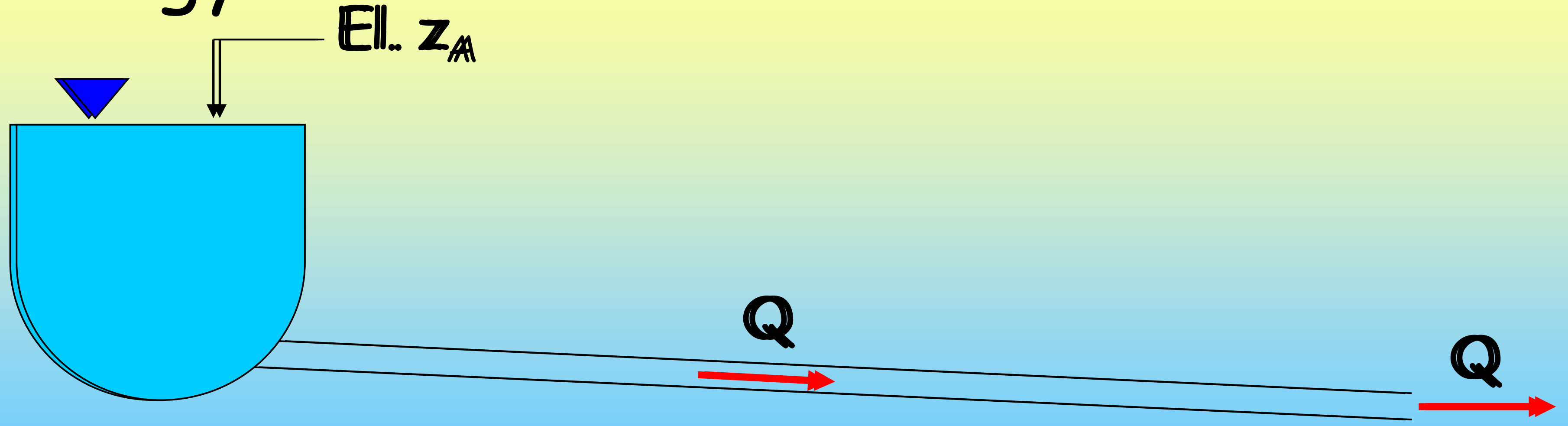
Water is to be transported from reservoir A to the junction, then to reservoirs B and C. Determine:

1. the discharge in each pipe
2. the head added to the system by the pump,
3. the head loss in each pipe for the computed discharges,
4. the power required by the pump if efficiency is $\eta = 0.8$ and
5. draw the piezometric line for the $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.015$ cm

GRAVITY PIPELINES

The pipelines through which flow is maintained by the action of gravity are known as **<u>gravity pipelines.</u>** In other words:

A gravity pipeline is the one in which flow takes place because of the potential energy in the reservoir.

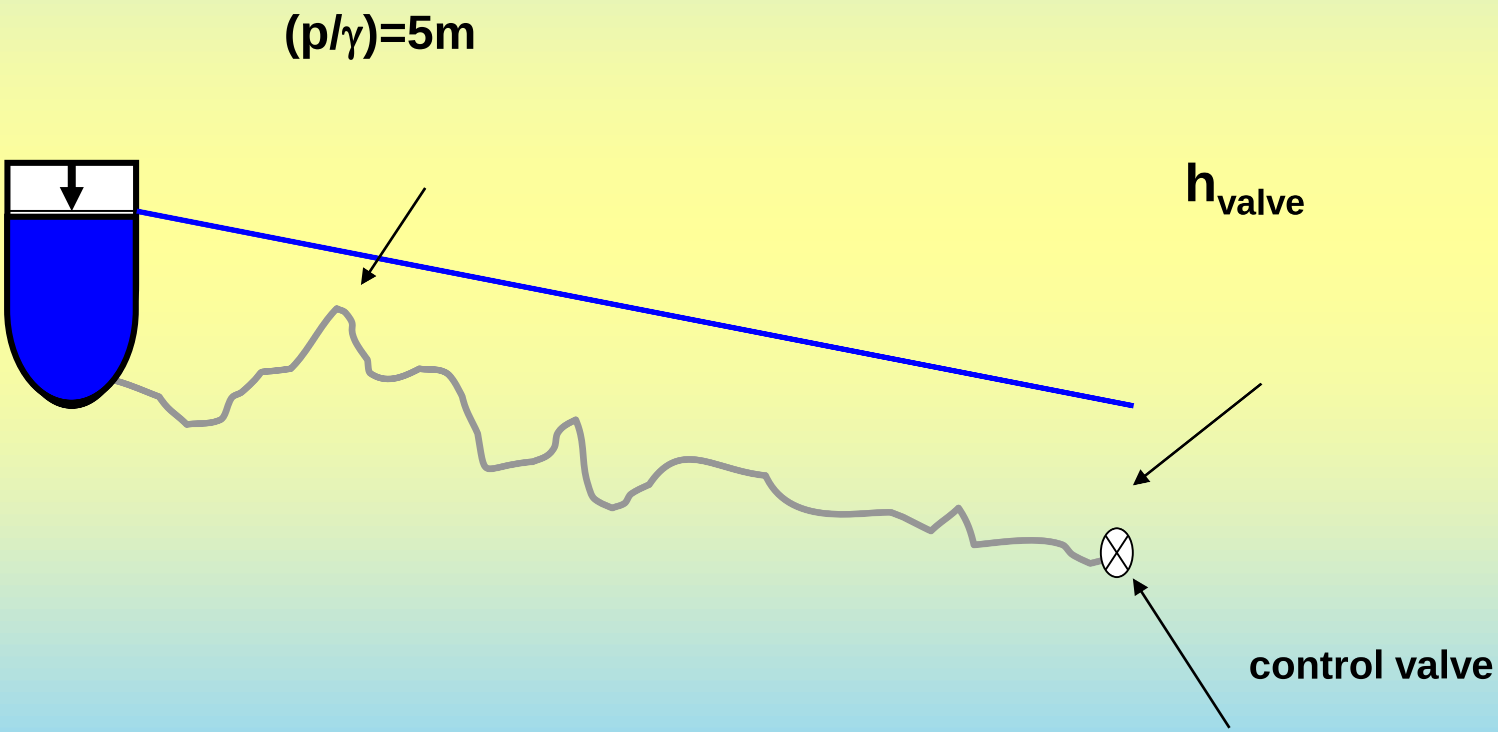


- In design of gravity pipelines, an optimum diameter is selected that minimizes the capital cost (The estimated total cost) and operation cost.
- Another criterion to select the pipe diameter is the average velocity in the pipe.
- There is also an upper and lower limits for the pressure in the pipe.
- Therefore, the hydraulic design criteria for gravity pipelines are:
- In the selection of the **diameter** of gravity pipe lines:
- Cost (capital + operation) is minimized;
- Depending on the type of pipe material a lower and an upper limits are set for the velocity as:

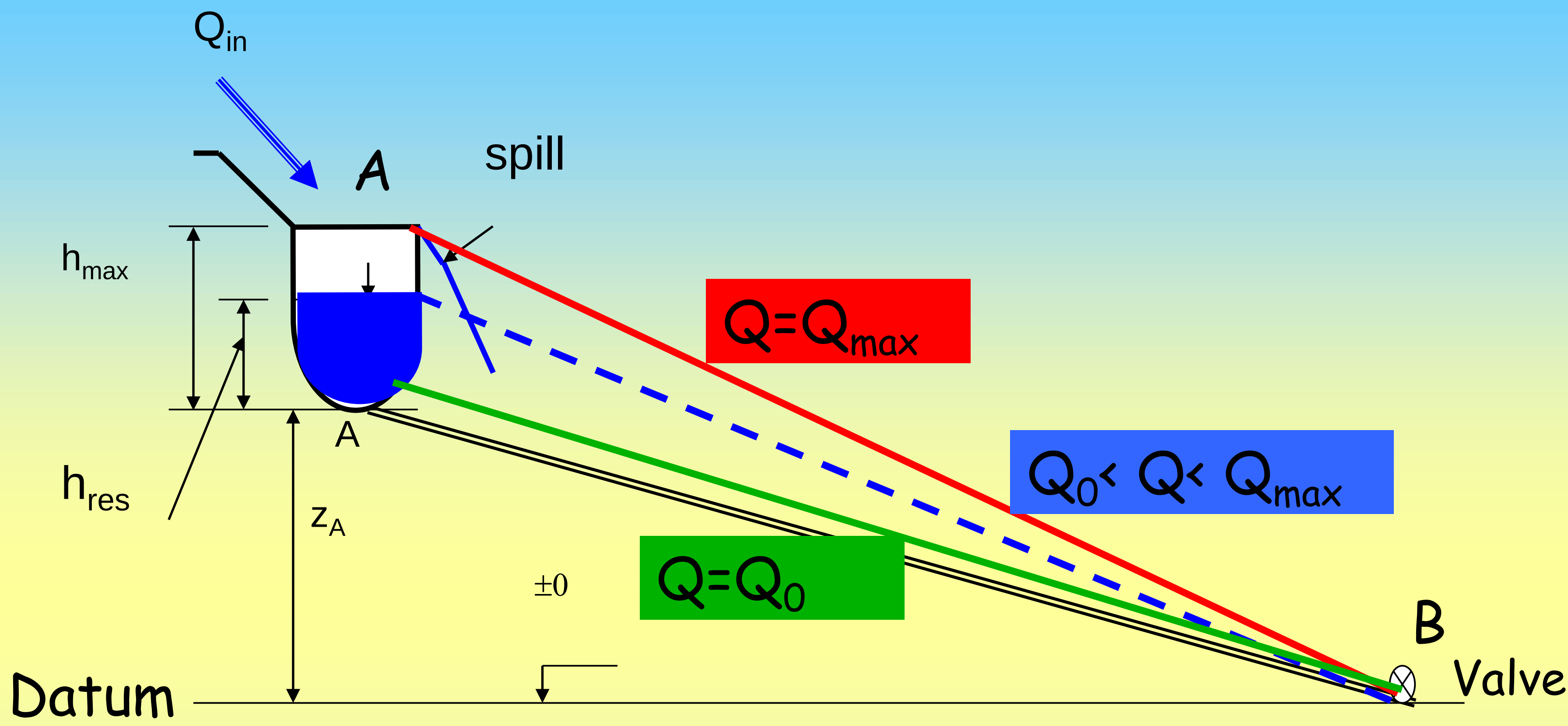
$$0.5 \text{ m/s} \leq V \leq 2 \text{ m/s};$$

- To prevent air entrainment minimum pressure head, $(p/\gamma)_{min}$ permitted along the pipeline is 5m.
- 3-5 m $\leq (p/\gamma) \leq$ 80 m for transmission lines
- 20-30 m $\leq (p/\gamma) \leq$ 80 m for city network

- A control valve might be used to control the discharge and pressure



Because of limits set forth on velocity and pressure head there are lower and upper bounds for the discharge through a gravity pipeline. Consider the following schematic representation of a gravity pipeline system shown below:



Energy Equation between points (A) & (B) is:

$H_A=H_B+h_L$, or in open form:

$$z_A + h_A + \frac{P_A}{\gamma} + \frac{V_A^2}{2g} = z_B + h_B + \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + h_L$$

- $P_A = P_B = 0$, $V_A = 0$, and $z_B = 0$.
- the maximum value of V=2 m/s. Therefore:

$$\frac{V_B^2}{2g} \leq \frac{2^2}{2x9.81} = 0.2 \text{ m} \quad \therefore \text{it can be neglected}$$

$$h_f = 8f \frac{L}{\pi^2 g D^5} Q^2 = KQ^2,$$

$$\text{where } K = 8f \frac{L}{\pi^2 g D^5} \Rightarrow h_f = KQ^2$$

Therefore, Energy Equation becomes:

$$h_A + z_A = h_f = KQ_{out}^2 \Rightarrow Q_{out} = \left[\left(h_A + z_A\right)/K\right]^{1/2}$$

(note that since V < 2 m/s velocity heads are neglected!)

A relationship between Q and the resrvoir level can be obtained as:

$$Q = \sqrt{\frac{h_A + z_A}{K}}$$

For a full reservoir $\longrightarrow$ $h_A = h_{max}$ $\longrightarrow$ $Q = Q_{max}$

$$h_A = h_{max} \qquad Q = Q_{max} \qquad \text{(since z and K are constants)}$$

$$Q_{max} = \sqrt{\frac{h_{max} + z_A}{K}}$$

= maximum discharge that may occur in the pipeline system is called

The system capacity

For empty reservoir $\longrightarrow$ $h_A = 0$ $\longrightarrow$ $Q = Q_{min} = Q_0$

Q_0 is the minimum flow rate, which will pressurize the pipe, in other words, it is the minimum flow rate for a full pipe flow.

$$\text{Therefore} \qquad Q_0 = \sqrt{\frac{z_A}{K}}$$

Remarks:

1. If $Q_{max} < Q_{in} \quad h = h_{max}$; Then there is a spill over the reservoir

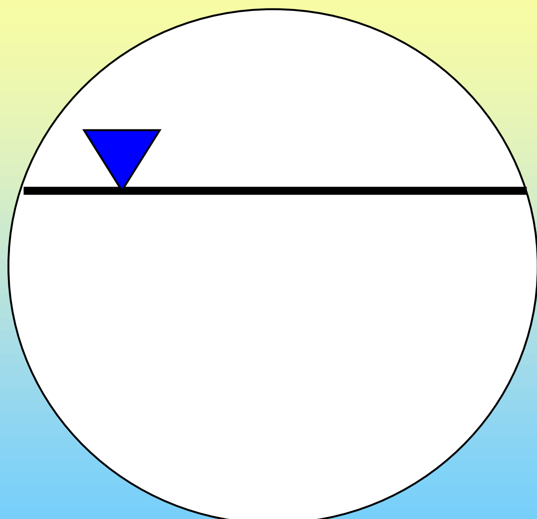
$$Q_{spill} = Q_{in} \qquad Q_{max}$$

2. If $Q_{min} < Q_{in} < Q_{max}$. then the reservoir level is :

$$0 < h_A < h_{max} ; \quad \text{and } Q_{spill} = 0$$

3. If $Q_{in} < Q_{min}$, Then the reservoir is empty,and free surface flow will occur in the pipe.

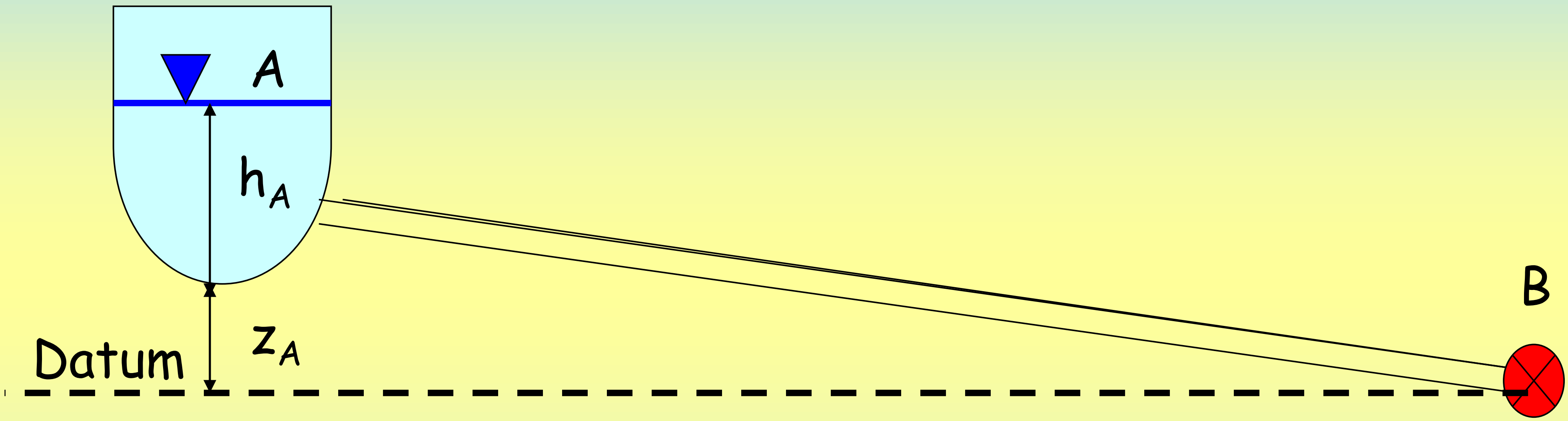
In other words, water will not fill the whole cross section of the pipe.



The free surface flow may be prevented by use of a valve at the pipe exit.

Valves control the flow rate by providing a mean to adjust the over all system loss coefficient to the desired value.

- Consider the same system with a valve at the end of the pipe:



Energy Equation between points (A) & (B):

$H_A = H_B + h_f + h_V$, or in open form:

$$z_A + h_A + \frac{P_A}{\gamma} + \frac{V_A^2}{2g} = z_B + h_B + \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + h_f + h_V$$

$P_A = P_B = 0$, $V_A = V_B = 0$, and $z_B = 0$. Also $h_f = KQ^2$ and the local loss can also be written as $h_V = K_V Q^2$

Therefore:

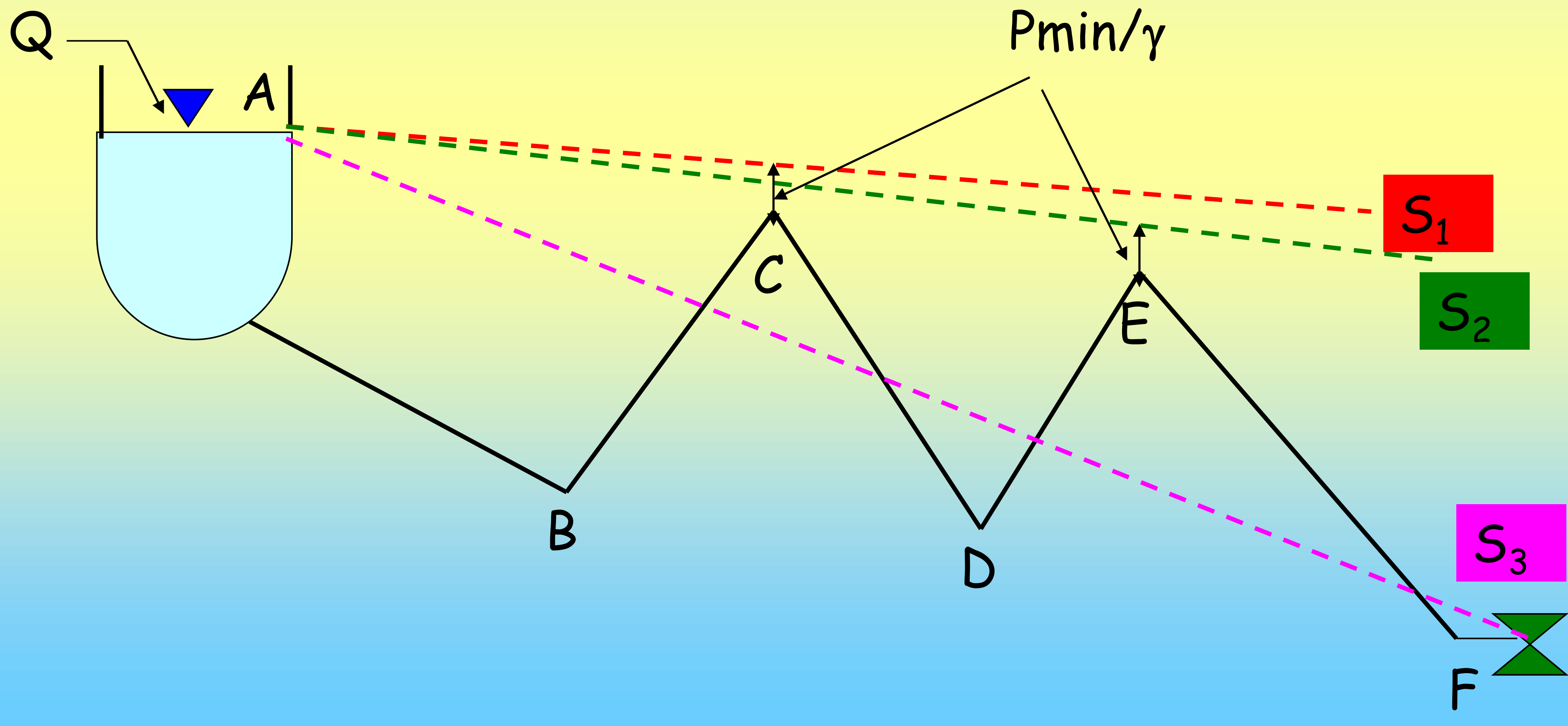
$z_A + h_A = (K + K_V)Q^2$, solving for Q:

$$Q = \sqrt{\frac{z_A + h_A}{K + K_V}}$$

- $\rightarrow$ when valve is closed $K_V \rightarrow \propto$ Therefore Q=0.
- Openining of the valve reduces the value of K_V, producing a desired flow rate.
- The head loss in valves is mainly a result of dissipation of kinetic energy of a high speed portion of flow.
- $\rightarrow$ when $Q<Q_0$, the control valve at the pipe exit will pressurize the flow in the pipe by dissipating the excess kinetic energy, h_V, and in turn increasing the water level in the reservoir.
- The magnitude of the valve loss depends on how much one needs to presurize the flow, that is the water depth in the reservoir.
- If the water level is to be kept constant to obtain a certain discharge, a control valve can be chosen accordingly.

Computation of Pipe Diameter for Gravity Pipelines

- Design Specifications:
- $V_{min} < V < V_{max}$
- $(P_{min}/\gamma) < P/\gamma < (P_{max}/\gamma)$



a) Determine the control points(C,E,F) and their topographic elevations z_C,z_E,z_F. Add the minimum required pressure head, P_{min}/γ, to these elevations (z_C & z_E), and then determine the minimum energy grade line slope, between the reservoir and control points. For the above case (z_C >,z_E >,z_F);

$$S_1 = \frac{H_A - \left(z_C + P_{\min}/\gamma\right)}{L_{AC}}$$

b) Compute the pipe diameter for line A-C using Darcy-Weisbach equation and select the nearest larger commercially available diameter.

$$D_{com} = \left(\frac{8fQ^2}{g\pi^2 S_1}\right)^{1/5} \Rightarrow \mathrm{D_{com}} = D_{available}$$

c) Compute velocity

$$V = \frac{4Q}{\pi D^2}$$

- If $V_{min} < V < V_{max}$, selected diameter is used in the project.

- If $V < V_{min}$, a booster pump must be installed at the reservoir site to increase the velocity to V_{min}. The additional head supplied by the booster pump, H_P, is computed from

$$H_A + H_P = z_C + \frac{P_{\min}}{\gamma} + f\,\frac{L}{D}\frac{V_{\min}^2}{2g}$$

- If $V > V_{max}$, reduce the velocity to V_{max} by increasing the pipe diameter from

$$\frac{\pi D^2}{4}\, x\, V_{\max} = Q$$

Since the velocity and hence head losses are now reduced, install a Pressure Reduction Valve (PRV), or allow increased pressures along the pipe as long as $P/\gamma < (P_{max}/\gamma)$.

d) After determining the diameter for pipe segment A-B-C and computing the piezometric level at point C (H_C), repeat the above procedure for the remaining segment C-D-E-F. Note that for the above example point E now becomes the control point. Therefore, first the diameter for segment C-D-E, and then the diameter of segment E-F is determined.

e) Finally, install a control valve at point F and determine the necessary head loss at the valve to maintain pressurized flow at segment E-F.

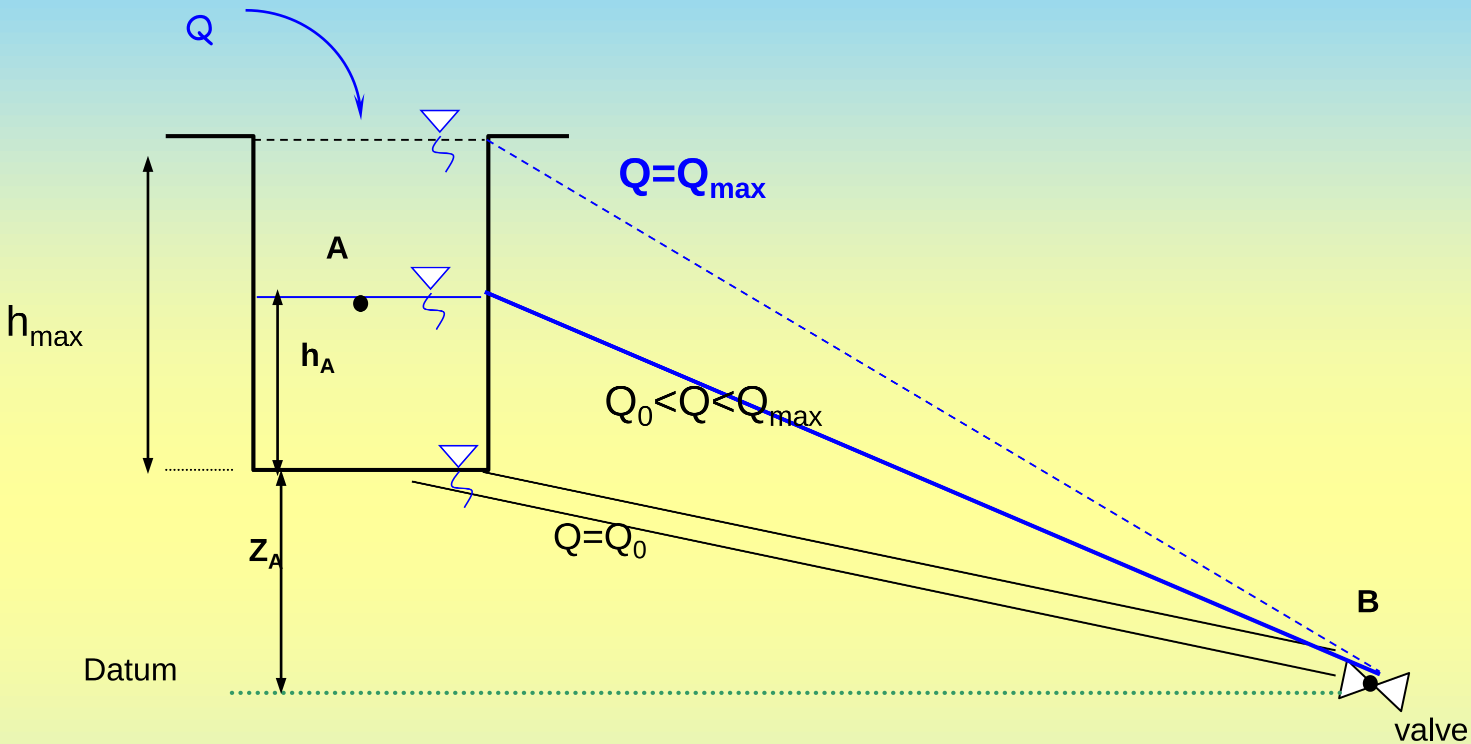
Example 2.12:

Consider the reservoir-pipe system given below, with following values.

Reservoir depth h_{max}=5m, z_A=5m, L=2000m, D=0.8m, f=0.02

Determine:

- a) The system capacity, Q_{max}.
- b) Minimum flow rate, Q_0.
- c) Spill flow rate, if Q=1.2m^3/s.
- D) Valve loss, h_v, if Q=0.5m^3/s.



Q
Q=Qmax
A
hmax
hA
Q0<Q<Qmax
ZA
Q=Q0
B
Datum
valve

Solution: if local losses except h_V (valve loss) are neglected:

$$Q_{max} = \sqrt{\frac{h_{max} + z}{K + K_V}} \qquad \text{where} \qquad K = \frac{8fL}{g\pi^2 D^5} = 10.09$$

K_V=0 when valve is fully open

a) $$Q_{max} = \left(\frac{5+5}{10.09}\right)^{1/2} \cong 1.0 \ \ m^3/s$$

b) $$Q_{min} = Q_0 = \sqrt{\frac{z}{K}} \longrightarrow Q_{min} = \left(\frac{5}{10.09}\right)^{1/2} \cong 0.70 \ \ m^3/s$$

c) Q=1.2 m^3/s > Q_{max} therefore spill over will occur. $Q_{spill} = 0.20\ m^3/s$

d)

Q=0.5 m^3/s, h_V=?

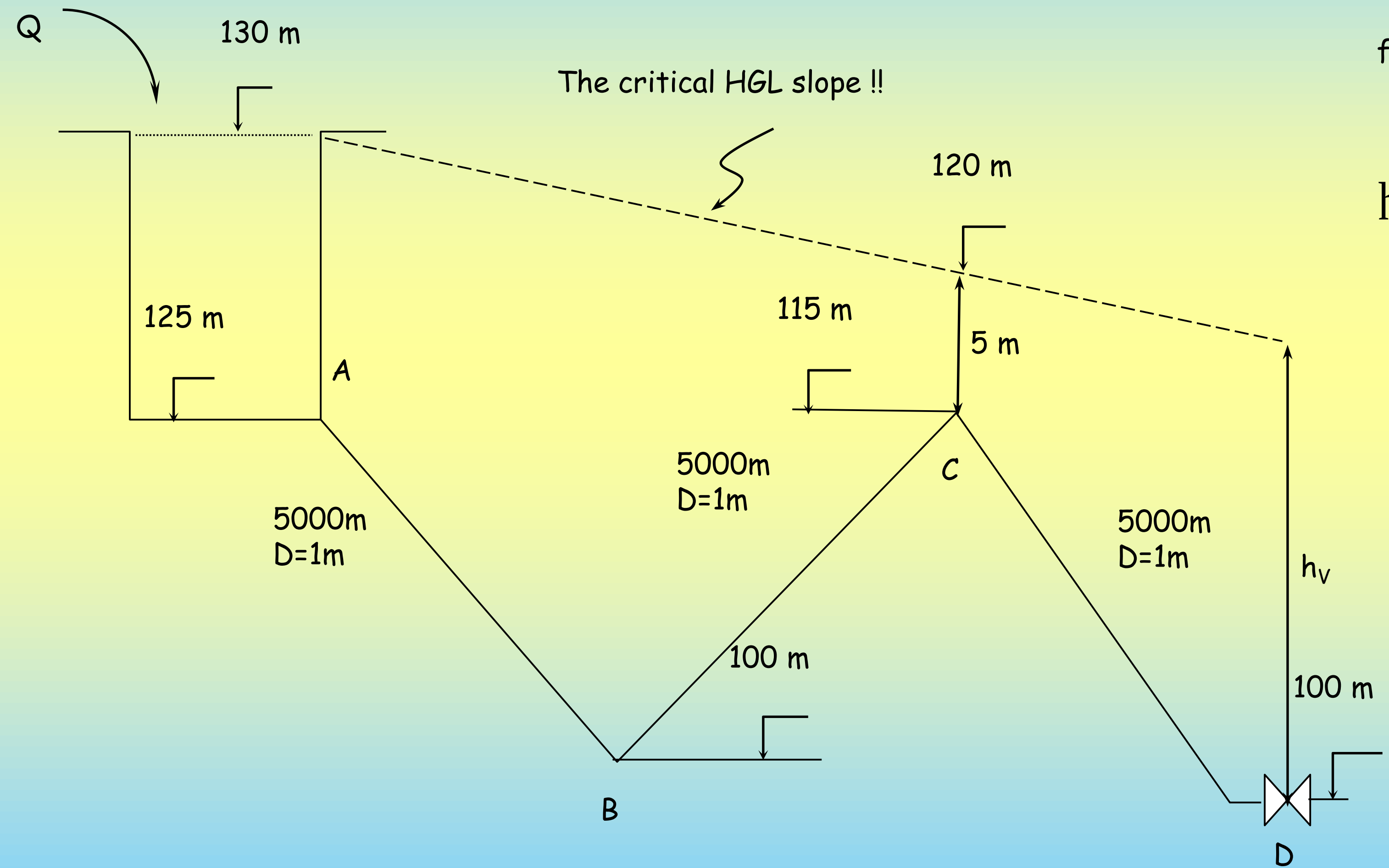
- The magnitude of the valve loss depends on the depth of water, we want to keep in the reservoir.
- If we need to maintain a water depth of 2 m, then h_A= 2 m., then:
- $H_A=H_B+h_{fAB}+h_V$
- $H_A=h_A+z=2+5=7$ m. $H_B \approx 0$, $h_{fAB}=KQ^2$
- $7=10.09Q^2+h_V \longrightarrow h_V=7-10.09(0.5)^2=4.48$ m

- Or the minimum value of the valve loss will be obtained when $h_A=0$:

$$h_{res} + z_A = h_V + KQ_{in}^2 \Rightarrow h_V \geq z_A - KQ_{in}^2$$

$$h_V \geq 5 - 10.09.(0.5)^2 \qquad h_V \geq 2.48 \text{ m}$$

- **Example 2.13:** Determine the flow capacity of the given gravity pipeline system. Use an air valve that operates under a pressure head of p/γ = 5m.



f=0.02

$$h_f = \frac{8fL}{g\pi^2 D^5} Q^2$$

- <u>Solution:</u>

$$S_{AC} = \frac{130-120}{10000} = 0.001 \qquad S_{AC} = \frac{130-100}{15000} = 0.002$$

Since $S_{AC} < S_{AD}$; $S_{AC}=0.001$ is the (milder) critical slope. Therefore at C an air valve is installed.

$$H_A = H_C + h_{\ell A-C} \; ; h_{\ell A-B} = h_{\ell B-C} = h_{\ell,5000} = 8.263Q^2$$

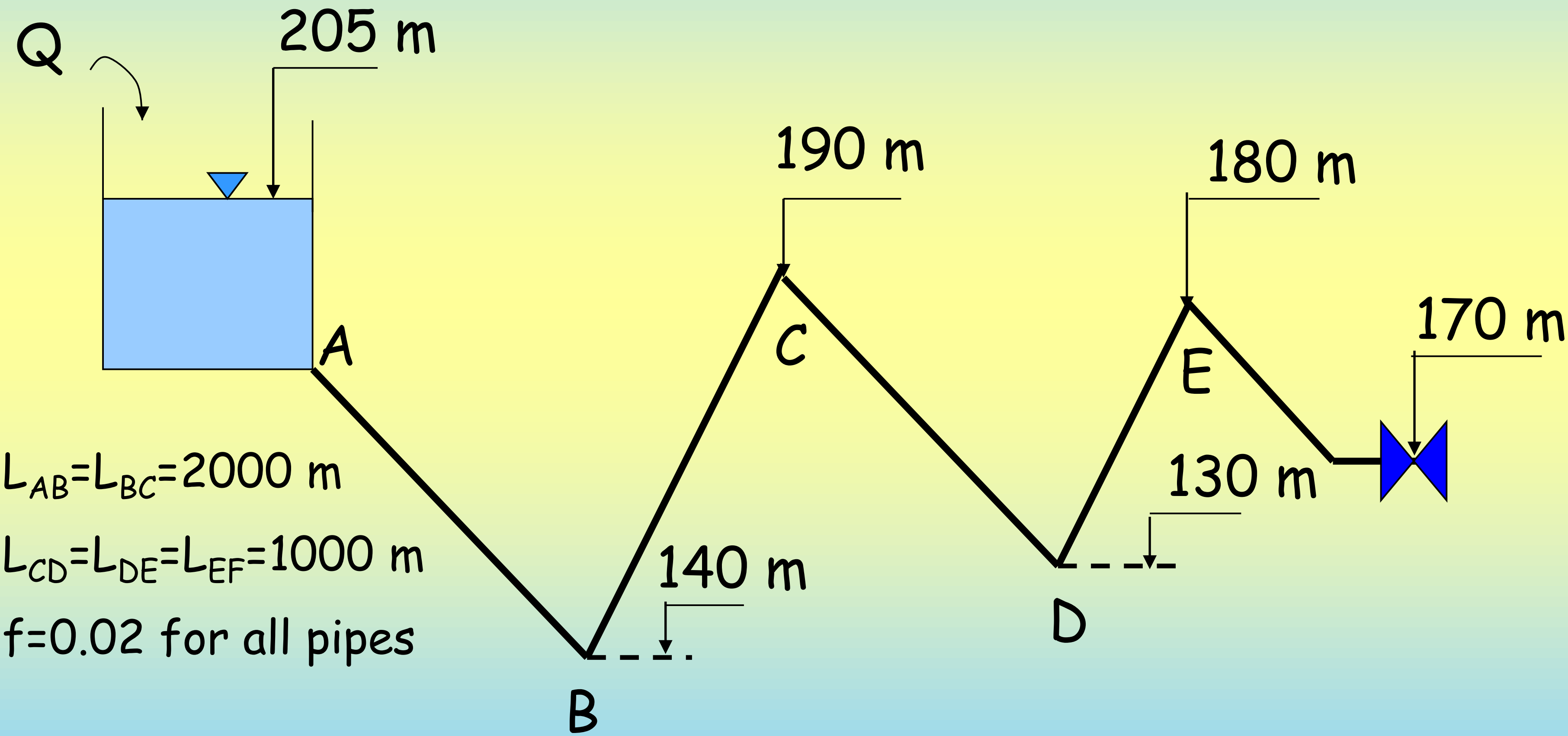
$$10 = 2x8.263Q^2 \Rightarrow Q = 0.78\,m^3/s$$

$$H_C = H_D + h_V \; ; h_{\ell C-D}$$

$$120 = 100 + h_V + 8.263x\,Q^2$$

h_V=15 m. (i.e at D at least 15 m of head must be dissipated.)

Example 2.14: Determine the pipe sizes of the following transmission line, which transmits $Q = 1\ m^3/s$



$L_{AB} = L_{BC} = 2000$ m

$L_{CD} = L_{DE} = L_{EF} = 1000$ m

f=0.02 for all pipes

- Design specifications:
- 0.5 m/s < V < 2.0 m/s
- 5 m < P/γ < 80 m

a) Control points:

$$\left.\begin{aligned}
S_{AC} &= \frac{205 \quad (190+5)}{4000} = 2.5\text{x}10^{3} \\
S_{AE} &= \frac{205 \quad (180+5)}{6000} = 3.33\text{x}10^{3} \\
S_{AF} &= \frac{205 \quad 170}{7000} = 5\text{x}10^{3}
\end{aligned}\right\} \longrightarrow S_{min}=S_{AC}$$

b) Diameter of line A-C, D_{AC}:

$$D_{AC} = {\frac{8x0.02x(1.0)^2}{g\pi^2(2.5x10^{3})}}^{1/5} = 0.92 \text{ m}$$

b) Take D_{AC}=1.0 m,

$V_{AC}=Q/A=4/(\pi x1^2)=1.27$ m/s O.K

$H_A=H_B+h_{fAB,}$ $H_B=H_A-h_{fAB}$

$$h_{fAB} = f\frac{L}{D}\frac{V_{AB}^2}{2g} = 002\frac{2000}{1}\frac{1.27^2}{2x9.81} = 3.3\,\text{m}$$

$H_B=205-3.3=201.7$ m $= z_B+P_B/\gamma+V_B^2/2g$

$P_B/\gamma=201.7-140=61.7$ m< 80 m **O.K**

c) Diameter of line C-E, D_{CE}

$$H_C = H_A \quad h_{fAC} = 205 \quad f\frac{L}{D}\frac{V_{AB}^2}{2g} = 205 \quad 0.02\frac{4000}{1}\frac{1.27^2}{19.62} = 205 \quad 6.6 = 198\,\mathrm{m}$$

$$\left.\begin{aligned} S_{CE} &= \frac{198 \quad (180+5)}{2000} = 6.5\mathrm{x}10^{\ \ 3} \\ S_{CF} &= \frac{198 \quad 170}{3000} = 9.33\mathrm{x}10^{\ \ 3} \end{aligned}\right\} \longrightarrow$$

S_{min}=S_{CE}=6.5x10^{-3}

Therefore Diameter of line C-E, D_{CE} becomes:

$$D_{CE} = \left(\frac{8\mathrm{x}0.02\mathrm{x}(1.0)^2}{g\pi^2(6.5\mathrm{x}10^{\ \ 3})}\right)^{1/5} = 0.76\ \mathrm{m}$$

Let us take D_{CE}=0.75 m,

V_{CE}=Q/A=4/(πx0.75^2)=2.26 m/s > 2.0 m/s

So re-compute the diameter taking V=2.0 m/s

$$A = \frac{Q}{V} = \frac{1}{2} = \frac{\pi D_{CE}^2}{4} \longrightarrow$$

D_{CE}= 0.80 m

$$H_E = H_C \quad h_{fCE} = 198 \quad f\frac{L}{D}\frac{V_{AB}^2}{2g} = 198 \quad 0.02\frac{2000}{0.8}\frac{2^2}{19.62} = 188\,\mathrm{m}$$

P_E/γ > 5 m.

For the last segment, line EF:

$$S_{EF} = \frac{188 \quad 170}{1000} = 18x10^{\;3} \qquad D_{EF} = \frac{8x0.02x(1.0)^2}{g\pi^2(18x10^{\;3})}^{1/5} = 0.62 \text{ m}$$

$V_{EF}=Q/A=4/(\pi x0.62^2)=3.31$ m/s > 2.0 m/s

Therefore take V_{EF} = 2.0 m/s

$$A = \frac{Q}{V} = \frac{1}{2} = \frac{\pi D^2_{EF}}{4} \longrightarrow \quad D_{EF} = 0.80 \text{ m}$$

For this velocity:

$$H_E = H_F + h_{fEF}$$

$$h_{fEF} = f\frac{L}{D}\frac{V^2_{EF}}{2g} = 0.02\frac{1000}{0.8}\frac{2^2}{2x9.81} = 5.10 \text{ m}$$

$H_F = H_E - h_{fEF}$

$H_F = 188 - 5.10 = 182.90$ m This is the total head required at point F. But the the total head that exist at point F is $H_F = 170$ m.

Therefore we have to install a valve which will reduce the head to 170 m. Therefore energy equation becomes:

$H_E = H_F + h_{fEF} + h_V \longrightarrow$

$h_V = H_E - (H_F + h_{fEF}) = 188 - (170 + 5.10) = 13$ m

Therefore supply a valve which produces a head loss of $h_V = 13$ m when $Q = 1\ m^3/s$.