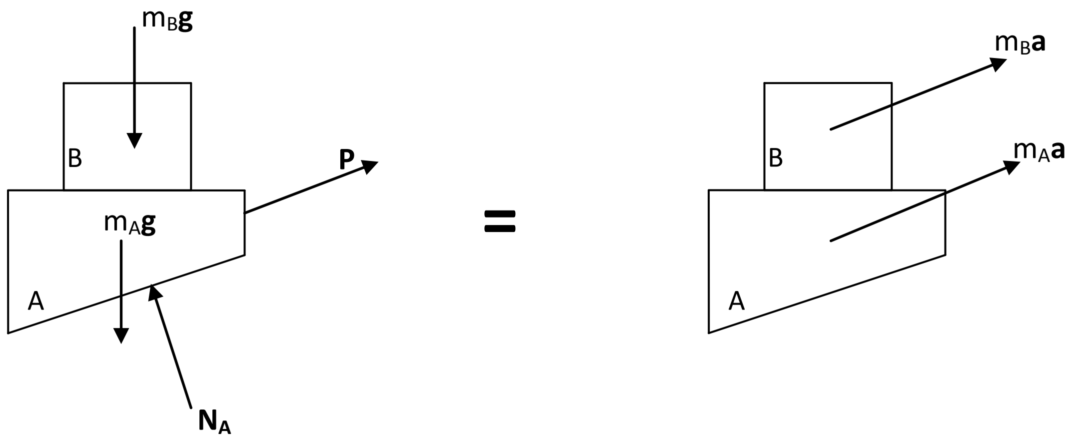
AE 262 DYNAMICS

2013-2014 SPRING SEMESTER

HOMEWORK #2 - SOLUTIONS

Solution to Question 1-)

In this figure vectors were defined by bold characters.



Both of the blocks move together, thus they have a common acceleration. Use blocks A and B together as a freebody as shown above.

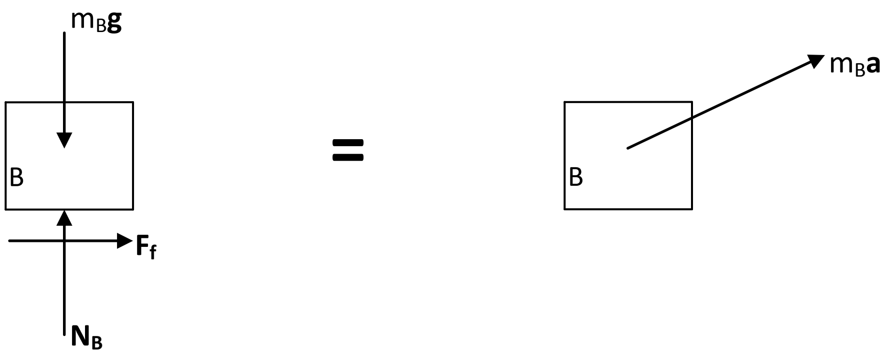
$$\overset{+}{\nearrow} \sum F = ma :$$

$$P - m_A\, g\sin(30) - \; m_B\; g\; \sin(30) = (m_A + \; m_B)\; a$$

$$a = \frac{P}{m_A + \; m_B} - \; g\; \sin(30) = \frac{500}{50} - \; 9.81\; x\, \sin(30)$$

$$= 5.095\; m/s^2$$

If block B is considered as a free body as shown below. In the figure below vectors are shown with bold letters:



$$\xrightarrow{+} \quad \sum F = m_B a \cos(30): \qquad F_f = m_B\, a\cos(30)$$

$$F_f = (10)(5.095)\cos(30) = 44.124\ N$$

$$+\uparrow \quad \sum F = m_B a\ \sin(30): \qquad N_B - \ m_B g\ = m_B a \sin(30)$$

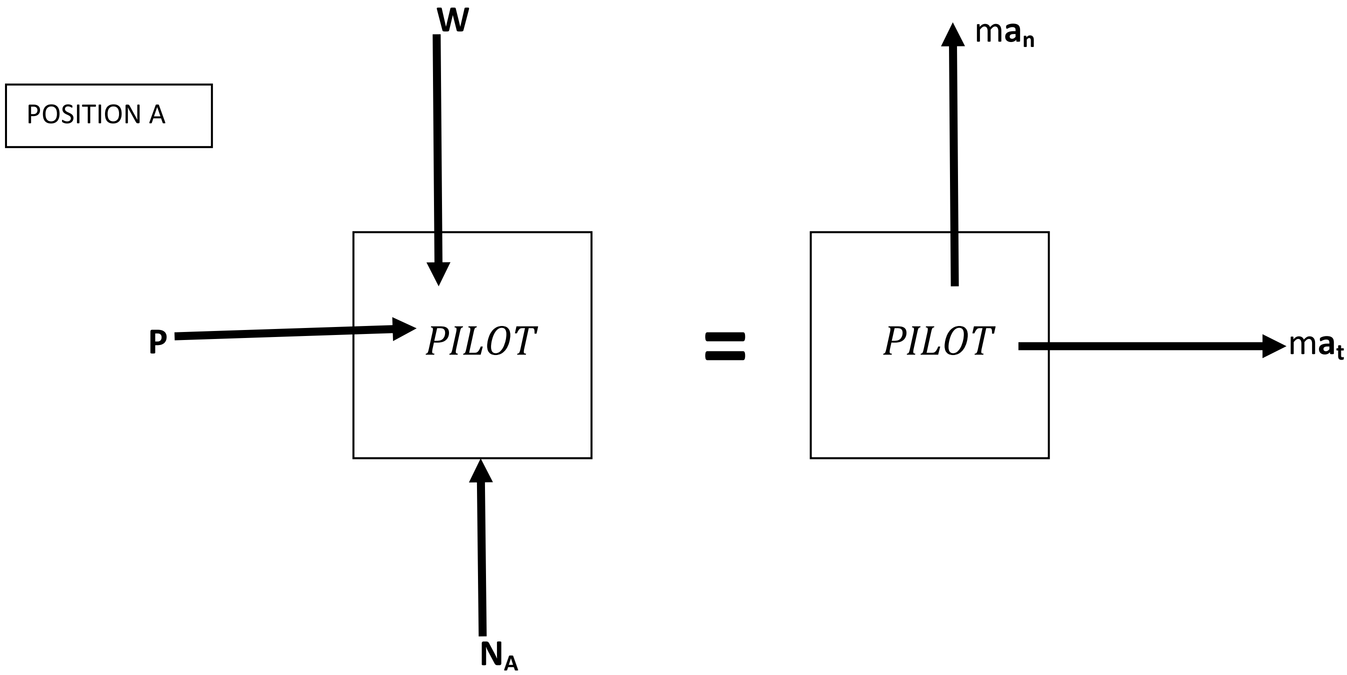
$$N_B = \ m_B\,[g + a\sin(30)] = 10\,[9.81 + 5.095\sin(30)]$$

$$= 123.575\ N$$

Minimum coefficient of static friction is:

$$\mu_{min} = \frac{F_f}{N_B} = \frac{44.124}{123.575} = 0.357$$

Solution to Question 2-)



The vectors are shown with bold characters in the figure above.

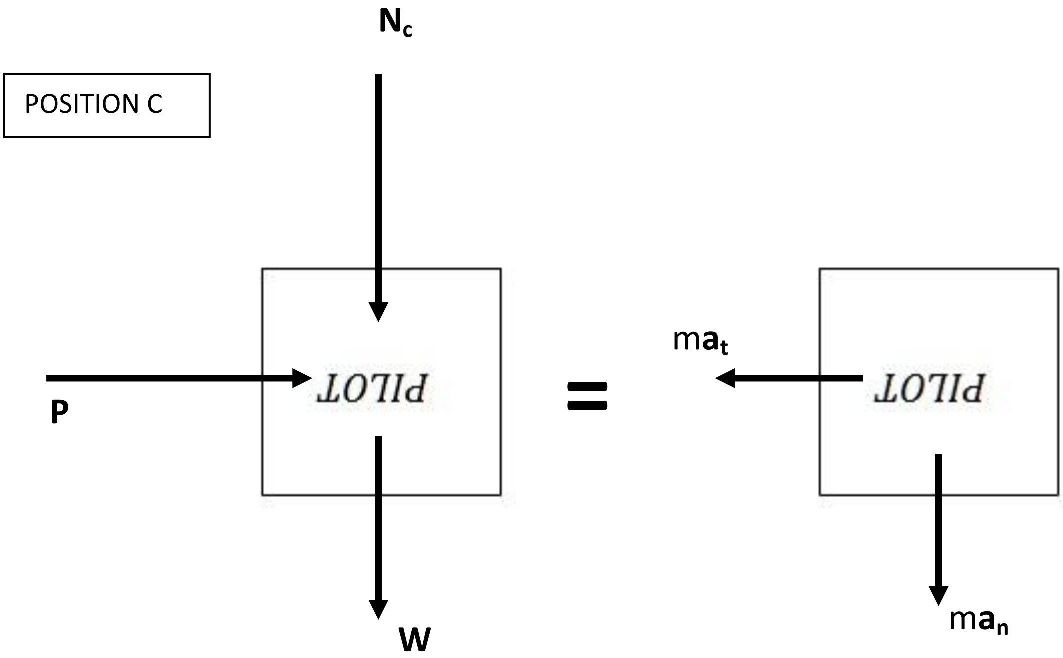
The pilot is shown at point A in the figure above. At position A, the vertical component of apparent weight is shown as $\mathbf{N_A}$.

$$+\uparrow \; \sum F = ma_N: \qquad N_A - W = \left({}^{W}\!/_{g}\right) a_N$$

$$1690 - (54.4)(9.81) = a_N(54.4)$$

$$\Rightarrow a_N = 21.256 \; {}^{m}\!/_{s^2}$$

$$v_a{}^2 = \rho a_N = (1097)(21.256) = 23317.832 \; {}^{m^2}\!/_{s^2}$$



The vectors are shown with bold characters in the figure above.

$$+\downarrow\ \sum F = m\,a_N: \qquad N_C + W = \left(W/g\right) a_N$$

$$355 + (54.4)(9.81) = 54.4a_N$$

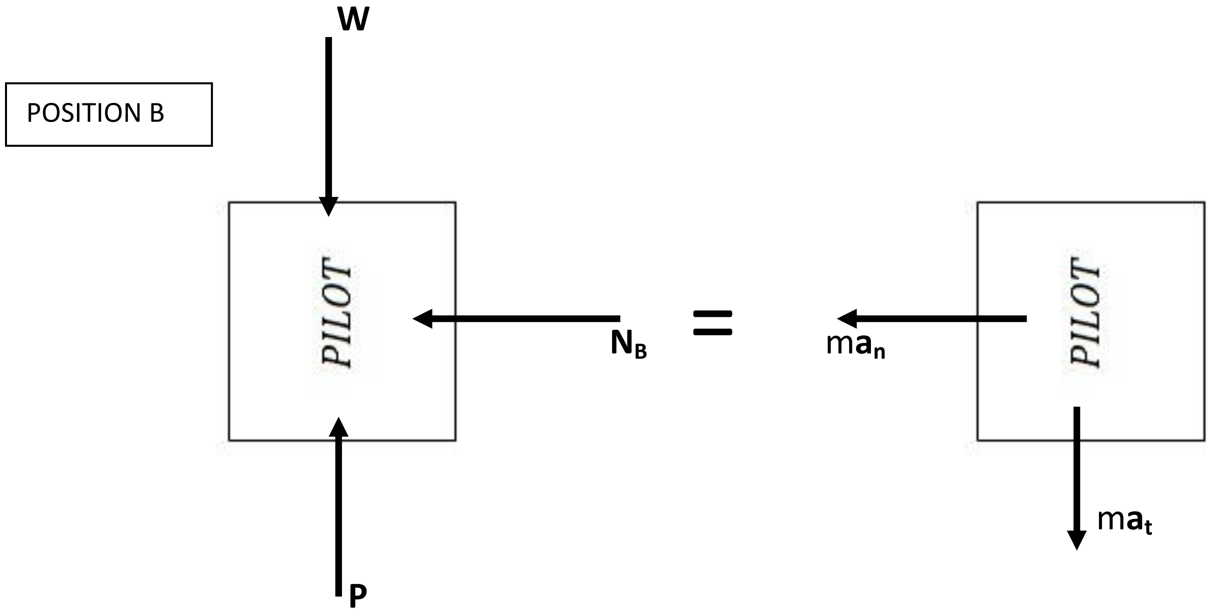
$$\Rightarrow a_N = 16.336\ {}^{m}/_{s^2}$$

$$v_c{}^2 = (1097)(16.336) = 17920.592\ {}^{m^2}/_{s^2}$$

Length of the arc ABC is; $s_{ABC} = \pi\rho = \pi(1097) = 3446.327\,m$

Then a_t can be calculated using $v_c{}^2 - v_A{}^2 = 2a_t s_{AC}$

$$a_t = \frac{v_c{}^2 - v_A{}^2}{2s_{AC}} = \frac{17920.592 - 23317.832}{(2)(3446.327)} = -0.783042\ {}^{m}/_{s^2}$$



The vectors are shown with bold characters in the figure above.

At position B

$$s_{AB} = \frac{\pi}{2}\rho = \frac{\pi}{2}1097 = 1723.16357m$$

$$v_B{}^2 = v_A{}^2 + 2a_t s_{AB} = 23317.832 \; + (2)(-0.783042)(1723.16357)$$
$$= 20619.2131 \; {m^2}/{s^2}$$

Effective forces at point B:

$$ma_N = m\frac{v_B{}^2}{\rho} = 54.4\frac{20619.2131}{1097} = 1022.502 \; N$$

$$ma_t = (54.4)(-0.783042) = -42.5974848 \; N$$

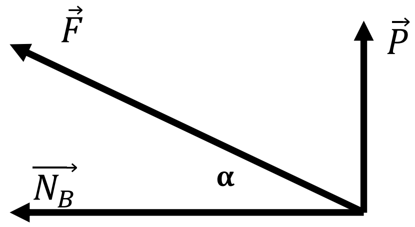
$$P - W = -ma_t \quad \Rightarrow \quad P = -ma_t \; + W$$

$$P = -(54.4)(0.783042) + (54.4)(9.81) = 491.066 \; N$$

$$\leftarrow + \ \sum F = m\,a_N; \qquad N_B = ma_N = (54.4)(18.796) = 1022.502\ N$$

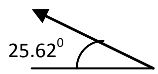
Then the force exerted by the seat:

$$F = \sqrt{{N_B}^2 + P^2} = \sqrt{1022.502^2 + 491.066^2} = 1134.308\ \text{N}$$



$$\alpha = tan^{-1}\left(\frac{491.066}{1022.502}\right) = 25.62^o$$

$$\vec{F} = 1134.308\ N$$



Solution to Question 3-)

$T_1 = 0,\ V_{1e} = \ V_{1g} = 0$ Constraint: $y_B = \ 2x_A$

$$T_2 = \frac{1}{2}M_A{v_A}^2 + \frac{1}{2}M_B{v_B}^2$$

$$= \frac{1}{2}\ (4\ kg)\left(\frac{v_B}{2}\right)^2 + \frac{1}{2}(1.5\ kg){v_B}^2 = 1.25{v_B}^2$$

Part (a)

$$y_B = 0.15\ m, \qquad x_A = 0.075\ m,$$

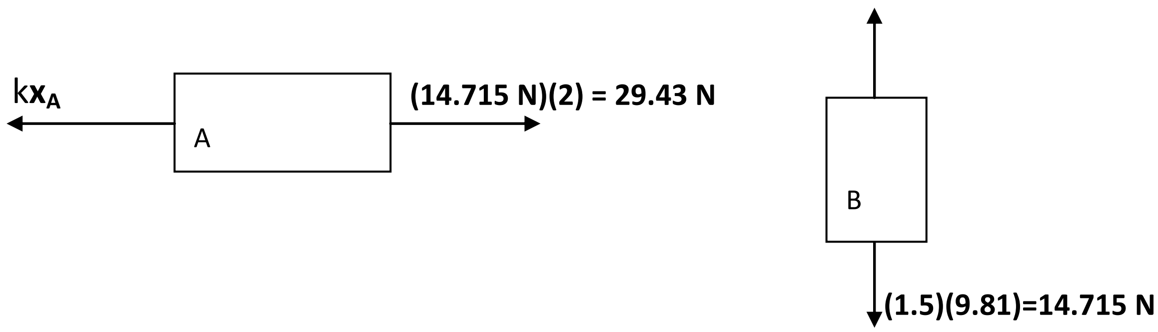
$$V_{2e} = \frac{1}{2}\ (300\ {}^{N}/_{m})(0.075\ m)^2 = 0.84375\ Nm$$

$$V_{2g} = -(1.5)(9.81)(0.15) = -2.20725\,J$$

$$T_1 + V_1 = T_2 + V_2\,; \qquad 0 = 1.25{v_B}^2 + 0.84375 - 2.20725$$

$$\Rightarrow\ v_B = 1.044\ {}^{m}\!/_{s}$$

Part (b)



$$\xrightarrow{+}\quad \sum F_x = m\,a_x; \qquad kx_A - 2T = 0; \qquad \vec{T} = cord\ tensile\ force$$

$$\frac{2T}{k} = \frac{2(14.715)}{300}; \qquad x_A = 0.0981\,m; \qquad y_B = 2\,x_A = 0.1962\,m$$

$$V_{2g} = (-14.715\,N)(0.1962\,m) = -2.8871\,Nm$$

$$V_{2e} = \frac{1}{2}\left(300\ {}^{N}\!/_{m}\right)(0.0981\,m)^2 = 1.4435\,Nm$$

$$T_1 + V_1 = T_2 + V_2; \qquad 0 = 1.25{v_B}^2 - 1.4435$$

$$\Rightarrow\ v_B = 1.075\ {}^{m}\!/_{s}$$

Part (c)

$$T_2 = 0; \qquad V_2 = 0 = \frac{1}{2}(300)\left(\frac{y_B}{2}\right)^2 - 14.715 y_B$$

$$\Rightarrow \; y_B = 0.392\,m = 392\,mm$$

$$\overrightarrow{y_B} = 392\,mm \;\downarrow$$

Solution to Question 4-)

Use conservation of energy and conservation of angular momentum

Conservation of angular momentum

$$r_1 m\, v_{1\theta} = r_2 m\, v_{2\theta} \qquad \Rightarrow \qquad v_{2\theta} = \frac{r_1\, v_{1\theta}}{r_2} = \frac{(0.2)(6)}{0.5} = 2.4\ {}^{m}/_{s}$$

Conservation of energy

$$T_1 + V_1 = T_2 + V_2$$

@ 1 $$T_1 = {}^{1}/_{2}\, m v_1{}^2 = {}^{1}/_{2}\,(4\,kg)(6\ {}^{m}/_{s})^2 = 72\,J$$

$$V_1 = {}^{1}/_{2}\, k x_1{}^2 = {}^{1}/_{2}\,(1500\ {}^{N}/_{m})(0.2\,m - 0.4\,m)^2 = 30\,J$$

@ 2 $$T_2 = {}^{1}/_{2}\, m v_{2\theta}{}^2 + {}^{1}/_{2}\, m v_{2r}{}^2 = {}^{1}/_{2}\,(4)(2.4)^2 + {}^{1}/_{2}\,(4) v_{2r}^2$$

$$= 11.52 + 2 v_{2r}{}^2$$

$$V_2 = {}^{1}/_{2}\, k x_2{}^2 = {}^{1}/_{2}\,(1500\ {}^{N}/_{m})(0.5\,m - 0.4\,m)^2 = 7.5\,J$$

Substituting into: $$T_1 + V_1 = T_2 + V_2$$

$$72 + 30 = 11.52 + 2 v_{2r}{}^2 + 7.5$$

$$\Rightarrow \qquad v_{2r} = 6.44\ {}^{m}/_{s}$$