<u>PAYBACK PERIOD ANALYSIS</u>

Payback analysis (also called payout analysis) is another form of sensitivity analysis that uses a PW equivalence relation. Payback can take two forms: one for $i > 0$ % (also called *discounted payback)* and another for $i = 0$ % (also called *no-return payback). The payback period* n_p is the time, usually in years, it will take for estimated revenues and other economic benefits *to recover the initial investment P and a specific rate of return i %.* The n_p value is generally not an integer.

The payback period should be calculated using a required return that is greater than 0 %. In practice, however, the payback period is often determined with a no-return requirement *(i* = 0%) to initially screen a project and determine whether it warrants further consideration.

To find the discounted payback period at a stated rate $i > 0\%$, calculate the years n_p that make the following expression correct.

$$0 = -P + \sum_{t=1}^{t=n_p} NCF_t \, (P/F, i, t)$$

NCF is the estimated net cash flow for each year t, where NCF = receipts – disbursements (Net Cash-Flow). If the NCF values are equal each year, the P/A factor may be used to find n_p.

$$0 = -P + NCF \, (P/A, i, n_p)$$

After n_p years, the cash flows will recover the investment and a return of i %. If, in reality, the asset or alternative is used for more than n_p years, a larger return may result; but if the useful life is less than n_p years, there is not enough time to recover the initial investment and the *i%* return. It is very important to realize that in payback analysis, *all net cash flows occurring after* n_p *years are neglected.* This is significantly different from the approach of all other evaluation methods (PW, AW (annual worth), ROR, B/C) where all cash flows for the entire useful life are included. As a result, payback analysis can unfairly bias the selection of alternatives. Therefore, the payback period, n_p, *should not be used as the primary measure of worth to select an alternative.* It provides initial screening or supplemental information in conjunction with an analysis performed using the PW or AW method.

No-return payback analysis determines n_p at i = 0%. This n_p value serves merely as an initial indicator that a proposal is viable and worthy of a full economic evaluation. To determine the payback period, substitute i = 0 % in the second equation and find n_p.

$$0 = -P + \sum_{t=1}^{t=n_p} NCF_t$$

For a uniform net cash flow series, the above equation is solved for n_p directly.

$$n_p = \frac{P}{NCF}$$

An example use of *no-return payback,* as an initial screening of proposed projects, can be the case where a president of a corporation, who absolutely insists that every project must recover the investment in 3 years or less. Therefore, no proposed project with $n_p > 3$ at i = 0% should be considered further.

As with n_p for i > 0 %, it is incorrect to use the no-return payback period to make final alternative selections. It *neglects any required return,* as the time value of money is omitted, and it *neglects all net cash flows after* time n_p, including positive cash flows that may contribute to the return on investment.

In general, the conclusions are: a 10% return requirement increases payback from 8 to 12 years; and when cash flows expected to occur after the payback period are considered, project return increases to 15 % per year.

If two or more alternatives are evaluated using payback periods to indicate initially that one may be better than the others, the primary shortcoming of payback analysis (i.e., neglection of cash flows after n_p) may lead to an economically incorrect decision. When cash flows that occur after n_p are neglected, it is possible to favor short-lived assets though longer-lived assets may produce a higher return. In these cases, PW or AW analysis should always be the primary alternative evaluation.

The benefits of the payback period can be identified in twofold. Firstly, the calculations can be easily made by people unfamiliar with economic analysis, especially in analysis of no-return payback period. One does not need to know how to use gradient factors, or even to have a set of compound interest tables. Second, payback period is a readily understood concept.

Moreover, payback period does give us a useful measure, telling us how long it will take for the cost of the investment to be recovered from the benefits of the investment. Businesses and industrial firms are often very interested in this time period: a rapid return of invested capital means that it can be re-used sooner for the other purposes by the firm.

In summary, the payback period gives some measure of the rate at which a project will recover its initial outlay. This piece of information is not available from the present value or the internal rate of return. The payback period may not be used as a direct figure of merit, but it may be used as constraint: no project may be accepted unless its payback is shorter than some specified period of time.

Example: A company has two machine alternatives whose economic lives are 6 years. The price and annual income of these machines are given in the following table. According to the no return payback period, determine the alternative the company should invest.

Alternatives	Cost (TL)	Annual income (TL/yr)
Machine A	200 000	45 000
Machine B	300 000	60 000

Payback Period$_1$= 200 000/45 000 = 4.4 years

Payback Period$_2$= 300 000/60 000 = 5 years

According to the payback periods, first alternative should be preferred.

Example: A company wants to buy a production device for their new factory. They have two alternatives, whose cash flows are given in the following table. According to these cash flows, determine the no return payback period of these alternatives.

	Alternative A	Alternative B
Cost	3 000 000 TL	3 500 000 TL
Annual income	1 200 000 TL first year, decreasing by 300.000 TL per year thereafter	100 000 TL for the first year, increasing 300 000 TL per year thereafter.
Useful life	4 years	8 years

Alternative A

Years	0	1	2	3	4
Cash Flow	-3 000 000	1 200 000	900 000	600 000	300 000
Cumulative value	-3 000 000	-1 800 000	-900 000	-300 000	0

PB_A= 4 years

Alternative B

Years	0	1	2	3	4	5
Cash Flow	-3 500 000	100 000	400 000	700 000	1 000 000	1 300 000
Cumulative value	-3 500 000	-3 400 000	-3 000 000	-2 300 000	-1 300 000	0

PB_B= 5 years

According to the payback periods, alternative A should be preferred.

Example: A construction company allocated a total of $18 million to a research on innovative construction methods that will improve their construction activities. The results are estimated to positively impact the net cash flow at the start of 6^{th} year, and for the foreseeable future, at an average level of $6 million per year. As an initial screening for economic viability, determine both the no-return and i = 10% payback periods.

The NCF for years 1 through 5 is $0 and $6 million thereafter. Let x = the number of years beyond 5 when NCF > 0. For no-return payback. In $ million units,

i = 0%: 0 = -18 + 5(0) + x (6)

$n_p = 5 + x = 5 + 3 = 8$ years

for i = 10%

i= 10%: 0= -18+ 6 (P/A, 10%, x) (P/F, 10%, 5)

(P/A, 10%, x)= $\frac{18}{6 *(0.6209)}$= 4.8319

x= 6.9 years

n_p= 5+7=12 years (rounded up)

Example: Compute the payback period of the following cash flow both for the no return and i = 10 % payback periods.

End of year	0	1	2	3	4	5	6
Cash Flow	-100	25	30	25	-10	45	45

a) For i= 0%, $n_p = 4 + \frac{100-(25+30+25-10)}{45}$ = 4. 66 years
b) 0 = -100 + 25 (P/A, 10 %, 3) + 5 (P/F, 10 %, 2) – 10 (P/F, 10 %, 4) + 45 (P/A, 10 %, x) (P/F, 10%, 4)

   100 = 25* (2. 4869) + 5* (0. 8264) -10 (0.6830) + 45 (P/A, 10 %, x) (0.6830)

   (P/A, 10 %, x) = $\frac{40.5255}{30.735}$ = 1.3185 $\Longrightarrow$

   (by interpolation)

   n=1 $\Longrightarrow$ (P/A, 10 %, 1) = 0.9091

   n=2 $\Longrightarrow$ (P/A, 10 %, 2) = 1.7355

   x= 1+ $\frac{1.3185-0.9091}{1.7355-0.9091}$ = 1.495

   n_p = 4+ 1.495 = 5.495 years

OR By trial and error

x = 1.483 years

n_p = 4+ 1.483 = 5.483 years

OR

Years	0	1	2	3	4	5	6
Cash Flow	-100	25	30	25	-10	45	45
PW	-100	22.73	24.79	18.78	-6.83	27.94	25.40
Cumulative present worth	-100	-77.27	-52.48	-33.70	-40.53	-12.59	12.82

$n_p = 5 + \frac{12.59}{12.59+12.82} = 5.495$ years

Example: Compute the payback period of the following cash flow for i = 10 % payback periods.

End of year	0	1	2	3	4	5	6
Cash Flow	-100	25	30	25	-10	45	50

Years	0	1	2	3	4	5	6
Cash Flow	-100	25	30	25	-10	45	50
PW	-100	22.73	24.79	18.78	-6.83	27.94	28.225
Cumulative present worth	-100	-77.27	-52.48	-33.70	-40.53	-12.59	15.635

$n_p = 5 + \frac{12.59}{12.59+15.635} = 5.446$ years

Example: A construction company wants to invest on a new regulator that produces electricity. The initial cost of the regulator is \$10.5 million. The regulator will produce 40 million kwh with 81 % efficiency per year. The government will buy the electricity at a price of 5 cent per kwh. After 30 years, the company will transfer the regulator to the government with no salvage value. If the MARR of the company is 12 %, determine the payback period of this project.

C= $10.5 * 10^6$ USD

A= $0.81 * 40 * 10^6 * 5/100 = 1.62 * 10^6$ USD

$-10.5*10^6 + 1.62*10^6(P/A, 12\%, PB) = 0$

$(P/A, 12\%, PB) = 10.5/1.62 = 6.4815$ (trial and error by looking from interest tables)

$PB \approx 13.05$ years

Example: A factory was bid at a cost of 25 million TL to a company. In addition, the company paid 5 million TL for the land of the factory and 16 million TL for the buildings associated with the factory. The useful life of the factory was determined as 30 years, and the salvage value of the factory was assumed to be 20 million TL. The average sales per year were determined as 2500 units. The cost of the each unit is tabulated as follows:

The price of the product : 10 000 TL/ unit

Cost	**TL/unit**
Material	2 000
Workmanship	1 500
Energy	1 000
Management	200
Marketing	150
Finance	100
General Expenses	110

By considering the MARR of the company as 25 % per year, determine discounted payback period of the project.

$C = 25*10^6 + 5*10^6 + 16*10^6 = (46*10^6)$ TL

$A = (10\,000 - 2\,000 - 1\,500 - 1\,000 - 200 - 150 - 100 - 110) * 2500 = 12.35*10^6$ TL/year

$46*10^6 = 12.35*10^6\ (P/A, 25\ \%, X)$

$(P/A, 25\ \%, X) = 3.725$ (trial and error by looking up interest tables)

$X \approx 12$ years

Example: Consider a three-year project that has a two-year no-return payback period

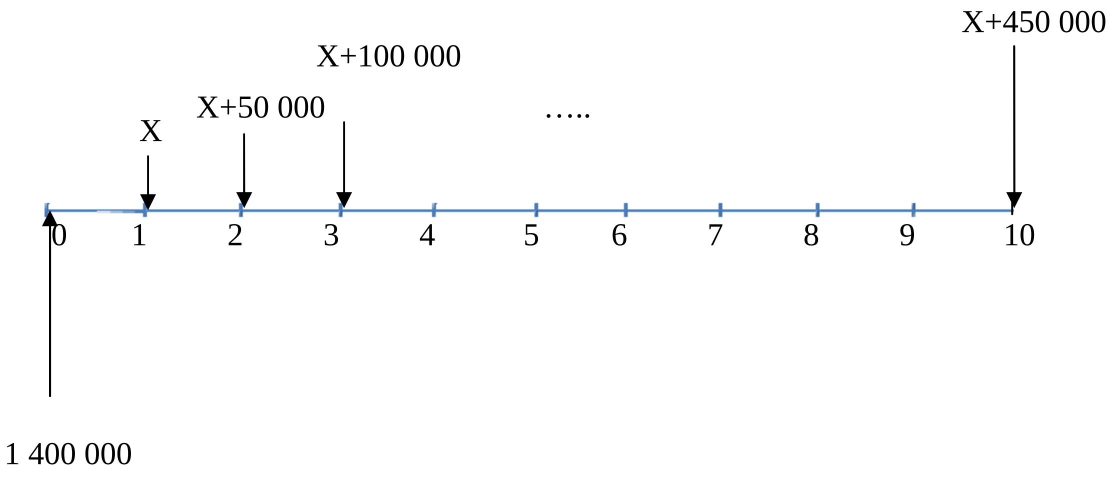
a. Will the project NPV be positive?

b. Will the project IRR be higher than the cost of capital?

c. Should the project be accepted?

The answer to all parts of the question is: “we cannot know with the information provided.” Simply because, the project’s payback period tells us nothing about whether it generates a net positive return or that the return is higher than the cost of capital. Payback does not include the time value of money. Therefore, we do not know if we should accept the project or not.

Example: A company invested 1 400 000 TL. The cash flow of the investment is illustrated below. The no-return payback period for this investment is determined as 8 years. Determine the MARR, if the profit obtained at the end of the life of the investment is 297 440 TL.



$$1\,400\,000 = X + (X+50\,000) + (X+100\,000) + (X+150\,000) + \ldots + (X+350\,000)$$

$$1\,400\,000 = 8X + 50\,000\,(1+2+3+4+5+6+7)$$

$$1\,400\,000 = 8X + 50\,000 * (28)$$

$$X = 0$$

$$297\,440 = -1\,400\,000\,(F/P, i\%, 10) + 50\,000\,(A/G, i\,\%, 10)\,(F/A, i\%, 10)$$

$$\mathbf{297\,440 + 1\,400\,000\,(F/P, i\%, 10) - 50\,000\,(A/G, i\,\%, 10)\,(F/A, i\%, 10) = 0}$$

With trial and error:

i= 8 %

297 440 + 1 400 000 *2.159- 50 000* 3.8713*14.487= 515 833.45

i= 6%

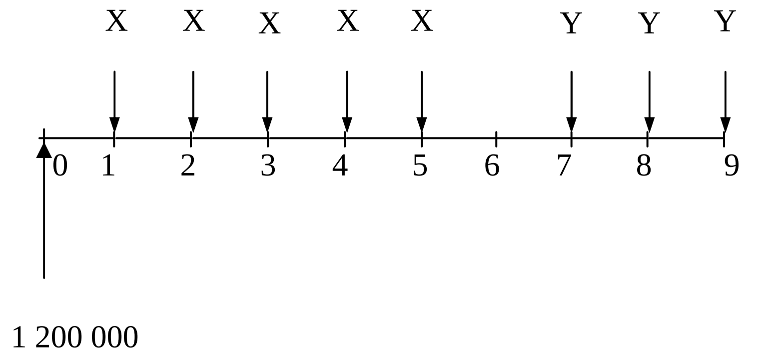
297 440 + 1 400 000* 1.791-50 000*4.0220*13.181= 146 964.32

i= 5 %

297 440 + 1 400 000*1.629-50 000* 4.0991*12.578= 0

MARR= 5%

Example: What must be the value of X and Y, according to the cash flow given below, if the discounted payback period of the cash flow is 7.6 years, and the profit at 9th year is 400 000 TL? The yearly interest rate is 10 %.



1 200 000 = X (P/A, 10%, 5) + Y (P/A, 10%, 1.6) (P/F, 10%, 6)

1 200 000 = 3.7908 X + Y (1.4144) (0.5645)

1 200 000 = 3.7908 X + 0.7984 Y (Equation 1)

400 000 = -1 200 000 (F/P,10%,9) + X(F/A,10%,5) (F/P,10%,4) + Y(F/A,10%,3)

400 000 = - 1 200 000 (2.358) + X (6.105) (1.464) +3.310Y

3 229 600 = 8.9377 X + 3.310 Y (Equation 2)

$$\left.\begin{aligned} 1\,200\,000 &= 3.7908\,X + 0.7984\,Y \\ 3\,229\,600 &= 8.9377\,X + 3.310\,Y \end{aligned}\right\} \quad \text{By solving these two equations simultaneously}$$

-2.3577*(1 200 000 = 3.7908 X + 0.7984 Y)

-2 829 281.41= -8.9377X-1.8824Y

3 229 600 = 8.9377 X + 3.310 Y

X= 257 496.09 TL Y= 280 416.07 TL

<u>Example:</u> Company A is considering three alternatives for investment, as shown below:

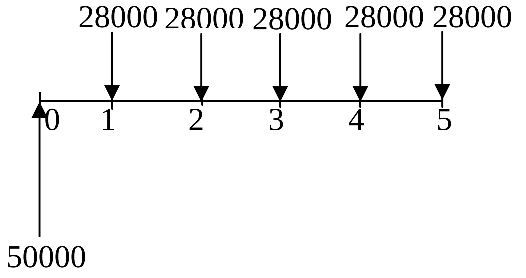
Alternative 1: P = -50 000 TL; n = 5 years; NCF = 28 000 TL per year

Alternative 2: P = -100 000 TL; n = 7 years; NCF = 70 000 TL for first and third year, decreasing by 10 000 TL per year thereafter

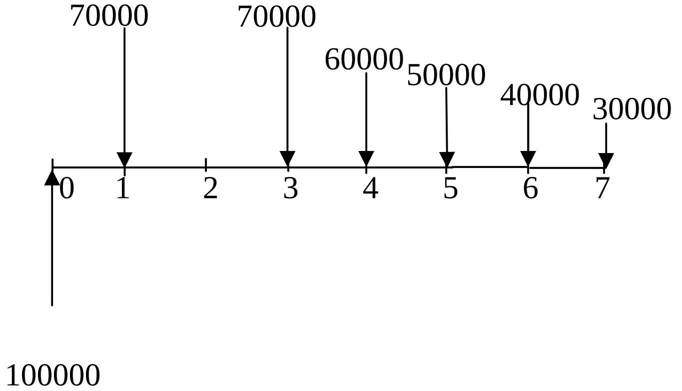
Alternative 3: P = -120 000 TL; n = 10 years; NCF = 40 000 TL for year 1, increasing by 2 500 TL per year thereafter

a) According to no-return payback period, which investment should be selected?
b) Calculate the internal rate of return for the cash flows of each project over its respective life. By considering the calculated internal rate of returns, which investment should be selected?
c) By considering the net present values (MARR of 12% per year), which investment should be selected?
d) According to the evaluations above, which alternative should the company invest? Why?

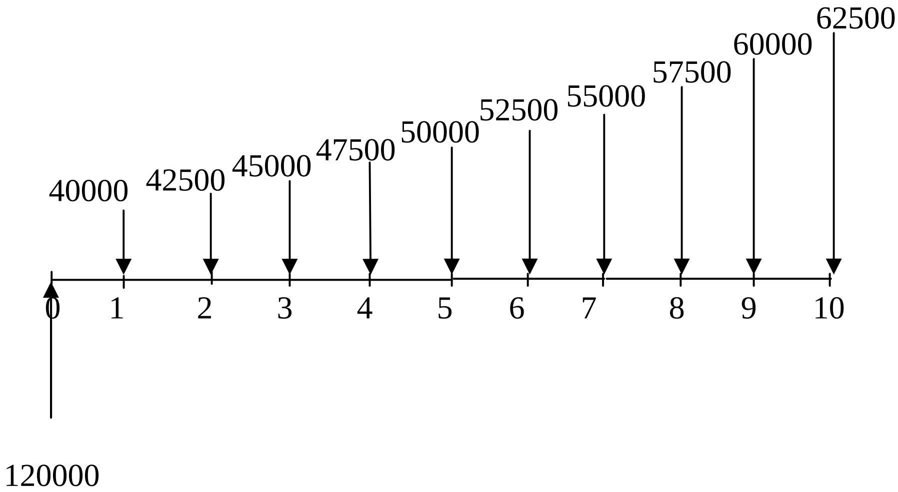
Alternative 1



Alternative 2



Alternative 3



a) $1+\frac{22\,000}{28\,000}$ = 1.786 years

$2+\frac{30\,000}{70\,000}$ = 2.428 years

$2+\frac{(120\,000-82\,500)}{45000}$ =2.833 years

Select Alternative 1

b) For Alternative 1:

PW=-50 000 + 28 000 (P/A, i%, 5) = 0

With trial and error:

i= 40%

50 000 + 28 000 * 2.0351= 6984.59

i= 45%

50 000 + 28 000 * 1.8755= 2514.77

i= 50%

50 000 + 28 000 * 1.7366= -1374.49

$45+\frac{2514.77}{(2514.77+1374.49)}$ *5=48.23 ROR= 48.23 %

For Alternative 2:

PW=-100 000+ (70 000- 10 000 (A/G, i%, 5))* (P/A, i%, 5)*(P/F, i%, 2) + 70 000 (P/F, i%, 1) =0

With trial and error:

i= 40%

-100 000+ (70 000- 10 000* 1.3580)* 2.0351* 0.5102+ 70000* 0.7143= 8583.77

i= 45%

-100 000+ (70 000- 10 000* 1.2980)* 1.8755* 0.4756+ 70000* 0.6896= -859.32

$40+\frac{8583.77}{(8583.77+859.32)}*5=44.54$ ROR= 44.54 %

For Alternative 3:

PW =-120 000 + (40 000 + 2 500 *(A/G, i%, 10))* (P/A, i%, 10) =0

With trial and error:

i = 40 %

-120 000+ (40 000+ 2 500 *2.1419)* 2.4136= -10 533.1

i = 35 %

-120 000+ (40 000+ 2 500 *2.3337)* 2.715= 4 442.37

$35+\frac{4442.37}{(10533.1+4442.37)} = 36.48$ ROR= 36.48 %

Select Alternative 1

c) For Alternative 1:

-50 000 + 28 000 (P/A, i%, 5)

-50 000 + 28 000 (P/A, 12%, 5)

-50 000+ 28 000 * 3.6048= 50 933.73

A= 50 933.73 (A/P, 12%, 5) = 50 933.73*0.2774=14 290.017 TL/yr

Common multiple of years: 70 years

PW (70) = 14 290.017 (P/A, 12%, 70)= 14 290.017*8.3303=117 698.95 TL

For Alternative 2:

-100 000+ (70 000- 10 000 (A/G, i%, 5))* (P/A, i%, 5)*(P/F, i%, 2) + 70 000 (P/F, i%, 1)

-100 000+ (70 000- 10 000 (A/G, 12%, 5))* (P/A, 12%, 5)*(P/F, 12%, 2) + 70 000 (P/F, 12%, 1)

-100 000+ (70 000- 10 000* 1.7746)* 3.6048* 0.8929+ 70000* 0.8929= 112 662.8

A= 112 662.8*(A/P, 12%, 7) = 112 662.8 *0.2191=24 684.42 TL/yr

PW (70) = 24 684.42 (P/A, 12%, 70) = 24 684.42*8.3303= 205 628.62 TL

For Alternative 3:

-120 000 + (40 000 + 2 500 *(A/G, i%, 10))* (P/A, i%, 10)

-120 000 + (40 000 + 2 500 *(A/G, 12%, 10))* (P/A, 12%, 10)

-120 000+ (40 000+ 2 500 *3.5846)* 5.6502= 156 644.1

A= 156 644.1 (A/P, 12%, 10)= 156 644.1 * 0.1770= 27 726 TL/yr

PW (70) = 27 726(P/A, 12%, 70)= 27 726 *8.3303= 230 965.95 TL

Select Alternative 3

d) Since the time value of money and cash flows that occur after PB are neglected in the no-return payback analysis, this method leads to a misleading alternative. Also, internal rate of return has a shortcoming in evaluation of mutually exclusive alternatives; therefore the projects should be evaluated by using incremental rate of return method. According to the other methods, the net present value method is a preferable method. Consequently, alternative 3 is determined as the best alternative among the alternatives.

Example: A family is planning to deposit 2 400 TL/year in a bank with a 25 % interest rate. How many years will they deposit this amount, in order to gain 17 000 TL/year continuously starting after the year of making last payment? (Hint P=$A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$)

$\lim_{n\to\infty}\frac{(1+i)^n-1}{i(1+i)^n}=\frac{1}{i}$

P= 17 000/i=17 000/0.25 = 68 000

2 400 (F/A, 25%, n) = 68 000

$\left[\frac{(1+0.25)^n-1}{0.25}\right] = 28.33$

$(1 + 0.25)^n - 1 = 7.0825$

$1.25^n = 8.0825$

$n \cdot \log(1.25) = \log(8.0825)$

$n = 9.37$ years