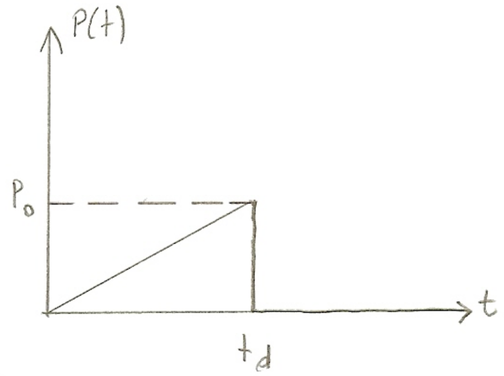
CE 487 FALL 2006 HOMEWORK 2

1a) Classical Solution



$$p(t) = \begin{cases} P_0 \cdot \dfrac{t}{t_d} & t \leq t_d \\ 0 & t > t_d \end{cases}$$

Solution for $t \leq t_d$ (Forced vibration phase)

$$m\ddot{u} + ku = \frac{p_0}{t_d} \cdot t$$

$$u = u_H + u_p$$

$$u_p = C \cdot t \qquad \dot{u}_p = C \qquad \ddot{u}_p = 0$$

$$kCt = \frac{p_0}{t_d} \cdot t \qquad C = \frac{p_0}{t_d \cdot k}$$

$$u_H = A\sin w_n t + B\cos w_n t$$

Hence;

$$u = A\sin w_n t + B\cos w_n t + \frac{p_0}{t_d \cdot k} t$$

Initial conditions are $u(0) = 0$ and $\dot{u}(0) = 0$

$$u(0) = 0 \Rightarrow B = 0$$

$$\dot{u} = Aw_n\cos w_n t - Bw_n\sin w_n t + \frac{p_0}{t_d \cdot k}$$

$$\dot{u}(0) = 0 \Rightarrow 0 = Aw_n + \frac{p_0}{t_d \cdot k} \qquad A = -\frac{p_0}{t_d \cdot k \cdot w_n}$$

$$u = -\frac{p_o}{t_d . k . w_n} \sin w_n t + \frac{p_o}{t_d . k} t$$

$$u = \frac{p_o}{k}\left(\frac{t}{t_d} - \frac{\sin w_n t}{w_n t_d}\right) \quad \text{when} \quad t \leq t_d$$

Solution for $t > t_d$ (Free vibration phase)

$$m\ddot{u} + ku = 0$$

$$u(t) = A\cos w_n(t - t_d) + B\sin w_n(t - t_d)$$

$$\dot{u}(t) = -Aw_n \sin w_n(t - t_d) + Bw_n \cos w_n(t - t_d)$$

Initial conditions (conditions at t_d) are those at the end of the forced vibration phase

At the forced vibration phase,

$$u(t) = \frac{p_o}{k}\left(\frac{t}{t_d} - \frac{\sin w_n t}{w_n . t_d}\right) \qquad u(t_d) = \frac{p_o}{k}\left(\overset{1}{\cancel{\frac{t_d}{t_d}}} - \frac{\sin w_n t_d}{w_n t_d}\right)$$

$$\dot{u}(t) = \frac{p_o}{k}\left(\frac{1}{t_d} - \frac{\cos w_n t}{t_d}\right) \qquad \dot{u}(t_d) = \frac{p_o}{k}\left(\frac{1}{t_d} - \frac{\cos w_n t_d}{t_d}\right)$$

Then

$$\frac{p_o}{k}\left(1 - \frac{\sin w_n t_d}{w_n t_d}\right) = A \Rightarrow A = \frac{p_o}{k}\left(1 - \frac{\sin w_n t_d}{w_n t_d}\right)$$

$$\frac{p_o}{k}\left(\frac{1}{t_d} - \frac{\cos w_n t_d}{w_n t_d}\right) = Bw_n \Rightarrow B = \frac{p_o}{kw_n}\left(\frac{1}{t_d} - \frac{\cos w_n t_d}{t_d}\right)$$

$$u(t) = \frac{p_o}{k}\left(1 - \frac{\sin w_n t_d}{w_n t_d}\right)\cos w_n(t - t_d) + \frac{p_o}{kw_n}\left(\frac{1}{t_d} - \frac{\cos w_n t_d}{t_d}\right)\sin w_n(t - t_d)$$

$$u(t) = \frac{p_o}{k}\left[\cos w_n(t-t_d) - \frac{\sin w_n t_d \cos w_n(t-t_d)}{w_n t_d} + \frac{\sin w_n(t-t_d)}{w_n t_d} - \frac{\cos w_n t_d \sin w_n(t-t_d)}{w_n t_d}\right]$$

$$u(t) = \frac{p_o}{k}\left[\cos w_n(t-t_d) + \frac{1}{w_n t_d}\sin w_n(t-t_d) - \frac{1}{w_n t_d}\left(\sin w_n t_d \cos w_n(t-t_d) + \cos w_n t_d * \sin w_n(t-t_d)\right)\right]$$

$$u(t) = \frac{p_o}{k}\left[\cos w_n(t-t_d) + \frac{1}{w_n t_d}\sin w_n(t-t_d) - \frac{1}{w_n t_d}\sin w_n t\right] \text{ when } t > t_d$$