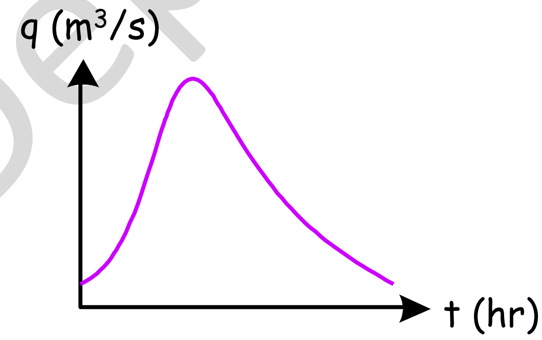
Chapter 3
Hydrograph Analysis

Hydrograph Analysis

- Components of Runoff
- Hydrograph Characteristics
- Unit Hydrograph Theory
- Synthetic Unit Hydrographs

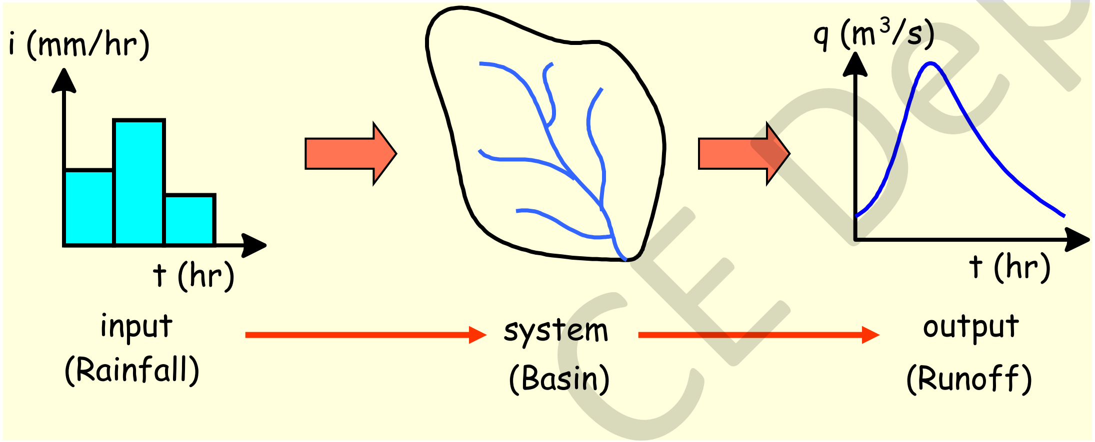


- The major characteristics of streamflow are:
  - its volume for a certain duration (month or year)

    → different uses & storage
  - its extreme values

    → drought & flood

Hydrograph Analysis



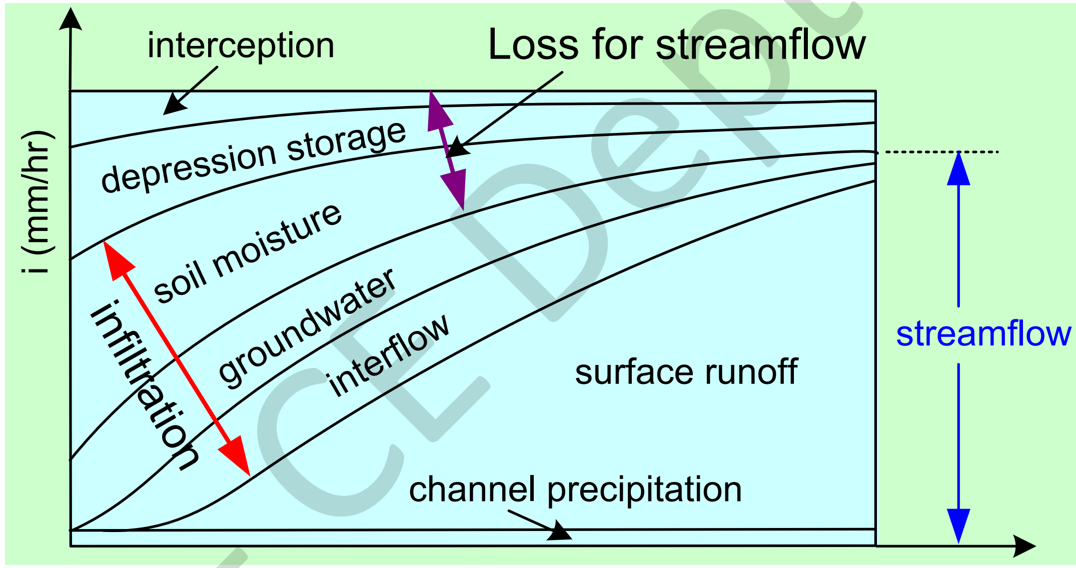
Components of Runoff

- Channel Precipitation
- Surface Runoff
- Interflow
- Groundwater Flow

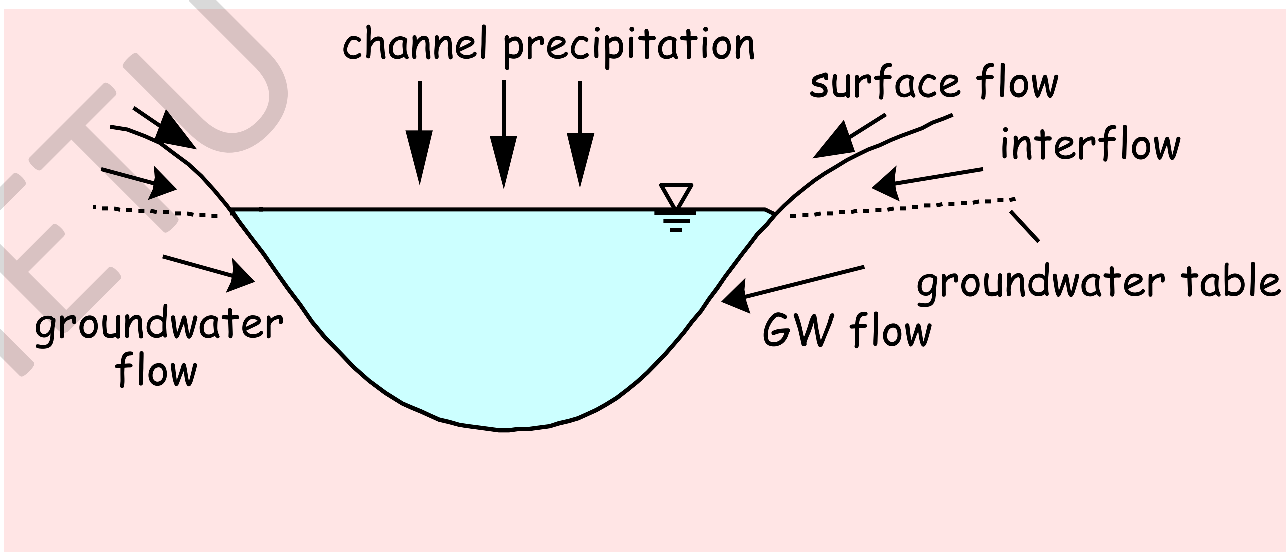
- Interception, depression storage, soil moisture are LOSSES for streamflow.
- The other portions of precipitation reach streams sooner or later.

COMPONENTS OF RUNOFF

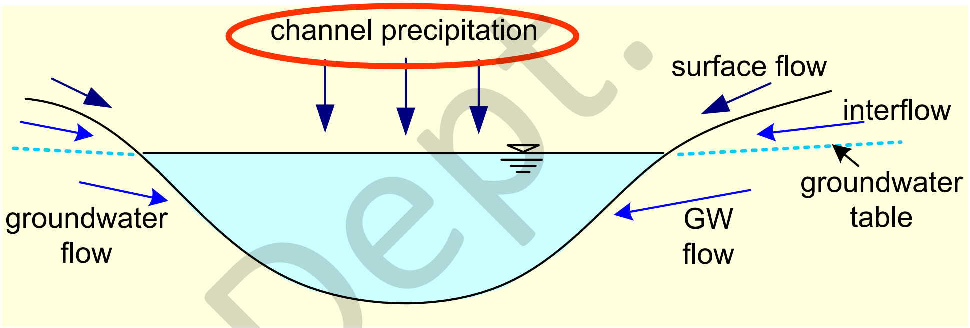
In a rainfall block



In the channel

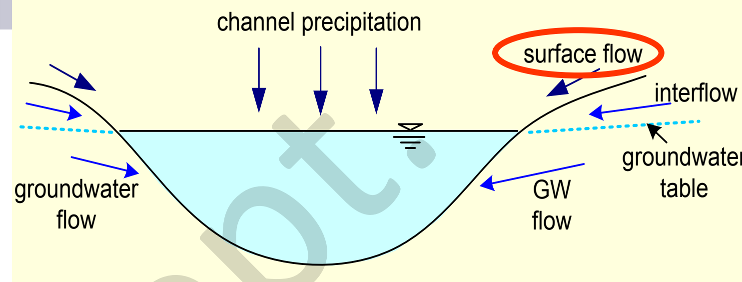


Channel Precipitation



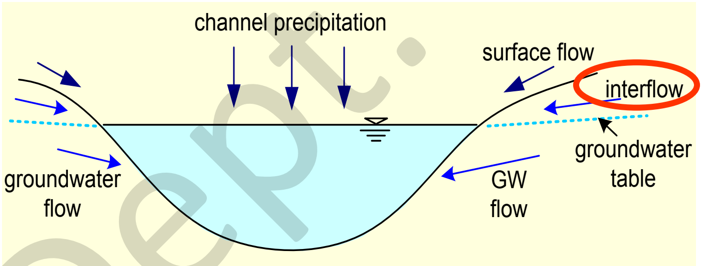
- The part of the precipitation which falls directly on the stream or lake surfaces.
- Usually very small → Surface area$_{\text{streams \& lakes}}$ < Basin area.
- Important when catchment has large lakes or reservoirs.
- Channel precipitation = rainfall depth x Surface area$_{\text{streams \& lakes}}$
- Channel precipitation$_{\text{streams}}$ < Channel precipitation$_{\text{lakes}}$

Surface Runoff



- The flow occurring due to gravity on the soil surface:
  surface runoff = overland flow + streamflow in channels
- At the beginning of the storm no surface runoff occurs for some time (i.e. all the precipitation infiltrates)
- When infiltration capacity < rainfall intensity:
  - water starts filling the depression then detention storages
  - a thin layer of water is formed on the soil surface & flows by gravity along the greatest slope (overland flow)
- After the overland flow reaches a stream channel, streamflow or channel flow begins to move toward the outlet of catchment.
- The rate of surface runoff depends on:
  - permeability & initial humidity of the soil,
  - rainfall intensity.

Interflow

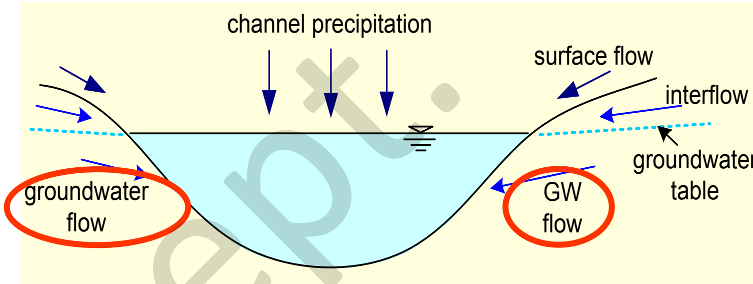


- Infiltrated water fills the voids in the soil & brings the water content to field capacity.
- After that, water may move in the soil vertically or horizontally in the upper soil layers

feeds groundwater	forms interflow (subsurface flow)

- Amount of interflow depends on the geological characteristics of the basin:
  - A thin soil cover over an impermeable layer ↑es interflow.
  - A thick and uniformly permeable soil ↓es interflow but ↑es percolation.

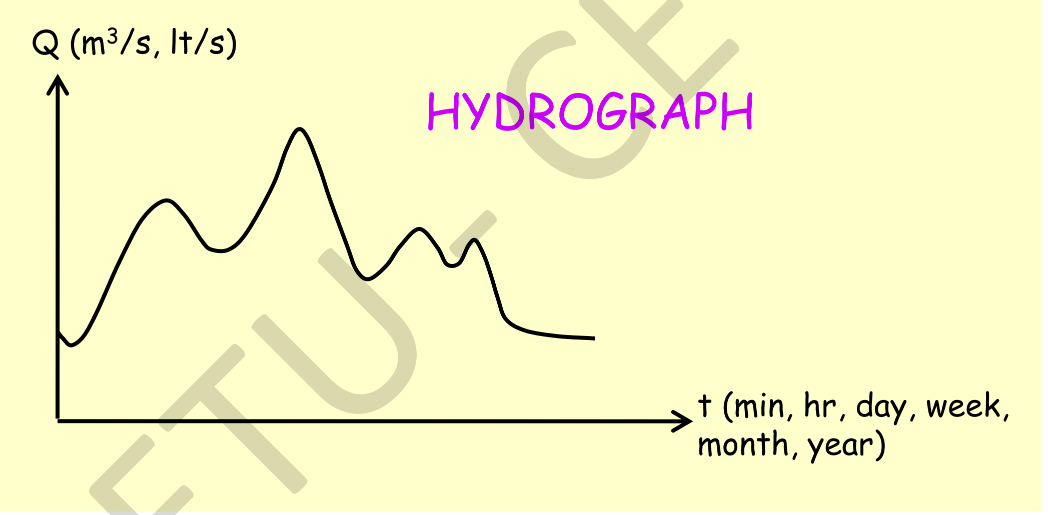
Groundwater Flow



- Infiltrated water may
  - fill the voids in the soil above the groundwater table
  - percolate down to feed groundwater (gw flow, base flow, dry weather flow)
- If GW table intersects with the river cross sections
  - gw may feed the river or
  - river may feed gw
- Since gw flow occurs through the soil particles with very small velocity, the rate of GW flow does not change rapidly.

Hydrograph

- Hydrograph → discharge vs time
  (m^3/s, lt/s) (min, hr, day, month, yr)
- Sometimes plotted as stage vs time.



- The comparison of hydrographs with the corresponding rainfall hyetographs provides a lot of information about the rainfall-runoff relation of the basin.

The Shape of Hydrograph

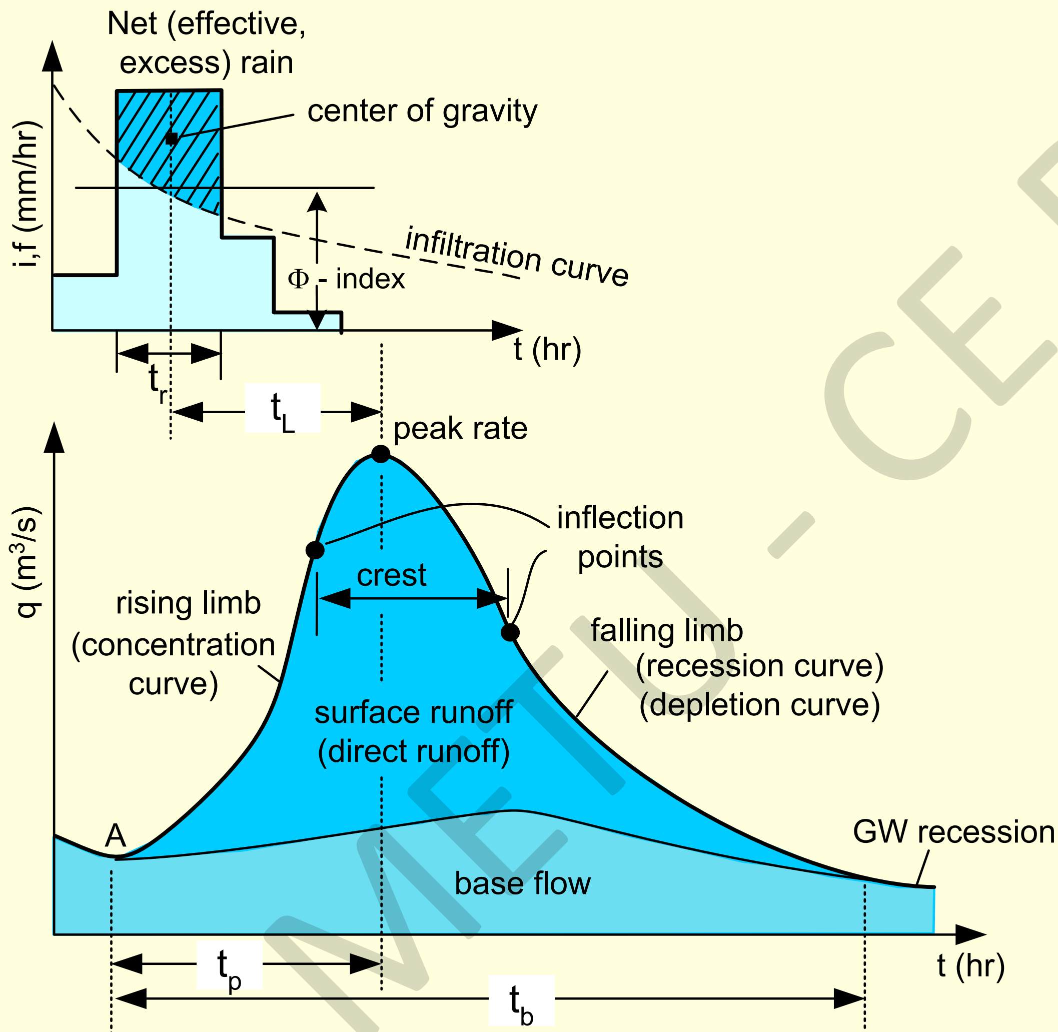


Figure 7.4 Hydrograph shape

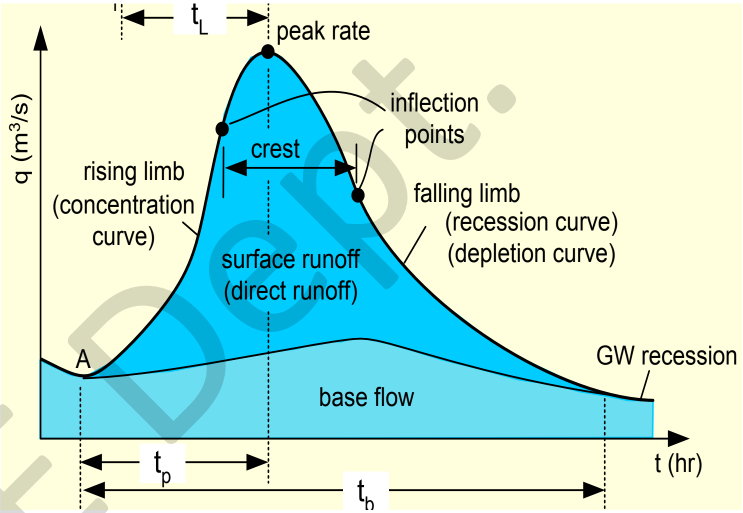
- Effective rainfall
- Infiltrated amount

- Surface runoff
- Baseflow

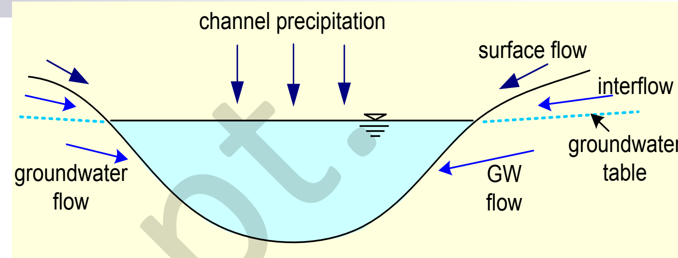
- Rising limb
- Crest
- Falling limb

- Duration of net rain (t_r)
- Basin lag (t_L)
- Time to peak (t_p)
- Base time (t_b)

Hydrograph

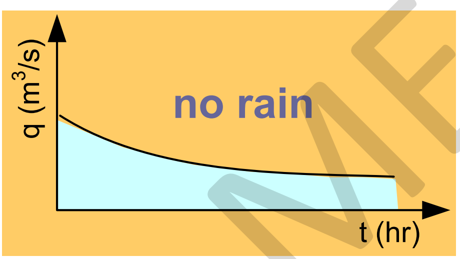


- The crest of the hydrograph is governed by the duration of rainfall.
- Shape of the rising limb = f(rainfall intensity & basin characteristics s.a. infiltration capacity, shape, slope, etc.)
- Shape of falling limb = f(basin characteristics s.a. surface storage, etc.)
- Characteristics of the rainfall do not impact the falling limb since recession starts after the end of the rainfall.

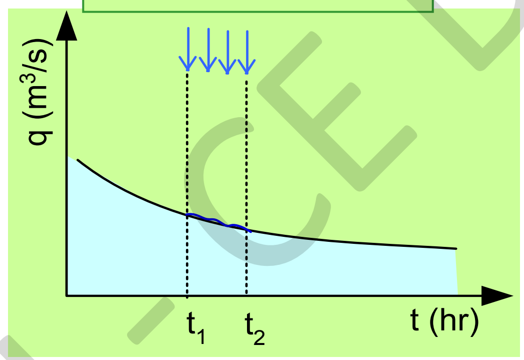


Hydrograph shapes for different conditions

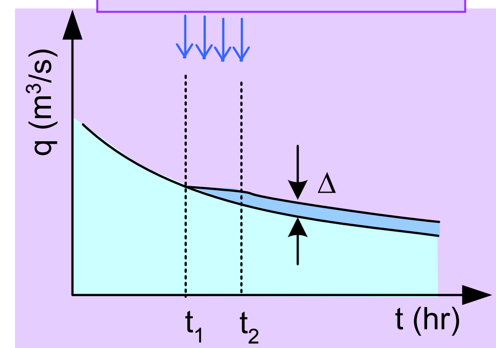
i	: rainfall rate
f	: infiltration rate
F	: total amount of infiltrated water
SMD	: soil moisture deficiency



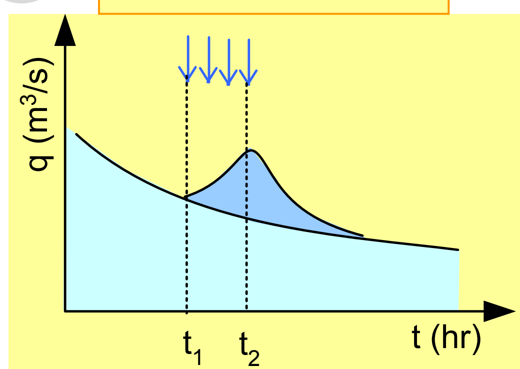
Case 1 $i<f$
$F<SMD$



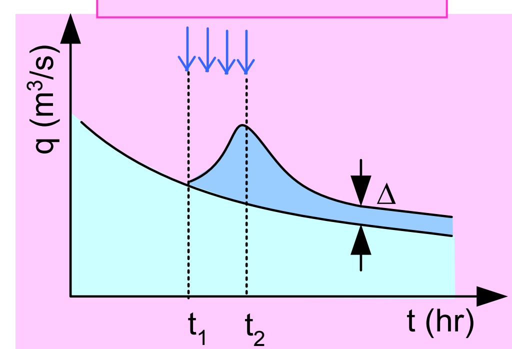
Case 2 $i<f$
$F>SMD$



Case 3 $i>f$
$F<SMD$

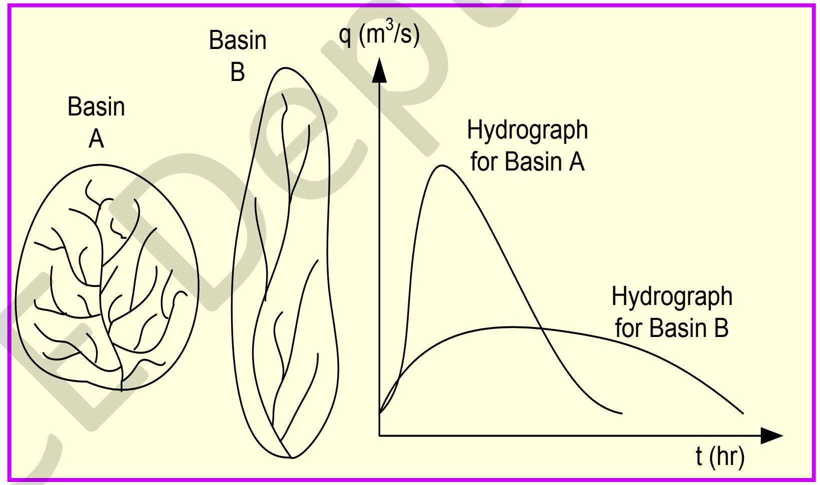


Case 4 $i>f$
$F>SMD$

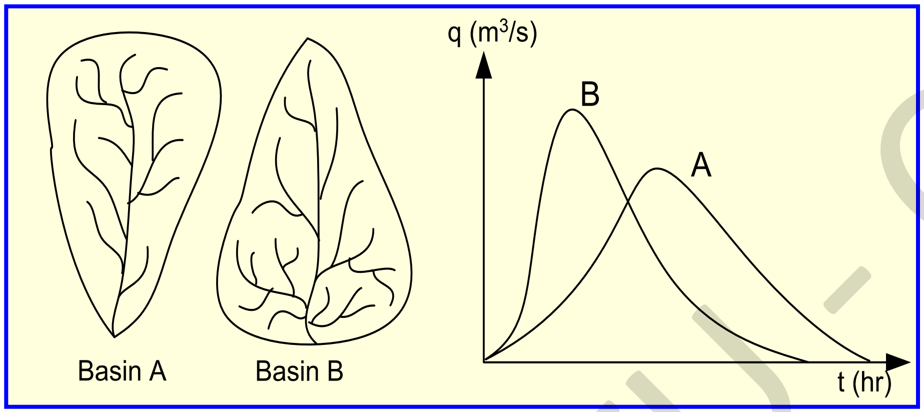


Hydrograph Analysis

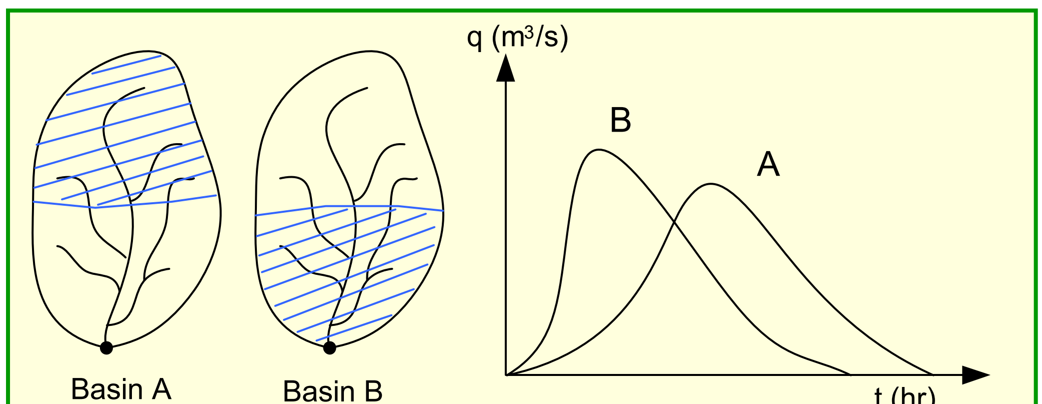
Effect of shape of the basin



Effect of different orientations of the same shape

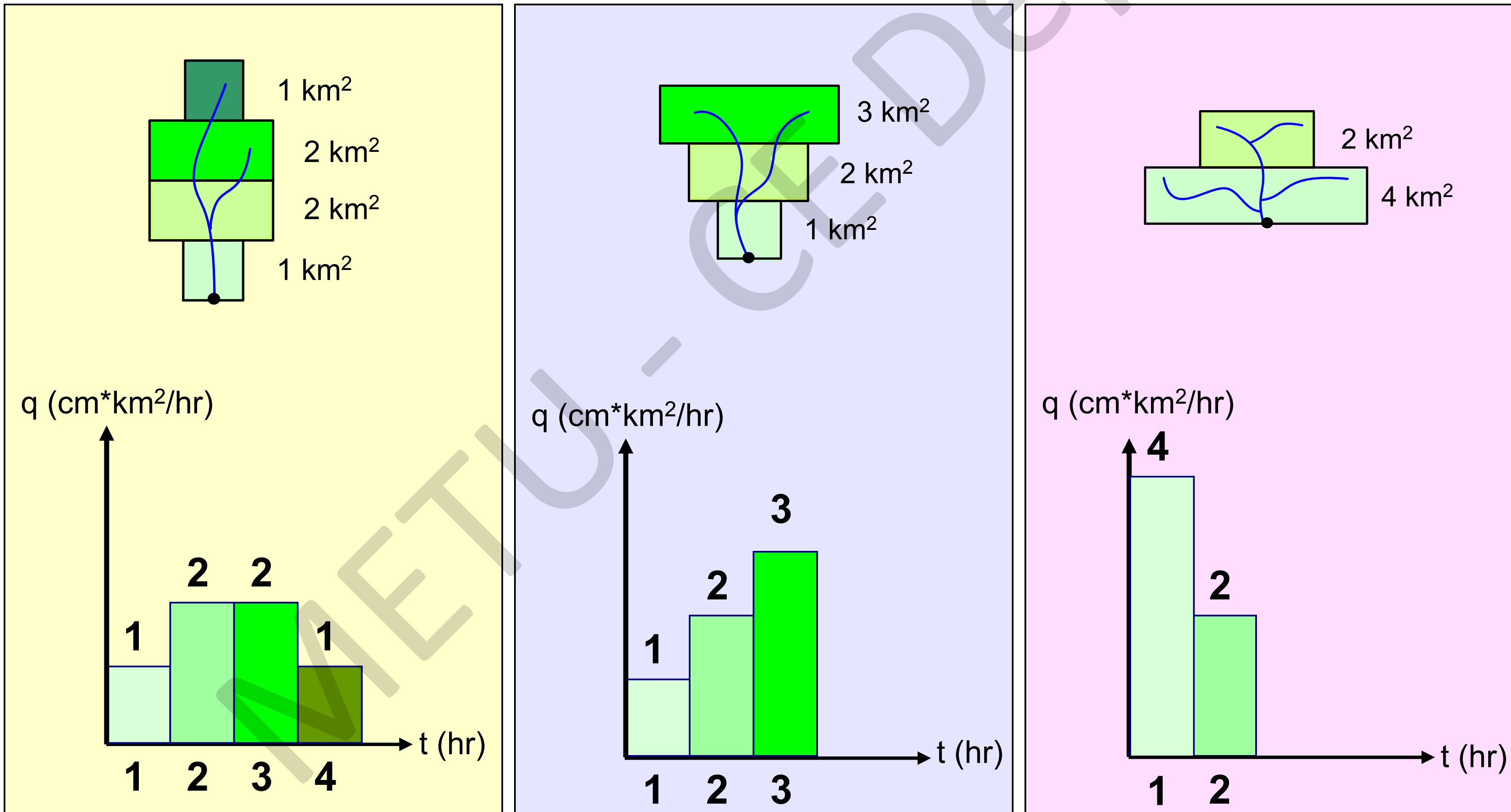


The shape of the hydrograph varies with the orientation of the storm in the basin.



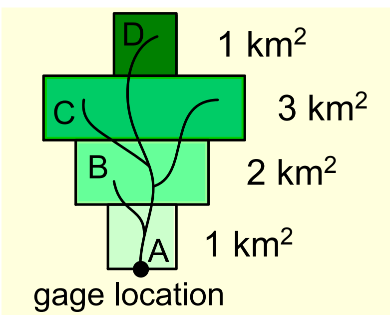
Time-area concept for hydrograph

After an effective rainfall of 1 cm depth in 1 hour

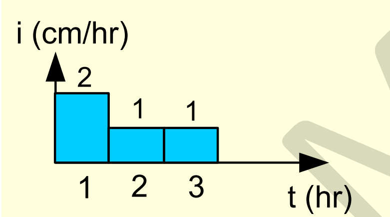


HYDROGRAPH ANALYSIS

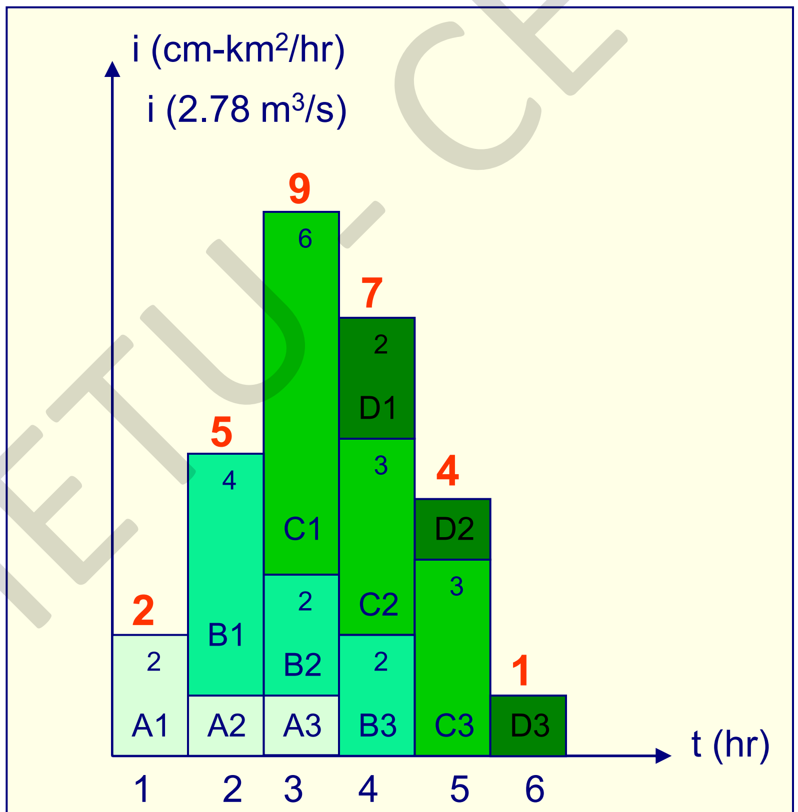
- Consider a rainfall with a certain pattern, which uniformly covers the entire basin.
- Assume there is no infiltration → all the rainfall will be effective, and it will produce only surface runoff.



a) Basin



b) Hyetograph



If the drainage area is divided into a larger number of subareas
&
hyetographs into smaller time intervals,
a better shaped hydrograph would be obtained.

Separation of Hydrograph Components

- Surface runoff & base flow are very different in character.
- Since baseflow represents the flow through soil pores, changes occur very slowly.
- There is a lag between cause and effect, (days, weeks, or months).
- This will depend on the transmissivity of the aquifers bordering the stream, as well as the climate.

Separation of Hydrograph Components

- GW flow from a basin can be simulated by the flow through outlet of a large tank.
- Assuming that there is a band of porous material in the outlet, the flow will be:

$$q = \frac{k\,p\,a}{L}\,h \qquad (1)$$

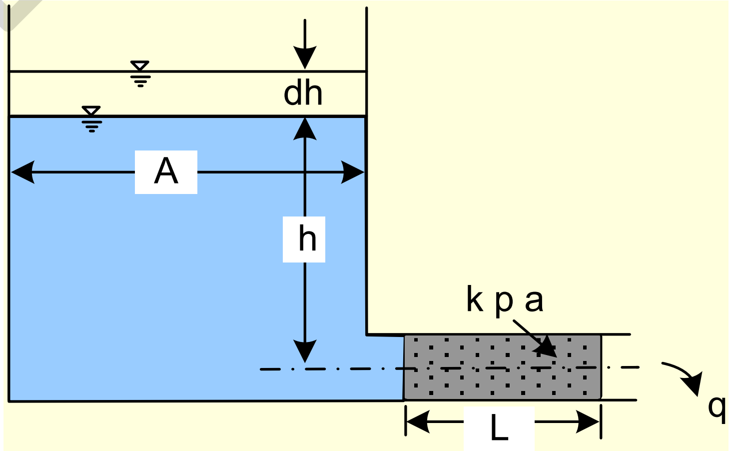


Figure 7.10 Baseflow simulation

q : discharge (m^3/s)

k : permeability (m/s)

p : porosity

a : cross-sectional area (m^2)

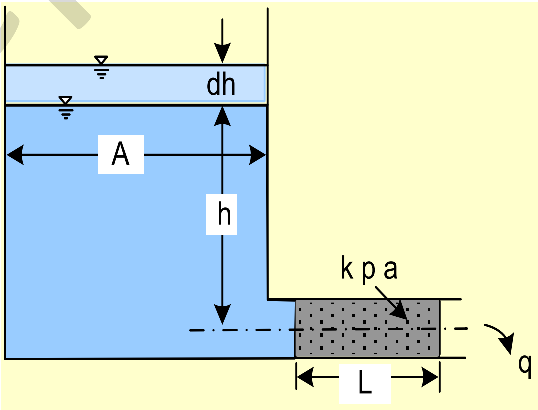
L : length of the porous material (m)

h : head (m) or depth of water in the tank

Separation of Hydrograph Components

- The amount of water that flows for a certain duration is:

$$A\,dh = -q\,dt \qquad (2)$$



- When an expression for dh is obtained from first eq. and introduced into second,

$$q = \frac{k\,p\,a}{L}\,h \qquad dh = \frac{L}{k\,p\,a}\,dq \qquad \frac{dq}{q} = -\frac{k\,p\,a}{A\,L}\,dt$$

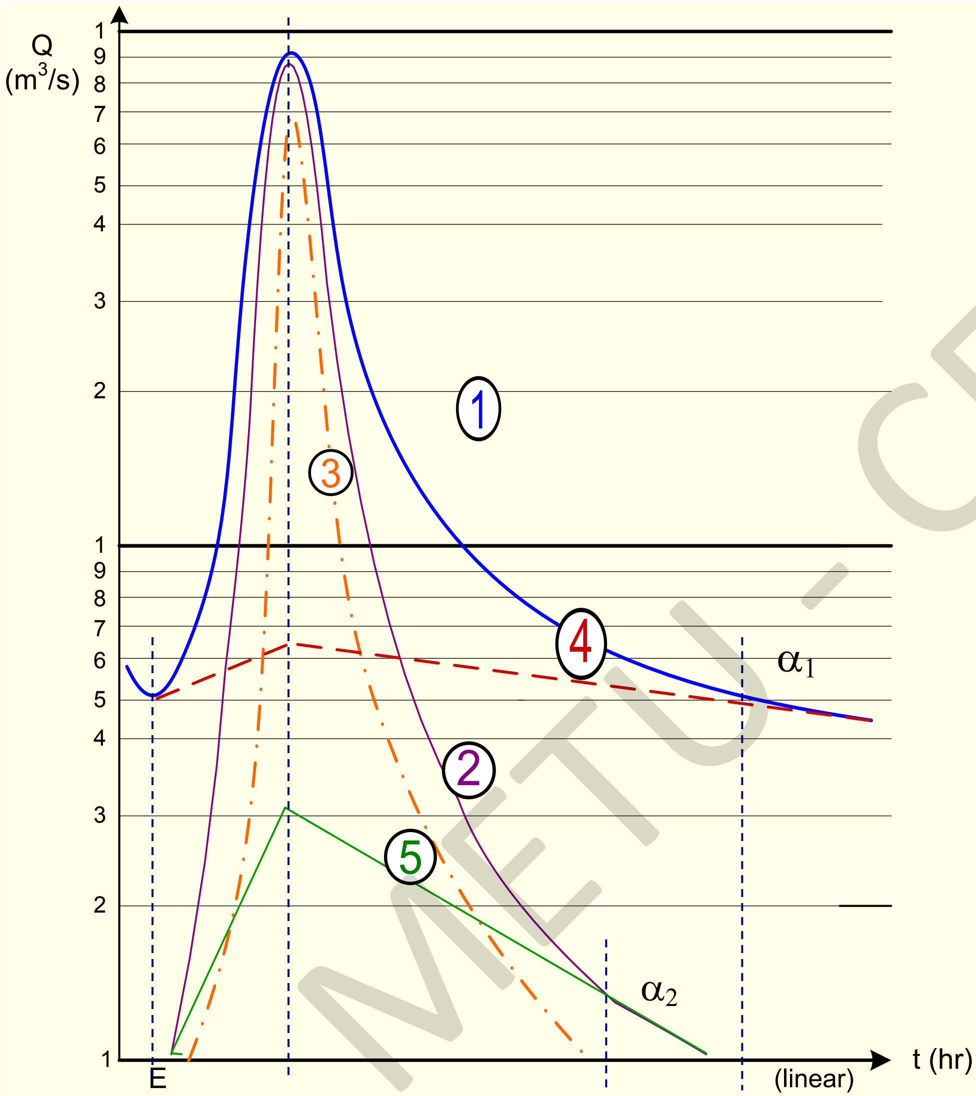
Let $\alpha = \dfrac{k\,p\,a}{A\,L}\left(\dfrac{LT^{-1}L^{2}}{L^{2}L} = \dfrac{1}{T}\right)$ which is a constant with T^{-1} unit.

$$\int_{q_0}^{q}\frac{dq}{q} = \int_{t_0}^{t} -\alpha\,dt \qquad q = q_0 e^{-\alpha(t-t_0)} \qquad q = q_0 e^{-\alpha t}$$

Hydrograph Separation Techniques

- Separation line between surface runoff & baseflow is not definite & varies widely depending on the existing conditions.
- Inaccuracies in separation are not very important for many storms, since the max. rate of discharge is only slightly affected by the base flow.
- Methods for seperation:
  - Simple Method
  - Approximate Method
  - Barnes (Semi-log) Method

Separation by Barnes (Semi-log) Method



1 Total flow (SF+SSF+BF)

$$q = q_0 \, e^{-\alpha_1 t}$$

4 Base flow (BF)

2 (SF+SSF) (1) – (4)

$$q = q_0 \, e^{-\alpha_2 t}$$

5 Subsurface flow (SSF)

3 Surface flow (SF) (2) – (5)

Unit Hydrograph (UH) Theory

- Hydrograph of surface runoff (direct runoff) resulting from 1 cm of excess rainfall which is uniformly distributed over basin area at a uniform rate during a specified period of time.
- Depth = 1 mm for arid or semi-arid regions or if basin is small.
- Given by Sherman in 1932.
- It is assumed that the UH is representative for the runoff process of a basin.
- Baseflow should be separated from total flow to find direct runoff, and all the losses should be subtracted from total precipitation before any analysis.

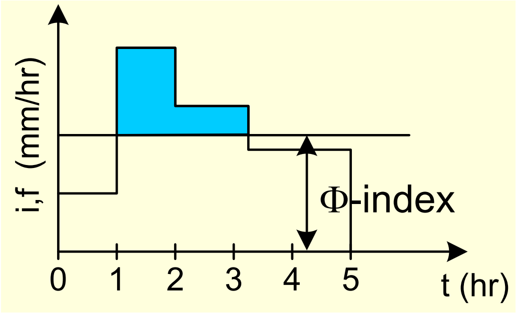
Unit Hydrograph (UH) Theory

- UH assumptions:

1. Excess rainfall is uniformly distributed within a specified period of time.
2. Excess rainfall is uniformly distributed over the basin area.
3. Base time of direct runoff is constant for a specified duration of rainfall.
4. Ordinates of direct runoff hydrograph are directly proportional to the total amount (depth) of direct runoff (= depth of excess rainfall) for the same duration rainfalls.
5. Unit hdrograph is unique for a basin.

Discussions of UH assumptions:

1. Short duration rainfalls



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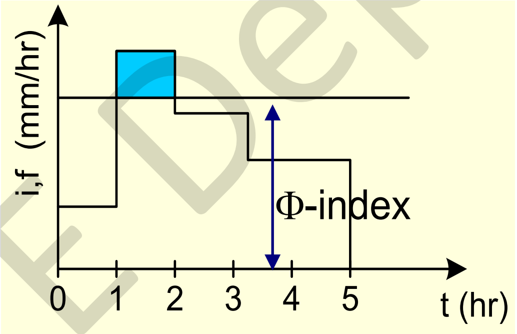

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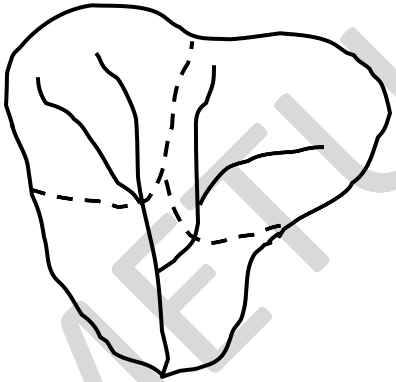
2. Small area < 5000 km^2 , otherwise sub-basins A, B, C.

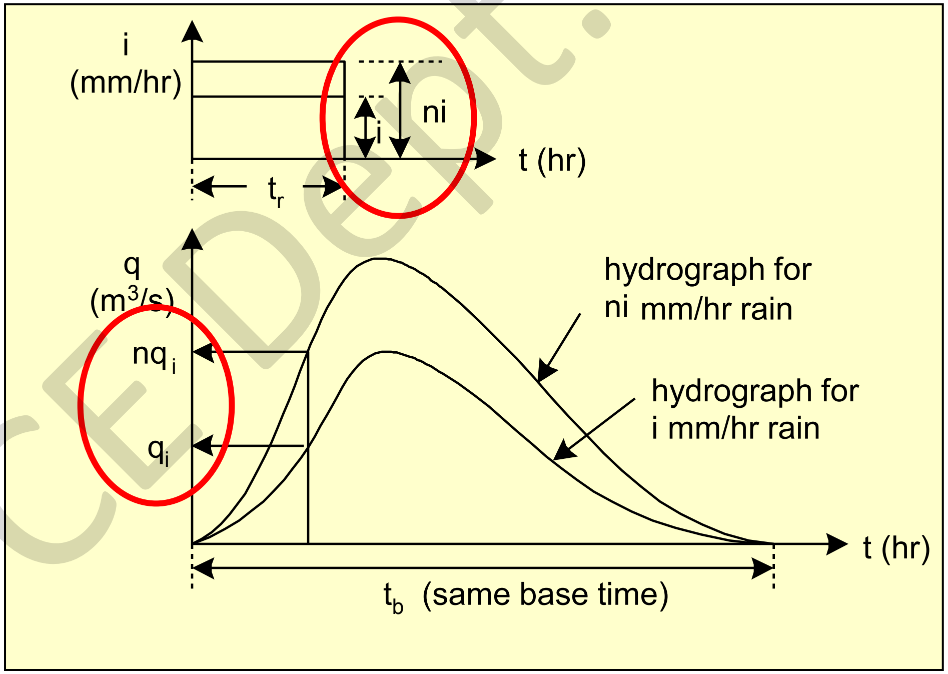


3. Baseflow separation methods.

4. Linearity assumption

(Principle of superposition,

Principle of proportionality)

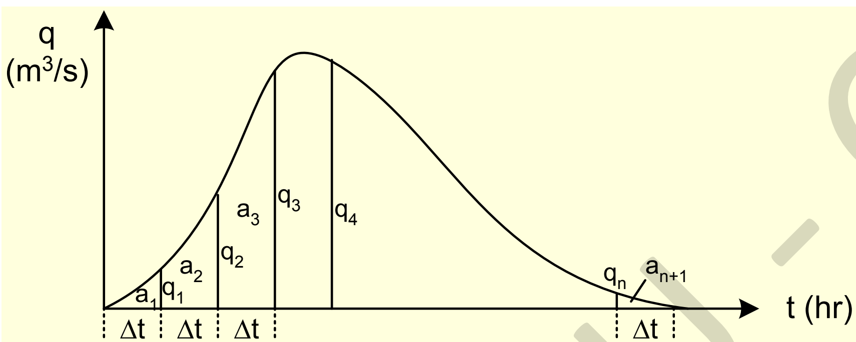


5. Principle of time-invariance

When basin characteristics change → UH changes

Unit Hydrograph

- The unit hydrograph is denoted as dUH_t (d in cm, t in hr)
- The depth of flow for a hydrograph → the area under the hydrograph.



$$a_1 = \frac{0 + q_1}{2}\Delta t$$

$$a_2 = \frac{q_1 + q_2}{2}\Delta t$$

$$a_3 = \frac{q_2 + q_3}{2}\Delta t$$

$$\vdots$$

$$a_n = \frac{q_{n-1} + q_n}{2}\Delta t$$

$$a_{n+1} = \frac{q_n + 0}{2}\Delta t$$

V = Σq*Δt

Figure 7.18 Determination of volume of runoff

$$d = \frac{V}{A} = \frac{\int q\, dt}{A} = \frac{\sum q \Delta t}{A}$$

$\sum q$ = sum of ordinates of the hydrograph (m³/s)

dt = time interval (s)

A = basin area (m²)

d = depth (m)

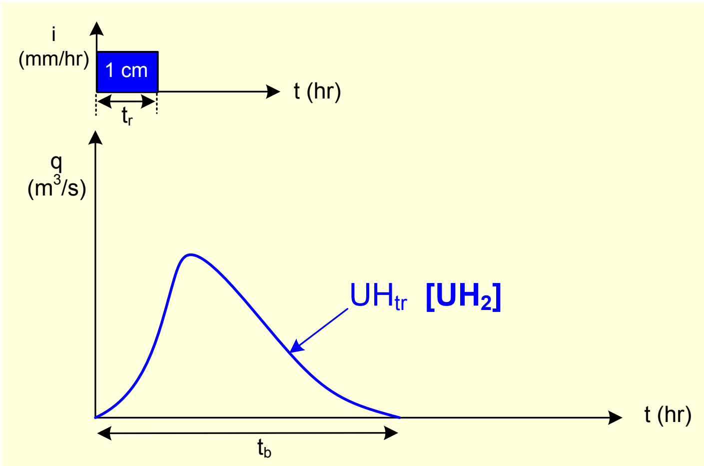
Unit Hydrographs of Different Durations

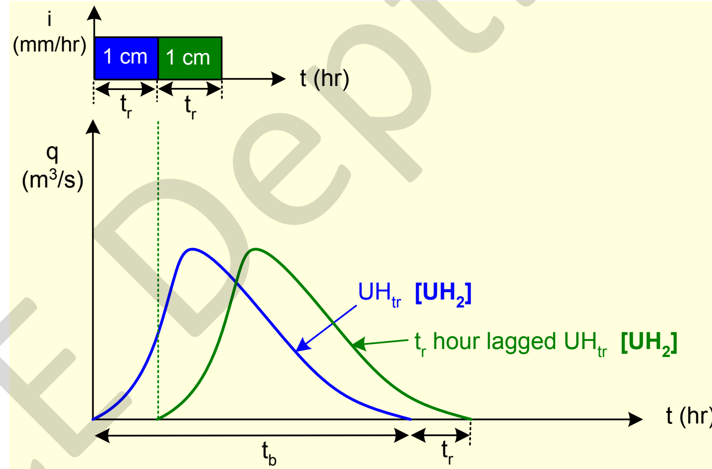
- There are 2 methods to obtain UHs of different durations for a basin when a UH of certain duration is known
  - The lagging method
  - S-curve method

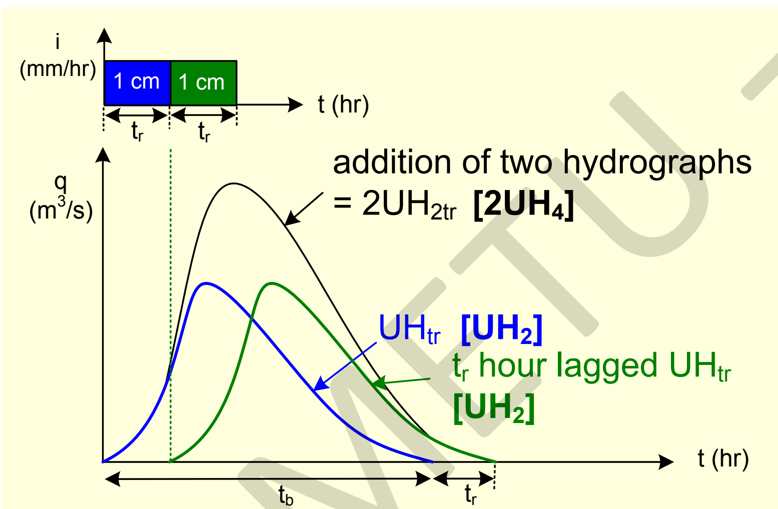
The Lagging Method

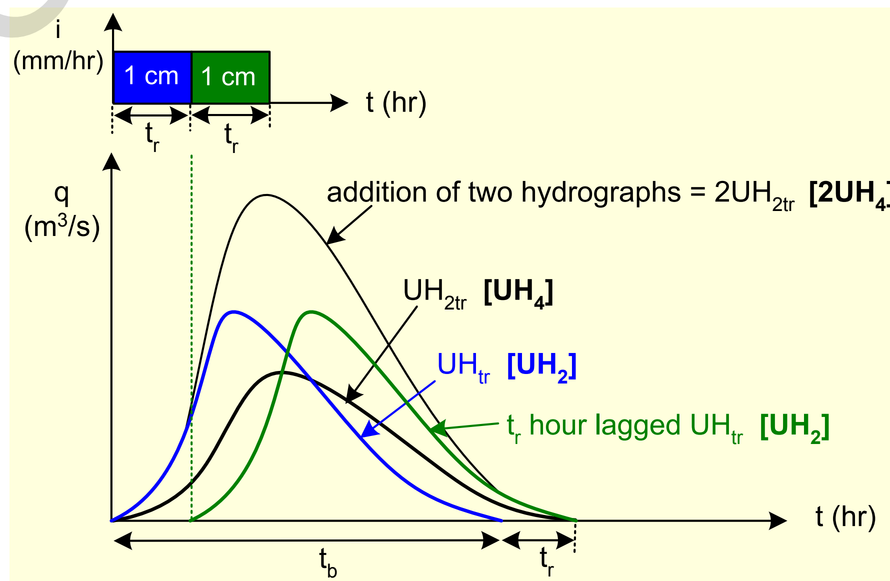
- A UH of certain duration can easily be obtained by using the lagging method if a UH of different duration is known for the basin.
- The only condition is that the durations be multiples of each other.

HYDROGRAPH ANALYSIS – lagging method



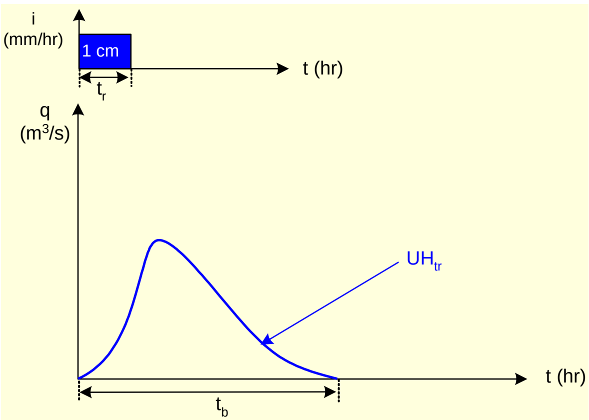


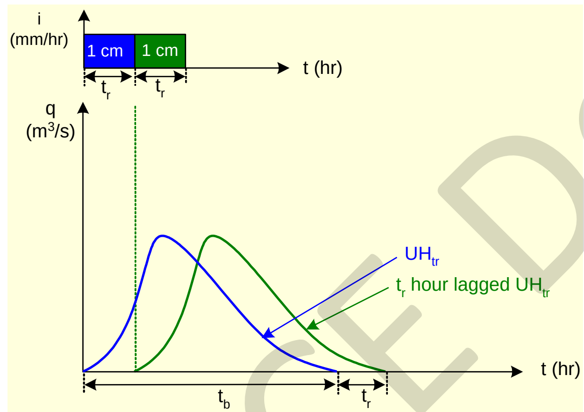


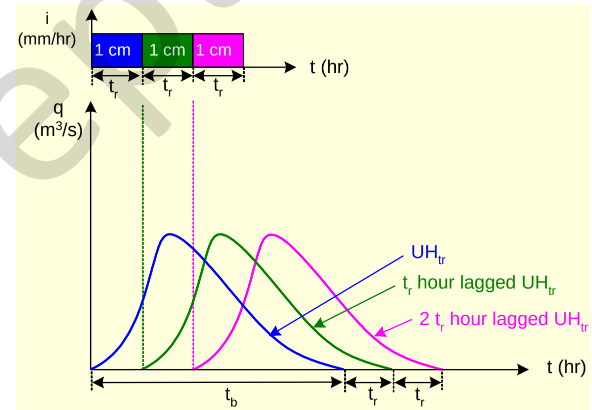


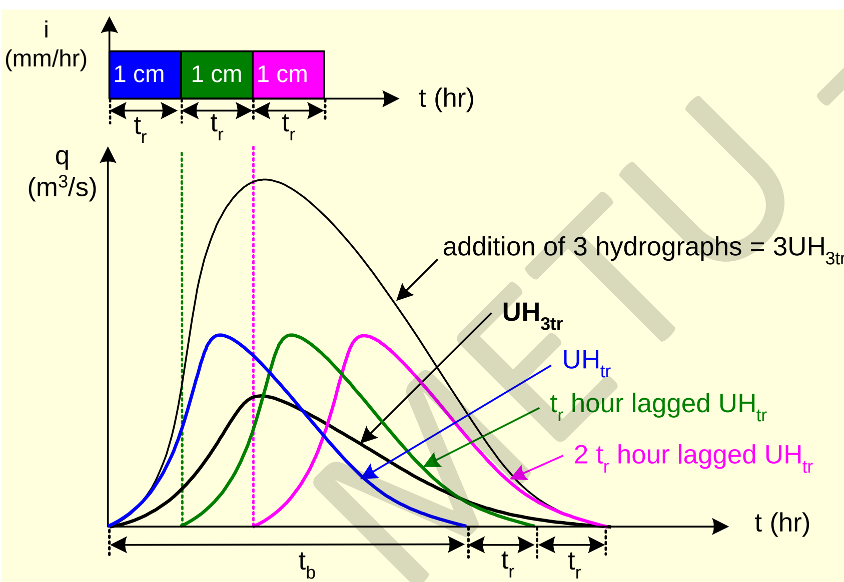
UH_2 + (2 hr lagged) UH_2 = $2UH_4$

HYDROGRAPH ANALYSIS – lagging method









$$UH_t + (t\ \text{hr. lag})\ UH_t + (2t\ \text{hr.lag})\ UH_t = 3UH_{3t}$$

As t_r ↑es

q_p ↓es

t_p ↑es

t_b ↑es

Example 5

UH_1 of a basin is given.

Determine UH_2 , UH_3 and area of this basin by lagging method

Time (hr)	UH_1 m^3/s	1 hr.lag UH_1	$2UH_2$	UH_2	2 hr.lag UH_1	$3UH_3$	UH_3
0	0		0	0		0	0
1	12	0	12	6		12	4
2	36	12	48	24	0	48	16
3	24	36	60	30	12	72	24
4	18	24	42	21	36	78	26
5	12	18	30	15	24	54	18
6	6	12	18	9	18	36	12
7	0	6	6	3	12	18	6
8		0	0	0	6	6	2
9					0	0	0
Σq	**108**			**108**			**108**

UH_1 + (1 hr. lag) UH_1 = **2 UH_2**

UH_1 + (1 hr. lag) UH_1 + (2 hr. lag) UH_1 = **3 UH_3**

$$d = \frac{\Sigma q * \Delta t}{A} \rightarrow A = \frac{\Sigma q * \Delta t}{d}$$

$$A = \frac{108*1*3600}{0.01} = 38.88*10^6 \ m^2 = 38.88 \ km^2$$

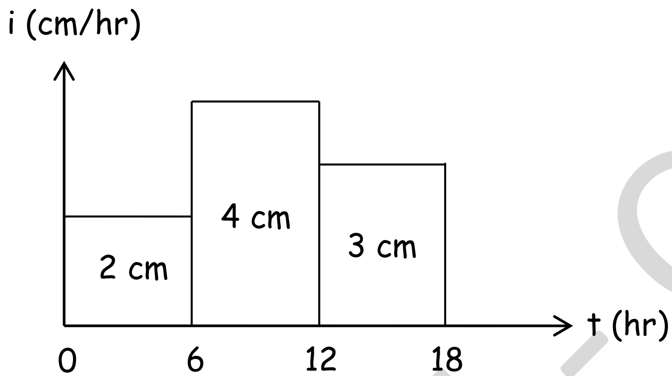
Example 6

Tabulated below are the ordinates of the total direct runoff hydrograph of an 18-hr complex storm. This storm is supposed to be uniformly distributed over the basin, and the excess rainfall has the following pattern: 2 cm in the first 6 hours, 4 cm in the following 6 hours and 3 cm in the last 6 hours. Catchment area is 732 km^2. Assuming there is no error in the data, determine the UH_6 of the basin.

Time (hr)	0	2	4	6	8	10	12	14	16	18	20	22	24	26
Flow (m^3/s)	0	266	544	656	944	1222	1334	1223	1084	1028	618	201	33	0

Solution

The hyetograph of the storm is given in figure below:



The equation of total storm hydrograph can be written as follows:

Total Storm = $2UH_6$ + (6 hr lagged) $4UH_6$ + (12 hr lagged) $3UH_6$

Solution

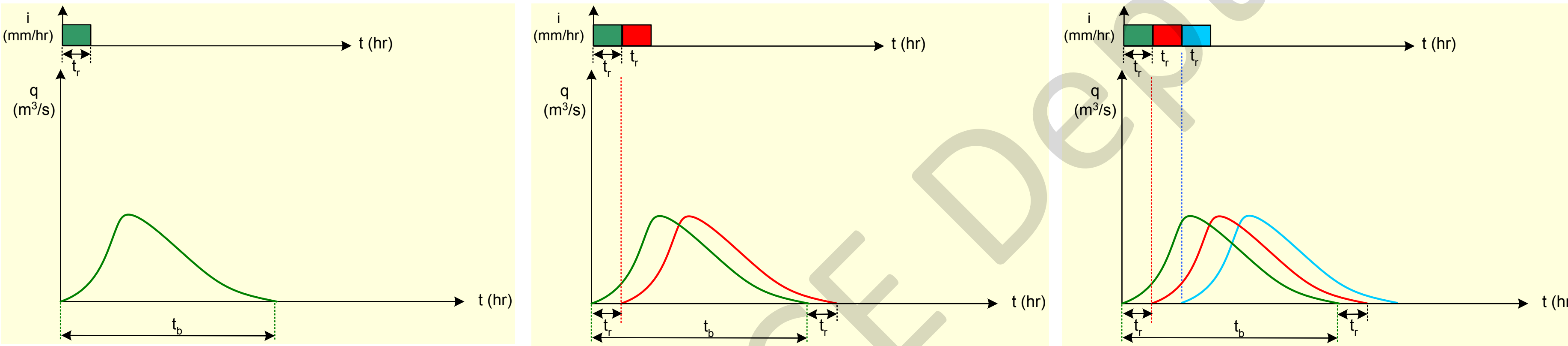
Since it is assumed that there are no errors in the data, lagging method can be used.

Time (hr)	Flow (m^3/s)	$2UH_6$	6 hr lagged $4UH_6$	12 hr lagged $3UH_6$	UH_6
0	0	0			0
2	266	266			133
4	544	544			272
6	656	656	0		328
8	944	412	532		206
10	1222	134	1088		67
12	1334	22	1312	0	11
14	1223	0	824	399	0
16	1084		268	816	
18	1028		44	984	
20	618		0	618	
22	201			201	
24	33			33	
26	0			0	
TOTAL	9153				

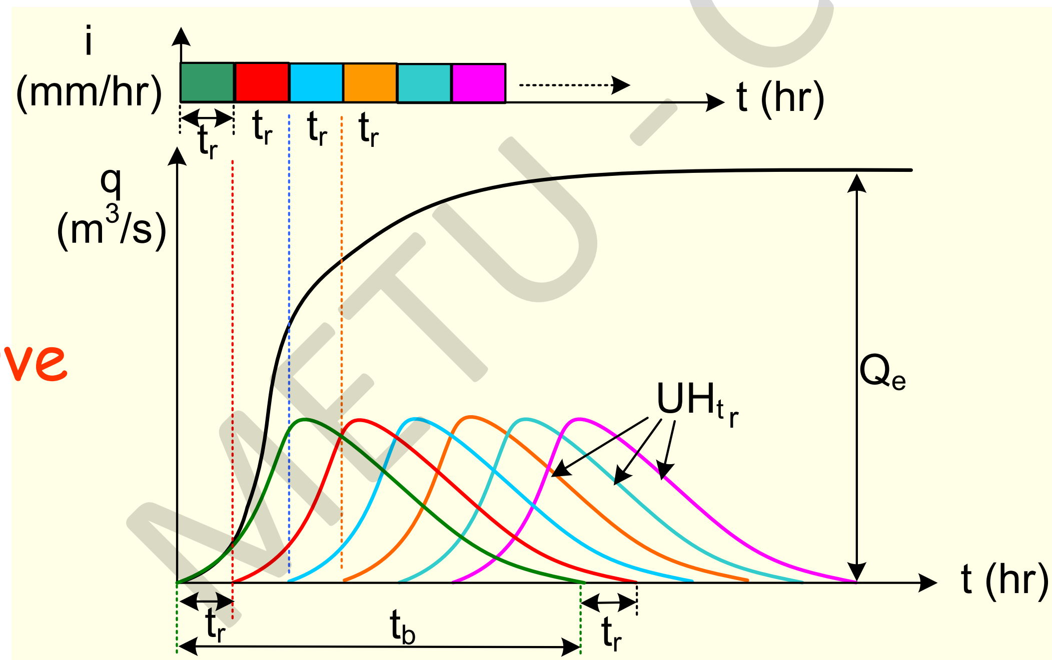
The S-Curve Method

- It is used to obtain UHs of different durations that are not multiples of each other.
- S-curve is the hydrograph that would result from an infinite series of UHs of t_r durations, each delayed t_r hours wrt the preceding one.
- In other words, it is the hydrograph of the runoff of continuous rainfall with an intensity of $1/t_r$.
- S-curve has the form of a mass curve, the discharge of the basin becoming constant around base time of UH of t_r.
- Thus each S-curve is unique for a specified UH duration, in a particular drainage basin.

S - Curve



S - curve



$$n = t_b/t_r$$

$Q_e = i \,.\, A$ (mm/hr . km^2)

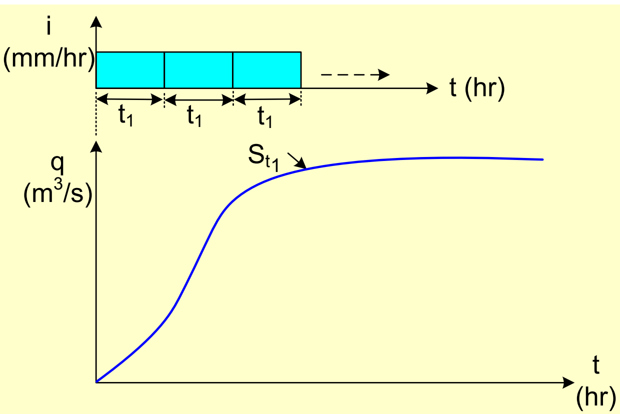
$Q_e = d/t_r \,.\, A = 1/t_r \,.\, A$

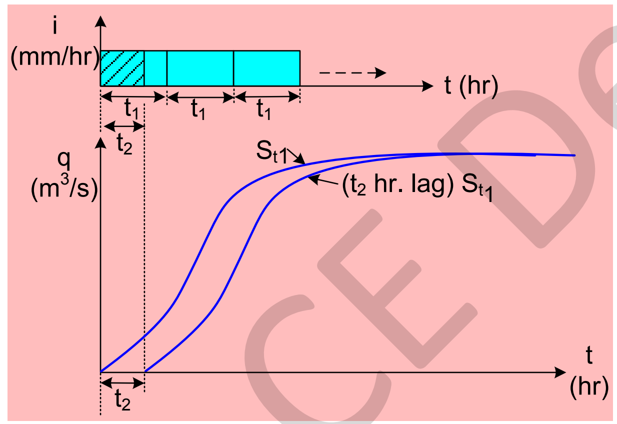
(d = 1 cm)

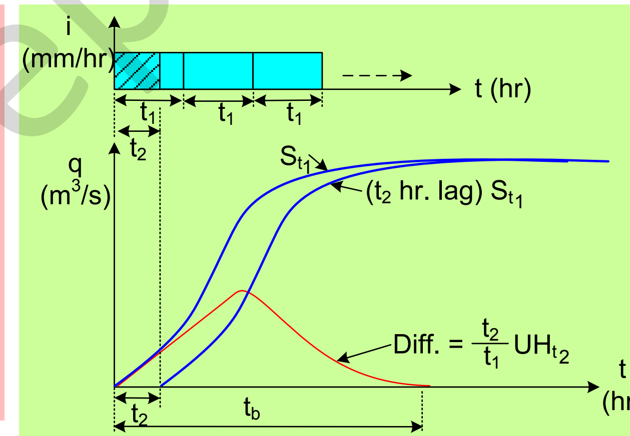
$$Q_e = 2.78 \frac{A}{t_r}$$

Q_e = constant outflow (m^3/s)
A = area of basin (km^2)
t_r = duration of UH (hr)

Obtaining UH using S-curve ($t_2 < t_1$)



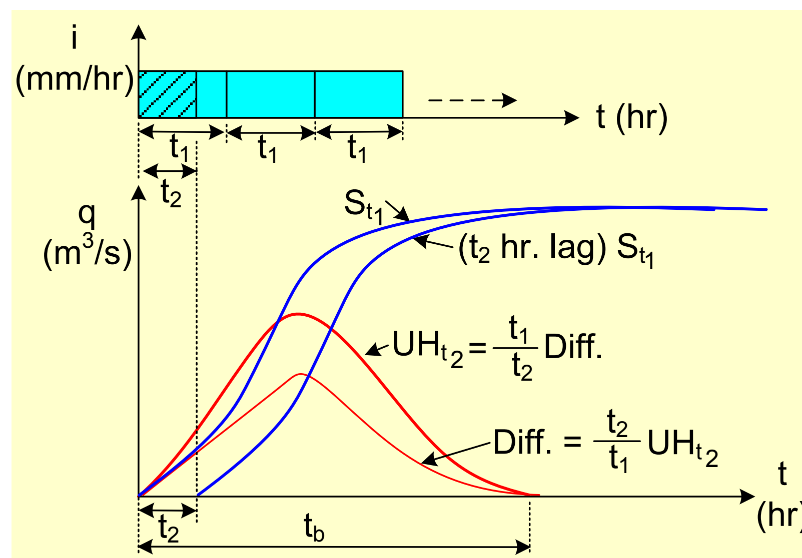




$$\text{Diff.} = S_{t1} - (t_2 \text{ hr. lag}) \ S_{t1}$$

$$\text{Diff.} = (t_2 \, . \, 1/t_1) \ UH_{t2} \ = \ t_2/t_1 \ UH_{t2}$$

$$UH_{t_2} = \frac{t_1}{t_2}\left[S_{t_1} - (t_2 \text{ hr. lag}) \ S_{t_1}\right]$$



Example 7

Excess rainfall blocks of a precipitation event over a basin are as follows: 150 mm in the first three hours, 90 mm in the consecutive two hours. The ordinates of the S3 curve of the related basin are given below.

i. Determine the ordinates of the surface runoff hydrograph resulting from the above given rainfall blocks,

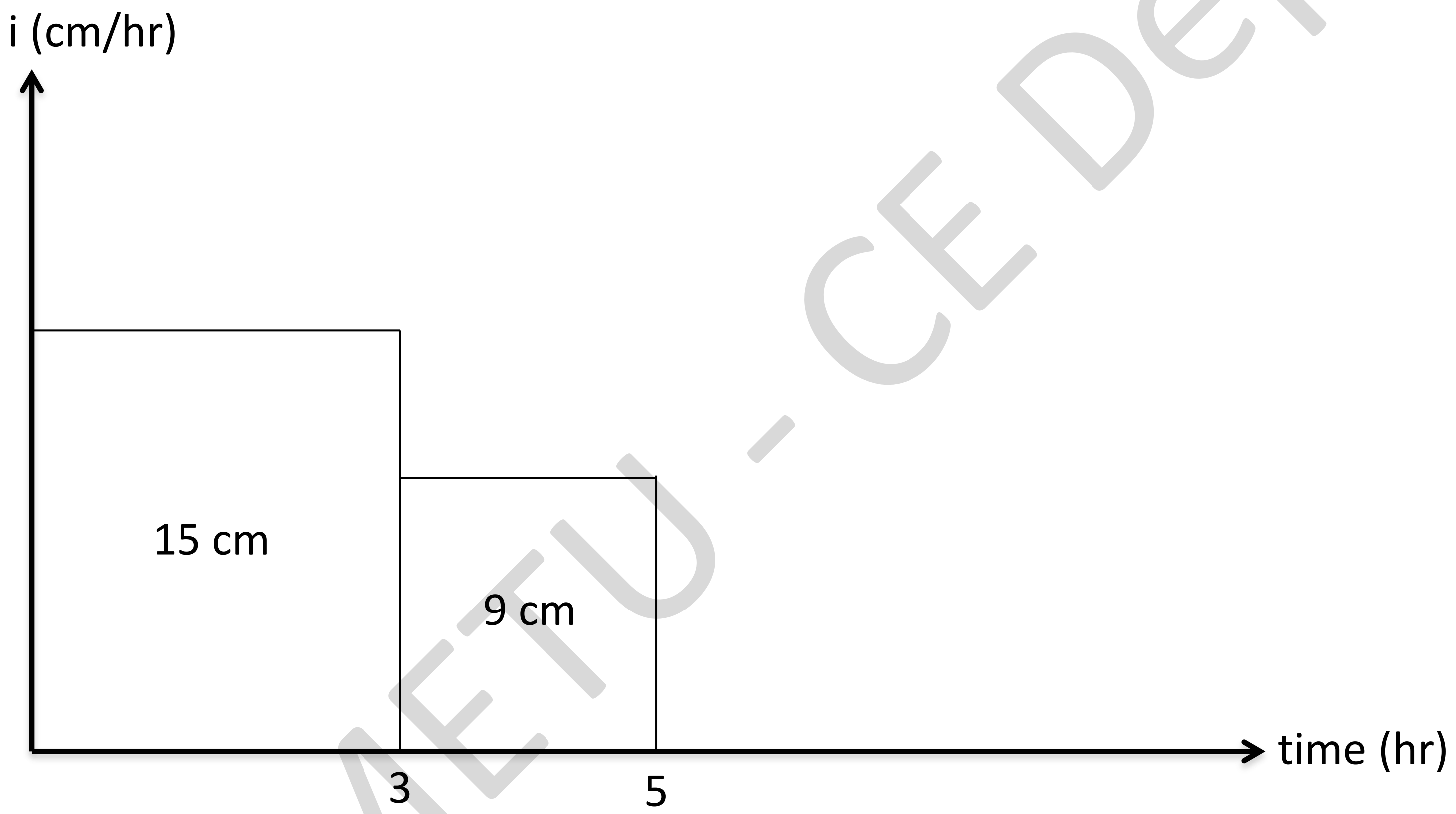
ii. Assuming constant baseflow of 30 m^3/s, determine the ordinates of resulting streamflow,

iii. Determine the size of the basin.

Time (hr)	0	1	2	3	4	5	6	7	8	9	10	11
S3 (m^3/s)	0	22	50	66	76	84	90	90	90	90	90	90

Solution

(i), (ii)



DR-equation = $15UH_3$ + (3hr lag) $9UH_2$

Time (hr)	S_3	2 hr lag S_3	Diff = 2/3 UH_2	(3/2) Diff = UH_2	3 hr lag S_3	Diff = (3/3) UH_3	(3/3) Diff = UH_3	$15UH_3$	3 hr lag $9UH_2$	DR hyd.	Total Hyd. (+30)
0	0		0	0		0	0	0		0	30
1	22		22	33		22	22	330		330	360
2	50	0	50	75		50	50	750		750	780
3	66	22	44	66	0	66	66	990	0	990	1020
4	76	50	26	39	22	54	54	810	297	1107	1137
5	84	66	18	27	50	34	34	510	675	1185	1215
6	90	76	14	21	66	24	24	360	594	954	984
7	90	84	6	9	76	14	14	210	351	561	591
8	90	90	0	0	84	6	6	90	243	333	363
9	90	90			90	0	0	0	189	189	219
10	90	90			90				81	81	111
11	90	90			90				0	0	30

Solution

(iii)

$$d_{SR} = (\Sigma\, Q_{SR}\, dt) / A_{basin}$$

$$d_{SR} = 15 + 9 = 24 \text{ cm} = 0.24 \text{ m}$$

$$A_{basin} = (\Sigma\, Q_{SR}\, dt) / d_{SR} = (6480 * 60 * 60) / 0.24$$

$$A_{basin} = 97.2 * 10^6 = 97.2 \text{ km}^2$$

Also, you can calculate A_{basin} using UH_2 (or UH_3).

Example 8

UH_1 of a basin is given.

Determine UH_2 and UH_3 of this basin by S-curve method

t (hr)	UH_1	1 h.l UH_1	2 h.l UH_1	3 h.l UH_1	4 h.l UH_1	5 h.l UH_1	6 h.l UH_1	S_1	2 h.l S_1	diff	UH_2	3 h.l S_1	diff	UH_3
0	0							0		0	0		0	0
1	12	0						12		12	6		12	4
2	36	12	0					48	0	48	24		48	16
3	24	36	12	0				72	12	60	30	0	72	24
4	18	24	36	12	0			90	48	42	21	12	78	26
5	12	18	24	36	12	0		102	72	30	15	48	54	18
6	6	12	18	24	36	12	0	108	90	18	9	72	36	12
7	0	6	12	18	24	36	12	108	102	6	3	90	18	6
8		0	6	12	18	24	36	108	108	0	0	102	6	2
9			0	6	12	18	24	108	108			108	0	0

$N = t_b / t_r = 7/1$
$= 7$

$$UH_2 = \frac{1}{2}[S_1 - (2\,\text{hr.lag})\,S_1]$$

$$UH_3 = \frac{1}{3}[S_1 - (3\,\text{hr.lag})\,S_1]$$

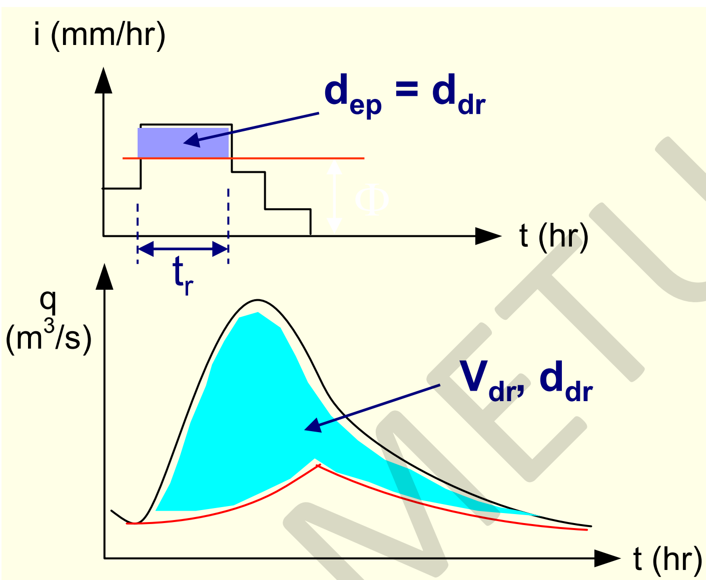
$$UH_{t_2} = \frac{t_1}{t_2}[S_{t_1} - (t_2\,\text{hr. lag})\,S_{t_1}]$$

Derivation of the Unit Hydrograph

- UH of a basin is derived from a hydrograph resulting from a storm of uniform intensity, which covers the basin uniformly.
- If the basin is very large, it may never be covered by a storm of uniform intensity.
- In such a case the basin should be divided into tributary subbasins, & a UH for each should be determined separately.
- In the derivation of the UH for a certain basin, it is necessary to study as many simple storms as possible, so as to obtain a representative UH.

The procedure:

- 1. Study rainfall records to select storms giving reasonably high excess rainfall depths (= direct runoff depths).
- 2. Separate direct runoff from the other components in the selected storm hydrographs.
- 3. Determine the volume & depth of direct runoff for each hydrograph.

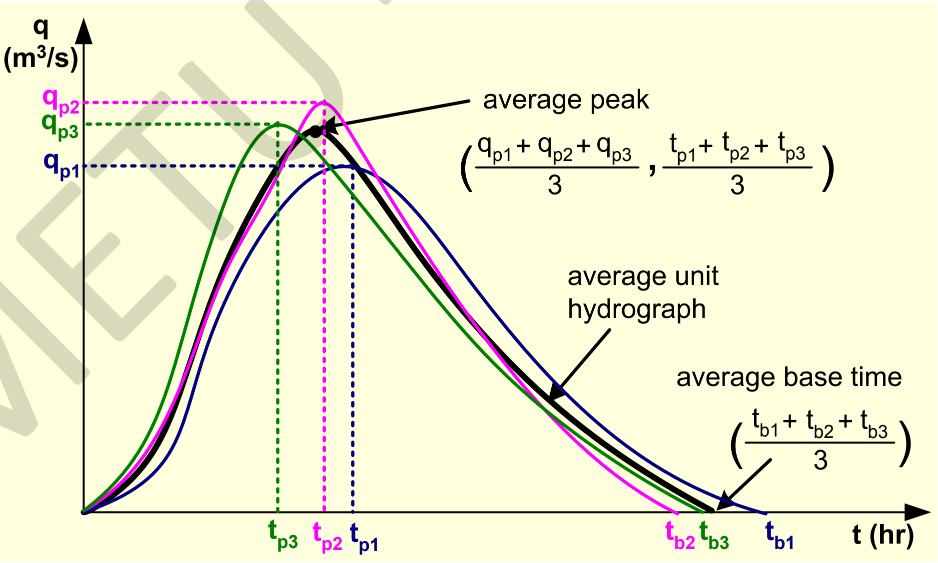


$DR = d_{dr}\ UH\ t_r$

- 4. Determine the Φ-index to find the duration of excess rainfall, which gives the duration of the hydrograph (t_r).
- 5. Divide ordinates of each of the direct runoff hydrograph by the corresponding DR depth to obtain the UHs
- 6. Obtain the same duration UHs from different duration UHs.

- 7. Determine the representative UH for the basin by averaging
  - the peak flows,
  - times to peak, and
  - time bases of the UHs.

  The average UH is then sketched following the shapes of the individual hydrographs.
- 8. Adjust the area under the curve to unit depth.



Synthetic Unit Hydrograph

- In case of lack rainfall-runoff data, a unit hydrograph can be generated synthetically.
- Synthetic unit hydrographs relate peak flow, time to peak, basin lag, & base time statistically to watershed characteristics like length, slope, area, & percent imperviousness, etc.
- Synthetic unit hydrographs may be synthesized on the basis of past experience in other areas & applied as first approximations to the ungaged catchments.

http://unithydro.sdsu.edu/

Synthetic Unit Hydrograph

- Snyder Method
- SCS Method

  (developed by Soil Conservation Services, 1957)
- Espey Method
- Mockus Method
- DSİ Synthetic Unit Hydrograph Method
- Time Area Method

SCS Method

- It is based on the evaluation of many UHs from basins ranging in size and geographic location.
- Then a dimensionless hydrograph, which has a simple triangular shape, is developed from these UHs.

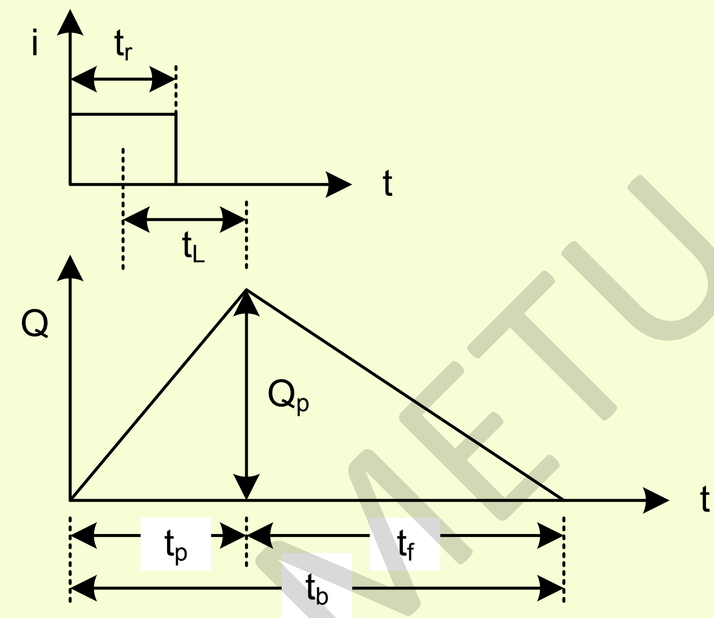


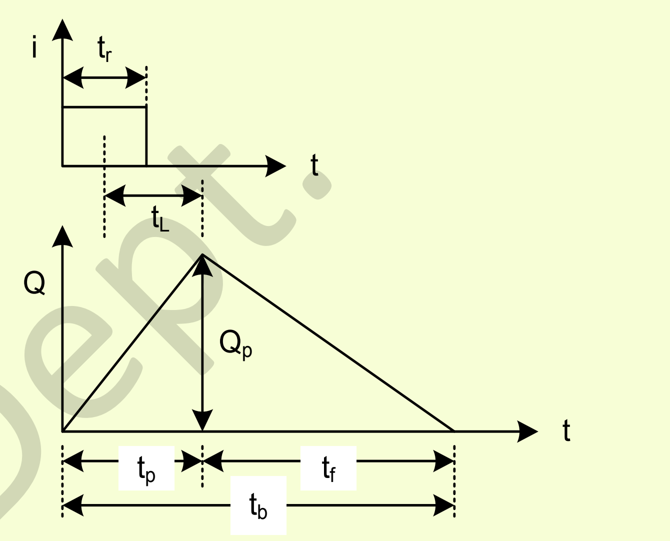
Figure 7.27 Triangular unit hydrograph

Q_p : peak flow (m^3/s)
t_r : duration of excess rainfall
t_L : lag time
t_p : time to peak or time of rise (hr)
t_f : time of fall or recession time (hr)
t_b : base time

SCS Method

$$Q_p = \frac{2.08A}{t_p}$$

A : basin area in (km^2)
t_p : time to peak (hr)
Q_p : peak discharge (m^3/s)



- Potential max. retention, S (inch): $S = \frac{1000}{CN} - 10$

CN = f (soil group, surface cover, & antecedent moisture condition)

- Basin lag (hr): $t_L = \frac{L^{0.8}(S+1)^{0.7}}{1900 S_h^{0.5}}$
- Time to peak (hr): $t_p = \frac{t_r}{2} + t_L$
- Base time (hr): $t_b = 2.67 t_p$

t_L: basin lag (hr)
L : the main stream length (ft)
S_h: the bed slope of the main stream.

Curve Number Table

Land Use Description on Input Screen	Description and Curve Numbers from TR-55					
	Cover Description		Curve Number for Hydrologic Soil Group			
	Cover Type and Hydrologic Condition	% Impervious Areas	A	B	C	D
Agricultural	Row Crops - Staight Rows + Crop Residue Cover- Good Condition (1)		64	75	82	85
Commercial	Urban Districts: Commerical and Business	85	89	92	94	95
Forest	Woods(2) - Good Condition		30	55	70	77
Grass/Pasture	Pasture, Grassland, or Range(3) - Good Condition		39	61	74	80
High Density Residential	Residential districts by average lot size: 1/8 acre or less	65	77	85	90	92
Industrial	Urban district: Industrial	72	81	88	91	93
Low Density Residential	Residential districts by average lot size: 1/2 acre lot	25	54	70	80	85
Open Spaces	Open Space (lawns, parks, golf courses, cemeteries, etc.)(4) Fair Condition (grass cover 50% to 70%)		49	69	79	84
Parking and Paved Spaces	Impervious areas: Paved parking lots, roofs, driveswways, etc. (excluding right-of-way)	100	98	98	98	98

With the use of the above relations, a triangular unit hydrograph is obtained. Using the dimensionless coordinates of SCS unit hydrograph, the actual coordinates can be obtained.

t/t_p	Q/Q_p	t/t_p	Q/Q_p
0.0	0.00	1.7	0.46
0.1	0.03	1.8	0.39
0.2	0.10	1.9	0.33
0.3	0.19	2.0	0.28
0.4	0.31	2.2	0.207
0.5	0.47	2.4	0.147
0.6	0.66	2.6	0.107
0.7	0.82	2.8	0.077
0.8	0.93	3.0	0.055
0.9	0.99	3.2	0.040
1.0	1.00	3.4	0.029
1.1	0.99	3.6	0.021
1.2	0.93	3.8	0.015
1.3	0.86	4.0	0.011
1.4	0.78	4.5	0.005
1.5	0.68	5.0	0.000
1.6	0.56		

Example 9

Given: t_r=2 hrs, A=150 km^2, CN=75, L=21 km, S_h=1%

- NOTE: 1 m=3.28 ft L=68800 ft

$$S = \frac{1000}{75} - 10 = 3.33\,\text{inch}$$

$$t_L = \frac{(68800)^{0.8}(3.33+1)^{0.7}}{1900\sqrt{1}} = 10.9\,\text{hr}$$

$$t_p = \frac{2}{2} + 10.9 = 11.9\,\text{hr}$$

$$Q_p = \frac{2.08 \times 150}{11.9} = 26.2\,\text{m}^3/s$$

$$t_b = 2.67 \times 11.9 = 31.77\,\text{hrs}$$

Let us check if the depth of runoff is 1 cm

$$d=\frac{V}{A}=\frac{0.5\times Q_p t_b}{150\times 10^6}=\frac{0.5\times 26.2\times 31.77\times 3600}{150\times 10^6}=0.01\,m=1\,\text{cm},\ \text{O.K.}$$