CE305 Numerical Methods for Civil Engineers — 2014 Fall

Hw 2

1.Answer the following questions.

$$50 = 5c - 7b$$
$$4b + 7c + 30 = 0$$
$$a - 7c = 40 - 3b + 5a$$

a. Write the set of equations in matrix form:

b. Is this system consistent? Show justification of your answer.If the system is consistent, solve the system by Gauss Elimination.

2.Determine if the following matrices are consistent or inconsistent? Is there a unique solution?

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \\ -2 & 6 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 5 & -2 \\ 3 & 0 & -1 \\ 2 & 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 \\ 3 & -5 \\ 3 & -13 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Try and solve the second system by Gaussian elimination.

3. Solve the following system of linear equations with LU decomposition using Doolittle's algorithm. Carry out your calculations with 3 decimal places. Make your solution by hand; afterwards check your result using MATLAB.
Hint: There is a special function in MATLAB for LU decomposition i.e. for decomposing matrix A you can use `[L U]=lu(A)` command.

$$8x_1 - x_2 + 3x_3 - 25 = 0$$
$$3x_1 + 24x_2 + 5x_3 - 87 = 0$$
$$x_1 - 8x_2 - 16x_3 + 108 = 0$$

4. Solve the following system of equations with an error tolerance of 10^{-2} and initial guess of $x^T = \lfloor x_1^0 \quad x_2^0 \quad x_3^0 \rfloor = [1.4 \quad 0.4 \quad 1.9]$ using

$$-x_1 + 3x_2 - 2x_3 = -4$$
$$2x_2 + 3x_3 = 7$$
$$3x_1 - x_2 + x_3 = 6$$

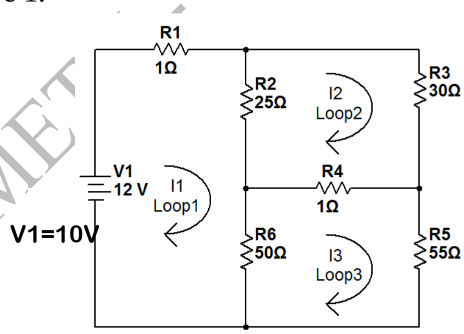
a) Gauss Jacobi method. b) Gauss Seidel method.
c) Compare the results of these two methods in terms of speed of convergence (or number of iterations) and briefly comment on the difference.

Hint : Error $= \max(|x_j^{k+1} - x_j^k|)$, $j = 1,2,3$. Use 4 decimal places in your calculations.

5.Kirchhoff's second circuit law deals with the potential difference (commonly known as voltage) in electrical circuits which can be stated as:

$$\sum_{k=1}^{n} V_k = 0 \text{ where n denotes number of loops.}$$

The following equations are obtained from loops shown in Figure 2. Values of $\mathbf{R_1}$ to $\mathbf{R_6}$ and $\mathbf{V_1}$ are also shown in Figure 1.



$$R_1 i_1 + R_2 (i_1 - i_2) + R_6 (i_1 - i_3) = V_1$$
$$R_2 (i_2 - i_1) + R_3 i_2 + R_1 (i_2 - i_3) = 0$$
$$R_6 (i_3 - i_1) + R_1 (i_3 - i_2) + R_5 i_3 = 0$$

a) Find i_1 i_2 and i_3 using Gauss-Seidel method with an initial guess $(i_1^0, i_2^0, i_3^0) = (0.3,\ 0.1,\ 0.1)$.

b) The Gauss-Seidel method could be accelerated by introducing so called *relaxation parameter*, which is denoted by **w.** The new approximation is taken as:

$$x_i^{k+1} = (1-w)\,x_i^k + \frac{w}{a_{ii}}\left(b_i - \sum_{j=1}^{i-1} a_{ij}\,x_j^k - \sum_{j=i+1}^{n} a_{ij}\,x_j^k\right) \text{ for } i = 1,\ldots,n.$$

a_{ii} : Elements of coefficient matrix of system of linear equations.

Note that for **w=1** the method is identical with the Gauss-Seidel method. Find i_1 i_2 and i_3 using accelerated version of Gauss-Seidel method taking **w=0.7**. Initial guess should be taken as given in Part (a).

$$\textbf{Hint:}\ \text{Error} = \max\left(\left|\frac{x_j^{k+1} - x_j^k}{x_j^{k+1}}\right|\right),\ j = 1,2,3.\ \text{Error tolerance is } 5*10^{-3}.\ \text{Use 4 decimal places in your calculations.}$$

6.The following set of equations is given. Solve the system using Newton-Jacobi method using initial guesses $(x_0, y_0) = (0.5, 0.4)$ in the first quadrant. Use an error tolerance of $5*10^{-2}$ and at least 6 decimal places in your calculations.

$x^2 + y^2 = 1$ $5x^2 + 21y^2 = 9$	$\textbf{Hint:}\ \text{Error} = \max\left(\left\lvert\frac{x_j^{k+1} - x_j^k}{x_j^{k+1}}\right\rvert\right),\ j = 1,2,3.$

Note: There are four solutions of the given system of equations in each quadrant. You may see these solutions by sketching these equations on the same graph.

7.The LORAN (LOng RAnge Navigation) system calculates the position of a boat at sea using signals from fixed transmitters. From the time differences of the incoming signals, the boat obtains differences of distances to the transmitters. This leads to two equations each representing hyperbolas defined by the coordinates of transmitters and boat.

$\frac{x^2}{x_1^2} - \frac{y^2}{y_1^2 - x_1^2} = 1$ $\frac{(y-b)^2}{x_2^2} - \frac{(x-a)^2}{y_2^2 - x_2^2} = 1$	where (x,y): Coordinates of the boat (x_1, y_1): Coordinates transmitter 1 (x_2, y_2): Coordinates transmitter 2 (a,b): Reflection coefficients	$\textbf{Hint:}\ \text{Error} = \max\left(\left\lvert\frac{x_j^{k+1} - x_j^k}{x_j^{k+1}}\right\rvert\right),\ j = 1,2,3.$

The coordinates of the boat can be estimated by solving these equations. If (x_1, y_1), (x_2, y_2) and (a,b) are given as $(186, 300)$, $(279, 500)$ and $(300, 500)$, respectively, find the coordinates of the boat (x, y). Initial guesses are given as $(x^0, y^0) = (100, 100)$ and $(5*10^{-3})$ should be taken as error tolerance. Use at least 6 decimal places in your calculations.