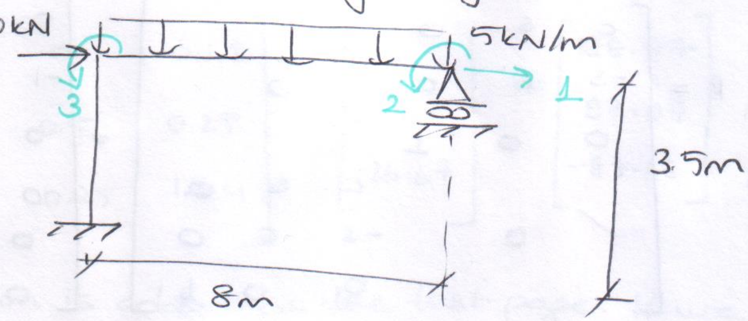
CE424

HW2 SOLUTIONS

Q1) For the given structure by using stiffness method:



Take $EI = 1 kN \cdot m^2$ for all elements and assume axial rigidity.

a) Discretize the structure and show degrees of freedom and local coordinate system of every element.

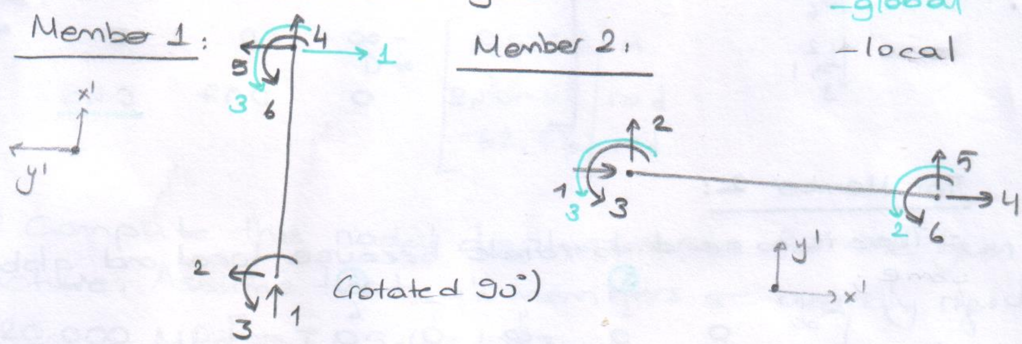
D.O.F in local coordinate system:

Member 1:

Member 2:

–global

–local



b) Calculate the element stiffness matrices.

For Member 1:

($EA/L \to \infty$, axially rigid)

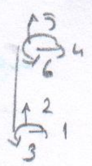
$$k' = \begin{bmatrix} EA/L & 0 & 0 & -EA/L & 0 & 0 \\ 0 & 12EI/L^3 & 6EI/L^2 & 0 & -12EI/L^3 & 6EI/L^2 \\ 0 & 6EI/L^2 & 4EI/L & 0 & -6EI/L^2 & 2EI/L \\ -EA/L & 0 & 0 & EA/L & 0 & 0 \\ 0 & -12EI/L^3 & -6EI/L^2 & 0 & 12EI/L^3 & -6EI/L^2 \\ 0 & 6EI/L^2 & 2EI/L & 0 & -6EI/L^2 & 4EI/L \end{bmatrix}$$

$$k' = \begin{bmatrix} \infty & 0 & 0 & -\infty & 0 & 0 \\ 0 & 0.28 & 0.49 & 0 & -0.28 & 0.49 \\ 0 & 0.49 & 1.14 & 0 & -0.49 & 0.57 \\ -\infty & 0 & 0 & \infty & 0 & 0 \\ 0 & -0.28 & -0.49 & 0 & 0.28 & -0.49 \\ 0 & 0.49 & 0.57 & 0 & -0.49 & 1.14 \end{bmatrix}$$

As the system is rotated $90°$, we have to use the rotation matrix, R;

$$R = \begin{bmatrix} c & s & 0 & & & \\ -s & c & 0 & & 0 & \\ 0 & 0 & 1 & & & \\ & & & c & s & 0 \\ & 0 & & -s & c & 0 \\ & & & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$k = R^T \cdot k' \cdot R =$ in global coordinate



$$\begin{array}{c|c} \begin{array}{cccccc} 1 & 2 & 3 & \overset{1}{4} & 5 & \overset{3}{6} \end{array} & \\ \begin{bmatrix} 0.28 & 0 & -0.49 & -0.28 & 0 & -0.49 \\ 0 & \infty & 0 & 0 & -\infty & 0 \\ -0.49 & 0 & 1.14 & 0.49 & 0 & 0.57 \\ -0.28 & 0 & 0.49 & \underline{0.28} & 0 & \underline{0.49} \\ 0 & -\infty & 0 & 0 & \infty & 0 \\ -0.49 & 0 & 0.57 & \underline{0.49} & 0 & \underline{1.14} \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \; 1 \\ 5 \\ 6 \; 3 \end{array} \end{array}$$

For Member 2:

There is no need to rotate because local and global axes are same;

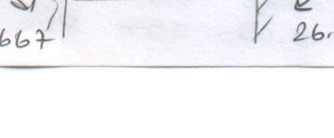
$$k = \begin{array}{c|c} \begin{array}{cccccc} 1 & 2 & \overset{3}{3} & 4 & 5 & \overset{2}{6} \end{array} & \\ \begin{bmatrix} \infty & 0 & 0 & -\infty & 0 & 0 \\ 0 & 0.023 & 0.084 & 0 & -0.023 & 0.084 \\ 0 & 0.084 & \underline{0.5} & 0 & -0.084 & \underline{0.25} \\ -\infty & 0 & 0 & \infty & 0 & 0 \\ 0 & -0.023 & -0.084 & 0 & 0.023 & -0.084 \\ 0 & 0.084 & \underline{0.25} & 0 & -0.084 & \underline{0.5} \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \; 3 \\ 4 \\ 5 \\ 6 \; 2 \end{array} \end{array}$$

c) Calculate the structural stiffness matrix and force vector for the given degrees of freedom;

$$K = \begin{bmatrix} 0.28 & 0 & 0.49 \\ 0 & 0.5 & 0.25 \\ 0.49 & 0.25 & 1.14 + 0.5 \end{bmatrix}$$

$$q = q_a - q_f = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ -26.67 \\ 26.67 \end{bmatrix} = \begin{bmatrix} 10 \\ 26.67 \\ -26.67 \end{bmatrix}$$

$\frac{wL^2}{12} =$

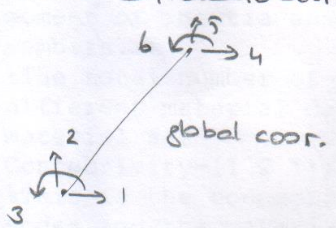


d) Compute the nodal displacements;

$K.D = Q \quad , \quad D = K^{-1}.Q$

$$D = \begin{bmatrix} 0.28 & 0 & 0.45 \\ 0 & 0.5 & 0.25 \\ 0.45 & 0.25 & 1.64 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 10 \\ 26.67 \\ -26.67 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 188.77 \\ 97.07 \\ -87.46 \end{bmatrix} \end{array} \begin{array}{l} m \\ rad \\ rad \end{array}$$

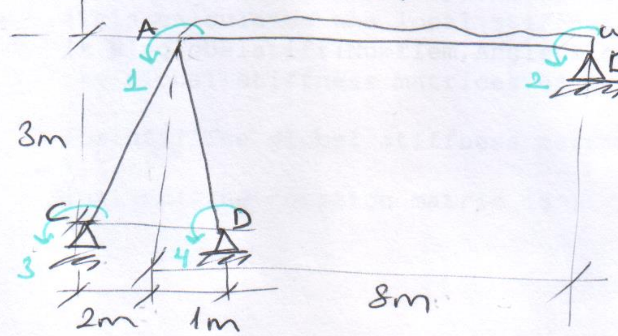
Q2) Algorithm is added as the last page. If we add the member shown to the structure shown in Q1, new structural matrix and new displacements would be;



$$K = \begin{bmatrix} 0.28 + 0.0125 & 0 + 0.0315 & 0.45 \\ 0 + 0.0315 & 0.5 + 0.46 & 0.25 \\ 0.45 & 0.25 & 1.64 \end{bmatrix}$$

$$D = \begin{bmatrix} 134.71 \\ 39.65 \\ -62.56 \end{bmatrix} \begin{array}{l} m \\ rad \\ rad \end{array}$$

Q3) Compute the nodal displacements for the given structure. Assume that all members are axially rigid.
$E = 20.000$ MPa, $I = 5.4 \times 10^5$ mm4.



* For the calculation of global stiffness matrices of AC and AD the algorithm in Question 2 was used. And AB is calculated using a similar procedure in Question 1. According to this;

$$K = \begin{bmatrix} 54.000 + 119.820 + 136610 & 27000 & 59910 & 68310 \\ 27000 & 54000 & 0 & 0 \\ 59910 & 0 & 119820 & 0 \\ 68310 & 0 & 0 & 136610 \end{bmatrix} \qquad Q = \begin{bmatrix} -53.33 \\ 53.33 \\ 0 \\ 0 \end{bmatrix}$$

$K.D = F$, $D = K^{-1}.F$, $D_1 = -0{,}00034$ rad, $D_2 = 0{,}00116$ rad, $D_3 = D_4 = 0{,}00017$ rad

Main Code:

```
clear all
clc
%CE 425 Fall 2014 HW2-Q2
%Prepared by Alper Aldemir for Fall 2013
%INPUT PHASE
%
XY=[0 0; 8 3.5];
%This matrix containes the x and y coordinates of the nodes, respectively. The total number of
rows is the total number of nodes.
%(NumNode) Data.Geometry.XY=[X Y] for each node
Materials=[0.1 1 1];
%This is a material matrix. Each row is composed of the area, the moment of inertia and the
modulus of elasticity of different members.
%The total number of rows is dependent on the total number of different material data sets.
Data.Materials.M=[A I E] for each material set
Connectivity=[1 2 1];
%This is the connectivity matrix. It connects the elements to the nodes and the materials. The
total number of rows is the total number of elements.
%(NumElem)This matrix is composed of starting node, ending node and the material set number in
each columns. Data.Geometry.C=[SN EN MS] for each elements
%
%MAIN PROGRAM
%
NumElem=size(Connectivity,1);
[Angle L]=framelength(NumElem,Connectivity,XY);     %This calculates the length and orientation
of the members.
klocal=stiffness(NumElem,Connectivity,Materials,L);             %This calculates the local
stiffness matrices of members.
[k R]=globalstiff(NumElem,Angle,klocal);        %This calculates the global stiffness matrices of
members.

fprintf('The global stiffness matrix is')
k{1,1}
fprintf('The rotation matrix is')
R
```

Functions:

```
function [Angle L]=framelength(NumElem,Connectivity,XY)
%This function calculates the length and orientation of members.

for i=1:1:NumElem
    Coord(i,1)=XY(Connectivity(i,1),1);
    Coord(i,2)=XY(Connectivity(i,1),2);
    Coord(i,3)=XY(Connectivity(i,2),1);
    Coord(i,4)=XY(Connectivity(i,2),2);
end

for i=1:1:NumElem
    L(i,1)=sqrt((Coord(i,3)-Coord(i,1))^2+(Coord(i,4)-Coord(i,2))^2);
    Angle(i,1)=atan((Coord(i,4)-Coord(i,2))/(Coord(i,3)-Coord(i,1)));
end

function [k, R]=globalstiff(NumElem,Angle,klocal)
%This function calculates the global stiffness matrices for members.

for i=1:1:NumElem

    R=[cos(Angle(i,1))  sin(Angle(i,1)) 0 0 0 0;
      -sin(Angle(i,1)) cos(Angle(i,1)) 0 0 0 0;
       0 0 1 0 0 0;
       0 0 0 cos(Angle(i,1)) sin(Angle(i,1)) 0;
       0 0 0 -sin(Angle(i,1)) cos(Angle(i,1)) 0;
       0 0 0 0 0 1];

    k{i,1}=R'*klocal{i,1}*R;
end
```

```matlab
function klocal=stiffness(NumElem,Connectivity,Materials,L)
%This function calculates the local stiffness matrix of an element.

for i=1:1:NumElem

    A=Materials(Connectivity(i,3),1);
    E=Materials(Connectivity(i,3),3);
    I=Materials(Connectivity(i,3),2);
    L=L(i,1);

    c1=E*A/L;
    c2=12*E*I/L^3;
    c3=6*E*I/L^2;
    c4=4*E*I/L;

    klocal{i,1}=[c1,0,0,-c1,0,0
                 0,c2,c3,0,-c2,c3
                 0,c3,c4,0,-c3,c4/2
                -c1,0,0,c1,0,0
                 0,-c2,-c3,0,c2,-c3
                 0,c3,c4/2,0,-c3,c4];
end
```