# CE 382 Homework 1

## Part A

i) Hognestad stress-strain model →

$E_c = 12680 + 460 f_c = 35680$ MPa ( $E_c = 12680 + 460 f_c$ (MPa) )

$f_c = 50$ MPa (given compressive strength)

$$\varepsilon_{co} = \frac{2 f_c}{E_c} = \frac{2 \times 50}{35680} = 2{,}8 \times 10^{-3}$$

Now $\sigma_c = f_c \left[ \frac{2\varepsilon_c}{\varepsilon_{co}} - \left( \frac{\varepsilon_c}{\varepsilon_{co}} \right)^2 \right] \rightarrow 40 = 50 \left[ \frac{2\varepsilon_c}{2{,}8\times 10^{-3}} - \left( \frac{\varepsilon_c}{2{,}8 \times 10^{-3}} \right)^2 \right]$

Hognestad equation

$0.8 = \left[ 714 \varepsilon_c - \varepsilon_c^2 \times 127551 \right]$

This one is used because it is smaller than $\varepsilon_{co}$ ← $\varepsilon_{c_1} = 1{,}55 \times 10^{-3}$ > solved by calculator

$\varepsilon_{c_1} = 4{,}05 \times 10^{-3}$

In order to find the strain at 15MPa, $\sigma = E\varepsilon$ (Hooke's Law) must be used. Because linear assumption is made.

$25 = \varepsilon E_c = \varepsilon_{25} \times 35680$ $\quad \varepsilon_{25} = 7 \times 10^{-4}$ (25MPa drop to 15MPa gives $\varepsilon_{15} = 7 \times 10^{-4}$ strain)

$\underbrace{1{,}55 \times 10^{-3}}_{\varepsilon_{c1}} - \underbrace{7 \times 10^{-4}}_{\varepsilon_{15}} = \boxed{8.49 \times 10^{-4}}$ → compressive strain in concrete

ii) $\sigma_c = f_c \left[ \frac{2\varepsilon_c}{\varepsilon_{co}} - \left( \frac{\varepsilon_c}{\varepsilon_{co}} \right)^2 \right] \rightarrow 45 = 50 \left[ \frac{2\varepsilon_c}{2{,}8\times 10^{-3}} - \left( \frac{\varepsilon_c}{2{,}8 \times 10^{-3}} \right)^2 \right]$

Since we are going from 15MPa to 45 MPa and linear loading-unloading is assumed, i can increase from 0 to 45 MPa instead of 15 to 45MPa

$\varepsilon_{c_1} = 3.68 \times 10^{-3}$ → This is bigger than $\varepsilon_{co}$

$\boxed{\underbrace{\varepsilon_{c_1}}_{\varepsilon_{c45}} = 1{,}91 \times 10^{-3}}$ → This one is used because it is bigger than $\varepsilon_{c_1}$

iii) Again Hooke's Law must be used.

$\sigma = E\varepsilon$

$45 = 35680\,\varepsilon$

$\varepsilon = 1{,}26 \times 10^{-3}$

$\varepsilon_{c2} - \varepsilon_c = 1{,}91 \times 10^{-3} - 1{,}26 \times 10^{-3} = \boxed{0{,}65 \times 10^{-3}}$

↳ Hooke's Law result

↳ strain left on the member

ii) result obtained in the previous part

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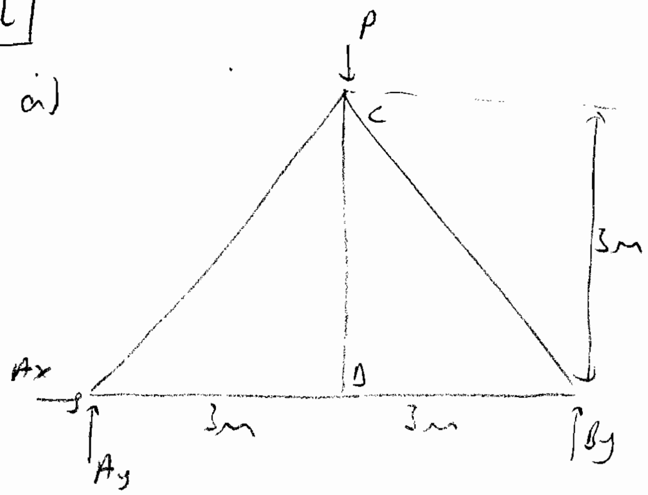
## Part B

## 1.2

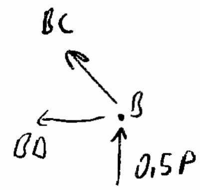
a)




$\Sigma M_A \Rightarrow P(3) - B_y(6) = 0$

$3P = 6B_y$

$B_y = 0{,}5P$




$\Sigma F_x \Rightarrow BD + BC\cos 45 = 0$

$BD = -0{,}71BC$

$\Sigma F_y \Rightarrow 0{,}5P + BC\sin 45 = 0$

$0{,}5P = -BC\, 0{,}71$

$BC = -0{,}70P$ (comp)

$BD = -0{,}71BC = 0{,}5P$

Since there is symmetry ; $AC = -0{,}70P$ (comp)

$AD = 0{,}5P$ (tension)

$f_{ck} = 20MPa$ $f_{ctk} = 1{,}6MPa$ 150x150mm $A = 22500\,mm^2$

I will check the axial capacity.

$T_{tension} = f_{ctk} \times A = 1{,}6(22500) = 36000N = 36kN$

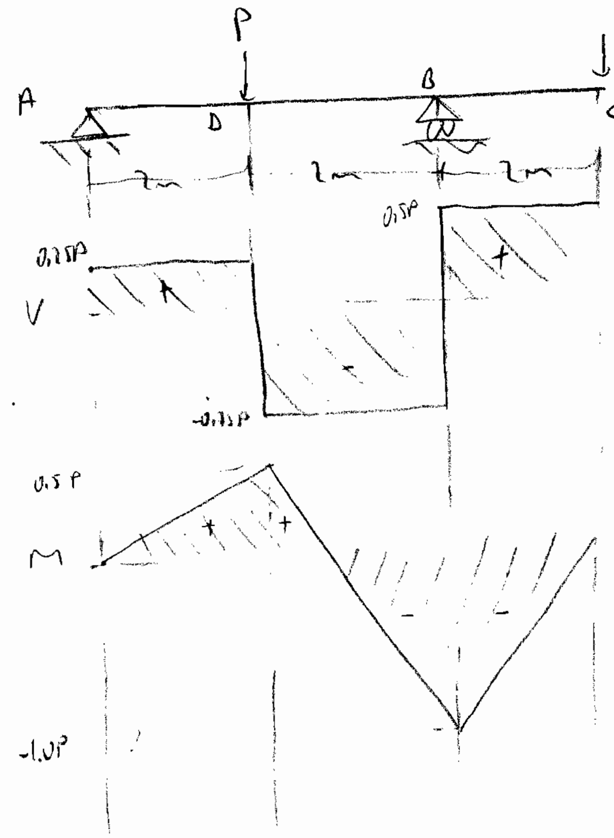
$T_{comp} = f_{ck} \times A = 20(22500) = 450000N = 450kN$

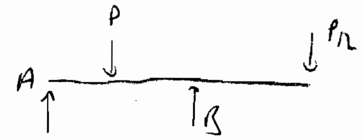
$BC \rightarrow 0{,}70P = 450\,kN$ $P = 642{,}9\,kN$

$BD \rightarrow 0{,}5P = 36kN$ $P = 72kN$ more critical

$\boxed{P_{max} = 72kN}$

b)




$\Sigma M_A \Rightarrow P(2) + P/2(6) - B(4) = 0$

$2P + 3P = 4B$

$5P = 4B$

$\boxed{B = 1.25P}$

$\Sigma F_y = 0 \Rightarrow P + P/2 - A - 1.25P = 0$

$1.5P - A - 1.25P = 0$

$\boxed{A = 0.25P}$

$f_{ctf} = 0.70\sqrt{f_c}$

$= 0.70(\sqrt{20})$

$= 3.13\,MPa$

$\sigma = \frac{Mc}{I}$ $\quad c = 200mm$

$I = \frac{1}{12}(200)(400^3) = 1066\,666\,667\,mm^4$

$M = \frac{\sigma I}{c}$

$= \frac{3.13(1066\,666\,667)}{200}$

$= 16\,693\,333\,Nmm$

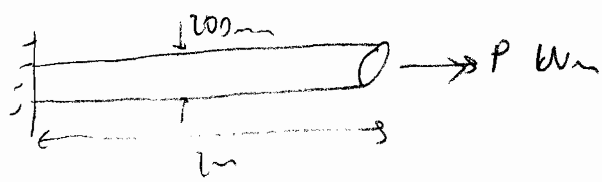
$= 16.7\,kNm$

$M_{max} = 1.0P$ (from the moment diagram)

$1.0P = 16.7\,kNm$

$\boxed{P = 16.7\,kN}$

c) $\phi = 200mm$




Torque = P kNm

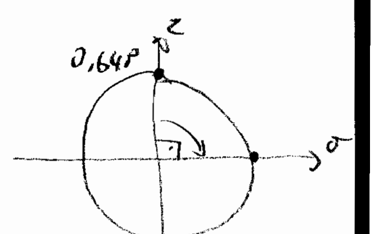
Torsion causes shear

$\tau = \frac{Tc}{J}$ $\quad c = 100mm$

$J = \frac{\pi}{2} r^4 = \frac{\pi}{2}(100^4) = 157\,079\,633\,mm^4$

$\tau = \frac{P(100)(1000)(1000)}{157\,079\,633}$

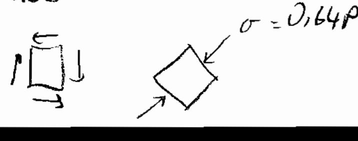
$= 0.64P$




$f_{cts} = 0.5\sqrt{f_c} = 2.24\,MPa$

In order to convert shear into tension → mohr circle is used

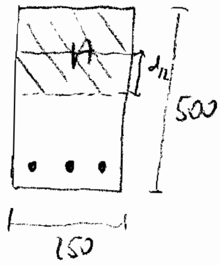
$2\theta = 90 \quad \theta = 45°$




$0.64P = 2.24$

$\boxed{P = 3.52\,kN}$

**1.4** Shear capacity must be checked $\tau = \frac{VQ}{It}$

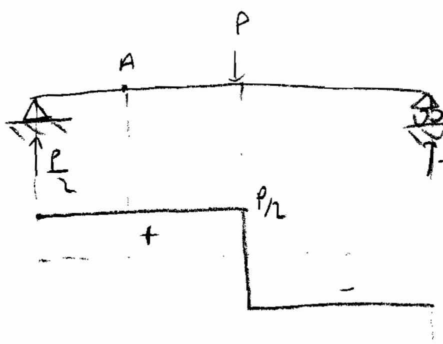


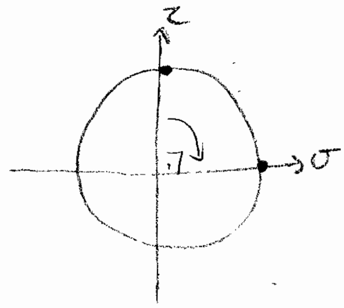

$I = \frac{1}{12} bh^3 = \frac{1}{12}(250)(500^3) = 2604166667\ mm^4$

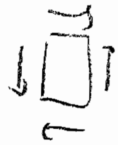
$Q = A \cdot d = 250 \times 250 \times 125$
$= 7812500\ mm^3$

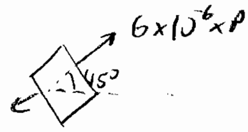
$\tau = \frac{VQ}{It} = \frac{P/2 \times 7812500}{2604166667 \times 250} = P \times 6 \times 10^{-6}$

$t = 250\ mm$ $\quad V_{max} = \frac{P}{2}$






$2\theta = 90°$ $\boxed{\theta = 45°}$ so $\tau_{max} = \sigma_{max}$

inclination angle of the crack

$f_{ctf} = 0{,}7\sqrt{f_c} = 3{,}13\ MPa$

$3{,}13 = 6 \times 10^{-6} \times P$

$\boxed{521{,}7\ kN = P_{max}}$

**1.6** $P = 1750\ kN$

$A_{st} = 8\left(\frac{\pi}{4} 26^2\right) = 4247\ mm^2$

$A_{conc} = A_{gross} - A_{st} = 400^2 - 4247 = 155753\ mm^2$

Creep Strain $= \varepsilon_{ce} = \frac{\sigma_{co}}{E_{c28}} \phi_{ce}$

$\sigma_{co}$ = stress in concrete $= 8\ MPa$
$E_{c28}$ = 28th day modulus of elasticity
$\phi_{ce}$ = creep coefficient

$\varepsilon_{ce} = \frac{8}{20000} 2 = 8 \times 10^{-4}$

compatibility $\rightarrow$ $\varepsilon_c + \varepsilon_s = \varepsilon_{ce}$

Force equilibrium $\rightarrow$ $F_s = F_s$ $\quad \sigma_s A_s = \sigma_c A_c$ $\quad \varepsilon_s E_s A_s = \varepsilon_c E_c A_c$

$\varepsilon_s \times 200000 \times 4247 = \varepsilon_c \times 27000 \times 155753$

S400 $\rightarrow$ $E_s = 200000\ MPa$

C20 $\rightarrow$ $E_c = 27000\ MPa$

$849400000 \times \varepsilon_s = 4205331000\ \varepsilon_c$

$\varepsilon_s = \varepsilon_c \times 4{,}95$

$\varepsilon_c + \varepsilon_s = \varepsilon_{ce} = 8 \times 10^{-4}$ $\quad \varepsilon_c + \varepsilon_c \times 4{,}95 = \varepsilon_c \times 5{,}95 = 8 \times 10^{-4}$

$\varepsilon_c = 1{,}34 \times 10^{-4}$ $\quad \varepsilon_s = 6{,}56 \times 10^{-4}$

$\sigma_s = E_s \varepsilon_s = 200000 \times 6{,}56 \times 10^{-4} = 131{,}2\,MPa$ $\boxed{\sigma_{st} = 120 + 131{,}2 = 251{,}2\,MPa}$

$\sigma_c = E_c \varepsilon_c = 27000 \times 1{,}34 \times 10^{-4} = 3{,}618\,MPa$ $\boxed{\sigma_c = 8 - 3{,}618 = 4{,}382\,MPa}$

$\boxed{11.7}$

$L_{tc}$ = change in concrete due to temperature change

$\Delta L = L_0 \Delta T \alpha_c = 1 \times 10^{-5} \times 10000 \times (-20) = -2mm$

$L_{ts}$ = change in steel due to temperature change

$\Delta L = L_0 \Delta T \alpha_s = -2mm$ again

Since $\alpha_c$ and $\alpha_s$ are equal, there exist no stress between concrete and steel due to temperature change.

Shrinkage → equivalent thickness: $l_e = 2\frac{A_c}{u} = 2\frac{250 \times 700}{2(250+700)} = \underline{184mm}$

u is take as the whole perimeter because all the faces are in contact with the environment.

| Curing | $l_e$ 150 | 600 |
|---|---|---|
| ☆ Inadequate | 0,0006 | 0,0005 |
| Adequate | 0,0004 | 0,0004 |

There is nothing said about curing so i will assume inadequate curing in order to be on the safe side.

Interpolation $l_e = 150 \to \varepsilon_{cs} = 0{,}0006$

$l_e = 600 \to \varepsilon_{cs} = 0{,}0005$

$l_e = 184 \to \varepsilon_{cs} = 5{,}92 \times 10^{-4} = \underline{0{,}000592}$

Force eqn → $F_s - F_c = 0$

$\sigma_s A_s - \sigma_c A_c = 0$

$E_s \varepsilon_s A_s - E_c \varepsilon_c A_c = 0$

$A_s = 10(\frac{\pi}{4} 16^2) = 2011mm^2$

$A_c = 250 \times 700 - 2011 = 172989mm^2$

$E_s = 206182\,MPa$ from TS 648 for S420

$E_c = 28500\,MPa$ from the book

$F_s = 206182 \times \varepsilon_s \times 2011 = 414632002\,\varepsilon_s$

$F_c = 28500 \times \varepsilon_c \times 172989 = 4930186500\,\varepsilon_c$

$F_s = F_c \to \varepsilon_s = \varepsilon_c \times 11{,}89$

$\varepsilon_{cs} = \varepsilon_s + \varepsilon_c = 12{,}89\,\varepsilon_c$

$\downarrow$

$0{,}000592$

$\varepsilon_c = 4{,}59 \times 10^{-5}$

$\boxed{\sigma_c = \varepsilon_c E_c = 1{,}31\,MPa}$

$\varepsilon_s = 5{,}46 \times 10^{-4}$

$\boxed{\sigma_s = \varepsilon_s E_s = 112{,}5\,MPa}$