Question 1)

P(winging project A) = P(A) = 0.5
P(B) = 0.3
$P(A|B) = 0.5x0.5 = 0.25$
$P(B|A) = 0.5x0.3 = 0.15$

(a) $P(A \cup B) = P(A)+P(B)-P(A|B)P(B)$
$= 0.5 + 0.3 - 0.25x0.3$
$= 0.725$

(b) $$P(A\overline{B}|A \cup B) = \frac{P(A\overline{B}(A \cup B))}{P(A \cup B)} = \frac{P(A\overline{B})}{P(A \cup B)} = \frac{P(\overline{B}|A)P(A)}{0.725} = \frac{0.85 \times 0.5}{0.725} = 0.586$$

(c) $$P(A|A\overline{B} \cup \overline{A}B) = \frac{P(A(A\overline{B} \cup \overline{A}B))}{P(A\overline{B} \cup \overline{A}B)} = \frac{P(A\overline{B})}{P(A\overline{B}) + P(\overline{A}B)} = \frac{P(\overline{B}|A)P(A)}{P(A \cup B) - P(AB)}$$

$$= \frac{0.85 \times 0.5}{0.725 - 0.15 \times 0.5} = 0.654$$

(d) $P(C|E) = 0.75$, $P(C|\overline{E}) = 0.5$

Where C denotes completion of project A on time

$P(C) = P(C|E)P(E)+P(C|\overline{E})P(\overline{E})$

$= 0.75x0.5 + 0.5x0.5 = 0.625$

(e) $$P(E|C) = \frac{P(C|E)P(E)}{P(C)} = \frac{0.75 \times 0.5}{0.625} = 0.6$$

Question 3)

Let A and B be water supply from source A and B are below normal respectively

$P(A) = 0.3, \quad P(B) = 0.15$

$P(B \mid A) = 0.3$

$P(S \mid A\overline{B}) = 0.2, \; P(S \mid \overline{A}B) = 0.25, \; P(S \mid \overline{A}\,\overline{B}) = 0, \qquad P(S \mid AB) = 0.8$

Where S denotes event of water shortage

(i) $P(A \cup B) = P(A)+P(B)- P(B \mid A)P(A)$

$= 0.3 + 0.15 - 0.3x0.3 = 0.36$

(ii) $P(A\overline{B} \cup \overline{A}B) = P(A \cup B) - P(AB)$

$= P(A)+P(B)-2P(B \mid A)P(A)$

$= 0.3 + 0.15 - 2x0.3x0.3$

$= 0.22$

(iii) $P(S) = P(S \mid A\overline{B})P(A\overline{B})+P(S \mid \overline{A}B)P(\overline{A}B)+P(S \mid \overline{A}\,\overline{B})P(\overline{A}\,\overline{B})+P(S \mid AB)P(AB)$

But $P(A|B) = \dfrac{P(B|A)P(A)}{P(B)} = \dfrac{0.3 \times 0.3}{0.15} = 0.6$

Hence $P(S) = 0.2x0.7x0.3 + 0.25x0.4x0.15 + 0 + 0.8x0.3x0.3 = 0.129$

(iv) $P(AB|S) = \dfrac{P(S|AB)P(AB)}{P(S)} = \dfrac{0.8 \times 0.3 \times 0.3}{0.129} = 0.558$

(v) $P(\overline{A} \mid \overline{S}) = P(\overline{A}B \mid \overline{S})+P(\overline{A}\,\overline{B} \mid \overline{S})$

$$= \frac{P(\overline{S}|\overline{A}B)P(\overline{A}B)}{P(\overline{S})} + \frac{P(\overline{S}|\overline{AB})P(\overline{AB})}{P(\overline{S})}$$

But $P(\overline{A}B) = [1-P(A \mid B)]\, P(B) = 0.4x0.15 = 0.06$

$P(\overline{A}\,\overline{B}) = 1-P(AB)-P(\overline{A}B)-P(A\overline{B})$

$= 1 - 0.3x0.3 - 0.06 - 0.7x0.3$

$= 0.64$

Hence $P(\overline{A}|\overline{S}) = \dfrac{0.75 \times 0.06}{0.871} + \dfrac{1 \times 0.64}{0.871} = 0.786$

Question 2

C : Cancer

P : Positive

$P(C) = \frac{4500}{170000} = 0.02647$

$P(P|C) = 0.95$

$P(P|\bar{C}) = 0.05$

ⓐ $P(C|P) = \frac{P(P,C)}{P(P)} = \frac{P(P|C)\,P(C)}{P(P|C)P(C) + P(P|\bar{C})P(\bar{C})}$

$= \frac{0.95 \times 0.02647}{0.95 \times 0.02647 + 0.05 \times (1 - 0.02647)} = \boxed{0.34}$

ⓑ $P(C|P) = 0.34 \rightarrow P(F|P) = 0.66$

Truly cancer after two positive results $= 1 - P(F|P)\,P(F|P)$

$= 1 - 0.66 * 0.66 = \boxed{0.56}$

ⓒ Truly cancer after three positive results $= 1 - P(F|P)^3$

$= 1 - 0.66\text{^}3 = \boxed{0.71}$

Question 4

Discharge $\sim N(675, 200)$

Dam fail if discharge > 1075

ⓐ $P(D > 1075) = P(Z > \frac{1075 - 675}{200}) = P(Z > 2) = 1 - P(Z < 2) = 1 - 0.977$

$= 2.3\,\%$

ⓑ $P(425 < D < 925) = P(\frac{425-675}{200} < Z < \frac{925-675}{200}) = P(-1.25 < Z < 1.25)$

$= P(Z < 1.25) - (1 - P(Z < 1.25))$

$= 0.8944 - (1 - 0.8944) = 0.788$

ⓒ $P(D > D_{0.33}) = 0.33 = P(Z > Z_{0.33}) \rightarrow P(Z < Z_{0.33}) = 1 - 0.33 = 0.67$

$\rightarrow Z_{0.33} = 0.44 = \frac{D_{0.33} - 675}{200} = 763$

ⓓ Standard Normal Dist mean & stdev are 0 & 1 respectively, while Normal Distribution can take any mean & stdev.