CE300 SUMMER PRACTICE I 2009-2010 Summer

MATLAB Assignment (Programming)

Due: 12.07.2010, Monday

Attention: You are expected to submit your solutions in MS Word format. Save your file named with your student ID. For instance, if your student ID is 1234567, your file name should be '1234567.doc'. If you can convert your document to Adobe PDF format, that will also be appreciated. Please submit your files electronically to the address ce300summer@gmail.com.

Write a Matlab code (M - File) that is capable of solving one of the following questions. (You have to solve **only one problem.**) Please, obey the following rules while writing your code.

- Your Matlab code should contain at least one user - defined function.
- At least two data should be entered by the user. (Use, at least two times, the input function.)
- Divide your code into parts so that it is more comprehensible. (Input part, solution part, explanation part, etc.)
- Please, keep in mind that this exercise is a good opportunity to solidify your Matlab talents. So, try to utilize as many different built-up functions as you can.
- Every student should generate a unique algorithm. (This is not a group work.)

1) Use the bisection method to determine the drag coefficient (c) needed for a parachutist of mass m=68.1 kg to have a velocity (v) of 40 m/s after free-falling for time t=10 s. (This is the solution of Equation 1.)

$$f(c) = \frac{mg}{c} * \left(1 - e^{\frac{-c*t}{m}}\right) - v \qquad Equation\ 1$$

Note : The acceleration due to gravity (g) is 9.8 m/s^2.

Hint : The bisection method is a numerical technique to find the root of an equation. This method starts with an interval selection, which should contain the root. After that, this interval is halved by monitoring the sign of the function at the interval boundaries and the mid-point of the chosen interval. This is because; the root lies on an interval, which contains both positive and negative values. Every step, the mid-point is assumed as the root of the equation. This procedure is carried on until the error between two consecutive root estimations is less than the chosen tolerance.

Example : Find one of the roots of equation x^2-5x+3 lying between 3 and 6 by using bisection method. The maximum acceptable error for this case may be taken as 5%.

Solution :

i. **Select the interval.**

[3, 6]

ii. **Calculate the values at the interval boundaries and the mid-point of the interval.**

$$f(3) = -3, f(4.5) = 0.75\,, f(6) = 9$$

iii. **Iteration 2: Determine the new interval. (Halved interval)**

The root should be between a positive and a negative value. Therefore, the new interval is [3, 4.5].

iv. **Calculate the values at the new interval boundaries and the mid-point of the new interval.**

$$f(3) = -3, f(3.75) = -1.69\,, f(4.5) = 0.75$$

The percentage error is

$$Error = \frac{|x_{new} - x_{old}|}{x_{new}} * 100 = \frac{|3.75 - 4.5|}{3.75} * 100 = 20\%$$

v. **Iteration 3: Determine the new interval.**

The new interval is [3.75, 4.5].

vi. **Calculate the values at the new interval boundaries and the mid-point of the new interval.**

$$f(3.75) = -1.69, f(4.125) = -0.609, f(4.5) = 0.75$$

The percentage error is

$$Error = \frac{|x_{new} - x_{old}|}{x_{new}} * 100 = \frac{|4.125 - 3.75|}{4.125} * 100 = 9.09\%$$

vii. Iteration 4: Determine the new interval.

The new interval is [4.125, 4.5].

viii. Calculate the values at the new interval boundaries and the mid-point of the new interval.

$$f(4.125) = -0.609, f(4.313) = 0.035, f(4.5) = 0.75$$

The percentage error is

$$Error = \frac{|x_{new} - x_{old}|}{x_{new}} * 100 = \frac{|4.313 - 4.125|}{4.313} * 100 = 4.35\%$$

The percentage error is less than the tolerance. Therefore, the root of the equation is 4.313. If this equation is solved analytically, the roots are calculated as 0.697 and 4.303, which means that the true percentage error is

$$True\ Error = \frac{|x_{true} - x_{estimated}|}{x_{true}} * 100 = \frac{|4.303 - 4.313|}{4.303} * 100 = 0.23\%$$

A more detailed explanation and examples about the bisection method may be found in the book called Numerical Methods for Engineers written by Steven C. Chapra and Raymond P. Canale.

2) A manufacturer for the quality control investigates 5 car parts and records the following frequencies for the number of defective car parts:

Number of Defective Car Parts, x	0	1	2	3	4	5
Frequencies	2	1	1	1	0	0

Can we say that underlying probability distribution of the defected car parts is a binomial distribution with n = 5 and θ = 0.10 at the 5% significance level of using both goodness of fit test (χ^2test)?

Note: In this question, you have to prepare a text file that contains the number of defective car parts and frequencies and read the data from this file. (You do not have to write the titles. Your file may be composed of only numbers.)

Hint: The hand solution of this question is given below.

The probabilities for the given x values are calculated by using the Binomial distribution.

$$P(x=0) = \binom{5}{0} * \theta^0 * (1-\theta)^{5-0} = 1 * 0.10^0 * (1-0.10)^5 = 0.591$$

$$P(x=1) = \binom{5}{1} * \theta^1 * (1-\theta)^{5-1} = 5 * 0.10^1 * (1-0.10)^4 = 0.328$$

$$P(x=2) = \binom{5}{2} * \theta^2 * (1-\theta)^{5-2} = 10 * 0.10^2 * (1-0.10)^3 = 0.073$$

$$P(x=3) = \binom{5}{3} * \theta^3 * (1-\theta)^{5-3} = 10 * 0.10^3 * (1-0.10)^2 = 0.008$$

$$P(x=4) = \binom{5}{4} * \theta^4 * (1-\theta)^{5-4} = 5 * 0.10^4 * (1-0.10)^1 = 4.5 * 10^{-4}$$

$$P(x=5) = \binom{5}{5} * \theta^5 * (1-\theta)^{5-4} = 1 * 0.10^5 * (1-0.10)^0 = 1 * 10^{-5}$$

The goodness of fit test is

X	**o_i**	**Probability**	**e_i**	**$(o_i-e_i)^2/e_i$**
0	2	0.591	2.955	0.309
1	1	0.328	1.640	0.250
2	1	0.073	0.365	1.105

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3	1	0.008	0.040	23.040
4	0	$4.5*10^{-4}$	0.002	0.002
5	0	$1*10^{-5}$	0.000	0.000
				24.705

n	5
k	6
r	0

$$df = k - r - 1 = 6 - 0 - 1 = 5$$

$$\chi_{0.05,5} = 11.07 < \sum \frac{(o_i - e_i)^2}{e_i} = 24.705$$

No Good

Therefore, the binomial distribution assumption is not reasonable.