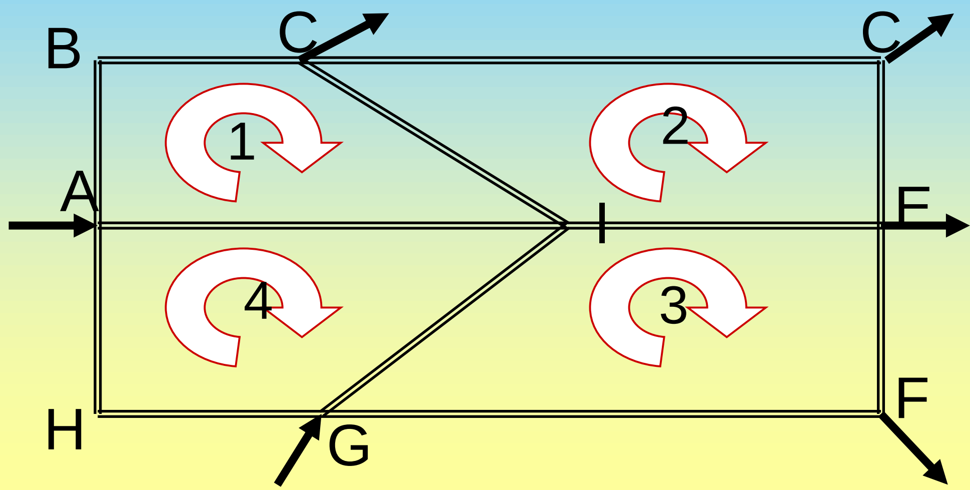
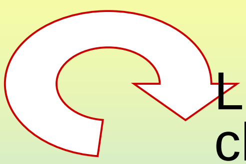
NETWORK OF PIPES



In general we know the discharges coming to a loop. However we don't know the discharge in each pipe. Therefore, it is necessary to compute them.

We have two equations:



LOOP “+”
clockwise

- **Conservation of Mass**: Flow into each junction must be equal flow out of the junction

A,...,I:JUNCTIONS/ NODES

EF:LINK/BRANCH

- **Energy Conservation**: Algebraic sum of head losses around each and every loop must be zero.

Darcy-Weicbach Eqution for head loss

$$h_L = f\frac{L}{D^5}\frac{8Q^2}{\pi^2 g} + K_m\frac{}{D^4}\frac{8Q^2}{\pi^2 g} = K_f Q^2 + K_m Q^2 = K_L Q^2$$

$$h_l = K_L Q|Q|$$

Where

$$K_f = 8f\frac{L}{D^5 g\pi^2}$$

K_m = Local Loss coefficient,

which can be wrítten in terms of equivalent length, and

K_L = combined loss coefícient

Conservation of Energy around any loop

$$\sum_{i=1}^{N} h_{l_{loop}} = \sum_{i=1}^{N} K_i Q_i^{\,n} = \sum_{i=1}^{N} K_i (Q_o + \Delta Q)^n$$

$$= \sum_{i=1}^{N} K Q_o^n + \sum_{i=1}^{N} n K \Delta Q Q_o^{n\ 1} + \ldots\ldots$$

where N = number of links

$$h_{loop} = 0 \Rightarrow \sum_{i=1}^{N} K Q_o^n + \sum_{i=1}^{N} n K \Delta Q Q_o^{n\ 1} = o$$

If $n = 2$

$$\Delta Q = \frac{\sum_{i}^{N} K Q_o |Q_o|}{\sum_{i}^{N} 2K |Q_o|}$$

Solution Procedure

1)Assume the best distribution of flow that satisfies the continuity equation at each junction.

Let Q_0 be the assumed discharge in a pipe, and Q be the actual discharge.

2) Then $Q=Q_0+\Delta Q$ for each pipe, where ΔQ is the error in estimation. Therefore, it is necessary to calculate the error, ΔQ, for each loop.

3) Calculaion of ΔQ:

The head loss can be written as: hf=KQn. For Darcy-Weisbach equation n=2, and

$$K = 8f \frac{L}{D^5 g \pi^2}$$

4) Around any loop, algebraic summation od head loss must be zero:

$$\sum h_f = \sum KQ^2 = \sum K(Q_0 + \Delta Q)^2 = \sum K(Q_0^2 + 2\Delta Q \bullet Q_0 + \Delta Q^2)$$

Assuming that ΔQ is small, ΔQ^2 can be neglected, and

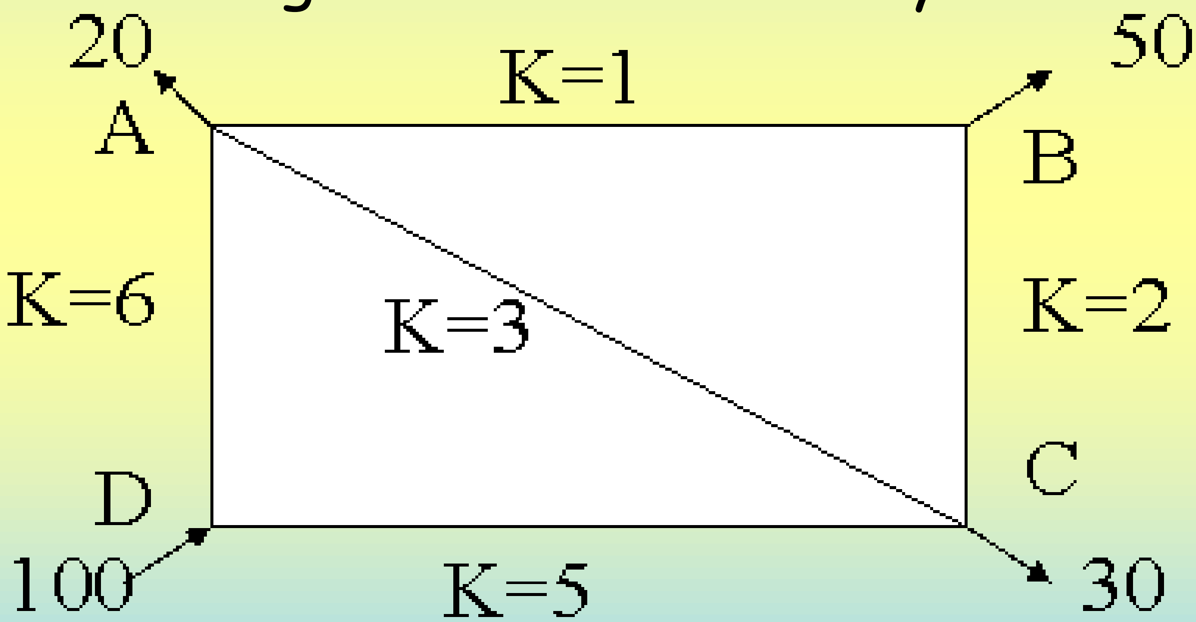
the above equation can be solved for ΔQ as :

$$\Delta Q = \frac{\sum KQ_0 |Q_0|}{\sum 2K|Q_0|}$$

- ΔQ for each and every loop must be smaller than a tolerable magnitude. Otherwise, it is necessary to iterate the solution until the error term ΔQ becomes acceptably small. The details can be explained best by an example:

Example 2.11

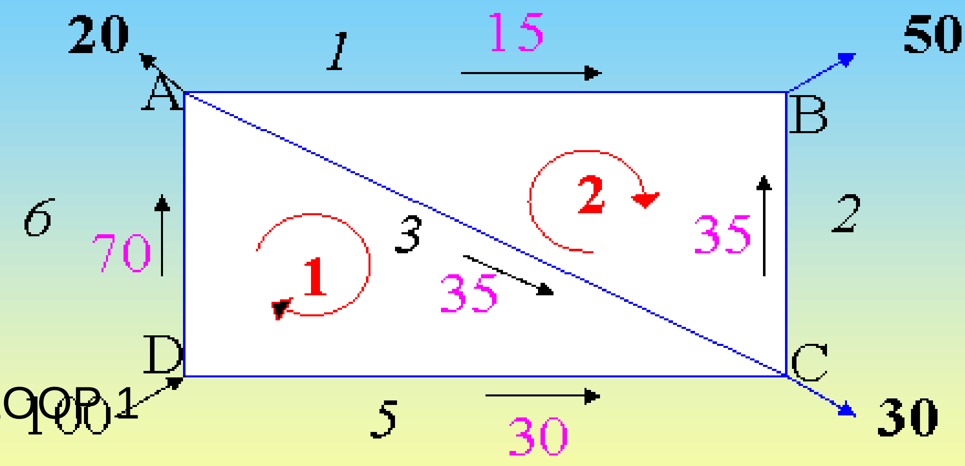
Given is the network shown in figure below. Find the discharges in each and every link.



Q's in m^3/sec

K's in "s^2/m^5"

Initial Guess



First Iteration

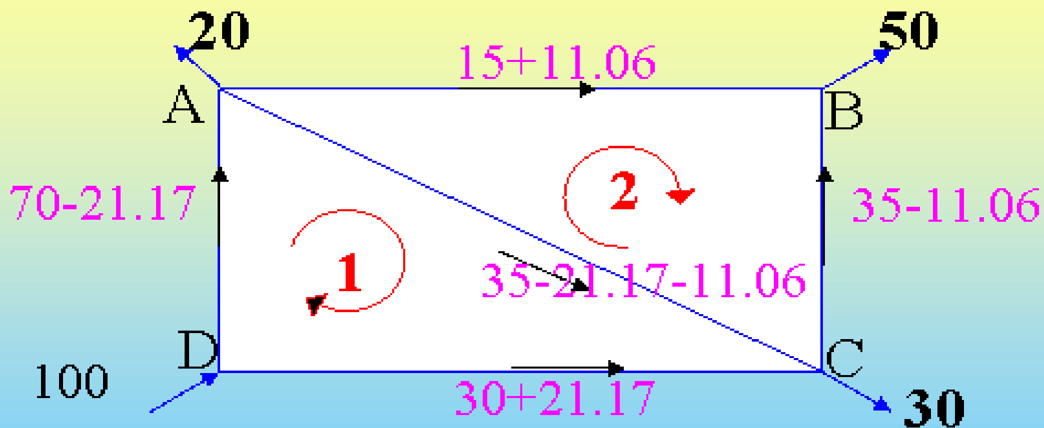
LOOP 1

K Q |Q|	2 K |Q|
AD 6*70*70=29400	2*6*70=840
AC 3*35*35 =3675	2*3*35=210
DC *-5*30*30 =-4500	2*5*30=300
Σ=28575	Σ=1350

$$\Delta Q_1 = -\frac{28575}{1350} = -21.17$$

AC 35-21.17

Flow Distribution After First Iteration

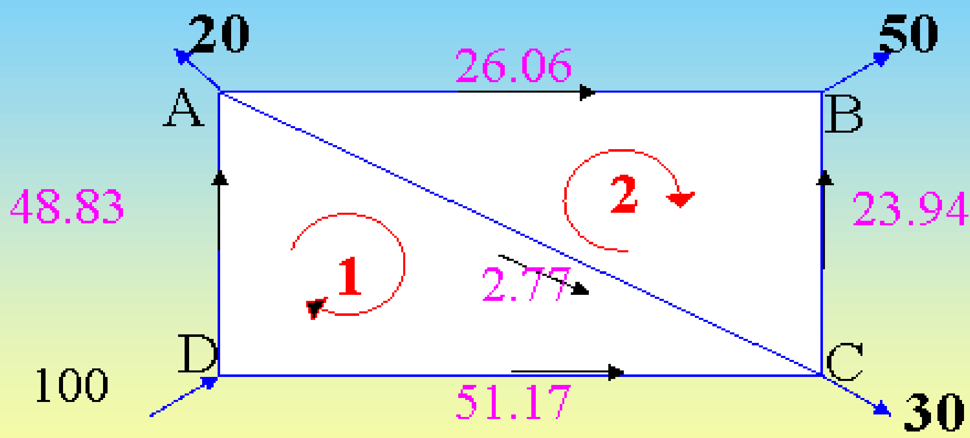


LOOP 2

K Q |Q|	2 K |Q|
AB 1*15*15 =225	2*1*15=300
BC 2*-35*35=-2450	2*2*35=140
AC 3*-*13.83**13.83 =-574	2*3*13.83=83
Σ=–2799	Σ=253

$$\Delta Q2 = \frac{2799}{253} = 11.06$$

Second iteration



LOOP 1

	K Q |Q|	2 K |Q|
AD	6*48.83*48.83 =14308	2*6*48.83 =586
AC	3*2.77*2.77 =23	2*3*2.77 =17
CD	5*-51.17*51.17 =-13090	2*5*51.17 =511
	Σ=1241	Σ=1114

$$\Delta Q_1 = -\frac{1241}{1114} = -1.114$$

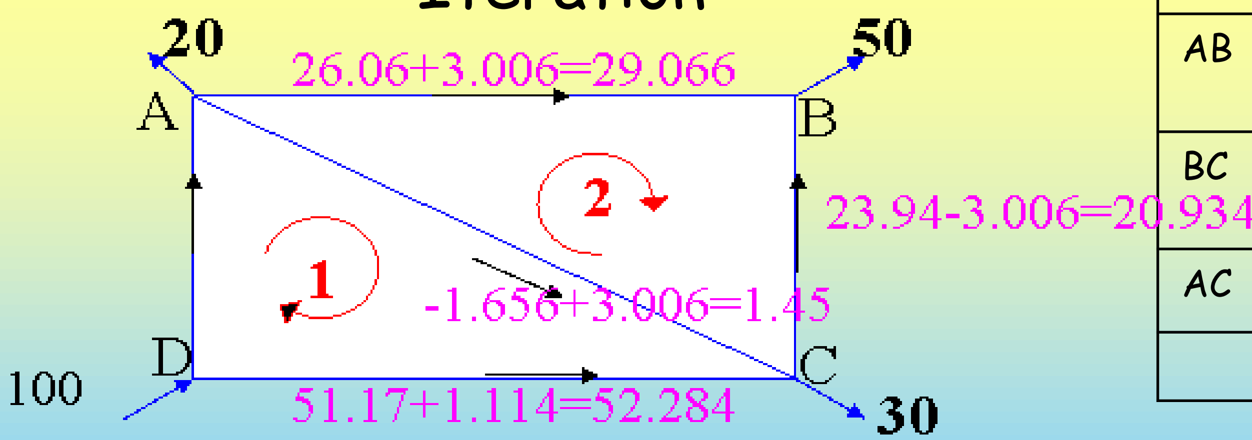
AC 2.77-1.114

LOOP **2**

	K Q |Q|	2 K |Q|
AB	1*26.06*26.06 =679	2*1*26.06=52
BC	2*-23.94*23.94=-1146	2*2*23.94=96
AC	3*-*1.656**1.656 =-8	2*3*1.656=10
	Σ=–475	Σ=158

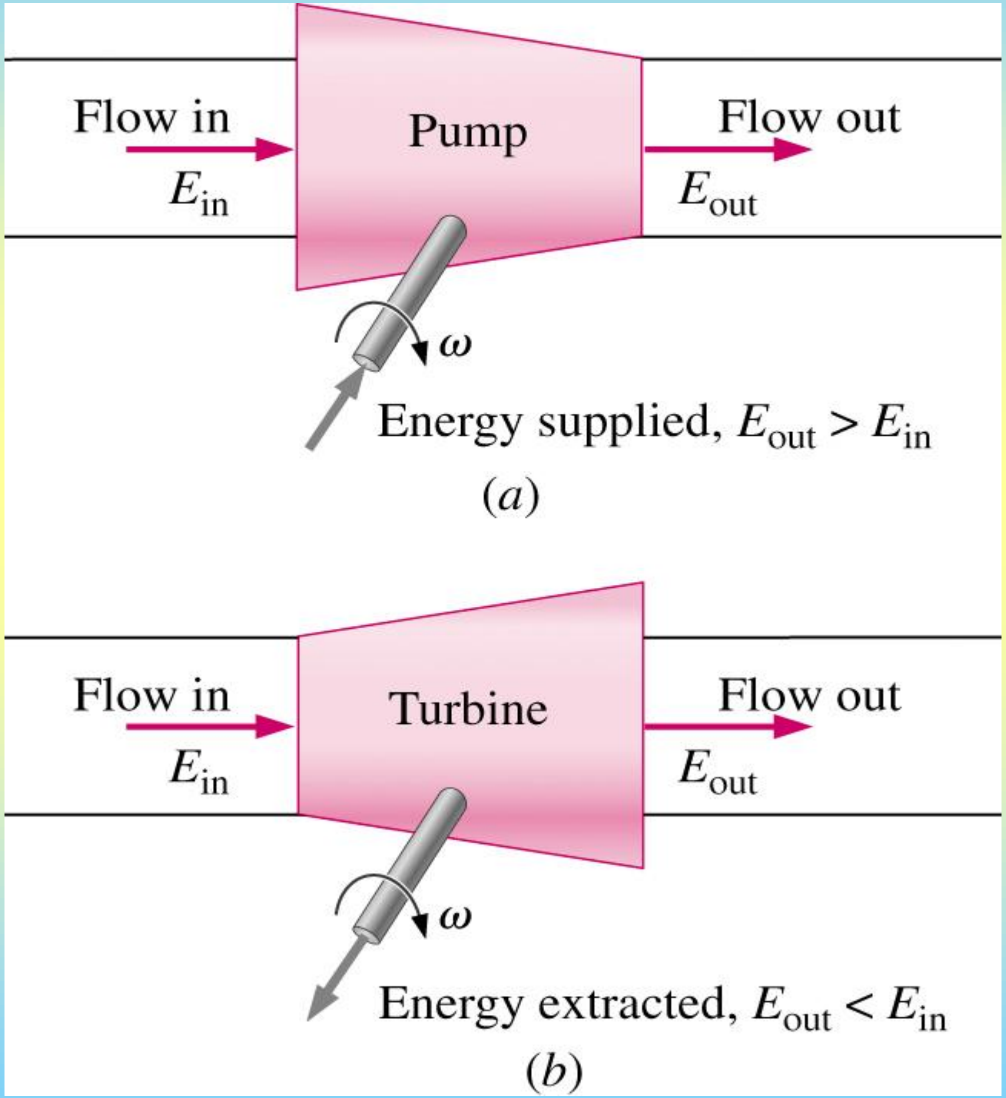
$$\Delta Q_2 = \frac{475}{158} = 3.006$$

Final Distribution of Second Iteration

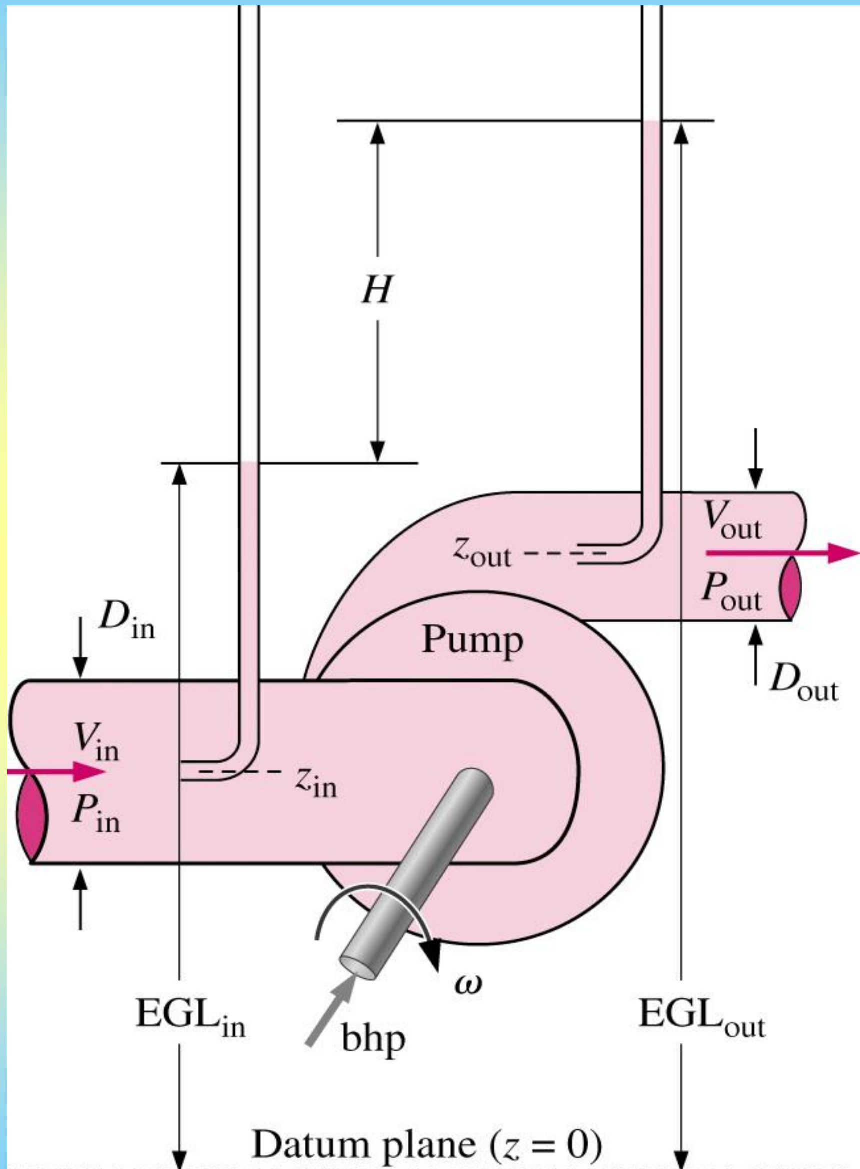


Attention: One more iteration is needed

HYDRAULICS AND OPERATION OF PUMPED DISCHARGE LINES



- Pump: adds energy to a fluid, resulting in an increase in pressure across the pump.
- Turbine: extracts energy from the fluid, resulting in a decrease in pressure across the turbine.



H
V_{out}
z_{out}
P_{out}
D_{in}
Pump
D_{out}
V_{in}
z_{in}
P_{in}
ω
EGL_{in}
bhp
EGL_{out}
Datum plane ($z = 0$)

Categories

For gases, pumps are further broken down into

- **Fans**: Low *pressure gradient*, High *volume flow rate*. Examples include ceiling fans and propellers.
- **Blower**: Medium *pressure gradient*, Medium *volume flow rate*. Examples include centrifugal and squirrel-cage blowers found in furnaces, leaf blowers, and hair dryers.
- **Compressor**: High *pressure gradient*, Low *volume flow rate*. Examples include air compressors for air tools, refrigerant compressors for refrigerators and air conditioners.

Categories

- Positive-displacement machines
  - Closed volume is used to squeeze or suck fluid.
  - Pump: human heart
  - Turbine: home water meter
- Dynamic machines
  - No closed volume. Instead, rotating blades supply or extract energy.
  - Enclosed/Ducted Pumps: torpedo propulsor
  - Open Pumps: propeller or helicopter rotor
  - Enlosed Turbines: hydroturbine
  - Open Turbines: wind turbine

A PUMP IS A MECHANICAL DEVICE WHICH ADDS ENERGY TO THE FLUID

The power required to derive the motor is bhp=ωT

ENERG
(Supply)

MOTO
R

S
H
A
F
T

BLADES

ROTO
R

TRANSFER

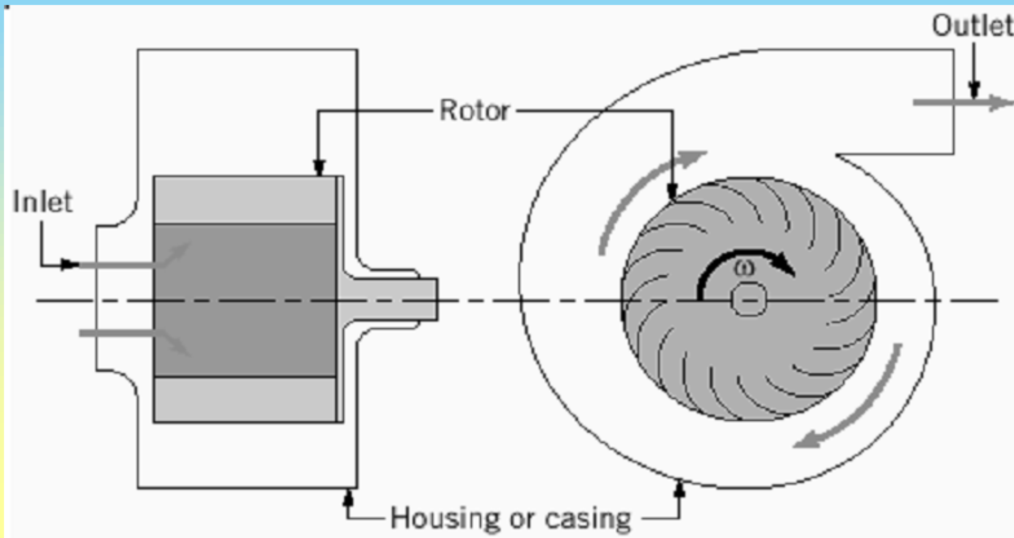
ENERG
Y
Into
fluid

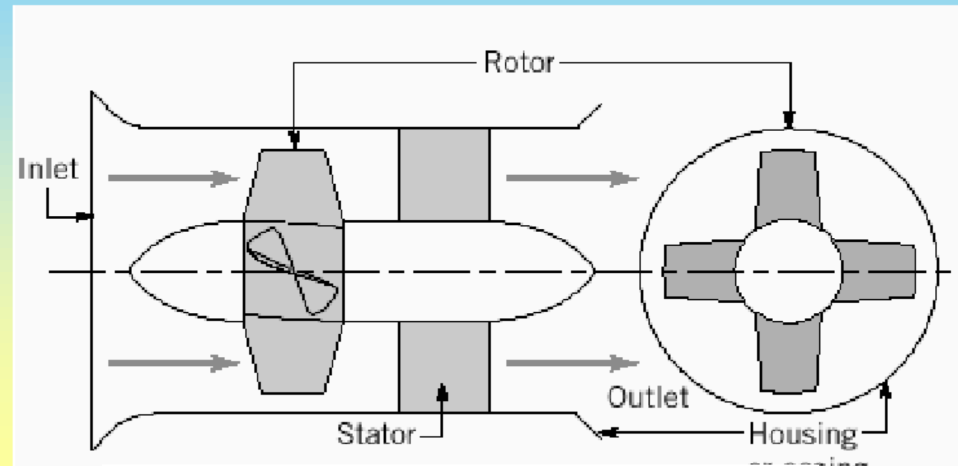
$P_f = \gamma\ Q\ H_p$

- Axial
- Radial
- Mixed

There is a casing which provides passageway to the fluid.

Types of Pumps

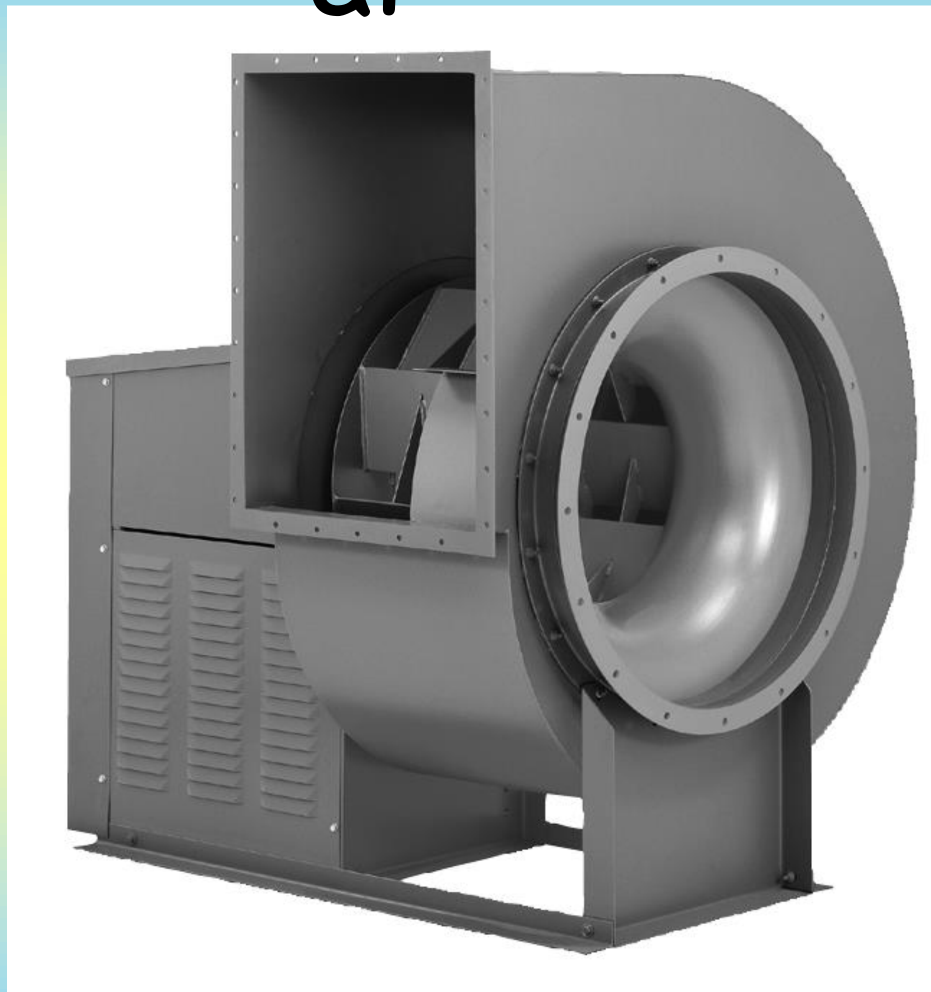




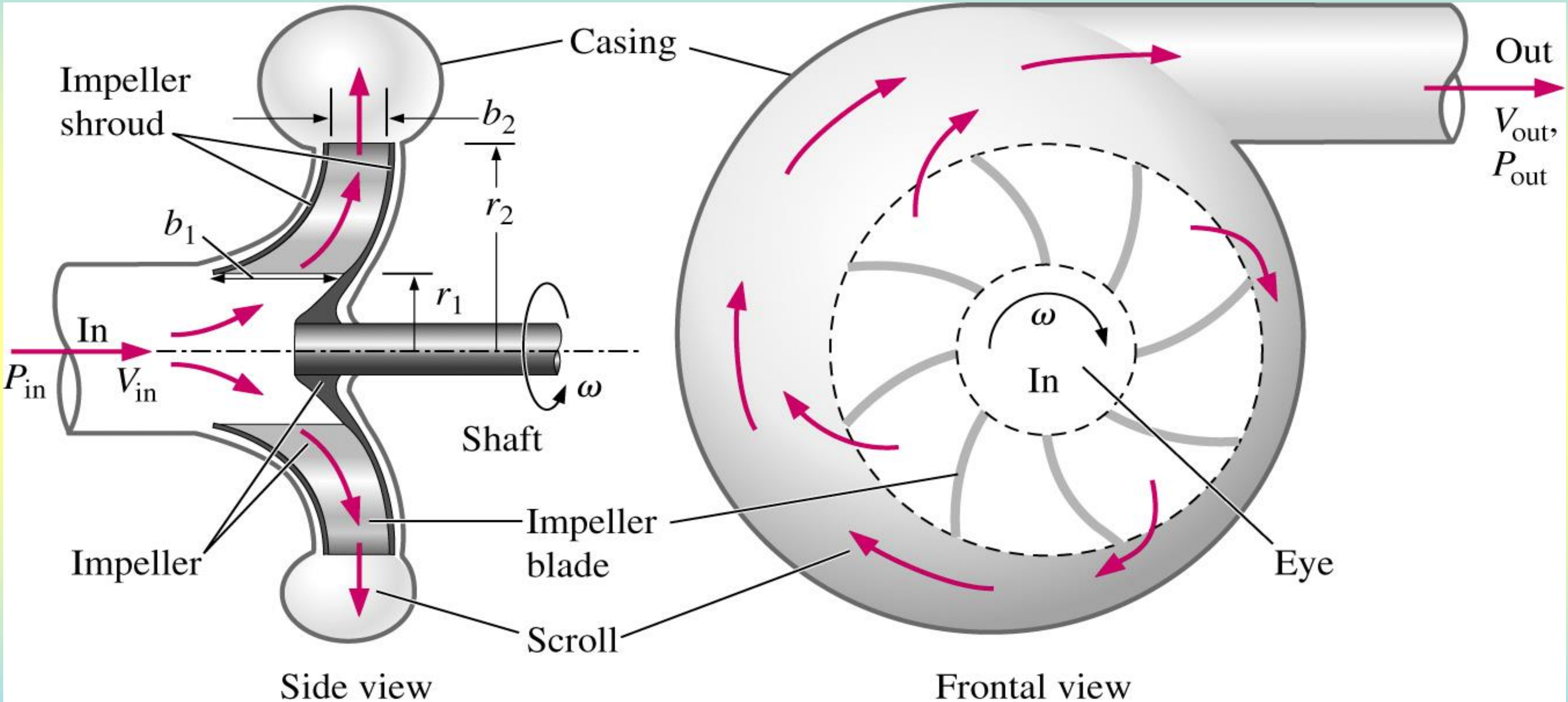
(b) Axial-flow pump

Axial

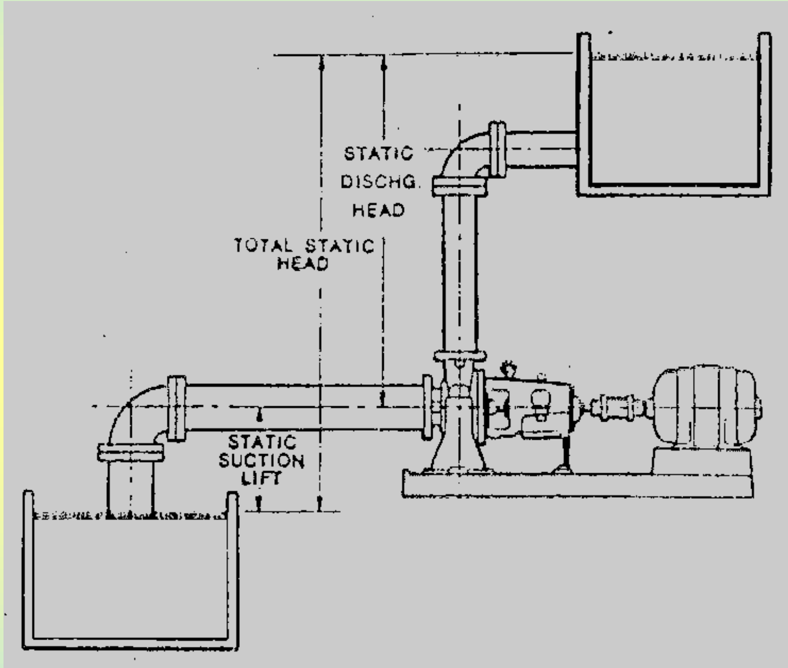
Centrifug al



Centrifugal Pumps

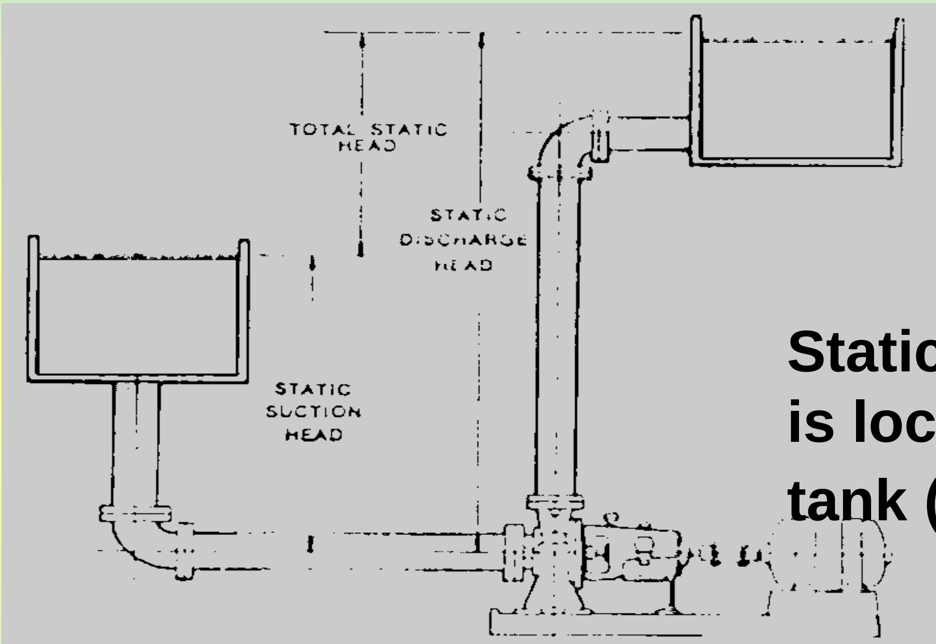


Suction Lift



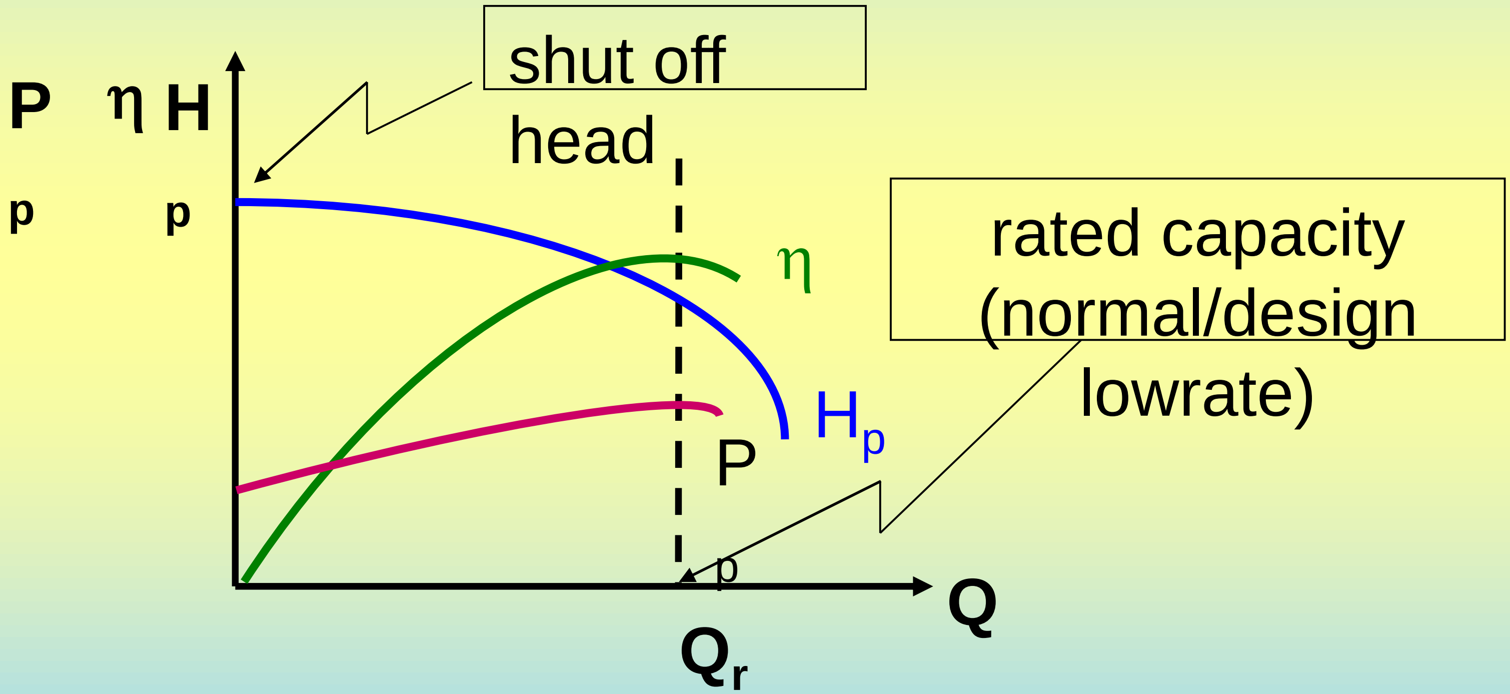
Static head when the pump is located above the suction tank (Static Suction Lift)

Suction Head

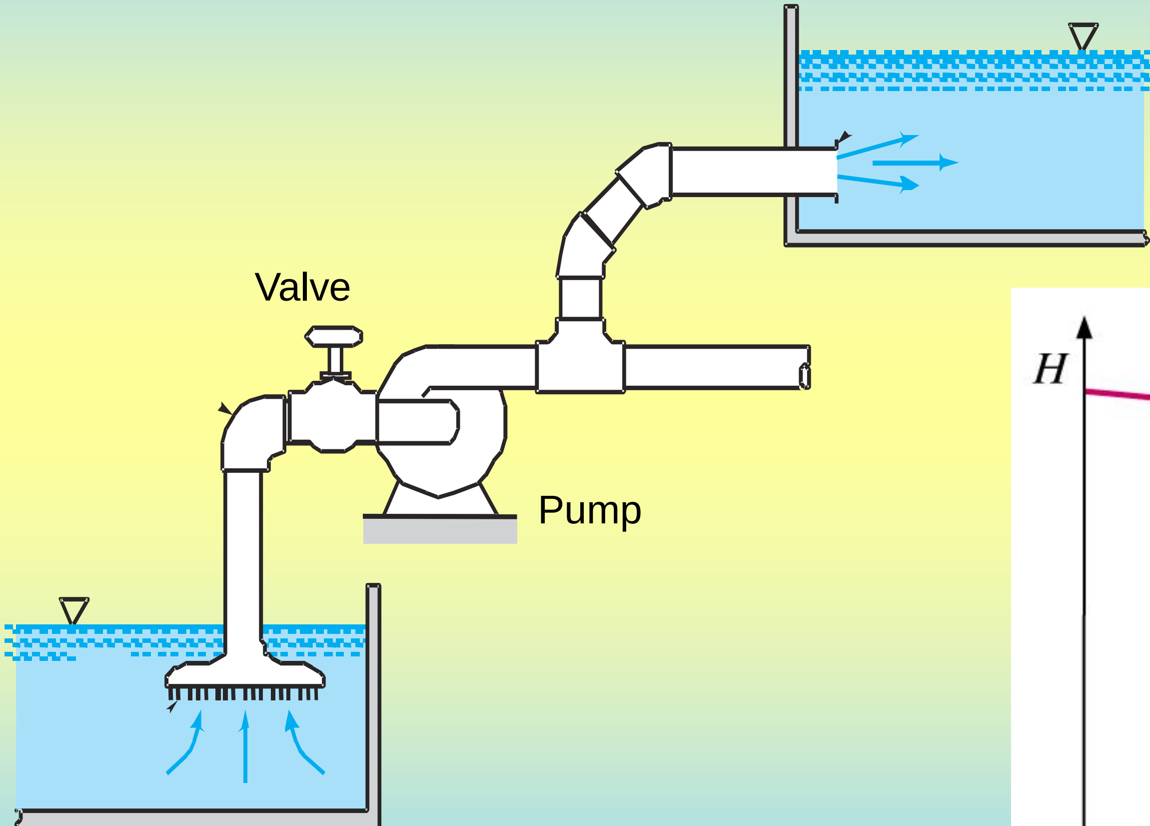


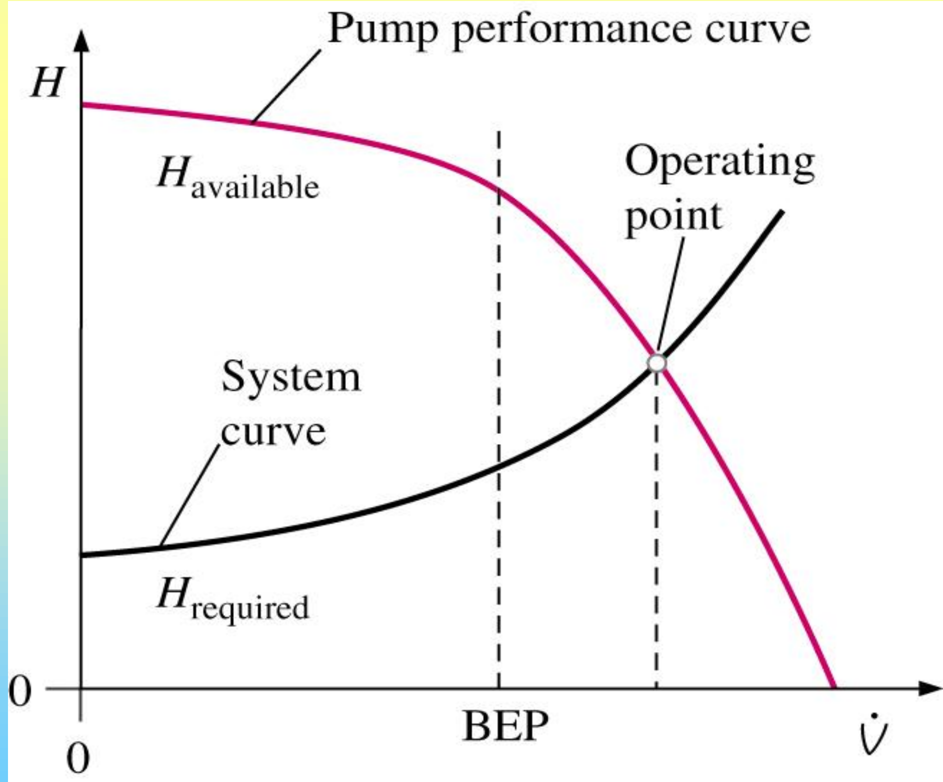
Static head when the pump is located below the suction tank (Static Suction Head)

Pump Performance

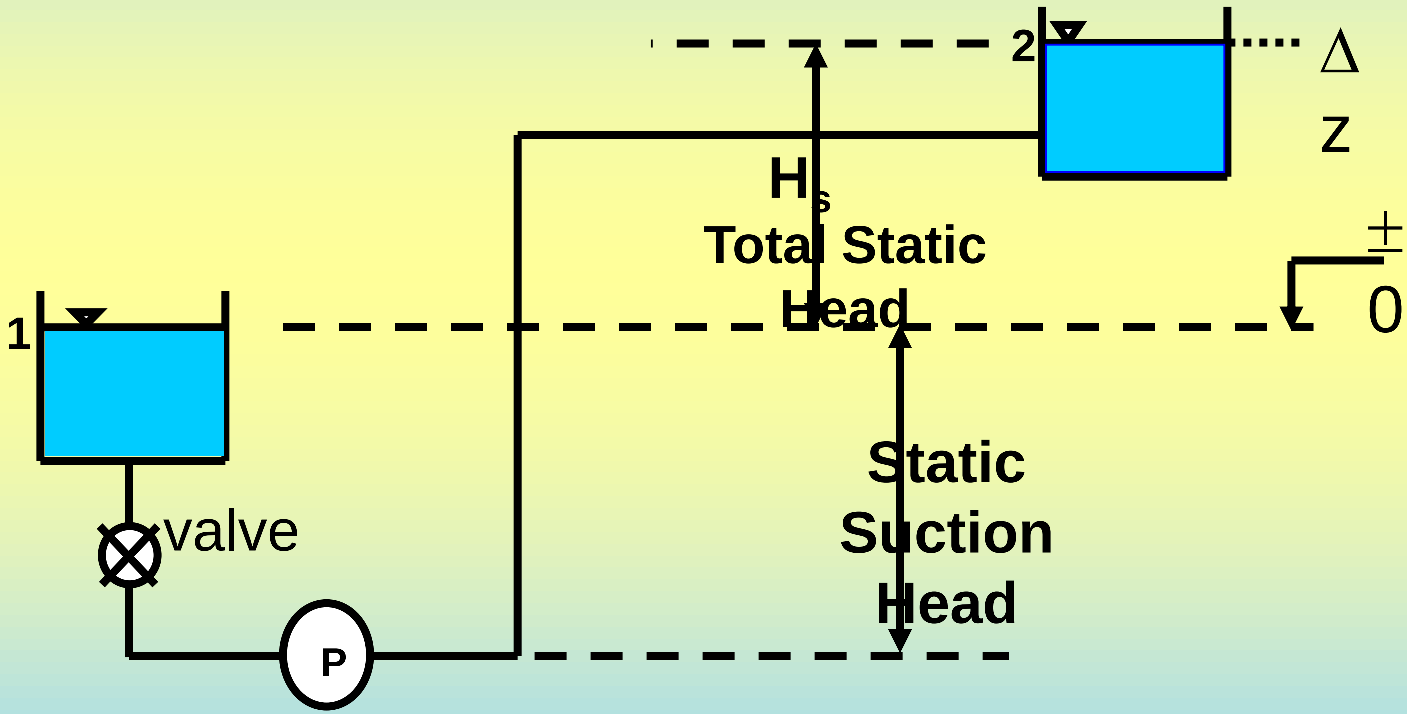


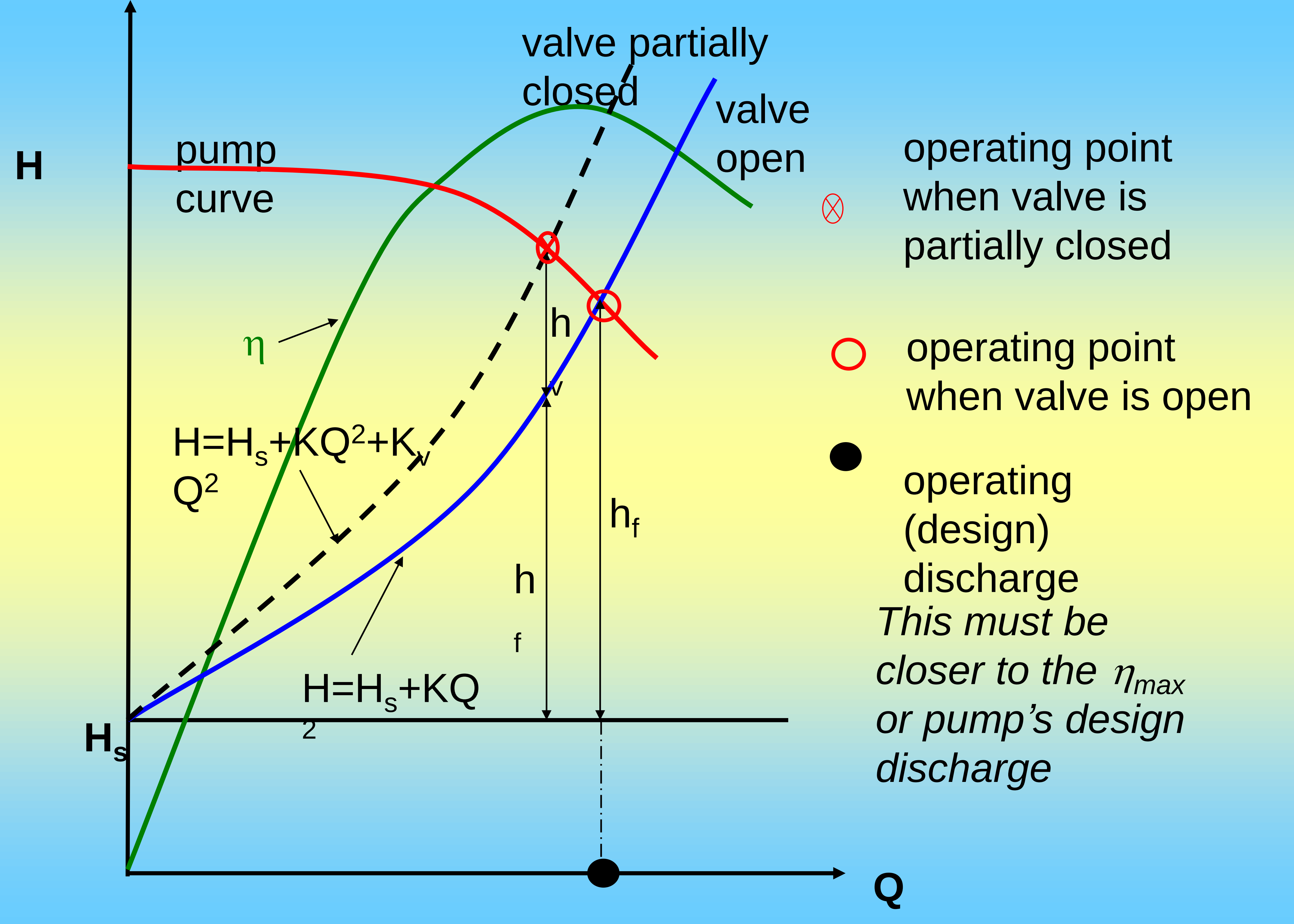
SELECTION OF A PUMP



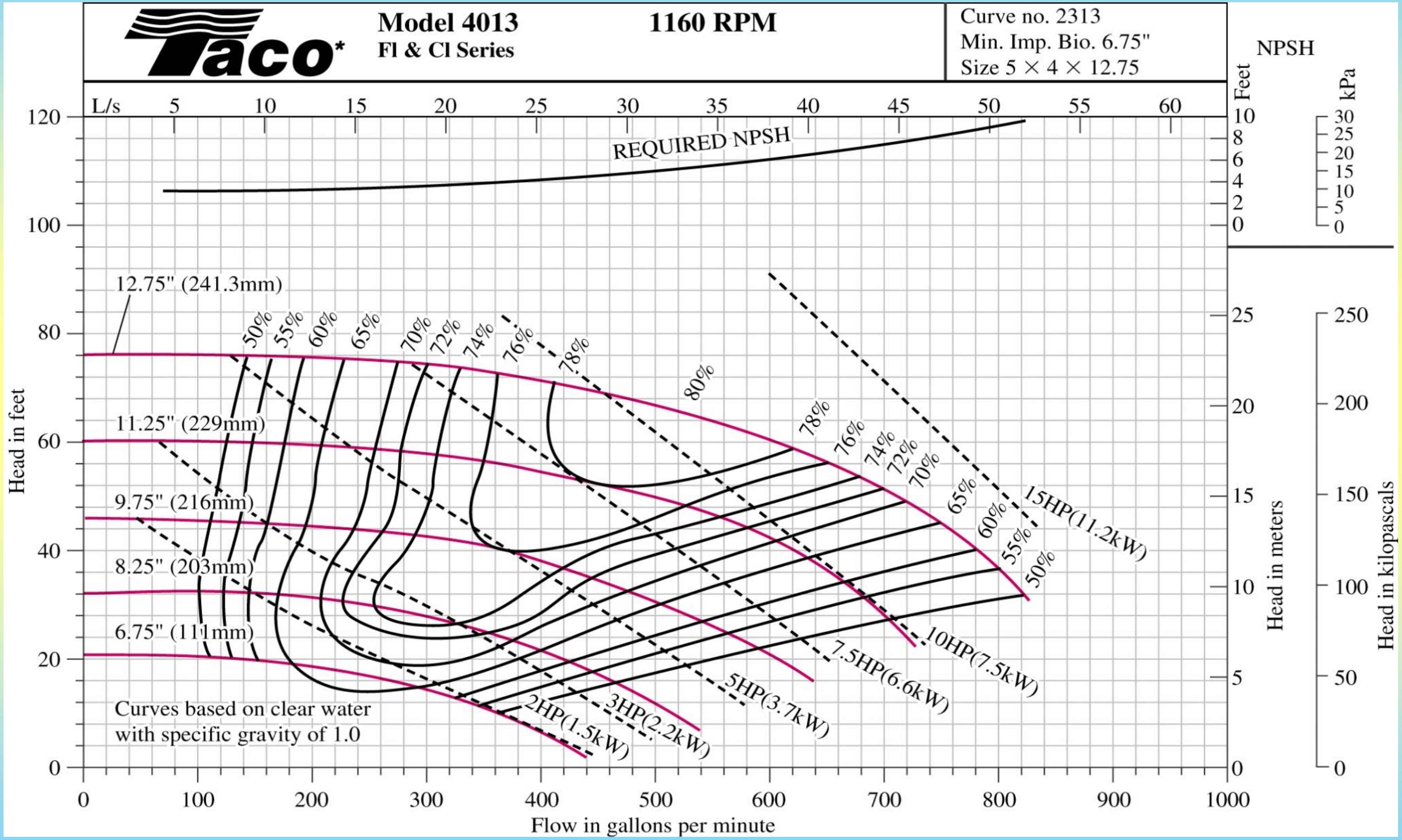


Matching Pump to System Demand

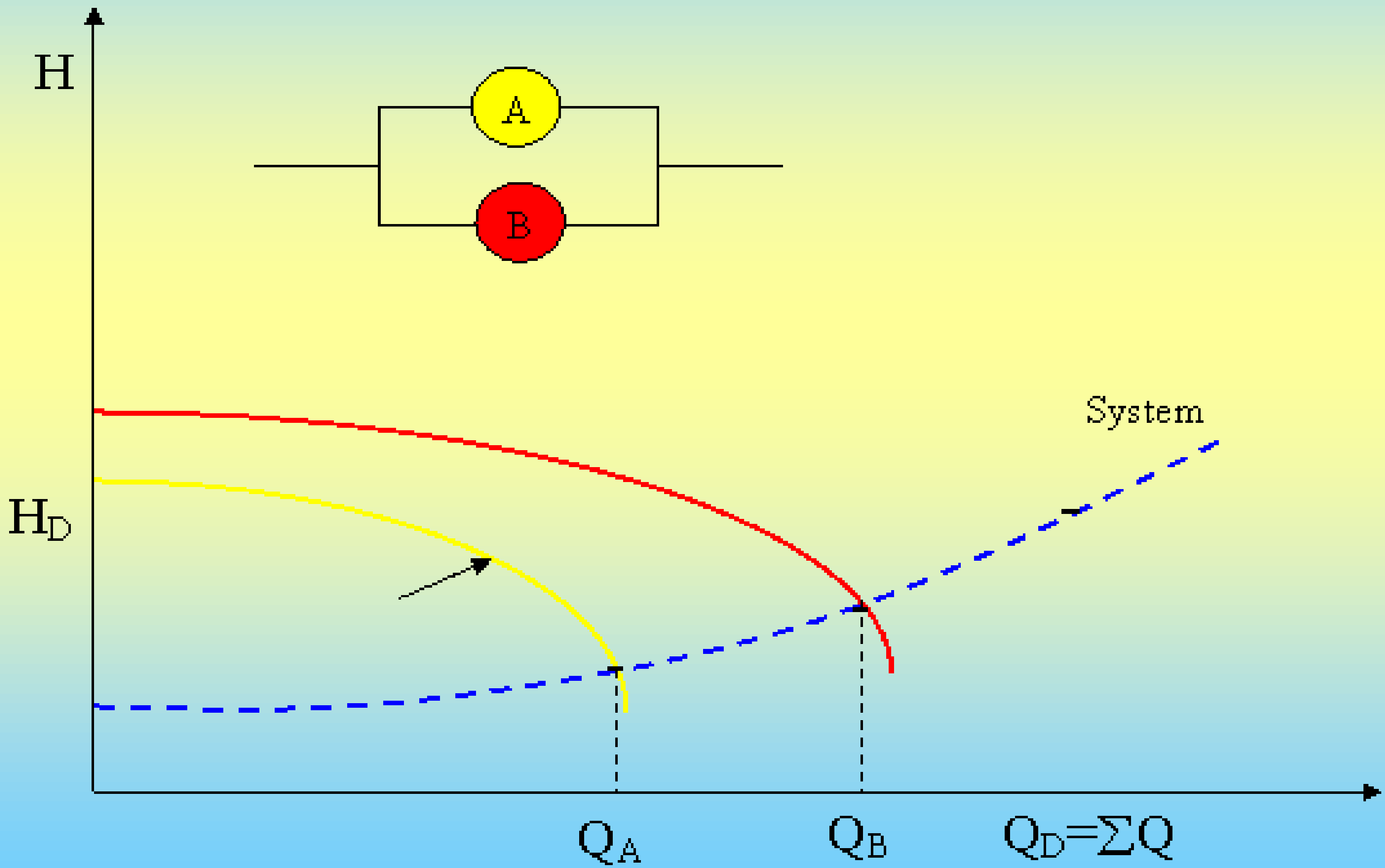



H
pump curve
valve partially closed
valve open
η
$H=H_s+KQ^2+K_v Q^2$
$H=H_s+KQ^2$
h_v
h_f
h_f
H_s
Q
operating point when valve is partially closed
operating point when valve is open
operating (design) discharge
This must be closer to the η_{max} or pump's design discharge

Manufacturer Performance Plot

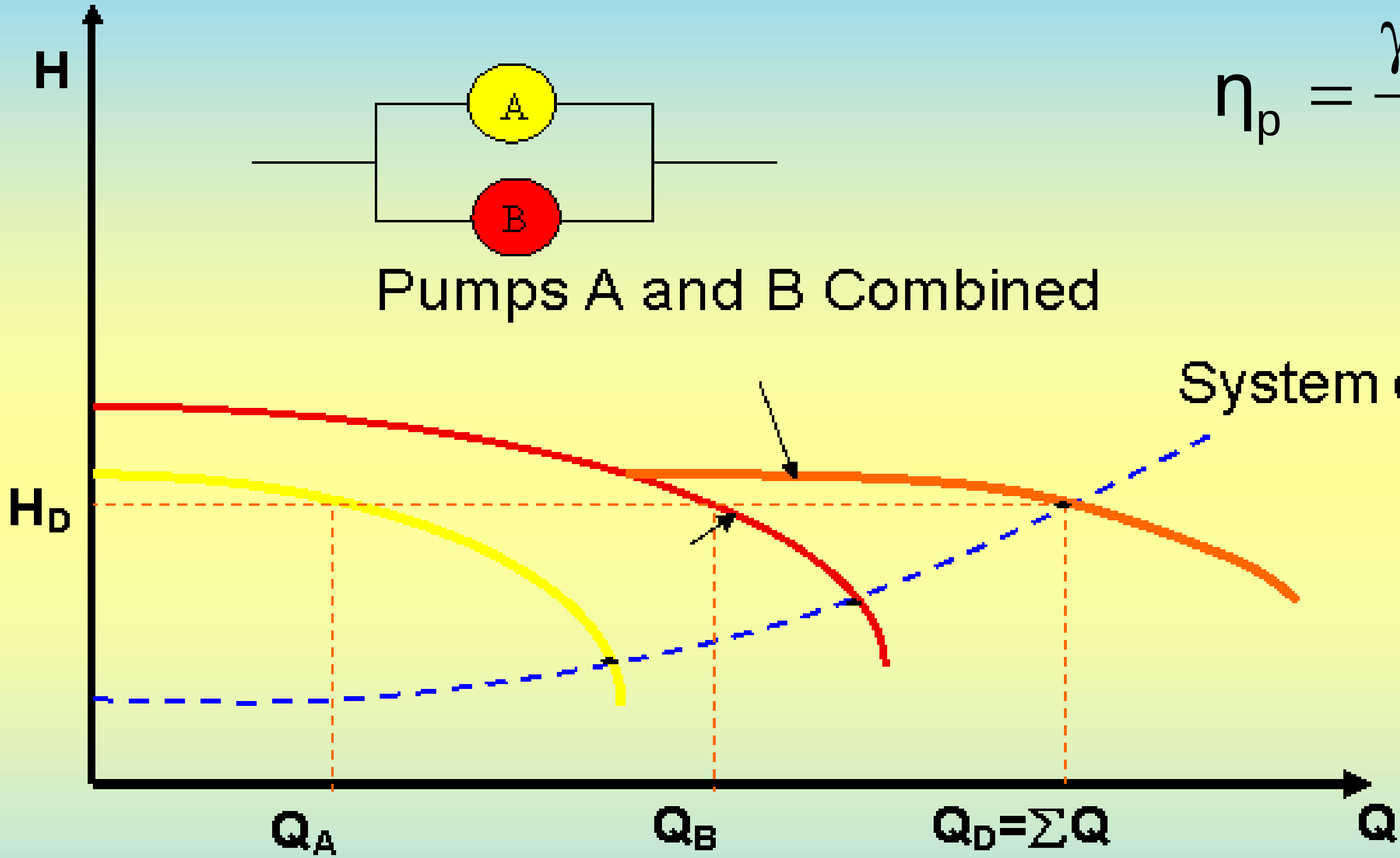


Pumps in Parallel

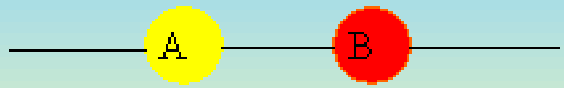


Pumps in Parallel

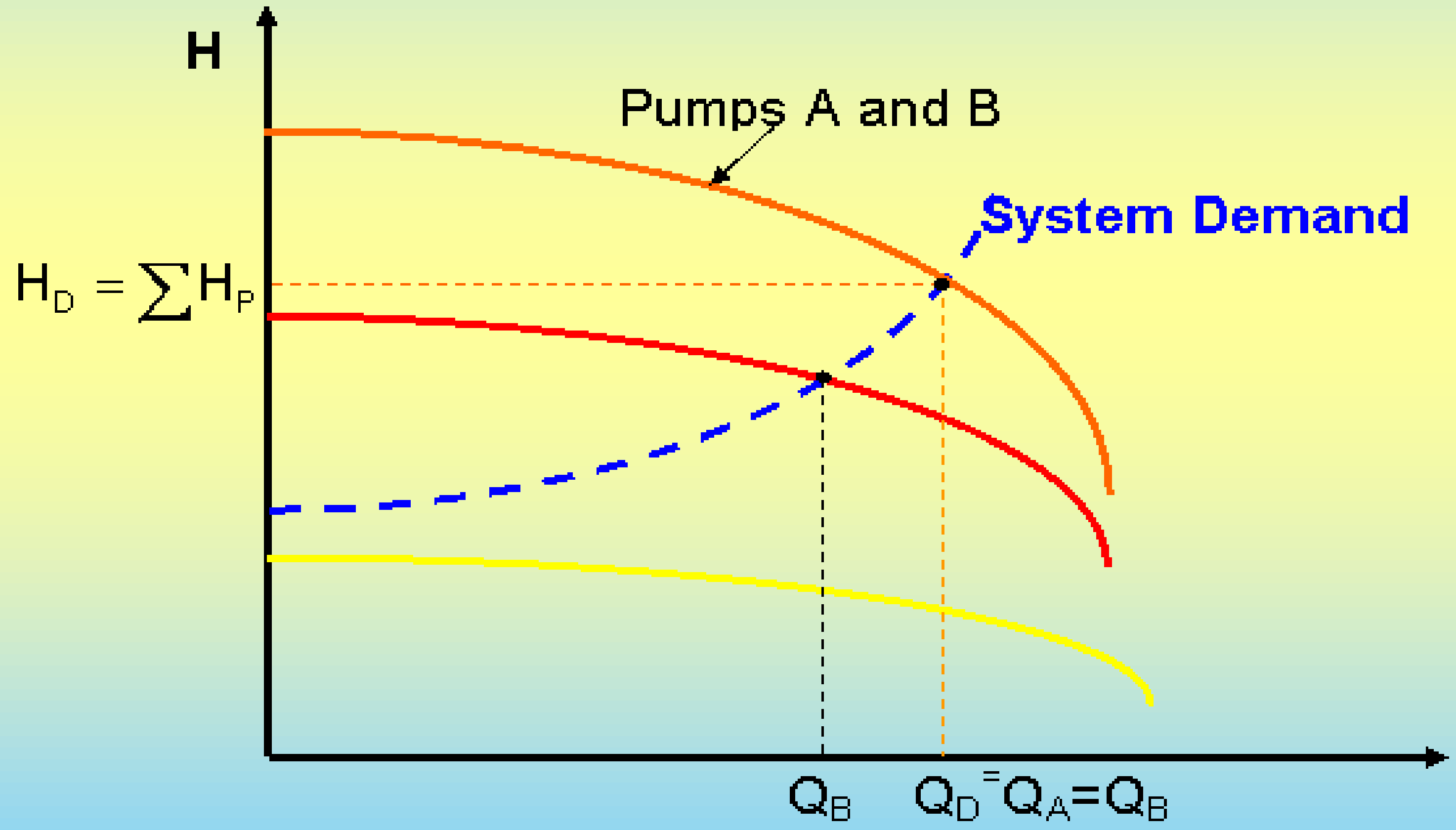
$$\eta_p = \frac{\gamma H_D \sum Q}{\sum P_P}$$



Pump in Series



$$\eta_P = \frac{\gamma\left(\sum H_P\right)Q_D}{\sum P_P}$$



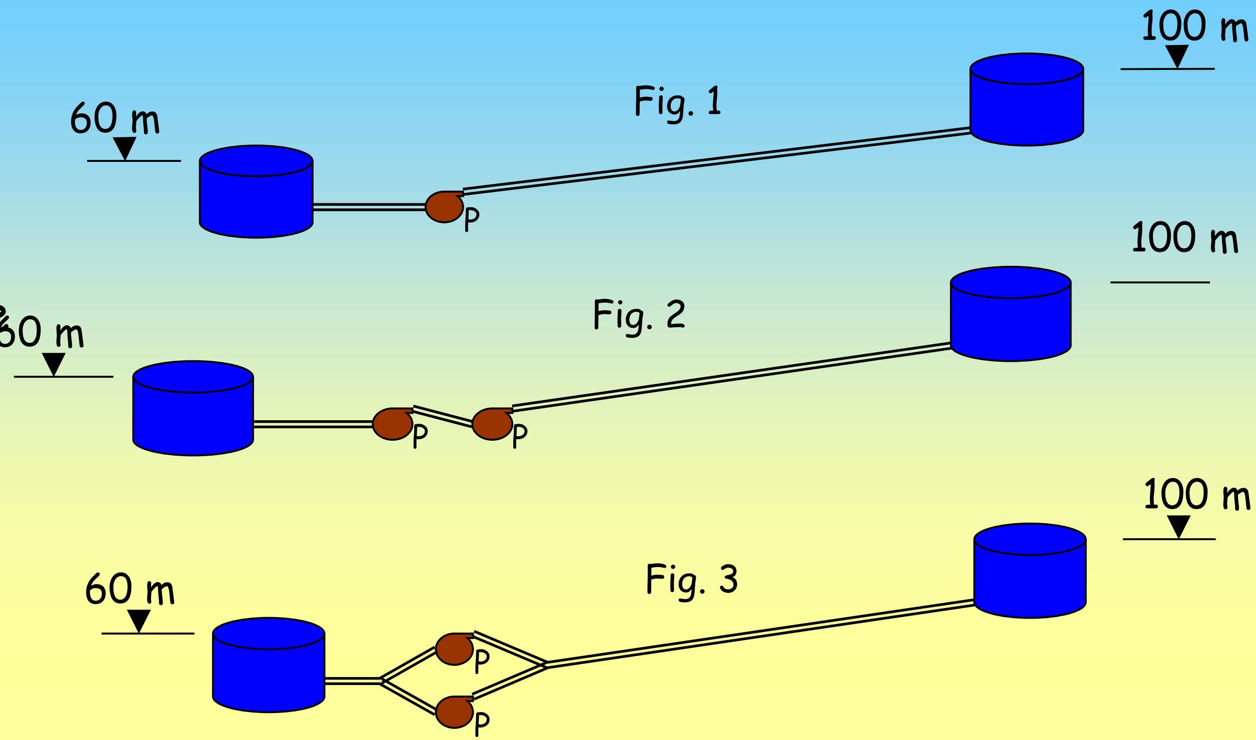
EXAMPLE 2.9:

The following pumped discharge pipelines are given;

a) Determine the discharge when only one pump is operating (as shown in figure 1.) and compute the power consumption.

b) Determine the discharge when both pumps are operating in series (as shown in figure 2.) and compute the power consumption.

c) Determine the discharge when both pumps are operating in parallel (as shown in figure 3.) and compute the power consumption.



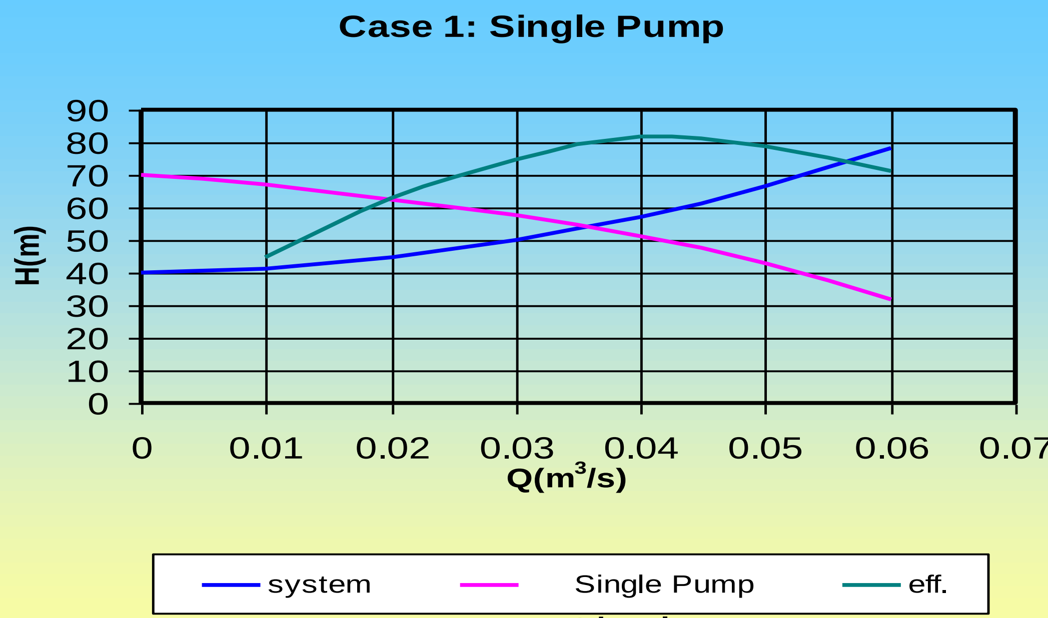
Pipeline Characteristics

L=2000 m; D=0.2 m; ε=0.0002m; ν=1* 10^{-6}m²/s

Pump Charecteristics

Q (lt/s)	**0**	**10**	**20**	**30**	**40**	**50**	**60**
H (m)	**70**	**67**	**62.5**	**57.5**	**51**	**43**	**32**
η (%)		**45**	**63**	**75**	**82**	**79**	**71**

Single Pump



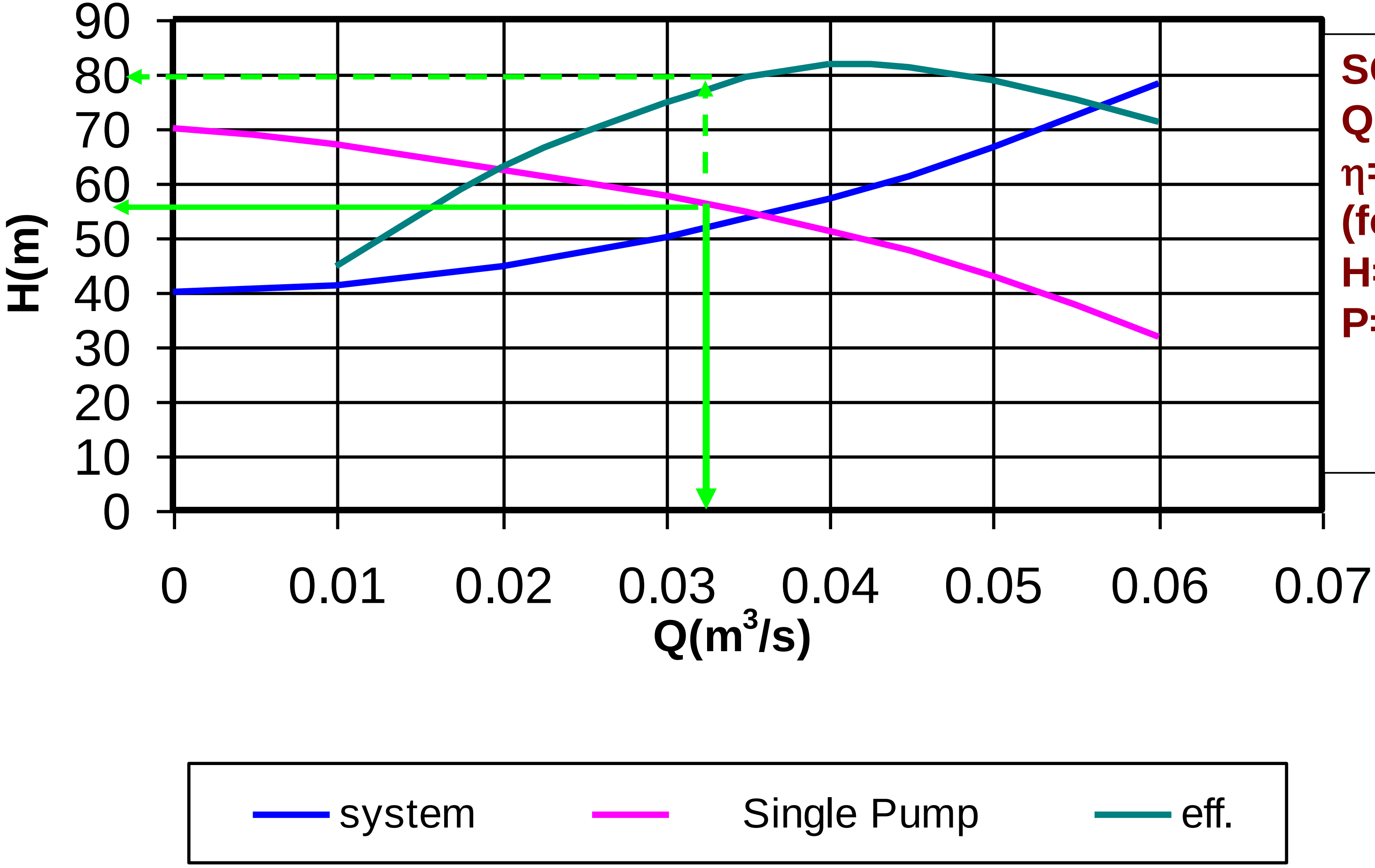
L (m)	2000
D (m)	0.2
ε (m)	0.0002
υ (m²/s)	0.000001

Q	Re	f	hl	system
0	0	0	0.00	40.00
0.01	63662	0.0234	1.21	41.21
0.02	127324	0.0219	4.52	44.52
0.03	190986	0.0212	9.88	49.88
0.04	254648	0.0209	17.28	57.28
0.05	318310	0.0207	26.72	66.72
0.06	381972	0.0205	38.19	78.19

Single Pump

H	Q
70	0
67	0.01
62.5	0.02
57.5	0.03
51	0.04
43	0.05
32	0.06

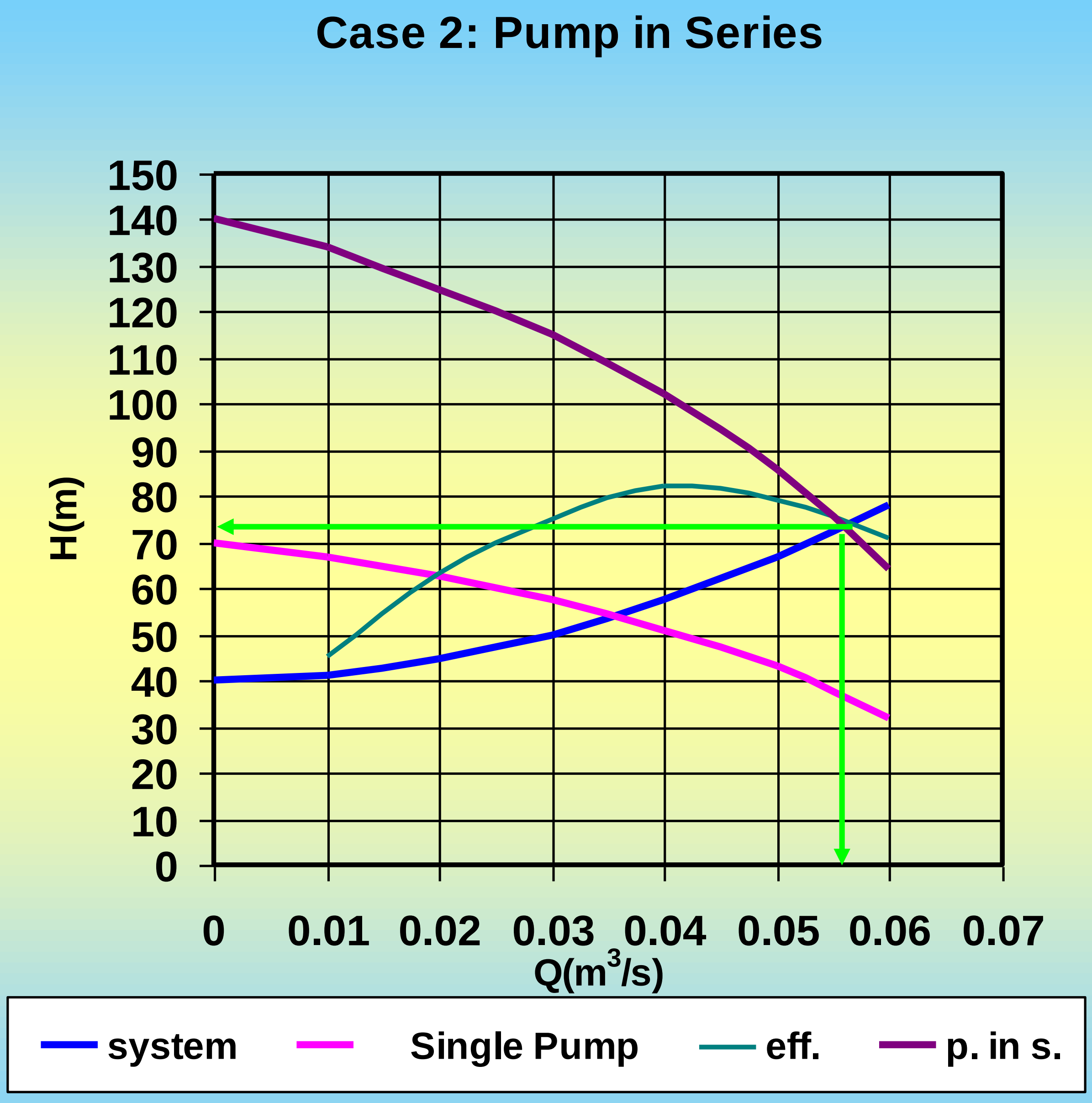
Case 1: Single Pump



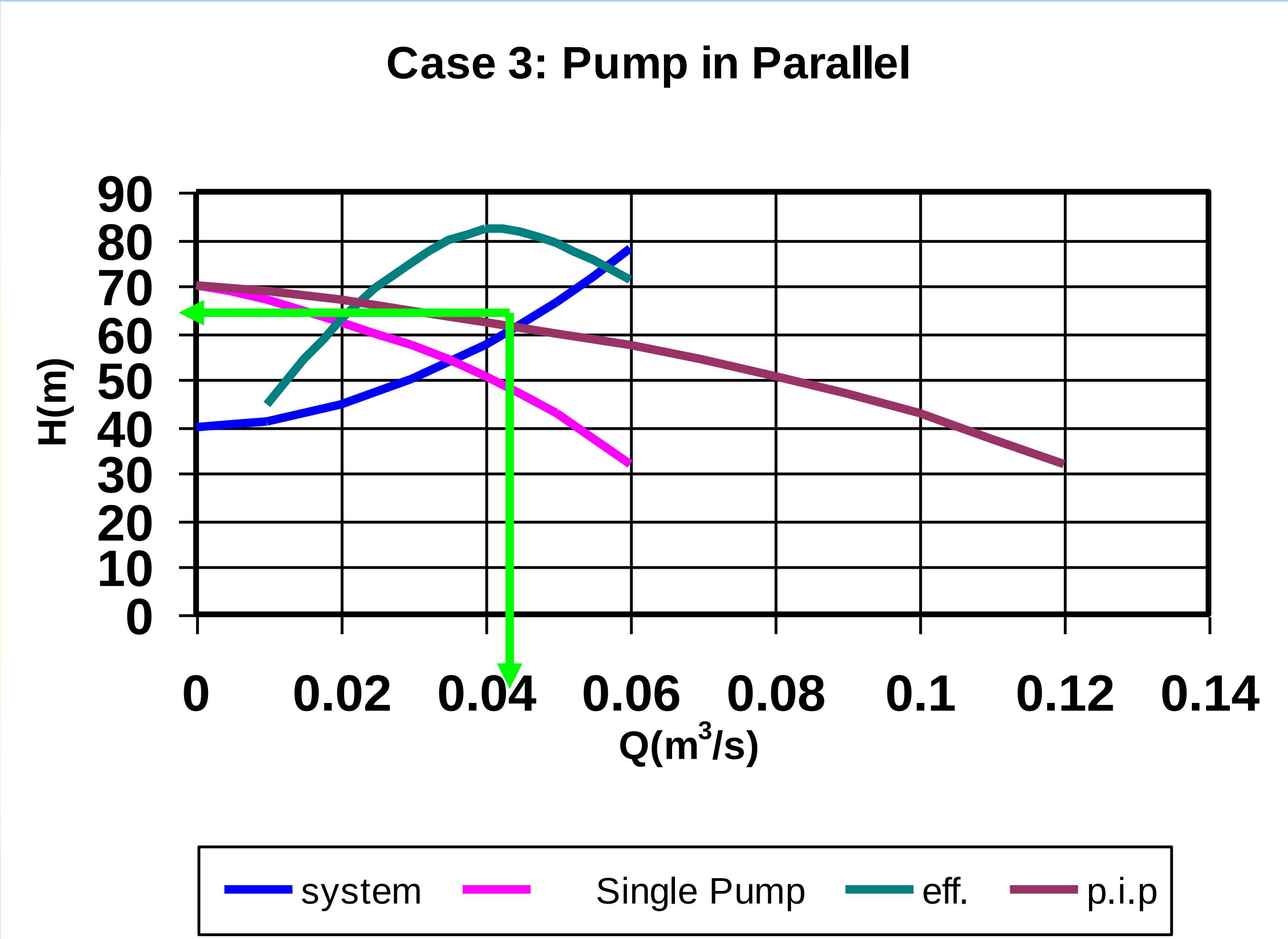
SOLUTION
Q=35 *lt*/s
η=80%
(for Q=35 *ℓ*/s)
H=54m
P=23.2kW

Pumps in Series

H(m)	Q(m³/s)
140	0
134	0.01
125	0.02
115	0.03
102	0.04
86	0.05
64	0.06



SOLUTION
Q=56 ℓ/s
η=75%
(for Q=56 ℓ/s)
∑H=72m
∑P=52.8kW



Parallel Pumps

H	Q
70	0
67	0.02
62.5	0.04
57.5	0.06
51	0.08
43	0.1
32	0.12

SOLUTION
Q=46 ℓ/s
η=68%
(for Q=23ℓ/s)
∑H=61m
∑P=40.5kW

summary

	single	series	paralel	
Q (lt/s)	35	56	46	(2*23)
η (%)	80	75	68	
H (m)	54	72	61	
P (kW)	23.2	52.7	40.5	(2*20.24)
P/Q (kW.s/lt)	0.66	0.94	0.88	

EXAMPLE 2.10

Water is pumped from reservoir A to reservoirs C and D. The system and pump characteristics are given below. Determine

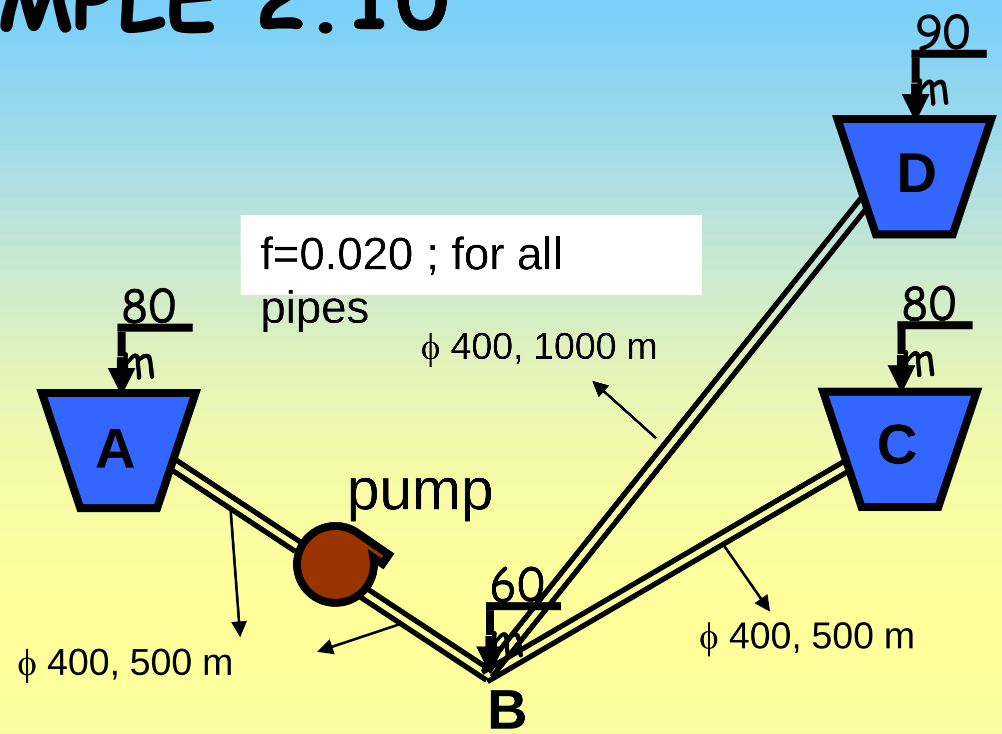
a) The discharge in each pipe,

b) The head added to the system by pump,

c) The head loss in each pipe for the computed discharges,

d) Complete the HGL,

e) Calculate the power of the pump for η=0.80.



Q (lt/s)	0	100	200	300	400	500
H_p (m)	50	48	45	38	30	18

SYSTEM CURVES

$$h_f = \frac{8fL}{g\pi^2 D^5} Q^2$$

Branch AB:

HA = HB-Hp+hfAB

80 = HB+hfAB-Hp

HB = 80+Hp-hfAB

$$h_f = 80.69Q^2$$

Branch BC:

HB = HC +hfBC

HB = 80+hfBC

Branch BD:

HB = HD +hfBD

HB = 90+hfBD

$$h_f = 161.38Q^2$$

QAB = QBC + QBD

Variation of head loss for each pipe

Q (lt/s)	h_{fAB} (m)	h_{fBD} (m)	h_{fBC} (m)
0	0	0	0
100	0.8	1.6	0.8
200	3.2	6.4	3.2
300	7.2	14.5	7.2
400	12.9	25.8	12.9
500	20.2	40.3	20.2

Head of Junction

Q (lt/s)	H_p (m)	from AB H_B (m)	from BC H_B (m)	from BD H_B (m)
0	50	130	80	90
100	48	127.2	80.8	91.6
200	45	121.8	83.2	96.5
300	38	110.8	87.2	104.5
400	30	97.1	92.9	115.8

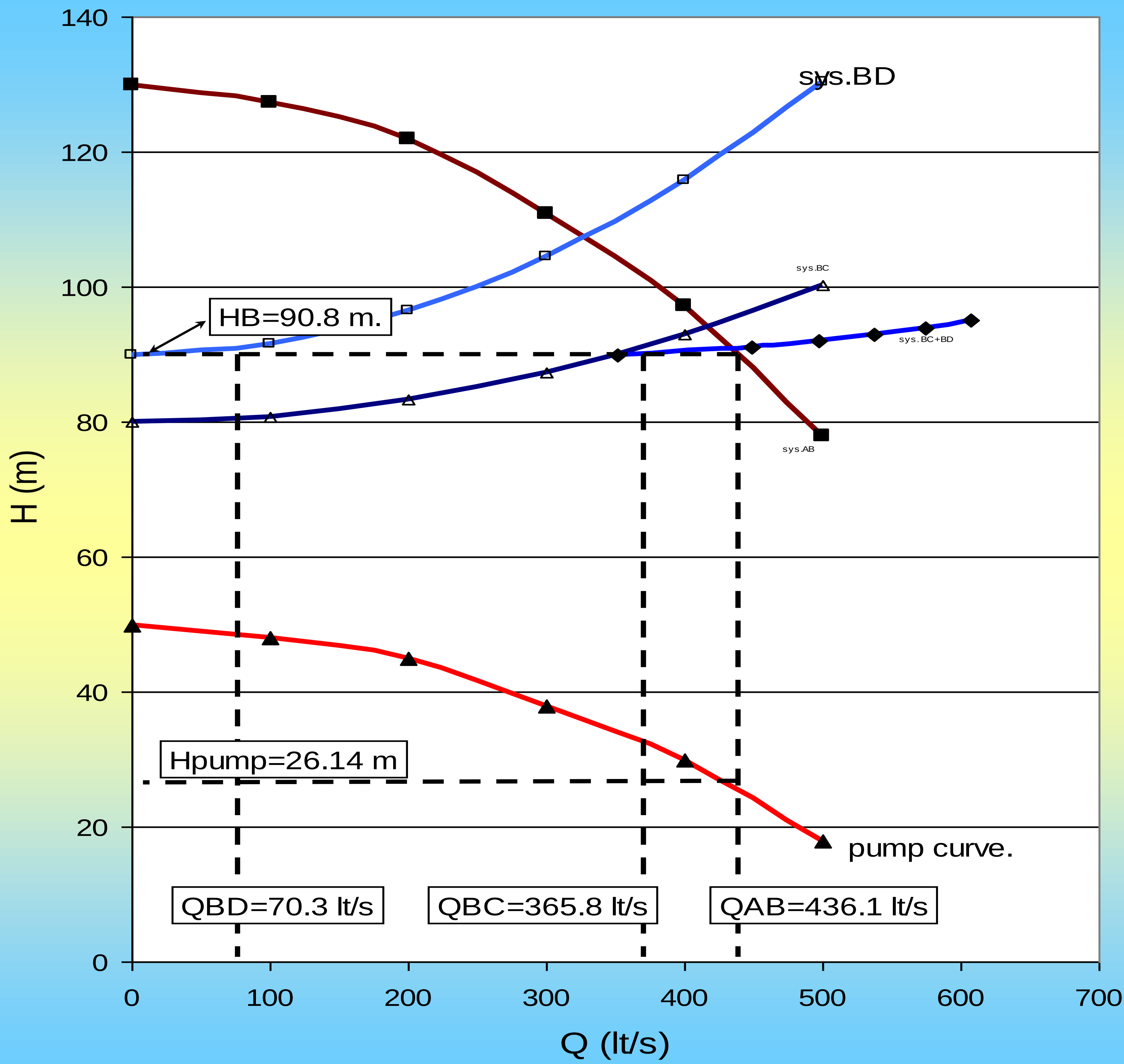
Attention:The minimum HB value is 90 m.;
above which flow in branch BC and BD will be in the assumed direction!

QAB = QBC + QBD

$$Q_{BC} = 1000.\left(\frac{H_B - 80}{80.69}\right)^{1/2}$$

$$Q_{BD} = 1000.\left(\frac{H_B - 90}{161.38}\right)^{1/2}$$

H_B (m)	Q_{BC} (lt/s)	Q_{BD} (lt/s)	$Q_{BC}+Q_{BD}$ (lt/s)
90	352	0	352
91	369.2	78.7	447.9
92	385.6	111.3	496.9
93	401.4	136.3	537.7
94	416.5	157.4	573.9
94	431.2	176	607.2

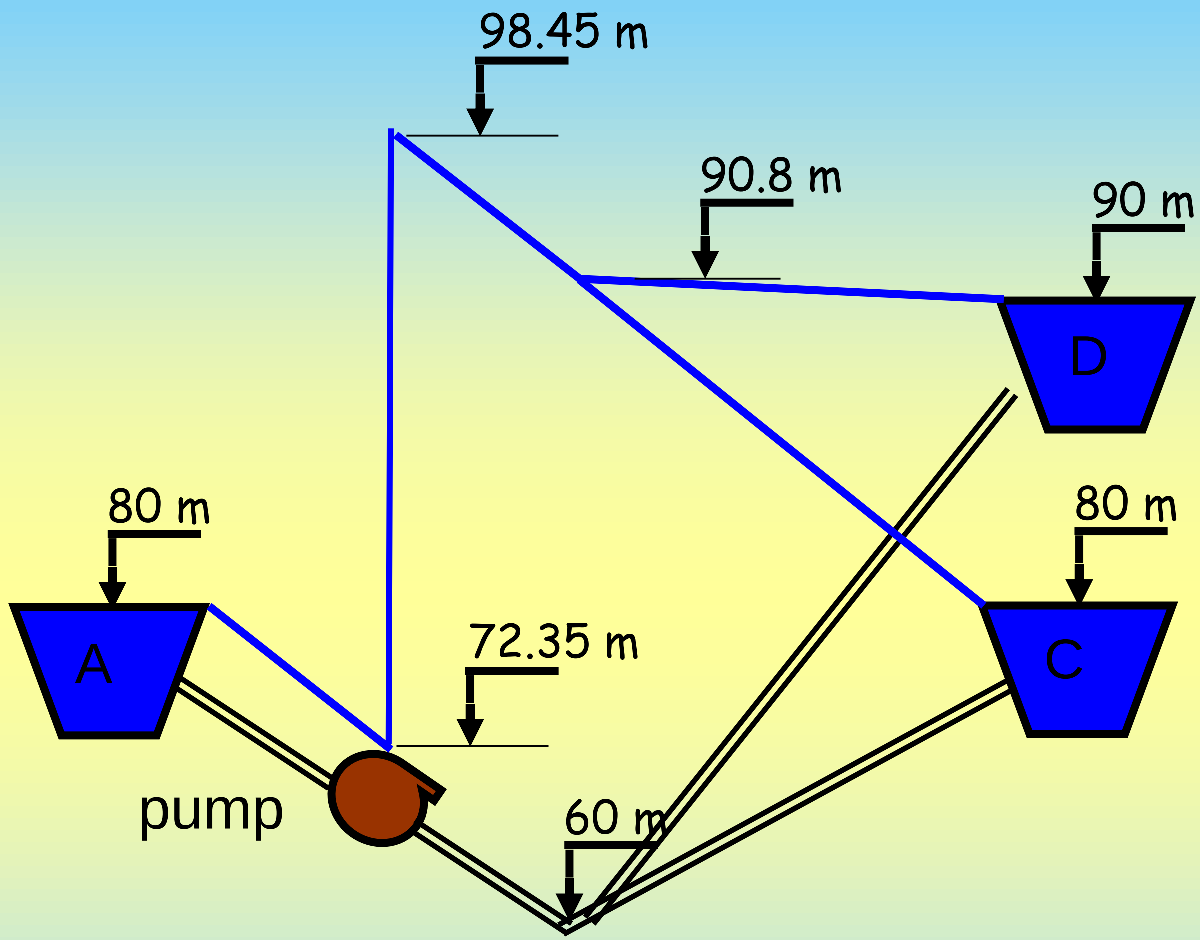


sys.BD
sys.BC
sys.BC+BD
sys.AB
pump curve.
HB=90.8 m.
Hpump=26.14 m
QBD=70.3 lt/s
QBC=365.8 lt/s
QAB=436.1 lt/s
H (m)
Q (lt/s)
0
20
40
60
80
100
120
140
0
100
200
300
400
500
600
700

SUMMARY OF RESULTS:

- From the graph, the intersection of HB vs Q_{AB} and H_B vs Q_{BC}+Q_{BD} is read to be at **H_B=90.8m** and **QAB=436.1 lt/s.** Furthermore, these values correspond to **Q_{BC}=365.8 lt/s** and **Q_{BD}=70.3 lt/s.**
- The pump head **H_{pump}=26.1 m.**
- **h_{fAB}=15.34 m.**
- **h_{fBC}=10.8 m.**
- **h_{fBD}=0.8 m.**

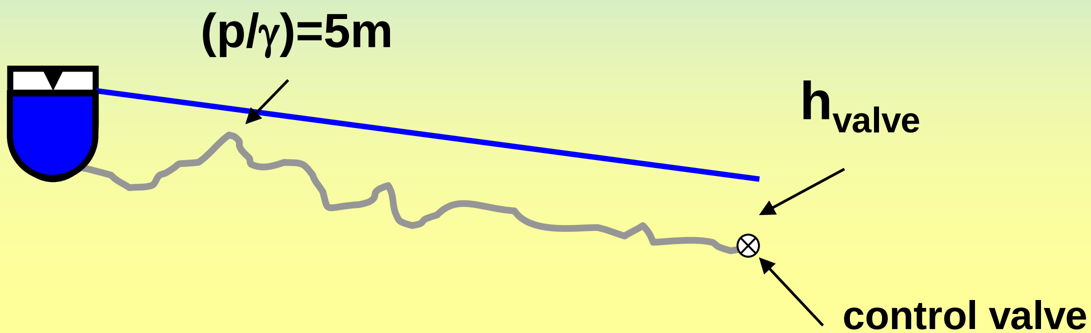
d)Hydraulic Grade Line



e) $P_{pump} = \gamma g Q H_{pump} / h$ = 139.6 kW

GRAVITY PIPELINES

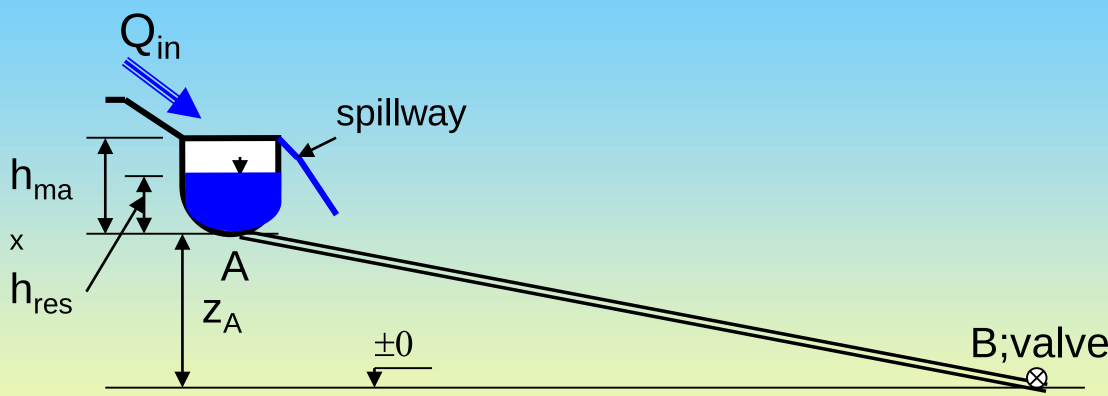
The pipelines through which flow is maintained by the action of gravity are known as **gravity pipelines.**



In the selection of the **diameter** of gravity pipe lines:

- Cost (capital + operation) is minimized;
- Depending on the type of pipe material a lower and an upper limits are set for the velocity, $0.5\ m/s < V < 2\ m/s$;
- To prevent air entrainment minimum pressure head, $(p/\gamma)_{min}$ permitted along the pipeline is 5m.

Because of limits set forth on velocity and pressure head there are lower and upper bounds for the discharge through a gravity pipeline. Consider the following schematic representation of a gravity pipeline system shown below



$$h_{res} + z_A = h_{lAB} = KQ_{out}^2 \Rightarrow Q_{out} = \left[\left(h_{res} + z_A\right)/K\right]^{1/2}$$

(note that since $V < 2$ m/s velocity heads are neglected!)

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$$h_{res} = h_{max} \quad Q_{out} = Q_{max} = \left[\left(h_{res} + z_A\right)/K\right]^{1/2}$$

$$h_{res} = 0 \quad Q_{out} = Q_{min} = \left[z_A / K\right]^{1/2}$$

if $Q_{max} < Q_{in}$ $h = h_{max}; Q_{spill} = Q_{in} - Q_{max}$

if $Q_{min} < Q_{in} < Q_{max}$ $0 < h_{res} < h_{max}; Q_{spill} = 0$

if $Q_{in} < Q_{min}$ to prevent free flow the valve is partially closed such that

$$h_{res} = KQ_{in}^2 + h_{valve} - z_A > 0; Q_{spill} = 0$$

Example 2.11:

Consider the reservoir-pipe system given below, with following values.

a) Reservoir depth h_{max}=5m, z_A=5m, L=2000m, D=0.8m, f=0.02
b) Determine:
c) The system capacity, Q_{max}.
d) Minimum flowrate, Q_0.
e) Spill flowrate, if Q=1.2m3/s.
f) Valve loss, h_v, if Q=0.5m3/s.

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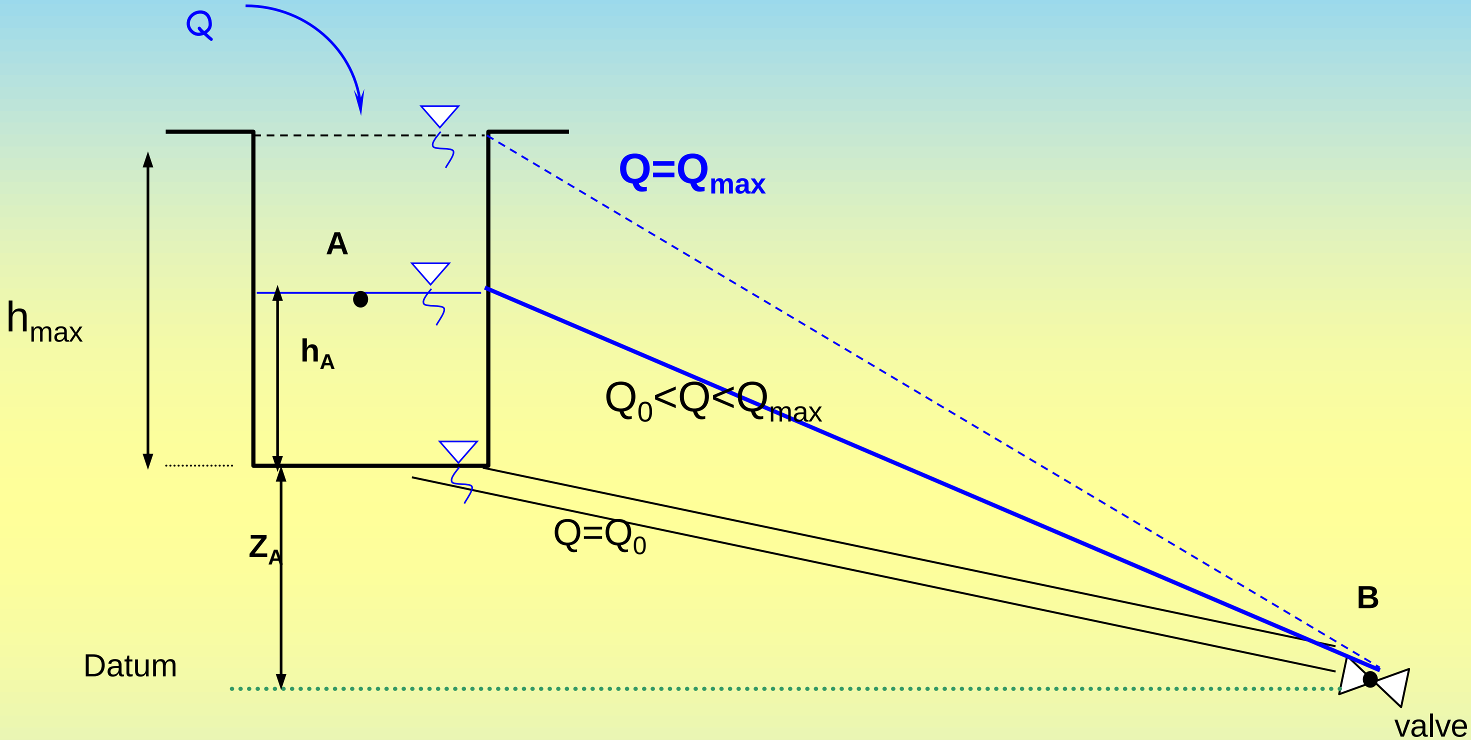
Q
Q=Qmax
A
hmax
hA
Q0<Q<Qmax
ZA
Q=Q0
B
Datum
valve

Solution

If minor losses except hv (valve loss) are neglected,

$$Q_{out} = \left(\frac{h_{res} + z_A}{K} \right)^{1/2} \qquad K = \frac{8fL}{g\pi^2 D^5} = 10.09$$

$$Q_{max} = \left(\frac{5+5}{10.09} \right)^{1/2} \cong 1.0 \ \ m^3/s \qquad Q_{min} = \left(\frac{5}{10.09} \right)^{1/2} \cong 0.70 \ \ m^3/s$$

$$Q_{spill} = 0.20 \ m^3 / s$$

$$h_{res} + z_A = h_V + KQ_{in}^2 \Rightarrow h_V \geq z_A - KQ_{in}^2$$

$$h_V \geq 5 - 10.09.(0.5)^2$$

$$h_V \geq 2.48 \ \ m$$