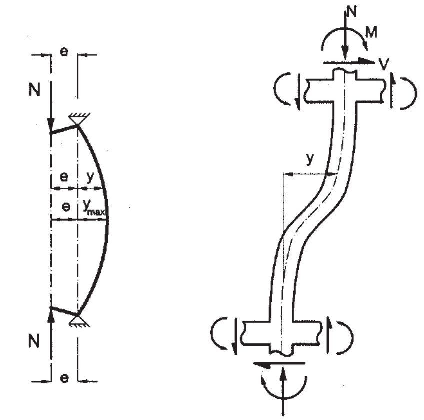
CE 382 Reinforced Concrete Fundamentals

Combined Flexure & Axial Load – Analysis of RC Columns

Introduction

- Beams are subjected to flexure, shear and axial load
  - Design practice $\rightarrow$ neglect axial force
- Columns are designed for combined axial load & flexure
  - Codes prohibit column design with zero moment
  - Minimum eccentricity
- Columns $\rightarrow$ slender members $\rightarrow$ second order moment

Second Order Moment

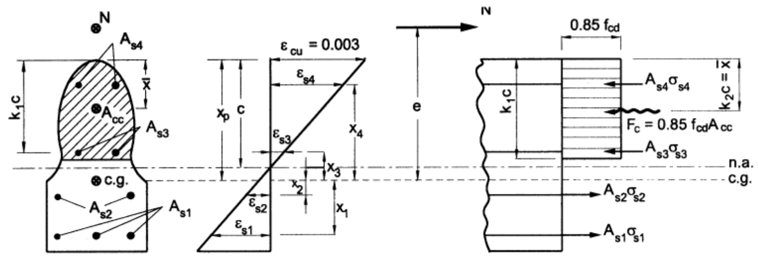


(a) Braced Non-sway frame

(b) Unbraced Sway frame

- $M_d = N \times e$
- $\Delta M = N \times y$
- $M_d' = N(e + y)$
  $= M_d + N \times y$

Ultimate strength of RC sections subjected to axial forces and flexure



(a) (b) (c)

- Cross-section is symmetrical about the plane of loading
- Equilibrium
  - $N = 0.85 f_{cd} A_{cc} + \sum_{i=1}^{n} A_{si} \sigma_{si}$
  - $M = Ne = 0.85 f_{cd} A_{cc} (x_p - \bar{x}) + \sum_{i=1}^{n} A_{si} \sigma_{si} x_i$

Ultimate strength of RC sections subjected to axial forces and flexure

- Compatibility:
  - $\frac{c}{0.003} = \frac{x_p - c - x_i}{-\varepsilon_{si}}$
- Force-deformation:
  - $\sigma_{si} = \varepsilon_{si} E_s \le f_{yd}$
  - $\Rightarrow \quad \sigma_{si} = 0.003 E_s \left(1 + \frac{x_i - x_p}{c}\right) \le f_{yd}$
- Given: geometry, steel area, material properties and M
- $\rightarrow$ Find: N

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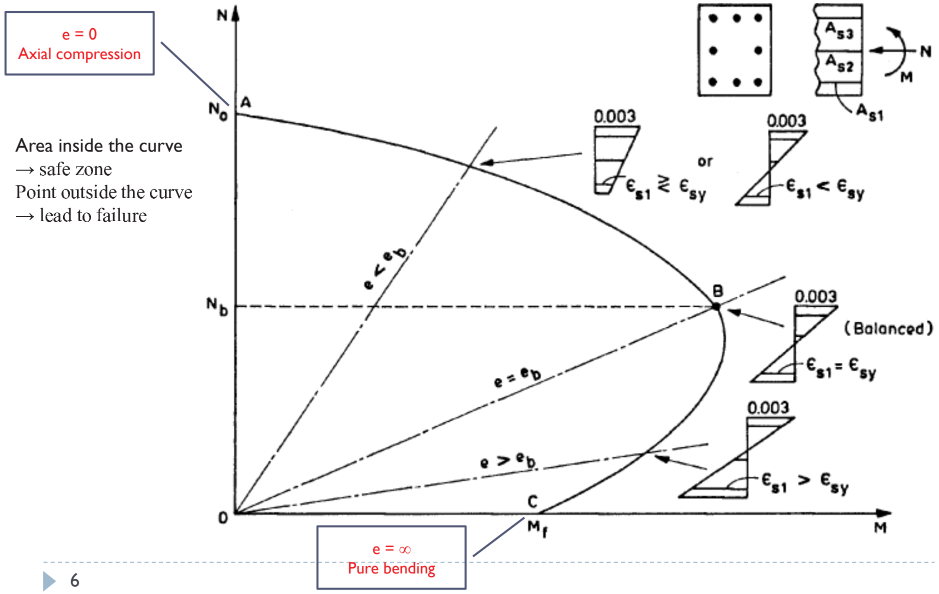
- $\rightarrow$ Find: M

Moment – Axial Load Interaction

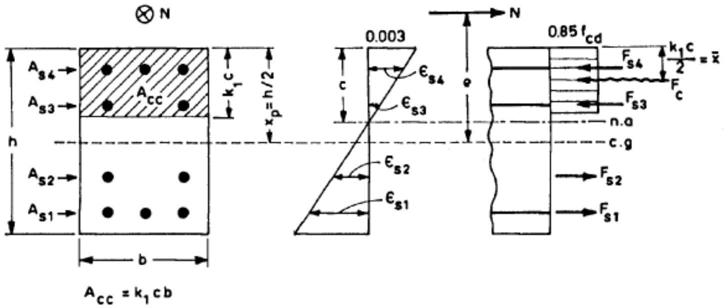


Moment – Axial Load Interaction

- Point A $\rightarrow$ uniaxial compression
- Point C $\rightarrow$ pure bending
- Point B $\rightarrow$ balanced failure
- Curve AB $\rightarrow$ compression failure
  - No tension on the section
  - With tension
- Curve BC $\rightarrow$ tension failure
  - Steel nearest to the tension face has already yielded
- Compression failure if $e < e_b$ or if $N > N_b$
- Tension failure if $e > e_b$ or if $N < N_b$

$$e = \frac{M}{N}$$

Ultimate strength of rectangular sections



- $N = 0.85 f_{cd} k_1 cb + \sum_{i=1}^{n} A_{si} \sigma_{si}$
- $M = N \times e = 0.85 f_{cd} k_1 cb \left(\frac{h}{2} - \frac{k_1 c}{2}\right) + \sum_{i=1}^{n} A_{si} \sigma_{si} x_i$
- $\sigma_{si} = 0.003 E_s \left(1 + \frac{x_i - \frac{h}{2}}{c}\right) \le f_{yd}$

Ultimate strength of rectangular sections

- Axial compression ($M = 0$)
  - $k_1 c = h \qquad \sigma_{si} = f_{yd} \qquad \sum A_{si} = A_{st}$
  - Due to symmetry, moment of steel forces cancel each other
  - $M = 0$
  - $N_r = 0.85 f_{cd} h b + A_{st} f_{yd}$
- Pure bending ($N = 0$)
  - For $A_{si} = A_{s1} = A_s$ & $x_i = d - \frac{h}{2}$ & $\sigma_{s1} = \sigma_s = -f_{yd}$
  - $0 = 0.85 f_{cd} k_1 c b_w - A_s f_{yd} \rightarrow k_1 c = \frac{A_s f_{yd}}{0.85 f_{cd} b_w}$
  - $M_r = 0.85 f_{cd} k_1 c b_w \left(\frac{h}{2} - \frac{k_1 c}{2}\right) + A_s f_{yd} \left(d - \frac{h}{2}\right)$
  - $M_r = A_s f_{yd} \left(d - \frac{k_1 c}{2}\right)$

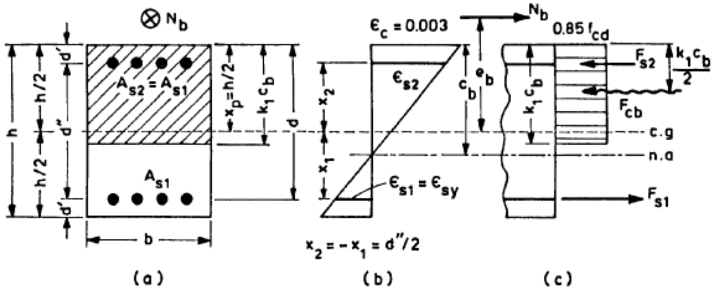
Drawing M-N Interaction Diagram

- Compute uniaxial strength ($N = N_{or}$ & $M = 0$)
- Compute balanced point ($N = N_b$ & $M = M_b$)
- $N_{or} = 0.85 f_{cd} A_c + A_{st} f_{yd}$
- Compute a set of (N,M) values
  - Assume «c» & $\varepsilon_{cu} = 0.003$
  - Compute strain → stress → force of steel at every level
  - Compute $F_c = 0.85 f_{cd} k_1 c b$
  - Compute $N = F_c + \sum F_{si}$
  - Compute M by taking moments of steel and concrete forces about the centroid

Study Example 6.3

Rectangular sections having symmetrical steel on two faces only

- A rare special case
- Reinforcement should placed symmetrically
  - Time dependent deformations
  - Biaxial bending
- Balanced case

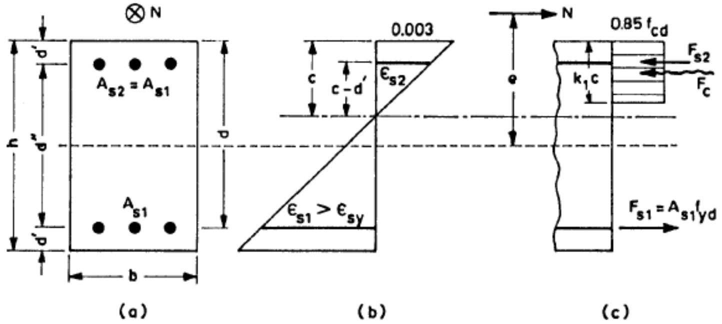


Rectangular sections having symmetrical steel on two faces only

- $\varepsilon_c = \varepsilon_{cu} = 0.003 \qquad \varepsilon_{s1} = \varepsilon_{sy} \qquad x_2 = x_1 = \frac{d''}{2} = \frac{h-2d'}{2}$
- $x_p = \frac{h}{2} \qquad A_{cc} = k_1 c b \qquad A_{s1} = A_{s2} = \frac{A_{st}}{2} \qquad \varepsilon_{s2} > \varepsilon_{sy}$
- $\frac{0.003}{c_b} = \frac{0.003+\varepsilon_{sy}}{d} \rightarrow c_b = \frac{0.003d}{0.003+\varepsilon_{sy}} = \frac{0.003 d E_s}{0.003 E_s + f_{yd}}$
- $N_b = 0.85 f_{cd} k_1 c_b b + \frac{A_{st}}{2} f_{yd} - \frac{A_{st}}{2} f_{yd} = 0.85 f_{cd} k_1 c_b b$
- $M_b = 0.85 f_{cd} k_1 c_b b \left(\frac{h}{2} - \frac{k_1 c_b}{2}\right) + \frac{A_{st}}{2} f_{yd} \frac{d''}{2} + \frac{A_{st}}{2} f_{yd} \frac{d''}{2}$
- $M_b = N_b \left(\frac{h}{2} - \frac{k_1 c_b}{2}\right) + \frac{A_{st}}{2} f_{yd} d''$
- $d'' = d - d' = h - 2d'$

Rectangular sections having symmetrical steel on two faces only

- Tension Failure:



  - $\varepsilon_{s1} > \varepsilon_{sy}$ & $\varepsilon_{s2} > \varepsilon_{sy}$ generally compression steel yields
  - $A_{s1} = A_{s2} = \frac{A_{st}}{2}$ $x_p = \frac{h}{2}$ $x_1 = x_2 = \frac{d''}{2}$
  - $A_{cc} = k_1 cb$ $F_{s1} = F_{s2}$

Rectangular sections having symmetrical steel on two faces only

- $\sum F = 0 = F_c - N + F_{s2} - F_{s1}$
- $N = F_c = 0.85 f_{cd} k_1 cb$
- $\sum M =$

$$Ne - F_c\left(\frac{h}{2} - \frac{k_1 c}{2}\right) - F_{s2}\left(\frac{h}{2} - d'\right) - F_{s1}\left(d - \frac{h}{2}\right) = 0$$

- $M = 0.85 f_{cd} k_1 cb\left(\frac{h}{2} - \frac{k_1 c}{2}\right) + \frac{A_{st}}{2} f_{yd} d''$

Rectangular sections having symmetrical steel on two faces only

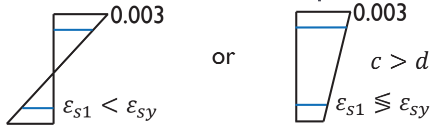
- If compression steel does not yield:
  - $N = 0.85 f_{cd} k_1 cb + \frac{A_{st}}{2}(\sigma_{s2} - f_{yd})$
  - $M = 0.85 f_{cd} k_1 cb\left(\frac{h}{2} - \frac{k_1 c}{2}\right) + \frac{A_{st}}{2}(f_{yd} + \sigma_{s2})\frac{d''}{2}$
  - $\sigma_{s2} = 0.003 E_s \frac{c-d'}{c} \leq f_{yd}$

Rectangular sections having symmetrical steel on two faces only

- Limiting case: $\varepsilon_{s2} = \varepsilon_{sy}$ & $\sigma_{s2} = f_{yd}$
  - $\frac{0.003}{c} = \frac{\varepsilon_{sy}}{c-d'} \quad \rightarrow \quad c = \frac{0.003 E_s d'}{0.003 E_s - f_{yd}}$
  - $N_c = 0.85 f_{cd} k_1 \frac{0.003 E_s d'}{0.003 E_s - f_{yd}} b + \frac{A_{st}}{2} f_{yd} - \frac{A_{st}}{2} f_{yd}$
  - $\frac{N}{bhf_{cd}} = \psi_c = 0.85 k_1 \frac{0.003 E_s\left(\frac{d'}{h}\right)}{0.003 E_s - f_{yd}}$
  - If $\frac{N_d}{bhf_{cd}} \geq \psi_c \rightarrow$ compression steel has yielded
  - If $\frac{N_d}{bhf_{cd}} < \psi_c \rightarrow$ compression steel has not yielded

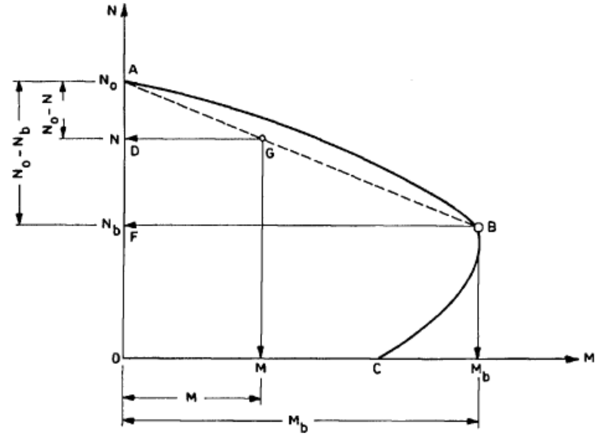
Rectangular sections having symmetrical steel on two faces only

- Compression failure:
  - Tension steel has not yielded & compression steel has yielded
  - $\varepsilon_{s1} < \varepsilon_{sy}$ & $\varepsilon_{s2} > \varepsilon_{sy} \rightarrow \sigma_{s2} = f_{yd}$
  - $N = 0.85 f_{cd} k_1 cb + \frac{A_{st}}{2}(f_{yd} + \sigma_{s1})$
  - $M = 0.85 f_{cd} k_1 cb\left(\frac{h}{2} - \frac{k_1 c}{2}\right) + \frac{A_{st}}{2}\frac{d''}{2}(f_{yd} - \sigma_{s1})$
  - $\sigma_{s1} = 0.003 E_s \left(\frac{c-d}{c}\right) \le f_{yd}$
  - Use proper sign for σ_{s1}
  - A_{s1} is either in tension or compression



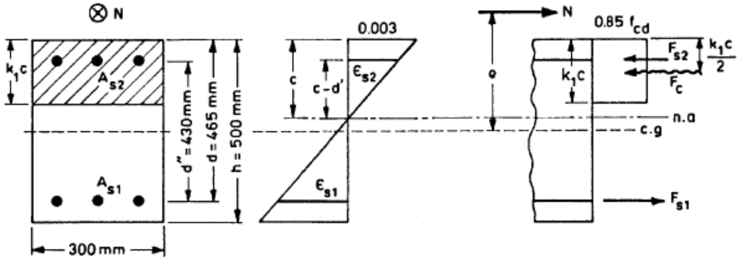
Rectangular sections having symmetrical steel on two faces only

- Compression failure:
  - Approximate Solution
  - $\frac{M}{M_b} = \frac{N_{or} - N}{N_{or} - N_b}$



Example 1

- $N_d = 247\ kN$
- C16 $\rightarrow f_{cd} = 11\ MPa$ S420 $\rightarrow f_{yd} = 365\ MPa$
- $A_{s1} = A_{s2} = 600\ mm^2$
- $M_r = ?$



$A_{s1} = A_{s2} = 600\ mm^2$ $\varepsilon_{sy} = \frac{365}{200000} = 0.001825$ $\frac{c_b}{d} = \frac{0.003}{0.003 + \varepsilon_{sy}}$ $c_b = 289\ mm$

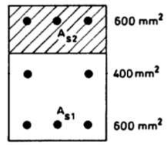
Example 1

- For balanced case both top and bottom steels yield
- $N_b = 0.85 f_{cd} k_1 c_b b = 0.85 \times 11 \times 0.85 \times 289 \times 300$
- $N_b = 689\ kN$
- $N < N_b \rightarrow$ tension failure
- Assume compression steel has yielded
- $N_d = 0.85 f_{cd} k_1 cb + A_{s2} f_{yd} - A_{s1} f_{yd}$
- $247000 = 0.85 \times 11 \times 0.85 \times c \times 300$
- $c = 103.6\ mm \rightarrow k_1 c = 88\ mm$

Example 1

- Check the assumption
- $\frac{0.003}{c} = \frac{\varepsilon_{s2}}{c-d'}$
- $\varepsilon_{s2} = 0.003\frac{103.6-35}{103.6} = 0.00199 > \varepsilon_{sy}$ OK✓
- $M_r = F_c\left(\frac{h}{2} - \frac{k_1 c}{2}\right) + \frac{A_{st}}{2} f_{yd} d''$
- $M_r = 247000\left(\frac{500}{2} - \frac{88}{2}\right) + 600 \times 365 \times 430$
- $M_r = 145\ kNm$

Example 2



- Same example with intermediate steel
- For $N_d = 247\ kN$
- $\rightarrow\ M_r = 164\ kNm$
- Difference:
- $in\ A_{st}$: 33% ↗
- $in\ M_r$: 13% ↗
- Intermediate steel does not affect the moment capacity significantly
- But, confine concrete & reduce the size of horizontal cracks due to shrinkage or other effects

Example 3

- Same as Example 1 but $N_d = 1200\ kN$
- $N > N_b \quad \rightarrow$ compression failure
- Use approximate solution
  - $N_{or} = 0.85 f_{cd} A_c + A_{st} f_{yd}$
  - $N_{or} = 0.85 \times 11 \times 150000 + 1200 \times 365 = 1840\ kN$
  - $M_b = F_{cb}\left(\frac{h}{2} - \frac{k_1 c_b}{2}\right) + \frac{A_{st}}{2} f_{yd} d''$
  - $M_b = 689000\left(\frac{500}{2} - \frac{0.85 \times 289}{2}\right) + 600 \times 365 \times 430 =$ $182\ kNm$
  - $\frac{M}{M_b} = \frac{N_{or}-N}{N_{or}-N_b} \quad \rightarrow \quad M = 182 \times \frac{1840-1200}{1840-689} = 101.3\ kNm$

Example 3

- Use exact solution
  - $N > N_b \quad \rightarrow$ compression failure
  - Assume top steel has yielded & bottom steel in tension
  - $\frac{0.003}{c} = \frac{\varepsilon_{s1}}{d-c} \quad \rightarrow \quad \varepsilon_{s1} = 0.003\frac{d-c}{c} \quad \rightarrow \quad \sigma_{s1} = 600\frac{d-c}{c}$
  - $N_d = 0.85 f_{cd} k_1 c b + A_{s2} f_{yd} - A_{s1}\sigma_{s1}$
  - $N_d = 0.85 \times 11 \times 0.85 \times c \times 300 + 600\left(365 - 600\frac{465-c}{c}\right)$
  - $c = 425.5\ mm \qquad c < d$ OK✓
  - $\frac{0.003}{c} = \frac{\varepsilon_{s2}}{c-d'} \quad \rightarrow \quad \varepsilon_{s2} = 0.00275 > \varepsilon_{sy} = 0.001825$
  - $\sigma_{s1} = 600\frac{465-425.5}{425.5} = 55.7\ MPa$
  - $M_r = 124.4\ kNm$

Analysis of rectangular sections with intermediate steel

- Trial & error solution is recommended
  - Assume «c»
  - From similar triangles compute $\varepsilon_{si} \rightarrow \sigma_{si} \rightarrow F_{si}$
  - Compute $F_c = 0.85 f_{cd} A_{cc}$
  - Check $N = F_c + \sum_{i=1}^{n} F_{si}$
  - If equilibrium not satisfied $\rightarrow$ assume another «c»
  - Compute moments of internal forces about the centroid

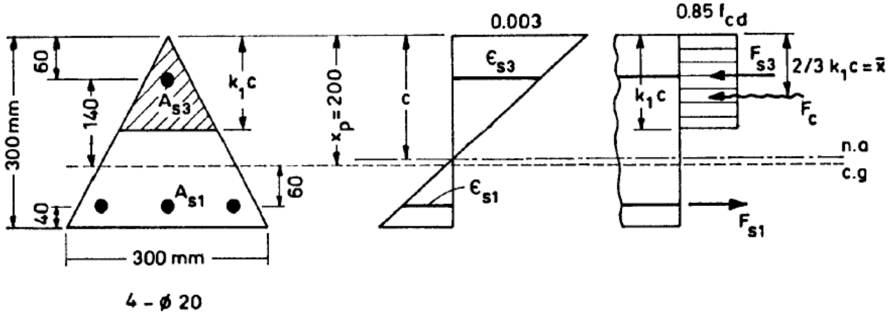
Study Example 6.6

Ultimate strength of Nonrectangular sections

- If there is intermediate steel
  - Use trial & error solution
- If there is no intermediate steel & the shape is simple
  - Try to write down closed form solution

Study Example 6.7

Example 4



- C20 & S420 ($f_{cd} = 13\ MPa$ & $f_{yd} = 365\ MPa$)
- $A_{s1} = 942\ mm^2$ & $A_{s3} = 314\ mm^2$
- $N_d = 100\ kN$
- $M_r = ?$

Example 4

- Balanced case:
  - $\varepsilon_{s1} = \varepsilon_{sy} = 0.001825$
  - $\frac{c_b}{0.003} = \frac{d}{0.003+0.001825} \rightarrow c_b = 161.7\ mm$
  - $\frac{\varepsilon_{s3}}{c_b - d'} = \frac{0.003}{c_b} \rightarrow \varepsilon_{s3} = 0.00188 > \varepsilon_{sy}$
  - $N_b = 0.85 f_{cd} \frac{(k_1 c)^2}{2} + A_{s3} f_{yd} - A_{s1} f_{yd}$
  - $N_b = 0.85 \times 13 \frac{(0.85 \times 161.7)^2}{2} + (314 - 942)365 = -124.7$
  - Balanced load tension !
  - $N = 100\ kN > N_b = -124.7\ kN \rightarrow$ compression failure

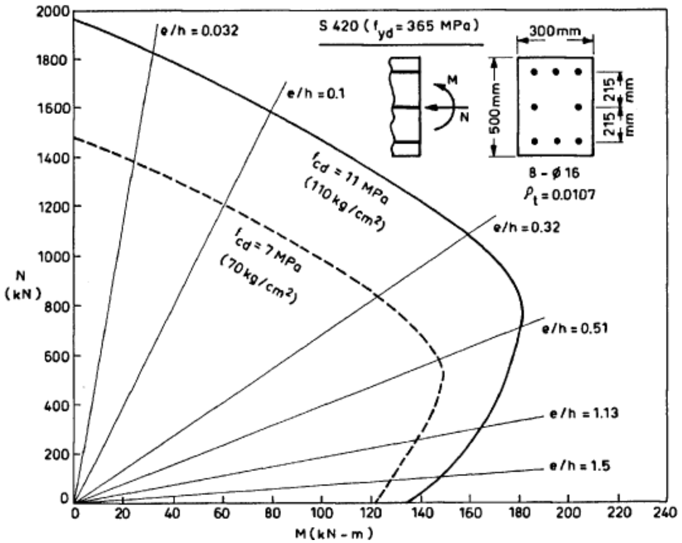
Example 4

- Use approximate method
  - $M_b = F_c\left(200 - \frac{2}{3}k_1 c_b\right) + F_{s3} \times 140 + F_{s1} \times 60 = 48\ kNm$
  - $N_{or} = 0.85 \times 13\frac{300\times300}{2} + 365(942 + 314) = 955\ kN$
  - $\frac{M}{M_b} = \frac{N_{or}-N}{N_{or}-N_b}$
  - $M = 48\frac{955-100}{955-(-124.7)} = 38\ kNm$

Example 4

- Use exact solution
  - Assume top steel yielded & bottom steel in tension ($c < d$)
  - $N_d = 0.85 f_{cd}\frac{(k_1 c)^2}{2} + A_{s3} f_{yd} - A_{s1}\sigma_{s1}$
  - $\frac{0.003}{c} = \frac{\varepsilon_{s1}}{d-c} \quad \rightarrow \quad \varepsilon_{s1} = 0.003\frac{d-c}{c} \quad \rightarrow \quad \sigma_{s1} = 600\frac{d-c}{c}$
  - $100000 = 0.85 \times 13\frac{(0.85c)^2}{2} + 314 \times 365 - 942 \times 600\frac{260-c}{c}$
  - $c = 198.4\ mm \quad < \quad d = 260\ mm$ ✓OK
  - $\varepsilon_{s3} = 0.003\frac{c-d'}{c} = 0.002 > \varepsilon_{sy} = 0.001825$ ✓OK
  - $M_r = 0.85 f_{cd}\frac{(k_1 c)^2}{2}\left(200 - \frac{2}{3}k_1 c\right) + A_{s3} f_{yd} 140 + A_{s1} f_{yd} 60$
  - $M_r = 50.4\ kNm$

Effect of Concrete Strength on the Ultimate Capacity



- 36% lower concrete strength
- At high levels of axial load both axial load & moment capacity influenced significantly
- At low levels of axial load the influence reduces
- The influence of concrete strength becomes more pronounced when S220 steel is used instead of S420