

(e) SHEAR STRENGTH (For this section, take $g = 9.81 \text{ m/s}^2$)

E1. Samples of compacted clean dry sand were tested in a 63 mm dia. shear box, and the following results obtained.

Normal load (kgf)	:	16	32	48
Peak shear load (N)	:	133.4	287.4	417.7
Ultimate shear load (N)	:	85.7	190.1	268.1

Determine the angle of shearing resistance of the sand (a) in the dense, and (b) in the loose state.

E2. In a mixed series of unconsolidated - undrained and consolidated - undrained triaxial tests with pore pressure measurement on the unsaturated, stiff, fissured Ankara Clay (average degree of saturation = 97 %), the following results have been obtained at failure.

Test no.	1	2	3	4	5	6
Pore pressure (kPa)	-17	75	-17	-14	-2	-5
Cell pressure (kPa)	53	220	81	178	158	201
Deviator stress ($\sigma_1 - \sigma_3$) (kPa)	234	210	374	378	450	462

Determine the shear strength parameters in terms of effective stress (a) by drawing the average tangent to the Mohr circles; (b) by calculation from the modified shear strength envelope. State which method is preferable for such variable test results, and why.

E3. (a) By considering the torque on the curved (cylindrical) surface, and integrating the torque on ring-shaped elements on the two circular ends (neglecting the presence of the vane rod) of the sheared cylinder of soil, derive the following equation for the torque T required to shear a soft, saturated clay of shear strength c_u , using a vane with rectangular blades of height h and diameter of circumscribing circle d .

$$T = \pi c_u \left(\frac{d^2 h}{2} + \frac{d^3}{6} \right)$$

(b) A vane 75 mm in diameter and 150 mm long was used to measure the undrained shear strength of a soft clay. A torque of 50 Nm was required to shear the soil. The vane was then rotated rapidly to remould the soil completely. The ultimate torque recorded was 19 Nm. Determine the undrained shear strength of the clay in the natural and remoulded states, and hence find the sensitivity of the clay.

(c) If a 36 mm dia. undistributed specimen of the same clay as in Part (b) were tested in an unconfined compression test, what would be the axial load at failure, if the initial height is 72 mm and the specimen fails at an axial strain of 18 % ?

E4. If a cylindrical specimen of saturated clay of initial height h_0 and initial cross-sectional area A_0 is subjected to an axial load under undrained conditions (either in the unconfined compression or in the triaxial compression test), it will undergo an axial shortening δh and its average cross-sectional area will increase to A , but its volume will remain unchanged. By equating the initial volume of the specimen to its intermediate volume, prove the relationship

$$A = A_o \left(\frac{1}{1 - \varepsilon_a} \right)$$

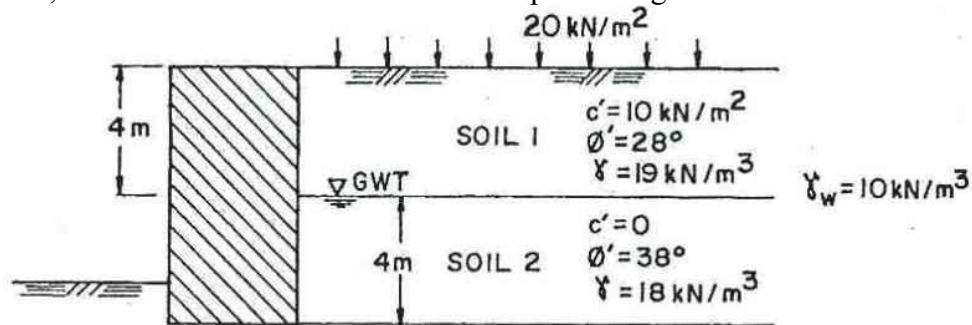
where ε_a = axial strain.

E5. In an unconfined compression test on a saturated clay, the maximum proving ring dial reading recorded was 240×10^{-3} mm, when the axial shortening of the specimen, having an initial height of 70 mm and an initial diameter of 36 mm, was 12 mm. If the calibration factor of the proving ring was $3.2 \text{ N}/10^{-3} \text{ mm}$, calculate the unconfined compressive strength and the undrained shear strength of the clay.

E6. The total vertical stress at a point P in a nearly saturated clay is 400 kPa and the pore pressure at P is 50 kPa. The pore pressure coefficients A and B of the clay have been measured as 0.4 and 0.8 respectively. Assuming the principal stress directions to remain horizontal and vertical, calculate the available shear strength on a horizontal plane at P when the load due to a structure results in an increase in total vertical stress at P of 80 kPa and an increase in total horizontal stress at P of 60 kPa. The shear strength parameters of the clay in terms of effective stress are $c' = 8 \text{ kPa}$; $\phi' = 24^\circ$.

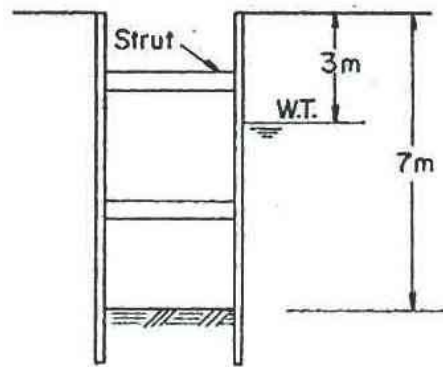
(f) LATERAL EARTH PRESSURE

F1. The depth of soil behind a retaining wall is 8 m, and the soil properties are given in the figure below. A surcharge of 20 kN/m^2 is applied on the horizontal ground surface. Using the Rankine theory, plot the active pressure distribution behind the wall, and determine the total active thrust per m length of the wall.



F2. A 7-m deep trench to be dug in a uniform, silty sand is supported by steel-sheet piling driven on either side of the trench, and supported by struts as shown. Such a system is normally in equilibrium if the total compression in the struts balances the active earth thrust, but if the compression in the struts continues to be increased, the sides may fail in passive resistance. The water table lies 3 m below the ground surface. The bulk unit weight of the soil is 16 kN/m^3 above and 18 kN/m^3 below the water table; the effective angle of friction $\phi' = 35^\circ$ and cohesion $c' = 12 \text{ kPa}$.

Plot the passive pressure distribution, and calculate the resultant compressive force in the struts per m length of the trench, for the sides to fail in passive resistance.



E1)

$$\text{area of shear box} = \frac{\pi * 0.063^2}{4} = 3.117 * 10^{-3} m^2$$

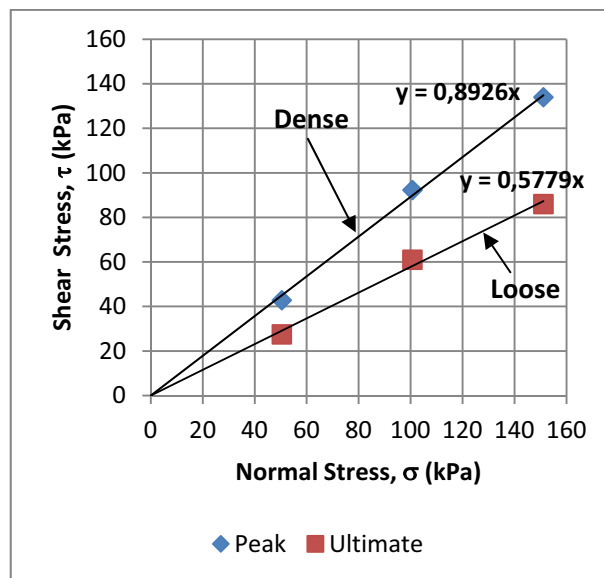
$$\text{normal stress} = \sigma = P * \frac{9.81}{3.117 * 10^{-3}} = 3147.2 P \frac{N}{m^2} = 3.147 P \text{ kN/m}^2$$

$$P = \text{normal load (kgf)}$$

$$\text{shear stress} = \tau = \frac{s}{3.117} * 10^{-3} = 0.3208 * s \text{ kN/m}^2$$

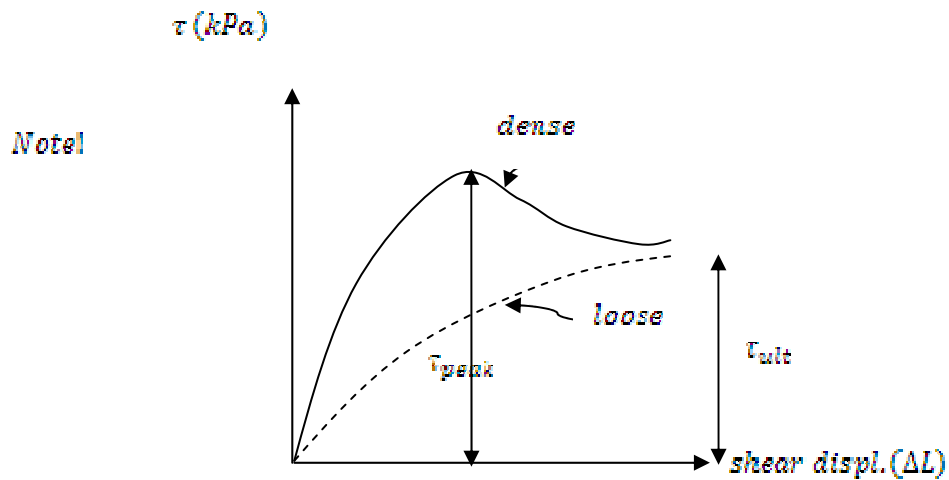
S: shear load (N)

Normal Load	Normal Stress	Peak Shear Load	Ultimate Shear Load	Peak Shear Stress	Ultimate Shear Stress
P(kgf) given	$\sigma(\text{kN/m}^2)$	(given)	(given)	$\tau(\text{kN/m}^2)$	$\tau(\text{kN/m}^2)$
16	50.4	133.4	85.7	42.8	27.5
32	100.7	287.7	190.1	92.3	61
48	151.1	417.7	268.1	134	86



$$\phi_{\text{dense}} = \arctan(0.8926) = 42^\circ$$

$$\phi_{\text{loose}} = \arctan(0.5779) = 30^\circ$$



E2)

$$\text{center of Mohr circles} = \frac{\sigma_1' + \sigma_3'}{2} = \frac{(\sigma_1 - u) + (\sigma_3 - u)}{2} = \frac{\sigma_1 + \sigma_3}{2} - u$$

$$\text{radius of Mohr circles} = \frac{\sigma_1' - \sigma_3'}{2} = \frac{(\sigma_1 - u) - (\sigma_3 - u)}{2} = \frac{\sigma_1 - \sigma_3}{2}$$

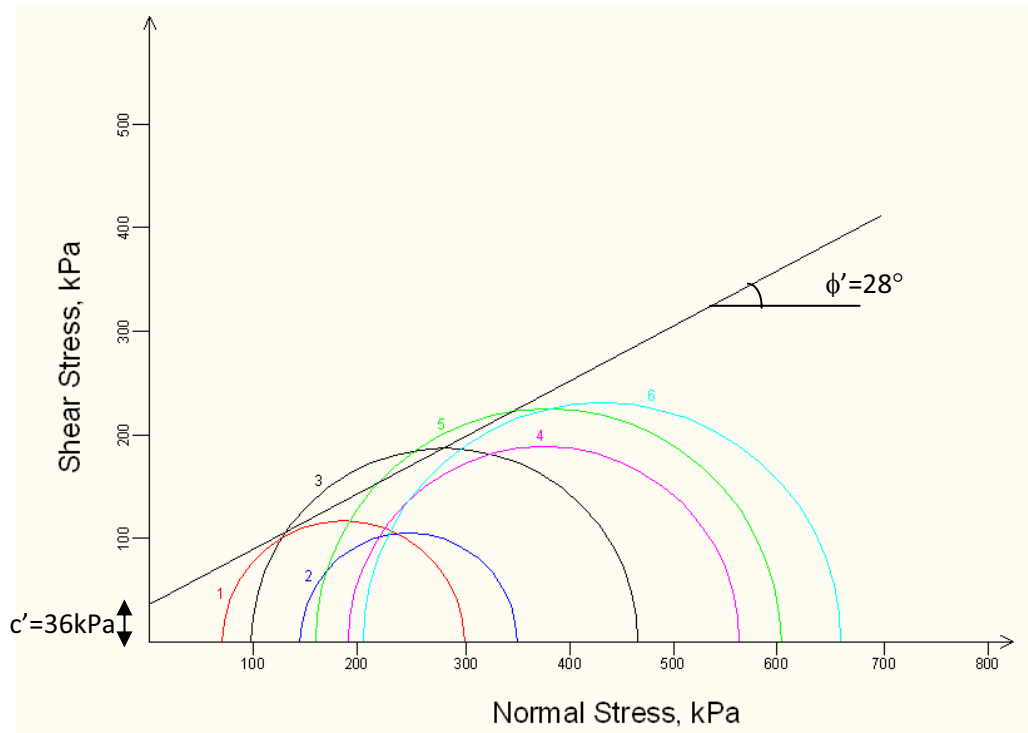
Test no	σ_3	$\sigma_1 - \sigma_3$	u	σ_1	σ_3'	σ_1'	$\frac{\sigma_1' + \sigma_3'}{2}$	$\frac{\sigma_1' - \sigma_3'}{2}$
1	53	234	-17	287	70	304	187	117
2	220	210	75	430	145	355	250	105
3	81	374	-17	455	98	472	285	187
4	178	378	-14	556	192	570	381	189
5	158	450	-2	608	160	610	385	225
6	201	462	-5	663	206	668	437	231

\swarrow cell pressure \downarrow deviator stress \swarrow pore pressure \downarrow center \downarrow radius

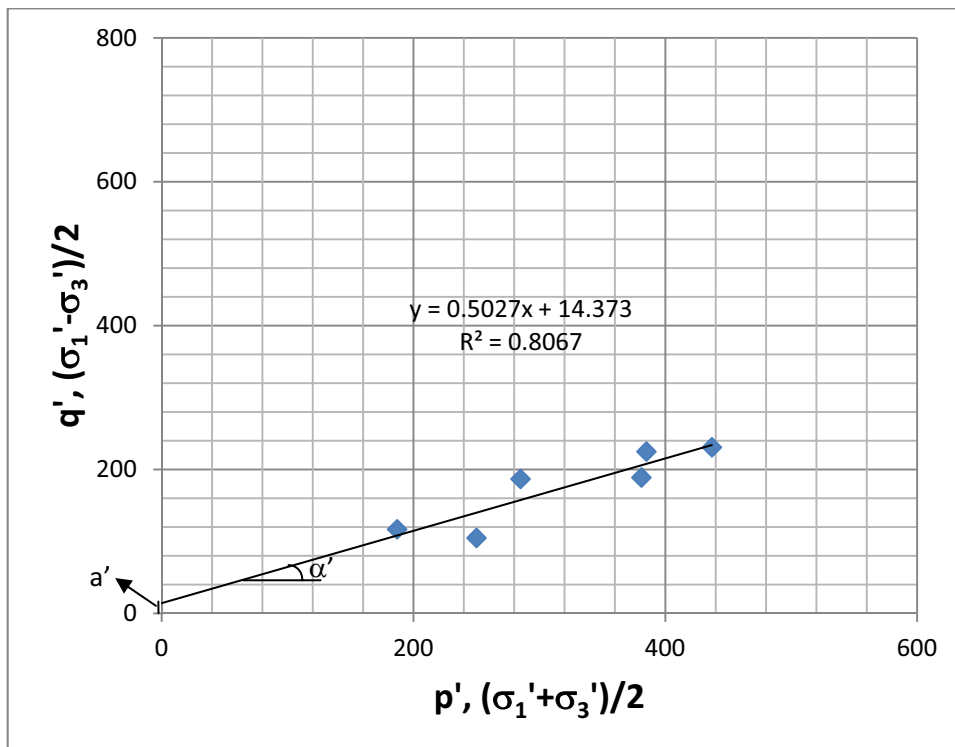
a. By drawing the mean tangent to the mohr circles

$$c' = 36 \text{ kPa}$$

$$\phi' = 28^\circ$$



b. By applying regression to the stress points, q-p plot



$$a' = 14.37 \text{ kPa}$$

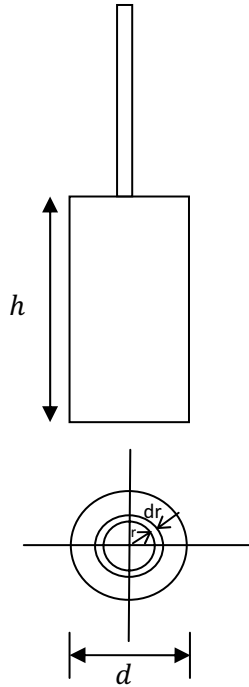
The assignment solutions are prepared by: Gizem CAN, Ezgi EROĞLU, Makbule ILGAÇ within the scope of undergraduate student assistantship programme

so

$$\phi' = \sin^{-1}(\tan \alpha') = 30.2^\circ \quad \text{and} \quad c' = \frac{a'}{\cos \phi'} = \frac{14.37}{\cos 30.6} = 16.6 \text{ kPa}$$

Method (b) is preferable because it enables use of statistical curve fitting techniques like linear regression.

E3)



a. Torque due to shear force on cylindrical surface:

$$\begin{aligned} T_1 &= \text{area} * c_u * \text{moment arm} \\ &= (\pi d h) * c_u * \frac{d}{2} \\ &= \frac{\pi d^2 h}{2} * c_u \end{aligned}$$

Torque on bottom end area:

$$\begin{aligned} T_2 &= \text{area} * c_u * \text{mom. arm} \\ &= \int_0^{d/2} 2\pi r dr c_u r \\ &= \frac{\pi d^3}{12} c_u \end{aligned}$$

Total torque: $T = T_1 + 2T_2 = \pi d^2 h c_u + 2 \frac{\pi d^3}{12} c_u$

$$T = \pi c_u \left(\frac{d^2 h}{2} + \frac{d^3}{6} \right)$$

b. $d = 0.075 \text{ m}$ $h = 0.150 \text{ m}$ $T = 50 \text{ Nm} = 0.050 \text{ kNm (initial)}$

$$T = 19 \text{ Nm} = 0.019 \text{ kNm (remoulded)}$$

$$T = \pi c_u \left(\frac{0.075^2 * 0.15}{2} + \frac{0.075^3}{6} \right) = 1.546 * 10^{-3} c_u$$

Initial, undisturbed, undrained shear strength $0.050 = 1.546 * 10^{-3} * c_u$

$$c_u = 32.34 \text{ kN/m}^2$$

Disturbed, remoulded shear strength $0.019 = 1.546 * 10^{-3} * c_{ur}$

$$c_{ur} = 12.29 \text{ kN/m}^2$$

$$\text{sensitivity} = \frac{\text{undisturbed strength}}{\text{remoulded strength}} = \frac{c_u}{c_{ur}} = \frac{32.34}{12.29} = 2.63 \text{ insensitive}$$

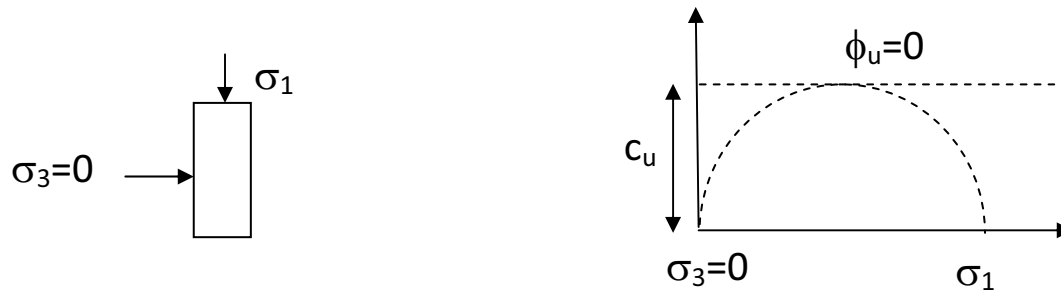
c.

$$d = 36 \text{ mm} \quad h = 72 \text{ mm}$$

Unconfined comp. test

axial load at failure ?

Failure axial strain=18%



$$\text{in unconfined comp. test: } \frac{q_u}{2} = \frac{\sigma_1}{2} = c_u$$

from part b: undisturbed, undrained shear strength : $c_u = 32.34 \text{ kPa}$

$$\sigma_1 = 2 * 32.34 = 64.68 \text{ kPa}$$

$$\sigma_1 = \frac{\text{axial load}}{\text{corrected area}} = \frac{P}{A}$$

$$A = \frac{A_0}{1 - \frac{\Delta l}{l_0}} = \frac{\frac{\pi * 0.036^2}{4}}{1 - 0.18} = 1.24 * 10^{-3} \text{ m}^2$$

$$P = \sigma_1 * A = 64.68 * 1.24 * 10^{-3} = 0.0802 \text{ kN} = 80.2 \text{ N}$$

E4) initial volume = intermediate volume $\rightarrow A_0 * L_0 = A * L$

$$A = A_0 * \left(\frac{L_0}{L} \right)$$

$$\varepsilon_a = \frac{\Delta L}{L_0} = \frac{L_0 - L}{L_0} = 1 - \frac{L}{L_0} \rightarrow \frac{L}{L_0} = \frac{1}{1 - \varepsilon_a}$$

($\varepsilon_a \rightarrow$ Axial strain)

$$\rightarrow A = A_0 * \left(\frac{L_0}{L} \right) = A_0 \frac{1}{1 - \varepsilon_a}$$

E5) $h_0 = 70 \text{ mm}$

Unconfined compressive strength=?

$d_0 = 36 \text{ mm}$

Undrained shear strength=?

$\Delta h = 12 \text{ mm}$

$$C_p = \frac{3.2N}{10^{-3}mm}$$

$$\varepsilon = \frac{\Delta h}{h_0} = \frac{12}{70} = 0.17 = 17\%$$

$$P_{axial} = (240 * 10^{-3}mm) * \frac{3.2N}{10^{-3}mm} = 768N$$

$$A = A_0 \left(\frac{1}{1 - \varepsilon} \right) = \frac{\pi * (0.036)^2}{4} * \frac{1}{1 - 0.17} = 1.226 * 10^{-3}m^2$$

$$\sigma_1 = \frac{P}{A} = \frac{0.768kN}{1.226 * 10^{-3}m^2} = 626.4 \text{ kPa} \quad (\text{unconfined compressive strength})$$

$$\frac{\sigma_1}{2} = C_u = 313.2 \text{ kPa} \quad (\text{undrained shear strength})$$

E6) Saturated clay

$$\sigma_v = 400 \text{ kPa} \quad A = 0.4$$

$$u = 50 \text{ kPa} \quad B = 0.8$$

$$\Delta\sigma_1 = 80 \text{ kPa} \quad c' = 8 \text{ kPa}$$

$$\Delta\sigma_3 = 60 \text{ kPa} \quad \phi' = 24^\circ$$

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)] = 54.4 \text{ kPa}$$

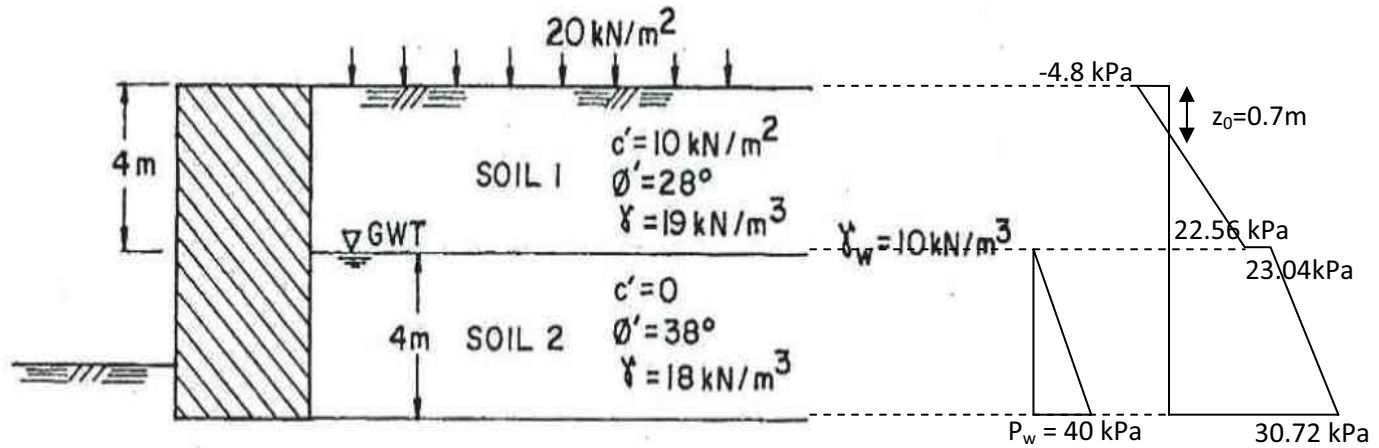
$$u_t = u + \Delta u = 50 + 54.4 = 104.4 \text{ kPa}$$

$$\sigma_v = 400 + 80 = 480$$

$$\sigma'_1 = 480 - 104.4 = 375.6 \text{ kPa}$$

$$\tau = c' + \sigma'_1 \tan \phi' = 8 + 375.6 \times \tan 24^\circ = 175.2 \text{ kPa}$$

F1)



Not to scale

$$K_A = \frac{1 - \sin \phi}{1 + \sin \phi}$$

$$K_{A1} = 0.36$$

$$K_{A2} = 0.24$$

$$P_A = K_A * (\gamma * z + q) - 2 * c * \sqrt{K_A}$$

Soil	Depth (m)	Active Pressure (kN/m ²)
1	0	$(0+20)*0.36-2*10*\sqrt{0.36}=-4.8$
1	4	$(20+4*19)*0.36-2*10*\sqrt{0.36}=22.56$
2	4	$(20+4*19)*0.24-0=23.04$
2	8	$((20+4*19)+4*(18-10))*0.24=30.72$

$$\text{Depth of tension cracks: } P_A = K_A * (\gamma * z + q) - 2 * c * \sqrt{K_{A1}}$$

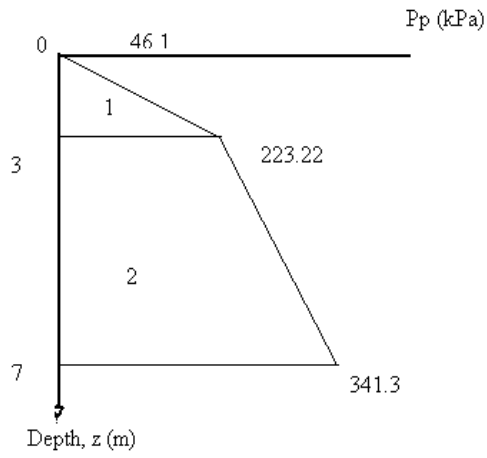
$$\text{For } P_A = 0 \quad z_0 = \frac{2 * c * \sqrt{K_{A1}} - K_{A1} * q}{\gamma * K_A} \rightarrow z_0 = 0.7m$$

“or z_0 can be calculated from the pressure diagram.”

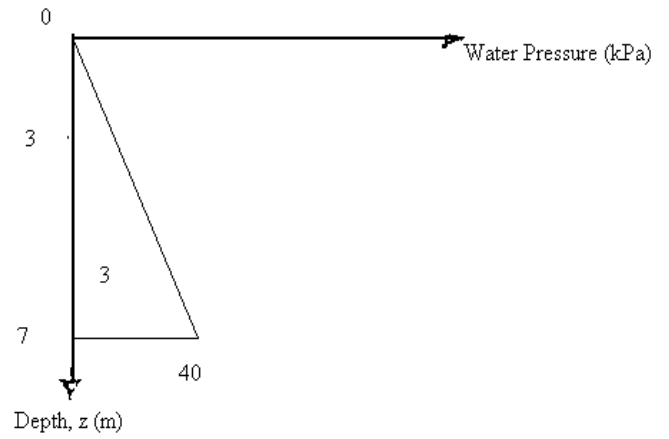
Total active thrust:

$$P_{A\text{total}} = P_A + P_w$$

$$P_{A\text{total}} = \frac{1}{2} * (22.56 * 3.3) + 4 * 23.04 + \frac{1}{2} * (4 * (30.72 - 23.04)) + \frac{1}{2} * (4 * 40) = 224.74 \text{ kN/m}$$



Effective Pressure Distribution



Water Pressure Distribution

$$total\ passive\ resistance = Area(1) + Area(2) + Area(3)$$

$$3 \times \frac{46.10 + 223.22}{2} + 4 \times \frac{223.22 + 341.3}{2} + 4 \times \frac{40}{2} = 1613\ kN/m$$