CE 382 Reinforced Concrete Fundamentals

Pure Bending - Analysis of RC Sections

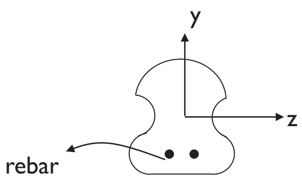
Basic Behavior of RC

- Non-linear
- Inelastic
- Time dependent

- Force equilibrium
- Geometric compatibility
- Non-linear σ-ϵ relationship for the materials

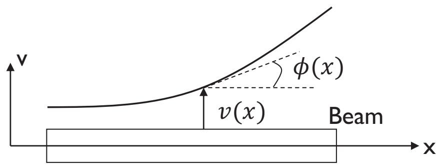
independent of material properties

Flexure: bending behavior of beams and slabs



- Assumptions
  - Axial load is zero (can be nonzero for columns)
  - Moment is applied about z axis
  - y and z are principle axes (I_y and I_z are min and max)
  - Plane sections remain plane (no shear deformations)

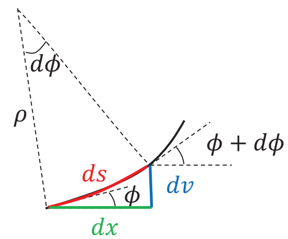
Kinematics of Beam Deflection



v: deflection
ϕ: slope
$\mathcal{K}$: curvature
ρ: radius of curvature

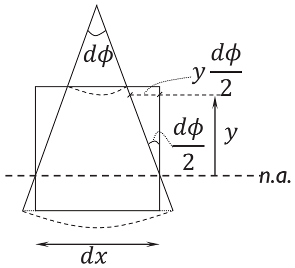
- $\phi = \frac{dv}{dx}$
- $ds = \rho \cdot d\phi$
- $\mathcal{K} = \frac{1}{\rho} = \frac{d\phi}{ds} \approx \frac{d\phi}{dx} = \frac{d^2v}{dx^2}$

for small deformations approximate



Strain variation across depth

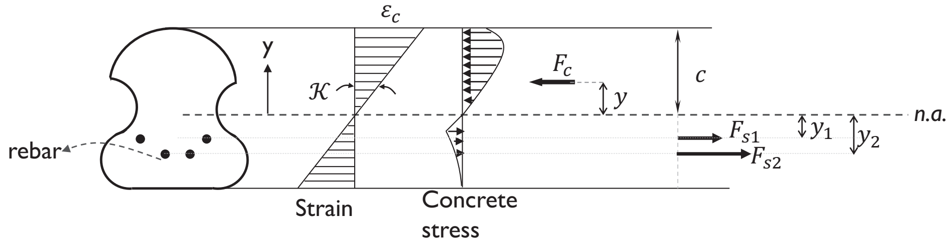
- Plane sections remain plane!



- $\varepsilon = \frac{change\ in\ length}{original\ length}$
- $\varepsilon = \frac{2y\frac{d\phi}{2}}{dx} = y\frac{d\phi}{dx} = y\mathcal{K}$
- Strain at any location along depth is proportional to y

Equilibrium conditions

- $\underbrace{v \to \phi \to \mathcal{K}}_{\text{Beam kinematics}} \to \underbrace{\varepsilon \to \sigma}_{\text{Material behavior}} \to \underbrace{N\ \&\ M}_{\text{Equilibrium}}$



Force Equilibrium

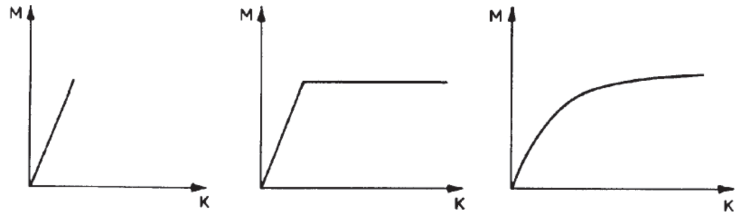
- $F_c + \sum F_s = 0$ (= N for non-zero axial load)
- $\int_{A_c} \sigma_c dA_c + \sum_{i=1}^{n} \sigma_{si} A_{si} = 0$
- Special Case: elastic material behavior
  - $\int \sigma dA = \int E\varepsilon dA = E\mathcal{K}\underbrace{\int y dA}_{=0} = 0$
  - → the first moment of the cross-sectional area about the neutral axis must vanish
  - → the neutral axis should pass through the centroid of the cross-section

Moment Equilibrium

- $\int_{A_c} \sigma_c y dA_c + \sum_{i=1}^{n} \sigma_{si} A_{si} y_i = M$
- If ε_{cu} is concrete top fiber maximum compressive strain
  - $\mathcal{K} = \frac{\varepsilon_{cu}}{c}$
- Special Case: elastic material behavior
  - $M = \int \sigma y dA = E\varepsilon \int y dA = E\mathcal{K} \int y^2 dA = EI\mathcal{K}$
  - $\mathcal{K} = \frac{M}{EI} \quad \to \quad \frac{\varepsilon}{y} = \frac{M}{EI} \quad \to \quad \frac{\sigma}{Ey} = \frac{M}{EI} \quad \to \quad \sigma = \frac{My}{I}$

Structural Analysis

- Linear Analysis
- Plastic or Limit Analysis
- Non-linear Analysis

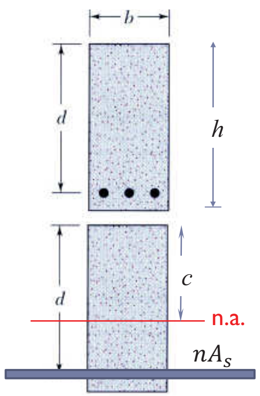


(a) (b) (c)

Design of a cross-section

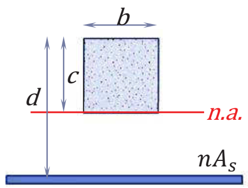
- Working Stress Design
  - Elastic Design
  - Both steel & concrete assumed linearly elastic
  - Steel area transformed into equivalent concrete area
  - $n = \frac{E_s}{E_c}$ $\qquad n \approx 10 - 15$
  - Stresses in steel and concrete have to be computed
  - Stresses are compared with «the allowable stresses»
  - Stresses in steel & concrete change significantly due to time dependent deformations

Elastic Behavior



- Prior to cracking of concrete, behavior is similar to an elastic composite beam.
- Note that concrete area is $(bh - A_s)$.We can assume it as bh while taking the steel area $(n-1)A_s$ for the transformed area.
- $\sigma_c = \frac{My}{I}$ & $\sigma_s = \frac{My}{I}n$ & $\mathcal{K} = \frac{M}{EI}$
- From first moment of area
- $bh\left(c - \frac{h}{2}\right) + (n-1)A_s(c-d) = 0$
- $\rightarrow \quad c = \frac{\frac{bh^2}{2} + (n-1)A_s d}{bh + (n-1)A_s}$
- $I = \frac{bh^3}{12} + bh\left(\frac{h}{2} - c\right)^2 + (n-1)A_s(d-c)^2$

Elastic-Cracked Behavior



- Upon cracking
  - Steel bars carry entire tensile load below the neutral axis
  - Concrete carries compressive load above the n.a.
- $\sigma_c = \frac{My}{I_{cr}}$ & $\sigma_s = \frac{My}{I_{cr}}n$ & $\mathcal{K} = \frac{M}{EI_{cr}}$
- $bc\frac{c}{2} + nA_s(c-d) = 0$
- $\rightarrow \quad c = \frac{-nA_s + \sqrt{(nA_s)^2 + 2bnA_s d}}{b}$
- $\rho = \frac{A_s}{bd}$
- $\rightarrow \quad c = \left(\sqrt{(\rho n)^2 + 2\rho n} - \rho n\right)d$
- $I_{cr} = \frac{bc^3}{3} + nA_s(d-c)^2$

Design of a cross-section

- Limit State Design
  - The Ultimate Limit States
    - correspond to the maximum load-carrying capacity
    - safety against failure
  - The Serviceability Limit States
    - related to the criteria describing the satisfactory performance under service loads
    - check for deformations, vibration & cracking

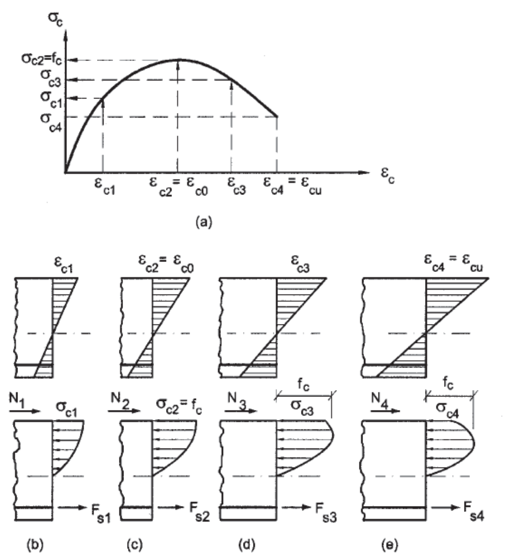
Ultimate Strength Theory

- Use nonlinear behavior of steel and concrete
- Compute the carrying capacity of the section at the ultimate stage
  - Equilibrium
  - Compatibility
  - Force-deformation (or actual σ–ε relationship)

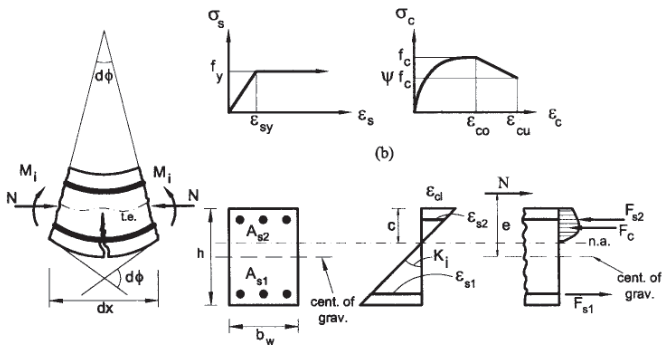
Ultimate Strength Theory

- Assumptions
  - Plane sections remain plane after bending
  - Concrete can not take any tension
  - Perfect bond between steel & concrete
  - Elasto-plastic σ–ε for reinforcing steel, $\sigma_s = E_s \varepsilon_s \leq f_{sy}$
  - Maximum strain in the extreme fiber of concrete in compression is ε_{cu}
  - Concrete stress distribution in the compression zone is assumed the same as the σ–ε from uniaxially loaded specimens

Stages of Loading Beyond Linear Range



Curvature

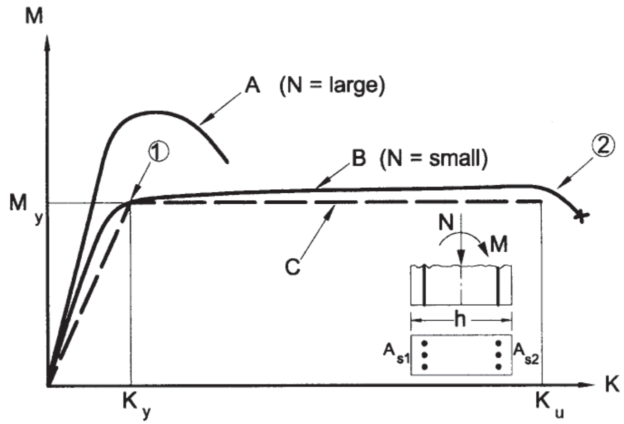


$$\frac{d^2y}{dx^2} = \frac{1}{\rho} = \mathcal{K} = \frac{d\phi}{dx}$$

$$\mathcal{K} = \frac{\varepsilon_x}{y} = \frac{\varepsilon_{ci}}{c}$$

Moment – Curvature relationship

- Curve A → high axial load
- Curve B → very low axial load



- $M - \mathcal{K}$ curve is nonlinear
- curve changes significantly with the level of axial load
- After yielding of tension steel, $\mathcal{K}_y$, curvature increases without any increase in the moment → **Plastic Hinge**

Moment – Curvature relationship

- *Classical hinge*: section rotates under zero moment
- *Plastic hinge*: rotation takes place under a constant moment
- *Ductility*: the capability of undergoing large deformations without a $\underbrace{\text{significant reduction}}_{\sim 15\%}$ in the strength.

- $\textit{curvature ductility ratio} = \frac{\mathcal{K}_u}{\mathcal{K}_y}$ related to cross-sectional properties & σ–ε of materials
- $\textit{displacement ductility}$ related to member properties

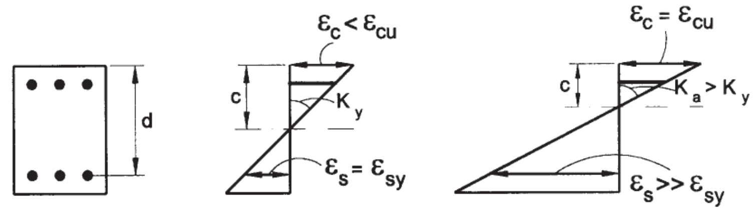
- Energy dissipation capacity → area under $M - \mathcal{K}$ diagram

Test results

Earthquake performance

Types of Failure

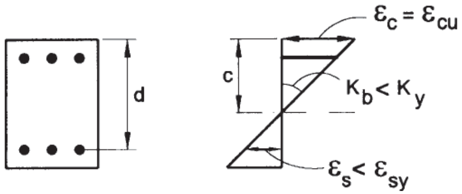
- Tension Failure



  - Steel reinforcement yields in tension, prior to the crushing of concrete → ductile behavior
  - Considerable deformation before failure
  - Ductility depends to the properties of steel
  - Desirable failure; warning before failure, not sudden

Types of Failure

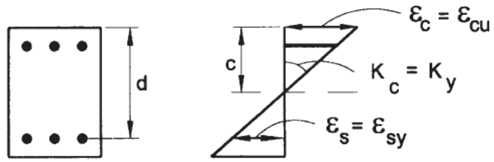
- Compression Failure



  - Concrete that is in compression crushes before the steel in tension yields
  - Brittle & sudden failure
  - Crushing strain of concrete is low
  - Energy dissipation capacity is low

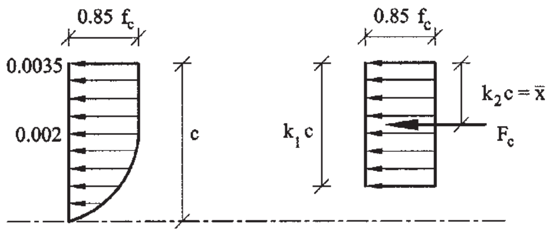
Types of Failure

- Balanced Failure



  - Crushing of concrete in the extreme fiber and yielding of the tension steel occur simultaneously

Rectangular Stress Block



(a) CEB (b) ACI (TS - 500)

Concrete Class	k_1
C12, C16, C20, C25	0.85
C30	0.82
C35	0.79
C40	0.76
C45	0.73
C50	0.70

- Rectangular stress block is the simplest block
- Have the same area and centroid with the real stress distribution

Design Codes

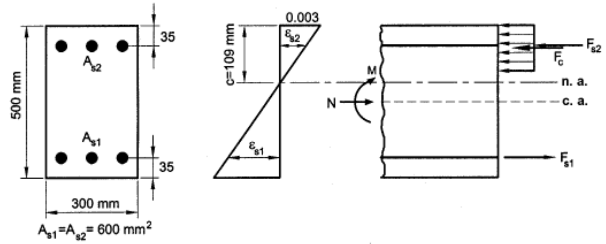
- Legal documents
- Represent the minimum requirements for obtaining safe structures
- Represent compromises rather than the best solutions
- Design engineers
  - should follow the current research from technical journals
  - remain up to date
  - able to understand the changes made in the design codes

Design

- Calculation accuracy
  - Approximate structural analysis methods
  - Some variables neglected
  - Reinforced concrete is nonhomogeneous, nonlinear, inelastic, time dependent
  - Variations in strength
  - high degree of precision in design computations is unnecessary
- Detailing
- Supervision

Example 1

- Compute the axial load and moment for the cross-section shown below, $f_c = 25\ MPa$ & $f_y = 420\ MPa$



(a) Cross-section (b) Strain Distribution (c) Internal Forces

$$\varepsilon_{sy} = \frac{420}{200000} = 0.0021$$

- $\dfrac{0.003}{109} = \dfrac{\varepsilon_{s1}}{500-109-35} \Rightarrow \varepsilon_{s1} = 0.00980$
- $\dfrac{0.003}{109} = \dfrac{\varepsilon_{s2}}{109-35} \Rightarrow \varepsilon_{s2} = 0.00204$

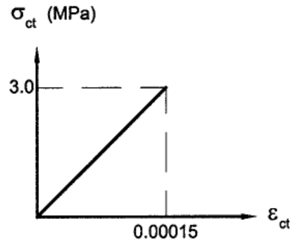
Example 1

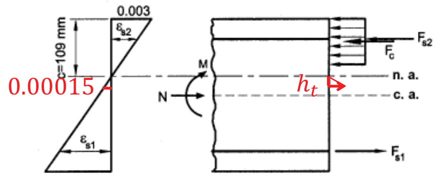
- $\varepsilon_{s1} > \varepsilon_{sy} \Rightarrow \quad F_{s1} = f_y A_{s1} = 420 \times 600 = 252\ kN$
- $F_{s2} = \sigma_{s2} A_{s2} = E_s \varepsilon_{s2} A_{s2} = \underbrace{200000 \times 0.00204}_{408\ MPa} \times 600$
- $F_{s2} = 244.8\ kN$
- $F_c = k_1 c \times 0.85 f_c \times b = 0.85 \times 109 \times 0.85 \times 25 \times 300$
- $F_c = 590.6\ kN$
- $N = F_{s2} + F_c - F_{s1} = 583.4\ kN$
- $M = F_c \left(\frac{h}{2} - \frac{k_1 c}{2}\right) + F_{s2}\left(\frac{h}{2} - 35\right) + F_{s1}\left(\frac{h}{2} - 35\right)$
- Moment about centroidal axis
- $M = 227.1\ kNm$
- At the ultimate stage $\varepsilon_{s1} > \varepsilon_{sy} \Rightarrow$ TENSION FAILURE

Example 2

- Solve the previous example by taking into account the tensile strength of concrete

$$\frac{0.003}{109} = \frac{0.0015}{h_t} \Rightarrow h_t = 5.45\ mm$$

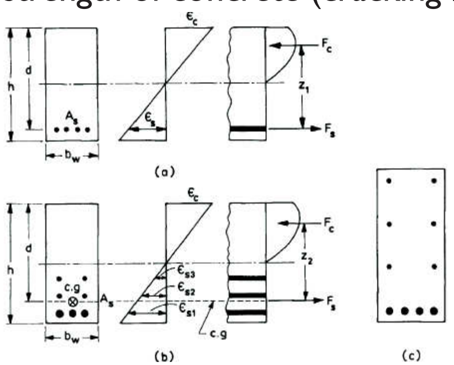




- $F_{ct} = \frac{\sigma_{ct} h_t}{2} b = \frac{3 \times 5.45}{2} 300 = 2.4\ kN$
- $N = F_{s2} + F_c - F_{s1} - F_{ct} = 581\ kN \quad 0.4\% \searrow$
- $M = 227.1 - 2.4 \frac{\left(250 - 109 - \frac{2}{3} 5.45\right)}{1000} = 226.8\ kNm \quad 0.13\% \searrow$

Ultimate Strength of Members Subjected to Flexure

- Beams & Slabs → flexural moment & shear force
  also axial load & torsional moment
- Plain concrete under flexure fails due to low tensile strength of concrete (cracking load)



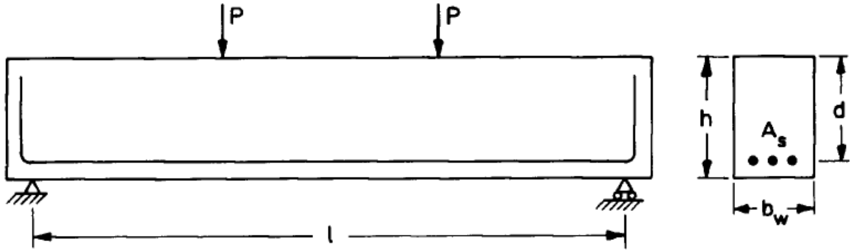
Neglect tensile strength of concrete
Use steel for tensile forces
Use web reinforcement if necessary
Use multi-layer tension steel for dense reinforcement

Members under Flexure

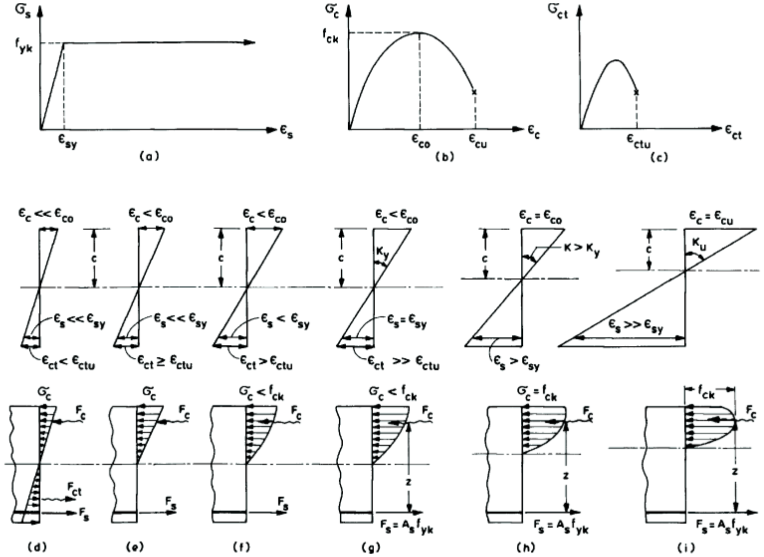
- Clear cover
  - Fire protection
  - Corrosion protection
  - Bond
- Steel in flexural member cannot prevent cracking but prevent sudden brittle failure when cracking occurs and keeps crack width small
- Stress level at cracking is almost the same for plain & RC beams

Flexural Behavior

- Rectangular beam
- Reinforced only in tension zone
- Small amount of reinforcement

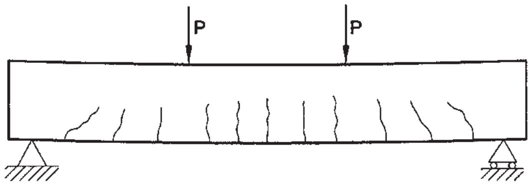


Flexural Behavior

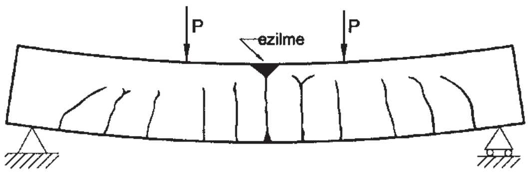


Flexural Behavior

- Beam looses its serviceability after yielding due to excessive deformations



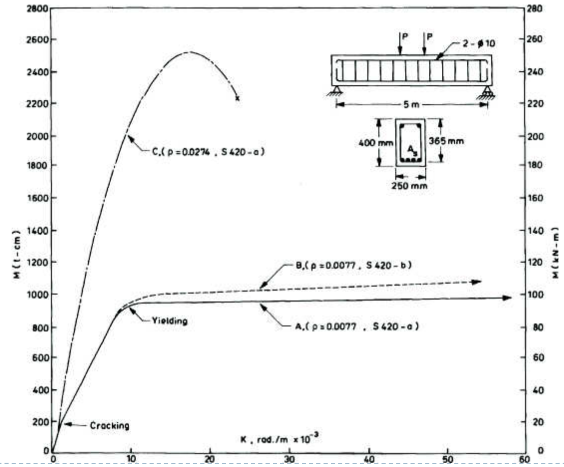
(a) Kiriş M1, akma konumunda



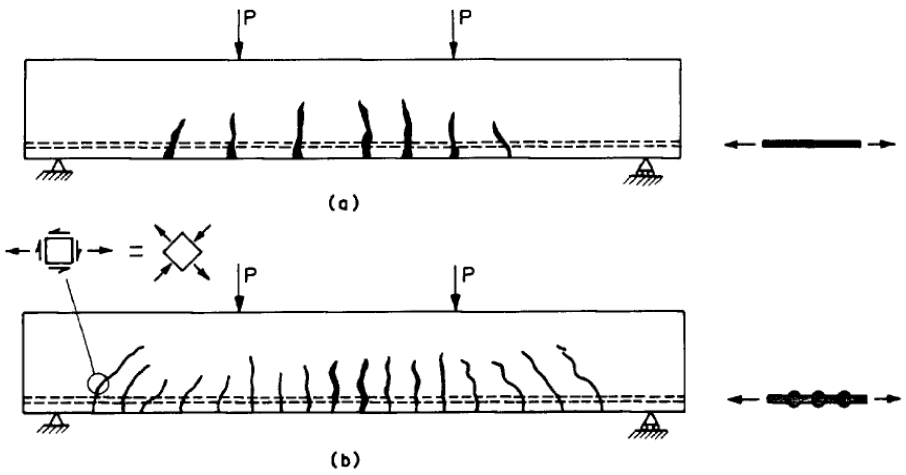
(b) Kiriş M1, taşıma gücü konumunda

Under- & over-reinforced beam behavior

- Tension failure → underreinforced beams
- Compression failure → overreinforced beams

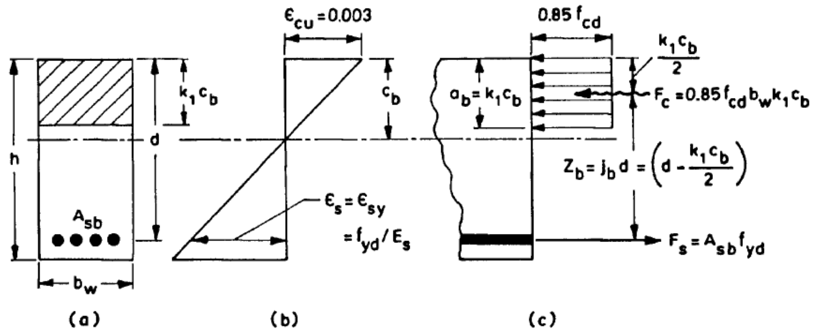


Plain & deformed bar difference on beam behavior



Analysis of Beams

- Balanced Case
  - ε_{cu} & ε_{sy} are reached simultaneously
  - Brittle failure; prohibited in the design codes
  - Rectangular beams reinforced for tension only



Analysis of Beams

- Balanced Case
  - Two equilibrium equations:
    - $\sum F = 0 \qquad A_{sb} f_{yd} - 0.85 f_{cd} b_w k_1 c_b = 0$
    - $\sum M = 0 \qquad M_b = A_{sb} f_{yd} (j_b d) = A_{sb} f_{yd} \left(d - \frac{k_1 c_b}{2}\right)$
  - Compatibility:
    - $\frac{\varepsilon_{sy}}{d - c_b} = \frac{0.003}{c_b}$ or $\frac{c_b}{d} = \frac{0.003}{0.003 + \varepsilon_{sy}}$
  - Stress-strain relationship:
    - $\varepsilon_{sy} = \frac{f_{yd}}{E_s}$
  - $\Longrightarrow \frac{c_b}{d} = \frac{0.003 E_s}{0.003 E_s + f_{yd}}$

Analysis of Beams

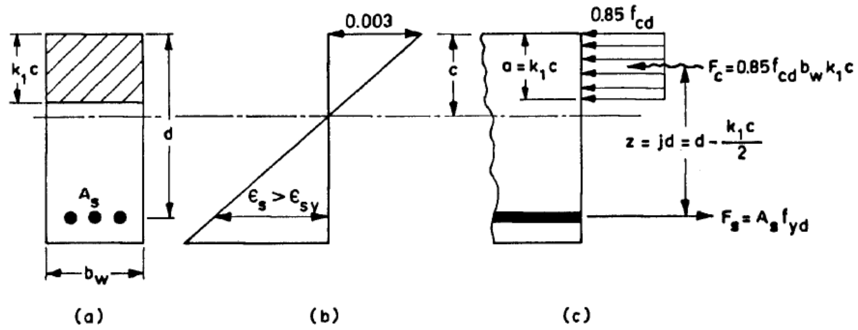
- Balanced Case
  - Divide force equilibrium by $b_w d$ & introduce $\rho_b = \frac{A_{sb}}{b_w d}$
  - $\Longrightarrow \rho_b = \frac{0.85 f_{cd}}{f_{yd}} k_1 \frac{c_b}{d}$
  - Divide moment equilibrium by $b_w d^2$
  - $\Longrightarrow \frac{b_w d^2}{M_b} = K_b = \frac{1}{\rho_b f_{yd} j_b}$
  - $j_b = 1 - \frac{k_1 c_b}{2d}$
- If $\rho < \rho_b$ or $K > K_b \Longrightarrow$ tension failure; underreinforced beam

Balanced Values; including $\gamma_{ms} = 1.15$ & $\gamma_{mc} = 1.5$

Class of Steel	Class of Concrete	K_b $\frac{cm^2}{t}$	$\left(\frac{mm^2}{kN}\right)$	$\frac{c_b}{d}$	j_b	ρ_b
S220	C16	24.5	(245)	0.759	0.678	0.0316
″	C18	22.5	(225)	″	″	0.0344
″	C20	20.7	(207)	″	″	0.0373
″	C25	15.8	(158)	″	″	0.0488
S420	C16	27.5	(275)	0.622	0.736	0.0135
″	C18	25.2	(252)	″	″	0.0148
″	C20	23.3	(233)	″	″	0.0160
″	C25	17.8	(178)	″	″	0.0209
″	C30	15.5	(155)	″	0.745	0.0237
″	C35	13.8	(138)	″	0.754	0.0263
″	C40	12.1	(121)	″	0.764	0.0297
″	C45	11.2	(112)	″	0.773	0.0317
″	C50	10.5	(105)	″	0.782	0.0334
S500	C16	28.8	(288)	0.580	0.754	0.0089
″	C18	26.4	(264)	″	″	0.0106
″	C20	24.4	(244)	″	″	0.0125
″	C25	18.6	(186)	″	″	0.0164
″	C30	16.2	(162)	″	0.762	0.0186
″	C35	14.5	(145)	″	0.771	0.0206
″	C40	12.7	(127)	″	0.780	0.0232
″	C45	11.8	(118)	″	0.788	0.0248
″	C50	11.0	(110)	″	0.797	0.0262

Analysis of Beams

- Underreinforced Case
  - $\varepsilon_s > \varepsilon_{sy}$ before failure
  - Rectangular beams reinforced for tension only



Analysis of Beams

- Underreinforced Case
  - Known: dimension of cross-section, steel area, material strength
  - Unknown: c & M_r
  - Force equilibrium:
  - $A_s f_{yd} - 0.85 f_{cd} b_w k_1 c = 0 \quad \Longrightarrow \quad k_1 c = \frac{A_s f_{yd}}{0.85 f_{cd} b_w}$
  - Moment equilibrium:
  - $M_r = A_s f_{yd} jd = A_s f_{yd}\left(d - \frac{k_1 c}{2}\right)$
  - Two equilibrium equation are adequate

Analysis of Beams

- Design Approach
  - Known: dimension of cross-section, material strength, M_d, underreinforced
  - Unknown: c & A_s
  - Force equilibrium:
  - $A_s f_{yd} - 0.85 f_{cd} b_w k_1 c = 0 \quad \Longrightarrow \quad A_s = \frac{0.85 f_{cd} b_w k_1 c}{f_{yd}}$
  - Moment equilibrium:
  - $M_r = A_s f_{yd} jd = \frac{0.85 f_{cd} b_w k_1 c}{f_{yd}} f_{yd}\left(d - \frac{k_1 c}{2}\right)$
  - Solve the quadratic equation above for $k_1 c$ & calculate A_s
  - Check $\varepsilon_s = 0.003 \frac{d-c}{c} \geq \varepsilon_{sy}$

Example 3

- Simply supported beam
  - span 5 m
  - $b_w = 230$ mm & $d = 460$ mm
  - Uniformly distributed load: $g = 10$ kN/m & $q = 5$ kN/m
  - Reinforcement 5ϕ20: $A_s = 5\pi \frac{20^2}{4} = 1570$ mm²
  - $\rho = \frac{A_s}{b_w d} = \frac{1570}{230 \times 460} = 0.0148$
  - Material: C16 ➔ $f_{cd} = \frac{16}{1.5} = 11$ MPa
  - Material: S220 ➔ $f_{yd} = \frac{220}{1.15} = 191$ MPa
  - From table for C16 & S220 ➔ $\rho_b = 0.0316$
  - $\rho < \rho_b$ ➔ underreinforced

Example 3

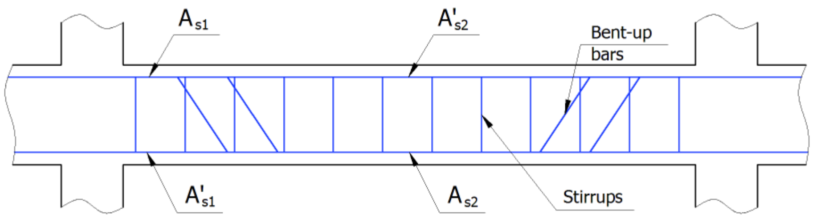
- or assume the beam is under-reinforced and check this assumption later on
  - $k_1 c = \frac{A_s f_{yd}}{0.85 f_{cd} b_w} = \frac{1570 \times 191}{0.85 \times 11 \times 230} = 139.4$ mm
  - $M_r = A_s f_{yd} jd = A_s f_{yd} \left(d - \frac{k_1 c}{2}\right) =$ $1570 \times 191 \left(460 - \frac{139.4}{2}\right) = 117$ kNm
  - External Moment
  - $M_g = \frac{g\ell^2}{8} = \frac{10 \times 5^2}{8} = 31.3$ kNm
  - $M_q = \frac{q\ell^2}{8} = \frac{5 \times 5^2}{8} = 15.6$ kNm
  - $M_d = 1.4 M_g + 1.6 M_q = 68.8$ kNm
  - $M_r > M_d$ ➔ safe

$$c = \frac{139.4}{0.85} = 164 \text{ mm}$$

$$\varepsilon_s = 0.003 \frac{d - c}{c} = 0.003 \frac{460 - 164}{164} = 0.00541$$

$$\varepsilon_{sy} = \frac{191}{200000} = 0.00096$$

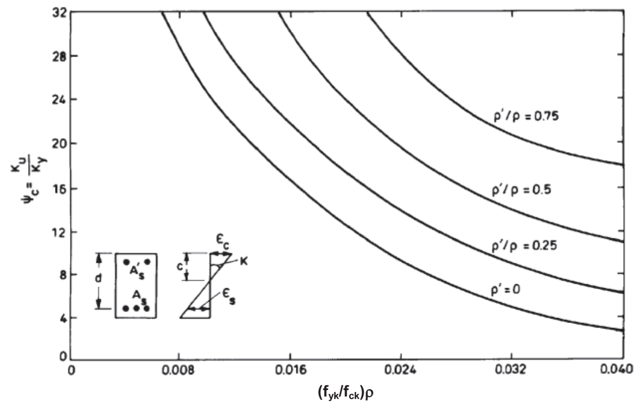
Double Reinforced Rectangular Beams



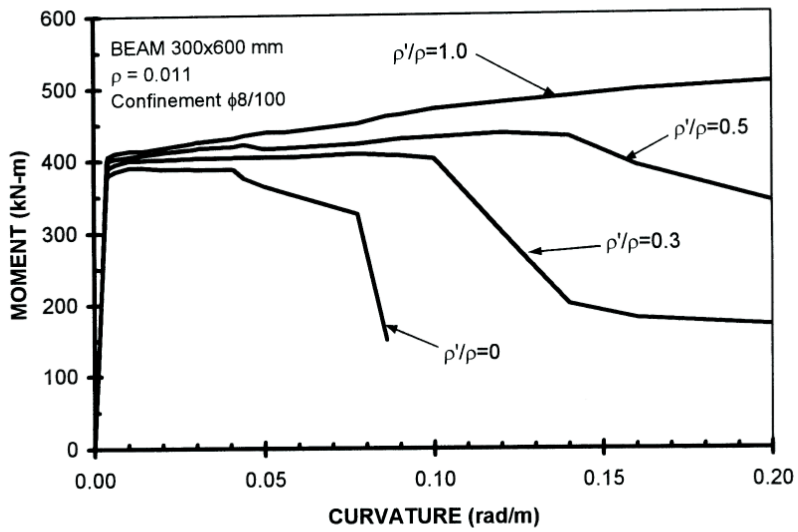
- Hang-up (hanger) bars: A'_{s2}; to hold the stirrups in place; usually 2ϕ12
- TS500: $A'_{s1} \geq \frac{1}{3} A_{s2}$
- TEC2007: 1st & 2nd earthquake zone $A'_{s1} \geq 0.5 A_{s1}$
- TEC2007: 3rd & 4th earthquake zone $A'_{s1} \geq 0.3 A_{s1}$
- TEC2007: $A'_{s2} \geq \frac{1}{4} A_{s1}$

Compression steel

- decreases time dependent deformations; steel in compression zone is not affected by creep.
- increases ductility

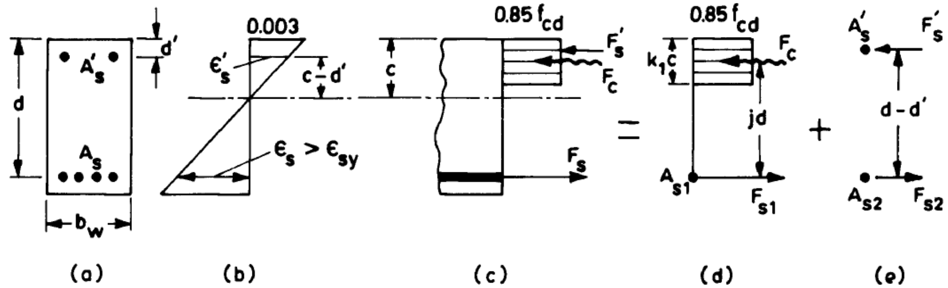


Compression steel



Double Reinforced Rectangular Beams

- $A_s' < A_s$
- Tension steel yielded
- Compression steel may or may not yield
- Moment can be decomposed into two couples



Double Reinforced Rectangular Beams

- Second couple composed of steel → ductile
- First couple $\rho_1 = \frac{A_{s1}}{b_w d} < \rho_b$
- Equilibrium:
  - $\sum F = 0 \qquad A_s f_{yd} - 0.85 f_{cd} b_w k_1 c - A_s' \sigma_s' = 0$
  - $\sum M = 0 \qquad \mathrm{M_r} = 0.85 f_{cd} b_w k_1 c \left(d - \frac{k_1 c}{2}\right) - A_s' \sigma_s' (d - d')$
- Compatibility:
  - $\frac{0.003}{c} = \frac{\varepsilon_s'}{c-d'} \qquad \Rightarrow \qquad \varepsilon_s' = 0.003 \frac{c-d'}{c}$
- Stress-strain relationship:
  - $\sigma_s' = \varepsilon_s' E_s \leq f_{yd}$

Double Reinforced Rectangular Beams

- Known: $A_s, A_s', b_w, d, d', f_{cd}, f_{yd}$
- Unknowns: $c, \sigma_s', \varepsilon_s', M$
- 4 equations & 4 unknowns
- If compression steel yields:
  - $\sigma_s' = f_{yd}$
  - $k_1 c = \frac{(A_s - A_s') f_{yd}}{0.85 f_{cd} b_w}$
  - $M_r = 0.85 f_{cd} b_w k_1 c \left(d - \frac{k_1 c}{2}\right) + A_s' f_{yd} (d - d')$
- Generally compression steel yields!
- Exception: shallow beams

Double Reinforced Rectangular Beams

- If compression steel has yielded
  - $\rho_1 = \rho - \rho'$
  - $(\rho - \rho') < \rho_b$ → underreinforced
- If compression steel has not yielded
  - $\rho_1 = \rho - \rho' \frac{\sigma_s'}{f_{yd}} < \rho_b$ → underreinforced

Example 4

- Given:
  - $b_w = 300\text{ mm}, d = 450\text{ mm}, d' = 30\text{ mm}$
  - C16 → $f_{cd} = 11\text{ MPa}$ & S420 → $f_{yd} = 365\text{ MPa}$
  - $A_s = 1580\text{ mm}^2$ & $A_s' = 520\text{ mm}^2$
- Find: Ultimate Moment M_r
- Assume both tension & compression steel have yielded
- $\varepsilon_{sy} = \frac{365}{200000} = 0.001825$
- $k_1 c = \frac{(A_s - A_s')f_{yd}}{0.85 f_{cd} b_w} = \frac{(1580-520)365}{0.85 \times 11 \times 300} = 138\text{ mm}$
- $c = \frac{138}{0.85} = 162.3\text{ mm}$

Example 4

- $\varepsilon_s' = 0.003 \frac{c-d'}{c} = 0.003 \frac{162.3-30}{162.3} = 0.00244 > \varepsilon_{sy}$ ✓
- $\varepsilon_s = 0.003 \frac{d-c}{c} = 0.003 \frac{450-162.3}{162.3} = 0.00532 > \varepsilon_{sy}$ ✓
- $M_r = 0.85 f_{cd} b_w k_1 c \left(d - \frac{k_1 c}{2}\right) + A_s' f_{yd}(d - d')$

  $= 0.85 \times 11 \times 300 \times 138 \left(450 - \frac{138}{2}\right) + 520 \times 365(450 - 30)$

  $= 227\text{ kNm}$

Example 5

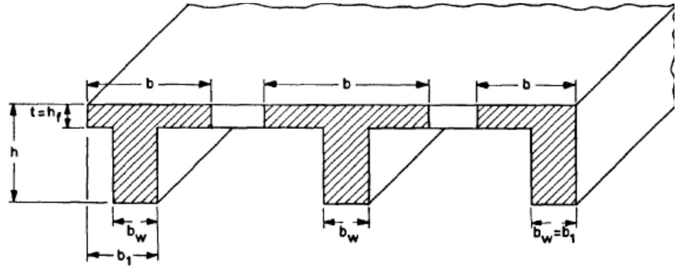
- same as Example 4 but $A_s' = 1200\text{ mm}^2$
- Tension steel has surely yielded
- Assume compression steel has yielded
- $c = \frac{(1580-1200)365}{0.85 \times 0.85 \times 11 \times 300} = 58.4\text{ mm}$
- $\varepsilon_s' = \frac{58.4-30}{58.4} = 0.00146 < \varepsilon_{sy}$ ✗ compression steel has not yielded
- Use general solution
- $\sigma_s' = \varepsilon_s' E_s$ & $\varepsilon_s' = 0.003 \frac{c-d'}{c}$ ⇒ $\sigma_s' = 0.003 E_s \frac{c-d'}{c}$
- $\sigma_s' = 600 \frac{c-30}{c}$

Example 5

- Force equilibrium:
- $A_s f_{yd} - 0.85 f_{cd} b_w k_1 c - A'_s \sigma'_s = 0$

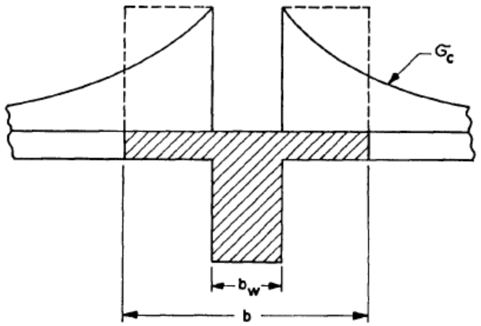
  $1580 \times 365 - 0.85 \times 11 \times 300 \times 0.85c - 1200 \times 600 \dfrac{c-30}{c} = 0$
- $c^2 + 60c - 9060 = 0$
- $c = 70 \text{ mm}$
- $\varepsilon'_s = 0.003 \frac{70-30}{70} = 0.00171 \quad \Rightarrow \quad \sigma'_s = 342 \text{ Mpa}$
- $M_r = 0.85 f_{cd} b_w k_1 c \left(d - \frac{k_1 c}{2}\right) + A'_s \sigma'_s (d - d')$
- $M_r = 243 \text{ kNm}$

Flanged Sections



- Beams & slabs cast monolithically
- T or L beams
- High compression area; generally no need for compression steel
- Cetroid of compression are shift up → Moment Capacity⇗

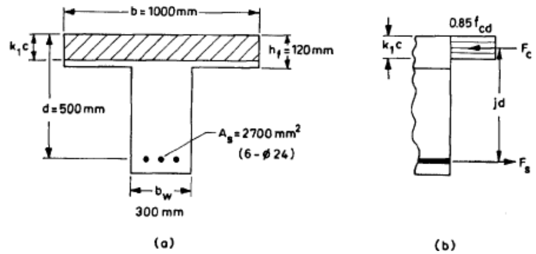
Effective Flange Width



- Generally $k_1 c < t$ → analyze as rectangle
- t: flange thickness

Example 6

- Given:

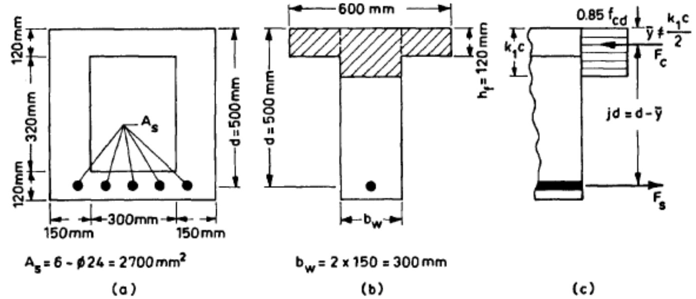


  - $b = 1000 \text{ mm}, \quad b_w = 300 \text{ mm}, \quad t = h_f = 120 \text{ mm}$
  - $d = 500 \text{ mm}, A_s = 6\emptyset 24 = 2700 \text{ mm}^2$
  - C20 → $f_{cd} = 13 \text{ MPa}$ S420 → $f_{yd} = 365 \text{ Mpa}$
- $M_r = ?$
- Assume $k_1 c = t = 120 \text{ mm}$ & steel yielded

Example 6

- $F_c = 0.85 f_{cd} b k_1 c = 0.85 \times 13 \times 1000 \times 120 = 1326 \text{ kN}$
- $F_s = A_s f_{yd} = 2700 \times 365 = 985 \text{ kN}$
- $F_c > F_s \quad \Rightarrow \quad k_1 c < t$ analyze as a rectangular section
- $k_1 c = \frac{A_s f_{yd}}{0.85 f_{cd} b} = \frac{2700 \times 365}{0.85 \times 13 \times 1000} = 89 \text{ mm}$
- $\varepsilon_s = 0.003 \frac{d-c}{c} = 0.003 \frac{500 - 89/0.85}{89/0.85} = 0.01133 > \varepsilon_{yd} = 0.001825$
- $M_r = A_s f_{yd} jd = 2700 \times 365 \times \left(500 - \frac{89}{2}\right) = 449 \text{ kNm}$

Example 7



- Convert the box section into a T-section
- Check if $k_1 c < t$
- $F_c = 0.85 \times 13 \times 600 \times 120 = 796 \text{ kN}$
- $F_s = 2700 \times 365 = 985 \text{ kN}$

$F_c < F_s$

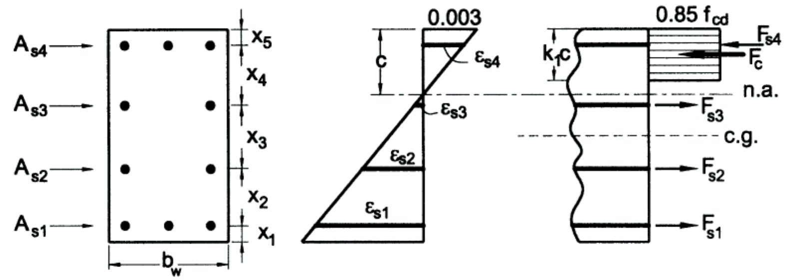
$\rightarrow \quad k_1 c > t$

Example 7

- $F_c = 0.85 \times 13[300 \times k_1 c + (600 - 300)120]$
- $\quad = 3315 k_1 c + 397800$
- $F_s = 985500 \text{ N}$
- $F_c = F_s \quad \Rightarrow \quad k_1 c = 178 \text{ mm} \quad \& \quad c = 209 \text{ mm}$
- Cetroid: $\bar{x} = \frac{300 \times 178 \times \frac{178}{2} + 300 \times 120 \times \frac{120}{2}}{300 \times 178 + 300 \times 120} = 77 \text{ mm}$
- $jd = d - \bar{x} = 500 - 77 = 423 \text{ mm}$
- $M_r = A_s f_{yd} jd = 2700 \times 365 \times 423 = 417 \text{ kNm}$

Beams with several layers of steel

- Neutral axis depth c is unknown
- Which steel layer is under compression?
- Which steel layer is under tension?
- Steel layers under tension yielded?
- Steel layers under compression yielded?



Beams with several layers of steel

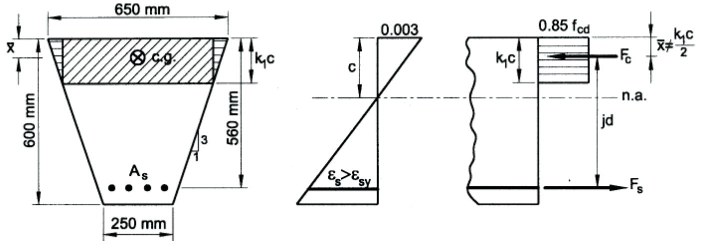
- Trial and error approach
  - Assume c
  - Compatibility; from similar triangles compute steel strains ε_{si}
  - Compute $F_{si} = A_{si}\sigma_{si}$ $\quad \sigma_{si} = \varepsilon_{si}E_s \le f_{yd}$
  - Compute $F_c = 0.85 f_{cd} b_w k_1 c$
  - Check $\sum F = 0$
    - if $\sum F \le 1 - 2\%\ \sum \text{compressive forces}$ → no further iteration
  - Change c & repeat steps until equilibrium is established
    - $\sum \text{tension} > \sum \text{compression}$ → increase c
    - $\sum \text{tension} < \sum \text{compression}$ → decrease c
  - Compute moment of forces about a convenient point (usually centroid)

Study Example 5.6-A

Beams with Nonrectangular Cross-Section

- Trial & error procedure can be used
- When cross-section can be divided into rectangles and/or triangles, a closed solution can be possible

Example 8



- $A_s = 1590\ \text{mm}^2$, C20 ($f_{cd} = 13\ \text{MPa}$), S420 ($f_{yd} = 365\ \text{MPa}$)
- $M_r = ?$
- Shaded area:
- $A_{cc} = \left(650 - 2\frac{1}{3}k_1 c\right)k_1 c + 2\frac{1}{2}k_1 c \frac{1}{3}k_1 c$
- $A_{cc} = 650 k_1 c - \frac{(k_1 c)^2}{3}$

Example 8

- $F_c = 0.85 f_{cd} A_{cc} = 0.85 \times 13 \times \left(650 k_1 c - \frac{(k_1 c)^2}{3}\right)$
- $F_c = 7182.5 k_1 c - 3.68 (k_1 c)^2$
- $F_s = A_s f_{yd} = 1590 \times 365 = 580350\ \text{N}$
- $F_c = F_s \quad \Rightarrow \quad (k_1 c)^2 - 1952 k_1 c + 159000 = 0$
- $k_1 c = 85\ \text{mm}$
- Centroid: $\dfrac{\left(650 - \frac{2}{3}85\right)85 \times \frac{85}{2} + 2\frac{1}{2}85\frac{85}{3}\frac{1}{3}85}{\left(650 - \frac{2}{3}85\right)85 + 2\frac{1}{2}85\frac{85}{3}} = 42\ \text{mm}$
- $jd = d - \bar{x} = 518\ \text{mm}$
- $M_r = A_s f_{yd} jd = 300.6\ \text{kNm}$