1. Derive the Trapezoidal Rule formula provided below.

Hint: There are many ways of obtaining Taylor Series derivation. You can use either of those two most frequently used ones.

1: Remember that in Trapezoidal Rule the function f(x) defined using only two points is a line. You can use the equation of a line to derive the formula.

2: Trapezoidal Rule can also be derived using the first order (linear) Taylor Series expansion.

2. Integrate the function below using trapezoidal rule with step size h=0.5 in the interval [-3, 4] and find the true error.

$$f(x) = 5x^4 - 8x^3 + 6x^2 + 4x - 20$$

3. A function is defined as follows:

$$f(x) = \begin{cases} 2x & \text{if } 1 \le x \le 5 \\ x^2 - 15 & \text{if } 5 \le x \le 14 \\ 0 & \text{o/w} \end{cases}$$

Calculate the integral of the function with step size = 1 in the interval [2, 10] using Simpson's 1/3 rule.

4. Calculate the integral of the function $f(x) = \frac{10}{\sqrt[3]{x^3 + 8}}$ in [0, 4.5]. Use composite Simpson's rule (use 6 slices, i.e., interval length = 0.5) in [0, 3] and using composite Trapezoidal rule (use 3 slices, i.e., interval length = 0.5) in [3, 4.5]. Find the true error for the integral in the interval [0, 4.5].

5. Use 3 points gauss-quadrature formula to integrate the function $f(x) = \frac{e^{-x^2}}{\sqrt{\pi}}$ in [-2.1, 2.5].

6. The following MATLAB code is part of a properly working code written for implementing Simpson's 1/3 Rule to calculate the integral of a function f(x) in [a,b]. Note that only some parts are shown below. What do you think an engineer would like to do when s/he writes for i=2:2:n-4? For which part of Simpson's Rule this code may be written for? Explain briefly.
Write the same code with using "while" loop instead of "for loop"?

```
disp(sprintf('\n\nSimulation of the Simpson"s 1/3rd Rule'))
% a, the lower limit of integration
% b, the upper limit of integration
% n, the number of segments. Note that this number must be even.
….%These parts are hidden
```

```
f= @(x) 2000*log(1400/21./x)-9.8*x;
n=12;
h=(b-a)/n;
….
sum = 0;
for i=1:2:n-3
sum=sum+f(a+i*h) ;
end
sum=4*sum ;
……
sum2=0;
for i=2:2:n-4
sum2=sum2+f(a+i*h) ; % f(x), the function to integrate
end
sum2=2*sum2 ;
….
```

7. Provide three example problems for which Civil Engineers use Numerical Integration techniques? Please provide citations.