CE382 – HOMEWORK 3

1) For C25 & S420 $f_{cd} = 17$ MPa, $f_{yd} = 365$ MPa

$j_M = 0.776$, $K_m = 133\ mm^2/kN$

$j_l = 0.86$, $K_l = 231\ mm^2/kN$

K101

$K = \frac{bd^2}{M_d} = \frac{1000 * 460^2}{275000} = 763 > K_l$ ✓ OK.

$\left.\begin{array}{l} j \cdot d = 0.9d = 414mm \\ j \cdot d = d - \frac{t}{2} = 390mm \end{array}\right\} jd = 414mm$

$A_s = \frac{M_d}{f_{yd} \cdot j \cdot d} = \frac{275000}{0.365 * 414} = 1820\ mm^2$ — 4Ø20 bent bars = $1256\ mm^2$

4Ø14 straight bars = $616\ mm^2$

$1872\ mm^2 > A_{s,req}$

Support 1

$M_d = 245 - \frac{250 * 0.4}{3} = 211.7$ kN.m

$K = \frac{300 * 460^2}{211700} = 300 > K_l$ ✓ OK

$A_s = \frac{211700}{0.365 * 0.86 * 460} = 1466\ mm^2$ — available 4Ø20 bent bars $= 1256\ mm^2$

2Ø12 hanger " $= 226\ mm^2$

$1486\ mm^2 > A_{s,req}$

K102

$K = \frac{1000 * 460^2}{100000} = 2116\ mm^2/kN > K_l$

$A_s = \frac{100000}{0.365 * 414} = 662\ mm^2$ — 2Ø14 straight bars $= 308\ mm^2$

2Ø20 bent " $= 628\ mm^2$

$936\ mm^2 > A_{s,req}$

Support 2

$M_d = 330 - \frac{270 \times 0.4}{3} = 234$ kN.m

$K = \frac{300 * 460^2}{234000} = 216 < K_l$ need double reinf. sect.

$M_1 = \frac{300 * 460^2}{231} = 218.14$ kN.m

$M_2 = M_d - M_1 = 75.86$ kN.m

$A_{s1} = \frac{218140}{0.365 * 0.86 * 460} = 1511\ mm^2$ $A_{s2} = A_s' = \frac{M_2}{f_{yd}(d-d')} = 495\ mm^2$

$A_s = 1511 + 495 = 2006\ mm^2$

Top — available 4Ø20 bent bars $= 1256\ mm^2$

" 2Ø20 " " $= 628\ mm^2$

2Ø12 hanger " $= 226\ mm^2$

$2110\ mm^2 > A_{s,req}$

Bottom — available 4Ø14 straight bars $= 616\ mm^2$

" 2Ø14 " " $= 308\ mm^2$

$924\ mm^2 > A'_{s,req}$

K103

$K = \frac{1000 * 460^2}{200000} = 1058 > K_l$ ✓ OK

$A_s = \frac{200000}{0.365 * 414} = 1324\ mm^2$ — 4Ø20 bent bars $= 1256\ mm^2$

2Ø14 straight " $= 308\ mm^2$

$1564\ mm^2 > A_{s,req}$

Support 3

$$Md = 300 - \frac{195*0.4}{3} = 276 \text{ kN.m}$$

$$K = \frac{300*460^2}{276000} = 230 < Kl \qquad \text{need double reinf. sect.}$$

$$M1 = \frac{300*460^2}{231} = 218.14 \text{ kN.m} \qquad M2 = Md - M1 = 57.86 \text{ kN.m}$$

$$As1 = \frac{218140}{0.365*0.86*460} = 1511 \text{ mm}^2 \qquad As2 = As' = \frac{57860}{0.365*420} = 378 \text{ mm}^2$$

$$As = As1 + As' = 1889 \text{ mm}^2$$

Top — Available 2Ø20 bent bars = 628 mm²
" 4Ø20 " " = 1256 mm²
2Ø12 hanger " = 226 mm²
2110 mm² > As,req

Bottom — available 2Ø14 straight bars = 308 mm²
" 2Ø14 " " = 308 mm²
616 mm² > As',req

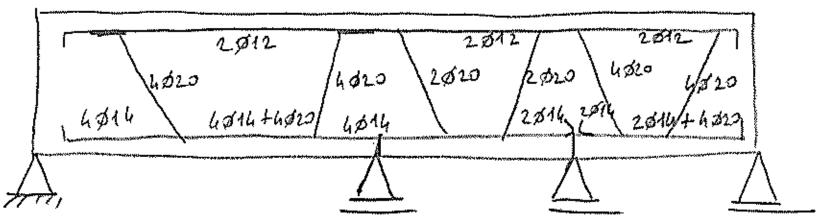
Support 4

$$Md = 177 - \frac{170*0.4}{3} = 154.3 \text{ kN.m}$$

$$K = \frac{300*460^2}{154300} = 411 > Kl \quad \checkmark \text{OK}$$

$$As = \frac{154300}{0.365*0.86*460} = 1069 \text{ mm}^2$$

available 4Ø20 bent bars = 1256 mm²
2Ø12 hanger " = 226 mm²
1482 mm² > As,req



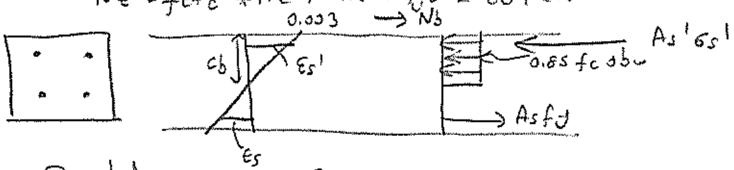
2) a.) $N_{or} = 0.85 f_{cd} A_c + A_s f_{yd} = 0.85*17*(400*400) + 4*\frac{\pi 24^2}{4}*365 = 2372.5 \text{ kN}$

b) Tension capacity

direct tensile — $f_{ctd} = 0.35\sqrt{17} = 1.44 \text{ MPa}$

$N_t = f_{ctd} * A_c + A_s * f_{yd} = 831 \text{ kN}$

c.)



For balance case $\varepsilon_s = \varepsilon_y = 365/200000 = 0.00183$

$$\frac{0.003}{cb} = \frac{0.00183}{360-cb} \quad — \quad cb = 223.6 \text{ mm}$$

$$\frac{0.003}{cb} = \frac{\varepsilon_s'}{cb-40} \quad — \quad \varepsilon_s' = 2.5*10^{-3} > \varepsilon_y \qquad \text{comp. steel yields.}$$

$$Nb + As f_y = 0.85 f_c a b_w + As' \sigma_s'$$

$$Nb = 1218 \text{ kN}$$

$$Mb = 2*(As f_y)*(160) + (0.85*f_c*a*b_w)*(200 - \tfrac{a}{2}) = 221 \text{ kN.m}$$

d) $F_c = 0.85 * 17 * 0.85c * 400 = 4913c$

$F_s = 2 * \frac{\pi 24^2}{4} * 365 = 330244 \ N$

$F_s' = 2 * \frac{\pi 24^2}{4} * \sigma_s' = 904.8 * 200000 * \left(0.003 - \frac{0.12}{c}\right) = 542880 - \frac{21715200}{c}$

$N = 0 = F_c + F_s' - F_s$ — $c = 48.35$ mm

$F_c = 237.5$ kN, $F_s = 330.2$ kN, $F_s' = 93.7$ kN

$M = F_c * \left(200 - \frac{0.85 * c}{2}\right) + F_s * 160 + F_s' * 160 = 110.4$ kN.m

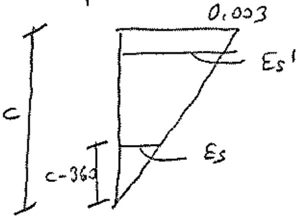
e) $N = 0.85 N_{or} = 2527$ kN

$N > N_b$ — comp. failure

assume all section in compression

$\varepsilon_s' > \varepsilon_y$

$\frac{0.003}{c} = \frac{\varepsilon_s}{c - 360}$



$F_c = 0.85 \times 17 \times 0.85c \times 400 = 4913c$

$F_s' = 365 \times 2 \times \frac{\pi 24^2}{4} = 330252$

$F_s = 2 \times \frac{\pi 24^2}{4} \times 200000 \times \left(0.003 - \frac{1.08}{c}\right) = 542880 - \frac{195436800}{c}$

$N = 2527000 = F_c + F_s' + F_s$ — $c = 429.3$ mm

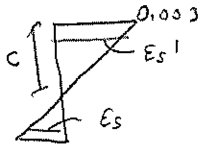
$c > h$ assumption is correct ✓

$F_c = 2109$ kN, $F_s = 87.7$ kN, $F_s' = 330.2$ kN $\Big\}$ $M = F_c \times \left(200 - \frac{a}{2}\right) + F_s \times 160 + F_s' \times 160 = 103.9$ kN.m

f) $N = 0.45 N_{or} = 1337.6$ kN

$N > N_b$ — comp. failure

assume bottom steel is under tension.



$\frac{0.003}{c} = \frac{\varepsilon_s}{360 - c}$

$F_c = 4913c$, $F_s' = 330252$, $F_s = \frac{195436800}{c} - 542880$ $\Big\}$ $N = F_c + F_s' - F_s$; $c = 252.2$ mm; $c < h$ ✓ assumption is correct

$F_c = 1239$ kN

$F_s = 330.2$ kN

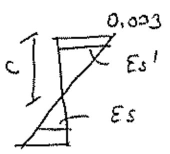
$F_s' = 232$ kN

$M = F_c \times \left(200 - \frac{a}{2}\right) + 160 \times F_s + F_s' \times 160 = 205$ kN.m

g) $N = 0.15 N_{or} = 446$ kN

$N < N_b$ — tension failure

assume $\sigma_s = f_y$ & $\sigma_s' < f_y$



$\frac{0.003}{c} = \frac{\varepsilon_s'}{c - 40}$ $\qquad \varepsilon_s' = 0.003 - \frac{0.12}{c}$

$F_c = 4913c$

$F_s = 330244$

$F_s' = \varepsilon_s' * 200000 * A_s' = 542880 - \frac{21715200}{c}$

$N = F_c + F_s - F_s'$ — $c = 75.7$ mm

from compatibility $\begin{cases} \varepsilon_s' = 0.0014 < \varepsilon_y \\ \varepsilon_s = 0.011 > \varepsilon_y \end{cases}$ assumption is correct.

$$\left.\begin{aligned} F_c &= 371.8\ \text{kN} \\ F_s &= 330.2\ \text{kN} \\ F_s' &= 256\ \text{kN} \end{aligned}\right\} \quad M = F_c * \left(200 - \frac{d}{2}\right) + 160 * F_s + F_s' * 160 = 156.2\ \text{kN.m}$$