# CE 383 Homework - 2

## Solutions

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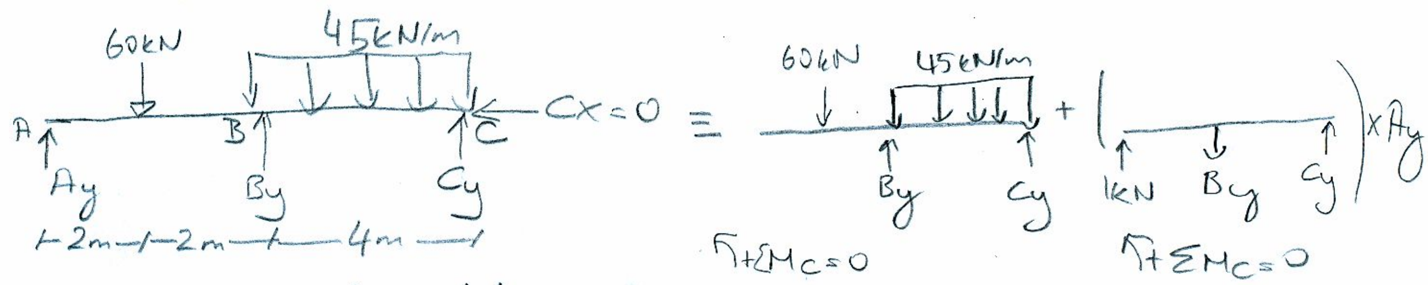



In this solution, $A_y$ is taken as redundant

$\curvearrowleft + \Sigma M_C = 0$

$60(6) - B_y(4) + 45(4)(2) = 0$

$B_y = 180 \text{ kN} (\uparrow)$

$+\uparrow \Sigma F_y = 0$

$-60 + 180 - 45(4) + C_y = 0$

$C_y = 60 \text{ kN} (\uparrow)$

$\curvearrowleft + \Sigma M_C = 0$

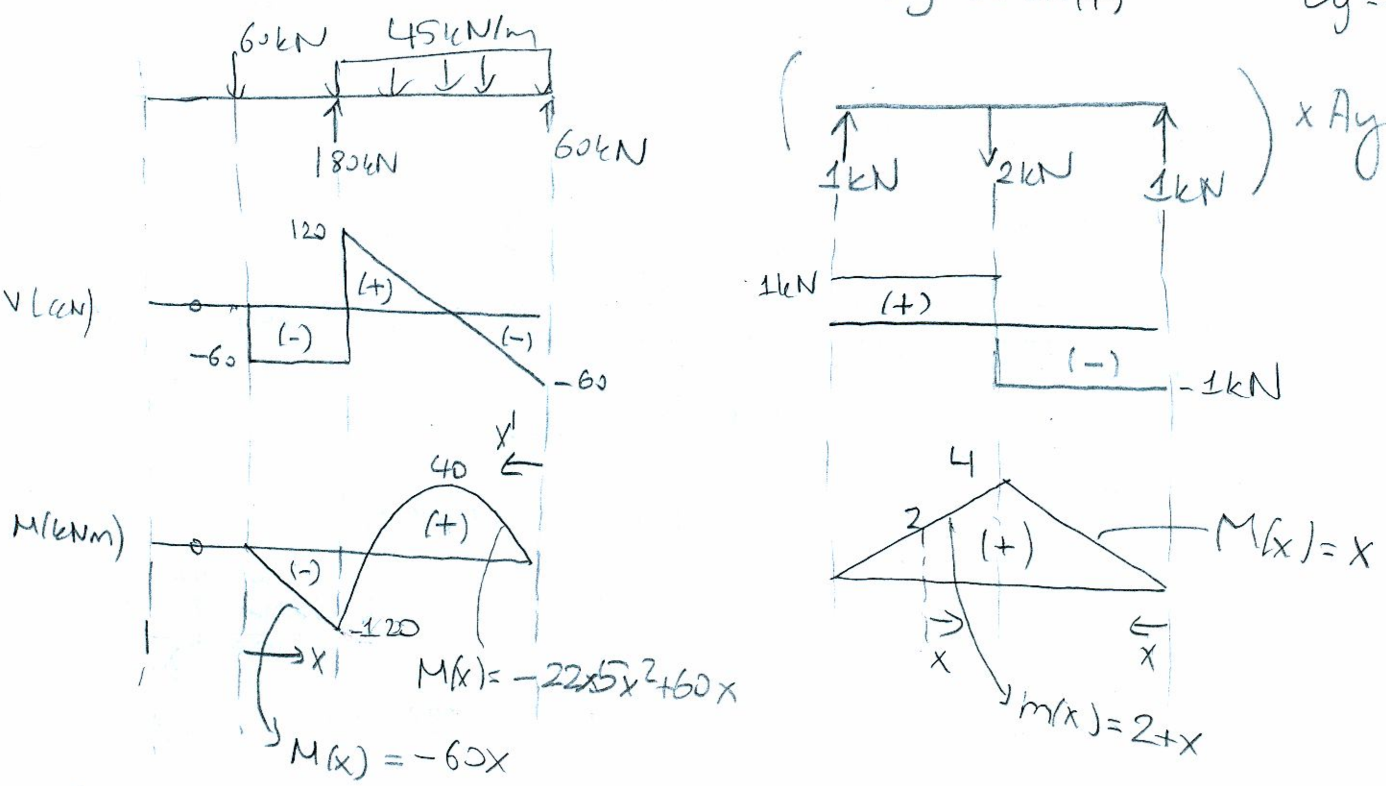
$-(1)(8) + B_y(4) = 0$

$B_y = 2 \text{ kN} (\downarrow)$

$\uparrow + \Sigma F_y = 0$

$1 - 2 + C_y = 0$

$C_y = 1 \text{ kN} (\uparrow)$



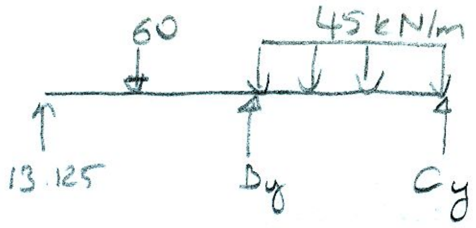

Compatibility equation. $\Delta_0 + f_1(A_y) = 0$

$$\Delta_0 = \int_0^2 \frac{(-60x)(2+x)\,dx}{EI} + \int_0^4 \frac{(-22.5x^2 + 60x)(x)\,dx}{EI}$$

$$\Delta_0 = \frac{1}{EI}\left(-60x^2 - 20x^3\right)\Big|_0^2 + \left[\frac{(-22.5)\frac{x^4}{4} + 20x^3}{EI}\right]\Big|_0^4 \Rightarrow \Delta_0 = \frac{-560}{EI}$$

$$f_1 = 2\left(\int_0^4 \frac{x^2}{EI}\,dx\right) = \frac{2}{EI}\frac{x^3}{3}\Big|_0^3 = \frac{42.67}{EI}$$

$$\Delta = \Delta_0 + f_1 A_y \Rightarrow \frac{-560}{EI} + \frac{42.67}{EI} A_y \Rightarrow A_y = 13.125 \text{ kN} (\uparrow)$$



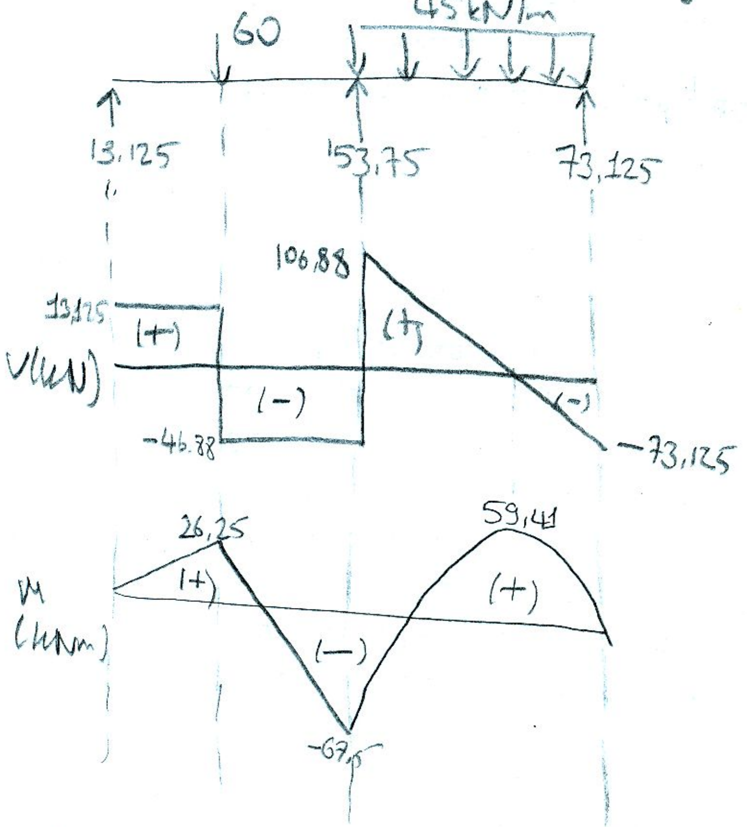

$+\circlearrowleft \sum M_C = 0$

$(-13{,}125)(8)+(60)(6)-B_y(4)+45(4)(2)=0$

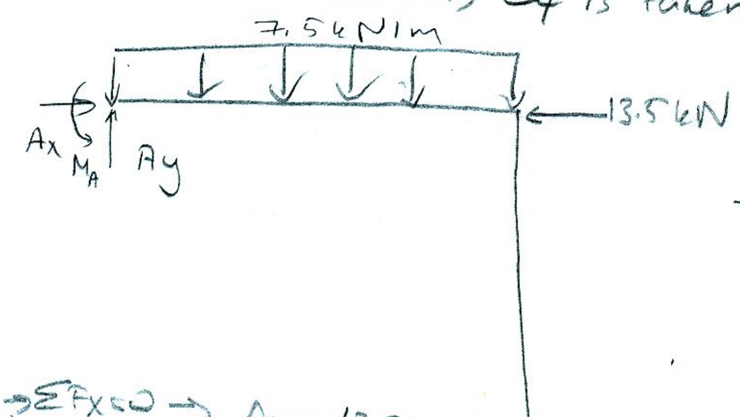
$B_y = 153{,}75 \text{ kN } (\uparrow)$

$\uparrow \sum F_y = 0 \Rightarrow 13{,}125+153{,}75-60-45(4)+C_y=0$

$C_y = 73{,}125 \text{ kN } (\uparrow)$




2. In this solution, $C_y$ is taken as redundant:

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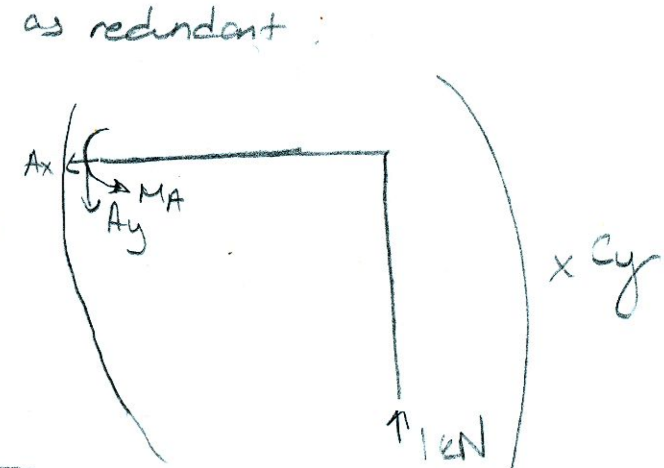



$\xrightarrow{+} \sum F_x = 0 \rightarrow A_x = 13.5 \text{ kN } (\rightarrow)$

$+\uparrow \sum F_y = 0 \quad (-7.5)(3)+A_y=0$

$A_y = 22.5 \text{ kN } (\uparrow)$

$+\circlearrowleft \sum M_A = 0 \quad (-7.5)(3)\left(\frac{3}{2}\right)+M_A=0$

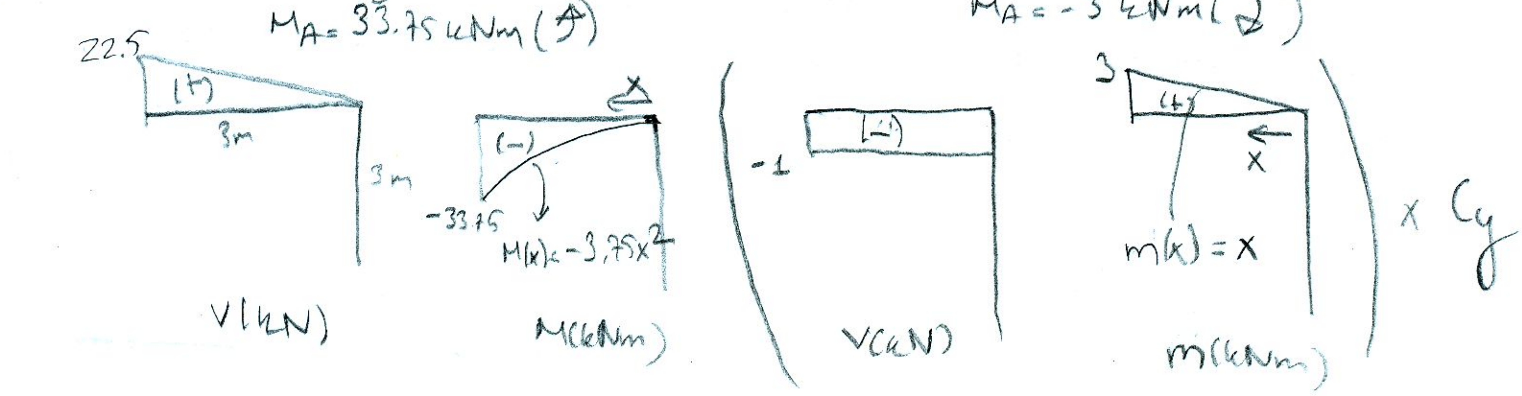
$M_A = 33.75 \text{ kNm } (\circlearrowleft)$

$\xrightarrow{+} \sum F_x = 0 \Rightarrow A_x = 0$

$+\uparrow \sum F_y = 0 \Rightarrow A_y = 1 \text{ kN } (\downarrow)$

$+\circlearrowleft \sum M_A = 0 \quad (1)(3)+M_A=0$

$M_A = -3 \text{ kNm } (\circlearrowright)$

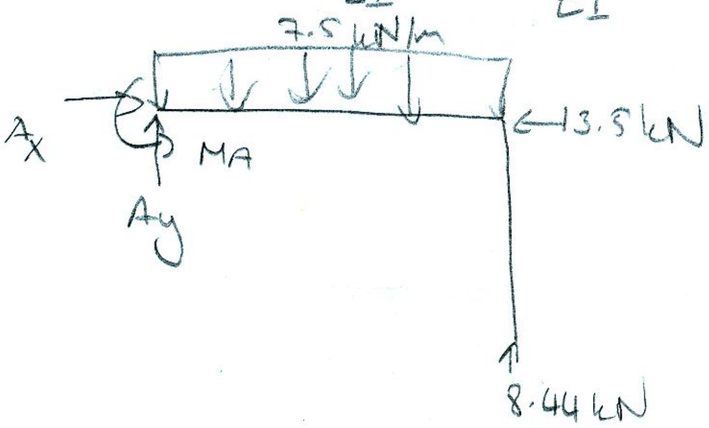


Compatibility equation $\Delta_0 + f_1 C_y = 0$

$$\Delta_0 = \int_0^3 \frac{(-3.75x^2)(x)}{EI}\,dx = \frac{-3.75x^4}{4EI}\Big|_0^3 = \frac{-75{,}9375}{EI}$$

$$f_1 = \int_0^3 \frac{x^2}{EI}\,dx = \frac{x^3}{3EI}\Big|_0^3 = \frac{9}{EI}$$

$$\frac{-75{,}9375}{EI} + \frac{9}{EI}(C_y) = 0 \qquad C_y = 8{,}4375\,kN\ (\uparrow)$$




$\Sigma F_x = 0 \Rightarrow A_x = 13.5\ kN\ (\rightarrow)$

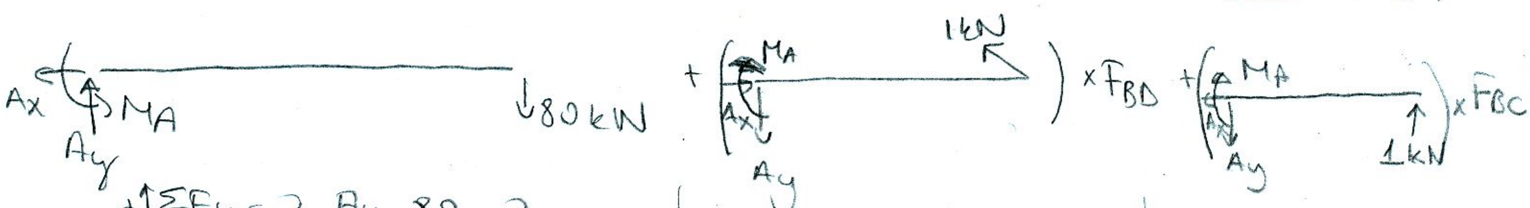
$\Sigma F_y = 0 \Rightarrow A_y = 14{,}06\ kN\ (\uparrow)$

$\Sigma M_A = 0$

$(8.4375)(3) - (7.5)(3)(1{,}5) - M_A = 0$

$M_A = 8.44\,kNm\ (\circlearrowleft)$

3- In this solution, $F_{BC}$ and $F_{BD}$ are taken as redundants.




$+\uparrow \Sigma F_y = 0 \quad A_y - 80 = 0$
$\Rightarrow A_y = 80\ kN\ (\uparrow)$

$\Sigma F_x = 0 \Rightarrow A_x = 0$

$\Sigma M_A = 0 \Rightarrow M_A = 320\ kNm\ (\uparrow)$

$\Sigma F_x = 0 \Rightarrow A_x = 0{,}8\ kN\ (\rightarrow)$

$\Sigma F_y = 0 \Rightarrow A_y = 0{,}6\ kN\ (\downarrow)$

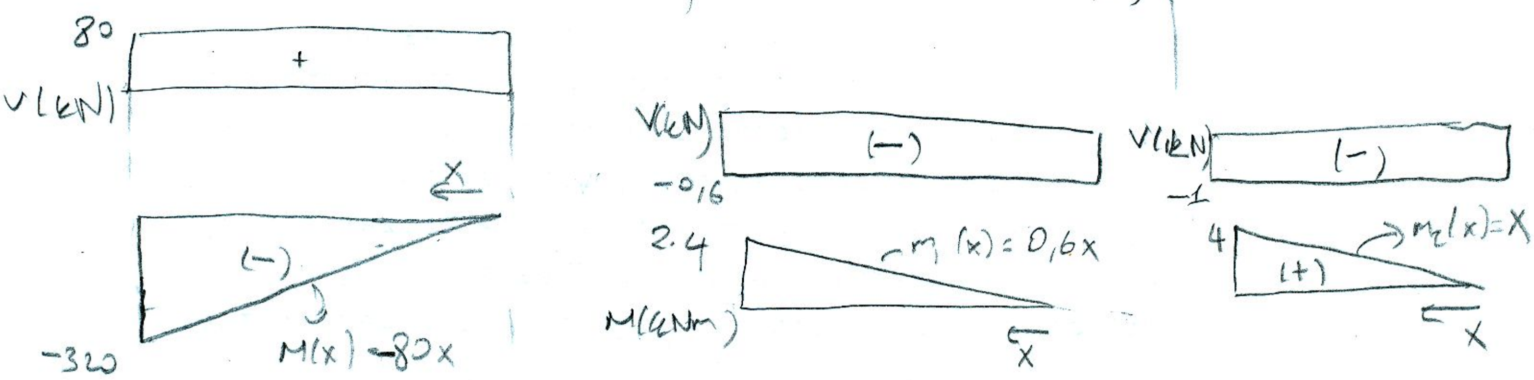
$\Sigma M_A = 0 \Rightarrow -M_A + (4)(0{,}6) = 0$

$M_A = 2{,}4\ kNm\ (\circlearrowright)$

$\Sigma F_x = 0 \Rightarrow A_x = 0$

$\Sigma F_y = 0 \Rightarrow A_y = 1\ kN\ (\downarrow)$

$\Sigma M_A = 0 \quad M_A = 4\ kNm\ (\circlearrowright)$




Compatibility equations
$\Delta_{10} + f_{11} F_{BD} + f_{12} F_{BC} = 0$
$\Delta_{20} + f_{21} F_{BD} + f_{22} F_{BC} = 0$

$$\Delta_{10} = \int \frac{M m_1}{EI}\,dx = \int_0^4 \frac{(-80x)(0,6x)}{EI} = -\frac{16x^3}{EI}\Big|_0^4 = -\frac{1024}{EI}$$

$$\Delta_{20} = \int \frac{M m_2}{EI}\,dx = \int_0^4 \frac{(-80x)(x)\,dx}{EI} = -\frac{80x^3}{3EI}\Big|_0^4 = -\frac{1706,667}{EI}$$

$$f_{11} = \int \frac{(m_1)^2}{EI}\,dx + \frac{n_1 n_1}{EA} L_{BD} = \frac{0,36x^3}{3EI}\Big|_0^4 + \frac{(1)(1)(5)}{EA} = \frac{7.68}{EI} + \frac{5}{EA} = \frac{17,68}{EI}$$

$$f_{22} = \int \frac{(m_2)^2}{EI}\,dx + \frac{n_2 n_2}{EA} L_{BC} = \frac{x^3}{3EI}\Big|_0^4 + \frac{(1)(1)(3)}{EA} = \frac{21,333}{EI} + \frac{3}{EA} = \frac{27,333}{EI}$$

$$f_{12} = \int \frac{(m_1)(m_2)}{EI} = \frac{0,6x^3}{3EI}\Big|_0^4 = \frac{12,8}{EI}$$

$$\frac{-1024}{\cancel{EI}} + \left(\frac{17,68}{\cancel{EI}}\right) \times F_{BD} + \left(\frac{12,8}{\cancel{EI}}\right) \times F_{BC} = 0$$

$$\frac{-1706,667}{\cancel{EI}} + \left(\frac{12.8}{\cancel{EI}}\right) \times F_{BD} + \left(\frac{27,333}{\cancel{EI}}\right) \times F_{BC} = 0$$

From these two equations $F_{BC} = 53,44$ kN

$F_{BD} = 19,23$ kN