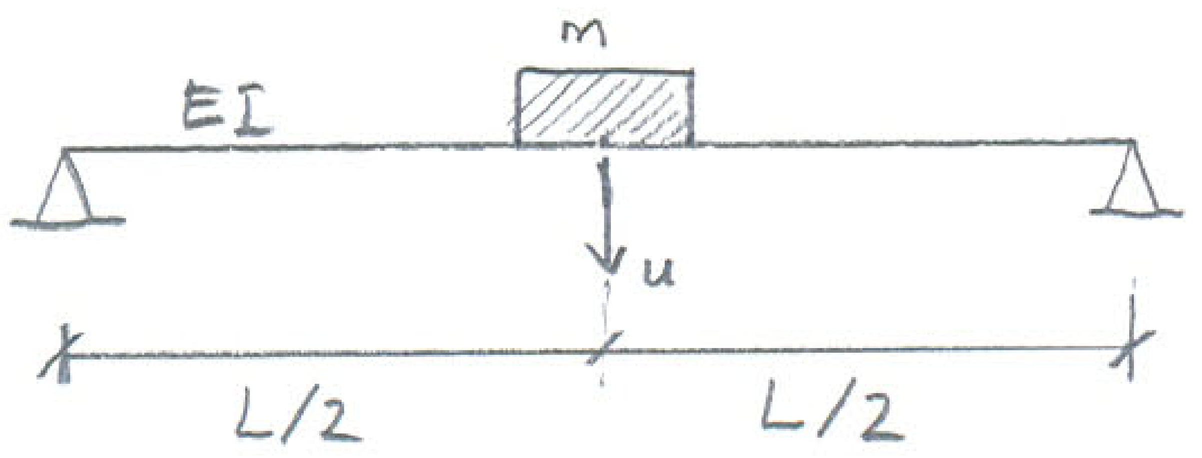
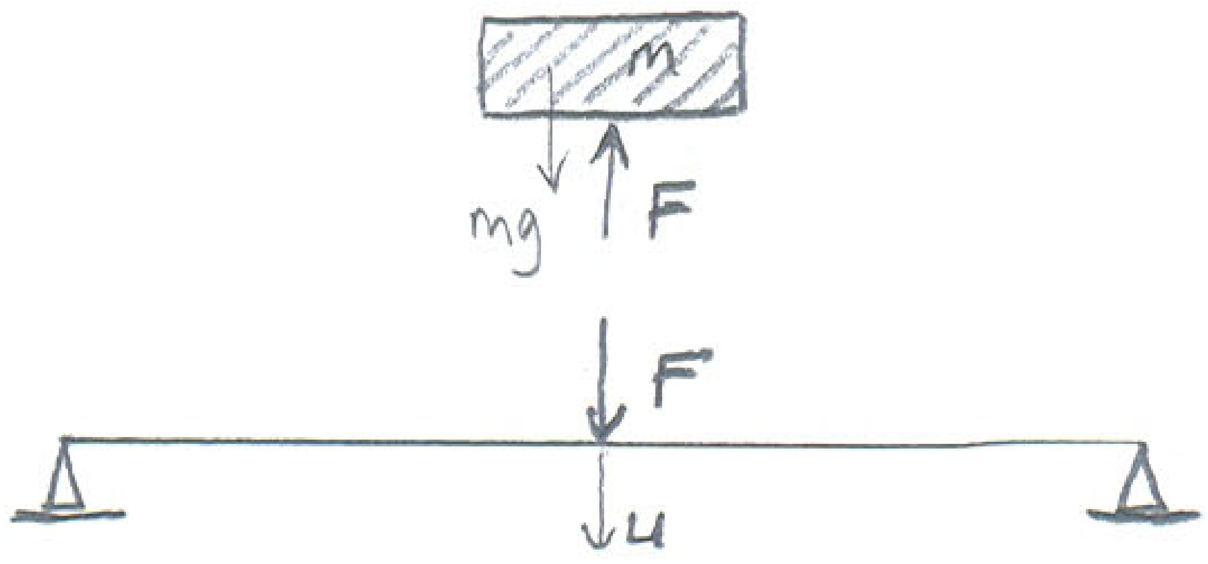
CE 487 HOMEWORK 1 SOLUTION

1 a)





$$mg - F = m\ddot{u}$$

$F = ku + ku^{st} \Rightarrow mg - ku - ku^{st} = m\ddot{u}$

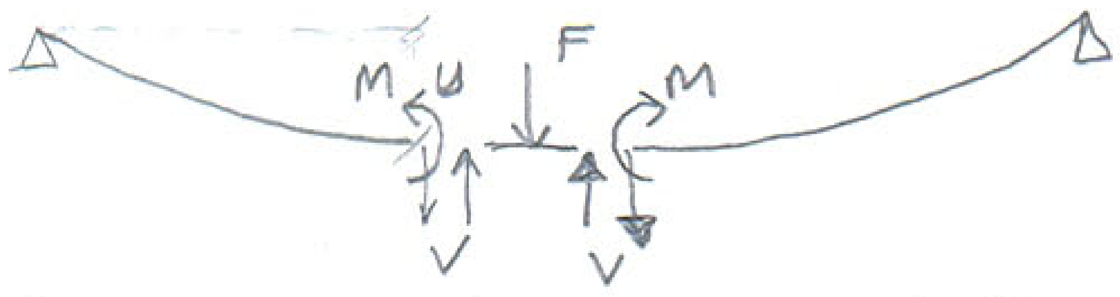
$mg = ku^{st}$; so

$m\ddot{u} + ku = 0$

u is the displacement from static equilibrium position.

Determination of k :

Apply a displacement "u" to the center of the beam



Using modified slope-deflection equation

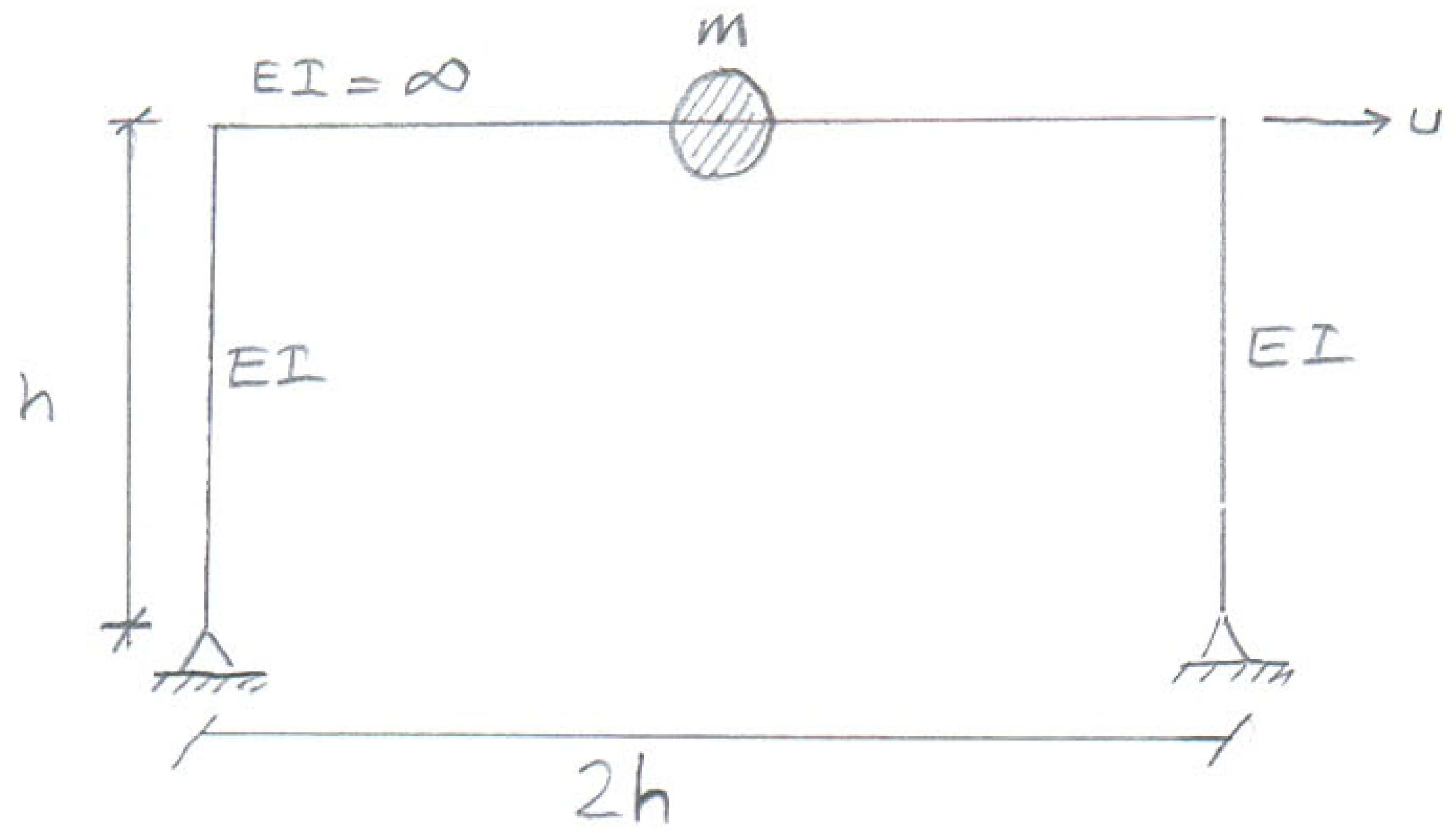
$$M = \frac{3EI}{(L/2)}\left(\theta + \frac{u}{(L/2)}\right) \qquad \theta = 0 \qquad M = \frac{12EI}{L^2}u$$

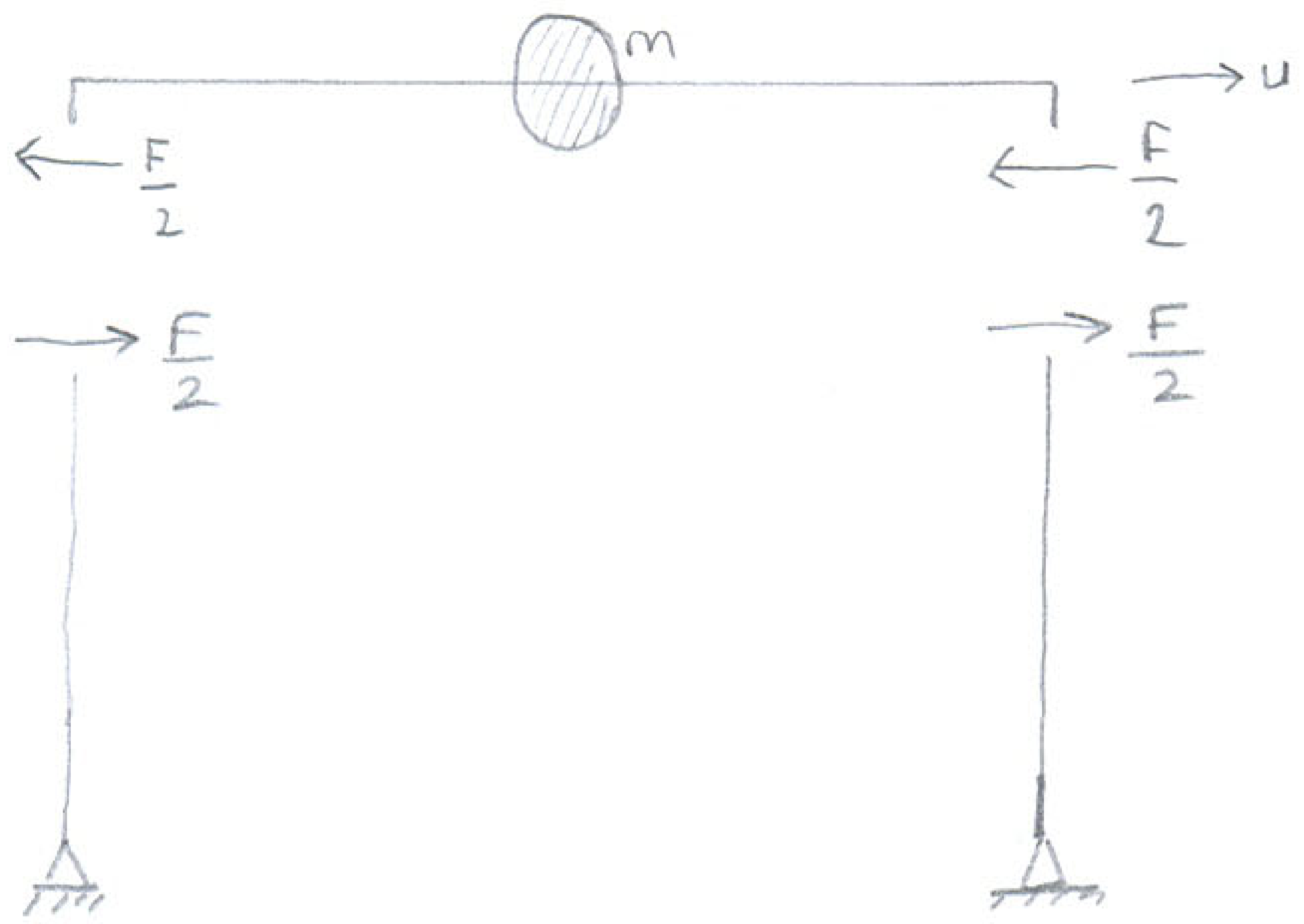
$$V = \frac{M}{L/2} = \frac{24EI}{L^3}u \qquad F = 2V \qquad F = \frac{48EI}{L^3}u$$

$$k = \frac{48EI}{L^3}$$

$$\boxed{\begin{array}{l}\text{Equation of motion}\\ m\ddot{u} + \dfrac{48EI}{L^3}u = 0\end{array}}$$

1 b)





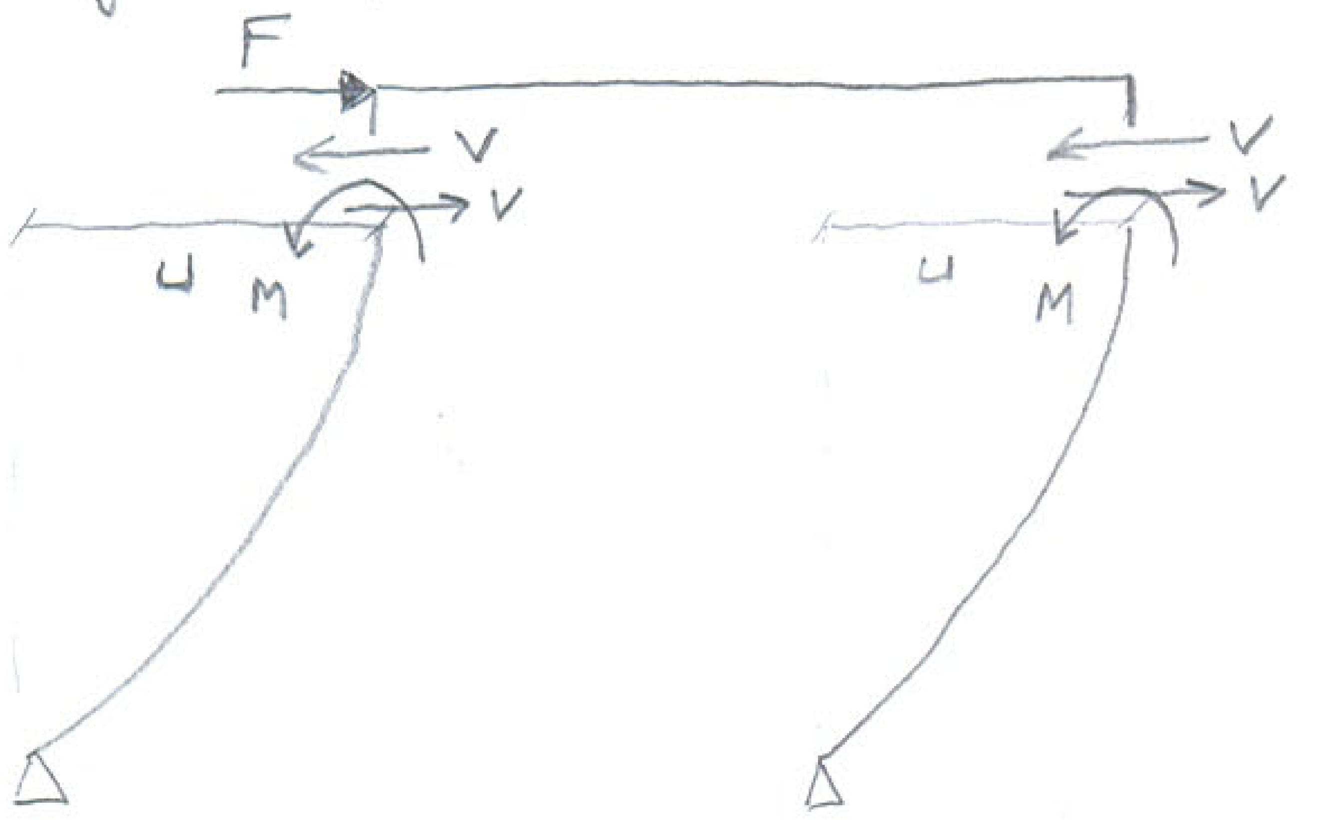
$$-F = m\ddot{u}$$

$$F = ku$$

$$m\ddot{u} + ku = 0$$

Determination of k :

Apply a displacement "u" to the frame



$$F = 2V$$

Applying modified slope-deflection equation;

$$M = \frac{3EI}{h}\left(\theta + \frac{u}{h}\right) \qquad \theta = 0$$

$$M = \frac{3EI}{h^2} \cdot u \qquad V = \frac{M}{h} = \frac{3EI}{h^3} u$$

$$F = 2V = 2\left(\frac{3EI}{h^3}\right) u = \frac{6EI}{h^3} u \qquad k = \frac{6EI}{h^3}$$

Equation of motion

$$\boxed{m\ddot{u} + \frac{6EI}{h^3} u = 0}$$

2) $T = 2\pi\sqrt{\frac{m}{k}}$

KN,m, ton and second are consistent units.

a) $m = 100$ tons $\quad k = \frac{48EI}{L^3}$

$L = 6m$

$I = 3*10^{-3}\ m^4$

$E = 25\ GPa = 25*10^9\ \frac{N}{m^2} = 25*10^6\ \frac{kN}{m^2}$

$$k = \frac{48*25*10^6*3*10^{-3}}{6^3} = 16666.67\ \frac{kN}{m}$$

$$T = 2\pi\sqrt{\frac{100}{16666.67}} = 0.487\ sec$$

b) $m = 100$ tons $\quad k = \frac{6EI}{h^3}$

$h = 3m$

$I = 3*10^{-3}\ m^4$

$E = 25\ GPa = 25*10^9\ \frac{N}{m^2} = 25*10^6\ \frac{kN}{m^2}$

$$k = \frac{6EI}{h^3} = \frac{6*25*10^6*3*10^{-3}}{3^3} = 16666.67\ kN/m$$

$$T = 2\pi\sqrt{\frac{100}{16666.67}} = 0.487\ sec$$

3) - Coulomb damping results from friction against sliding of two dry surfaces. Friction force is independent of velocity. In viscous damping, damping force is proportional to velocity.

- In Coulomb damping, direction of damping force is in the direction against motion. In viscous damping, direction of damping force is also in the direction against motion.

- In Coulomb damping free vibration, the duration to complete a full cycle is equal to the natural period of vibration (T_n).
In viscous damping free vibration, the duration to complete a full cycle is longer than the natural period of vibration.

- In Coulomb damping, maximum displacement reduce linearly in successive cycles. In viscous damping, maximum displacement reduce exponentially in successive cycles.

- The free vibration motion of a system with Coulomb damping stops at the end of the half-cycle for which the maximum displacement is less than u_F, where u_F is equal to the friction force μN divided by the spring constant, k. At the end of this half-cycle, when the velocity becomes zero, spring force acting on the mass is less than the friction force, hence the motion stops.
In purely viscous damping, free vibration motion theoritically continues forever, although at infinitesmally small amplitudes.

4) $m\ddot{u}+c\dot{u}+ku=p_0 \sin \omega t$ - - - - - - - - - - - - ①

General solution of ① is;

$$u(t)=e^{-\zeta\omega_n t}(A\cos\omega_D t+B\sin\omega_D t)+C\sin\omega t+D\cos\omega t$$

$$\omega_D=\omega_n\sqrt{1-\zeta^2}$$

$$C=\frac{p_0}{k}\,\frac{1-(\omega/\omega_n)^2}{[1-(\omega/\omega_n)^2]^2+[2\zeta(\omega/\omega_n)]^2}$$

$$D=\frac{p_0}{k}\,\frac{-2\zeta\,\omega/\omega_n}{[1-(\omega/\omega_n)^2]^2+[2\zeta(\omega/\omega_n)]^2}$$

$p_0 = 1\ kN$

$k=16666.67\ \frac{kN}{m}$ $\quad \frac{\omega}{\omega_n}=0.25 \quad \zeta=0.1 \quad \omega_n=\frac{2\pi}{T_n}=\frac{2\pi}{0.487}$

$\omega_n = 12.902\ rad/sec$

$\omega_D=12.902\sqrt{1-0.1^2}=12.837\ rad/sec$

$\omega=0.25\omega_n=3.226\ rad/sec$

$$C=\frac{1}{16666.67}\,\frac{1-0.25^2}{[1-0.25^2]^2+[2(0.1)(0.25)]^2}=6.382*10^{-5}$$

$$D=\frac{1}{16666.67}\,\frac{-2(0.1)(0.25)}{[1-0.25^2]^2+[2(0.1)(0.25)]^2}=-3.404*10^{-6}$$

$$u(t)=e^{-(0.1)(12.902)t}(A\cos 12.837t+B\sin 12.837t)+6.382*10^{-5}\sin 3.226t$$
$$-3.404*10^{-6}\cos 3.226t$$

$$\dot{u}(t)=e^{-1.2902t}(-12.837A\sin 12.837t+12.837B\cos 12.837t)-1.2902e^{-1.2902t}$$
$$(A\cos 12.837t+B\sin 12.837t)+(6.382*10^{-5})(3.226)\cos 3.226t+$$
$$3.404*10^{-6}*3.226*\sin 3.226t$$

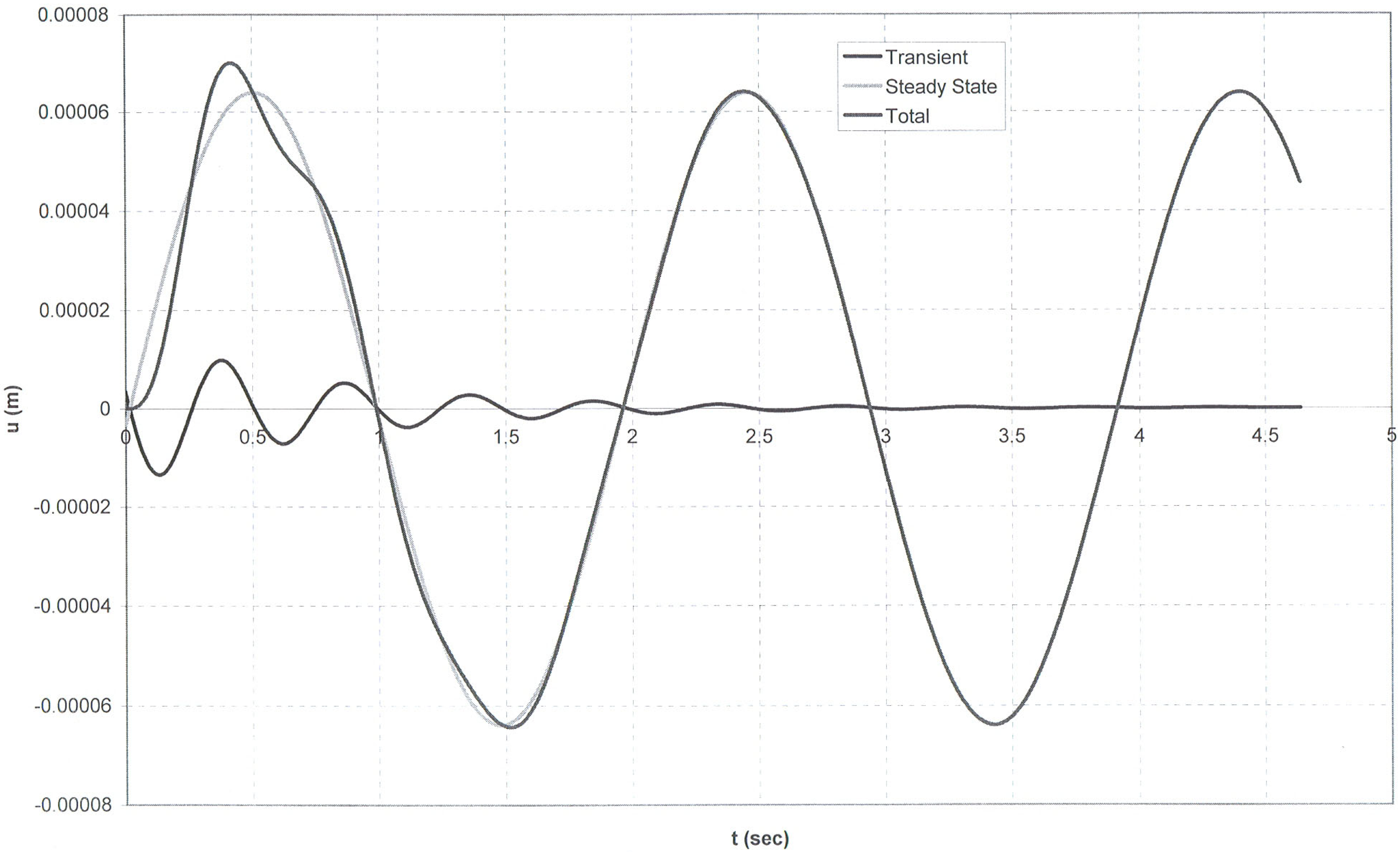
$u(0)=0 \qquad 0 = A - 3.404 * 10^{-6} \qquad A = 3.404 * 10^{-6}$

$\dot{u}(0)=0 \qquad 0 = 12.837B - 1.2902A + (6.382 * 10^{-5})(3.226)$

$$B = -1.569 * 10^{-5}$$

Displacement response,

$$u(t) = e^{-1.2902t}(3.404 * 10^{-6} \cos 12.837t - 1.569 * 10^{-5} \sin 12.837t)$$
$$+ 6.382 * 10^{-5} \sin 3.226t - 3.404 * 10^{-6} \cos 3.226t$$


u (m)
0.00008
0.00006
0.00004
0.00002
0
-0.00002
-0.00004
-0.00006
-0.00008
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
t (sec)
Transient
Steady State
Total