



**Additional Example on Solving Nonlinear System of Equations**

Solve the following set of nonlinear equations using;

- (i) Graphical Method,
- (ii) Fixed Point Iteration, and
- (iii) Newton Jacobi Method,

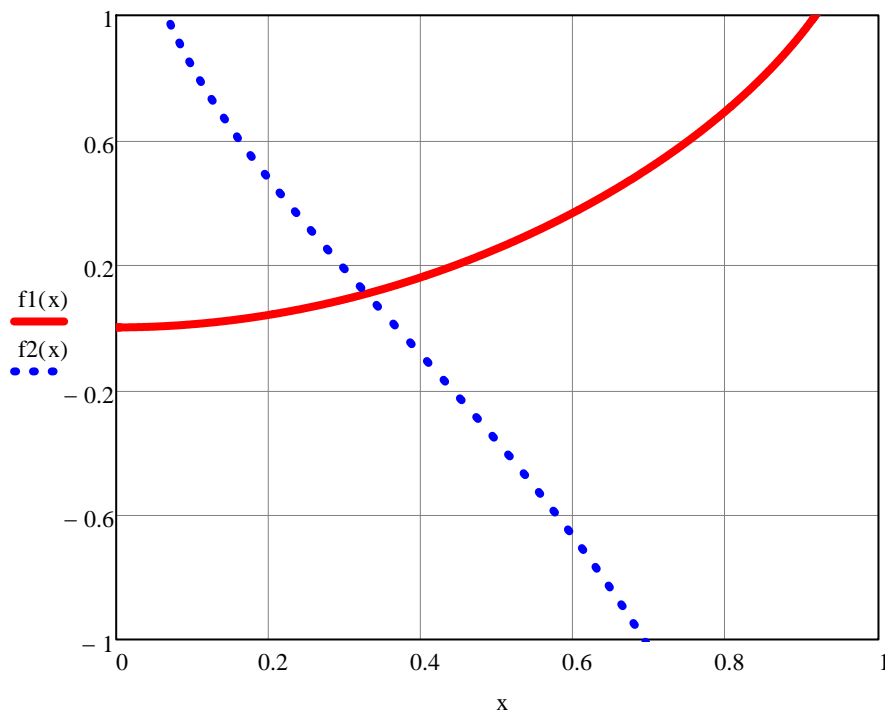
given the tolerance = 0.005 and  $(x_0, y_0) = (0.5, 0.5)$

$$x^2 = \sin(y)$$
$$\ln\left(\frac{1}{x}\right) = e^y$$

**(i) Graphical Method:**

$$f1(x) := \sin(x^2)$$

$$f2(x) := \ln\left(\ln\left(\frac{1}{x}\right)\right)$$



## (ii) Fixed Point Iteration

$$x = \sqrt{\sin(y)}$$

$$x := 0.5 \quad f1(x) = 0.6924$$

$$x := -0.3665 \quad f1(x) = 0.5986i$$

**Complex Number**

$$y = \ln(\ln(1/x))$$

$$y := 0.5 \quad f2(y) = -0.3665$$

$$y := 0.6924 \quad f2(y) = -1.0008$$

$$y := -1.0008 \quad f2(y) = 1.1447 + 1.5711i$$

**Complex Number**

The solution is **divergent** when the equations are expressed using the formulas above, rewriting the same equation given below will produce:

$$x = \frac{1}{e^{e^y}}$$

$$x0 := 0.5 \quad f1(x0) = 0.1923$$

$$x1 := 0.2527 \quad f1(x1) = 0.276$$

$$x2 := 0.0361 \quad f1(x2) = 0.3546$$

$$x3 := 0.0762 \quad f1(x3) = 0.3399$$

$$x4 := 0.1261 \quad f1(x4) = 0.3216$$

$$x5 := 0.1158 \quad f1(x5) = 0.3254$$

$$x6 := 0.1036 \quad f1(x6) = 0.3298$$

$$x7 := 0.1061 \quad f1(x7) = 0.3289$$

$$y = \arcsin(x^2)$$

$$y0 := 0.5 \quad f2(y0) = 0.2527$$

$$y1 := 0.1923 \quad f2(y1) = 0.037$$

$$y2 := 0.276 \quad f2(y2) = 0.0762$$

$$y3 := 0.3546 \quad f2(y3) = 0.1261$$

$$y4 := 0.3399 \quad f2(y4) = 0.1158$$

$$y5 := 0.3216 \quad f2(y5) = 0.1036$$

$$y6 := 0.3254 \quad f2(y6) = 0.1061$$

$$y7 := 0.3298 \quad f2(y6) = 0.1061$$

$$\epsilon_{AA\_x\_1} := |x1 - x0| \quad \epsilon_{AA\_y\_1} := |y1 - y0| \quad \max(\epsilon_{AA\_x\_1}, \epsilon_{AA\_y\_1}) = 0.3077$$

$$\epsilon_{AA\_x\_2} := |x2 - x1| \quad \epsilon_{AA\_y\_2} := |y2 - y1| \quad \max(\epsilon_{AA\_x\_2}, \epsilon_{AA\_y\_2}) = 0.2166$$

$$\epsilon_{AA\_x\_3} := |x3 - x2| \quad \epsilon_{AA\_y\_3} := |y3 - y2| \quad \max(\epsilon_{AA\_x\_3}, \epsilon_{AA\_y\_3}) = 0.0786$$

$$\epsilon_{AA\_x\_4} := |x4 - x3| \quad \epsilon_{AA\_y\_4} := |y4 - y3| \quad \max(\epsilon_{AA\_x\_4}, \epsilon_{AA\_y\_4}) = 0.0499$$

$$\epsilon_{AA\_x\_5} := |x5 - x4| \quad \epsilon_{AA\_y\_5} := |y5 - y4| \quad \max(\epsilon_{AA\_x\_5}, \epsilon_{AA\_y\_5}) = 0.0183$$

$$\epsilon_{AA\_x\_6} := |x6 - x5| \quad \epsilon_{AA\_y\_6} := |y6 - y5| \quad \max(\epsilon_{AA\_x\_6}, \epsilon_{AA\_y\_6}) = 0.0122$$

$$\epsilon_{AA\_x\_7} := |x7 - x6| \quad \epsilon_{AA\_y\_7} := |y7 - y6| \quad \max(\epsilon_{AA\_x\_7}, \epsilon_{AA\_y\_7}) = 4.4 \times 10^{-3}$$

Please note that the equations are calculated on the basis of updated guesses from the previous iteration, i.e., the updated guesses are not used until all the equations in the system are calculated. The absolute approximate error is calculated using the infinity  $\infty$  norm.

### (iii) Newton-Jacobi Method

$$F(x_1, x_2) := \begin{pmatrix} x_1^2 - \sin(x_2) \\ \ln\left(\frac{1}{x_1}\right) - e^{x_2} \end{pmatrix}$$

$$J(x_1, x_2) := \begin{pmatrix} \frac{d}{dx_1}F(x_1, x_2)_1 & \frac{d}{dx_2}F(x_1, x_2)_1 \\ \frac{d}{dx_1}F(x_1, x_2)_2 & \frac{d}{dx_2}F(x_1, x_2)_2 \end{pmatrix} \quad J(x_1, x_2) := \begin{pmatrix} 2 \cdot x_1 & -\cos(x_2) \\ -\frac{1}{x_1} & -e^{x_2} \end{pmatrix}$$

$X(x_1, x_2)$

$F(x_1, x_2)$

$J(x_1, x_2)$

$$X_0 := \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$

$$F(0.5, 0.5) = \begin{pmatrix} -0.2294 \\ -0.9556 \end{pmatrix}$$

$$J(0.5, 0.5) = \begin{pmatrix} 1 & -0.8776 \\ -2 & -1.6487 \end{pmatrix}$$

$$J(0.5, 0.5)^{-1} = \begin{pmatrix} 0.4844 & -0.2578 \\ -0.5876 & -0.2938 \end{pmatrix}$$

$$X_1 := X_0 - J(0.5, 0.5)^{-1} \cdot F(0.5, 0.5)$$

$$X_1 = \begin{pmatrix} 0.3648 \\ 0.0845 \end{pmatrix}$$

$$X_1 := \begin{pmatrix} 0.3648 \\ 0.0845 \end{pmatrix} \quad F(0.3648, 0.0845) = \begin{pmatrix} 0.0487 \\ -0.0798 \end{pmatrix}$$

$$J(0.3648, 0.0845) = \begin{pmatrix} 0.7296 & -0.9964 \\ -2.7412 & -1.0882 \end{pmatrix}$$

$$J(0.3648, 0.0845)^{-1} = \begin{pmatrix} 0.3087 & -0.2826 \\ -0.7776 & -0.207 \end{pmatrix}$$

$$X_2 := X_1 - J(0.3648, 0.0845)^{-1} \cdot F(0.3648, 0.0845)$$

$$X_2 = \begin{pmatrix} 0.3272 \\ 0.1058 \end{pmatrix}$$

$$X_2 := \begin{pmatrix} 0.3272 \\ 0.1058 \end{pmatrix} \quad F(0.3272, 0.1058) = \begin{pmatrix} 1.4571 \times 10^{-3} \\ 5.5841 \times 10^{-3} \end{pmatrix}$$

$$J(0.3272, 0.1058) = \begin{pmatrix} 0.6544 & -0.9944 \\ -3.0562 & -1.1116 \end{pmatrix}$$

$$J(0.3272, 0.1058)^{-1} = \begin{pmatrix} 0.2951 & -0.264 \\ -0.8114 & -0.1737 \end{pmatrix}$$

$$X_3 := X_2 - J(0.3272, 0.1058)^{-1} \cdot F(0.3272, 0.1058)$$

$$X_3 = \begin{pmatrix} 0.3282 \\ 0.108 \end{pmatrix}$$