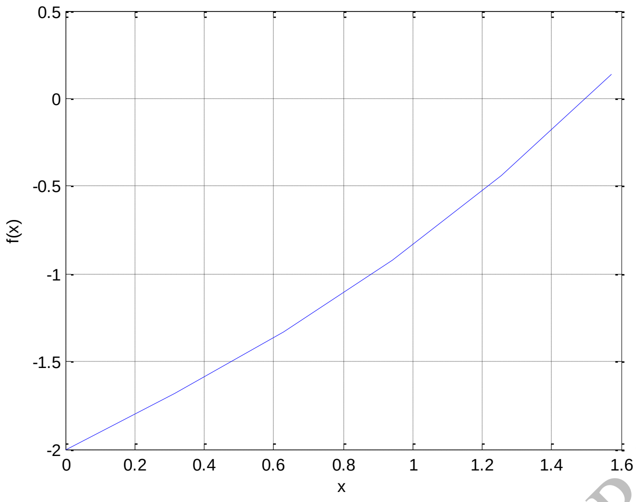
1. (a)



(b) After 4th iteration, x_{root} = 1.4987 with relative approximate percent error 0.05%.

(c) After 4th iteration, x_{root} = 1.4987 with relative approximate percent error 0%.

2. 1st alternative: g(x) = 1 + (sinx)/2

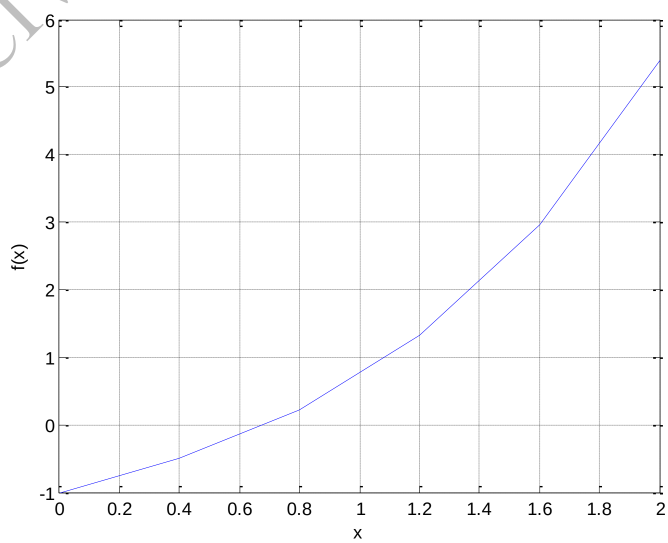
2nd alternative: g(x) = 3x – 2 – sinx

1st alternative: after 4th iteration, x_{root} = 1.4987 with relative approximate percent error 0.03%.

2nd alternative: after 4th iteration, x_{root} = -33.8 with relative approximate percent error 69%.

In the 2nd alternative, the | g'(x) | < 1 condition cannot be satisfied, hence divergence occurs.

3. i.



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ii. (a) When solved with a tolerance of 10^{-4}, $x_{root} = 0.6932$ at 15^{th} iteration.

(b) When solved with a tolerance of 10^{-4}, $x_{root} = 0.6931$ at 15^{th} iteration.

(c) When solved with a tolerance of 10^{-4}, $x_{root} = 0.6931$ at 4^{th} iteration.

iii. Newton-Raphson method is the fastest since it has quadratic convergence. Regula-Falsi is generally faster than bisection because the algorithm is improved.

4. (a) 1st alternative: $g(x) = (x/2)^4 + 3x^2 + x - 5$

2nd alternative: $g(x) = \sqrt{\frac{5}{3} - \frac{1}{3}\frac{x^4}{16}}$

3rd alternative: $g(x) = \sqrt[4]{80 - 48\,x^2}$

(b) 1st alternative is selected, after the third iteration $x_{root} = 30.410$, divergence occured.

(c) By satisfying the $|\, g'(x) \,| < 1$ condition, the solution converges to a root.

5. Rate of convegence: $\frac{|p_{n+1} - p|}{|p_n - p|^k} < \lambda$

where p is the real root, p_n and p_{n+1} are n^{th} and the $n{+}1^{th}$ estimations.

k = 1 linear convergence

k = 2 quadratic convergence

6. In the turbulent flow of fluid in a smooth pipe, the frictional force f can be calculated as follows:

$$\sqrt{\frac{1}{f}} = 2\log_{10}\left(Re\sqrt{f}\right) - 0.8$$

Where Re is the constant Reynolds number and f is a positive value which is smaller than 0.1.

For Re = 1000, the frictional force is found by bisection method in the interval [0.01 0.99]. f is found as 0.035.

7. $\ln(3) \cong 1.0942$

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