CE483 STRUCTURAL ANALYSIS

THREE DIMENSIONAL FRAME ELEMENT STIFFNESS MATRIX INCLUDING SHEAR DEFORMATIONS

$$\Phi = \frac{12EI}{GA_s L^2}$$

$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ F_{z1} \\ M_{x1} \\ M_{y1} \\ M_{z1} \\ \hline F_{x2} \\ F_{y2} \\ F_{z2} \\ M_{x2} \\ M_{y2} \\ M_{z2} \end{Bmatrix} = E \left[\begin{array}{cccccc|cccccc}
\frac{A}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{A}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12\,I_z}{L^3(1+\Phi)} & 0 & 0 & 0 & \frac{6\,I_z}{L^2(1+\Phi)} & 0 & -\frac{12\,I_z}{L^3(1+\Phi)} & 0 & 0 & 0 & \frac{6\,I_z}{L^2(1+\Phi)} \\
0 & 0 & \frac{12\,I_y}{L^3(1+\Phi)} & 0 & -\frac{6\,I_y}{L^2(1+\Phi)} & 0 & 0 & 0 & -\frac{12\,I_y}{L^3(1+\Phi)} & 0 & -\frac{6\,I_y}{L^2(1+\Phi)} & 0 \\
0 & 0 & 0 & \frac{J}{2L(1+\nu)} & 0 & 0 & 0 & 0 & 0 & -\frac{J}{2L(1+\nu)} & 0 & 0 \\
0 & 0 & -\frac{6\,I_y}{L^2(1+\Phi)} & 0 & \frac{(4+\Phi)\,I_y}{L(1+\Phi)} & 0 & 0 & 0 & \frac{6\,I_y}{L^2(1+\Phi)} & 0 & \frac{(2-\Phi)\,I_y}{L(1+\Phi)} & 0 \\
0 & \frac{6\,I_z}{L^2(1+\Phi)} & 0 & 0 & 0 & \frac{(4+\Phi)\,I_z}{L(1+\Phi)} & 0 & -\frac{6\,I_z}{L^2(1+\Phi)} & 0 & 0 & 0 & \frac{(2-\Phi)\,I_z}{L(1+\Phi)} \\
\hline
-\frac{A}{L} & 0 & 0 & 0 & 0 & 0 & \frac{A}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{12\,I_z}{L^3(1+\Phi)} & 0 & 0 & 0 & -\frac{6\,I_z}{L^2(1+\Phi)} & 0 & \frac{12\,I_z}{L^3(1+\Phi)} & 0 & 0 & 0 & -\frac{6\,I_z}{L^2(1+\Phi)} \\
0 & 0 & -\frac{12\,I_y}{L^3(1+\Phi)} & 0 & \frac{6\,I_y}{L^2(1+\Phi)} & 0 & 0 & 0 & \frac{12\,I_y}{L^3(1+\Phi)} & 0 & \frac{6\,I_y}{L^2(1+\Phi)} & 0 \\
0 & 0 & 0 & -\frac{J}{2L(1+\nu)} & 0 & 0 & 0 & 0 & 0 & \frac{J}{2L(1+\nu)} & 0 & 0 \\
0 & 0 & -\frac{6\,I_y}{L^2(1+\Phi)} & 0 & \frac{(2-\Phi)\,I_y}{L(1+\Phi)} & 0 & 0 & 0 & \frac{6\,I_y}{L^2(1+\Phi)} & 0 & \frac{(4+\Phi)\,I_y}{L(1+\Phi)} & 0 \\
0 & \frac{6\,I_z}{L^2(1+\Phi)} & 0 & 0 & 0 & \frac{(2-\Phi)\,I_z}{L(1+\Phi)} & 0 & -\frac{6\,I_z}{L^2(1+\Phi)} & 0 & 0 & 0 & \frac{(4+\Phi)\,I_z}{L(1+\Phi)}
\end{array}\right] \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ \theta_{x1} \\ \theta_{y1} \\ \theta_{z1} \\ \hline u_2 \\ v_2 \\ w_2 \\ \theta_{x2} \\ \theta_{y2} \\ \theta_{z2} \end{Bmatrix}$$