

**Homework 1- Date Due: March 20th, 2013 Wednesday till 16.00**

**IMPORTANT NOTICE:**

- You are allowed to collaborate with other students (or ask questions to your assistants/ instructors) on homework provided that you stay away from plagiarizing (according to dictionaries “to plagiarize” means to steal and pass off ideas and/or words/ solutions of another as one’s own without citing the source). That is, collaboration is accepted if you write and give your own solutions. If you are caught on plagiarizing or cheating by handing in “too similar” homework, you will be graded by zero on this homework.**

1. In your own words please explain the following terms and give at least two examples to clarify your explanation for each case.

- aleatory uncertainty/**
- epistemic uncertainty**
- probability**
- mutually exclusive events**
- probabilistically/statistically independent events**

2. Deformations that may develop over concrete bridge girders will cause a vertical harmonic excitation on a vehicle travelling over the bridge with constant speed. When the bridge deflection exceeds 1.6 cm vertical motion may occur on the vehicle depending on the speed of the vehicle. If the speed is less than 40 km/hr no vertical excitation will occur, otherwise, vertical harmonic excitation will occur (this is important for the design of shock absorbers to provide vibration-isolation system). The probability that the bridge deflection exceeds 1.6 cm is 3% and if the deflection exceeds 1.6 cm, the probability that the speed of the vehicle will be higher than 40 km/hr is 45%.

- Define all the events related with your problem, give a Venn Diagram.
- Find the probability that the vehicle over the bridge will be excited by vertical motion.
- If it is known that no vertical excitation has occurred on the vehicle find the probability that the deflection has not exceeded 1.6 cm.

3. Let  $E_1$ ,  $E_2$ , and  $E_3$  denote the events of excessive snowfall in the first, second, and third winters, respectively. Statistical records of snowfalls indicate that during any winter, the probability of excessive snowfall is 0.10. However, if excessive snowfall occurred in the previous winter, the probability of excessive snowfall in the following winter is increased to 0.40, whereas if the preceding two winters are both subjected to excessive snowfalls, the probability of excessive snow in the following winter will be 0.20.

- From the information given above, determine probability of the following:  
 $P(E_1)$ ,  $P(E_2)$ ,  $P(E_2|E_1)$ ,  $P(E_3|E_1 E_2)$ ,  $P(E_3|E_2)$
- What is the probability that excessive snowfall will occur in at least one of the next two winters?
- What is the probability that excessive snowfall will occur in each of the next three winters?
- If the preceding winter did not experience excessive snowfall, what is the probability that the subsequent winter will not suffer excessive snowfall? In other words, determine  $P(\bar{E}_2|\bar{E}_1)$ .

Hint: Start out with the following relationship:

$$P(\bar{E}_1 \cup \bar{E}_2) = 1 - P(\bar{E}_1 \cap \bar{E}_2) = 1 - P(E_1 E_2)$$

Note that notations for complement and intersection are as used in your text book.

E.g.  $\bar{A} = A'$  and  $AB = A \cap B$

4. Before the design of a tunnel through a rocky region, geological exploration was conducted to investigate the joints and potential slip surfaces that exist in the rock strata. For economic reasons, only portions of the strata are explored. In addition, the measurements recorded by the instruments are not perfectly reliable. Thus the geologist can only conclude that the condition of the rock may be either highly fissured (H), intermediately fissured (M), or slightly fissured (L) with relative likelihoods of 1 : 2 : 7. Based on this information, the engineer designs the tunnel and estimates that if the rock condition is L, the reliability of the proposed design is 95 %. However, if it turns out that the rock condition is M, the probability of failure will be doubled; similarly, if the rock condition is H, the probability of failure is 8 times that for condition L.

- What is the expected reliability of the proposed tunnel design?
- A more reliable device is subsequently used to improve the prediction of rock condition. Its results indicate that a highly fissured condition for the rock around the tunnel is practically impossible, but it cannot give better information on the relative likelihood between rock conditions M and L. In light of this information, what would be the revised reliability of the proposed tunnel design?
- If the tunnel collapsed, what should be the updated probabilities of M and L?