



Solve the following system of equations using

1. Gauss-Jacobi method
2. Gauss-Seidel method

with **4 decimal place accuracy** in both cases.

$$6x_1 + 2x_2 - x_3 = 5$$

$$-x_1 + 4x_2 + x_3 = 3$$

$$x_1 - 2x_2 + 5x_3 = 10$$

$$A = \begin{bmatrix} 6 & 2 & -1 \\ -1 & 4 & 1 \\ 1 & -2 & 5 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 5 \\ 3 \\ 10 \end{bmatrix}$$

1. Gauss-Jacobi Method:

$$x_1^{k+1} = \frac{5 - 2x_2^k + x_3^k}{6}, \quad x_2^{k+1} = \frac{3 + x_1^k - x_3^k}{4}, \quad x_3^{k+1} = \frac{10 - x_1^k + 2x_2^k}{5}$$

Initial guess: $X_0^T = (0 \ 0 \ 0)$

Error: $\max(|x_j^{k+1} - x_j^k|), j = 1, 2, 3$ (i.e., $\max(|x_1^{k+1} - x_1^k|, |x_2^{k+1} - x_2^k|, |x_3^{k+1} - x_3^k|)$)

$$\text{Error tolerance: } \varepsilon = \frac{1}{2} 10^{-4} = 0.00005$$

Iteration #	x_1	x_2	x_3	ε_{\max}
0	0.000000	0.000000	0.000000	
1	0.833333	0.750000	2.000000	2.000000
2	0.916667	0.458333	2.133333	0.291667
3	1.036111	0.445833	2.000000	0.133333
4	1.018056	0.509028	1.971111	0.063194
5	0.992176	0.511736	2.000000	0.028889
6	0.996088	0.498044	2.006259	0.013692
7	1.001695	0.497457	2.000000	0.006259
8	1.000848	0.500424	1.998644	0.002967
9	0.999633	0.500551	2.000000	0.001356
10	0.999816	0.499908	2.000294	0.000643
11	1.000080	0.499881	2.000000	0.000294
12	1.000040	0.500020	1.999936	0.000139
13	0.999983	0.500026	2.000000	0.000064
14	0.999991	0.499996	2.000014	0.000030

**2. Gauss-Seidel Method:**

$$x_1^{k+1} = \frac{5 - 2x_2^k + x_3^k}{6}, \quad x_2^{k+1} = \frac{3 + x_1^{k+1} - x_3^k}{4}, \quad x_3^{k+1} = \frac{10 - x_1^{k+1} + 2x_2^{k+1}}{5}$$

Initial guess: $X_0^T = (0 \ 0 \ 0)$

Error: $\max(|x_j^{k+1} - x_j^k|), j = 1, 2, 3$ (i.e., $\max(|x_1^{k+1} - x_1^k|, |x_2^{k+1} - x_2^k|, |x_3^{k+1} - x_3^k|)$)

Error tolerance: $\varepsilon = \frac{1}{2} 10^{-4} = 0.00005$

Iteration #	x_1	x_2	x_3	ε_{\max}
0	0.000000	0.000000	0.000000	
1	0.833333	0.958333	2.216667	2.216667
2	0.883333	0.416667	1.990000	0.541667
3	1.026111	0.509028	1.998389	0.142778
4	0.996722	0.499583	2.000489	0.029389
5	1.000220	0.499933	1.999929	0.003498
6	1.000011	0.500020	2.000006	0.000210
7	0.999994	0.499997	2.000000	0.000023