The CE305 Ltd. is a company whose one of the working area is to design and analyze the truss systems. Your task as an employee of this company is to check whether the displacements of truss members (δ_1 and δ_2) are below the allowable limit, which is defined as 3 cm (a very large value!!!) in this case. You are required to find the displacements using Newton-Jacobi method. The parameters of the truss system are given as follows:

l_{01} = 3 m, l_{02} = 4 m,

F_1 = 60 kN, F_2 = 80 kN

A_1 = 2 cm^2, A_2 = 2 cm^2

E_1 = 1E5 MPa, E_2 = 2E5 MPa

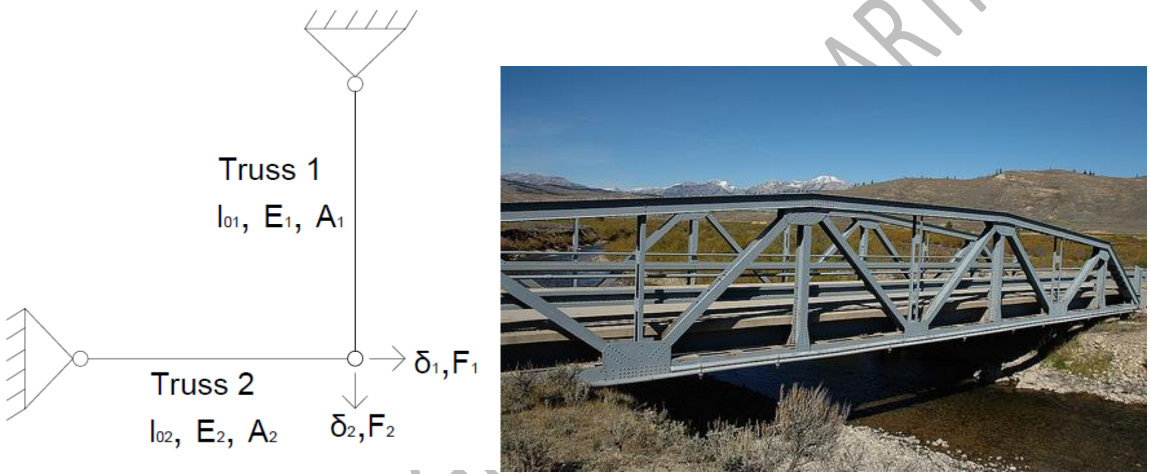

Figure 1 Truss Systems

Hint 1:

The truss system given above in Figure 1 needs to be modeled by a non-linear material model: Neo-Hookean material model (in 1-D), which is generally used to model large deformations.

The stresses in the truss members can be calculated by taking the derivative of the free energy function, which is defined for this specific model as follows:

$\Psi_i(\lambda_i) = \frac{1}{2} E_i (\lambda_i^2 - 1)^2$ for i=1,2

where E_i are the moduli of elasticity, and λ_1 and λ_2 denote the stretch in each bar. The stretch is defined through:

$\lambda = 1+\varepsilon$

where ε is the engineering strain.

Consequently, stretch in each truss can be calculated by:

$\lambda_i = \frac{l_i}{l_{oi}}$ for i=1,2 ………………………………………………………………….(1)

l_{oi} is the initial length of the trusses and l_i are the final lengths.

According to the above configuration of the truss system, l_1 and l_2 are calculated as:

$l_1 = \sqrt{(l_{01} + \delta_2)^2 + {\delta_1}^2}$ ……………………………………………………………….(2)

$l_2 = \sqrt{(l_{02} + \delta_1)^2 + {\delta_2}^2}$ ……………………………………………………………….(3)

The engineering stress in each structural member is found by taking the first derivative of free energy function with respect to stretch:

$\frac{\partial \Psi i}{\partial \lambda i} = \sigma_i = 2E_i \lambda_i (\lambda_i^2 - 1)$ for i=1,2

Similarly, the force in each truss is calculated as follows:

$F_i = 2E_i \lambda_i A_i (\lambda_i^2 - 1)$ for i=1,2 …………………………………………………………(4)

where A_i is the cross-sectional area.

Based on the above equations,

- stretch values for each truss can be calculated using Equation (4).
- the final lengths of trusses can be calculated from Equation (1).

From Equations (2) and (3), the only unknowns are the displacements of trusses, i.e. δ_1 and δ_2.

Then, the system of nonlinear equations to be solved is formed as follows:

$$\boxed{\begin{aligned} {\delta_1}^2 + {\delta_2}^2 + 2l_{01}\delta_2 + l_{01}^2 - l_1^2 = 0 \\ {\delta_1}^2 + {\delta_2}^2 + 2l_{02}\delta_1 + l_{02}^2 - l_2^2 = 0 \end{aligned}}$$

Hint 2: In the system of nonlinear equations, the only unknowns should be δ_1 and δ_2. Therefore, you first need to calculate stretch values from Equation (4). Then calculate the final lengths of trusses from Equation (1).

Note that Equations (2) and (3) are used to determine the system of nonlinear equations.

Let's take the error tolerance as 0.5E-6 and use the error definition: Error = $\max(|X^{k+1} - X^k|)$.

Initial guess can be chosen as as $X_0 = [0\ 0]^T$.