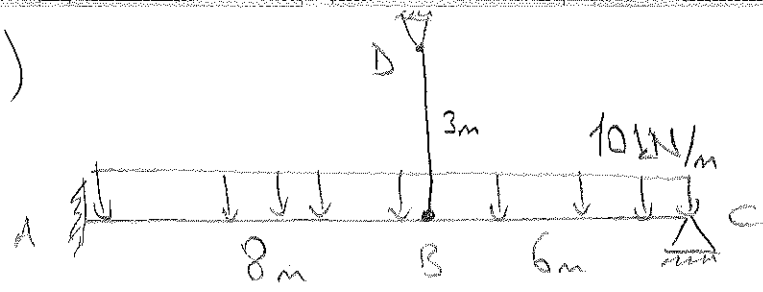


1)

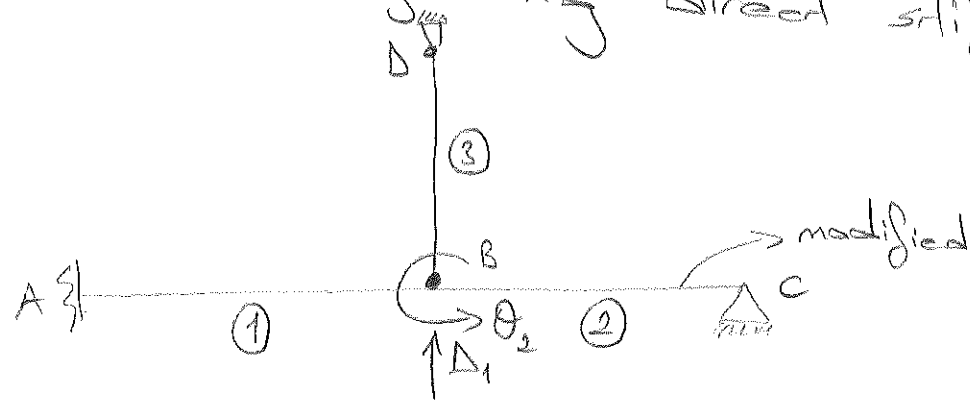


$$EI_{beam} = 100,000 \text{ kN.m}^2$$

$$EA_{druss} = 10,000 \text{ N}$$

Find support reactions at

support A by using direct stiffness method.



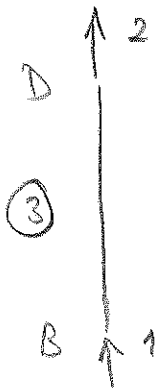
$$k_1 = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ \text{Symmetrical} & & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ & & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

$$k_1 = \begin{bmatrix} 2,344 & 3,375 & -2,344 & 3,375 \\ & 50 & -3,375 & 25 \\ \boxed{2,344} & \boxed{-3,375} & & \\ \boxed{-3,375} & \boxed{50} & & \end{bmatrix} \cdot 10^3$$

$$k_2 = \begin{bmatrix} \frac{3EI}{L^3} & \frac{3EI}{L^2} & -\frac{3EI}{L^3} \\ \frac{3EI}{L^2} & \frac{3EI}{L} & -\frac{3EI}{L^2} \\ \frac{3EI}{L^3} & \frac{3EI}{L^2} & \frac{3EI}{L} \end{bmatrix} = \begin{bmatrix} \boxed{1,383} & \boxed{8,333} & -1,383 \\ \boxed{8,333} & \boxed{50} & -8,333 \\ & & 1,383 \end{bmatrix} \cdot 10^3$$



$$k_3 = \frac{EA}{L} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} \boxed{3,333} & -3,333 \\ -3,333 & 3,333 \end{bmatrix} \cdot 10^3$$

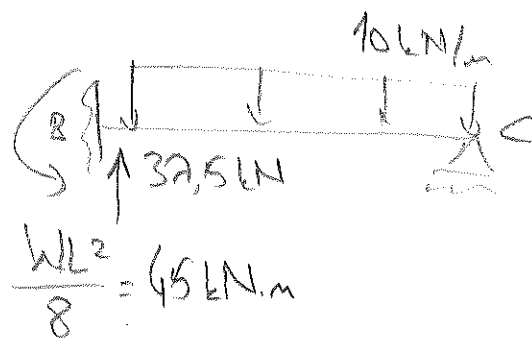
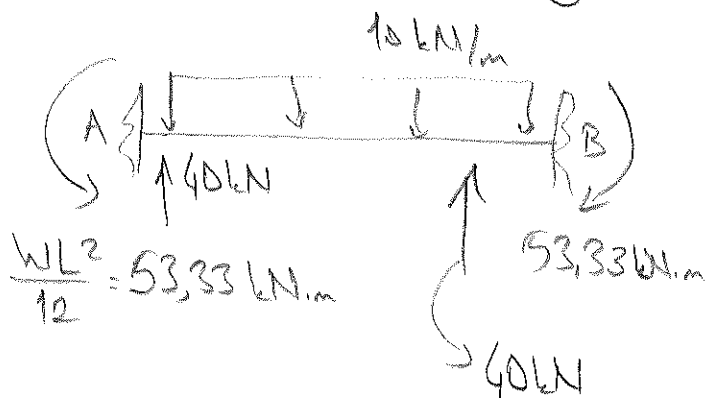


Form the structural stiffness matrix.

$$K_{str} = \begin{bmatrix} 2,334 + 1,389 + 3,333 & -9,375 + 8,333 \\ -9,375 + 8,333 & 50 + 50 \end{bmatrix} \cdot 10^3$$

$$K_{str} = \begin{bmatrix} 7,056 & -1,042 \\ -1,042 & 100 \end{bmatrix} \cdot 10^3$$

Form structural force vector.



$$F_{sdr} = \begin{bmatrix} -1 \cdot (40 + 37,5) \\ -1 \cdot (45 - 53,33) \end{bmatrix} = \begin{bmatrix} -77,5 \text{ kN} \\ 8,33 \text{ kN.m} \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

$$\Delta = K_{sdr}^{-1} \cdot F_{sdr} = \begin{bmatrix} -0,01099 \text{ m} \\ -3,12 \cdot 10^{-5} \text{ rad} \end{bmatrix}$$

$$F_A = FEM + k_{i,u} = \begin{bmatrix} 40 \\ 53,33 \\ 40 \\ -53,33 \end{bmatrix} + \begin{bmatrix} 2,344 & 3,375 & -2,344 & 3,375 \\ & 50 & -3,375 & 25 \\ \text{Symmetrical} & 2,344 & -3,375 & 50 \end{bmatrix} \cdot 10^3$$

$$F_A = \begin{bmatrix} 65,47 \text{ kN} \\ 155,58 \text{ kN.m} \\ 14,53 \text{ kN} \\ 48,14 \text{ kN.m} \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ -0,01099 \\ -3,12 \cdot 10^{-5} \end{bmatrix}$$

Displacement:

$$M_{AB} = FEM_{AB} + \frac{2EI}{8} \cdot \left( 2\theta_A + \theta_2 + \frac{3\Delta_1}{8} \right)$$

$$M_{AB} = 53,33 + \frac{2 \cdot 100.000}{8} \cdot \left( -3,12 \cdot 10^{-5} + \frac{3 \cdot 0,01099}{8} \right)$$

$$M_{AB} = 155,58 \text{ kN.m}$$

$$M_{BA} = FEM_{BA} + \frac{2EI}{8} \cdot \left( 2\theta_2 + \theta_A + \frac{3\Delta_1}{8} \right)$$

$$M_{BA} = -53,33 + \frac{2 \cdot 100.000}{8} \cdot \left( 2 \cdot -3,12 \cdot 10^{-5} + \frac{3 \cdot 0,01099}{8} \right) = 48,14 \text{ kN.m} \quad (3)$$

