CE - 366

CE - 366

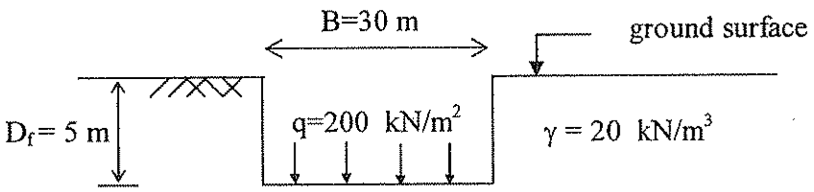
P & S

CE 366 – SITE INVESTIGATION (Problems & Solutions)

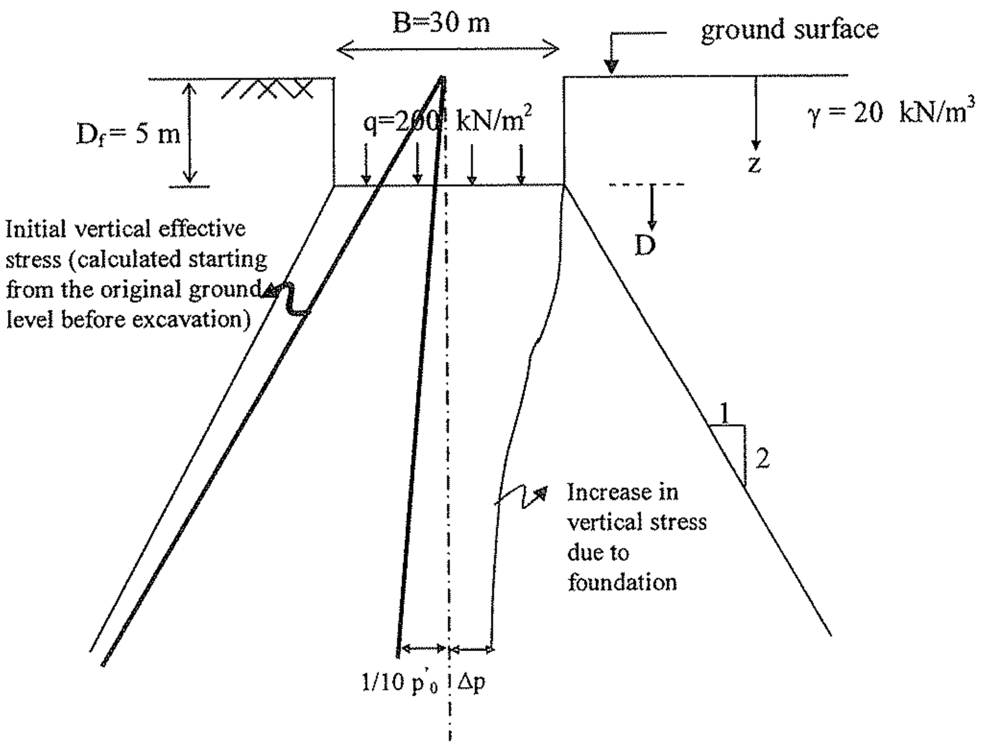
P1. DEPTH OF EXPLORATION

Question:

Find the depth of exploration from ground level for the 30x50 m, 15 storey building.



Solution:



Depth of exploration should reach such a depth where vertical stress increase due to weight of the structure would approximately be equal to the 10% of the initial effective overburden pressure:

De Beer's rule $\Rightarrow \therefore \frac{1}{10} p_0' = \Delta p$

$$\Delta p = \frac{200x30x50(kN)}{(30+D)(50+D)}$$

$$\frac{1}{10}p_0^{'} = \frac{1}{10}(20(D+5))$$

$$\frac{200x30x50}{(30+D)(50+D)} = \frac{1}{10}x20(D+5)$$

$$2D^3 + 170D^2 + 3800D = 285000$$

$$D=28 \text{ m}$$

Thus, depth of exploration, z =28+5=**33 m** from ground level

P2. CONE PENETRATION TEST (CPT)

Question:

Table 1. CPT Data

Depth (m)	q_c (MPa)	q_s (kPa)
0.5	1.86	22.02
1.5	1.16	28.72
2.5	2.28	24.89
3.5	0.29	12.44
4.5	0.38	15.32
5.5	0.40	14.74
6.5	6.90	28.72
7.5	9.20	26.81
8.5	8.45	43.09
9.5	9.50	34.60

- Indicate the soil classification by depth.
- Plot the cone penetration test data given in Table 1 including friction ratio F_R.
- Estimate undrained shear strength at depth 5.5 m assuming the cone factor N_k=18.
- Estimate angle of shearing resistance of the soil ($\phi^{'}$) at depth 7.5 m using the graph given in Figure 1.

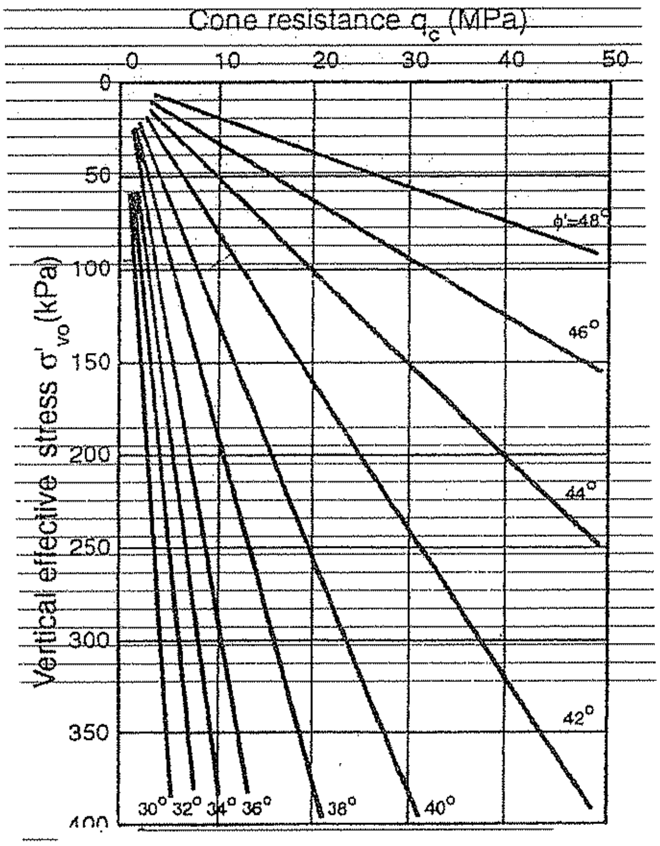


Figure 1. Correlation between cone data and angle of internal friction (Robertson and Campanella, 1983)

Assume an average γ=16.5 kN/m^3 to GWT at depth 3 m, and γ_{sat}=19.8 kN/m^3 for below GWT.

Solution:

Depth (m)	q_c (MPa/kPa)	q_s (kPa)	F_R (%) *[1]	Soil Classification*[2]
0.5	1.86 / 1860	22.02	1.19	Silt Mixtures – clayey silt to silty clay
1.5	1.16 / 1160	28.72	2.48	Clay – silty clay to clay
2.5	2.28 / 2280	24.89	1.09	Sand Mixtures – silty sand to sandy silt
3.5	0.29 / 290	12.44	4.29	Clay – silty clay to clay
4.5	0.38 / 380	15.32	4.03	Clay – silty clay to clay
5.5	0.40 / 400	14.74	3.69	Clay – silty clay to clay
6.5	6.90 / 6900	28.72	0.42	Sands – clean sand to silty sand
7.5	9.20 / 9200	26.81	0.29	Sands – clean sand to silty sand
8.5	8.45 / 8450	43.09	0.51	Sands – clean sand to silty sand
9.5	9.50 / 9500	34.60	0.36	Sands – clean sand to silty sand

*[1]: Note that; F_R (%) $= \frac{q_s}{q_c} x100$

*[2]: Soil classification is obtained by using Figure 2.19 in lecture notes.

- Plot of the Cone Penetration Test data (CPT) is shown in Figure 2.

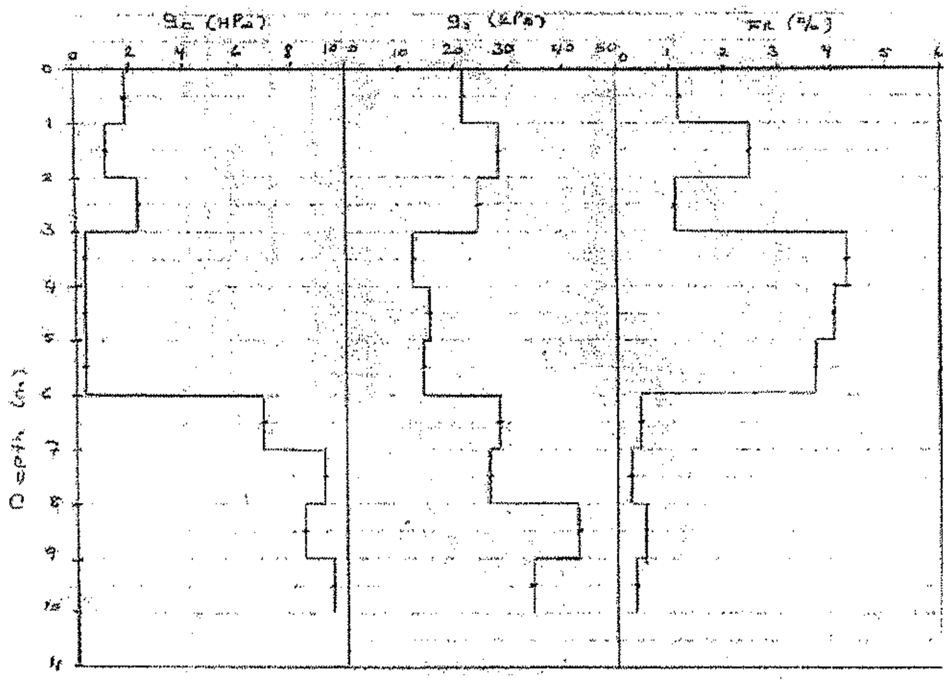


Figure 2. Plot of CPT data

- Estimate undrained shear strength at depth 5.5 m assuming the cone factor N_k=18.

$$c_u = \frac{q_c - p_o}{N_k}$$

where; p_o is total overburden pressure at the level of cone tip

Since γ_d=16.5 kN/m^3 and γ_{sat}=19.8 kN/m^3 then,

$$p_o = 3x16.5 + 2.5x19.8 = 99 \text{ kPa}$$

$$c_u = \frac{400-99}{18} = 16.7 \; kPa \text{ (@ z = 5.5 m)}$$

- Estimate ϕ' at depth 7.5 m from Figure 1 where p'_o= effective overburden pressure;

$$\left.\begin{array}{l}(p'_o @ z = 7.5\text{ m}) = [3 \times 16.5] + [4.5 \times (19.8\text{-}9.8)] = 94.5\text{ kPa} \\ \\ (q_c @ z = 7.5\text{ m}) = 9.2\text{ MPa}\end{array}\right\} \quad \phi = 41°$$

P3. STANDARD PENETRATION TEST (SPT)

Question:

Referring to Table 2 given below estimate the SPT-N_1 value you would use for a footing which is 2 x 2 m in dimensions and located at 2 m depth. Assume the unit weight of both sands is 18.1 kN/m^3 and 19.7 kN/m^3 above and below ground water table, respectively. GWT is at 6 m depth. Assume that the given N values are corrected for energy efficiency and field procedures.

Table 2. SPT Data

Depth (m)	N_{field}	Soil Type
1	6	Coarse Sand
2	9	Coarse Sand
3	10	Coarse Sand
4	8	Coarse Sand
5	7	Coarse Sand
6	9	Coarse Sand
7	22	Silty fine sand
8	28	Silty fine sand
9	31	Silty fine sand

Solution:

Depth (m)	**N**	**Soil Type**	**Silty Sand Correction, N' [1]**	**p'$_0$ (kPa) [2]**	**C_N [3]**	**N_1 [4]**
1	6	Coarse Sand	6 (no correction)	18.1	2.00	12
2	9	Coarse Sand	9 (no correction)	36.2	1.63	15
3	10	Coarse Sand	10 (no correction)	54.3	1.33	13
4	8	Coarse Sand	8 (no correction)	72.4	1.15	9
5	7	Coarse Sand	7 (no correction)	90.5	1.03	7
6	9	Coarse Sand	9 (no correction)	108.6	0.94	8
7	22	Silty fine sand	18.5≈19	118.3	0.90	17
8	28	Silty fine sand	21.5≈22	128.0	0.86	19
9	31	Silty fine sand	23	137.7	0.83	19

[1]: **<u>Silty sand correction</u>**: SPT-N values should be corrected for the increased resistance due to negative excess pore water pressure.

Applied when all three conditions are satisfied simultaneously:

(i) When the test is carried out in very fine sand or silty sand

(ii) When the test is carried out below ground water table

(iii) When N is greater than 15 ($N > 15$)

$$N' = 15 + \tfrac{1}{2}(N-15)$$

Where;

N': Silty sand corrected SPT-N value

Note that N' values should always be reported as rounded to the nearest integer.

[*2]: Overburden correction (C_N):

$$N_1 = N \times C_N$$

Where;

C_N: values are determined using equation 2.3 in Lecture Notes (page 31)

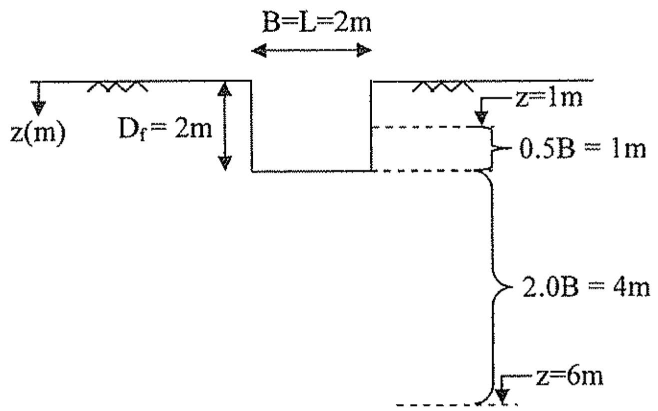
$$C_N = 9.78 \cdot \sqrt{\frac{1}{\sigma'_v (kN/m^2)}} \leq 2.00$$

N: is equal to N' if silty sand correction is applicable.

Note that N_1 values should always be reported as rounded to the nearest integer.

In order to estimate SPT-N_1 value to be used in calculations for a foundation, the SPT-N_1 values in depth interval of 0.5B above and 2B below the foundation level should be considered (where B = foundation width).

0.5B above 0.5 x 2 = 1 m above foundation level
2B below 2 x 2 = 4 m below foundation level
Depth interval is in between z = 1 m and z = 6 m.



Within this depth interval the weighted average of SPT-N_1 can be used solely for the analysis of given footing.

$$N_{1,av} = \frac{(12+15+13+9+7+8)}{6} = 11$$

P4. PLATE LOAD TEST (FIELD LOAD TEST)

a. Footing on Clay (load test on clay)

Question:

The results of a plate load test in stiff clay are shown in the Figure 3. The size of the test plate is 0.305 m x 0.305 m. Determine the size of a square column footing that should carry a load of 2500 kN. (FS = 2.0; maximum permissible settlement is 40 mm)

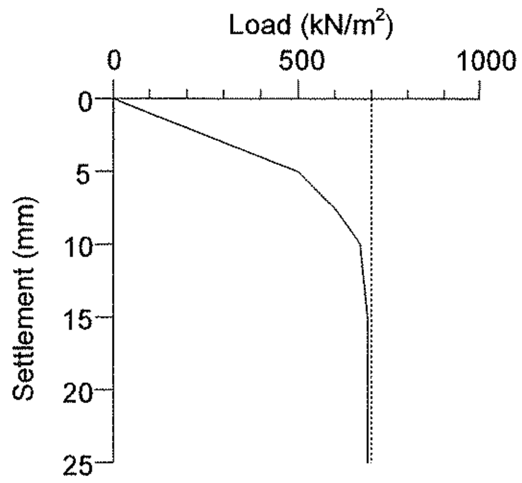
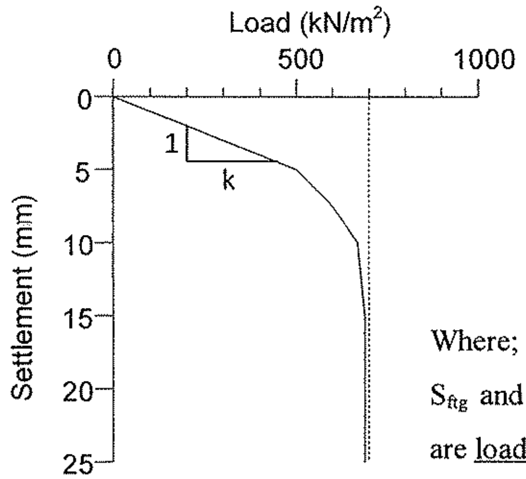


Figure 3. Plate load test in stiff clay

Solution:



Cohesive Soil

$(q_{ult})_{ftg} = (q_{ult})_{test}$

$$S_{ftg} = S_{test} \times \frac{B}{b}$$

Where;

S_{ftg} and S_{test} : settlements of footing and test plate which are <u>loaded with the same pressure,</u> respectively.

B (or B_F such as in Lecture Notes): width of footing

b (or B_P such as in Lecture Notes): width of test plate

$$(q_{ult})_{test} = 700\ kN/m^2 = (q_{ult})_{ftg}$$

$$q_{all} = \frac{700}{FS = 2.0} = 350kPa \Rightarrow \frac{2500}{BxB} = 350 \Rightarrow B = 2.7m$$

$$S_{ftg} = \underbrace{\frac{350x5}{500}}_{S_{test} \text{ under 350 kPa loading}} x \frac{2.7}{0.305} = 31mm < 40mm$$

Settlement calculation, however, is not very reliable; because it can not represent consolidation settlement. Generally, bearing capacity criteria governs, not the settlement, in the design of foundations resting on clays.

Coefficient of subgrade reaction: $k = \frac{q}{S}$ (kN / m^3) (k is the slope of q vs S graph, for more information about "k" see Lecture Notes p.112-113 or Ordemir p. 28)

$$k_{test} = \frac{500}{0.005} = 100000\ kN/m^3 = 100 MN/m^3$$

b. Load test on sand

Question:

The results of a plate load test in a sandy soil are shown in the Figure 4. Size of the test plate: 0.305 m x 0.305 m. Determine the size of square footing that should carry a load of 2500 kN with a maximum permissible settlement of 40 mm.

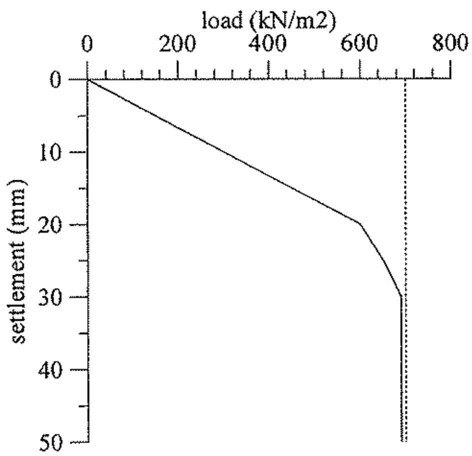
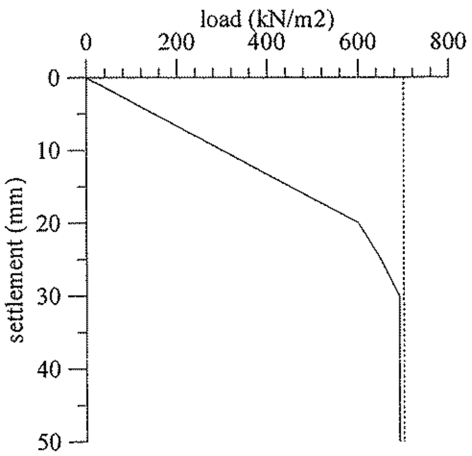


Figure 4. Plate load test in sandy soil

Solution:



Cohesionless Soils

$(q_{ult})_{ftg} = (q_{ult})_{test} \times \frac{B}{b}$

$S_{ftg} = S_{test}\left[\frac{B(b+0.3)}{b(B+0.3)}\right]^2$ (For typical plate dimension, b=0.3 m, this becomes equation 2.13 page 52 in Lecture Notes)

B and b are in meters!

Q (kN)	**Assumed B (m)**	**$q=Q/B^2$ (kPa)**	**S_{test} (mm)**	**S_{ftg} (mm)**
2500	4	156	5.2 (20/600x156)	17.7<40
	3	278	9.3	30.2<40
	2.5	400	13.3	41.7>40
	2.6	370	12.3	38.9
	2.55	384	12.8	40.4

Square column footing of 2.55 x 2.55 m dimensions will be appropriate.

-OR-

$$S_{ftg} = 40 = \underbrace{\frac{2500}{B^2} x \frac{20}{600}}_{S_{test}} \times \left[\frac{B(0.305+0.3)}{0.305(B+0.3)}\right]^2 \Rightarrow B = 2.56m$$

Thus,

$$q_{ftg} = q_{test} x \frac{B}{b} = 700x\frac{2.55}{0.305} = 5852 kN/m^2$$

$$F.S. = \frac{5852}{384.5} = 15.2 \quad (\textit{settlement governs design})$$

Thus, factor of safety against bearing capacity (*F.S.*) is,

Thus, coefficient of subgrade reaction (*k*) is,

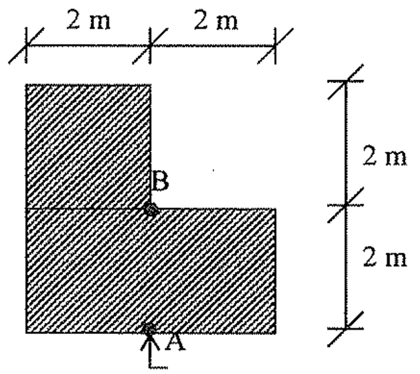
$$k = \frac{600}{0.020} = 30 MN/m^3$$

CE 366 – SETTLEMENT (Problems & Solutions)

P. 1) LOAD UNDER A RECTANGULAR AREA (1)

Question:

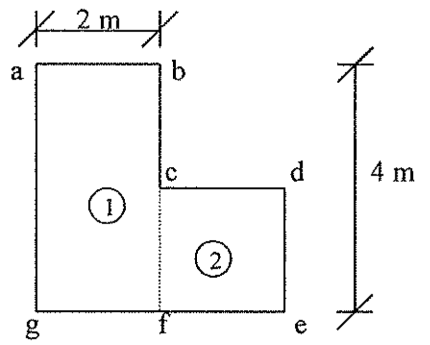
The footing shown in the figure below exerts a uniform pressure of 300 kN/m^2 to the soil. Determine **vertical stress increase due to uniform pressure,** at a point of 4 m directly under; (a) point A, (b) point B.



L – Shaped Footing (Plan view)

Solution:

a) Point A:



$\Delta\sigma_z = q.I_r$

By the use of Figure 1.6 in Lecture Notes, page 10;

- *For area 1 : A(abcfg)*

$z = 4$ m $\longrightarrow$ $mz = 4$, $nz = 2$ $\longrightarrow$ $m = 4/4 = 1$, $n = 2/4 = 0.5$ $\longrightarrow$ $I_r = 0.12$

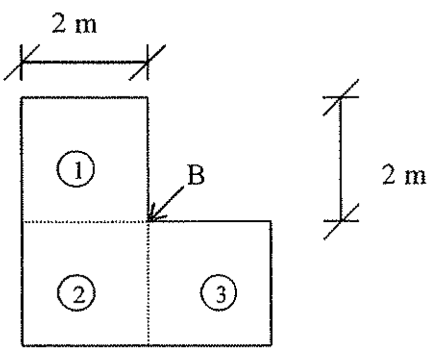
- *For area 2 : A(cdef)*

$z = 4$ m $\longrightarrow$ $mz = 2$, $nz = 2$ $\longrightarrow$ $m = 2/4 = 0.5$, $n = 2/4 = 0.5$ $\longrightarrow$ $I_r = 0.085$

$\Delta\sigma_z = 300\ (0.12 + 0.085) = 61.5$ kPa

↳ the stress at 4 m depth under point A due to 300 kN/m^2 uniform pressure

b) Point B;



Area 1 = Area 2 = Area 3

$\longrightarrow$ $mz = nz = 2$ $\longrightarrow$ $m = n = 2/4 = 0.5$ $\longrightarrow$ $I_r = 0.085$

$\Delta\sigma_z = 300\ (3 \times 0.085)$

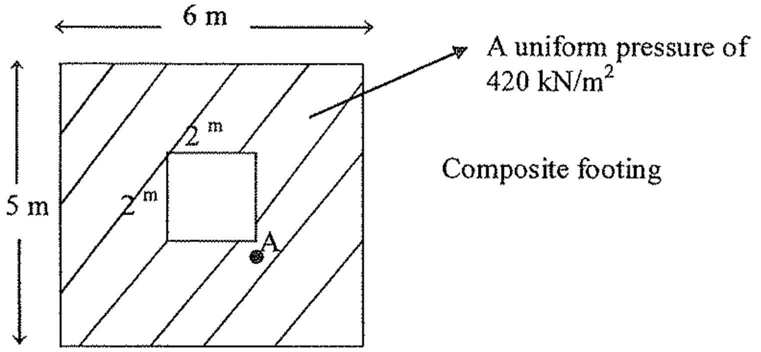
$= 76.5$ kPa

↳ the stress at 4 m depth under point B due to 300 kN/m^2 uniform pressure

P. 2) LOAD UNDER A RECTANGULAR AREA (2)

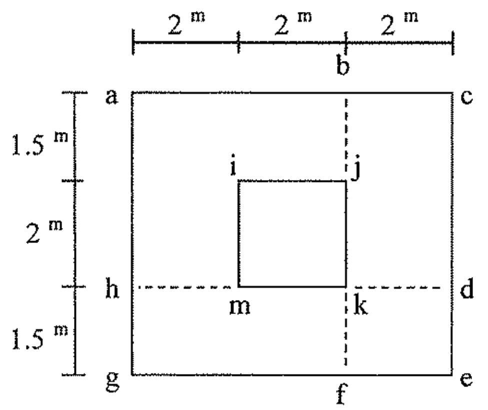
Question:

A rectangular footing as shown in figure below exerts a uniform pressure of 420 kN/m^2. Determine the vertical stress due to uniform pressure at point A for a depth of 3 m.



Plan View

Solution:



For area (abkh) :

$z = 3^m$ $\quad \left.\begin{matrix} mz = 4 \\ nz = 3.5 \end{matrix}\right\} \quad \left.\begin{matrix} m = 4 / 3 = 1.33 \\ n = 3.5 / 3 = 1.17 \end{matrix}\right\} \quad I_r = 0.195$

For area (bcdk) :

$\left.\begin{matrix} mz = 3.5 \\ nz = 2 \end{matrix}\right\} \quad \left.\begin{matrix} m = 3.5 / 3 = 1.17 \\ n = 2 / 3 = 0.67 \end{matrix}\right\} \quad I_r = 0.151$

For area (defk) :

$\left.\begin{matrix} mz = 2 \\ nz = 1.5 \end{matrix}\right\} \quad \left.\begin{matrix} m = 2 / 3 = 0.67 \\ n = 1.5 / 3 = 0.5 \end{matrix}\right\} \quad I_r = 0.105$

For area (fghk) :

$$\left.\begin{array}{l} mz = 4 \\ nz = 1.5 \end{array}\right\} \quad \left.\begin{array}{l} m = 4 / 3 = 1.33 \\ n = 1.5 / 3 = 0.5 \end{array}\right\} \qquad I_r = 0.133$$

For area (ijkm) :

$$mz = nz = 2 \longrightarrow m = n = 2 / 3 = 0.67 \longrightarrow I_r = 0.117$$

$$\Delta\,\sigma_z = \sigma \cdot I_r$$

$$= 420\,[\,I_{r\,1} + I_{r\,2} + I_{r\,3} + I_{r\,4} - I_{r\,5}\,]$$

$$= 420\,[\,0.195 + 0.151 + 0.105 + 0.133 - 0.117\,]$$

$\boldsymbol{\Delta\,\sigma_z = 196.14}$ **kPa**

Note: Where do we use the vertical stress increase, $\Delta\sigma_z$, values?
For example, in a consolidation settlement problem, stress increase, $\Delta\sigma_z$, values are needed to calculate settlement under a foundation loading. We make the following calculations for a point located under the foundation at a certain depth (for example, at the mid-depth of the compressible layer):

(1) First, calculate the initial effective vertical stress, $\sigma'_{v,o}$, before the building was constructed,
(2) Then, find the vertical stress increase $\Delta\sigma_z$ at that depth, by using Boussinesq stress distribution or by approximate methods (for example 2V: 1H approximation)
(3) Find the final effective vertical stress, $\sigma'_{v,f} = \sigma'_{v,o} + \Delta\sigma_z$, after the building is constructed.
(4) Use these values in calculating the settlement under the foundation.

P. 3) IMMEDIATE SETTLEMENT

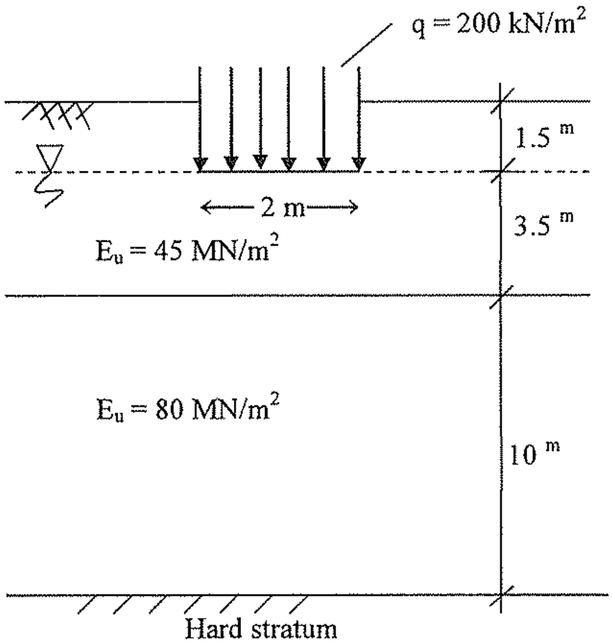
Question:

A foundation 4 ^{m} × 2 ^{m}, carrying a net uniform pressure of 200 kN/m^2, is located at a depth of 1.5 m in a layer of clay 5 m thick for which the value of E_u is 45 MN/m^2. The layer is underlain by a second layer, 10 m thick, for which the value of E_u is 80 MN/m^2. A hard stratum lies below the second layer. Ground water table is at depth of foundation. Determine the average immediate settlement under the foundation.

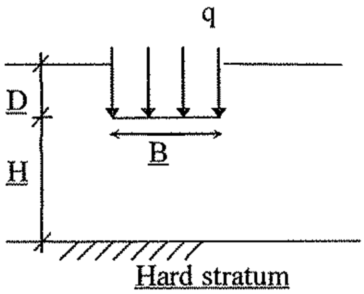
Hint: Since soil is SATURATED CLAY, v_s=0.5. So the following equation can be used:

$$S_i = \mu_0 \cdot \mu_1 \cdot \frac{q \cdot B}{E_u}$$

Solution:



$$S_i = \mu_0 \cdot \mu_1 \cdot \frac{q \cdot B}{E_u}$$

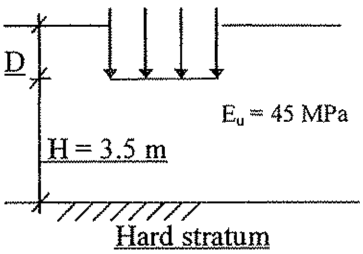


B is the smaller dimension !

We obtain,
μ_0 from D / B
μ_1 from H / B and L / B

D / B = 1.5 / 2 = 0.75 → $\mu_0 = 0.95$ (Figure 3.3, p.62 Lecture Notes)

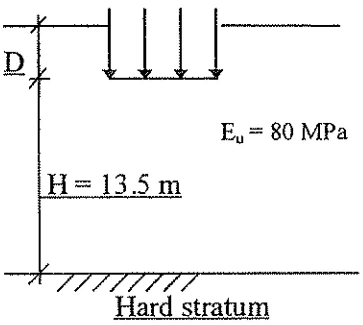
(1) Consider the upper layer with E_u = 45 MPa.



$$\left.\begin{array}{l} H / B = 3.5 / 2 = 1.75 \\ L / B = 4 / 2 = 2 \end{array}\right\} \mu_1 = 0.65$$

$$S_{i1} = \mu_0 \cdot \mu_1 \cdot \frac{q \cdot B}{E_u} = (0.95) \cdot (0.65) \cdot \frac{(200) \cdot 2}{45} = 5.49mm$$

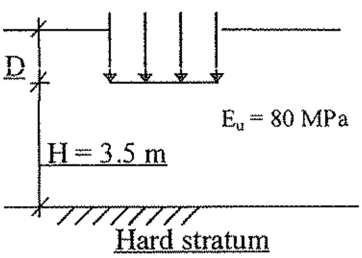
(2) Consider the two layers combined with E_u = 80 MPa.



$$\left.\begin{array}{l} H / B = (3.5 + 10) / 2 = 6.75 \\ L / B = 4 / 2 = 2 \end{array}\right\} \mu_1 = 0.9$$

$$S_{i2} = \mu_0 \cdot \mu_1 \cdot \frac{q \cdot B}{E_u} = (0.95) \cdot (0.9) \cdot \frac{(200) \cdot 2}{80} = 4.28mm$$

(3) Consider the upper layer with E_u = 80 MPa.



$$\left.\begin{array}{l} H / B = 3.5 / 2 = 1.75 \\ L / B = 4 / 2 = 2 \end{array}\right\} \mu_1 = 0.65$$

$$S_{i3} = \mu_0 \cdot \mu_1 \cdot \frac{q \cdot B}{E_u} = (0.95) \cdot (0.65) \cdot \frac{(200) \cdot 2}{80} = 3.08mm$$

Using the principle of superposition, the settlement of the foundation is given by;

$S_i = S_{i\,1} + S_{i\,2} - S_{i\,3}$

$S_i = 5.49 + 4.28 - 3.08$

$S_i = \mathbf{6.69\ mm}$

P. 4) SCHMERTMAN

Question:

A soil profile consists of deep, loose to medium dense sand (γ_{dry} = 16 kN/m^3 , γ_{sat} = 18 kN/m^3). The ground water level is at 4 m depth. A 3.5 m x 3.5 m square footing rests at 3 m depth. The **total (gross) load** acting at the foundation level (footing weight + column load + weight of soil or footing) is 2000 kN. Estimate the elastic settlement of the footing 6 years after the construction using influence factor method (Schmertman, 1978).

End resistance values obtained from static cone penetration tests are;

Depth (m)	q_c (kN/m^2)
0.00 – 2.00	8000
2.00 - 4.75	10000
4.75 - 6.50	8000
6.50 – 12.00	12000
12.00 – 15.00	10000

Note that;

- for square footing;

z (depth)(from foundation level)	I_z (strain factors)
0	0.1
B/2	0.5
2B	0.0

Where; B : width of footing

- $E_s = 2.0\ q_c$

Solution:

$$S_i = C_1\, C_2\, q_{net}\, \Sigma\, \frac{I_z}{E}\Delta z$$

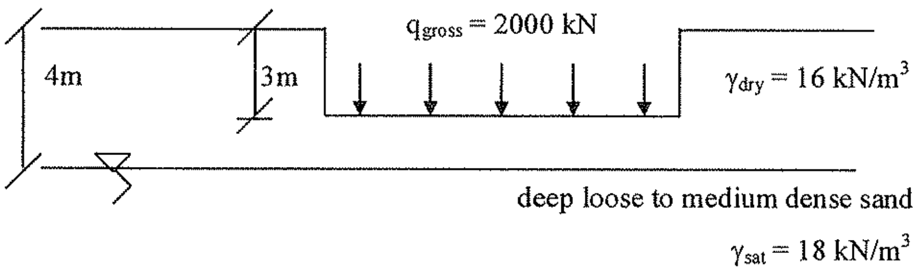
q_{net} = net foundation pressure

$$C_1 = 1 - 0.5\frac{\sigma'_o}{q_{net}} \longrightarrow \text{correction factor for footing depth}$$

σ'_o = effective overburden pressure at foundation level

$$C_2 = 1 + 0.2\,\log\frac{t}{0.1} \longrightarrow \text{correction factor for creep}$$

t = time at which the settlement is required (in years)

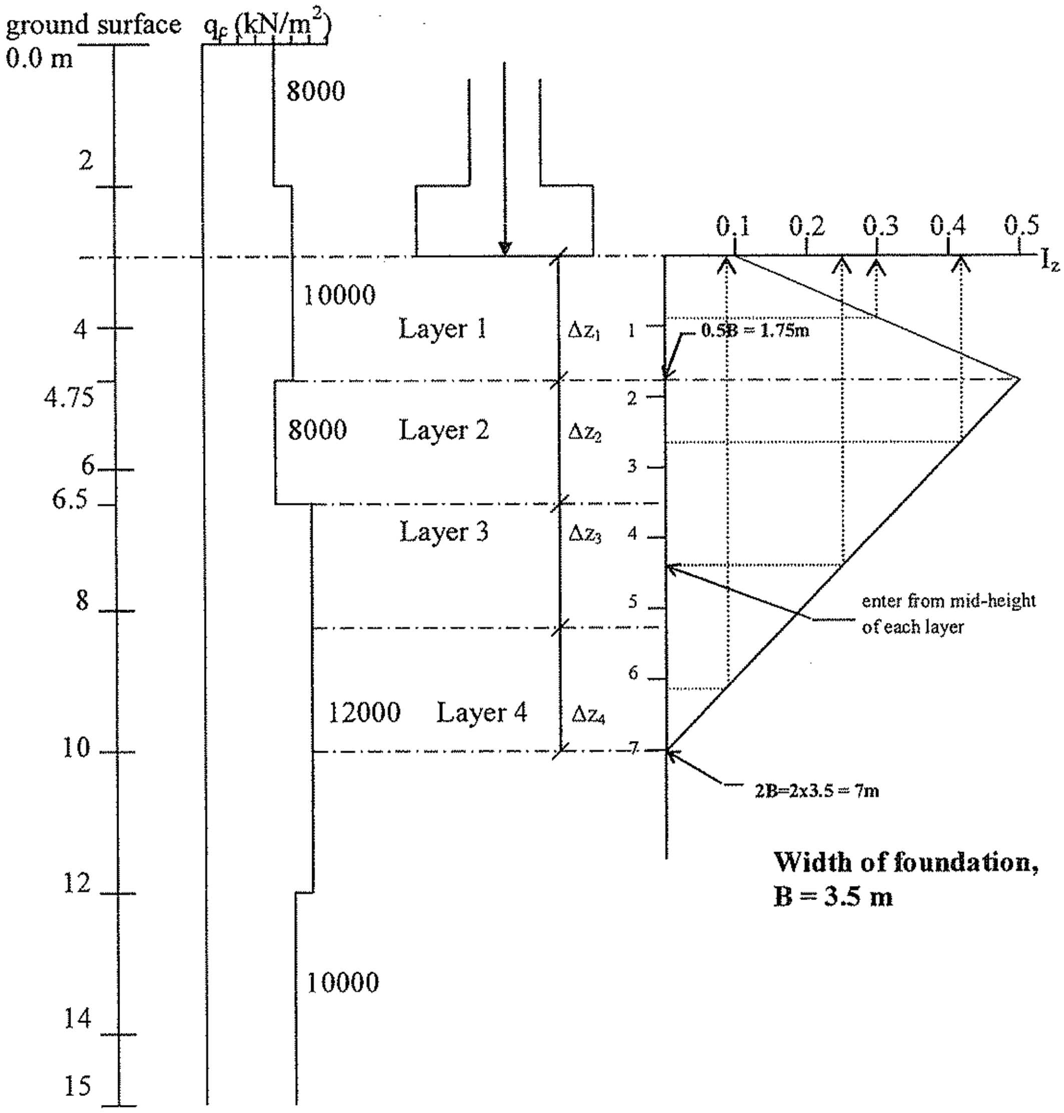


$$q_{net} = \underbrace{\frac{2000}{3.5x3.5}}_{\text{gross pressure.}} - \underbrace{3x16}_{\text{initial effective overburden pressure}} = 115.26\, kPa$$

$\sigma_o' = 3x16 = 48$ kPa

$$C_1 = 1 - 0.5\frac{48}{115.26} = 0.792$$

$$C_2 = 1 + 0.2\,\log\frac{6}{0.1} = 1.356$$



$E_s = 2.0\ q_c$

Layer No	Depth(m)	Δz(m)	q_c(kPa)	Es(kPa)	I_z	$(I_z/E_s)\ \Delta z$
1	3.00-4.75	1.75	10.000	20.000	0.3	$2.65\text{x}10^{-5}$
2	4.75-6.50	1.75	8.000	16.000	0.416	$4.55\text{x}10^{-5}$
3	6.50-8.25	1.75	12.000	24.000	0.249	$1.82\text{x}10^{-5}$
4	8.25-10.00	1.75	12.000	24.000	0.083	$0.605\text{x}10^{-5}$

$\Sigma = 9.625\text{x}10^{-5}$

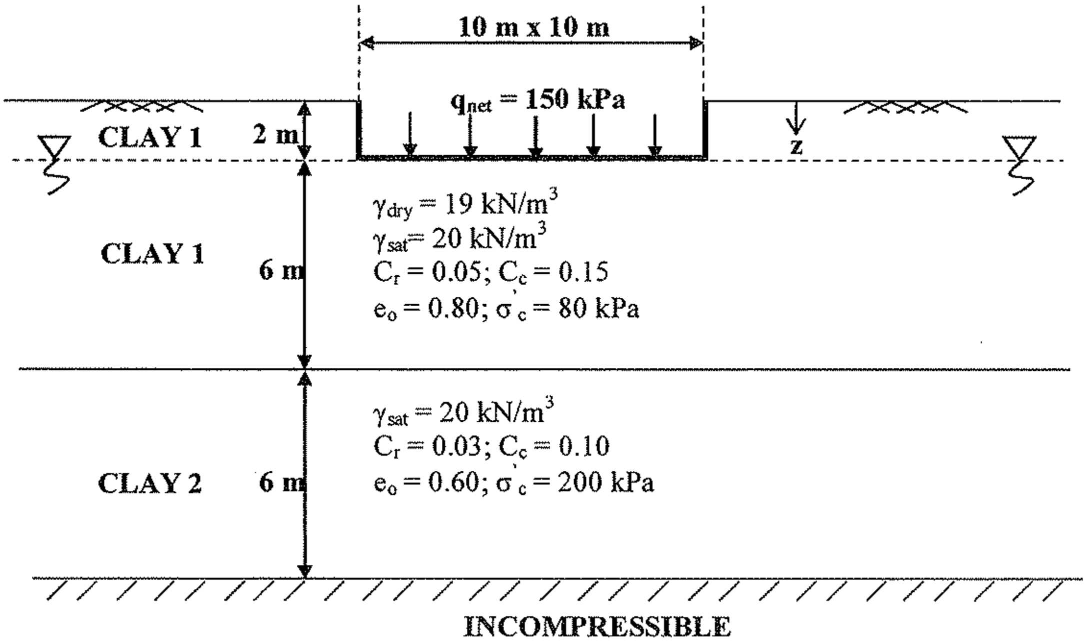
$S_i = (0.792)\,(1.356)\,(115.26)\,(9.625\text{x}10^{-5})$

$= 0.01191$ m $\longrightarrow$ $\mathbf{S_i = 11.91\ mm}$

P5. CONSOLIDATION SETTLEMENT

Question:

Ignore the immediate settlement, and calculate total consolidation settlement of soil profile composed of two different types of clay, i.e. Clay 1 and Clay 2 due to 150 kPa net foundation loading. Take unit weight of water as 10 kN/m^3 and assume that Skempton-Bjerrum Correction Factor is $\mu = 0.7$ for both clay layers. Note that σ'_c (or sometimes shown as σ'_p) is the preconsolidation pressure.



Solution:

Settlement will take place due to loading (q_{net} = 150 kPa) applied at a depth of 2 m. Thus, all (consolidation) settlement calculations will be performed for clayey soil beneath the foundation (z > 2 m).

Reminder: General equation of 1D consolidation settlement (one dimensional vertical consolidation) for an overconsolidated clay is;

$$S_{c,1D} = \frac{C_r}{1+e_o} H \log\left(\frac{\sigma'_c}{\sigma'_{v,o}}\right) + \frac{C_c}{1+e_o} H \log\left(\frac{\sigma'_{v,f}}{\sigma'_c}\right)$$

*Note that, all calculations are done for the **mid-depth** of the compressible layers under the loading.*

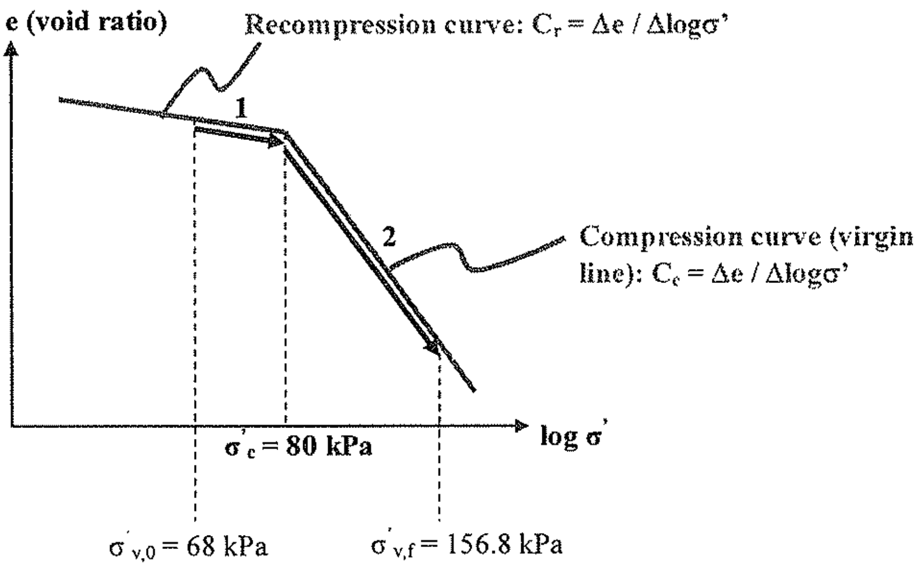
Consolidation settlement in Clay 1:

Initial effective overburden stress, $\sigma'_{v,o}$ = (2*19) + (3*(20-10)) = 68 kPa

Stress increment due to foundation loading, $\Delta\sigma$= [150*(10*10)] / [(10+3)*(10+3)] = 88.8 kPa

Final stress, $\sigma'_{v,f}$ = 68 + 88.8 = 156.8 kPa

This is an overconsolidated clay (overconsolidation ratio OCR = σ'_c / $\sigma'_{v,o}$ = 80 / 68 > 1.0) ; and the final stress, $\sigma'_{v,f}$ is greater than σ'_c ($\sigma'_{v,f} > \sigma'_c$) therefore we should use both C_r and C_c in consolidation settlement calculation (see figure below).



$$S_{c,1D} = \underbrace{\left\{\frac{0.05}{1+0.80}(6)\log\left(\frac{80}{68}\right)\right\}}_{1} + \underbrace{\left\{\frac{0.15}{1+0.80}(6)\log\left(\frac{156.8}{80}\right)\right\}}_{2} = 0.158m = 15.8\ cm$$

Consolidation settlement in Clay 2:

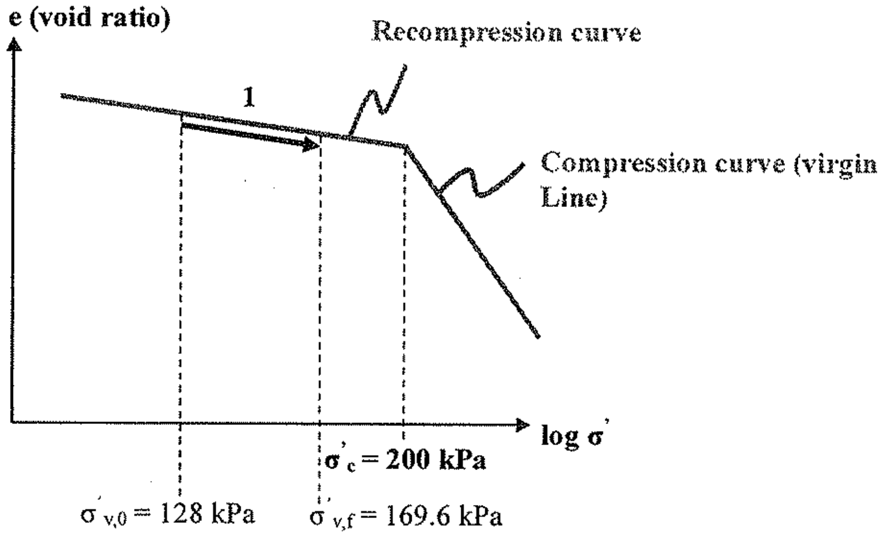
Initial effective overburden stress, $\sigma'_{v,0}$ = (2*19) + (6*(20-10)) + (3*(20-10)) = 128 kPa

Stress increment due to foundation loading, $\Delta\sigma$= [150*(10*10)] / [(10+9)*(10+9)] = 41.6 kPa

Final stress, $\sigma'_{v,f} = 128 + 41.6 = 169.6$ kPa

This is an overconsolidated clay (overconsolidation ratio OCR = $\sigma'_c / \sigma'_{v,o} = 200 / 128 > 1.0$) ; and the final stress, $\sigma'_{v,f}$ is less than σ'_c ($\sigma'_{v,f} < \sigma'_c$) therefore we should use only C_r in consolidation settlement calculation (see figure below).

[Note: If a soil would be a normally consolidated clay (OCR = $\sigma'_c / \sigma'_{v,o} = 1.0$), we would use only C_c in consolidation settlement calculation.]



$$\underbrace{S_{c,1-D} = \left\{\frac{0.03}{1+0.60}(6)\log\left(\frac{169.6}{128}\right)\right\}}_{1} = 0.014\,m = 1.4\,cm$$

Total Consolidation Settlement (1D):

$$S_{c,1D} = 15.9 + 1.4 = 17.3cm$$

Corrected Settlement for 3D Consolidation (Skempton-Bjerrum Factor):

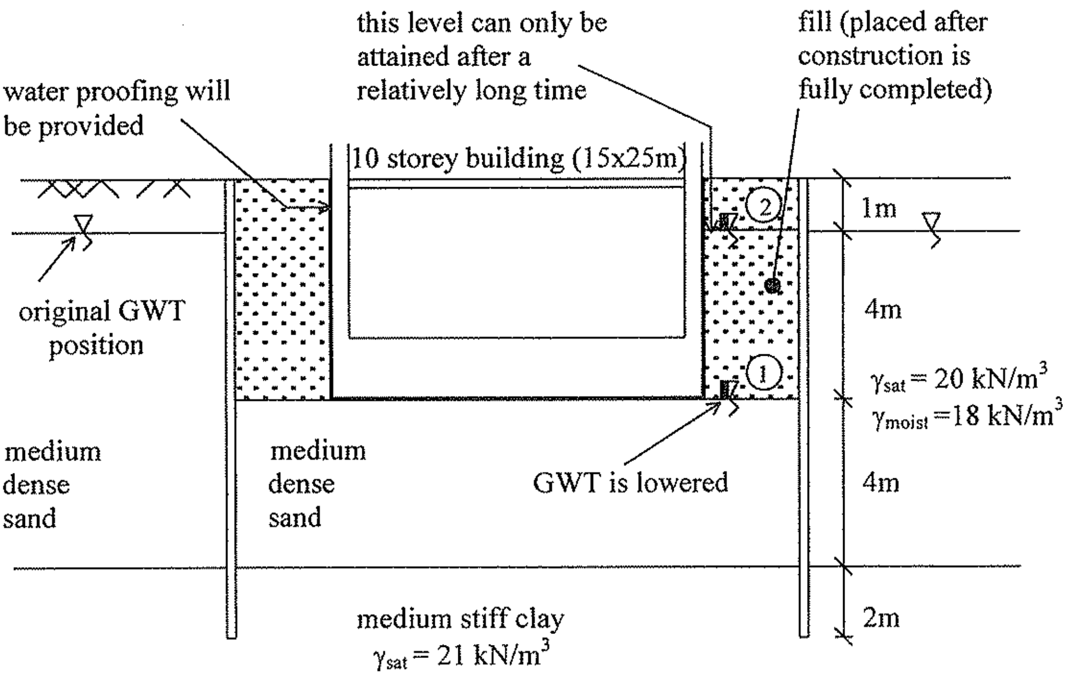
$$S_{c,3D} = S_{c,1D} * \mu = 17.3 * 0.7 = \mathbf{12.1\ cm}$$

CE 366 – BEARING CAPACITY (Problems & Solutions)

P1

Question:

An excavation will be made for a ten storey 15x25 m building. Temporary support of earth pressure and water pressure will be made by deep secant cantilever pile wall. The gross pressure due to dead and live loads of the structure and weight of the raft is 130 kPa (assume that it is uniform).

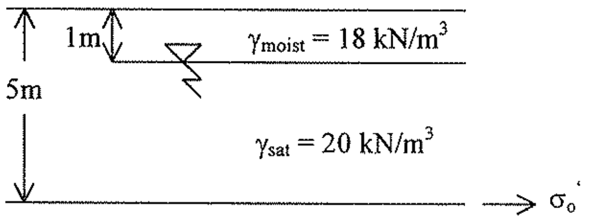


a) What is net foundation pressure at the end of construction but before the void space between the pile wall and the building has been filled, and *there is no water inside the foundation pit yet* (water level at the base level) (GWT position 1).

b) What is net foundation pressure long after the completion of the building, i.e. water level is inside the pile wall and the backfill between the building and the pile wall is placed (GWT position 2). What is the factor of safety against uplift?

Solution:

a) $q_{net} = \begin{bmatrix} \text{final effective stress} \\ \text{at foundation level} \end{bmatrix} - \begin{bmatrix} \text{initial effective stress} \\ \text{at foundation level} \end{bmatrix}$



<u>$\sigma_o' = 18x1 + 4x(20\text{-}9.8) = 58.8$ kPa</u>

(gross pressure – uplift pressure) = final effective stress at foundation level, σ_f'

gross pressure =130 kPa (given)

uplift pressure = 0 kPa (Since GWT is at foundation level (1), it has no effect on structure load)

$\sigma_f' = 130 - 0 = 130$ kPa
$q_{net} = 130 - 58.8$
$= \mathbf{71.2\ kPa}$

b) $\sigma_f' = 130 - \underbrace{4x9.8}_{\text{uplift pressure}} = 90.8$ kPa

$\sigma_o' = 58.8$ kPa (same as above)

$q_{net} = 90.8 - 58.8$
$= \mathbf{32.0\ kPa}$

<u>OR</u>

$q_{net} = q_{gross} - \gamma_{sat} D = 130\text{-}(18x1+4x20)$
$= \mathbf{32.0\ kPa}$

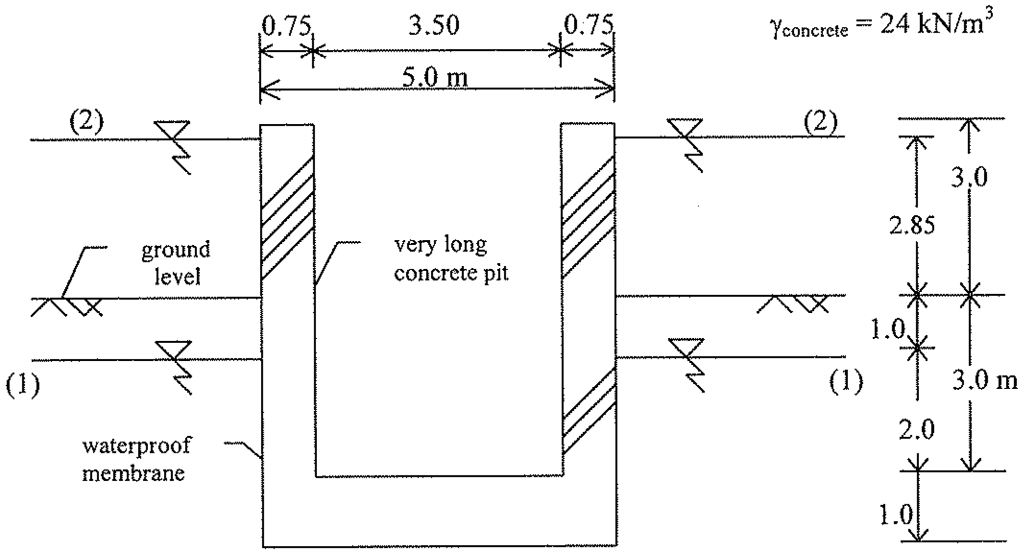
<u>Factor of safety against uplift is:</u>

$(FS)_{uplift}$ = weight of structure / uplift
= (130x15x25) / (4x9.8x15x25)
= **3.3**

P2

Question:

Calculate the FS against uplift and calculate effective stress at the base level for water level at (1) and (2) for the canal structure given below. Note that the canal is very long into the page.



Solution:

- **water table at (1)**

Factor of Safety against uplift = $\underbrace{(2x6x0.75 + 5x1)x24}_{\Sigma\text{weight of pit}}$ / $\underbrace{(3x5)x9.8}_{\text{uplift}}$

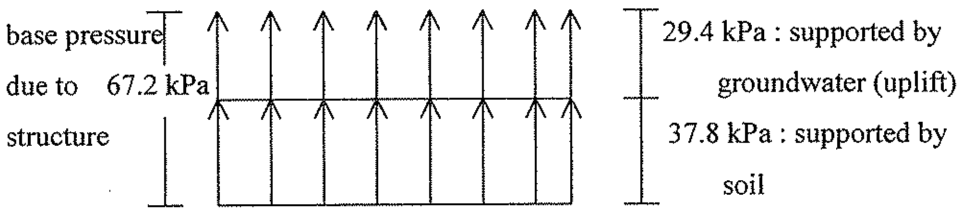
= 336 / 147

= **2.28**

Base pressure = 336 / 5 = 67.2 kN/m^2 due to weight of structure.(per meter of canal)

147 / 5 = 29.4 kN/m^2 is supported by groundwater

67.2 – 29.4 = 37.8 kN/m^2 is supported by soil (effective stress at the base)



- <u>**water table at (2)**</u>

FS = 336 / (6.85x5x9.8)

= 1.0 < 1.5 NOT OKEY

⇒ base pressure = 67.2 kPa is supported by ground water

uplift = weight of structure

<u>Soil does not carry any load, structure tends to float</u>

P3

Question:

A residential block will be constructed on a clay deposit. The building will rest on a mat foundation at 2m depth and has 20mx20m dimensions in plan.

The clay deposit is 26m deep and overlies limestone. The groundwater level is at 2m depth. The bulk unit weights are 18 and 20 kN/m^3 above and below water table respectively.

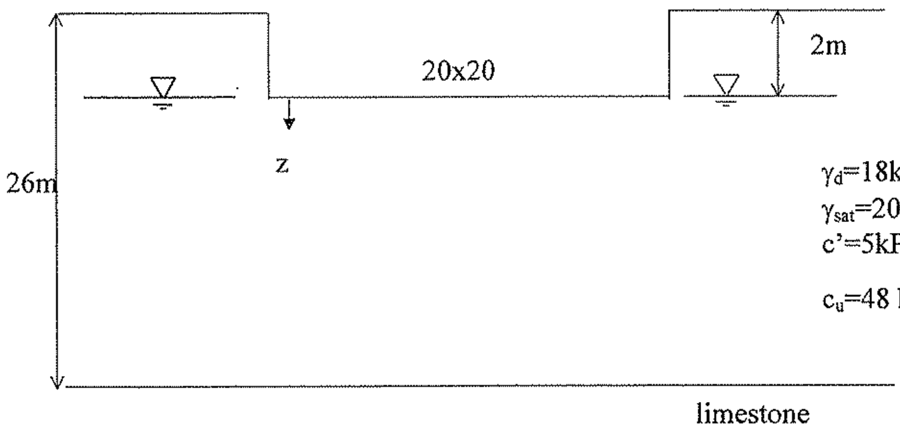
The clay has c'=5 kN/m^2, ϕ'=20^0, c_u=48 kN/m^2, ϕ_u=0. The coefficient of volume compressibility is 1.00x10^{-4} m^2/kN at the ground surface and decreases with depth at a rate of 0.02x10^{-4} m^2/kN per meter. Use E_u/c_u = constant = 1250 and I_s = 1.2

a) Calculate ultimate bearing capacity of the foundation in the short term?

b) For the foundation described above what is the (gross) allowable bearing capacity?

NOTE: For ϕ_u=0 case use Skempton values, use <u>a safety factor of 3.00</u> against shear failure of the foundation. Use sublayers. <u>Maximum allowable total settlement of the building is 15 cm.</u>

Solution:



Skempton expression for ϕ_u=0 is : $q_f = c_u N_c + \gamma_{sat} D$ (total stress analysis)

$q_{nf} = c_u N_c$

Short Term :

$$\frac{D}{B} = \frac{2}{20} = 0.1 \quad N_{c\ square} = 6.4 \quad \text{(Skempton Chart, page 73 Fig.4.6 in Lecture Notes)}$$

$$q_f = 48x6.4 + 18x2 = 343.2 \text{ kPa}$$

$$q_{nf} = q_f - \gamma D = c_u N_c = 307.2 \text{ kPa}$$

Settlement Check :

$S_t = S_i + S_c$

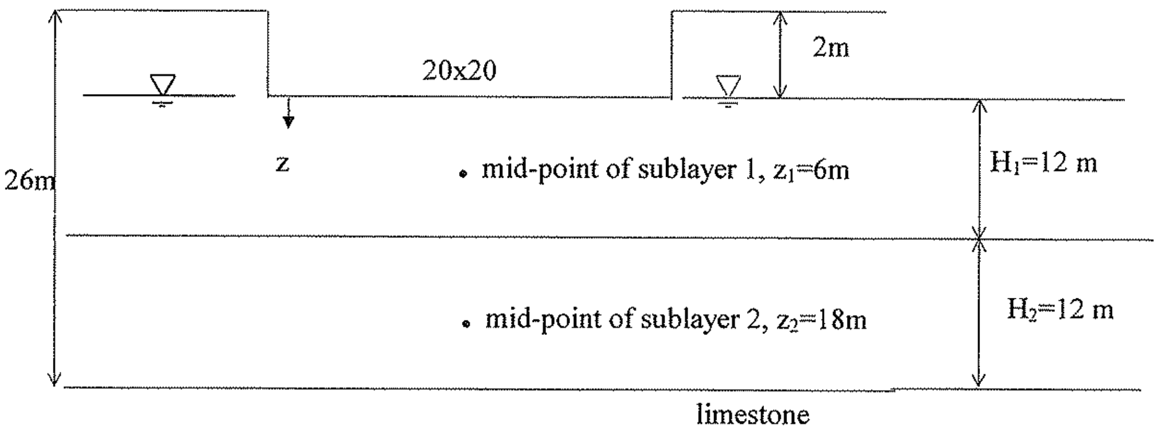
IMMEDIATE SETTLEMENT IN CLAY, S_i:

$$S_i = \frac{qB}{E}(1-\mu^2)I_s \quad \text{where } q = q_{net} \text{ (net foundation presure)} = \frac{q_{nf}}{FS} = \frac{307.2}{3} = 102.4 \text{ kPa}$$

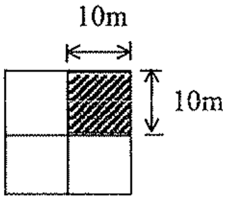
- Note that in clay for UNDRAINED CASE $\rightarrow \mu = 0.5$
- undrained modulus, $E_u = 60000$ kPa
- $I_s = 1.2$ (given)

$$S_i = \frac{102.4x20}{60x10^3}(1-0.5^2)x1.2 = 0.031\text{m} = 31\text{mm}$$

CONSOLIDATION SETTLEMENT IN CLAY, S_c:



- Vertical Stress due to q_{net} should be determined at the mid-point of each sublayer



$S_{oed} = m_v \, \Delta\sigma H$

$\Delta\sigma = 4qI_r$; $q = q_{net} = 102.4$ kPa

$m_v = [1 - 0.2(2+z)] \times 10^{-4}$

Layer no	z	m=n=10/z	I_r	$\Delta\sigma$	$m_v(m^2/kN)$
1	6	1.67	0.2	81.9	0.84×10^{-4}
2	18	0.55	0.093	38.1	0.6×10^{-4}

$S_{oed} = (0.84\times10^{-4} \times 81.9 \times 12) + (0.6\times10^{-4} \times 38.1 \times 12) = 0.110m = 110mm$

$S_t = 31 + 110 \cong 141mm < 150mm$ (allowable) OK.

$\therefore$ GENERALLY IN CLAY SHEAR FAILURE CONTROLS THE DESIGN, SETTLEMENT IS NOT CRITICAL. BUT IT SHOULD BE CHECKED ALSO

$(q_{all})_{net} = 102.4$ kPa

$(q_{all})_{gross} = 102.4 + 2\times18 = 138$ kN/m^2

P4

Question:

A footing of 4mx4m carries a uniform gross pressure of 300 kN/m^2 at a depth of 1.5m in a sand. The saturated unit weight of the sand is 20 kN/m^3 and the unit weight above the water table is 17 kN/m^3. The shear strength parameters are c'=0, ϕ'=32^0. Determine the factor of safety with respect to shear failure for the following cases;

a) The water table is at ground surface

b) The water table is 1.5m below the surface

Solution:

$$FS = \frac{(q_{ult})_{net}}{q_{net}} = \frac{q_{nf}}{q_n} = \frac{q_{ult} - \gamma D}{q_{gross} - \gamma D} = \frac{q_f - \gamma D}{q_n}$$

For square footing:

$$q_f = q_{ult} = 0.4\gamma B N_\gamma + 1.2cN_c + \gamma D N_q$$

$c' = 0$ and $\phi' = 32^0$ $\quad N_\gamma = 26, N_q = 29$ (see page 69 Figure 4.3 in Lecture Notes)

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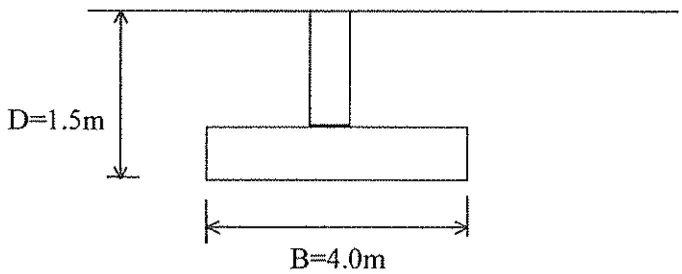


a)

$$q_f = 0.4B\gamma' N_\gamma + \gamma' DN_q = 0.4x4x(20-10)x26 + (20-10)x1.5x29 = 851kPa$$

$$q_{nf} = q_f - \gamma' D = 851 - (20-10)x1.5 = 836kPa$$

$$q_{gross} = 300kPa$$

i. $q_{net} = 300 - 20x1.5 = 270kPa$ OR

ii. $q_{net} = (300 - 1.5x10) - 1.5(20-10) = 270kPa$

$$FS = \frac{836}{270} = 3.1$$

b)

$$q_f = 0.4B\gamma' N_\gamma + \gamma_d DN_q = 0.4x4x(20-10)x26 + 17x1.5x29 = 1156\,kPa$$

$$q_{nf} = q_f - \gamma D = 1156 - 17x1.5 = 1130\,kPa$$

$$q_{gross} = 300\,kPa$$

$$q_{net} = 300 - 17x1.5 = 275\,kPa$$

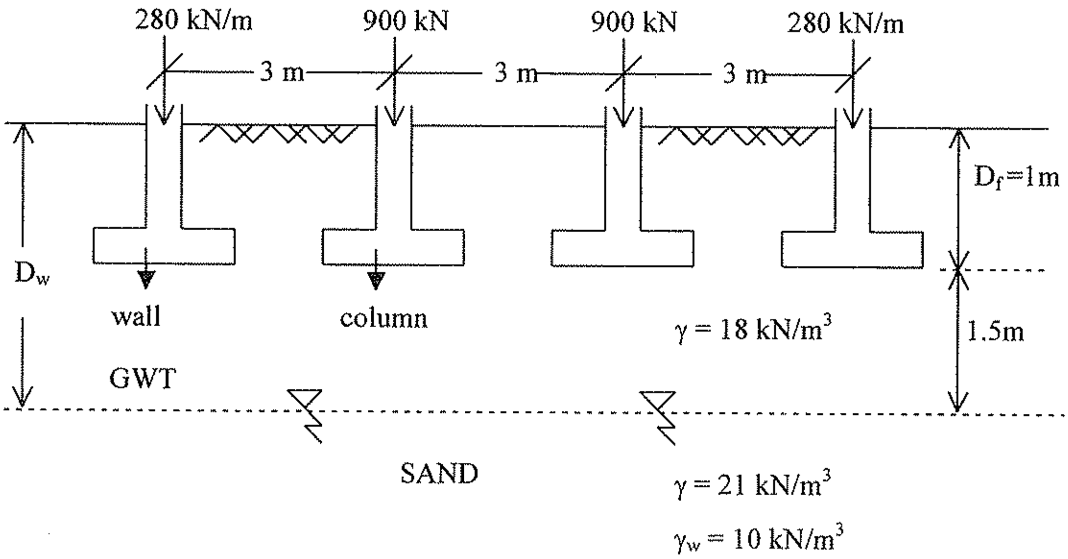
$$FS = \frac{1130}{275} = 4.1$$

P5 FOOTING ON SAND

Question:

The column loads, wall loads and the pertinent soil data for a proposed structure is given below. Design the square column and wall footings for a permissible settlement of 30 mm, using Peck & Hanson & Thornburn charts. Make a reasonable assumption to obtain an average N value below the footing.

depth	1	2	3	4	5	6	7	8	9	10	11	12
N	8	14	11	16	18	11	9	13	18	20	50/11	50/7



Footing on Cohesionless Soils:

Assumptions:

- significant depth: 0.5 B above, 2 B below the footing
- weight of excavated soil ≅ weight of (footing + column) in the soil
  column load / area ≅ q_{net}
- footings to be designed for the largest q_{net} (i.e. column ftg)

Solution:

NOTE: For Peck-Hanson-Thorburn, N values should be corrected for overburden stress

Depth	N_{field}	σ_o'	C_N	N_1
1	8	18	2.0	16
2	14	36	1.63	23
3	11	50.5	1.38	15
4	16	61.5	1.25	20
5	18	72.5	1.15	21
6	11	83.5	1.07	12
7	9	94.5	1.01	9
8	13	105.5	0.95	12
9	18	116.5	0.91	16
10	20	127.5	0.87	17
11	50/11			-
12	50/7			-

C_N (overburden correction) values are calculated by using eq.2.3 (page 31) in Lecture Notes. ($C_N = 9.78x(1/\sigma_v')^{0.5} \leq 2$)

<u>**Square column footings**</u> Peck & Hanson & Thornburn charts:Fig 4.8 in Lecture Notes

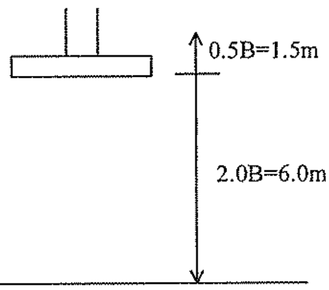
⇒ **assume B=3.0 m**

⇒ To obtain the average N value to be used in the calculations

Consider 0.5B=0.5x3=1.5m above

2.0B=2.0x3=6.0m below the foundation level

Depth	N_1
1	16
2	23
3	15
4	20
5	21
6	12
7	9
8	12
9	16
10	17



$N_{1,av}$ = (16+23+15+20+21+12+9) / 7 = 17

$C_w = 0.5 + 0.5x[2.5/(1+3)] = 0.81$

$(q_n)_{all}$=11x $N_{1,av}$ x c_w (kN/m^2) for 25 mm settlement (page 78 in Lecture Notes) $(q_n)_{all}$=11x17x0.81 = 151 kPa

$$(q_n)_{all} = (q_n)_{all}\, x \frac{S_{all}(mm)}{25}$$

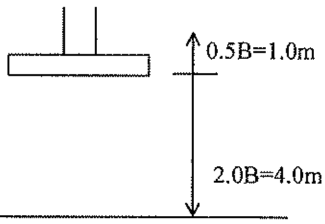
q_{all} = 151x(30/25) = 181 kPa

q_{net} = 900/(3x3) = 100 kPa

181 >>100 $\longrightarrow$ overdesign

⇒ **assume B = 2.0 m**

Depth	N_{cor}
1	16
2	23
3	15
4	20
5	21
6	12
7	9
8	12
9	16
10	17



N_{av} = (16+23+15+20+21) / 5 = 19

$C_w = 0.5 + 0.5x[2.5/(1+2)] = 0.92$

$(q_n)_{all}$=11x19x0.92 = 192 kPa

q_{all} = 192x(30/25) = 230 kPa

q_{net} = 900/(2x2) = 225 kPa

230 $\cong$ 225 OK

$\boxed{B = 2.0\ m}$

Wall footings

⇒ Use $q_{net} = 225$ kPa

$$B = \frac{280}{225} = 1.25m$$

Check B value

$N_{av} = (16+23+15) / 3 = 18$

$C_w = 0.5 + 0.5x[2.5/(1+1.25)] \leq 1.0 \Rightarrow C_w = 1.0$

$(q_n)_{all} = 11x18 = 198$ kPa

$q_{all} = 198x(30/25) = 238$ kPa

238 > 225 OK

P6 FOOTING ON CLAY

Question:

A public building consists of a high central tower which is supported by four widely spaced columns. Each column carry a combined dead load and representative sustained load of 2500 kN inclusive of the substructure (gross load). Trial borings showed that there is a 7.6m of stiff fissured Ankara clay (c_u=85 kPa, E_u = 30 MN/m^2 and m_v = 1x10^{-4} m^2/kN) followed by dense sand which is relatively incompressible. Determine the required foundation width (assume square foundation) and allowable bearing pressure for the tower footings.

Assume $\gamma_{wet} = \gamma_{sat}$ = 18.6 kN/m^3 (above and below GWT)

γ_w = 10 kN/m^3

Consider immediate and consolidation settlements. Divide the clay layer into 4 equal sublayers.

The foundation depth can be taken as 2m.

⇒ D=2.0m, c_u = 85 kPa, Skempton-Bjerrum factor:μ=0.5, D_w = 1.2 m, F.S. = 2.5

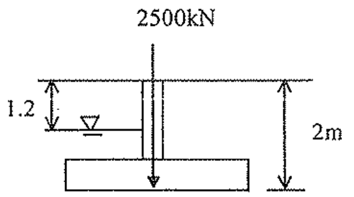
Solution:

- Assume B=2.0m

$D_f/B=1$ ⇒ N_c = 7.7 (Skempton)

$q_{nf} = (q_{ult})_{net} = c_u N_c$ = 85x7.7 = 654.5 kPa

for FS=2.5 $(q_{net})_{safe}$ = 654.5/2.5 = 261.8 kPa



q_{net} = 2500/(2x2) – 2x18.6 = 587.8 kPa

OR

q_{net} = (2500/(2x2) – 0.8x10)-(1.2x18.6+0.8x8.6)

= 587.5 kPa

$(q_{net})_{safe} << q_{net}$ NOT ACCEPTED

- **Assume B=3.0m**

$D_f/B=0.67 \Rightarrow N_c = 7.4$ (Skempton)

$q_{nf} = (q_{ult})_{net} = c_u N_c = 85 \times 7.4 = 629$ kPa

for FS=2.5 $\quad (q_{net})_{safe} = 629/2.5 = 251.6$ kPa

$q_{net} = 2500/3 \times 3 - 2 \times 18.6 = 241$ kPa

$(q_{net})_{safe} \approx q_{net}$ OK

$\therefore$ **B=3.0m**

Settlements

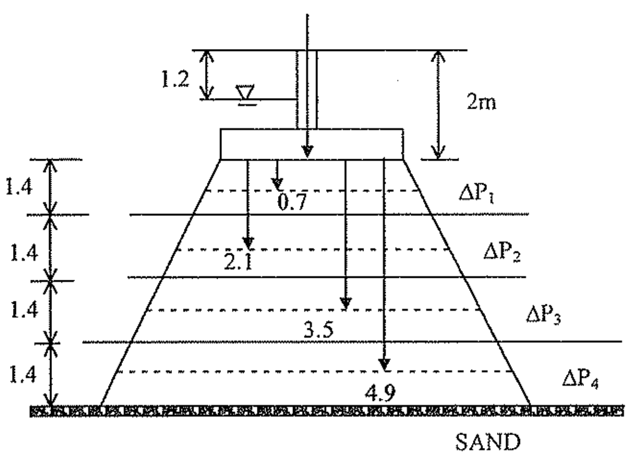
B=3.0m $E_u = 30000$ kPa D_f=2.0m

Compressible layer thickness H=7.6-2=5.6m

$$S_i = \mu_0 \mu_1 \frac{qB}{E_u}$$

$$\frac{H}{B} = 1.87 \quad \frac{D}{B} = 0.67 \quad \Rightarrow \mu_0 = 0.95 \quad \mu_1 = 0.57$$

$$S_i = 0.57 \times 0.95 \times \frac{241 \times 3}{30000} = 0.013\text{m} = 13\text{mm}$$



Sand is relatively incompressible
(also = 2B)

q_{net}=241 kPa

$$\Delta P = \frac{q_{net}BL}{(B+z)(L+z)}$$

(Use 2:1 approximation)

Layer no	Thickness,H (m)	ΔP
1	1.4	158
2	1.4	83.4
3	1.4	51.3
4	1.4	34.8

Note that:

⇒ ΔP= vertical stress due to q_{net} at the mid-point of each sublayer

S_{oed}=mv.Δσ.H

$S_{oed}=1x10^{-4}x1.4x(158+83.4+51.3+34.8)=4.585x10^{-2}m=45.85mm$

Apply Skempton-Bjerrum factor μ=0.5

$S_c = S_{oed}\ \mu = 45.85x0.5 = 22.9mm$

$S_{total} = S_I + S_c = 13+22.9 = 35.9mm$

P7 RAFT FOUNDATION ON DEEP CLAY LAYER

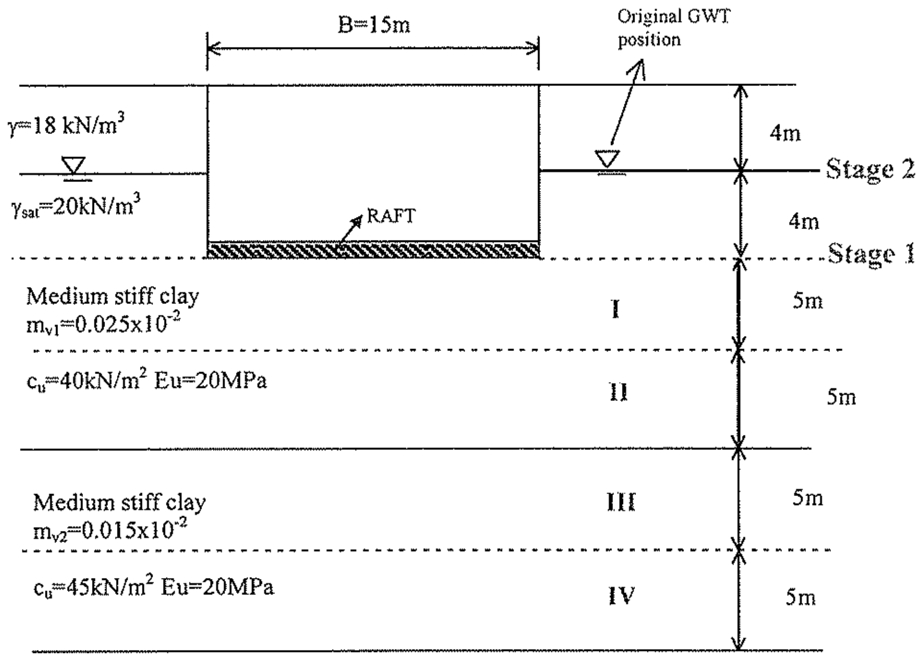
Question:

A 16-storey apartment block is to be constructed at a site. The soil profile consists of a deep clay layer. The ground water table is at 4m depth. The base of the raft under the building is 8m deep from the ground surface. The profile and the soil properties are shown in the figure below.

The dimensions of the building and the raft are the same (15mx30m). Total weight of the building (dead+live+raft) is 90 000 kN.

Find the net foundation pressure and check the factor of safety against bearing capacity and calculate the total settlement of the building.

No secondary settlements are expected. Take the Skempton-Bjerrum correction factor μ= 0.75. Consider the compressions of the soil within 20m distance from the foundation level. The G.W.T. is at the "Stage 2" level prior to construction, lowered to "Stage1" level during the construction and rises back to "Stage 2" level in the long term.



Solution:

Total weight of the building (dead+live+raft)=Q_{gross}=90000 kN

q_{gross}= 90000/(15x30) = 200 kPa

Stage 1 (GWT is lowered to the foundation level)

Uplift = 0

σ_o'=4x18+4(20-9.8) = 112.8 kPa

q_{net}=(200-0)-112.8 = 87.2 kPa (net foundation pressure)

Stage 2 (GWT is raised to its original position)

Uplift= 4x9.8= 39.2 kPa

σ_o'=4x18+4(20-9.8) = 112.8 kPa

q_{net}=(200-39.2)-112.8 = 48 kPa

q_{net}= 87.2 kPa is MORE CRITICAL

Net bearing capacity of the foundation : $q_{nf} = q_f - \gamma D = c_u N_c + \gamma D - \gamma D = c_u N_c$

c_u = 40 kPa

D_f/B=8/15=0.53 $(N_c)_{square}$=7.1

$(N_c)_{rect.}=(N_c)_{square}$ (0.84+0.16B/L) = 7.1(0.84+0.16x15/30) = 6.5

q_{nf} = 6.5x40 = 260 kPa

Safety factor against shear $FS = \dfrac{q_{nf}}{q_{net}} = \dfrac{260}{87.2} = 3.0$ OK

Settlement Analysis:

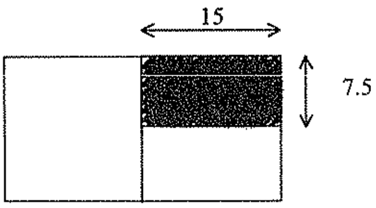
Total settlement= $S_t = S_I + S_c$

Consider the compressions of the soil within 20m distance from the foundation level.

Initial settlement $:S_i = \mu_0\mu_1 \dfrac{qB}{E_u} = 0.95x0.5x\dfrac{87.2x15}{20000} = 3.1cm$

Consolidation Settlement : $S_c = m_v \Delta\sigma H$

For consolidation settlement; consider 5m thick sublayers.



$\Delta\sigma = 4qI_r$

q_{net} = 48 kPa since consolidation is a LONG TERM situation

n=B/z	m=L/z	I_r	$\Delta\sigma = 4qI_r$	m_v (m^2/kN)
7.5/2.5	15/2.5	0.245	47	$0.025x10^{-2}$
7.5/7.5	15/7.5	0.2	38.4	$0.025x10^{-2}$
7.5/12.5	15/12.5	0.145	27.8	$0.015x10^{-2}$
7.5/17.5	15/17.5	0.102	19.6	$0.015x10^{-2}$

$S_c = 0.025x10^{-2}x47x5 + 0.025x10^{-2}x38.4x5 + 0.015x10^{-2}x27.8x5 + 0.015x10^{-2}x19.6x5$

S_c = 0.142m=14.2cm

$\Rightarrow$ μ=0.75 (Skempton-Bjerrum)

S_c = 14.2x0.75= 10.7cm

S_t = 3.1+10.7 = 13.8 cm

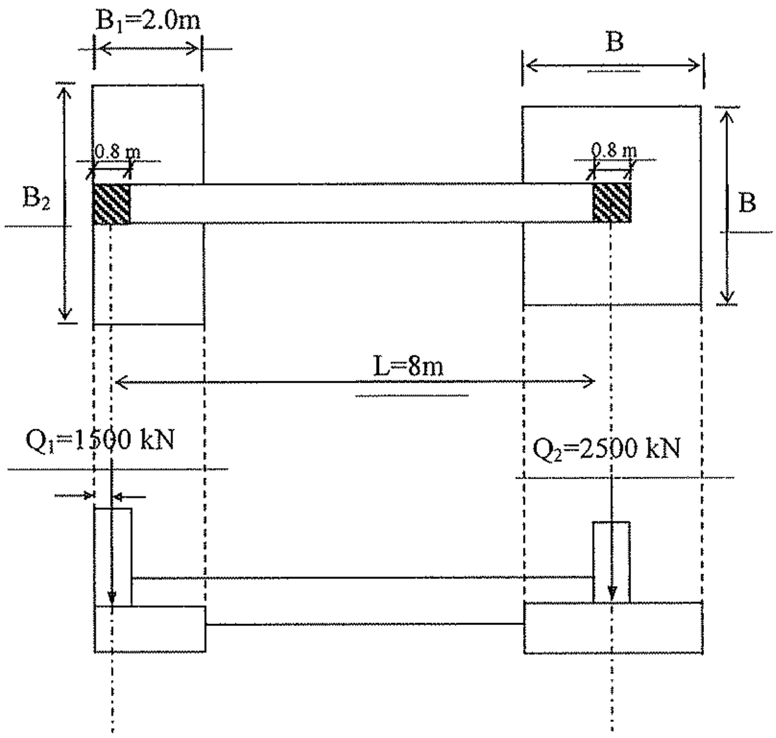
CE 366 – SHALLOW FOUNDATIONS

P.1) CANTILEVER FOOTING

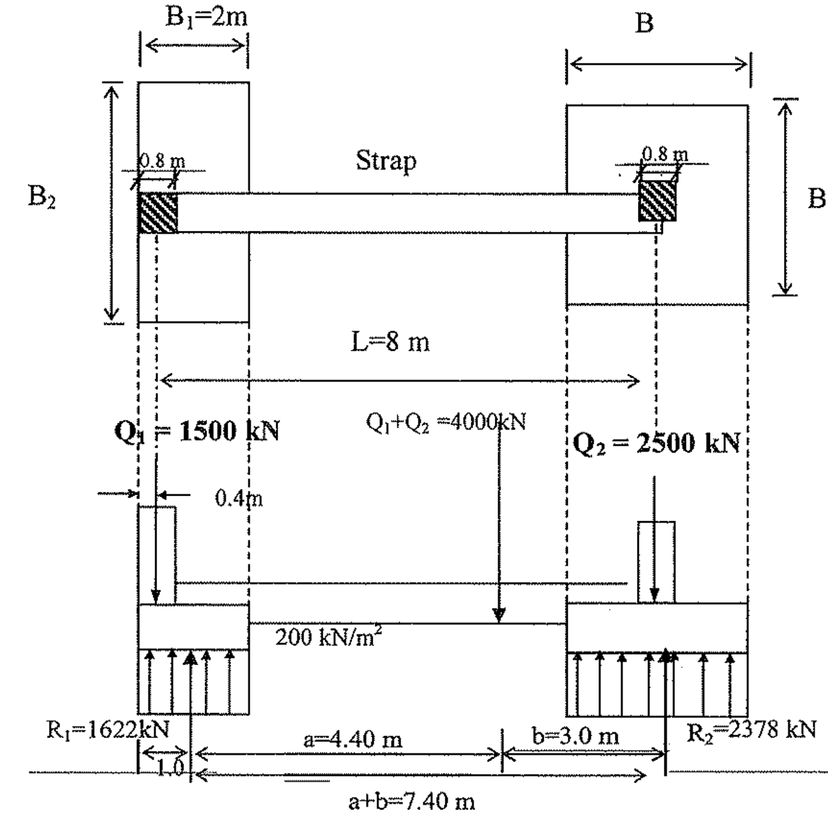
Question:

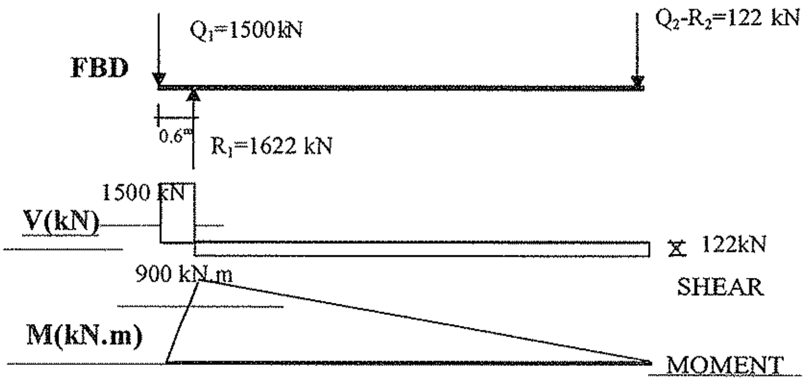
Given: $Q_1 = 1500$ kN, $Q_2 = 2500$ kN, $q_{all} = 200$ kN/m^2

Ignore the weight of footings and find dimensions B and B_2 of a cantilever footing for a uniform soil pressure distribution. Draw shear and bending moment distributions.



Solution:





Locate $\Sigma Q = Q_1 + Q_2$

b=1500 x 8 / 4000=3

For B_1=2m

a = 8.0 + 0.4 – 1.0 – 3.0

→ a = 4.40m

R_1= (4000 x 3) / 7.4 = 1622kN

R_2 = 4000 – 1622 = 2378 kN

Determine B_2,

$q_{all} = Q / (2 \times B_2)$

200 = 1620 / (2 x B_2) → B_2 = 4 m

OR

Without considering resultant (Q_1 +Q_2)

Moment w.r.t Q_2 or (R_2);

Q_1x8- R_1x7.4=0 →R_1=1622 kN

From force equilibrium;

$\Sigma F_{vertical} = 0$

1500+25000-1622-R_2 =0

→R_2=2378 kN

Determine B_2,

$q_{all} = Q / (2 \times B_2)$

200 = 1622 / (2 x B_2) → **$B_2 \approx 4$ m**

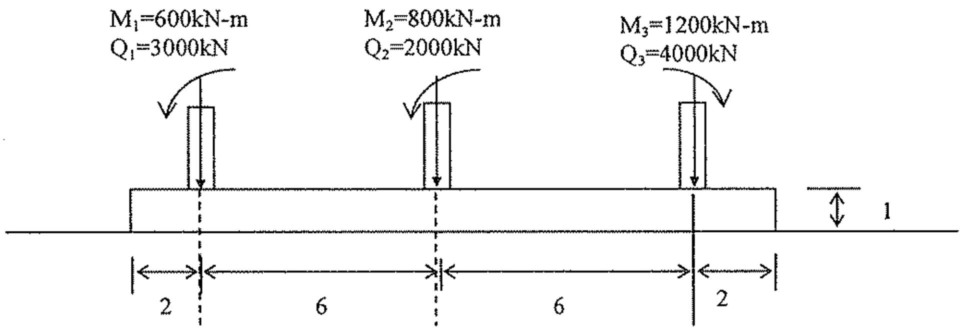
Similary,

200 = 2378 / B^2 → **$B \approx 3.45$ m**

P.2) TRAPEZOIDAL FOOTING

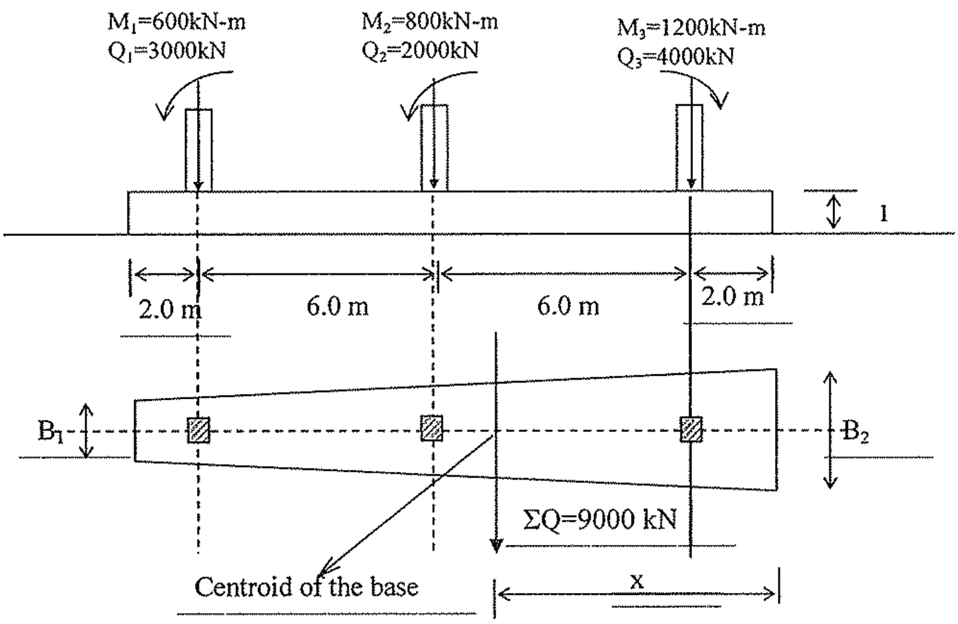
Question:

Determine B_1 and B_2 of a trapezoidal footing for a uniform soil pressure of 300 kN/m^2. (γ_{conc} = 24kN/m^3)



Solution:

After finding the location of resultant force, you can decide whether the wider side of the trapezoid will be at the left or right side.



General formulation for finding coordinates of a centroid;

$\bar{x} = \frac{\overline{x_1}A_1 + \overline{x_2}A_2}{A_1 + A_2}$ $\bar{y} = \frac{\overline{y_1}A_1 + \overline{y_2}A_2}{A_1 + A_2}$

ΣQ=9000kN

Weight of footing= 16x24x(B_1+B_2)/2 = 192(B_1+B_2)

Area of footing = 8(B_1+B_2)

<u>ΣF_y=0</u> 9000+192(B_1+B_2)=8(B_1+B_2)x300

B_1+B_2 = 4.07m ....(1)

<u>ΣM=0</u> (moment about centroid of the base)

3000(14-x)+600+2000(8-x)+800-4000(x-2)-1200+(wght of ftg)x0=(base pressure)x0

x = 7.36m

x=7.36=$\frac{1}{3}x16x\frac{2B_1+B_2}{B_1+B_2}$ $B_2 = 1.63B_1$ ....(2)

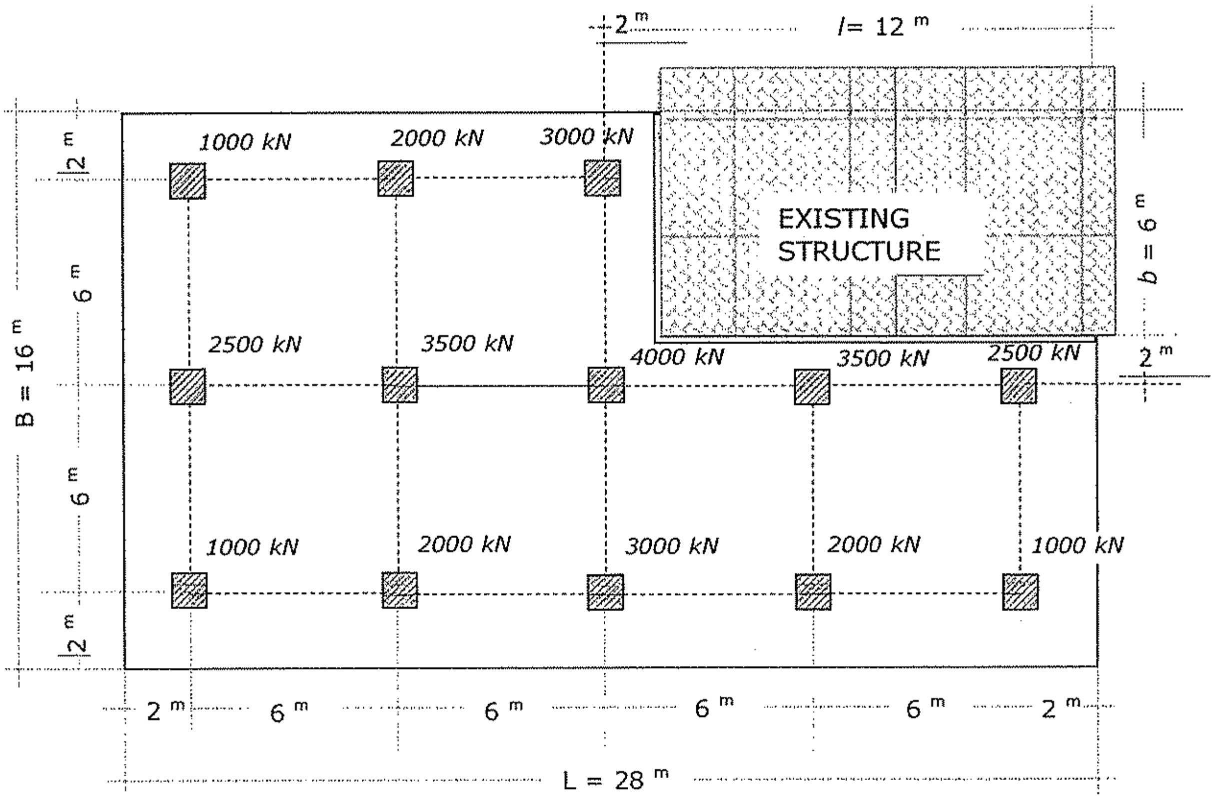
From (1) and (2) B_1= 1.55m ; B_2 = 2.52m

Note: You can also take moment about the left or right side but keep in mind that weight and base pressure will also have moment about left or right side.

P.3) MAT FOUNDATION

Question:

A mat foundation rests on a sand deposit whose allowable bearing value is 150 kN/m^2. Column loads are given in the figure. The thickness of the mat is 2.0 m ($\gamma_{concrete}$ = 24 kN/m^3). Calculate base pressures assuming that the lines passing through the centroid of the mat and parallel to the sides are principal axes. Find the base pressure distribution beneath the base and check whether the mat foundation given is safe?



Solution:

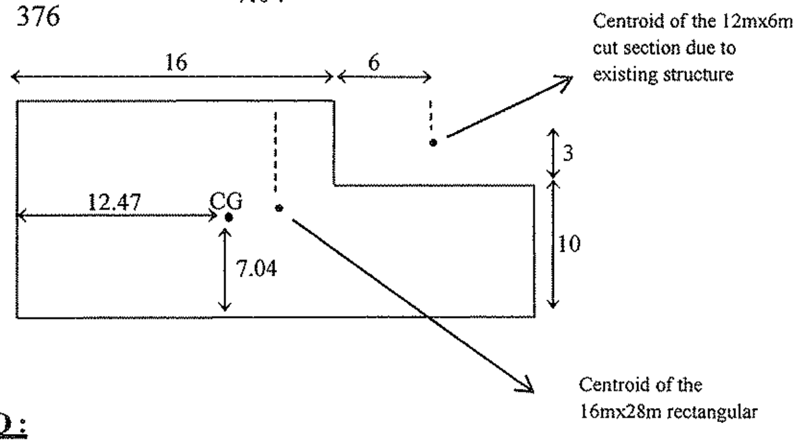
Area of foundation = $28 \times 16 - 12 \times 6 = 376\ m^2$

Total vertical load = ΣV = Column loads + Weight of mat=31000+(376) × 24 × 2 = 49048 kN

* **Center of gravity (CG) of mat:**

$$\frac{28 \times 16 \times 14 - 6 \times 12 \times (16 + 6)}{376} = 12.47 \quad \text{m from left}$$

$$\frac{28 \times 16 \times 8 - 6 \times 12 \times (3 + 10)}{376} = 7.04 \quad \text{m from bottom}$$



* **Location of ΣQ :**

→ Take moment about the left side:

= (1 / 49048) . [2 × (1000+2500+1000) + 8 × (2000+3500+2000) + 14 × (3000 +

+ 4000 + 3000) + 20 × (3500+2000) + 26 × (2500+1000) + 376 × 2 × 24 × 12.47]

= 12.95 m from left

→ Take moment about bottom side :

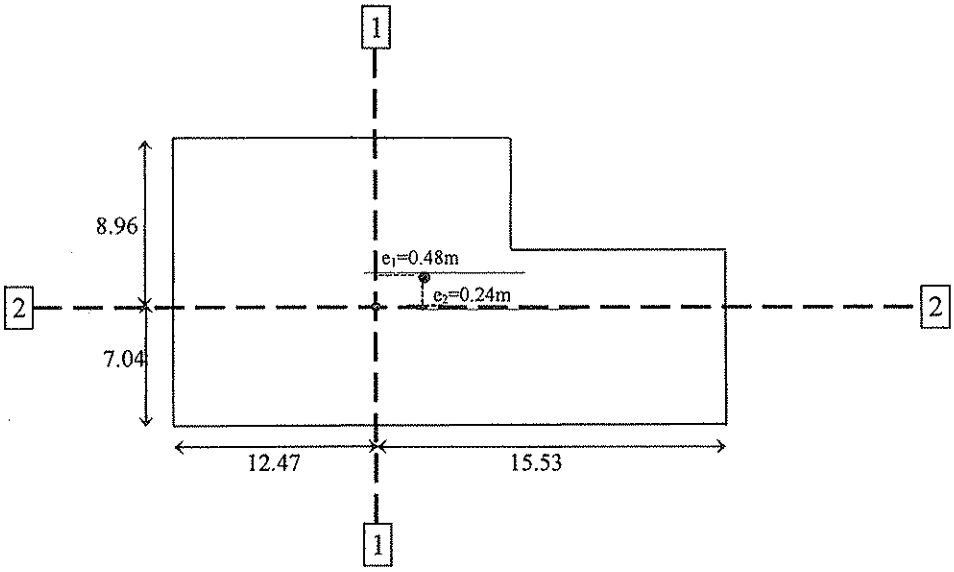
= (1 / 49048) . [2 × (1000+2000+3000+2000+1000) + 8 × (2500 + 3500 + 4000 +

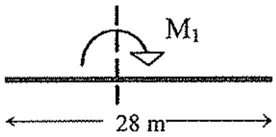
+ 3500 + 2500) + 14 × (1000+2000+3000) + 376 × 2 × 24 × 7.04]

= 7.28 m from bottom

Eccentricity :

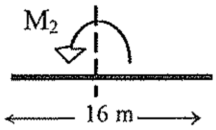
$$e_1 = 12.95 - 12.47 = 0.48 \text{ m}$$
$$e_2 = 7.28 - 7.04 = 0.24 \text{ m}$$





M_1 about 1-1 axis:

$$M_1 = \Sigma Q \cdot e_1 = 49048 \cdot (0.48) = 23543 \text{ kN.m}$$



M_2 about 2-2 axis:

$$M_2 = \Sigma Q \cdot e_2 = 49048 \cdot (0.24) = 11772 \text{ kN.m}$$

$$I_{1-1} = \left[\frac{B \cdot L^3}{12} + B \cdot L \cdot (D_1)^2\right] - \left[\frac{b \cdot l^3}{12} + b \cdot l \cdot (d_1)^2\right] =$$

$$= \left[\frac{16 \times 28^3}{12} + 16 \times 28 \times (14 - 12.47)^2\right] - \left[\frac{6 \times 12^3}{12} + 6 \times 12 \times (22 - 12.47)^2\right] = 22915 \quad \text{m}^4$$

$$I_{2-2} = \left[\frac{L \cdot B^3}{12} + B \cdot L \cdot (D_2)^2\right] - \left[\frac{l \cdot b^3}{12} + b \cdot l \cdot (d_2)^2\right] =$$

$$= \left[\frac{28 \times 16^3}{12} + 16 \times 28 \times (8 - 7.04)^2\right] - \left[\frac{12 \times 6^3}{12} + 12 \times 6 \times (13 - 7.04)^2\right] = 7197 \quad \text{m}^4$$

Note: In soil mechanics compression is taken as positive (+)

$$q = \frac{\Sigma Q}{Area} \pm \frac{M_1 \cdot y_1}{I_{1-1}} \pm \frac{M_2 \cdot y_2}{I_{2-2}}$$

$$q = \frac{49048}{376} \pm \frac{23543 \cdot y_1}{22915} \pm \frac{11772 \cdot y_2}{7197} = 130.4 \pm 1.03 y_1 \pm 1.64 y_2$$

$q_A = 130.4 \pm 1.03 y_1 \pm 1.64 y_2 = 130.4 + 1.03 \cdot (3.53) + 1.64 \cdot (2.96) = 138.9$ kN/m²

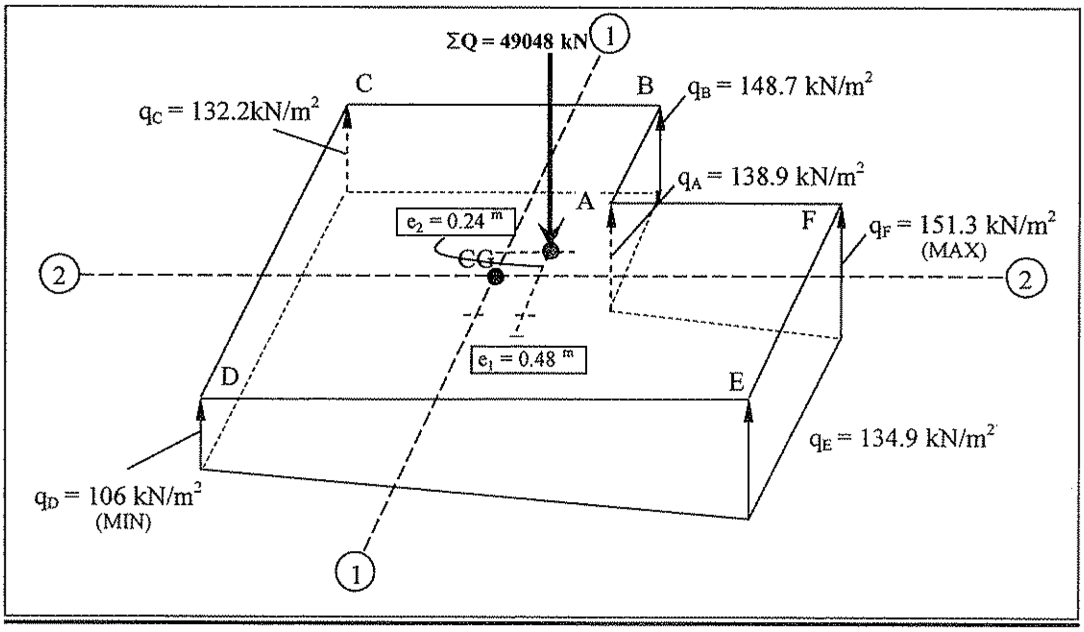
$q_B = 130.4 + 1.03 \cdot (3.53) + 1.64 \cdot (8.96) = 148.7$ kN/m²

$q_C = 130.4 - 1.03 \cdot (12.47) + 1.64 \cdot (8.96) = 132.2$ kN/m²

$q_D = 130.4 - 1.03 \cdot (12.47) - 1.64 \cdot (7.04) = 106$ kN/m²

$q_E = 134.9$ kN/m²

$q_F = 151.3$ kN/m²

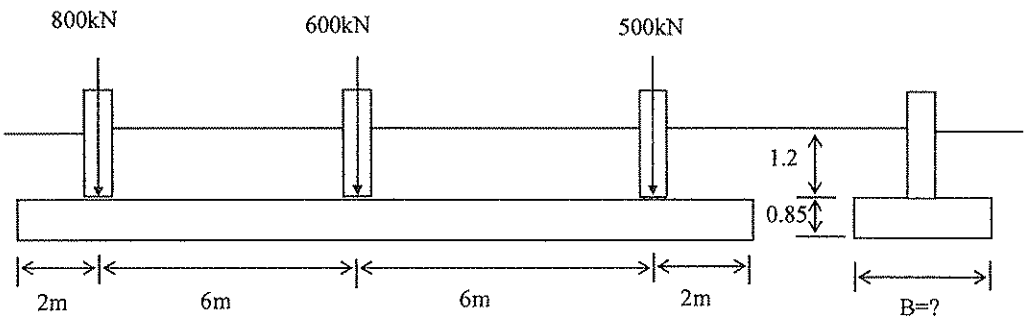


Since at all critical points stress values are almost ≈< $q_{all} = 150$ kN/m² given mat foundation is safe.

P.4) COMBINED FOOTING ANALYZED BY RIGID METHOD

Question:

A rectangular combined footing which supports three columns is to be constructed on a sandy clay layer *whose allowable bearing value (base pressure) is 84 kN/m²*. The thickness of the concrete footing is 0.85m. There is a 1.20m thick soil fill having same unit weight with sandy clay on the footing. Unit weight of the sandy clay and the concrete are 20 kN/m^3 and 24 kN/m^3 respectively. Analyze the footing by rigid method and plot shear and moment diagrams.



Solution:

Neglecting column weights;

ΣQ_{net}=800+600+500+(24-20)x16xBx0.85 = 1900+54.4B

$$e = \frac{\Sigma M}{\Sigma V} = \frac{800x6 - 500x6}{1900 + 54.4B} = \frac{1800}{1900 + 54.4B}$$

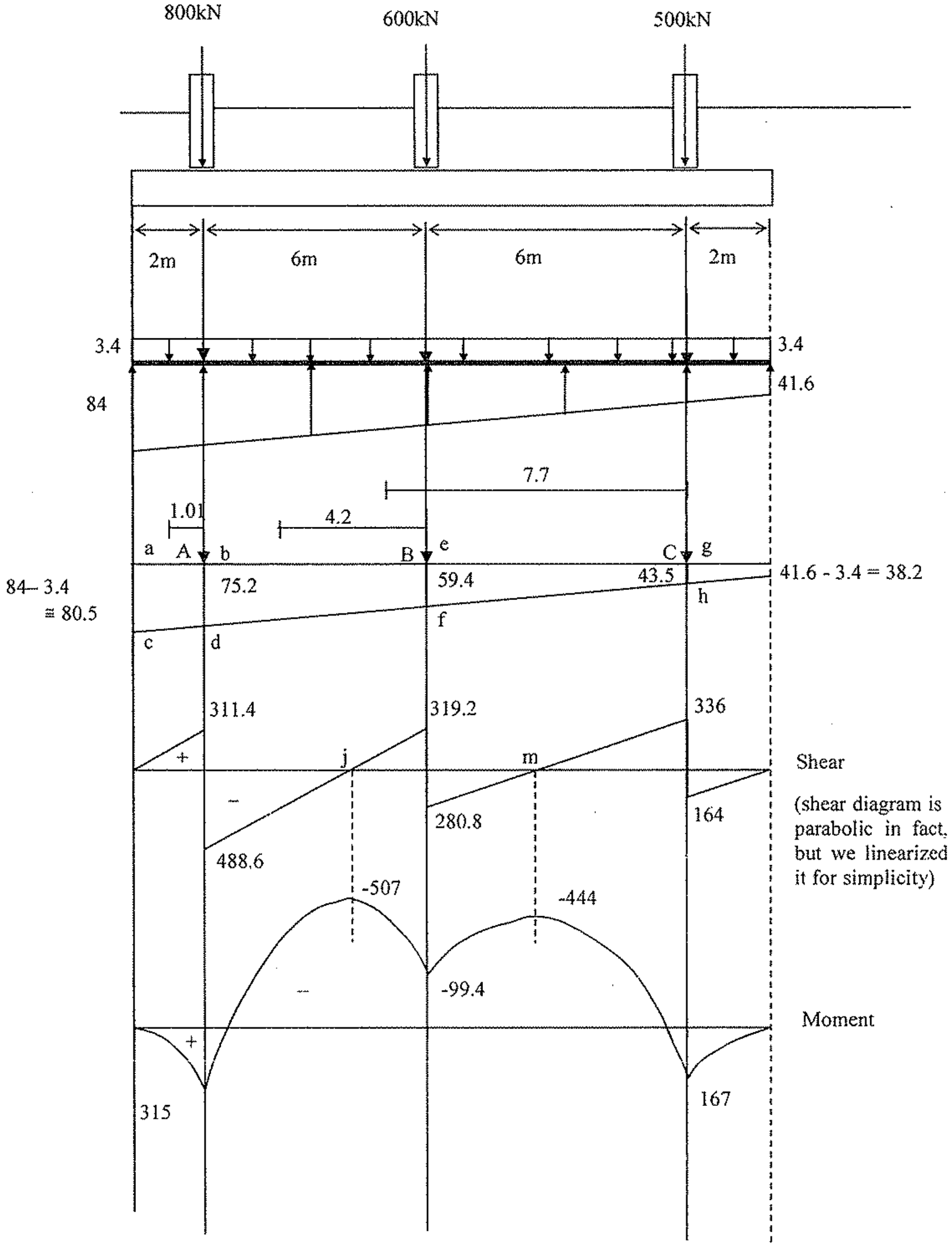
$$q_{max} = \frac{\Sigma V}{BxL}(1 + \frac{6e}{L}) = 84kN/m^2$$

$$84 = \frac{1900 + 54.4B}{Bx16}(1 + \frac{6x1800}{(1900 + 54.4B)x16}) \quad \Rightarrow B = 2.0m$$

Use B=2.0m q_{max} = 84 kN/m^2 ; q_{min} = 41.6 kN/m^2

Downward uniform pressure = 0.85 x (24-20) = 3.4 kN/m^2

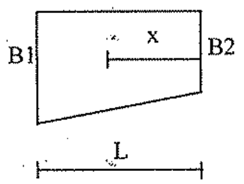
These are the diagrams related to forces and moments acting on the foundation. Explanations are at the next page;



$$V_A = \underbrace{\frac{80.5+75.2}{2}}_{\text{area (abcd)}} \text{ x } 2 \text{ x } 2 = 311.4\,\text{kN} \qquad B = 2$$

$$V_{A'} = 311.4 - 800 = -488.6\ \text{kN}$$

To find the centroid of a trapezoid on the horizontal axis;



$$x = \frac{L}{3}\,\frac{2B1+B2}{B1+B2}$$ x: distance from shorter dimension

$$x\,(\text{abcd}) = \frac{2}{3}\,\frac{2\text{x}80.5+75.2}{80.5+75.2} = 1.01\,\text{m}$$

$$M_A = 311.4 \text{ x } 1.01 = 315\ \text{kN.m}$$

$$V_{BA} = \frac{80.5+59.4}{2} \text{ x } 8 \text{ x } 2 - 800 = 319.2\,\text{kN}$$

$$V_{BC} = 319.2 - 600 = -280.8\ \text{kN}$$

$$x\,(\text{aefc}) = \frac{8}{3}\,\frac{2\text{x}80.5+59.4}{80.5+59.4} = 4.2\,\text{m}$$

$$M_B = \left[\frac{80.5+59.4}{2} \text{ x } 8 \text{ x } 2\right] \text{ x } 4.2 - 800 \text{ x } 6 = -99.4\,\text{kN.m}$$

$$V_{CB} = \frac{80.5+43.5}{2} \text{ x } 14 \text{ x } 2 - 800 - 600 = 336\,\text{kN}$$

$$V_C = 336 - 500 = -164\ \text{kN}$$

Check the end point; $\frac{80.5+38.2}{2} \text{ x } 16 \text{ x } 2 - 800 - 600 - 500 = 0 \longrightarrow$ OK

$$x\,(aghc) = \frac{14}{3}\,\frac{2x80.5+43.5}{80.5+43.5} = 7.7\text{ m}$$

$$M_C = \left[\frac{80.5+43.5}{2} x\, 14\, x\, 2\right] x\, 7.7 - 800\, x\, 12 - 600\, x\, 6 = 167\text{ kN.m}$$

Slope of V b/w x = 2 and x = 8 m Æ (488.6+319.2)/6=134.6 kN/m

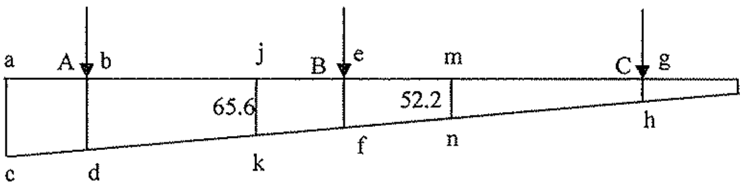
Equation of V b/w x = 2 and x = 8 m Æ V(x) = -488.6 + 134.6 (x) for V(x)=0 Æ x=3.6 m

Base pressure @ x = 2 + 3.6 = 5.6 m Æ 65.6 kN/m^2

Similarly @ x = 10.7 m V = 0

Base pressure @ x =10.7 m Æ 52.2 kN/m^2

maximum points of the moment diagram;



j and m are the points where shear forces are equal to zero (i.e. moment is max)

<u>the distance between point b and point j</u>

$$x\,(ajkc) = \frac{(2+3.6)}{3}\,\frac{2x80.5+65.6}{80.5+65.6} = 2.9\text{ m}$$

$$M_{AB} = \left[\frac{80.5+65.6}{2} x\,(2+3.6)\, x\, 2\right] x\, 2.9 - 800\, x\, 3.6 = -507\text{ kN.m}$$

$x\,(amnc) = 5.73\text{ m}$

M_{BC} = -444 kN.m

RETAINING WALL PROBLEMS

P1. CANTILEVER RETAINING WALL

Question

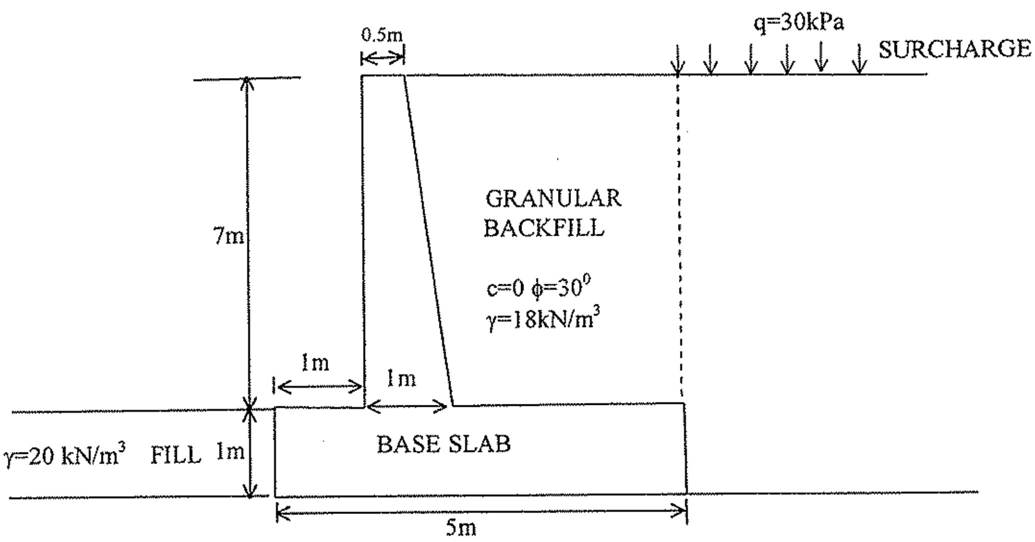
For the retaining wall and the profile shown below, calculate:

a. The safety factor against overturning,

b. The safety factor against sliding (minimum required F.S. =1.5)
   Do not consider the passive resistance of the fill in front of the wall.

c. If the overturning safety is not satisfactory, extend the base to the right and satisfy the overturning stability requirement.

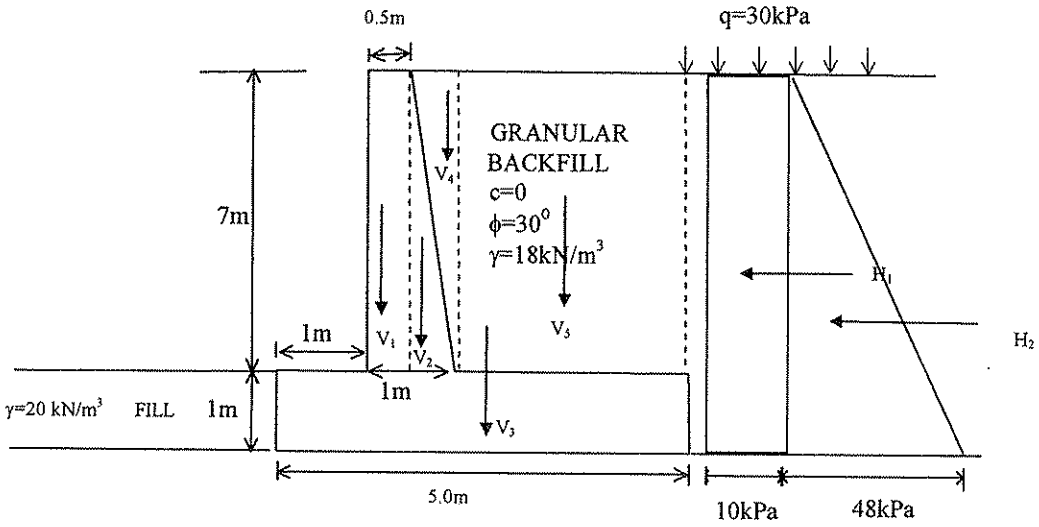
   If the sliding is not satisfactory, design a shear key (location, thickness, depth) under the base slab to satisfy the sliding stability. Take advantage of passive resistance of the foundation soil. Calculate the vertical stress starting from the top level of the base but consider the passive resistance starting from the bottom level of the base slab (i.e. in the sand). Use a factor of safety of 2.0 with respect to passive resistance.



MEDIUM DENSE SILTY SAND

c=0, $\phi=32^0$, $\gamma=20kN/m^3$, $\tan\delta=0.5$(base friction), $\gamma_{conc}=24kN/m^3$

Solution:



$K_a = \tan^2(45 - \phi/2)$

For granular backfill $\Rightarrow K_a = \tan^2(45-30/2) = 0.333$

Active pressure, $p_a = (q + \gamma z)K_a - 2c\sqrt{Ka}$

$z=0 \Rightarrow p_a = 30x0.333 = 10\ kN/m^2$

$z=8 \Rightarrow \sigma_a = (30+18x8)0.333 = 58\ kN/m^2$

Force(kN/m)	Arm,about toe(m)	Moment(kN.m/m)
V_1=0.5x7x24=84	1.25	105
V_2=0.5x7x1/2x24=42	1.67	70
V_3=1x5x24=120	2.5	300
V_4=0.5x7x1/2x18=31.5	1.83	57.75
V_5=3x7x18=378	3.5	1323
ΣV=655.5		ΣM_r=1855.75
H_1=10x8=80	4	320
H_2=(58-10)x8x1/2=192	8/3	512
ΣH=272.0		ΣM_{ov}=832

a)

$$(F.S.)_{ov}=\frac{\sum M_r}{\sum M_{ov}}=\frac{1855.75}{832}=2.23$$

$(F.S.)_{ov}$=2.23>2.0 O.K.

c=0(at the base) do not consider

b)

$$(F.S.)_{sliding}=\frac{\sum V.\tan\delta+(2/3cB)+P_p}{\sum H}$$

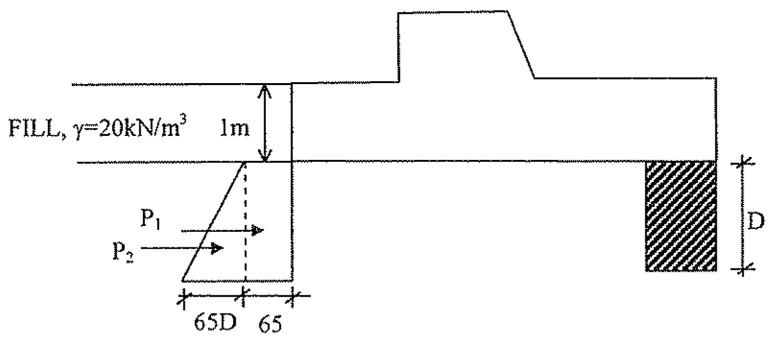
$$(F.S.)_{sliding}=\frac{\sum V.\tan\delta}{\sum H}=\frac{655.5x0.5}{272}=1.20$$

$(F.S.)_{sliding}$=1.20<1.5 NOT O.K. <u>DESIGN BASE KEY</u>

c)

<u>Base key design:</u>

Passive resistance at the base key;



$K_p=\tan^2(45+32/2)=3.25$

$z=0 \Rightarrow p_p=1x20x3.25=65$ kPa

$z=D \Rightarrow p_p=(1x20+20xD)x3.25=65+65D$ kPa

$P_p=P_1+P_2=65D+1/2x65D^2$

Use F.S.=2.0 w.r.t. passive resistance $\Rightarrow P_p=1/2(65D+1/2x65D^2)$

$$(F.S.)_{sliding}=\frac{\sum V.\tan\delta+P_p}{\sum H}=\frac{655.55x0.5+1/2(65D+1/2x65D_2)}{272}=1.5$$

Then, $65D+32.5D^2=160.5 \quad \Rightarrow \underline{D=1.43m}$

<u>If passive resistance (with a F.S. of 2.0) is subtracted from the driving horizontal forces, (i.e. used in the denominator)</u>

Use F.S.=2.0 w.r.t. passive resistance $\Rightarrow P_p=1/2(65D+1/2x65D^2)$

$$(F.S.)_{sliding} = \frac{\sum V.\tan\delta}{H\text{-}Pp} = 1.50$$

Then, $\underline{D=1.07m}$

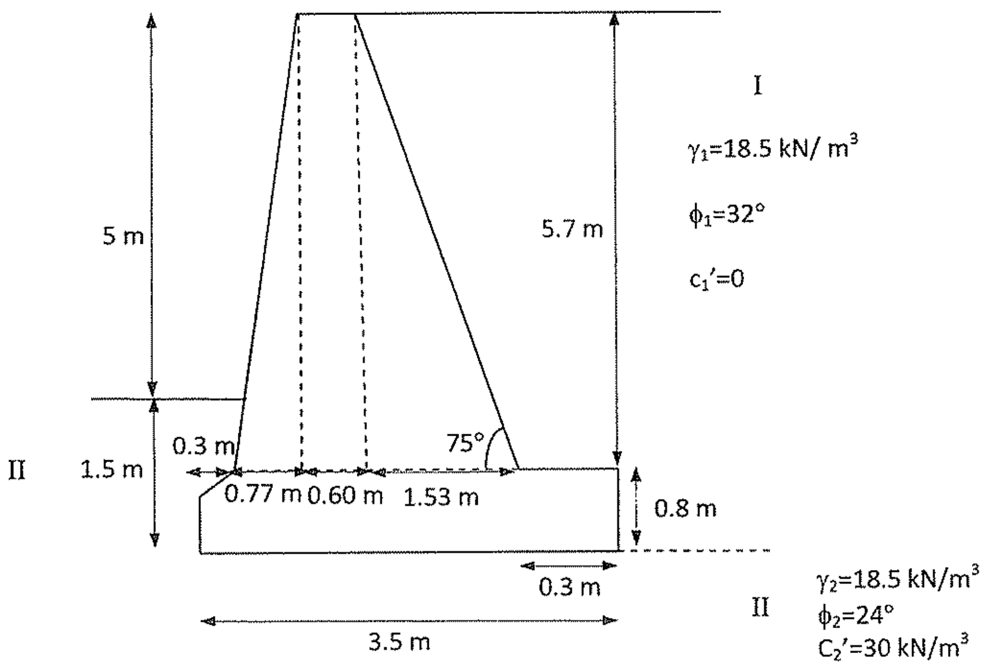
Take **D=1.43m** as it is on safe side.

P2. GRAVITY RETAINING WALL

Question

A gravity retaining wall is shown below. Use $\delta = 2/3\ \phi$ and Coulomb active earth pressure theory. Determine

a. The factor of safety against overturning
b. The factor of safety against sliding
c. Calculate base pressures for both cases;
   - considering the passive pressure, and
   - neglecting it.

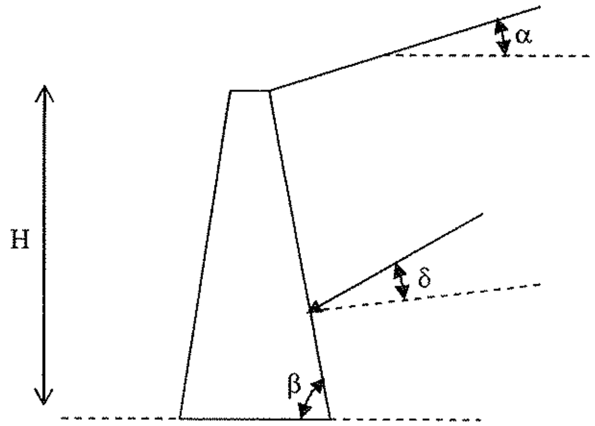


Soil properties: **I)** γ_1=18.5 kN/m^3 , ϕ_1=32^0, c_1= 0

II) γ_2=18.0 kN/m^3 , ϕ_2=24^0, c_2= 30 kN/m^2

$\gamma_{concrete}$=24 kN/m^3

Note: In Coulomb's active earth pressure theory, the forces to be considered are **only** P_a(Coulomb) and weight of the wall i.e. the weight of the soil above the back face of the wall is not taken into account.



Coulomb active forces;

$$P_a = \frac{1}{2}\gamma H^2 K_a$$ where

H =Height of the wall

K_a = Coulomb's active earth pressure coefficient

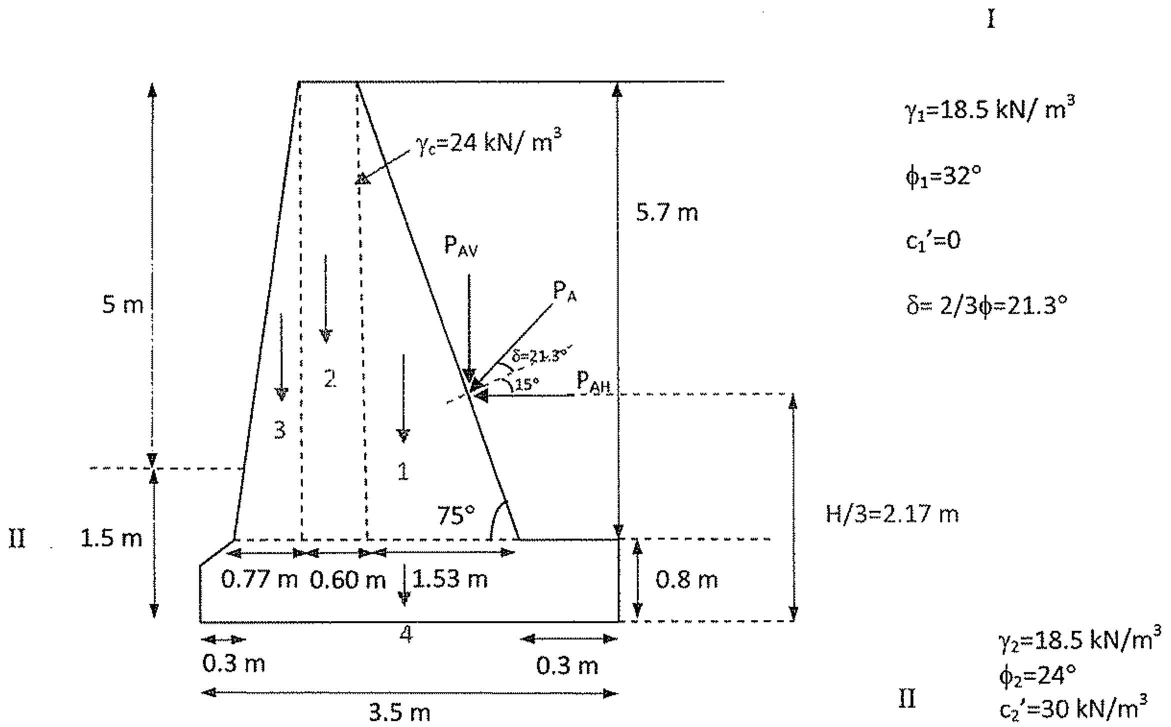
$$K_a = \frac{\sin^2(\beta+\phi)}{\sin^2\beta.\sin(\beta-\delta).(1+\sqrt{\frac{\sin(\phi+\delta).\sin(\phi-\alpha)}{\sin(\beta-\delta).\sin(\alpha+\beta)}})^2}$$

With horizontal backfill; α=0°

With vertical retaining wall; β=90°

δ: friction between the wall and adjacent soil

Solution:



For $\alpha=0^0$

$\beta=75^0$ $\Rightarrow K_a=0.4023$ (use eqn. 1)

$\phi=32^0$

$\delta=(2/3)x32=21.3^0$

$$P_a=\frac{1}{2}.\gamma.H^2.K_a=\frac{1}{2}x18.5x6.5^2x0.4023=157.22\text{ kN/m}$$

$$P_h=P_a.\cos(15+\delta)=157.22\,x\cos 36.3=126.65\text{ kN/m}$$

$$P_v=P_a.\sin(15+\delta)=157.22\,x\sin 36.3=93.15\text{ kN/m}$$

Force (kN/m)	Moment arm about pt. A (m)	Moment (kN.m/m)
1) $(\frac{1}{2}x1.53x5.7)x24 = 104.65$	2.18	228.14
2) $(0.6x5.7)x24 = 82.08$	1.37	112.45
3) $(\frac{1}{2}x0.77x5.7)x24 = 52.67$	0.81	42.66
4) $(3.5x0.8)x23.58 = 67.20$	1.75	117.60
P_v= 93.15	2.83	263.61
ΣV=399.75		**$\Sigma M_{resisting}$ =764.46**

$\Sigma M_{overturning}$= P_h x H/3 = 126.65x 2.17 = **274.83 kN.m/m**

a) $(F.S.)_{overt.} = \frac{\sum M_r}{\sum M_o} = \frac{764.46}{274.83} = 2.78 > 2.0$ O.K.

Note: if there is cohesionless soil at the base (c=0) ignore this term

if passive pressure is considered

b) $(F.S.)_{sliding} = \dfrac{\sum V.\tan\delta + (\frac{2}{3}.c_2\ .B) + P_p}{\sum H}$

$\delta=(2/3)x\phi_2$

- **P_p is ignored**

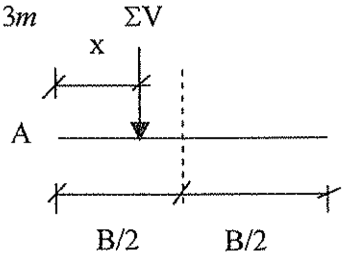
$$(F.S)_{sliding} = \frac{399.75x\tan(\frac{2}{3}x24) + (\frac{2}{3}x30x3.5)}{126.65} = 1.46$$

c. Pressure on soil at toe and heel

-If P_p is ignored

ΣM_{net}=764.46-274.83=489.63 kN.m/m

$$x = \frac{\sum M_{net}}{\sum V} = \frac{489.63}{399.75} = 1.22m \qquad \Rightarrow e = \frac{B}{2} - x = \frac{3.5}{2} - 1.22 = 0.53m$$



$$q_{min}^{max} = \frac{399.75}{3.5}\left[1 \pm \frac{6x0.53}{3.5}\right]$$

q_{max}=217.99 kN/ m^2 / m (toe)

q_{min}=10.44 kN/ m^2 / m (heel)

-If P_p is considered

K_p = (1+sin24°)/(1-sin24°)=2.37

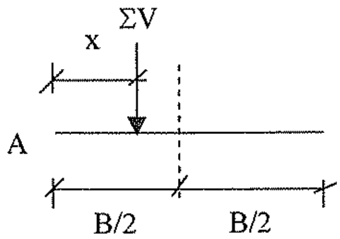
p_p @z=0 = $K_p(\gamma z)$+ 2c$(K_p)^{0.5}$=2x30x2.37$^{0.5}$=92.40 kPa

p_p @z=1.5 = $K_p(\gamma z)$+ 2c$(K_p)^{0.5}$=2.37x18.5x1.5+ 92.40=65.80+92.40=155.20 kPa

M_{res} (due to P_p)=92.4x1.5^{2}x0.5+0.5x65.80x(1/3)x1.5^2=128.63 kN.m/m

ΣM_{res}=764.46+128.63 = 893.09 kN.m/m

ΣM_{net}=893.09-274.83=618.26 kN.m/m



$$x = \frac{\sum M_{net}}{\sum V} = \frac{618.26}{399.75} = 1.55m \qquad \Rightarrow e = \frac{3.5}{2} - 1.55 = 0.20m$$

$$q_{min}^{max} = \frac{399.75}{3.5}\left[1 \pm \frac{6x0.20}{3.5}\right]$$

q_{max}=153.37 kN/ m^2 / m (toe)

q_{min}=75.05 kN/ m^2 / m (heel)

P3. REINFORCED EARTH WALL

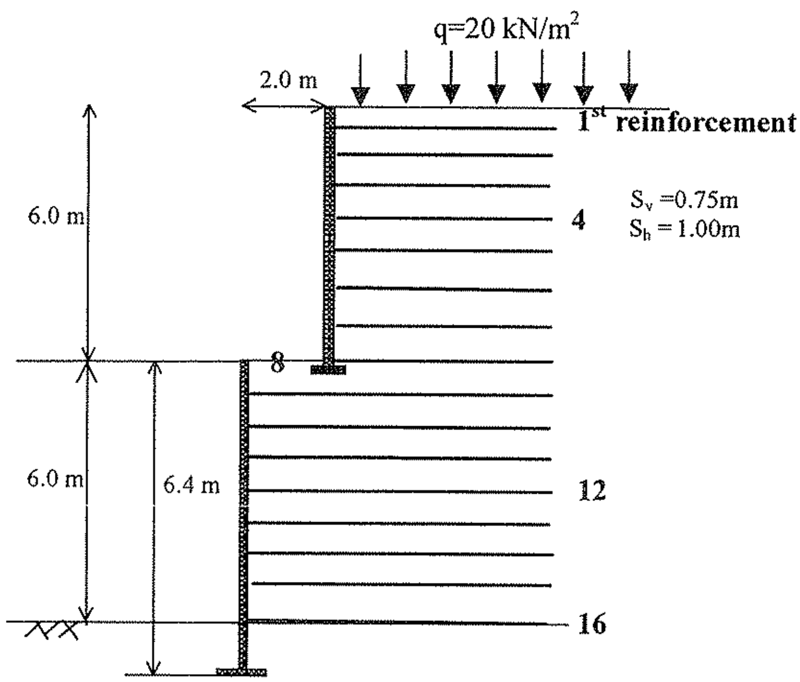
Question:

A reinforced earth wall is to be constructed as shown in the figure below. The material that will be used as backfill shall have the following properties, $\gamma = 17$ kN/m^3, $\phi = 30^0$, c=0. The strips will be galvanized steel and will have a width of 75mm. The yield stress for strip material is $f_y = 3 \times 10^5$ kN/m^2.

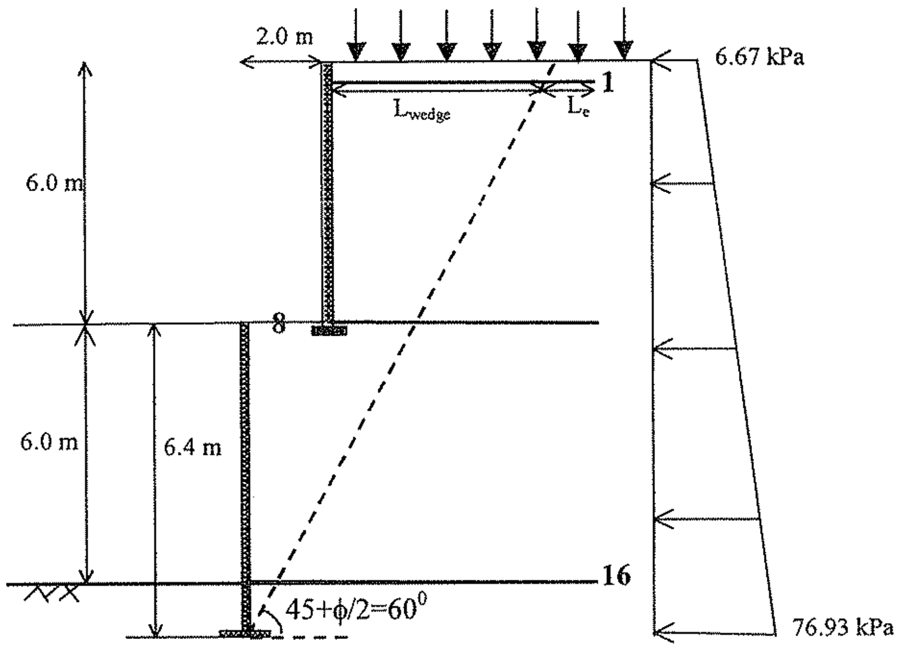
a. Design the reinforcements (i.e. determine the length and thickness) by using a factor of safety of 3.0 for both tie-breaking and pull-out.

b. Find the factor of safety along sliding on the base and calculate the base pressures for the foundation soil.

- Design life for structure 50 yrs.
- Corrosion=0.025 mm/yr
- Use Rankine Earth Pressure Theory and take the friction angle between soil and reinforcement as 20^0



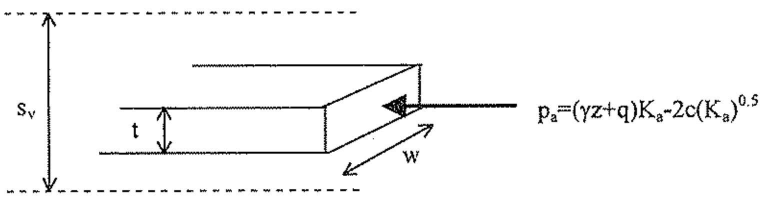
Solution:



$\phi=30^0 \Rightarrow K_a = 1/3$

a) **Design of reinforcement**

As far as the tie breaking is concerned, bottom reinforcement (16) is the most critical one since the lateral pressure is maximum at that level.



$$(F.S.)_{breaking} = \frac{w.t.f_y}{T_{max}} = 3.0$$

$$T = S_v.S_h.(\gamma z + q)\, K_a$$

$$T_{max} = 0.75x1.0x\,(12x17+20)x\frac{1}{3} = 56\ kN$$

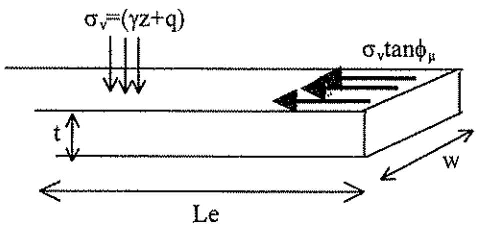
$$(\text{F.S.})_{\text{breaking}} = \frac{0.075 \times t \times 3 \times 10^5}{56} = 3.0 \qquad \Rightarrow t = 7.46 \text{ mm}$$

Corrosion rate $\Rightarrow$ 0.025mm/yr. x 50 =1.25mm

t=7.46+1.25 = 8.71mm

USE t_{design} =9mm

- As far as tie pull-out is concerned,



Frictional resistance is available on both surface (top and bottom)

Friction angle between soil and reinforcement

$$(\text{F.S.})_{\text{pull-out}} = \frac{2\cancel{(\gamma z + q)} \tan\phi_\mu L_e w}{\cancel{(\gamma z + q)} K_a S_v S_h} = \frac{2\tan\phi_\mu L_e w}{K_a S_v S_h} = 3.0$$

$$(\text{F.S.})_{\text{pull-out}} = \frac{2 \times 0.075 \times L_e \times \tan 20}{\frac{1}{3} \times 0.75 \times 1.0} = 3.0 \qquad \Rightarrow L_e = 13.7\text{m}$$

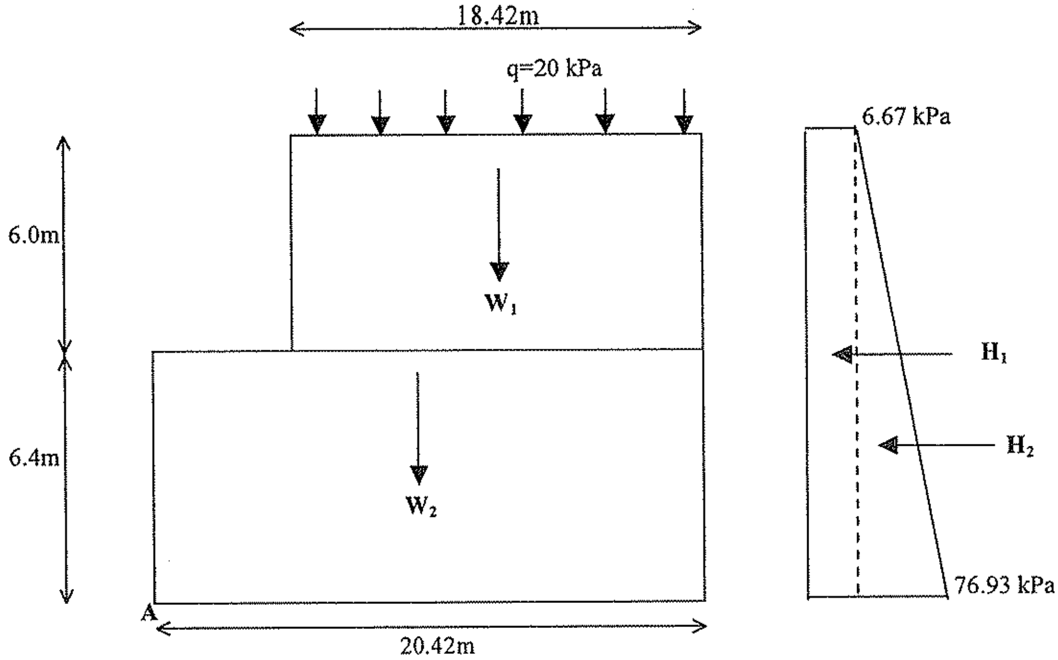
<u>Since first reinforcement (1) is the most critical one when the pull-out criterion is concerned,</u>

$$\tan(45-\phi/2) = \frac{L_{\text{wedge}} + 2}{12.4 - 0.75} \qquad \Rightarrow L_{\text{wedge}} = 4.72\text{m}$$

- Total tie length $L = L_{wedge} + L_e$=13.7+4.72=18.42m for upper 6m of the wall

- For lower 6m of the wall , L=20.42m

b) $\underline{(F.S.)_{sliding}}$ and Base Pressure



Forces (kN/m)	Moment arm, about A (m)	Moment (kN.m/m)
W_1=18.42x6.0x17=1878.8	11.21	21061
W_2 = (18.42+2)x6.4x17 = 2221.7	10.21	22684
Load = 20x18.42 = 368.4	11.21	4130
ΣF_v = 4469		**ΣM_r = 47875**
H_1 = 6.67x12.4 = 82.7	12.4 /2	512.7
H_2 = (76.93-6.67)x12.4x(1/2) = 435.7	12.4 /3	1800
ΣF_h = 518		**ΣM_{ov} = 2313**

$(FS)_{sliding} = (\Sigma F_v \cdot \tan\delta) / \Sigma F_h$

In gravity or cantilever retaining walls, at the base of the wall, we would use tan δ for soil-wall friction. However in this problem, we see that, at the bottom of the wall, there is soil-soil interface, therefore we should use the friction angle of the soil in the F.s. sliding equation. (If two soils have different internal friction angles, the lower value should be used.

$(FS)_{sliding} = (4469 \times \tan 30) / 518$

$= \mathbf{4.98}$

$X = \Sigma M_{net} / \Sigma F_v = (47875 - 2313) / 4469 = 10.2$ m

$e = B/2 - X = (18.42 + 2)/2 - 10.2 = 0.01 \sim 0$ $\Rightarrow$ no eccentricity

$q_{max} = \Sigma F_v / B = 4469 / 20.42$

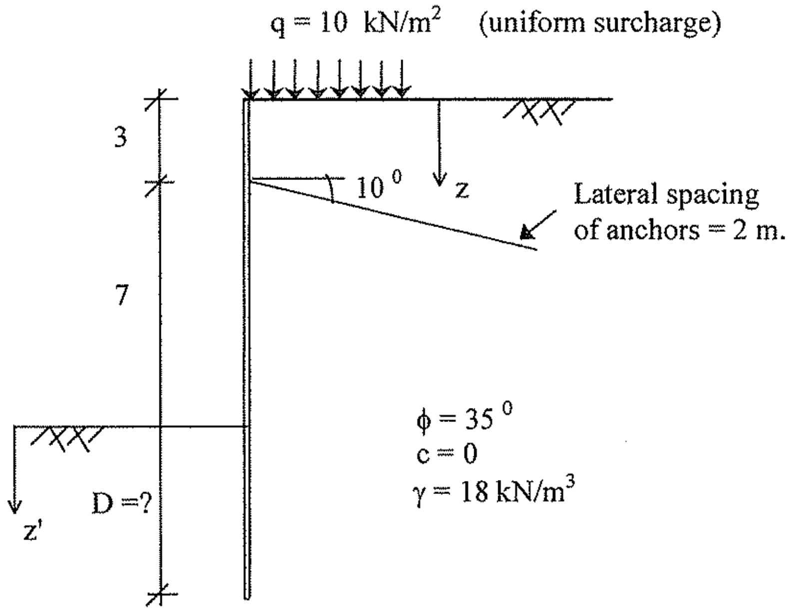
$= \mathbf{218.85\ kN/m^2 / m}$

P1. ANCHORED SHEET PILE WALL

Question:

An anchored sheet-pile wall is constructed as shown in the figure below. By using Rankine's Earth Pressure Theory and free earth support method, determine:

a. Depth of penetration.
b. Axial anchor force if center to center spacing of two successive anchors is 2 meters.
c. Maximum bending moment in the sheet pile.



Solution:

$$K_a = \tan^2(45 - \frac{\phi}{2}) = \tan^2(45 - \frac{35}{2}) = 0.27$$

$$K_p = \frac{1}{K_a} = 3.69$$

Active Pressure:

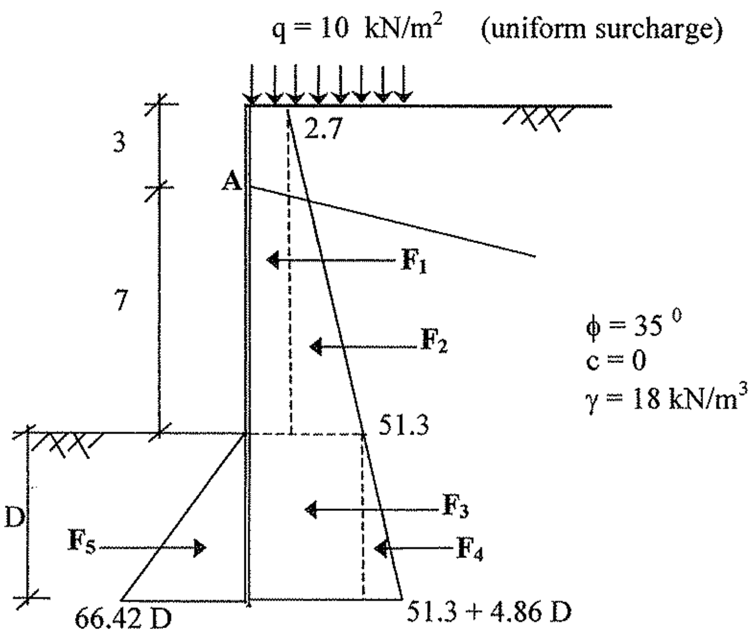
$$P_a = (\gamma z + q).K_a - 2.c.\sqrt{K_a}$$

z = 0 m	$p_a = 10 \times 0.27 = 2.7$ kPa
z = 10 m	$p_a = (10 \times 18 + 10) \times 0.27 = 51.3$ kPa
z = 10 + D	$p_a = [\,(10 + D) \times 18 + 10\,] \times 0.27 = 51.3 + 4.86\,D$ kPa

Passive Pressure:

$$P_p = (\gamma z + q).K_p + 2.c.\sqrt{K_p}$$

z' = 0 m	$p_p = 0$ kPa
z' = D m	$p_p = 18 \times D \times 3.69 = 66.42\,D$ kPa



Force (kN/m)	Moment arm about point A (m)	Moment, M_A (kN.m / m)
$F_1 = 2.7 \times 10 = 27$	2	54
$F_2 = (51.3-2.7) \times 10 \times 0.5 = 243$	3.67	889.38
$F_3 = 51.3 \times D = 51.3\,D$	$7 + D/2$	$359.1\,D + 25.6\,D^2$
$F_4 = 4.86\,D \times D \times 0.5 = 2.43\,D^2$	$7 + 2D/3$	$17.01\,D^2 + 1.62\,D^3$
$-F_5 = 66.42\,D \times D \times 0.5 = -33.21\,D^2$	$7 + 2D/3$	$-232.47\,D^2 - 22.14\,D^3$

$\Sigma F_H = 270 + 51.3\,D - 30.78\,D^2$

$\Sigma M_A = 0$

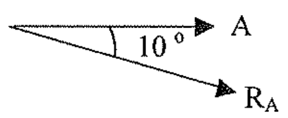
$\Sigma M_A = 943.38 + 359.1\,D - 189.86\,D^2 - 20.52\,D^3 = 0$ $\Rightarrow D = 2.80$ m.

a) Depth of penetration : $1.2 \times 2.80 = 3.36$ m.

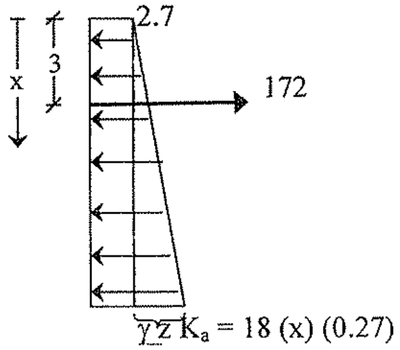
b) Anchor Force : $\Sigma H = 0$ (force equilibrium)

$\Sigma H = 270 + 51.3\,(2.80) - 30.78\,(2.80)^2 - A = 0$ $\Rightarrow A = 172$ kN / m

$R_A = (A / \cos 10) \times 2 = 350$ kN (2 m is the lateral spacing of anchors)



c) Max. Bending Moment : (when shear , V= 0)



To find the location of M_{max}, determine the point at which shear force is equal to 0

$2.7\ (x) + [18.(x).(0.27)].(x).0.5 - 172 = 0$

$$2.7\ x + 2.43\ x^2 - 172 = 0 \qquad x = 7.88\ m\ \text{(distance from top)}$$

$M_{max} = 2.7\ (7.88)\ (7.88 / 2) + 18\ (7.88)(0.27)\ (7.88 / 2)\ (7.88 / 3) - 172\ (7.88 - 3)$

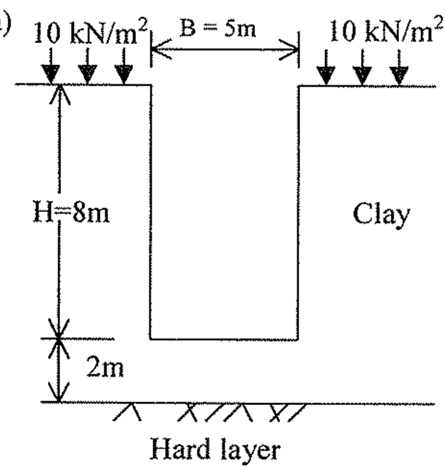
$$M_{max} = 359.24\ kN.m / m$$

P2. BRACED CUTS

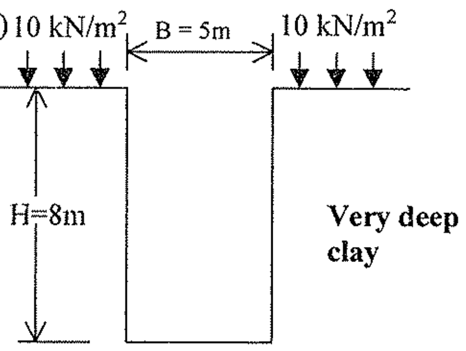
Question:

For the very long braced systems shown in the figures (a) and (b), when c_u=40 kN/m^2, ϕ_u=0, γ=19 kN/m^3, and there is no water, what is the factor of safety of the bottom against heave?

(a)

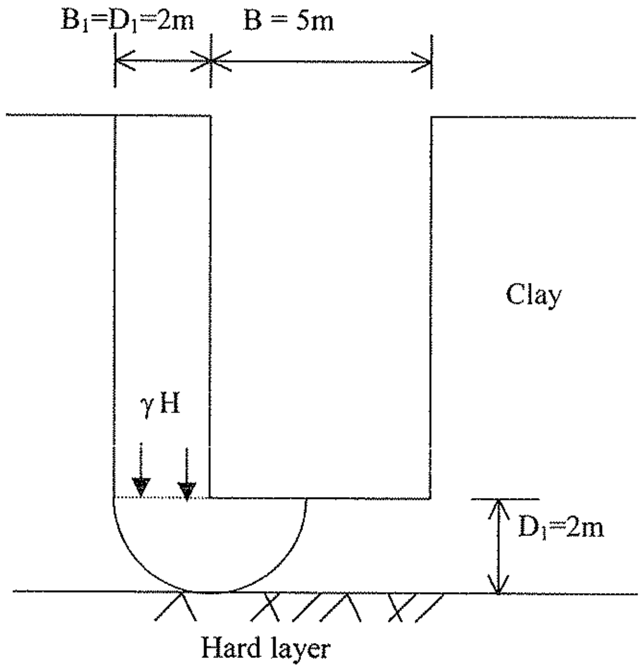


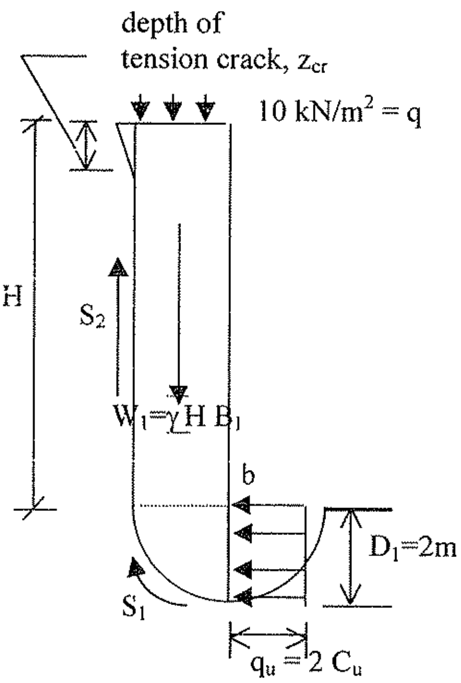
(b)



Solution:

(a)





Depth of tension crack;

$$P_{active} = (\gamma z + q)K_a - 2C_u\sqrt{K_a}$$

$\phi = 0° \longrightarrow K_a = 1$

$$p_{active} = (\gamma z+q) - 2C_u = 0$$

$$(\gamma z+q) = 2C_u$$

$$z_{cr} = \frac{2C_u - q}{\gamma}$$

For ; $C_u = 40$ kPa ; $q=10$ kPa
$\gamma = 19$ kN/m^3

$$z_{cr} = \frac{2C_u - q}{\gamma} = \frac{2x40 - 10}{19} = \frac{70}{19} = 3.68m$$

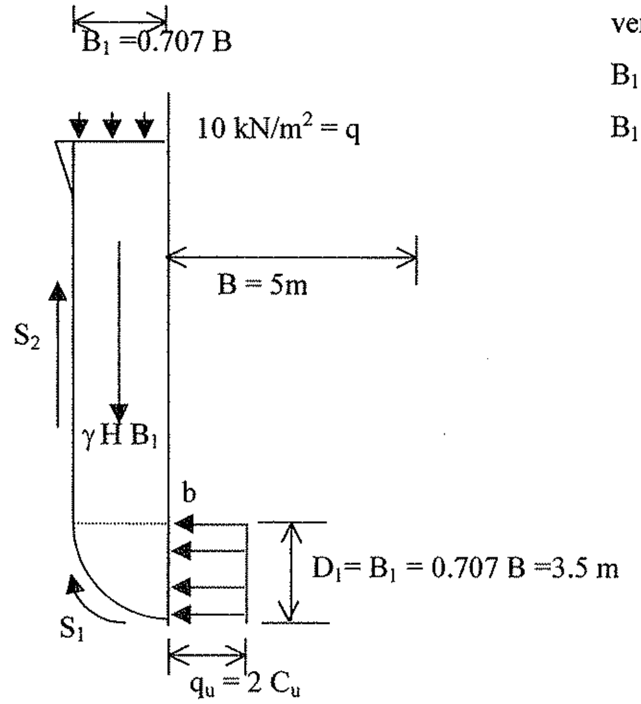
For ; $B_1 = D_1 = 2$m
$q_u = 2\ C_u = 2x40 = 80$ kPa
$H = 8$m ; $\gamma = 19$ kN/m^3

taking moment about b;

Force (kN/m)	Moment arm (m), about point b	Moment, M_b (kN.m / m)
$S_1 = (0.5x\pi xB_1)xCu= (0.5x\pi x2)x40 =125.60$	$B_1=2$	251.20
$S_2 = (H-z_{cr})xC_u = (8-3.68)x40= 172.80$	$B_1=2$	345.60
$P_1 = q_u xB_1=2xC_u xB_1=2x40x2=160$	$0.5xB_1=1$	160
$W_1 = \gamma xHxB_1=19x8x2=304$	$0.5xB_1=1$	-304
$W_2 = qxB_1=10x2=20$	$0.5xB_1=1$	-20

$$FS = \frac{251.20 + 345.60 + 160}{304 + 20} = 2.34$$

(b)



very deep clay;

B_1 =0.707 B

B_1 =0.707 x 5 = 3.5 m

For ; $B_1 = D_1 = 3.5$ m
$q_u = 2\ C_u = 2x40 = 80$ kPa
$H = 8m\ ;\ \gamma = 19\ kN/m^3$

taking moment about b;

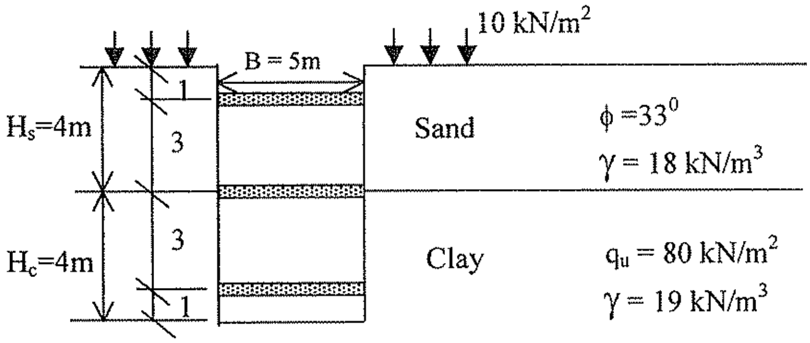
Force (kN/m)	Moment arm (m), about point b	Moment, $\mathbf{M_b}$ (kN.m / m)
$S_1 = (0.5x\pi xB_1)xCu= (0.5x\pi x3.5)x40=219.80$	B_1=3.5	769.30
$S_2 = (H-z_{cr})xC_u = (8-3.68)x40= 172.80$	B_1=3.5	604.80
$P_1 = q_uxB_1=2xC_u\ xB_1=2x40x3.5=280$	$0.5xB_1$=1.75	490
$W_1 = \gamma xHxB_1=19x8x3.5=532$	$0.5xB_1$=1.75	-931
$W_2 = qxB_1=10x3.5=35$	$0.5xB_1$=1.75	-61.25

$$FS = \frac{769.30 + 604.80 + 490}{931 + 61.25} = 1.88$$

P3. BRACED CUTS

Question:

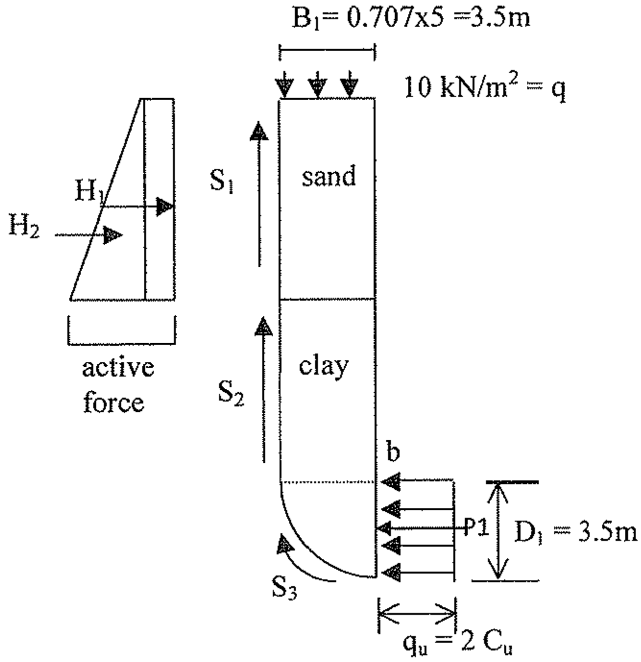
Determine the factor of safety of the bottom against heave for the very long braced system shown below (hint: make reasonable assumptions).



Solution:

For sand, consider active earth pressure, not earth pressure at rest, because of some lateral displacement during excavation.

$K_a = \tan^2(45 - \phi/2) = \tan^2(45 - 33/2)$
$K_a = 0.29$



z = 0 $\qquad p_a = 10 \times 0.29 = 2.9$ kPa

z = 4 $\qquad p_a = (10 + 4 \times 18) \times 0.29 = 23.8$ kPa

$H_1 = 2.9 \times 4 = 11.6$ kN/m

$H_2 = (23.8 - 2.9) \times 4 \times (1/2) = 41.8$ kN/m

$\Sigma = 53.4$ kN/m

Force (kN/m)	Moment arm about point A (m)	Moment, M_A (kN.m / m)
$S_1 = \sigma_n \tan\phi = 53.4 \times \tan 33 = 35$	3.5	122.5
$S_2 = 4\ C_u = 4 \times 40 = 160$	3.5	560
$S_3 = 0.5 x \pi x B_1 x C_u = 0.5 x \pi x 3.5 x 40 = 220$	3.5	770
$P_1 = 80x3.5 = 280$	1.75	490
$W_1 = 4x18x3.5=252$	1.75	-441
$W_2 = 4x19x3.5=266$	1.75	-465.5
$W_3 = 10x3.5=35$	1.75	-61.25

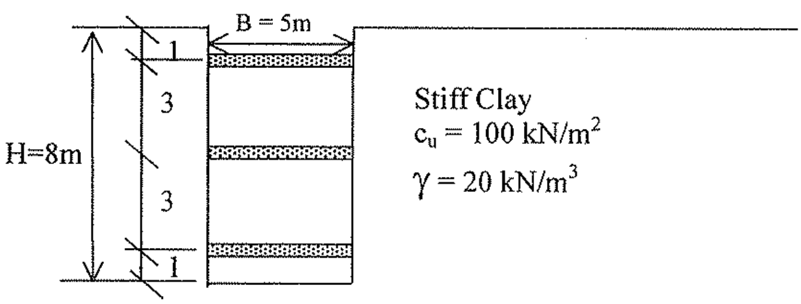
$$FS = \frac{122.5 + 560 + 770 + 490}{441 + 465.5 + 61.25} = 2.0$$

P4. BRACED CUTS

Question:

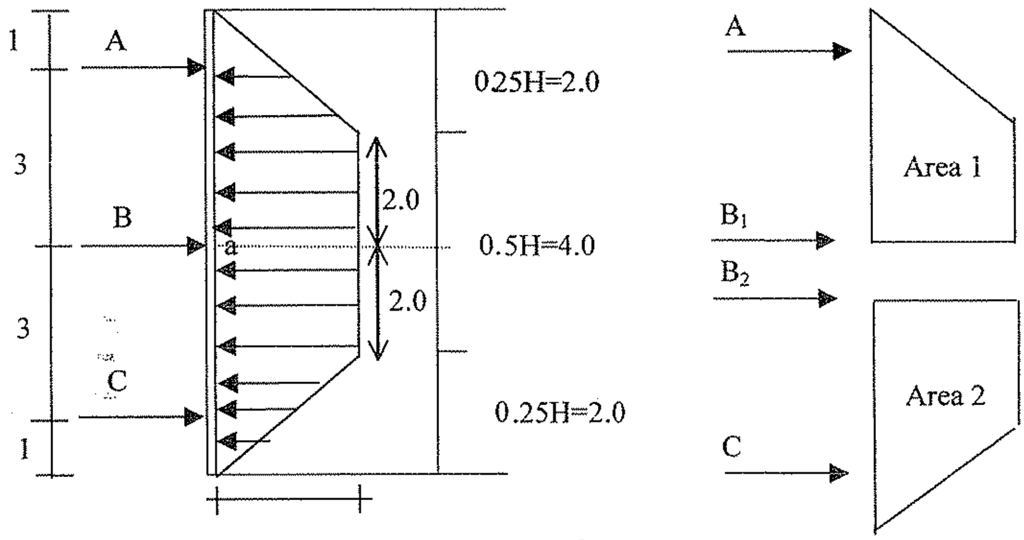
Find the strut loads for each level for the long braced system given below.

Horizontal struts are spaced at every 5 m. No ground water.



Solution:

- for strut loads, the earth pressure distribution is



$0.3\gamma_t H$ =0.3x20x8=48 kN/m^2 per linear meter

area 1 ⟶ 2.0x48x(1/2) + 2.0x48 = 144 kN/m

area 2 ⟶ 2.0x48 + 2.0x48x(1/2) = 144 kN/m

taking moment wrt. point a;

⟶ 3.0 A = 2.0 x 48 x (2.0 / 2) + 2.0 x 48 x (1/2) x (2.0 /3 + 2.0)

A = 74.7 kN/m

B_1 = 144 – 74.7 = 69.3 kN/m

⟶ 3.0 C = 2.0 x 48 x (2.0 / 2) + 2.0 x 48 x (1/2) x (2.0 /3 + 2.0)

C = 74.7 kN/m

B_2 = 144 – 74.7 = 69.3 kN/m

spacing

Strut loads; A = 74.7x 5 = 373.5 kN

B = (69.3 + 69.3) x 5 = 693 kN

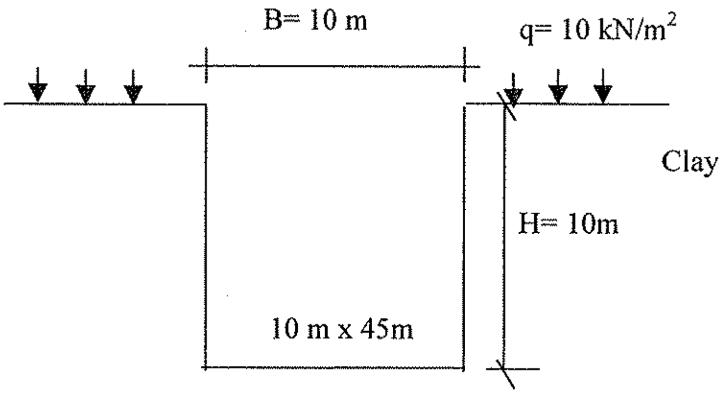
C = 74.7 x 5 = 373.5 kN

P5. BRACED CUTS

Question:

For a braced system constructed in a 10 m deep rectangular excavation in a clay, when length L= 45m ; width B= 10m ; surcharge $q = 10 kN/m^2$; unit weight $\gamma = 19 kN/m^2$ and unconfined compressive strength $q_u = 80 kN/m^2$; and there is no water, what is the factor of safety at the bottom against heave?

Solution:



If the excavation is not very long $(L/B \leq 10) \longrightarrow$ square, rectangular or circular exc.

Assumption $\longrightarrow$ braced cut is a deep footing

$$F.S. = \frac{N_c C_u}{(\gamma H + q)} = \frac{N_c q_u}{2(\gamma H + q)} = \frac{\text{ultimate bearing capacity}}{\text{applied load}}$$

N_c : bearing capacity factor

(from Fig 4.6, pp 73 of Lecture Notes)

$H/B = 10 / 10 = 1$ $\quad\quad$ $N_c\,(\text{square}) = 7.7$

$N_{c\,(rect)} = (0.84 + 0.16\, B/L)\, N_c\,(\text{square})$

$= (0.84 + 0.16 \times 10 / 45) \times 7.7$

$= 6.8$

$$FS = \frac{6.8x40}{(19x10 + 10)} = 1.36$$

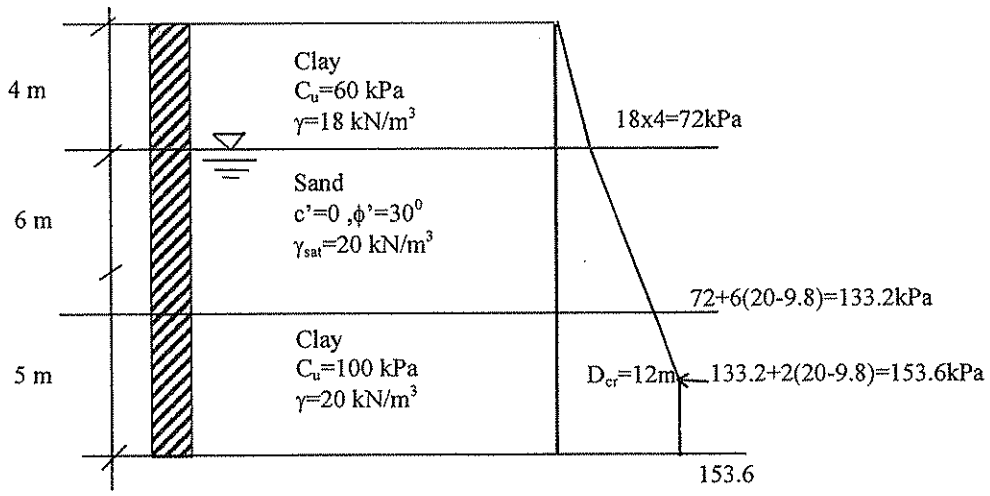
CE 366 – PILE FOUNDATIONS

Q1) ULTIMATE BEARING CAPACITY OF SINGLE PILES

Question

Determine the ultimate bearing capacity of the 800mm diameter concrete, bored pile given in the figure below. Assume D_{cr} = 15*diameter; and the pile-friction angle =0.75ϕ'

D_{cr}=15*0.8=12m



Solution

Ultimate capacity of pile:

$Q_{ult}=Q_p+Q_s$

- $Q_p = N_c.c_u.A_p = 9c_uA_p = 9x100x[\pi x(0.8)^2/4]$ = <u>452 kN</u>

  N_c : Fig 4.6, pp 73 of Lecture Notes
- $Q_s = Q_{s1} + Q_{s2} + Q_{s3}$
- where Q_{s1} ($0 < z < 4$m), Q_{s2} ($4 < z < 10$m), and Q_{s3} ($10 < z < 15$m),

$Q_{s1} = \alpha.c_u.A_s = 0.8x60x(\pi x0.8x4)$ = <u>483kN</u>

α : pp 215 of Lecture Notes

$Q_{s2} = K_s.\sigma_{vo}'.\tan\delta.A_s$; where

K_s = 0.5 (pp 215 of Lecture Notes), δ=0.75, ϕ'=22.5^0

A_s = p.L = $(\pi x0.8)x6$ =15.08 m^2

Q_{s2}=0.5x[(72+133.2)/2]xtan(22.5)x15.08 =<u>320 kN</u>

$Q_{s3} = \alpha.c_u.A_s = 0.58x100x(\pi x0.8x5)$ = <u>729kN</u>

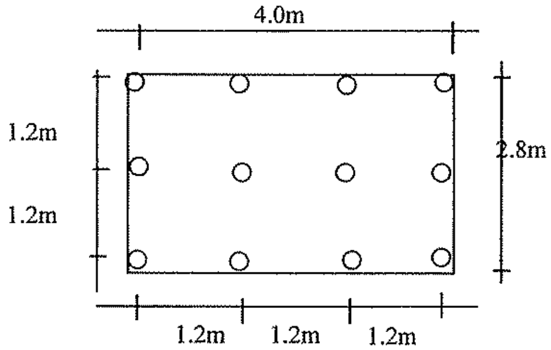
Q_{ult}=452+(483+320+729)=**1984 kN**

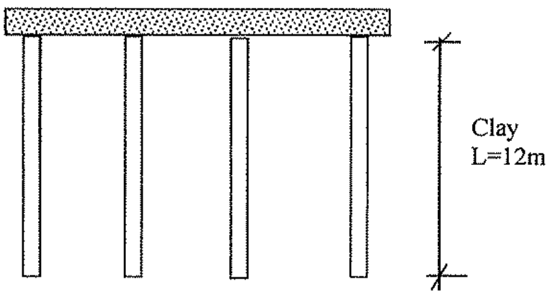
Q2) PILE GROUP CAPACITY

Question

Find the allowable bearing capacity of a single pile in the group of piles given below, by using: (Diameter : 0.4m, C_u=50kPa, γ=18kN/m^3, and use F.S.=2.5)

a) Converse-Labarre Formula
b) Terzaghi-Peck Method





Solution

a)

$Q_{ult}=Q_p+Q_s$

- $Q_p=9c_uA_p=9x50x\pi x(0.4)^2/4 = 56.5$ kN
- $Q_s=\alpha.c_u.A_s=0.84x50x\pi x0.4x12 = 633$kN

$Q_{ult} = 56.5+633 = 689.5$ kN

$Q_{all} = 689.5 / 2.5 \cong$ **275.8 kN**

Converse-Labarre(Group action reduction)

$$E = 1 - \theta\left[\frac{(n-1)m + (m-1)n}{90mn}\right]$$

m= # of rows=3

n= # of piles in a row=4

θ = arctan(D/s)=arctan(0.4/1.2)=18.4^0

D=diameter

s=spacing(center to center)

$$E = 1-18.4\left[\frac{(4-1)3+(3-1)4}{90x3x4}\right] = 0.71$$

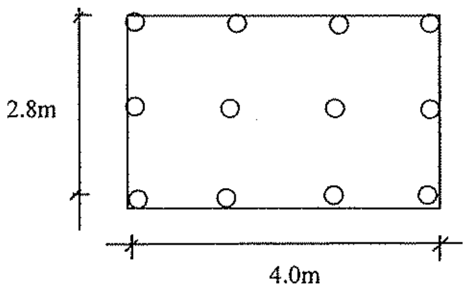
$(Q_{all})_{\text{single pile in the group}}=275.8x0.71 = \mathbf{195.8\ kN}$

b)

<u>Terzaghi Peck Group Reduction Method:</u>

$$(Q_g)_{ult} = \overbrace{pD_f c_u}^{\text{Skin fric.}} + \overbrace{A(q_{ult})_{net}}^{\text{Tip resist.}} \qquad ;\ (q_{ult})_{net} = q_{nf} = c_u N_c$$

$$= pD_f c_u + Aq_{ult} - AD_f\gamma$$



$p=2(2.8+4.0)=13.6m$

$A=2.8x4= 11.2m^2$

$Q_g=13.6x12x50+11.2(50x8.6)=12976kN$

$(Q_{ult})_{\text{for single pile}} = 12976/12 = 1081\ kN$

$(Q_{all})_{\text{for single pile}} = 1081/2.5 = 432.5\ kN > 275.8\ kN$

$> 195.8\ kN$

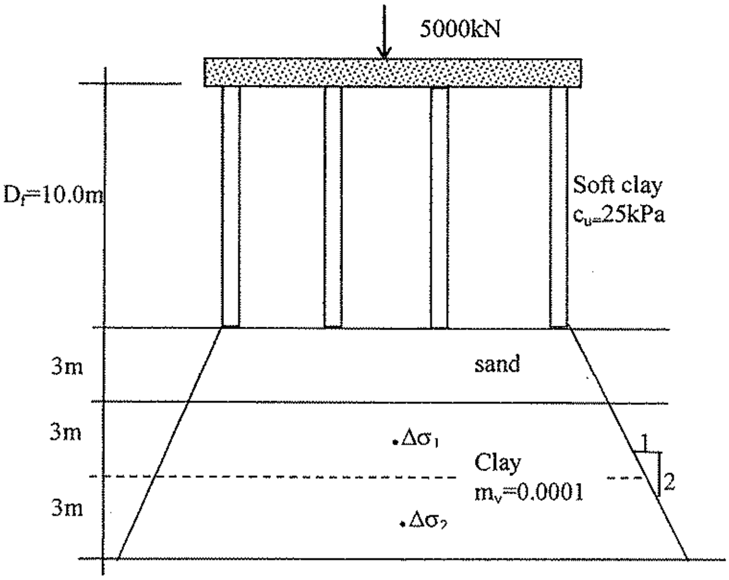
Bearing capacity is not controlled by Q_{group}; therefore:

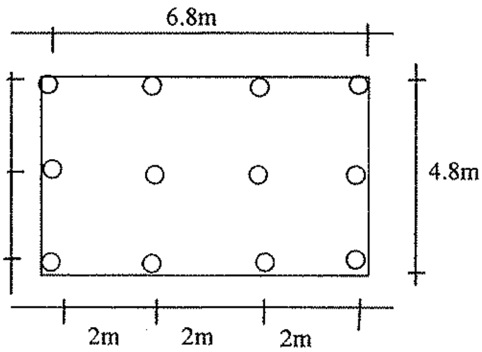
$(q_{all})_{\text{single pile}} = \underline{\mathbf{195.8\ kN}}$

Q3) SETTLEMENT OF PILE GROUPS

Question

Calculate the consolidation settlement of the 12 pile group consisting of 10 m long end bearing piles. Piles are 80 cm in diameter and spaced at 2m center to center in either direction. Pile group carries a vertical load of 5000 kN including the weight of the pile cap. Use 2:1(V:H) pressure distribution and divide the layer into two equal layers.





Solution

Stress distribution begins at the bottom of the pile group in stiff clay media. In the case of soft clay the distribution should be started at depth of 2/3 of L where L is the length of the pile.

$s = H \times m_v \times \Delta\sigma$

$\Delta\sigma_1 = 5000/((6.8+4.5)x(4.8+4.5)) = 47.6\ kN/m^2$

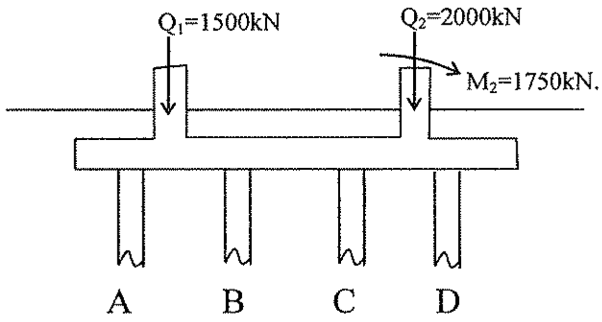
$\Delta\sigma_2 = 5000/((6.8+7.5)x(4.8+7.5)) = 28.4\ kN/m^2$

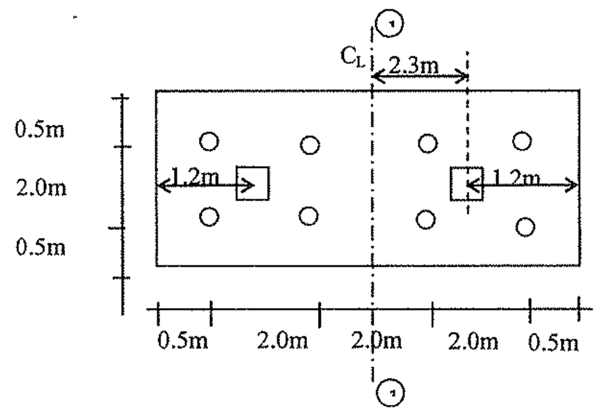
s=3x0.0001x(47.6+28.4)=0.0228m=**2.28cm**

Q4) LOAD ON PILES

Question

Find the maximum and minimum vertical load in the pile group given below





Solution

$\Sigma Q = 3500kN$

$$e = \frac{2000 * 2.3 + 1750 - 1500 * 2.3}{3500} = 0.83m$$

$M = \Sigma Q x e = 3500 x 0.83 = 2900 kN.m$

$I_{1-1} = 2x(2x1^2 + 2x3^2) = 40 pile\text{-}m^4$

due to piles

$Q_i = \Sigma Q / n \pm M x d_i / I$

$Q_D = 3500/8 + 2900x3.0/40$ = **655 kN(max)**

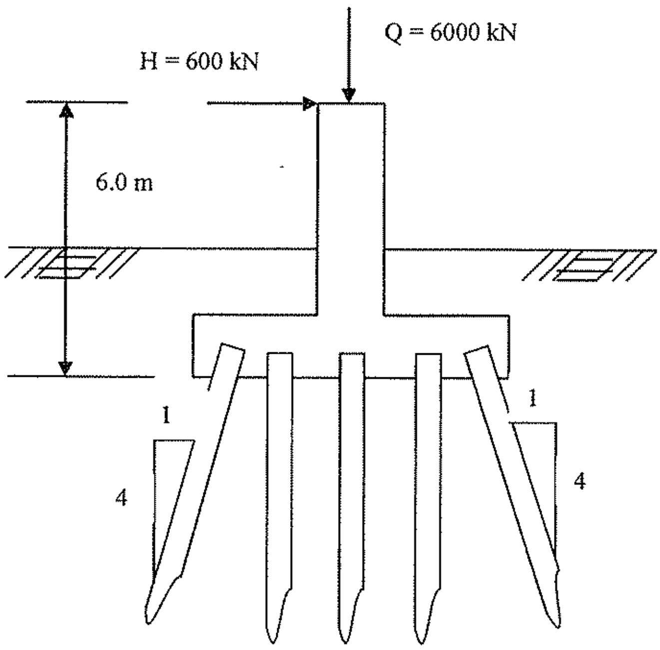
$Q_A = 3500/8 - 2900x3.0/40$ = **220 kN(min)**

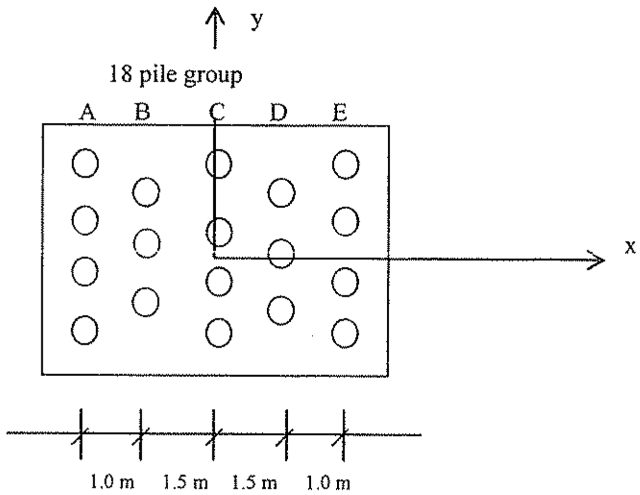
Q5) BATTER PILES

Question

One of the legs of a steel structure rests on a concrete pedestal footing which is supported by a group of 18 piles. Outside rows are formed from batter piles (1 to 4) as shown below. Each pile is permitted to resist an horizontal force of 25 kN.

a) Calculate axial loads on all piles.
b) Could the unbalanced horizontal force be overcome?
c) If the allowable bearing value of single pile is equal to 600 kN, state whether or not this pile foundation is safe.





Solution

a)

- $M = 6 * 600 = 3600 kN - m$
- $I = 2 * \left(3 * 1.5^2 + 4 * 2.5^2\right) = 63.5 pile - m^2$

- Vertical Component of pile loads:

$$Q_i = \frac{Q}{n} \pm \frac{M}{I} * x_i$$

$$Q_A = \frac{6000}{18} - \frac{3600}{63.5} * 2.5 = 192 kN$$

$$Q_B = \frac{6000}{18} - \frac{3600}{63.5} * 1.5 = 248 kN$$

$$Q_C = \frac{6000}{18} = 333 kN$$

$$Q_D = \frac{6000}{18} + \frac{3600}{63.5} * 1.5 = 418 kN$$

$$Q_E = \frac{6000}{18} + \frac{3600}{63.5} * 2.5 = 475 kN$$

- Axial loads:

$$\tan\alpha = \frac{1}{4} \rightarrow \cos\alpha = 0.970$$

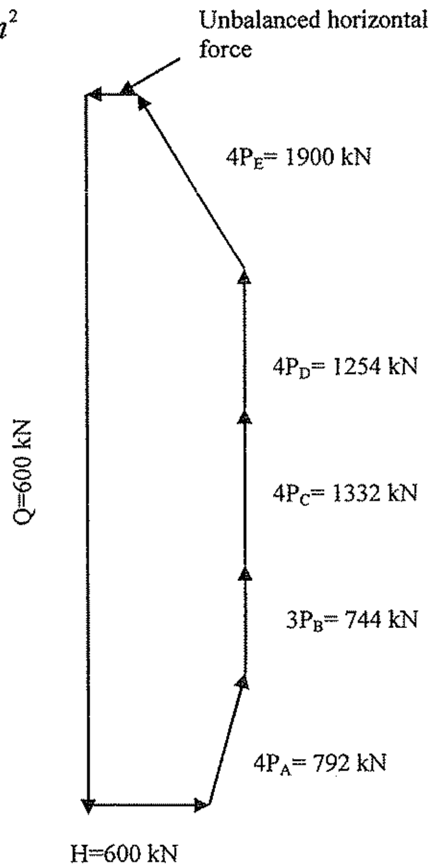
$$P_A = \frac{Q_A}{\cos\alpha} = \frac{192}{0.970} = 198$$

$$P_B = Q_B = 248 kN$$

$$P_C = Q_C = 333 kN$$

$$P_D = Q_D = 418 kN$$

$$P_E = \frac{Q_E}{\cos\alpha} = \frac{475}{0.970} = 490 kN$$



NOT SCALED

b)

- Unbalanced horizontal force:

$$H_u = 600 + 4 * 192 * \frac{1}{4} - 4 * 475 * \frac{1}{4} = 317 kN$$

c)

- Available horizontal resistance:

$H_r = 18 * 25 = 450 kN$. Since $H_u < H_r$ $\therefore$ Unbalanced force is overcome.

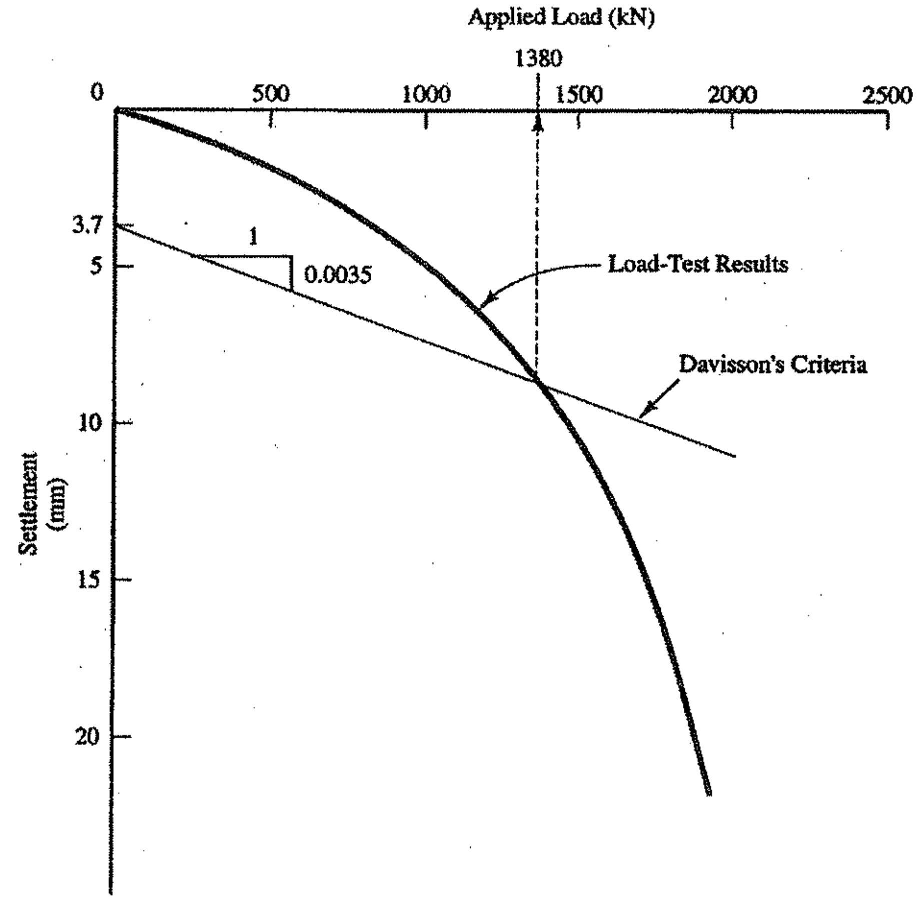
- Maximum axial load:

$P_{max} = 490 kN < P_{all} = 600 kN$ **(Foundation is safe).**

Q6) INTERPRETATION OF PILE LOAD TEST

Question

The load-settlement data shown in the figure were obtained from a full-scale load test on a 400 mm square, 17 m long concrete pile (28-day compressive strength of concrete, $f_c' = 40$ MPa). Use Davisson's method to compute the ultimate downward load capacity.



Solution

Davisson's method defines the ultimate capacity as that which occurs at a settlement of $0.012B_r + 0.1B/B_r + PL/(AE)$. The last term in this formula is the elastic compression of a pile that has no skin friction.

B= Diameter of the pile	B_r= Reference width = 300 mm
P= Applied load	L= Length of the pile

A= Cross-section area of the pile

E= Modulus of elasticity of the pile

200,000 MPa for steel

11,000 MPa for timber

$15{,}200\sigma_r\,(f_c'/\sigma_r)^{0.5}$ for concrete where reference stress, $\sigma_r = 0.1$ MPa

$E = 15{,}200\sigma_r\,(f_c'/\sigma_r)^{0.5} = (15{,}200)(100\text{ kPa})(40\text{ MPa} / 0.10\text{ MPa})^{0.5}$

$= 30.4 * 10^6$ kPa

$$\text{Settlement} = 0.012B_r + \frac{0.10B}{B_r} + \frac{PL}{AE}$$

$$= 0.012*300 + \frac{0.10(400)}{300} + \frac{P(17{,}000)}{400^2(30.4*10^6)(1*10^{-6}\,m^2/mm^2)}$$

$$= 3.7mm + 0.0035P$$

Plotting this line on the load-displacement curve produces $\mathbf{P_{ult}}$ **=1380 kN**

Q7) PILE DRIVING ENERGY CORRECTION

Question

400 mm square concrete piles are to be driven to depth of 12.5. m. What should be the stroke of a 1.5 ton rammer, due to the Danish Formula, if the amount of penetration should be about 5 mm/blow for the safe working load of 2000 kN on the pile? ($E_{conc} = 30*10^6$ kN/m^2)

Solution

$$A_{pile} = 0.4^2 = 0.16\,\text{m}^2$$

$$Q = \sqrt{\frac{W_r * H}{s + 0.5 * c_2}}, c_2 = \sqrt{\frac{2\cdot Wr\cdot \text{H}\cdot L}{A\cdot E}}$$

$$c_2 = \sqrt{\frac{2\cdot(1.5\cdot 9.8)\cdot \text{H}\cdot 12.5}{0.16\cdot 30\cdot 10^6}} = \sqrt{\frac{\text{H}}{13100}}$$

$$Q = 2000 = \frac{14.7\cdot \text{H}}{0.005 + 0.5\cdot\sqrt{\frac{\text{H}}{13100}}} \Rightarrow$$

$$\text{H} = 1.4\,\text{m}$$

where
Wr: weight of rammer
H: stroke
L: Length of bored pile
s: average set
A: Area
E: Modulus of elasticity