Horizontal Alignment

- Constructing the horizontal alignment is simply the task of locating the centerline of the proposed highway with due consideration of design standards. However it is a complex process since there are no deterministic methods to achieve the best and the most feasible route.
- The zero line method might be used to provide a guidance to locate the centerline of the highway between two successive topographic control points.
- The purpose of zero line method is to develop a route with selected grades not exceeding the maximum such that the amount of excavation or embankment is negligible along the centerline. Even though the final alignment will certainly deviate from the zero line, the zero line will be guideline to construct the alignment such that the amount of earthwork will be minimized as much as possible.

Zero-line Method

- **The first step: s**elect two topographic control points such that the topographic characteristics of the area between these points do not change considerably and the final route is expected to be continuously rising or falling.
- **The second step**: measure the aerial distance, ΔL, between these points.

Then the uniform grade (G) that will satisfy a straight line route connecting these points will be calculated as follows:

$$G = 100\ \Delta H/\Delta L \tag{3.1}$$

Where,

ΔH= Elevation difference between start and end points of zero line (m)
ΔL = Aerial distance between start and end points of zero line (m)

Zero-line Method

- Construct zero line by starting from one of the points by applying the grade (G) sequentially from one contour line to the next until reaching nearby the next point.
- By applying the calculated grade "G" as it is may not result with a zero line reaching nearby the second point since because the path traversed by the zero line application will not be a straight line but a combination of broken lines as seen in Fig. 3.10.
- If the topography is highly irregular then zero line might diverge away from the target point. For this reason, as the first trial grade it is better to select a milder slope (G^*) than the one calculated for straight path (i.e. when you calculate G as 5.0 % by Eq. 3.1 take $\mathbf{G}^*$ as 4.5% or even smaller).

Zero-line Method

- After selecting the trial grade G^* the distance required to traverse from one contour line to the next is calculated by:

$$d = 100\ h/G^* \tag{3.2}$$

where,

d = horizontal distance from one contour line to the next (m)

h = contour interval, elevation difference between two successive contour lines (m)

G^* = selected trial gradient (%)

Divider opening (DO) to be applied on a given map of scale S is calculated by:

$$DO = 100\ d\ S \tag{3.3}$$

- Construct zeroline on the given map by applying the divider opening from one contour line to the next in a stepwise manner until the contour line nearby the point on the other ends is reached as shown in Fig. 3.10.

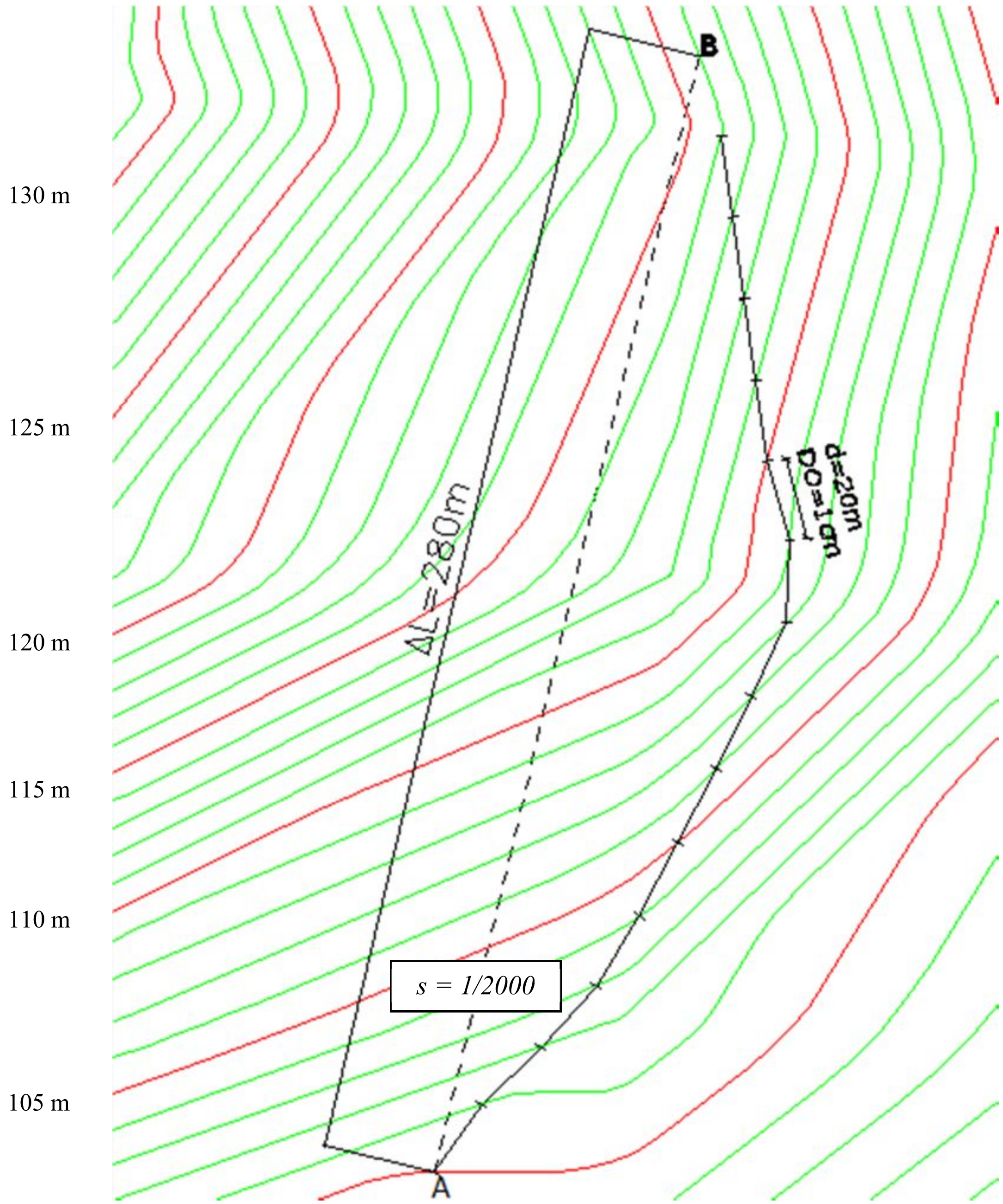


Figure 3.10 Demonstration of zero line application on a sample topographic map

Example 3.1

For the zero line application demonstrated on Fig. 3.10, if the scale of the map (s) is 1/2000, calculate the divider opening (DO) for the values that can be read from the map.

Solution: The following data is readily obtained from the map:

$\Delta L = 280$ m
$H_1 = 100$ m (Elevation of point A)
$H_2 = 114$ m (Elevation of point B)
$h = 1.0$ m

Then,

$$\Delta H = H_2 - H_1 = 114.0 - 100.0 = 14.0 \text{ m}$$
$$G \;\; = 100 * \Delta H / \Delta L = 100 * 14/280 = 5.0 \%$$

If $G^* = G$ is applied as the first trial grade, the horizontal distance to be traversed from one contour line to the next will be:

$$d = 100 * h / G = 100 * 1.0/5.0 = 20 \text{ m}$$
$$DO = 100 * d * S = 100 * 40.0 * (1/2000) = 1.0 \text{ cm}$$

Elements of Horizontal alignment

- Horizontal Alignment of a highway is represented by the centerline route which is normally constructed by combining straight lines and horizontal curves.
- The alignment formed by only straight lines without curves is called the polygonal alignment.
-

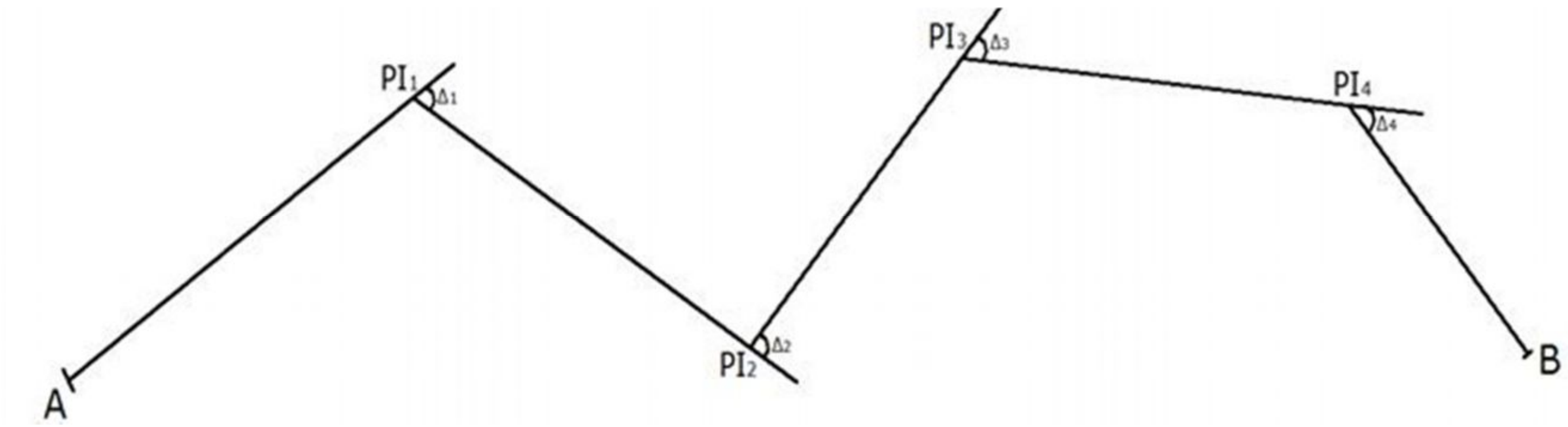


Figure 3.11 A sample polygonal alignment

Elements of Horizontal alignment

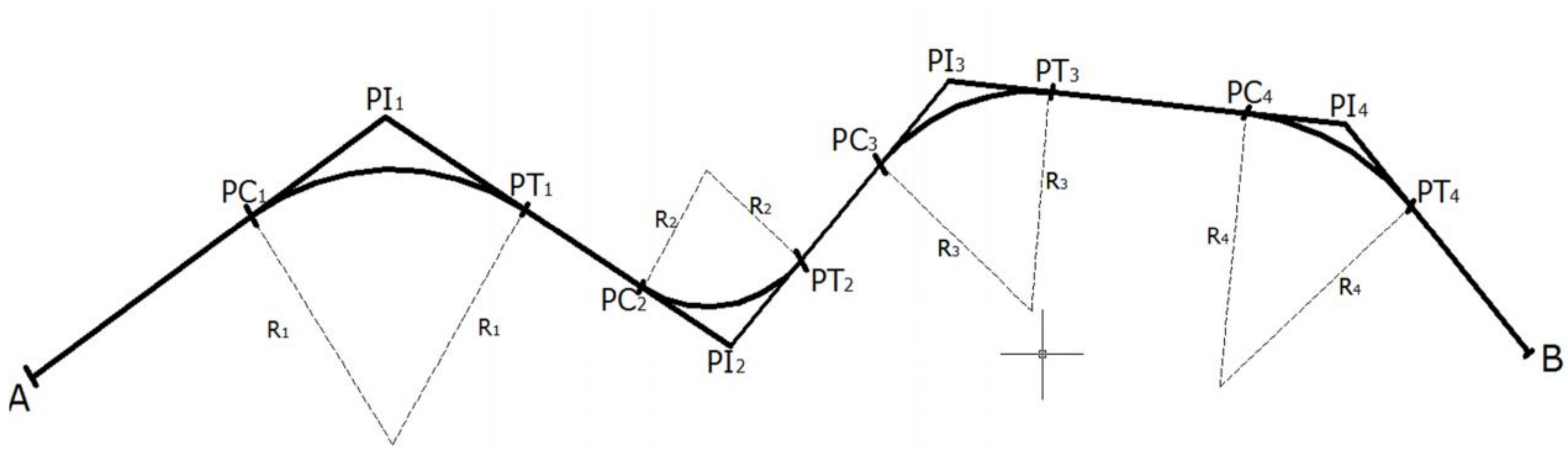


Figure 3.12 Elements of horizontal alignment with circular curves

- For most of highways and streets horizontal curves will be circular.
- The route will be marked at stations having definite distances in between and every station will be addressed by its station km with respect to the starting point of the project.

Elements of Horizontal alignment

- The successive stations are normally labeled by numbers (1, 2, 3 etc). If needed, some stations can be labeled by capital letters (A, B etc.) to indicate special reference points.

- Some specific points on the centerline route like starting and end points of a curve are labeled by definite abbreviations of the naming of those points. For example, the starting point of a simple circular curve is called as "Point of Curve" and labeled as "PC" point, and the ending point is called as "Point of Tangent" and labeled as "PT". Straight portions of the alignment are also referred to as tangents. The remaining parameters in Fig. 3.11 and 3.12 are as follows:

PI = point of intersection

Δ = Intersection angle (degrees or grads)

PC = point of curve

PT = point of Tangent

Circular curves

- Circular curves used in horizontal alignment are simply a portion of a circle of radius (R) subtended by the central angle (Δ).

- Since this portion of the circle is located in between two successive straight lines (tangent lines) and the circle is tangent to these lines, the central angle of the circle and the intersection angle at that point are exactly the same and both are labeled by "Δ".

Circular curves

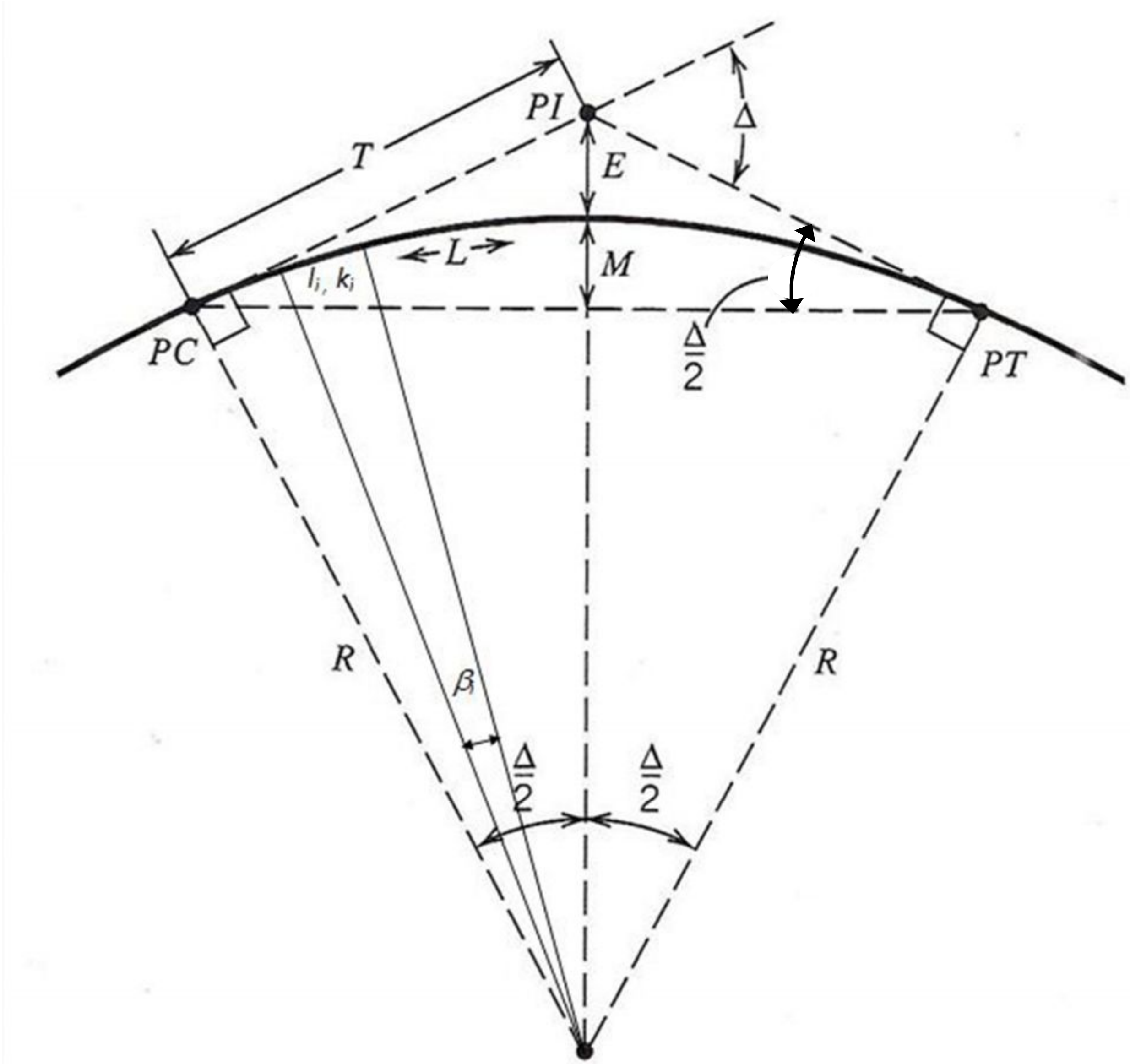


Figure 3.13 Elements of a simple circular horizontal curve

Circular curves

In Fig. 3.13,

Δ = central angle or intersection angle) (in degrees: Δ^o, in radians: Δ^r)
R = radius of the curve (m)
L = curve length (m)
PI = point of intersection
PC = beginning point of horizontal curve (point of curve)
PT = ending point of horizontal curve (point of tangent)
T = tangent length (m)
E = External distance (m)
M = middle ordinate (m)
K = long chord (m)
l_i = any arc length (m)
k_i = any cord length (m)
β_i = central angle subtending any arc length l_i (m)

Basic Relations for circular highway curves

Degree of Curve (curvature): the central angle subtended by unit length of arc (1 m or 100 m) of a circle. For 1 m arc length and curve radius R:

$$D^r = \frac{1}{R} \qquad \text{or} \qquad D^o = \frac{180}{\pi R} \tag{3.4}$$

Where,

D^r = degree of curve (in radians)

D^o = degree of curve (in degrees)

Curve length:

$$L = R\Delta^r \qquad \text{or} \qquad L = R\frac{\Delta^o \pi}{180} \tag{3.5}$$

Basic Relations for circular highway curves

Tangent length:

$$T = R\tan\left(\frac{\Delta}{2}\right) \tag{3.6}$$

External distance:

$$E = T\tan\left(\frac{\Delta}{4}\right) \quad \text{or} \quad E = R\left[\frac{1}{\cos(\Delta/2)} - 1\right] \tag{3.7}$$

Middle ordinate:

$$M = R\left(1 - \cos(\Delta/2)\right) \tag{3.8}$$

Long chord:

$$K = 2R\sin(\Delta/2) \tag{3.9}$$

Basic Relations for circular highway curves

$$\beta_\mathrm{i} = l_i D \tag{3.10}$$

$$l_i = R\beta_i^r \quad \text{or} \quad l_i = R\frac{\beta_i^o \pi}{180} \tag{3.11}$$

$$k_i = 2R\sin(\beta_i / 2) \quad \text{or} \quad k_i = 2R\sin\left(l_i D / 2\right) \tag{3.12}$$

Stationing (Polygonal and true station kilometers)

- The stations are marked on the centerline route
- The station kilometer (St. Km.) of any point on the project is expressed in the format of ### + ###.## where the digits before "+" gives the kilometer, three digits after "+" indicates meters and two digits after "." indicate centimeters.
- For polygonal alignment the stationing is based on lengths of straight lines forming the polygon.
- The reference points for a polygonal alignment are starting and ending points of the project and the intersection points in between.

Stationing (Polygonal and true station kilometers)

Example 3.1

The intermediate distances between reference points of the polygonal alignment shown in Figure 3.11 are given as follows:

$$\overline{A - PI_1} = 298.00 \text{ m}$$

$$\overline{PI_1 - PI_2} = 306.20 \text{ m}$$

$$\overline{PI_2 - PI_3} = 272.20 \text{ m}$$

$$\overline{PI_3 - PI_4} = 331.72 \text{ m}$$

$$\overline{PI_4 - B} = 373.13 \text{ m}$$

Determine the polygonal stations of all reference points taking station km of point A (St. A) as 0+000.00.

Stationing (Polygonal and true station kilometers)

Solution:

P. St. PI_1= P. St. A + $\overline{A - PI_1}$ = (0 + 000.00) + 298.00 = 0 + 298.00

P. St. PI_2= P. St. PI_1 + $\overline{PI_1 - PI_2}$ = (0 + 298.00) + 306.20 = 0 + 604.20

P. St. PI_3 = P. St. PI_2 + $\overline{PI_2 - PI_3}$ = (0 + 604.20) + 272.20 = 0 + 876.40

P. St. PI_4 = P. St. PI_3 + $\overline{PI_3 - PI_4}$ = (0 + 876.40) + 331.72 = 1 + 208.12

P. St. B = P. St. PI_4 + $\overline{PI_4 - B}$ = (1 + 208.12) + 373.13 = 1 + 581.25

Stationing (Polygonal and true station kilometers)

- Stationing for a horizontal alignment with circular curves runs along the centerline route of the highway formed by straight lines (tangent lines) and circular curves.

- For this case the particular stations for the horizontal alignment will be the points A, PC_1, PT_1, PC_2, PT_2 and so on.

- In order to determine the true station kilometers of these stations, it is necessary to calculate curve lengths (L_i's), and tangent lengths (T_i's) of the curves. Then true stations of succeeding reference points can be calculated in sequential manner starting from a given reference station.

Stationing (Polygonal and true station kilometers)

Example 3.2

Calculate the true stations of reference points (PC_1, PT_1, PC_2, PT_2, PC_3, PT_3) of the alignment shown in Fig. 3.12 for the same polygon of Example 3.1 and the below given data.

Radius (R_i)	Int. Angle (Δ_i)
250.00 m	45.00^o
200.00 m	60.00^o
250.00 m	35.00^o

Stationing (Polygonal and true station kilometers)

Solution:

Curve Lengths and tangent Lengths:

Curve	R (m)	Δ^o	$T = R\tan(\Delta/2)$	$L = \Delta^o R(\pi/180)$
1	250.00	45.00	103.553 m	196,350 m
2	200.00	60.00	115.470 m	209.440 m
3	250.00	35.00	78.825 m	152.716 m

Distances of straight portions:

$$\overline{A-PC_1} = \overline{A-PI_1} - T_1 = 298.00 - 103.553 = 194.447 \text{ m}$$

$$\overline{PT_1-PC_2} = \overline{PI_1-PI_2} - T_1 - T_2 = 306.20 - 103.553 - 115{,}470 = 87.177 \text{ m}$$

$$\overline{PT_2-PC_3} = \overline{PI_2-PI_3} - T_2 - T_3 = 272.00 - 115.470 - 78.825 = 77.705 \text{ m}$$

Then,

St Km. PC_1 = St. Km. A + $\overline{A - PC_1}$ = (0 + 000.00) + 194.447 = 0 + 194.447

St. Km. PT_1 = St. Km. PC_1 + L_1 = (0 + 194.447) + 196.350 = 0 + 390.796

St Km. PC_2 = St. Km. PT_1 + $\overline{PT_1 - PC_2}$ = (0 +390.796) + 87.177= 0 + 477.973

St. Km. PT_2 = St. Km. PC_2 + L_2 = (0 + 477.973) + 209.440 = 0 + 687.412

St Km. PC_3 = St. Km. PT_2 + $\overline{PT_2 - PC_3}$ = (0 + 687.412) + 77.705 =0+765.317

St. Km. PT_3 = St. km. PC_3 + L_3 = (0 + 765.317) + 152.716= 0 + 918.033

Transition curves

- When a vehicle moving on a straight enters a circular curve of radius (R_c) and continues to move on curved path, it is affected by the centrifugal force (F_c) given by:

$$F_c = m\frac{V^2}{R_c} \tag{3.13}$$

Where,

F_c = centrifugal force (N)
V = speed of the vehicle (m/s)
m = mass of the vehicle (kg)
R_c = radius of the curve (m)

- This force tends to push the vehicle out of its course and also to overturn the vehicle. In such cases the driver is forced to either slow down the vehicle or leave its course.

Transition curves

- For most curves, drivers can follow a suitable transition path within the limits of normal lane width.

- However, high speeds and small radii lead to longer transition paths resulting in shifts in lateral positions and sometimes encroachment on adjoining lanes which may be quite dangerous.

- Two precautions may be taken to limit the adverse effects of the centrifugal force:

  a) Superelevate the curve (will be seen later)

  b) Place a transition curve between the straight and the circular curve

- Transition curve is a curve with radius decreasing through its length from infinity down to the radius (R_c) of the circular curve. This provides opportunity to driver to adjust and adopt the centrifugal force gradually.

Transition curves

- The principle advantages of transition curves in horizontal alignments of highways are:

1. A properly designed transition curve minimizes encroachment of vehicles on adjoining traffic lanes and simulates the natural turning path of the vehicle.

2. The transition curve length provides a suitable location for the superelevation.

3. A transition curve also facilitates curve widening.

4. The appearance of the highway or street is enhanced by the application of transition curves.

Transition curves

- There are three different curve types commonly used as transition curves in horizontal alignment,
  a) Lemniscate,
  b) Cubic parabola
  c) Clothoid (Euler’s spiral).

- Because of its analytical simplicity and relatively easier application at site, the most commonly used one in highway design is the clothoid (Euler’s spiral) among the transition curves mentioned above
- For spirals the radius of the curvature varies as the inverse of the distance along the curve measured from its beginning. A double-end Euler’s spiral is shown in Figure 3.14.

Transition curves

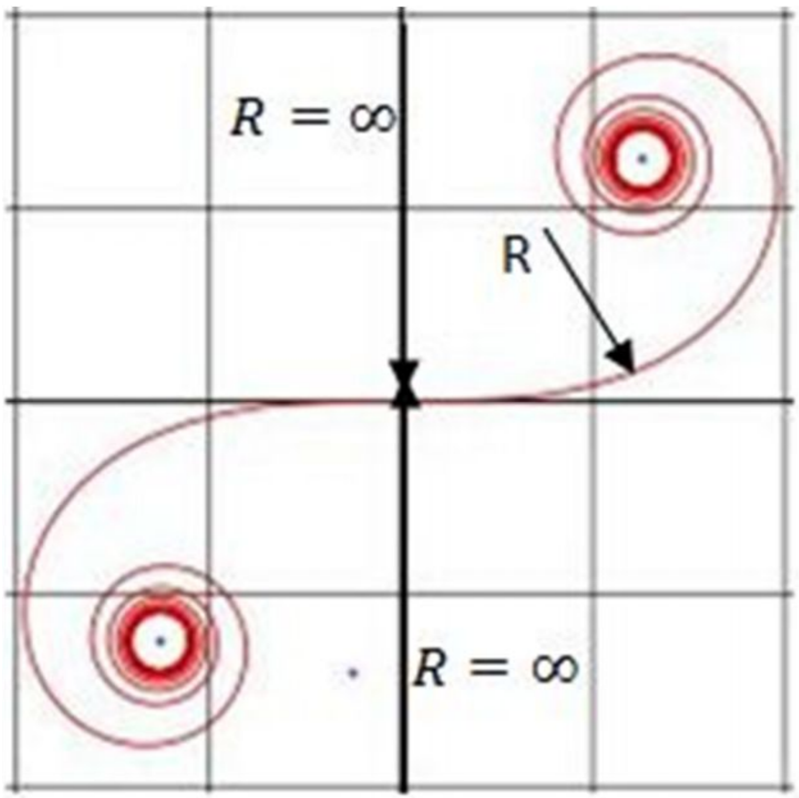


Figure 3.14 Double-end Euler spiral

Transition curves

- A specific length of the spiral (L_s) starting from the origin of zero curvature point up to a point where spiral attains the radius equal to the radius of the circular curve (R_c) is taken and applied as the transition curve.

- The basic parameter of the spiral (A^2)

$$A^2 = R_c L_s = RL \tag{3.14}$$

Where R_c and L_s are as defined before and,

R = Radius at any point of the spiral (m)
L = Distance from starting point to the point where radius is R (m)

- The application of spiral as a transition curve before and after a circular curve in horizontal alignment is shown in Fig. 3.15.

Transition curves

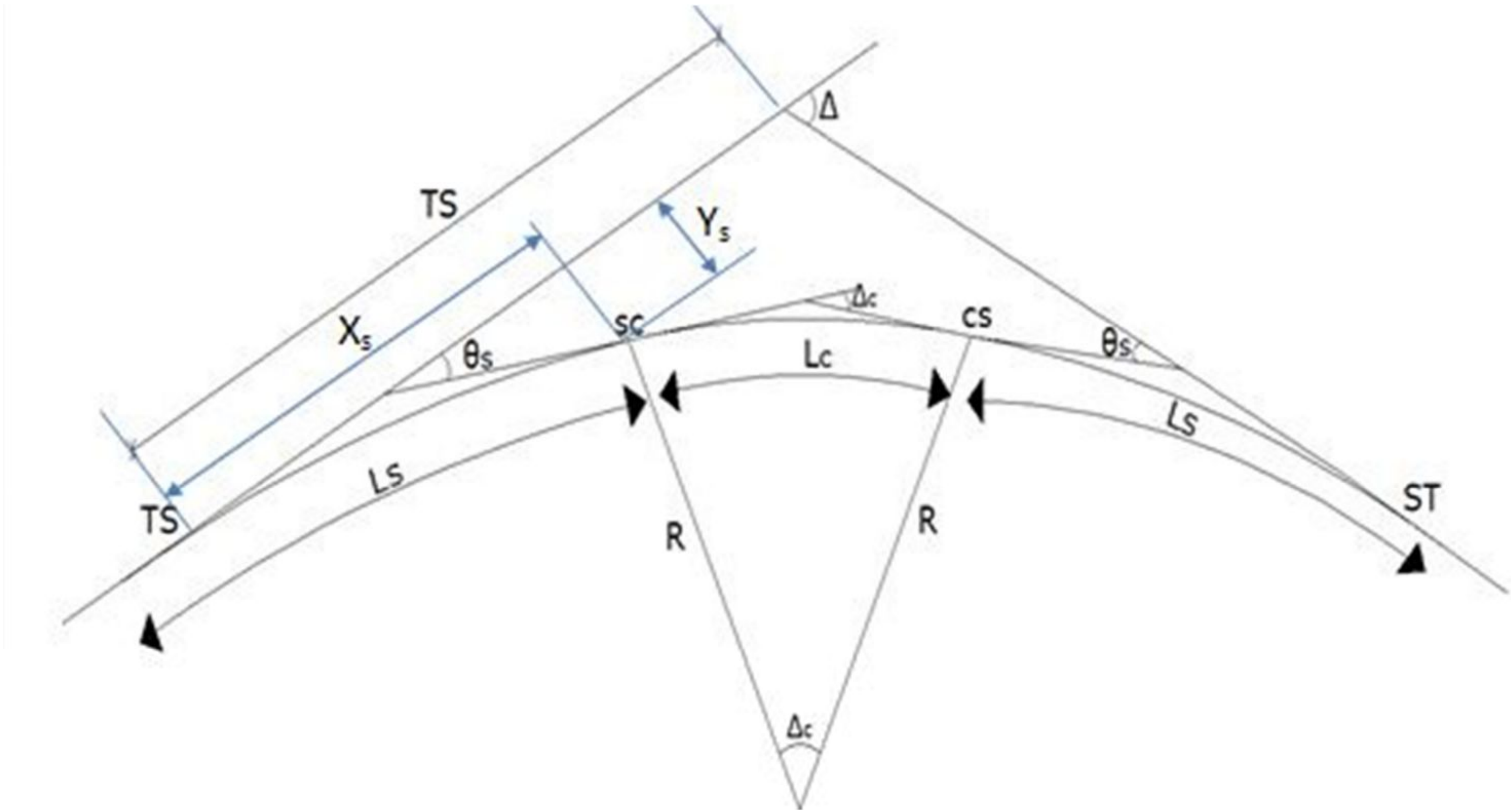


Figure 3.15 Spiral – circular curve – spiral application at a highway bent

Transition curves

The parameters of such a symmetric spiral – circular curve – spiral application shown in figure 3.15 are:

Δ = intersection angle (Δ^{r}: in radians, Δ^{o}: in degrees)

Δ_c = central angle of inner circular curve

TS = tangent to spiral

SC = spiral to curve

CS = curve to spiral

TS = spiral to tangent

θ_s = maximum spiral angle (θ_s^r: in radians, θ_s^o : in degrees)

R_c = radius of inner circular curve

Y_s, X_s = Local coordinates of SC with respect to point TS

Transition curves

- The use of spiral transition is found to increase safety and obtain operational benefits only when they are used for radii less than some suggested maximum values.

- It is recommended that the maximum radius for use of a spiral should be based on a minimum lateral acceleration rate of 1.3 m/s^2 (AASHTO, 2004). In other words, The use of spiral transition is not found to be beneficial for curves with radii generating lateral acceleration less than 1.3 m/s^2 for the design speed under consideration.

- The table 3.1 gives the maximum radii above which the safety benefits of spiral curve transitions are likely to be negligible.

Transition curves

Table 3.1 Maximum Radius for Use of a Spiral Transition (AASHTO 2004)

Design Speed (km/h)	Maximum radius (m)
20	24
30	54
40	95
50	148
60	213
70	290
80	379
90	480
100	592
110	716
120	852
130	1000

Setting out horizontal circular curves

- It is necessary to locate points (stations) along the centerline of the horizontal alignment.
- It is straightforward to locate stations along the straight portions of the alignment.
- On the other hand, in order to locate the stations along a circular curve special treatment involving some calculations and special site work are required.
- The process is called as either setting out or staking out the curve.
- In practice today, this work is very simple with the use of available precise surveying instruments.
- However, in order to understand the basic calculations necessary to develop data tables needed for use of such instruments it is valuable to study two old fashioned method commonly called as deflection angles method and coordinates method.

Deflection angles method

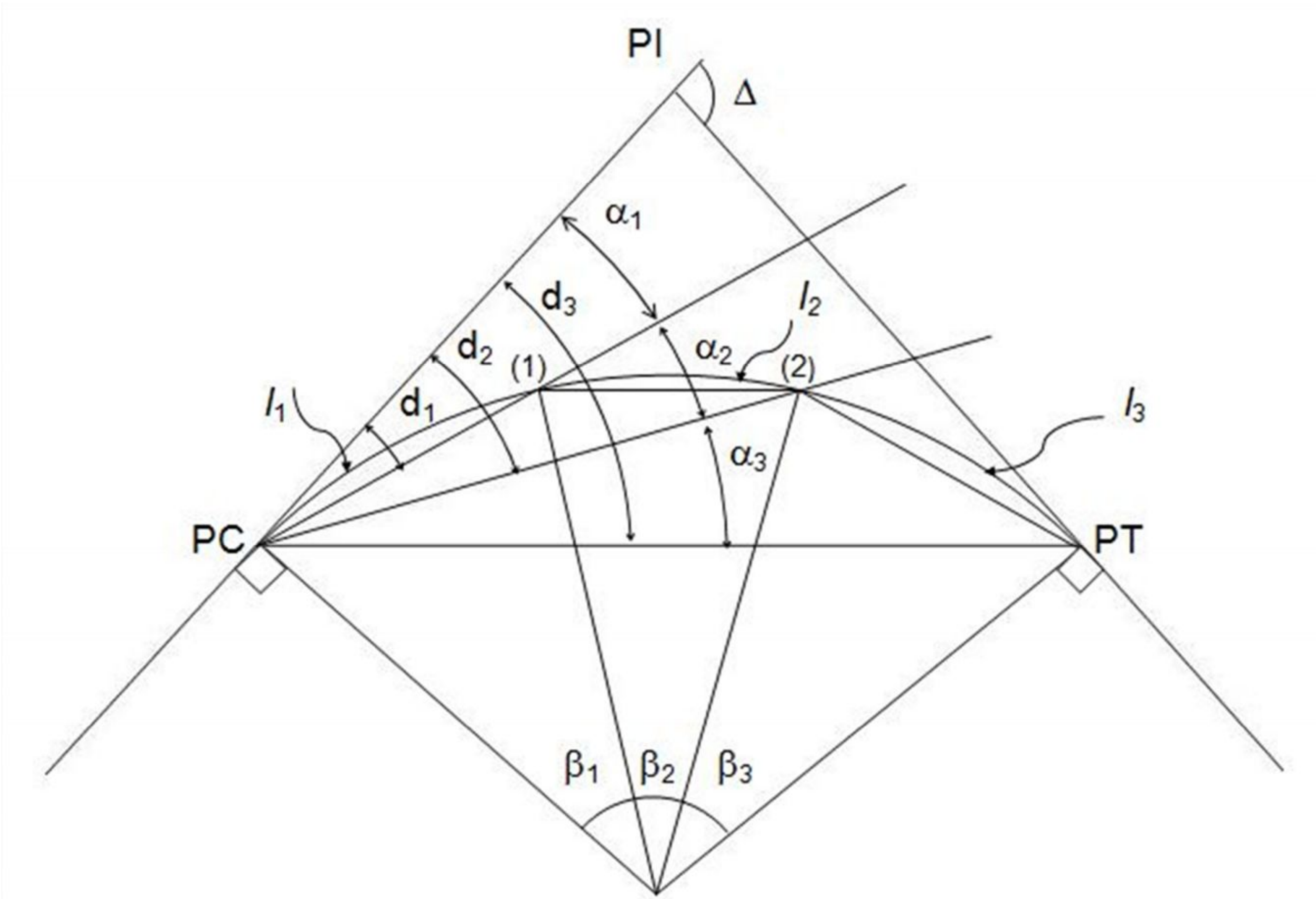


Figure 3.16 Deflection angles method.

Deflection angles method

- In the figure,

  d_i = deflection angles

  l_i = arc lengths

  β_i = central angle subtending arc lengths $l_i^{'}$s

- Deflection angles are readily calculated as follows:

$$d_1 = \alpha_1 = \frac{\beta_1}{2} = l_1 \frac{D}{2}$$

$$d_2 = \alpha_1 + \alpha_2 = \frac{\beta_1 + \beta_2}{2} = (l_1 + l_2)\frac{D}{2}$$

- The relation for deflection angle can be generalized as:

$$d_i = \sum_i l_i \frac{D}{2} \tag{3.15}$$

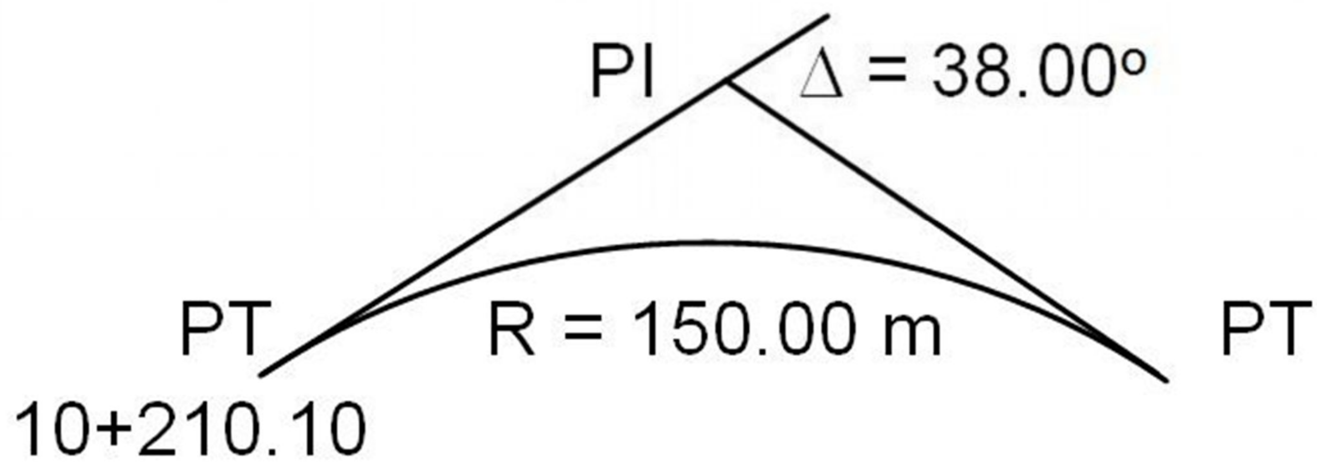
Deflection angles method

- For a given circular curve the steps for staking out the points on the curve with known l_i distances inbetween are as follows:
  - Calculate d_i and k_i
  - Set theodolite on *PC*, sight *PI* with angle 0.00 reading
  - Turn theodolite by angle d_1, measure k_1 distance along line of sight, set first point (1)
  - Complete angle to d_2, measure k_2 from point (1), intersection of line of sight and distance k_2 is point (2)
  - Repeat the process for all remaining points.
- Generally, the curves are staked out at regular stations of project unless otherwise indicated. For l_i less than $0.1R$ than k_i can be taken equal to l_i to facilitate computations.

Deflection angles method

Example 3.3

The following circular curve will be set by staking the centerline at regular stations. Calculate deflection angles and cord lengths for the stations on the curve to set the curve. (Note: use 20 m even stations)



Deflection angles method

Solution:

$$L = \frac{\pi}{180} R\Delta^{O} = \frac{\pi}{180} 150.00 * 38.00 = 99.48m$$

St. PT = St PC + L = (10 + 210.10) + 99.48 = 10 + 309.58

$$D = \frac{180}{\pi R} = \frac{180}{\pi * 150.00} = 0.381972$$

Calculations in tabular form:

Station	St. Km.	l_i (m)	Σl_i (m)	$k_i = 2R\sin(l_i D/2)$ (m)	d_i (degrees) $= \sum_i l_i \frac{D}{2}$
PC	10 + 210.10	0	0	0	0
1	10 + 220.00	9.90	9.90	9.90	1.8908
2	10 + 240.00	20.00	20.90	19.99	5.7105
3	10 + 260.00	20.00	40.90	19.99	9.5302
4	10 + 280.00	20.00	60.90	19.99	13.3499
5	10 + 300.00	20.00	80.90	19.99	17.1696
PT	10 + 309.58	9.58	99.48	19.99	19.0000

Coordinates method

- In this method, PC point is selected as the origin of local x-y coordinate system as shown in Figure 3.17.

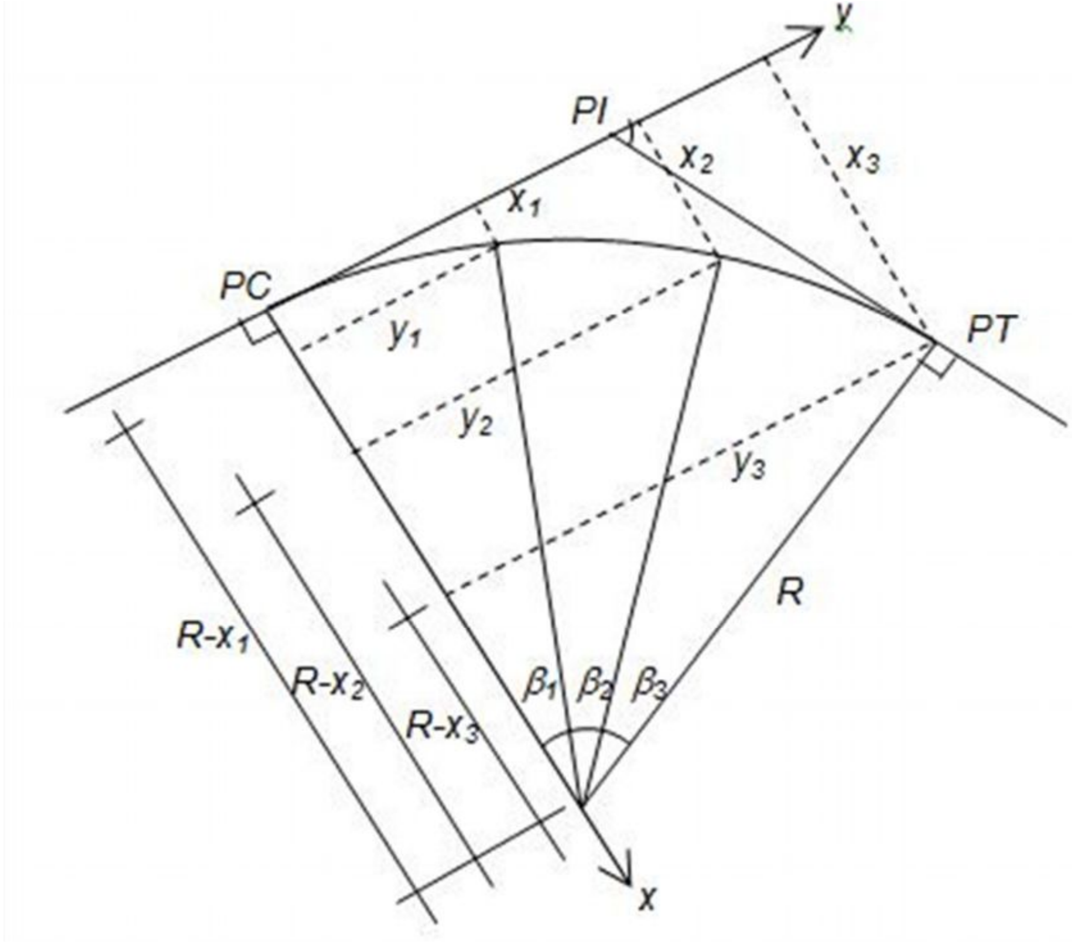


Fig.3.17 Setting out circular curve by coordinates method.

Coordinates method

- x-y coordinates of the stations on the curve are calculated with respect to the seleceted coordinate system as shown in Figure 3.17. Then, y coordinates are measured along the tangent and x coordinates (offsets) are measured perpendicular to tangent line to lay out.

- Referring Fig. 3.17, y coordinates are calculated as follows:

$$y_1 = R\sin(\beta_1) = R\sin(l_1 D)$$

$$y_2 = R\sin(\beta_1 + \beta_2) = R\sin\left[(l_1 + l_2)D\right]$$

$$y_3 = R\sin(\beta_1 + \beta_2 + \beta_3) = R\sin\left[(l_1 + l_2 + l_3)D\right]$$

Thus,

$$y_i = R\sin\left(\sum_i l_i D\right) \tag{3.16}$$

Coordinates method

And x coordinates are calculated as follows:

$$x_1 = R(1 - \cos\beta_1) = R[1 - \cos(l_1 D)]$$
$$x_2 = R[1 - \cos(\beta_1 + \beta_2)] = R[1 - \cos D(l_1 + l_2)]$$
$$x_3 = R[1 - \cos(\beta_1 + \beta_2 + \beta_3)] = R[1 - \cos D(l_1 + l_2 + l_3)]$$

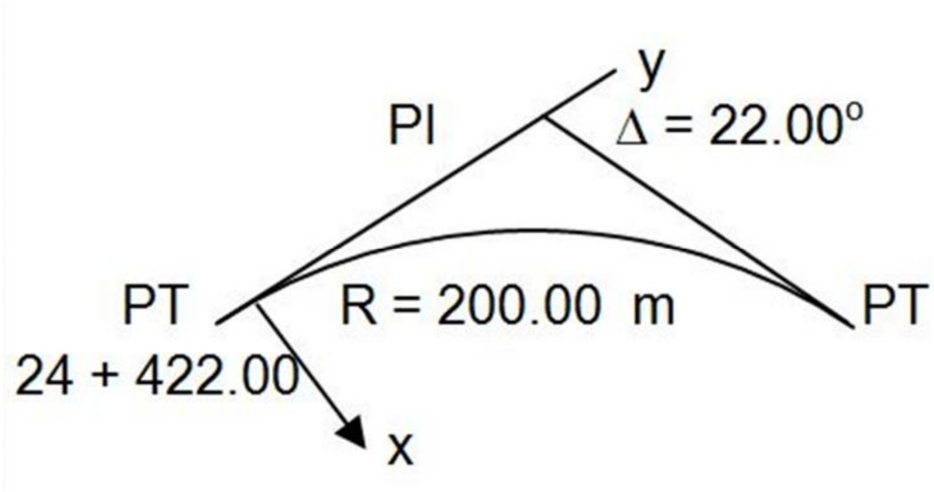
Thus

$$x_i = R\left[1 - \cos\left(D\sum_i l_i\right)\right] \tag{3.17}$$

Coordinates method

Example 3.4

Given the following curve, calculate and tabulate local coordinates for setting out the curve at site. (Note: use 20 m even stations)



Coordinates method

Solution:

$$L = \frac{\pi}{180} R\Delta^{O} = \frac{\pi}{180} 200.00 * 22.00 = 76.79m$$

St. PT = St PC + L = (24 + 422.00) + 76.79 = 24 + 498.79

$$D = \frac{180}{\pi R} = \frac{180}{\pi * 200.00} = 0.28648^{o}$$

Calculations in tabular form:

Station	St. Km.	l_i (m)	Σl_i (m)	$y_i = R\sin\left(\sum_i l_i D\right)$ (m)	$x_i = R\left[1-\cos\left(D\sum_i l_i\right)\right]$ (m)
PC	24 + 422.00	0	0	0	0
1	24 + 440.00	18.00	18.00	17.96	0.81
2	24 + 460.00	20.00	38.00	37.77	3.60
3	24 + 480.00	20.00	58.00	57.19	8.35
PT	24 + 498.79	18.79	76.79	74.92	14.56