EXAMPLE 1:

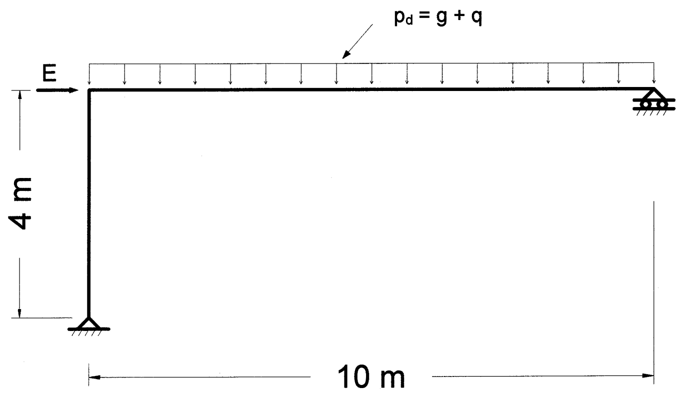
Given: The simple frame shown in figure below. The cross-sectional dimensions of the beam are, 350×1000 mm, and of the column are 350×350 mm. The beam has a rectangular cross-section (no flange)

Materials: C25 and S420

E = 50 kN

g = 50 kN/m and q = 20 kN/m

Required: Using 1.0G+1.0Q+1.0E load combination, find the column reinforcement. Consider second order effects.



Solution:

Frame is statically determinate. Internal forces calculated for the column are given below.

V_d = 50 kN

N_d = 330 kN

M_{d2} = 200 kN·m (top)

M_{d1} = 0 (bottom)

Since there is no bracing element, the frame is an unbraced frame (sway frame). All the vertical members in the floor have to be considered. However there is only one column in this case!

Column: ℓ = 4 m, ℓ_n = 3.0 m

Here, ℓ_n is the distance from the top of the footing to the bottom of the beam.

i = 0.3h = 0.30×0.35 = 0.105

$I = (1/12) \times (0.35)^4 = 0.00125 \text{ m}^4$

$(I/\ell)_{col} = 0.00125/4 = 0.000312$

$E_c = 30\times10^6 \text{ kN/m}^2$ (for C25 concrete grade)

$I_c E_c = 1250\times10^{-6}\times30\times10^6 = 37500 \text{ kN·m}^2$

Beam: $\ell = 10$ m

$$I_{cr} = 0.5\left(1/12\right)\times 0.35\times(1.0)^3 = 0.0146$$

$$\left(I/\ell\right)_{beam} = 0.00146 \text{ (cracked)}$$

$$\alpha_{top} = \frac{\sum\left(I/\ell\right)_{col}}{\sum\left(I/\ell\right)_{beam}} = \frac{1\times 0.000312}{1\times 0.00146} = 0.21$$

$\alpha_{bott.} = \infty$ (because there is a hinge), $\sum\left(I/\ell\right)_{beam} = 0$)

$\alpha_2 = \infty$ and $\alpha_1 = 0.21$

For one-end pinned frames, Eq. (6.48) is used to calculate "k". Clear length of the column is, $\ell_n = 3$ m.

k = 2+0.3×α = 2+0.3×0.21 = 2.06 ≈ 2.1

$\ell_k = k\ell_n = 2.1\times 3.0 = 6.3$ m

$\ell_k/i = 6.3/0.105 = 60>22$. Second order moments should be calculated.

$V_{gd} = 0$, therefore $R_m = 0$. However, authors recommend $R_m \geq 0.5$. Effective EI will be calculated using Eq. (6.54)

$$EI = \frac{E_c I_c}{2.5(1+R_m)} = \frac{37500}{2.5(1+0.5)} = 10000\ kN-m^2$$

$$N_{cr} = \frac{\pi^2 EI}{\ell_k^2} = \frac{\pi^2\times 10000}{(6.3)^2} = 2484\ kN$$

$$N_d/N_{cr} = 330/2484 = 0.133 = \left(\sum N_d / \sum N_{cr}\right)$$

$$\beta_s = \beta = \frac{1.0}{1-1.3\left(N_d/N_{cr}\right)} = \frac{1.0}{1-1.3\times 0.133} = 1.2$$

$$M'_d = \beta M_d = 1.2\times 200 = 240\ kN.m$$

$$\frac{N_d}{bhf_{cd}} = \frac{330}{350\times 350\times 0.017} = 0.16$$

$$\frac{M'_d}{bh^2 f_{cd}} = \frac{240000}{350\times(350)^2\times 0.017} = 0.33$$

Assuming λ=1/4 and d''/h=0.8

$\rho_t m$ = 0.8 is found from the charts

m = 365/17 = 21.5

ρ_t = 0.8/21.5 = 0.037

This ratio is close to the maximum ratio specified in the code, max.ρ_t=0.04. It is recommended to increase the cross-sectional dimensions of the column. Columns with high steel ratios are not economical.

Since the frame was statically determinate, it is possible to calculate the displacement at the top of the column and find the second order moment as,

$$\Delta M = N_d \times \delta$$

The deflection calculated using reduced EI for the column (EI=10000 kN·m^2) is, δ=0.15 m

$$\Delta M = 330 \times 0.15 = 50\ kN - m$$
$$M'_d = 200 + 50 = 250\ kN - m$$

The moment value found using the TS500-2000 approach was 240 kN·m. The difference is about 4%.

Example 2:

Given: The four storey building shown in figure.

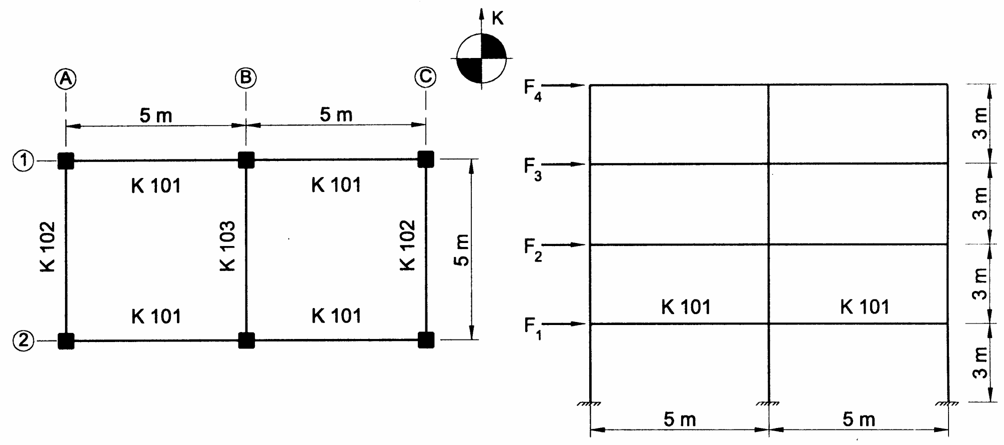
Materials : C20 and S420 (f_{cd} = 13 MPa, f_{yd} = 365 MPa)

Building is located in Seismic Zone-1 (zone of high seismicity).

Required: (a) Preliminary design of columns B1 and B2

(b) Final design of columns B1 and B2

Consider the seismic action in east-west direction only.



(a) Typical Floor Plan (b) Frames 1 and 2

Solution:

(a) Preliminary Design :

Tributory area A_{tr} = 5×5/2 = 12.5 m^2

Weight (including live load) ≈ 15 kN/m^2

N_d = n×15×12.5 = 4×15×12.5 = 750 kN

Since there are no structural walls, frames are unbraced. Also the building is located in a highly seismic region. For these reasons, the constant in the denominator of Eq. (6.28) will reduced from 0.75 to 0.5.

$$A_c \geq \frac{N_d}{0.5 f_{cd}} = \frac{750000}{0.5 \times 13} = 115380 \; mm^2 > \min A_c = 75000 \; mm^2$$

Column size is chosen as 350×350 mm.

350×350 = 122500 mm^2 > 115380 mm^2

In seismic regions lateral stiffness is very important. Therefore it is recommended to be generous in chosing the column sizes.

(b) Final Design:

The following dimensions have been obtained in the preliminary design stage :

All columns : 350×350 mm (all floors)

All beams : 250×400 mm

All slabs : 120 mm

Since there are no structural walls, there is no need to check sidesway. The frames are unbraced.

Frame analyses have been carried out for; (a) dead load (on all spans), (b) live load, five different arrangements and (c) lateral load (earthquake).

For each column, design axial loads and design moments have been obtained putting the results of the analyses into the following load combinations.

(A) 1.4G + 1.6Q (five different live load combinations)

(B) 1.0G + 1.0Q + 1.0E (seismic)

(C) 0.9G + 1.0E (seismic)

In total seven N_d and M_d combinations have been obtained for each column. In this example only the design values found for the seismic load combination (1.0G+1.0Q+1.0E) will be treated. In practice all seven have to be considered.

$R_m = \Sigma V_{gd} / \Sigma V_d$ will come out to be zero, since $V_{gd} = 0$. However in line with authors' recommendation, R_m will be taken as 0.5.

The results of analysis for seismic load combination for all columns are summarized in Table 6.4.

Table 1.1 Summary of Results for Seismic Load Combination

(Columns below the First Story)

Design Internal Force	Column No.	
	A1, A2, C1 and C2	B1 and B2
N_d (kN)	493	690
M_{d1} (kN·m)	53.0	71.8
M_{d2} (kN·m)	138.8	148.7
min. M_d (kN·m)	12.6	17.6
design M_d (kN·m)	138.8	148.7

Minimum moments in the table have been calculated using the minimum eccentricitiy. $\min.M_d=N_d(15+0.03\times350)/1000=0.0255N_d$. Since the frame is unbraced, minimum moments were much smaller than the design moment found from analyses.

$I_{col} = (1/12)(350)^4 = 1.25\times10^9 \text{ mm}^4 = 0.00125 \text{ m}^4$

$\ell = 3.0$ m and $\ell_n = 3.0-0.4 = 2.6$ m (beam depth = 0.4 m)

$I_{beam} = (1/12) \times 250 \times (400)^3 = 1.33 \times 10^9\ mm^4 = 0.00133\ m^4$

In east-west direction the beam span is, $\ell = 5$ m.

For columns B1 and B2 in east-west direction effective length will be calculated.

At the top joint, $$\alpha = \frac{\sum \left(I/\ell \right)_{col}}{\sum \left(I/\ell \right)_{beam}} = \frac{2 \times \left(0.00125/3 \right)}{2 \times \left(0.00133/5 \right)} = 1.56$$

At the bottom joint sum of beam stiffnesses should be taken as infinity since the joint is fixed against rotation. Therefore, $\alpha = 0$

$\alpha_1 = 0$ and $\alpha_2 = 1.56$

$\alpha_m = (\alpha_1 + \alpha_2)/2 = 0.78$

"k" will be calculated from Eq. (6.46), since $\alpha_m < 2.0$.

$$k = \frac{20 - \alpha_m}{20} \sqrt{1 + \alpha_m} = 1.28$$

$$\ell_k = k \ell_n = 1.28 \times 2.6 = 3.3\ m$$

For columns A1, A2, C1 and C2 :

At the upper joint there are two columns and only one beam.

$$\alpha_1 = 0 \ \text{ and } \ \alpha_2 = \frac{2 \times \left(0.00125/3 \right)}{1 \times \left(0.00133/5 \right)} = 3.12$$

$$\alpha_m = \left(\alpha_1 + \alpha_2 \right)/2 = 1.56$$

Since $\alpha_m < 2.0$

$$k = \frac{20 - \alpha_m}{20} \sqrt{1 + \alpha_m} = 1.47$$

$$\ell_k = 1.47 \times 2.6 = 3.8\ m$$

For columns ($E_c = 28000$ MPa $= 2.8 \times 10^7$ kN/m^2),

$E_c I_c = 2.8 \times 10^7 \times 0.00125 = 35000$ kN·m^2

Effective flexural rigidity EI,

$$EI = \frac{E_c I_c}{2.5\left(1 + R_m\right)} = \frac{35000}{2.5\left(1 + 0.5\right)} = 9333\ kN - m^2$$

For columns A1, A2, C1 and C2:

$$N_{cr} = \frac{\pi^2 EI}{\ell_k^2} = \frac{\pi^2 \times 9333}{(3.8)^2} = 6372\ kN$$

For columns B1 and B2:

$$N_{cr} = \frac{\pi^2 \times 9333}{(3.3)^2} = 8450\ kN$$

For all column in the story:

$\Sigma N_d = 4\times493+2\times690 = 3352$ kN

$\Sigma N_{cr} = 4\times6372+2\times8450 = 42390$ kN

According to TS500-2000, the following requirement has to be satisfied:

$\Sigma N_d < 0.45\Sigma N_{cr}$

3352/42390 = 0.08<0.45, OK.

$$\beta_s = \frac{1.0}{1-1.3\left(\sum N_d \big/ \sum N_{cr}\right)} = \frac{1.0}{1-1.3\left(3352 \big/ 42390\right)} = 1.11$$

For the individual column (column B1 and B2),

$$\beta = \frac{1.0}{1-1.3\left(690 \big/ 8450\right)} = 1.12$$

Note that for edge columns A1, A2, C1 and C2 individual β = 1.08. Since $\beta_s > \beta$, in the design of these columns β_s = 1.11 shall be used.

β governs, $\beta > \beta_s$.

$$M'_d = \beta M_d = 1.12\times148.7 = 166.5\ kN-m$$

$$\frac{N_d}{bhf_{cd}} = \frac{690}{350\times350\times0.013} = 0.433$$

$$\frac{M'_d}{bh^2 f_{cd}} = \frac{166500}{350\times(350)^2\times0.013} = 0.3$$

Assuming λ=1/4 and d"/h=0.8, $\rho_t m$ is found from the charts.

$\rho_t m = 0.65$ and m = 365/13 = 28

$\rho_t = 0.65/28 = 0.023$

$A_{st} = \rho_t bh = 0.023 \times (350)^2 = 2818\ mm^2$

8-ϕ22 which provides a total area of 3040 mm^2 (3 bars on each face).

3 bars on each face so, λ=1/4 which is same with the assumed value.

If $\phi 8$ ties are used and if the clear cover is 20 mm,

$d' = 20 + 8 + 22/2 = 39$ mm ≈ 40 mm

$d'' = h - d' = 350 - 2 \times 40 = 270$ mm

$d''/h = 270/350 = 0.77$.

This is close to 0.8 assumed.

No need for recalculation because assumed reinforcement pattern, λ and d''/h have not changed.