CE 382 Reinforced Concrete Fundamentals

Pure Bending: Analysis of RC Sections

Example 4

- Given:
  - $b_w = 300\text{ mm}, d = 450\text{ mm}, d' = 30\text{ mm}$
  - C16 → $f_{cd} = 11\text{ MPa}$ & S420 → $f_{yd} = 365\text{ MPa}$
  - $A_s = 1580\text{ mm}^2$ & $A_s' = 520\text{ mm}^2$
- Find: Ultimate Moment M_r
- Assume both tension & compression steel have yielded
- $\varepsilon_{sy} = \frac{365}{200000} = 0.001825$
- $k_1 c = \frac{(A_s - A_s')f_{yd}}{0.85 f_{cd} b_w} = \frac{(1580-520)365}{0.85\times 11\times 300} = 138\text{ mm}$
- $c = \frac{138}{0.85} = 162.3\text{ mm}$

Example 4

- $\varepsilon_s' = 0.003\,\frac{c-d'}{c} = 0.003\,\frac{162.3-30}{162.3} = 0.00244 > \varepsilon_{sy}$ ✓
- $\varepsilon_s = 0.003\,\frac{d-c}{c} = 0.003\,\frac{450-162.3}{162.3} = 0.00532 > \varepsilon_{sy}$ ✓
- $M_r = 0.85 f_{cd} b_w k_1 c\left(d - \frac{k_1 c}{2}\right) + A_s' f_{yd}(d - d')$

$$= 0.85 \times 11 \times 300 \times 138\left(450 - \frac{138}{2}\right) + 520 \times 365(450 - 30)$$

$$= 227\ \text{kNm}$$

Example 5

- same as Example 4 but $A_s' = 1200 \text{ mm}^2$
- Tension steel has surely yielded
- Assume compression steel has yielded
- $c = \frac{(1580-1200)365}{0.85\times0.85\times11\times300} = 58.4 \text{ mm}$
- $\varepsilon_s' = \frac{58.4-30}{58.4} = 0.00146 < \varepsilon_{sy}$ ✘ compression steel has not yielded
- Use general solution
- $\sigma_s' = \varepsilon_s' E_s$ & $\varepsilon_s' = 0.003\frac{c-d'}{c}$ $\Rightarrow$ $\sigma_s' = 0.003E_s\frac{c-d'}{c}$
- $\sigma_s' = 600\frac{c-30}{c}$

Example 5

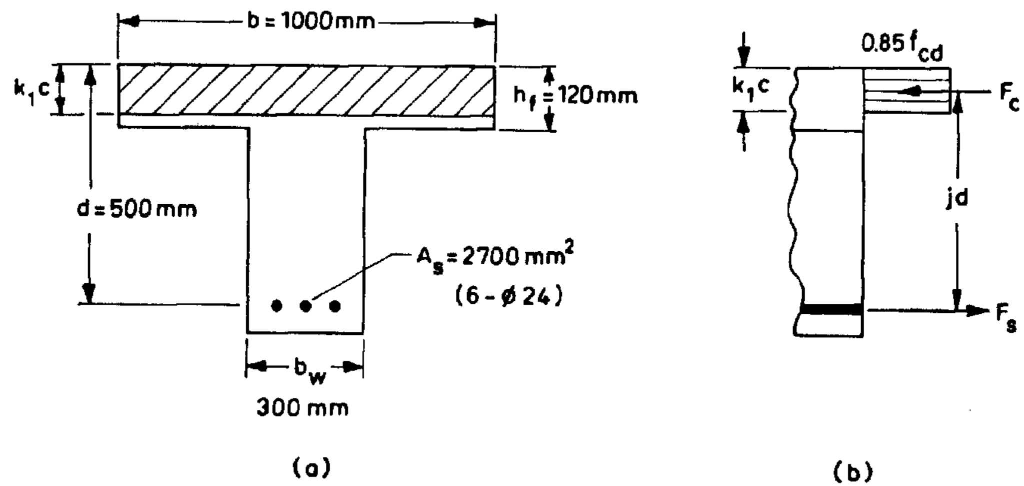
- Force equilibrium:
- $A_s f_{yd} - 0.85 f_{cd} b_w k_1 c - A'_s \sigma'_s = 0$

$$1580 \times 365 - 0.85 \times 11 \times 300 \times 0.85c - 1200 \times 600 \frac{c-30}{c} = 0$$

- $c^2 + 60c - 9060 = 0$
- $c = 70 \text{ mm}$
- $\varepsilon'_s = 0.003 \frac{70-30}{70} = 0.00171 \quad \Rightarrow \quad \sigma'_s = 342 \text{ Mpa}$
- $M_r = 0.85 f_{cd} b_w k_1 c \left(d - \frac{k_1 c}{2}\right) + A'_s \sigma'_s (d - d')$
- $M_r = 243 \text{ kNm}$

Example 6

- Given:

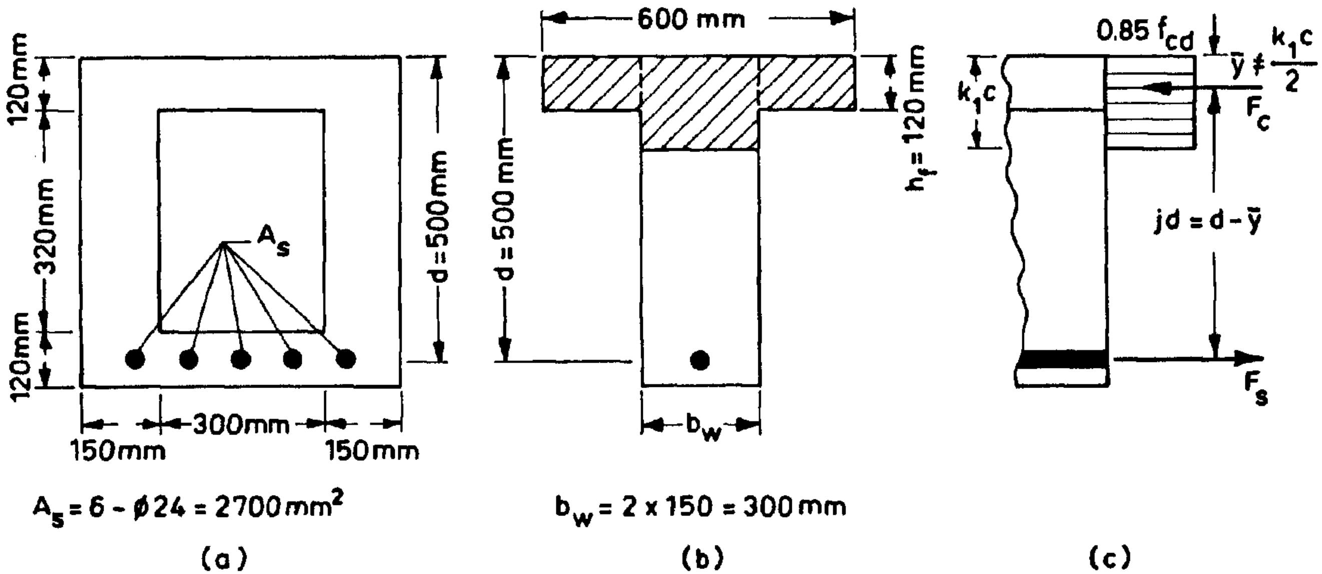


  - $b = 1000 \text{ mm}, \quad b_w = 300 \text{ mm}, \quad t = h_f = 120 \text{ mm}$
  - $d = 500 \text{ mm}, A_s = 6Ø24 = 2700 \text{ mm}^2$
  - C20 ➔ $f_{cd} = 13 \text{ MPa}$ S420 ➔ $f_{yd} = 365 \text{ Mpa}$
- M_r =?
- Assume $k_1c = t = 120 \text{ mm}$ & steel yielded

Example 6

- $F_c = 0.85 f_{cd} b k_1 c = 0.85 \times 13 \times 1000 \times 120 = 1326 \text{ kN}$
- $F_s = A_s f_{yd} = 2700 \times 365 = 985 \text{ kN}$
- $F_c > F_s \quad \Rightarrow \quad k_1 c < t$ analyze as a rectangular section
- $k_1 c = \frac{A_s f_{yd}}{0.85 f_{cd} b} = \frac{2700 \times 365}{0.85 \times 13 \times 1000} = 89 \text{ mm}$
- $\varepsilon_s = 0.003 \frac{d-c}{c} = 0.003 \frac{500-89/0.85}{89/0.85} = 0.01133 > \varepsilon_{yd} = 0.001825$
- $M_r = A_s f_{yd} jd = 2700 \times 365 \times \left(500 - \frac{89}{2}\right) = 449 \text{ kNm}$

Example 7



- Convert the box section into a T-section
- Check if $k_1 c < t$
- $F_c = 0.85 \times 13 \times 600 \times 120 = 796 \text{ kN}$
- $F_s = 2700 \times 365 = 985 \text{ kN}$

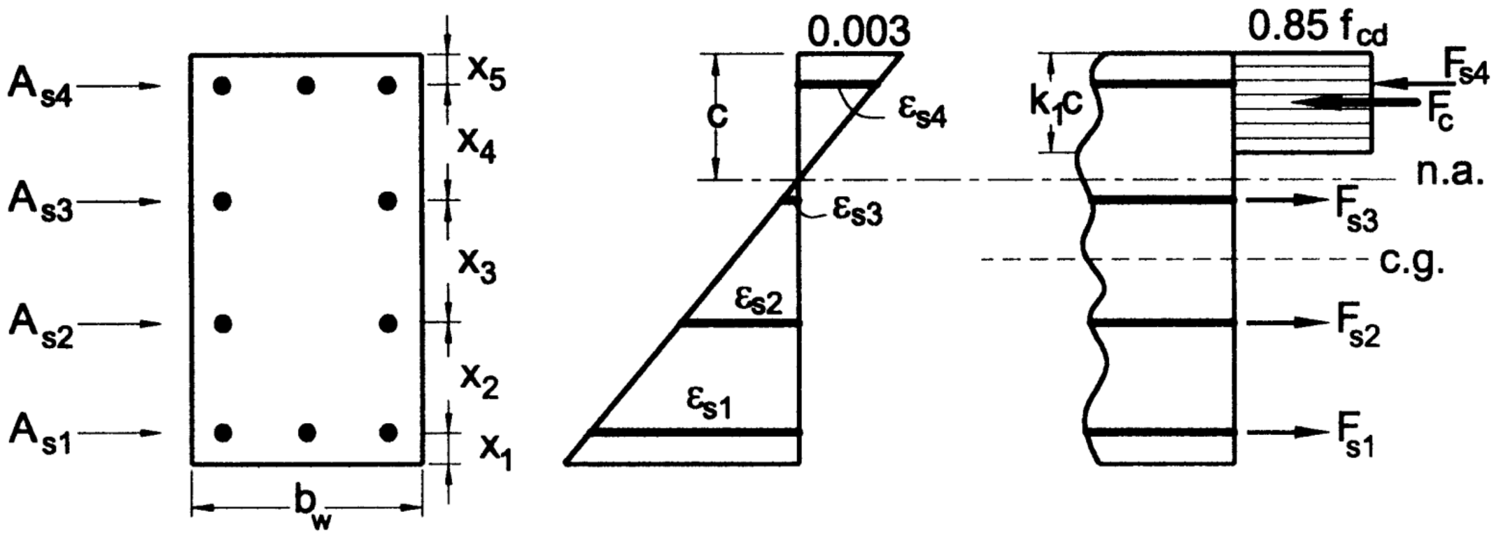
$$F_c < F_s$$
$$\rightarrow \; k_1 c > t$$

Example 7

- $F_c = 0.85 \times 13[300 \times k_1 c + (600 - 300)120]$
- $\quad = 3315 k_1 c + 397800$
- $F_s = 985500 \text{ N}$
- $F_c = F_s \quad \Rightarrow \quad k_1 c = 178 \text{ mm} \quad \& \quad c = 209 \text{ mm}$
- Cetroid: $\bar{x} = \dfrac{300\times178\times\frac{178}{2}+300\times120\times\frac{120}{2}}{300\times178+300\times120} = 77 \text{ mm}$
- $jd = d - \bar{x} = 500 - 77 = 423 \text{ mm}$
- $M_r = A_s f_{yd} jd = 2700 \times 365 \times 423 = 417 \text{ kNm}$

Beams with several layers of steel

- Neutral axis depth c is unknown
- Which steel layer is under compression?
- Which steel layer is under tension?
- Steel layers under tension yielded?
- Steel layers under compression yielded?



Beams with several layers of steel

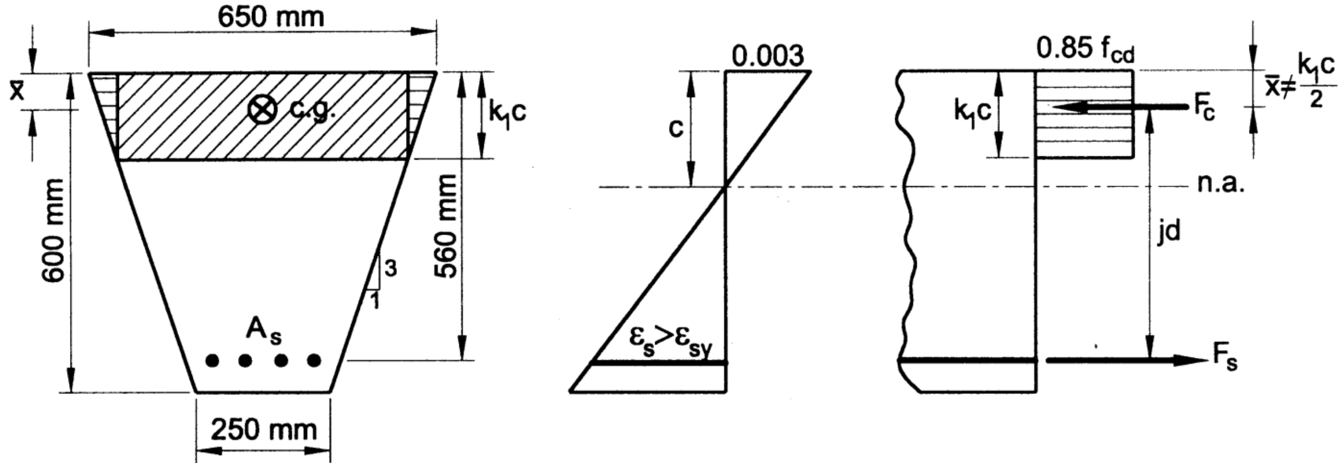
- Trial and error approach
  - Assume c
  - Compatibility; from similar triangles compute steel strains ε_{si}
  - Compute $F_{si} = A_{si}\sigma_{si}$ $\quad \sigma_{si} = \varepsilon_{si}E_s \leq f_{yd}$
  - Compute $F_c = 0.85 f_{cd} b_w k_1 c$
  - Check $\sum F = 0$
    - if $\sum F \leq 1 - 2\% \sum \text{compressive forces}$ → no further iteration
  - Change c & repeat steps until equilibrium is established
    - $\sum \text{tension} > \sum \text{compression}$ → increase c
    - $\sum \text{tension} < \sum \text{compression}$ → decrease c
  - Compute moment of forces about a convenient point (usually centroid)

Study Example 5.6-A

Beams with Non-rectangular Cross-Section

- Trial & error procedure can be used
- When cross-section can be divided into rectangles and/or triangles, a closed solution can be possible

Example 8



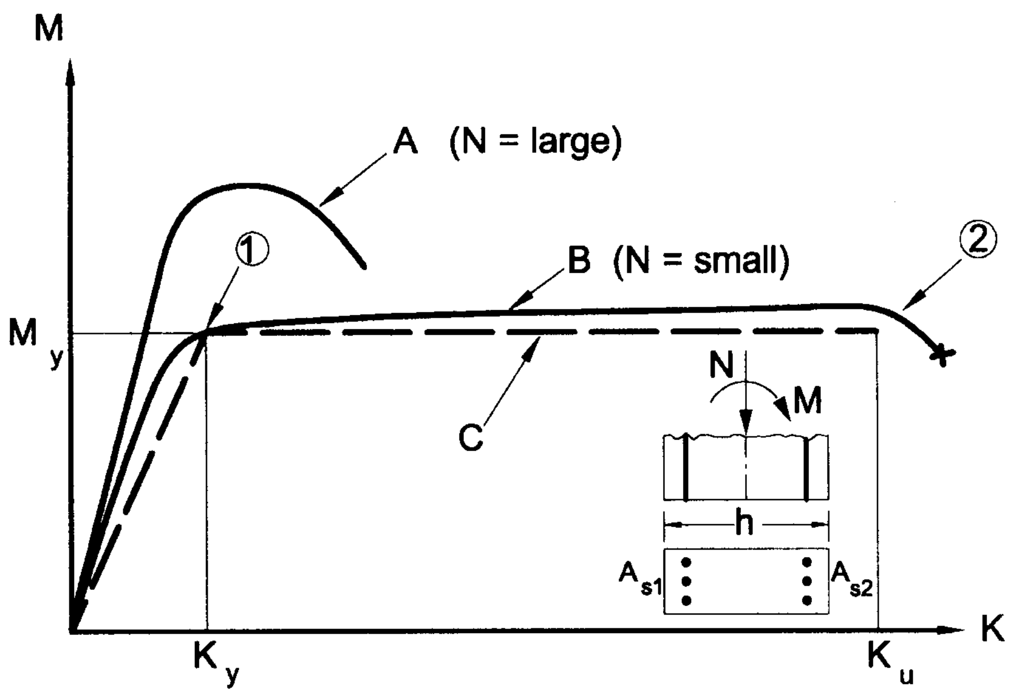
- $A_s = 1590\ \text{mm}^2$, C20 ($f_{cd} = 13$ MPa), S420 ($f_{yd} = 365$ MPa)
- $M_r = ?$
- Shaded area:
- $A_{cc} = \left(650 - 2\frac{1}{3}k_1c\right)k_1c + 2\frac{1}{2}k_1c\frac{1}{3}k_1c$
- $A_{cc} = 650k_1c - \frac{(k_1c)^2}{3}$

Example 8

- $F_c = 0.85 f_{cd} A_{cc} = 0.85 \times 13 \times \left(650 k_1 c - \frac{(k_1 c)^2}{3}\right)$
- $F_c = 7182.5 k_1 c - 3.68 (k_1 c)^2$
- $F_s = A_s f_{yd} = 1590 \times 365 = 580350 \text{ N}$
- $F_c = F_s \quad \Rightarrow \quad (k_1 c)^2 - 1952 k_1 c + 159000 = 0$
- $k_1 c = 85 \text{ mm}$
- Centroid: $\frac{\left(650 - \frac{2}{3}85\right) 85 \times \frac{85}{2} + 2 \frac{1}{2} 85 \frac{85}{3} \frac{1}{3} 85}{\left(650 - \frac{2}{3}85\right) 85 + 2 \frac{1}{2} 85 \frac{85}{3}} = 42 \text{ mm}$
- $jd = d - \bar{x} = 518 \text{ mm}$
- $M_r = A_s f_{yd} jd = 300.6 \text{ kNm}$

Section Response: Moment – Curvature

- Provides Full Range Behavior
- Multiple steel layers, axial force+moment, complicated section geometry requires use of computers.

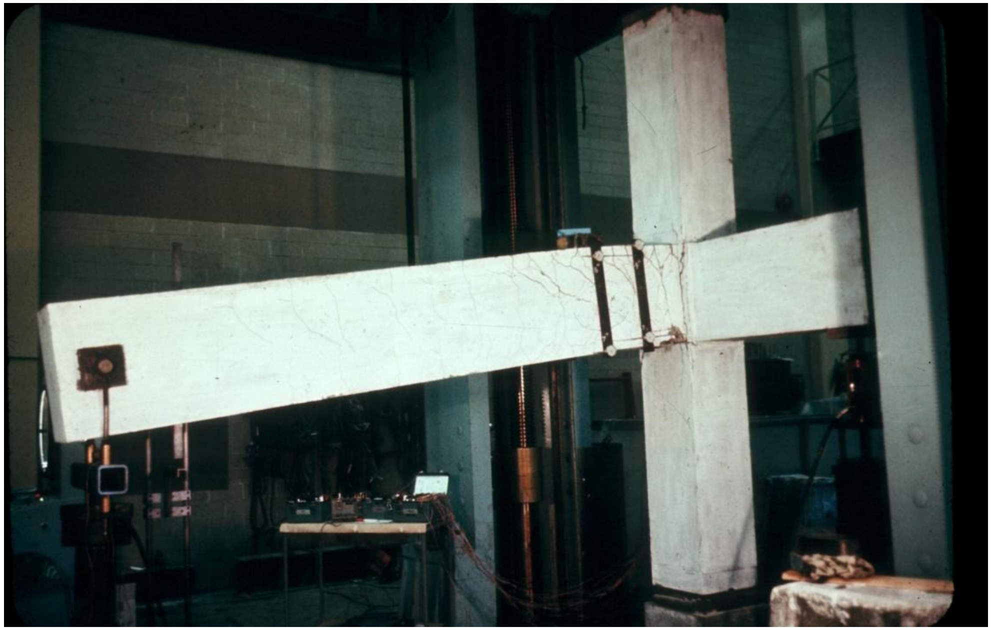


- $\mathrm{M} - \mathcal{K}$ curve is nonlinear
- curve changes significantly with the level of axial load
- After yielding of tension steel, $\mathcal{K}_y$, curvature increases without any increase in the moment → **Plastic Hinge**

Moment – Curvature relationship

- *Classical hinge*: section rotates under zero moment
- *Plastic hinge*: rotation takes place under a constant moment
- *Ductility*: the capability of undergoing large deformations without a significant reduction in the strength.

- $curvature\ ductility\ ratio = \dfrac{\mathcal{K}_u}{\mathcal{K}_y}$ — related to cross-sectional properties & σ–ε of materials
- $displacement\ ductility$ — related to member properties




P = 260 K
10th CYCLE
INELASTIC
2
6
5


Drift : 3%