## Integration Formulas

**a)** <u>Trapezoidal Rule:</u>

$$I \approx \frac{h}{2}\left[ f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n) \right]$$

or

$$I \approx \frac{(b-a)}{2n}\left[ f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n) \right];$$

$$E = -\frac{(b-a)^3}{12n^3}\sum_{i=1}^{n} f^{(2)}(\varepsilon_i) \quad \text{or} \quad E = -\frac{(b-a)^3}{12n^2}\overline{f}^n \quad \text{for n segments (or intervals).}$$

**b)** <u>Simpson's 1/3 Rule:</u>

$$I \approx \frac{h}{3}\left[ f(x_0) + 4\sum_{i=1,3,5,\ldots}^{n-1} f(x_i) + 2\sum_{i=2,4,6,\ldots}^{n-2} f(x_i) + f(x_n) \right]$$

or

$$I \approx \frac{b-a}{3n}\left[ f(x_0) + 4\sum_{i=1,3,5,\ldots}^{n-1} f(x_i) + 2\sum_{i=2,4,6,\ldots}^{n-2} f(x_i) + f(x_n) \right]$$

$$E = -\frac{(b-a)^5}{90n^5}\sum_{i=1}^{n/2} f^{(4)}(\varepsilon_{2i}) = -\frac{(b-a)^5}{180n^4} \quad \text{for n segments (or intervals).}$$
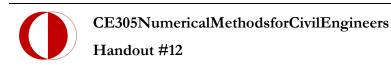
**c)** <u>Gauss-Legendre Formulas:</u>

Let your original integral be of the form:

$$\int_a^b g(t)dt \tag{1}$$

Gauss Legendre Formulation has the variable x for $\int_{-1}^{1} f(x)dx$

Now, change (transform) t to x by;

$$t = \frac{b+a}{2} + \frac{b-a}{2}x \tag{2}$$

and dt to dx by;

$$dt = \frac{b-a}{2}dx \qquad (3)$$

Now substitute equations (2) and (3) into equation (1) and get ready to use Gauss-Legendre formula $\int_{-1}^{1} f(x)\,dx \approx \sum_{i=0}^{n} w_i f_i$ with

| N | Abscissas, $x_{N,k}$ ($x_i$) | Weights, $w_{N,k}$ ($w_i$) | Truncation Error |
|---|---|---|---|
| 2 | -0.5773502692<br>0.5773502692 | 1.0000000000<br>1.0000000000 | $\dfrac{f^{(4)}(\varepsilon)}{135}$ |
| 3 | ±0.7745966692<br>0.0000000000 | 0.5555555556<br>0.8888888888 | $\dfrac{f^{(6)}(\varepsilon)}{15,750}$ |
| 4 | ±0.8611363116<br>±0.3399810436 | 0.3478548451<br>0.6521451549 | $\dfrac{f^{(8)}(\varepsilon)}{3,472,875}$ |
| 5 | ±0.9061798459<br>±0.5384693101<br>0.0000000000 | 0.2369268851<br>0.4786286705<br>0.5688888888 | $\dfrac{f^{(10)}(\varepsilon)}{1,237,732,650}$ |