CE 383 STRUCTURAL ANALYSIS

2012 Spring Semester

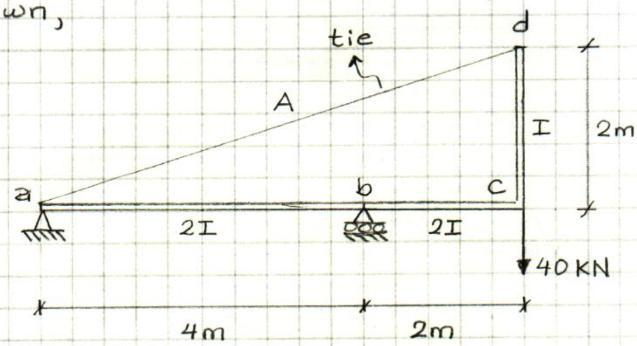
RECITATION NO:3

Q. For the compound structure shown,

a) Find the force in tie ad

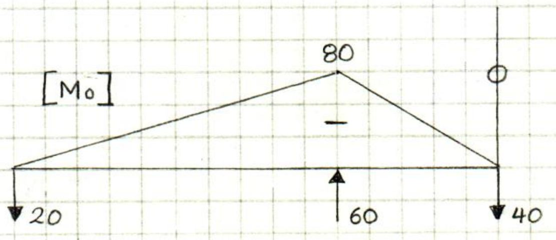
b) Draw moment diagram for the arm abcd

c) Find the vertical displacement at C by Reduction Theorem.



$E = 2e8$ kN/m^2, $I = 21e{-5}$ m^4, $A = 3e{-4}$ m^2

a) Tie force is taken as redundant

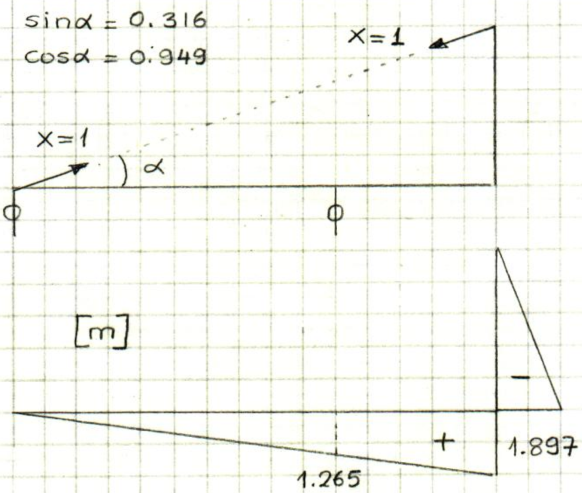


$$\Delta = \Delta_o + fX = 0$$

$$EI\,\Delta_o = \Sigma \int m\,M_o\,dx$$

{Using charts}

$$= -\frac{1}{6}(1.897)(80)\left(1+\frac{4}{6}\right)\left(\frac{1}{2}\right)6 = -126.49$$



$$EIf = \Sigma \int m^2\,dx + (1)^2 \frac{I}{A} L$$

{Using charts}

$$= \frac{1}{3}(1.897)^2\left(\frac{1}{2}\right)6$$

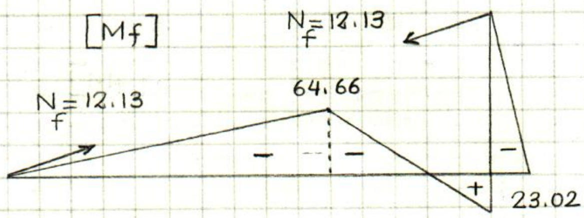
$$+ \frac{1}{3}(1.897)^2(1)(2)$$

$$+ \frac{21e{-5}}{30e{-5}}(6.32) = 10.427$$

$$\Rightarrow 0 = \frac{-126.49}{EI} + \frac{10.427}{EI}(X)$$

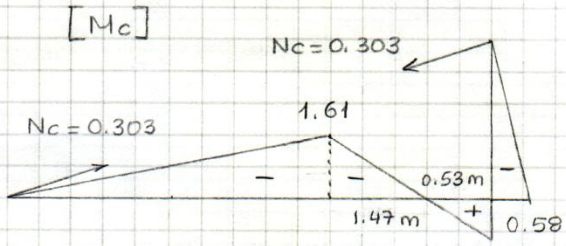
$$\Rightarrow \underline{\underline{X = 12.13 \text{ kN}}}$$

Final moment diagram can easily be obtained by using statics



c) Since the required displacement is in the same location with the applied force of 40 kN, the moment diagram of the structure with a unit load at C is simply 1/40 th scale of [Mf] diagram.

Therefore [Mc] diagram is obtained by scaling down the values of [Mf] diagram by 1/40.



$$EI\,\delta_{cv} = \Sigma \int Mf\,Mc\,dx + Nf\,Nc\,\frac{I}{A}\,L$$

$$EI\,\delta_{cv} = \frac{1}{3}(64.66)(1.61)\left(\frac{1}{2}\right)(4) + \frac{1}{3}(64.66)(1.61)(1.47)\left(\frac{1}{2}\right) +$$

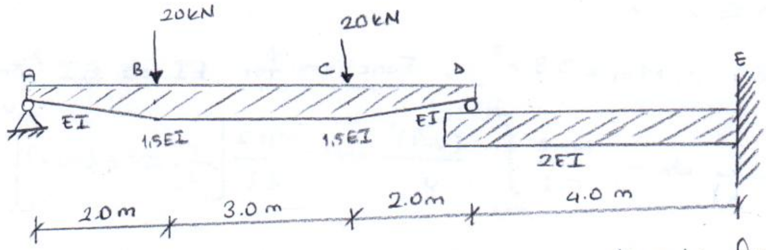
$$\frac{1}{3}(23.02)(0.58)(0.53)\left(\frac{1}{2}\right) + \frac{1}{3}(23.02)(0.58)(1)(2) +$$

$$(12.13)(0.303)(21/30)(6.32)$$

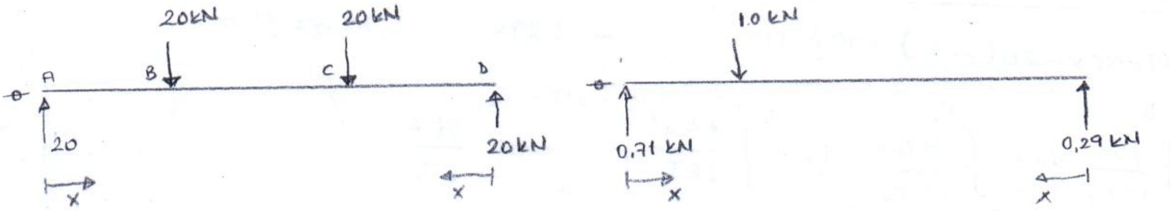
$$EI\,\delta_{cv} = 121.6$$

$$\underline{\underline{\delta_{cv} = 2.9 \times 10^{-3}\ m \downarrow}}$$

Q2)



The EI value of the beam ABCD varies linearly from EI at the supports A and D to 1.5 EI at B and C respectively and is constant between B and C. Determine the value of the vertical deflection at B given that $EI = 15.0 \times 10^3$ kNm^2



EI is not linear between A-B & C-D.

$$\delta_B = \int_A^B \frac{Mm}{EI}\,dx + \int_D^C \frac{Mm}{EI}\,dx + \int_C^B \frac{Mm}{EI}\,dx + \int_D^E \frac{Mm}{EI}\,dx$$

Between A & B $\quad 0 \leq x \leq 2.0\,m$

$M = 20x \quad ; \quad m = 0.71x \qquad \therefore \; Mm = 14.2x^2$

Function for EI $\Rightarrow EI(1+0.25x)$

$$\int_A^B \frac{Mm}{EI}\,dx = \int_0^2 \frac{14.2x^2}{EI(\underbrace{1+0.25x}_{v})}\,dx$$

$v = (1+0.25x) \Rightarrow x = 4.(v-1) \qquad dx = 4\,dv \quad$ and $\quad x^2 = 16(v-1)^2$

$x=0 \Rightarrow v = 1.0 \quad ; \quad x = 2 \Rightarrow v = (1+0.5) = 1.5$

$$Mm\,dx = 14.2\,x^2\,dx = 14.2\,(16\cdot(v-1)^2)\cdot 4\,dv = 908.8\,(v-1)^2\,dv$$

$$\int_A^B \frac{Mm}{EI}\,dx = \frac{908.8}{EI}\int_{1.0}^{1.5} \frac{(v-1)^2}{v}\,dv = \frac{908.8}{EI}\int_{1.0}^{1.5}\left(v - 2.0 + \frac{1}{v}\right)dv = \frac{908.8}{EI}\left[\frac{v^2}{2} - 2v + \ln(v)\right]_{v=1}^{v=1.5}$$

$$\int_A^B \frac{Mm}{EI}\,dx = \frac{27.69}{EI}$$

Between C&D $\quad 0 \leq x \leq 2.0\text{ m}$

$M = 20x \quad ; \quad m = 0{,}29x \quad \therefore Mm = 5.8x^2 \quad$; Function for $EI \Rightarrow EI(1+0{,}25x)$

$$\int_C^D \frac{M.m}{EI}dx = \int_0^2 \frac{5.8x^2}{EI(1+0{,}25x)}dx = \frac{371{,}2}{EI}\int_{v=1}^{v=1.5}\frac{(v-1)^2}{v}dv = \frac{371.2}{EI}\left[\frac{v^2}{2} - 2v + \ln(v)\right]_{v=1.0}^{v=1.5}$$

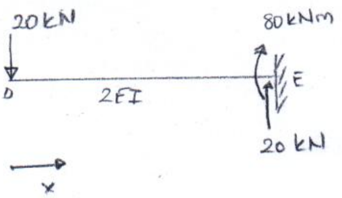
$$\int_C^D \frac{Mm}{EI}dx = \frac{11.31}{EI}$$

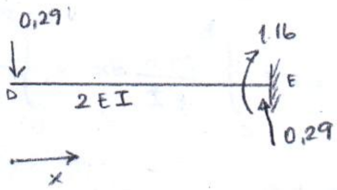
Between C&B $\quad 2 \leq x \leq 5\text{ m}$ (From C to B)

$M = 20x - 20(x-2) = 40\text{ kNm} \quad ; \quad m = +0{,}29x \quad \therefore Mm = 11.6x$

$$\int_C^B \frac{Mm}{1.5EI}dx = \int_2^5 \frac{11.6x}{1.5EI}dx = \left[\frac{11.6x^2}{3EI}\right]_{x=2}^{x=5} = \frac{81.2}{EI}$$

Consider the cantilever beam DE





$M = -20x \quad ; \quad m = -0{,}29x \quad \therefore Mm = 5{,}8x^2$

$$\int_D^E \frac{Mm}{EI}dx = \int_0^4 \frac{5.8x^2}{2EI}dx = \left[\frac{5{,}8x^3}{6EI}\right]_{x=0}^{x=4} = \frac{61.87}{EI}$$

$$\delta_B = \frac{27{,}69}{EI} + \frac{11{,}31}{EI} + \frac{81{,}2}{EI} + \frac{61{,}87}{EI} = \frac{182.07}{EI} = 12.14\text{ mm} \downarrow$$