RULES

1. The answers of the exercises will be posted on our website at 17:00 PM on 10.12.2012: www2.ce.metu.edu.tr/~ce305 .
2. This is the **version 3.1**. In case there are any corrections for this exercise, we will post an updated version on our website. You can follow the changes in the exercises by the **Version History** section below.

Version History

V3.0 Exercise is released.
V3.1 In question 2 "y" values corresponding to "x=6" and "x=8" are changed from "21.0 and 50.0" to "19.0 and 48.0".

1. Solve the following system of nonlinear equations by
   a. graphical method (simply plot the equations on the same coordinate system and observe the intersection points)
   b. Newton-Jacobi method using $x_0 = 1$ and $y_0 = -1.7$, and error tolerance with respect to maximum norm as 10^{-4}.

$$x^2 + y^2 = 4$$
$$e^x + y = 1$$

2. We measured the value of a function $f(x)$ at the following values, i.e. $x = 1,3,5,6,7,8$. Using the table given below, answer the following questions:

x	1	3	5	6	7	8
y	-0.70	0.45	11.0	19.0	29.0	48.0

   a. Plot your points. You may use Matlab or do it manually.
   b. Fit a second order Lagrange polynomial to the data series using 1,3 and 5 as your points.
   c. For this polynomial function, determine the % deviation (error) from the known f(x) values at $x = 1,6,8$.
   d. Determine $f(3.6)$ and $f(9.0)$.
   e. Now we are going to use polynomial functions and linear regression analysis for interpolation and extrapolation. Using functions 1 and x, determine the parameters c_0 and c_1 of the function $f(x) = c_0 + c_1 x$. Determine the root mean square error for all the data points you are given. Plot the estimated function on your graph.
   f. Using functions $1, x$ and x^2, determine the parameters c_0, c_1 and c_2 of the function $f(x) = c_0 + c_1 x + c_2 x^2$. Determine the root mean square error for all the data points you are given. Estimate $f(3.6)$ and $f(9.0)$. Plot the estimated function on your graph.

g. Using functions $1, x, x^2$ and x^3, determine the parameters c_0, c_1, c_2 and c_3 of the function $f(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$. Determine the root mean square error for all the data points you are given. Estimate $f(3.6)$ and $f(9.0)$. Plot the estimated function on your graph.

h. How did the linear, quadratic and cubic estimations work? Which one gave the smallest RMS error?

i. We used two quadratic functions (b) a Lagrange polynomial and (f) a quadratic regression function to estimate the value of y at $x = 9$. Which one works better? Both are quadratic, why don't they work similarly?