INTRODUCTION

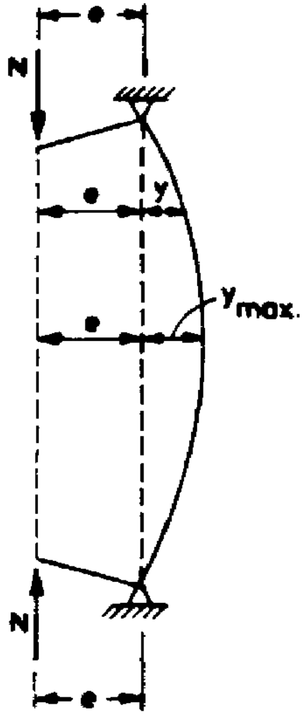
Due to monolithic nature of reinforced concrete construction, a great many of reinforced concrete members are subject to axial loads in addition to flexure. These RC members are named as columns.

Moments are present due to initial crookedness of columns, unsymmetrically placed floor loads and due to non-homogeneity of concrete along any cross-section.

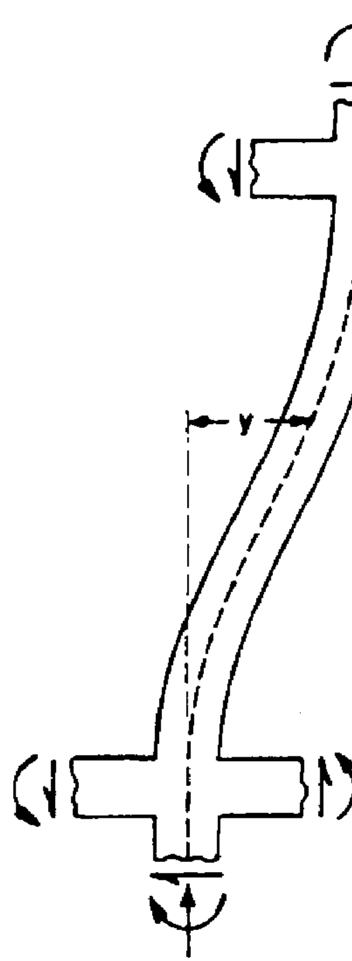
Therefore, most of the design codes prohibit column design with no moment and specify a minimum eccentricity even when the analysis results in zero moment.

In general, cross-sectional dimensions of columns are small as compared to their height, i.e. they are slender members. Therefore, additional moments are created due to deformations.

The presence of the second order moments can be illustrated by the simple example shown in figure below.



(a)



(b)

$$M'_d = N(e + y) = N \cdot e + N \cdot y$$

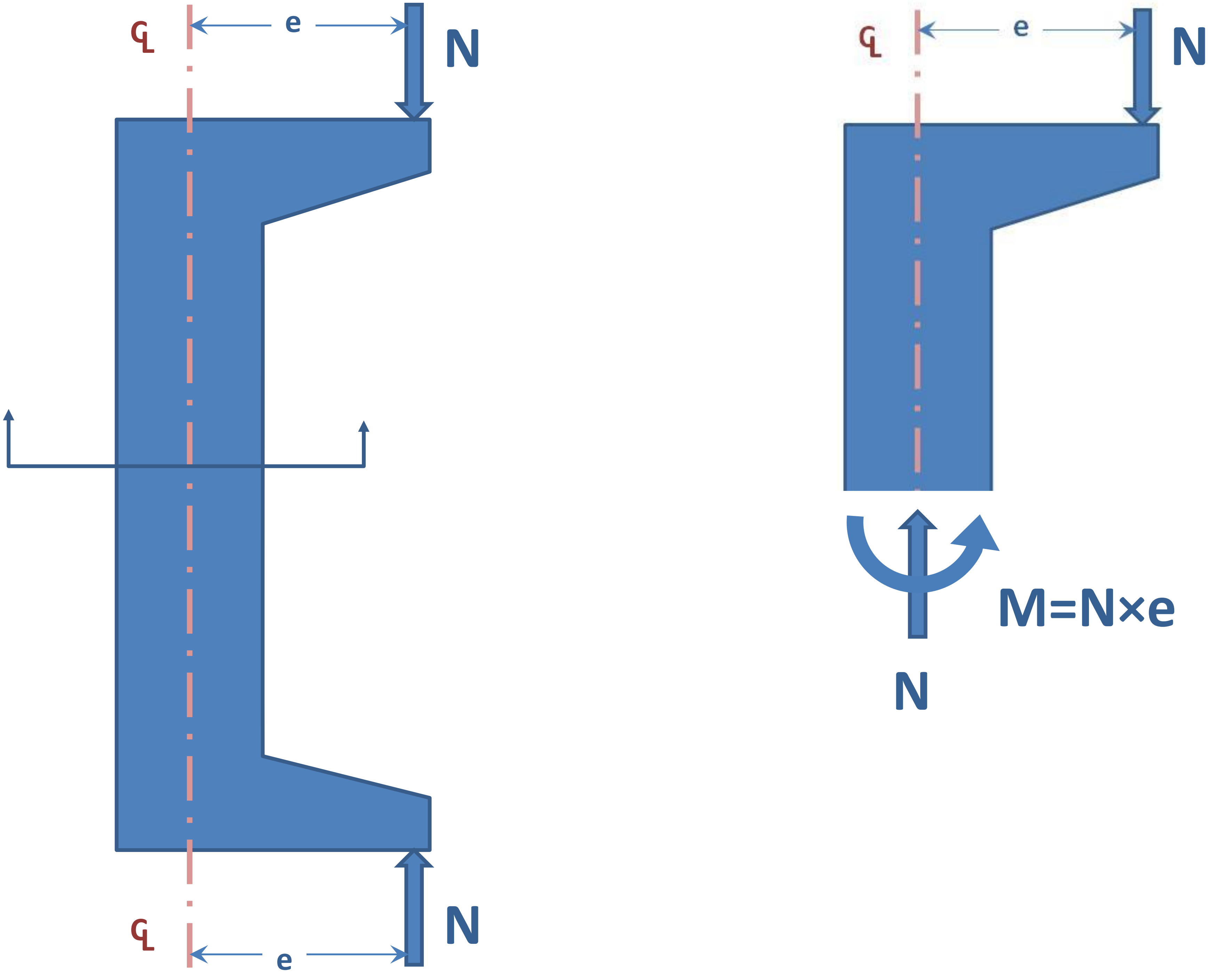
$$M'_d = M_d + N \cdot y_{\max}$$

In sway frames, the second order moments become more critical due to displacement of column ends relative to each other.

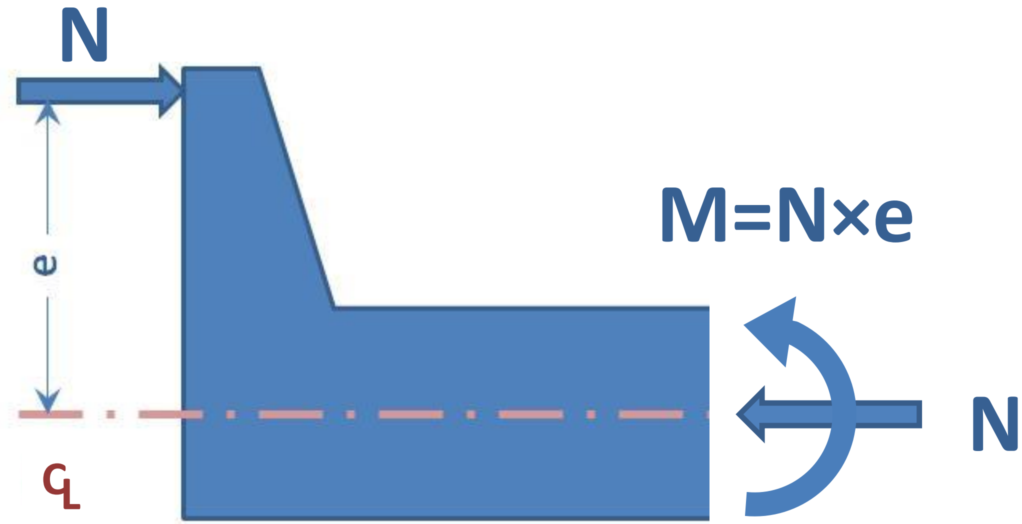
General Equations

The basic assumptions made in ultimate strength theory are summarized below:

- Plane sections remain plane after bending
- Failure occurs by crushing of concrete in the extreme fiber in compression
- Crushing strain of concrete is $\varepsilon_{cu} = 0.003$
- Perfect bond between steel and concrete
- Tension in concrete is neglected
- Stress-strain curve of steel is assumed to be elasto-plastic, $\sigma_{si} = \varepsilon_{si} E_s \leq f_{yd}$
- Concrete stress block can be replaced by an equivalent rectangular block having a width of $0.85f_{cd}$ and depth of k_1c, where c is the depth of the neutral axis.



CL
e
N
N
CL
e
N
CL
e
N
M=N×e
N


N
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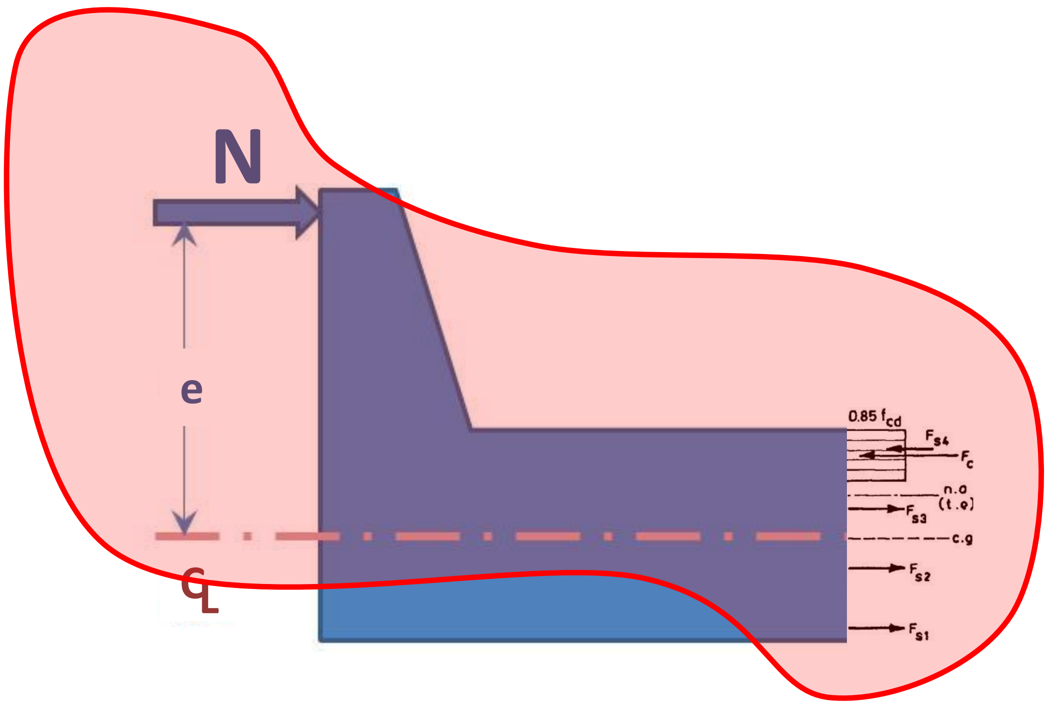
N
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CL
0.85 f_{cd}
F_{s4}
F_c
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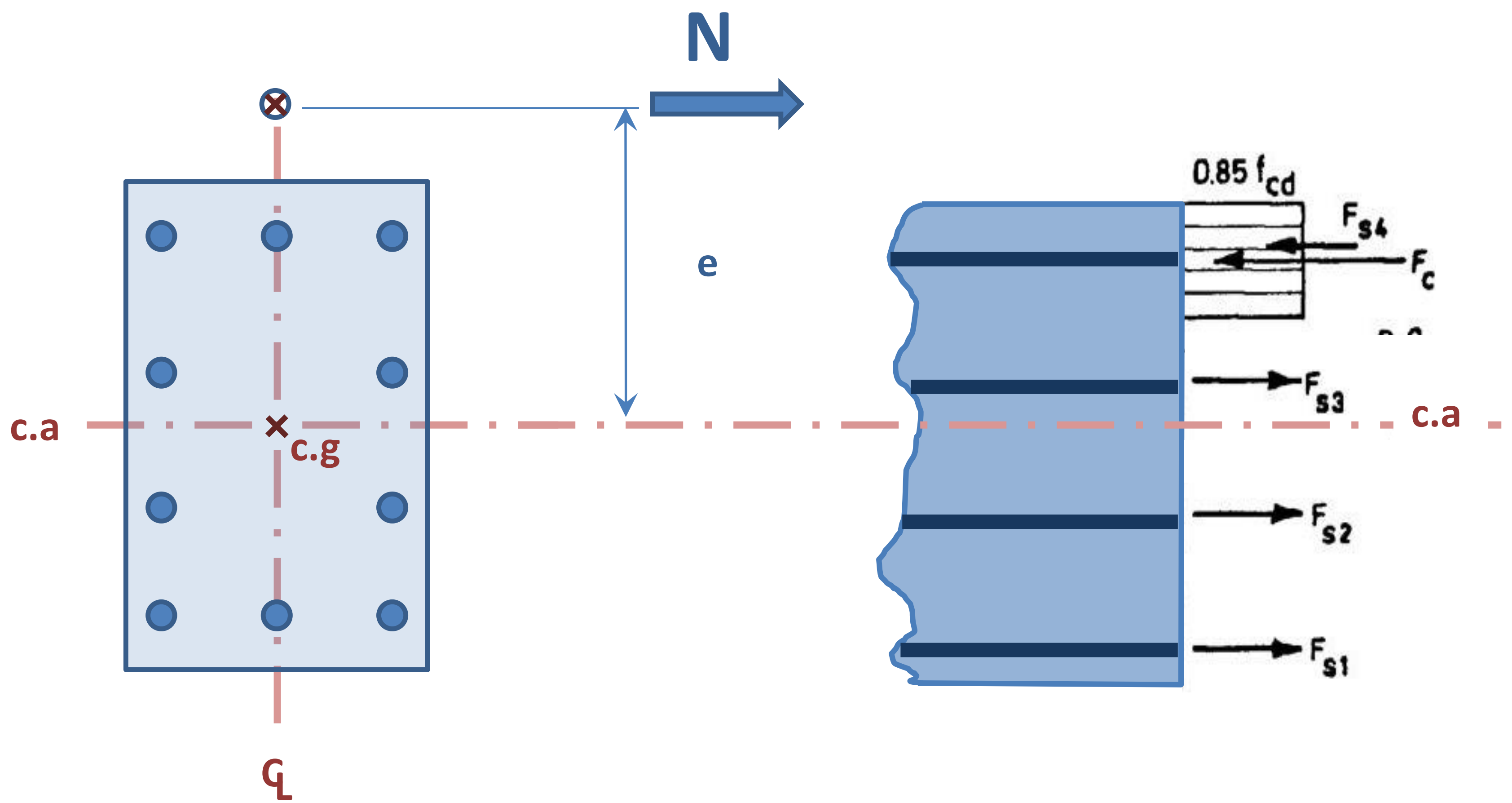
(t.a)
F_{s3}
c.g
F_{s2}
F_{s1}



N
e
0.85 f_cd
F_s4
F_c
F_s3
F_s2
F_s1
c.a
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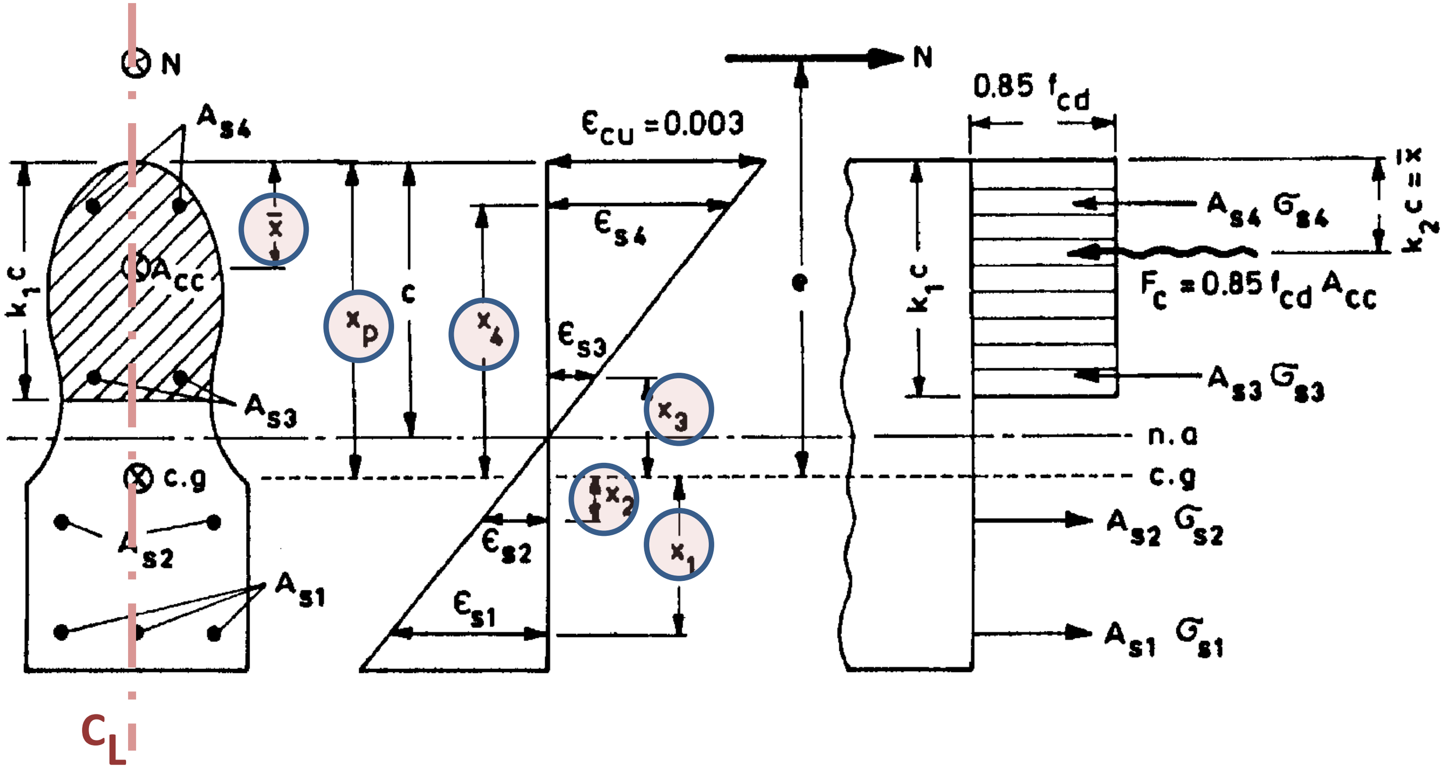
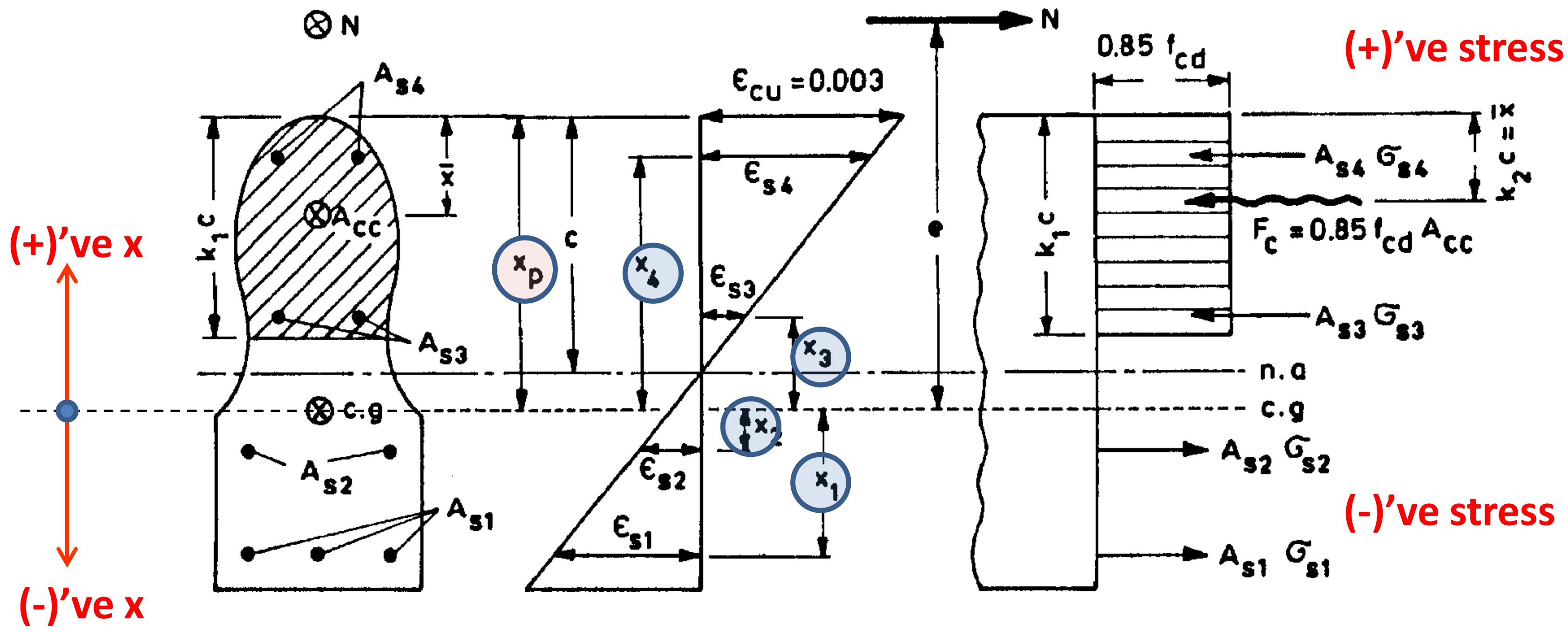
c.a
CL



Equilibrium:

$$N = 0.85 f_{cd} A_{cc} + \sum_{i=1}^{n} A_{si}\sigma_{si}$$

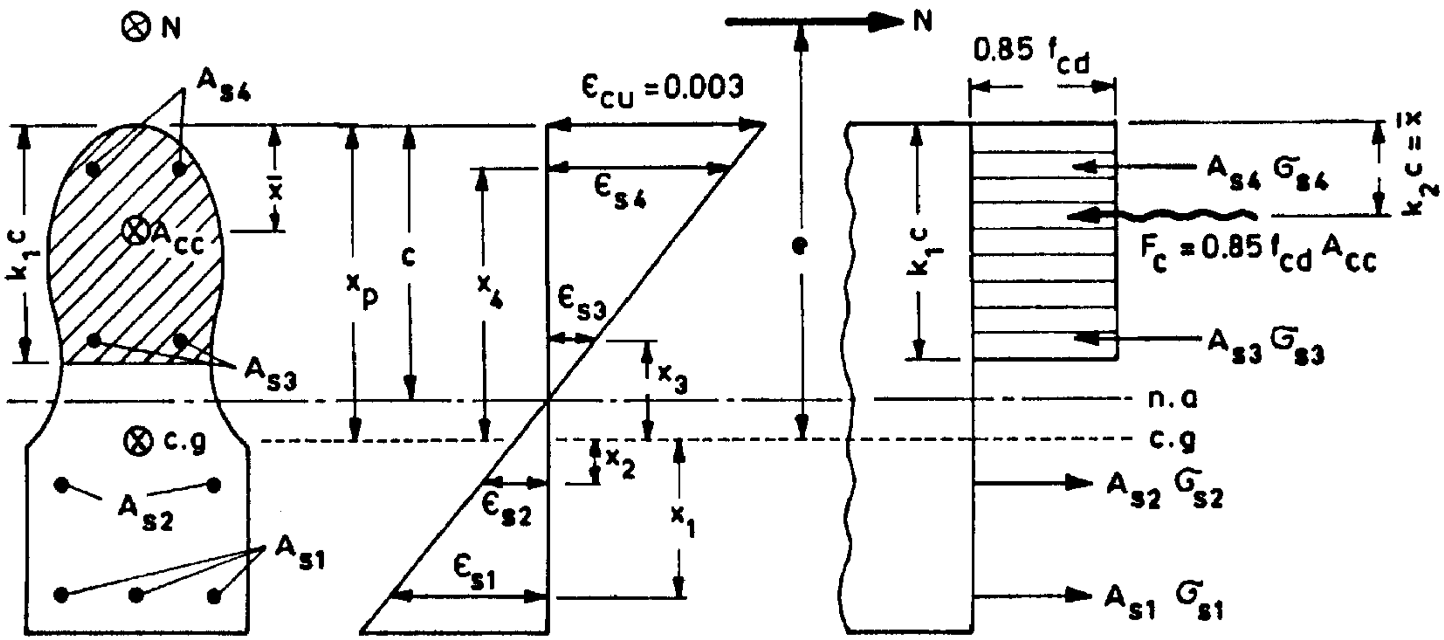
$$M = N \cdot e = 0.85 f_{cd} A_{cc}(x_p - \bar{x}) + \sum_{i=1}^{n} A_{si}\sigma_{si} x_i$$



Compatibility:

$$\frac{c}{0.003} = \frac{x_p - c - x_i}{-\varepsilon_{si}} \quad \text{or} \quad \varepsilon_{si} = 0.003\,\frac{x_i + c - x_p}{c}$$

1 ≤ i ≤ number of steel layers



Force Deformation:

$$\sigma_{si} = \varepsilon_{si} E_s \leq f_{yd}$$

by substituting ε_{si} *in this expression we get*

$$\sigma_{si} = \left\{ 0.003 E_s \left(1 + \frac{x_i - x_p}{c} \right) \right\} \leq f_{yd}$$

If all geometric and material properties are known, These three equations are adequate to determine the ultimate capacity of a cross-section. Types of analyses are:

(a) *Given:* *Geometry of cross-section and steel areas,*
f_{cd} and f_{yd}
M or e

Find: ***N***

There exist three unknowns N, σ_{si}, c and three equations. Solution is available.

(b) *Given*: *Geometry of cross-section and steel areas,*
f_{cd} and f_{yd}
N

Find: ***M*** *or* ***e***

There exist three unknowns M, σ_{si}, c and three equations. Solution is available.

The equations derived are very general. It should be noted that no restrictions have been placed on the geometry of the cross-section ***(except symmetry about plane of loading)*** and on the arrangement of steel. It should also be realized that no restrictions have been stated on the magnitude of the eccentricity.

Therefore the equations are applicable all the way from **e = 0** *(axial compression)* to **e = ∞** *(pure bending)*.

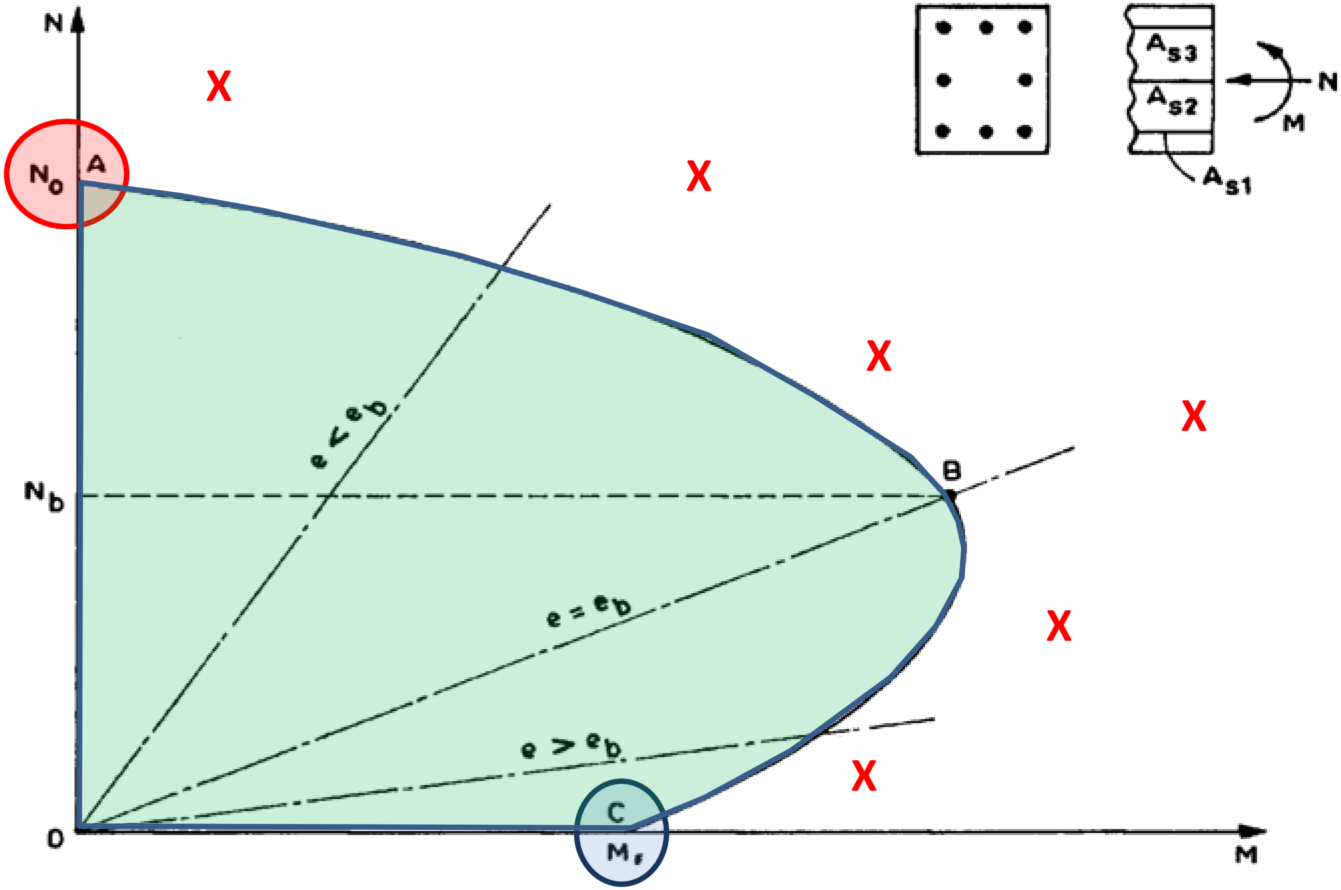
In other words, these three simple equations represent the ultimate capacity of a reinforced cross-section subjected to axial load and flexure in all ranges of eccentricity. Then, for a given cross-section the plot of ***N*** versus ***M*** obtained from these equations will represent the ***strength envelope***.

Strength envelopes are usually called ***"interaction diagrams"*** or ***"interaction curves".***

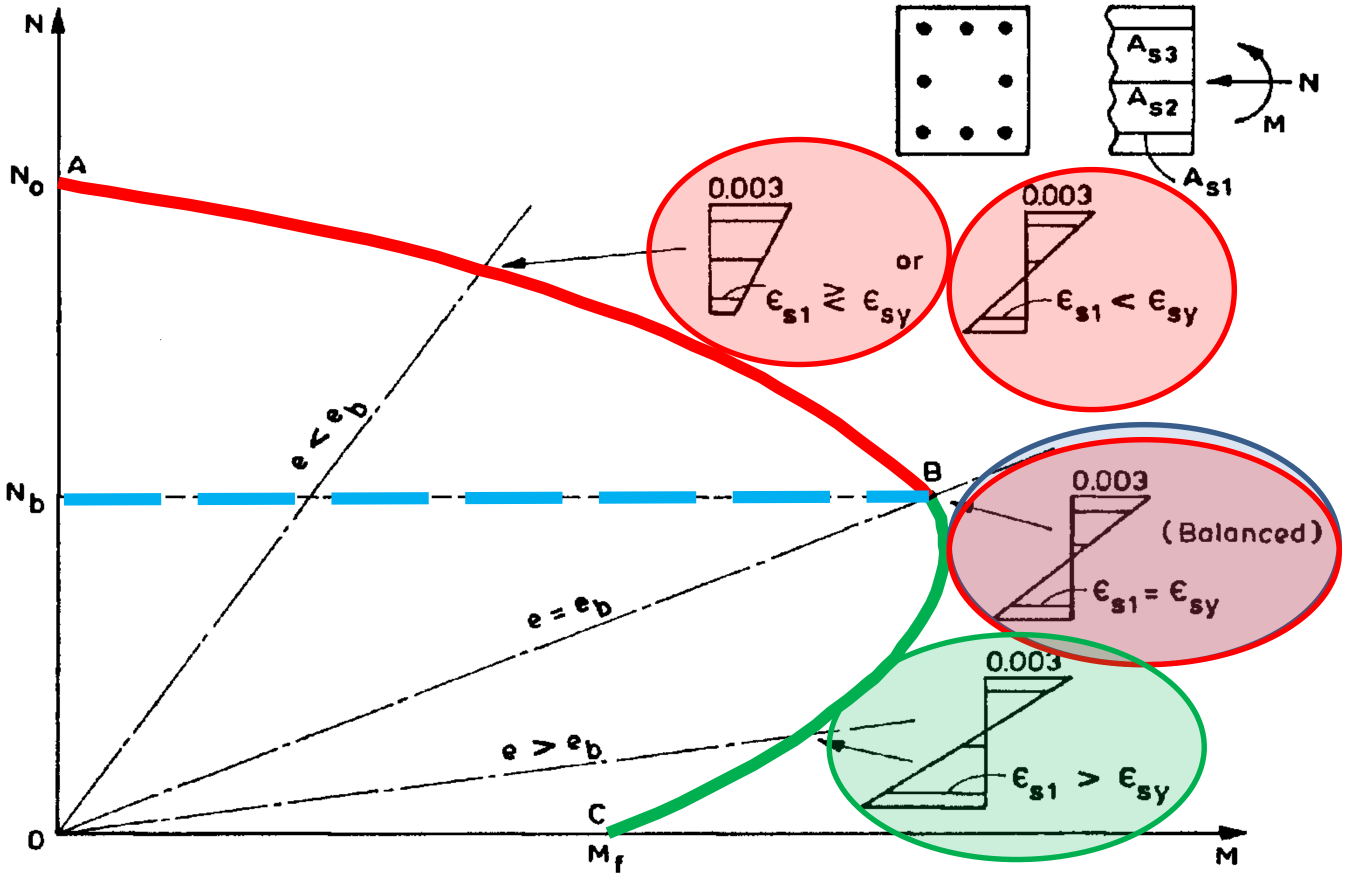
For a given cross-section with specified material properties, such a plot can be obtained easily by assuming c values and computing corresponding N and M using the equations derived.

N and M obtained for each assumed c value will represent a point on the strength envelope.

In each case, there will be three unknowns, N, M and σ_{si} and three equations.



On this curve, point A represents uniaxial compression and point C represents pure bending. **Area inside the strength envelope is the safe zone**, i.e. any N, M combination remaining in this area will not cause failure. <u>Loadings with M and N combinations falling outside the envelope represent failure.</u>

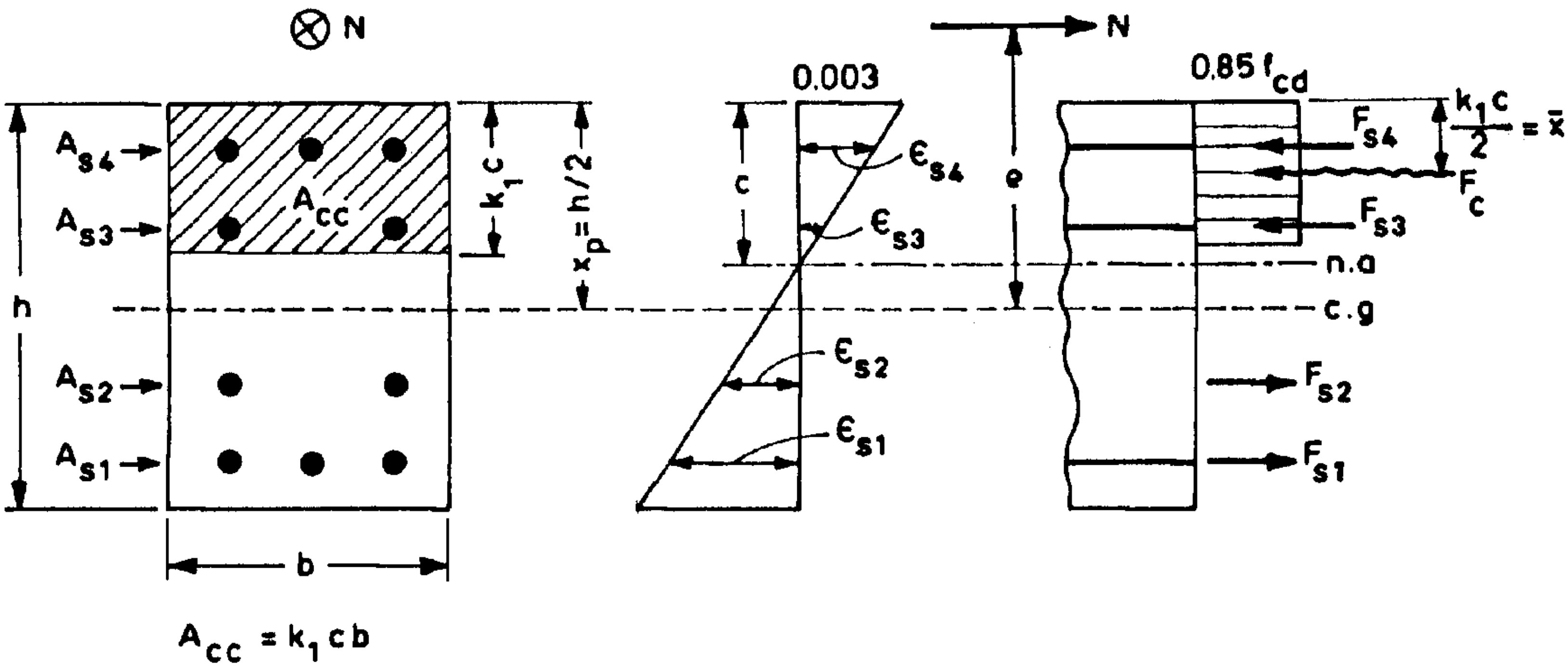


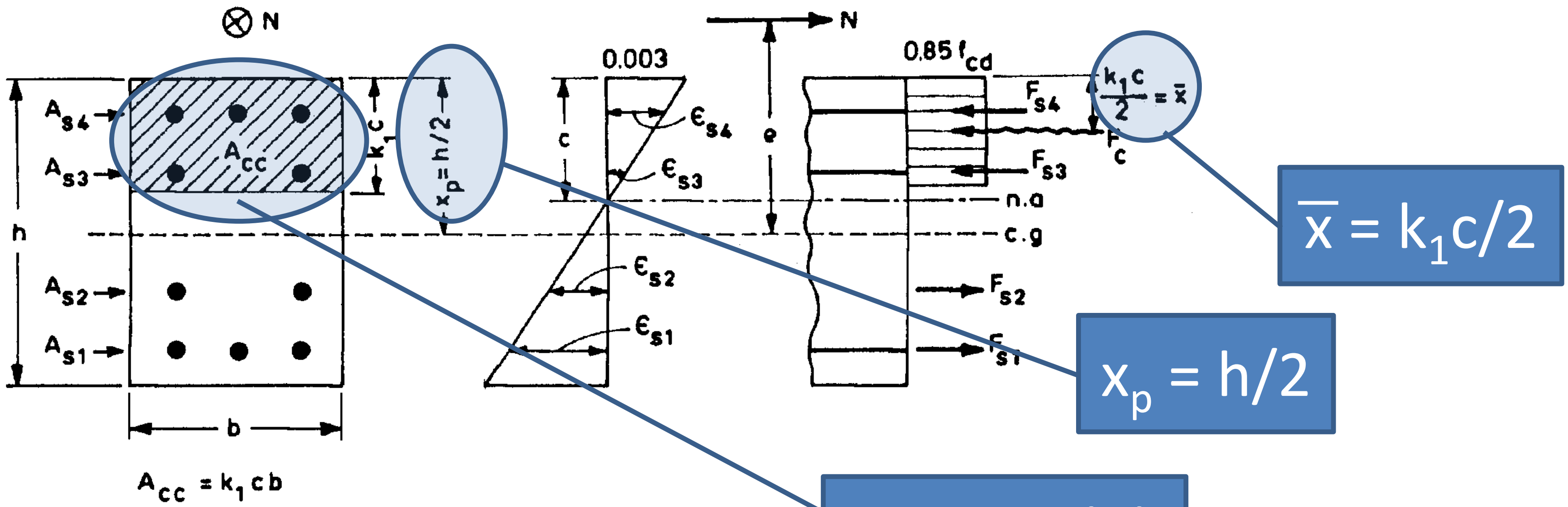
compression failure if $e < e_b$ or if $N > N_b$

tension failure if $e > e_b$ or if $N < N_b$

ULTIMATE STRENGTH OF RECTANGULAR COLUMNS

Most of the columns in our structures have rectangular cross-sections. Therefore, the general equations derived will be rewritten for rectangular sections, which will have a wide application.



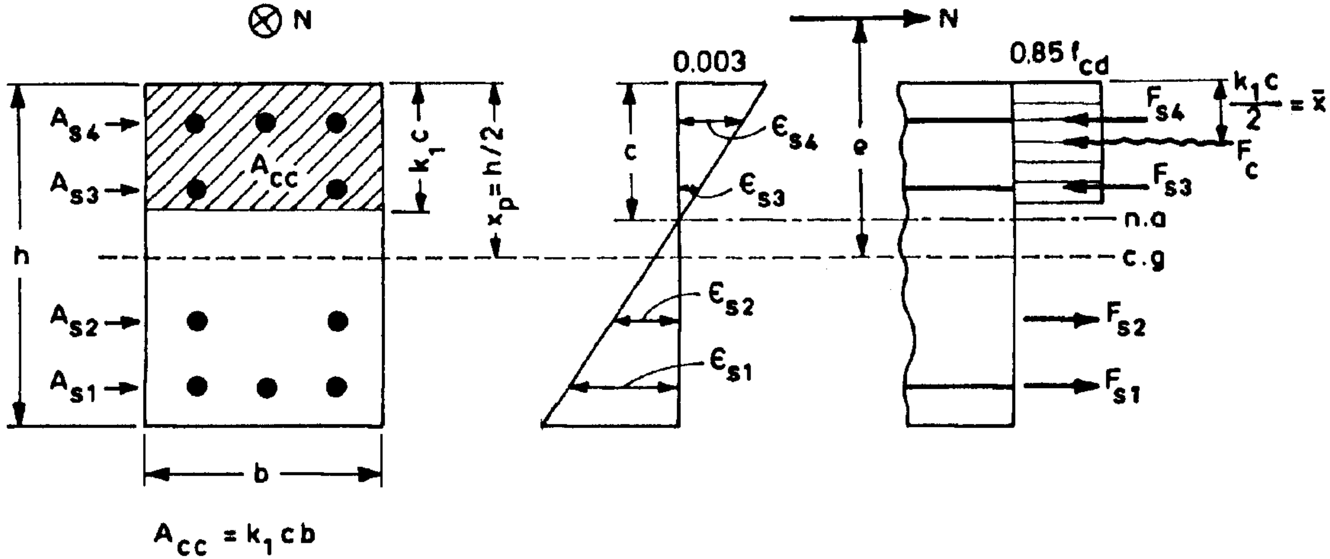


$$N = 0.85 f_{cd} A_{cc} + \sum_{i=1}^{n} A_{si} \sigma_{si}$$

$$M = N \cdot e = 0.85 f_{cd} A_{cc} (x_p - \bar{x}) + \sum_{i=1}^{n} A_{si} \sigma_{si} x_i$$

$$\sigma_{si} = \left\{ 0.003 E_s \left(1 + \frac{x_i - x_p}{c} \right) \right\} \leq f_{yd}$$

Above equations may be simplified further by considering the followings:

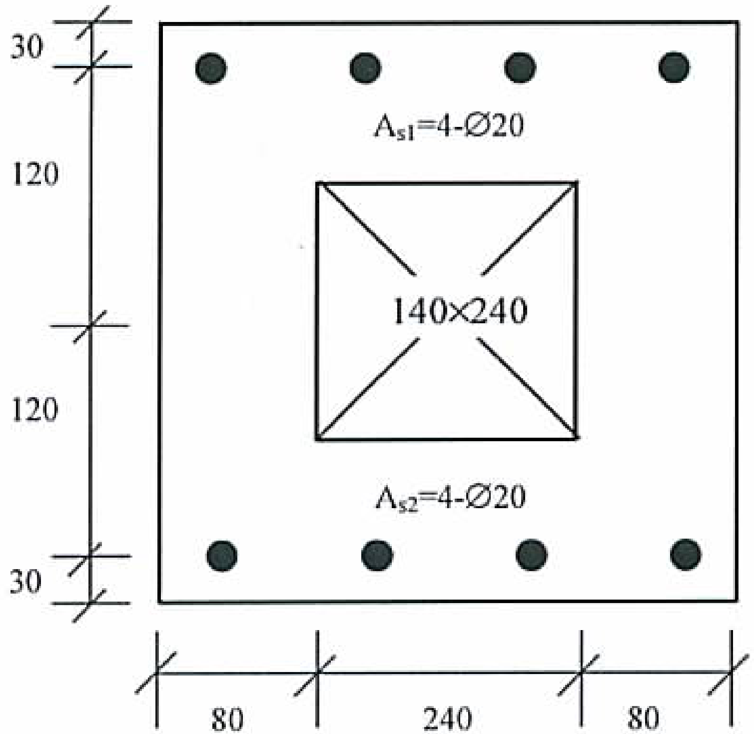


$$N = 0.85 f_{cd} k_1 cb + \sum_{i=1}^{n} A_{si} \sigma_{si}$$

$$M = N \cdot e = 0.85 f_{cd} k_1 cb \left(\frac{h}{2} - \frac{k_1 c}{2} \right) + \sum_{i=1}^{n} A_{si} \sigma_{si} x_i$$

$$\sigma_{si} = 0.003 E_s \left(1 + \frac{x_i - h/2}{c} \right) \leq f_{yd}$$

Example 1: M-N Diagram

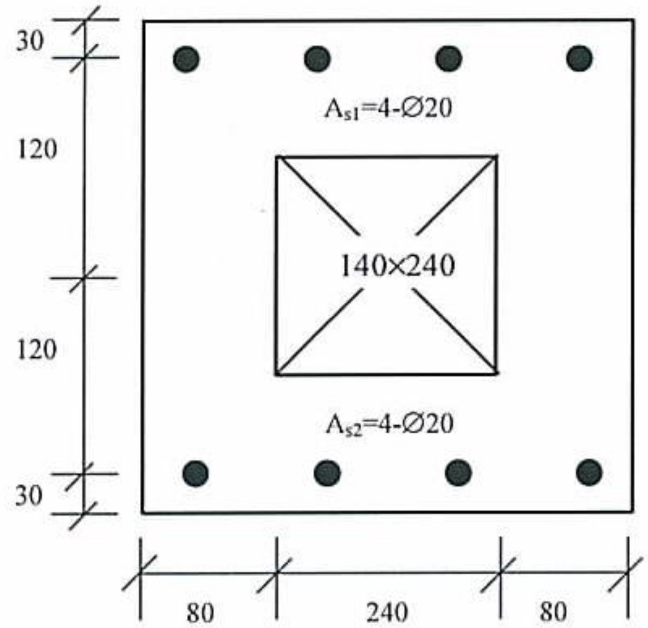


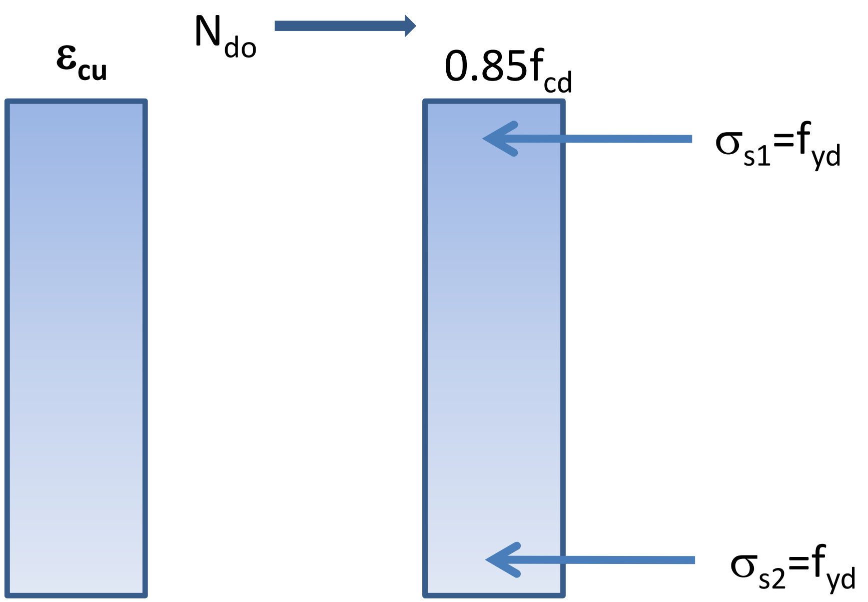
Wall thickness=80 mm everywhere

C25 (f_{cd}=17 MPa)

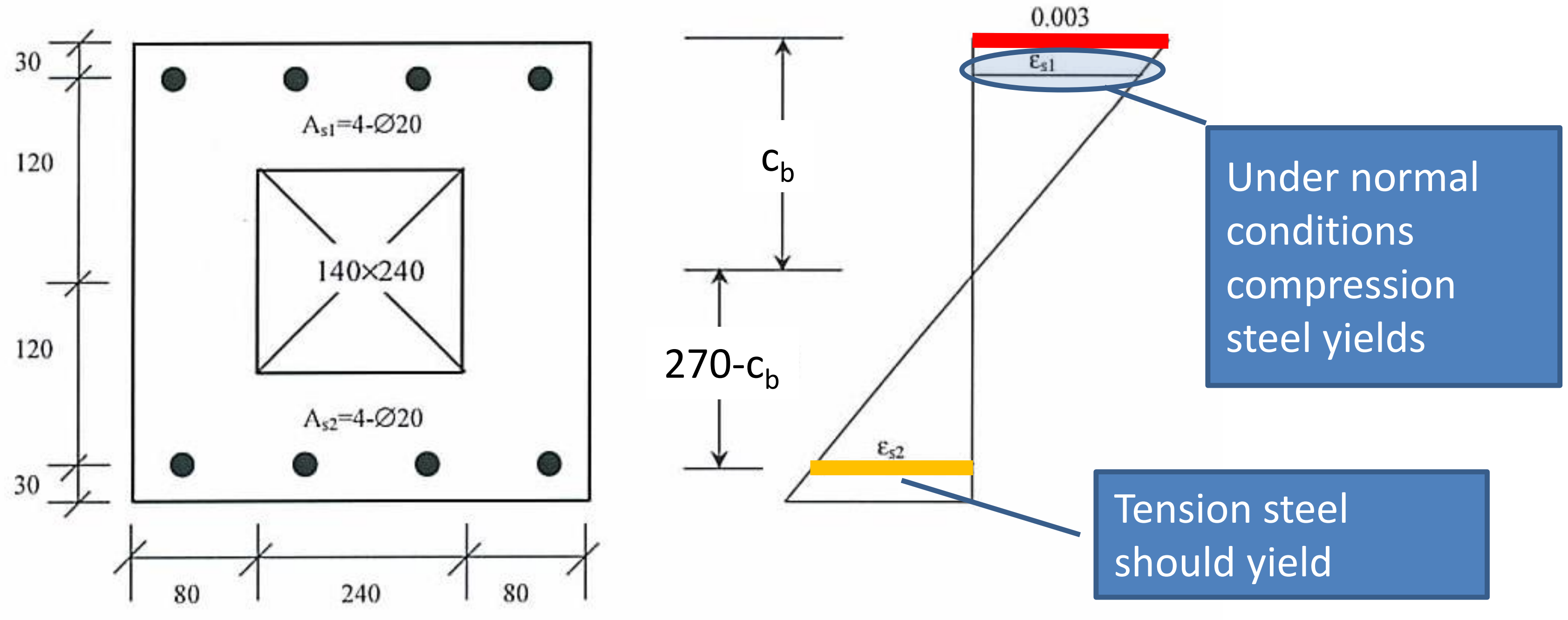
S420 (f_{yd}=365 MPa)

Compute the necessary M-N values to sketch the interaction diagram.

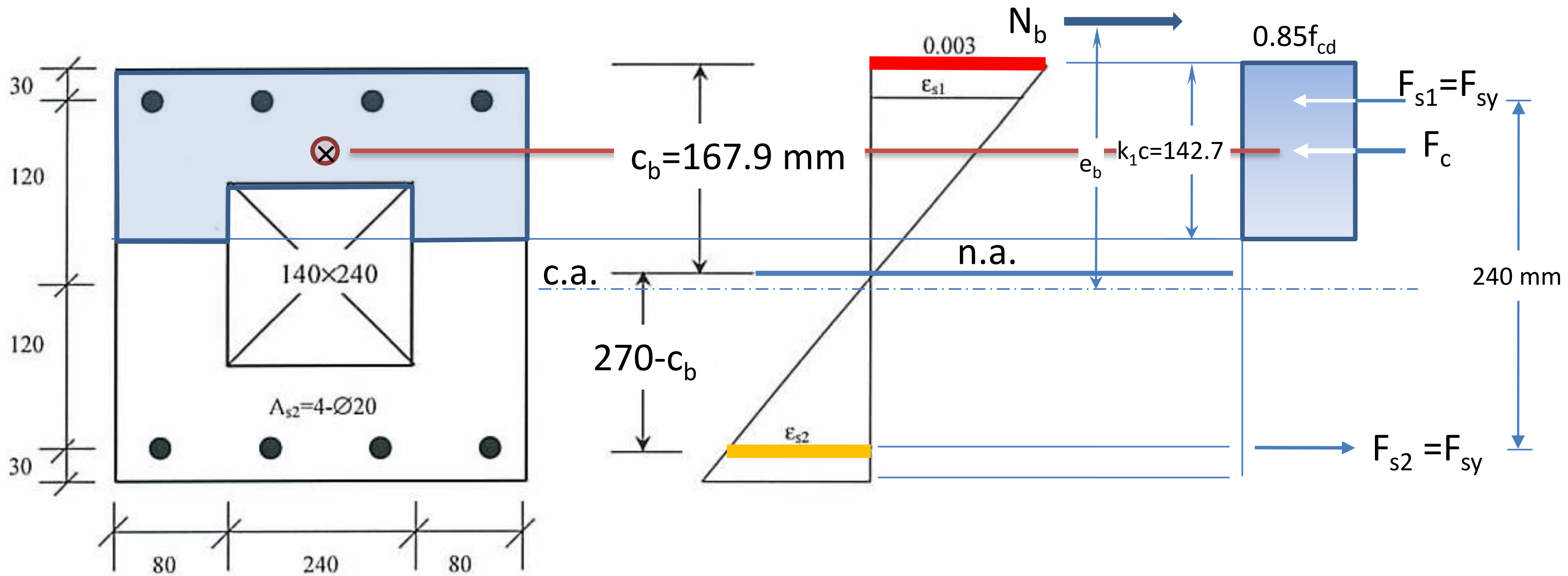




a) What is N_{do} for $M_d=0$?



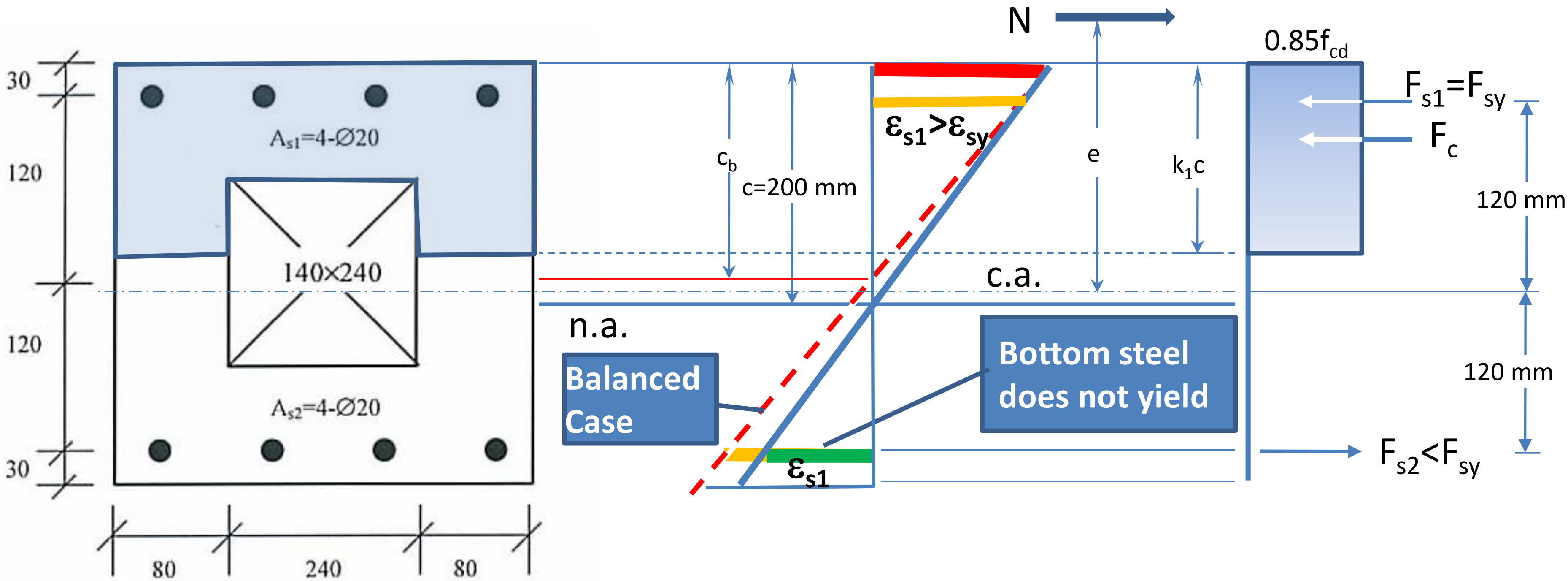
b) What are the balanced $(N_d, M_d)_b$?



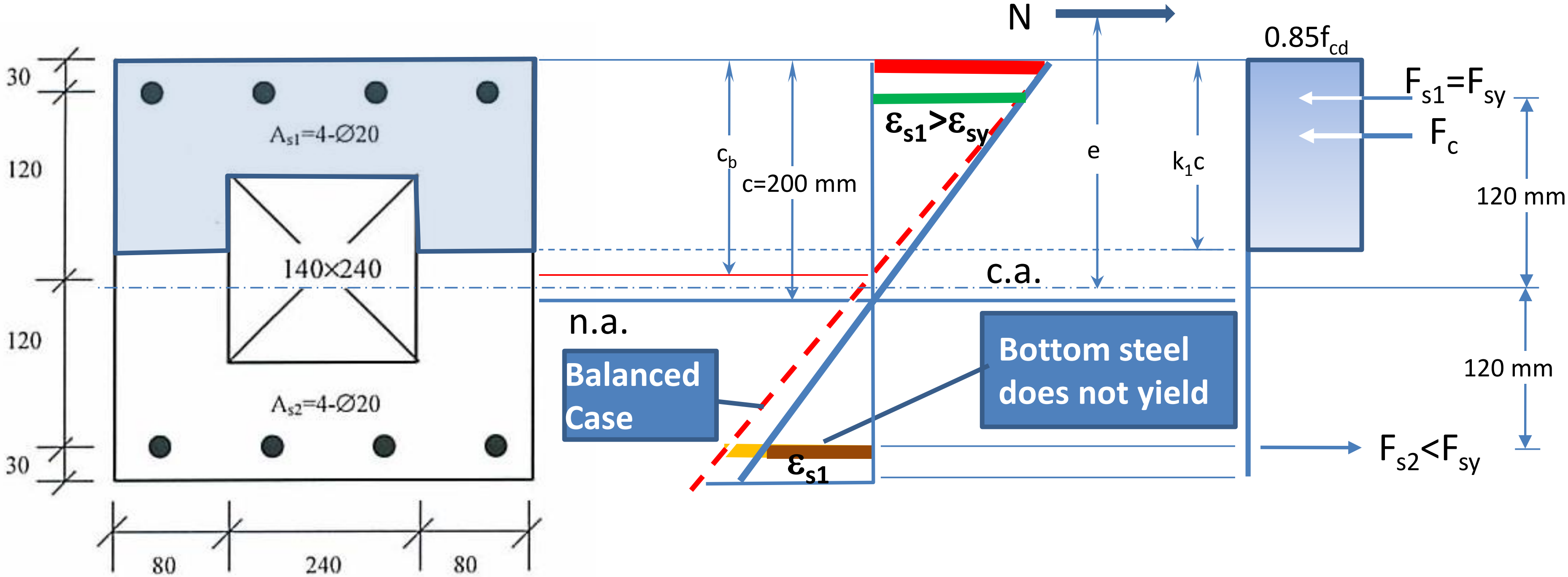
Equilibrium:

$$\sum F = N_b = [0.85\times 17\times (80\times 400 + (142.7-80)\times (2\times 80)) + 4\times 314\times 365 - 4\times 314\times 365]\times 10^{-3}$$

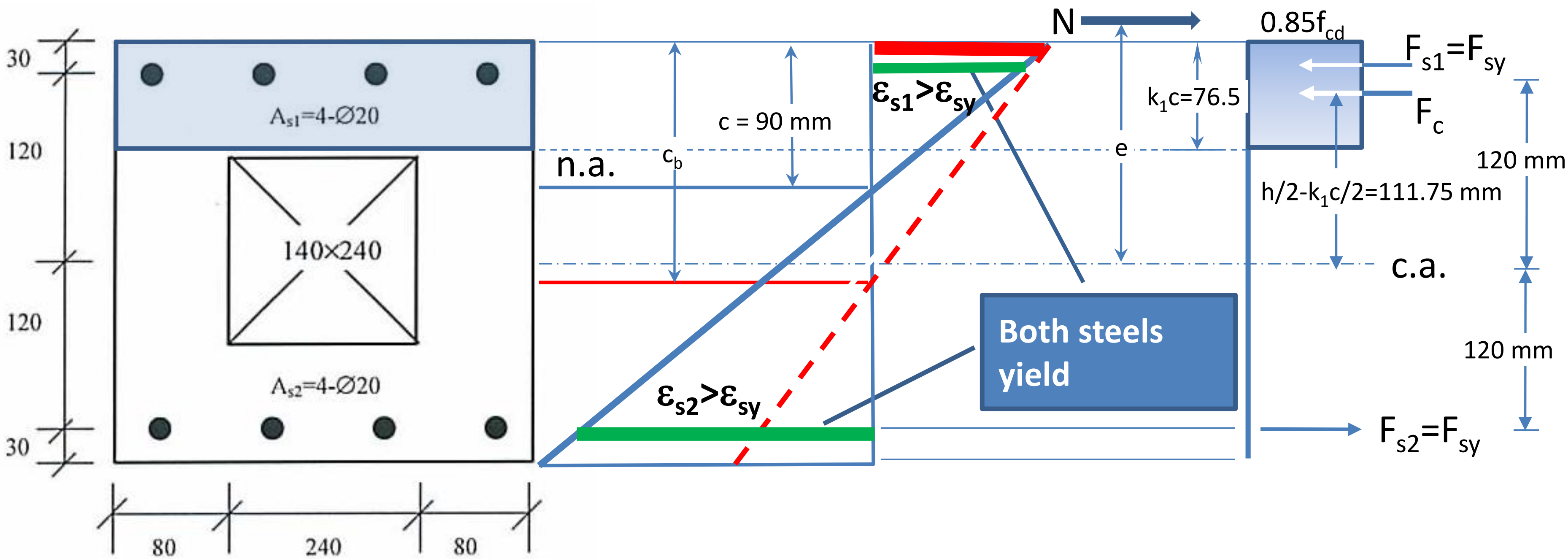
$$N_b = 607.4\ kN$$



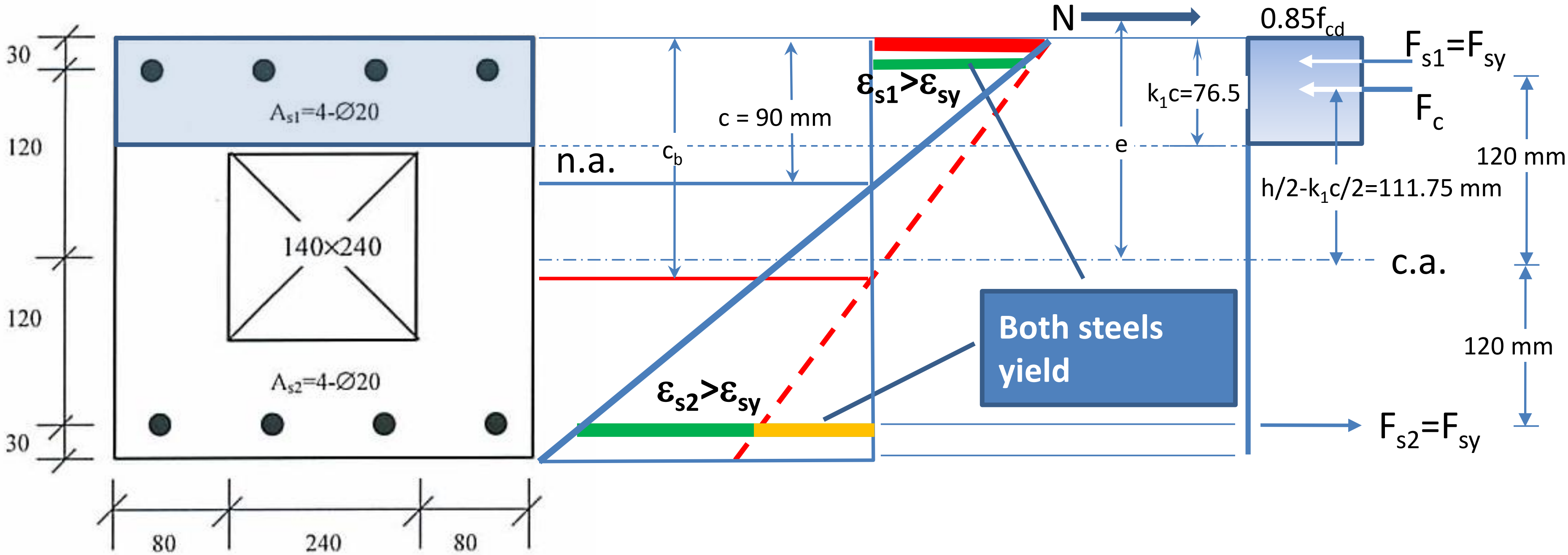
c) Obtain a point in compression failure region. (i.e. $e<e_b$ or $c>c_b$)



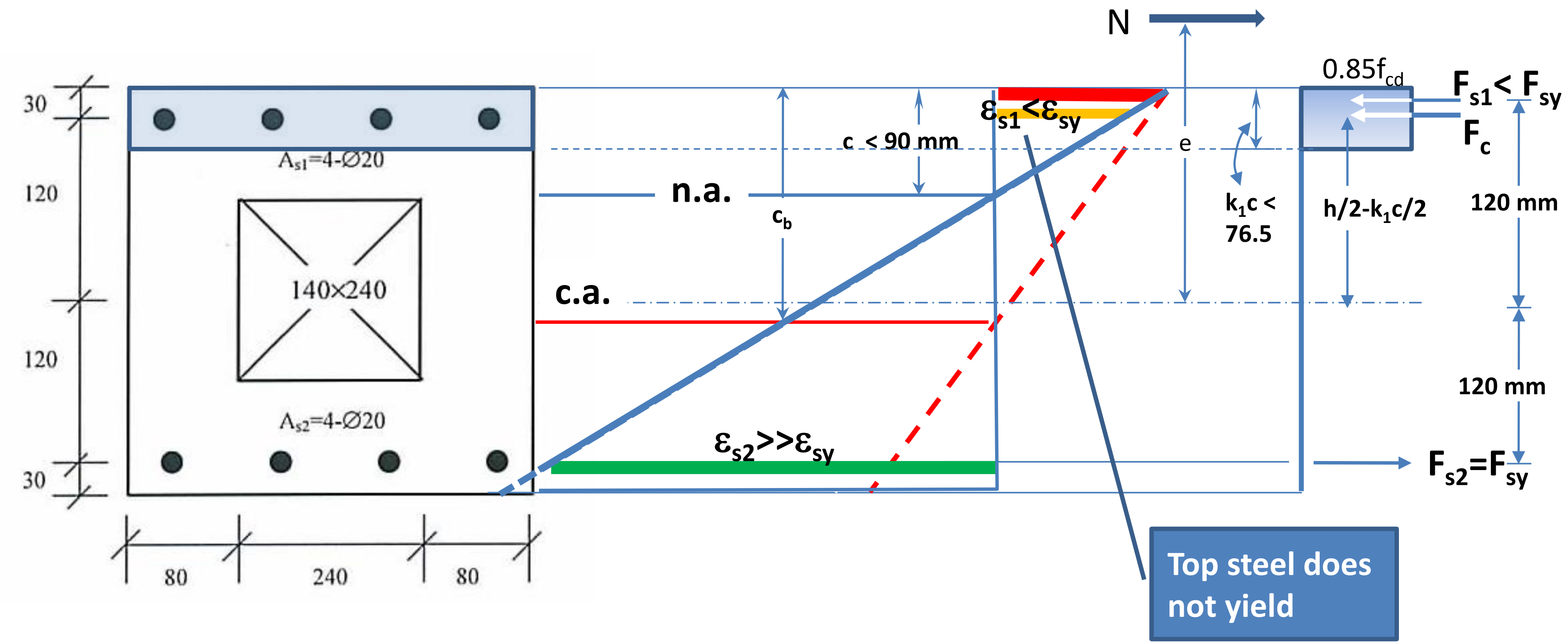
Equilibrium:



d) Obtain a point in tension failure zone. (i.e. $e>e_b$ & $c<c_b$)



Equilibrium:

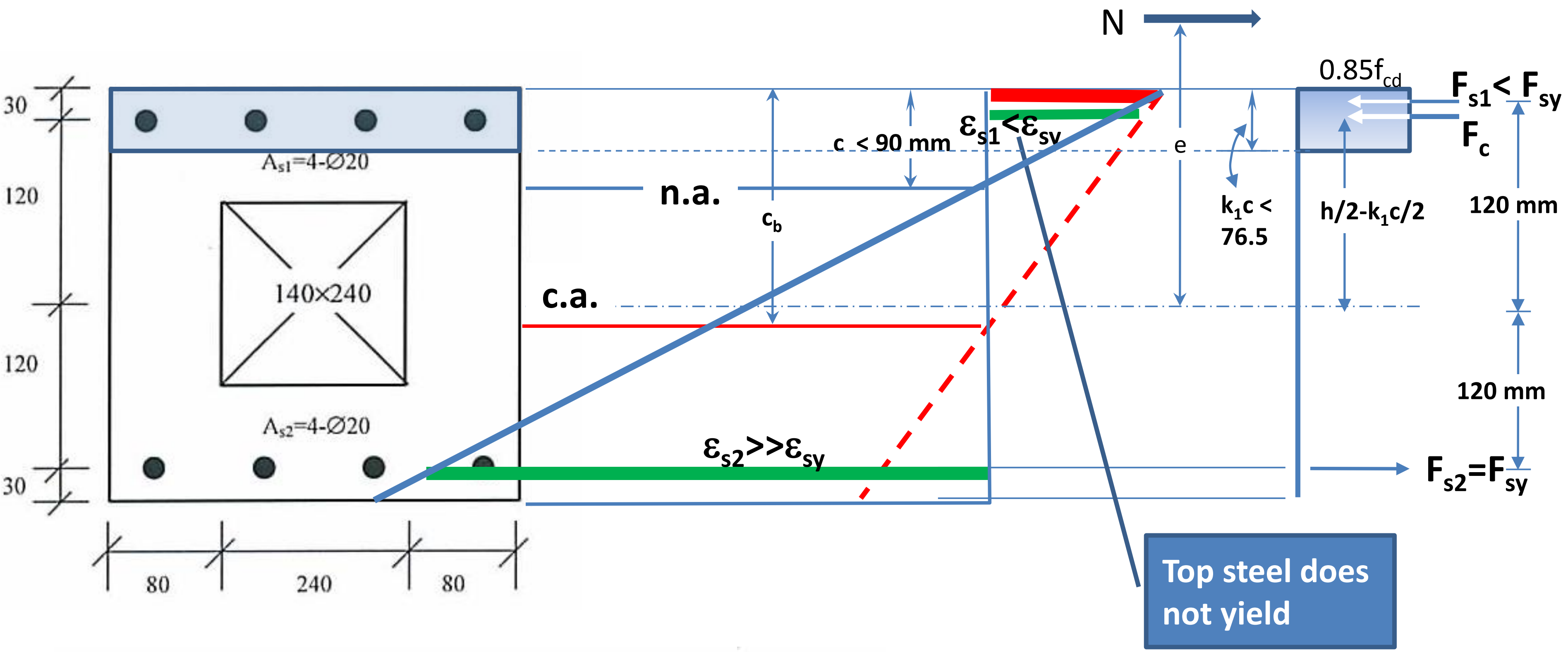


e) What is the pure bending capacity (M_d=? For N_d=0) ?

What we know so far?

- N_b=607.4 kN, c_b=167.4 mm (Compression failure)
- N_d=442.2 kN, c=90 mm, $\varepsilon_{s1} = 0.002 \approx \varepsilon_{sy}$, $\varepsilon_{s2} = -0.006 >> \varepsilon_{sy}$

To have N_d=0 we should reduce "c" further. This approach would result in non-yielding compression steel



Therefore, following assumption seems to be reasonable.

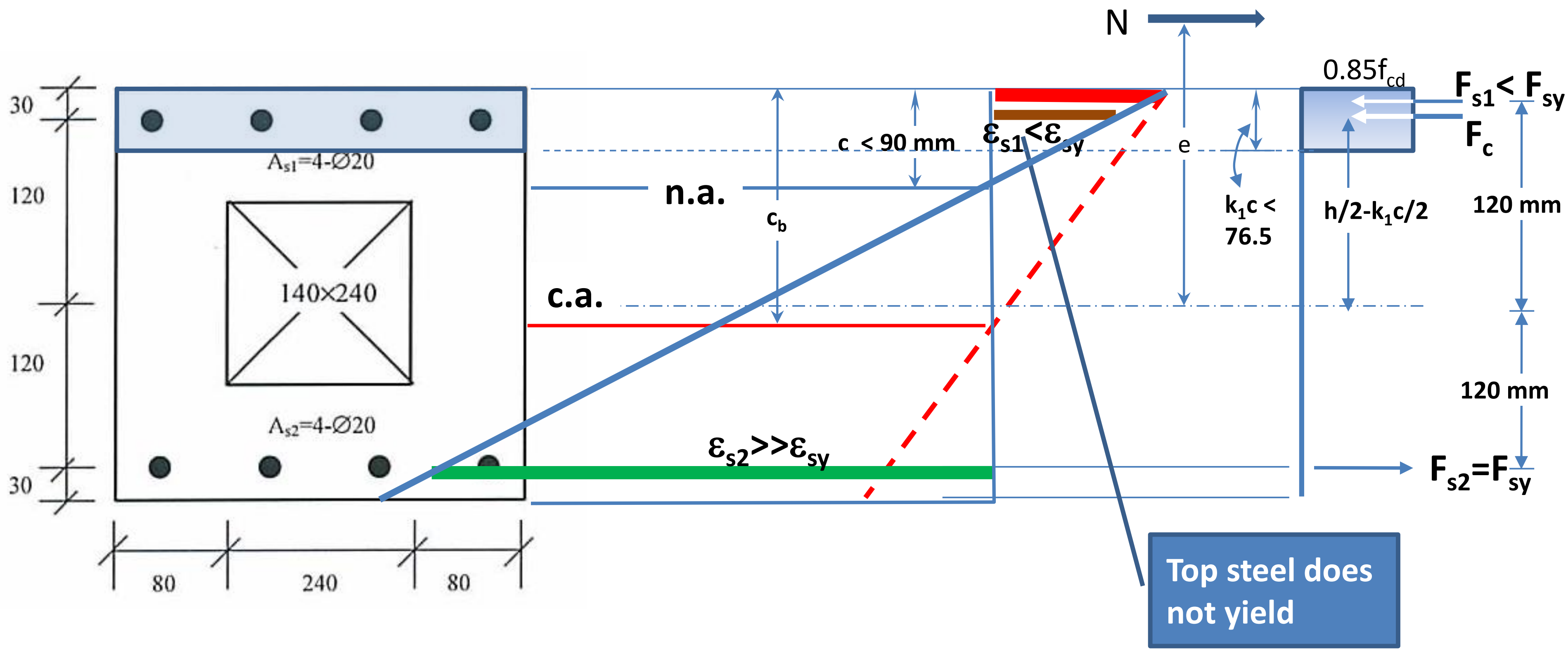
$$\varepsilon_{s1} < \varepsilon_{sy} \quad \therefore \varepsilon_{s1} = \frac{c-30}{c} \times 0.003$$

$$\varepsilon_{s2} >> \varepsilon_{sy}$$

which leads to:

$$\sigma_{s1} = \varepsilon_{s1} \times E_s = \frac{c-30}{c} \times 600$$ (note: E_s=200 GPa)

$$\sigma_{s2} = -365\ MPa$$



Equilibrium:

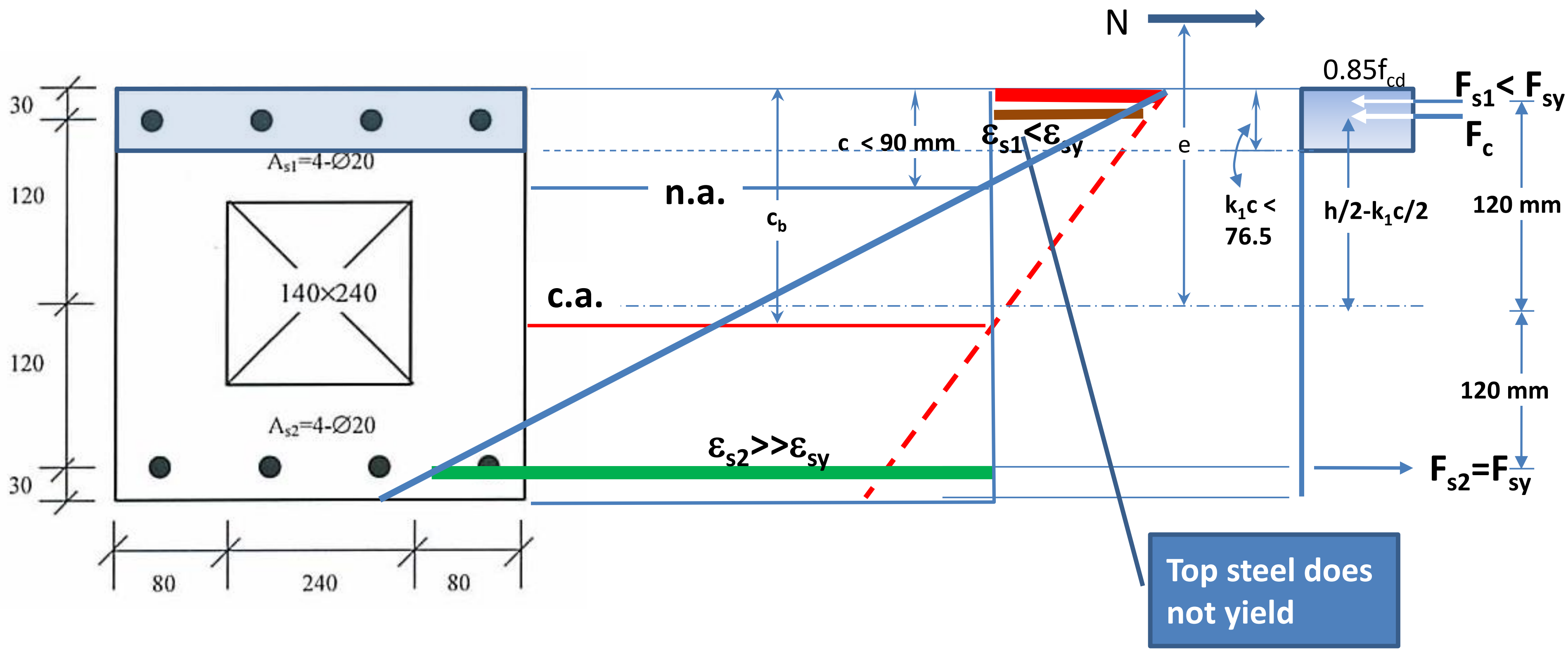
Noting that neutral axis will fall in the upper flange

$$\sum F = N = 0.85\times 17\times 0.85\times c\times 400 + \frac{c-30}{c}\times 600\times 4\times 314 - 365\times 4\times 314 = 0$$

Solving for "c" we get c=44.15 mm, k_1c=37.53 mm.

Note that for c=44.15 mm, $\varepsilon_{s1} = 0.00096$, $\sigma_{s1} = 192\ MPa$

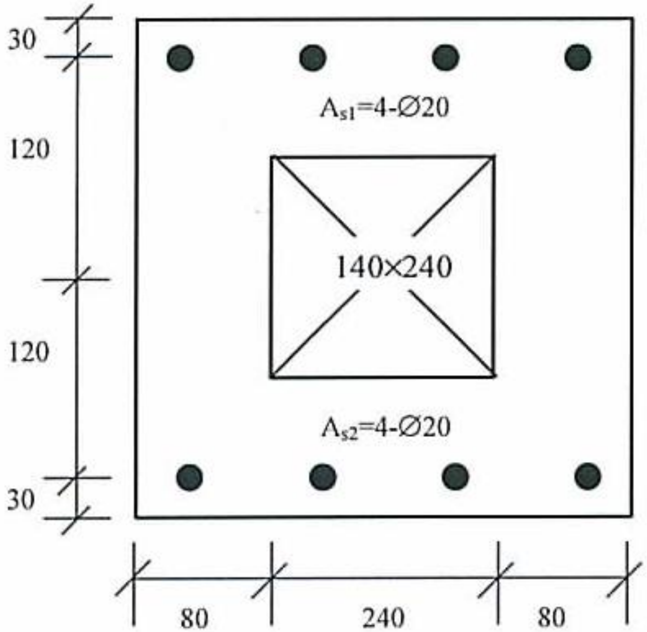
$$\varepsilon_{s2} = -0.01535,\ \sigma_{s2} = -365\ MPa$$



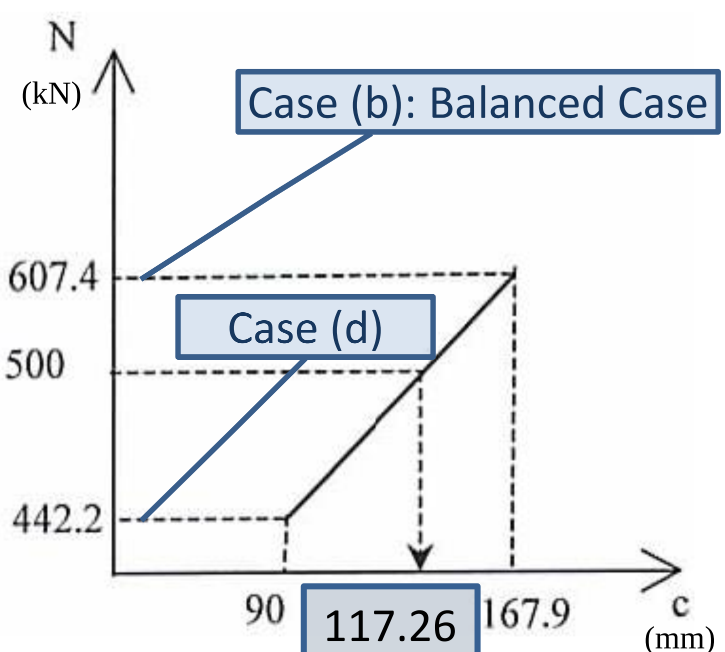
Taking moment about c.a.

$$M_d = 0.85 \times 17 \times 37.53 \times 400 \times \left(150 - \frac{37.53}{2}\right) + 4 \times 314 \times 192.5 \times 120 + 4 \times 314 \times 365 \times 120$$

$$M_d = 112.5\ kN.m,\ e = \infty,\ c = 44.15 < c_b.$$



f) Determine e_{max} for N_d=500 kN?



By linear interpolation

c=117.26 mm, ε_{s1}=0.0023, σ_{s1}=365 MPa

k_1c=99.67 mm, ε_{s2}=-0.00391, σ_{s2}=-365 MPa

$$\sum F = N_d$$

$$=[0.85 \times 17 \times (99.67 \times 400 - 19.67 \times 240) + 4 \times 314 \times (365 - 365)] \times 10^{-3}$$

N_d=508 kN → not in equilibrium!

Note both steels are yielding, therefore:

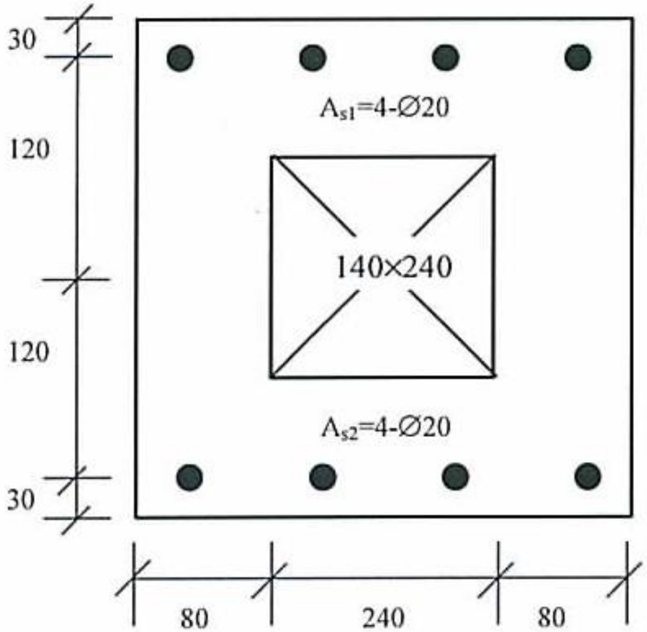
$N_d = N_c$ = 500 kN

$$\sum F = N_d = 0.85 \times 17 \times A_{cc} = 500{,}000\ N$$

$$\sum A_{cc} = 34{,}600\ mm^2$$

$$= [80 \times 400 + 240 \times (k_1 c - 80)]$$

k_1c=90.8 mm and c= 107 mm



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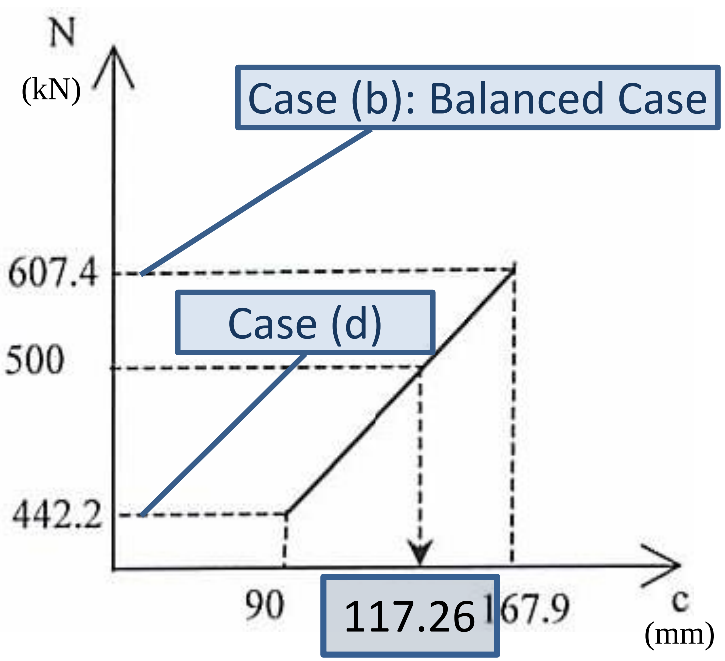


Taking moment about c.a.

$$\bar{x} = \frac{80 \times 400 \times 40 + 2 \times 10,8 \times 80 \times \left(80 + \frac{10.8}{2}\right)}{80 \times 400 + 2 \times 10.8 \times 80} =$$

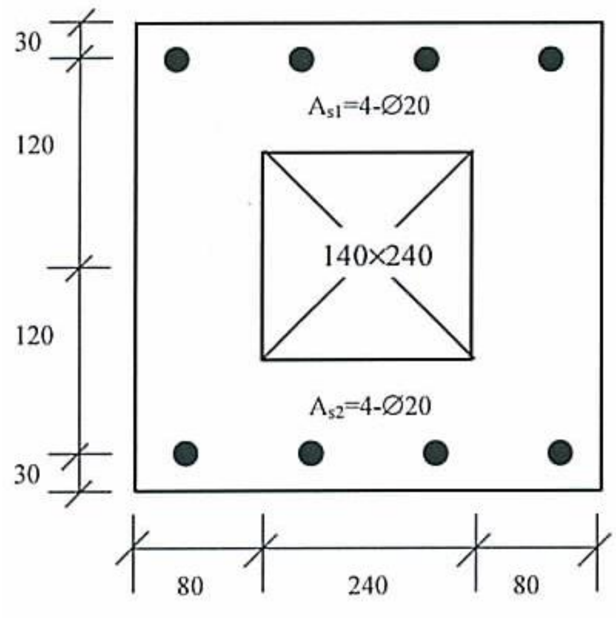
$$\bar{x} = 42.3 \text{ mm}$$

$$\sum M_{rd} = 0$$

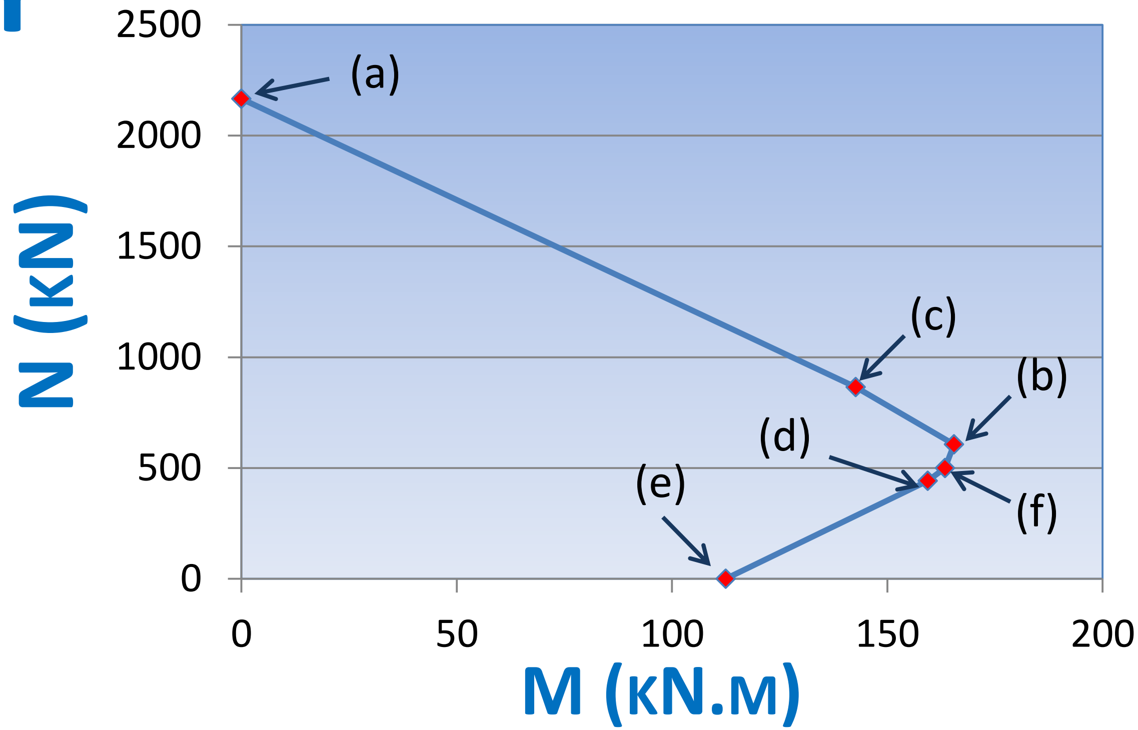
$$= 500,000 \times (150 - 42.3) + 4 \times 314 \times 365 \times 240$$

M_{rd} = 163.9 kN.m and e=163.9/500 = 0.328 m

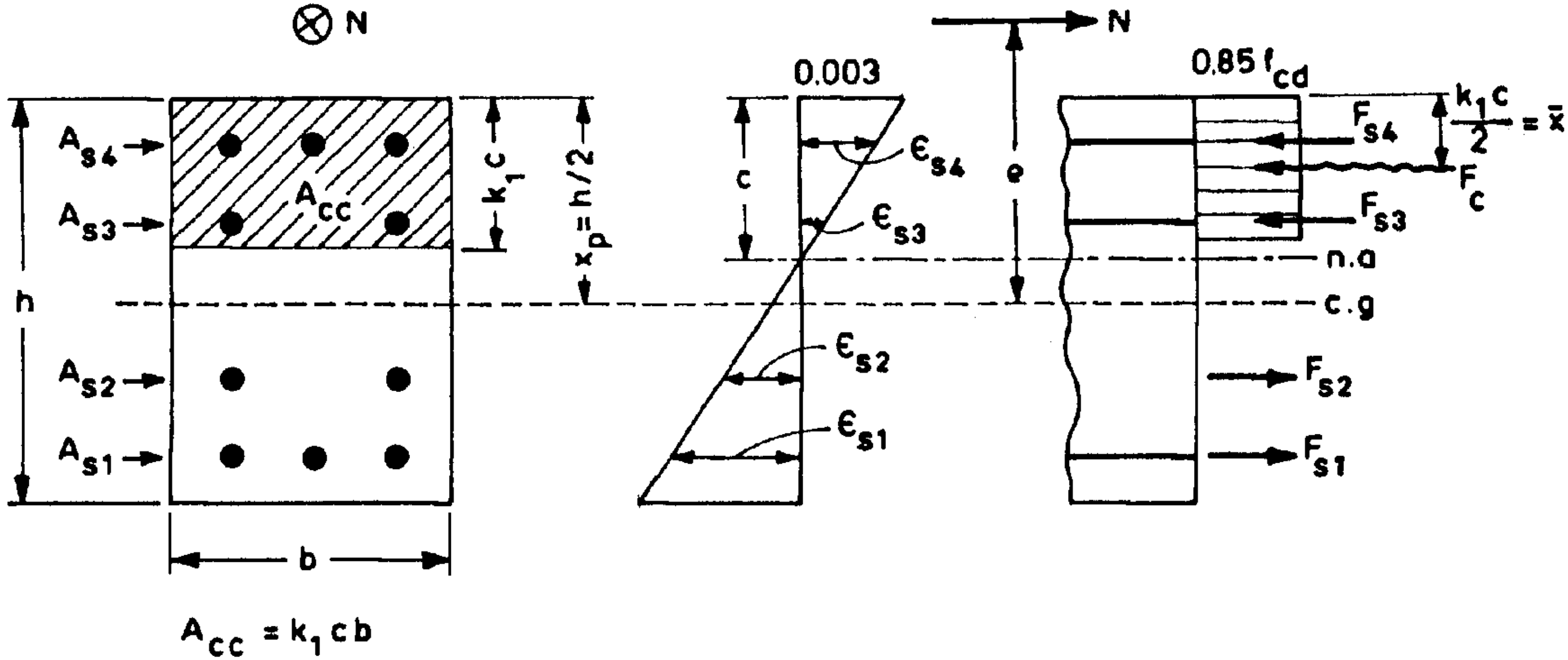
Summary



M-N Diagram



Case	c	e	K	N_d	M_d	Remarks
(a)	∞	0	0.00E+00	2166.0	0	N_o , compression failure
(c)	200.0	0.165	1.50E-05	865.2	142.7	Compression failure
(b)	167.9	0.274	1.79E-05	607.4	165.5	Balanced Case
(f)	107.0	0.328	2.80E-05	500.0	163.9	Tension failure
(d)	90.0	0.361	3.33E-05	442.2	159.4	Tension failure
(e)	44.2	∞	6.80E-05	0.0	112.5	Pure bending, tension failure



$A_{cc} = k_1 c b$

When steel is located at more than two levels, closed form solution is no longer feasible. For such cases a trial and error solution is recommended

a. Assume a value for c.
b. From strain triangles (with $\varepsilon_{cu} = 0.003$) compute the steel strains, stresses and forces.
c. Compute resultant concrete compressive force and check equilibrium.
d. If equilibrium is not satisfied change the value of c and go to (b); otherwise compute moment of forces about the centroid.