Solve the following system of equations using

1. Gauss-Jacobi method
2. Gauss-Seidel method

with ***4 decimal place accuracy*** in both cases.

$$\begin{aligned} 6x_1 + 2x_2 - x_3 &= 5 \\ -x_1 + 4x_2 + x_3 &= 3 \\ x_1 - 2x_2 + 5x_3 &= 10 \end{aligned}$$

$$A = \begin{bmatrix} 6 & 2 & -1 \\ -1 & 4 & 1 \\ 1 & -2 & 5 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 5 \\ 3 \\ 10 \end{bmatrix}$$

1. Gauss-Jacobi Method:

$$x_1^{k+1} = \frac{5 - 2x_2^k + x_3^k}{6}, \qquad x_2^{k+1} = \frac{3 + x_1^k - x_3^k}{4}, \qquad x_3^{k+1} = \frac{10 - x_1^k + 2x_2^k}{5}$$

Initial guess: $X_0^T = (0 \quad 0 \quad 0)$

Error: $\max\left(\left|x_j^{k+1} - x_j^k\right|\right), j = 1,2,3$ (i.e., $\max\left(\left|x_1^{k+1} - x_1^k\right|, \left|x_2^{k+1} - x_2^k\right|, \left|x_3^{k+1} - x_3^k\right|\right)$)

Error tolerance: $\varepsilon = \frac{1}{2}10^{-4} = 0.00005$

Iteration #	x_1	x_2	x_3	ε_{max}
0	0.000000	0.000000	0.000000	
1	0.833333	0.750000	2.000000	2.000000
2	0.916667	0.458333	2.133333	0.291667
3	1.036111	0.445833	2.000000	0.133333
4	1.018056	0.509028	1.971111	0.063194
5	0.992176	0.511736	2.000000	0.028889
6	0.996088	0.498044	2.006259	0.013692
7	1.001695	0.497457	2.000000	0.006259
8	1.000848	0.500424	1.998644	0.002967
9	0.999633	0.500551	2.000000	0.001356
10	0.999816	0.499908	2.000294	0.000643
11	1.000080	0.499881	2.000000	0.000294
12	1.000040	0.500020	1.999936	0.000139
13	0.999983	0.500026	2.000000	0.000064
14	0.999991	0.499996	2.000014	0.000030

2. Gauss-Seidel Method:

$$x_1^{k+1} = \frac{5 - 2x_2^k + x_3^k}{6}, \qquad x_2^{k+1} = \frac{3 + x_1^{k+1} - x_3^k}{4}, \qquad x_3^{k+1} = \frac{10 - x_1^{k+1} + 2x_2^{k+1}}{5}$$

Initial guess: $X_0^T = (0 \quad 0 \quad 0)$

Error: $\max\left(\left|x_j^{k+1} - x_j^k\right|\right), j = 1,2,3$ (i.e., $\max\left(\left|x_1^{k+1} - x_1^k\right|, \left|x_2^{k+1} - x_2^k\right|, \left|x_3^{k+1} - x_3^k\right|\right)$)

Error tolerance: $\varepsilon = \frac{1}{2}10^{-4} = 0.00005$

Iteration #	x_1	x_2	x_3	ε_{max}
0	0.000000	0.000000	0.000000	
1	0.833333	0.958333	2.216667	2.216667
2	0.883333	0.416667	1.990000	0.541667
3	1.026111	0.509028	1.998389	0.142778
4	0.996722	0.499583	2.000489	0.029389
5	1.000220	0.499933	1.999929	0.003498
6	1.000011	0.500020	2.000006	0.000210
7	0.999994	0.499997	2.000000	0.000023