NOMINAL AND EFFECTIVE INTEREST RATE STATEMENTS

- The primary difference between simple and compound interest is that compound interest includes interest on the interest earned in the previous record, while simple does not.
- The nominal and effective interest rates have also the same basic relationship.
- The difference here is that the concepts of nominal and effective must be used when interest is compounded more than once each year.
- **Nominal interest rate,r,** is an interest rate that does not include any consideration of compounding.

$$\boxed{r = \text{interest rate per period x number of periods}}$$

A nominal rate r may be stated for any time period, 1 year, 6 months, quarter, month, week, day, etc.

EX: The nominal rate of r= 1.5% per month is the same as each of the following rates;

1^{st} case	= 1.5% per month x 24 months	= 36% per 2-year period
2^{nd} case	= 1.5% per month x 12 months	= 18% per year
3^{rd} case	= 1.5% per month x 6 months	= 9 % per semiannual period
4^{th} case	= 1.5% per month x 3 months	= 4.5% per quarter

!!! None of these nominal rates make mention of the compounding frequency. They all have the format " r% per time period t."

- **Effective interest rate** is the actual rate that applies for a stated period of time. The compounding of interest during the time period of the corresponding nominal rate is accounted for by the effective interest rate.
- It is commonly expressed on an annual basis as the effective annual rate i_a, but any time basis can be used.
- An effective rate has the compounding frequency attached to the nominal rate statement.

EX: 12% per year, compounded monthly
12% per year, compounded quarterly
3% per quarter, compounded monthly
6% per 6 months, compounded weekly
3% per quarter, compounded quarterly (compounding same as time period)

!!! All examples are nominal rate statements; however, they will not have the same effective interest rate value over all time periods, due to the different compounding frequencies.
!!! In the last example, the nominal rate of 3% per quarter is the same as the effective rate of 3% per quarter, compounded quarterly.

Time units associated with an interest rate statement

- Time period: The basic time unit of the interest rate
- Compounding period (CP): The time unit used to determine the effect of interest.
- The compounding frequency, m: The number of times that compounding occurs within the time period t.

EX: 8% per year, compounded monthly, has a compounding frequency of m=12 times per year.

A rate of 8% per year, compounded weekly, has a frequency of m=52.

$$\textbf{Effective rate per CP} = \frac{\textbf{r\% per time period t}}{\textbf{m compounding periods per t}} = \frac{\textbf{r}}{\textbf{m}}$$

EX: Determine the effective rate on the basis of the compounding period for each interest rate.

a) 9% per year, compounded yearly
b) 6% per year, compounded quarterly
c) 8% per year, compounded monthly
d) 5% per 6- months, compounded weekly

Nominal r% per t	Compounding Period	m	Effective Rate per CP
9% per year	Year	1	9/1 = 9%
6% per year	Quarter	4	6/4 = 1.5%
8% per year	Month	12	8/12 = 0.667%
5% per 6- months	Week	26	5/26 = 0,192%

Various Ways to Express Nominal and Effective Interest Rates

!!! Sometimes it is not obvious whether a stated rate is nominal or an effective rate. Basically there are three ways to express interest rates.

Format of Rate Statement	Examples of Statement	What about the Effective Rate?
Nominal rate stated, compounding period stated	8% per year, compounded quarterly	Find effective rate
Effective rate stated	Effective 2.5% per year, compounded quarterly	Use effective rate directly
Interest rate stated, no compounding period stated	5% per year or 3% per quarter or 4.5 & per month	Rate is effective only for time period stated, find effective rate for all other time periods

EFFECTIVE ANNUAL INTEREST RATES

- The effective interest rate is the one rate that truly represents the interest earned in a year.
- Like compound interest, the effective interest rate at any point during the year includes (compounds) the interest rate at any point during the year.
- The following formula should be used for the future worth calculation;

$$\boxed{F = P(1+i)^m}$$

Where; F is the future worth
P is the present worth
i is the effective interest rate per compounding period (CP) = r/m
m is the number of compounding periods per year

EX: Suppose you deposit \$10,000 in a savings account that pays you at an interest rate of 9% compounded quarterly. Here, 9% represents the nominal interest rate, and the interest rate per quarter is 2,25% (9%/4). Below table shows the example of how the interest is compounded when it is paid quarterly.

End of period	**Present Amount**	**Interest Earned**	**New (future) Amount**
First Quarter	\$10,000.00	2.25% x \$10,000.00 =\$225.00	\$10,225.00
Second Quarter	\$10,225.00	2.25% x \$10,255.00 =\$230.06	\$10,455.06
Third Quarter	\$10,455.06	2.25% x \$10,455.06 =\$235.24	\$10,690.30
Fourth Quarter	\$10,690.30	2.25% x \$10,690.30 =\$240.53	\$10,930.83

- The total amount annual interest payment for a principal amount of \$10,000 can also be calculated with the formula.
  $F = P(1+i)^m$
  $P = \$10,000$ $i = 2.25\%$ $m= 4$
  $F= \$10,000\ (1+0.0225)^4$
  $F= \$10,930.83$

- You are earning more than 9% on your original deposit. In fact, you are earning 9,3083% (\$930,83/\$10,000).
- Effective annual interest rate per year, ia, = 930,83/ 10,000 = 0,093083 = 9,3083%

!!! Earning 2.25% interest per quarter for four quarters is equivalent to earning 9,3083% interest just one time each year.

$$\boxed{\textbf{Effective annual interest rate},\ i_a = (1+i)^m - 1}$$

- Again for the same example, but this time by the formula;
  $i_a = (1+i)^m - 1 = (1+0,0225)^4 - 1 = 0,093083 = 9,3083\ \%$

EX: For a $1000 balance at the beginning of the year, if the stated rate is 18% per year, compounded monthly, find the effective annual rate and the total amount owned after 1 year.

There are 12 compounded periods per year, thus m= 12.
i = 18% 12 = 1,5% per month
$i_a = (1+i)^m - 1 = (1+0,015)12 - 1 = 1,19562 - 1 = 0,19562$
$F = P(1+i)^m = \$1000(1,19562) = \$1195,62$

Hence, 19,562% or $195.62 owned!!

Following table shows the rate of 18% per year, compounded over different times (yearly to weekly) to determine the effective annual interest rates over these various compounding periods.

r= 18% per year, compounded monthly			
Compounding period	**Times Compounded per year,m**	**Rate per Compound Period,i (r/m)**	**Effective Annual Rate, i_a**
Year	1	18%	$(1+0,18)^1-1=$ 18%
6 month	2	9%	$(1+0,09)^2-1=$ 18,81%
Quarter	4	4,5%	$(1+0,045)^4-1=$ 19,252%
Month	12	1,5%	$(1+0,015)^{12}-1=$ 19,562
Week	52	0,34615%	$(1+0,0034615)^{52}-1=$ 19,684%

EFFECTIVE INTEREST RATES FOR ANY TIME PAYMENT PERIOD

- The payment period is the frequency of the payment or receipts in other words, is cash flow transaction period.
- It is important to distinguish the compounding period and the payment period!

EX: If company deposits money each month into an account that pays a nominal interest rate of 14% per year, compounded semiannually, the payment period is 1 month while the compounding period is 6 months.

- The effective annual interest rate formula is easily generalized to any nominal rate by substituting r/m for the period interest rate

$$\boxed{\textbf{Effective } i = (1+r/M)^c - 1} \qquad \boxed{\textbf{Effective } i = (1+r/CK)^c - 1}$$

Where; M is the number of interest per year
C is the number of interest periods per payment period
K is the number of payment periods per year

!!! Note that M= CK. For the special case of annual payments with annual compounding, we obtain i=ia with C=M and K=1.

EX: Suppose that you make quarterly deposits in a savings account that earns 9% interest compounded monthly. Compute the effective interest rate per quarter.

R=9%, C= three interest periods per quarter, K= four quarterly payments per year, M= 12 interest periods per year

Using Effective $i = (1+r/CK)^{c} -1$

$i = (1+ 0,09/12)^{3}-1 = 2,27\ \%$

EX: A dot-com company plans to place money in a new venture capital that fund that currently returns 18% per year, compounded daily. What effective rate is this a) yearly and b) semiannually?

a) r= 18%, C= 365 interest periods per year, K= 1 payment per year, M= 365 interest periods per year
   Using Effective $i = (1+r/CK)^{c} -1$
   $i = (1+ 0,18/365)^{365}-1 = 19,716\ \%$
b) r= 18%, C= 182 interest periods per semiannual, K= 2 payment per year, M= 365 interest periods per year
   Using Effective $i = (1+r/CK)^{c} -1$
   $i = (1+ 0,18/365)^{182}-1 = 9,415\ \%$

EX: Find the effective interest rate per quarter at a nominal rate of 8% compounded a) quarterly, b) monthly, c) weekly, d) daily if r=8% and there are 4 payment per year (K=4).

a) Quarterly compounding:
   r= 8%, C= 1 interest period per quarter, K= 4 quarterly payments per year, M= 4 interest period per year
   $i = (1+ 0,08/4)^{1} -1 = 2,00\ \%$

b) Monthly compounding:
   r= 8%, C= 3 interest period per quarter, K= 4 quarterly payments per year, M= 12 interest period per year
   $i = (1+ 0,08/12)^{3} -1 = 2,013\ \%$

c) Weekly compounding:
   r= 8%, C= 13 interest period per quarter, K= 4 quarterly payments per year, M= 52 interest period per year
   $i = (1+ 0,08/52)^{13} -1 = 2,0186\ \%$

d) Daily compounding:
   r= 8%, C= 91,25 interest period per quarter, K= 4 quarterly payments per year, M= 365 interest period per year
   $i = (1+ 0,08/365)^{91,25} -1 = 2,0199\ \%$

EQUIVALENCE RELATIONS: COMPARING PAYMENT PERIOD AND COMPOUNDING PERIOD LENGHTS

- All the examples up to here assumed annual payments and annual compounding. However, a number of situations involve cash flows that occur at intervals that are not the same as the compounding intervals often used in practice.
- Whenever payments and compounding periods differ from each other, one or the other must be transformed so that both conform to the same unit of time.

Equivalence Relations: Single Amounts with PP ≥ CP

- When only single-amount cash flows are involved, there are two equally correct ways to determine I and n for P/F and F/P factors.
- Method 1 is easier to apply, because the interest tables can usually provide the factor value.
- Method 2 likely requires a factor formula calculation, because the resulting effective interest rate is not an integer.

Method 1

Determine the effective interest rate over the compounding period CP, and set n equal to the number of compounding periods between P and F. The relations to calculate P and F are

P = F (P/F, effective i% per CP, total number of periods n)

F = P (F/P, effective i% per CP, total number of periods n)

!!! The CP is the best because only over the CP can the effective rate have the same numerical value as the nominal rate over the same time period as the CP.

Method 2

Determine the effective interest rate for the time period t of the nominal rate, and set n equal to the total number of periods using this same time period

The P and F relations are the same as in above equations with the term effective i% per t substituted for the interest rate.

Equivalence Relations: Series with PP ≥ CP

- When uniform or gradient series are included in the cash flow sequence, the procedure is basically the same as method 2 above, except that PP is now defined by the frequency of the cash flows.
- This also establishes the time unit of the effective interest rate.
- For example, if cash flows occur on a quarterly basis, PP is a quarter and the effective quarterly rate is necessary.
- Then n value is the total number of quarters. If PP is a quarter, 5 years translates to an n value of 20 quarters.

This is a direct application of the following general guideline:

- When cash flows involve a series (i.e. A,G, g) and the payment period equals or exceeds the compounding period in length,
- Find the effective I per payment period.
- Determine n as the total number of payment periods.

!!! Following table shows the correct formulation for several cash flow series and interest rates. Note that n is always equal to the total number of payment periods and I is an effective rate expressed over the same time period as n.

Examples of n and I values where PP=CP or PP>CP			
Cash Flow Series	**Interest Rate**	**What to Find?**	**Standard Notation**
\$500 semiannually for 5 years	16% per year, compounded semiannually	Find P, given A	P= 500(P/A,8%,10)
\$75 monthly for 3 years	24% per year, compounded monthly	Find F,given A	F= 75(F/A,2%,36)
\$180 quarterly for 15 years	5% per quarter	Find F, given A	F=180(F/A,5%,60)
\$25 per month increase for 4 years	1% per month	Find P, given G	P= 25(P/G,1%, 48)
\$5000 per quarter for 6 years	1% per month	Find A, given P	A= 5000(A/P, 3,03%, 24)

Equivalence Relations: Single Amounts and Series with PP < CP

The computational procedure for establishing economic equivalence is as follows.

Step 1: Identify the number of compounding periods per year (M), the number payment periods per year (K); and the number of interest periods per payment period (C):
Step 2: Compute the effective interest rate per payment period.

- For discrete compounding, compute
  $i = (1+r/M)^c - 1$
- For continuous compounding, compute
  $i = e^{r/K} - 1$

Step 3: Find the total number of payment periods:
N= K x (number of years)
Step 4: Use i and N in the appropriate formulas from the table.

EX: Suppose you make equal quarterly deposits of \$1500 into a fund that pays interest at a rate of 6% compounded monthly. Find the balance at the end of year 2.

Given: A= \$1500 per quarter, r= 6% per year, M= 12 compounding periods per year and N= 8 quarters
Find: F!
Step 1: Identify the parameter values for M,K and C, where
M= 12 compounding periods per year
K= 4 payment periods per year
C= 3 interest periods per payment period (quarter)

Step 2: Compute the effective interest:

$i = (1+r/M)^{c} -1$

$= (1+ 0,06/12)^{3} -1$

$= 1,5075\%$ per quarter

Step 3: Find the total number of payment periods, N:

N= K(number of years) = 4(2) = 8 quarters

Step 4: Use I and N in the appropriate equivalence formulas:

F=$1500(F/A, 1,5075%,8) = $12,652.61

!!!No 1,5075% value appears in the interest tables, but F can still evaluated using F=$1500(A/F,0,5%,3)(F/A,0,5%,24), where the first interest factor finds its equivalent monthly payment and the second interest factor converts the monthly payment series to an equivalent lump-sum future payment.