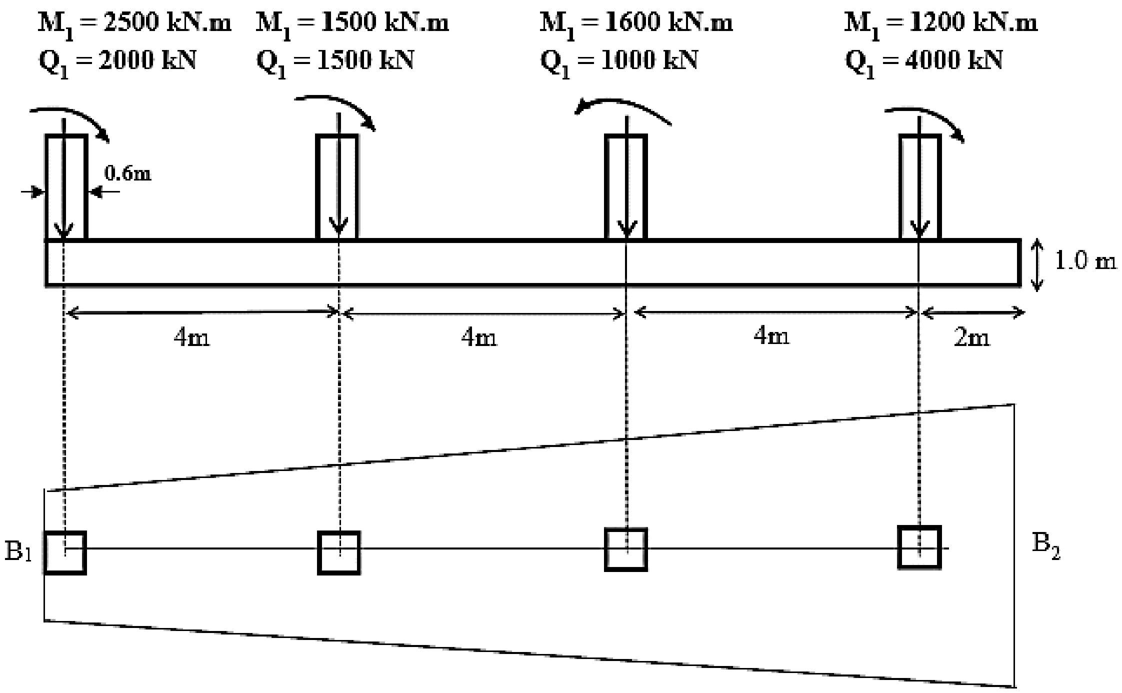
SOLUTION OF HOMEWORK 2

Question 1 (25%)

Determine the dimensions B1 and B2 of a rigid trapezoidal combined footing to achieve a uniform soil pressure of 200 kN/m^2.

($\gamma_{conc.}$ = 24 kN/m^3)

Note: Assume that the footing rests on top of the ground surface.



Solution 1

$$\sum Q = 8500\ kN$$

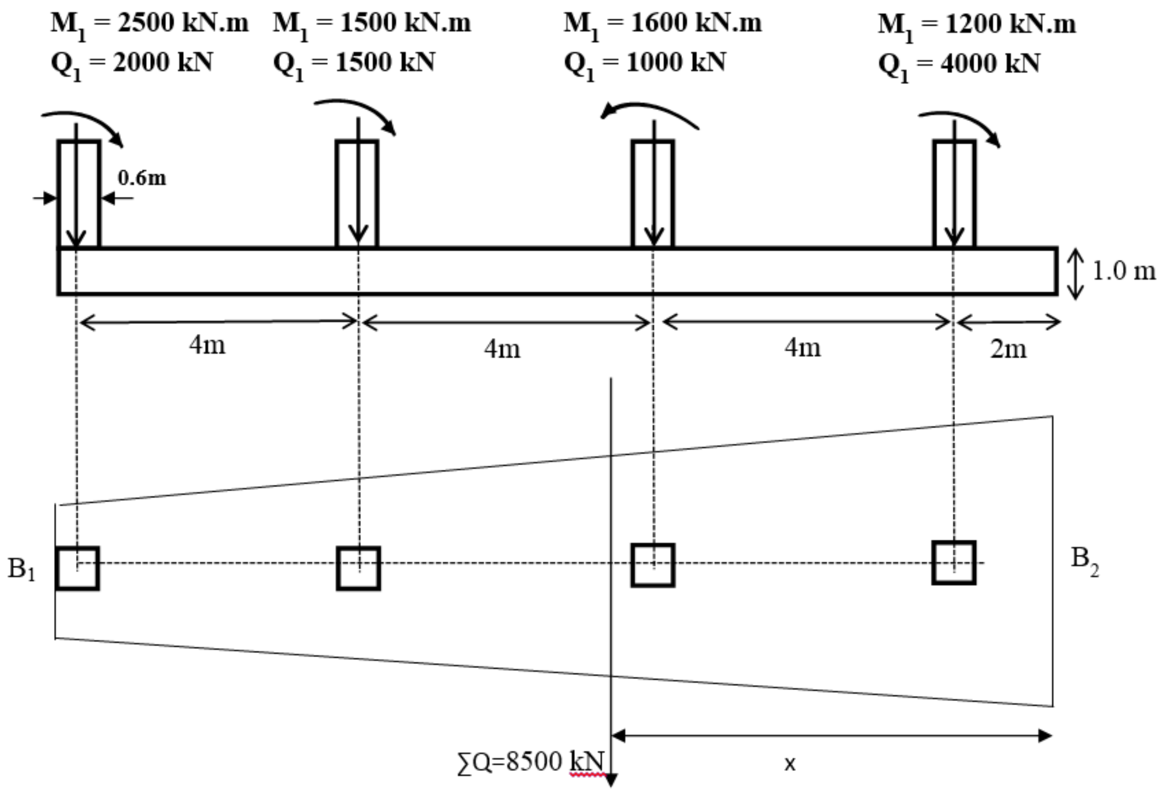
$$Weight\ of\ footing = (14 + 0.3) * 1 * \frac{B_1 + B_2}{2} * 24 = 171.6 * (B_1 + B_2)$$

$$Area\ of\ footing = (14 + 0.3) * \frac{B_1 + B_2}{2} = 7.15 * (B_1 + B_2)$$

$$By\ using \sum F_v = 0$$

$$8500 + 171.6 * (B_1 + B_2) = 7.15 * (B_1 + B_2) * 200$$

$$From\ above;\ B_1 + B_2 = 6.75\ m \ldots\ldots\ldots\ldots (1)$$



$$By\ using\ \sum M = 0\ (about\ centroid\ of\ the\ base)$$

$$2000(14-x) - 2500 + 1500(10-x) - 1500 - 1000(x-6) + 1600 - 4000(x-2) - 1200 + (wght\ of\ ftg) * 0 = (base\ pressure * area\ of\ ftg) * 0$$

$$Solving\ for\ x\ gives\ \longrightarrow\ x \cong 6.3\ m$$

$$x = \frac{1}{3} * L * \frac{2B_1 + B_2}{B_1 + B_2} = 6.3\ m$$

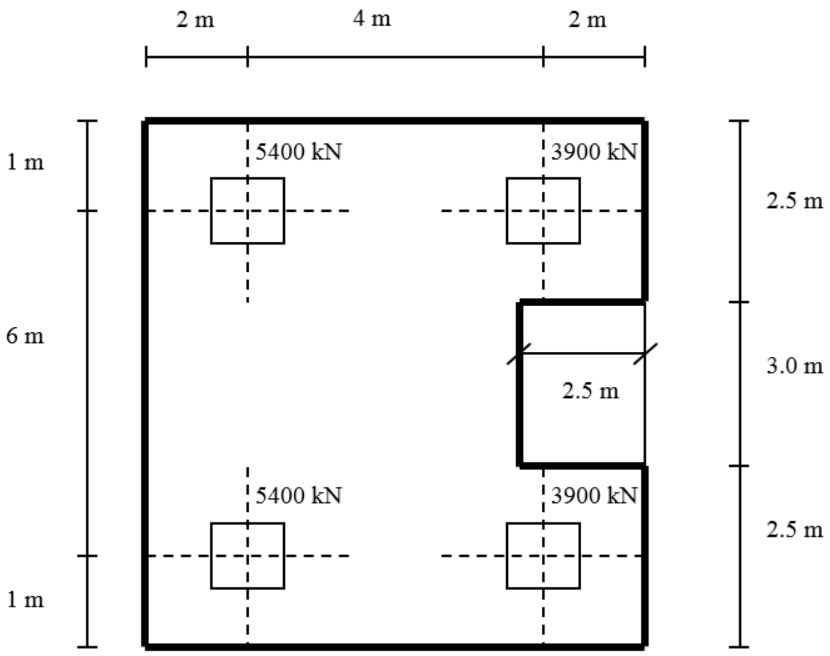
$$2.125B_1 = B_2 \ldots\ldots\ldots\ldots(2)$$

$$By\ solving\ Equ1\ and\ Equ2 \longrightarrow\ B_1 = 2.16\ m\ and\ B_2 = 4.59\ m$$

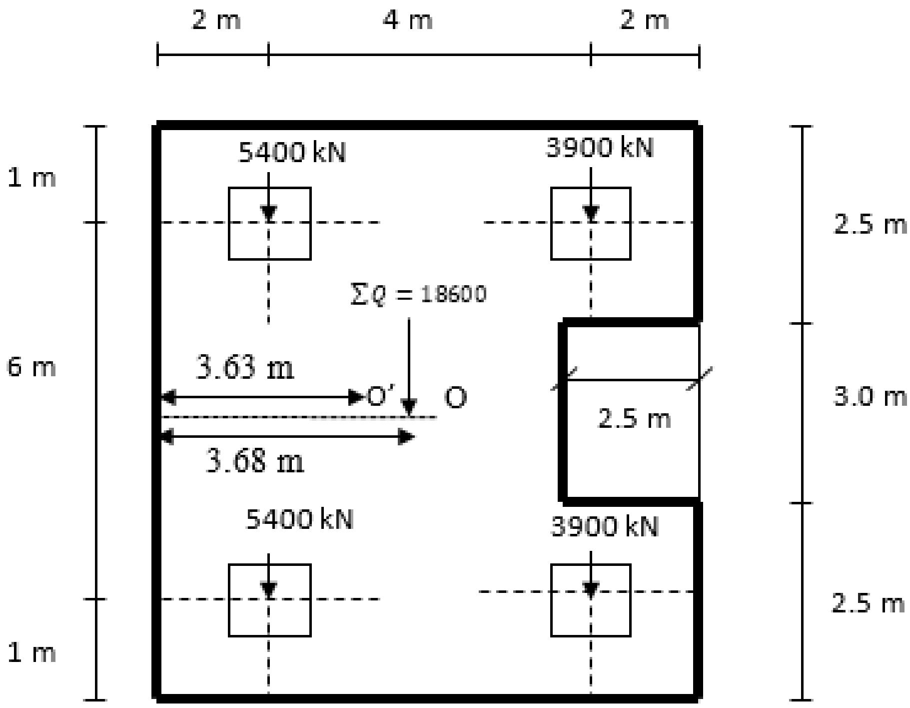
Question 2 (25%)

Calculate the base pressures at the corners of the mat foundation shown below. 4 square columns apply 5400, 5400, 3900 and 3900 kN's load. Do not consider weight of the mat.

Note: Assume that the mat foundations rests on top of the ground surface.



Solution 2



$$\sum Q = 2 * 5400 + 2 * 3900 = 18600\ kN$$

$$O: Center\ of\ uncut\ base$$

$$O': Center\ of\ gravity\ of\ cut\ base$$

$$Area\ of\ base = 8 * 8 - 3 * 2.5 = 56.5\ m^2$$

$$Center\ of\ gravity\ of\ base = \frac{8 * 8 * 4 - 3 * 2.5 * 6.75}{56.5} = 3.63\ m\ from\ left\ side$$

$$Point\ of\ application\ of\ the\ resultant\ force$$

$$5400 * 2 * 2 = 21600\ kN.m$$

$$3900 * 6 * 2 = 46800\ kN.m$$

$$\sum F = 18600\ kN$$

$$\sum M = 68400\ kN.m$$

$$b = \frac{68400}{18600} = 3.68\ m\ from\ left\ side$$

$$e = 3.68 - 3{,}63 = 0.05\ m \rightarrow M_{1-1} = 0.05 * 18600 = 930\ kN.m$$

Note: Axis 1-1 passes through center of gravity of the cut base (e.g. O')

$$I_{1-1} = \left[\frac{1}{12} * 8 * 8^3 + (8^2 * (4 - 3.63)^2)\right] - \left[\frac{1}{12} * 2.5 * 3^3 + (2.5 * 3 * (8 - 3.63 - 2.5/2)^2)\right] = 271.46\ m^4$$

$$Bottom\ Left\ Corner\ (BLC): q_{BLC} = \frac{18600}{56.5} - \frac{930}{271.46} * 3.63 = 316.8\ kPa$$

$$Top\ Left\ Corner = BLC = 316.8\ kPa$$

$$Bottom\ Right\ Corner\ (BRC): q_{BRC} = \frac{18600}{56.5} + \frac{930}{271.46} * 4.37 = 344.2\ kPa$$

$$Top\ Right\ Corner = BRC = 344.2\ kPa$$

Question 3 (25%)

Do a "selfie (özçekim)" with a retaining wall!

Please take a look at your close vicinity to see different types of retaining walls in Ankara. For this question, we would like you to take a photo of yourself with a retaining wall. (Please note that group pictures are not allowed in this question. Photos from your summer practice are not allowed, either.)

i) (10%) Put at most 4 photos showing the general overview of the wall.
ii) (3%) Try to identify the type of retaining wall and report the approximate location.
iii) (12%) Prepare a sketch (hand drawing) showing the approximate dimensions and construction details of the wall to the best of your knowledge.
(A sample answer is given below for you.)

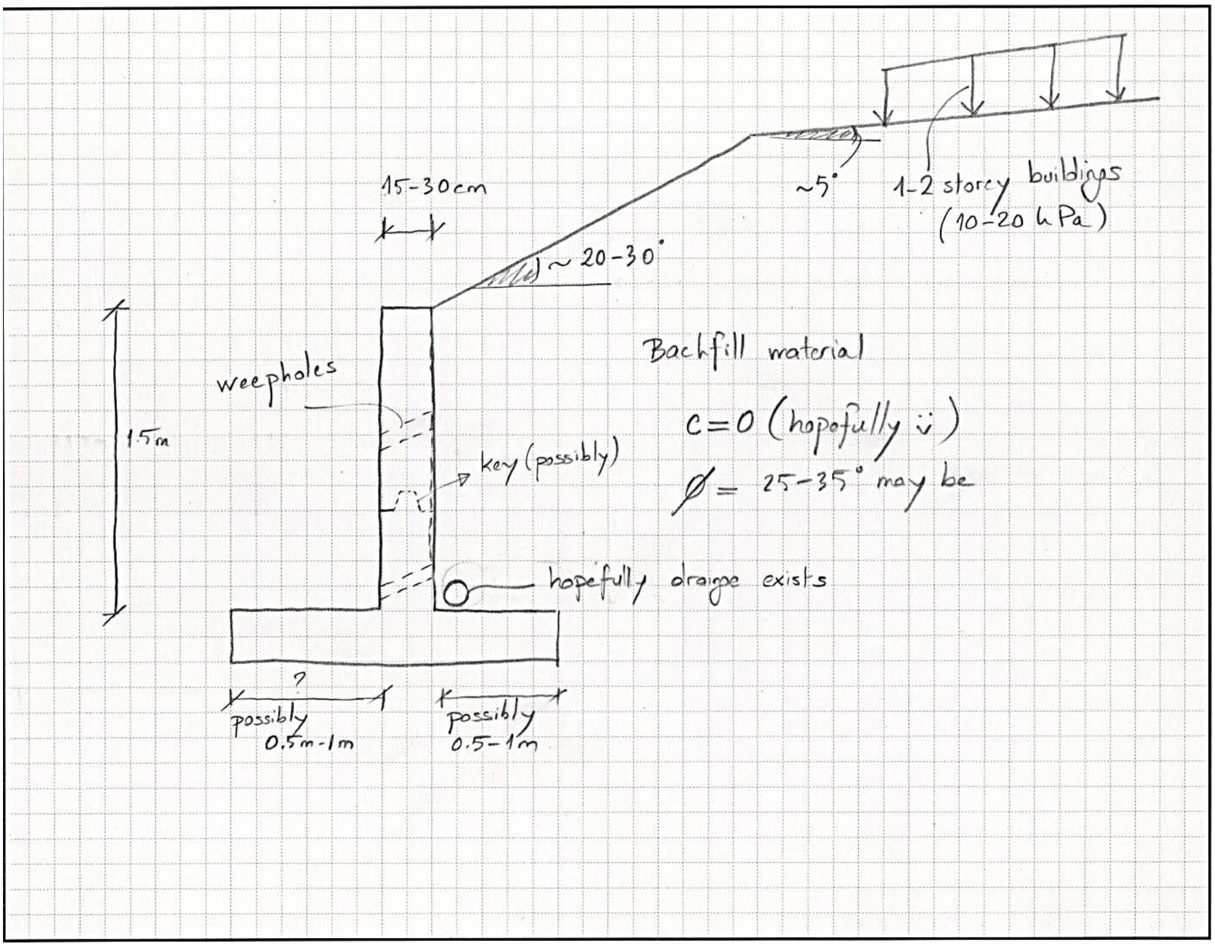
Solution 3

(i)

	
Pic 1: Selfie with the wall	Pic 2: Weepholes along the height of the wall
	
Pic 3: Cracks along the height of wall	Pic 4: Cracks along the height of wall

ii) I guess the wall is cantilever wall. It is located in METU Campus, Teknokent area where METU faculty members reside.

iii)



Question 4 (10%)

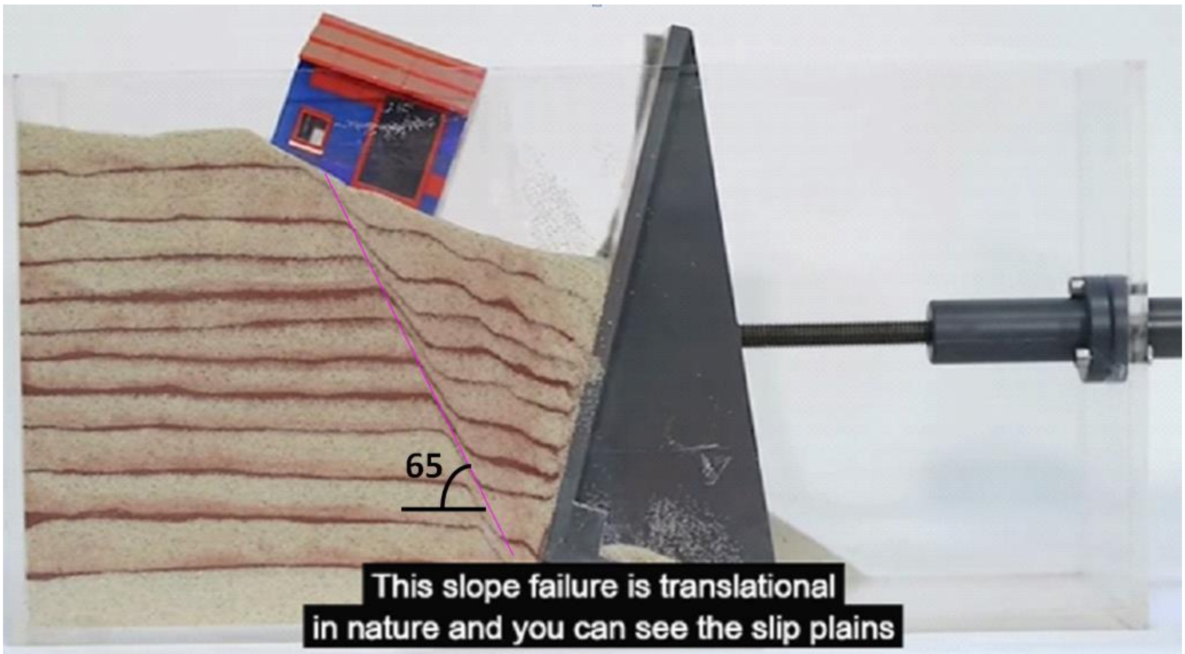
Watch the video at the link below, and answer the questions.

https://www.youtube.com/watch?v=fYjIdsJXE5k

a) What is the angle of the failure plane at the back of the retaining wall? Capture an image and measure this angle. This angle must be related to the friction angle of the sand. What is the friction angle of this sand?
b) At the time of failure, assume the factor of safety against overturning as equal to 1.00 and find the surcharge, q (kPa), applied by the building at the top. You need to draw the problem to scale. The height of the wall is 50 cm. The unit weight of the clean sand and the wall material can be assumed as 19 and 27 kN/m^3, respectively. Use Rankine earth pressure theory.

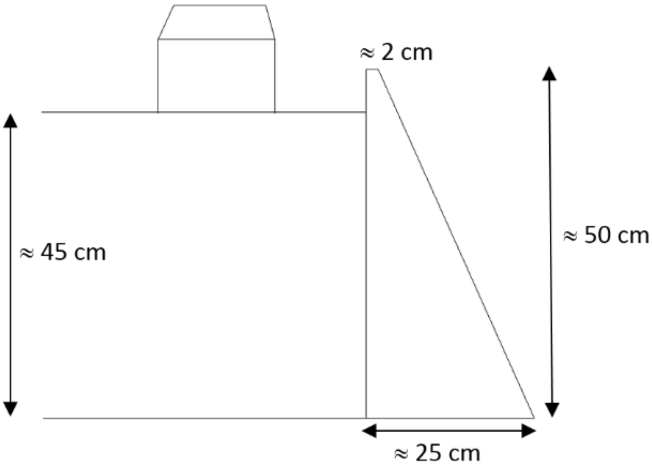
Solution 4

a) The image captured from the video shows the development of the failure plane.



The angle of the failure plane from the horizontal plane is approximately **62 to 67 degrees**, let's use 65 degrees. This is the Rankine active failure wedge, which is typically at an angle of **45+ϕ/2** from the horizontal plane. The friction angle of the clean sand is approximately **40 degrees**.

b)

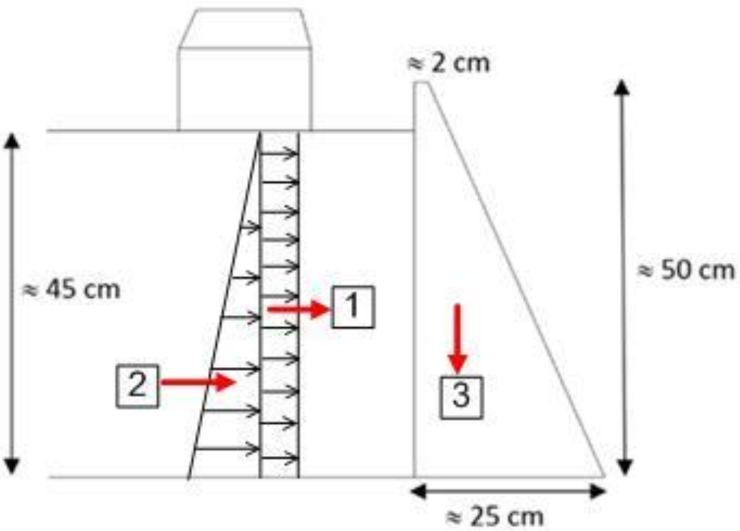


Friction angle of the sand = 40 degrees (from part (a), by measuring the angle of the failure plane)

Clean sand, therefore c' = 0

Ka = (1-sin ϕ) / (1+sin ϕ) = (1-sin 40) / (1+sin 40) = 0.217

Active lateral earth pressure (kPa)	σ_v Ka – 2 c √Ka = σ_v Ka
At depth z=0 m	(q) · 0.217 = 0.217 q
At depth z=0.45 m	(q+0.45·19) · 0.217 = 0.217 q + 1.85



Forces	Force magnitude (kN/m)	Moment arm about toe (m)
1	0.217q · 0.45 = 0.098 q	0.45 / 2 = 0.225
2	1.85 · 0.45 / 2 = 0.416	0.45 / 3 = 0.15
3	[(0.25 + 0.02)·0.50 / 2]· 27 = 1.82	0.25 · 2 / 3 = 0.167

At the time of failure, $FS_{overturning} = 1.00$, find q=?

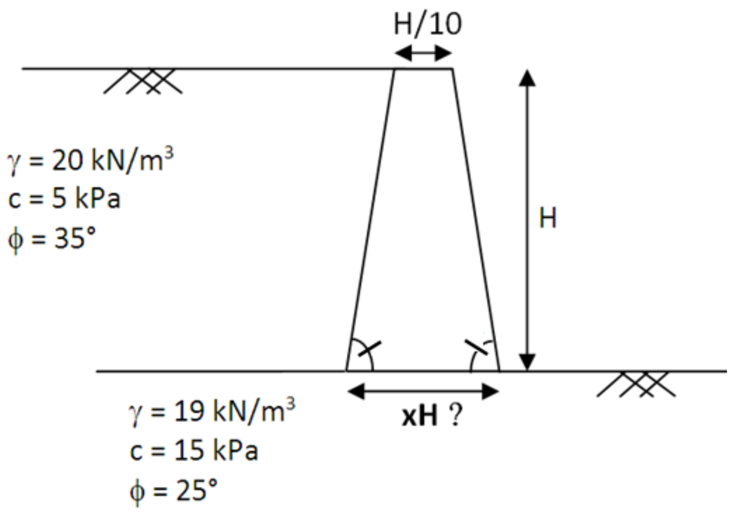
$$FS_{overturning} = \frac{1.82 \cdot 0.167}{(0.098\, q \cdot 0.225) + (0.416 \cdot 0.15)} = 1.00$$

q = 11 kPa (≈ 1 storey building)

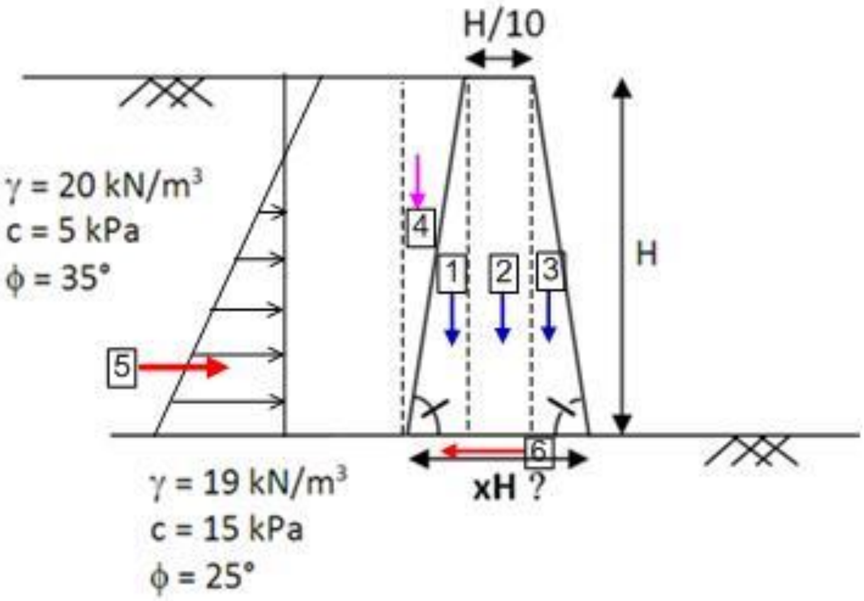
Question 5 (15%)

In order to satisfy $F.S_{sliding} \geq 1.5$ and $F.S_{overturning} \geq 2.0$, what should be the minimum width of the gravity retaining wall shown below in terms of wall height, H. Wall is made of a material with unit weight 24 kN/m^3. Note that, for economical and green/sustainable construction purposes, a local soil with c=5 kPa and ϕ=35° will be used as backfill material (instead of bringing many truckloads of clean gravel from far away). Backfill material is freely draining and all proper drainage measures are taken. $c_{wall\text{-}soil} = 2/3 * c_{soil}$ and $\tan \delta = 2/3 * \tan \phi$.

Note: Solve by using Rankine earth pressure theory.



Solution 5



Backfill soil, c = 5 kPa, $\phi = 35°$

Ka = (1-sin ϕ) / (1+sin ϕ) = (1-sin 35) / (1+sin 35) = 0.271

Active lateral earth pressure (kPa)	σ_v Ka – 2 c √Ka
At depth z=0 m	0 – 2 · 5 · √0.271 = - 5.2
At depth z=H m	(20 · H) 0.271 – 2 · 5 · √0.271 = 5.4 · H -5.2

The depth of tension crack : σ_v Ka – 2 c √Ka = 0 ; z = 0.96 m ≈ 1 m

To calculate FS against overturning:

Forces	Force magnitude (kN/m)	Moment arm about toe (m)
1	[(xH – H/10)/2] · H / 2 · 24 = (6x – 0.6) H^2	xH – 2/3 (xH – H/10)/2
2	(H/10) · H · 24 = 2.4 H^2	xH/2
3	[(xH – H/10)/2] · H / 2 · 24 = (6x – 0.6) H^2	2/3 (xH – H/10)/2
4	[(xH – H/10)/2] · H/2 · 20 = (5x – 0.5) H^2	xH – 1/3 (xH – H/10)/2
5	(5.4 H - 5.2) (H - 1) / 2 = 2.7 H^2 – 2.6 H – 0.1	(H – 1) · 1/3
6	$c_{wall\text{-}soil}$ · base length + ΣF_v · tan δ =(2/3)·15·xH+ (F_1+ F_2+ F_3+ F_4)·2/3 (tan 25°) = 10xH + [2·(6x – 0.6) H^2+2.4 H^2+(5x – 0.5) H^2] · 0.31	-

$$FS_{overturning} = \frac{\sum(M_{resisting})\ due\ to\ forces\ 1,2,3,4}{\sum(M_{disturbing})\ due\ to\ force\ 5} \geq 2.0$$

$$FS_{overturning} = \frac{(6x-0.6)H^2 \cdot \mathrm{x}\mathrm{H} + 2.4H^2 \cdot xH/2 + (5x-0.5)H^2 \cdot (5xH+0.1H)/6}{(2.7H^2-2.6H-0.1)\cdot(H-1)/3} \geq 2.0$$

$$\boldsymbol{After\ re-arranging}: \quad FS_{overturning} = \frac{H^3(x-0.1)\left(\frac{61x+0.5}{6}\right)+1.2xH^3}{(2.7H^2-2.6H-0.1)\cdot(H-1)/3} \geq 2.0$$

$$FS_{sliding} = \frac{\sum(F_{resisting})\ due\ to\ force\ 6}{\sum(F_{disturbing})\ due\ to\ force\ 5} \geq 1.50$$

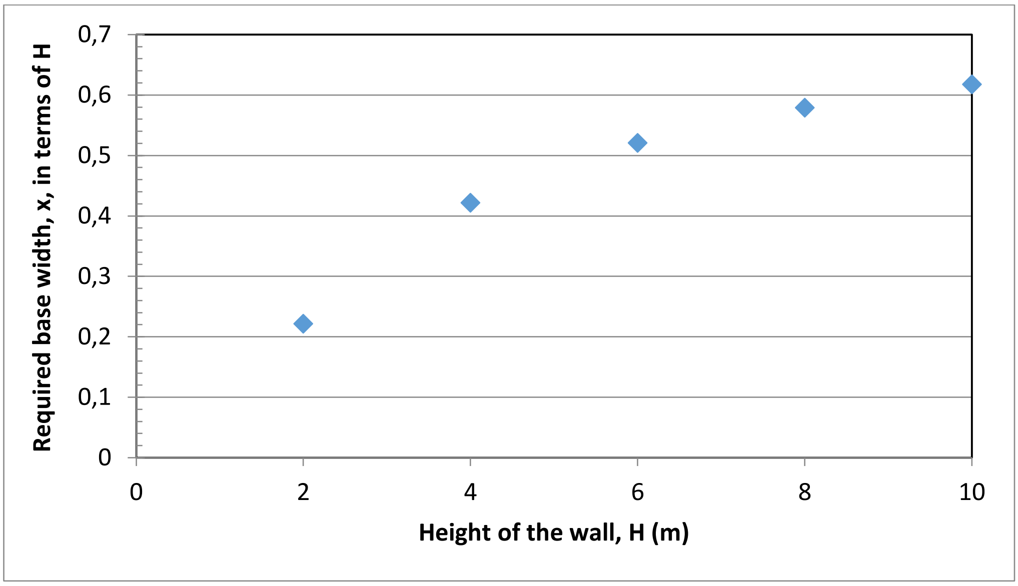
$$FS_{sliding} = \frac{10\mathrm{xH} + [2\cdot(6\mathrm{x}-0.6)\,\mathrm{H}^2 + 2.4\,\mathrm{H}^2 + (5\mathrm{x}-0.5)\,\mathrm{H}^2]\cdot 0.31}{2.7\mathrm{H}^2 - 2.6\,\mathrm{H} - 0.1} \geq 1.50$$

$$\boldsymbol{After\ re-arranging}: \quad FS_{sliding} = \frac{10\mathrm{xH} + 0.31H^2(17x-0.7)}{2.7\mathrm{H}^2 - 2.6\,\mathrm{H} - 0.1} \geq 1.50$$

The required base length, xH will depend on the value of H. For different H values, factor of safety against overturning and sliding is calculated, and presented in the table and graph below. The xH value that satisfies both overturning and sliding conditions can be seen in **bold** font in the table below. To find the x values, MS Excel's Goal Seek option is used.

Wall height, H (m)	Base length (in terms of H) x	$FS_{overturning}$ $FS_{sliding}$	Base length (in terms of H) x	$FS_{overturning}$ $FS_{sliding}$
2	**0.222**	2.41 1.50	0.202	2,00 1.35
4	**0.422**	3.76 1.50	0.306	2.00 1.06
6	**0.521**	4.59 1.50	0.340	2.00 0.95
8	**0.579**	5.14 1.50	0.357	2.00 0.89
10	**0.618**	5.52 1.50	0.368	2.00 0.86

Following graph is obtained by using the x values that satisfies both conditions (e.g. 2nd column of the above table).



NOTES:

1) It is seen from the figure that, the required base width in terms of wall height, H, for gravity retaining walls, to satisfy both $F.S_{sliding} \geq 1.5$ and $F.S_{overturning} \geq 2.0$, is not a constant value, but it changes with the wall height.
2) Gravity retaining wall is one of the oldest retaining wall type. It can be made of stone masonry, unreinforced concrete or reinforced concrete. Gravity walls require significant volume of material to construct the wall. It is still used today for short wall heights (less than 10 m). For greater than 10 m wall heights, gravity wall becomes uneconomical and other wall types are preferred (cantilever wall, reinforced soil wall etc).
3) From the graph and table, we can see that for wall heights less than 10 m, the required base width, xH, is in the range of 0.32H to 0.67H. These values are more or less in agreement with the "typical dimensions of gravity retaining walls" given on page 125 of CE366 Lecture Notes. The "typical base widths used in the design", for gravity retaining walls, are in the range of 0.5H to 0.67H. When giving these typical dimensions, check for base-pressures, construction practice, and other factors may also have been considered.
4) We see from our calculations that when $FS_{sliding}$ is satisfied, $FS_{overturning}$ is already satisfied. Therefore the critical failure mode seems to be the sliding.