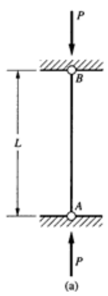
CE 388 – FUNDAMENTALS OF STEEL DESIGN

CHAPTER 3A: COMPRESSION MEMBERS

Compression members

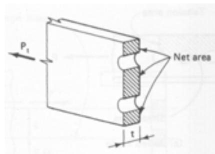
- In any member which is squeezed and shortened, compressive stresses are produced, and such a member is called a compression member



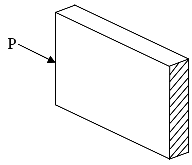
a compression member

Compression members

- Two significant differences between tension and compression members:
  - In tension members, rivets and bolt holes are subtracted from the gross area when determining the net area. In compression members it is assumed that the rivets and bolts fill the holes completely, so the entire gross area is available for resisting compressive loads



tension member
(net area)



compression member
(gross area)

Compression members

- Tension members remain straight under tensile loads, whereas compressive members tend to buckle under compression loads. In buckling the members bend out of the plane of the loading

Buckling of a
compression member

Compression members

Buckling of columns in New York
Metro Station

Failure Modes of a Compression Member

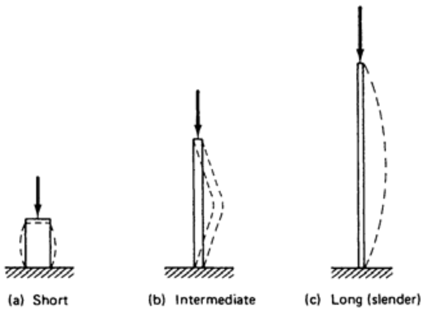
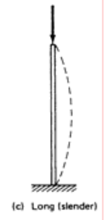
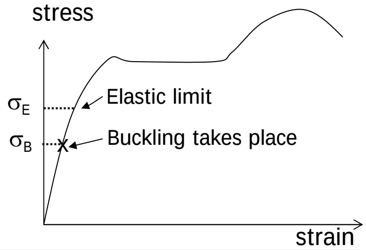


FIGURE 3-2 Columns types and failure modes.

Failure Modes of a Compression Member

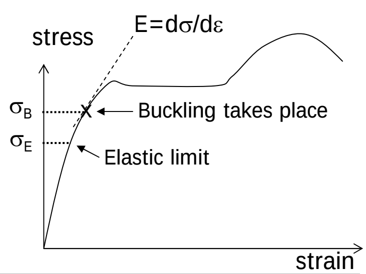
- Long and slender column (elastic buckling):
  - The column fails due to buckling
  - The buckling takes place in linear portion of the stress-strain diagram (the member will buckle before the elastic/proportional limit is reached)
  - When buckling occurs, the elasticity modulus is equal to initial elasticity modulus





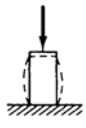
Failure Modes of a Compression Member

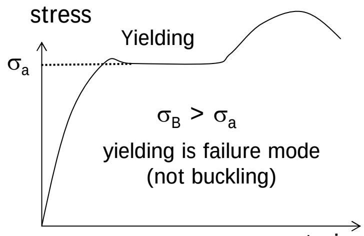
- Intermediate column (inelastic buckling):
  - A column with intermediate size also buckles
  - However, since the member is stronger, it will resist further to the applied load and buckling occurs when the elastic limit is exceeded
  - The modulus of elasticity is no longer a constant value



Failure Modes of a Compression Member

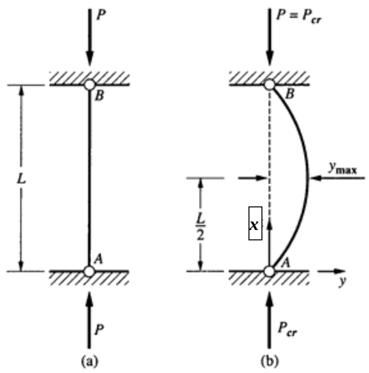
- Short and stocky column (yielding):
  - Very short and stocky members will fail due to yielding
  - The buckling strength of these members is higher than the yield stress and thus they yield before the inception of buckling





Elastic Buckling of Columns – Derivation of Euler Buckling Formula

- Consider a pin ended straight column, which remains straight under service loads
- At a particular load level called critical load or buckling load (P_{cr}), the column will displace laterally.



column under service loads

column under critical load

Elastic Buckling of Columns – Derivation of Euler Buckling Formula

- Summing the moments about point A,

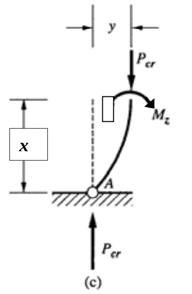
$$\sum M_A = 0; \quad M = -P_{cr}.y \qquad (1)$$

- From mechanics of materials,

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = -\frac{P_{cr}y}{EI} \qquad (2)$$

- Substituting (1) into (2),

$$\frac{d^2y}{dx^2} + \frac{P_{cr}}{EI}y = 0 \qquad (3)$$



FBD of the lower portion of the column at the time of buckling

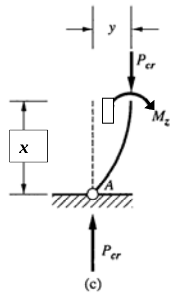
Elastic Buckling of Columns – Derivation of Euler Buckling Formula

- Defining $k^2 = P_{cr}/EI$ in (3)

$$\frac{d^2y}{dx^2} + k^2y = 0 \qquad (4)$$

- The solution of (4) is

$$y = \underbrace{ACoskx + BSinkx}_{\text{constants to be determined from boundary conditions}} \qquad (5)$$



FBD of the lower portion of the column at the time of buckling

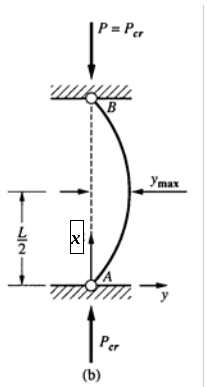
Elastic Buckling of Columns – Derivation of Euler Buckling Formula

- Boundary conditions are
  - at x=0, y=0
  - at x=L, y=0
- Using the first B.C in (5), we find

$$\underbrace{y = ACoskx + BSinkx}_{A=0}$$

⇓

$$y = BSinkx \qquad (6)$$



Elastic Buckling of Columns – Derivation of Euler Buckling Formula

- Using the second B.C in (6), we find

$$0 = \underbrace{BSinkL}_{SinkL=0}$$

(for non-trivial solution)

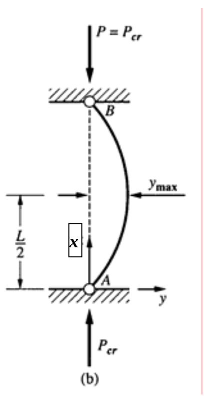
⇓

$$kL = n\pi \qquad (7)$$

- Substituting $k^2=P_{cr}/EI$ in (7) and rearranging

$$P_{cr} = \frac{n^2 \pi^2 EI}{L^2} \qquad (8)$$

integer value
(buckling mode number)



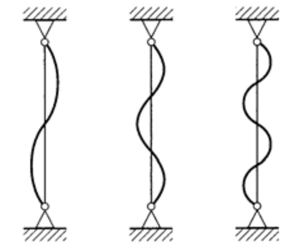
Elastic Buckling of Columns – Derivation of Euler Buckling Formula

- For n=1 (the first mode):

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad \text{and} \quad y = BSin\frac{\pi}{L}x$$

the first mode of buckling
(n=1)

- For n>1 (the higher modes):



the higher modes of buckling
(n>1)

Elastic Buckling of Columns – Derivation of Euler Buckling Formula

- The critical load corresponding to the first mode is lowest. This load is referred to as ***Euler Buckling Load*** or ***Critical Buckling Load***

$$\underbrace{P_{cr}}_{\text{Euler Buckling load}} = \frac{\pi^2 EI}{L^2} \quad \text{(for n=1)} \qquad (9)$$

- Defining λ (slenderness ratio)

$$\lambda = \frac{L}{\underbrace{i}_{\text{radious of gyration}}} \quad \text{where} \quad i = \sqrt{\frac{I \leftarrow \text{moment of inertia}}{A \leftarrow \text{area}}}$$

Elastic Buckling of Columns – Derivation of Euler Buckling Formula

- Equation (9) becomes

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 Ei^2 A}{L^2} = \frac{\pi^2 EA}{(L/i)^2} = \frac{\pi^2 EA}{\lambda^2} \qquad (10)$$

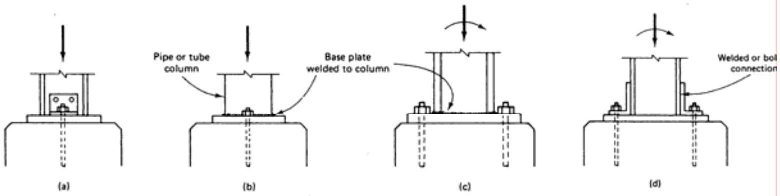
- Euler Buckling Strength (σ_{cr}):

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{\lambda^2} \qquad (11)$$

(Euler buckling strength for pin-ended columns)

Elastic Buckling of Columns – Euler Buckling for Other End Restraints

- Euler's formula gives the critical buckling load or buckling strength of a pin ended column
- A practical column may have different boundary conditions

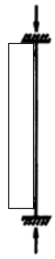


Pinned end supports

Fixed end supports

Elastic Buckling of Columns – Euler Buckling for Other End Restraints

- Euler formula can also be applied to columns with different end conditions
- For a column with fixed ends,

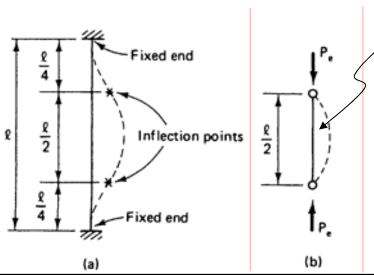


$$P_{cr} = \frac{\pi^2 EI}{\underbrace{(0.5L)}^2}$$

Identical to critical buckling of a pin-ended column except that the length of the column is multiplied by 0.5

Elastic Buckling of Columns – Euler Buckling for Other End Restraints

- The buckled column has two points of inflection, the points of zero moments
- Theoretically, points of inflections can be replaced with pins



Behaves as a pin ended column with length L/2

Euler critical load is the same as a pin ended column with length L/2

$$\frac{1}{2}L$$

Effective length

Effective length factor

Elastic Buckling of Columns – Euler Buckling for Other End Restraints

- In general,

KL ← Effective length

↑ Effective length factor

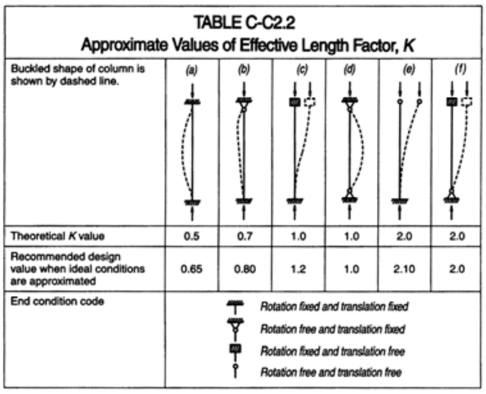


TABLE C-C2.2
Approximate Values of Effective Length Factor, K

Buckled shape of column is shown by dashed line.	(a)	(b)	(c)	(d)	(e)	(f)
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0
End condition code	Rotation fixed and translation fixed; Rotation free and translation fixed; Rotation fixed and translation free; Rotation free and translation free					

Other restraint conditions

Elastic Buckling of Columns – Euler Buckling for Other End Restraints

- Generalized Euler formulas can now be written as follows:

Critical Buckling Load ⟶ $$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

Critical Buckling Stress ⟶ $$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2} = \frac{\pi^2 E}{\left(\frac{KL}{i}\right)^2}$$

Elastic Buckling of Columns –
Euler Buckling for Other End Restraints

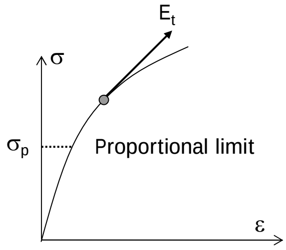
Example Problem

Inelastic Buckling of Columns

- Buckling occurs after elastic limit is exceeded.
- Above the elastic limit, E is not constant but depends on the stress
- Buckling at stresses above the elastic limit is referred to as inelastic buckling
- The two theories developed for inelastic buckling:
  - Tangent modulus theory
  - Reduced modulus theory

Inelastic Buckling of Columns

- Tangent Modulus Theory:
  - Tangent modulus E_t is used in buckling strength formula



$$E_t = \frac{d\sigma}{d\varepsilon}$$

Tangent modulus

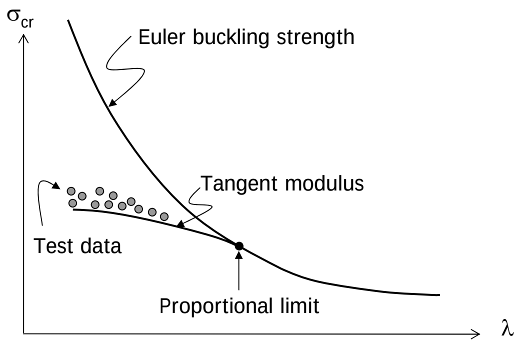
Slope of stress-strain diagram

  - Buckling strength is given by

$$\sigma_{cr} = \frac{\pi^2 E_t}{\lambda^2}$$

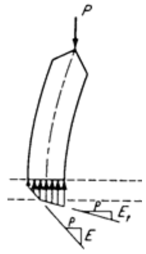
Inelastic Buckling of Columns

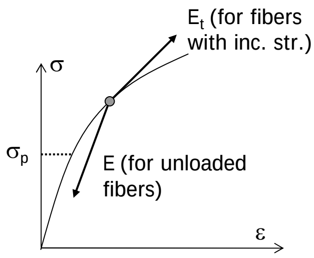
  - The test results show that tangent modulus theory underestimates the buckling strength of members



Inelastic Buckling of Columns

- Reduced Modulus Theory:
  - At the instant of inelastic buckling, the member changes from a straight to bent form.
  - Bending causes increased strain on side side, whereas it causes decreased strain (or unloading) on the other side
  - For fibers with increased strain E_t should be used, whereas for unloaded fibers, E should be used

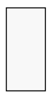




Inelastic Buckling of Columns

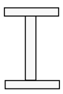
- A combined value is used for elasticity modulus (E_r)
  - For rectangular sections,

$$E_r = \frac{4EE_t}{\left(\sqrt{E}+\sqrt{E_t}\right)^2}$$



  - For wide flange sections,

$$E_r = \frac{2EE_t}{\left(E+E_t\right)}$$

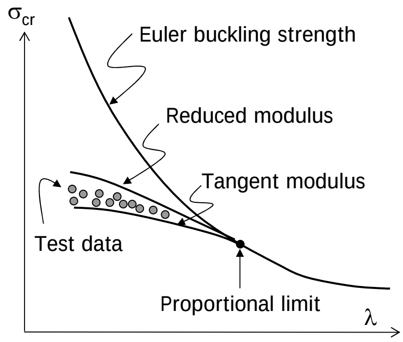


- The critical buckling stress is

$$\sigma_{cr} = \frac{\pi^2 E_r}{\lambda^2}$$

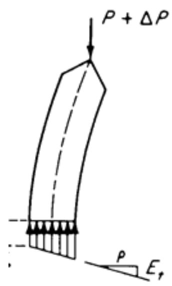
Inelastic Buckling of Columns

- Although reasonable, the reduced modulus theory gives overestimated results
- The test resuts fall in between the values given by tangent and reduced modulus theories
- In fact, the test results tend to be closer to tangent modulus theory



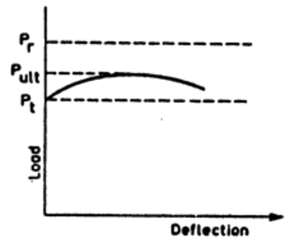
Inelastic Buckling of Columns

- The Shanley Contribution-True Column Behaviour
  - Why does the apparently accurate reduced theory lead to overestimated buckling loads?
  - Why do the experimental results tend to be closer to less accurate tangent-modulus values?
  - Shanley showed that the bending in a column is accompanied by increase in axia load, $P+\Delta P$.
  - The additional axial force ΔP creates strains that offest effect of bending



Inelastic Buckling of Columns

- The tangent modulus theory is the lower bound for column strength whereas the reduced modulus is the upper bound
- The ultimate load of a column lies between these two bounds
- The increase of capacity from P_t (tangent modulus load) to P_{ult} (ultimate load) can be neglected and P_t may be treated as critical buckling load



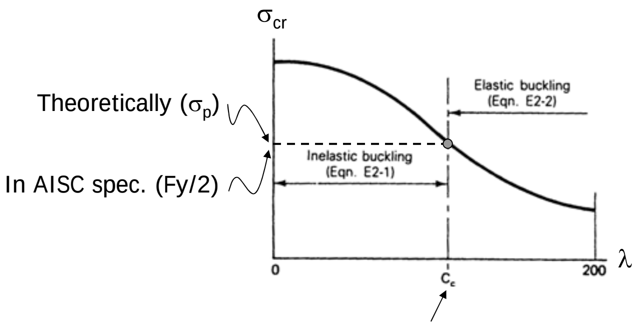
True column behaviour

Inelastic Buckling of Columns

Example Problem

AISC Column Formulas

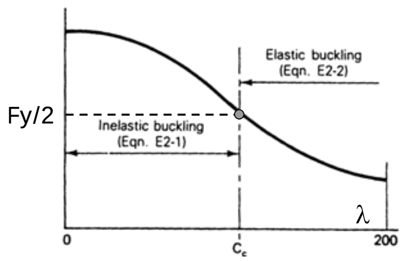
- Theoretically, the upper limit of elastic buckling is proportional limit (σ_p)
- To be on the safe side, the upper limit of elastic buckling is assumed to be half of the yield point (F_y)



Slenderness ratio between elastic and inelastic buckling

AISC Column Formulas

- C_c can be determined as,



$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2} \longleftarrow \text{Euler formula}$$

$\sigma_{cr} \leftarrow \frac{F_y}{2}$, $\lambda \leftarrow C_c$

$$\Downarrow$$

$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$$

- If $\lambda \le C_c$ (inelastic buckling) and $\lambda > C_c$ (elastic buckling)

$$\lambda = \frac{KL}{i} \le C_c \Rightarrow \text{inelastic buckling}$$

$$\lambda > C_c \Rightarrow \text{elastic buckling}$$

AISC Column Formulas

- For inelastic buckling ($\lambda \le C_c$), the following formula is used in AISC,

Allowable stress $\rightarrow$

$$F_a = \frac{\overbrace{\left[1-\frac{(\lambda)^2}{2C_c^2}\right]F_y}^{\text{Buckling strength}}}{\underbrace{\frac{5}{3}+\frac{3(\lambda)}{8C_c}-\frac{(\lambda)^3}{8C_c^3}}_{\text{Factory of safety (F.S)}}} = \frac{\left[1-\frac{(KL/r)^2}{2C_c^2}\right]F_y}{\frac{5}{3}+\frac{3(KL/r)}{8C_c}-\frac{(KL/r)^3}{8C_c^3}}$$

For $\lambda \cong 0$, F.S = 5/3
For $\lambda \cong C_c$, F.S = 23/12

For slenderer columns larger safety factor is used

AISC Column Formulas

- For elastic buckling ($\lambda > C_c$), the following formula is used in AISC,

Allowable stress $\rightarrow$

$$F_a = \frac{12\pi^2 E}{23\lambda^2} = \frac{12\pi^2 E}{23(KL/r)^2}$$

Euler buckling stress with F.S.= 23/12

- According to AISC, slenderness ratio of compression members shall not exceed 200

$$\lambda = \frac{KL}{i} \le 200$$

Column Analysis as to TS648

- Two methods that can be used for the column analysis are
  - Buckling formulas method
  - Buckling coefficient method

Column Analysis as to TS648

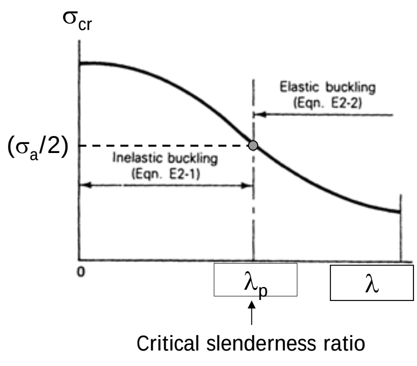
- Buckling formulas method:
  - Similar to AISC, the upper limit of elastic buckling is assumed to be half of the yield point

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2}$$ ← Euler formula

$\frac{\sigma_a}{2}$ λ_p

$$\Downarrow$$

$$\lambda_p = \sqrt{\frac{2\pi^2 E}{\sigma_a}}$$



Column Analysis as to TS648

Critical slenderness ratio:

$$\lambda_p = \sqrt{\frac{2\pi^2 E}{\sigma_a}}$$

$E = 2.1 \times 10^6$ kgf/cm^2

St37 ($\sigma_a = 2400$ kgf/cm^2) → $\lambda_p = 131.4$

St52 ($\sigma_a = 3700$ kgf/cm^2) → $\lambda_p = 107.3$

Column Analysis as to TS648

- If $\lambda \leq 20$ (no buckling):
  - Failure takes place because of yielding, not buckling → no reduction in allowable stress due to buckling

$$\sigma_{bem} = 0.6\sigma_a \qquad (\sigma_{bem} = \sigma_{çem})$$

- If $20 < \lambda \leq \lambda_p$ (inelastic buckling):

$$\sigma_{bem} = \left\{1 - 0.5\left(\frac{\lambda}{\lambda_p}\right)^2\right\}\frac{\sigma_a}{n} \qquad n = 1.5 + 1.2\left(\frac{\lambda}{\lambda_p}\right) - 0.2\left(\frac{\lambda}{\lambda_p}\right)^3$$

Factory of safety

Column Analysis as to TS648

- If $\lambda > \lambda_p$ (elastic buckling):

$$\sigma_{bem} = \frac{2\pi^2 E}{5\lambda^2} \quad \text{or} \quad \sigma_{bem} = \frac{8{,}290{,}000}{\lambda^2} \quad \text{(for } E=2.1*10^6 \text{ kgf/cm}^2\text{)}$$

Euler buckling stress with
F.S.= 5/2=2.5

- TS648 and AISC are similar except safety factors. In TS648, increased safety factors are used
- In TS648, slenderness ratio of compression members shall not exceed 250, same as a member in tension

$$\lambda = \frac{KL}{i} \leq 250$$

Column Analysis as to TS648

- Buckling coefficient method:
  - It is simply a tabularized form of Buckling formulas method
  - The buckling strength is computed from

$$\sigma_{bem} = \frac{\sigma_{çem}}{w}$$

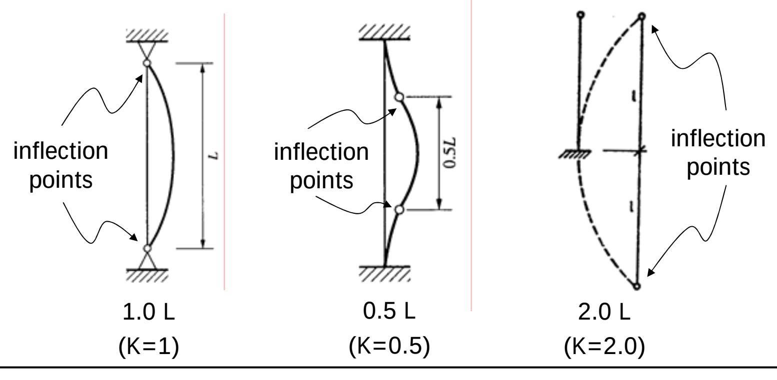
Buckling coefficient
(function of the slenderness ratio λ and directly read from the table)

Column Analysis as to TS648

Example Problem

Effective Lengths of Columns in Frames

- The effective length of a column is defined as the distance between the inflection points of the column

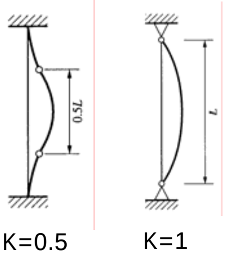


Effective Lengths of Columns in Frames

- Columns can be classified into two groups depending on their effective lengths
  - Swaying columns:
    - The ends of the column displace relative to each other
  - Non-swaying columns:
    - The ends of the column **do not** displace relative to each other

Effective Lengths of Columns in Frames

- Non-swaying columns:

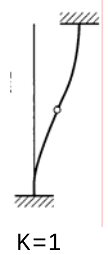


K=0.5 K=1

two extreme cases

$0.5 \leq K \leq 1.0$

- Swaying columns:



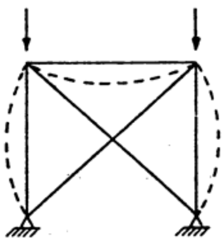
K=1

The most rigid case

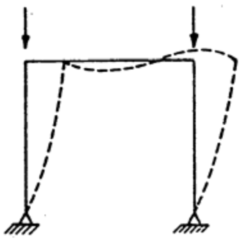
$K \geq 1.0$

Effective Lengths of Columns in Frames

- In structural frames, the effective length of a column depends on the type of the frame



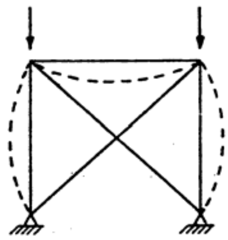
Braced frame



Unbraced frame

Effective Lengths of Columns in Frames

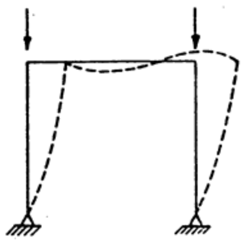
- Braced frames (sideaway prevented frames):
  - The sidesway (lateral movement) is effectively prevented or considerably reduced by some means, e.g., diagonla bracing, shear walls, etc.
  - The relative displacement of columns are prevented
  - $0.5 \leq K \leq 1.0$ for columns



Braced frame

Effective Lengths of Columns in Frames

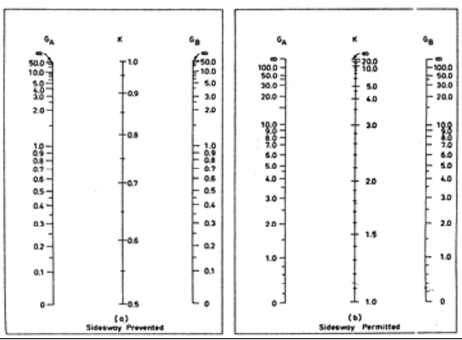
- Unbraced frames (sideaway permitted frames):
  - The lateral stability of the frames depends on bending stiffness of beams and columns
  - Sidesway (lateral displacement)
  - K ≥ 1.0 for columns



Unbraced frame

Effective Lengths of Columns in Frames

- Alignment charts:
  - To estimate the effective length of columns in frames, alignment charts are used.



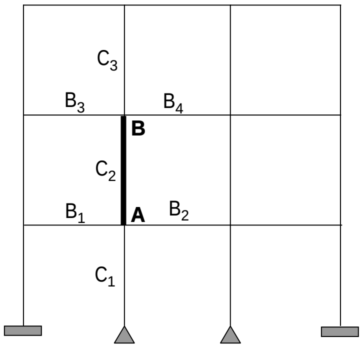
Sidesway prevented   Sidesway permitted

Effective Lengths of Columns in Frames

- Calculating K for column C_2:
  - Calculate the relative stiffness at both ends (A and B) of C_2
  - The relative stiffness at a joint is defined as follows:

$$G = \frac{\sum (I/L)_{columns}}{\sum (I/L)_{beams}}$$

Relative stiffness



Example frame

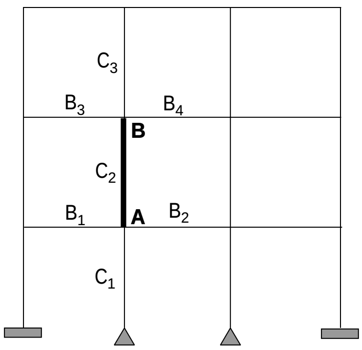
Effective Lengths of Columns in Frames

- Accordingly, G_A and G_B

$$G_A = \frac{\sum (I_c/L_c)_A}{\sum (I_b/L_b)_A} = \frac{(I/L)_{C1} + (I/L)_{C2}}{(I/L)_{B1} + (I/L)_{B2}}$$

$$G_B = \frac{\sum (I_c/L_c)_B}{\sum (I_b/L_b)_B} = \frac{(I/L)_{C2} + (I/L)_{C3}}{(I/L)_{B3} + (I/L)_{B4}}$$

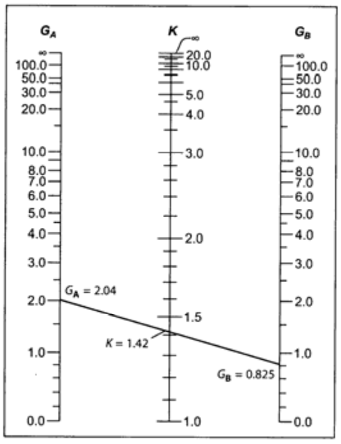
- Important points:
  - Consider only members that are rigidly connected to the joint in the plane of buckling
  - I is taken normal to the plane of buckling



Example frame

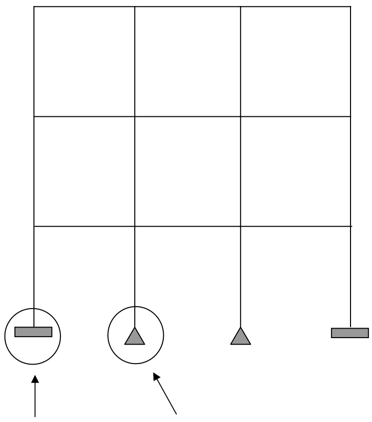
Effective Lengths of Columns in Frames

- Having determined G_A and G_B the points representing the values of G_A and G_B are connected with a straight line
- The value of K is read at intersection of this line with central vertical K reference line



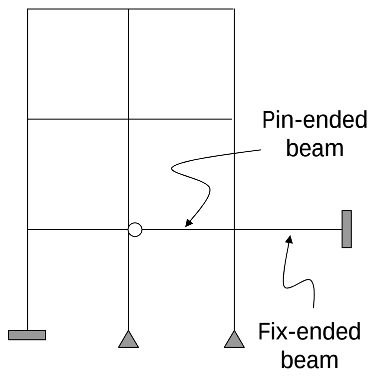
Effective Lengths of Columns in Frames

- Additional considerations:
  - Column ends:
    - For a pin-connected column end, G is theoretically infinity. For practical design, G can be taken as 10.0
    - For a fixed end column, G is taken as 1.0 for practical design



Effective Lengths of Columns in Frames

- Conditions at the far ends of the beam:
  - If the far end of a beam is pin or fixed connected, the stiffness (I/L) of that beam is multiplied by a factor (n)
  - Sidesway permitted:
    - Far end pinned, n=0.5
  - Sidesway prevented:
    - Far end pinned, n=1.5
    - Far end fixed, n=2.0



Effective Lengths of Columns in Frames

Example Problem