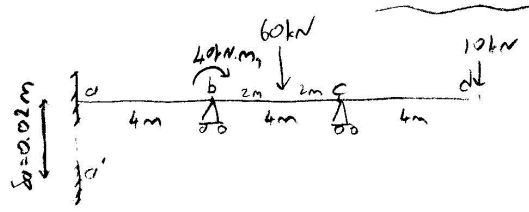
CE 383 Recitation 5 Fall 2011

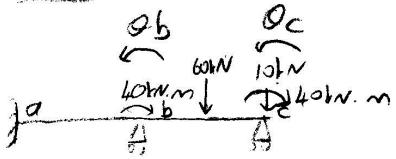


Support a settles downward 0.02 m.

$EI = 4000\ kN\cdot m^2$, axially rigid members.

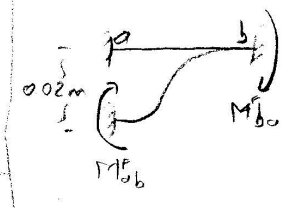
Draw M-diagram using 5-D Method.

1-) DOF



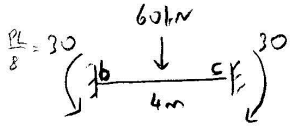
2 DOF (θ_b, θ_c)

2-) FEM's



$$M'_{ab} = \frac{2EI}{L}\left(-3\times\frac{0.02}{4}\right) = -30\ kN.m\ (cw)$$

$$M'_{ba} = \frac{2EI}{L}\left(-3\times\frac{0.02}{4}\right) = -30\ kN.m\ (cw)$$



$M'_{bc} = 30\ kN.m$

$M'_{cb} = -30\ kN.m$

3-) S-D Equations

$$M_{ab} = \frac{2EI}{L_{ab}}\left(2\theta_a + \theta_b - 3\psi\right) + FEM_{ab}$$

$$= \frac{2EI}{L}\left(\theta_b\right) - 30$$

$$M_{ba} = \frac{2EI}{L_{ab}}\left(2\theta_b + \theta_a - 3\psi\right) + FEM_{ba}$$

$$= \frac{2E}{4}\left(2\theta_b\right) - 30$$

$$M_{bc} = \frac{2EI}{L_{bc}}\left(2\theta_b + \theta_c - 3\psi\right) + FEM_{bc}$$

$$= \frac{2EI}{4}\left(2\theta_b + \theta_c\right) + 30$$

$$M_{cb} = \frac{2EI}{L_{bc}}\left(2\theta_c + \theta_b - 3\psi\right) + FEM_{cb}$$

$$= \frac{2EI}{4}\left(2\theta_c + \theta_b\right) - 30$$

4-) Equilibrium Equations

@ Joint b

$$M_{ba} + M_{bc} + 40 = 0 \quad \cdots (1)$$

@ Joint c

$$M_{cb} + 40 = 0 \quad \cdots (2)$$

from (1)

$$EI\theta_b - 30 + EI\theta_b + 0.5EI\theta_c + 30 + 40 = 0$$

$$2EI\theta_b + 0.5EI\theta_c = -40$$

from (2)

$$0.5EI\theta_b + EI\theta_c = -10$$

$$EI\begin{bmatrix} 2 & 0.5 \\ 0.5 & 1 \end{bmatrix}\begin{Bmatrix} \theta_b \\ \theta_c \end{Bmatrix} = \begin{Bmatrix} -40 \\ -10 \end{Bmatrix}$$

$$\begin{Bmatrix} \theta_b \\ \theta_c \end{Bmatrix} = \frac{1}{EI}\begin{bmatrix} 2 & 0.5 \\ 0.5 & 1 \end{bmatrix}^{-1}\begin{Bmatrix} -40 \\ -10 \end{Bmatrix} = \frac{1}{EI}\begin{Bmatrix} -20 \\ 0 \end{Bmatrix}$$

5-) Back Substitution

$M_{ab} = 0.5EI\times\frac{-20}{EI} - 30 = -40\ kN.m\ (cw)$

$M_{ba} = EI\times\frac{-20}{EI} - 30 = -50\ kN.m\ (cw)$

$M_{bc} = EI\times\frac{-20}{EI} + 0.5EI\times 0 + 30 = +10\ kN.m\ (ccw)$

$M_{cb} = EI\times 0 + 0.5EI\times\frac{-20}{EI} - 30 = -40\ kN.m\ (cw)$

M-diagram