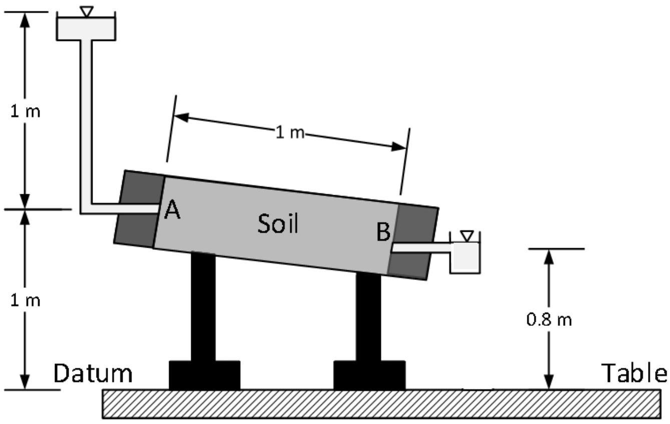
HOMEWORK 2 SOLUTION

Question 1 ((a): 6%; (b): 6%; (c): 6%; (d): 6%))

A soil sample 10 cm in diameter is placed in a tube 1 m long. A constant supply of water is allowed to flow into one end of the soil at A, and the outflow at B is collected by a beaker. The average amount of water collected is 1 cm^3 for every 10 seconds. The tube is inclined as shown in Figure. Determine (a) hydraulic gradient, (b) flow rate , (c) average velocity, (d) hydraulic conductivity.



Solution 1

The total heads at A (inflow) and B (outflow)

$$H_A = (h_P)_A + (h_Z)_A = 1 + 1 = 2\ m$$

$$H_B = (h_P)_B + (h_Z)_B = 0 + 0.8 = 0.8\ m$$

a. *Hydraulic Gradient*

$$\Delta H = H_A - H_B = 2 - 0.8 = 1.2\ m$$

$$L = 1m\ ; i = \frac{\Delta H}{L} = \frac{1.2}{1} = 1.2$$

b. *Flow Rate*

Volume of water collected = 1 cm^3 @ t = 10 seconds

$$q = \frac{1}{10} = 0.1\ cm^3/s$$

c. *Average Velocity*

$$q = A * v$$

$$A = \frac{\Pi * (diam)^2}{4} = \frac{\Pi * 10^2}{4} = 78.54\ cm^2$$

$$v = \frac{q}{A} = \frac{0.1}{78.54} = 0.0013\ cm/s$$

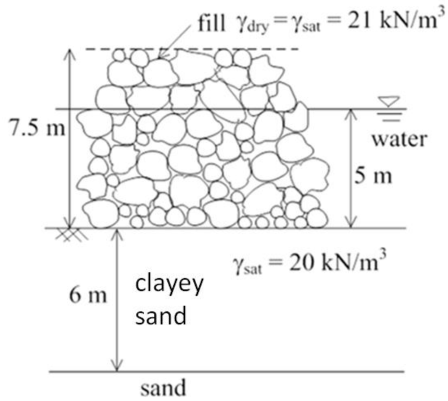
d. *Hydraulic Conductivity*

From Darcy's Law , $v = k * i$

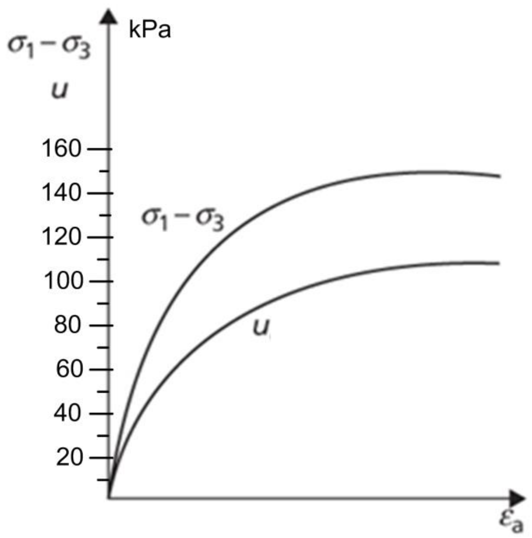
$$k = \frac{v}{i} = \frac{0.0013}{1.2} = 10.8 * 10^{-4} cm/s$$

Question 2 ((a): 8%; (b): 8%; (c): 8%)

For the construction of "Palm Island" in the sea, 7.5-m-thick fill made of boulders and gravel is constructed on top of the sea bottom sediments composed of clayey sand. The fill is constructed rapidly and long time after the construction of the fill, someone wanted to check the shear strength of the clay. They took soil samples from 12.5-m depth measured from the top of the fill.



The soil samples are brought to the soil mechanics laboratory for consolidated undrained triaxial testing with pore pressure measurement. The first specimen is consolidated to an equal all around pressure that is equal to its in-situ effective vertical stress. This sample is sheared by increasing the axial stress in undrained conditions, and the following graphs are obtained.



The second specimen is consolidated to a cell pressure of 440 kPa, and when sheared in undrained conditions, the measured principal stress difference at failure was 395 kPa and the pore pressure at failure was 290 kPa.

Find c′ and ϕ′ of this soil

a) Graphically, by drawing Mohr's Circles and using Mohr-Coulomb failure envelope,
b) Graphically, by using Modified Mohr-Coulomb failure envelope,
c) Analytically, by using trigonometry and computations.

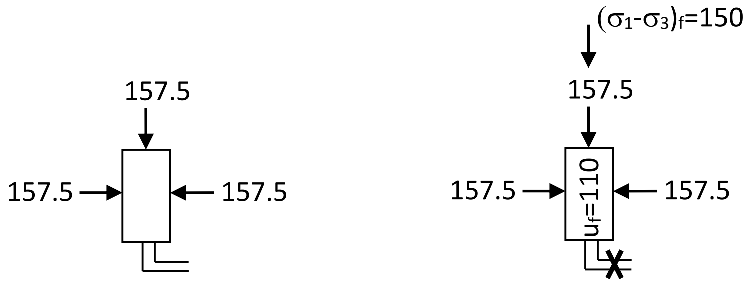
Solution 2

Specimen 1:

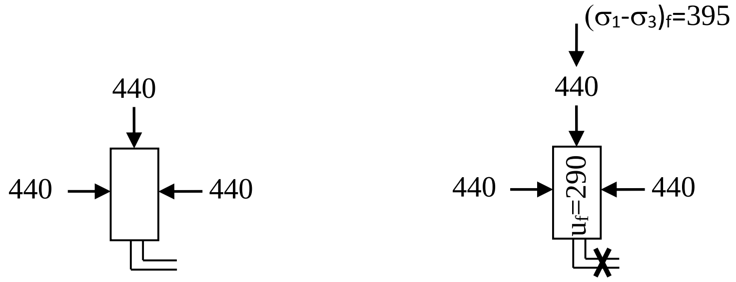
Specimen 1 is consolidated to its in-situ effective vertical stress:

$\sigma' = 2.5\times21+5\times(21-10)+5\times(20-10)=157.5$ kPa

From the graph, at the time of failure, $(\sigma_1-\sigma_3)_f = 150$ kPa and $u_f = 110$ kPa.



Specimen 2:



a) Since the question is asking effective shear strength parameters c' and ϕ', effective major and minor principle stress values should be calculated at failure, and used to draw Mohr Circles.

For specimen 1:

At the time of failure, σ_{3f} = 157.5 kPa and $(\sigma_1 - \sigma_3)_f$ = 150 kPa.

From these two, σ_{1f} = 157.5 + 150 = 307.5 kPa

Using pore water pressure at failure, effective stress values can be calculated as follows;

σ_{1f}' = 307.5 – 110 = 197.5 kPa and σ_{3f}' = 157.5 – 110 = 47.5 kPa

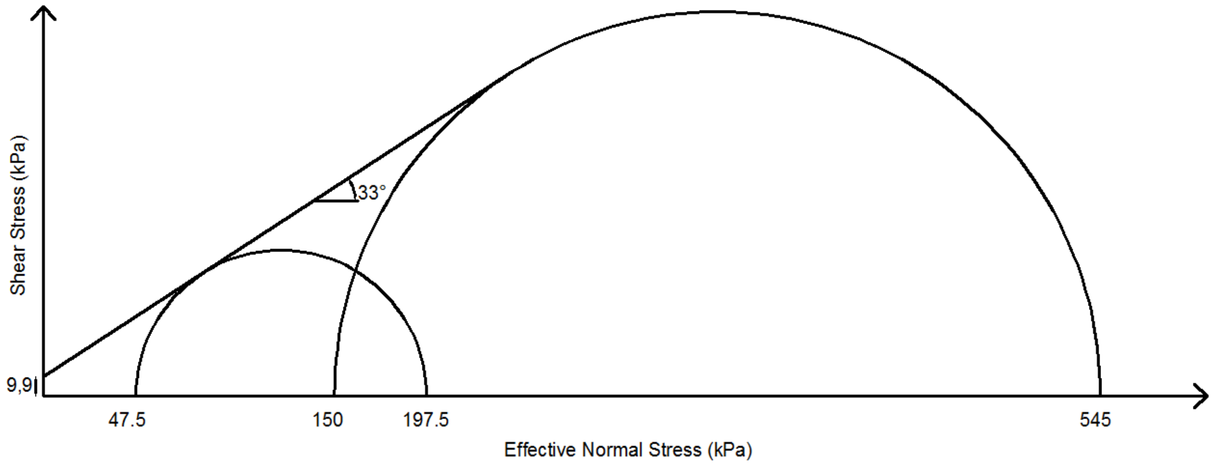
For specimen 2:

σ_{3f} = 440 kPa and $(\sigma_1 - \sigma_3)_f$ = 395 kPa. From these two, σ_{1f} = 440 + 395 = 835 kPa

Using pore water pressure at failure, effective stress values can be calculated as follows;

σ_{1f}' = 835 – 290 = 545 kPa and σ_{3f}' = 440 – 290 = 150 kPa

Using the calculated values, following Mohr's Circles are drawn and Mohr-Coulomb failure envelope is drawn tangent to these two circles. From the slope of this line we will obtain φ' while intercept value will be c'.



Hence, c' = 9.9 and φ' = 33°

b) Modified Mohr-Coulomb failure envelope is drawn by using the centers and radii of the Mohr's Circles in x and y axis, respectively. These values can be expressed by minor and major principle stresses as follows:

Center: $\frac{\sigma'_{1f}+\sigma'_{3f}}{2}$

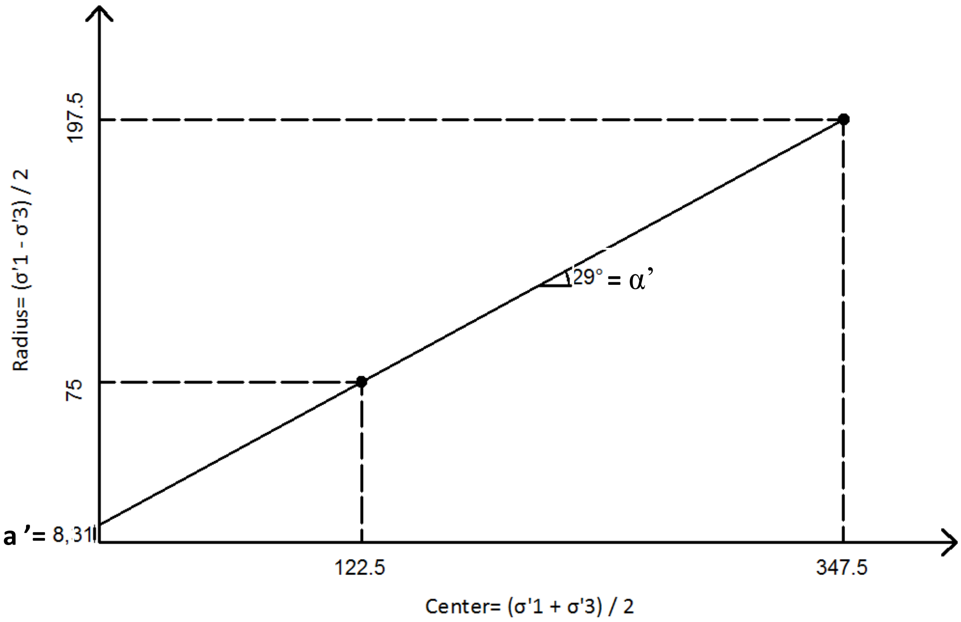
Radius: $\frac{\sigma'_{1f}-\sigma'_{3f}}{2} = \frac{\sigma_{1f}-\sigma_{3f}}{2}$

For specimen 1: Center: (197.5 + 47.5) / 2 = 122.5 kPa, and Radius: (197.5 – 47.5) / 2 = 75 kPa

For specimen 2: Center: (545 + 150) / 2 = 347.5 kPa, and Radius: (545 – 150) / 2 = 197.5 kPa

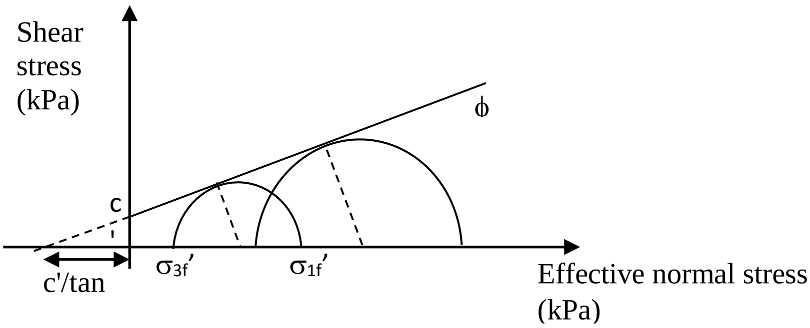
After plotting centers and radii values on x and y axes, best line (in this case line connecting these two points) are drawn. Following correlations are known between intercept & slope of this line and c' & φ';

c' = a' / cos φ' and tan α' = sin φ' where tan α' = slope of Modified Mohr-Coulomb failure envelope, a' = intercept of Modified Mohr-Coulomb failure envelope



Using above correlations and graph, **c' = 9.9 kPa and φ' = 33°**

c) Following trigonometric relation can be obtained from Mohr's circle.



$$sin\emptyset' = \frac{\left(\frac{\sigma'_{1f} - \sigma'_{3f}}{2}\right)}{\frac{c'}{tan\emptyset'} + \frac{\sigma'_{1f} + \sigma'_{3f}}{2}}$$

For Specimen 1:

$$sin\emptyset' = \frac{\left(\frac{197.5 - 47.5}{2}\right)}{\frac{c'}{tan\emptyset'} + \frac{197.5 + 47.5}{2}}$$

For Specimen 2:

$$sin\emptyset' = \frac{\left(\frac{545 - 150}{2}\right)}{\frac{c'}{tan\emptyset'} + \frac{545 + 150}{2}}$$

We have two equations and two unknowns (c' and φ'). Solve together. For easier calculation, let's call (c'/tanφ') = x, and equate the two equations to find x:

$$\frac{\left(\frac{197.5 - 47.5}{2}\right)}{x + \frac{197.5 + 47.5}{2}} = \frac{\left(\frac{545 - 150}{2}\right)}{x + \frac{545 + 150}{2}}$$

x = 15.26 and

$$sin\emptyset' = \frac{\left(\frac{197.5 - 47.5}{2}\right)}{15.26 + \frac{197.5 + 47.5}{2}}$$

φ' = 33 degrees

x = c'/tan φ' = 15.26

c'=9.9 kPa

<u>OR</u>

If we reorganize the terms in:

$$sin\emptyset' = \frac{\left(\frac{\sigma'_{1f} - \sigma'_{3f}}{2}\right)}{\frac{c'}{tan\emptyset'} + \frac{\sigma'_{1f} + \sigma'_{3f}}{2}}$$

$$\sigma'_{1f} - \sigma'_{3f} = 2 \cdot c' \cdot cos\emptyset' + (\sigma'_{1f} + \sigma'_{3f}) \cdot sin\emptyset'$$

When σ_{1f}' and σ_{3f}' values of specimen 1 and specimen 2 are put into this equation, following 2 equations are obtained:

$$197.5 - 47.5 = 2 \cdot c' \cdot cos\emptyset' + (197.5 + 47.5) \cdot sin\emptyset' \text{ .....} \qquad \text{Eqn.1}$$

$$545 - 150 = 2 \cdot c' \cdot cos\emptyset' + (545 + 150) \cdot sin\emptyset' \text{......} \quad \text{Eqn.2}$$

If we subtract Eq.1 from Eq.2, following is obtained:

$$245 = 450 \cdot sin\emptyset'$$

From this, φ' value can be calculated as 33°. By putting φ' = 33° into Eq.1 or Eq.2, c' can be calculated as 9.9 kPa.

Question 3 (10%)

For a sandy soil sample with relative density of 65 %, a 101.6 mm x 101.6 mm square shear box apparatus is intended to be used. Unfortunately, only available dead load is 100 N and the maximum shear load capacity of the apparatus is 50 N. If the shear strength parameters of the sandy soil sample are estimated as c= 1kPa and ϕ=33˚, discuss if the sample can be loaded until failure under the constraints of available dead load and equipment capacity.

Solution 3

$$\sigma_v = \frac{P}{A} = \frac{100 * 10^{-3}\ kN}{(101.6 * 10^{-3})^2\ m^2} = 9.7 \text{ kPa}$$

$$\tau = c + \sigma_v \tan\emptyset = 1 + 9.7 * \tan 33 = 7.3 \text{ kPa}$$

$$T = \tau.A = 7.3 * (101.6 * 10^{-3})^2 = 0.075 \text{ kN} = 75.4 \text{ N}$$

$$T = 75 \text{ N} < 50 \text{ N (Sample cannot be loaded or failure)}$$

Question 4 ((a): 6%; (b): 6%))

For a 15 m thick, fully-saturated clay layer, which is subjected to 1-D loading and overlain and underlain by permeable silty sand layers, the final consolidation settlement was estimated as 20 cm. Answer the following questions:

a) What is the degree of consolidation at the mid-depth of this clay layer when the amount of settlement reaches to 10 cm?

b) When the degree of consolidation at the mid-depth of this clay layer reaches up to 40 %, what is the average degree of consolidation of the saturated clay layer?

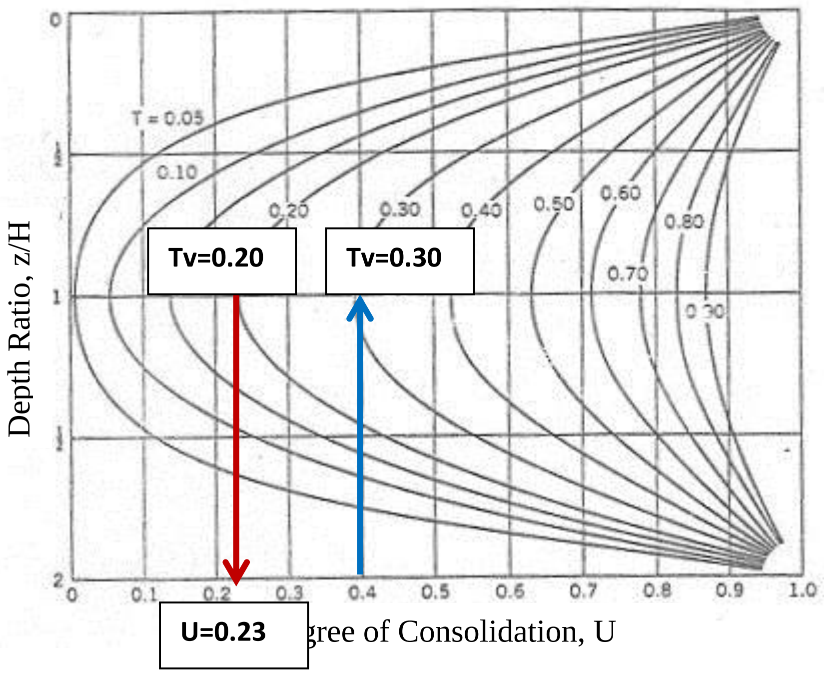
Useful formula: $T_v = \dfrac{C_v \cdot t}{d^2}$

Tv = time factor, t = time, d = drainage distance,

z = depth below the top of the clay layer,

H = one half of the thickness of the clay layer.

$\bar{U}$ %	T_v
0	0.000
10	0.008
15	0.018
20	0.031
25	0.049
50	0.196
55	0.238
60	0.286
65	0.342
70	0.403
75	0.477



Solution 4

a)

$$\bar{u}_{av} = \frac{10}{20} = 50\% \rightarrow T_v = 0.196 \approx 0.20$$

$u = 0.23 \rightarrow 23$ % from the graph, red arrow

b)

$u_{mid-depth} = 40\% \rightarrow T_v = 0.3$ from the graph, blue arrow

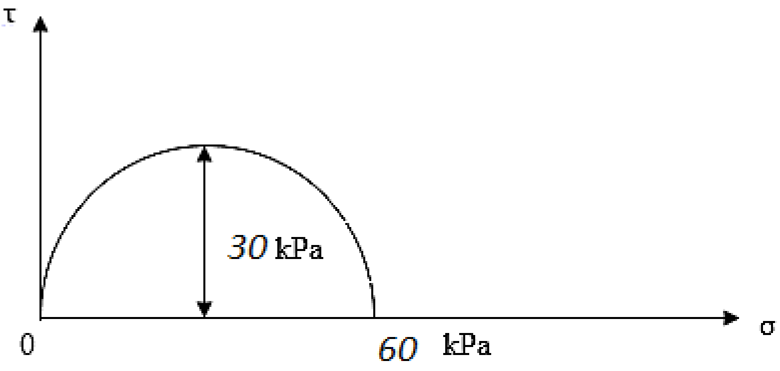
$T_v = 0.3 \rightarrow \bar{u}_{av} = 61$ % from the table, linear interpolation between 60 %- 65 %

Question 5 (6%)

If the unconfined compressive strength (UCS) of a fully saturated, fissured clayey soil sample is given as 60 kPA, estimate the major (σ_1) and minor (σ_3) total stresses at the time of failure.

Discuss how the shear strength value would change if you perform the test under a confining pressure of 50 kPA.

Solution 5



Unconfined compression strength, q_c = 60 kPa; $c_u=q_c/2$ = 30 kPa

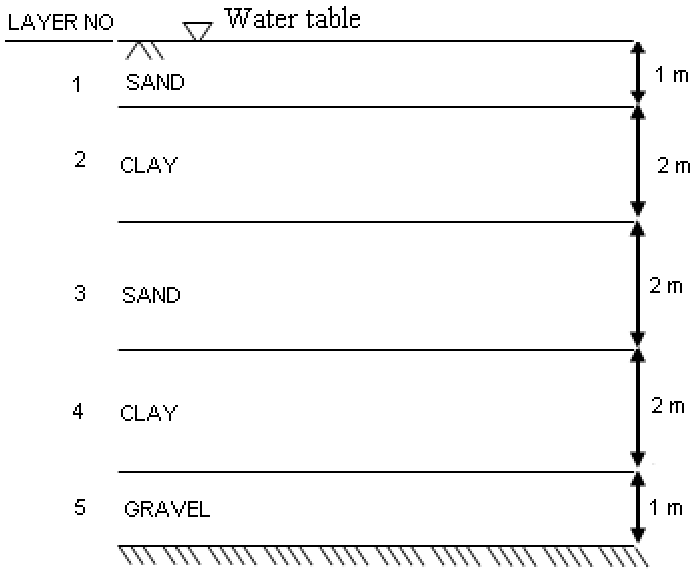
$\sigma_3 = 0$ kPa; $\sigma_1 = 60$ kPa,

since it is fissured if it tested under $\sigma_3 = 50$ kPa ;

the fissures in clay will be closed under confining pressure and most likely the the shear stregth value will increase.

Question 6 ((a): 8%; (b): 8%; (c): 8%)

A 60 kN/m^2 surcharge was applied over a large area on the soil profile given below.



a) Find the total final consolidation settlement for this profile assuming all layers for which consolidation settlement may be observed have the same coefficient of volume compressibility (0.0015 m^2/kN)

b) What is the average degree of consolidation when the total settlement is 5 cm for this soil profile?

c) Determine the coefficient of consolidation if 5 cm settlement was observed 2 years after the load was placed.

Solution 6

a) Layer 2 and Layer 4

$$Sc = Mv * \Delta\sigma * H$$

Mv_2=Mv_4= 0.0015 m^2/ kN $\quad$ $H_2 = H_4 = 2$m $\quad$ $\Delta\sigma$= 60 kN/m^2

$$S_{Final} = Mv * \Delta\sigma * H = 0.0015 * 60 * (2+2) = 0.36m = 36\ cm$$

b)

$$S_c = U * S_{Final}$$

$$U = \frac{S_C}{S_{Final}} = \frac{5}{36} = 0.138$$

c)

$$T_v = \frac{C_v \cdot t}{d^2}$$

S_{final}= $\frac{36}{2}$ =18 cm final settlement for each layer since they are identical.

S_c = $\frac{5}{2}$ =2.5 cm settlement observed at each layer after 2 years.

$U = \frac{S_C}{S_{Final}} = \frac{2.5}{18} = 0.138$ (average settlement of one layer)

U =0.138 < 0.60 Tv= $\left(\frac{\pi}{4}\right)*$ U^2 =$\left(\frac{\pi}{4}\right)*$ 0.138^2 = 0.015

$$C_V = \frac{Tv * \ d^2}{t} = \ \frac{(0.015) * (1)^2}{2} = 0.0075 \ m^2/years$$