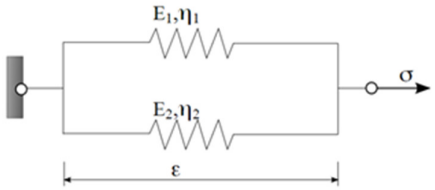
Example Problem: Numerical Modeling of Non-linear Elasticity

Consider the following rheological element of non-linear elasticity composed of two non-linear springs connected in parallel.



Owing to the equilibrium, the total stress σ is equal to the sum of stresses in the springs; that is,

$$\sigma = \sigma_1 + \sigma_2$$

where the stresses σ_1 and σ_2 are defined through the following constitutive equations

$$\sigma_i = E_i \, \varepsilon \, / \, [\eta_i^2 - \varepsilon^2] \qquad \text{for i=1, 2}$$

in terms of the material parameters E_1=100 MPa, η_1=0.08, E_2=50 MPa, and η_2=0.05.

For σ=1000 MPa, calculate the corresponding strain ε in the device and the stresses in each spring by using the Newton-Raphson method until the tolerance 10^{-6} is fulfilled for the percent relative approximate error. Start your iterations with ε^0= 0.

Solution Code:

```
clear all;
clc;
format long
sig = 1000;E1 = 100;E2 = 50;
eta1 = 0.08;eta2 = 0.05;
%
eps= 0.0;
era= 1.;
tol = 1e-6;
i =0;
fprintf('\n\n-------------------------------------------------------------\n');
while (era>tol && i<100)
sig1 = E1*eps/(eta1^2-eps^2);
sig2 = E2*eps/(eta2^2-eps^2);
tang1 = -E1*(eta1^2 + eps^2) / (eta1^2-eps^2)^2;
tang2 = -E2*(eta2^2 + eps^2) / (eta2^2-eps^2)^2;
res = sig - sig1 - sig2;
epsn = eps;
eps = eps - res/(tang1+tang2);
era = abs((eps-epsn)/eps)*100;
fprintf('Iteration:%2i, Residual= %10.8e, eps=%10.8e, era=%10.8e\n',...
        i, abs(res), eps,era);
i=i+1;
end
fprintf('-------------------------------------------------------------\n');
fprintf('eps= %10.8e, sig1=%10.8e, sig2=%10.8e\n', eps, sig1, sig2);
```

Output:

```
-------------------------------------------------------------
Iteration: 0, Residual= 1.00000000e+03, eps=2.80701754e-02, era=1.00000000e+02
Iteration: 1, Residual= 3.19950781e+02, eps=2.40154445e-02, era=1.68838472e+01
Iteration: 2, Residual= 3.67482414e+01, eps=2.34242925e-02, era=2.52367061e+00
Iteration: 3, Residual= 5.48083652e-01, eps=2.34152077e-02, era=3.87985524e-02
Iteration: 4, Residual= 1.23732228e-04, eps=2.34152057e-02, era=8.76289575e-06
Iteration: 5, Residual= 6.48014975e-12, eps=2.34152057e-02, era=4.59329108e-13
-------------------------------------------------------------
eps= 2.34152057e-02, sig1=4.00141721e+02, sig2=5.99858279e+02
```