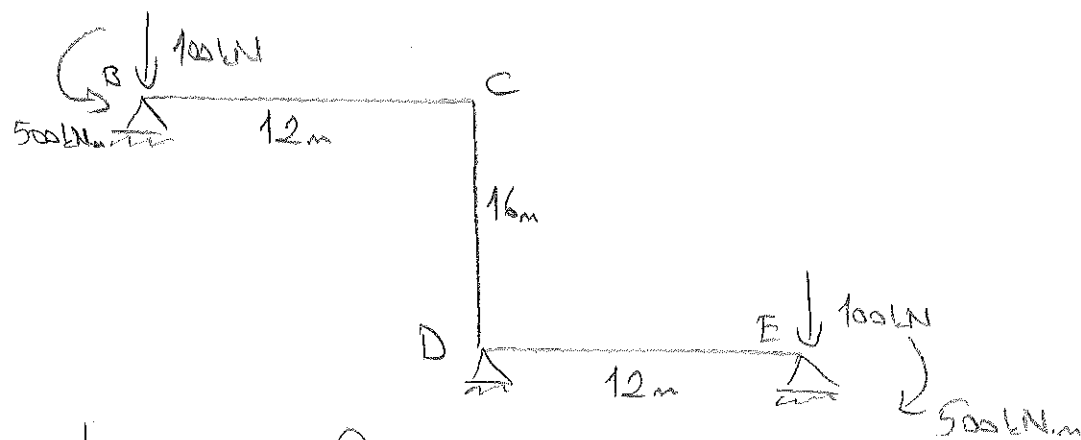


↓ Simplify it.



$k_{BC} = 0$ // for pin support

$$k_{CB} = \frac{4EI}{12} = \frac{EI}{3}$$

$$k_{CD} = \frac{4EI}{16} = \frac{EI}{4}$$

$$k_{DC} = \frac{4EI}{16} = \frac{EI}{4}$$

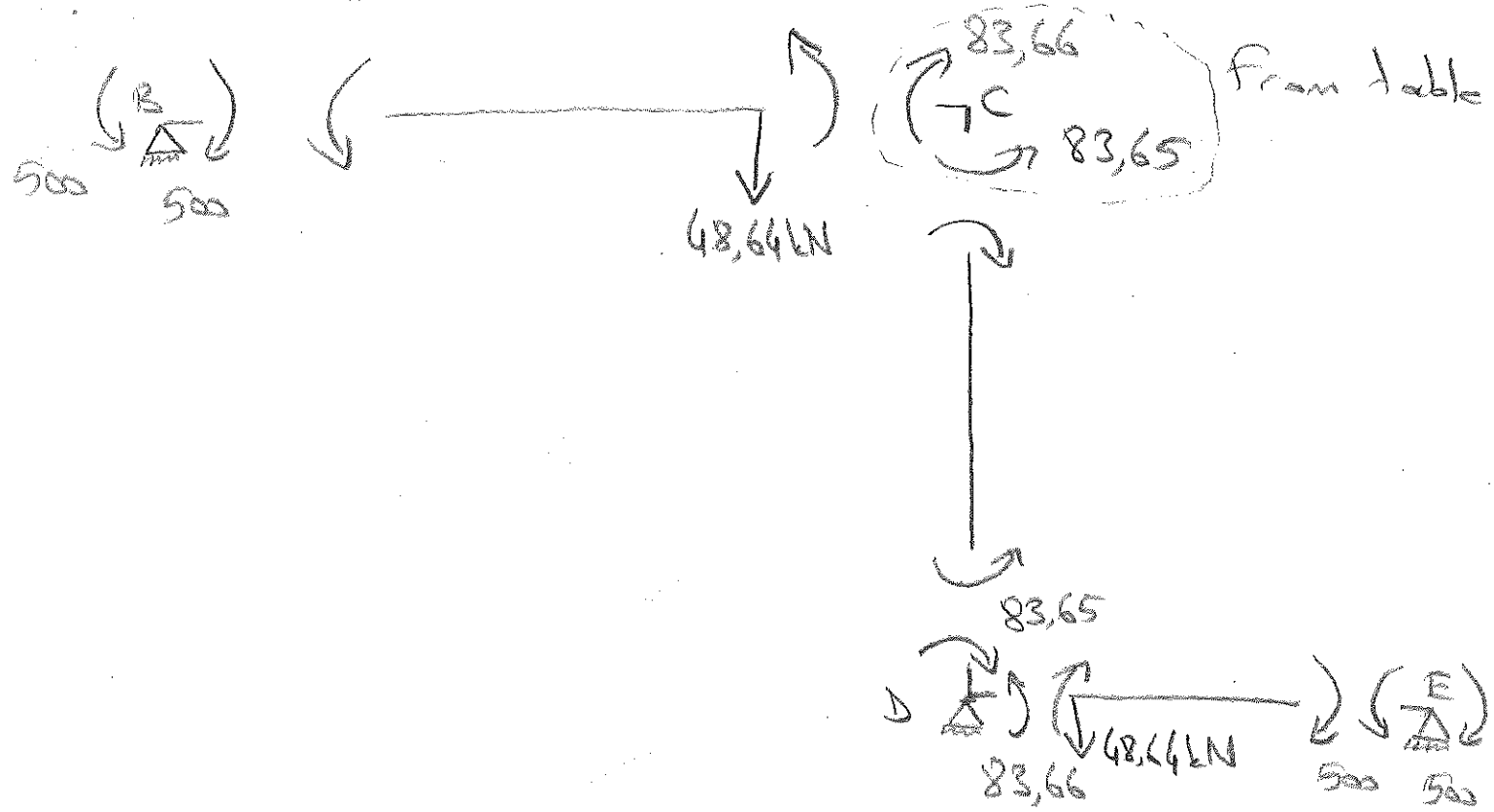
$$k_{DE} = \frac{4EI}{12} = \frac{EI}{3}$$

Calculate the distribution factors.

$$D_{CB} = \frac{EI/3}{EI/3 + EI/4} = 0,57 // \Rightarrow D_{CD} = 1 - D_{CB} = 0,43 //$$

$$D_{DC} = \frac{EI/4}{EI/3 + EI/4} = 0,43 // \Rightarrow D_{DE} = 1 - D_{DC} = 0,57 //$$

R		D		E									
DF	<table><tr><td>0</td><td>1</td></tr></table>	0	1	<table><tr><td>0,57</td><td>0,43</td></tr></table>	0,57	0,43	<table><tr><td>0,43</td><td>0,57</td></tr></table>	0,43	0,57	<table><tr><td>1</td><td>0</td></tr></table>	1	0	
0	1												
0,57	0,43												
0,43	0,57												
1	0												
P _{in}	BC	CB	CD	DC	DE	ED	P _{in}						
FEM	<table><tr><td>500</td><td>0</td></tr></table>	500	0	0	0	0	0	<table><tr><td>0</td><td>500</td></tr></table>	0	500			
500	0												
0	500												
Dist.	500	0	0	0	0	-500							
Co	0	250	0	0	-250	0							
Dist.	0	-142,5	107,5	107,5	142,5	0							
Co	-71,25	0	53,75	-53,75	0	71,25							
Dist.	71,25	-30,64	23,11	23,11	30,64	-71,25							
Co	-15,32	35,63	11,56	-11,56	-35,63	15,32							
Dist.	15,32	-26,3	-20,3	20,3	26,3	-15,32							
Co	-13,45	7,66	10,15	-10,15	-7,66	13,45							
Dist.	13,45	-10,15	-7,66	7,66	10,15	-13,45							
Co	-5,08	6,73	3,83	-3,83	-6,73	5,08							
Dist.	5,08	-6,02	-4,54	4,54	6,02	-5,08							
Co	-3,01	2,54	2,27	-2,27	2,54	3,01							
Dist.	3,01	-2,74	-2,07	2,07	2,74	-3,01							
Co	-1,37	1,51	1,04	-1,04	-1,51	1,37							
Dist.	-1,37	-1,45	-1,1	1,1	1,45	-1,37							
Co	-0,73	0,6	0,55	-0,55	-0,6	0,73							
Dist.	0,73	-0,71	-0,53	0,53	0,71	-0,73							
Co	-0,36	0,37	0,27	-0,27	0,37	0,36							
Dist.	0,36	-0,36	-0,26	0,26	0,36	-0,36							
M _A	500	83,66	-83,65	83,65	-83,66	-500							



$$R_D = 2 \cdot 48.64 = 97.28 \text{ kN} \quad \downarrow$$

II. way: Use modified stiffness coefficients while calculating distribution factors.

$$k_{CB} = \frac{3EI}{12} = \frac{EI}{4}$$

$$k_{CD} = \frac{4EI}{16} = \frac{EI}{4}$$

$$k_{DC} = \frac{EI}{4}$$

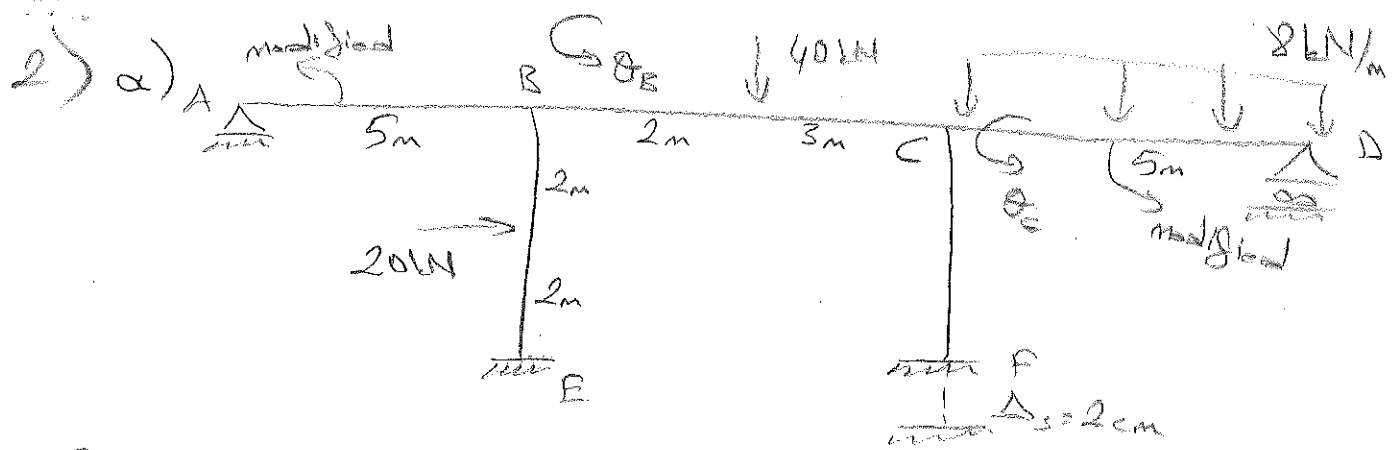
$$k_{DE} = \frac{3EI}{12} = \frac{EI}{4}$$

$$D_{CB} = \frac{EI/4}{EI/4 + EI/4} = 0.5 \Rightarrow D_{CD} = 1 - 0.5 = 0.5$$

$$D_{DC} = \frac{EI/4}{EI/4 + EI/4} = 0.5 \Rightarrow D_{DE} = 1 - 0.5 = 0.5$$

DF	B		C		D		E	
	0	1	0,5	0,5	0,5	0,5	1	0
	P _{in}	BC	CB	CD	DC	DE	ED	P _{in}
FEM	-500	0	0	0	0	0	0	500
Dist.	500		0	0	0	0	-500	
CO	N ₀		250	0	0	-250		N ₀
Dist.	-		-125	-125	125	125	-	-
CO	-		0	62,5	-62,5	0	-	-
Dist.	-		-31,25	-31,25	31,25	31,25	-	-
CO	-		0	15,63	-15,63	0	-	-
Dist.	-		-7,81	-7,81	7,81	7,81	-	-
CO	-		0	3,91	-3,91	0	-	-
Dist.	-		-1,96	-1,96	1,96	1,96	-	-
CO	-		0	0,98	-0,98	0	-	-
Dist.	-		-0,49	-0,49	0,49	0,49	-	-
CO	-		0	0,25	-0,25	0	-	-
Dist.	-		-0,13	-0,13	0,13	0,13	-	-
CO	-						-	-
Dist.	-						-	-
M _A	500		83,36	-83,37	83,37	-83,36	-500	

$$R_D = 2 \cdot \frac{500 + 83,36}{12} = 97,23 \text{ kN} \downarrow$$



There are two unknown displacements θ_B and θ_C .

$$\sum M_B = 0 \Rightarrow M_{BA} + M_{BC} + M_{BE} = 0$$

$$\underbrace{\frac{3EI}{5} \cdot (\theta_B)}_{M_{BA}} + \underbrace{\frac{2EI}{5} \cdot \left(2\theta_B + \theta_C + \frac{3 \cdot 0,02}{5} \right)}_{M_{BC}} + \frac{40 \cdot 3^2 \cdot 2}{5^2} +$$

$$\underbrace{\frac{2EI}{4} \cdot (2\theta_B + 0) - \frac{20 \cdot 4}{8}}_{M_{BE}} = 0$$

$$48.000 \cdot \theta_B + 8000 \cdot \theta_C = -114,8 \quad (1)$$

$$\sum M_C = 0 \Rightarrow M_{CB} + M_{CD} + M_{CF} = 0$$

$$\underbrace{\frac{2EI}{5} \cdot \left(2\theta_C + \theta_B + \frac{3 \cdot 0,02}{5} \right) - \frac{40 \cdot 2^2 \cdot 3}{5^2}}_{M_{CB}} + \underbrace{\frac{3EI}{5} \cdot \left(\theta_C - \frac{0,02}{5} \right) +}_{M_{CD}}$$

$$\underbrace{\frac{8 \cdot 5^2}{8}}_{M_{CF}} + \underbrace{\frac{2EI}{4} \cdot (2\theta_C + 0)}_{M_{CF}} = 0$$

$$8000 \cdot \theta_B + 48.000 \cdot \theta_C = -53,8 \quad (2)$$

Solve ① and ② simultaneously.

$$\theta_B = -2,268 \cdot 10^{-3} \text{ rad} //$$

$$\theta_C = -7,429 \cdot 10^{-4} \text{ rad} //$$

$$M_{AB} = 0 \text{ N.m} //$$

$$M_{BA} = -27,216 \text{ kN.m} //$$

$$M_{BC} = 82,569 \text{ kN.m} //$$

$$M_{BE} = -55,36 \text{ kN.m} //$$

$$M_{EB} = -12,68 \text{ kN.m} //$$

$$M_{CB} = 46,77 \text{ kN.m} //$$

$$M_{CD} = -31,915 \text{ kN.m} //$$

$$M_{DC} = 0 \text{ N.m} //$$

$$M_{CF} = -14,858 \text{ kN.m} //$$

$$M_{FC} = -7,429 \text{ kN.m} //$$

b) Resolve the same structure by using moment distribution method.

$$k_{BA} = \frac{3EI}{5} //$$

$$k_{BE} = \frac{4EI}{6} //$$

$$k_{BC} = \frac{4EI}{5} //$$

$$\left. \begin{array}{l} k_{BA} = \frac{3EI}{5} \\ k_{BE} = \frac{4EI}{6} \\ k_{BC} = \frac{4EI}{5} \end{array} \right\} \begin{array}{l} DF_{BA} = 0,25 \\ DF_{BE} = 0,417 \\ DF_{BC} = 0,333 \end{array} //$$

$$DF_{AB} = 1 // \text{ for pin support}$$

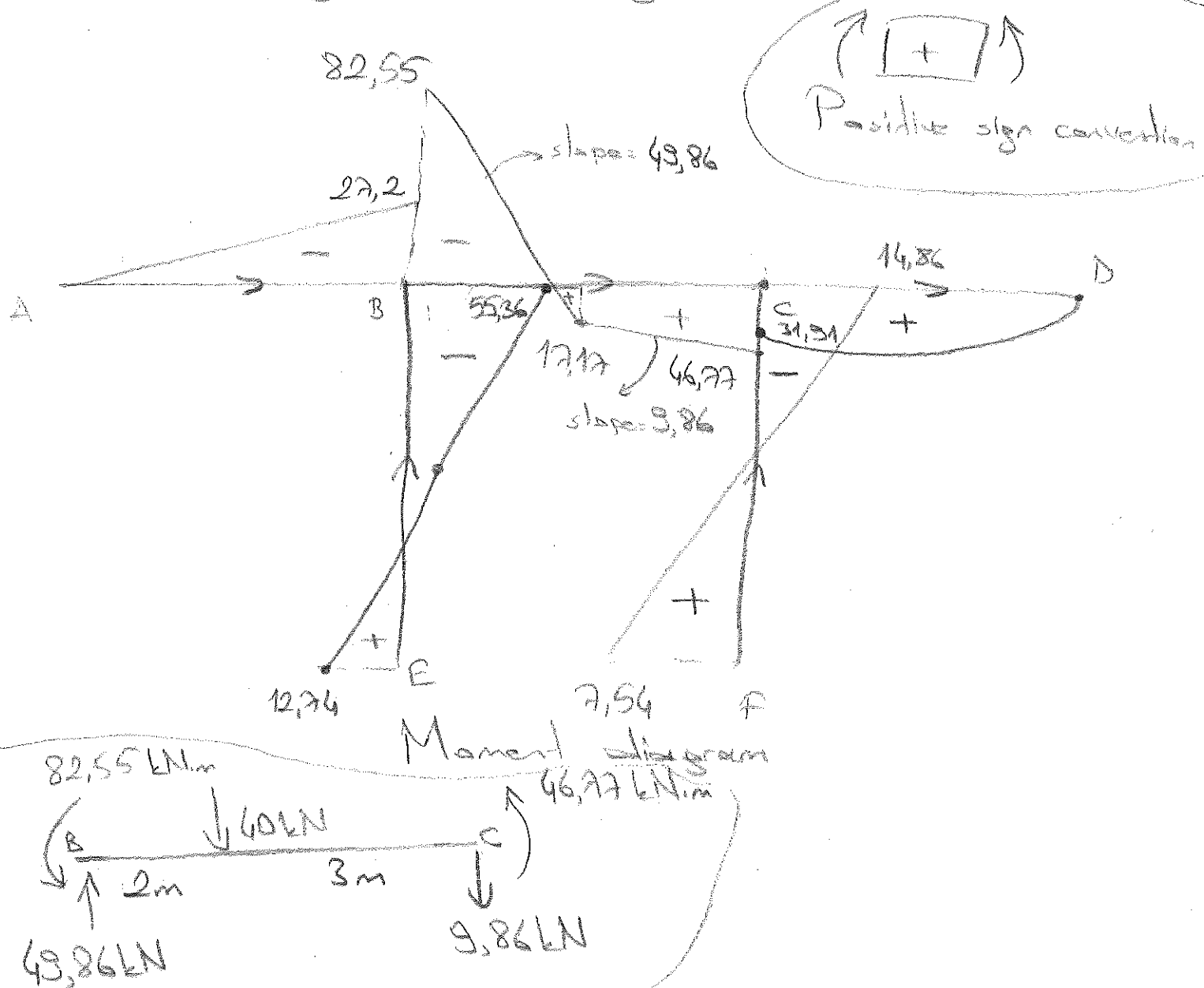
$$DF_{DC} = 1 // \text{ for pin support}$$

A		B		C		D		E	
DF	1011	03	033	033	033	033	033	033	033
P _n		BA	BC	CB	CD	DC	DE	ED	FE
FEM	0101	0	124,8	76,8	-23	0	0	0	0
Dist	0	-28,7	-38,23	-19,92	-13,45	0	0	0	0
CO	N ₀	0	-8,36	-13,12	0	0	0	0	0
Dist	—	2,24	2,98	6,37	4,98	—	—	—	—
CO	—	0	3,13	1,42	0	—	—	—	—
Dist	—	-0,80	-1,06	-0,50	-0,37	—	—	—	—
CO	—	0	-0,25	-0,53	0	—	—	—	—
Dist	—	0,06	0,08	0,18	0,13	—	—	—	—
M _t	0	-27,2	82,55	46,77	-31,91	0	0	0	0

-55,36	-12,74 M _t	0	0	0	0	0	0	0	0
0,10	0	0	0	0	0	0	0	0	0
-1,33	0	0	0	0	0	0	0	0	0
3,74	0	0	0	0	0	0	0	0	0
-47,87	0	0	0	0	0	0	0	0	0
-10	0	0	0	0	0	0	0	0	0
0,47	0	0	0	0	0	0	0	0	0
0,47	0	0	0	0	0	0	0	0	0
-22,43	0	0	0	0	0	0	0	0	0
0,33	0	0	0	0	0	0	0	0	0
0,29	0	0	0	0	0	0	0	0	0
-14,86	0	0	0	0	0	0	0	0	0
-7,54 M _t	0	0	0	0	0	0	0	0	0

The member end moments are shown as M_t in the above table.

c) The bending moment diagram will be



Note The positive sign convention for SAP2000 is that M_3 is positive when it creates compression on the positive 2 axis.

