Determining engineering properties of construction materials plays a crucial role when designing structures against natural hazards such as earthquakes, landslides, etc. In the last two decades, rubber has been used in many engineering applications such as base isolator against earthquake. As a matter of fact, the technology of seismic isolation using rubber has become the preferred structural design technique for buildings and bridges in earthquake prone regions. Figure 1 shows a typical application of rubber as seismic isolator between steel platens.

Figure 1: Application of Rubber Seismic Isolation

The videos “Earthquake Protective Foundation - base isolation” and “LBR Test” are a typical application of rubber under shear:

http://www.youtube.com/watch?v=ChaqMDc4ces and

http://www.youtube.com/watch?NR=1&feature=endscreen&v=2yXgu4aS8HE

To properly determine the mechanical characteristics of an engineering material, laboratory tests are required, through which stress-strain relations can be obtained under controlled conditions. Uniaxial tensile test is one of the most common experimental techniques to obtain mechanical properties of materials.

These types of experiments are extremely expensive to perform. In addition, as an engineer you need to know the stress-strain behavior of materials to be used in simulations.

In this assignment, *CE305 Ltd.* hires you as a consultant. You are required to develop a material model for the uniaxial tensile test performed on a rubber specimen. To this end, the following constitutive model of non-linear elasticity has been chosen.

$$\sigma = \sigma_\infty (1 - \exp(-\tau\varepsilon)) \qquad \text{(Equation 1)}$$

In this saturation-type model, σ_∞ denotes the saturated stress value as the uniaxial strain approaches to infinity and the parameter τ controls the rate of non-linear saturation.

In this lab session your specific objective is to determine the material properties σ_∞ and τ by using ***the non-linear regression analysis.***

PART A:

Note: *Please use MATLAB editor when coding and do not forget to write "clear all" and "clc" at first.*

You should download the archive file 'lab8_data.zip' containing stress-strain data from CE305 website, unzip and copy it to the current directory of MATLAB (which is usually MATLAB folder under MY DOCUMENTS). The copied data file (`'treolar_uni.dat'`) consists of Treolar's Uniaxial Tensile Test data. The first column and second columns contain "Strain ε [-]" and "Stress σ [MPa]" values, respectively. To analyze the given data, use `'dlmread'` command described last week and assign the data to an array called `'data'`. Note that the data are separated by semi-colons `';'`.

You now have the array 'data' which has 12 rows and 2 columns.

PART B:

In this part, you are expected to perform the algorithm for non-linear regression which is also called *Gauss-Newton method.* The formulation is given below:

$$\{\Delta A\} = \left[\left[Z_j\right]^T \left[Z_j\right]\right]^{-1} \left[\left[Z_j\right]^T \left[D\right]\right] \qquad \text{(Equation 2)}$$

$$\left[Z_j\right] = \begin{bmatrix} \partial f_1 / \partial a_0 & \partial f_1 / \partial a_1 \\ . & . \\ . & . \\ \partial f_n / \partial a_0 & \partial f_n / \partial a_1 \end{bmatrix} \qquad \text{(Equation 3)}$$

$$\left[D\right] = \begin{Bmatrix} y_{1,observ.} - f(x_1) \\ y_{2,observ.} - f(x_2) \\ . \\ . \\ y_{n,observ.} - f(x_n) \end{Bmatrix} \qquad \text{(Equation 4)}$$

$$\{\Delta A\} = \begin{bmatrix} \Delta a_0 \\ \Delta a_1 \end{bmatrix} \qquad \text{(Equation 5)}$$

For the problem at hand, you can take $a_0 = \sigma_\infty$, $a_1 = \tau$; $f(x) = \sigma$ and $x = \varepsilon$, hence, Equation 1 becomes:

$$f(x) = a_0(1 - \exp(-a_1 x)) \qquad \text{(Equation 6)}$$

Partial derivatives given below will be used to form the matrix $\left[Z_j\right]$.

$$\partial f_1 / \partial a_0 = 1 - \exp(-a_1 x) \qquad \text{(Equation 7)}$$
$$\partial f_1 / \partial a_1 = a_0 x \exp(-a_1 x) \qquad \text{(Equation 8)}$$

In the non-linear regression analysis, a first step of the procedure is three-fold: First, we assume initial values for a_0 and a_1. We then update a_0 and a_1 by adding the increment $\{\Delta A\}$ through Equations 2 and 9. In the last step, as in Equation 10, the percentage relative error is calculated. The second and third steps are repeated until the error falls below an acceptable error threshold.

$$\begin{aligned} a_{0,j+1} &= a_{0,j} + \Delta a_0 \\ a_{1,j+1} &= a_{1,j} + \Delta a_1 \end{aligned} \qquad \text{(Equation 9)}$$

$$\left|\varepsilon_a\right|_k = \max\left(\left(\left|\frac{a_{k,j+1} - a_{k,j}}{a_{k,j+1}}\right| * 100\right)\%\right) \qquad \text{(Equation 10)}$$

Now, use initial guesses of a_0 =10.0 , a_1 =1.0 and $\left|\varepsilon_a\right|_{tolerance} = 10^{-15}$. Please report "Error, a0 and a1" for each iteration using the command `fprintf` in the following format:

```
Error= XXX a0= XXX a1= XXX
```

PART C:

In your code, report σ_∞, saturated stress, and τ , rate of non-linear saturation, in the following format using the command `fprintf`.

```
Saturated stress is XXX.
The rate of non-linear saturation is XXX.
```

PART D:

Plot the scattered data and fitted line on the same graph (*You can use the command 'hold on' to plot both)*. Use the command `scatter` for plotting the given data as follows.:

```
scatter(x,y)
```