CE383 STRUCTURAL ANALYSIS

SPRING 2015

HOMEWORK 2

DUE: 14.04.2015 @ 13.00

Homework assignments submitted past the deadline will be accepted subject to a 20% deduction per day.

Q1)

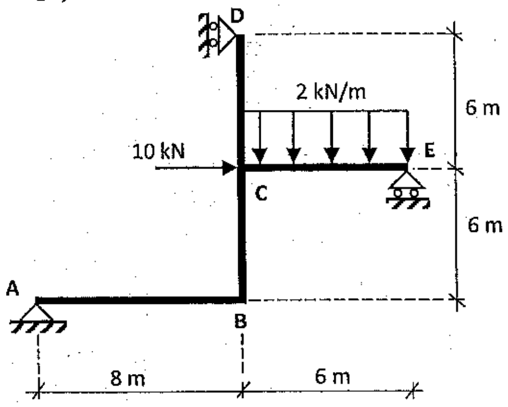


Figure 1
(All members with EI)

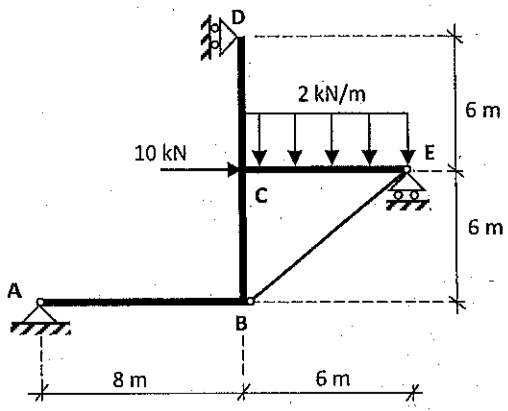


Figure 2
(Brace with EA inserted)

The structure in Figure 1 has four members that are rigidly connected to each other as shown. These four members have flexural rigidity EI and negligible axial deformations. This structure is then braced as shown in Figure 2 with axial stiffness EA. You are asked to answer the following listed questions.

a) Calculate the support reactions of the structure in Figure 1 by using force method of analysis.

b) Calculate the support reactions of the structure in Figure 2 by using slope deflection method.

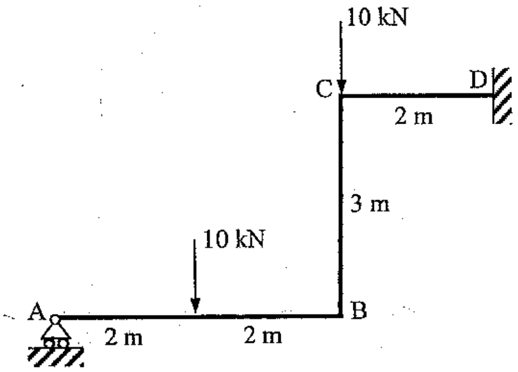
c) For structure in Figure 2, determine the support reactions by using force method of analysis. Take $EI_{frame} = 4EA_{truss}$

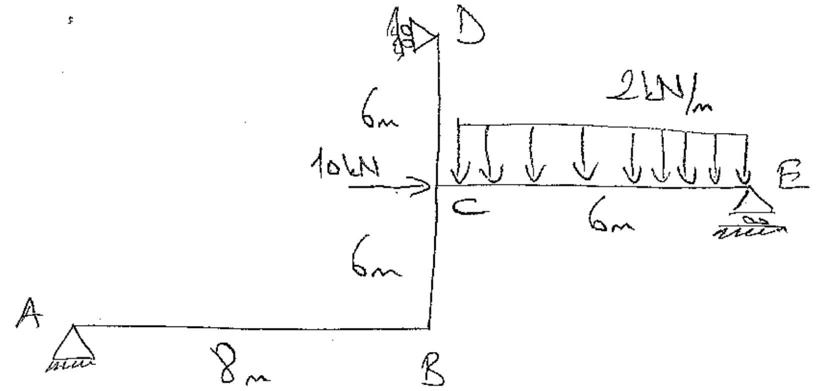
Q2) For the given structure with axially rigid members and constant EI, you are asked to answer the following:

a) Calculate the support reactions of the structure by using force method of analysis.

b) Calculate the support reactions of the structure by using slope deflection method.

c) Plot axial force, shear force and bending moment diagrams for the entire structure. Clearly show your sign convention for your plots.



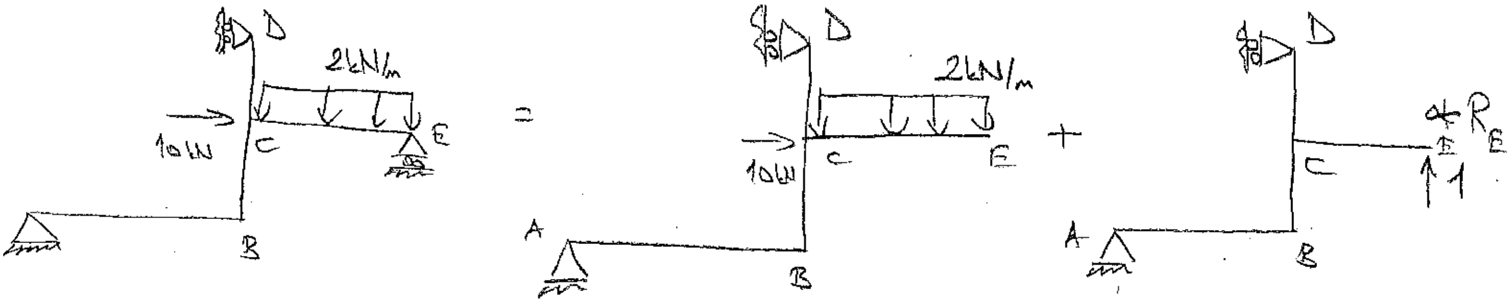


$$D_T = 3n + r - e - x$$

$$D_T = 3.0 + 4 - 3 - 0$$

$$D_T = 1 \Rightarrow 1^{st} \text{ degree indeterminate}$$

Remove support at E.

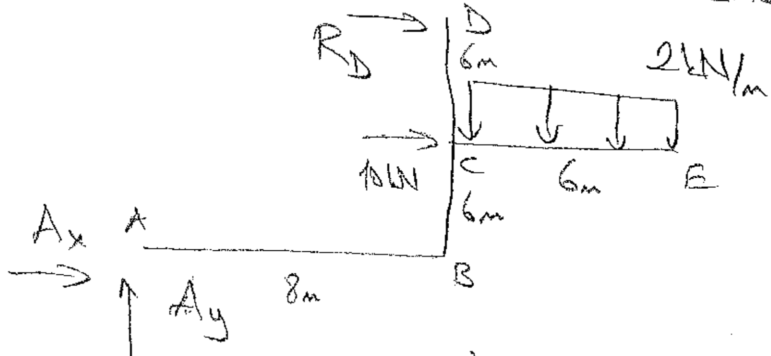


Basic structure Fictitious structure

The compatibility equation is

$$\Delta_E = \Delta_{E_0} + f \cdot R_E = 0$$

Solve both structures to obtain moment diagrams.



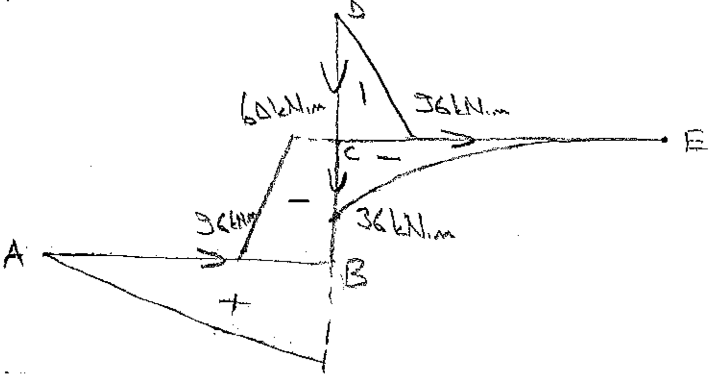
$$\Sigma F_y = 0 \Rightarrow A_y = 2.6 = 12\text{kN} \uparrow$$

$$\Sigma M_A = 0 \Rightarrow 6.10 + 12.R_D + 2.6.11 = 0$$

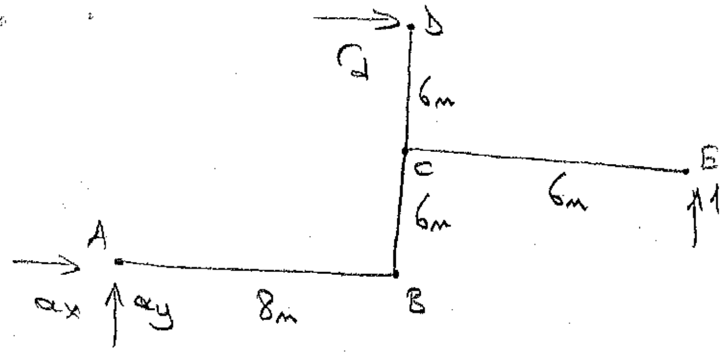
$$R_D = -16\text{kN} \leftarrow$$

$$\Sigma F_x = 0 \Rightarrow -16 + 10 + A_x = 0$$

$$A_x = 6\text{kN} \rightarrow$$



M



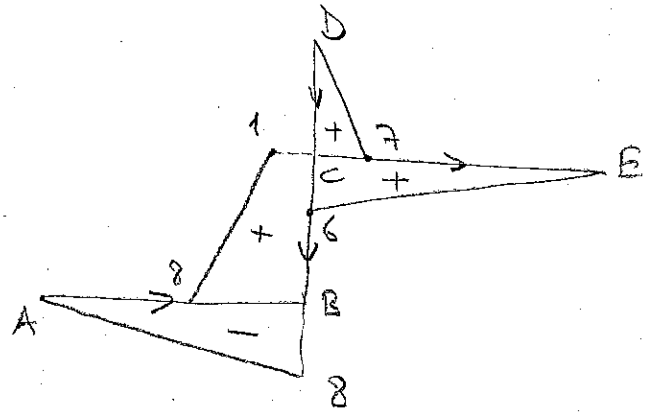
$\Sigma F_y = 0 \Rightarrow a_y = -1$ ↓

$\Sigma M_A = 0 \Rightarrow 12 \cdot r_d - 1 \cdot 14 = 0$

$r_d = 1{,}17$ →

$\Sigma F_x = 0 \Rightarrow a_x + r_d = 0$

$a_x = -1{,}17$ ←

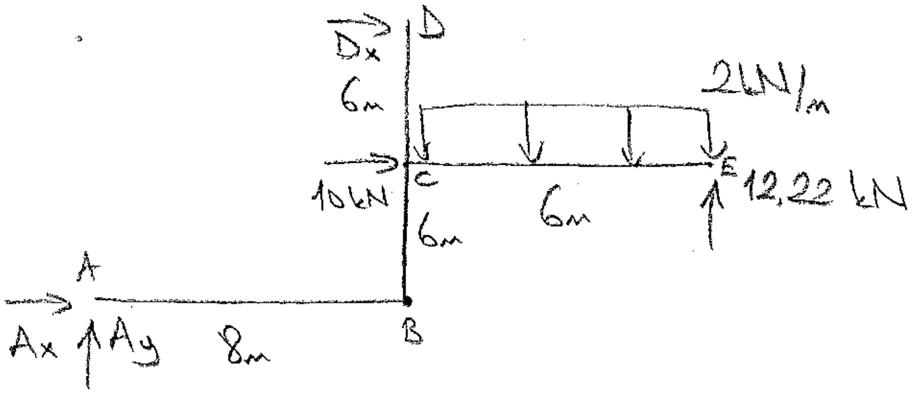


$$\Delta_{E_0} = \int \frac{M \cdot \underline{m} \cdot dx}{EI} = \Big[\underbrace{\tfrac{1}{3} \cdot 8 \cdot -8 \cdot 96}_{} + \underbrace{\Big(\tfrac{1}{6} \cdot 6 \cdot \big(2 \cdot 1 \cdot -60 + 1 \cdot -96 + 8 \cdot -60 + 2 \cdot 8 \cdot -96\big)\Big)}_{} + \underbrace{\tfrac{1}{3} \cdot 6 \cdot 7 \cdot -96}_{} + \underbrace{\tfrac{1}{4} \cdot 6 \cdot 6 \cdot -36}_{}\Big] \cdot \frac{1}{EI} = \frac{-5.948}{EI}$$

$$\delta = \int \frac{m \cdot m \cdot dx}{EI} = \Big[\underbrace{\tfrac{1}{3} \cdot 8 \cdot -8 \cdot -8}_{} + \underbrace{\Big(\tfrac{1}{6} \cdot 6 \cdot \big(2 \cdot 1 \cdot 1 + 1 \cdot 8 + 8 \cdot 1 + 2 \cdot 8 \cdot 8\big)\Big)}_{} + \underbrace{\tfrac{1}{3} \cdot 6 \cdot 7 \cdot 7}_{} + \underbrace{\tfrac{1}{3} \cdot 6 \cdot 6 \cdot 6}_{}\Big] \cdot \frac{1}{EI} = \frac{486{,}67}{EI}$$

$$\Delta_{E_0} + \delta \cdot R_E = 0$$

$$\frac{-5.948}{EI} + \frac{486{,}67}{EI} \cdot R_E = 0 \Rightarrow R_E = 12{,}22 \text{ kN} \uparrow$$



$$\Sigma F_y = 0 \Rightarrow A_y + 12{,}22 - 2.6 = 0$$

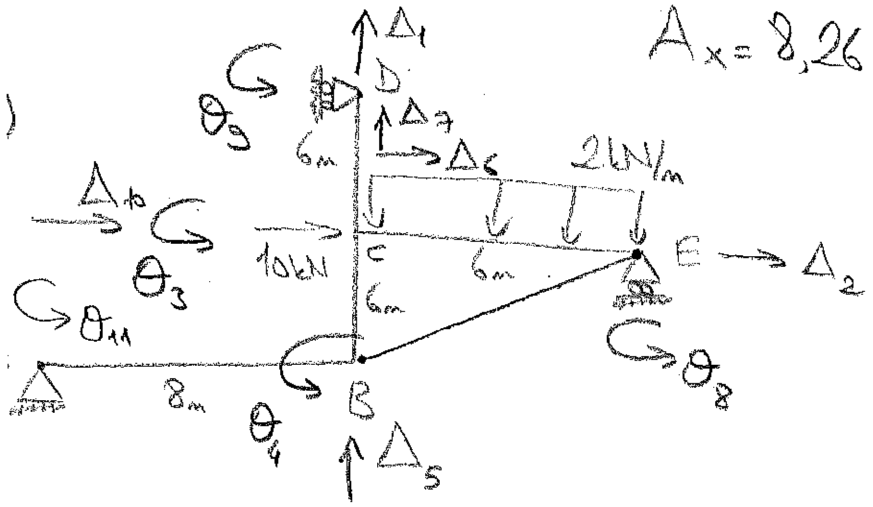
$$A_y = -0{,}22\ kN \ \downarrow$$

$$\Sigma M_A = 0 \Rightarrow 6.10 + D_x.12 + 2.6.11 - 12{,}22.14 = 0$$

$$D_x = -1{,}74\ kN \ \leftarrow$$

$$\Sigma F_x = 0 \Rightarrow A_x - 1{,}74 + 10 = 0$$

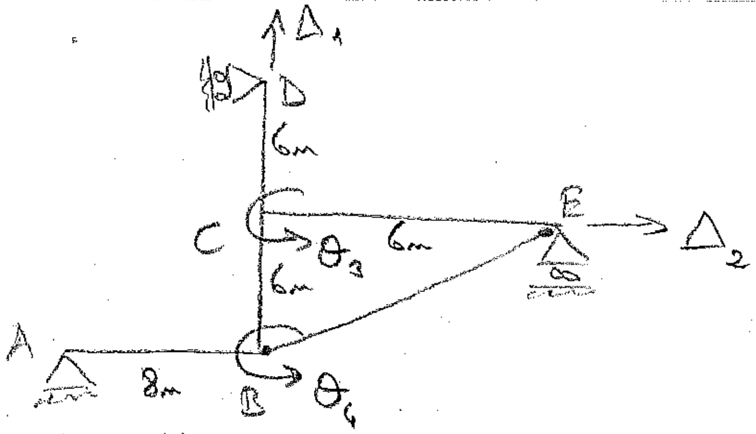
$$A_x = 8{,}26\ kN \ \leftarrow$$



$\Delta_5 = \Delta_7 = \Delta_1$ as member BC and CD are axially rigid.

$\Delta_{10} = \Delta_2$ as member CE is axially rigid.

θ_{11}, θ_9 and θ_8 can be eliminated by utilizing modified equations for members AB, CD and CE, respectively. Therefore, the active dof's are Δ_1, Δ_2, θ_3 and θ_4.



2 translational dof's require 2 shear equilibriums.
2 rotational dof's require 2 moment equilibriums.

Moment equilibrium at joint B =

$$M_{BA} = \frac{3EI}{8} \cdot \left(\theta_4 - \frac{\Delta_1}{8}\right)$$

$$M_{BC} = \frac{2EI}{6} \cdot \left(2\theta_4 + \theta_3 + \frac{3\Delta_2}{6}\right)$$

At joint B, the equilibrium is satisfied as follows:

$$M_{BA} + M_{BC} = 0$$

$$\frac{3EI}{8} \cdot \left(\theta_4 - \frac{\Delta_1}{8}\right) + \frac{2EI}{6} \cdot \left(2\theta_4 + \theta_3 + \frac{3\Delta_2}{6}\right) = 0$$

$$EI \cdot (-0{,}047 \cdot \Delta_1 + 0{,}167 \cdot \Delta_2 + 0{,}333 \cdot \theta_3 + 1{,}042 \cdot \theta_4) = 0 \quad (1)$$

Moment equilibrium at joint C =

$$M_{CB} = \frac{2EI}{6} \cdot \left(2\theta_3 + \theta_4 + \frac{3\Delta_2}{6}\right)$$

$$M_{CE} = \frac{3EI}{6} \cdot \left(\theta_3 + \frac{\Delta_1}{6}\right) + \frac{2 \cdot 6^2}{8}$$

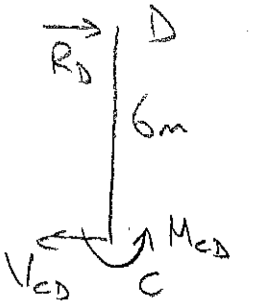
$$M_{CD} = \frac{3EI}{6} \cdot \left(\theta_3 - \frac{\Delta_2}{6}\right)$$

At joint C, the equilibrium is satisfied as follows:

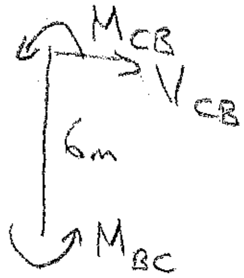
$$M_{CB} + M_{CD} + M_{CE} = 0$$

$$EI \cdot (0{,}083 \cdot \Delta_1 + 0{,}083 \cdot \Delta_2 + 1{,}667 \cdot \theta_3 + 0{,}333 \cdot \theta_4) = -9 \quad (2)$$

$$V_{CD} = \frac{M_{CD}}{6} = \frac{3EI}{36}\cdot\left(\theta_3 - \frac{\Delta_2}{6}\right)$$



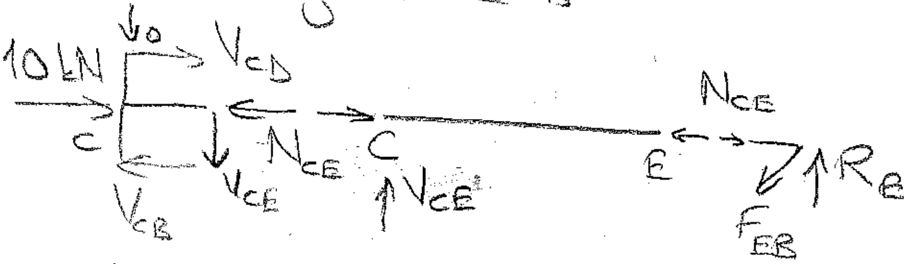
$$V_{CB} = \frac{M_{BC}+M_{CB}}{6}$$



$$V_{CB} = \frac{1}{6}\cdot\left(\frac{2EI}{6}\cdot\left(2\theta_4+\theta_3+\frac{3\Delta_2}{6}\right) + \frac{2EI}{6}\cdot\left(2\theta_3+\theta_4+\frac{3\Delta_2}{6}\right)\right)$$

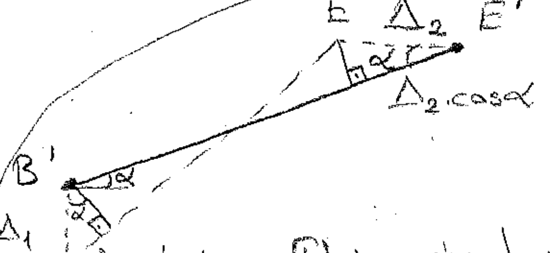
$$V_{CB} = \frac{EI}{36}\cdot\left(6\theta_3+6\theta_4+2\Delta_2\right)$$

The force equilibrium at joint C is



$$\Sigma F_x = 0 \Rightarrow 10 + V_{CD} - V_{CB} - N_{CE} = 0$$

$$10 + \frac{3EI}{36}\cdot\left(\theta_3 - \frac{\Delta_2}{6}\right) - \frac{EI}{36}\cdot\left(6\theta_3+6\theta_4+2\Delta_2\right) - \frac{EA}{6\sqrt{2}}\cdot\left(\Delta_2\cdot\cos\alpha - \Delta_1\cdot\sin\alpha\right)\cdot\cos\alpha = 0$$



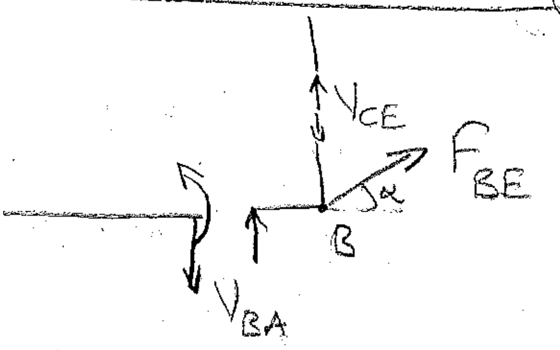
It is assumed as tensile.

$$F_{EB} = \frac{EA}{L}\cdot\left(\Delta_2\cdot\cos\alpha - \Delta_1\cdot\sin\alpha\right)$$

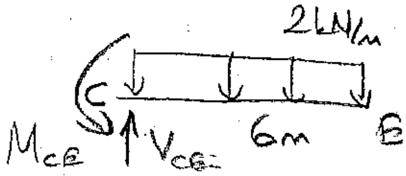
$$EI\cdot(0{,}015\cdot\Delta_1 - 0{,}084\cdot\Delta_2 - 0{,}083\cdot\theta_3 - 0{,}167\cdot\theta_4) = -10 \quad (3)$$

$$N_{CE} = F_{EB}\cdot\cos\alpha$$

Force equilibrium at joint B =



$$V_{BA} = \frac{M_{BA}}{8} = \frac{3EI}{64}\cdot\left(\theta_4 - \frac{\Delta_1}{8}\right)$$



$$V_{CE} = \frac{M_{CE}+26.3}{6} = \frac{3EI}{36}\cdot\left(\theta_3 + \frac{\Delta_1}{6}\right) + \frac{9}{6} + 6$$

$$\Sigma F_y = 0 \Rightarrow V_{BA} + F_{BE}\cdot\sin\alpha - V_{CE} = 0$$

$$\frac{3EI}{64}\cdot\left(\theta_4 - \frac{\Delta_1}{8}\right) + \frac{EI}{4.6\sqrt{2}}\cdot\left(\Delta_2\cdot\cos\alpha - \Delta_1\cdot\sin\alpha\right)\cdot\sin\alpha -$$

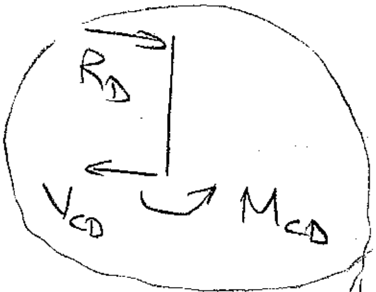
$$\frac{3EI}{36}\cdot\left(\theta_3 + \frac{\Delta_1}{6}\right) - 7{,}5 = 0$$

$$-0{,}034\cdot EI\cdot\Delta_1 + 0{,}015\cdot EI\cdot\Delta_2 - 0{,}083\cdot EI\cdot\theta_3 + 0{,}047\,EI\,\theta_4 = 7{,}5$$

Compute unknown displacements from (4)

$$EI\cdot\begin{bmatrix} -0{,}047 & 0{,}167 & 0{,}333 & 1{,}042 \\ 0{,}083 & 0{,}083 & 1{,}667 & 0{,}333 \\ -0{,}015 & 0{,}084 & 0{,}083 & 0{,}167 \\ 0{,}034 & -0{,}015 & 0{,}083 & -0{,}047 \end{bmatrix}\cdot\begin{bmatrix}\Delta_1 \\ \Delta_2 \\ \theta_3 \\ \theta_4\end{bmatrix} = \begin{bmatrix} 0 \\ -9 \\ 10 \\ -7{,}5\end{bmatrix}$$

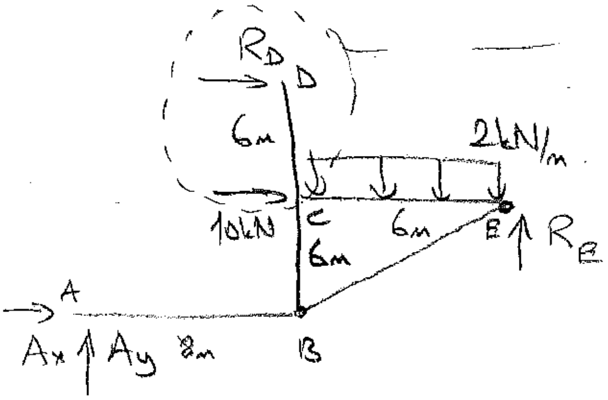
$$\begin{bmatrix}\Delta_1\\ \Delta_2\\ \theta_3\\ \theta_4\end{bmatrix} = \frac{1}{EI}\cdot\begin{bmatrix}-214,607\\ 144,301\\ 4,983\\ -34,342\end{bmatrix}$$



$$R_D = V_{CD} = \frac{3EI}{36}\cdot\left(\theta_3 - \frac{\Delta_2}{6}\right) = -1,59\ kN \leftarrow$$

$$\xrightarrow{+}\sum F_x = 0 \Rightarrow A_x - 1,59 + 10 = 0$$

$$A_x = -8,41\ kN \leftarrow$$



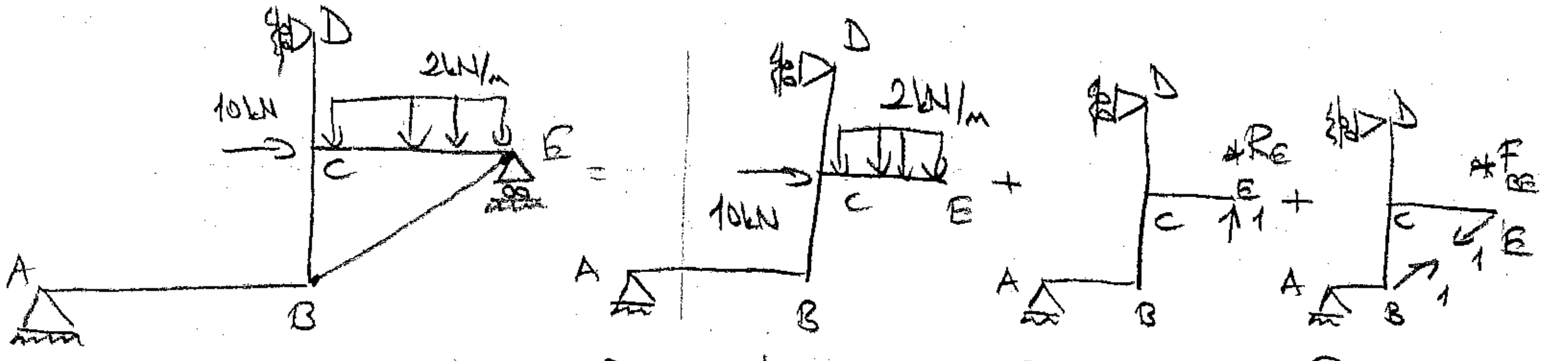
$$\overset{+}{\circlearrowleft}\sum M_A = 0 \Rightarrow 14\cdot R_E + 1,59\cdot 12 - 10\cdot 6 - 12\cdot 11 = 0$$

$$R_E = 12,35\ kN \uparrow$$

$$\uparrow^{+}\sum F_y = 0 \Rightarrow A_y + R_E - 2\cdot 6 = 0$$

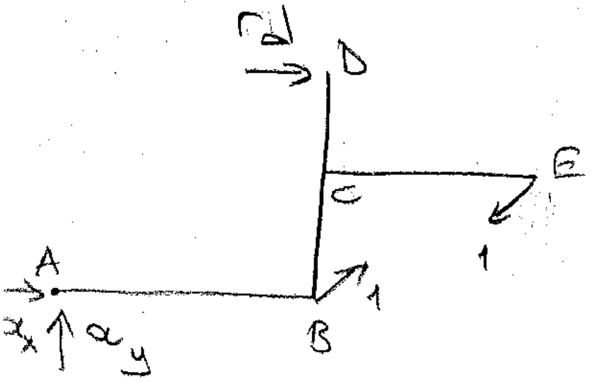
$$A_y = -0,35\ kN \downarrow$$

c) The structure is 2nd order indeterminate. Remove support E and truss member BE.



Basic str. Fig. 1 Fig. 2

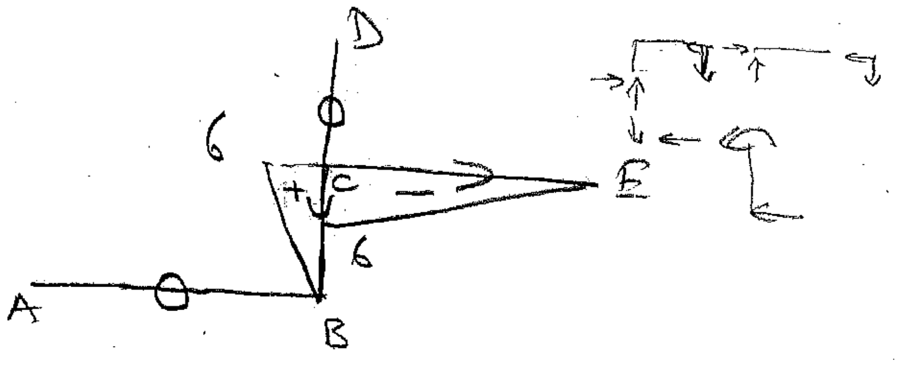
Solutions from part (a)



$$\Sigma M_A = 0 \Rightarrow r_d = 0$$

$$\Sigma F_x = 0 \Rightarrow a_x = 0$$

$$\Sigma F_y = 0 \Rightarrow a_y = 0$$



m_2

$$\Delta_E = \Delta_{E_0} + \delta_{11} \cdot R_E + \delta_{12} \cdot F_{BE} = 0 \quad (1)$$

$$\Delta_{BE} = \Delta_{BE_0} + \delta_{21} \cdot R_E + \delta_{22} \cdot F_{BE} = 0 \quad (2)$$

$$\Delta_{E_0} = \frac{-5.968}{EI}$$

$$\delta_{11} = \frac{486,67}{EI}$$

$$\delta_{12} = \int \frac{m \cdot m_2 \cdot dx}{EI} = \underbrace{\frac{1}{6} \cdot 6 \cdot 6 \cdot (2 \cdot 1 + 8)} + \underbrace{\frac{1}{3} \cdot 6 \cdot -6 \cdot 6}$$

$$= \frac{-12}{EI}$$

$$\delta_{21} = \delta_{12}$$

$$\delta_{22} = \int \frac{m_2 \cdot m_2 \cdot dx}{EI} + \Sigma \frac{n_2 \cdot n_2 \cdot L}{EA} = \underbrace{\frac{1}{3} \cdot 6 \cdot 6 \cdot 6} + \underbrace{\frac{1}{3} \cdot 6 \cdot -6 \cdot -6} + \frac{1 \cdot 1 \cdot 6\sqrt{2}}{EI/4} = \frac{177{,}94}{EI}$$

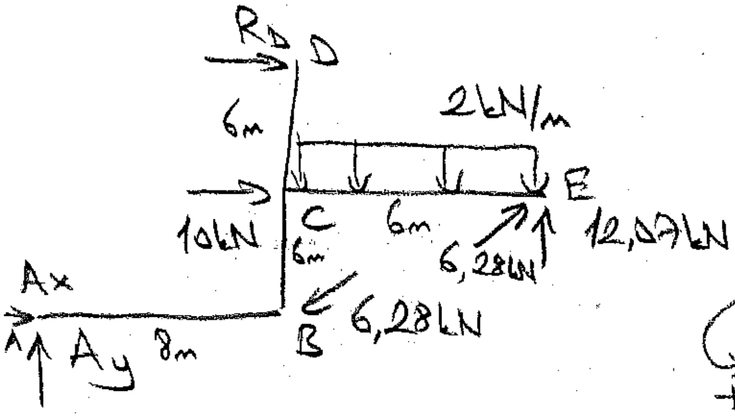
$$\Delta_{BE_0} = \int \frac{M \cdot m_2 \cdot dx}{EI} = \underbrace{\frac{1}{6} \cdot 6 \cdot 6 \cdot (2 \cdot -60 - 96)} + \underbrace{\frac{1}{4} \cdot 6 \cdot -36 \cdot -6}$$

$$= \frac{-972}{EI}$$

$$\frac{1}{EI} \cdot \begin{bmatrix} 486{,}67 & -12 \\ -12 & 177{,}94 \end{bmatrix} \cdot \begin{bmatrix} R_E \\ F_{BE} \end{bmatrix} = \begin{bmatrix} 5348/EI \\ 972/EI \end{bmatrix}$$

$$\begin{bmatrix} R_E \\ F_{BE} \end{bmatrix} = \begin{bmatrix} 12{,}07\,kN \\ -6{,}28\,kN \end{bmatrix}$$

Solve the whole structure with the help of known reaction and truss force.



$\Sigma F_y = 0 \Rightarrow A_y + 12,07 - 2.6 = 0$

$A_y = -0,07 kN$ ↓

$\circlearrowleft + \Sigma M_A = 0 \Rightarrow 12,07.14 - 2.6.11 - 10.6 + 12.R_D = 0$

$R_D = -1,92 kN$ ←

$\Sigma F_x = 0 \Rightarrow -1,92 + A_x + 10 = 0$

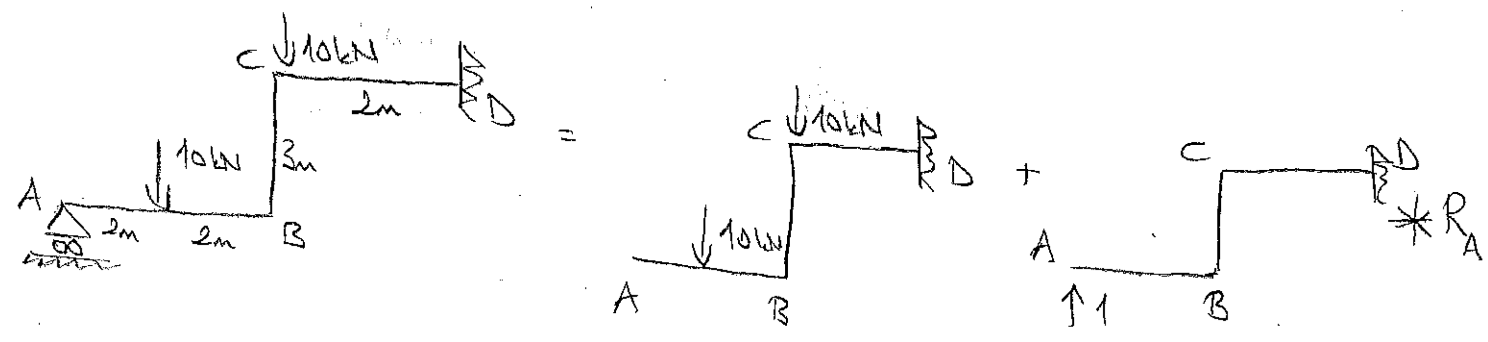
$A_x = -8,08 kN$ ←

The discrepancy between solutions (b) and (c) is due to lack of proper digits in solution (c).

-) a) Determine the degree of indeterminacy.

$$D_I = 3n + r - e - x = 3.0 + 4 - 3 - 0 = 1$$

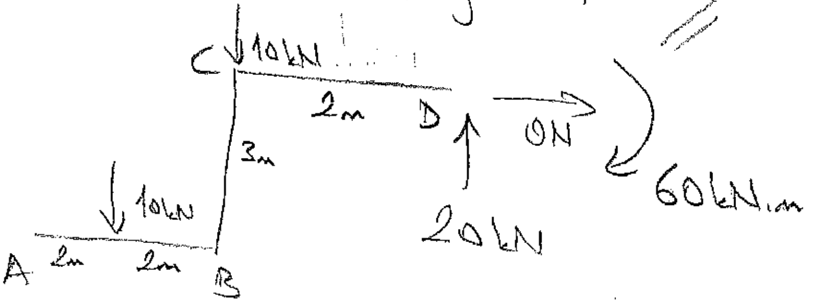
Remove the support at A.

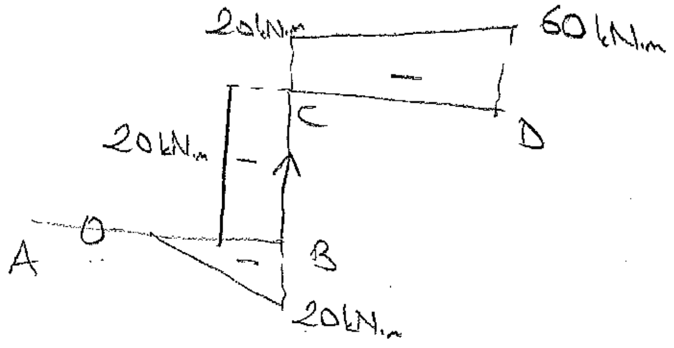


Basic structure   Fictitious structure

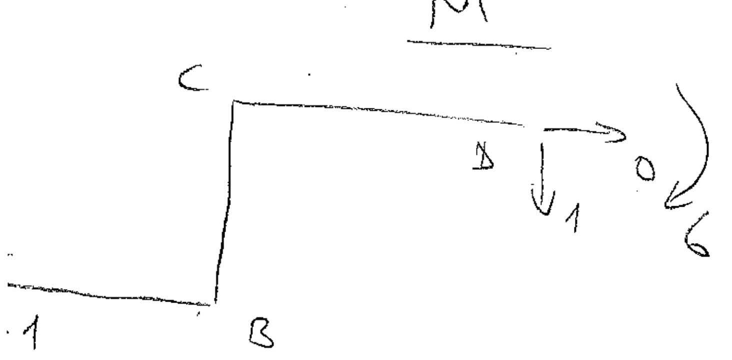
$$\Delta_A = \Delta_{A_0} + f \cdot R_A = 0$$

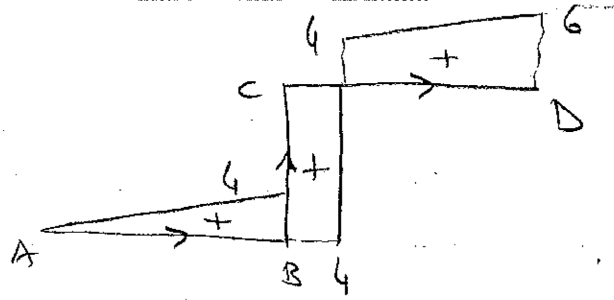
Compatibility equation





M



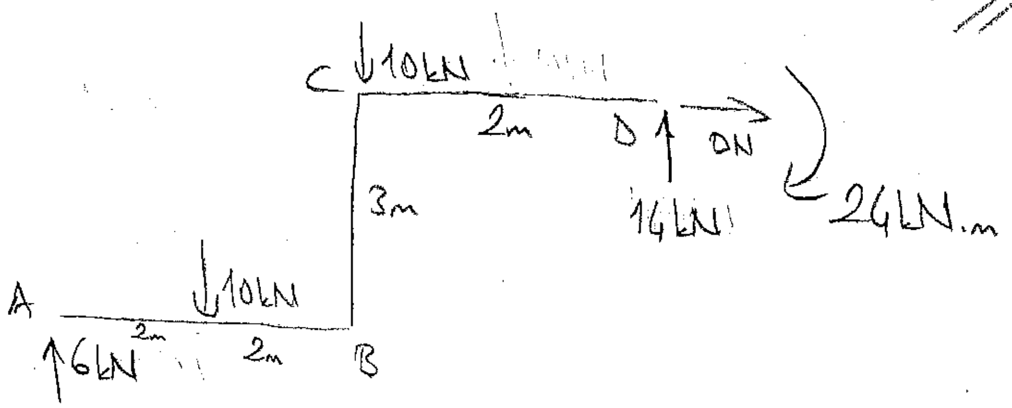


$$\Delta_A = \int \frac{M \cdot m \cdot dx}{EI} = \underbrace{\frac{1}{6} \cdot 2 \cdot (2 + 2 \cdot 4) \cdot -20}_{} + \underbrace{3 \cdot 4 \cdot -20}_{} + \underbrace{\frac{1}{6} \cdot 2 \cdot (2 \cdot 4 \cdot -20 + 4 \cdot -60 + 6 \cdot -20 + 2 \cdot 6 \cdot -60)}_{} = \frac{-720}{EI}$$

$$f = \int \frac{m \cdot m \cdot dx}{EI} = \underbrace{\frac{1}{3} \cdot 4 \cdot 4 \cdot 4}_{} + \underbrace{3 \cdot 4 \cdot 4}_{} + \underbrace{\frac{1}{6} \cdot 2 \cdot (2 \cdot 4 \cdot 4 + 4 \cdot 6 + 6 \cdot 4 + 2 \cdot 6 \cdot 6)}_{}$$

$$= \frac{120}{EI}$$

$$\Delta_A = \frac{-720}{EI} + \frac{120}{EI} \cdot R_A = 0 \Rightarrow R_A = 6\ \text{kN} \uparrow$$



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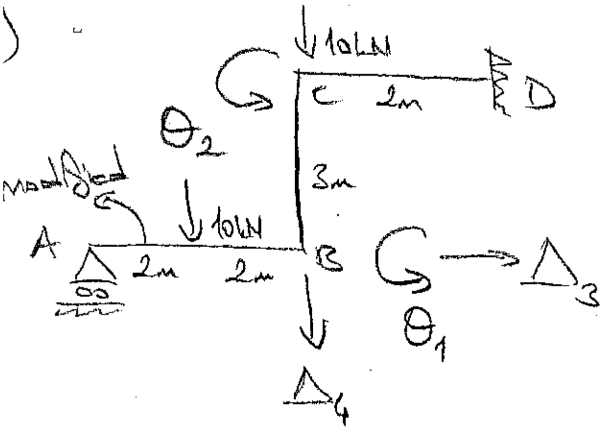
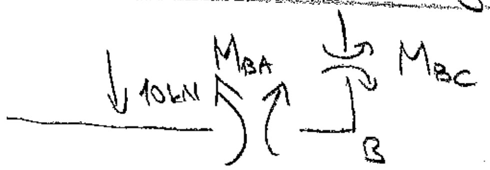


Detect the active d.o.f's.

Moment equilibrium at joint B =

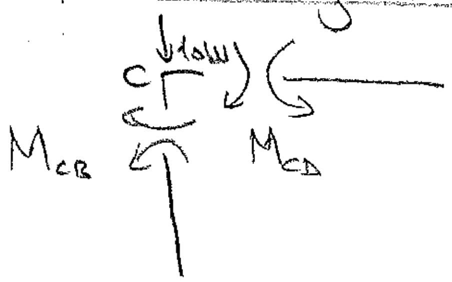


$$M_{BA} + M_{BC} = 0$$

$$\frac{3EI}{4}\cdot\left(\theta_1 + \frac{\Delta_4}{4}\right) - \frac{3.10.4}{16} + \frac{2EI}{3}\cdot\left(2\theta_1 + \theta_2 - \frac{3\Delta_3}{3}\right) = 0$$

$$EI.(2{,}083.\theta_1 + 0{,}667.\theta_2 - 0{,}667.\Delta_3 + 0{,}188.\Delta_4) = 7{,}5 \quad (1)$$

Moment equilibrium at joint C =



$$M_{CB} + M_{CD} = 0$$

$$\frac{2EI}{3}\cdot\left(2\theta_2 + \theta_1 - \frac{3\Delta_3}{3}\right) + \frac{2EI}{2}\cdot\left(2\theta_2 + 0 - \frac{3.\Delta_4}{2}\right) = 0$$

$$EI.(0{,}667.\theta_1 + 3{,}333.\theta_2 - 0{,}667.\Delta_3 - 1{,}5.\Delta_4) = 0 \quad (2)$$

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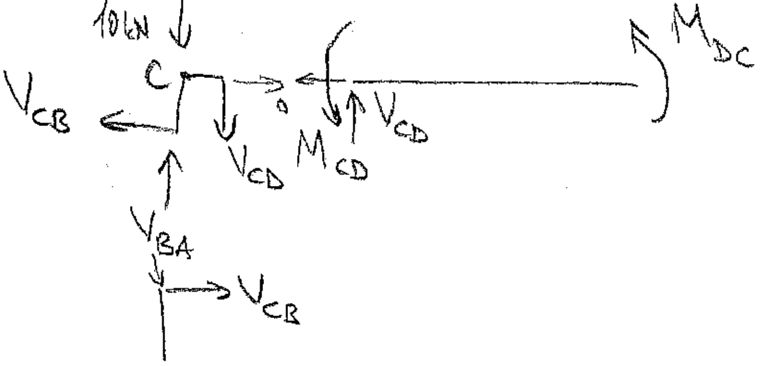
Force equilibrium at joint C =



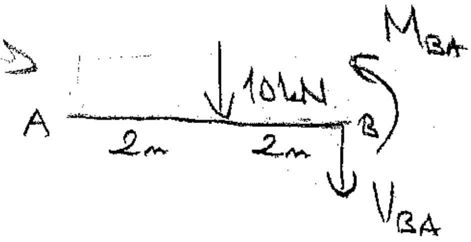
$\sum F_x = 0 \Rightarrow V_{CB} = 0$

$$\frac{M_{CB}+M_{BC}}{3} = 0$$

$$\frac{1}{3}\cdot\left(\frac{2EI}{3}\cdot\left(2\theta_2+\theta_1-\frac{3\Delta_3}{3}\right)+\frac{2EI}{3}\cdot\left(2\theta_1+\theta_2-\frac{3\Delta_3}{3}\right)\right)=0$$

$$EI\cdot(0{,}667\cdot\theta_1+0{,}667\cdot\theta_2-0{,}444\cdot\Delta_3)=0 \quad (3)$$

$\sum F_y = 0 \Rightarrow V_{BA} - 10 - V_{CD} = 0$



$$\frac{M_{BA}-2\cdot10}{4}-10-\frac{M_{CD}+M_{DC}}{2}=0$$

$$\frac{1}{4}\cdot\left(\frac{3EI}{4}\cdot\left(\theta_1+\frac{\Delta_4}{4}\right)-7{,}5\right)-5-10-\frac{1}{2}\cdot\left(\frac{2EI}{2}\cdot\left(2\cdot\theta_2+0-\frac{3\Delta_4}{2}\right)+\frac{2EI}{2}\cdot\left(2\cdot0+\theta_2-\frac{3\Delta_4}{2}\right)\right)=0$$

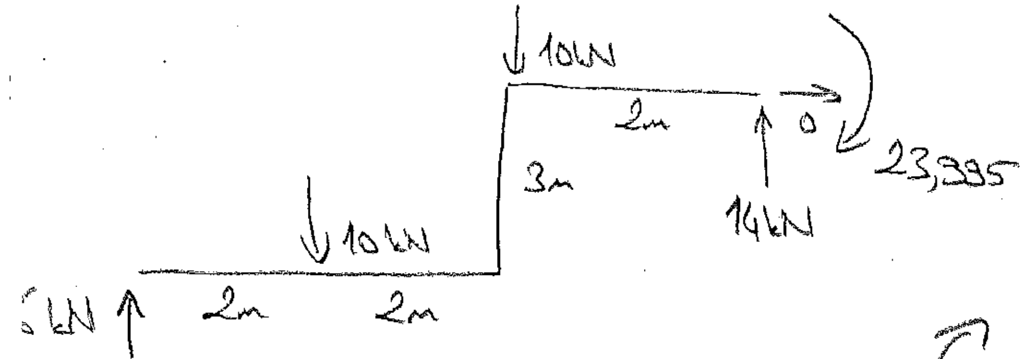
$$EI\cdot(0{,}188\cdot\theta_1-1{,}5\cdot\theta_2+0{,}5\cdot\Delta_4)=16{,}875 \quad (4)$$

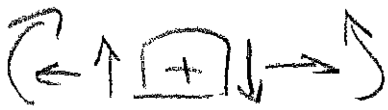
Solve the following equations simultaneously.

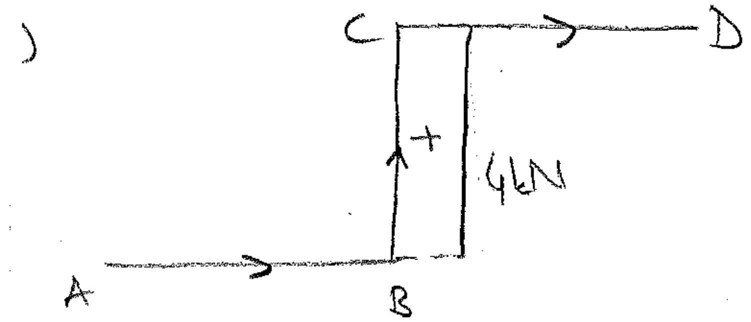
$$EI\cdot\begin{bmatrix} 2{,}083 & 0{,}667 & -0{,}667 & 0{,}188 \\ 0{,}667 & 3{,}333 & -0{,}667 & -1{,}5 \\ -0{,}667 & -0{,}667 & 0{,}444 & 0 \\ 0{,}188 & -1{,}5 & 0 & 1{,}5 \end{bmatrix}\cdot\begin{bmatrix}\theta_1\\ \theta_2\\ \Delta_3\\ \Delta_4\end{bmatrix}=\begin{bmatrix}7{,}5\\ 0\\ 0\\ 16{,}875\end{bmatrix}$$

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \Delta_3 \\ \Delta_4 \end{bmatrix} = \begin{bmatrix} 8,00/EI \\ 20,00/EI \\ 42,00/EI \\ 29,33/EI \end{bmatrix}$$

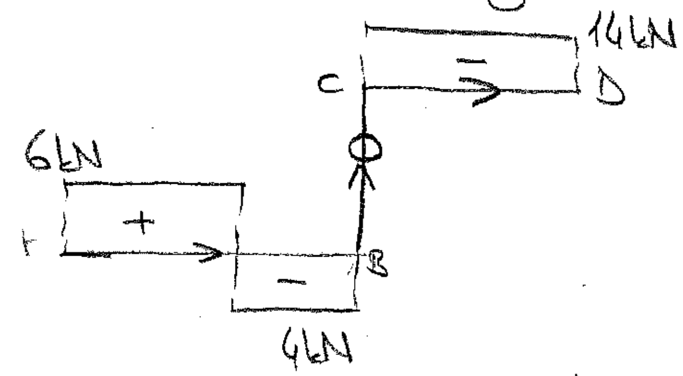
$$M_{DC} = \frac{2EI}{2} \cdot \left(2 \cdot 0 + \theta_2 - \frac{3\Delta_4}{2} \right) = -23,995 \text{ kN.m}$$



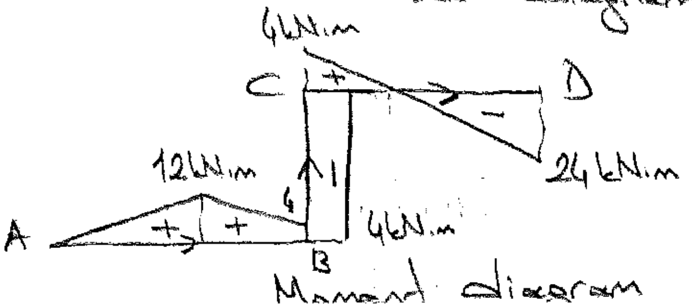




Axial force diagram



Shear diagram



Moment diagram