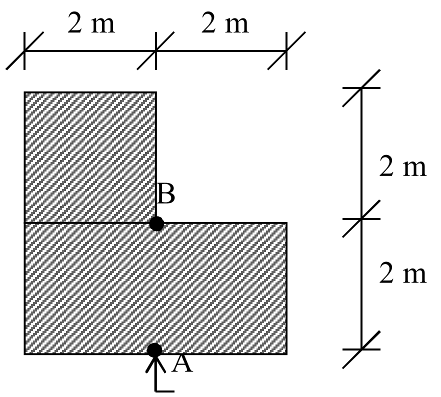
<u>CE 366 – SETTLEMENT (Problems & Solutions)</u>

<u>P. 1) LOAD UNDER A RECTANGULAR AREA (1)</u>

Question:

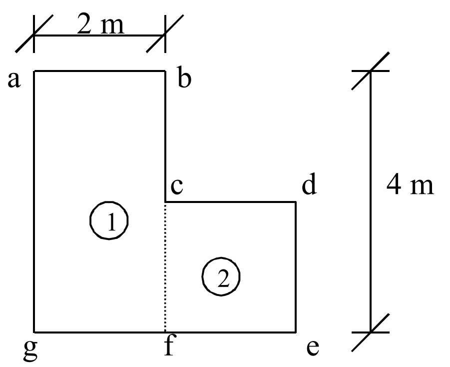
The footing shown in the figure below exerts a uniform pressure of 300 kN/m^2 to the soil. Determine **<u>vertical stress increase due to uniform pressure</u>**, at a point of 4 m directly under; (a) point A, (b) point B.



L – Shaped Footing (Plan view)

Solution:

a) <u>Point A;</u>



$\Delta\sigma_z = q.I_r$

By the use of Figure 1.6 in Lecture Notes, page 10;

- *For area 1 : A(abcfg)*

$z = 4$ m $\longrightarrow$ $\left.\begin{array}{l} mz = 4 \\ nz = 2 \end{array}\right] \longrightarrow \left.\begin{array}{l} m = 4/4 = 1 \\ n = 2/4 = 0.5 \end{array}\right] \longrightarrow I_r = 0.12$

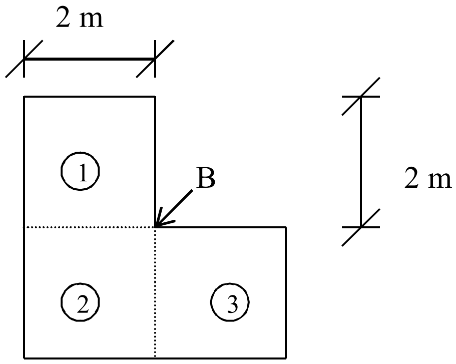
- *For area 2 : A(cdef)*

$z = 4$ m $\longrightarrow$ $\left.\begin{array}{l} mz = 2 \\ nz = 2 \end{array}\right] \longrightarrow \left.\begin{array}{l} m = 2/4 = 0.5 \\ n = 2/4 = 0.5 \end{array}\right] \longrightarrow I_r = 0.085$

$\Delta\sigma_z = 300\,(0.12 + 0.085) = 61.5$ kPa

$\hookrightarrow$ the stress at 4 m depth under point A due to 300 kN/m^2 uniform pressure

b) <u>Point B;</u>



Area 1 = Area 2 = Area 3

$\longrightarrow mz = nz = 2 \longrightarrow m = n = 2/4 = 0.5 \longrightarrow I_r = 0.085$

$\Delta\sigma_z = 300\,(3 \times 0.085)$

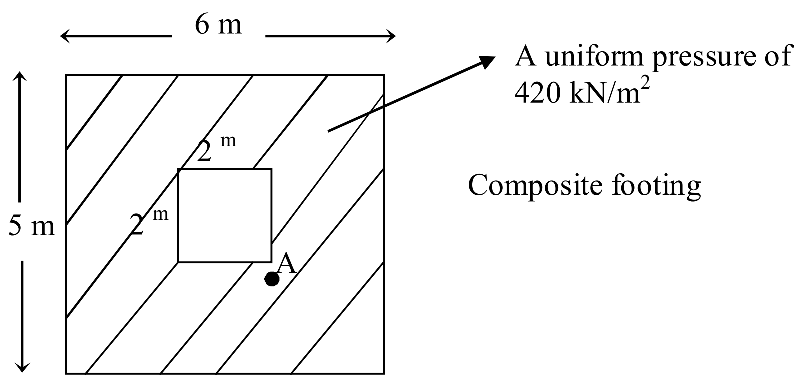
$= 76.5$ kPa

$\hookrightarrow$ the stress at 4 m depth under point B due to 300 kN/m^2 uniform pressure

P. 2) LOAD UNDER A RECTANGULAR AREA (2)

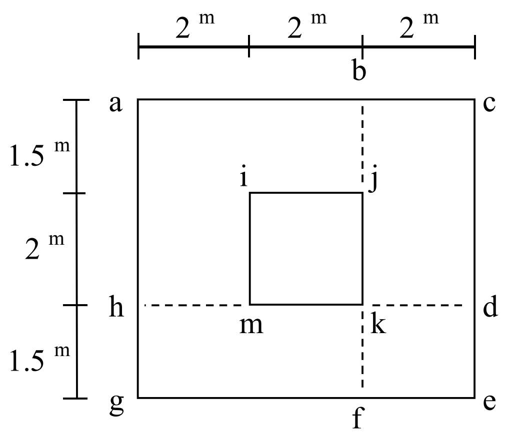
Question:

A rectangular footing as shown in figure below exerts a uniform pressure of 420 kN/m^2. Determine the vertical stress due to uniform pressure at point A for a depth of 3 m.



Plan View

Solution:



For area (abkh) :

$z = 3^m$ $\quad \left.\begin{matrix} mz = 4 \\ nz = 3.5 \end{matrix}\right\} \quad \left.\begin{matrix} m = 4 / 3 = 1.33 \\ n = 3.5 / 3 = 1.17 \end{matrix}\right\} \quad I_r = 0.195$

For area (bcdk) :

$\left.\begin{matrix} mz = 3.5 \\ nz = 2 \end{matrix}\right\} \quad \left.\begin{matrix} m = 3.5 / 3 = 1.17 \\ n = 2 / 3 = 0.67 \end{matrix}\right\} \quad I_r = 0.151$

For area (defk) :

$\left.\begin{matrix} mz = 2 \\ nz = 1.5 \end{matrix}\right\} \quad \left.\begin{matrix} m = 2 / 3 = 0.67 \\ n = 1.5 / 3 = 0.5 \end{matrix}\right\} \quad I_r = 0.105$

For area (fghk) :

$$\left.\begin{array}{l} mz = 4 \\ nz = 1.5 \end{array}\right\} \quad \left.\begin{array}{l} m = 4 / 3 = 1.33 \\ n = 1.5 / 3 = 0.5 \end{array}\right\} \qquad I_r = 0.133$$

For area (ijkm) :

$$mz = nz = 2 \longrightarrow m = n = 2 / 3 = 0.67 \longrightarrow I_r = 0.117$$

$$\Delta \sigma_z = \sigma \cdot I_r$$

$$= 420 \, [I_{r\,1} + I_{r\,2} + I_{r\,3} + I_{r\,4} - I_{r\,5}]$$

$$= 420 \, [0.195 + 0.151 + 0.105 + 0.133 - 0.117]$$

$$\boldsymbol{\Delta \sigma_z = 196.14 \text{ kPa}}$$

Note: Where do we use the vertical stress increase, $\Delta\sigma_z$, values?
For example, in a consolidation settlement problem, stress increase, $\Delta\sigma_z$, values are needed to calculate settlement under a foundation loading. We make the following calculations for a point located under the foundation at a certain depth (for example, at the mid-depth of the compressible layer):

(1) First, calculate the initial effective vertical stress, $\sigma'_{v,o}$, before the building was constructed,
(2) Then, find the vertical stress increase $\Delta\sigma_z$ at that depth, by using Boussinesq stress distribution or by approximate methods (for example 2V: 1H approximation)
(3) Find the final effective vertical stress, $\sigma'_{v,f} = \sigma'_{v,o} + \Delta\sigma_z$, after the building is constructed.
(4) Use these values in calculating the settlement under the foundation.

P. 3) IMMEDIATE SETTLEMENT

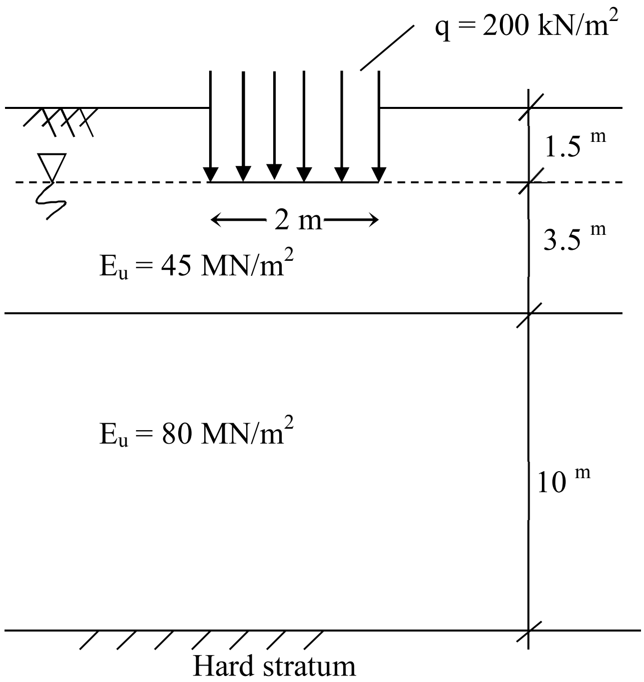
Question:

A foundation 4 ^{m} × 2 ^{m}, carrying a net uniform pressure of 200 kN/m^2, is located at a depth of 1.5 m in a layer of clay 5 m thick for which the value of E_u is 45 MN/m^2. The layer is underlain by a second layer, 10 m thick, for which the value of E_u is 80 MN/m^2. A hard stratum lies below the second layer. Ground water table is at depth of foundation. Determine the average immediate settlement under the foundation.

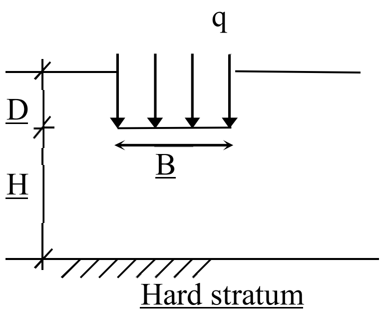
Hint: Since soil is SATURATED CLAY, ν_s=0.5. So the following equation can be used:

$$S_i = \mu_0 \cdot \mu_1 \cdot \frac{q \cdot B}{E_u}$$

Solution:



$$S_i = \mu_0 \cdot \mu_1 \cdot \frac{q \cdot B}{E_u}$$

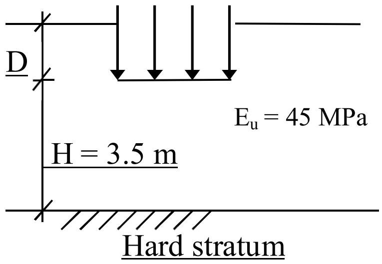


B is the smaller dimension !

We obtain,
μ_0 from D / B
μ_1 from H / B and L / B

D / B = 1.5 / 2 = 0.75 → μ_0 = 0.95 (Figure 3.3, p.62 Lecture Notes)

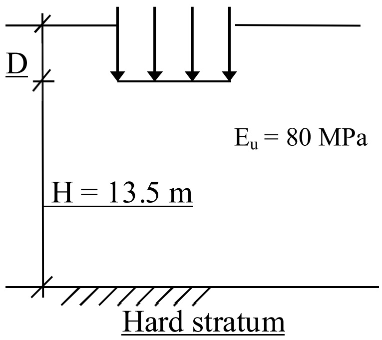
(1) Consider the upper layer with E_u = 45 MPa.



$$\left.\begin{array}{l} H / B = 3.5 / 2 = 1.75 \\ L / B = 4 / 2 = 2 \end{array}\right\} \mu_1 = 0.65$$

$$S_{i1} = \mu_0 \cdot \mu_1 \cdot \frac{q \cdot B}{E_u} = (0.95) \cdot (0.65) \cdot \frac{(200) \cdot 2}{45} = 5.49 mm$$

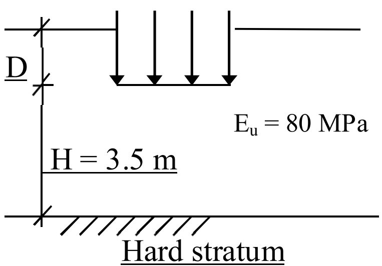
(2) Consider the two layers combined with E_u = 80 MPa.



$$\left.\begin{array}{l} H / B = (3.5 + 10) / 2 = 6.75 \\ L / B = 4 / 2 = 2 \end{array}\right\} \mu_1 = 0.9$$

$$S_{i2} = \mu_0 \cdot \mu_1 \cdot \frac{q \cdot B}{E_u} = (0.95) \cdot (0.9) \cdot \frac{(200) \cdot 2}{80} = 4.28 mm$$

(3) Consider the upper layer with E_u = 80 MPa.



$$\left.\begin{array}{l} H / B = 3.5 / 2 = 1.75 \\ L / B = 4 / 2 = 2 \end{array}\right\} \mu_1 = 0.65$$

$$S_{i3} = \mu_0 \cdot \mu_1 \cdot \frac{q \cdot B}{E_u} = (0.95) \cdot (0.65) \cdot \frac{(200) \cdot 2}{80} = 3.08 mm$$

Using the principle of superposition, the settlement of the foundation is given by;

$S_i = S_{i\,1} + S_{i\,2} - S_{i\,3}$

$S_i = 5.49 + 4.28 - 3.08$

$\mathbf{S_i = 6.69\ \ mm}$

P. 4) SCHMERTMAN

Question:

A soil profile consists of deep, loose to medium dense sand (γ_{dry} = 16 kN/m^3 , γ_{sat} = 18 kN/m^3). The ground water level is at 4 m depth. A 3.5 m x 3.5 m square footing rests at 3 m depth. The **total (gross) load** acting at the foundation level (footing weight + column load + weight of soil or footing) is 2000 kN. Estimate the elastic settlement of the footing 6 years after the construction using influence factor method (Schmertman, 1978).

End resistance values obtained from static cone penetration tests are;

Depth (m)	q_c (kN/m^2)
0.00 – 2.00	8000
2.00 - 4.75	10000
4.75 - 6.50	8000
6.50 – 12.00	12000
12.00 – 15.00	10000

Note that;

- for square footing;

z (depth)(from foundation level)	I_z (strain factors)
0	0.1
B/2	0.5
2B	0.0

Where; B : width of footing

- $E_s = 2.0\ q_c$

Solution:

$$S_i = C_1\, C_2\, q_{net}\, \Sigma\, \frac{I_z}{E} \Delta z$$

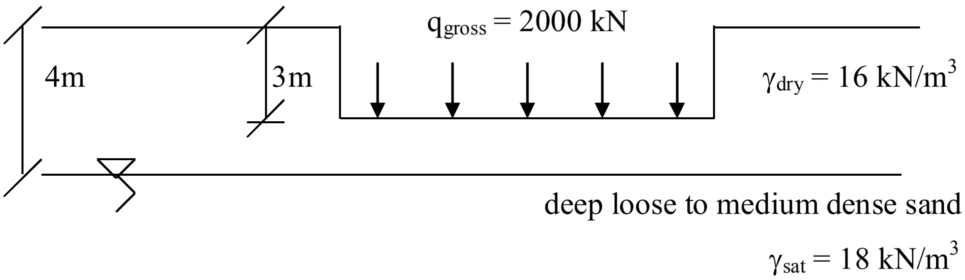
q_{net} = net foundation pressure

$$C_1 = 1 - 0.5 \frac{\sigma'_o}{q_{net}} \longrightarrow \text{correction factor for footing depth}$$

σ'_o = effective overburden pressure at foundation level

$$C_2 = 1 + 0.2 \log \frac{t}{0.1} \longrightarrow \text{correction factor for creep}$$

t = time at which the settlement is required (in years)

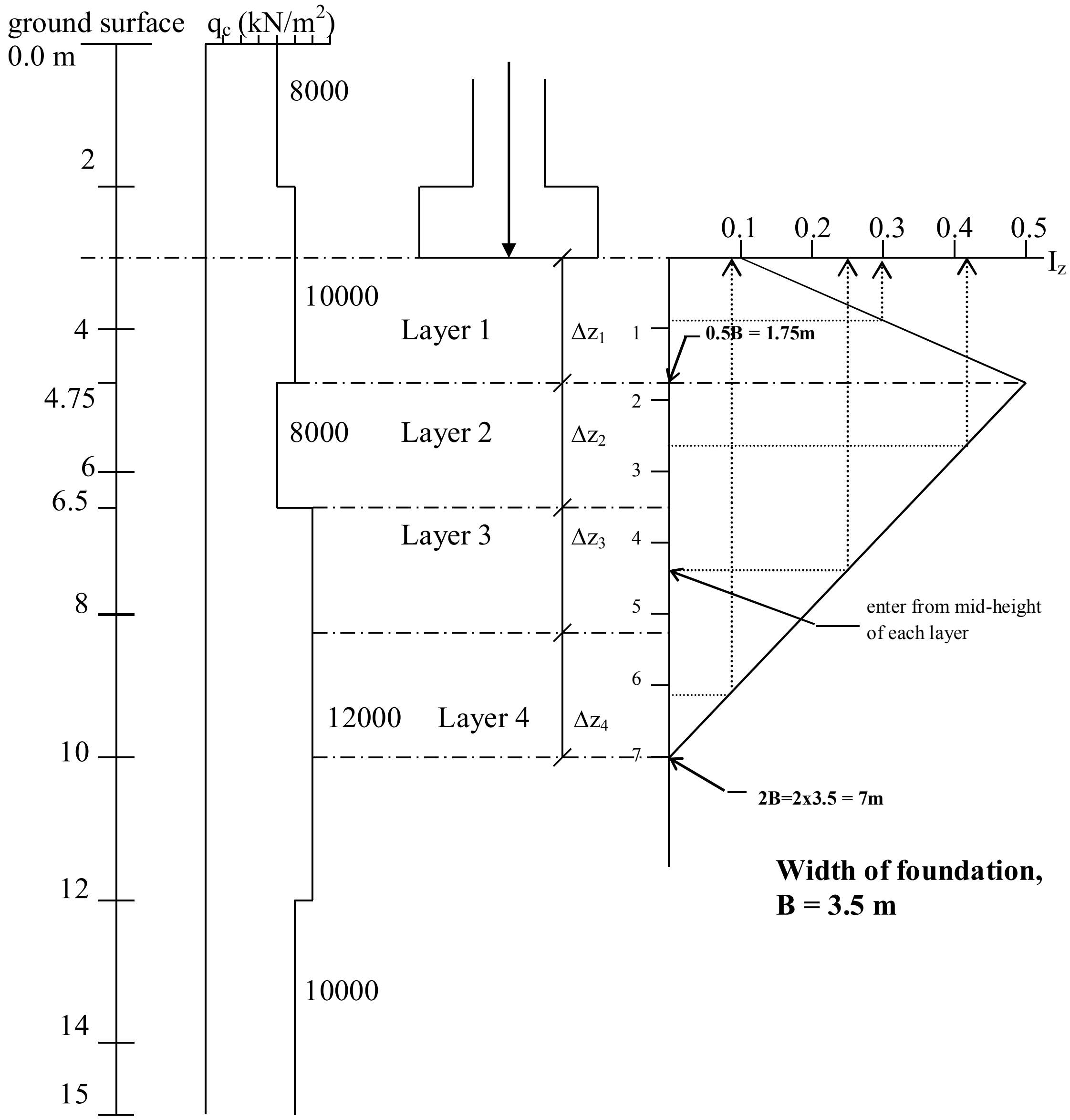


$$q_{net} = \underbrace{\frac{2000}{3.5x3.5}}_{\text{gross pressure.}} - \underbrace{3x16}_{\text{initial effective overburden pressure}} = 115.26\, kPa$$

$\sigma_o' = 3x16 = 48$ kPa

$$C_1 = 1 - 0.5 \frac{48}{115.26} = 0.792$$

$$C_2 = 1 + 0.2 \log \frac{6}{0.1} = 1.356$$



$E_s = 2.0\, q_c$

Layer No	Depth(m)	Δz(m)	q_c(kPa)	Es(kPa)	I_z	$(I_z/E_s)\,\Delta z$
1	3.00-4.75	1.75	10.000	20.000	0.3	$2.65\text{x}10^{-5}$
2	4.75-6.50	1.75	8.000	16.000	0.416	$4.55\text{x}10^{-5}$
3	6.50-8.25	1.75	12.000	24.000	0.249	$1.82\text{x}10^{-5}$
4	8.25-10.00	1.75	12.000	24.000	0.083	$0.605\text{x}10^{-5}$
						$\Sigma = 9.625\text{x}10^{-5}$

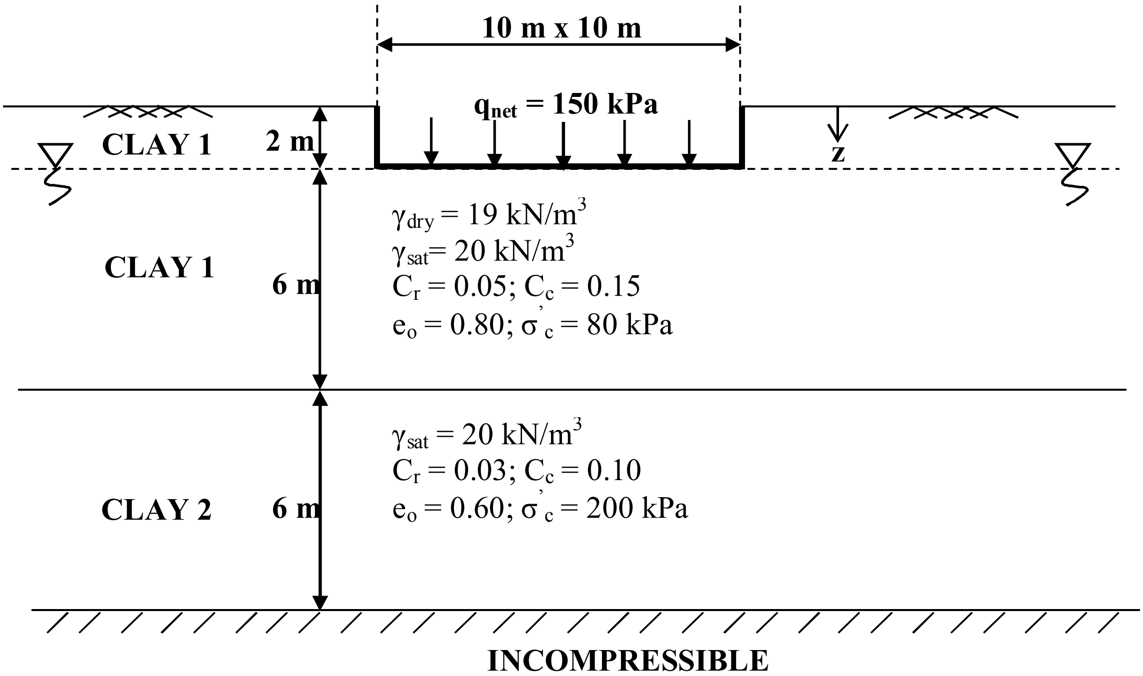
$S_i = (0.792)\,(1.356)\,(115.26)\,(9.625\text{x}10^{-5})$

$= 0.01191$ m $\longrightarrow$ **S_i = 11.91 mm**

P5. CONSOLIDATION SETTLEMENT

Question:

Ignore the immediate settlement, and calculate total consolidation settlement of soil profile composed of two different types of clay, i.e. Clay 1 and Clay 2 due to 150 kPa net foundation loading. Take unit weight of water as 10 kN/m^3 and assume that Skempton-Bjerrum Correction Factor is $\mu = 0.7$ for both clay layers. Note that σ'_c (or sometimes shown as σ'_p) is the preconsolidation pressure.



Solution:

Settlement will take place due to loading (q_{net} = 150 kPa) applied at a depth of 2 m. Thus, all (consolidation) settlement calculations will be performed for clayey soil beneath the foundation (z > 2 m).

Reminder: General equation of 1D consolidation settlement (one dimensional vertical consolidation) for an overconsolidated clay is;

$$S_{c,1D} = \frac{C_r}{1+e_o} H \log\left(\frac{\sigma'_c}{\sigma'_{v,o}}\right) + \frac{C_c}{1+e_o} H \log\left(\frac{\sigma'_{v,f}}{\sigma'_c}\right)$$

Note that, all calculations are done for the ***mid-depth*** *of the compressible layers under the loading.*

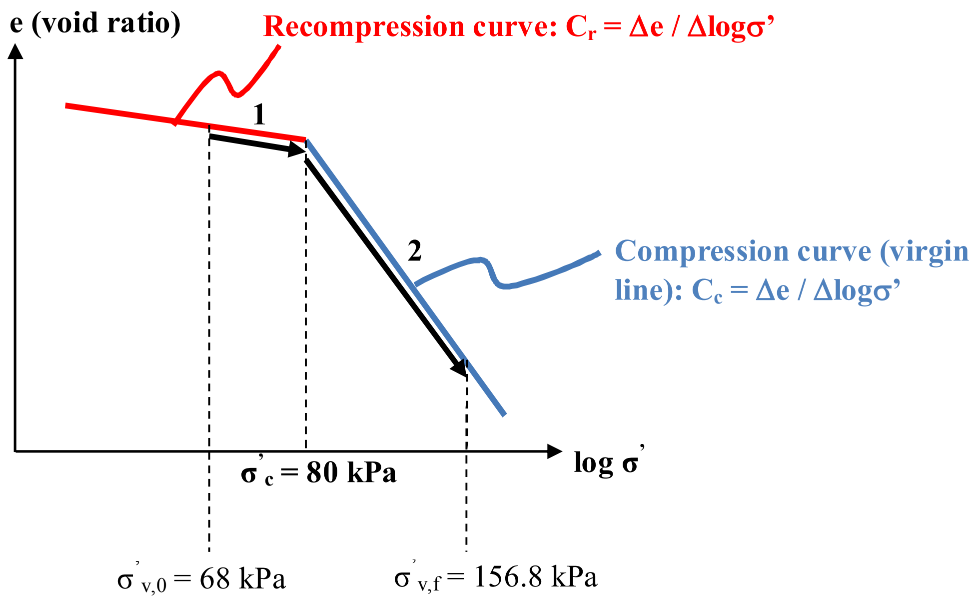
Consolidation settlement in Clay 1:

Initial effective overburden stress, $\sigma'_{v,o}$ = (2*19) + (3*(20-10)) = 68 kPa

Stress increment due to foundation loading, $\Delta\sigma$= [150*(10*10)] / [(10+3)*(10+3)] = 88.8 kPa

Final stress, $\sigma'_{v,f}$ = 68 + 88.8 = 156.8 kPa

This is an overconsolidated clay (overconsolidation ratio OCR = σ'_c / $\sigma'_{v,o}$ = 80 / 68 > 1.0) ; and the final stress, $\sigma'_{v,f}$ is greater than σ'_c ($\sigma'_{v,f} > \sigma'_c$) therefore we should use both C_r and C_c in consolidation settlement calculation (see figure below).



$$S_{c,1D} = \underbrace{\left\{\frac{0.05}{1+0.80}(6)\log\left(\frac{80}{68}\right)\right\}}_{\mathbf{1}} + \underbrace{\left\{\frac{0.15}{1+0.80}(6)\log\left(\frac{156.8}{80}\right)\right\}}_{\mathbf{2}} = 0.158m = 15.8\ cm$$

Consolidation settlement in Clay 2:

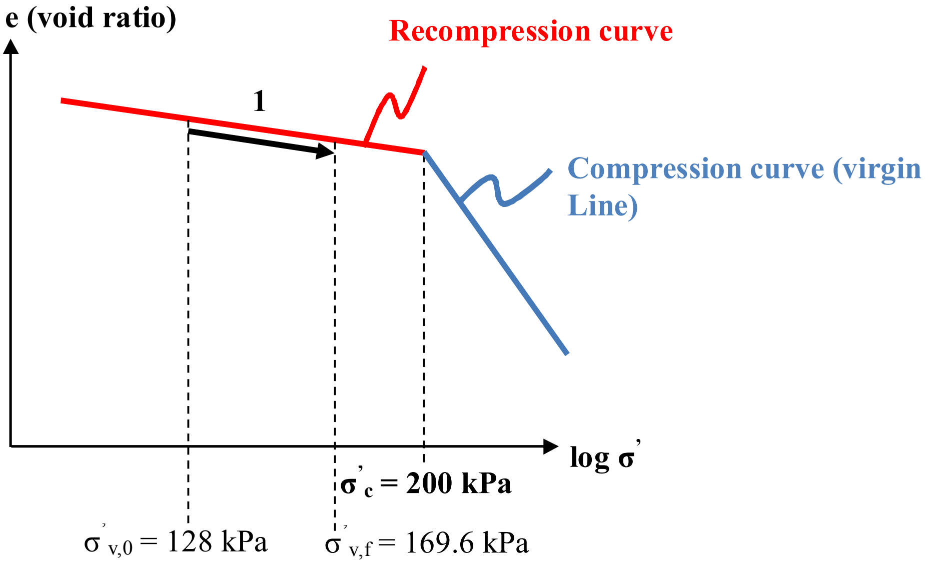
Initial effective overburden stress, $\sigma'_{v,0}$ = (2*19) + (6*(20-10)) + (3*(20-10)) = 128 kPa

Stress increment due to foundation loading, $\Delta\sigma$= [150*(10*10)] / [(10+9)*(10+9)] = 41.6 kPa

Final stress, $\sigma'_{v,f} = 128 + 41.6 = 169.6$ kPa

This is an overconsolidated clay (overconsolidation ratio OCR = $\sigma'_c / \sigma'_{v,o} = 200 / 128 > 1.0$) ; and the final stress, $\sigma'_{v,f}$ is less than σ'_c ($\sigma'_{v,f} < \sigma'_c$) therefore we should use only C_r in consolidation settlement calculation (see figure below).

[Note: If a soil would be a normally consolidated clay (OCR = $\sigma'_c / \sigma'_{v,o} = 1.0$), we would use only C_c in consolidation settlement calculation.]



$$S_{c,1-D} = \underbrace{\left\{\frac{0.03}{1+0.60}(6)\log\left(\frac{169.6}{128}\right)\right\}}_{1} = 0.014\,m = 1.4\,cm$$

<u>Total Consolidation Settlement (1D):</u>

$$S_{c,1D} = 15.9 + 1.4 = 17.3cm$$

<u>Corrected Settlement for 3D Consolidation (Skempton-Bjerrum Factor):</u>

$$S_{c,3D} = S_{c,1D} * \mu = 17.3 * 0.7 = \mathbf{12.1\ cm}$$