

Using the principle of virtual work, calculate vertical component of the deflection at C for the frame shown.

$$(EI_{\text{beam}})_{ABC} = 20000 \text{ kNm}^2$$

$$(EI_{\text{bar}})_{BD} = 10000 \text{ kN}$$



$$+\circlearrowleft \sum M_A = 0 \quad 5 \times 7 + 8 \times 2 + 6 \times 15 - 4 F_{BD} = 0$$

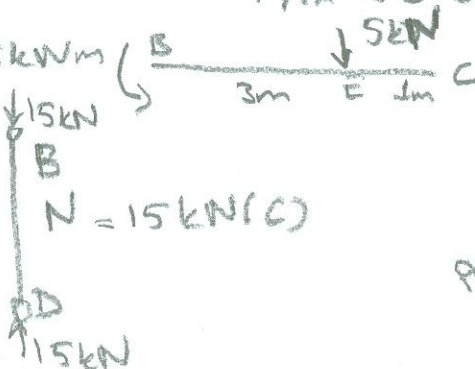
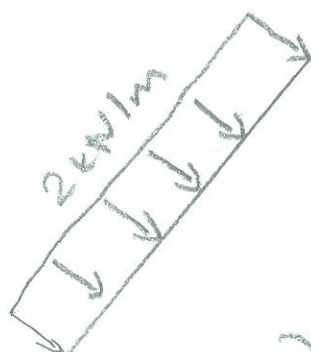
$$\Rightarrow F_{BD} = 15 \text{ kN} (\uparrow)$$

$$+\uparrow \sum F_y = 0 \quad 15 - 5 - 8 - R_{AY} = 0$$

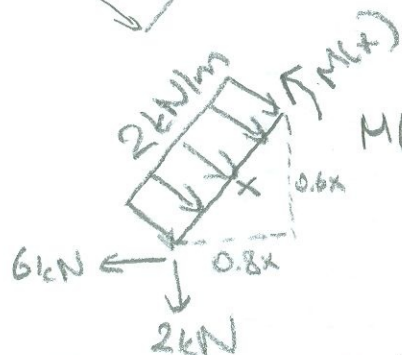
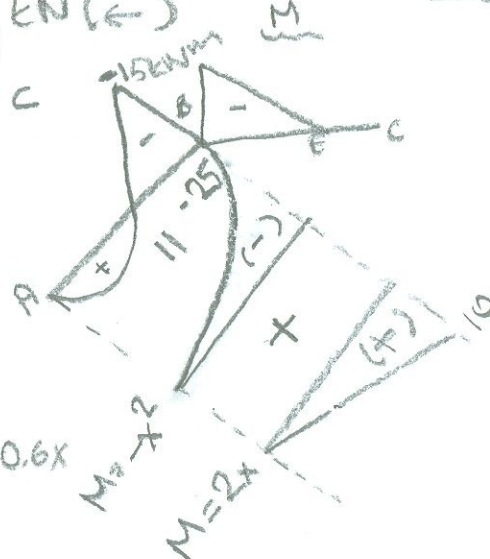
$$R_{AY} = 2 \text{ kN} (\downarrow)$$

$$\Rightarrow \sum F_x = 0 \quad 6 - R_{AX} = 0$$

$$R_{AX} = 6 \text{ kN} (\leftarrow)$$



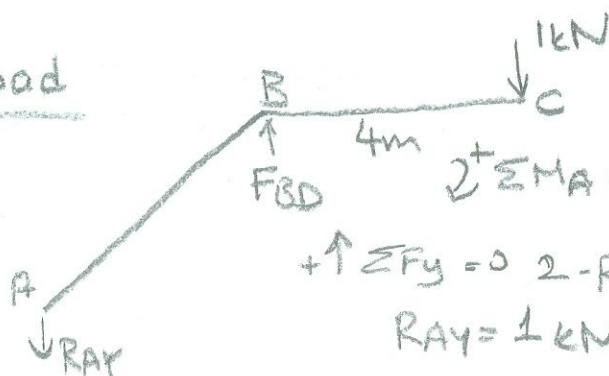
$$N = 15 \text{ kN} (\text{C})$$



$$M(x) = -2 \times x + \frac{x^2}{2} - 2 \times 0.8x + 6 \times 0.6x$$

$$= -x^2 + 2x$$

FBD of unit load



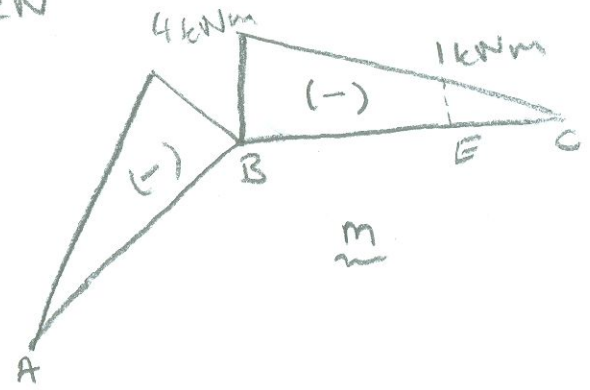
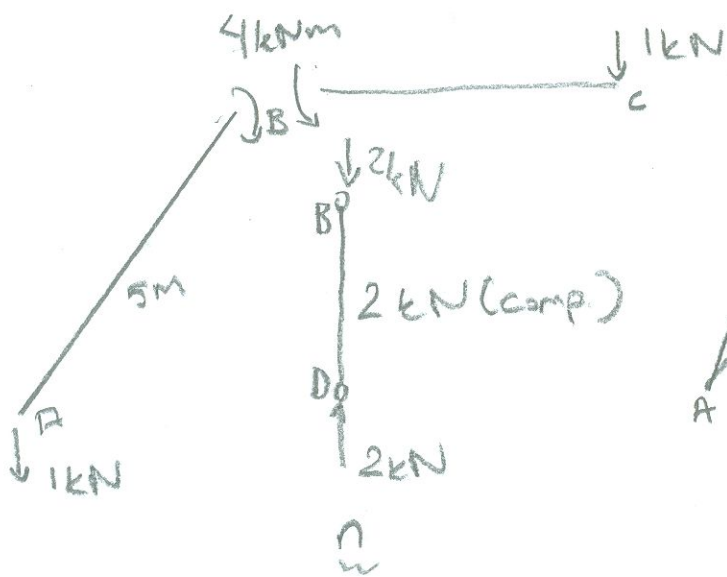
$$+\circlearrowleft \sum M_A = 0 \quad 1 \times 8 - F_{BD} \times 4 = 0$$

$$F_{BD} = 2 \text{ kN} (\uparrow)$$

$$+\uparrow \sum F_y = 0 \quad 2 - R_{AY} - 1 = 0$$

$$R_{AY} = 1 \text{ kN} (\downarrow)$$





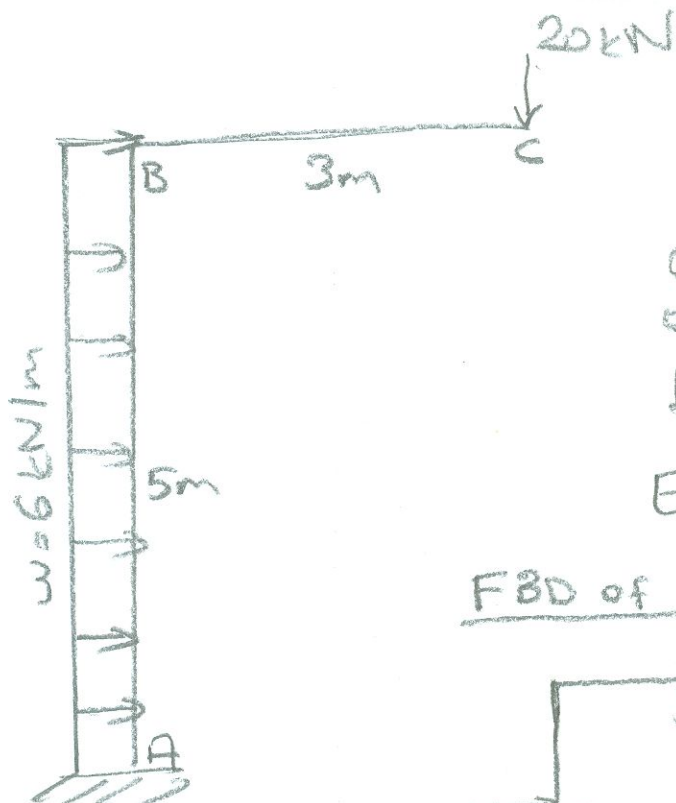
$$1 \times \Delta_{cy} = \frac{\int m M dx}{EI} + \sum \frac{n N L}{EA}$$

$$= \frac{\frac{1}{4} \times 5 \times 4 \times 25 - \frac{1}{3} \times 5 \times 4 \times 10 + \frac{1}{6} \times 3 \times 15 \times (1+2+4)}{EI} + \frac{-2 \times -15 \times 3}{EA}$$

$$= \frac{20000}{EI} + \frac{90}{EA}$$

\downarrow
 $0,00529 \text{ m}$
 \downarrow
 $0,009 \text{ m}$

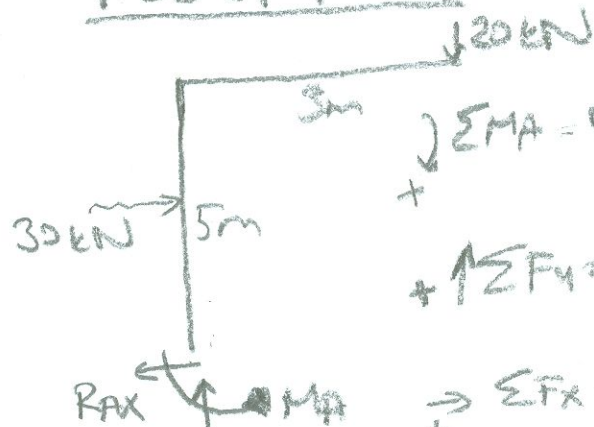
$$\Delta_{cy} = 0,01529 \text{ m}$$



Using the principle of virtual work
calculate the horizontal
component of the deflection at C
for the frame shown left.

$$E = 200 \text{ GPa} \quad I = 240 \times 10^6 \text{ mm}^4$$

FBD of frame



$$+\circlearrowleft \sum M_A = 0 \quad 20 \times 3 + 30 \times 2.5 - M_A = 0$$

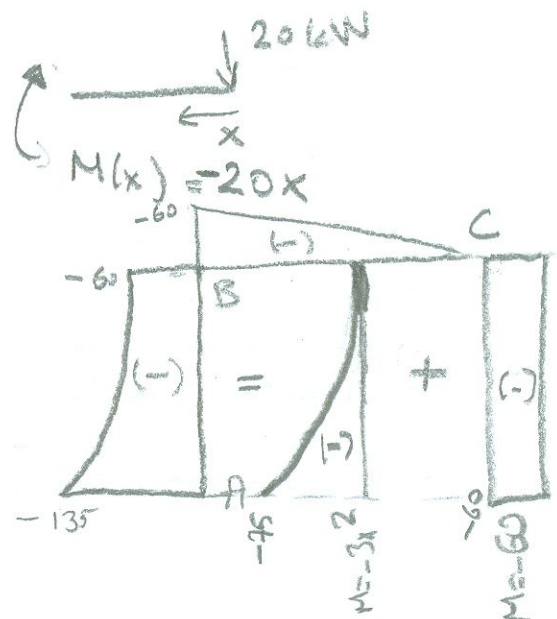
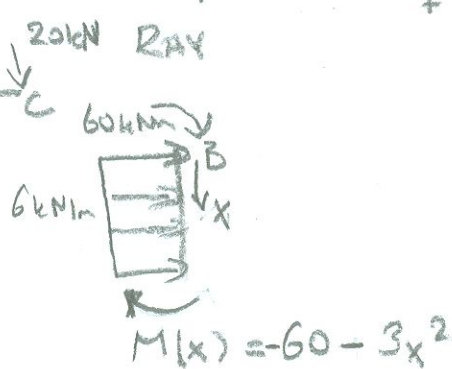
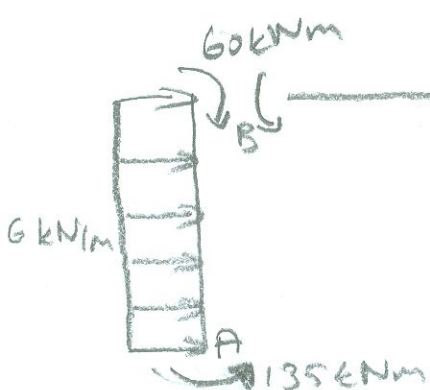
$$M_A = 135 \text{ kNm} (\uparrow)$$

$$+\uparrow \sum F_y = 0 \quad R_{AY} - 20 = 0$$

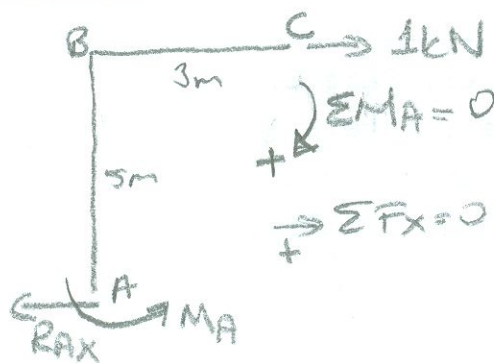
$$R_{AY} = 20 \text{ kN} (\uparrow)$$

$$+\rightarrow \sum F_x = 0 \quad 30 - R_{AX} = 0$$

$$R_{AX} = 30 \text{ kN} (\leftarrow)$$



FBD of unit load

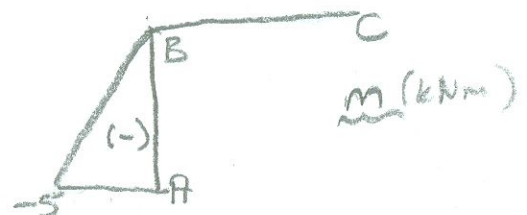
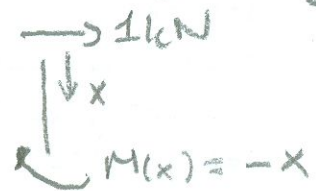


$$+\circlearrowleft \sum M_A = 0 \quad 1 \times 5 - M_A = 0$$

$$M_A = 5 \text{ kNm} (\uparrow)$$

$$+\rightarrow \sum F_x = 0 \quad 1 - R_{AX} = 0$$

$$R_{AX} = 1 \text{ kN} (\leftarrow)$$



$$1 * \Delta_{cx} = \frac{\int m M dx}{EI}$$

$$= \frac{\left[\begin{array}{c} \text{Diagram 1: Triangle with base } 5m, \text{ height } -5, \text{ area } (-) \\ \text{Diagram 2: Triangle with base } 5m, \text{ height } -75, \text{ area } (-) \\ \text{Diagram 3: Rectangle with base } 5m, \text{ height } -60, \text{ area } (-) \end{array} \right] + 0 \text{ (no M)}}{EI}$$

$$1 * \Delta_{cx} = \frac{\frac{1}{4} (5)(75)(5) + \frac{1}{2} (5)(5)(60)}{EI} = \frac{1248.75}{EI}$$

$$\Delta_{cx} = 25.4 \text{ mm} \quad (\rightarrow)$$