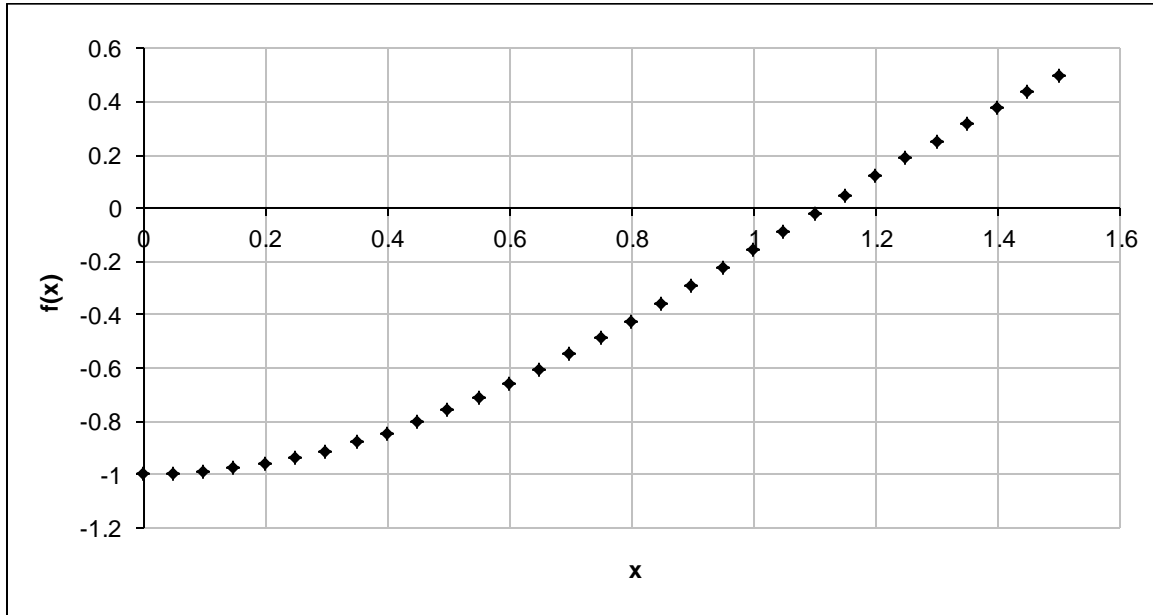Consider the function $f(x) = x\sin(x) - 1$. Find the root of the function that is located in the interval $[0.5, 1.5]$ using a percent relative error of 0.5×10^{-2} as tolerance. Carry out your solutions with <u>at least</u> 5 decimal places.

Graph of x*sin(x)-1 in the interval [0.5,1.5]

Bisection Method:

Step #	x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	ε_{RA} (%)
0	0.50000	-0.76029	1.50000	0.49624	1.00000	-0.15853	---
1	1.00000	-0.15853	1.50000	0.49624	1.25000	0.18623	2.00E+01
2	1.00000	-0.15853	1.25000	0.18623	1.12500	0.01505	1.11E+01
3	1.00000	-0.15853	1.12500	0.01505	1.06250	-0.07183	5.88E+00
4	1.06250	-0.07183	1.12500	0.01505	1.09375	-0.02836	2.86E+00
5	1.09375	-0.02836	1.12500	0.01505	1.10938	-0.00664	1.41E+00
6	1.10938	-0.00664	1.12500	0.01505	1.11719	0.00421	6.99E-01
7	1.10938	-0.00664	1.11719	0.00421	1.11328	-0.00122	3.51E-01
8	1.11328	-0.00122	1.11719	0.00421	1.11523	0.00150	1.75E-01
9	1.11328	-0.00122	1.11523	0.00150	1.11426	0.00014	8.76E-02
10	1.11328	-0.00122	1.11426	0.00014	1.11377	-0.00054	4.38E-02
11	1.11377	-0.00054	1.11426	0.00014	1.11401	-0.00020	2.19E-02
12	1.11401	-0.00020	1.11426	0.00014	1.11414	-0.00003	1.10E-02
13	1.11414	-0.00003	1.11426	0.00014	1.11420	0.00006	5.48E-03
14	1.11414	-0.00003	1.11420	0.00006	1.11417	0.00001	2.74E-03

False-position (Regula Falsi) Method:

Step #	x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	ε_{RA} (%)
0	0.50000	-0.76029	1.50000	0.49624	1.10507	-0.01263	---
1	1.10507	-0.01263	1.50000	0.49624	1.11487	0.00099	8.79E-01
2	1.10507	-0.01263	1.11487	0.00099	1.11416	0.00000	6.38E-02
3	1.10507	-0.01263	1.11416	0.00000	1.11416	0.00000	2.28E-05

Newton Raphson Method: ($x_0 = 0.5$)

Step #	x_i	$f(x_i)$	$f'(x_i)$	x_{i+1}	ε_{RA} (%)
1	0.50000	-0.76029	0.91822	1.32800	6.23E+01
2	1.32800	0.28905	1.28994	1.10392	2.03E+01
3	1.10392	-0.01422	1.38985	1.11415	9.18E-01
4	1.11415	-0.00001	1.38881	1.11416	3.26E-04

Secant Method: ($x_0 = 0.5$, $x_1 = 0.6$)

Step #	x_{i-1}	x_i	$f(x_{i-1})$	$f(x_i)$	x_{i+1}	ε_{RA} (%)
1	0.50000	0.60000	-0.76029	-0.66121	1.26740	5.27E+01
2	0.60000	1.26740	-0.66121	0.20952	1.10681	1.45E+01
3	1.26740	1.10681	0.20952	-0.01021	1.11427	6.69E-01
4	1.10681	1.11427	-0.01021	0.00016	1.11416	1.01E-02
5	1.11427	1.11416	0.00016	0.00000	1.11416	2.96E-06

Question: Why did Secant method converge in 5 iterations whereas Newton Raphson converged in 4 iterations?

Fixed Point Iteration: ($x_0 = 0.5$)

Step #	x_{i-1}	$x_i = 1/\sin(x_{i-1})$	ε_{RA} (%)
1	0.50000	2.08583	76.02872
2	2.08583	1.14906	81.52481
3	1.14906	1.09603	4.83793
4	1.09603	1.12435	2.51846
5	1.12435	1.10866	1.41509
6	1.10866	1.11719	0.76325
7	1.11719	1.11250	0.42110
8	1.11250	1.11506	0.22948
9	1.11506	1.11366	0.12591
10	1.11366	1.11443	0.06883
11	1.11443	1.11401	0.03770
12	1.11401	1.11424	0.02063
13	1.11424	1.11411	0.01129
14	1.11411	1.11418	0.00618
15	1.11418	1.11414	0.00338
16	1.11414	1.11416	0.00185

Question: Why is convergence so slow with Fixed Point Iteration Method?