Concrete = is composed of - cement
- water
- sand
- aggregate

Properties of Concrete:

Pro's:	Con's
* High compressive strength	* low tensile strength
* can be molded into any shape	Monier, 1857, reinforced concrete (concrete flower pot with wires)
* low maintanence cost	* requires use of formwork
* good fire resistance	* high unit weight (heavier than steel)
	* undergoes time dependent deformations

Mixing, Placing and Curing of Concrete:

Mix design is used to obtain the required strength and workability.

Water to cement ratio (w/c) is the main parameter in determining the strength of concrete.

As w/c ↑ → strength ↓
workability ↑

Workability is important in transporting and casting

During transportation, high workability prevents segregation. If segregation occurs, larger pieces seperate from the bulk of the mass and settle down → honeycombs are observed on the surface of the finished concrete.

During casting, high workability eases the finishing of concrete.

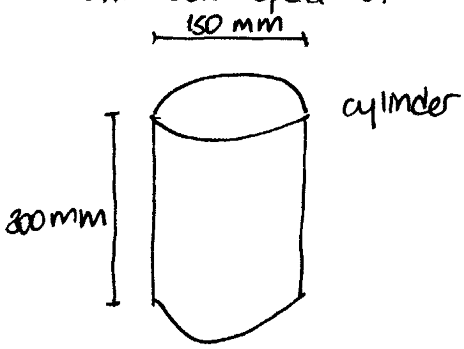
Concrete strength also depends on curing. For proper curing, water evaporation should be minimized after casting

Mechanical Properties of Concrete:

Uniaxial Compressive strength:

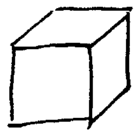
In design, 28 day strength of concrete (f_{c28}) under uniaxial compression is used. This value is obtained by testing some standard size specimens.

Universal Specimen



In Turkey: cylinder

+



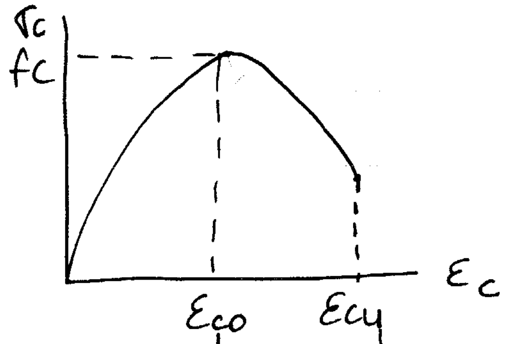
200x200x200 mm
or
150x150x150 mm } cube

$$\frac{(f_c)\,\text{cylinder}}{(f_c)\,\text{cube}} = 0.7 \sim 1.1 \text{ with a mean value of } 0.8 \sim 0.85$$

Variables affecting the strength:

- Shape of the specimen affects the strength. Cube specimens have larger cross sectional areas, therefore, require test machines with greater capacities. Moreover, due to sharp corners of the cube, residual stresses caused by the shrinkage of concrete become more significant.
In cube specimens, failure cracks are inclined due to frictional forces.
Lubricating top and bottom faces of the cube prevents this type of failure but reduces the strength up to 50%.
In cylinder specimens, capping top and bottom faces reduce the frictional forces and pure compression can be observed at the mid-section.
- Size of the specimen
size ↑ → strength ↓ due to increased imperfection
- Height / cross-sectional dimension ↑ – strength ↓
- Since concrete is a time-dependent material, rate of loading influences the strength
rate ↑ → strength ↑

Compressive Stress- Strain Characteristics:



ε_{cu} → ultimate

ε_{co} → ~0.002 for normal strength concrete (higher for confined and high strength concrete)

As the concrete strength increases, initial modulus of elasticity, which is the initial tangent to the $\sigma_c - \varepsilon_c$ curve, increases.

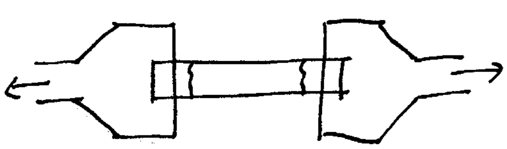
$\sigma_c - \varepsilon_c$ curve depends on :
- concrete strength
- confinement
- rate of loading

To obtain a mathematical model to represent the $\sigma_c - \varepsilon_c$ curve, the initial portion is approximated by a second degree curve and the descending portion is simplified to a straight line. A commonly used model is the Hognestad parabola.

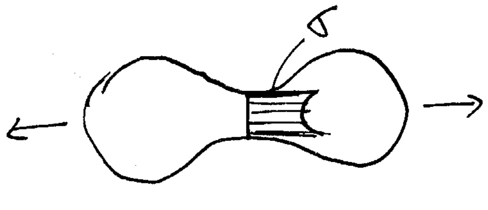
Tensile Strength

Brittle material → low tensile strength (10-15% of comp. strength)

Uniaxial Tension test:



local failure and lower strength due to stresses caused by the clamps of the machine.



fail below actual tensile strength due to stress concentration near the edges

⇒ both tests result in lower than actual strengths

⇒ need to develop other testing methods.

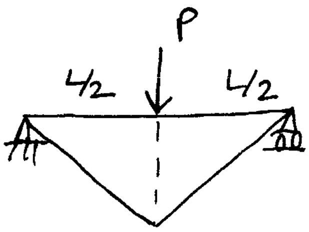
Indirect Tests:

Modulus of Rupture Test:

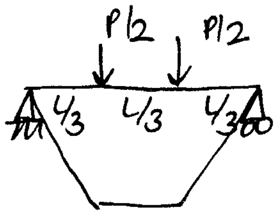
$$f_{ctf} = \frac{My}{I}$$ ⇒ for linear elastic materials.

150x150 mm   Modulus of Rupture
100 x 100 mm

Beam is tested under midspan or third point loading.



Midspan loading

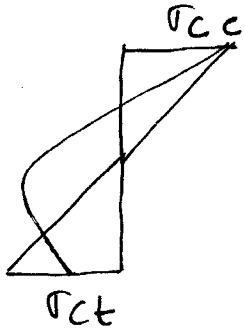


Third Point Loading

Third point loading gives lower average tensile strength than midspan loading. Because probability of having defects at a finite length is higher than at a single section.

$$\therefore \underbrace{f_{ctf}}_{\text{tensile strength obtained from flexural tests}} > \underbrace{f_{ct}}_{\text{true tensile strength.}}$$

due to redistribution

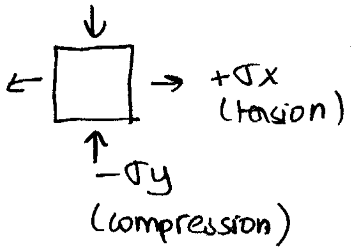


Split Cylinder Test:

Standard concrete cylinder (150 x 300 mm) is loaded under compression along two diametrically opposite lines. Tensile stress occurs perpendicular to the loading direction.

$$\underbrace{f_{cts}}_{\text{split tensile strength}} = \frac{2P}{\pi l d}$$

* Stresses are not uniaxial but biaxial.



Compression in the perpendicular direction would decrease the tensile strength compared to direct tension. However, since there is only one possible plane of failure, the possibility of having defects at the failure section is much smaller compared to direct tension test.

Generally $f_{cts} > f_{ct}$

Direct Tension Test: by Prof. Rüsch

Special specimen → uniform cross section at the middle and hunched at the ends (dog bone).

Direct tensile strength $= f_{ct} = 0.35\sqrt{f_c}$ (MPa)

Split tensile strength $= f_{cts} = 0.5\sqrt{f_c}$ (MPa)

Flexural tensile strength $= f_{ctf} = 0.64\sqrt{f_c}$ (MPa) (third point)

Flexural tensile strength $= f_{ctf} = 0.7\sqrt{f_c}$ (MPa) (midspan)

TS 500 : $f_{ct} = 0.35\sqrt{f_c}$ (MPa)

- σ − ε curve for concrete under tension: almost linear up to peak stress.
- With steel, plastic or carbon fibers, tensile strength of concrete can be increased.

Shear Strength:

$\underbrace{\text{Shear strength}}_{\cong 35\% - 80\% \text{ of } f_c}$ > Tensile strength for concrete

⇒ even under pure shear, a concrete section fails due to principal tensile stresses.

Modulus of Elasticity:

Modulus of Elasticity is the slope of the $\sigma - \varepsilon$ curve.

E_c = instantaneous Elastic modulus (time effect is not considered):

- initial modulus
- secant modulus (line between origin and $0.5 f_c$) ← commonly used
- tangent modulus (slope at $0.4 - 0.5 f_c$)

TS 500 → $E_{cj} = 3250 \sqrt{f_{cj}} + 14\,000$ (MPa)

j refers to age of concrete, $j = 28$

⇒ E_{c28}

When time dependent deformations are considered, under sustained load $E_c \downarrow$ up to $1/2 - 1/3$ of the initial value.

Shear Modulus:

$$G_c = \frac{E_c}{2(1+\mu_c)}$$, TS 500 → $G_c = 0.4\, E_c$

Poisson's ratio:

$$\mu = \frac{\text{transverse strain}}{\text{longitudinal strain}}$$

At lower stress levels ($\sigma_c / f_c \neq 0.3 - 0.7$) → $\mu_c = 0.15 - 0.25$

At higher stress levels μ increases significantly

TS 500 → $\mu_c = 0.20$, in design μ is neglected.

Coefficient of Thermal Expansion

$\alpha_c = 1 \times 10^{-5}$ mm/mm/°C $= \alpha_s$

(α_c ↳ concrete, α_s ↳ steel)

Behavior under multiaxial stresses:

* Under biaxial stresses:

- Tension in both directions: tensile strength is same as that of uniaxial tension.

- Tension in one direction : comp. in the orthogonal direction → Tensile strength is less than that of uniaxial tension.

- Compression in both directions: compressive strength is greater than that of uniaxial comp.

  max difference: 27% (for $k = 1.92$)

  If $\sigma_1 = \sigma_2$ $(k=1)$ → difference is 16%.

* Under triaxial stresses:

  More common state of stress in RC members.

  Compressive stresses: σ_1, $\sigma_2 = \sigma_3$

  due to monotonically increasing comp. load

  If $\sigma_2 = \sigma_3$ ↑ → strength and strain capacity of concrete ↑ (Richart)

* Failure theory by Zia and Cowan ⇒ revised version of Mohr's circle.

  For all compressive stresses, when $\sigma_2 = \sigma_3$

⇒ $f_{c1} = f_c + 4.0\,\sigma_2$ ⇒ equation gives failure envelope.
(f_{cc})

Time Dependent Deformation of Concrete =

Even under the same load or stress, deformations increase with time.

- shrinkage
- creep

$\}$ * cause significant deformations and stresses.
   * can effect both strength and serviceability

Shrinkage:

After concrete casting, the excess water that is not used in hydration evaporates → volume ↓ (shrink)

Shrinkage is a function of:

- temperature
- humidity
- area of exposed surface.
- water content of the mix.
- time

For long term shrinkage (at the end of 3 years) shrinkage coefficients, ε_{cs} are obtained from Table 1.3 (TS 500),

$l_c = (2A_c)/u$ → cross-sectional area

↳ equivalent thickness

→ perimeter in contact with the environment

If a plain concrete member is not restrained by others, it will shrink, but there won't be any stresses due to shrinkage.

For reinforced concrete members, steel won't shrink, therefore, as concrete dries, shrinkage will cause compression in reinforcement and tension in concrete.

For long walls and buildings, expansion joints are used to reduce internal forces (moment, shear, axial force) due to shrinkage and temperature drop.

effects are very similar

Creep:

Time dependent deformations under sustained load.

Depends on:

- age of concrete (for older concrete, less creep)
- w/c ratio (w/c ↑ → creep ↑)
- humidity and temperature (humidity ↑, creep ↓)
- level of sustained stress (stress ↑ → creep ↑)
- time (creep rate decreases with time, no effect after 3 years)

If concrete specimens are loaded up to 75% of comp. strength ($\sigma_c / f_c \leq 0.75$) → the load will be carried infinitely

$\sigma_c / f_c \geq 0.80$ → specimen fails under this load after a certain time.

⇒ strength is reduced 20% due to creep.

Δ_i = instantaneous deformation

= time - dependent deformation $= \Delta_t - \Delta_i$

Δ_{re} = elastic recovery (immediate)

Δ_p = never recovered $> \Delta_i$

creep strain $= \varepsilon_{ce} = \dfrac{\sigma_{co}}{E_{c28}} \phi_{ce}$ → creep coefficient under sustained loading, Table 1.4

Δ_{rc} = creep recovery

ex: Compute long term creep & shrinkage

$A_c = 90\,000\ mm^2$, u (perimeter) = 660 mm, $\sigma_c = 5$ MPa.

The member is loaded when concrete is 30 days old.

$E_{c28} = 27\,000$ MPa, $T_{average} = +15°C$, 75% relative humidity, curing is inadequate.

$$l_e = \frac{2A_c}{u} = \frac{2(90\,000)}{660} = 273\ mm$$

From Table 1.3 → $\varepsilon_{cs} = 0.00037$ (linear interpolation)

from Table 1.4 → $\phi_{ce} = 1.67$ (linear interpolation)

$$\varepsilon_{ce} = \frac{\sigma_{co}}{E_{c28}} \phi_{ce} = \frac{5}{27\,000} \times 1.67 = 0.00031$$

Creep can reduce stresses caused by imposed deformations such as differential settlement.

Steel Reinforcement:

Concrete is weak under tension → steel bars are needed.

- plain bars
- deformed bars (prevent slip)

In slabs & walls → welded wire fabric rectangular or square grid welded at each joint.

Ø12 → steel bar with a diameter of 12 mm.

↳ generally between Ø6 and Ø40 → Table 1.5

#5 → steel bar with a diameter of 5/8 inches.

Behavior under Monotonic Loading:

a) hot-rolled → definite yield point, significant yield plateau, strain hardening

b) cold worked → not suitable in seismic regions
high strength but low strain
not good for welding.

yield : $\varepsilon_s = 0.002$

Important properties of $\sigma - \varepsilon$ diagram:

- yield strength
- ultimate strength
- strain capacity

$E_s = 200\,000$ MPa (2×10^6 kgf/cm^2)

1.72

<u>Behavior under Repeated and Reversed Loading:</u>

Unloading before proportional limit → same curve

" after " " → // to the original curve

<u>Bauschinger Effect:</u> under reversed loading $\sigma - \varepsilon$ curve becomes nonlinear before reaching the yield level. This type of behavior is influenced by the strain history under earthquake loading.

<u>Concrete and Steel Grades:</u>

<u>Concrete</u>

C16 → cylinder compressive strength at 28 days in MPa.
↓
16 MPa

TS 500 → C16 – C50 (After C40 → high strength)

<u>Table 1.6</u>

f_{ck}
↳ characteristic

$0.85\sqrt{f_c}$

f_{ctk}

cube strength

E_{c28}

Zeynep

<u>Steel</u>

S420 → yield strength of steel in MPa
↓
420 MPa

<u>Table 1.7</u>

f_{yk} = min yield strength

f_{su} = max strength

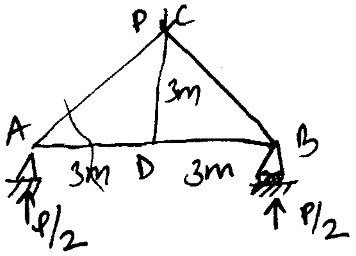
ε_{su} = min strain capacity (ultimate strain)

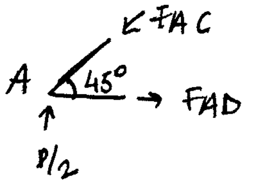
TS 500 → S220, S420, S500

ex: **Prob 1.2**

$f_c = 20$ MPa . Find max P carried by the structures below.

a) $A_c = 150 \times 150 = 22\,500 \text{ mm}^2$





$F_{AC} \sin 45 = P/2 \rightarrow F_{AC} = \frac{\sqrt{2}}{2} P$ (C)

$F_{AC} \cos 45 = F_{AD} \rightarrow F_{AD} = \frac{P}{2}$ (T)

$F_{AD} = F_{BD} = \frac{P}{2}$

$F_{CD} = 0$

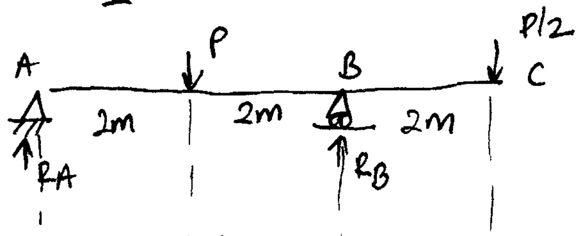
Direct tension $\rightarrow f_{ct} = 0.35\sqrt{f_c}$

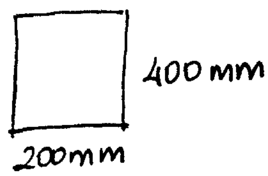
$F_{AD} = \frac{P}{2} = f_{ct} A_c$

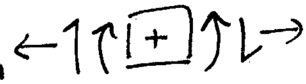
$\Rightarrow \frac{P}{2} = 0.35\sqrt{20} \times 22\,500 \rightarrow P = 70.4$ kN

$F_{AC} = \frac{\sqrt{2}}{2} P = f_c A_c$

b)

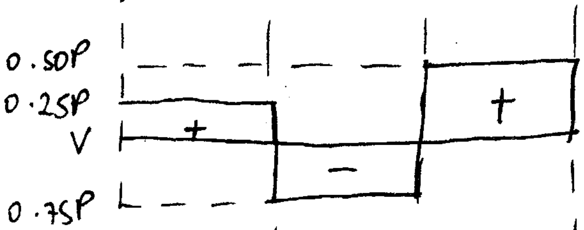


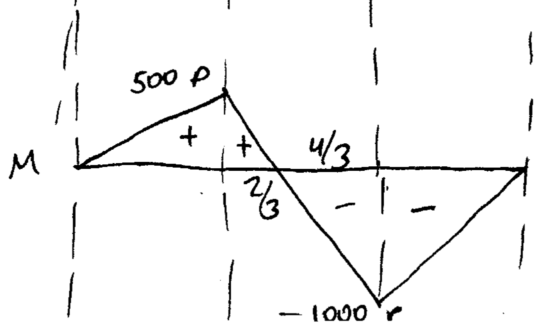




$\Sigma M @ A = 0 \; +\circlearrowleft \quad 4R_B - 2P - 3P = 0 \rightarrow R_B = 1.25\,P \uparrow$

$\Sigma F_y = 0 \; \uparrow + \quad R_A + R_B - P - \frac{P}{2} = 0 \rightarrow R_A = 0.25\,P \uparrow$



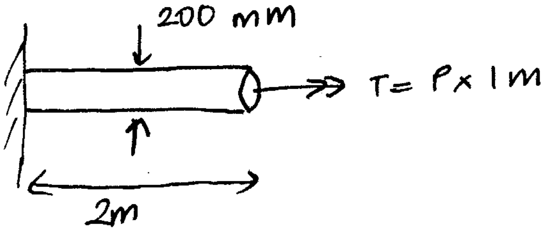


$\Rightarrow$ mid span

$$\underbrace{\sigma_{ctf}}_{\text{due to flexure}} = \frac{My}{I} = \frac{1000\, P \times 400/2}{\frac{1}{12}(200)(400)^3} = f_{ctf} = 0.7\sqrt{fc}$$

$\Rightarrow P = 16.7$ kN

c)



$\underbrace{f_{cts}}$ = $0.5\sqrt{fc}$ = σ_{cts}

due to torsion $\rightarrow \sigma_{cts} = Mr/J$

or shear stress $\rightarrow \tau_{cts} = VQ/It$

$$\sigma_{cts} = \frac{Mr}{J} \quad r \to d/2, \quad J \to \frac{\pi d^4}{32} \quad = \frac{1\times10^3\, P\,(N\,mm) \times 200/2}{\frac{\pi(200)^4}{32}}$$

$\Rightarrow P = 3.5$ kN

Structural Safety $\Rightarrow$ Chapter 2

Main objective of structural design is to prevent failure $\rightarrow$ the structure should remain functional under applied loading.

Functional $\rightarrow$ no collapse, remain serviceable.

Failure $\rightarrow$ collapse or become unserviceable.

↳ loss of money and life.

unserviceable $\rightarrow$ excessive deformations

" vibrations

" cracking

depends on the function of the structure or structural member.

Structure is safe (won't fail) if:

$R \geq F$

external action

Resistance $\geq$ Load Effect

↳ strength

↳ resistance against deflection or cracking ex: moment capacity ≥ applied moment

shear capacity ≥ applied shear

A factor of safety is used in design to be conservative (on the safe side) and to account for all the variations in resistance and load effect.

Variations in Resistance:

There are variations in resistance due to uncertainties some of which are listed below:

- variations in actual strength of the material

  steel reinf → ± %5

  concrete → ± %20

- variations in actual dimensions of structural members may be critical depending on the member.

  15 mm error in 800 mm deep beam → not critical

  15 mm error in 100 mm deep slab → critical

- variation in steel reinf. area

  If a Ø8 bar has a nominal diameter of 7.7 mm in steel of 8 mm → area is 7% less than the required value.

- real behavior of the structure may differ from ideal behavior. (ex: linear elastic analysis)

- approximations and assumptions made for analysis.

- behavior and strength change as a function of time.

Variations in the Design Load:

For some types of loading (ex: earthquake loading), statistical data is limited → actual load may be different from design load.

Variations in resistance can be represented by normal distribution. In most cases, loading has an unsymmetrical distribution, but for simplicity assume it also has normal distribution.

For normal distribution,

$$\text{mean value} = \frac{1}{n}\sum_{i=1}^{n} x_i = x_m$$

$$\text{standard distribution} = \bar{\sigma} = \sqrt{\frac{\Sigma (x_i - x_m)^2}{n-1}}$$

R and F are random variables.

$R_m, F_m \rightarrow$ mean values

$R_k, F_k \rightarrow$ characteristic values

R_k = characteristic resistance is the resistance below which only a small percentage will fail.

$\rightarrow$ probability of having a resistance less than R_k is low.

F_k = characteristic load effect is the load which has a low probability of exceedence.

$$\left.\begin{aligned} F_k &= F_m + u\bar{\sigma} \\ R_k &= R_m - u\bar{\sigma} \end{aligned}\right\}$$ u is a coefficient related to probability obtained from a table.

ex: Failure probability of 10% to satisfy characteristic resistance.

$1-F = 0.1 \Rightarrow u = 1.282$ (from table)

To be more specific,

$R_k \geq F_k \rightarrow$ not safe enough

$$\Rightarrow \frac{R_k}{\gamma_m} \geq F_k \gamma_f$$

$\gamma_m \geq 1.0$ material factor

$\gamma_f \geq 1.0$ load factor

In working stress design γ_m & γ_f are combined into FS.

$$\cancel{\frac{R_k}{\gamma_m \gamma_f} = \frac{R_k}{FS}} \geq F_k$$ → correct if R_k and F_k are independent but they are dependent for RC.

γ_m and γ_f depend on: → probability of failure
→ type and purpose of the structure
→ type of the material
→ type of loading

Limit State Design:

In limit state design, the local or overall behavior at all stages are considered.
– elastic, plastic, cracked, ultimate

In TS 500 → ultimate limit state
→ serviceability limit state

Ultimate Limit State: may be reached due to:

– loss of equilibrium
– rupture of critical sections
– formation of a mechanism
– instability
– fatigue

For ultimate limit state, characteristic loads are considered.

$F_k = F_m + u\sigma$

Nominal values given in the building codes can be used as characteristic values.

$$F_d = \gamma_g G + \gamma_q \{ Q_{1k} + \Sigma \psi_{0i} Q_{ik} \}$$

F_d → design load
G → dead load
Q_{1k} → basic live load
Q_{ik} → other live load
ψ_{0i} → combination factor
γ_g → load factor for the dead load
γ_q → load factor for the live load

$\gamma_g < \gamma_q$, because γ_g can be determined more precisely

$\psi_{0i} < 1$, because when several loads act at the same time the probability of each load reaching its characteristic value decreases.

$$f_d = \frac{f_k \rightarrow \text{characteristic strength}}{\gamma_m \rightarrow \text{material factor} \geq 1.0}$$

f_d ↓ design strength (material strength used in ultimate limit state)

For reinf. if $f_{yd} = \dfrac{f_{yk}}{\gamma_{ms}}$

For concrete, $f_{cd} = \dfrac{f_{ck}}{\gamma_{mc}}$

$\gamma_{mc} > \gamma_{ms}$, because the variation in f_c is greater than the variation in f_y.

<u>Serviceability Limit State</u>: is checked to verify that structure is functional.

Load factor = 1.0

Material factors $\geq$ 1.0

<u>Safety Provisions in TS 498 (TS 500):</u>

Load Factors and Load Combinations:

* For gravity loads only:

$*F_d = 1.4G + 1.6Q$

G ↓ dead load; Q ↳ live load

T: load effect due to imposed deformations such as temperature change, shrinkage, support settlement etc.

$* F_d = 1.0G + 1.2Q + 1.2T$

if T can be neglected, only 1st equation is checked.

* For wind load: Always consider first two eqns.

$* F_d = 1.0G + 1.3Q + 1.3W$

W ↳ load effect due to wind.

$* F_d = 0.9G + 1.3W$

* For seismic load:

* $F_d = 1.0G + 1.0Q + 1.0E$

* $F_d = 0.9G + 1.0E$

* For earth pressure:

$F_d = 1.4G + 1.6Q + 1.6H$
↳ load effect due to earth pressure

$F_d = 0.9G + 1.6H$

⇒ use max F_d obtained

Material factors:

Steel → $f_{yd} = (f_{yk}) / \gamma_{ms}$ (design; characteristic)

$\gamma_{ms} = 1.15$

$f_{ywd} = f_{ywk} / \gamma_{ms}$ (transverse reinforcement)

For characteristic values ⇒ Table 1.7

Concrete → $f_{cd} = f_{ck} / \gamma_{mc}$

$f_{ctd} = f_{ctk} / \gamma_{mc}$ (design tensile strength)

For characteristic values ⇒ Table 1.6

$$\gamma_{mc} = \begin{cases} \boxed{1.5}, & \text{cast is place} \\ 1.4, & \text{precast} \\ 1.7, & \text{inadequate quality control for cast in place} \end{cases}$$

Order ready-mix concrete wrt f_{ck}

In design f_{cd}

For mix-design use $f_{cm} = f_{ck} + u\bar{\sigma}$ (f_{cm}: Mean)
↳ usually, $u = 1.28$

of μ & $\bar{\sigma}$ are not known, $f_{cm} = f_{ck} + \Delta f_c$

$$\Delta f_c = \begin{cases} 4 \text{ MPa for C16/C18} \\ 6 \text{ MPa for C20 to C30} \\ 8 \text{ MPa for C35 to C50} \end{cases}$$

Live load Arrangements:

DL → on all spans of the structure

LL → arranged to produce max internal force

(checker board loading)

Quality Control:

Essential in the construction state to ensure safety.

For concrete:

- From each production unit take at least one group (3 standard cylinders) of tests specimens.
- From each cast take at least 3 groups (9 cylinders).

For ready mix concrete take 3 groups from each different concrete mixer. Accept concrete only if:

Average of each batch, $f_{cm} \geq f_{ck} + 1.0$ MPa

Min group average in each batch, $f_{cmin} \geq f_{ck} - 3.0$ MPa

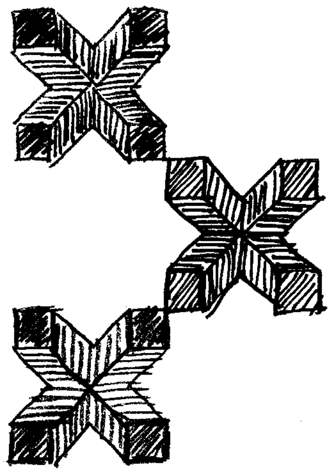
<u>Uni-Axial loading:</u>

* <u>Behavior of concrete confined by lateral Reinforcement</u>:

Rectangular hoops (stirrups) and continuous spirals are the most commonly used types of lateral reinforcement.

At high compressive stresses, concrete cover, which is not confined by lateral reinforcement, reaches its limiting strain nature, crushes and starts to spall off. Concrete inside the lateral reinforcement tries to expand, but this deformation is prevented by the lateral reinforcement. Due to this confining pressure, the concrete is under triaxial stress rather than uniaxial one.

* Rectangular hoops are not as effective as the circular hoops or spiral reinforcement.



* Confinement $\nearrow$ $\rightarrow$ strength & strain capacity $\nearrow$

Material to Material Redistribution:

RC column subjected to uniaxial compression.

Equilibrium : $N = N_c + N_s = A_c \sigma_c + A_s \sigma_s$

Compatibility: $\varepsilon_s = \varepsilon_c = \varepsilon$

$\sigma - \varepsilon$ Relationships: Concrete is not linearly elastic!

$$\sigma_c \neq E_c \varepsilon_c$$

$$\sigma_c = f_c \left[\frac{2\varepsilon_c}{0.002} - \left(\frac{\varepsilon_c}{0.002} \right)^2 \right]$$

$$\sigma_s = \varepsilon_s E_s \leq f_y$$

Assume $\varepsilon = 0.001 \rightarrow \sigma_c = 20 \left[\frac{2 \times 0.001}{0.002} - \left(\frac{0.001}{0.002} \right)^2 \right] = 15 \text{ MPa}$

C20 20x20 cm

$\rightarrow N_c = A_c \sigma_c = 15 \times 40000 = 600 \text{ kN}$

$\sigma_s = 0.001 \times 200\,000 = 200 \text{ MPa} < f_y$

$\rightarrow N_s = 200 \times 1600 = 320 \text{ kN}$

After concrete reaches the peak stress, when the strain increase, concrete starts to unload and N_c/N_s reduces $\rightarrow$ redistribution

For $\varepsilon = 0.00262 \rightarrow N_s = 840$ kN, $N_c = 732$ kN, $N_c/N_s = 0.86$

$\varepsilon = 0.003 \rightarrow N_s = 840$ kN, $N_c = 600$ kN, $N_c/N_s = 0.71$

Time dependent deformations like shrinkage and creep also cause significant redistribution between steel & concrete.

Axially Loaded Members:

Columns: Vertical members which support the floor load transferred either directly or through beams. Columns transfer the load to the foundation.

Uniaxially loaded members are not permitted in the design codes, but uniaxial compression is the limiting case for combined flexure and axial compression.

Behavior of Axially Loaded Columns:

If reinforcement reaches its yield strength first, the increased deformations cause build up stresses in concrete until crushing strength is reached.

If concrete reaches its ultimate strength before steel yields, the increased deformations force the stresses on reinforcement to build up until yield strength is reached.

In any case, column won't fail until both materials reach their limiting values.

Compressive strength of concrete in the columns is about 85% of fc for a standard test cylinder. Because, the columns are casted vertically, hence, concrete is not as compacted as in a cylinder.

Strength of Confined Concrete:

* $\underbrace{f_{cl} = f_{cc}}_{\text{strength of concrete under triaxial compression}} = f_{ck} + 4\,\sigma_2$

Under high compressive stresses → shell concrete spalls off

q = radial pressure ← core will expand laterally ← Poisson's ratio ↑

From Strength of Materials:

$$\sigma_{sp} = \frac{\overset{\text{radial pressure}}{q}\, r}{t}$$

$\hookrightarrow$ stress in a thin pipe

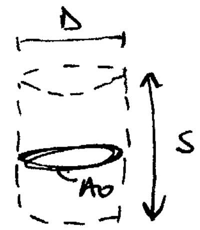
$$r = \frac{D}{2}, \quad q = \sigma_2, \quad t = \frac{A_o}{s}$$

$A_o \rightarrow$ area of spiral

$s \rightarrow$ spiral spacing

$$\sigma_{sp} = \frac{\sigma_2 D s}{2 A_o}$$

From equilibrium $\Rightarrow \sigma_2 . D . s = \overset{\text{2 legs}}{2} A_o f_{ywk}$



$$\sigma_{sp} = f_{ywk} \Rightarrow \sigma_2 = \frac{2 A_o f_{ywk}}{D . s}$$

$$\text{Volumetric ratio} = \rho_s = \frac{\overbrace{A_o \pi D}^{\text{volume of spiral}}}{\underbrace{\frac{\pi D^2}{4} s}_{\text{volume of concrete}}} = \frac{4 A_o}{D s}$$

The strength at the second peak of the load-deformation curve should not be less than the one at the first peak.

Loss of strength is due to the spalling off of cover concrete.

$$-\Delta F = \underbrace{0.85}_{\text{reduction for column}} f_c \,(A_c - A_{ck})$$

$A_{ck} \hookrightarrow$ core area

Strength gain is due to confinement of the core concrete:

$$+\Delta F = (f_{cc} - f_{ck}) A_{ck}$$

$$= [(\cancel{f_{ck}} + 4\sigma_2) - \cancel{f_{ck}}] A_{ck} = (4\sigma_2) A_{ck}$$

$$= 4\left(\frac{2 A_o f_{ywk}}{D . s}\right) A_{ck} = \frac{8 A_o f_{ywk}}{D . s} A_{ck}$$

To have equal peaks: $-\Delta F = +\Delta F$

$$0.85 f_{ck} (A_c - A_{ck}) = \underbrace{\left(\frac{8 A_o}{D . s}\right)}_{2\rho_s} A_{ck} f_{ywk}$$

$$\rho_s = \frac{0.85 f_{ck} (A_c - A_{ck})}{2 A_{ck} f_{ywk}}$$

$\Rightarrow$ This is the minimum spiral volumetric ratio to have equal first and second peaks.

$$\min \rho_s = 0.425 \frac{f_{ck}}{f_{ywk}} \left(\frac{A_c}{A_{ck}} - 1 \right)$$

In TS 500:

$$* \min \rho_s = 0.45 \frac{f_{ck}}{f_{ywk}} \left(\frac{A_c}{A_{ck}} - 1 \right) \quad \ldots\ldots (1)$$

In codes a second equation is given to prevent $\min \rho_s \to 0$ when $\frac{A_c}{A_{ck}} \to 1$.

$$* \min \rho_s = 0.12 \left(\frac{f_{ck}}{f_{ywk}} \right) \quad \ldots (2)$$

$\min \rho_s$ is the larger value obtained from (1) & (2)

* The deformation on the rectangular stirrups are similar to the deflection of a beam. max deflection occurs at midspan where the confinement approaches to zero. Effective confinement is at the corners of the stirrup or at the cross-tie which hold the longitudinal reinf. together.

Flexural stiffness < Axial stiffness

$\Rightarrow$ Rectangular ties are less effective than spiral ones.

<u>Strength of Columns under Uniaxial Compression</u>:

$$N_{or} = 0.85 f_{ck} (A_c - A_{st}) + A_{st} f_{yk}$$

A_{st} is generally small (not in ACI!)

$\Rightarrow N_{or} = \underbrace{0.85}_{} f_{ck} A_c + A_{st} f_{yk}$ ← ultimate strength of tied columns and spiral columns wrt the first peak.

$- A_{st} f_{yk}$ $= N_{or}$ (tension)

difference between column & standard cylinder

Strength of spiral columns wrt the second peak:

Replace A_c with A_{ck} and $0.85 f_{ck}$ by f_{cc} → strength of confined concrete

$$N_{or} = (0.85 f_{ck} + 4\sigma_2) A_{ck} + A_{st} f_{yk}$$

$$N_{or2} = (0.85 f_{ck} + 4\sigma_2) A_{ck} + A_{st} f_{yk}$$

$$\sigma_2 = \frac{2 A_o f_{ywk}}{D \cdot s}$$

$$\Rightarrow N_{or2} = 0.85 f_{ck} A_{ck} + 8 \frac{A_{ck} f_{ywk} A_o}{D.s} + A_{st} f_{yk}$$

$$\rho_s = \frac{4 A_o}{D s}$$

$$\Rightarrow N_{or2} = A_{ck} (0.85 f_{ck} + 2 \rho_s f_{ywk}) + A_{st} f_{yk}$$

Second peak is only reached at very high deformations. This strength is relied upon only under severe dynamic loading which causes excessive inelastic deformations.

<u>Minimum Requirements for Column Design:</u>

Minimum cross-sectional dimension.

- to provide minimum stiffness against lateral loading.
- to provide adequate spacing between longitudinal reinf.
- to ease concrete casting.

Minimum longitudinal reinf. ratio:

- accidental eccentricities (bending)
- time dependent deformations

Minimum diameter of the longitudinal reinf:

- to have a minimum stiffness against buckling.

Maximum longitudinal reinf. ratio:

- to ease concrete casting

* Longitudinal reinf ratio of 1% to 2% is common

Minimum diameter and max spacing for the lateral reinf:

- to provide confinement
- to prevent buckling of longitudinal bars
- to hold the longitudinal bars in place.

In seismic regions use closed stirrups

<u>Strength of Columns under Uniaxial tension:</u>

For a symmetrically reinforced prismatic member,

Strength upto cracking $= N = A_c \sigma_c + A_{st} \sigma_s$

Approximate cracking load $= N_{cr} = A_c f_{ctd}$

$f_{ctd} = \frac{0.35}{1.5} \sqrt{f_{ck}}$ MPa.

After cracking, concrete resistance $= 0 \Rightarrow$

$N = A_{st} \sigma_s$

ultimate load $= N_r = A_{st} f_{yd}$

Minimum reinf. is required to prevent sudden brittle failure.

Plain concrete $\rightarrow N = A_c f_{ctk}$

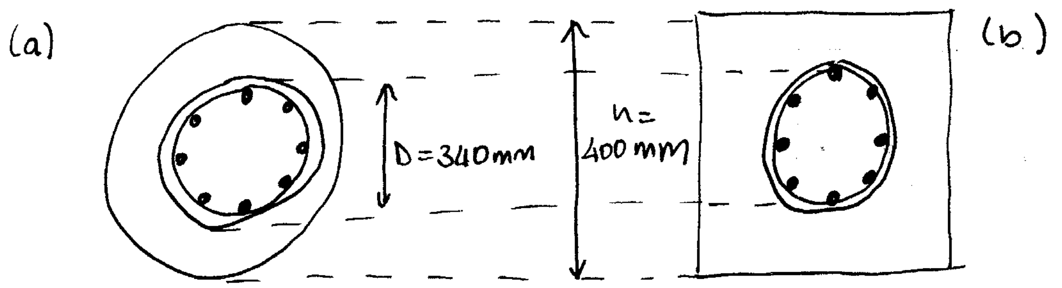
Reinforced concrete $\rightarrow N = A_{st} f_{yk}$

min $A_{st} = \frac{f_{ctk}}{f_{yk}} A_c$

min $\rho_t = \frac{f_{ctk}}{f_{yk}}$

Take F.S. $= 1.5 \rightarrow$ min $\rho_t = 1.5 \frac{f_{ctk}}{f_{yk}}$

ex: Calculate min spiral reinf.



C25, S220, 8Ø16 bars

C25 $\rightarrow f_{ck} = 25$ MPa

S220 $\rightarrow f_{yk} = f_{ywk} = 220$ MPa

8Ø16 $\rightarrow A_{st} = 8 \times \pi \frac{16^2}{4} = 1608$ mm^2

a) Both circular, $D = 340$ mm, $h = 400$ mm

$A_c = \pi \frac{400^2}{4} = 125664$ mm^2, $A_{ck} = \pi \frac{340^2}{4} = 90792$ mm^2

$\min \rho_s = 0.45 \frac{f_{ck}}{f_{ywk}} \left(\frac{A_c}{A_{ck}} - 1 \right) = 0.0196$

For $\phi 8 \rightarrow A_o = 50\ mm^2$, $\rho_s = \frac{4A_o}{Ds} \Rightarrow s^{-1} = \frac{\rho_s D}{4A_o}$

$$s = \frac{4(50)}{340(0.0196)} = 30\ mm$$

For $\phi 10 \rightarrow A_o = 78.5\ mm^2$, $s = \frac{4(78.5)}{340(0.0196)} = 47\ mm$

$\Rightarrow$ use $\phi 10/40\ mm$

b) Square column with spiral, $D_c = 340\ mm$, $h = 400\ mm$

$A_c = 400 \times 400 = 160000\ mm^2$

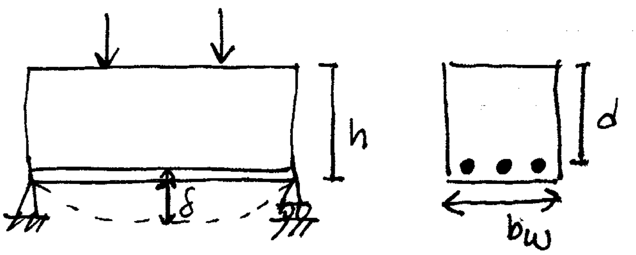
$A_{ck} = 90792\ mm^2$

$$\min \rho_s = 0.45\left(\frac{25}{220}\right)\left(\frac{160000}{90792} - 1\right) = 0.039$$

$\phi 10 \rightarrow A_o = 78.5\ mm^2$, $s = \frac{4(78.5)}{340(0.039)} = 23.7\ mm$

$\phi 12 \rightarrow A_o = 113\ mm^2$, $s = \frac{4(113)}{340(0.039)} = 34\ mm$

Pure Bending:

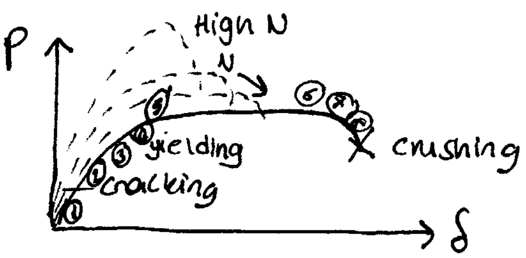


h = total depth

d = effective depth

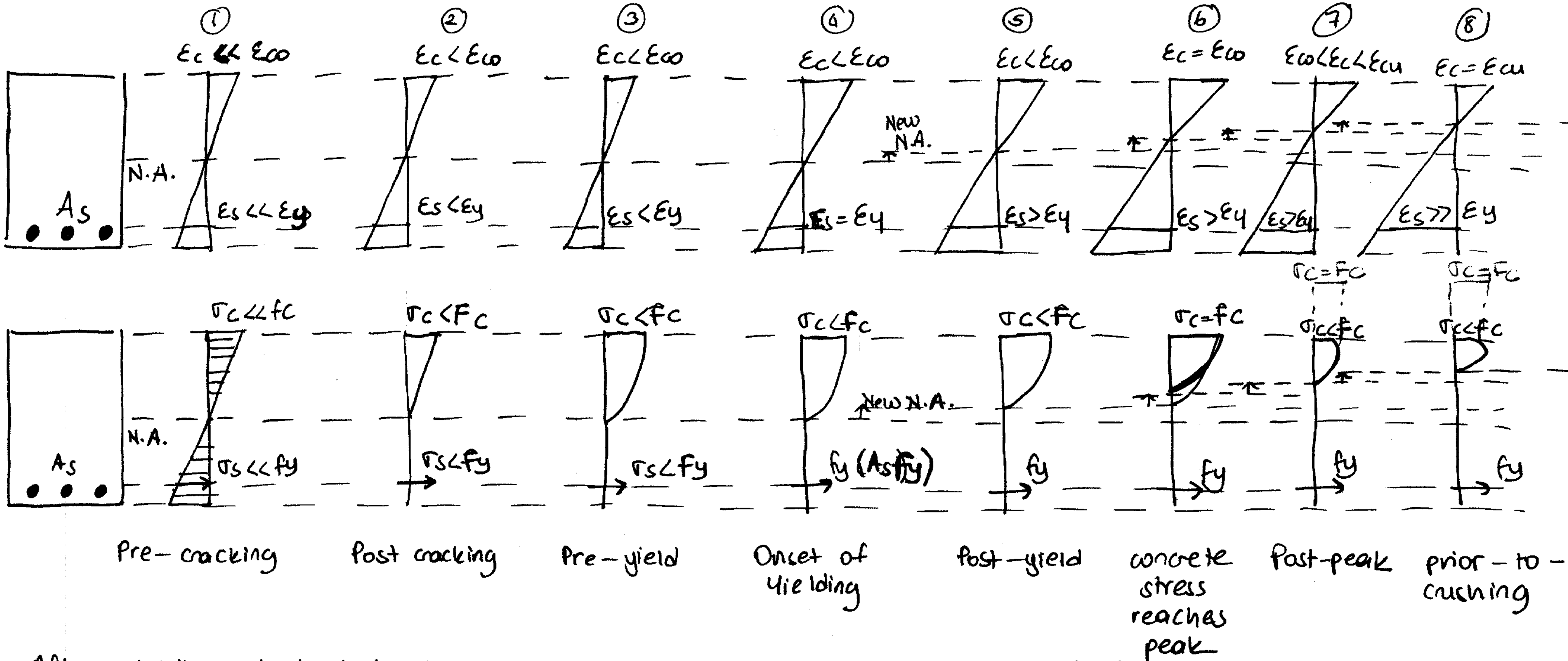
Clear cover is required for:

* Bond
* Fire resistance
* Corrosion protection

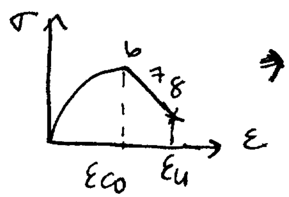


② → when inelastic activity starts

Beam Behavior:



- After yielding, steel strain increases much more rapidly when compared to concrete strain.
  To satisfy compatibility neutral axis rises. (c gets smaller).
- The ratio of ultimate moment to yield moment is 1.0 to 1.15.



→ Hognestad parabola

Moment – Curvature ($M - K$) Relationship:

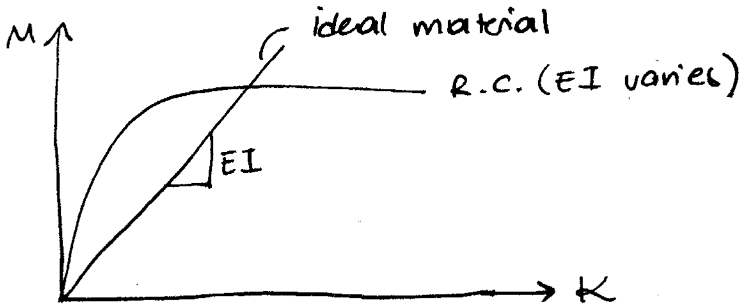
* Load – Deflection Curve gives a general idea, but Moment – Curvature Relationship is the correct indication of flexural behavior.
* Moment is the load effect for bending.
* Curvature is the measure of bending deformation.

<u>Curvature:</u> is the rate of change of the slope (i.e, second derivative) or the rotation per unit length.

From strength of materials:

$$K = \phi = \frac{1}{\rho} = \frac{d^2y}{dx^2} = \frac{M}{EI}$$

→ EI is the slope of the M–K diagram and is called the flexural rigidity.



For the same cross section area, the shape and properties of the M–K diagram changes significantly with the level of axial load.

K_y shows the yielding of tension reinf. After this point, curvature increases without an increase in the moment ⇒ plastic hinge is formed.

hinge: rotates under zero moment.

plastic hinge (PH): section rotates under a constant member.

<u>Ductility:</u> is the capability of undergoing large deformations without a $\underbrace{\text{significant reduction}}_{\leq 15\%}$ in the strength.

Curvature Ductility (Ratio): $\frac{K_u}{K_y}$ related to the properties of the cross section & the $\sigma-\varepsilon$ relationship of the materials.

Displacement Ductility: $\frac{\delta_u}{\delta_y}$ related to the member properties. Different from curvature ductility.

Area under M-K diagram ↑ Energy Dissipation Capacity ↑

⇒ Ductility ↑ Energy Dissipation Capacity ↑

Types of Failure in RC Members:

Section	Behavior	Failure
Balanced	Balanced	Balanced Failure
Under-reinforced	Ductile	Tension Failure
Over-reinforced	Brittle	Compression Failure

Tension Failure: Steel reinf. yields under tension prior to the crushing of concrete ⇒ ductile behaviour → considerable deformation before failure. Desirable, because not sudden, warning before failure and excessive deformations.

At K_y, $\varepsilon_s = \varepsilon_y$, $\varepsilon_c < \varepsilon_{cu}$

At Failure, K_u, $\varepsilon_s \gg \varepsilon_y$, $\varepsilon_c = \varepsilon_{cu}$

Compression Failure: Concrete crushes under compression, before reinf. yields under tension. Crushing strain of concrete is low, therefore, the failure is brittle & sudden and the energy dissipation capacity is low.

At Failure, $K_{(c)} < K_y$, $\varepsilon_s < \varepsilon_y$, $\varepsilon_c = \varepsilon_{cu}$

Balanced Failure: Crushing of concrete in extreme fiber occurs simultaneously with the yielding of tension reinf.

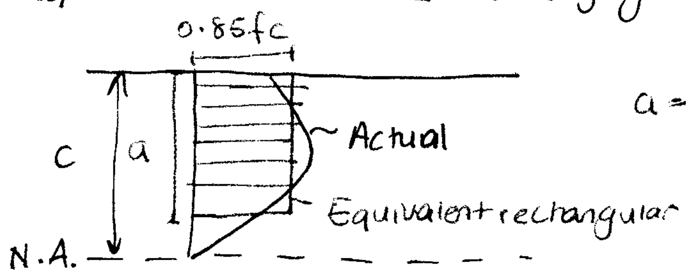
At Failure, $K_b > K_{y(+)}$, $\varepsilon_c = \varepsilon_{cu}$, $\varepsilon_s = \varepsilon_y$

Assumptions:

* Plane sections remain plane after bending.
* Concrete cannot take any tension (negligible tension in conc.).
* Perfect bond between steel and concrete.
* Elastoplastic $\sigma - \varepsilon$ for reinforcing steel. (realistic for hot-rolled steel)

$\sigma_s = E_s \varepsilon_s \leq f_y$

* maximum strain in the extreme fiber of concrete in compression is ε_{cu} ($\varepsilon_{cu} = 0.003 \Rightarrow$ an approximate, average, small effect on the result).
* Concrete stress distribution in the compression zone is represented by an equivalent stress block satisfying the area and centroid.



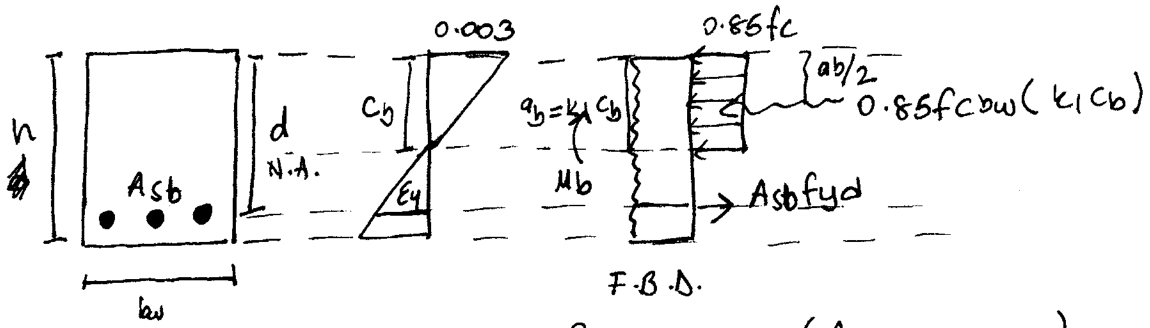
$a = k_1 c$ ⟶ depends on compressive strength of concrete, for $f_c \leq 25$ MPa $k_1 = 0.85$, $f_c \uparrow$ $k_1 \downarrow$

Note: For each 5MPa increase in f_c, k_1 reduces by 0.03.

Analysis of Beams:

* Ballanced Case, Rectangular Beams Reinforced for Tension only:

$\varepsilon_{cu} = 0.003$ and ε_y are reached simultaneously.



F.B.D.

3 unknowns (A_{sb}, C_b, M_b)

$\Rightarrow$ 2 eqns of equilibrium

+1 compatibility eqn.

Equilibrium eqns : $\sum F = 0 \rightarrow A_{sb} f_{yd} = 0.85 f_c b_w (k_1 c_b)$

$$\sum M = 0 \rightarrow M_b = A_{sb} f_{yd} \underbrace{\left(d - \frac{k_1 c_b}{2} \right)}_{\gamma_b \cdot d}$$

Compatibility: $\dfrac{c_b}{d} = \dfrac{0.003}{0.003+\varepsilon_y} = \dfrac{0.003}{0.003+\dfrac{f_{yd}}{E_s}} = \dfrac{0.003\,E_s}{0.003E_s + f_{yd}}$

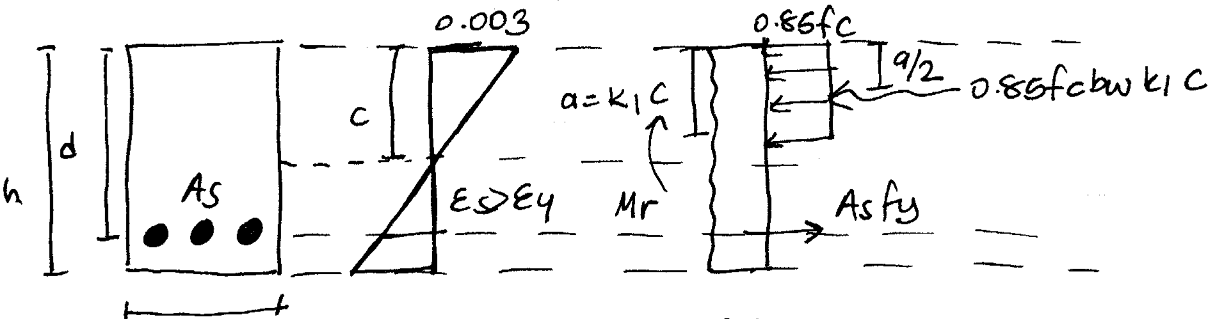
Divide $\Sigma f = 0$ by $b_w d$, $\rho_s = \dfrac{A_s}{b_w d} \rightarrow \rho_b = \dfrac{0.85 f_{cd}}{f_{yd}}\, k_1 \dfrac{c_b}{d}$

Divide $\Sigma M = 0$ by $b_w d^2$, $\dfrac{b_w d^2}{M_b} = K_b = \dfrac{1}{\rho_b f_{yd} j_b}$

$j_b = \left(1 - \dfrac{k_1 c_b}{2d}\right)$

All of them are functions of material properties $\rightarrow$ Table 5.1 is obtained by $\gamma_{ms} = 1.15$ & $\gamma_{mc} = 1.5$.

* Under-reinforced Rectangular Beams Reinforced for Tension Only:



bw   ε-Distribution   F.B.D.

2 unknowns c, M_r

2 eqns of equilibrium are sufficient.

Design Coefficient, K

$\Sigma F = 0 \longrightarrow a = \dfrac{A_s f_{yd}}{0.85 f_{cd} b_w}$

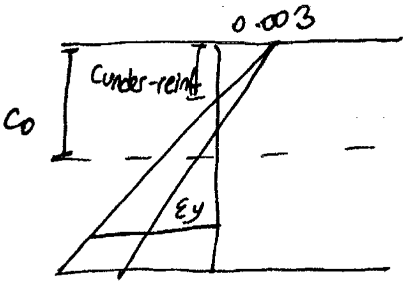
$\Sigma M = 0 \longrightarrow M_r = A_s f_{yd}\left(d - \dfrac{k_1 c}{2}\right) = A_s f_{yd}\left(d - \dfrac{A_s f_{yd}}{1.7 f_{cd} b_w}\right)$

$\rho = \dfrac{A_s}{b_w d} \rightarrow M_r = \rho\, b_w d\, f_{yd}\left(d - \dfrac{\rho b_w d f_{yd}}{1.7 f_{cd} b_w}\right)$

$= \rho\, b_w d^2 f_{yd} \left(1 - 0.59\, \rho \dfrac{f_{yd}}{f_{cd}}\right)$

$K = \dfrac{b_w d^2}{M_d} = \dfrac{1}{1 - 0.59\,\rho \dfrac{f_{yd}}{f_{cd}}}$ $\Rightarrow$ function of material properties.

For tension failure (under-reinforced beam)



Under-reinf beam if:

$A_s < A_{so}$ or $\rho < \rho_b$

$c < c_b$ or $a < a_b$

$k > k_b$

ex: Simply supported beam

$L = 5m$, $b_w = 230$ mm, $d = 460$ mm.

Uniformly distributed load $g = 10$ kN/m & $q = 5$ kN/m

Can the beam carry the given load?

$5\phi 20$, C16, S220.

$5\phi 20 \rightarrow A_s = 1570\ mm^2 \rightarrow \rho = \frac{A_s}{b_w d} = \frac{1570}{(230)(460)} = 0.0148$

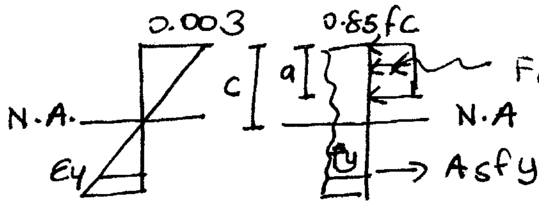
$C16 \rightarrow f_{cd} = \frac{16}{1.5} = 11$ MPa

$\hookrightarrow k_1 = 0.85$

$S220 \rightarrow f_{yd} = \frac{220}{1.15} = 191$ MPa

From Table 5.1 $\rightarrow \rho_b = 0.0316$

OR (if the table is not given)



strain diagram FBD

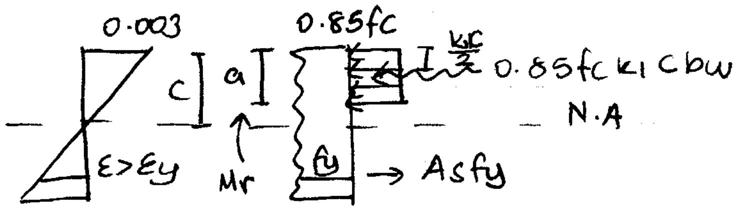
Equilibrium $\Rightarrow 0.85 f_c k_1 c b_w = A_{sb} f_{yd} \rightarrow \rho_b = \frac{0.85 f_{cd} k_1 c_b}{f_{yd} \cdot d}$

$\frac{c_b}{d} = \frac{0.003}{0.003 + \varepsilon_y} \rightarrow c_b = 348.9$ mm

$\varepsilon_y = \frac{191}{200\,000} \Rightarrow f_{yd} = \sigma_y = E_s \varepsilon_y$

$\rho_b = \frac{0.85 \times 11 \times 0.85 \times 348.9}{191 \times 460} = 0.0316$

$\rho < \rho_b \rightarrow$ under - reinforced

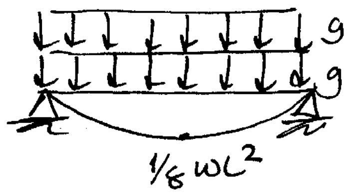


$$0.85 f_c k_1 c b_w = A_s f_y \rightarrow k_1 c = \frac{(1570)\times 191}{0.85(11)(230)} = 139.4 \text{ mm}$$

$$M_r = A_s f_{yd} \left(d - \frac{k_1 c}{2} \right)$$

$$M_r = 1570 \times 191 \left(460 - \frac{139.4}{2} \right) \times 10^{-3} \times 10^{-3} = 117 \text{ kN.m}$$

<u>Applied moment:</u>



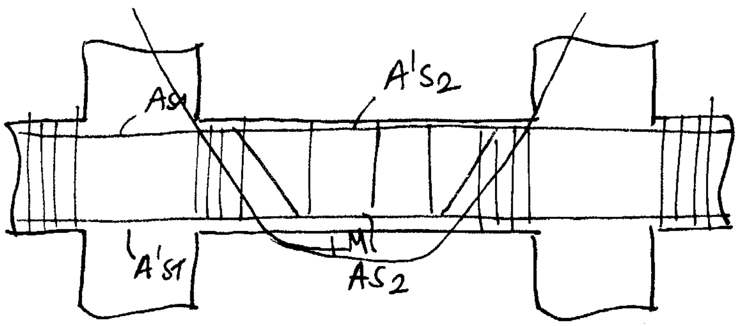
$$M_g = \frac{1}{8} g L^2 = \frac{1}{8}(10)(5)^2 = 31.3 \text{ kN.m}$$

$$M_q = \frac{1}{8} q L^2 = \frac{1}{8}(5)(5)^2 = 15.6 \text{ kN.m}$$

$$M_d = 1.4 M_g + 1.6 M_q = 1.4(31.3) + 1.6(15.6) = 68.8 \text{ kNm}$$

$M_r = 117$ kNm $> M_d = 68.8$ kNm $\rightarrow$ <u>safe</u>
can carry the load.

Doubly (Double) Reinforced Rectangular Beams:



At support $\rightarrow A'_{s1} \rightarrow$ code requires that some portion of A_{s2} should be extended as compression reinf.

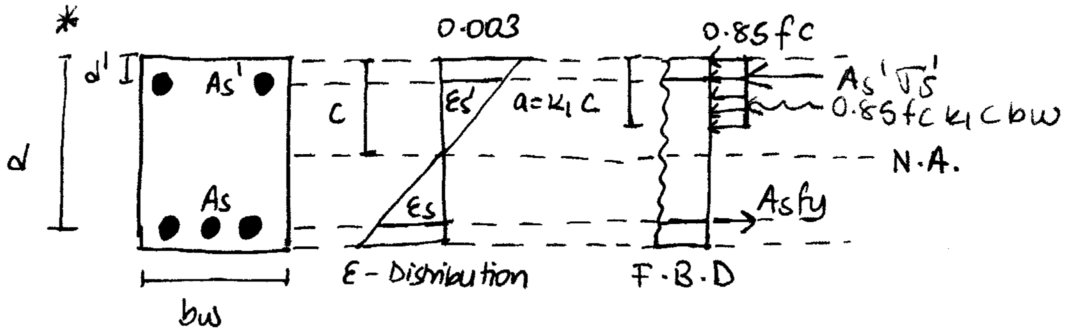
$\Rightarrow$ $\frac{1}{2} A_{s2}$ for seismic zones 1 & 2

$\frac{1}{3} A_{s2}$ otherwise

At span $\rightarrow A'_{s2} \rightarrow$ usually 2Ø12 to hold the stirrups in place.

* The time dependent deformation creep takes place in the compression zone. Creep affects only concrete, the reinf. in the comp. zone is not affected by creep. If comp. reinf is used, some part of the compressive force is resisted by steel instead of concrete. Therefore, when comp. reinf. is used, the time dependent deformation will decrease and ductility will increase.

*



Equilibrium $\Rightarrow A_s f_y = 0.85 f_c k_1 c b_w + A_s' \sigma_s'$

$$+\circlearrowleft \; M_r = 0.85 f_c k_1 c b_w \left(d - \frac{k_1 c}{2} \right) + A_s' \sigma_s' (d - d')$$

Compatibility $\Rightarrow \dfrac{\varepsilon_s'}{0.003} = \dfrac{c-d'}{c} \rightarrow \varepsilon_s' = 0.003 \dfrac{c-d'}{c} \rightarrow \sigma_s' = E_s \varepsilon_s' \leq f_y$

if needed

$$\frac{\varepsilon_s}{0.003} = \frac{d-c}{c}$$

3 unknowns : M_r, c, σ_s'

2 equilibrium eqns + 1 compatibility eqn

- Most of the time, comp. reinf. yields $\rightarrow \sigma_s' = f_y$
- When comp. reinf. yields, the beam is under-reinf. if $(\rho - \rho') < \rho_b$
- When compression reinf. does not yield, the beam is under-reinf if $\left(\rho - \rho' \dfrac{\sigma_s'}{f_{yd}} \right) < \rho_b$

Recomended Procedure:

1) Assume $\varepsilon_s' > \varepsilon_y \rightarrow \sigma_s' = f_y$ → if the assumption is correct, compatibility eqn is not needed.

2) $c = \dfrac{(A_s - A_s')\, f_y}{0.85 f_c\, b_w\, k_1}$

3) Calculate $\varepsilon_s' = 0.003 \dfrac{c - d'}{c}$

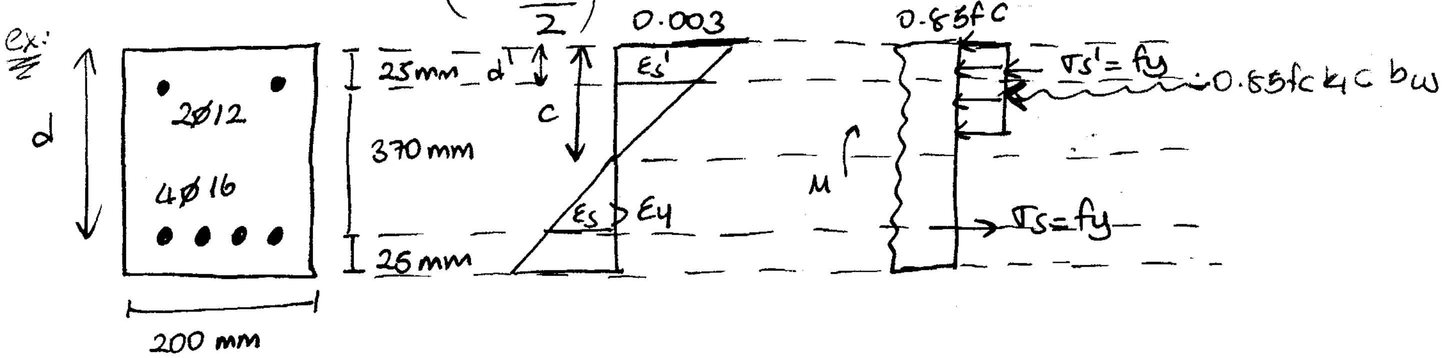
4) Calculate $\sigma_s' = E_s\, \varepsilon_s' \leq f_y$

$\sigma_s' = f_y$

$\sigma_s' < f_y$ → *use general solution

$* M_r = 0.85 f_c k_1 c b_w \left(d - \dfrac{k_1 c}{2}\right) + A_s' \sigma_s' (d - d')$

ex:



$f_c = 20$ MPa

$f_y = 420$ MPa

$4Ø16 \longrightarrow A_s = 804\ mm^2$

$2Ø12 \longrightarrow A_s' = 226\ mm^2$

Assume under-reinf.

Assume $\sigma_s' = f_y$

$0.85 \times 20 \times 200 \times \overset{0.85}{k_1} . c = (804 - 226) \times 420 \rightarrow c = 84$ mm

$\varepsilon_s' = 0.003 \dfrac{c - d'}{c} = 0.00211$

$\sigma_s' = \varepsilon_s' E_s = 421.4$ MPa $> f_y$ **OK**

$\varepsilon_s = 0.003 \dfrac{d - c}{c}$, $\sigma_s = E_s \varepsilon_s = 2221.4$ MPa $>> f_y$ → under-reinf. OK

$M_r = 0.85(20)\underbrace{(71.4)}_{k_1 c}(200)\left(395 - \dfrac{71.4}{2}\right) + 226(420)(395 - 25)$

$M_r = 122.3$ kN.m

same example with

$A_s' = 402\ mm^2\ (2\varnothing 16)$

Assume under-reinf. $\rightarrow \sigma_s = f_y$

Assume $\sigma_s' = f_y$

$0.85\,(20)\,(200)\,k_1 c = (804-402)\times 420 \Rightarrow k_1 c = 49.7\ mm$

$c = 58.4\,mm$

$\varepsilon_s' = 0.003\ \frac{c-d'}{c} = 0.0017 < \varepsilon_y$

$\sigma_s' = \varepsilon_s' E_s = 343.2\ MPa < f_y \Rightarrow$ assumption not correct
$\Downarrow$
general solution

$\varepsilon_s' = 0.003\ \frac{c-d'}{c}$, $\sigma_s' = \varepsilon_s' E_s$

$\sigma_s' = 600\ \frac{c-25}{c}$

$\Sigma F = 0 \Rightarrow 0.85 f_c b_w k_1 c + A_s' \sigma_s' = A_s f_y$

$0.85\,(20)\,(200)\,(0.85c) + 402\times 600\ \frac{c-25}{c} - 804\times 420 = 0$

$2890c^2 + 241200c - 337680c - 6030000 = 0$

$c^2 - 33.4c - 2086 = 0 \longrightarrow c = 65.3\ mm$

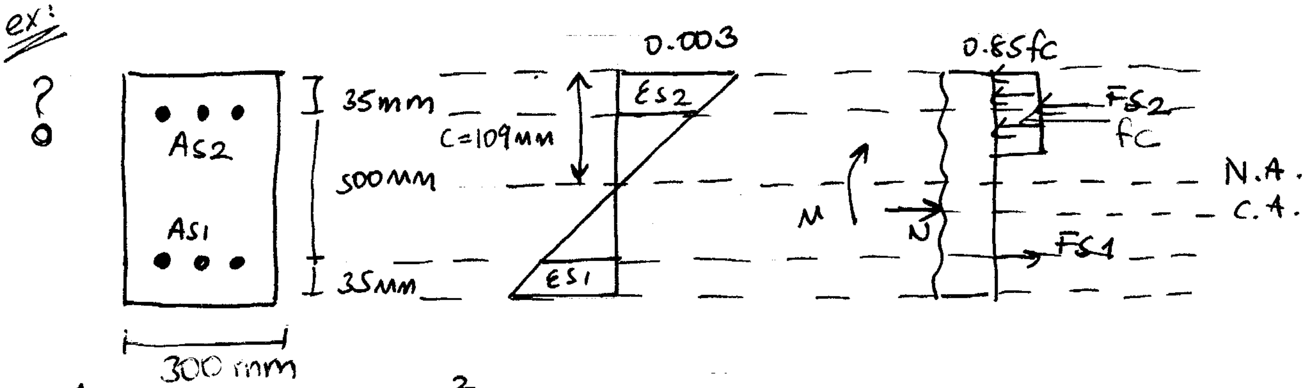
$\varepsilon_s' = 0.003\ \frac{65.3-25}{65.3} = 0.00185 \rightarrow \sigma_s' = 370.4\ MPa$

$\sigma_s > f_y$ $\underline{\underline{ok}}$

$M_r = 0.85\,(20)\,(0.85)\,(65.3)\,(200)\left(395 - \frac{0.85\times 65.3}{2}\right) + 402(370.4)(395-25)$

$M_r = 124.4\ kN.m$

ex:



$As_1 = As_2 = 600 \text{ mm}^2$

$f_c = 25 \text{ MPa} \quad , \quad f_y = 420 \text{ MPa}$

$$\frac{0.003}{109} = \frac{\varepsilon_{s2}}{109-35} \Rightarrow \varepsilon_{s2} = 0.00204 \rightarrow \sigma_{s2} = 200\,000\,(0.00204) = 408 \text{ MPa} < f_y$$

$$\frac{0.003}{109} = \frac{\varepsilon_{s1}}{500-109+35} \Rightarrow \varepsilon_{s1} = 0.0107 \rightarrow \sigma_{s1} = f_y$$

$F_{s2} = \sigma_{s2} A_{s2} = 408\,(600) = 244.8 \text{ kN}$

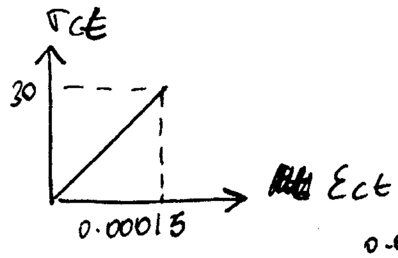
$F_{s1} = \sigma_{s1} A_{s1} = 420(600) = 252 \text{ kN}$

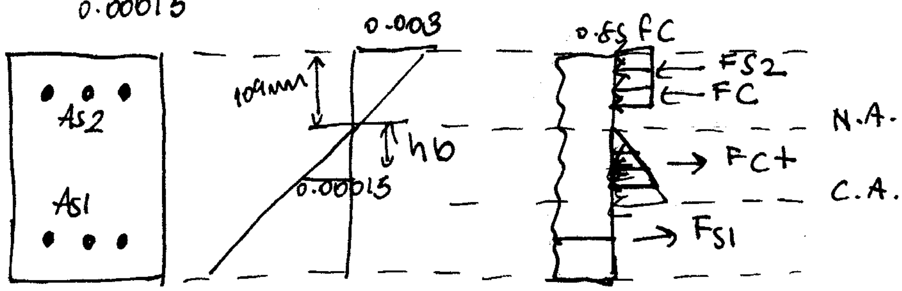
$F_c = 0.85 f_c k_1 c b_w = 0.85\,(25)(0.85)(109)(300) = 590.6 \text{ kN}.$

$N + F_{s1} - f_c - F_{s2} = 0 \longrightarrow N = 583.4 \text{ kN (comp.)}$

$$M = F_c\left(\frac{h}{2} - \frac{k_1 c}{2}\right) + F_{s2}\left(\frac{h}{2} - 35\right) + F_{s1}\left(\frac{h}{2} - 35\right) = 227.1 \text{ kNm}$$

ex: Same as previous example, but take into accant the tensile capacity of concrete.





$$\frac{0.003}{109} = \frac{0.00015}{h_t} \rightarrow h_t = 5.45\,mm$$

$$F_{ct} = \frac{\sigma_{ct} h_t}{2} \times b_w = \frac{3 \times 5.45}{2} \times 300 = 2.4\,kN$$

$$N = F_c + F_{s2} - F_{s1} - F_{ct} = 581\,kN.\ (comp.)$$

$$M = 227.1 - 2.4 \times \underbrace{\left[\frac{285}{1000} - \left(\frac{2}{3} \times \frac{5.45}{1000} + \frac{109}{1000}\right)\right]}_{0.33 \rightarrow \text{negligible}}$$

?
0

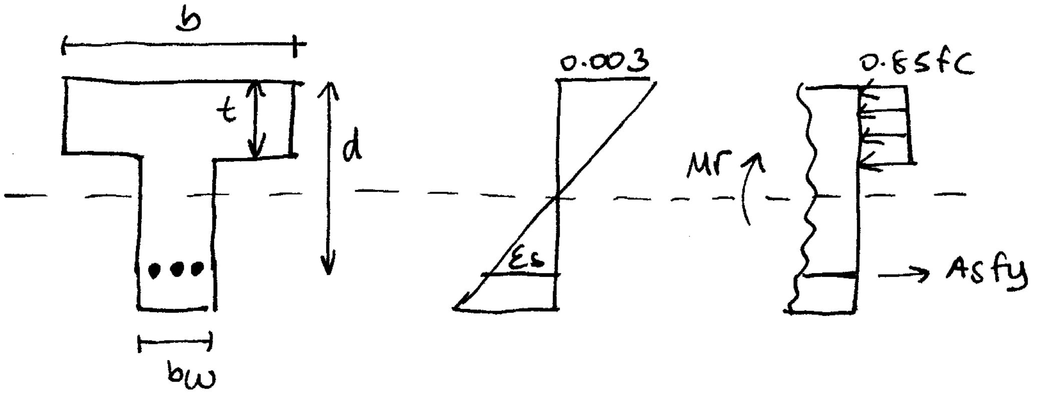
$$M \cong 227.1\ kN.m$$

~~Flanged sections:~~

T and L beams

* large compression area (generally no need for comp. rei'nf. to increase moment capacity)

* centroid of the area shifts up → moment capacity ↗



A_{sb} is very high as a result of the large compression area in the balanced case where the location of the N.A. is fixed. Therefore generally a flanged section is under-reinforced.

2 unknowns : c & Mr ⇒ 2 equilibrium eqns are sufficient.

* Test: For $k_1 c = t$ → $C = 0.85 f_c b t$ (compressive force), $T = A_s f_y$

1) If $C \geq T$ → $k_1 c \leq t$ → analyze as a rectangular section
(C: compressive force, T: tensile force)

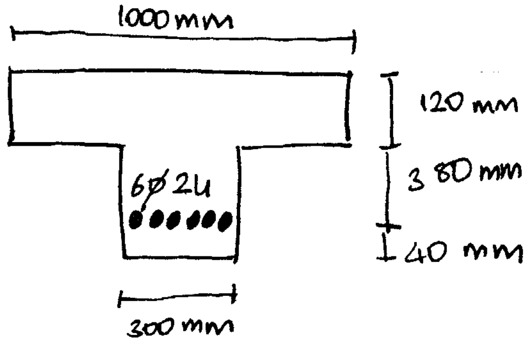
→ $M_r = A_s f_y \left(d - \frac{k_1 c}{2}\right)$ where $k_1 c = \frac{A_s f_y}{0.85 f_c b}$

2) If $C < T \rightarrow k_1 c > t \rightarrow$ analyze as a T-section.

Equilibrium: $0.85 f_c \left[k_1 c b_w + t(b - b_w) \right] = A_s f_y$

$$M_r = 0.85 f_c \left[k_1 c b_w \left(d - \frac{k_1 c}{2}\right) + t(b - b_w)\left(d - \frac{t}{2}\right) \right]$$

ex2



$f_c = 200 \ kg/cm^2$

$f_y = 4200 \ kg/cm^2$

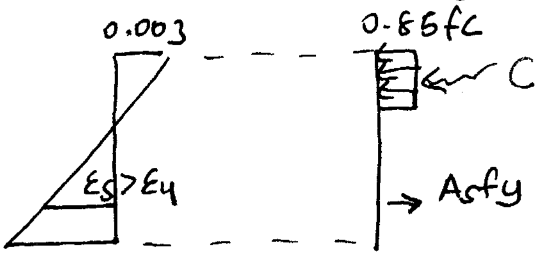
M_r?

Assume $k_1 c = t = 120$ mm.

$C = 0.85 f_c b k_1 c = 0.85(200)(100)(12 \times 10^{-3}) = 204 t$
↳ compressive force

$T = 6(4.52)(4200) 10^{-3} = 114 t$

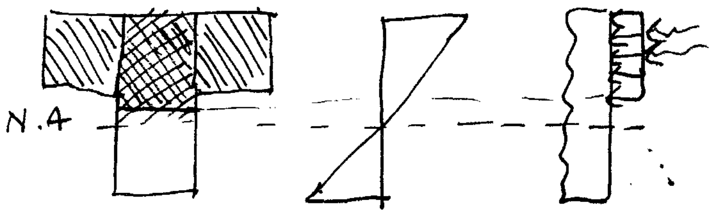
$C > T \rightarrow k_1 c < t \Rightarrow$ analyze as a rectangular section



$0.85 f_c k_1 c b = A_s f_y \rightarrow k_1 c = \dfrac{6 \times 4.52 \times 4200}{0.85(200)(100)} = 6.7$ cm

$M_r = 6 \times 4.52 \times 4200 \times 10^{-3} \left(50 - \frac{6.7}{2}\right) = 5318$ t.cm

ex3 Same section with 10ø26.



$A_s = 53.1 \ cm^2$

Assume $k_1 c = t = 12$ cm

$0.85 f_c k_1 c b + 0.85 f_c t(b - b_w) = A_s f_y$

$C = 204$ t

$T = 53.1\,(4200)\,10^{-3} = 223$ T

$C < T \rightarrow k_1 c > t \Rightarrow$ analyze as a T-section

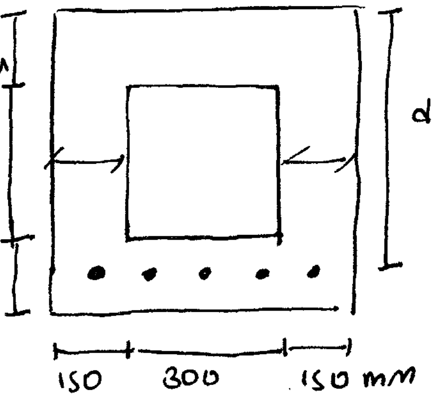
$$0.85\,(200)\left[30\,k_1 c + 12(100-30)\right] = 53.1 \times 4200$$

$\Rightarrow k_1 c = 15.7$ cm

$$M_r = \left[0.85 \times 200 \times 30 \times 15.7\left(50 - \frac{15.7}{2}\right) + 0.85 \times 200 \times 12 \times (100-30)\left(50 - \frac{12}{2}\right)\right]10^{-3}$$

$= 9662$ t cm

ex: 5.5



$A_s = 6\varnothing 24 = 2700\ mm^2$

C20 $\rightarrow f_{cd} = 13$ MPa

S420 $\rightarrow f_{yd} = 365$ MPa

M_r?

$b = 600$ mm

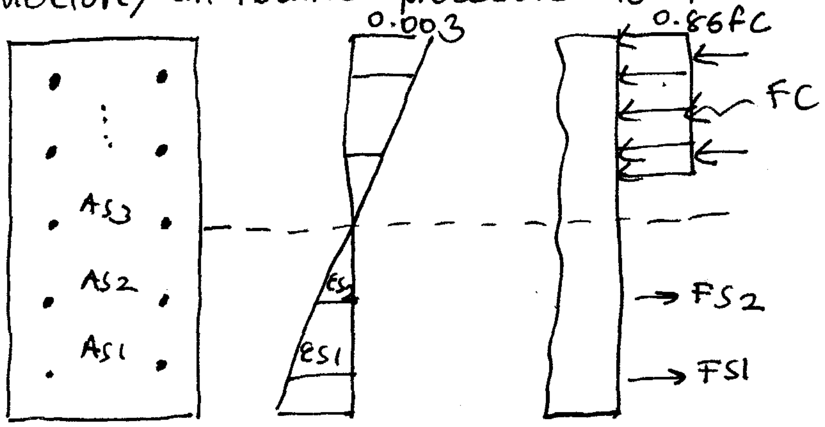
$b_w = 150 + 150 = 300$ mm

$d = 500$ mm

Beams with Multi-Layered Reinforcement:

The equilibrium and compatibility equations cannot be uncoupled.

Therefore, an iterative procedure is recomended.



Procedure:

* Assume a value for "c".
* Construct ε – diagram.
* Compute ε_{si}, $\sigma_{si} = \varepsilon_{si} E_s \leq f_y$, $F_{si} = A_{si}\sigma_{si}$
* Compute $F_c = 0.85 f_c b_w k_1 c$
* Check if $\Sigma F = 0$
* Change "c" and repeat steps until $\Sigma F = 0$

  If $\Sigma T > \Sigma C \rightarrow$ increase c

  $\Sigma T < \Sigma C \rightarrow$ decrease c
* Compute M_r with $\Sigma M = 0$ wrt any point (generally wrt the centroid)

Doubly Reinforced Rectangular Sections:

Dimensions are fixed, no need for the preliminary design.

Final

Case 1: Given: materials, M_d, b_w & d

Find: A_s & A_s'

* Compute $K = \dfrac{b_w d^2}{M_d}$
* If $K \geq K_\ell \rightarrow A_s = \dfrac{M_d}{f_{yd} \cdot j_\ell \cdot d}$ (single reinforced section)

* If $\left.\begin{array}{l} K < K_\ell \\ \geq K_m \end{array}\right\}$ To avoid deflection check, double reinf. is required.

Assume $\sigma_s' = f_{yd}$

$$M_1 = \frac{b_w d^2}{K_\ell} \rightarrow A_{s1} = A_{s\ell} = \frac{M_1}{f_{yd} \cdot j_\ell \cdot d}$$

$$\rightarrow c = \frac{A_{s1} f_{yd}}{0.85 f_{cd} k_1 b_w}$$

$$\rightarrow \varepsilon_s' = 0.003 \left(\frac{c - d'}{c}\right)$$

$$\rightarrow \sigma_s' = \varepsilon_s' E_s \leq f_{yd}$$

$M_2 = M_d - M_1 \rightarrow A_{s2} = A_s' = \dfrac{M_2}{f_{yd}(d-d')}$

If $\varepsilon_s' < \varepsilon_y \rightarrow A_s' = \dfrac{f_{yd}}{\sigma_s'} A_{s2}$

$A_s = A_{s\ell} + A_s'$

<u>Case 2:</u> Given: materials, M_d, b_w, d & A_s'

Find: A_s

* Assume $\sigma_s' = f_{yd}$ & compute $M_2 = A_s' f_{yd} (d - d')$
* $M_1 = M_d - M_2$
* $A_{s1} = \dfrac{M_1}{f_{yd}\, j_\ell\, d} \rightarrow c = \dfrac{A_{s1} f_{yd}}{0.85 f_{cd} k_1 b_w}$ & $\varepsilon_s' = 0.003 \left(\dfrac{c-d'}{c}\right)$
* If $\varepsilon_s' \geq \varepsilon_y \rightarrow A_s = A_{s\ell} + A_s'$
* If $\varepsilon_s' < \varepsilon_y \rightarrow$ follow the steps in case 1 ignoring given A_s'.

If A_s' obtained $> A_s'$ given $\rightarrow$ use additional reinf.

exp. Rectangular section, $b_w = 20$ cm, $h = 40$ cm ($d = 36.5$ cm)

$M_d = 11$ tm

C25 & S420 $\rightarrow$ $f_{cd} = 170$ kg/cm² , $f_{yd} = 3650$ kg/cm² $\Big\}$ $k_\ell = 29.1$ cm²/t, $j_\ell = 0.861$; $k_m = 19.9$ cm²/t, $j_m = 0.776$

<u>Final</u> Check $K = \dfrac{20 \times 36.5^2}{1100} = 24.2$ cm²/t $< k_\ell$
$> k_m$

$\Rightarrow \therefore$ Double reinf. is required to avoid deflection check.

$M_1 = \dfrac{20 \times 36.5^2}{29.1} = 915.6$ tcm

$M_2 = 1100 - 915.6 = 184.4$ t.cm

$A_{s1} = \dfrac{915600}{3650 \times 0.861 \times 36.5} = 7.98$ cm²

$k_1 c = \dfrac{7.98 \times 3650}{0.85 \times 170 \times 20} = 10.08$ cm.

$\rightarrow \sigma_s' = 0.003 \times 2000000 \left(1 - \dfrac{3.5}{10.08/0.85}\right) = 4230$ kg/cm² $> f_{yd}$

$$A_{s2} = A'_s = \frac{184400}{3650\,(36.5 - 3.5)} = 1.53 \text{ cm}^2$$

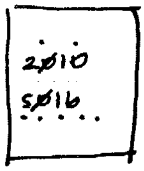
$A_s = 7.98 + 1.53 = 9.51 \text{ cm}^2$

$\Rightarrow$ use $2\phi10$ Spacer (Hanger) $\rightarrow 1.56 \text{ cm}^2$

use $2\phi16$ straight $\rightarrow 4.0 \text{ cm}^2$

$3\phi16$ bent $\rightarrow 6.0 \text{ cm}^2$

10.0 cm^2



Alternative Solution:

$k = 24.2 \text{ cm}^2/t < k_l$ single reinf.
$> k_m$ or with deflection check

$$A_s = \frac{1100000}{3650\,\underbrace{(0.776)}_{j_m}(36.5)} = 10.64 \text{ cm}^2$$

$\Rightarrow$ use $2\phi18$ straight (tension) $\rightarrow 5.08 \text{ cm}^2$

$3\phi16$ bent $\rightarrow 6.00 \text{ cm}^2$

$+$

11.08 cm^2

compare: $2\phi10$ spacers will be used in any case.

10 % waste of reinf.

Time consuming deflection check.

Flanged Sections:

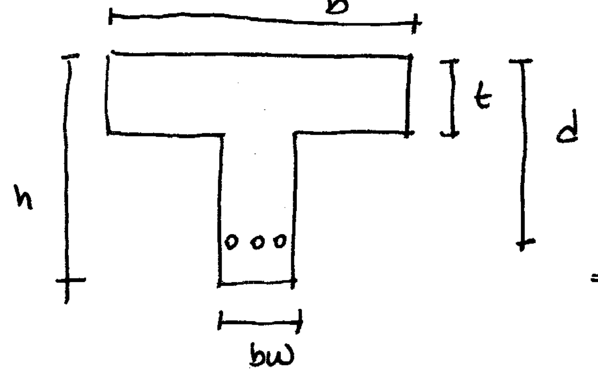
In monolithic construction, the floor slab acts as a flange for beams.

at span $\rightarrow$ T-section

at support $\rightarrow$ rectangular (may be double reinf.)

Generally moment at the support $>$ moment at the span

$\Rightarrow$ dimensions of the beam are governed by the moment at the support.



t is fixed (from slab design)

b is fixed (from code requirements)

b_w & h chosen at the support.

⇒ section dimensions are fixed.

∴ No need for preliminary design.

Final design:

Check is usually not necessary because A_{sb} is very high.

Compute the area of reinf = $A_s = \dfrac{M_d}{f_{yd}\, jd}$

Approximate moment arm : $jd = d - \dfrac{t}{2}$ \
$jd = 0.9d$ } use the larger one

or use the charts given in appendix A.

Select the proper chart wrt b/b_w

j is a function of $\bar{k} f_{cd}$ & t/d

$\bar{k} f_{cd} = \dfrac{b d^2}{M_d} f_{cd}$

* Cutting Off or Bending Reinforcing Bars:

The required area of reinforcement is calculated for the maximum moment. Therefore, it can be reduced where the applied moment is lower → either cut off or bend the bar into the compressive zone.

$A_s = \dfrac{M_d}{f_{yd}\, jd}$

f_{yd} = constant

$jd \approx$ constant (variation in j is insignificant)

⇒ reinf. need varies almost proportionally with the moment.

∴ $A_s \propto M_d$

→ moment diagram $\propto A_{sreq}$ diagram

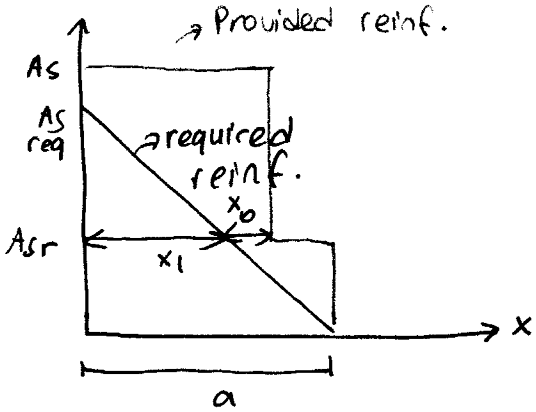
Due to the stress concentration at the cut off or bent point there would be problems with bond and diagonal tension.

⇒ Do not cut off or bend the bars at the theoretical cut off points, but extend them. x_0 is the extension required by the code considering anchorage and stress concentration.

For cut off → $x_0 = 20\phi$ or d whichever is larger.

bent → $x_0 = 8\phi$ or $d/3$

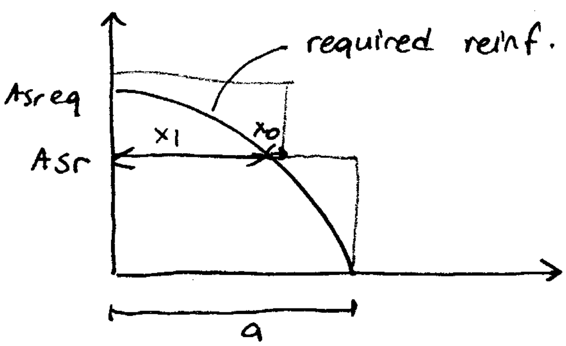
Concentrated Load → Triangular M. Diagram



$$\frac{x_1}{a} = \frac{As_{req} - A_{sr}}{As_{req}}$$

$$l_1 = x_1 + x_0$$

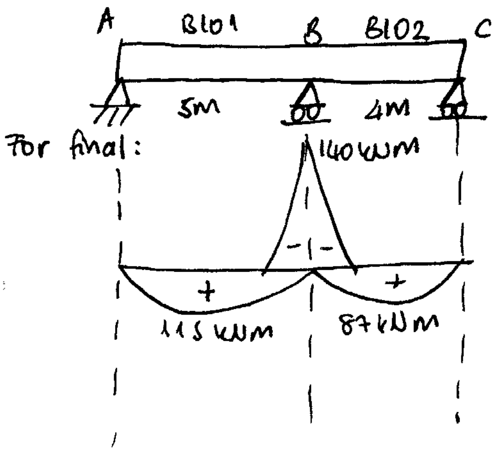
Uniformly Distributed Load → Parabolic M-diagram



$$\left(\frac{x_1}{a}\right)^2 = \frac{As_{req} - A_{sr}}{As_{req}}$$

$$l_1 = x_1 + x_0$$

ex: (Rect. Section)



C16 & S420

$f_{cd} = 11$ MPa, $f_{yd} = 365$ MPa

$k_1 = 0.85$

$K_l = 449\ mm^2/kN$, $j_l = 0.861$

$K_m = 307\ mm^2/kN$, $j_m = 0.776$

Preliminary:

$g = 20$ kN/m , $q = 10$ kN/m (estimated values)

$P_d = 1.4(20) + 1.6(10) = 44$ kN/m .

$M_d = \frac{1}{8}(44)\left(\frac{5+4}{2}\right)^2 = 111.4$ kNm (support)

$k = \frac{b_w d^2}{M_d} \geq k_\ell \longrightarrow b_w = \overset{250}{\cancel{2.50}}$ mm , $d = 447$ mm

$\Rightarrow$ choose $b_w = 250$mm , $h = 600$ mm ($d = 460$ mm)

Final:

Using more accurate loads , the given envelope diagram is obtained.

B101 :

$M_d = 115$ kNm

$k = \frac{250 \times 460^2}{115000} = 460$ mm²/kN $> k_\ell \rightarrow$ single reinf.

$A_s = \frac{115 \times 10^6}{365\,(0.861)(460)} = 796$ mm²

$A_{s\,min} = \underbrace{0.8\,\frac{f_{ctd}}{f_{yd}}}_{\rho min}\,b_w d = 0.8\left(\frac{0.9}{365}\right) \times 250 \times 460 = 227$ mm² $< A_s$ ✓

$\Rightarrow$ use $2\phi 14$ straight $\longrightarrow$ 308 mm²

$2\phi 18$ bent $\longrightarrow$ 508 mm²

$+$

816 mm²

B102

$M_d = 87$ kNm $\rightarrow k > k_\ell$ ok

$A_s = 796 \times \frac{87}{115} = 602$ mm² $> A_{smin}$

$\Rightarrow$ use $2\phi 12$ straight $\rightarrow$ 226 mm²

$2\phi 16$ bent $\rightarrow$ 402 mm²

$+$

628 mm² $> A_{s\,min}$ ok

B101 - B102

$M_d = 140$ kNm

$$k = \frac{250 \times 460^2}{140\,000} = 467 \ mm^2/kN > k_m \ (< k_e) \Rightarrow \text{double reinf.}$$

$$M_1 = \frac{250(460)^2}{449} \times 10^{-3} = 117.8 \ kNm.$$

$$M_2 = 140 - 117.8 = 22.2 \ kNm$$

$$A_{s1} = \frac{117.8\,(10^6)}{365 \times 0.861 \times 460} = 815 \ mm^2$$

$$k_1 c = \frac{815\,(365)}{0.85(11)(250)} = 127.3 \ mm$$

$$\sigma'_s = 0.003 \times 200\,000 \left(1 - \frac{40}{127.3/0.85}\right) = 440 \ MPa > f_{yd}$$

$$A_{s2} = A'_s = \frac{22.2 \times 10^6}{365 \times 420} = 145 \ mm^2$$

$A_s = 815 + 145 = 960 \ mm^2 > A_{s\,min}$ ok $\Rightarrow$ Avaliable comp. reinf. (B102)

$2\varnothing12 = 226 \ mm^2 > 145 \ mm^2$ ok

B101 $\rightarrow$ $2\varnothing14 \rightarrow$ St, $2\varnothing18 \rightarrow$ St

B102 $\rightarrow$ $2\varnothing12$ St, $2\varnothing16$ St

$\rightarrow$ Avaliable comp. reinf (B102) $\rightarrow$ $2\varnothing12 = 226 \ mm^2 > 145 \ mm^2$ ok

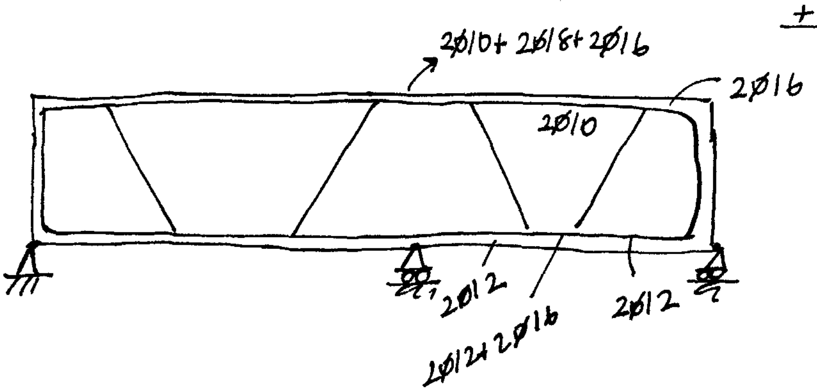
Avaliable tension reinf $2\varnothing18$ bent (B101) $\rightarrow$ 508 mm^2

$2\varnothing16$ bent (B102) $\rightarrow$ 402 mm^2

$2\varnothing10$ spacer $\rightarrow$ 156 mm^2

+

1066 mm^2 > 960 mm^2 ok



TS 500 Requirements for Beam Design

* Max tension reinf $\rightarrow \rho$ or $(\rho - \rho') \leq \rho_m = 0.85\, \rho_b \leq 0.02$
* Min tension reinf $\longrightarrow \rho \geq \rho_{min} = 0.8 \dfrac{f_{ctd}}{f_{yd}}$

  $\phi \geq 12$ mm (for tension)

* If $h > 600$ mm, use two web bars at the mid depth with $A_{sl} \geq 0.01\, b_w d$ with $\phi \geq 10$ mm and $s_u \leq 300$ mm

  → For each additional 300 mm add an extra row of web longitudinal reinf.

* Extension beyond theoretical cut off point $= x_0$

  $x_0 \geq 20\phi$ or d for cut bars

  $x_0 \geq 8\phi$ or $d/3$ for bent bars.

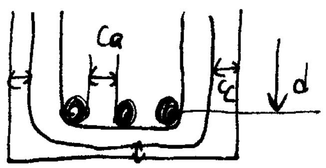
* Bottom bars of span extending to support

  (as comp. reinf) $\geq A_s/3$ for continuous beam

  $\geq A_s/2$ for simply supported beam

Dimensions:

* Clear cover, $c_c \geq 20$ mm (int), 25 mm (ext)



* Clear space between bars, $c_a \geq 20$ mm, ϕ, D_{agr}

  ϕ → bar diameter

  D_{agr} → diameter of the largest aggregate

* Effective flange width:

  Symmetrical Flange on two sides (i.e. symm T-sect)

  → $b = b_w + 0.2\, l_p$

  Unsymmetrical Flange on two sides (ex: L-sect, unsymm T-sect)

  → $b = b_w + 0.1\, l_p$

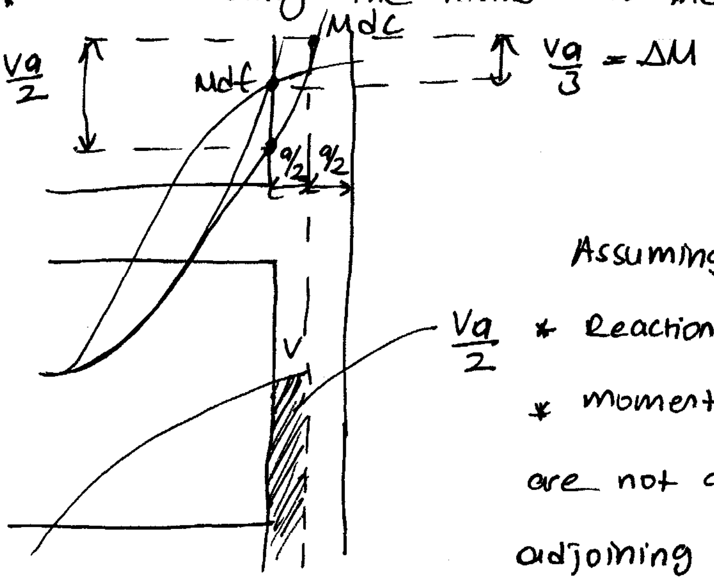
  l_p → distance between points of inflection

$$
\ell_p = \begin{cases} 0.8\ \ell_n & \text{exterior span of a cont. beam} \\ 0.6\ \ell_n & \text{interior} \quad '' \quad '' \quad '' \quad '' \quad '' \\ 1.0\ \ell_n & \text{simply supported beam} \\ 1.5\ \ell_n & \text{cantilever} \end{cases}
$$

ℓ_n = clear span

Support moment Capacity:

* structural analysis gives the moment at the center-line of the members (both beams and columns)
* Need to carry the moment to the column face. → actual max is lower



Assuming:

* Reactions are concentrated at the column axes.
* moment of inertia at the ends of the beams are not affected by the stiffening effect of the adjoining supports.

$M_{df} = M_{dc} - \Delta M$, $\Delta M = \frac{Va}{3}$

design face center-line

Effect of material Strength on the moment Capacity:

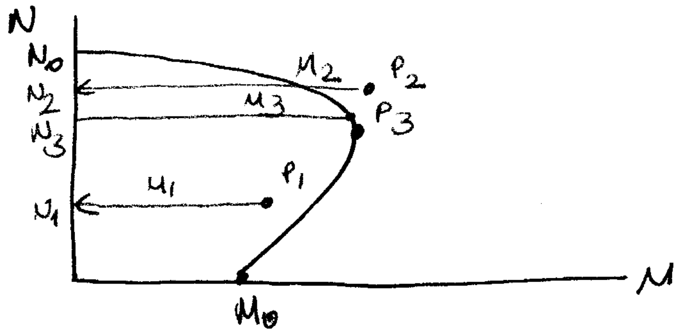
$$
M_r = A_s f_y \left(d - \frac{k_1 c}{2} \right) = A_s f_y \left(d - \frac{A_s f_y}{2 \times 0.85 \times f_c \times b_w} \right)
$$

Combined Flexure and Axial Load:

R/C columns

Ultimated strength under combined N & M:

* when $M=0$, ultimate strength $= N_r = N_0$
* when $N=0$, ultimate strength $= M_r = M_0$
* when combined N & M
  - ultimate strength is not N_0, M_0
  - ultimate strength is not a single pair (N_r, M_r)
  - interaction diagram is the best representation of ultimate strength.

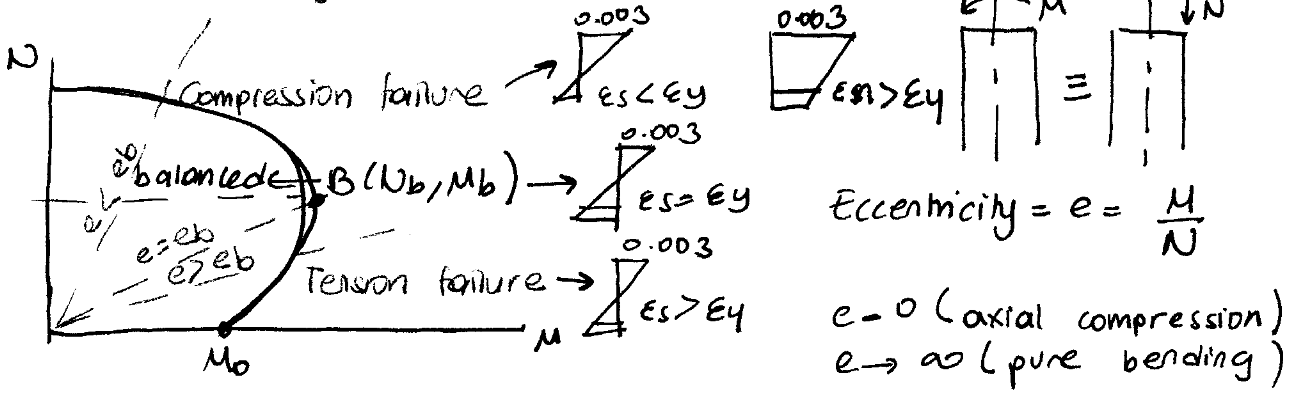


- interaction diagram is a strength envelope.
- A combination N_i, M_i * can be safely carried if P_1 is inside the envelope.

* can not be carried if P_2 is outside the envelope.

corresponds to failure if P_3 is on the envelope.

Interaction Diagram



Eccentricity $= e = \dfrac{M}{N}$

$e \to 0$ (axial compression)
$e \to \infty$ (pure bending)

* Ductile behavior at M_0 and nearby.
* Brittle behavior at N_0 and nearby
* Between M_0 and $N_b/M_b \rightarrow$ tension failure

Between N_0 and $N_b/M_b \rightarrow$ compression failure

Balanced behavior at N_b/M_b

$\Rightarrow$ If $N < N_b$ or $e > e_b \rightarrow$ ductile behavior, tension failure.

If $N > N_b$ or $e < e_b \rightarrow$ brittle behavior, comp. failure

If $N = N_b$ or $e = e_b \rightarrow$ balanced behavior, balanced failure

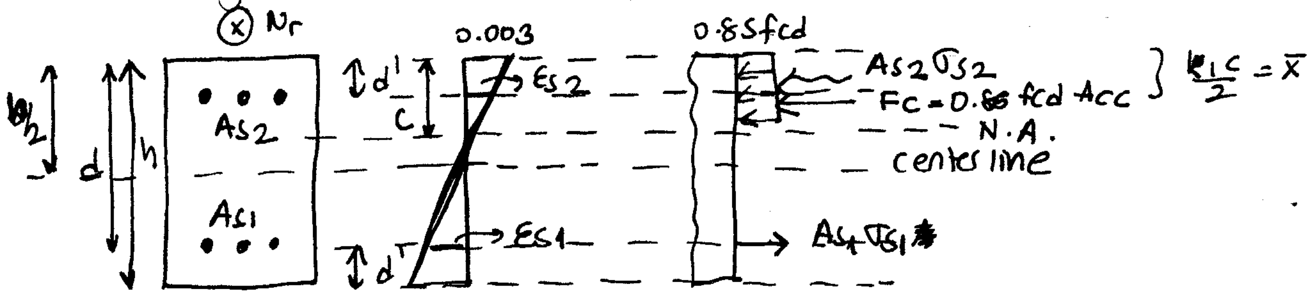
$\Rightarrow$ In combined bending and axial load the behavior is controlled by N or e, while in pure bending it is controlled by the tension reinforcement area (A_s).

* In compression failure zone $\rightarrow N\uparrow \rightarrow M\downarrow$

In tension failure zone $\rightarrow N\uparrow \rightarrow M\uparrow$

In tension failure zone, increasing N has the same effect as increasing A_s in pure bending. It causes increase in flexural capacity, but reduction in ductility. (length of yield plateau $\downarrow$)

<u>Rectangular Section with two layers of reinf.</u>



(Generally $A_{s1} = A_{s2}$)

4 unknowns
N or e
c or a
σ_{s1} and σ_{s2}

$\therefore$ 2 comp. eqns

Compatibility:

$$\frac{\varepsilon_{s1}}{0.003} = \frac{d-c}{c} \rightarrow \sigma_{s1} = \varepsilon_{s1} E_s \leq f_{yd}$$

$$\frac{\varepsilon_{s2}}{0.003} = \frac{c-d'}{c} \rightarrow \sigma_{s2} = \varepsilon_{s2} E_s \leq f_{yd}$$

Equilibrium:

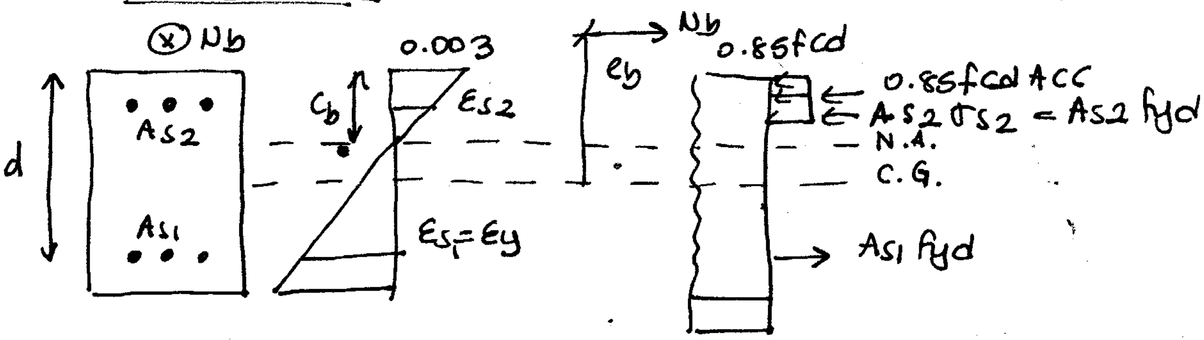
$$N_r = 0.85 f_{cd} \underbrace{k_1 c b_w}_{A_{cc}} + A_{s2}\sigma_{s2} - A_{s1}\sigma_{s1}$$

$$\underset{\text{w.r.t C.G.}}{M_r} = N_r \cdot e = 0.85 f_{cd} k_1 c b_w \left(\frac{h}{2} - \frac{k_1 c}{2}\right) + A_{s2}\sigma_{s2}\left(\frac{h}{2} - d'\right) + A_{s1}\sigma_{s1}\left(\frac{h}{2} - d'\right)$$

If balanced values are avaliable →

* if $N < N_b$ or $e > e_b$ → tension failure → $\sigma_{s1} = f_y$
* if $N > N_b$ or $e < e_b$ → compression failure
  * if σ_{s1} is under tension → $\sigma_{s1} < f_y$ & $\sigma_{s2} = f_y$
  * if σ_{s1} is under comp → $\sigma_{s1} \leq f_y$ & $\sigma_{s2} = f_y$

Balanced case:



⇒ 3 unknowns → N_b or e_b ∴ 1 compatibility eqn

c_b or a_b

usually $\sigma_{s2} = f_{yd}$ (check)

Compatibility :

$$\frac{c_b}{d} = \frac{0.003}{0.003+\varepsilon_y}$$

$$\frac{c_b}{c_b - d'} = \frac{0.003}{\varepsilon_{s2}} \rightarrow \text{check } \varepsilon_{s2} \geq \varepsilon_y$$

Equilibrium.

$$N_b = 0.85 f_{cd} k_1 c_b b_w + A_{s2} f_{yd} - A_{s1} f_{yd}$$

$$M_b = N_b \cdot e_b = 0.85 f_{cd} k_1 c_b b_w \left(\frac{h}{2} - \frac{k_1 c_b}{2}\right) + A_{s2}\left(\frac{h}{2} - d'\right) f_{yd} + A_{s1} f_{yd}\left(\frac{h}{2} - d'\right)$$

$\downarrow$

wrt C.G.

General solution

For multilayered reinf and for computer applications:

Procedure:

* Assume c
* construct ε-diagram
* compute ε_{si}, σ_{si}, f_{si}, F_c

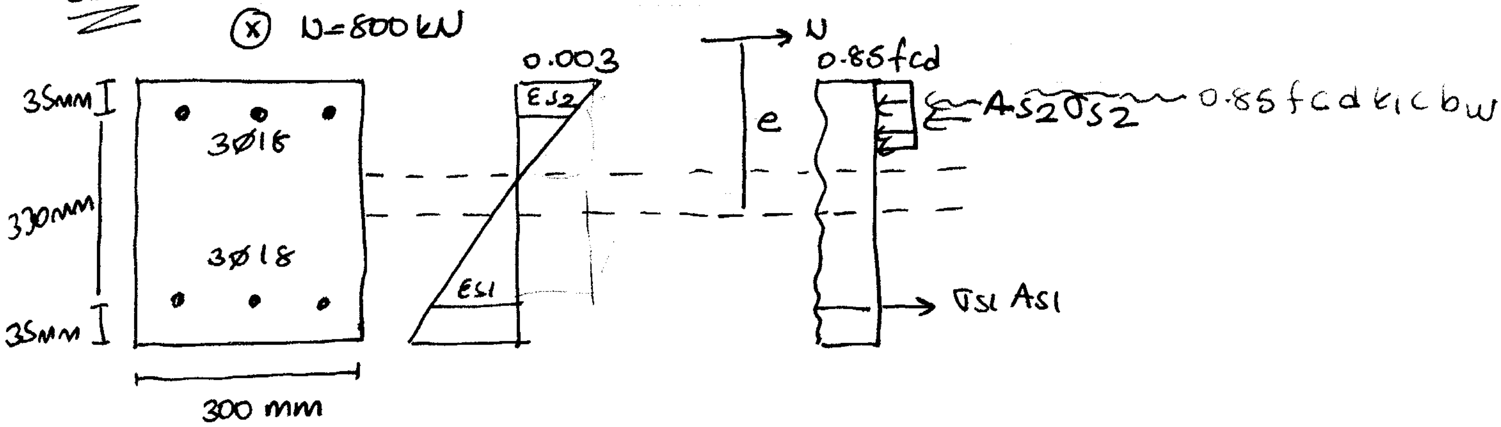
if N is given:

* check with $\Sigma F = 0$
  - if $C > T \rightarrow$ reduce c
  - if $C < T \rightarrow$ increase c
* repeat until $C \simeq T$
* Compute M_r with $\Sigma M = 0$

if e is given

* compute N_r with $\Sigma F = 0$
* check with $\Sigma M = 0$
* Repeat until $M_r = N_r \cdot e$

ex:



Find Mr corresponding to axial load level at N= 800 kN.

$f_{cd} = 17$ MPa ($k_1 = 0.85$)

$f_{yd} = 191$ MPa

Balanced case:

Compatibility = $c_b = \dfrac{0.003}{0.003+\varepsilon_y} d \rightarrow k_1 c_b = 235.3$ mm

Equilibrium = $N_b = 0.85 \times 17 \times 235.3 \times 300 \times 10^{-3} = 1020.2$ kN

Check assumption $\rightarrow \varepsilon_{s2} > \varepsilon_y$ ✓

$N < N_b \rightarrow$ tension failure $\rightarrow \sigma_{s1} = f_{yd}$

Assume $\sigma_{s2} = f_{yd} \rightarrow A_{s1}\sigma_{s1} = A_{s2}\sigma_{s2}$

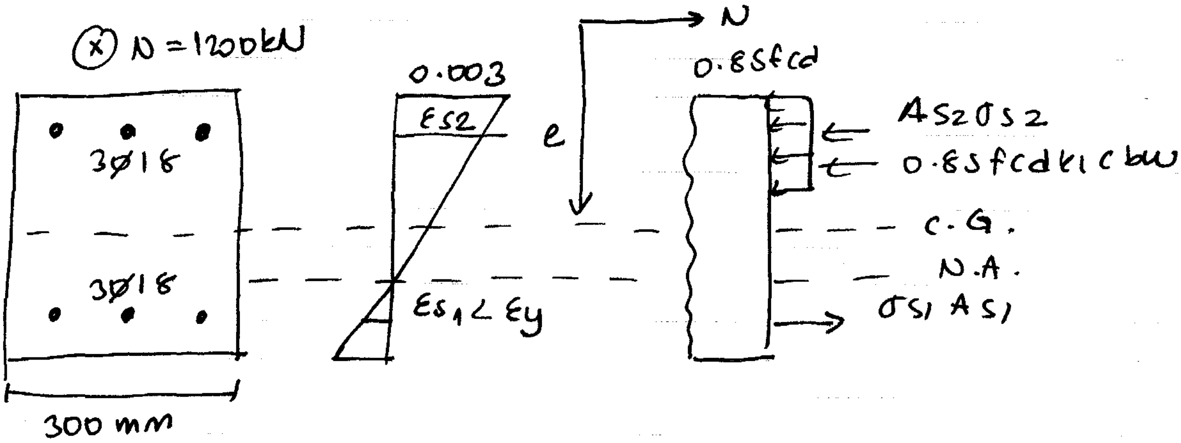
Equilibrium : $k_1 c = \dfrac{800 \times 10^3}{0.85 \times 17 \times 300} = 184.5$ mm

check: $\sigma_{s2} = 0.003 (200\,000) \left(1 - \dfrac{35}{184.5/0.85} \right) = 503$ MPa $> f_{yd}$ ok

Equilibrium : $M_r = N \cdot e = \left[0.85 \times 17 \times 300 \times 184.5 \left(200 - \dfrac{184.5}{2} \right) + 3 \times 254 \times 191 \times 165 \times 2 \right] \times 10^{-6} = 134.2$ kNm

$\Rightarrow e = \dfrac{134.2}{800} \times 10^3 = 167.8$ mm

ex:



$N > N_b \Rightarrow$ comp failure $\Rightarrow$ compatibility : $\varepsilon_{s1} = 0.003\left(\frac{d}{c} - 1\right)$

Equilibrium :

$$N = 1200\times10^3 = 0.85(17)(300 k_1 c) + 3\times 254\times\overset{\sigma_{s2} = f_{yd}}{\widehat{191}}$$

$$- 3\times 254\times 200\,000 \times 0.003\left(\frac{365}{k_1 c/0.85} - 1\right)$$

$\rightarrow k_1 c = 262.5$ mm $\rightarrow \sigma_{s1} = 200000 \times 0.003\left(\frac{365}{262.5/0.85} - 1\right)$

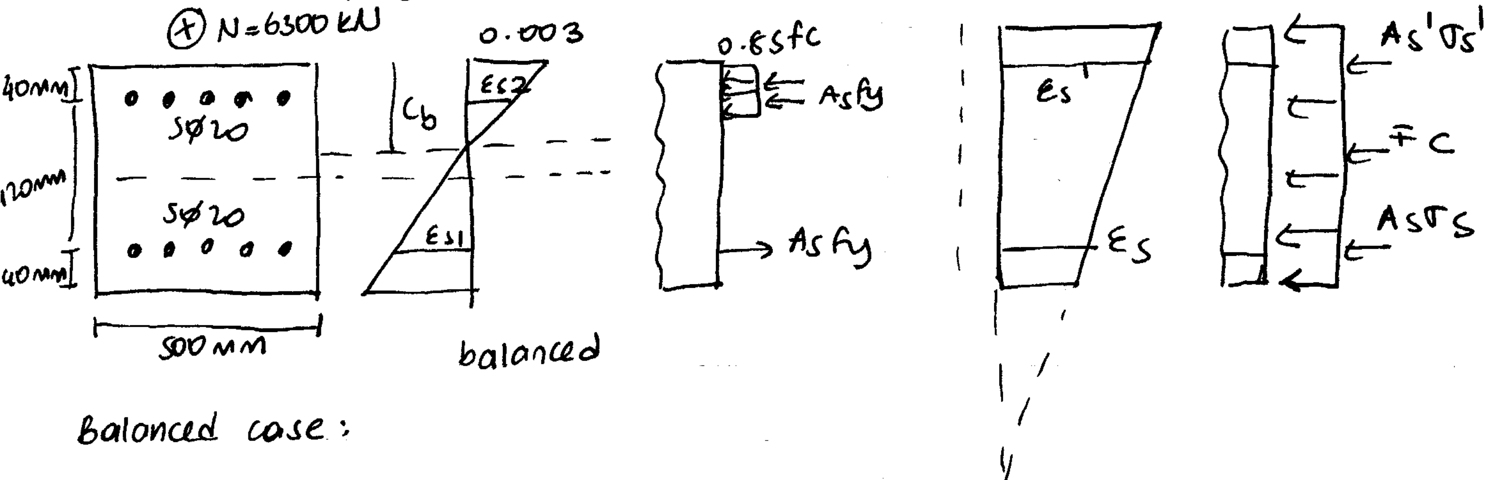
$\sigma_{s1} = 109.3$ MPa $< f_{yd}$ ok

$\sigma_{s2} > f_{yd}$

$$M_r = N.e = \left[0.85\times17\times300\times262.5\times\left(200 - \frac{262.5}{2}\right) + 3(254)(191)(165) + 3\times 254\times109.3\times165\right]\times10^{-6} = 116 \text{ kN}.$$

$$e = \frac{116\times10^3}{1200} = 967 \text{ mm}$$

ex: Determine Mr corresponding to N = 6300 kN axial comp. $f_c = 25$ MPa ($k_1 = 0.85$), $f_y = 420$ MPa.



Balanced case :

comp. $c_b = \frac{0.003}{0.003 + \varepsilon_y}\times 460 \rightarrow k_1 c_b = 230$ mm $\rightarrow \varepsilon_s' > \varepsilon_y$

equilibrium: $N_b = 0.85(25)(500)(230)\times10^{-3} = 2443.8$ kN

Given case:

$N > N_b \Rightarrow$ compression failure $\rightarrow$ Assume $\sigma_s' = f_y$ & $\sigma_s < f_y$

$$\frac{c}{c-d} = \frac{0.003}{\varepsilon_s} \longrightarrow \sigma_{s1} = 200\,000 \times 0.003 \left(\frac{c - 460}{c}\right)$$

equilibrium: $6300\times10^3 = 0.85\,(25)\,(500\,k_1c) + 5(314)\left[420 - 600\left(\dfrac{\frac{k_1c}{0.85} - 460}{\frac{k_1c}{0.85}}\right)\right]$

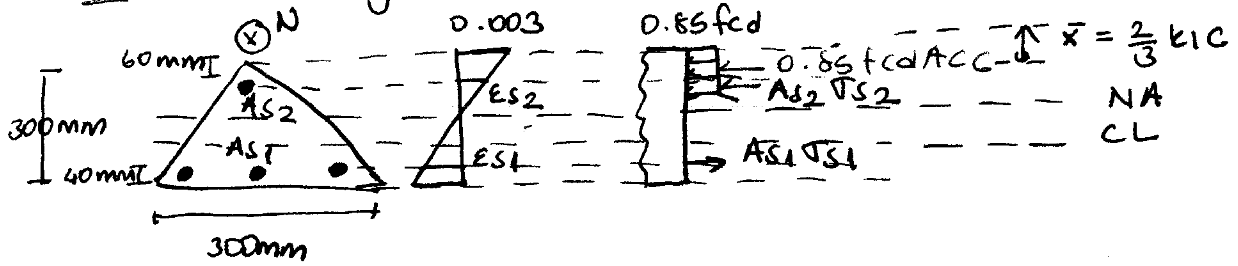
$\Rightarrow k_1c = 510$ mm $> h = 500$ mm $\therefore$ As in comp & take $\boxed{k_1c = h}$

equilibrium: $6300\times10^3 = 0.85\times25\times500\times500 + 5(314)(420+\sigma_s)$

$\Rightarrow \sigma_s = 209$ MPa

equilibrium: $M_r = [fc \times 0 + 5(314)(420-209)(210)] \times 10^{-6} = 69.6$ kNm

ex: Non-rectangular section



C 20 & S420 ($f_{cd} = 13$ MPa , $f_{yd} = 365$ MPa)

$A_{s1} = 942\ mm^2$, $A_{s2} = 314\ mm^2$ 4Ø20

$N = 100$ kN , $M = ?$

Balanced case: $\varepsilon_{s1} = \varepsilon_y = 0.001825$

$$\frac{c_b}{260} = \frac{0.003}{0.003 + 0.001825} = 161.7\ mm, \quad k_1c_b = 137.4\ mm$$

$$\frac{c_b}{c_b - d'} = \frac{0.003}{\varepsilon_{s2}} \rightarrow \varepsilon_{s2} = 0.003$$

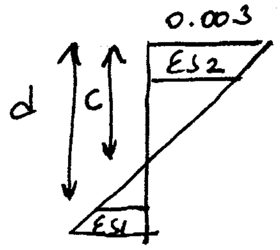
$$\frac{161.7 - 60}{161.7} = 0.00188 > \varepsilon_y$$

$$N_b = 0.85\,f_{cd}\,\frac{1}{2}(k_1c)(k_1c) + A_{s2}f_{yd} - A_{s1}f_{yd}$$

$$= 0.85\times13\times\frac{(137.4)^2}{2} + 314(365) - 942(365) = \ominus 124.7\ kN$$

(balanced load is tension)

$N = 100\ kN > N_b = 124.7\ kN \Rightarrow$ compression failure



Compatibility: $\varepsilon_{s1} = 0.003\left(\frac{d-c}{c}\right)$

Equilibrium : $100\times10^3 = 0.85 \times \frac{13\ (k_1 c)^2}{2} + 314(365)$
$- 942\times 200\,000 \times 0.003\left(\frac{260}{c} - 1\right)$

$\Rightarrow 3992\,c^3 + 579810\,c - 146952000 = 0$

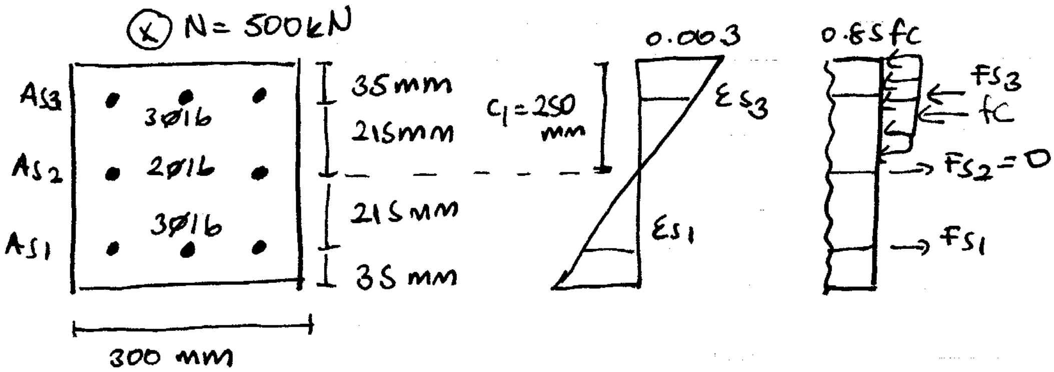
$\Rightarrow c = 199.1\ mm$

$M_r = N\cdot e = \left[0.85 \times 13 \times \frac{(0.85\times199.1)^2}{2}\left(200 - \frac{2}{3}(0.85)(199.1)\right)\right]$
$+ 314(365)(140) + 942\times 200\,000 \times 0.003\left(\frac{260}{199.1} - 1\right)\times 60\Big] = 40.2\ kN.m$

ex: Multilayer reinf. $f_c = 11$ MPa & $f_y = 365$ MPa

$M_r = ?$ for $N = 500$ kN.

$N = 500$ kN



Trial 1 : Assume $c = 250$ mm to avoid $f_{s2} \rightarrow k_1 c = 212.5$ mm

$\varepsilon_{s1} = \varepsilon_{s3} = 0.003 \times \frac{215}{250} = 0.0026 > \varepsilon_y = \frac{365}{200\,000} = 0.001825$

$\varepsilon_{s2} = 0$

$\sigma_{s1} = \sigma_{s3} = 365$ MPa, $\sigma_{s2} = 0$.

$F_{s1} = F_{s3} = 3\times201\times365\times10^{-3} = 219$ kN, $F_{s2} = 0$.

$F_c = 0.85 \times 11 \times 212.5 \times 300 \times 10^{-3} = 596$ kN.

check, $\Sigma F = 500 - \cancel{219} - 596 + 0 + \cancel{219} = -96\ kN \neq 0$
may be $-500 + \cancel{219} + 596 - 0 - \cancel{219} = +96\ kN \neq 0$
$\Rightarrow c < 250$ mm

Trial 2 = Assume $c = 220$ mm $\rightarrow k_1 c = 187$ mm

$\varepsilon_{s1} > \varepsilon_y$, $\varepsilon_{s2} = 0.003 \times \frac{30}{220} = 0.00041 < \varepsilon_y$

$\varepsilon_{s3} = 0.003 \times \frac{(185)}{220} = 0.0025 > \varepsilon_y$

$\sigma_{s1} = \sigma_{s3} = 365$ MPa , $\sigma_{s2} = 0.00041 \times 200\,000 = 82$ MPa

$F_{s1} = F_{s3} = 3 \times 201 \times 365 \times 10^{-3} = 219$ kN , $F_{s2} = 2 \times 201 \times 82 \times 10^{-3} = 32.8$ kN

$F_c = 0.85 \times 11 \times 187 \times 300 \times 10^{-3} = 524$ kN.

check, $\Sigma F = -500 + 219 + 524.5 - 32.8 - 219 = -8.3$ kN $\neq 0$

$\Rightarrow c > 220$ mm

Trial 3 = Assume $c = 222$ mm $\rightarrow k_1 c = 189$ mm

$\varepsilon_{s1} > \varepsilon_y$, $\varepsilon_{s2} = 0.003 \times \dfrac{28}{222} = 0.00038 < \varepsilon_y$, $\varepsilon_{s3} > \varepsilon_y$

$\sigma_{s1} = \sigma_{s3} = 365$ MPa , $\sigma_{s2} = 75.7$ MPa

$F_{s1} = F_{s3} = 219$ kN, $F_{s2} = 30.1$ kN

$F_c = 530$ kN.

check, $\Sigma F_c = -500 + 219 + 530 - 30.1 - 219 = -0.1$ kN ≈ 0.0 OK

$M = 500 \cdot e = \left[530\left(250 - \frac{189}{2}\right) + 219 \times 215 \times 2 + 0\right] \times 10^{-3}$

$M = 176.6$ kNm

$\Rightarrow e = \dfrac{176\,600}{500} = 353$ mm

Constructing the interaction diagram =

* Compute uniaxial strength ($N = N_{or}$, & $M = 0$)
  pure bending strength ($M = M_r$, $N = 0$)
  balanced point ($N = N_b$ & $M = M_b$)

* Compute a set of (N, M) pairs.
  - Choose a c value
  - Construct ε-diagram ($\varepsilon_{cu} = 0.003$)
  - Compute ε_{si}, σ_{si}, F_{si}, F_c
  - Compute N_i from $\Sigma F = 0 \Rightarrow N_i = F_{ci} + \Sigma F_{si}$
  - Compute M_i with $\Sigma M = 0$ wrt the centroid
  - Plot and choose the next c.
  - Repeat a sufficient number of times.

Column Design:

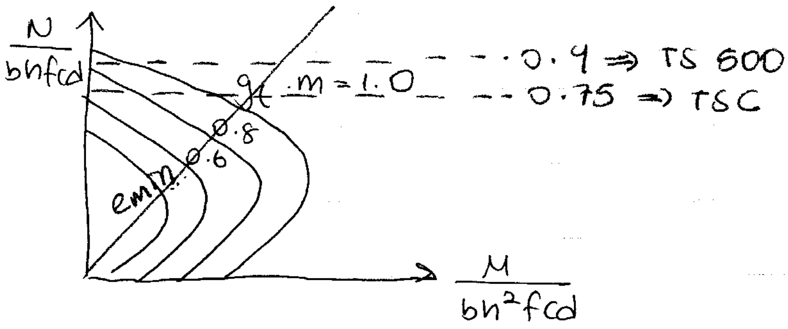
Short Column Design.

* Choose section dimensions assigning N_d to concrete.

$$A_c \geq \frac{N_d}{0.75 f_{cd}} \geq 75\,000 \text{ mm}^2$$

* Provide the reinf. considering M_d and using design charts.

Design Charts:

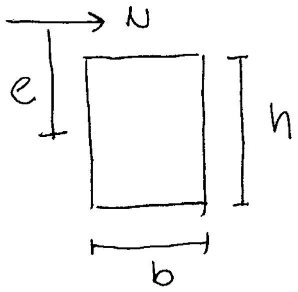


$$\rho_t = \frac{A_{st}}{bh}$$

$$m = \frac{f_{yd}}{f_{cd}}$$

$e_{min} = 15 \text{ mm} + 0.03\,h$

h = dimension in the direction of eccentricity



* Family of non-dimensional interaction diagrams.
* Depend on f_{yd} & d'' (Not other cross-sectional dimensions or f_{cd}) and arrangement of reinf.
* Are chosen wrt f_{yd} & d''/h ratios & arrangement.

Procedure:

* Compute $\dfrac{N}{bhf_{cd}}$ & $\dfrac{M_d}{bh^2 f_{cd}}$
* Obtain $\rho_t \cdot m$ by interpolation.
* Compute $A_{st} = (\rho_t \cdot m)\,\dfrac{f_{cd}}{f_{yd}} \cdot bh$

ex: Short column design:

$N_d = 72\ t$, $M_d = 664\ t\,cm$

$C25/S420 \rightarrow f_{cd} = 170\ kg/cm^2$ & $f_{yd} = 3650\ kg/cm^2$

Preliminary:

$A_c \geq \frac{72\,000}{0.75 \times 170} = 564.7\ cm^2 \Rightarrow$ if $25\ cm \times 25\ cm$ is chosen $625\ cm^2 < 750\ cm^2$

$\Rightarrow$ choose $30 \times 30\ cm \Rightarrow 900\ cm^2 > 750\ cm^2$ ✓

For $d' = 3\ cm \Rightarrow \frac{d''}{h} = \frac{30 - 2\times3}{30} = 0.8$

Final:

$\frac{N_d}{bhf_{cd}} = \frac{72\,000}{30\times30\times170} = 0.471$

$\frac{M_d}{bh^2 f_{cd}} = \frac{664\,000}{30\times30^2\times170} = 0.145$

From the chart for S420 & $d''/h = 0.8$ $\Rightarrow (\rho_t \cdot m) = 0.19$

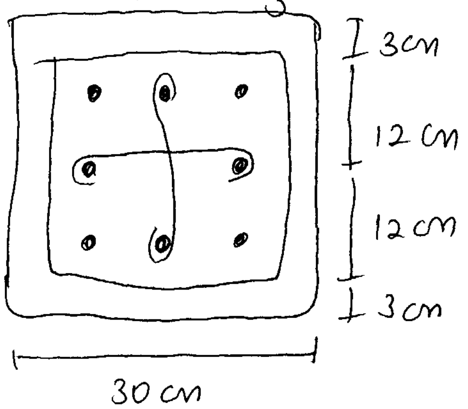
$\rho_t = (\rho_t \cdot m)\frac{f_{cd}}{f_{yd}} = 0.19 \times \frac{170}{3650} = 0.009 < 0.01$ use 1% reinf.

Arrange two-layer symmetrical reinf and intermediate reinf.

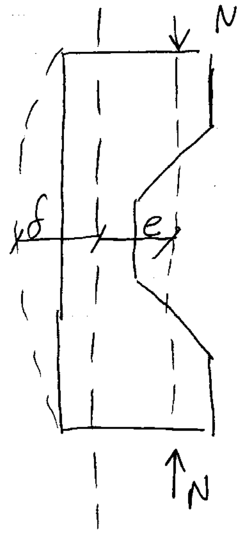
A_s outermost tension $= \frac{3}{8}(0.01)(30)(30) = 3.38\ cm^2 = A_s$ comp

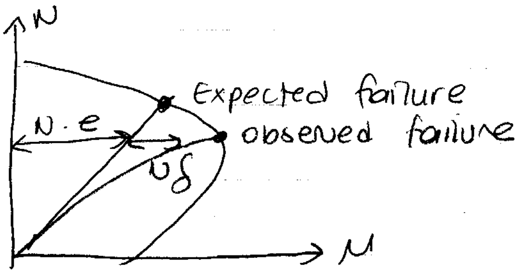
$A_{s\,int} = \frac{2}{3}(3.38) = 2.25\ cm^2$

→ use 3Ø14 < 3Ø12 are enough but code requirement.



Slenderness Effect:





* N . e is the first order moment (undeformed geometry).
* N . δ is the second order moment (deformed geometry).
* Structural analysis is based on undeformed geometry.

→ First order moments.

* When deformed geometry is considered, second order moments have to be taken into account.
* Most practical methods consider undeformed geometry.
* To consider second order effects, iterative procedures are required.
* Since practical methods are first order, obtained moments are magnified to include second order effects.

either by

$M_d = \underbrace{\beta}_{\text{moment magnifier}} M_2$

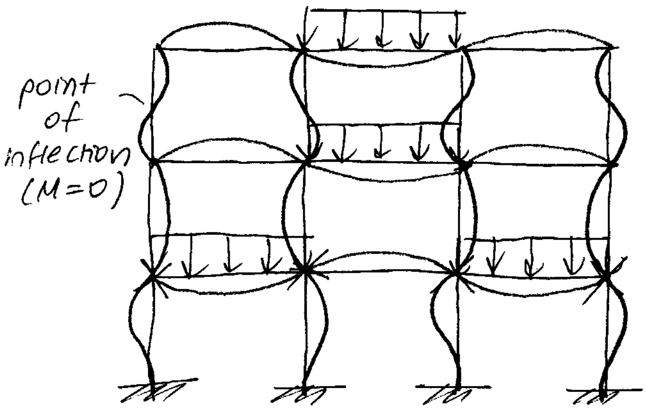
OR by

$M_d = \cancel{M_2 + \Delta M}$

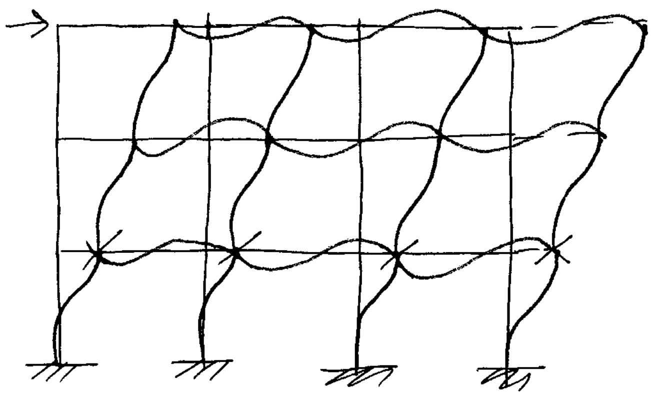
⇒ TS 500 recommends the use of moment magnifier.

Factors Affecting Slenderness:

* System Properties Affecting Slenderness =
  Consider frame deformations.

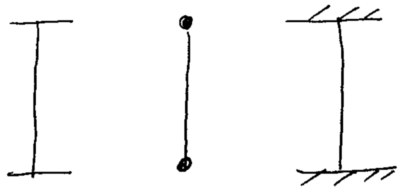


no sidesway



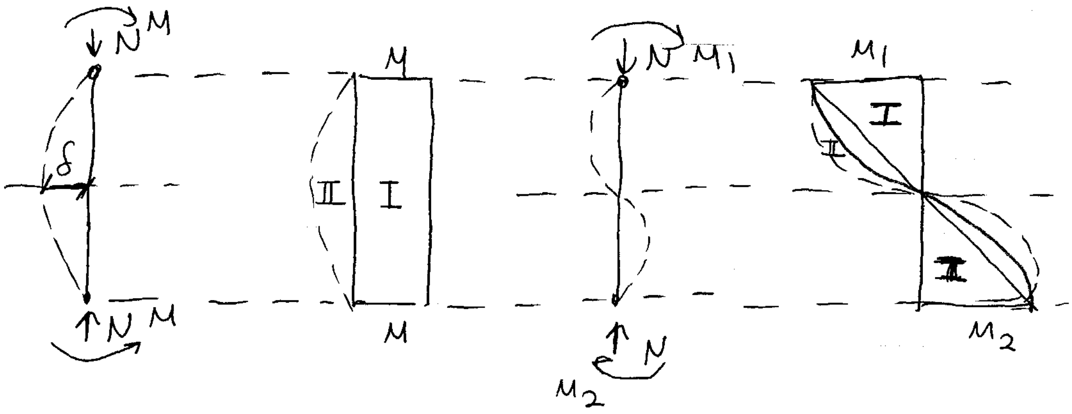
sidesway

→ Isolate a column,



* if the beams are infinitely flexible → rotation is free → pin connected.
* if the beams are infinitely rigid → rotation is prevented → fixed ended.
* Actual behavior is in between these two limiting cases.

* Pin-connected columns in the sidesway frame:



<u>Single curvature:</u>

* even if the second order moment is small, it will increase the design moment.

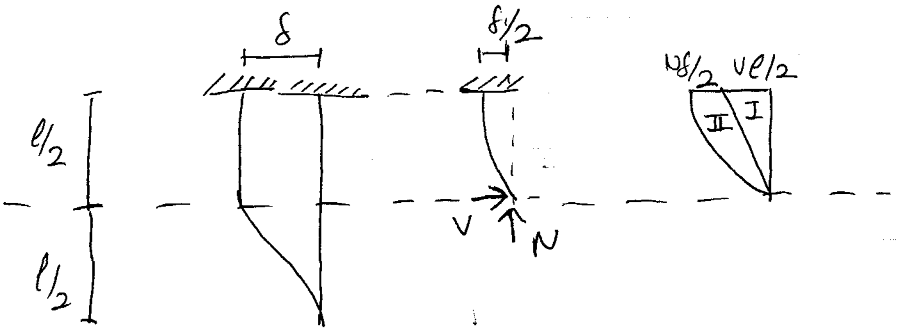
⇒ Slenderness is always critical.

<u>Double curvature:</u>

* second order moment may or may not increase the design moment.

⇒ Slenderness effect is critical for large deflections.

Fixed - ended columns in the sidesway frame:



* Second order moment always increases the design moment even if it is small.

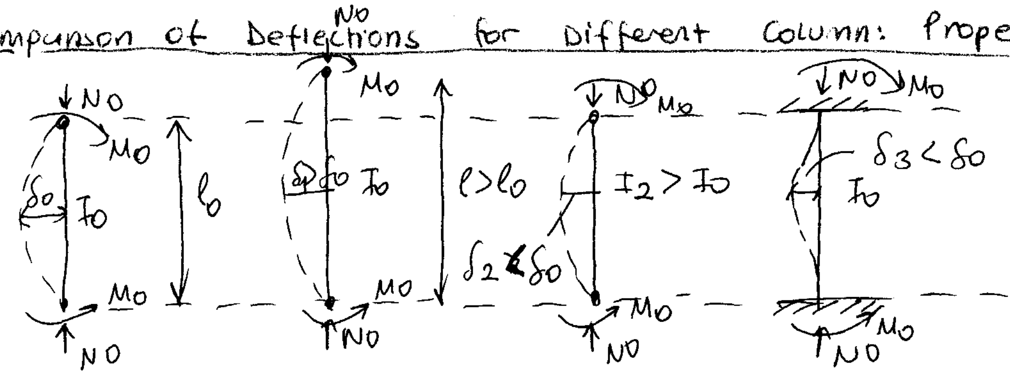
⇒ slenderness is always critical.

Column Properties Affecting Slenderness:

Slenderness and the formation of second order moments are related to deflection. Deflection is related to column properties such as:

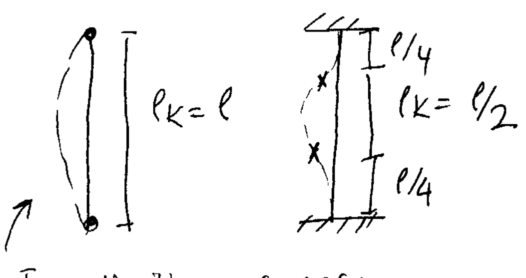
* length
* support conditions (degree of fixity) } ⇒ effective length
* sectional properties ⇒ flexural rigidity.

Comparison of Deflections for Different Column Properties:



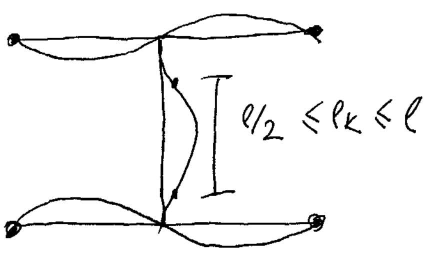
* Effective length (l_k) = is the distance between two points of inflections.

without sidesway:



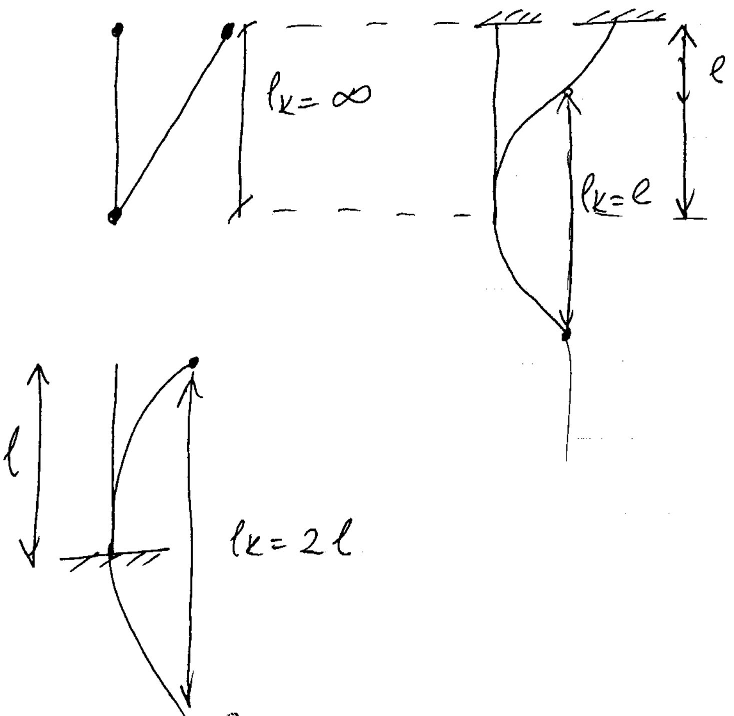
Two limiting cases:

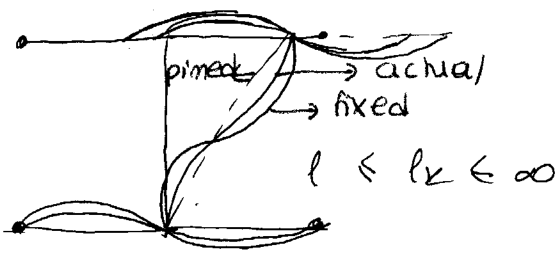
General case:



With sidesway:

Two limiting cases: General cases:

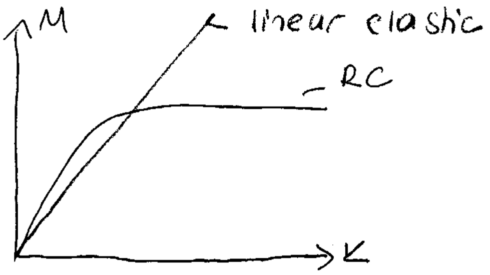




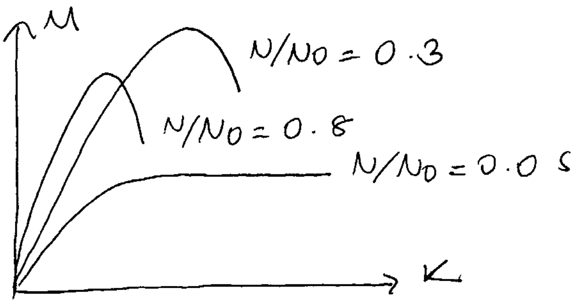
Flexural Rigidity = is the resistance against flexural deformations, EI.

* For a linear elastic material, $\kappa = \frac{M}{EI} \rightarrow EI = \frac{M}{\kappa}$

* For reinforced concrete, EI is the slope of the moment-curvature diagram. However, the slope is different for different zones. The diagram can be idealized in two zones: pre-yield and post-yield.



Moreover, there is no unique $M-\kappa$ diagram for a given cross-section. Therefore, EI also depends on the level of axial load.



EI is difficult to estimate. Because,

* E is influenced by creep.
* I may be reduced due to cracking or severe seismic loading.

Moment Magnification method (TS 500)

* Sway Check =

$$\psi = 1.5 \, \Delta_i \, \frac{\Sigma (N_{di} / l_i)}{V_{fi}}$$

where ⇒ V_{fi} = sum of the horizontal shear forces in the i^{th} floor.

Δ_i = relative displacement of the i^{th} floor.

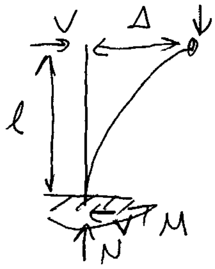
l_i = center to center length of each member on the i^{th} floor.

N_{di} = axial force on each member on the i^{th} floor.

If $\psi \leq 0.05$ ⇒ sway is prevented, non-sway (braced) frame.

> 0.05 ⇒ sway is permitted, sway (unbraced) frame.

If the length of all members are identical →



$$\psi = 1.5 \, \frac{(\Sigma N_{di}) \, \Delta_i}{V_{fi} \cdot l_i}$$

$$M_f = V l \;\; \& \;\; M_s = N \cdot \Delta$$

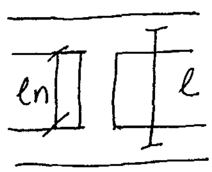
$$\Rightarrow \psi = \underbrace{1.5}_{\text{for uncertainities}} \, \frac{M_s}{M_f}$$

* Braced Frame =

* Compute the relative stiffness, α, at the joints at the top and bottom of the member. Denote larger as α_2 and smaller as α_1.

$$\alpha_{1,2} = \frac{\Sigma (I_g / l)_c}{\Sigma (I_{cr} / l)_b} , \quad \alpha_2 \geq \alpha_1$$

$I_{cr} \sim \frac{1}{2} I_g$



$$\alpha_m = \frac{1}{2} (\alpha_1 + \alpha_2)$$

* Compute $l_k = k \, l_n$, where k is the smaller of:

Take min
$$\begin{cases} k = 0.7 + 0.05 (\alpha_1 + \alpha_2) \\ k = 0.85 + 0.05 \alpha_1 \leq 1.0 \end{cases}$$

l_n = clear length.

* Compute $\frac{l_k}{i}$, where i = radius of gyration $= 0.3h$ (rect. sect)

$= 0.25\,h$ (circ. sect)

* Check if $\left(\frac{l_k}{i}\right) \overset{?}{\leq} 34-12\left(\frac{M_{d1}}{M_{d2}}\right)$

M_{d1}, M_{d2} = first order design end moments, $M_{d2} \geq M_{d1}$

Double curvature $\rightarrow$ $\frac{M_{d1}}{M_{d2}}$ is negative.

Single curvature $\rightarrow$ $\frac{M_{d1}}{M_{d2}}$ is positive

If smaller $\rightarrow$ short column, second order moments can be neglected.

$$M_d = M_{d2} \geq N_d\, e_{min}$$

If larger $\longrightarrow$ slender column $\rightarrow$ continue

* Compute $N_{cr} = \frac{\pi^2 EI}{l_k^2}$ = Euler's buckling load.

where, $EI = \frac{0.4\, E_c\, I_g}{1+R_m}$

R_m reflect the effect of creep $= \frac{N_{gd}}{N_d}$

$N_{gd} = N_g \times$ load factor = design sustained axial load.

N_d = total design axial load.

* Compute $C_m = 0.6 + 0.4\left(\frac{M_{d1}}{M_{d2}}\right) \geq 0.4$

\- For double curvature.
\+ For single curvature.

* Compute $\beta = \frac{C_m}{1-1.3\frac{N_d}{N_{cr}}} \geq 1.0$

* $M'_d = \beta\, M_{d2}$ $\xrightarrow{}$ moment magnifier

* <u>Unbraced frame</u> =

* Compute $\alpha_{1,2}$, α_m.

* Compute l_k

* where, $k = \frac{20-\alpha_m}{20}\sqrt{1+\alpha_m}$, if $\alpha_m < 2$

$k = 0.9\sqrt{1+\alpha_m}$, if $\alpha_m \geq 2$

Note: For a column with one pinned end,

$k = 2 + 0.3\alpha$ ↳ computed on the opposite end.

15 mayıs ⇒ 8:00
mid summer night's dream

* Compute $\frac{l_k}{i}$ and check $\frac{l_k}{i} \leq 22$

smaller → short column, larger → slender column.

* Compute N_{cr}

$$R_m = \frac{\Sigma V_{gd}}{\Sigma V_d}$$

ΣV_{gd} = sum of the design shear forces due to sustained load in a floor.

ΣV_d = sum of the total design shear forces.

unless there is earth pressure, $V_{gd} = 0 \Rightarrow R_m = 0$

However, take $R_m \geq 0.5$ when $V_{gd} = 0$ to account for the effect of creep on EI.

* $C_m = 1.0$

* $$\beta_s = \frac{1.0}{1 - 1.3(\Sigma N_d / \Sigma N_{cr})} \geq 1.0 \Rightarrow \text{for the whole story}$$

$$\beta = \frac{C_m}{1 - 1.3(N_d / N_{cr})} \geq 1.0 \Rightarrow \text{for each individual column.}$$

In design use the larger of β_s and β

If $$\left(\frac{l_k}{i}\right) > \frac{35}{\sqrt{\frac{N_d}{f_{ck} A_c}}} \Rightarrow M'_d = \beta \cdot \beta_s \cdot M_{d2}$$

* $M'_d = (\beta \text{ or } \beta_s) M_{d2}$

TS 500 Requirements for Column Design:

Dimensions:

* Rectangular section size ≥ 250 mm.
* For I, T, L, etc → Flange thickness ≥ 200 mm.
* For box section → wall thickness ≥ 120 mm
* circular column diameter ≥ 300 mm.

* For all columns → $A_c \geq \dfrac{N_d}{0.9 f_{cd}}$ (TS 500)

$$A_c \geq \frac{N_d}{0.75 f_{cd}} \quad (TSC)$$

* minimum eccentricity, $e \geq 0.003h + 15$ mm.

Reinforcement:

* min. longitudinal reinf → $\rho_t \geq 0.01$, $\phi_\ell \geq 14$ mm.

  if $\rho_t \geq 1.3\, \rho_{req}$ → $\rho_t \geq 0.005$ can be used.

* max longitudinal reinf. → $\rho_t \leq 0.04$

  for lap splice region → $\rho_t \leq 0.06$

* lateral reinforcement, $\phi_t \geq \phi_{\ell max}/3 \geq 8$ mm

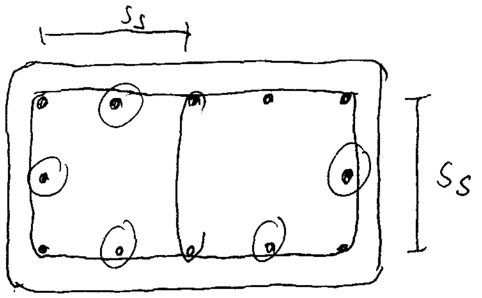
  $s \leq b/3 \leq 100$ mm (tied column, confined region)

  $s \leq b/2 \leq 12\phi_\ell \leq 200$ mm (tied column)

  $s \leq D/5 \leq 80$ mm (spiral column)

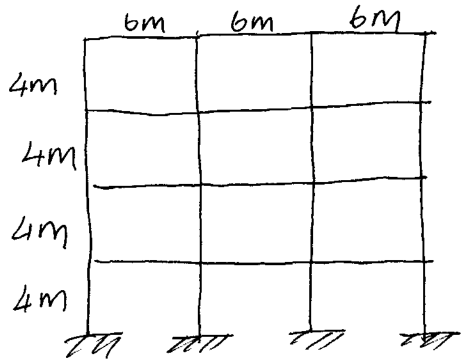
  For spiral reinf, $\rho_{s\,ideal} \leq \rho_s \leq 0.02$

  $S_s \leq 300$ mm



* length of confined regions at column ends → h, $\ell_n/6$, 500 mm
* confinement over lap slice $\geq 6\phi 8$ stirrups, $s \leq h/4 \leq 200$ mm.

ex: Design the column shown below. Use C30/S420



From first order structural analysis:

Effect	Exterior Cols	Interior Cols
N_g	500 kN	800 kN
N_q	260 kN	500 kN
$M_{g1} = M_{g2}$	80 kNm	0 kNm
$M_{q1} = M_{q2}$	60 kNm	10 kNm
V_g	16 kN	12 kN
V_q	21 kN	18 kN

$\Delta_2 = 8.2$ mm

$f_{cd} = 20$ MPa
$E_c = 3.18 \times 10^4$ MPa
$f_{yd} = 365$ MPa

Column = 400 x 400 mm $\rightarrow$ $\frac{I_g}{\ell} = \frac{400 \times 400^3}{12 \times 4000} = 533.3 \times 10^3 \ mm^3$

Beams = 300 x 500 mm $\rightarrow$ $\frac{I_{cr}}{\ell} = \frac{300 \times 500^3}{2 \times 12 \times 6000} = 260.4 \times 10^3 \ mm^3$

$\left(I_{cr} = \frac{1}{2} I_g\right)$

Sway check: $\psi = 1.5 \ \Delta_i \ \frac{\Sigma N_{di}/\ell_i}{V_{fi}} =$

Exterior =

$N_d = 1.4 \times 500 + 1.6 \times 260 = 1116 \ kN$

$M_{d_2} = 1.4 \times 80 + 1.6 \times 60 = 208 \ kNm$

$e = \frac{208\,000}{1116} = 186 mm > e_{min}$ <u>OK</u>

Interior =

$N_d = 1.4 \times 800 + 1.6 \times 500 = 1920 \ kN$

$M_d = 1.4 \times 0 + 1.6 \times 10 = 16 \ kNm$

$\Rightarrow e = \frac{16\,000}{1920} = 8.3 \ mm < e_{min} = (0.003 \times 400) + 15 = 27 mm$

$\Rightarrow M_{d_2} = 1920 (0.027) = 51.8 \ kNm.$

$\psi = 1.5 \times 8.2 \times \frac{2(1116 + 192)}{4000 \times 2 (\underbrace{56}_{1.4(16)+1.6(21)} + \underbrace{45}_{1.4(12)+1.6(18)})} = 0.092 > 0.05 \Rightarrow$ unbraced frame (sway is permitted)

$\Downarrow$

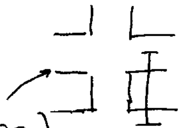
* dealing with both exterior & interior columns

Exterior:

$N_d = 1116 \ kN$, $M_d = 208 \ kNm$, $e > e_{min}$

$\alpha_1 = \alpha_2 = \alpha_m = \frac{2 \times 533.3}{1 \times 260.4} = 4.10 > 2$

$k = 0.9\sqrt{1 + \alpha_m} = 2.03$



$\left(\frac{\ell_k}{i}\right) = \frac{k \ell_n}{i} = \frac{2.03 \ (4000 - 500)}{0.3 \times 400} = 59.3 > 22 \Rightarrow$ slender

$R_m = \frac{V_{gd}}{V_d} = \frac{1.4 \ (16)}{1.4(16) + 1.6(21)} = \frac{22.4}{56} = 0.4$

$EI = \frac{0.4 \ E_c \ I_g}{1 + R_m} = \frac{0.4(31\,800) \ 400 \times 400^3}{12(1 + 0.4)} = 19.38 \times 10^{12} \ Nmm^2$

$N_k = \frac{\pi^2 EI}{\ell_k^2} = \frac{\pi^2 (19.38) \ 10^{12}}{((2.03)(3500))^2} \times 10^{-3} = 3789 \ kN.$

Interior =

$N_d = 1920 \ kN$, $e < e_{min} \Rightarrow M_{d2} = 51.8 \ kNm$

$\alpha_1 = \alpha_2 = \alpha_m = \frac{2 \times 533.3}{2 \times 260.4} = 2.05$

$k = 0.9\sqrt{1 + \alpha_m} = 1.57$

$\left(\frac{l_k}{i}\right) = \frac{1.57 \ (4000 - 500)}{0.3 \ (400)} = 45.8 > 22 \Rightarrow$ slender

$$R_m = \frac{1.4(12)}{1.4(12)+1.6(18)} = \frac{16.8}{45.6} = 0.368$$

$$EI = \frac{0.4(31800)\times 400\times 400^3}{12\,(1+0.368)} = 19.84\times 10^{12}\ Nmm^2$$

$$N_K = \frac{\pi^2 (19.84)10^{12}}{(1.57\times 3500)^2}\times 10^{-3} = 6485\ kN.$$

$$\Sigma N_d = 2(1116+1920) = 6072\ kN$$

$$\Sigma N_k = 2(3789+6485) = 20548\ kN > 2.2\times 6072 = 13358\ kN$$

ok No change in dimension

For the story $\rightarrow \beta_s = \dfrac{1.0}{1-1.3\dfrac{\Sigma N_d}{\Sigma N_k}} = \dfrac{1.0}{1-1.3\left(\dfrac{6072}{20548}\right)} = 1.624 > 1$ ok

For the individual column $\rightarrow \beta = \dfrac{1.0}{1-1.3\dfrac{N_d}{N_k}} = \dfrac{1.0}{1-1.3\left(\dfrac{1116}{3789}\right)} = 1.621 > 1$ ok

For the column $\rightarrow \dfrac{l_k}{i} = 59.3 \overset{?}{>} \dfrac{35}{\sqrt{\dfrac{N_d}{A_c f_{ck}}}} = \dfrac{35}{\sqrt{\dfrac{1116000}{400(400)(20)}}} = 59.3$

$\Rightarrow M_d = (\underbrace{\beta \text{ or } \beta_s}_{\text{use larger one}})\, M_{d2} = 1.624\,(208) = 337.8\ kNm$

$\rightarrow$ if larger $M_d = \beta\beta_s M_{d2}$

$$\frac{N_d}{bhf_{cd}} = \frac{1116000}{400\times 400\times 20} = 0.35$$

$$\frac{M_d'}{bh^2 f_{cd}} = \frac{337.8\times 10^6}{400\times 400^2\times 20} = 0.264$$

Use the chart for S420 &

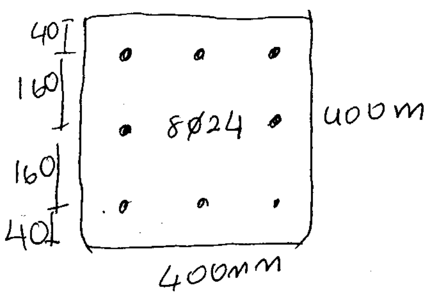
$$\frac{d''}{h} = \frac{400-2\times 40}{400} = 0.8$$

$\Rightarrow \rho_{tm} = 0.38 \rightarrow \dfrac{f_{yd}}{f_{cd}}$

$$\Rightarrow A_{st} = 0.38\left(\frac{20}{365}\right)400^2 = 3332\ mm^2$$

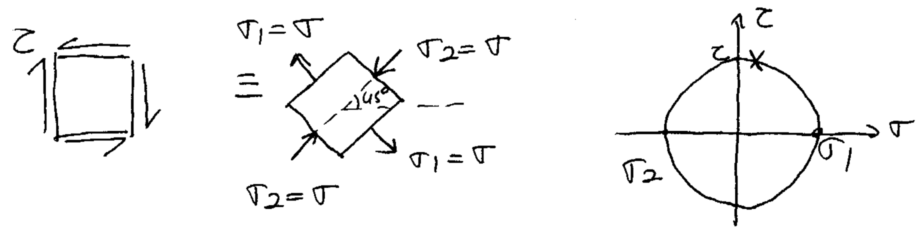
use 3Ø24 on each face and 2Ø24 at the mid-section.

$\Rightarrow$ 8 Ø24 = 3619 mm^2



Shear (Diagonal Tension)

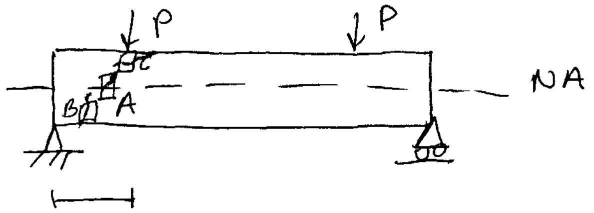
Consider an element in pure shear (on the neutral axis)



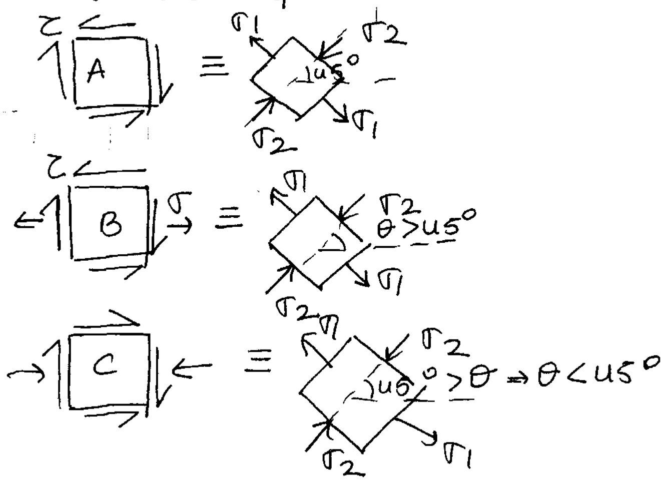
Shear strength of concrete $\approx 0.4 - 0.8 f_c$
Tensile strength of conc $\approx 0.1 f_c$

} Always tension failure under principal tension

Consider a beam under three-point loading:

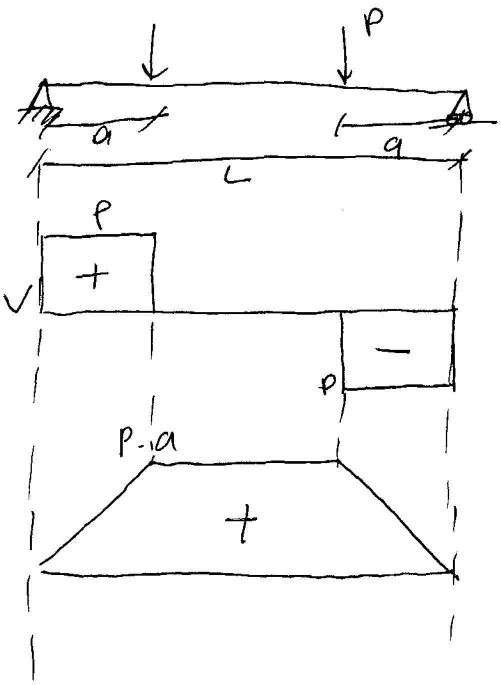


a → shear span



→ Shear crack is formed by the combined action of shear and bending. Shear failure is caused by principal tensile stresses.

Consider a beam with a moment capacity of M_r.



$V = P$ & $M = P.a$

moment capacity $= M_r$
Shear capacity $= V_r$

* when a is large → P is small → V is small
  ⇓
  Bending failure

* when a is small → P is large → V is large
  ⇓
  Shear failure

⇒ Failure mode depends on the shear span, a

$a/d = M/Vd$ → effective depth of member

Behavior of a Reinforced Concrete Beam without web reinf:

$\frac{a}{d} > 7-8 \Rightarrow$ Flexural failure (for an under reinf section first longitudinal tensile reinf. yields, then concrete crushes)

$7 > \frac{a}{d} > 3 \Rightarrow$ Diagonal tension failure

$3 > \frac{a}{d} > 1.5 \Rightarrow$ shear compression failure

$\frac{a}{d} \leq 1 \Rightarrow$ tied arch behavior

Diagonal Tension failure:

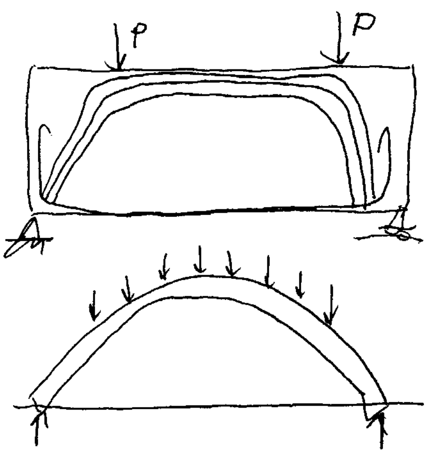
* First flexural cracks appear at the maximum moment region and vertical cracks due to combined shear and bending at the shear span. 1
* The crack is incline towards the load. 2 & 3
* One of these inclined cracks kicks back (kicking back action) i.e. the inclined crack progresses downward to the level of tens. reinf. 4
* beam fails suddenly with the formation of 5 & 6.

$\Rightarrow$ Extremely brittle shear failure.

Shear - Compression failure:

* First flexural cracks appear at the max moment region and shear cracks at the shear span as in Diagonal Tension failure. 1
* Shear cracks get inclined. 2
* Crack extends towards the load 3 and the reaction 4.
* Beam continues to carry the increasing load with fully developed diagonal cracks.
* Failure is due to concrete crushing 5 under the load or over the support.
* Flexural capacity > max moment reached at Shear - comp. failure > max moment reached at Diagonal tension failure.
* Diagonal Tens. Failure is more sudden and brittle than shear - comp failure.
* Shear - comp failure is a brittle shear failure.

Tied Arch Behavior =

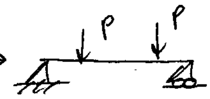


* Shear is transferred to the support through a compression strut forming in the web.
* Tension reinf becomes the bar after cracking horizontally reactions.
* Failure is due to concrete crushing in the compression zone or crushing of the web (if web width is small) or bond (anchorage) failure near the support.

In addition to a/d ratio, ρ and f_{ct} affects the behavior and strength of beams w/o web reinf.

Shear valley (Strength envelope)

* For a beam under three-point loading →

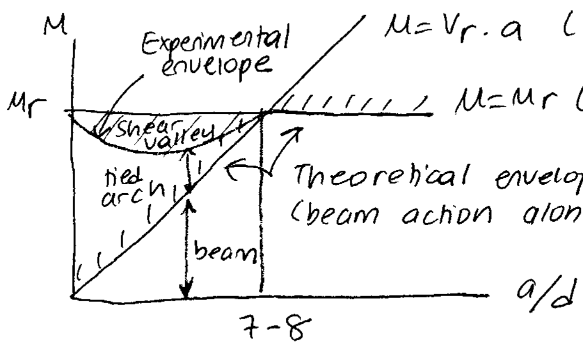


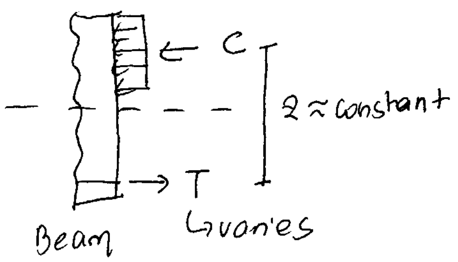
$V = P$ & $M = P \cdot a$

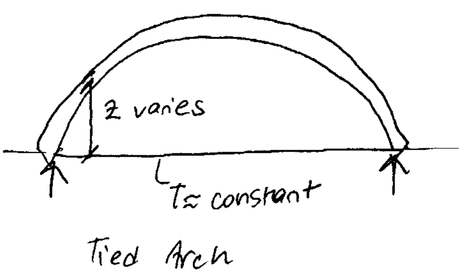
Constant moment capacity, M_r

Constant shear capacity, V_r

* Flexural failure when $M = Pa = M_r$ = constant
* Shear failure when $V = P = \frac{M}{a} = V_r$ = constant







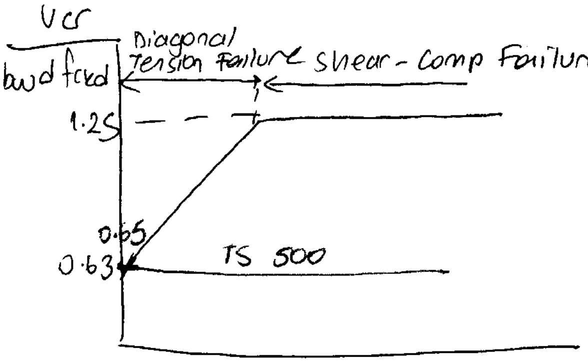
Equilibrium → $M = Tz$.

$$\Rightarrow V = \frac{\partial M}{\partial x} = \underbrace{\frac{\partial T}{\partial x}}_{\text{Beam}} z + \underbrace{\frac{\partial z}{\partial x}}_{\text{Tied Arch}} T$$

Strength of Reinforced Concrete Beams without Web Reinf.

* Design of beams without web reinf is not allowed by the codes.
* However, the ultimate strength of beams without web reinf = cracking strength of beams with web reinf.
* Viest equation (lower band)

$$V_{cr} = \left[\left(0.16\sqrt{f_{ck}} + 17\rho \frac{Vd}{M}\right) b_w d \right] \leq 0.3\sqrt{f_{ck}}\, b_w d$$



$$\frac{M}{Vd} = \frac{Pa}{Pd} = \frac{a}{d}$$

$$\frac{\rho}{f_{ctd}} \times \frac{Vd}{M} \leftarrow \frac{d}{a}$$

TS 500 $\rightarrow V_{cr} = 0.65\, f_{ctd}\, b_w d$

If there is axial force on the beam $\rightarrow V_{cr} = 0.65\, f_{ctd}\, b_w d \left(1 + \gamma \frac{N_d}{A_c}\right)$

For tension $\rightarrow \gamma = -0.3$

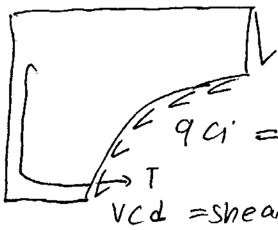
For comp $\rightarrow \gamma = 0.07$

if $N_d/A_c < 0.5$ MPa $\rightarrow \gamma = 0$

Mechanism of Shear Failure for RC Beams without Web Reinf:

FBD after cracking

$$V = V_{cc} + F_{ci} + V_{cd}$$

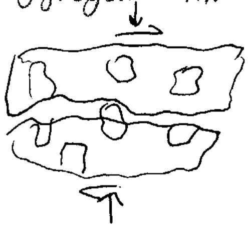


V_{cc} = shear resistance of the uncracked compression zone.

q_{ci} = shear carried by aggregate interlock

V_{cd} = shear transmitted by dowel action.

Aggregate interlock =



Crack interfaces (surfaces of cracks) are not smooth. The coarse agg particles projecting across the cracks produce roughness. Up to 60 % of the total shear can be resisted by agg interlock.

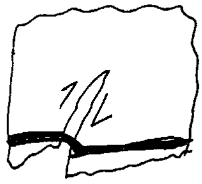
$f_t = (1 + R_3 + R_t \log \frac{t}{3})$

time correction for $t > 3$ years

R_3 = settlement occurring in first 3 years after construction
R_t = " " during each log cycle of time in excess of 3 years

$t > 30$ years $f_t = 1.5$ for static loads
$f_t = 2.5$ for fluctuating loads.

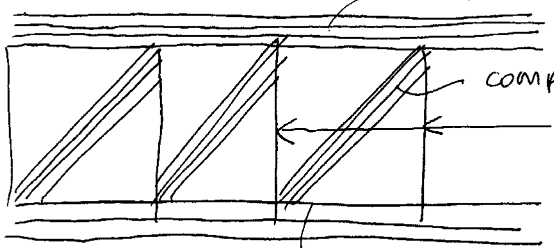
Dowel action:



Longitudinal reinf. resists the shear deformation. Dowel capacity depend on:
* tensile strength of concrete
* crack width
* concrete cover
* diameter of tension reinf.

Up to 20% of the total shear can be resisted by dowel action. Close to ultimate storage, cracks get wider → shear resistance by agg interlock and dowel action decrease significantly.

Behavior and Strength of RC Beams with Web Reinf.



ultimate strength of beams with web reinf.

$V_r = V_c + V_w$

Unlike bending, concrete contribution continues after shear cracking up to failure.

$V_c = \underbrace{V_{cc}}_{\text{uncracked}} + \underbrace{F_{ci}}_{\text{agg interlock}} + \underbrace{V_{cd}}_{\text{dowel}}$

Non-seismic → $V_c = 0.8\, V_{cr}$
$V_{cr} = 0.65\, f_{ctd}\, b_w d\, (1 + \gamma\, N_d / A_c)$
$\gamma = 0.07$ (comp)
-0.3 (tens)

$V_w = \frac{A_{sw}}{s} f_{ywd} \cdot d$ (modified truss analogy)

number of legs

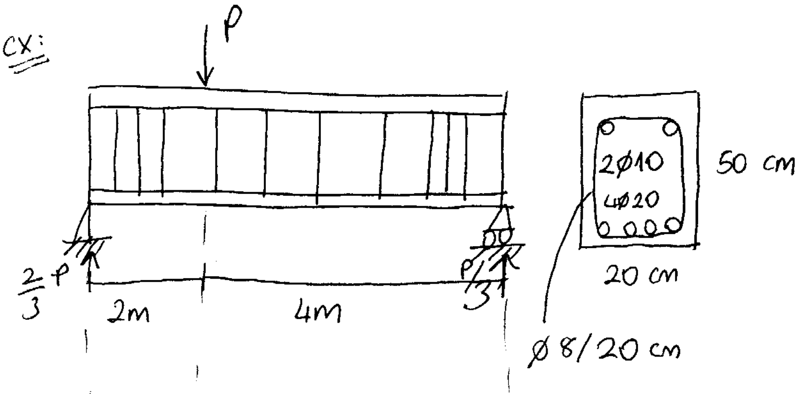
$A_{sw} = n A_0$ = cross sectional area of stirrups

s = stirrup spacing

V_w = shear carried by web reinf.

d = effective depth

ex:



Rect. Section

20×50 cm

(d = 46 cm)

What is the max load that beam can carry (P_{allw}) before it fails under shear?

C 25/ S420; $f_{cd} = 170$ kg/cm^2

$f_{ctd} = 11.5$ kg/cm^2

$f_{yd} = f_{ywd} = 3650$ kg/cm^2

$N_d = 0 \rightarrow V_{cr} = 0.65\,(11.5)(20)(46\times10^{-3}) = 6.88\,t$

$A_{sw} = 2\times 0.5 = 1\ cm^2 \rightarrow V_w = \frac{1}{20}(3650)(46\times10^{-3}) = 8.4t$

$V_d = \frac{2}{3}P_{allw} = 0.8\,(6.88) + 8.4 = 13.9\ t$

$\Rightarrow P_{allw} = 20.9\ t$

Shear Design of Beams (TS 500)

* $V_r \geq V_d$

* If $V_d \leq V_{cr} \rightarrow$ use min $\frac{A_{sw}}{s}$ to prevent brittle failure

$$\frac{A_{sw}}{s} \geq 0.3\,\frac{f_{ctd}}{f_{ywd}}\,b_w$$

* If $V_d > V_{cr} \longrightarrow V_w = V_d - V_c$, where $V_c = 0.8\,V_{cr}$

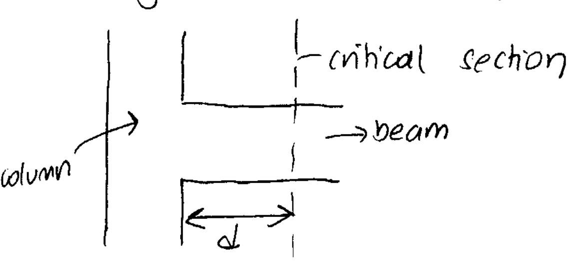
* Use $\frac{A_{sw}}{s} = \frac{V_w}{f_{ywd}\,d} \geq \min \frac{A_{sw}}{s}$

* To prevent crushing in the slender webs,

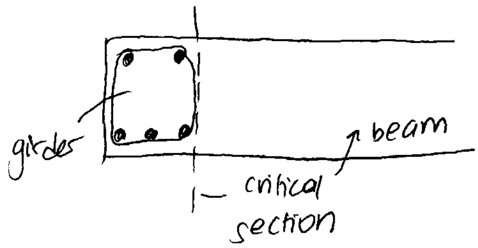
$V_d \leq 0.22\,f_{cd}\,b_w\,d$

* Calculate V_d at the critical section!

If the beam is supported by a column, calculate V_d at a distance d away from the column face.



If the beam is supported by a girder, calculate Vd at the girder face (indirect support).



* Stirrup Spacing

$s \leq \frac{d}{2}$, if $V_d > 3\ V_{cr} \rightarrow s \leq \frac{d}{4}$

In the end zones ($l = 2d$)

$$s \leq \begin{cases} d/4 \\ 8\phi \\ 150\ mm \end{cases}$$

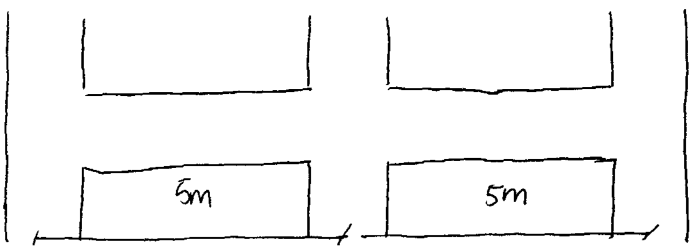
* Preliminary Design

flexural $\rightarrow b_w d^2 = M_d K_\ell$

shear $\rightarrow b_w\ d = \frac{0.9\ V_d}{f_{ctd}}$

ex: Design the given 2 span beam.

C 20, S420, stirrups S220, columns 400x400 mm.



* Preliminary Design

Estimated loads $\rightarrow g = 30$ kN/m, $q = 15$ kN/m

$P_d = 1.4 \times 30 + 1.6 \times 15 = 66$ kN/m

Max moment $= \frac{1}{9} P_d L^2 = \frac{1}{9}(66)5^2 = 183$ kNm

max shear $= \frac{P_d L}{2} = \frac{66 \times 5}{2} = 165$ kN

Bending $\rightarrow K_\ell = 380$ mm²/kN

$b_w d^2 = M_d K_\ell = 183\,000 \times 380 = 69.54 \times 10^6$ mm³

↗ if $b_w = 250$ mm $\rightarrow d = 527$ mm

↘ if $b_w = 300$ mm $\rightarrow d = 481$ mm

Shear

$$b_w d = \frac{0.9 V_d}{f_{ctd}} = 0.9 \frac{(165\,000)}{1.0} = 148\,500 \text{ mm}^2$$

if $b_w = 250$ mm $\rightarrow$ $d = 594$ mm

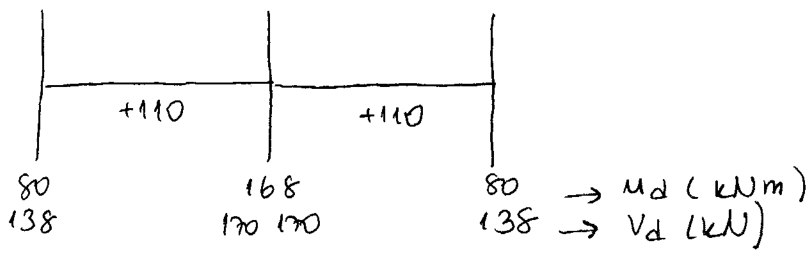
if $b_w = 300$ mm $\rightarrow$ $d = 495$ mm

$\Rightarrow$ choose 300 x 500 mm beam

* Final Design

Loads $g = 28$ kN/m, $q = 14$ kN/m $\rightarrow$ $P_d = 61.6$ kN/m

From analysis (centerline values)



For supports $\Delta M = \frac{V_d a}{3} = \frac{170(0.4)}{3} = 22$ kNm (Int)

$$\Delta M = \frac{138(0.4)}{3} = 18 \text{ kNm (Ext)}$$

$\Rightarrow$ Exterior support $\rightarrow$ $M_d = 80 - 18 = 62$ kNm

Interior support $\rightarrow$ $M_d = 168 - 22 = 146$ kNm

Span:

$M_d = 110$ kNm , T- Beam , $d = 460$ mm

$$A_s = \frac{M_d}{f_{yd}\, j\, d} = \frac{10^6 \times 110}{365 \times 0.9 \times 460} = 728 \text{ mm}^2$$

$$\min A_s = 0.8 \frac{f_{ctd}}{f_{yd}} b_w d = 302 \text{ mm}^2$$

2 Ø 16 straight — 400 mm²

2 Ø 16 Bent — 400 mm²

\+ 800 mm²

Supports

Ext support (rect beam)

$M_d = 62$ kNm , $K = \frac{b w d^2}{M_d} = 1023$ mm²/kN $> K_\ell$ ok single reinf.

$$A_s = \frac{M_d}{f_{yd}\, j d} = \frac{62 \times 10^6}{365(0.86)(460)} = \cancel{}\ 429 \text{ mm}^2 > A_{s\,min} \ \text{ok}$$

Avaliable → 2Ø12 Hanger – 226 mm²

2Ø16 Bent – 400 mm²
+
626 mm² > 429 mm²

Int support

$M_d = 146$ kNm , $K = 435$ mm²/kN $> K_\ell$ ok single reinf.

$$A_s = \frac{146 \times 10^6}{365 \times 0.\cancel{86} \times 460} = \cancel{}\ 1011 \text{ mm}^2$$

Avaliable : 2Ø12 Hanger – 226 mm²

4Ø16 Bent – 800 mm²
+
1026 mm² > ~~[illegible]~~ 1011 mm² ok

Shear Design

Critical shear = $V_d' = V_d - P_d\left(\frac{a}{2} + d\right) = 170 - 61.6(0.2 + 0.46) = 130$ kN

$V_{cr} = 0.65\, f_{ctd}\, b_w\, d = 0.65(1.0)(300)(460) = 89.7$ kN

$V_c = 0.8\, V_{cr} = 72$ kN

$V_{max} = 0.22\, f_{cd}\, b_w\, d = 0.22 \times 13 \times 300 \times 460 = 394$ kN

$V_{cr} < V_d' < V_{max}$

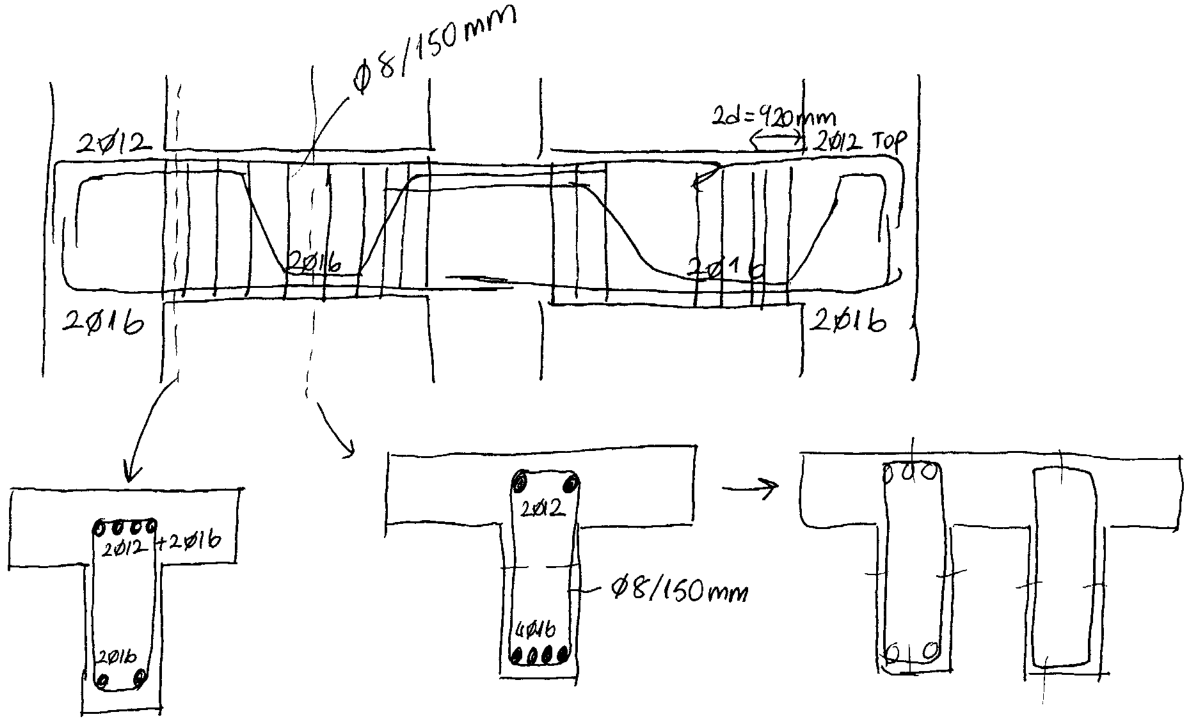
$$\min \frac{A_{sw}}{s} = 0.3\frac{f_{ctd}}{f_{ywd}} b_w = 0.3\frac{f_{ctd}}{f_{ywd}} b_w = 0.3 \times \frac{1.0}{191} \times 300 = 0.47 \text{ mm}$$

$$\frac{A_{sw}}{s} = \frac{V_d' - V_c}{f_{ywd} \times d} = \frac{130 - 72}{0.191 \times 460} = 0.66 \text{ mm} > \min \frac{A_{sw}}{s} \ \text{ok}$$

if Ø8 is used → $A_{sw} = 2 \times 50 = 100$ mm² → $s = 151$ mm

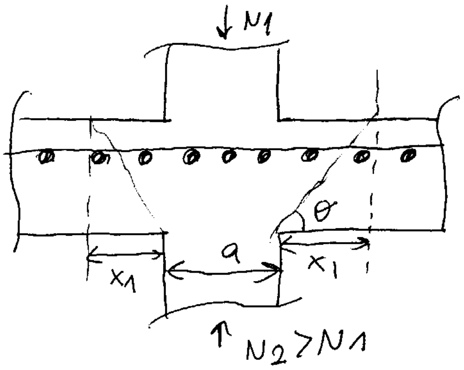
max $s = d/2 = 230$ mm → use Ø8/150 mm.

end zones : $\begin{cases} d/4 = 115 \text{ mm} \\ 8Ø = 64 \text{ mm} \\ 150\text{mm} = 150 \text{mm} \end{cases}$ → Ø8/60 mm.



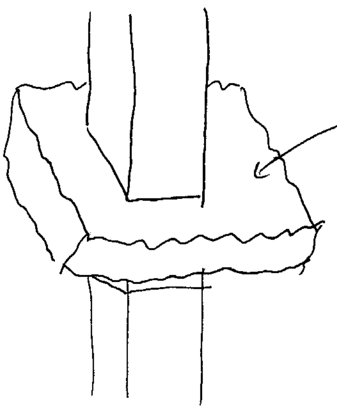
Punching Shear:

Puncning is a 3D shear problem common to flat plates and individual footings. The failure due to punching shear is very brittle, but it can be delayed by proper reinforcement.



$\theta \simeq 25-45°$ (depends on the reinf)

generally close to $45°$.



inverted pyramid or cone.

Punching Strength of Slabs and Footing Without (Punching) Shear Reinf.

$V_{pc} = \gamma f_{ctd} u_p d$ (similar to $V_{cr} = 0.65 f_{ctd} b_w d$)

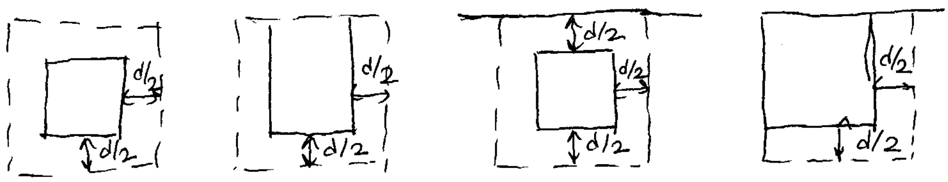
where u_p = critical perimeter (measured at $d/2$ from the column face)

γ = factor to account for effect of bending

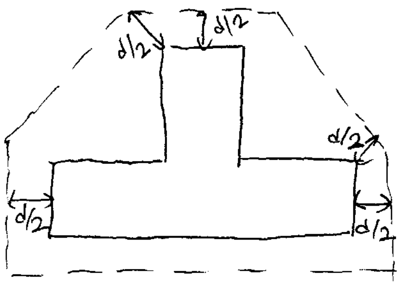
$\gamma = 1.0$ for uniaxial loading (M negligible)

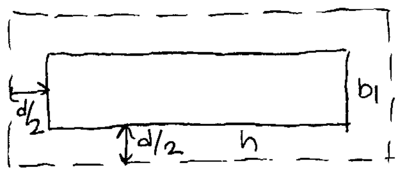
< 1.0 if bending is considered depends on e_x & e_y

Critical Perimeter



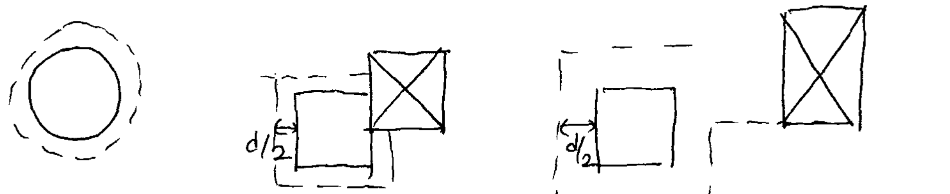
Interior Edge Corner





If aspect ratio > 3

Take h = 3b



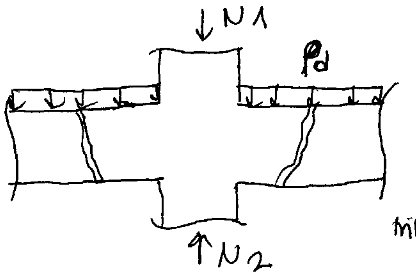
opening

opening closer than 5d

The shortest possiple u_p is the most likely one.

$V_{pc} > V_{pd}$

where $V_{pd} = F_d - F_a = N_2 - N_1 - \underbrace{F_a}_{\text{inside } u_p}$



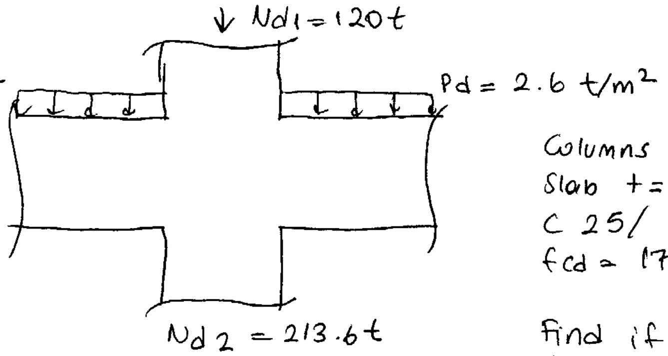
tributary area

Equilibrium $\Rightarrow N_2 = N_1 + P_d A_t$

$F_a = p_d A_p$ ← area defined by u_p

$F_a = p_d \cdot \underbrace{b_1 \cdot b_2}_{\text{side lengths of the } u_p} = p_d (h+d)(b+d)$ (int column)

ex:



Columns 40x40 cm
Slab t = 30 cm (d = 28 cm)
C 25/ S 420
$f_{cd} = 170\ kg/cm^2$, $f_{ctd} = 11.5\ kg/cm^2$

Find if this column is safe under punching shear, if not improve.

$$V_{pc} = 1.0 \times 11.5 \times 4\,(40+28) \times 28 \times 10^{-3} = 87.6\ t$$

$$V_{pd} = 213.6 - 120 - 2.6 \times 0.68^2 = 92.4\,t > V_{pc} \rightarrow \text{unsafe!}$$

To improve V_{pc} :

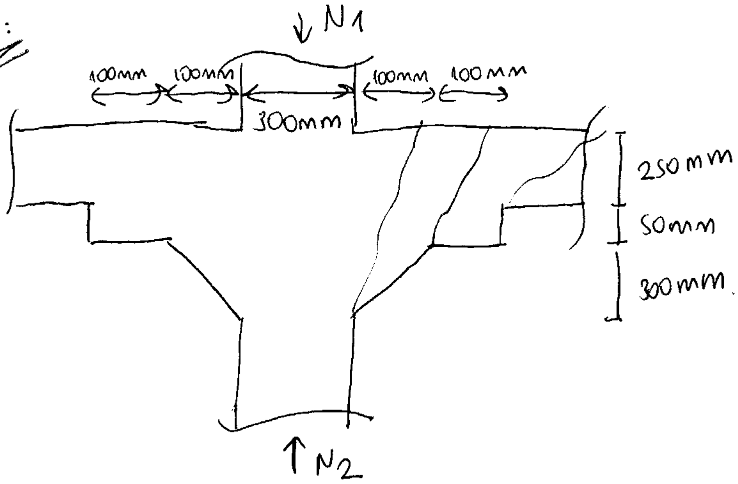
* increase d → Vpc increases proportionally but Vpd also increases due to the increase in DL.
* increase column dimensions → structurally not desirable.
* introduce column capital → formwork problem
* introduce drop panels → formwork problem
* use punching reinf. → limited use

To improve make the lower story columns → 45x45 cm

$$V_{pc} = 1.0 \times 11.5 \times 4 \times (45+28) \times 28 \times 10^{-3} = 94.0\ t$$

$$V_{pd} = 213.6 - 120 - 2.6 \times 0.73^2 = 92.2\,t < V_{pc} \rightarrow \text{slab is safe under punching}$$

ex:



Circular columns : D = 300 mm

Drop panels: $D_{dp} = 700$ mm, $t_{dp} = 50$ mm

Column capitals : $D_{cc} = 500$ mm , $t_{cc} = 300$ mm

$N_{d1} = 2880$ kN & $N_{d2} = 3840$ kN

is this slab-column connection safe?

$P_d = 15\ kN/m^2$

Slab = $h_s = 250$ mm & $d_s = 230$ mm

$f_{cd} = 20$ MPa

$f_{ctd} = 1.5$ MPa, $f_{yd} = 365$ MPa.

* Failure at drop panel edge

$d_s = 230$ mm , $u_p = (700 + 230)\pi = 2922$ mm

$V_{pr} = 1.0 \times 1.5 \times 2922 \times 230 \times 10^{-3} = 1008$ kN

$V_{pd} = 3840 - 2880 - \frac{\pi}{4}(0.7 + 0.23)^2 15 = 950$ kN

$V_{pr} > V_{pd} \rightarrow$ safe

* Failure at column capital edge

$d_{dp} = 300 - 20 = 280$ mm

$u_p = (500 + 280)\pi = 2450$ mm

$V_{pr} = 1.5 \times 2450 \times 280 \times 10^{-3} = 1029$ kN

$V_{pd} = 3840 - 2880 - (0.5 + 0.28)^2 \frac{\pi}{4} \times 15 = 953$ kN

$V_{pr} > V_{pd} \rightarrow$ safe

* Failure at column edge

$d = 580$ mm , $u_p = (300 + 580)\pi = 2765$ mm

$V_{pr} = 1.5 \times 2765 \times 580 \times 10^{-3} = 2405$ kN

$V_{pd} = 3840 - 2880 - (0.3 + 0.58)^2 \times \frac{\pi}{4} \times 15 = 951$ kN

$V_{pr} > V_{pd} \rightarrow$ safe

$\therefore$ The slab-to-column connection is safe under punching shear.

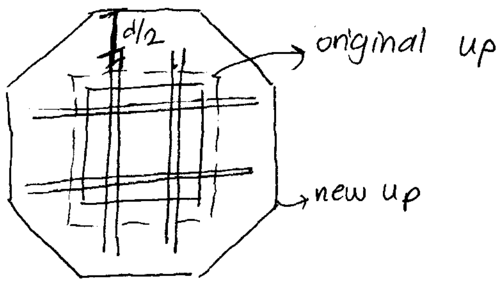
<u>Punching Reinforcement:</u>

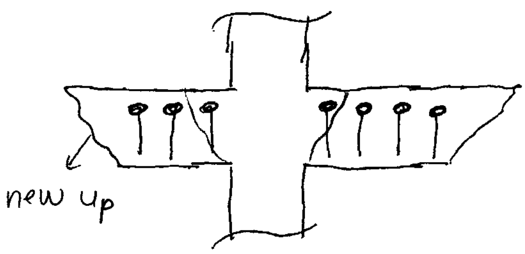
* is not allowed if $h < 250$ mm

* ineffective if remains inside the punching pyramid.

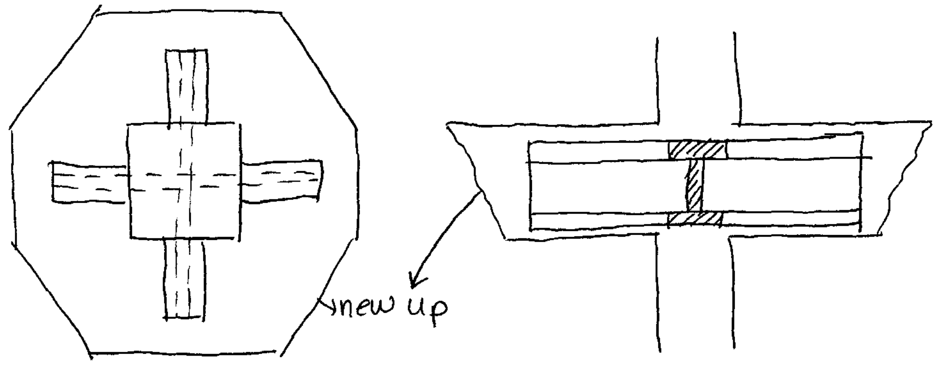
<u>Types:</u>

* shear-head (preferred in North America)

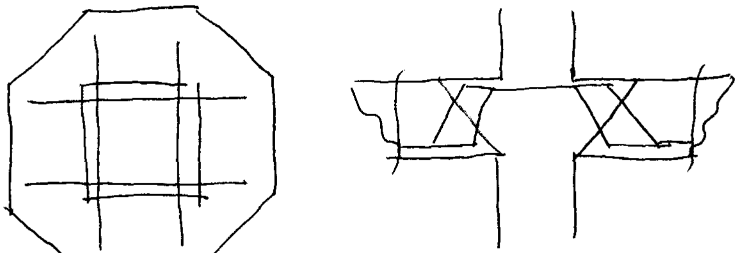




* Steel I-beams



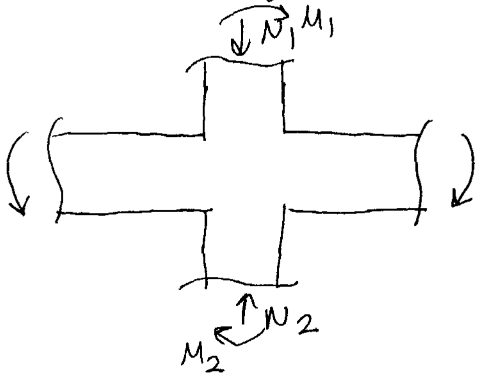
* Bar type punching reinf. (Turkey and Europe)



Anchorage problem!

<u>Note:</u> V_{pc} cannot be increased more than 50% by the addition of punching shear reinf. (higher increase is not allowed!)

<u>Combined Bending and Axial Load</u>:



$$e = \frac{0.4\,\overbrace{(M_1 + M_2)}^{\text{unbalanced moment}}}{N_2 - N_1}$$

* Rectangular column:

$$\gamma = \frac{1}{1 + 1.5\,\frac{e_x + e_y}{\sqrt{b_1 \cdot b_2}}}$$

* Circular column:

$$\gamma = \frac{1}{1 + \frac{2e}{d+h}}$$, where h = diameter of the circular column