PIPE FLOW

Scope of the Course

- **In many water systems, transportation of water from one location to another is the main concern.**
- **Two main modes of transportation are:**
- **Closed conduits with pressurized flow inside**
- **Open conduits with free surface flow inside**
- **The main objective in this course is to study the flow in closed conduits (mainly pipes) and in open channels**

Examples include:

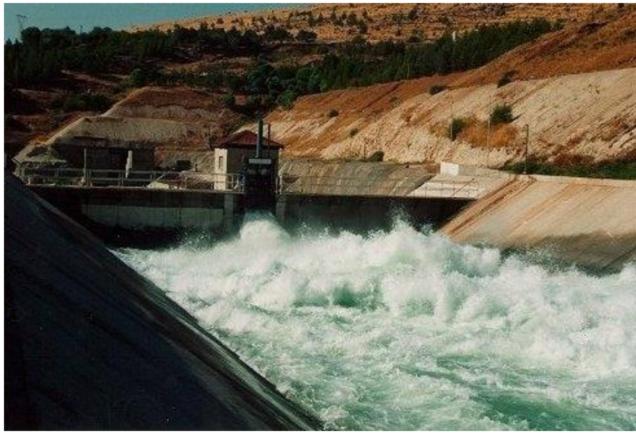
- **Water distribution networks in urban areas**
- **Water transmission line from Çamlıdere Dam to İvedik Water Treatment Plant**

  (ϕ = 3.4 m, L = 15.5 km)
- **Urfa Tunnels from Atatürk Dam to Harran Plain (ϕ = 7.62 m, L = 2 x 26.4 km)**
- **Main irrigation canal in Harran Plain (L=118 km, Q = 80 m3/s)**

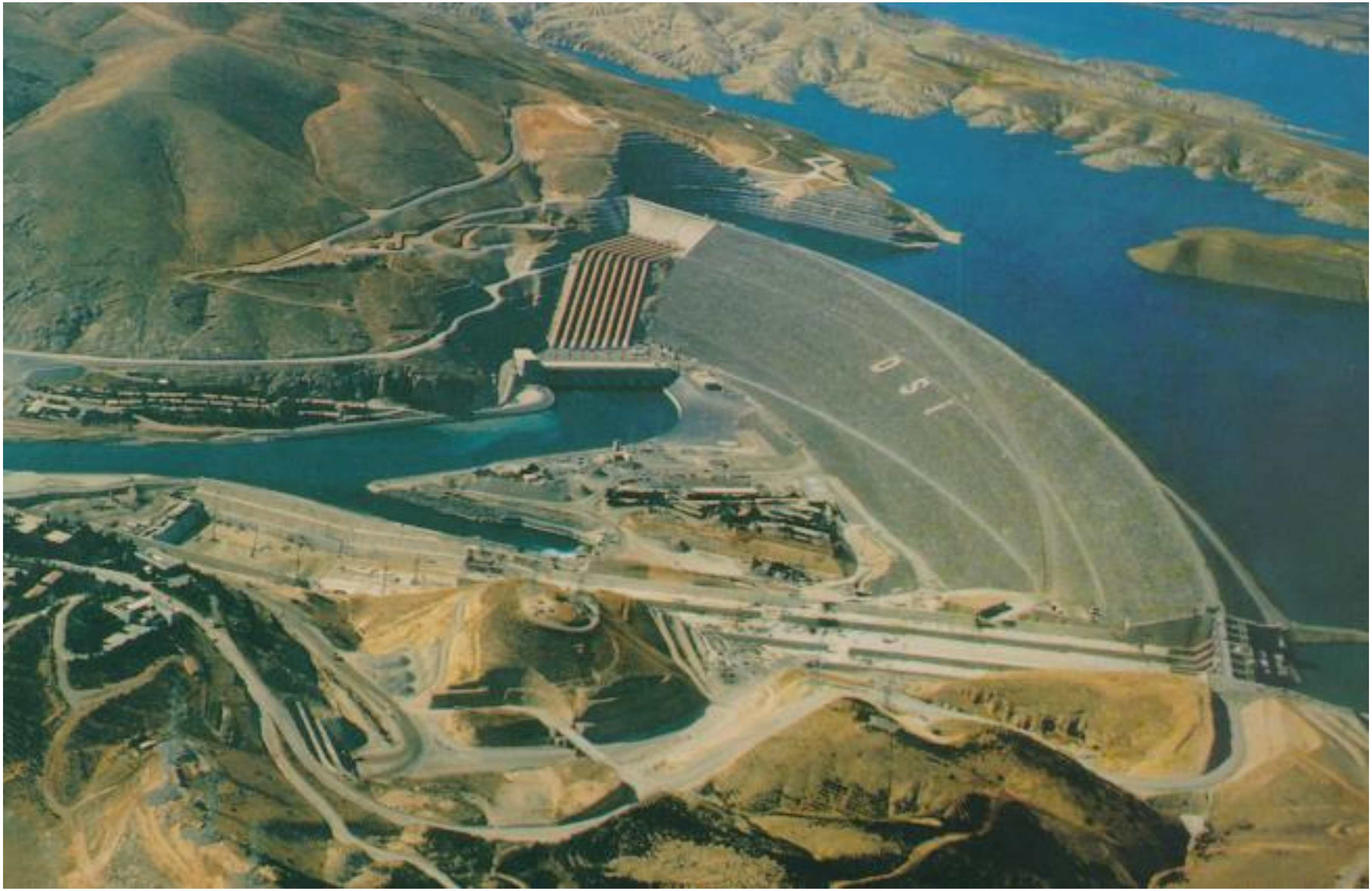
Urfa Tunnels from Atatürk Dam to Harran Plain

- **ϕ = 7.62 m,**
- **L = 2 x 26.4 km**
- **Q=80 m^3/s**

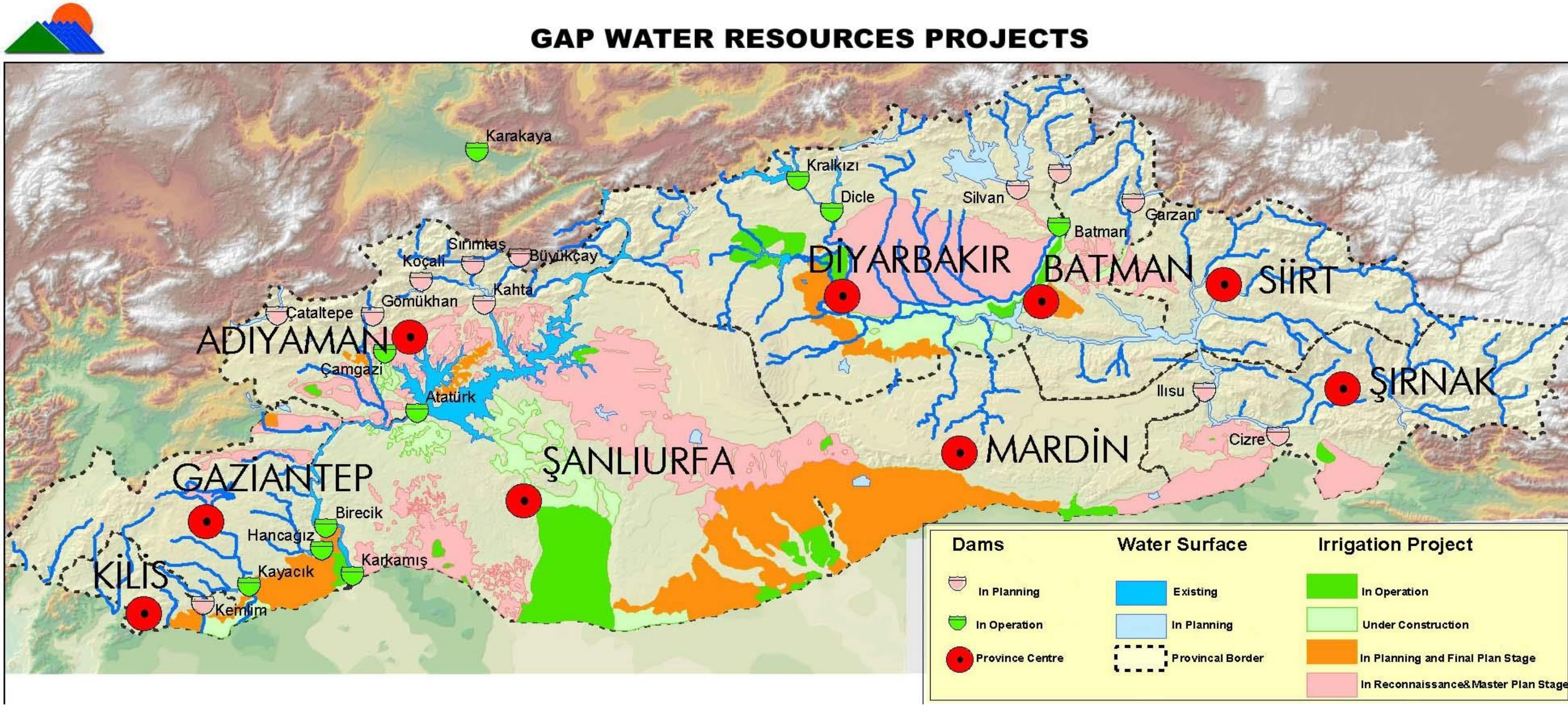
Main irrigation canal in Harran Plain (L=118 km, Q = 80 m3/s)



The View of Atatürk Dam



GAP WATER RESOURCES ROJECTS



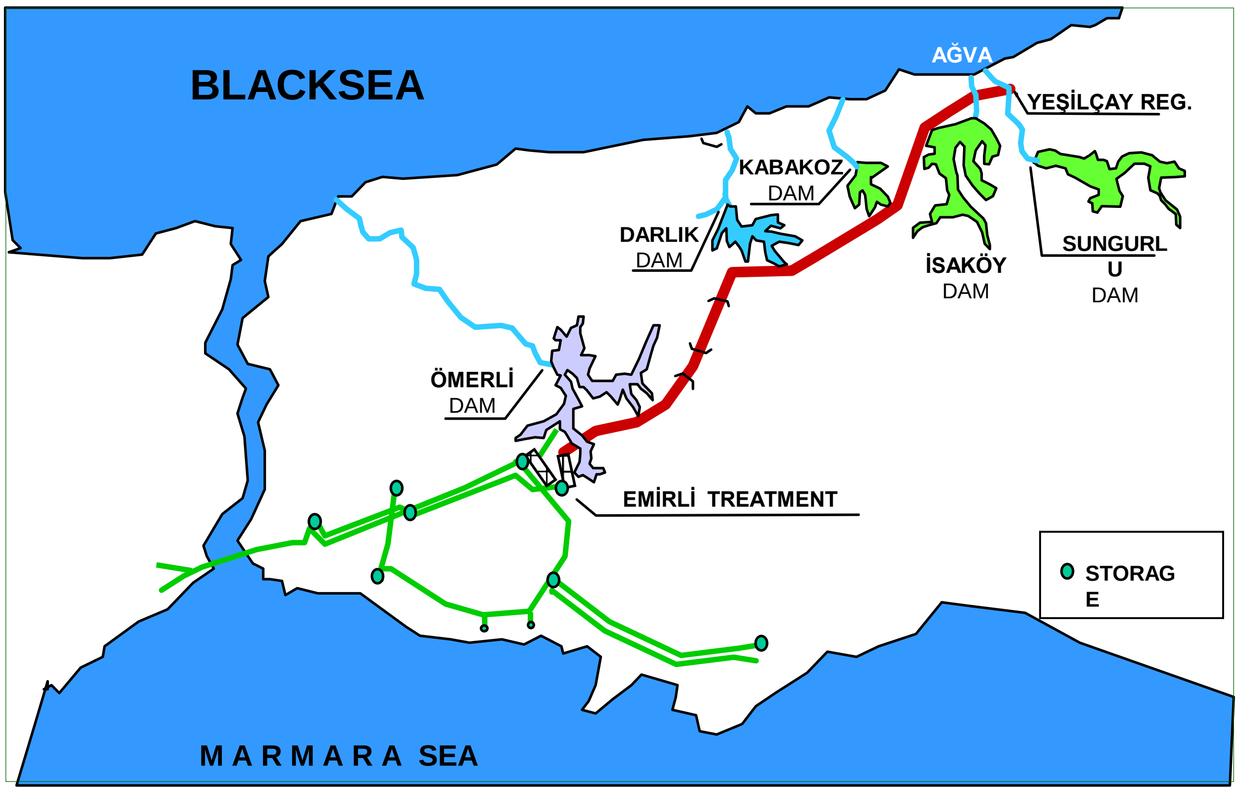
Total 22 dams, 19 HPP

1.7 million ha, 7485 MW, 27 billion kWh


Before 1995


HARRAN PLAIN

YEŞİLÇAY SYSTEM



YEŞİLÇAY SYSTEM CHARACTERISTICS

Length of transmission lines: 723 712 m

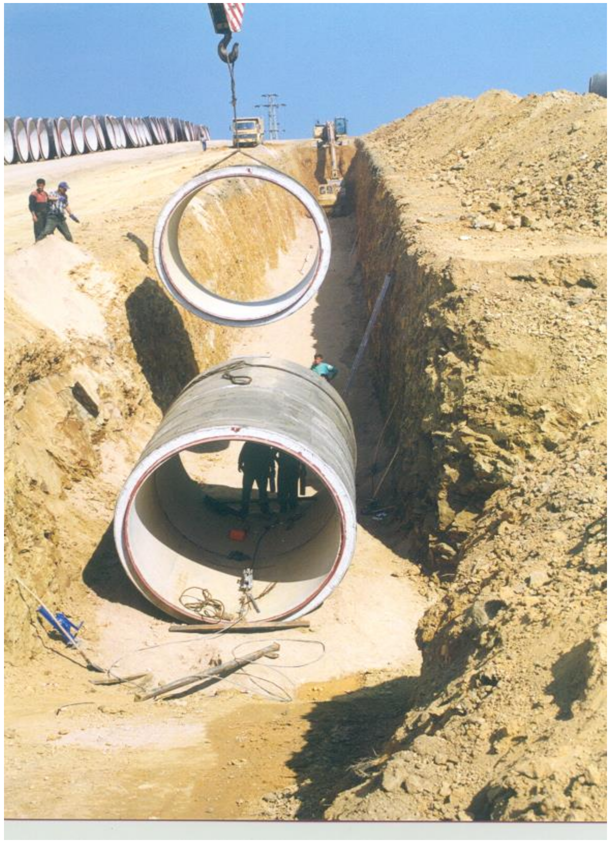
Length of water Network : 11 738 km

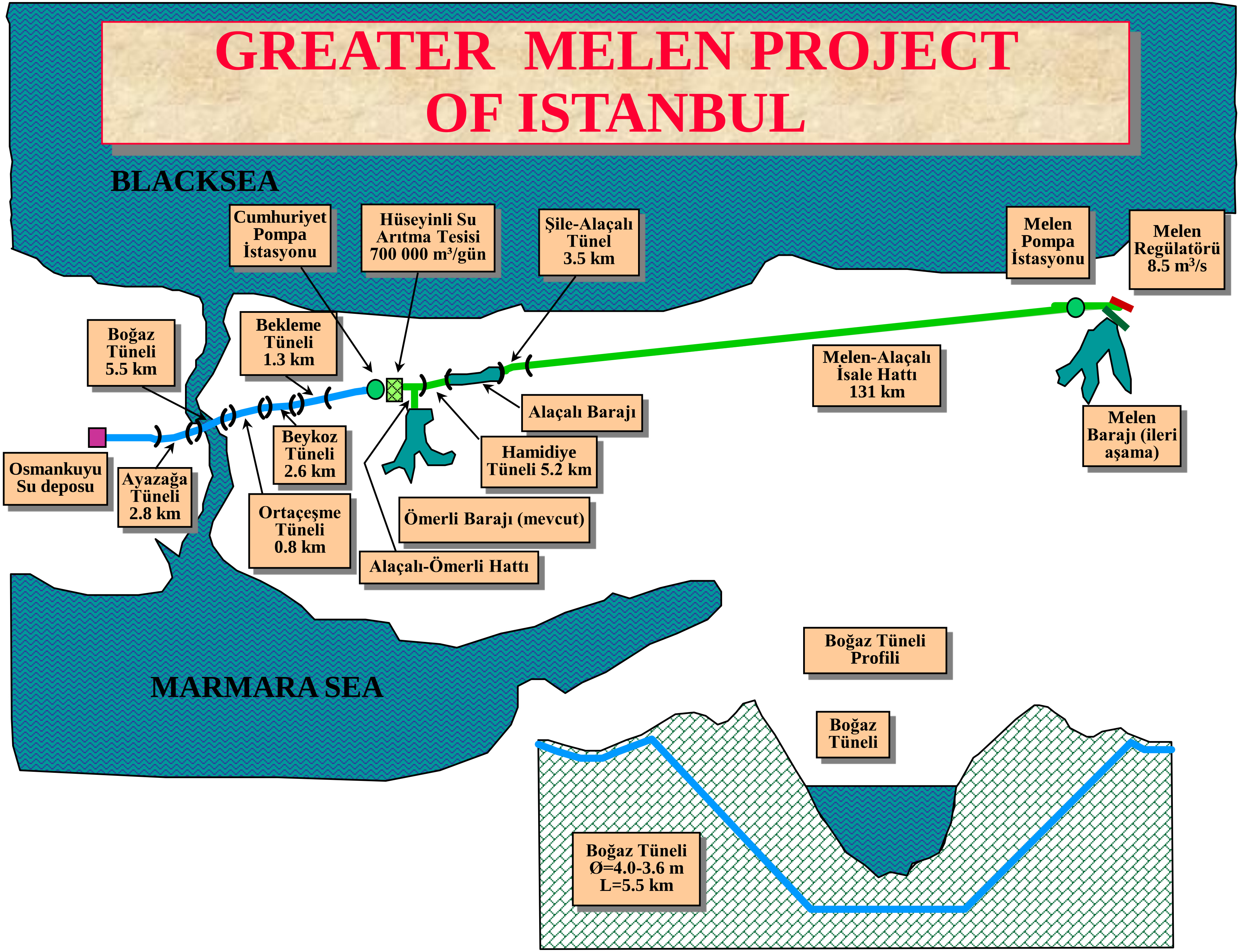
Volume of water reservoir : 914 000 m^3

Water Supplied (2003) : 920 million m^3/year

Water treatment capacity : 3.5 million m^3/day

Ø 3 000 mm Prestressed Concrete Cylinder Pipes



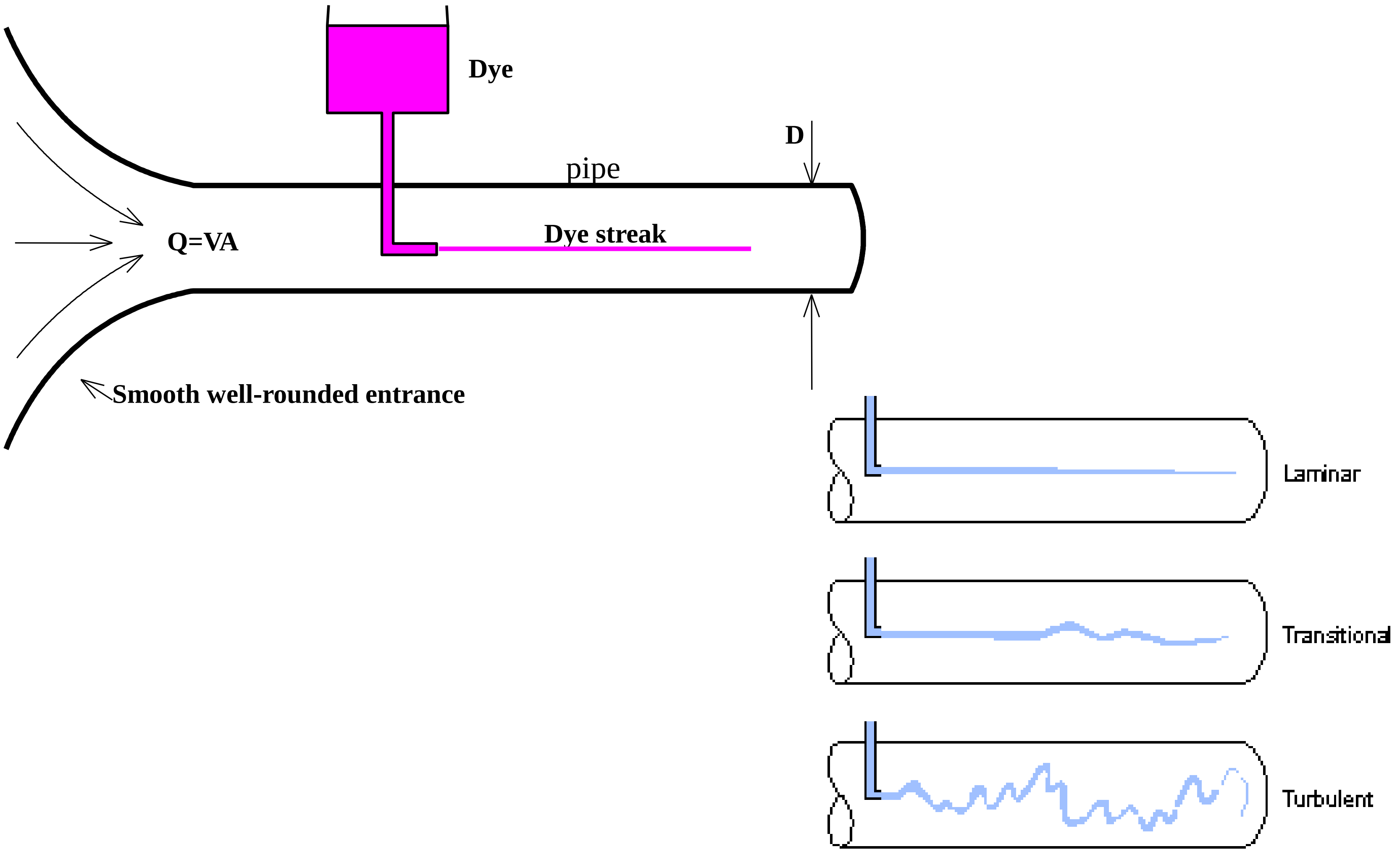

GREATER MELEN PROJECT OF ISTANBUL
BLACKSEA
Cumhuriyet Pompa İstasyonu
Hüseyinli Su Arıtma Tesisi 700 000 m³/gün
Şile-Alaçalı Tünel 3.5 km
Melen Pompa İstasyonu
Melen Regülatörü 8.5 m³/s
Boğaz Tüneli 5.5 km
Bekleme Tüneli 1.3 km
Melen-Alaçalı İsale Hattı 131 km
Alaçalı Barajı
Melen Barajı (ileri aşama)
Beykoz Tüneli 2.6 km
Osmankuyu Su deposu
Ayazağa Tüneli 2.8 km
Hamidiye Tüneli 5.2 km
Ortaçeşme Tüneli 0.8 km
Ömerli Barajı (mevcut)
Alaçalı-Ömerli Hattı
MARMARA SEA
Boğaz Tüneli Profili
Boğaz Tüneli
Boğaz Tüneli Ø=4.0-3.6 m L=5.5 km

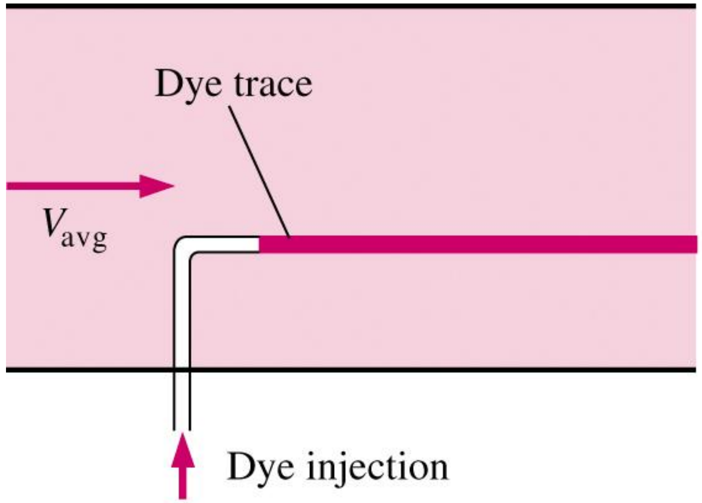
GREAT MELEN PROJECT
TECHNICAL SPECIFICATIONS

System Length : 185 600 m

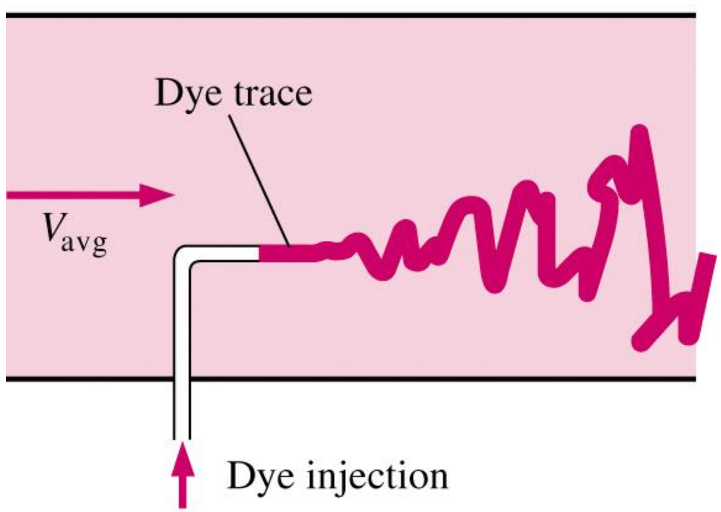
Ø 2 500 mm Steel Pipe	:	**163 950 m**
Ø 4 500 mm		
tunnel length	:	**8 700 m**
Ø 4 000 mm		
tunnel length	:	**11 550 m**
Ø 3 600 mm		
tunnel length	:	**1 400 m**

Reynolds Experiment



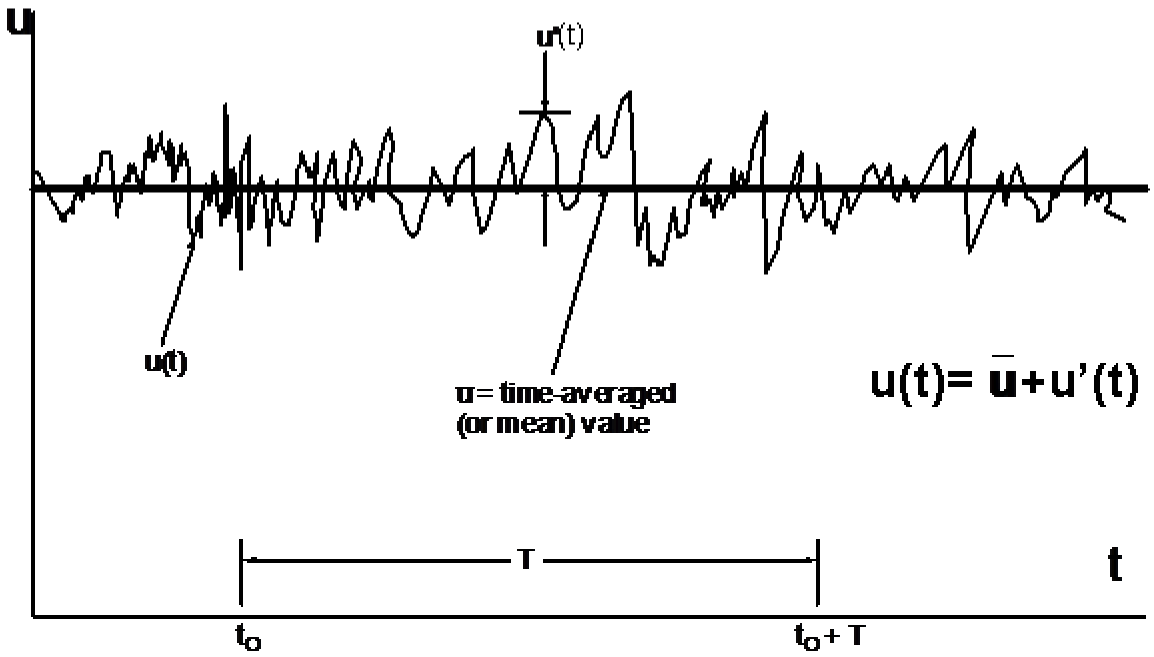


(*a*) Laminar flow



(*b*) Turbulent flow

Characteristics of Turbulent Flow



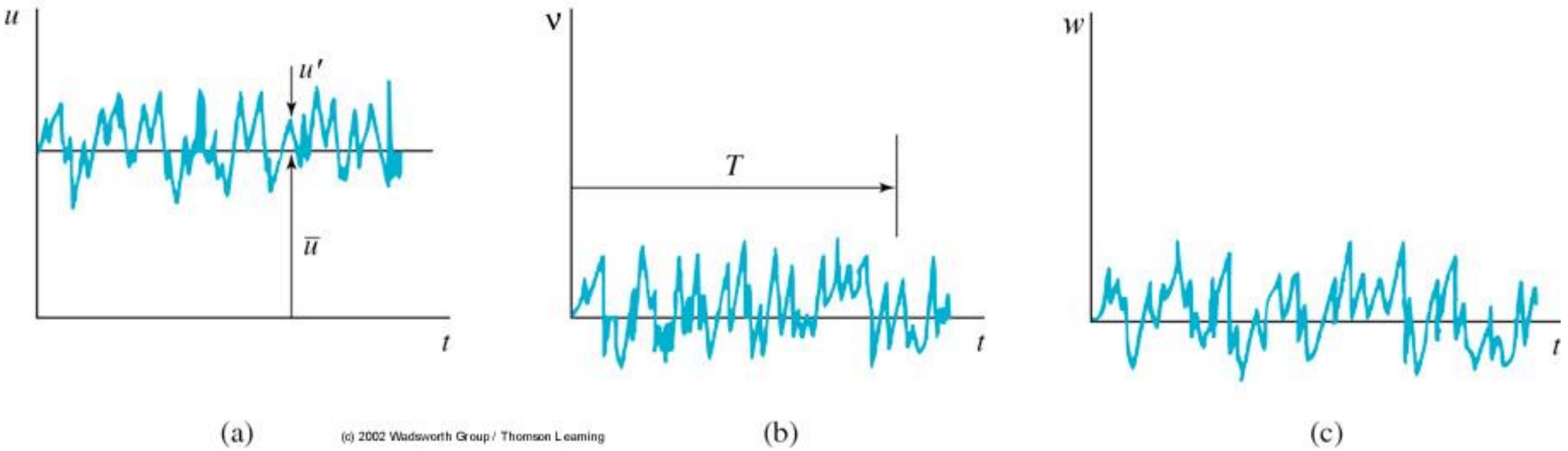
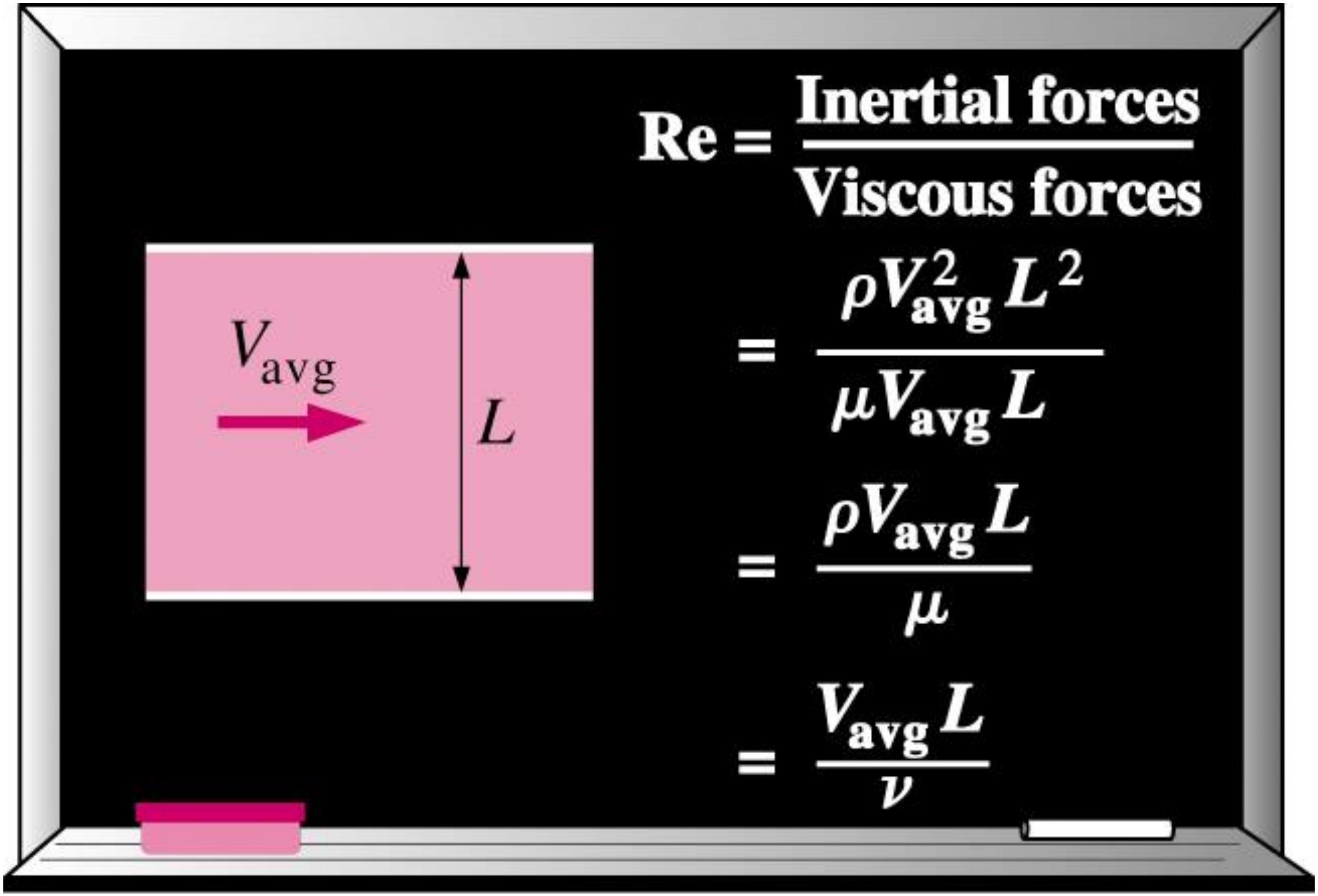


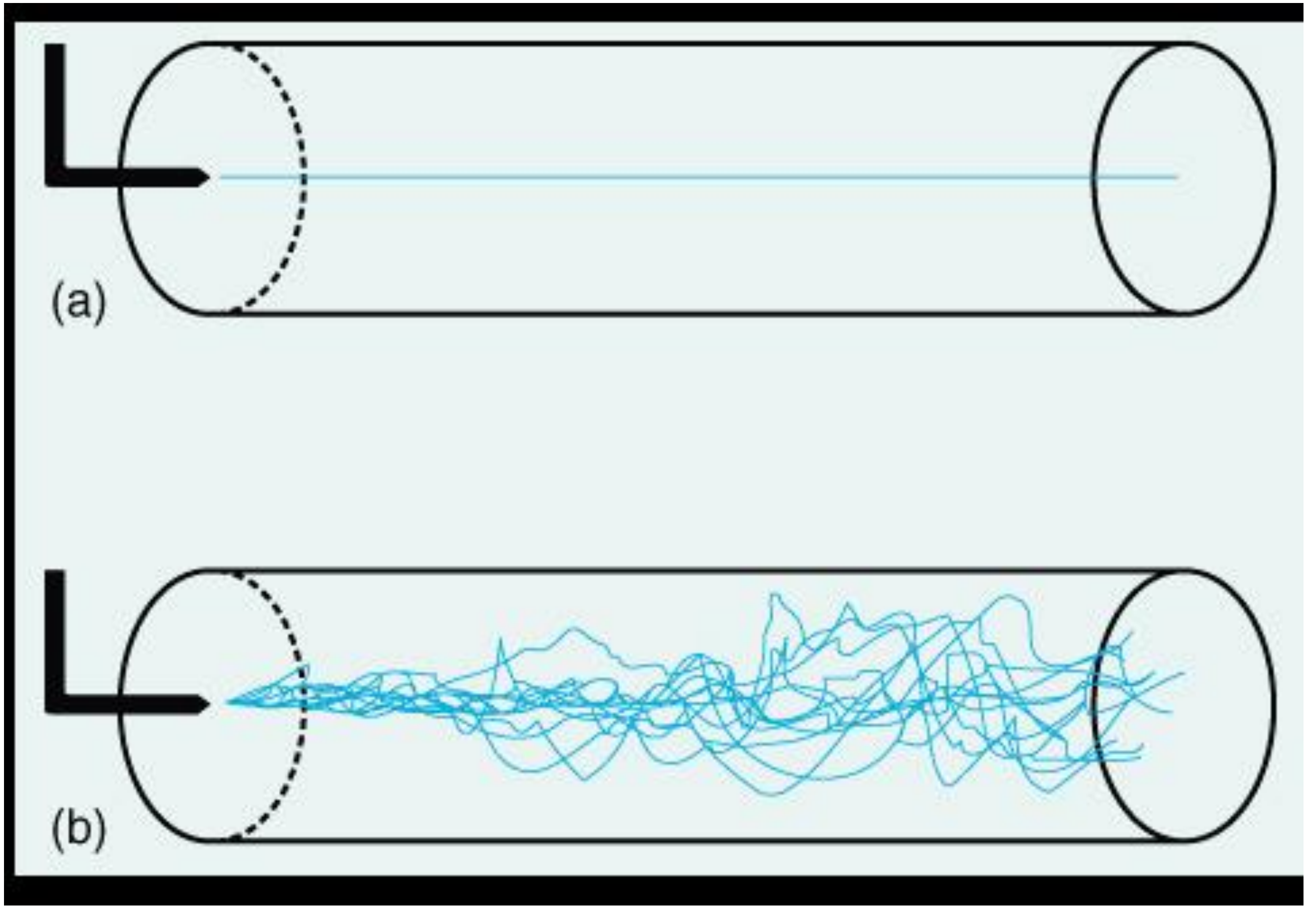
Figure 7.7 – Velocity components in a turbulent pipe flow: (a) *x*-component velocity; (b) *r*-component velocity; (c) θ-component velocity.

Type of Flow

- **Laminar flow: Re < 2000**
- **Transitional flow: 2000 < Re < 4000**
- **Turbulent flow: Re > 4000**


V_{avg}
L
$\mathrm{Re} = \dfrac{\textbf{Inertial forces}}{\textbf{Viscous forces}}$
$= \dfrac{\rho V_{avg}^2 L^2}{\mu V_{avg} L}$
$= \dfrac{\rho V_{avg} L}{\mu}$
$= \dfrac{V_{avg} L}{\nu}$

Laminar and Turbulent Flows

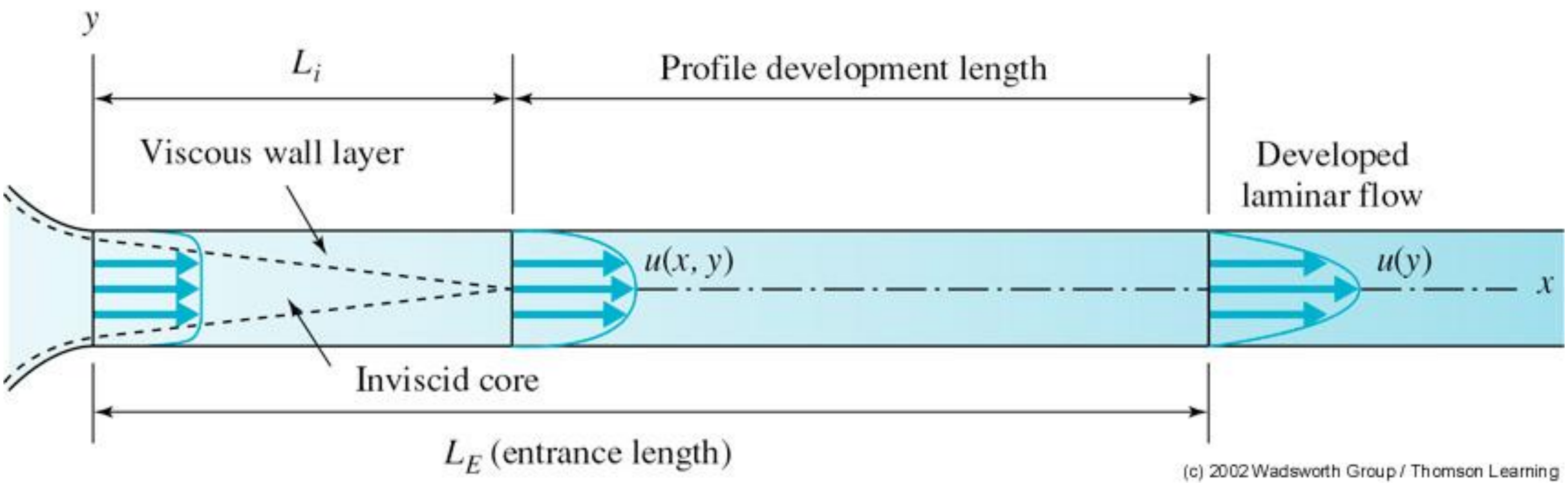


View of Turbulent and Laminar Flows

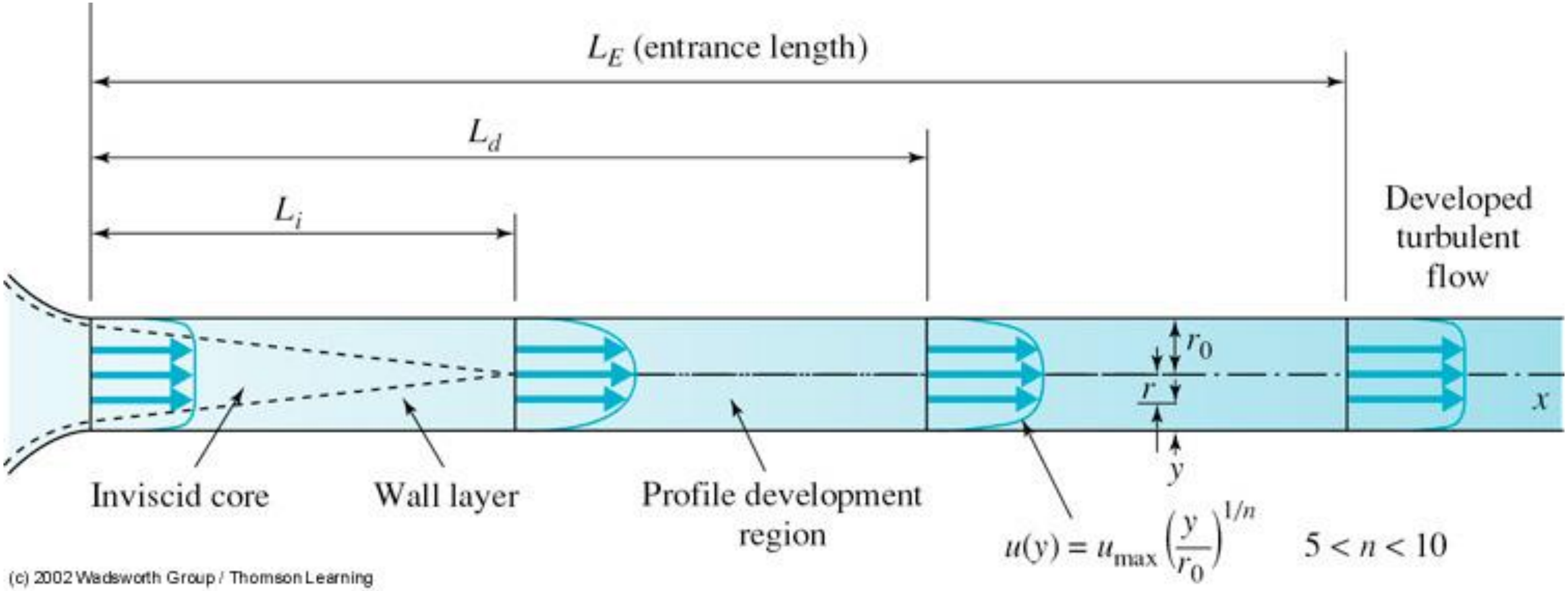
- Turbulent flow

Laminar flow

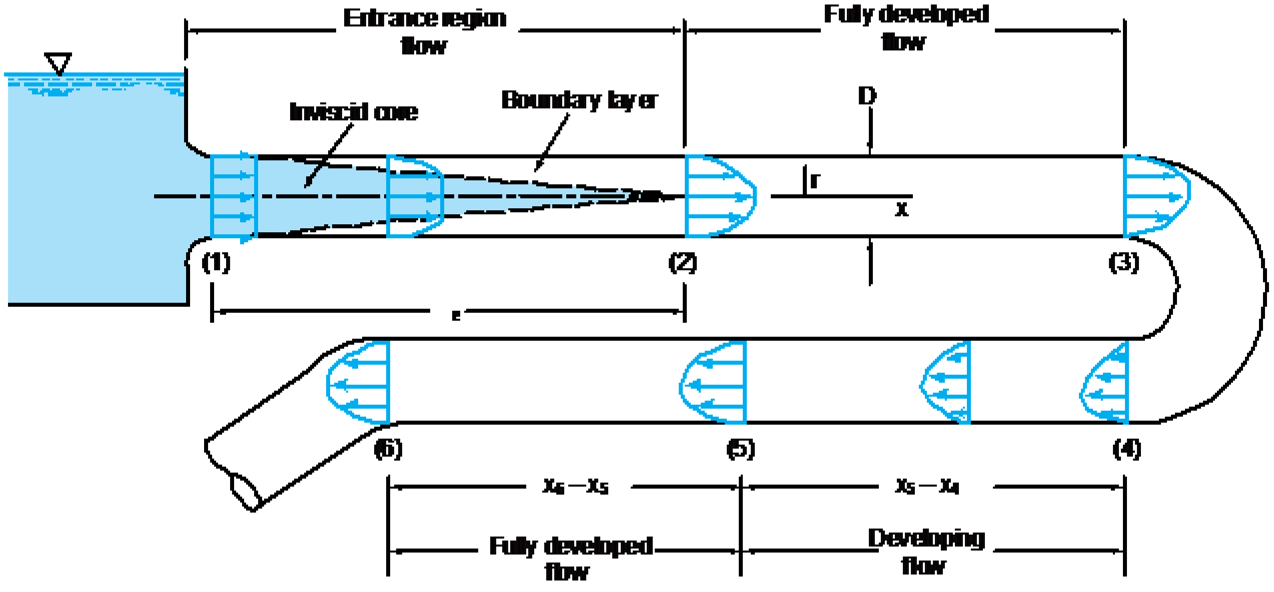
Entrance region of a laminar flow in a pipe or a wide rectangular channel.



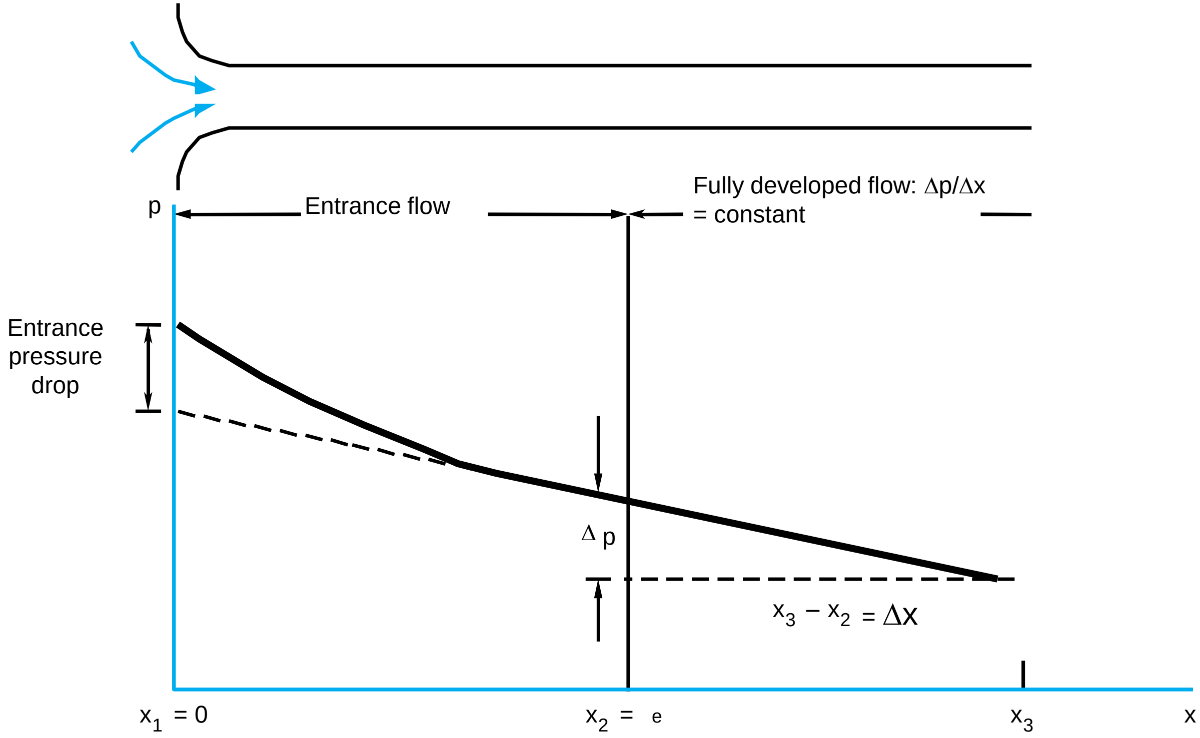
Velocity profile development in a turbulent pipe flow.

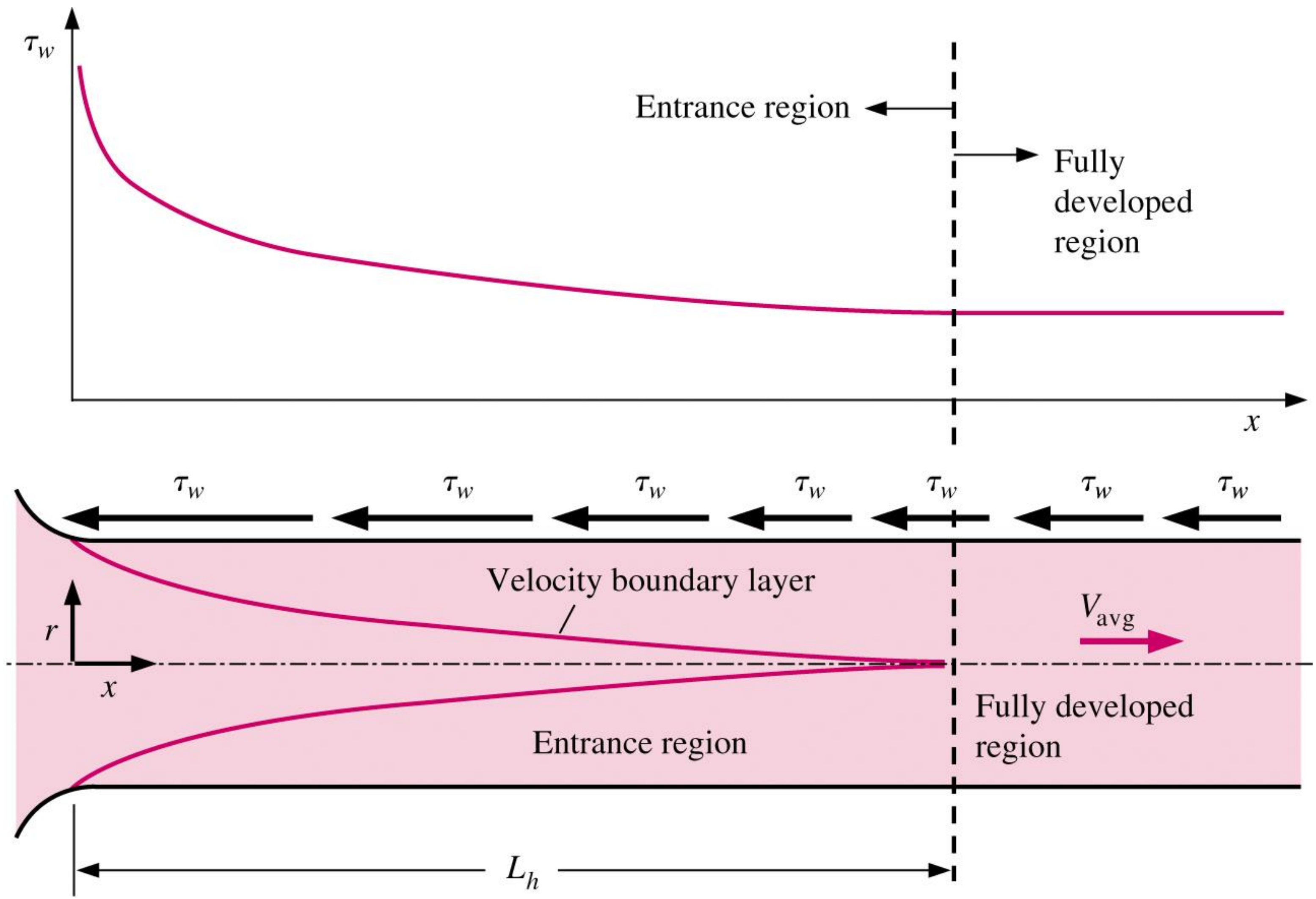


Fully Developed Flow



Entrance Region





τ_w
Entrance region
Fully
developed
region
x
τ_w
Velocity boundary layer
V_{avg}
r
x
Fully developed
region
Entrance region
L_h

Entrance Length

- For a laminar flow in a circular pipe with a uniform flow at the inlet, the entrance length is given by:

$$\frac{L_E}{D} = 0.065\,\mathrm{Re} \qquad \mathrm{Re} = \frac{VD}{\upsilon}$$

- For a turbulent flow, where Re>10^5

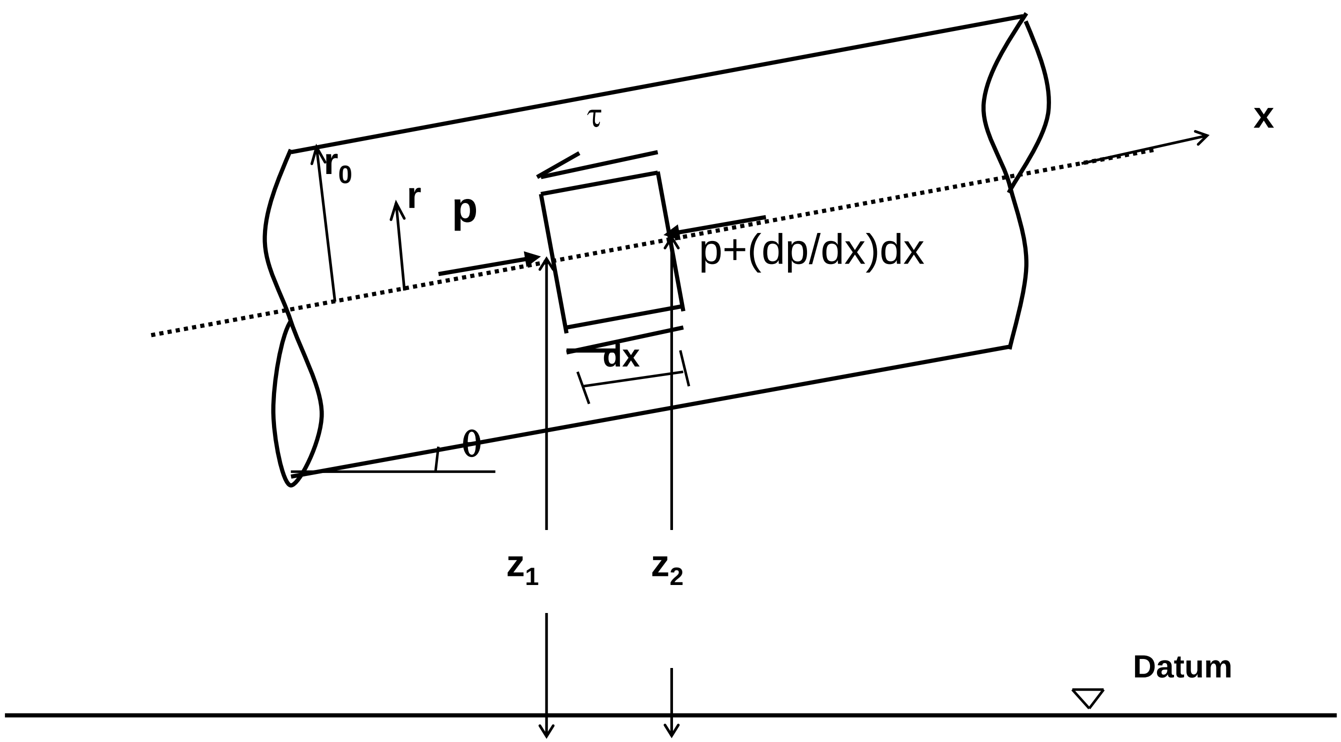
$$\frac{L_E}{D} = 120$$

Laminar Flow in Pipes

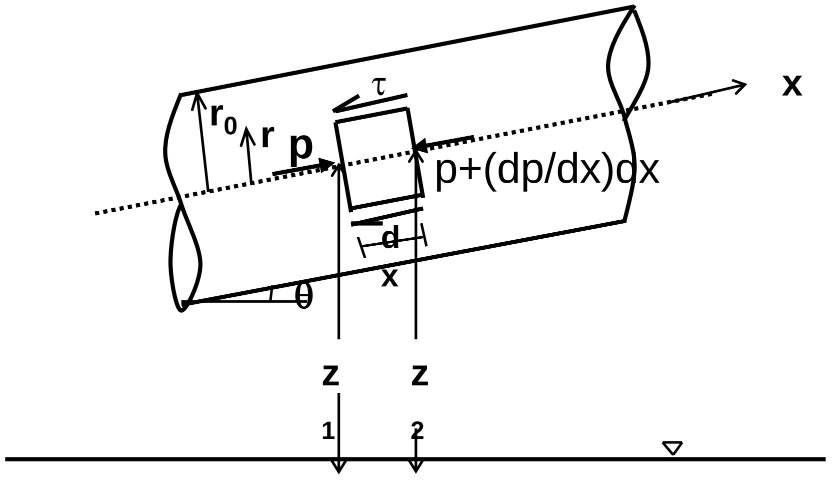
- Fluid is incompressible and Newtonian.
- Flow is steady, fully developed, parallel and, symmetric with respect to pipe axis.
- Pipe is straight pipe and has a constant diameter.

Laminar Flow in Pipes

- *Momentum Equation along x direction*



$$pA - \left(p + \frac{dp}{dx} dx \right) A - \gamma A dx \sin\theta - \tau 2\pi r dx = 0$$



$$-\frac{dp}{dx} dx A - \gamma dx \frac{dz}{dx} A - \tau 2\pi r dx = 0$$

(Divide both sides by $A = \pi r^2$)

$$-\frac{d(p + \gamma z)}{dx} = \frac{2\tau}{r}$$

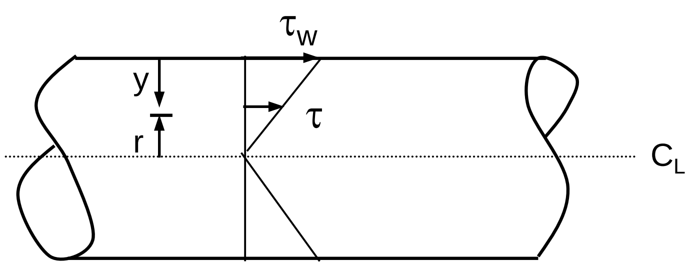
$$-\frac{dh}{dx} = -\frac{d(p + \gamma z)}{\gamma dx} = +\frac{2\tau}{\gamma r} \qquad \left(\text{since } h = \frac{p}{\gamma} + z \right)$$

Boundary Conditions

$$-\frac{dh}{dx} = -\frac{d(p+\gamma z)}{\gamma dx} = +\frac{2\tau}{\gamma r}$$

- when r = 0 , τ = 0
- r = r_o , $\tau = \tau_w$

$$-\frac{dh}{dx} = \frac{2\tau}{\gamma r} = \frac{2\tau_w}{\gamma r_o}$$



equations are equally applicable to both laminar and turbulent flow in pipes

Laminar Flow

$$\tau = +\mu \frac{du}{dy} = -\mu \frac{du}{dr} \qquad (1)$$

$$\tau = -\frac{d(p + \gamma z)}{dx} \frac{r}{2} \qquad (2)$$

$$\frac{du}{dr} = +\frac{d(p + \gamma z)}{dx} \frac{r}{2\mu}$$

Boundary Conditions

$$\frac{du}{dr} = +\frac{d(p+\gamma z)}{dx}\frac{r}{2\mu}$$

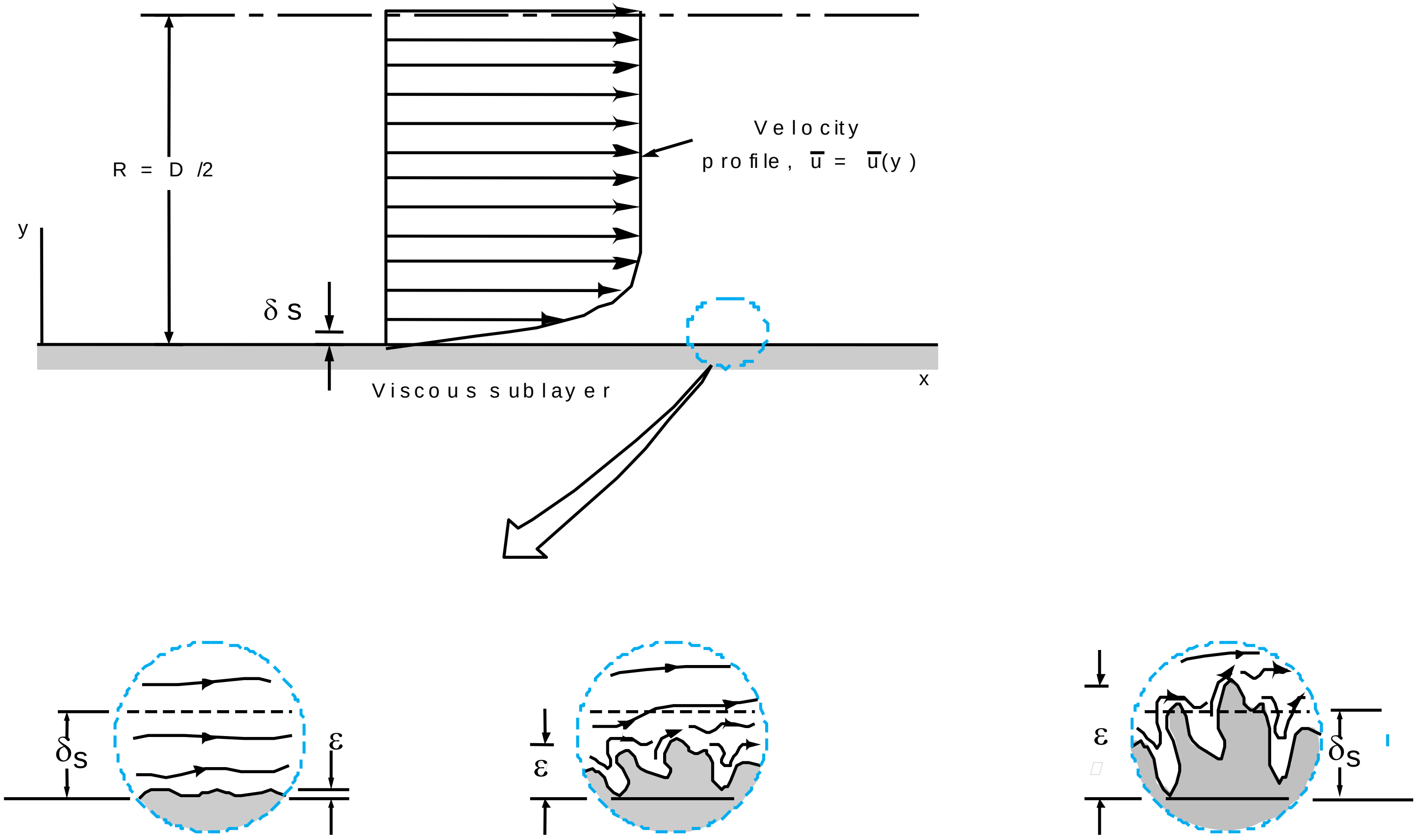
$r = 0$, $u = u_{max}$

$r = r_o$; $u = 0$

$u = u(r)$ may be solved by integration

- **Velocity:** $$u = u_{max}\left[1-\left(\frac{r}{r_o}\right)^2\right] = -\frac{d(p+\gamma z)}{dx}\frac{r_o^2}{4\mu}\left[1-\left(\frac{r}{r_o}\right)^2\right]$$
- **Average velocity:** $$V = \frac{Q}{A} = \frac{\int u da}{A} = \frac{u_{max}}{2} = -\frac{d(p+\gamma z)}{dx}\frac{r_o^2}{8\mu}$$
- **Maximum velocity:** $$u_{max} = -\frac{d(p+\gamma z)}{dx}\frac{r_o^2}{4\mu}$$
- **Wall shear stress:** $$\tau_w = \frac{4\mu V}{r_o}$$
- **Shear stress:** $$\tau = -\mu\frac{du}{dr} = \tau_w \frac{r}{r_o}$$
- **Flow rate:** $$Q = V\ A = -\frac{\pi r_o^4}{8\mu}\frac{d(p+\gamma z)}{dx}$$
- **Head loss:** $$\frac{h_f}{L} = -\frac{dh}{dx} = -\frac{d(p+\gamma z)}{\gamma dx}$$

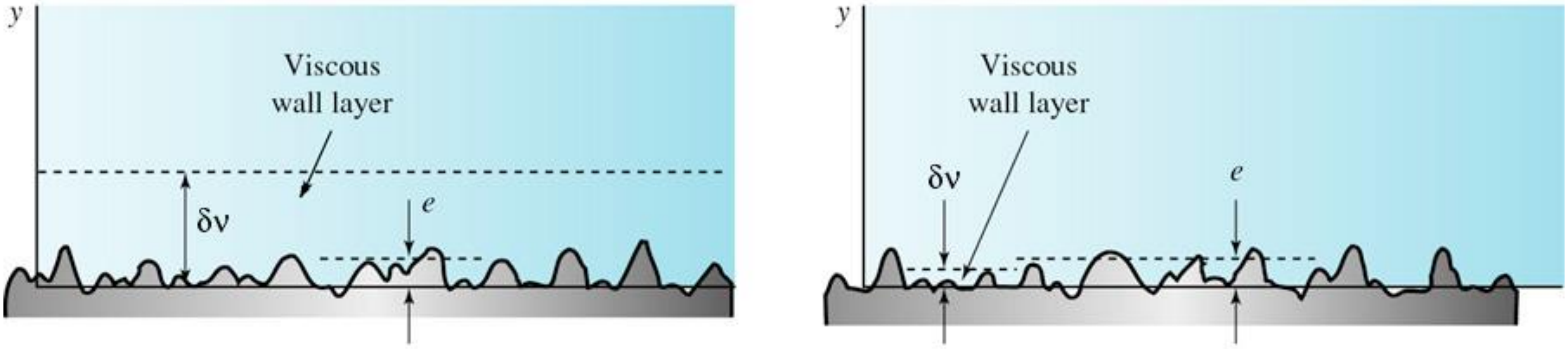
Turbulent Flow : Re ≥ 4000



(a) Smooth wall

(b) Transitional flow

(c) Rough wall



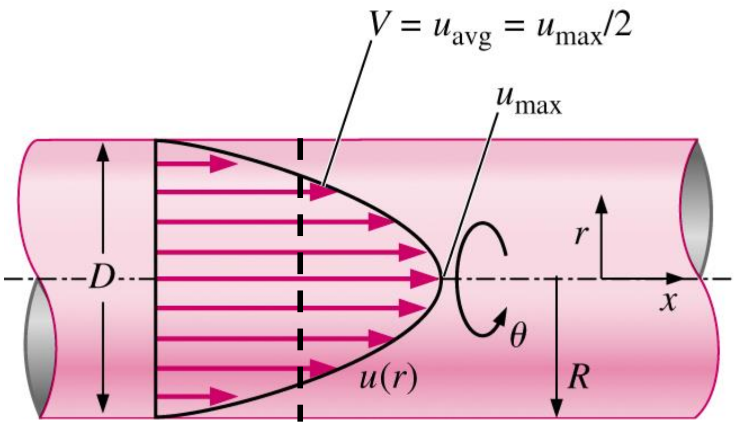
(a) (b)

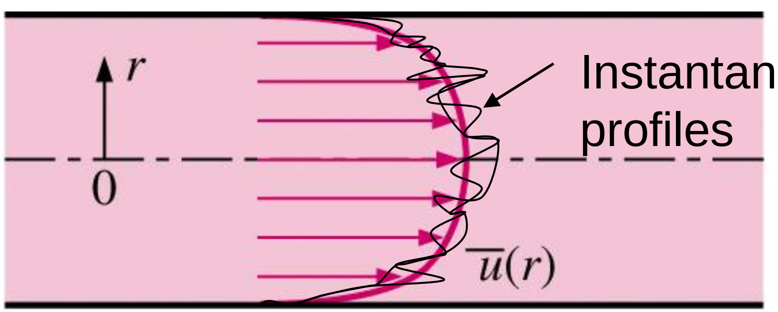
A smooth wall and (b) a rough wall.

Comparison of laminar and turbulent flow

Laminar

Can solve exactly
Velocity profile is parabolic
Pipe roughness does not affect the flow

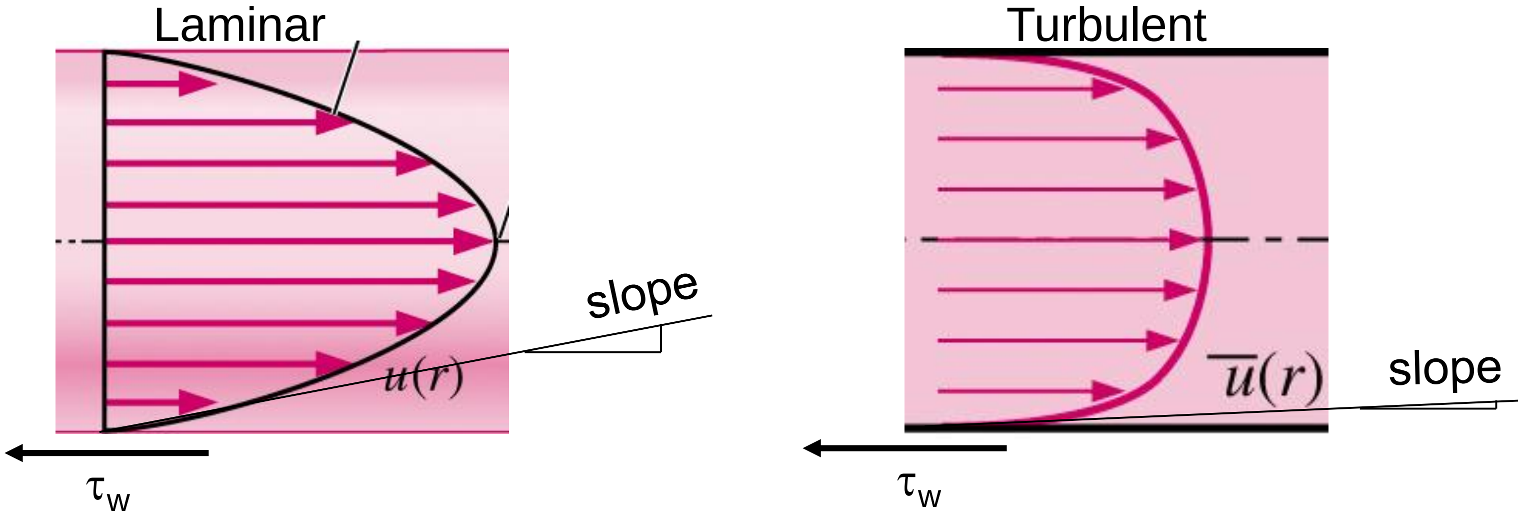




Turbulent

- *Cannot* solve exactly (too complex)
- Flow is unsteady , but it is steady in the mean
- Mean velocity profile is fuller
- Pipe roughness is very important
- V_{avg} 85% of U_{max} (and depends on Re)
- No analytical solution, but there are some good semi-empirical expressions that approximate the velocity profile shape.

Laminar and Turbulent



$$\tau_{w,turb} > \tau_{w,lam}$$

Total Head Loss, h_ℓ

- ***Total Head Loss,*** $h_\ell = h_f + h_m$
- h_f – **Frictional loss (Viscous, Major)**
- h_m – **Local loss (Minor)**

Determination of Frictional Loss (h_f):

- Darcy - Weisbach Equation

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D^5}\frac{16}{\pi^2}\frac{Q^2}{2g} = KQ^2 \quad \text{where} \quad K = \frac{8fL}{g\pi^2 D^5}$$

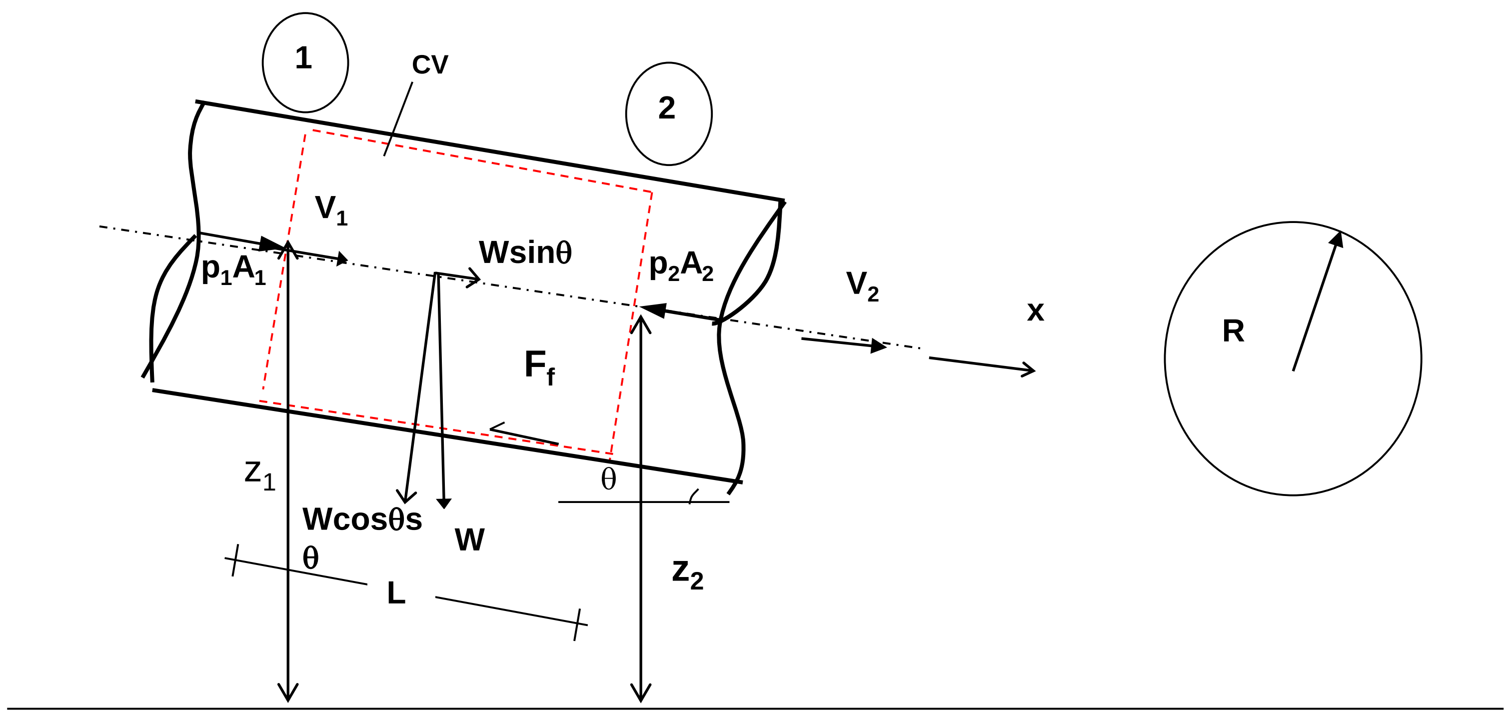
- Hazen-Williams Equation

$$h_f = \frac{6.8}{C^{1.85}}\frac{L}{D^{1.165}}V^{1.85} = \frac{10.6}{C^{1.85}}\frac{L}{D^{4.87}}Q^{1.85} = KQ^{1.85}$$

- Where
- D=diameter of pipe
- V= average velocity
- g=acceleration of gravity
- Q=discharge
- L=length of pipe over which head loss occurs
- f=friction factor
- C=Hazen-William roughness coefficient

Derivation of Darcy Weisbach Equation

- Consider a steady fully developed flow in a prismatic pipe, that is A = constant along the flow direction



The momentum equation along the flow direction gives:

$$p_1 A_1 - p_2 A_2 + W\sin\theta - F_f = \rho Q(\beta_2 V_2 - \beta_1 V_1)$$

$W\sin\theta = \gamma AL\sin\theta = \gamma A(z_1 - z_2)$ $\quad F_f = \tau_w PL$ $\quad Q = V_1 A_2 = V_2 A_2 = \text{Constant}$

$$P_1 A_1 - P_2 A_2 + \gamma A(z_1 - z_2) - \tau_w PL = 0 \qquad \text{dividing by } \gamma A$$

$$z_1 + \frac{p_1}{\gamma} - z_2 - \frac{p_2}{\gamma} = \frac{\tau_w LP}{\gamma A} = \frac{\tau_w L}{\gamma R_H}$$

$R_h = A/P$

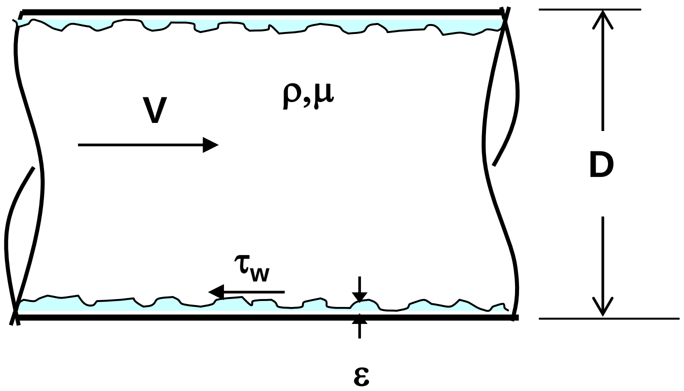
$R_h = D/4$

$$z_1 + \frac{p_1}{\gamma} - z_2 - \frac{p_2}{\gamma} = h_f$$

$$h_f = \frac{\tau_w L}{\gamma R_H} = \frac{2\tau_w L}{\gamma R} = \frac{4\tau_w L}{\gamma D}$$

this equation is applicable to both laminar and turbulent flows.

Relation between wall shear stress and average velocity



τ_w= F(V, D, ρ, μ, ε)
k = 6 parameters
r = 3 basic dimensions,
n = k-r = 6-3 =3π terms

$$\pi_1 = \frac{\tau_w}{\rho V^2} = f' \Rightarrow \left[\frac{\text{shear stress}}{\text{dynamic pressure}}\right]$$

$$\pi_2 = \frac{\rho V D}{\mu} = \frac{VD}{\nu} = R_e, \text{ Reynolds Number}$$

$$\pi_3 = \frac{\varepsilon}{D}, \text{ Relative Roughness}$$

$$\therefore \pi_1 = \phi(\pi_2, \pi_3)$$

$$\frac{\tau_w}{\rho V^2} = \phi\left(\frac{\rho V D}{\mu}, \frac{\varepsilon}{D}\right) = f'$$

$$\tau_w = f' \rho V^2$$

Relation between wall shear stress and head loss

$$f' = \frac{\tau_w}{\rho V^2} = \text{func.}(R_e, \varepsilon / D)$$

$$\tau_w = f'\rho V^2$$

$$h_f = \frac{4\tau_w L}{\gamma D} = \frac{4f'\rho V^2 L}{\gamma D}$$

let $8f' = f$ and $g = \gamma/\rho$ then

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

$f \rightarrow$ Darcy-Weisbach friction factor

$$\left[f = \text{funct}\left(\frac{\rho V D}{\mu}, \frac{\varepsilon}{D} \right) = f\left(R_e, \frac{\varepsilon}{D} \right) \right]$$

Darcy-Weisbach Equation

$$h_f = \frac{4\tau_w L}{\gamma D}$$

- **Relation between wall shear stress and head loss**

$$\tau_w = f'\rho V^2$$

- **Relation between wall shear stress and head loss**

$$\tau_w = f'\rho V^2$$

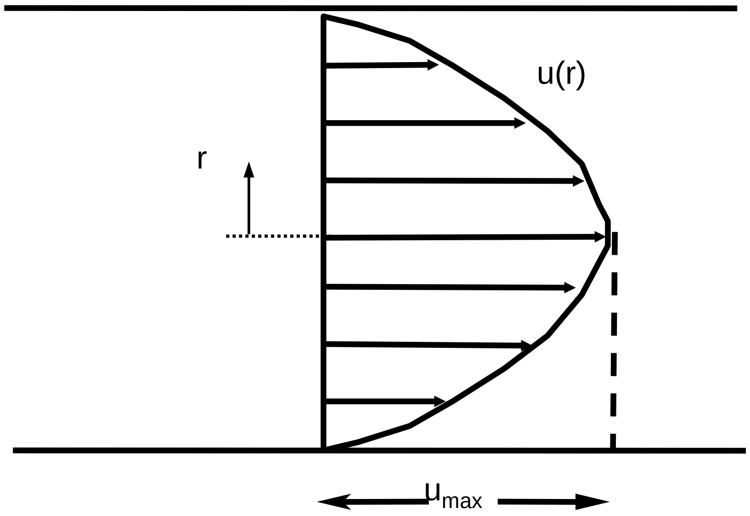
$$h_f = \frac{4\tau_w L}{\gamma D} = \frac{4f'\rho V^2 L}{\gamma D}$$

$$8f' = f \qquad g = \gamma/\rho \text{ then}$$

$$h_f = f\,\frac{L}{D}\,\frac{V^2}{2g}$$

Darcy – Weisbach Friction Factor

Laminar Flow: Re ≤ 2000



$$h_f = f\frac{LV^2}{D2g}$$

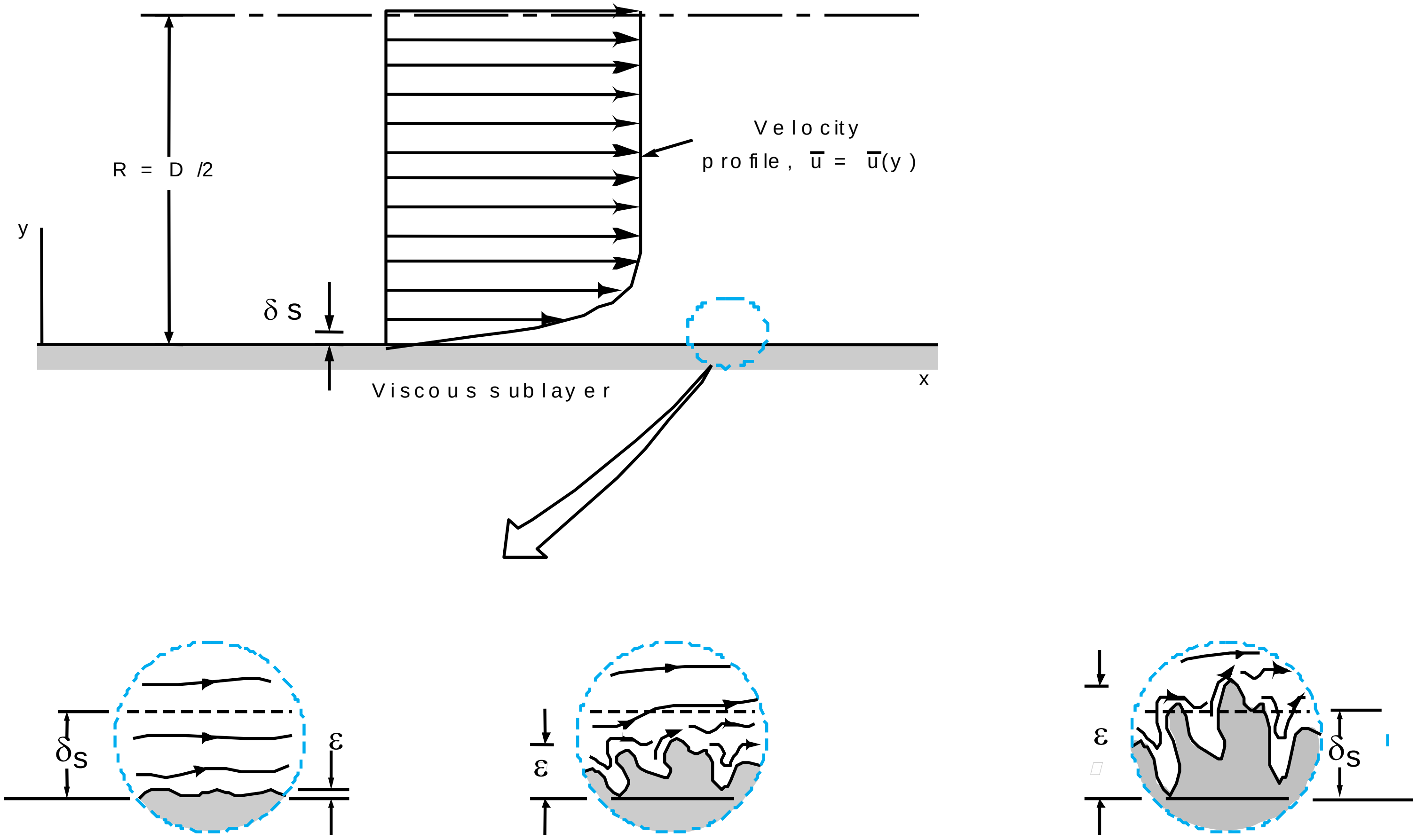
$$h_f = \frac{4\tau_w L}{\gamma D} \quad \text{and}$$

$$\tau_w = \frac{4\mu V}{r_0} = \frac{8\mu V}{D} \quad \text{for Laminar flow}$$

$$h_f = \frac{4\,\dfrac{8\mu V}{D}L}{\gamma D} = \frac{32\mu VL}{\rho g D^2} = f\,\frac{L}{D}\frac{V^2}{2g} \text{ solving for f} \longrightarrow f = \frac{64}{R_e}$$

- In turbulent flow, if the pipe is smooth, the $\varepsilon = 0$. However, if the pipe is rough, then although $\varepsilon \neq 0$, the pipe might act like a smooth pipe:
- If the roughness height ε is smaller than the viscous sublayer thickness, δ_s, then pipe is hydraulically smooth pipe.
- But if the roughness height ε is much greater than the viscous sublayer thickness, δ_s, then pipe is fully rough pipe.

Turbulent Flow : Re ≥ 4000



(a) Smooth wall

(b) Transitional flow

(c) Rough wall

Friction factor for Turbulent Flows

In general $f=f(R_e, \varepsilon/D)$

For Turbulent flow, there are 3 cases:

1. Smooth pipe, $\varepsilon=0$, Therefore $f=f(R_e)$ only
2. Rough pipe, and for rough pipe there are 3 cases depending on the magnitude of roughness:

Friction Factor

1. For hydraulically smooth pipe,
   $f=f(R_e)$ only
2. For frictionally transition zone:
   $f=f(R_e, \varepsilon/D)$
3. For fully rough pipe:

   $f=f(\varepsilon/D)$ only.

Formula for friction factors in Turbulent flows

Smooth Pipe and Hydraulically Smooth Flow

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{2.51}{R_e\sqrt{f}}\right)$$

$$f = \mathrm{func}(R_e)$$

Colebrook - White – Transitional Flow

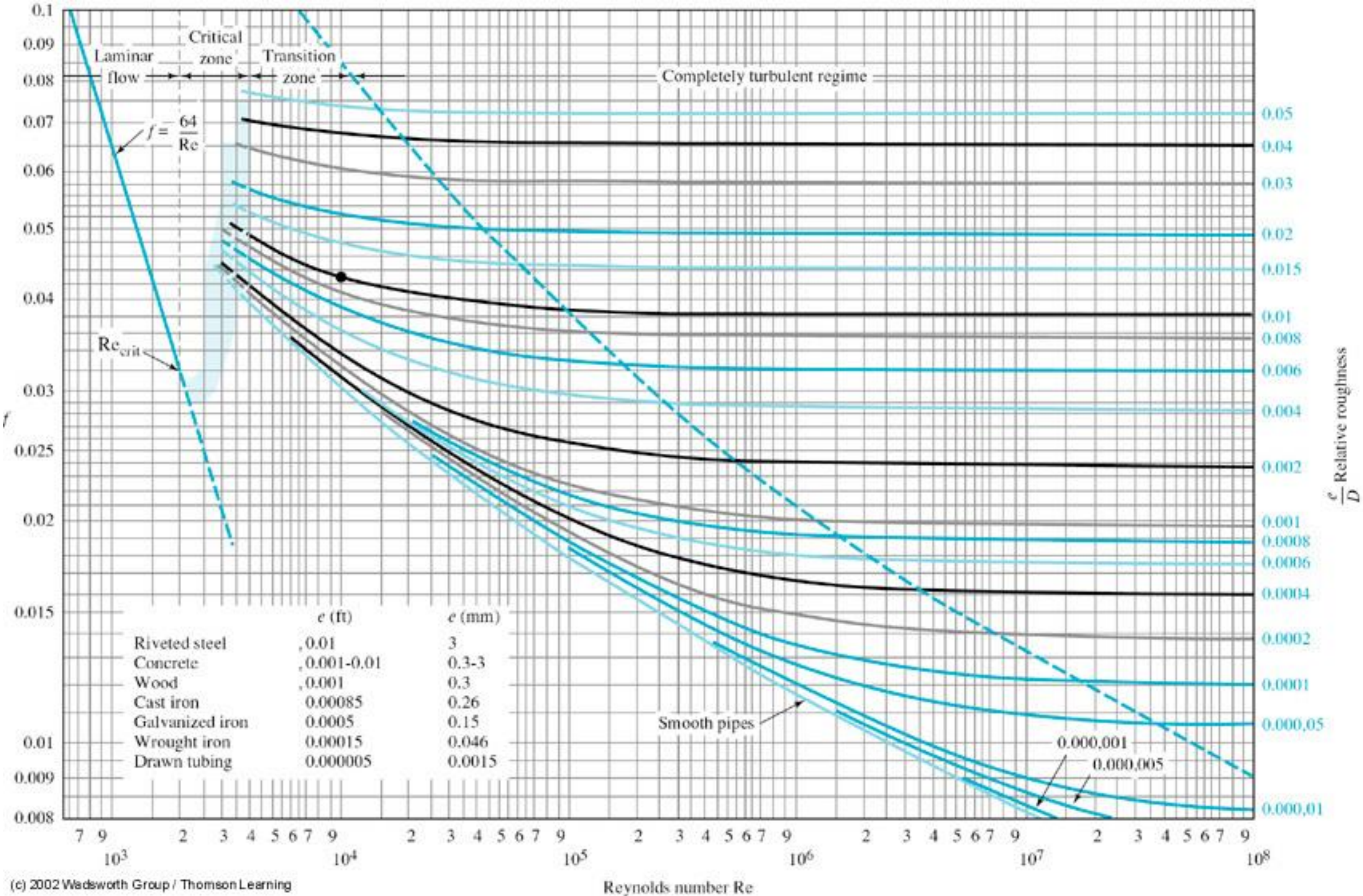
$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{2.51}{R_e\sqrt{f}} + \frac{\varepsilon}{3.7D}\right)$$

$$f = \mathrm{func}\left(R_e, \frac{\varepsilon}{D}\right)$$

Rough Pipe- Hydraulically Rough Flow

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon}{3.7D}\right)$$

$$f = \mathrm{func}\left(\frac{\varepsilon}{D}\right)$$



Moody diagram. (From L.F. Moody, *Trans*. *ASME*, Vol.66,1944.) (Note: If *e/D* = 0.006 and *Re* = 10^4, the dot locates *f* = 0.043.)

Swamee – Jain Formula *(Explicit)*

$$f = \frac{1.325}{\left[\ln\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{R_e^{0.9}}\right)\right]^2}$$

for the range of $10^{-6} < \varepsilon / D < 10^{-2}$ and $5000 < R_e < 10^8$

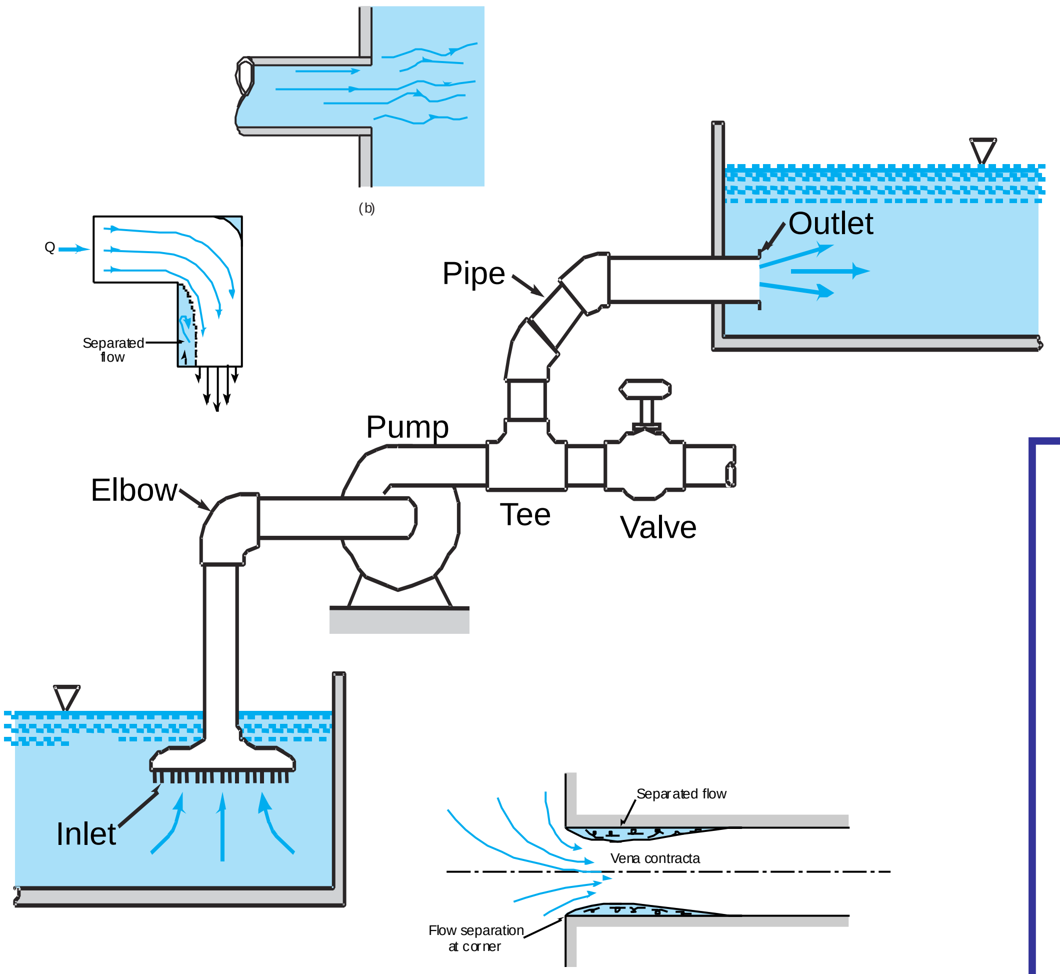
Roughness Coefficients

Material	Hazen-Williams C	Manning's Coefficient n	Darcy-Weisbach Roughness Height ε (mm)
Asbestos cement	140	0.011	0.0015
Brass	135	0.011	0.0015
Brick	100	0.015	0.6
Cast-iron, new	130	0.012	0.26
Concrete:	140	0.011	0.18
Steel forms	120	0.015	0.6
Wooden forms	135	0.013	0.36
Centrifugally spun	135	0.011	0.0015
Copper	---	0.022	45
Corrugated metal	120	0.016	0.15
Galvanized iron	140	0.011	0.0015
Glass	135	0.011	0.0015
Lead	150	0.009	0.0015
Plastic	148	0.010	0.0048
Steel:	145	0.011	0.045
Coal-tar enamel	110	0.019	0.9
New unlined	120	0.012	0.18
Riverted			
Wood stave			

$$h_f = \frac{6.8}{C^{1.85}} \frac{L}{D^{1.165}} V^{1.85}$$

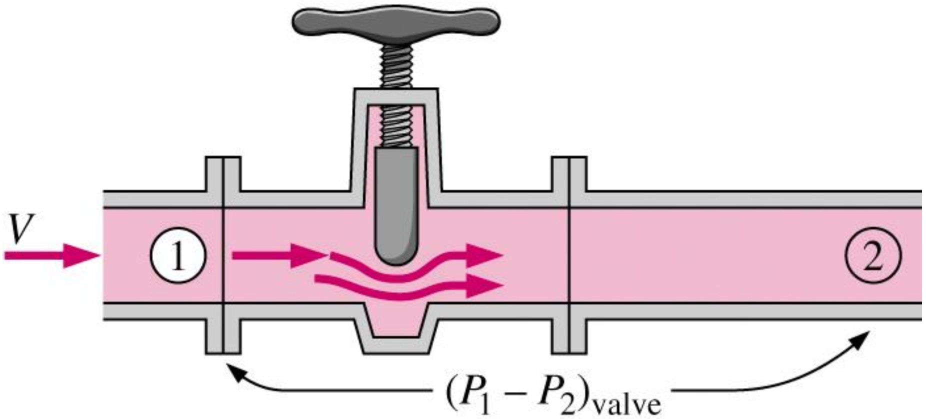
$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

Minor Losses

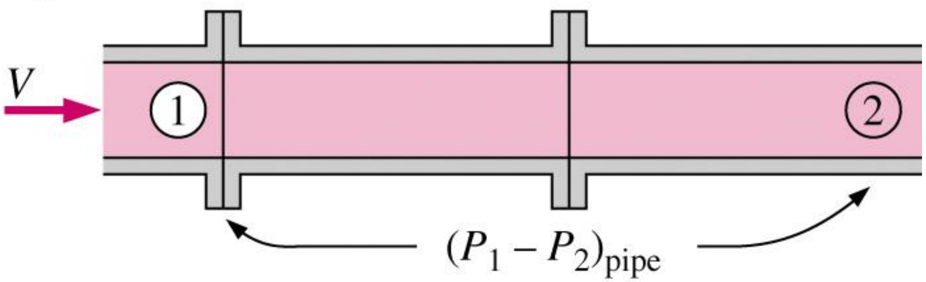


- Pipe entrance or exit
- Sudden expansion or contraction
- Bends, elbows, tees, and other fittings
- Valves, open or partially closed
- Gradual expansions or contractions

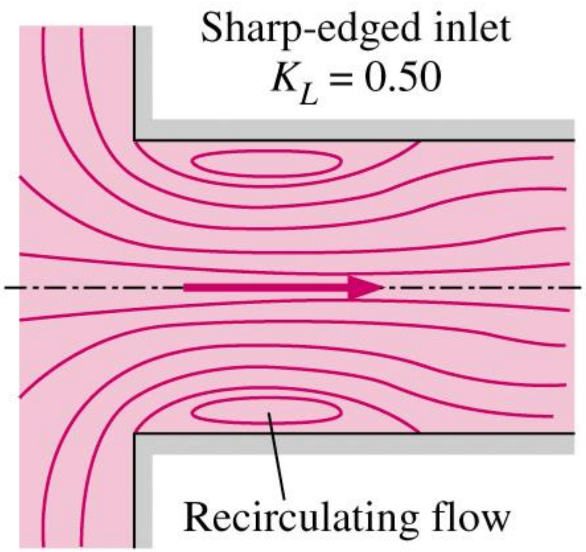
Pipe section with valve:

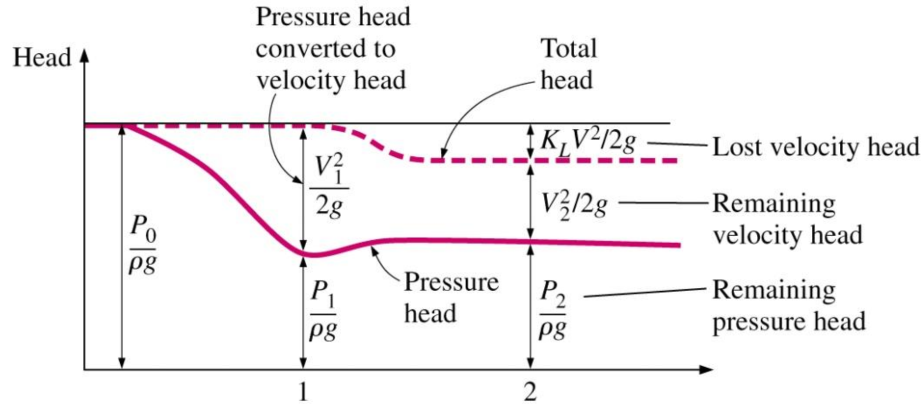


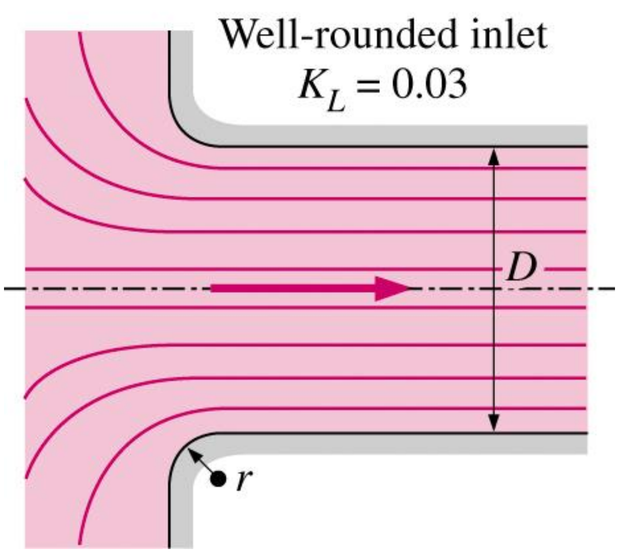
Pipe section without valve:

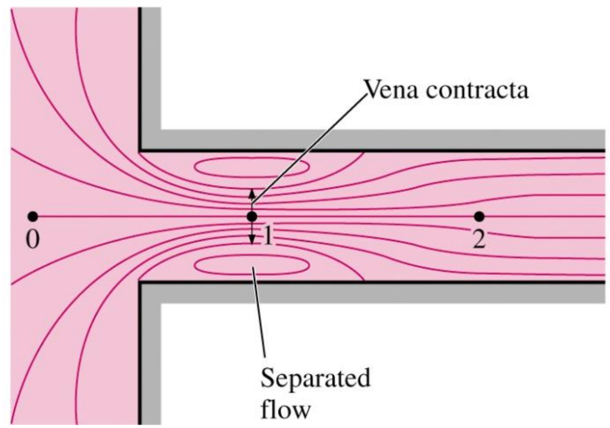


$$\Delta P_L = (P_1 - P_2)_{\text{valve}} - (P_1 - P_2)_{\text{pipe}}$$


Sharp-edged inlet
$K_L = 0.50$
Recirculating flow


Head
Pressure head converted to velocity head
Total head
$K_L V^2/2g$
Lost velocity head
$\frac{V_1^2}{2g}$
$V_2^2/2g$
Remaining velocity head
$\frac{P_0}{\rho g}$
Pressure head
$\frac{P_1}{\rho g}$
$\frac{P_2}{\rho g}$
Remaining pressure head
1
2


Well-rounded inlet
$K_L = 0.03$
D
r


Vena contracta
0
1
2
Separated flow

$$h_\ell = h_f + h_m$$

total loss = $H_1 - H_2$

friction loss: $h_f = f\,(L/D)(V^2/2g)$

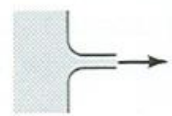
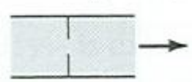
minor loss: $h_m = K_m\,(V^2/2g)$

K_m is the loss coefficient

For each pipe segment (i.e. reaches along which pipe diameter remains constant) there may be several minor losses.

Determination of Local Loss (h_m):

$$h_m = K_m \frac{V^2}{2g}$$

Type of fitting	*Screwed*			*Flanged*	
Diameter	2.5 cm	5 in.	10 cm	5 cm	10 cm
Globe valve (fully open)	8.2	6.9	5.7	8.5	6.0
(half open)	20	17	14	21	15
(one-quarter open)	57	48	40	60	42
Angle valve (fully open)	4.7	2.0	1.0	2.4	2.0
Swing check valve (fully open)	2.9	2.1	2.0	2.0	2.0
Gate valve (fully open)	0.24	0.16	0.11	0.35	0.16
Return bend	1.5	.95	.64	0.35	0.30
Tee (branch)	1.8	1.4	1.1	0.80	0.64
Tee (line)	0.9	0.9	0.9	0.19	0.14
Standard elbow	1.5	0.95	0.64	0.39	0.30
Long sweep elbow	0.72	0.41	0.23	0.30	0.19
45° elbow	0.32	0.30	0.29		
Square-edged entrance 			0.5		
Reentrant entrance 			0.8		
Well-rounded entrance 			0.03		
Pipe exit			1.0		
Area ratio					
Sudden contraction[b]  2:1			0.25		
5:1			0.41		
10:1			0.46		
Area ratio A/A_0					
Orifice plate  1.5:1			0.85		
2:1			3.4		
4:1			29		
$\geq 6:1$			$2.78\left(\frac{A}{A_0} - 0.6\right)^2$		
Sudden enlargement[c] 			$\left(1 - \frac{A_1}{A_2}\right)^2$		
90° miter bend (without vanes) 			1.1		

Computation of Flow in Single Pipes

- the flow computation in single pipes requires solution of three equations simultaneously:

1. The energy equation:

$$z_1 + \frac{P_1}{\gamma} + \alpha\frac{V_1^2}{2g} = z_2 + \frac{P_2}{\gamma} + \alpha\frac{V_2^2}{2g} + h_f$$

2. Equation of Contunity: $Q=V_1A_1=V_2A_2$
3. Darcy-Weissbach Equation:

$$h_f = f\,\frac{L}{D}\,\frac{V^2}{2g}$$

- In general, there are three types of problems depending on the information we have:

1. Head Loss problem:

   Given: $Q, L, D, \nu, \varepsilon$ $\rightarrow$ Find h_f

2. Discharge problem

   Given: $h_f, L, D, \nu, \varepsilon$ $\rightarrow$ Find Q

3. Diameter problem (Design problem)

   Given: $h_f, L, Q, \nu, \varepsilon$ $\rightarrow$ Find D

COMPUTATION OF FLOW IN SINGLE PIPES

Variable	Type of the problem		
	Type I	**Type II**	**Type III**
a)Fluid • *Density • ***Viscosity**	**•G** **•G**	**•G** **•G**	**•G** **•G**
a)Pipe • *Diameter • ***Length** • ***Roughness**	**•G** **•G** **•G**	**•G** **•G** **•G**	**•D (G)** **•G** **•G (D)**
a)Flow **•*<u>Flowrate</u>, *or*** **•Average velocity**	**•G**	**•D**	**•G**
a)Pressure *<u>**Pressure Drop**</u>**,** ***or* Head loss**	**D**	**G**	**G**

G- Given,

D- Determined

1) Determination of Head Loss (Type I)

$$\boxed{H_1 = H_2 + h_\ell \quad \text{and} \quad h_\ell = h_f \qquad \text{since } h_m = 0}$$

$$\boxed{h_f = H_1 - H_2}$$

$$\boxed{H = z + \frac{p}{\gamma} + \frac{V^2}{2g}}$$

$$\boxed{h_f = f\frac{L}{D}\frac{V^2}{2g} \quad \text{since } V = Q/A \text{ then } h_f = \frac{8fL}{g\pi^2 D^5}Q^2}$$

- **Given : Q (or V), L, D, ν,ε**

  Find : h_f

1) $V = \dfrac{4Q}{\pi D^2}$ (or $Q = \dfrac{V\pi D^2}{4}$)
2) $R_e = \dfrac{VD}{\nu} = \dfrac{4Q}{\pi D \nu}$
3) ε/D
4) $f(Re, \varepsilon/D)$ is determined (from Moody Chart or Eqs.)
5) h_f is computed

Example 2.1 (Type-I problem):

- A galvanized iron pipe with a roughness height of 5x10-6 m with a diameter of 0.05 m and a length of 100 m carries a discharge of 0.003 m3/s. Calculate the head loss.

	π	-	3.141592	
EXAMPLE	g	m/s^2	9.81	
for water	ρ	kg/m^3	1000	
for water	μ	kg/m.s	0.001	
for water	ν	m^2 /s	0.000001	
A galvanized iron pipe with a roughness height of			0.000005	m
with a diameter of			0.05	m
and a length of			100	m
carries a discharge of			0.003	m^3 /s
its x-section is			0.001963	m^2
and water flows with a velocity of			1.528	m/s
Flow Reynolds number is			76394	
and the relative roughness is			0.0001	
using S-J formula the friction factor is obtained as			0.01941	
The head loss in the pipe is found to be			4.620	m

Example 2.1

EXAMPLE	π	-	3.141592	
	g	m/s^2	9.81	
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and water flows with a velocity of			1.528	m/s
Flow Reynolds number is			76394	
and the relative roughness is			0.0001	
using S-J formula the friction factor is obtained as			0.01941	
The head loss in the pipe is found to be			4.620	m

2)Determination of average velocity (Type II)

$$H_1 = H_2 + h_\ell \quad , \quad h_\ell = h_f = f\,\frac{L}{D}\,\frac{V^2}{2g}$$

$$(h_m \approx \;\; 0)$$

$$V = \sqrt{(H_1 - H_2)\frac{2gD}{fL}} = \sqrt{h_f\,\frac{2gD}{fL}}$$

- **Given: h_f,L,D,υ,ε.**
- **Find : V**

Since f depends on V through Re, and V is unknown apriori, iteration is needed

V,D,ν

$\downarrow$

R_e=VD/ν $\rightarrow$ Moody Chart or Eqs.

$\downarrow$

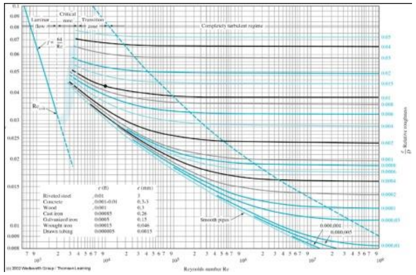
ε/D $\rightarrow$ Moody Chart or Eqs.

Solution procedure (Type II):

1. **Calculate relative roughness** $\frac{\varepsilon}{D}$
2. **Select friction factor,** **f(i) = f(0)**

   ***(assume completely rough turbulent flow)*; f(i) = f(0)**

3. **Calculate velocity;** $V = \sqrt{\frac{h_f D 2g}{fL}}$
4. **Calculate Reynolds number;** $R_e = \frac{VD}{\nu}$
5. **Determine f by using data from Step1 and 4; f(i+1) (use Moody Chart, or Equation)**



6. **Check if f(i+1) = f(i);?**
7. **no, go to step 3 with f(i+1)**
8. **yes, continue**
9. **Calculate Q or V** $Q = V\frac{\pi D^2}{4}$

Iteration Table

$f^{(i)}$	V	R_e	$f^{(i+1)*}$
$f^{(0)}$ Assumed	calculated	calculated	$f^{(1)}$-determined
$f^{(1)}$	calculated	calculated	$f^{(2)}$ -determined
$f^{(2)}$	calculated	calculated	$f^{(3)}$ -determined
. . ***iteration is*** .	. . ***stopped*** .	. . ***when*** .	. . $f^{(i)}=f^{(i+1)}$.
$f^{(i)}$			$f^{(i+1)}$

** obtained from Moody Chart, or determined using equations.*

Example 2.2 (Type-II problem):

- **Example 2.2 (Type-II problem):** A galvanized iron pipe with a roughness height of 5x10-6 m with a diameter of 0.05 m and a length of 100 m has experienced head loss of 10 m. Calculate the flow rate.

EXAMPLE

	e	-	2.718282
	π	-	3.141592
	g	m/s^2	9.81
for water	ρ	kg/m^3	1000
for water	μ	kg/m.s	0.001
for water	ν	m^2 /s	0.000001

A galvanized iron pipe with a roughness height of	ε	0.000005	m
with a diameter of	D	0.05	m
and a length of	L	100	m
has experienced head loss of	hf	10	m

its x-section is	A	0.00196	m^2
and the relative roughness is	ε/D	0.0001	

	i	f_i	V	Re	f_{i+1}	Δf
carrying out an iterative solution procedure	0	0.02	2.215	110736.2	0.018105	-0.00189
	1	0.01811	2.328	116387.2	0.017944	-0.00016
	2	0.01794	2.338	116909.4	0.017929	-1.4E-05
	3	0.01793	2.339	116956.2	0.017928	-1.3E-06
	4	0.01793	2.339	116960.4	0.017928	-1.1E-07
one obtains the friction factor and velocity (m/s)	5	0.01793	2.339	116960.8	0.017928	-1E-08

hence the discharge is obtained as	Q	0.00459	m^3 /s

3) Determination of Diameter (Type III)

$$D = \sqrt[5]{f\,\frac{8LQ^2}{h_\ell \pi^2 g}} = \left(\frac{8LQ^2}{h_\ell \pi^2 g}\right)^{1/5} f^{1/5}$$

- **Given: $h_f, L, Q, \nu, \varepsilon$.**
- Find : D

1. **Assume f(i) = f(0) (arbitrarily 0.02)**
2. **Calculate pipe diameter**
3. **Calculate Reynolds number**
4. **Calculate relative roughness**
5. **Determine friction factor, f(i+1) use Moody Chart or Equations**
6. **Check if f(I+1) = f(i) ; ?**
7. **if no, go to step 2 with f(i+1)**
8. **if yes, stop**
9. **Diameter Calculated at Step 2 is the result. Select the next larger commercially available pipe diameter size**

$$R_e = \frac{VD}{\nu} = \frac{4Q}{\pi D \nu}$$

$$\frac{\varepsilon}{D}$$

Iteration Table

$f^{(i)}$	D	R_e	ε/D	$f^{(i+1)*}$
$f^{(0)}$ Assumed	calculated	calculated	calculated	$f^{(1)}$-determined
$f^{(1)}$	calculated	calculated	calculated	$f^{(2)}$-determined
$f^{(2)}$	calculated	calculated	calculated	$f^{(3)}$ -determined
.. .	. . ***iteration is*** .	. . ***stopped*** .	. . ***when*** .	. . $\underline{f^{(i)}=f^{(i+1)}}$.
$f^{(i)}$	calculated	calculated	calculated	$f^{(i+1)}$- determined

Example 2.3 (Type-III problem):

- A galvanized iron pipe with a roughness height of 0.00005 m and a length of 100 m under the head loss of 10 m delivers discharge of 0.003 m3/s. Calculate pipe diameter.

EXAMPLE

e	-	2.71828
π	-	3.14159
g	m/s^2	9.81
ρ	kg/m^3	1000
μ	kg/m.s	0.001
ν	m^2/s	1E-06

A galvanized iron pipe with a roughness height of	ε	m	0.00005
with a length of	L	m	100
under the head loss of	hf	m	10
delivers discharge of	Q	m^3/s	0.003

carrying out an iterative solution procedure

i	fi	D	A	V	Re	ε/D	fi+1	Δf
0	0.02	0.0431	0.00146	2.05624	88624.3	0.00116	0.02312	0.00312
1	0.02312	0.0444	0.00155	1.94025	86088.5	0.00113	0.02308	-4E-05
2	0.02308	0.0444	0.00155	1.94169	86120.4	0.00113	0.02308	5.2E-07
3	0.02308	0.0444	0.00155	1.94168	86120	0.00113	0.02308	-6E-09
4	0.02308	0.0444	0.00155	1.94168	86120.1	0.00113	0.02308	7.7E-11
5	0.02308	0.0444	0.00155	1.94168	86120.1	0.00113	0.02308	-9E-13

one obtains the friction factor and velocity (m/s)

hence the discharge is obtained as D 0.0444 m

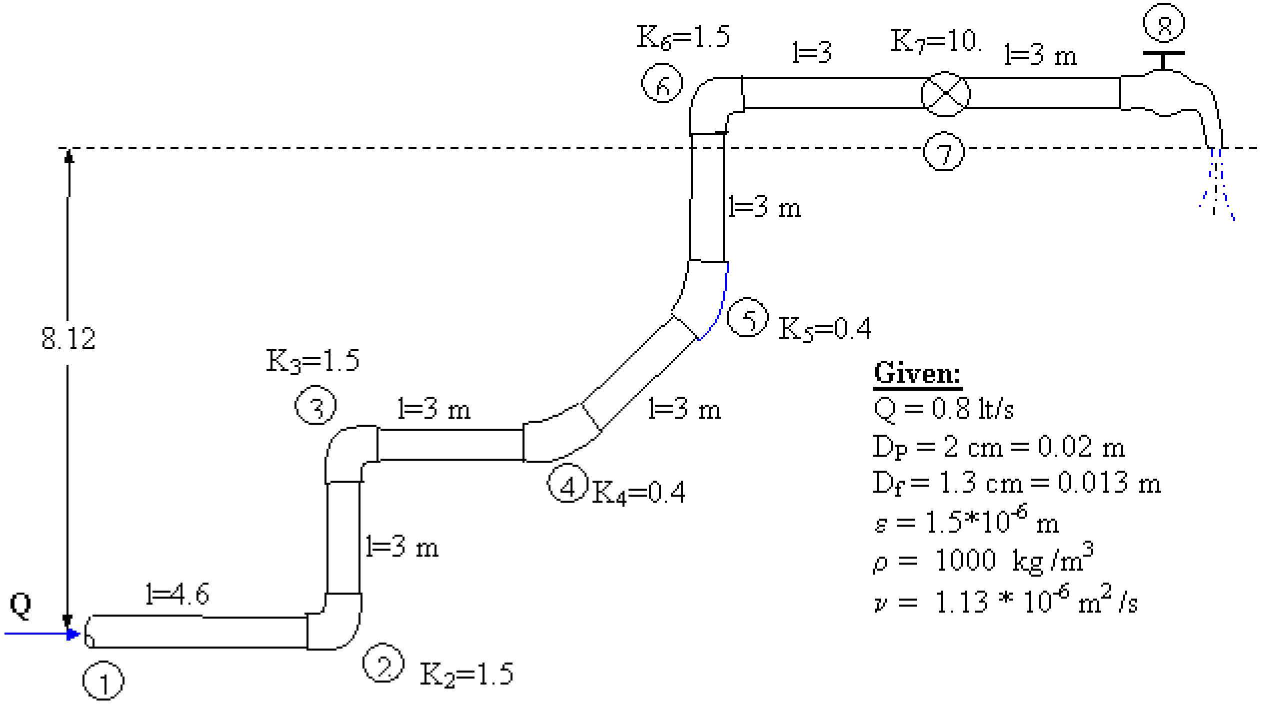
Example 2.4

Water flows from the basement to the second floor through the 2 cm diameter copper pipe (a drawn tubing) at a rate of Q = 0.8 lt/s and exits through a faucet of diameter 1.3 cm as shown in figure. Determine the pressure at point 1 if :

a)viscous effects are neglected,

b)the only losses included are major losses

c) all losses are included



Given:

Q = 0.8 lt/s

D_P = 2 cm = 0.02 m

D_f = 1.3 cm = 0.013 m

$\varepsilon = 1.5*10^{-6}$ m

ρ = 1000 kg/m^3

ν = 1.13 * 10^{-6} m^2/s

$$V = \frac{Q}{A} = \frac{0.0008}{3.1416*10^{-4}} = 2.546\ m/s$$

$$Re = \frac{VD}{\nu} = \frac{2.546*0.02}{1.13*10^{-6}} = 45062 = 4.5*10^{4}$$ flow is turbulent

a) Energy equation between (1) and (9)

$$H_1 = H_9$$

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{V_9^2}{2g} + \frac{P_9}{\gamma} + z_9 \qquad V_1 = V = 2.546\ m/s\ ,\ \ V_9 = \frac{Q}{A_9} = 6.027\ m/s$$

$$P_1 = \gamma\left[\frac{V_9^2 - V_1^2}{2g} + z_9\right]$$

$$P_1 = 9810\left[\frac{6.027^2 - 2.546^2}{2*9.81} + 8.12\right] = 94578.5\ Pa = 94.6\ kPa$$

$$\frac{P_1}{\gamma} = 9.64\,m \quad \text{Compare with } z = 8.12\ m$$

b) $H_1 - h_f = H_9 \qquad P_1 = \gamma\left[\frac{V_9^2 - V_1^2}{2g} + z_9 + f\frac{L}{D}\frac{V^2}{2g}\right]$

drawn tubing, ε = 0.00016 cm $\underbrace{\varepsilon/D = 8.10^{-5}\ ,\ \ Re = 45062}_{f = 0.0215}$

L_{total} = 4.6 + 3*6 = 22.6 m

$$P_1 = 9810\left[\frac{6.027^2 - 2.546^2}{2 * 9.81} + 8.12 + 0.0215\frac{22.6}{0.02} * \frac{2.546^2}{2 * 9.81}\right]$$

$$P_1 = 173320\,\text{Pa} = 173.3\,\text{kPa} \qquad \frac{P_1}{\gamma} = 17.67\,\text{m}$$

c) $$H_1 - h_L - \sum_2^8 K_i \frac{V^2}{2g} = H_9$$

$$P_1 = \gamma\left[\frac{V_9^2 - V_1^2}{2g} + z_9 + f\frac{L}{D}\frac{V^2}{2g} + \sum K_i \frac{V^2}{2g}\right]$$

$$\sum K_i = 1.5 + 1.5 + 0.4 + 0.4 + 1.5 + 10 + 2 = 17.3$$

$$P_1 = 9810\left[\frac{6.027^2 - 2.546^2}{2 * 9.81} + 8.12 + 0.0215\frac{22.6}{0.02} \cdot \frac{2.546^2}{2 * 9.81} + 17.3\frac{2.546^2}{2 * 9.81}\right]$$

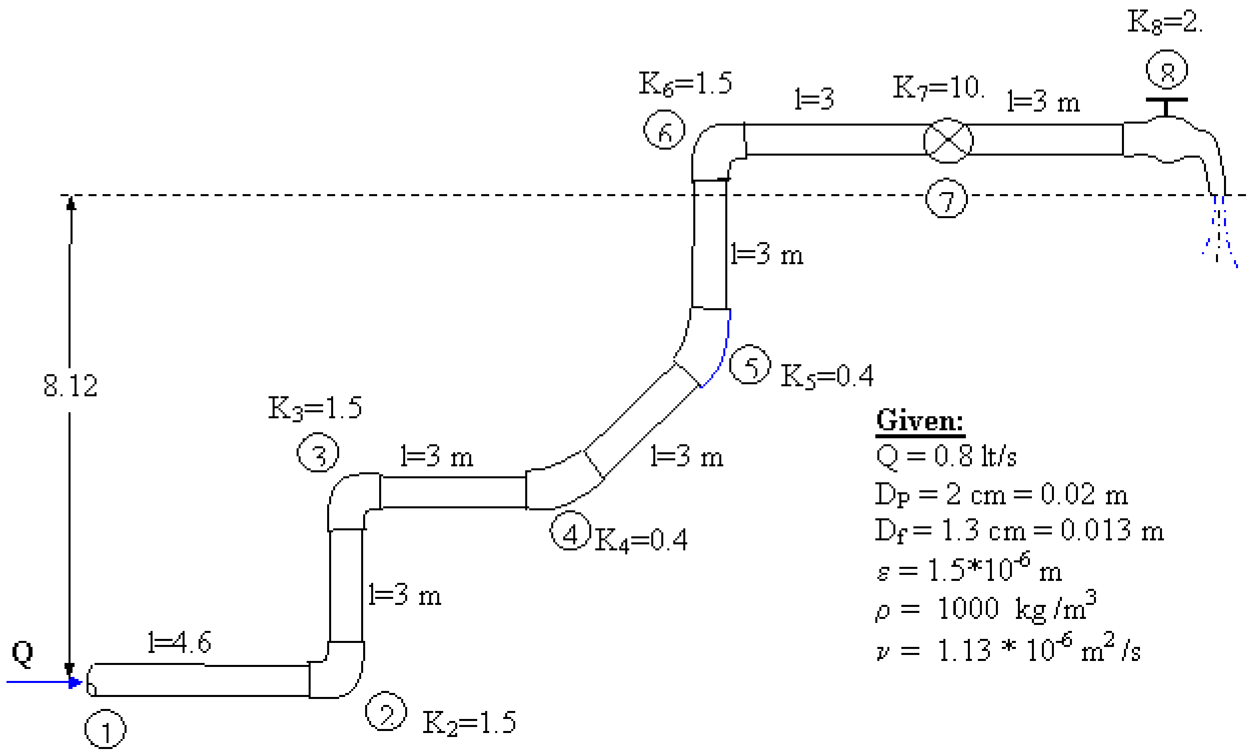
$$P_1 = 229390.3\,\text{Pa} = 229.4\,\text{kPa} \qquad \frac{P_1}{\gamma} = 23.38\,\text{m}$$

This pressure drop (229.4 kPa) calculated by including all losses should be the most realistic answer of the three cases considered.

Comparison:

	$h_l=0$	$h_l=h_f$	$h_l=h_f+h_m$
P_1 (kPa)	**94.6**	**173.3**	**229.4**
P_1/γ (m)	**9.64**	**17.67**	**23.38**

Sketch EGL and HGL.



Given:

$Q = 0.8$ lt/s

$D_P = 2$ cm $= 0.02$ m

$D_f = 1.3$ cm $= 0.013$ m

$\varepsilon = 1.5*10^{-6}$ m

$\rho = 1000$ kg /m^3

$\nu = 1.13 * 10^{-6}$ m^2 /s

Friction loss for non-circular conduits

Circular

$$R_h = \frac{A}{P}$$

$$D_h = D$$

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

$$R_e = \frac{D\ V}{\nu}$$

$$f = f\left(R_e, \frac{\varepsilon}{D}\right)$$

Non-circular

$$R_h = \frac{A}{P}$$

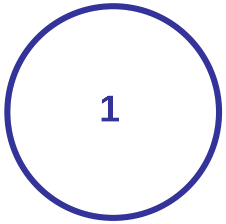
$$D_h = 4R_h$$

$$h_f = f\frac{L}{D_h}\frac{V^2}{2g}$$

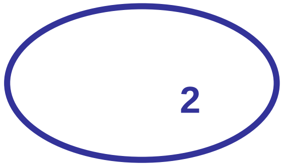
$$R_e = \frac{D_h V}{\nu}$$

$$f = f\left(R_e, \frac{\varepsilon}{D_h}\right)$$

Friction loss for non-circular conduits



$$\text{for } P_2 = P_1$$



$$A_2 < A_1$$

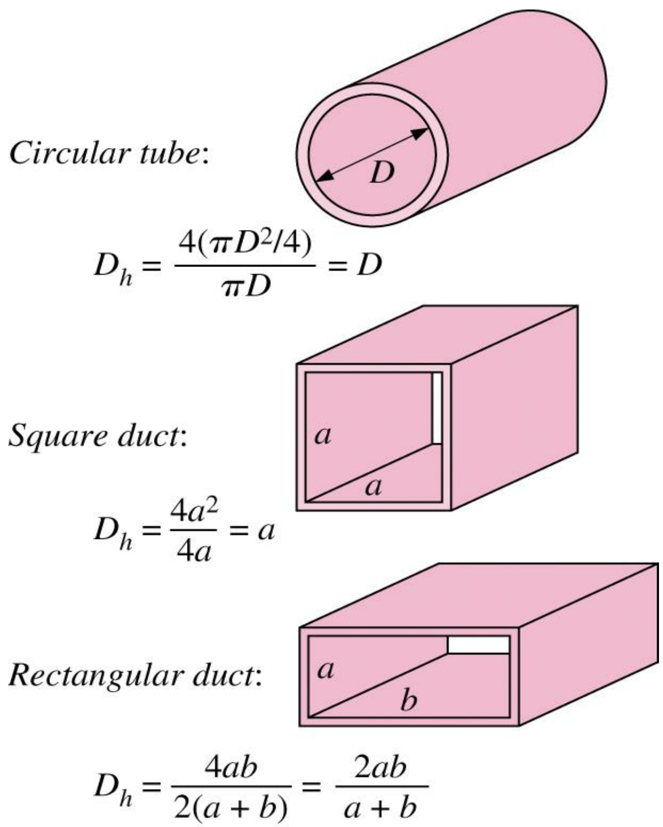
$$D_h < D$$

$$V_2 > V_1$$

$$\frac{\varepsilon}{D_h} > \frac{\varepsilon}{D} \text{ and } R_{e2} \approx R_{e1} \Rightarrow f_2 > f_1$$

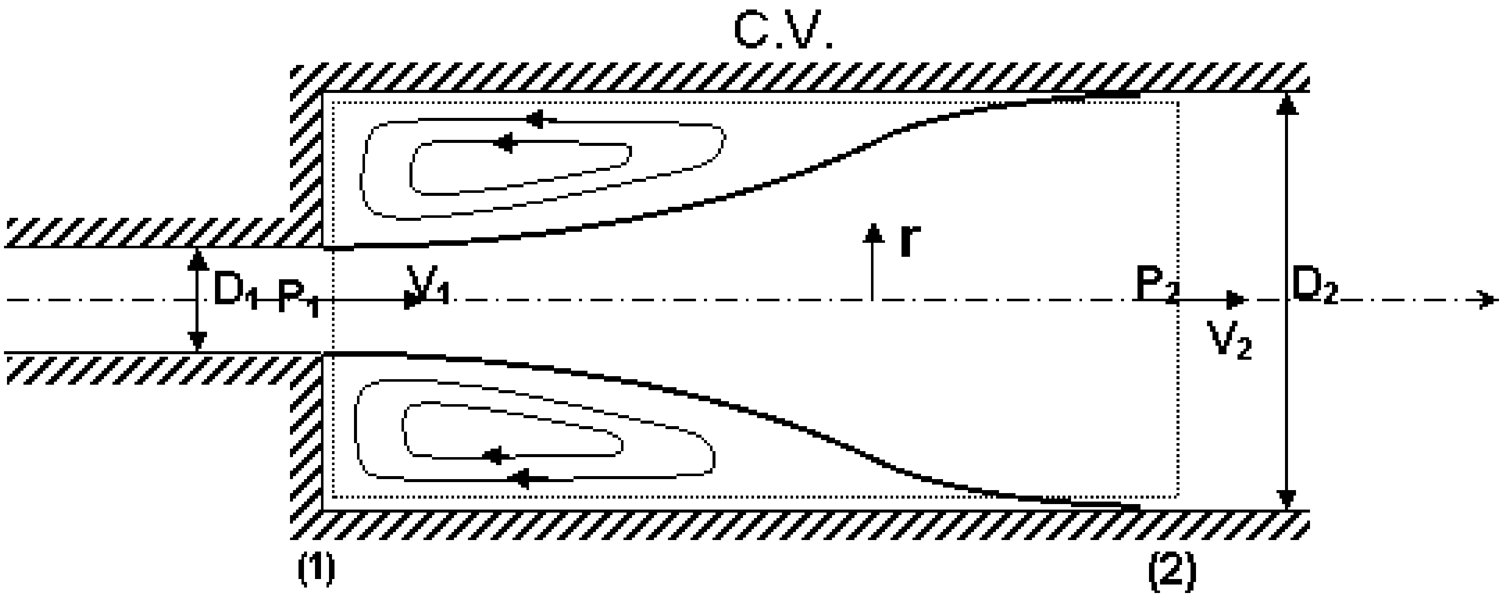
$$h_{f_2} > h_{f_1}$$

Equivalent Diameter



- For <u>non-circular</u> pipes, define an equivalent diameter as:
- $D_h = 4A / P$
  - A = cross-section area
  - P = wetted perimeter
- Example: open channel with b=0.5 m and y=0.15 m
  - $A = 0.15 * 0.5 = 0.075\text{m}^2$
  - $P = 0.15 + 0.15 + 0.5 = 0.8\text{m}$
  - Don't count free surface, since it does not contribute to friction along pipe walls!
  - $D_h = 4A/P = 4*0.075/0.8 = 0.375\text{m}$
  - <u>What does it mean?</u> This channel flow is equivalent to a circular pipe of diameter 0.375m (approximately), as far as frction factor is concerned.

Expanding Flows:



Expanding flow

Continuity eq.: $$Q = V_1A_1 = V_2A_2$$

$$(p_1 - p_2)A_2 = \rho Q(V_2 - V_1)$$

Momentum eq. In x-direction: $$\frac{(p_1 - p_2)}{\gamma} = \frac{1}{g}(V_2^2 - V_1V_2)$$

Energy eq.:

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^{\ 2}}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{V_2^{\ 2}}{2g} + h_m$$

$$h_m = \frac{p_1 - p_2}{\gamma} + \frac{V_1^{\ 2} - V_2^{\ 2}}{2g}$$

$$h_m = \frac{(V_2 - V_1)^2}{2g} \quad , \quad V_2 = \frac{V_1A_1}{A_2} = \frac{V_1D_1^{\ 2}}{D_2^{\ 2}}$$

$$h_m = \left[\left(\frac{D_1}{D_2}\right)^2 - 1\right]^2 \frac{V_1^{\ 2}}{2g} \quad , \ h_m = K_m \frac{V_1^{\ 2}}{2g}$$

FLOWMETERS

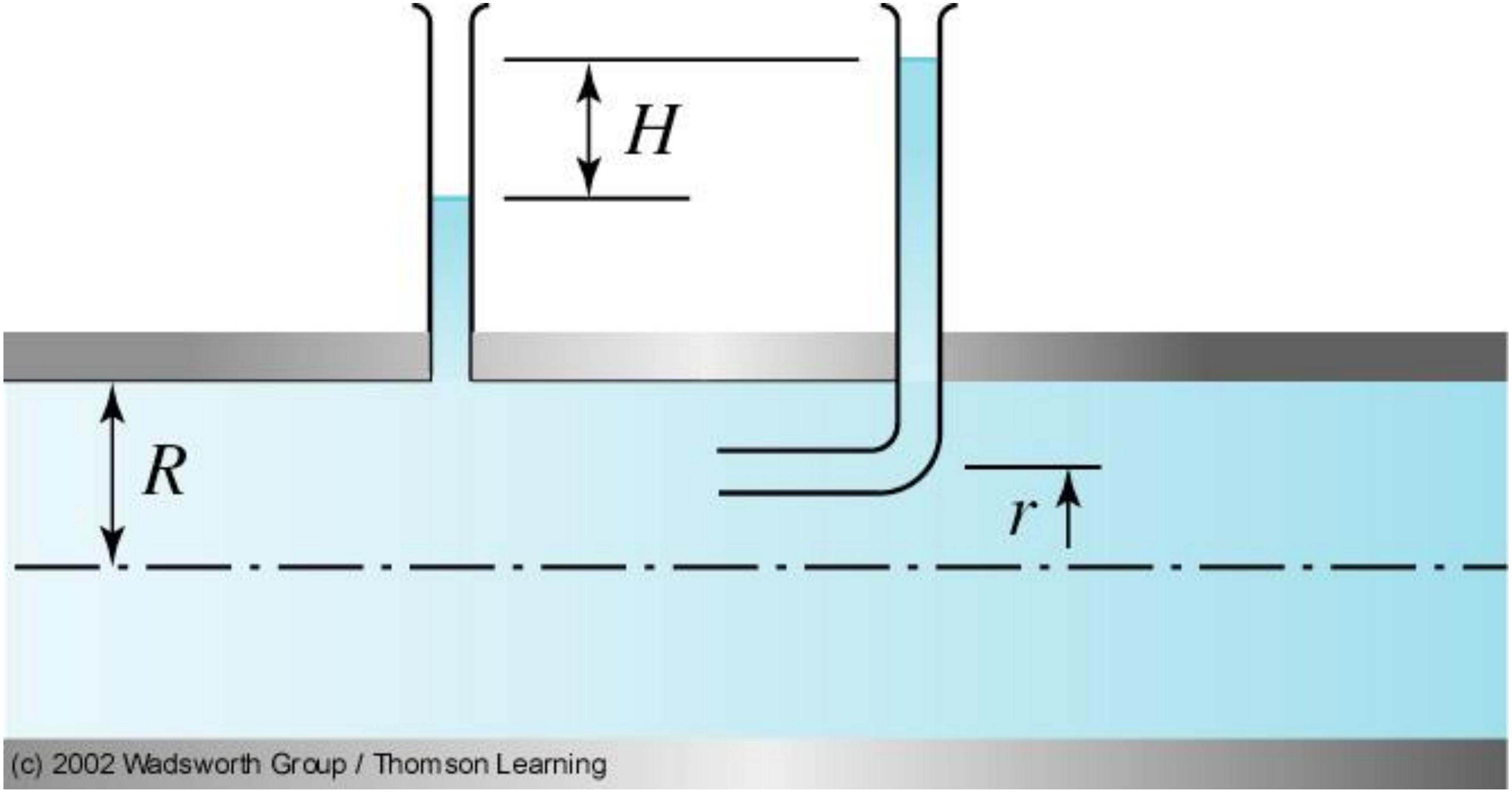
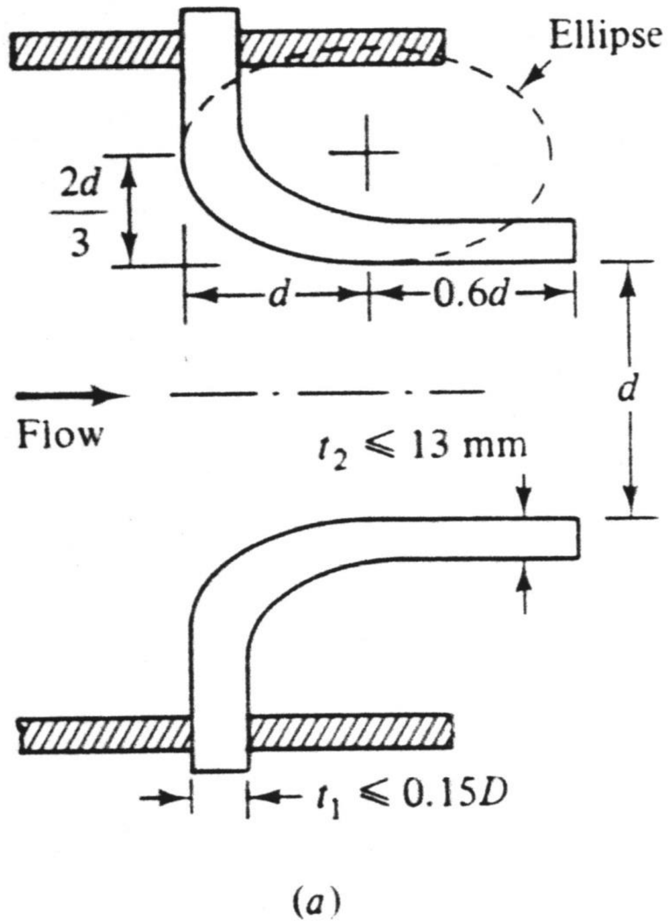
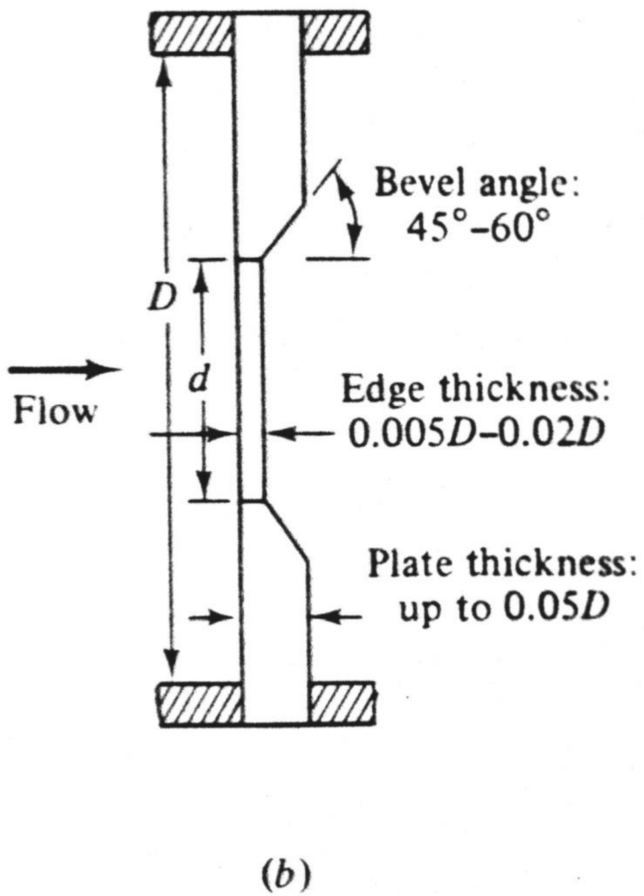


Figure P7.28

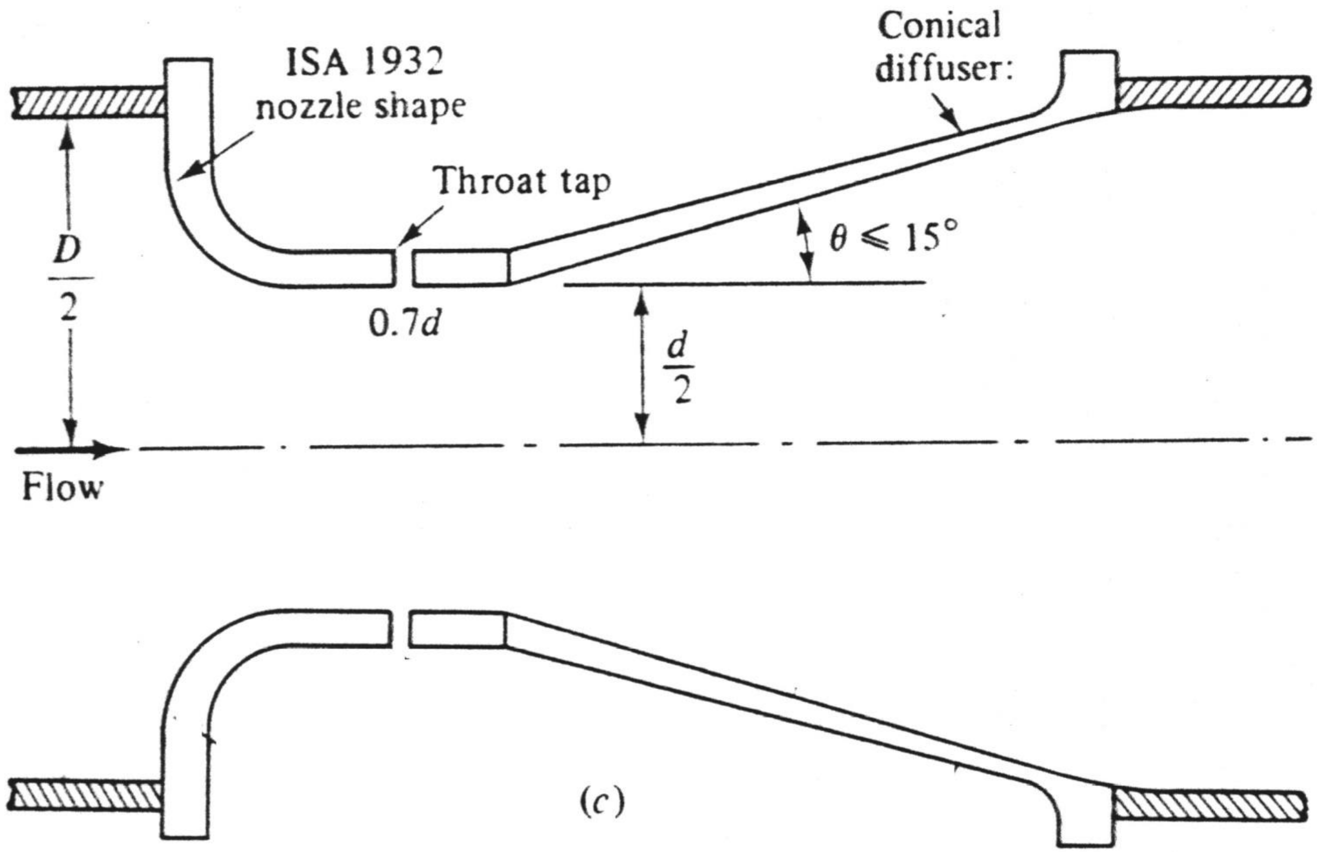
FLOWMETERS





long-radius nozzle

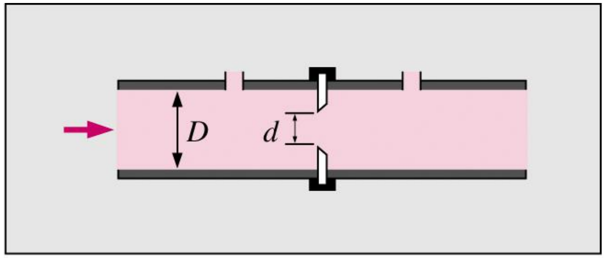
thin-plate orifice



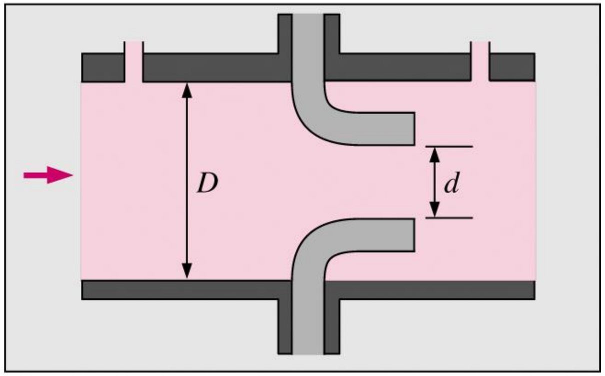
(c)

venturi nozzle

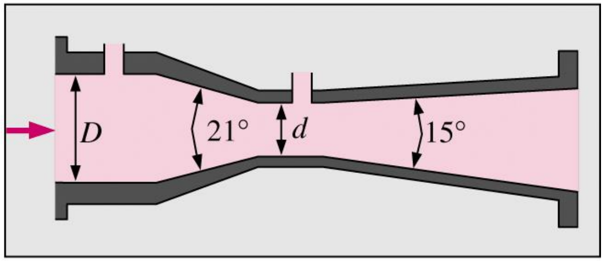
Flowmeters:



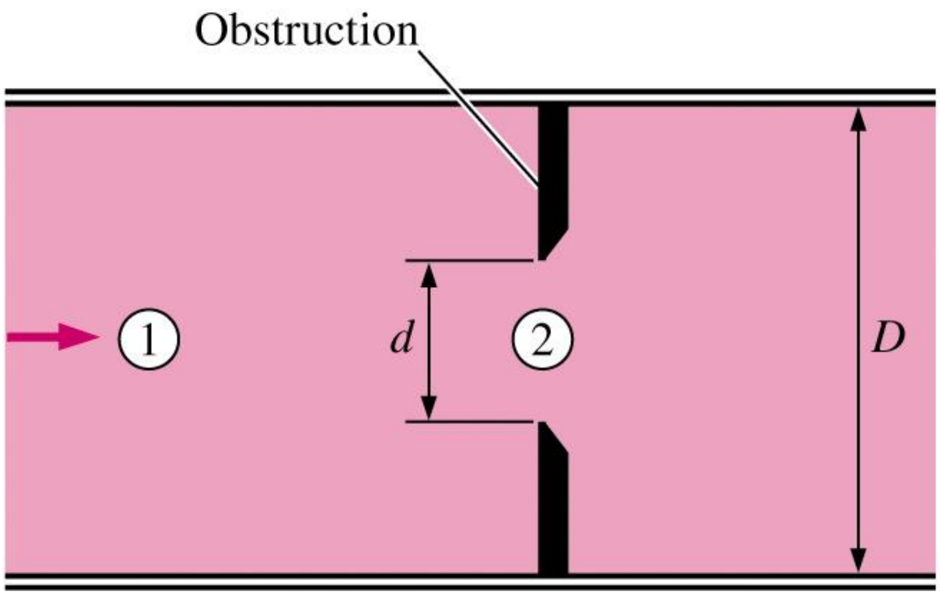
(*a*) Orifice meter

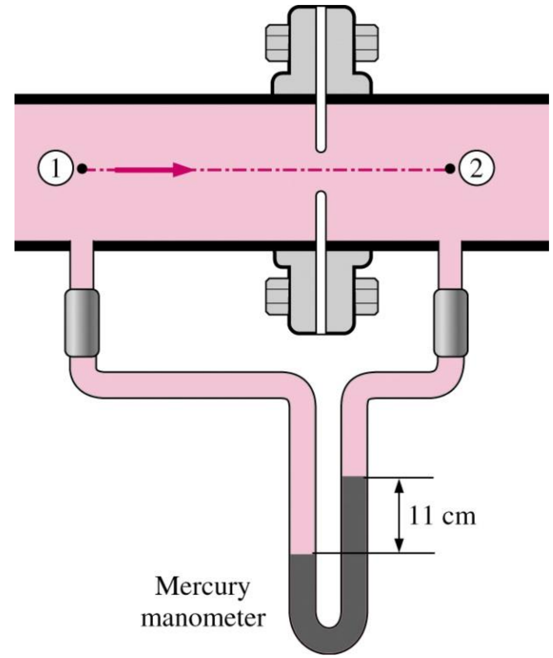


(*b*) Flow nozzle

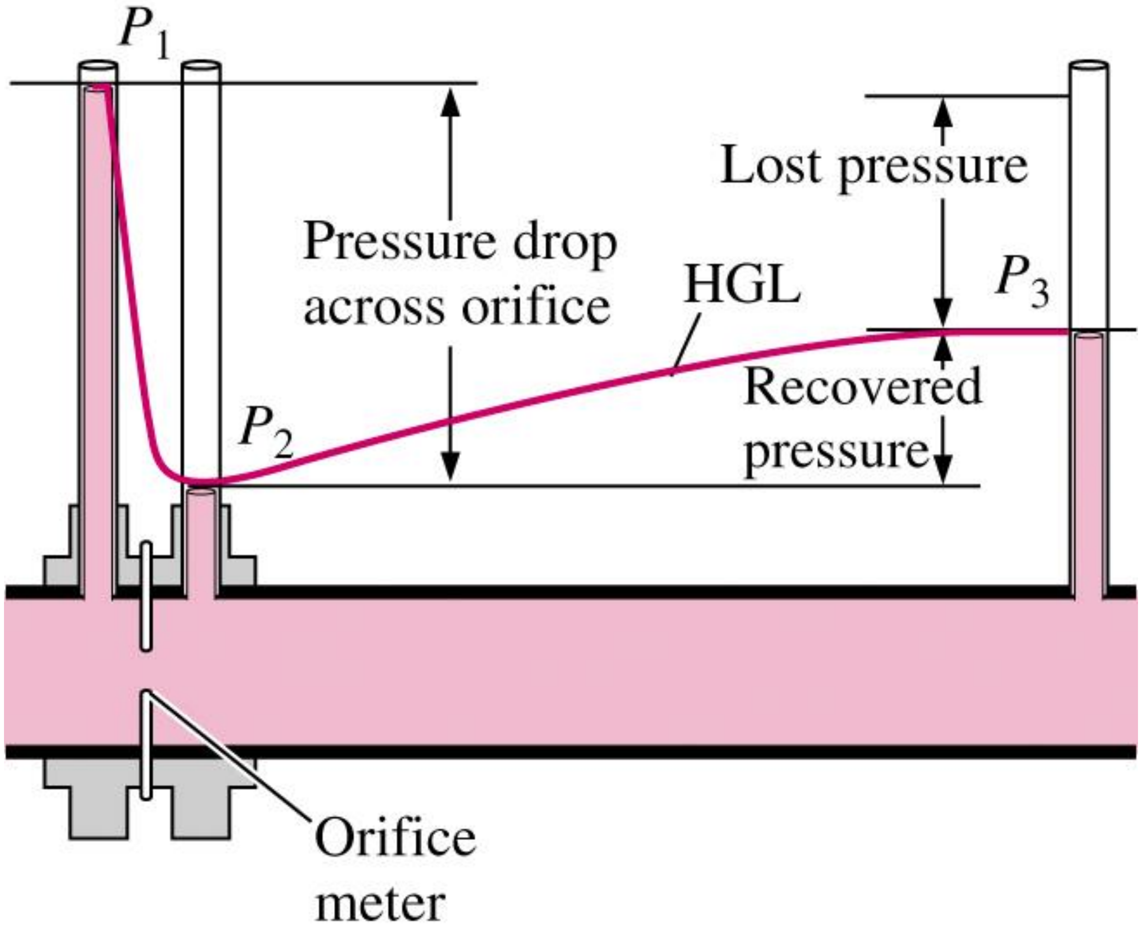


(*c*) Venturi meter

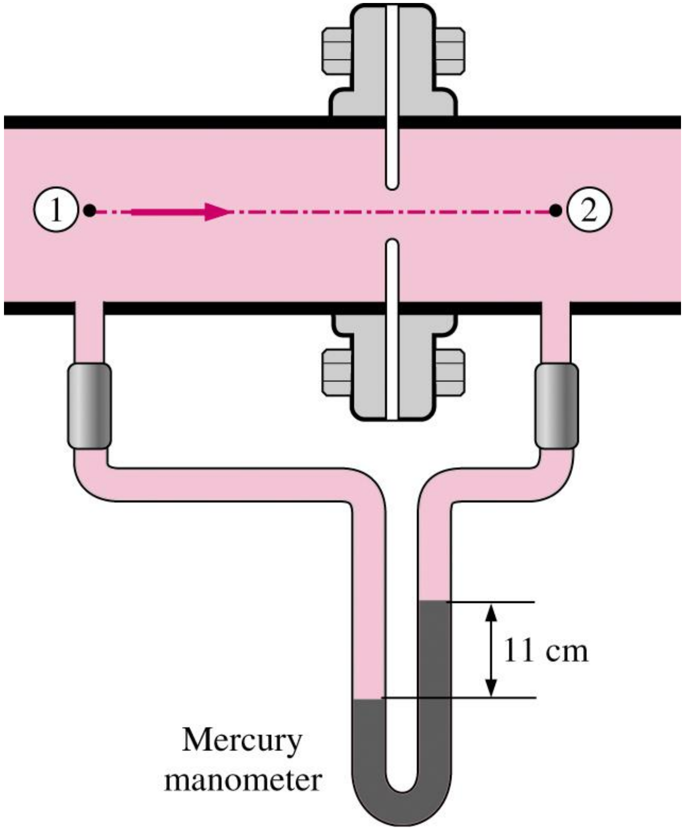

Obstruction
1
d
2
D


1
2
11 cm
Mercury
manometer

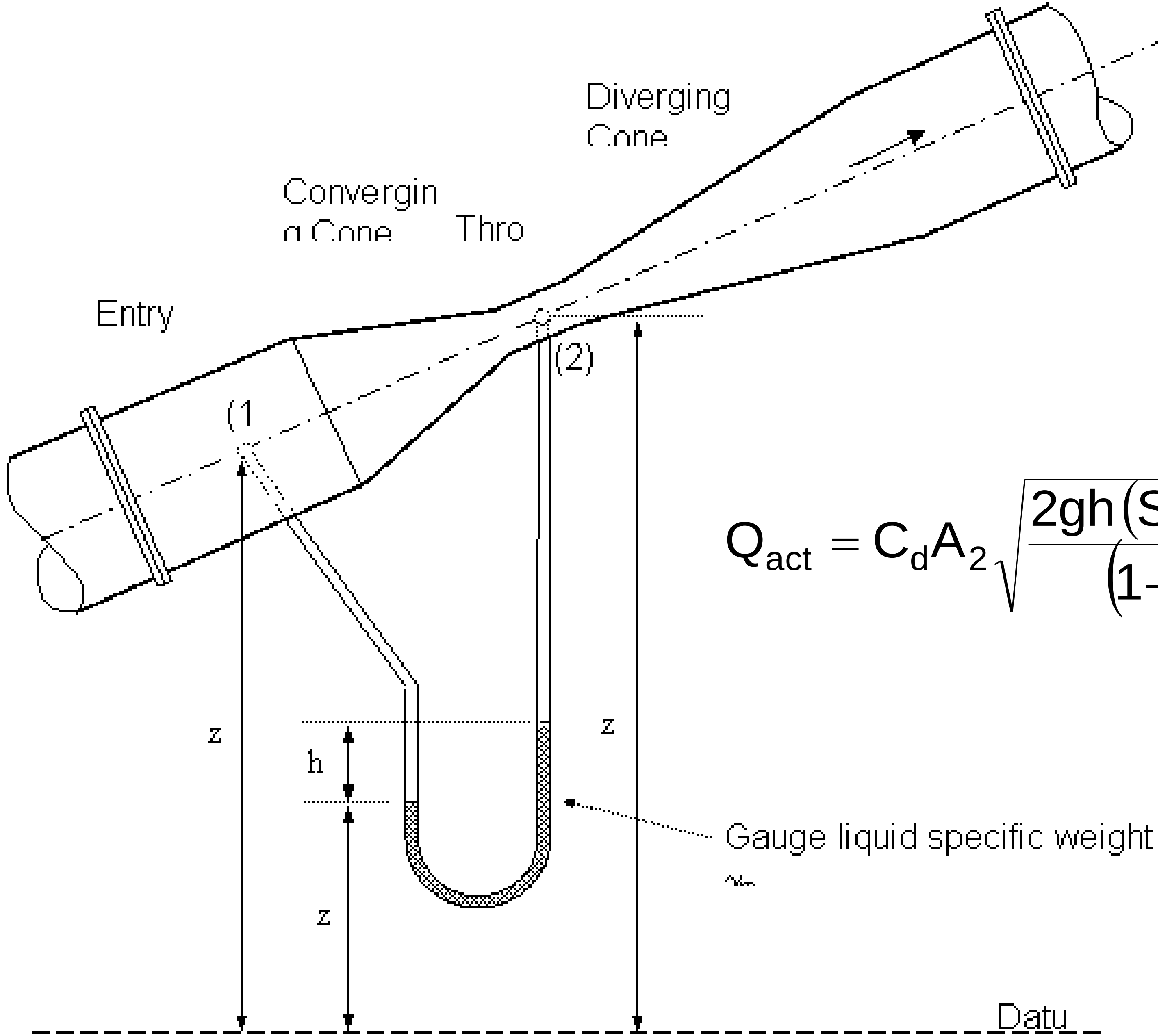
Orifice meter



- Detail of Orifice meter

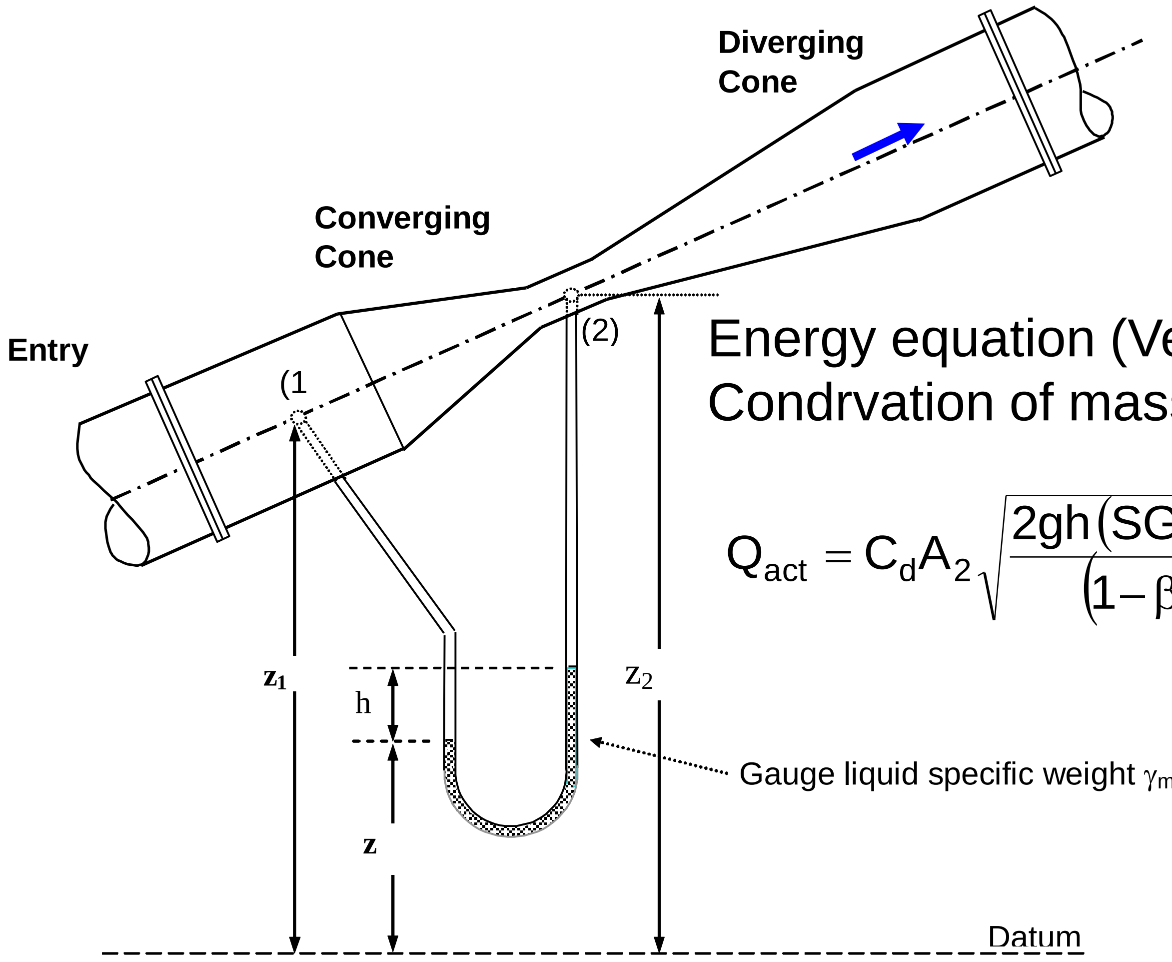


VENTURIMETER



$$Q_{act} = C_d A_2 \sqrt{\frac{2gh(SG_m - 1)}{(1 - \beta^4)}}$$

Venturimeter



Energy equation (Velocity-pressure)
Condrvation of mass

$$Q_{act} = C_d A_2 \sqrt{\frac{2gh(SG_m - 1)}{(1 - \beta^4)}}$$

Energy and Continuity Equations for sections 1 and 2

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g}$$

$$V_1 = \frac{Q}{A_1} \quad \text{and} \quad V_2 = \frac{Q}{A_2}$$

$$z_1 + \frac{p_1}{\gamma} + \frac{Q^2}{2gA_1^2} = z_2 + \frac{p_2}{\gamma} + \frac{Q^2}{2gA_2^2}$$

$$\Delta h = \frac{p_1 - p_2}{\gamma} + (z_1 - z_2) = h_m\left(\frac{\gamma_m}{\gamma} - 1\right)$$

$$\Delta h = \frac{p_1 - p_2}{\gamma} + (z_1 - z_2) = h_m\left(\frac{\gamma_m}{\gamma} - 1\right)$$

$$Q = \frac{A_2}{\left(1 - \beta^4\right)^{1/2}}\sqrt{2g\Delta h}$$

$$\Delta h = \frac{p_1 - p_2}{\gamma} + (z_1 - z_2)$$

Pressure Equation in Manometerr

$$p_1 + \gamma(z_1 - z) = p_2 + \gamma(z_2 - z - h) + \gamma_m h$$

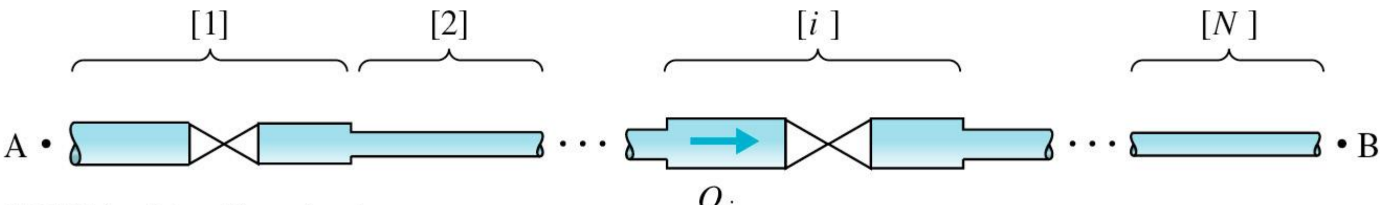
$$\Delta h = \frac{p_1 - p_2}{\gamma} + (z_1 - z_2) = h_m\left(\frac{\gamma_m}{\gamma} - 1\right)$$

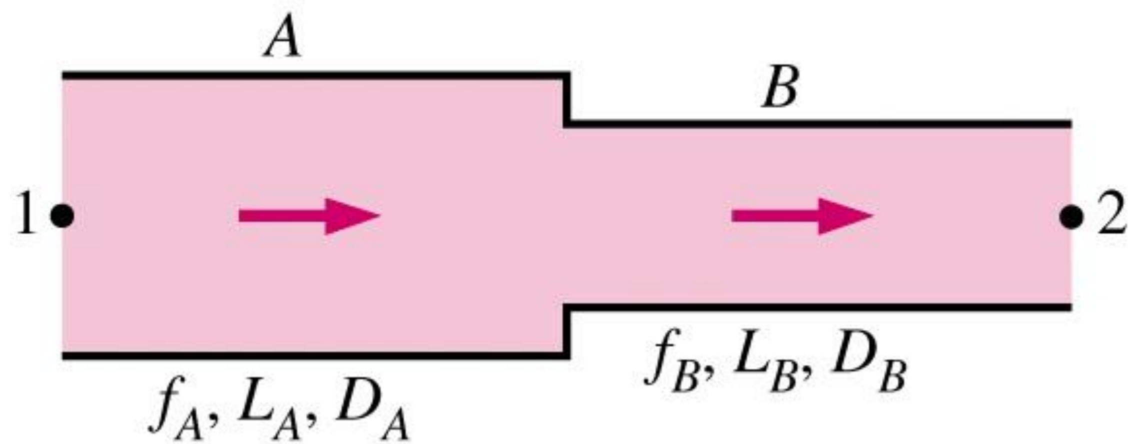
$$Q_{act} = C_d A_2 \sqrt{\frac{2gh_m\left(SG_m - 1\right)}{\left(1 - \beta^4\right)}}$$

$$Q_{act} = \frac{C_d A_2}{\left(1 - \beta^4\right)^{1/2}}\sqrt{2g\Delta h}$$

$$\beta = \frac{D_2}{D_1}$$

Pipes in Series





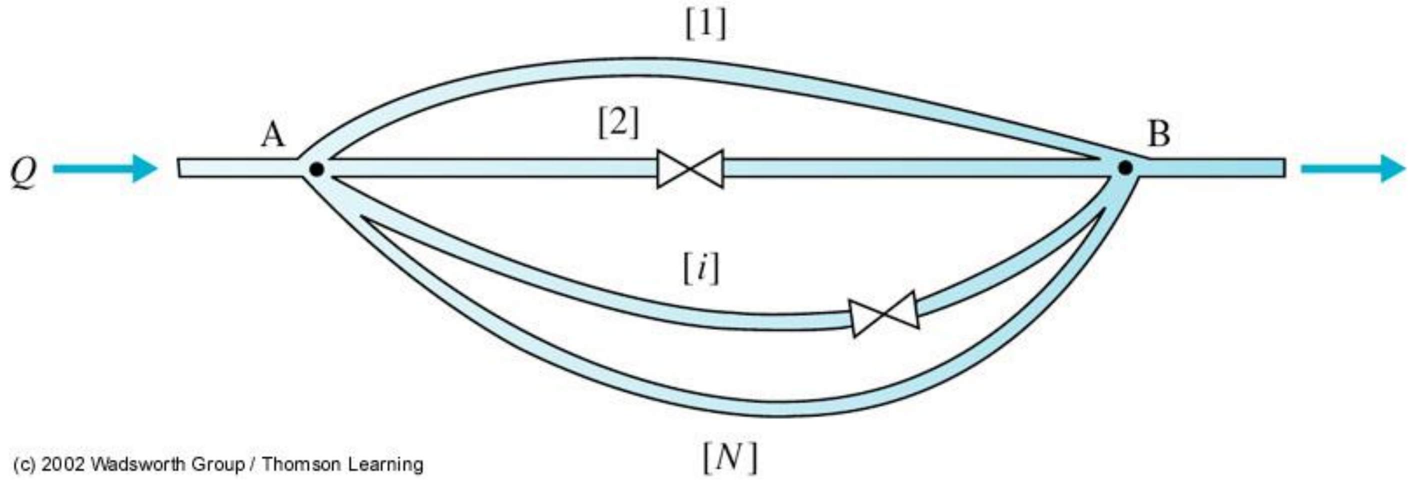
$$\dot{V}_A = \dot{V}_B$$

$$h_{L,\,1\text{-}2} = h_{L,\,A} + h_{L,\,B}$$

Pipes in Series

- For pipes connected in series:
- Q=constant $Q_A = Q_1 = Q_2 = \text{-----} = Q_n = Q_B$
- But the head loss is additive:
- $h_\ell = \sum h_{fi} + \sum h_{mi}$, $i = 1, 2, \ldots, n$
- Where h_{fi} is the frictional loss in i-th pipe, and h_{mi} is the local loss in i-th pipe.

Pipes in Paralel



- In order to increase the capacity of a pipeline system, pipes might be connected in parallel.

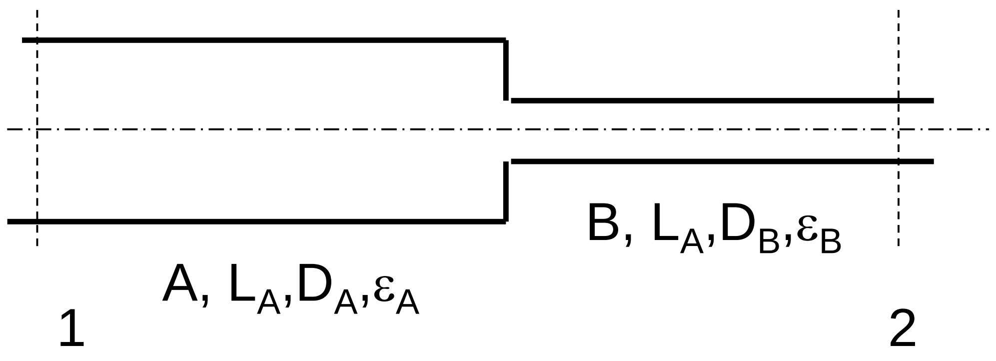
  For pipes connected in parallel:

- The discharges are additive:
- $Q_A = Q_B = \sum Q_i$, i=1,2,....,n
- The Total head at junctions must have single value. Therefore the head loss in each branch must be the same:
- $h_{f1} = h_{f2} = h_{f3}$ = ----- $= h_{fn}$

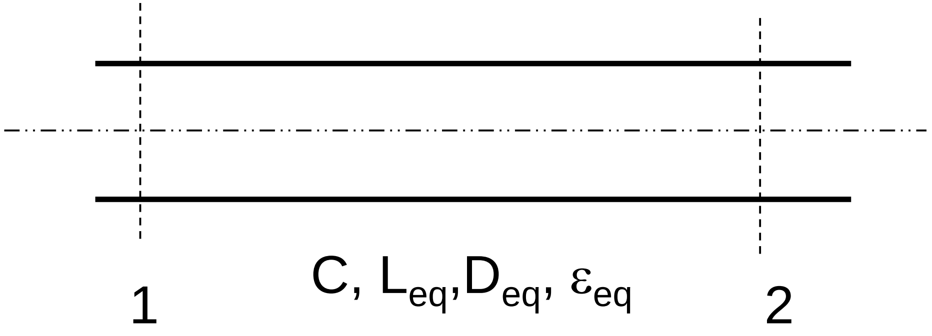
Equivalent Pipe Concept

For pipes in series:

- Consider two pipes connected in series:



- We want to replace these two pipes with an equivalent pipe C:



- The head loss between sections (1) and (2) is:
- $h_{f12}=h_{fA}+h_{fB}=h_{fC}$ (1)
- and
- $Q_A=Q_B=Q_C$ (2)

- The Darcy-Weissbach Equation:

$$h_f = f\frac{L}{D}\frac{V^2}{2g} \quad \text{and inserting} \quad V = \frac{4Q}{\pi D^2}:$$

$$h_f = 8f\frac{L}{D^5}\frac{Q^2}{g\pi^2}$$

- Therefore Eq.(1) can be written as:

$$8f_A\frac{L_A}{D_A^5}\frac{Q_A^2}{g\pi^2} + 8f_B\frac{L_B}{D_B^5}\frac{Q_B^2}{g\pi^2} = 8f_C\frac{L_C}{D_C^5}\frac{Q_C^2}{g\pi^2} \quad \text{or}$$

$$f_A\frac{L_A}{D_A^5} + f_B\frac{L_B}{D_B^5} = f_C\frac{L_C}{D_C^5} = f_{eq}\frac{L_{eq}}{D_{eq}^5}$$

- If $f_A=f_B=f_{C,}$ then

$$\frac{L_{eq}}{D_{eq}^5} = \frac{L_A}{D_A^5} + \frac{L_B}{D_B^5}$$

- Generalizing for n pipes connected in series

$$\frac{L_{eq}}{D_{eq}^5} = \sum_{1}^{n} \frac{L_i}{D_i^5}$$

- We choose either D_{eq}, or L_{eq}, then compute the other from the equation.

- If desired minor losses may be expressed in terms of equivalent lengths and added to the actual length of pipe as:

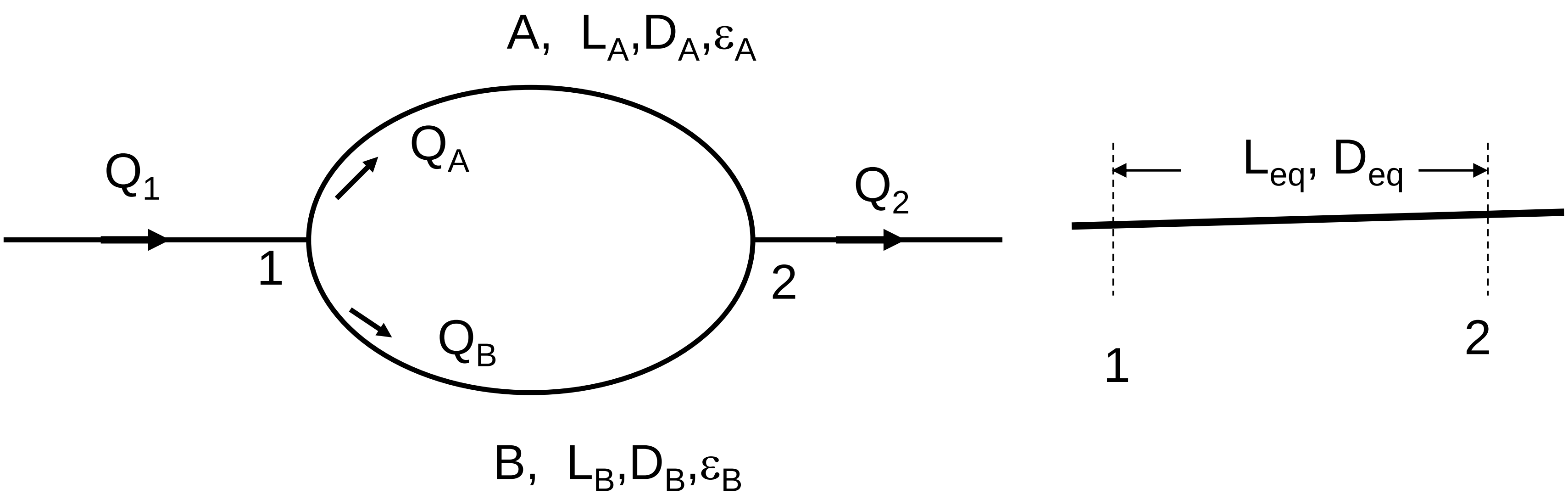
$$h_m = K_m \frac{V^2}{2g} = f \frac{L_{eq}}{D} \frac{V^2}{2g} \quad \text{Hence} \quad L_{eq} = K_m \frac{D}{f}$$

Where

- K_m=local loss coefficient, and
- f=fricton factor of the pipe
- Then the pipe length should be taken as:
- $L=L_{ac}+L_{eq}$

Equivalent pipe concept for parallel pipes

- Consider two pipes, A and B, connected in parallel:



- For the equivalent pipe with length, L_{eq}, and diameter, D_{eq}:

- $h_{f1\text{-}2}=h_{fA}=h_{fB}$ and $Q_1=Q_A+Q_B=Q_2$
- From the Darcy-Weissbach equation:

$$Q=\sqrt{\frac{h_f g\pi^2 D^5}{8fL}}$$

- Therefore:

$$Q_A=\sqrt{\frac{h_{fA} g\pi^2 D_A^5}{8f_A L_A}} \text{ and } Q_B=\sqrt{\frac{h_{fB} g\pi^2 D_B^5}{8f_B L_B}} \text{ and hence}$$

$$\sqrt{\frac{h_{fC} g\pi^2 D_C^5}{8f_C L_C}}=\sqrt{\frac{h_{fA} g\pi^2 D_A^5}{8f_A L_A}}+\sqrt{\frac{h_{fB} g\pi^2 D_B^5}{8f_B L_B}}$$

- Simplifying:

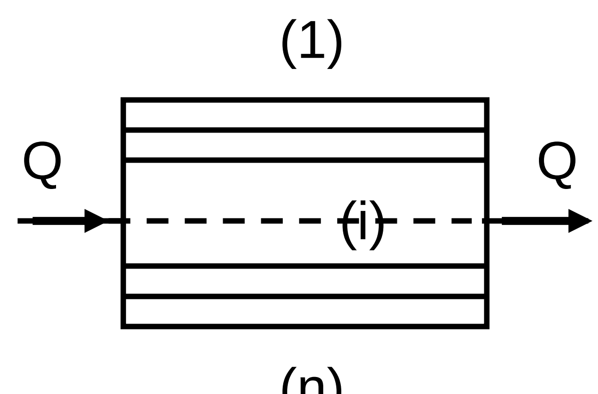
$$\sqrt{\frac{D_C^5}{f_C L_C}} = \sqrt{\frac{D_A^5}{f_A L_A}} + \sqrt{\frac{D_B^5}{f_B L_B}}$$

Furthermore if $f_C = f_A = f_B$, then

$$\sqrt{\frac{D_C^5}{L_C}} = \sqrt{\frac{D_A^5}{L_A}} + \sqrt{\frac{D_B^5}{L_B}} \qquad \sqrt{\frac{D_{eq}^5}{L_{eq}}}$$

Generalizing for n pipes connected in parallel:

$$\sqrt{\frac{D_{eq}^5}{L_{eq}}} = \sum_1^n \sqrt{\frac{D_i^5}{L_i}}$$

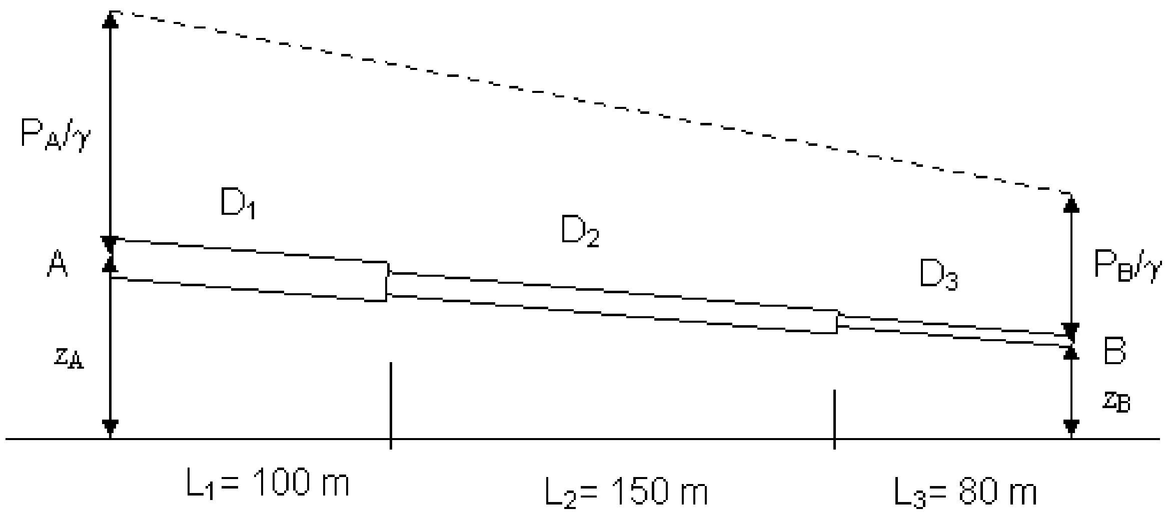


Example 2.5:

- Given a three-pipe series system as shown below. The total pressure drop is p_A-p_B = 150 kPa, and the elevation drop is z_A-z_B = 5 m. The pipe data are

pipe	L (m)	D (cm)	ε (mm)
1	100	8	0.24
2	150	6	0.12
3	80	4	0.20

- The fluid is water (ρ= 1000 kg/m3 , ν= 1.02x10-6 m2/s). Calculate the flow rate Q (m3/hr). <u>Neglect minor losses.</u>



$$H_A = H_B + \sum h_f + \sum h_m$$

$$\overset{\frac{V_1^2}{2g}}{\cancel{\frac{V_A^2}{2g}}} + \frac{P_A}{\gamma} + z_A = \overset{\frac{V_3^2}{2g}}{\cancel{\frac{V_B^2}{2g}}} + \frac{P_B}{\gamma} + z_B + f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} + f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} + f_3 \frac{L_3}{D_3}\frac{V_3^2}{2g}$$

$$\left(\frac{p_A}{\gamma} - \frac{p_B}{\gamma}\right) + (z_A - z_B) = \left(f_1 \frac{L_1}{D_1} - 1\right)\frac{V_1^2}{2g} + f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} + \left(f_3 \frac{L_3}{D_3} + 1\right)\frac{V_3^2}{2g}$$

from continuity: $V_1 A_1 = V_2 A_2 = V_3 A_3$

$$A_1 = \left(\frac{8}{6}\right)^2 A_2 = \left(\frac{8}{4}\right)^2 A_3 \text{ and } \qquad A_1 = 1.78 A_2 = 4 A_3$$

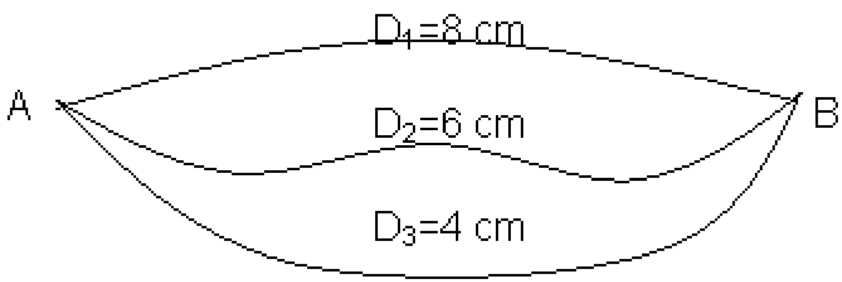
$$V_2 = 1.78 V_1 \text{ and } V_3 = 4 V_1$$

$$\left(\frac{p_A}{\gamma} - \frac{p_B}{\gamma}\right) + (z_A - z_B) = \left(f_1 \frac{L_1}{D_1} - 1\right)\frac{V_1^2}{2g} + (1.78)^2 f_2 \frac{L_2}{D_2}\frac{V_1^2}{2g}$$
$$+ (4)^2 \left(f_3 \frac{L_3}{D_3} + 1\right)\frac{V_1^2}{2g}$$

$$20.3 = (1250 f_1 + 7920 f_2 + 32000 f_3 + 15)\frac{V_1^2}{2g}$$

Example 2.6:

- Assume that the same three pipes of previous example are now in parallel with the same total loss of 20.3 m. Compute the total rate Q(m3/hr), neglecting the minor losses.



- **<u>Solution-I</u>:**

Energy equation b/w A and B:

$H_A = H_B + h_L = H_B + h_f + h_m$

no matter which route is followed b/w A and B

$$H_A - H_B = 20.3 = f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} = f_3 \frac{L_3}{D_3}\frac{V_3^2}{2g}$$

- Substituting the known L's and D's ,

$$20.3 = 1250\, f_1 \frac{V_1^2}{2g} = 2500\, f_2 \frac{V_2^2}{2g} = 2000\, f_3 \frac{V_3^2}{2g}$$

Since V_i's and f_i's are not known, assume hydraulically rough regime

Pipe	ε/D	f_0	V (m/s)	Re	f1
1	0.003	0.0262	3.49	273726	0.268
2	0.002	0.0234	2.61	153529	0.247
3	0.005	0.0304	2.56	100392	0.315

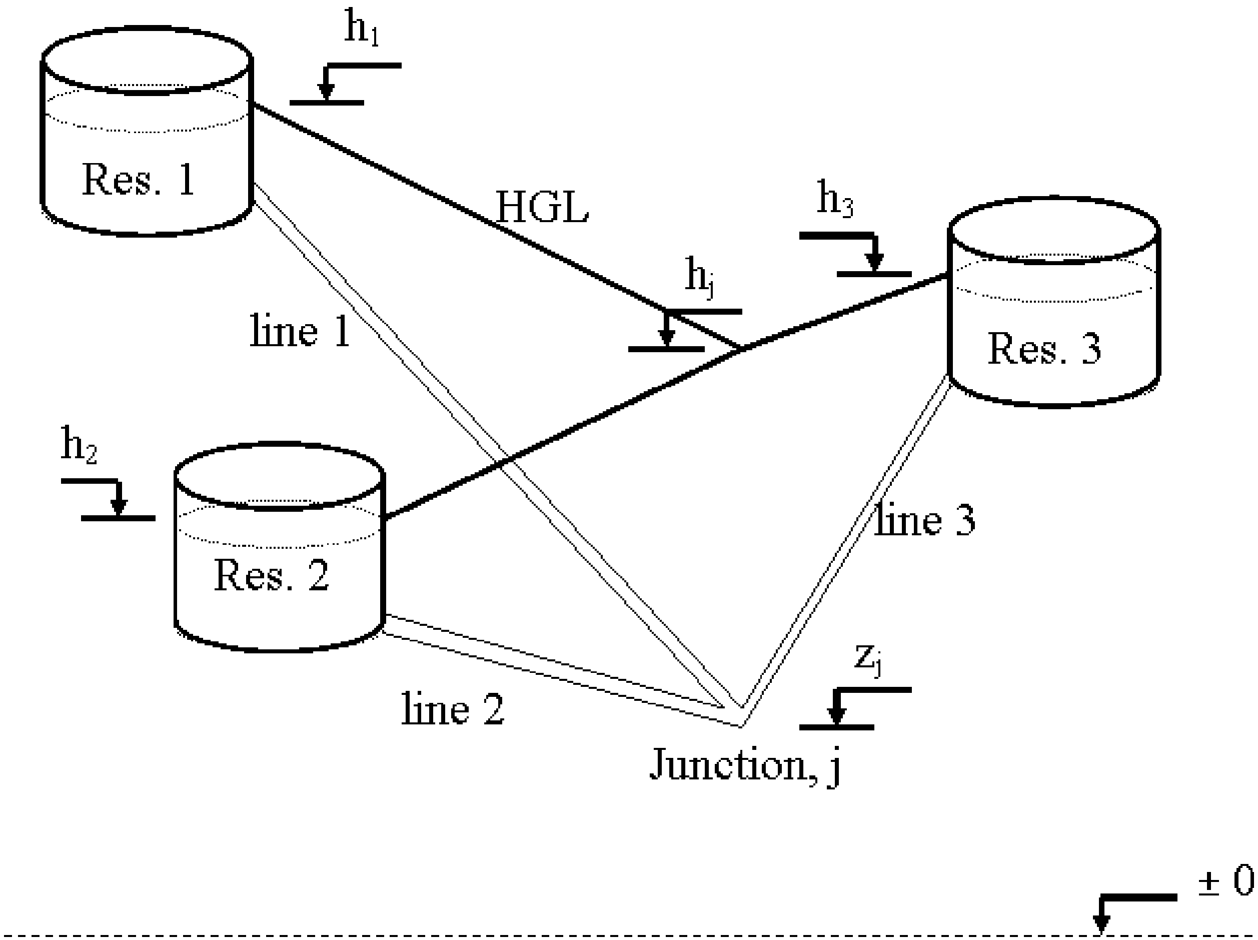
Pipe	ε/D	f_1	V (m/s)	R_e	f_2
1	0.003	0.268	3.46	271373	0.268
2	0.002	0.247	2.55	150000	0.247
3	0.005	0.315	2.52	98823	0.315

f's have converged

Pipe	V (m/s)	Q (m^3/s)	Q (m^3/hr)
1	3.46	0.0174	62.6
2	2.55	0.0072	26.0
3	2.52	0.0032	11.4
TOTAL			100

Q= 100 m^3/hr

BRANCHING PIPES
Junction Problems



$Q_1+Q_2+Q_3=0$

$h_j=z_j+p_j/\gamma$

$h\ell,i= h_{f,i} + h_{m,i}=z_i-h_j$

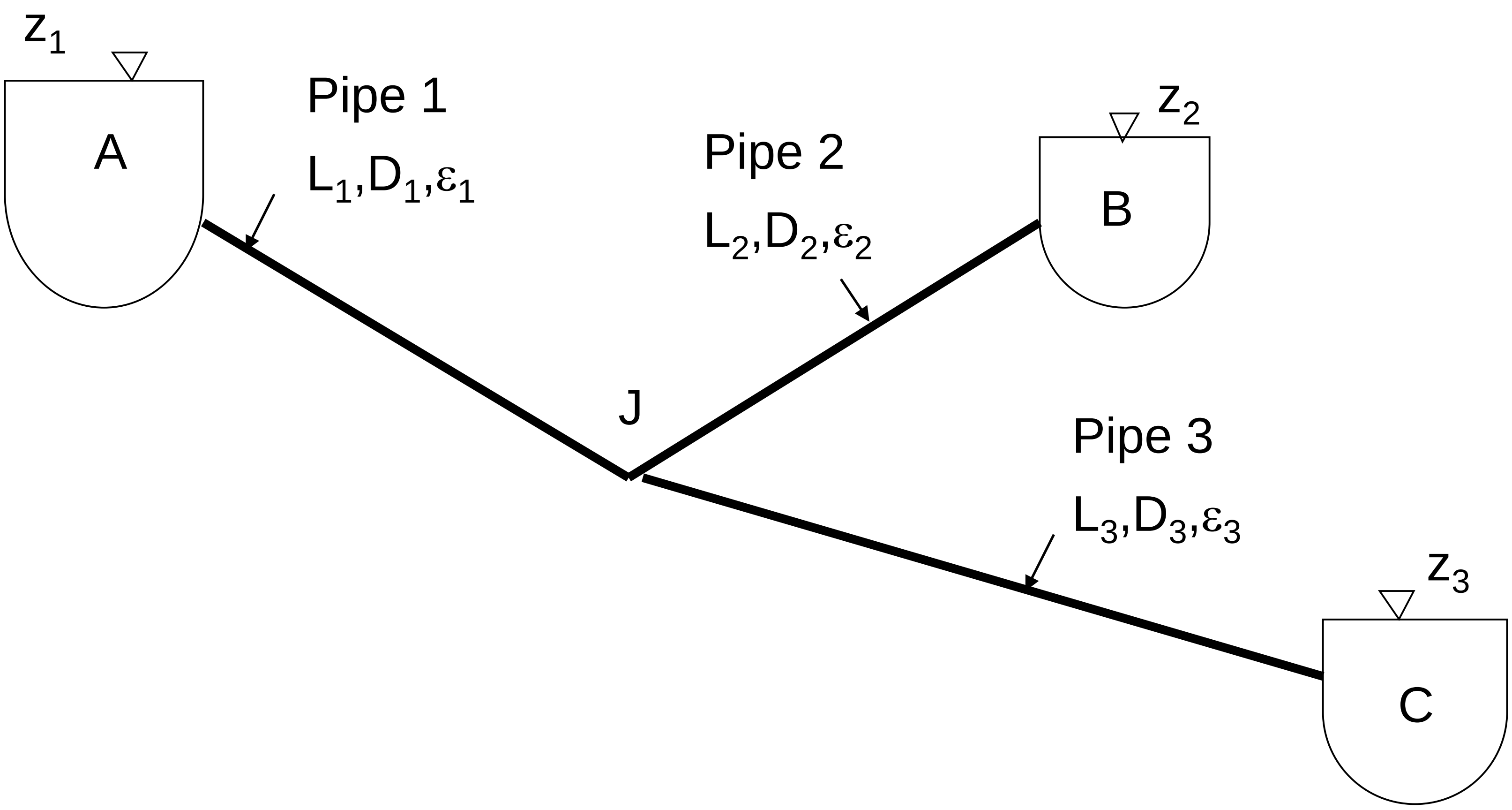
Three-Reservoir Problem (Junction Problem)

In a three reservoir problem, in general the data known are:

1. Elevations of reservoirs, (z_1, z_2, z_3),
2. The pipe characteristics such as length, L, diameter, D, and the roughness height, ε.

   The problem is to determine the discharge in each pipe line.

- Depending on the total head at junction, H_J, there are 3 possibilities:



Case 1:

- If $z_1 > H_J > z_2$, and z_3, the flow is
  $A \rightarrow J$, $J \rightarrow B$, and $J \rightarrow C$

The energy equation gives:

$H_A = H_J + h_{f1}$, or $h_{f1} = H_A - H_J$

$H_J = H_B + h_{f2}$, or $h_{f2} = H_J - H_B$, and

$H_J = H_C + h_{f3}$, or $h_{f3} = H_J - H_C$

2. The continuity equation becomes:

   $Q_1 = Q_2 + Q_3$

There are 4 unknowns, H_J, Q_1, Q_2, and Q_3, and 4 equations. Therefore we can determine them.

Case 2:

- If $z_1 > H_J$, and $z_2 > H_J$, and $H_J > z_3$, the flow is $A \rightarrow J$, $B \rightarrow J$, and $J \rightarrow C$

The energy equation gives:

$H_A = H_J + h_{f1}$, or $h_{f1} = H_A - H_J$

$H_B = H_J + h_{f2}$, or $h_{f2} = H_B - H_J$, and

$H_J = H_C + h_{f3}$, or $h_{f3} = H_J - H_C$

2. The continuity equation becomes:

   $Q_1 + Q_2 = Q_3$

There are 4 unknowns, H_J, Q_1,Q_2, and Q_3, and 4 equations. Therefore we can determine them.

Case 3:

- If $z_1 > H_J$, and $H_J > z_2$, and $H_J = z_3$, the flow is $A \rightarrow J$, $J \rightarrow B$, and no flow in pipe 3.

The energy equation gives:

$H_A = H_J + h_{f1}$, or $h_{f1} = H_A - H_J$

$H_J = H_B + h_{f2}$, or $h_{f2} = H_J - H_B$, and

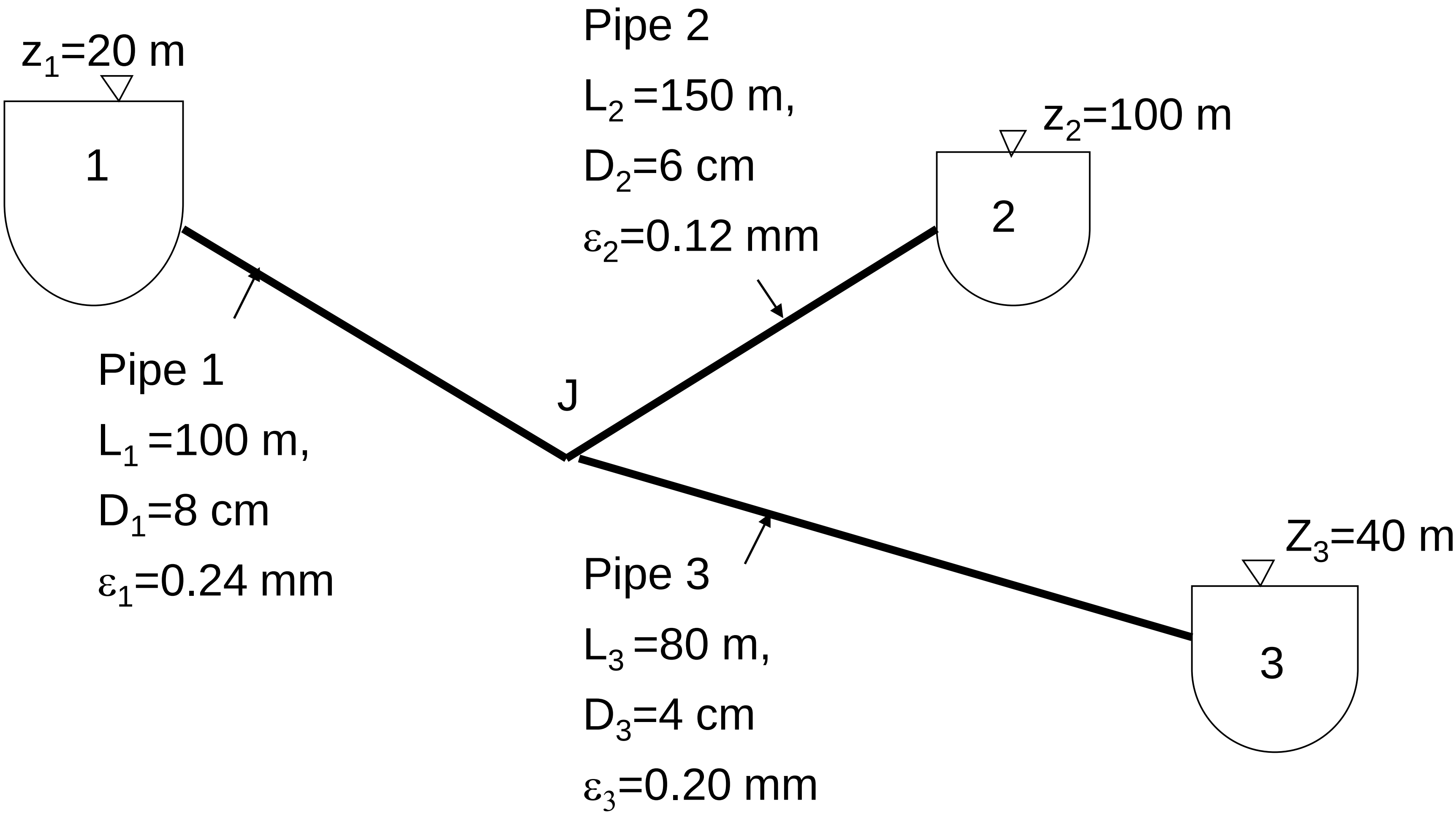
2. The continuity equation becomes:

   $Q_1 = Q_2$

There are 3 unknowns, H_J, Q_1,and Q_2, and 3 equations. Therefore we can determine them.

- In solution since energy equation gives nonlinear equations in terms of the discharges, it is usually difficult to solve the simultaneous equations.
- Therefore, we usually use a trail and error method:
- Assume a junction head. Then determine the flow directions, and the discharge in each pipeline by using the energy equation. (Type 2 problem)
- Then check if the equation of continuity is satisfied at junction.
- To assume a junction head equal to the elevation of one of the reservoirs will lead to case (3) problem. The magnitude of the discharges determined in two pipes will give the actual flow directions.

- **Example 2.7:** Find the flow rate in each pipe, neglecting the minor losses. (ν=1.02x10^{-6})



- Assume junction head H_J, and check if the continuity equation is satisfied.
- Assume that $H_J=z_J+P_J/\gamma=40$ m.
- Then the flow is:
- $(2)\rightarrow J$, and $J\rightarrow (1)$
- The energy equation between (J) and (1) gives:
- $H_J=z_1+h_{f1} \rightarrow h_{f1}=H_J-z_1=40-20=20$ m

$$20 = f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} = f_1 \frac{100}{0.08}\frac{V_1^2}{2x9.81} \quad \text{and hence } V_1 = \sqrt{\frac{0.3139}{f_1}}$$

f_i	V_1 (m/s)	Re_1	ε/D	f_{i+1}
0.0262	3.46	271372.6	0.003	0.0268
0.0268	3.42	268235.4	0.003	0.0268

f values converged, therefore

$Q_1=3.42x5.027x10^{-3}x3600= 61.89\ m^3/hr$ (outflow)

The energy equation between (2) and (J) gives: $z_2=H_J+h_{f2}$

$h_{f2}= z_2-H_J=100-40=60$ m

$$60 = f\,2\frac{L_2}{D_2}\frac{V_2^2}{2g} = f_2\,\frac{150}{0.06}\,\frac{V_2^2}{2x9.81} \quad \text{and hence}\;\; V_2 = \sqrt{\frac{0.4709}{f_2}}$$

f_i	V_2 (m/s)	Re_2	ε/D	f_{i+1}
0.0234	4.49	264117.6	0.002	0.0243
0.0243	4.40	258823.5	0.002	0.0243

f values converged, therefore

Q_2=4.40x2.827x10^{-3}x3600= 44.78 m^3/hr (inflow)

- $H_J = z_3 \rightarrow Q_3 = 0$, Therefore equation of continuity at junction: $Q_1 = Q_2$, but
- $Q_1 = 61.89\ m^3/hr > Q_2 = 44.78\ m^3/hr$
- Therefore H_J must be reduced.
- Assume $H_J = 30$ m, and repeat the procedure.

- $H_J = z_1 + h_{f1} \rightarrow \; h_{f1} = H_J - z_1 = 30 - 20 = 10$ m

$$10 = f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = f_1 \frac{100}{0.08} \frac{V_1^2}{2x9.81} \quad \text{and hence } V_1 = \sqrt{\frac{0.157}{f_1}}$$

f_i	V_1 (m/s)	Re_1	ε/D	f_{i+1}
0.0268	2.42	189804	0.003	0.0270
0.0270	2.41	189019	0.003	0.0270

$Q_1 = 43.6$ m^3/hr

- The energy equation between (2) and (J) gives: $z_2=H_J+h_{f2}$
- $h_{f2}= z_2-H_J=100-30=70$ m

$$70 = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} = f_2 \frac{150}{0.06}\frac{V_2^2}{2x9.81} \quad \text{and hence} \quad V_2 = \sqrt{\frac{0.5494}{f_2}}$$

f_i	V_2 (m/s)	Re_2	ε/D	f_{i+1}
0.0243	4.75	279411	0.002	0.0242

$Q_2=48.4\ m^3/hr$

- The energy equation between (3) and (J) gives: $z_3=H_J+h_{f3}$
- $h_{f3}= z_3-H_J=40-30=10$ m

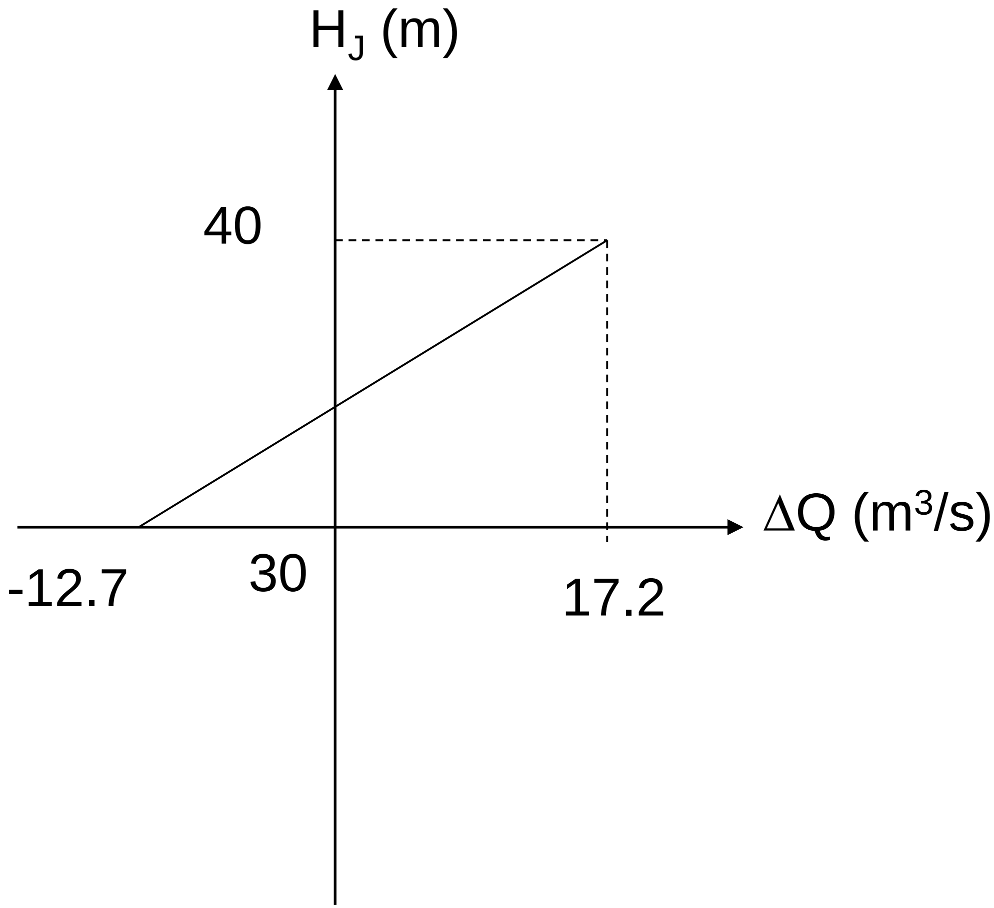
$$10 = f_3 \frac{L_3}{D_3}\frac{V_3^2}{2g} = f_3 \frac{80}{0.04}\frac{V_3^2}{2x9.81} \quad \text{and hence} \quad V_3 = \sqrt{\frac{0.0981}{f_3}}$$

f_i	**V_3 (m/s)**	**Re_3**	**ε/D**	**f_{i+1}**
0.0304	1.80	70588.24	0.005	0.0320
0.0320	1.75	68627.46	0.005	0.0320

Q_2=7.9 m^3/hr

- The equation of continuity at junction: $Q_1=Q_2+Q_3$,
- $Q_1=43.6\ m^3/hr$
- $Q_2+Q_3=48.4+7.9=56.3\ m^3/s > Q_1$
- $H_J > 30$ m.
- We can make interpolation:

$$H_J = 30 + \frac{40-30}{17.2-(\ \ 12.7)} x12.7 = 34.25$$



H_J (m)
40
ΔQ (m^3/s)
-12.7
30
17.2

- Assume H_J=34.5 m
- For pipe (1):

$$V_1 = \sqrt{\frac{0.2237}{f_1}}$$

f_i	V_1 (m/s)	Re_1	ε/D	f_{i+1}
0.0270	2.88	225757	0.003	0.0269

Q_1= 52.1 m^3/hr

For pipe (2):

$$V_2 = \sqrt{\frac{0.516}{f_2}}$$

f_i	V_2 (m/s)	Re_2	ε/D	f_{i+1}
0.0242	4.62	271765	0.002	0.0242

Q_2= 47 m^3/hr

For pipe (3):

$$V_3 = \sqrt{\frac{0.0564}{f_3}}$$

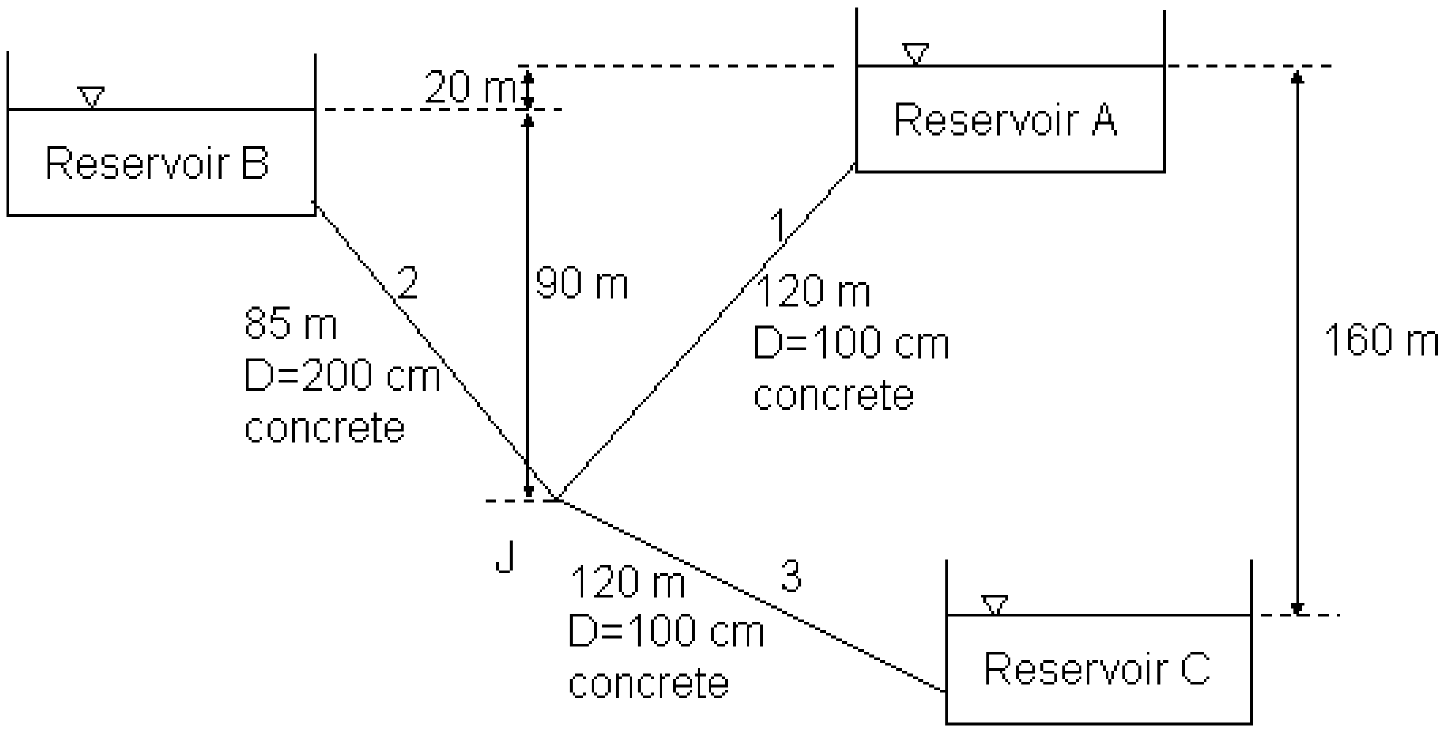
f_i	V_2 (m/s)	Re_2	ε/D	f_{i+1}
0.032	1.33	52157	0.005	0.0324
0.0324	1.32	51764	0.005	0.0324

Q_3= 5.97 m^3/hr

- The equation of continuity at junction: $Q_1=Q_2+Q_3$,
- $Q_1=52.1\ m^3/hr$
- $Q_2+Q_3=47+5.97=52.97\ m^3/hr \approx Q_1$
- $H_J= 34.5$ m.

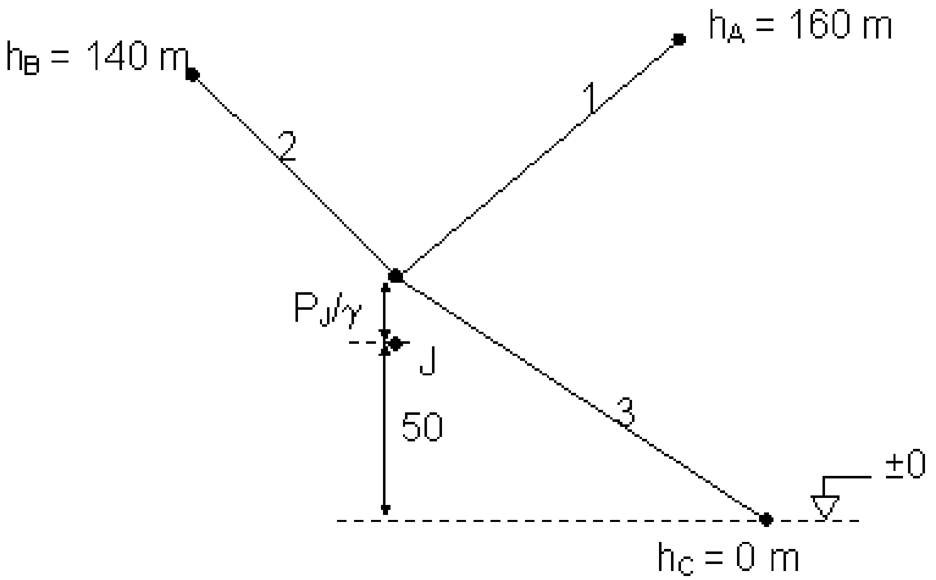
- % Error:(52.1-52.97)x100/(52.97)=1.6%

Example 2.7:



Reservoir B
Reservoir A
Reservoir C
20 m
90 m
160 m
1
120 m
D=100 cm
concrete
2
85 m
D=200 cm
concrete
J
3
120 m
D=100 cm
concrete

- Assume Q1 and Q2 “+” and Q3 “-“
- Neglect all minor losses



Energy conservation b/w A&J:	$h_A = h_J + h_{l,AJ} = h_J + h_{f,AJ}$
Energy conservation b/w B&J:	$h_B = h_J + h_{l,BJ} = h_J + h_{f,BJ}$
Energy conservation b/w J&C:	$h_J = h_C + h_{l,JC} = h_C + h_{f,JC}$

•Assume h_J and check if $Q_1+Q_2+Q3 = 0$. If not, iterate by assuming new h_J

•until $\sum Qi = 0$ checks.

- h_J = 100m (elevation+50 m of pressure head)

$$h_{f,AJ} = \frac{8fLQ_1^2}{\pi^2 g D_1^5} = 9.92\, f_1\, Q_1^{\,2}$$

$$h_{f,BJ} = 0.22\, f_2\, Q_2^{\,2}$$

$$h_{f,BJ} = 9.92\, f_3\, Q_3^{\,2}$$

hence b/w A and J $\quad 160 - 100 = 60 = 9.92\, f_1\, Q_1^{\,2}$
b/w B and J $\quad 140 - 100 = 40 = 0.22\, f_2\, Q_2^{\,2}$
b/w J and C $\quad 100 - 0 = 100 = 9.92\, f_3\, Q_3^{\,2}$

* Assume hydraulically rough flow, $f=f(\varepsilon/D)$

ε/D	f_i	Q_i
0.001	0.02	17.39
0.0005	0.018	100.50
0.001	0.02	22.57

17.39 + 100.50 >> 22.57
Therefore increase h_J

- h_J = 130m

f_i	Q_i
0.02	**12.30**
0.018	**50.25**
0.02	**25.59**

12.30 + 50.25 > 25.29
Therefore increase h_J

- h_J = 139m

f_i	Q_i
0.02	**10.29**
0.018	**15.89**
0.02	**26.47**

$10.29 + 15.89 = 26.18 \cong 26.47$