Reduced Matrix

Reduced matrix can be obtained by performing elementary row operations. A matrix is a reduced matrix if it satisfies the following conditions:

1) Reading from left to right, the first non-zero element in any non-zero row (if there exists at least one non-zero element in a row, it is called a non-zero row) must be 1.
2) All matrix elements directly above or below any leading entry (the leading entry of a non-zero row is first non-zero element reading from left to right) are zero.
3) All zero rows (if all the elements in a row are zero then it is a zero row) lie below all non-zero rows.
4) The leading entries move downward to the right as one looks at the matrix.

The following matrices are examples of reduced matrix.

$$\begin{bmatrix} 1 & -4 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 3 & 1 \\ 0 & 1 & 0 & -2 & 4 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Elementary Row Operations

Let A be a n x m matrix. We define 3 types of elementary row operations on rows of A.

1) Interchange 2 rows of A
2) Multiply a row of A by a non-zero scalar
3) Add a scalar multiple of one row to another row of A

Row Equivalence of Matrices

If A & B are n x m matrices, we say that A is a row equivalent of B if B can be obtained from A by a sequence of elementary row operations.

METU - CIVIL ENGINEERING DEPARTMENT