CE382 - HOMEWORK 2

1) a) $\sigma_c = 13200\,\varepsilon_c$

$N_c + N_{st} = 1500\ kN$

$N_c = \sigma_c . A_c$, $N_{st} = \sigma_{st} . A_{st}$

$\sigma_c = 13200\,\varepsilon$, $\sigma_{st} = 200000\,\varepsilon$

$N_c = (13200\varepsilon) \times (300 \times 300) = 1728000\,\varepsilon\ kN$

$N_{st} = (200000\varepsilon) \times (6 \times \frac{\pi 20^2}{4}) = 377000\,\varepsilon\ kN$

$2105000\,\varepsilon = 1500$

$\varepsilon = 7.126 \times 10^{-4}$

$\sigma_{st} = 200000\,\varepsilon = 200000 * 7.126 \times 10^{-4} = 142.5\ MPa < f_y$ ✓

$N_c = 1728000 * 7.126 \times 10^{-4} = 1231.4\ kN$

$N_{st} = 377000 * 7.126 \times 10^{-4} = 268.6\ kN$

b) $\sigma_c = 26480\,\varepsilon_c [1 - 221\,\varepsilon_c]$ $\varepsilon_c = \varepsilon_{st} = \varepsilon$

$N_c = (26480\,\varepsilon_c - 3642080\,\varepsilon_c^2) \times (300 \times 300) = 2383200\,\varepsilon - 327787200\,\varepsilon^2\ kN$

$N_{st} = 377000\,\varepsilon\ kN$

$-327787200\,\varepsilon^2 + 276200\,\varepsilon - 1500 = 0$

$\varepsilon_1 = 7.84 \times 10^{-3}$ ✗ ↳ it does not satisfy $\sigma_{st} < f_y$

$\varepsilon_2 = 5.84 \times 10^{-4}$. ✓

$N_c = 1280\ kN$

$N_{st} = 220\ kN$

c) Assuming linear-elastic behaviour for concrete provides less stress on it. In non-linear elastic behavior for same stress according to linear-elastic behaviour, it gains more strain.

2) $N_{or} = 0.85 f_{ck} A_c + A_{st} f_{yk}$

a) $N_{or} = 0.85 \times 20 \times (300 \times 300) + 8 \times \frac{\pi \times 12^2}{4} \times 420 = 1310\ kN$

b) $N_{or} = 0.85 \times 40 \times (300 \times 300) + 8 \times \frac{\pi \times 12^2}{4} \times 420 = 3440\ kN$

c) $N_{or} = 0.85 \times 20 \times (300 \times 300) + 8 \times \frac{\pi \times 24^2}{4} \times 420 = 3050\ kN$

d) $N_{or} = 0.85 \times 40 \times (300 \times 300) + 8 \times \pi \times \frac{24^2}{4} \times 420 = 4580\ kN$

e) High in concrete strength and having larger much more steel reinforcement area provides higher axial load capacity.

3) a) $f_{cc} = 0.85 f_c + 6\sigma_2$, $\sigma_2 = 2 f_{yw} \frac{A_o}{D.s}$

$N_{or} = N_{or2}$

$0.85 f_c A_c + f_y A_{st} = f_{cc} . A_{ck} + f_y A_{st}$

$0.85 f_c A_c + f_y A_{st} = 0.85 f_c A_{ck} + 6 \left(2 f_{yw} \frac{A_o}{D.s}\right) A_{ck} + f_y A_{st}$

$0.85 \times 35 \times (550 \times 300) + 420 \times 12 \times \frac{\pi 20^2}{4} = 0.85 \times 35 \times \left(2 \times \frac{\pi 250^2}{4}\right) + 6 \left(2 \times 420 \times \frac{\pi 12^2}{4} \times \frac{1}{250 \times s}\right) \times \left(2 \times \frac{\pi 250^2}{4}\right) + 420 \times 12 \times \frac{\pi 20^2}{4}$

$6432 = 4504 + \frac{223838.5}{s}$ — $s = 112.6\ mm$

b) $N_{or} = 6432\ kN$ & $N_{or2} = 6432\ kN$

4) a) i. for first spiral column $\quad \sigma_2 = \dfrac{2 A_o f_{yw}}{D S}$

$N_{or} = 0.85 \times 20 \times \dfrac{\pi 500^2}{4} + 8 \times \dfrac{\pi 20^2}{4} \times 420 = 4333.5 \text{ kN}$

$f_{cc} = 0.85 f_c + 4\sigma_2 = 0.85 \times 20 + 8 \times \dfrac{\pi 10^2}{4} \times \dfrac{220}{300 \times 40} = 28.5 \text{ MPa}$

$N_{or2} = 28.5 \times \dfrac{\pi 300^2}{4} + 8 \times \dfrac{\pi 20^2}{4} \times 420 = 3070 \text{ kN}$

ii. for second spiral column

$N_{or} = 0.85 \times 20 \times \dfrac{\pi 350^2}{4} + 8 \times \dfrac{\pi 20^2}{4} \times 420 = 2691 \text{ kN}$

$f_{cc} = 28.5$ MPa (from part i)

$N_{or2} = 28.5 \times \dfrac{\pi 300^2}{4} + 8 \times \dfrac{\pi 20^2}{4} \times 420 = 3070 \text{ kN}$ (same as part i because core area is not changed.)

iii. for third spiral column

$N_{or} = 0.85 \times 20 \times (350 \times 350) + 8 \times \dfrac{\pi 20^2}{4} \times 420 = 3138 \text{ kN}$

$N_{or2} = 28.5 \times \dfrac{\pi 300^2}{4} + 8 \times \dfrac{\pi 20^2}{4} \times 420 = 3070 \text{ kN}$ (spiral core area is not changed, then N_{or2} is not changed)

b) i. $N_{or2} = 1.2 N_{or} = 4333.5 \times 1.2 = 5272.2 \text{ kN}$

$5272.2 = f_{cc} \times \dfrac{\pi 300^2}{4} \times \dfrac{1}{1000} + 1055.6 \rightarrow f_{cc} = 59.6 \text{ MPa}$

$53.6 = 0.85 \times 20 + 8 \times \dfrac{\pi 10^2}{4} \times \dfrac{220}{300 \times S} \rightarrow S = 10.8 \text{ mm}$

ii. $N_{or2} = 1.2 N_{or} = 3229.2 \text{ kN}$

$3229.2 = f_{cc} \times \dfrac{\pi 300^2}{4} \times \dfrac{1}{1000} + 1055.6 \rightarrow f_{cc} = 30.75 \text{ MPa}$

$30.75 = 0.85 \times 20 + 8 \times \dfrac{\pi 10^2}{4} \times \dfrac{220}{300 \times S} \rightarrow S = 33.5 \text{ mm}$

iii. $N_{or2} = 1.2 N_{or} = 3765.6 \text{ kN}$

$3765.6 = f_{cc} \times \dfrac{\pi 300^2}{4} \times \dfrac{1}{1000} + 1055.6 \rightarrow f_{cc} = 38.3 \text{ MPa}$

$38.3 = 0.85 \times 20 + 8 \times \dfrac{\pi 10^2}{4} \times \dfrac{220}{300 \times S} \rightarrow S = 21.6 \text{ mm}$

5) a) $\varepsilon_{st} = \varepsilon_c = \varepsilon$ $\quad$ Hognested model $\quad \sigma_c = f_c \left[\dfrac{2\varepsilon_c}{\varepsilon_{co}} - \left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right)^2 \right]$

$\sigma_{st} = 200000\,\varepsilon$

$\sigma_c = 8$ MPa

$E_c = 12680 + 460 f_c = 28780 \text{ MPa}$

$\varepsilon_{co} = \dfrac{2 f_c}{E_c} = 2.43 \times 10^{-3}$

$\sigma_c = 8 = 35 \left[\dfrac{2\varepsilon}{2.43 \times 10^{-3}} - \left(\dfrac{\varepsilon}{2.43 \times 10^{-3}}\right)^2 \right] \rightarrow \varepsilon = 3 \times 10^{-4}$

$\sigma_{st} = 60$ MPa, $\sigma_c = 8$ MPa

$N = 8 \times (400 \times 600) + 60 \times \left(10 \times \dfrac{\pi 20^2}{4}\right) = 2108.5 \text{ kN}$

b) $\sigma_c = 28$ MPa, $E_c = 28780$ MPa, $\varepsilon_{co} = 2.43 \times 10^{-3}$

$28 = 35 \left[\dfrac{2\varepsilon}{2.43 \times 10^{-3}} - \left(\dfrac{\varepsilon}{2.43 \times 10^{-3}}\right)^2 \right] \rightarrow \varepsilon = 1.34 \times 10^{-3}$

$\sigma_{st} = 268$ MPa, $\sigma_c = 28$ MPa

$N = 8 \times (400 \times 600) + 268 \times 10 \times \dfrac{\pi 20^2}{4} = 7562 \text{ kN}$

c) $\sigma_{st} = 420$ MPa

$420 = \varepsilon_{st} \times 200000 \rightarrow \varepsilon_{st} = 2.1 \times 10^{-3}$

$\sigma_c = 35 \left[\dfrac{2 \times 2.1 \times 10^{-3}}{2.43 \times 10^{-3}} - \left(\dfrac{2.1 \times 10^{-3}}{2.43 \times 10^{-3}}\right)^2 \right] = 34.35 \text{ MPa}$

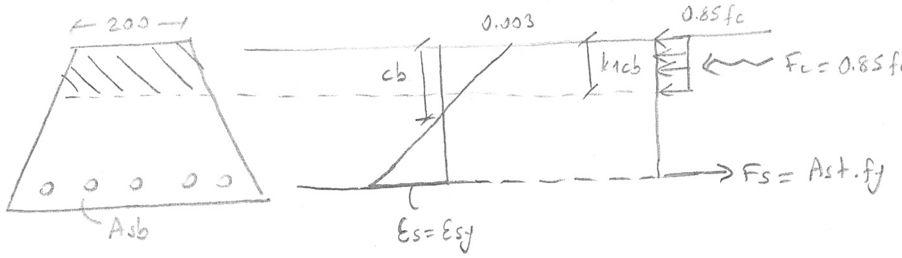
$N = 34.35 \times (400 \times 600) + 420 \times 10 \times \dfrac{\pi 20^2}{4} = 9563.5 \text{ kN}$

d) $N_{or} = 35 \times (400 \times 600) + 420 \times \left(10 \times \dfrac{\pi 20^2}{4}\right) = 9719.5 \text{ kN}$

e) $f_{ct} = 0.35 \sqrt{f_c}_{\rightarrow \text{MPa}} = 2.1 \text{ MPa}$

$N = 2.1 \times (400 \times 500) = 504 \text{ kN}$

6)



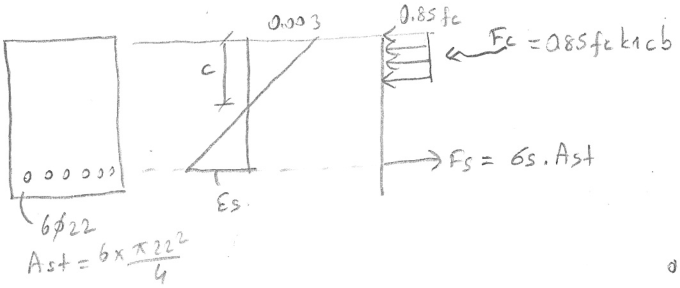
$\varepsilon_{sy} = 0.0021$ (S420) → clear cover is 40 mm.

$$\frac{0.003}{0.0021} = \frac{c_b}{560 - c_b} \qquad c_b = 323.4 \text{ mm}$$

$$F_c = 0.85 \times 25 \times \left(\frac{200 + 386.7}{2} \times 230\right) = 1745.4 \text{ kN}$$

$$1745.4 \times 10^3 = A_{st} \times 420 \longrightarrow A_{st} = 4156 \text{ mm}^2$$

7)



assume it is underreinforced $\sigma_s = f_y$

$$\frac{c}{660} = \frac{0.003}{0.003 + \varepsilon_s}$$

$0.85 f_c k_1 c b = A_s f_y$

$0.85 \times 30 \times 0.85 \times c \times 400 = 6 \times \frac{\pi 22^2}{4} \times 420$

$c = 110.5$ mm

$\varepsilon_s = 0.015$

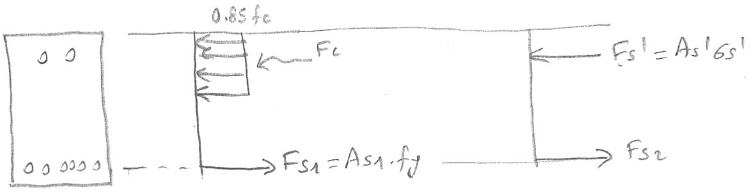
$\varepsilon_{sy} = 0.0021$.

$\varepsilon_s > \varepsilon_{sy}$

assumption is correct. ✓

$$M_r = A_{st} \cdot f_y \cdot jd = 6 \times \frac{\pi 22^2}{4} \times 420 \times \left(660 - \frac{0.85 \times 110.5}{2}\right) \qquad M_r = 587.3 \text{ kN.m}$$

8)



assume $\sigma_s' = f_y$

$A_s' \cdot \sigma_s' = A_{s2} \cdot f_y \rightarrow A_s' = A_{s2}$

$A_s' = 2 * \frac{\pi 26^2}{4} = 1062 \text{ mm}^2$

$A_s = A_{s1} + A_{s2} \rightarrow A_{s1} = 6 \times \frac{\pi 22^2}{4} - 1062 = 1219 \text{ mm}^2$

$0.85 \times 30 \times 0.85 \times c \times 400 = 1219 \times 420 \qquad c = 59$ mm

$$\frac{0.003}{c} = \frac{\varepsilon_s'}{c - 40} \qquad \varepsilon_s' = 0.001 < 0.021$$

assumption is wrong X

assume $\sigma_s = f_y$

$$\frac{0.003}{c} = \frac{0.003 + \varepsilon_s}{660} \qquad \frac{0.003}{c} = \frac{\varepsilon_s'}{c - 40}$$

$$(0.85 \times 30 \times 0.85 \times c \times 400) + \left[2 \times \frac{\pi 26^2}{4} \times 200000 \times \left(\frac{0.003 \times (c - 40)}{c}\right)\right] = 6 \times \frac{\pi 22^2}{4} \times 420$$

$$8760c + 637115 - \frac{25484600}{c} = 357934 \longrightarrow c = 75.79 \text{ mm}$$

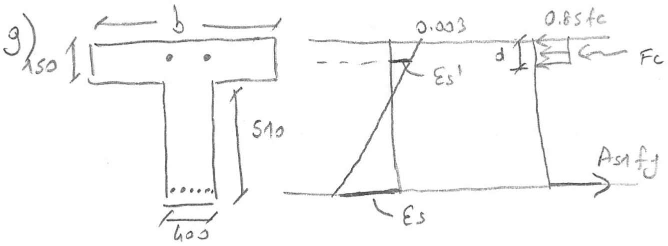
$\varepsilon_s' = 1.62 \times 10^{-3}$

$\varepsilon_s = 0.023 > 0.021$ assumption is correct ✓

$$M_r = \left[(0.85 \times 30 \times 0.85 \times 75.79 \times 400) \times \left(600 - \frac{0.85 \times 75.79}{2}\right)\right] + \left[2 \times \frac{\pi 26^2}{2} \times 284 \times (660 - 40)\right]$$

$M_r = 593.5$ kN.m

9)



assume $\sigma_s' = f_y$ and $a < h_f$

$A_s = A_{s1} + A_{s2}$

$A_s' f_y = A_{s2} f_y \longrightarrow A_s' = A_{s2} = 2 * \frac{\pi 26^2}{4} = 1062 \text{ mm}^2$

$A_{s1} = 6 * \frac{\pi 22^2}{4} - 1062 = 1219 \text{ mm}^2$

From force equilibrium $\longrightarrow F_c = 1219 \times \frac{420}{1000} = 512$ kN

$M_r = 593.5$ kN.m (from question 8)

$$M_r = F_c \times \left(660 - \frac{a}{2}\right) + 1062 \times 420 \times (660 - 40)$$

$a = 58.4$ mm $\qquad c = 68.7 \text{ mm} < h_f$ ✓

$$\frac{0.003}{c} = \frac{\varepsilon_s'}{c - 40} \longrightarrow \varepsilon_s' = 0.00125 < \varepsilon_{sy}$$

assumption is wrong X

assume $a < h_f$

$$\frac{0.003}{c} = \frac{\varepsilon_s'}{c - 40} \qquad \varepsilon_s' = 0.003 - \frac{0.12}{c} \qquad \sigma_s' = 600 - \frac{24000}{c}$$

$$(0.85 \times 30 \times 0.85c \times b) + \left[1062 \times \left(600 - \frac{24000}{c}\right)\right] = 2281 \times 420 \qquad (1)$$

$$\left[(0.85 \times 30 \times 0.85c \times b) \times \left(660 - \frac{0.85c}{2}\right)\right] + \left[\left(1062 \times \left(600 - \frac{24000}{c}\right)\right) \times (660 - 40)\right] = 593 \times 10^6 \qquad (2)$$

Solving (1) & (2) together:

$c = 73.51$ mm

$b = 419$ mm