RULES

1. This is the **version 7.0**. In case there are any corrections for the solutions of Exercise 7, we will post an updated version on our website. You can follow the changes in the exercises by the **Version History** section below.

Version History

V7.0 Solutions of Exercise 7 are released.

1. Let

$$\frac{dy}{dz} = t = g = g(t)$$
$$\frac{dt}{dz} = \frac{f(z)}{2EI}(L-z)^2 = h = h(z)$$

Thus initial conditions will be:

$$y_0 = y(0) = 0$$
$$t_0 = t(0) = 0$$

Predictors can be written as:

$$y_{i+1}^* = y_i + stepsize * g(z_i, y_i, t_i)$$
$$t_{i+1}^* = t_i + stepsize * h(z_i, y_i, t_i)$$

Correctors can be written as:

$$y_{i+1} = y_i + stepsize * \frac{g(z_i, y_i, t_i) + g(z_{i+1}, y_{i+1}^*, t_{i+1}^*)}{2}$$
$$t_{i+1} = t_i + stepsize * \frac{h(z_i, y_i, t_i) + h(z_{i+1}, y_{i+1}^*, t_{i+1}^*)}{2}$$

Define the function $h(z)$ since it is a complicated one:

```
function out=h(z)
L=30;
E=1.25*10^6;
I=0.05;
f=200*z*exp(-2*z/30)/(5+z);
out=f*((L-z)^2)/(2*E*I);
end
```

The following code performs the numerical solution using Heun's method (one-cycle):

```
clear all
clc
format long
initial=0;
final=30;
stepsize=0.5;
n=(final-initial)/stepsize;
z=(0:0.5:30)'; %z started from 0 up to L=30.
y(1,1)=0; %initial condition
t(1,1)=0; %initial condition
for i=1:n
   %predictors
   y_star(i+1,1)=y(i,1)+stepsize*t(i,1);
   t_star(i+1,1)=t(i,1)+stepsize*h(z(i,1));
   %correctors
   y(i+1,1)=y(i,1)+stepsize*(t(i,1)+t_star(i+1,1))/2;
   t(i+1,1)=t(i,1)+stepsize*(h(z(i,1))+h(z(i+1,1)))/2;
end
```

Finally deflection can be found as $\frac{d^2y}{dz^2} = t(30) = 4.0960$.

2. O(h^2) finite difference formula is Central Difference Formula. Using this formulation:

$$\frac{d^2x}{dt^2} = \frac{x_{i+1} - 2x_i + x_{i-1}}{(\Delta x)^2}$$

$$\frac{dx}{dt} = \frac{x_{i+1} - x_{i-1}}{2\Delta x}$$

Replacing the terms in the ODE with the given formulas:

$$\frac{x_{i+1} - 2x_i + x_{i-1}}{(\Delta x)^2} + 3*\frac{x_{i+1} - x_{i-1}}{2\Delta x} + 4*x_i = 0$$

For i=1, 2 and 3, the following equations can be formed. Note that $\Delta x = 0.5$.

i=1 $$\frac{x_2 - 2x_1 + x_0}{0.5^2} + 3*\frac{x_2 - x_0}{2*0.5} + x_1 = 0$$

i=2 $$\frac{x_3 - 2x_2 + x_1}{0.5^2} + 3*\frac{x_3 - x_1}{2*0.5} + x_2 = 0$$

i=3 $$\frac{x_4 - 2x_3 + x_2}{0.5^2} + 3 * \frac{x_4 - x_2}{2 * 0.5} + x_3 = 0$$

Simplifying the equations we will get,

$$-7x_1 + 7x_2 = -3$$
$$x_1 - 7x_2 + 7x_3 = 0$$
$$x_2 - 7x_3 = 14$$

Solving these equations simultaneously,

$$x_1 = x(1.0) = -2.9643$$
$$x_2 = x(1.5) = -3.3929$$
$$x_3 = x(2.0) = -2.9694$$