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4 AXIALLY LOADED MEMBERS

4.1 MEMBERS SUBJECTED TO AXIAL COMPRESSION (COLUMNS)

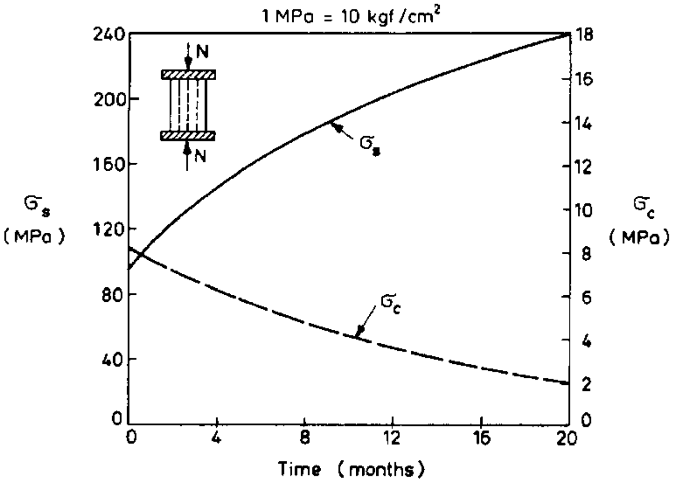
The columns are vertical members, which support the floor loads, transferred either directly or through beams. Their function is to resist the floor loads and other action effects (shear, moment, torsion produced by lateral loads, settlement, shrinkage etc.) and transfer them to the foundation.

Uniaxially loaded compression members are not permitted in the design codes. Although this is the case, the behavior and strength of reinforced concrete members under uniaxial compression will be discussed, since it represents a limiting case for combined flexure and axial compression.

4.2 Behavior of Axially Loaded Columns

Early in 20^{th} century engineers realized that the stresses in steel and concrete of a column subjected to sustained loads could not be calculated with reasonable accuracy.

In about 1930, a very extensive research project on reinforced concrete columns was initiated at Universities of Illinois and Lehigh.



1 MPa = 10 kgf/cm²
240
200
160
120
80
40
0
18
16
14
12
10
8
6
4
2
0
N
N
σs
σc
σs
(MPa)
σc
(MPa)
0
4
8
12
16
20
Time (months)

As a result of these tests, it was concluded that the stresses in axially loaded reinforced concrete columns cannot be calculated with reasonable accuracy using the elastic theory, because such stresses were significantly influenced by time dependent deformations.

The same test results revealed that although the stresses cannot be computed, the ultimate strength of an axially loaded column could be computed with reasonable accuracy. In the final report, it was pointed out that the ultimate capacity could be computed by summing the internal force carried by concrete and the internal force carried by steel at the onset of ultimate limit state.

In these studies, it was also found out that the column do not fail until both materials reach their limiting values, regardless of loading history.

If the steel reaches its yield strength first, further straining of the column section causes increase in concrete stress until it reaches its peak strength.

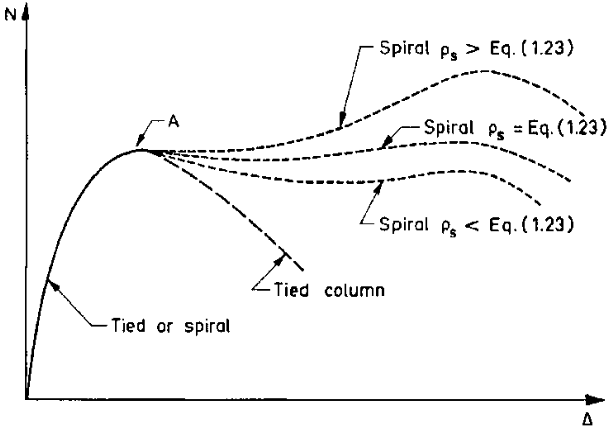
If the concrete approaches its ultimate strength before the steel reaches its yield strength, the increased deformations in concrete near the maximum stress force the steel stresses to build up more rapidly until the yield strength of steel is reached.

The conclusion about the ultimate strength of axially loaded columns reached in 1932 was not original, it was just a return to the forgotten principles. At the turn of 19th century, A French scientist A. Considére stated that, "*The ultimate strength of tied columns varied little from the sum of the resistances offered by the crushing strength of concrete and yield strength of longitudinal steel*".

Another important finding of this study was that the strength of concrete in the columns was about 85% that of the concrete in a standard test cylinder, probably due to slower application of the load, size of the specimens and better compaction in the test cylinder.

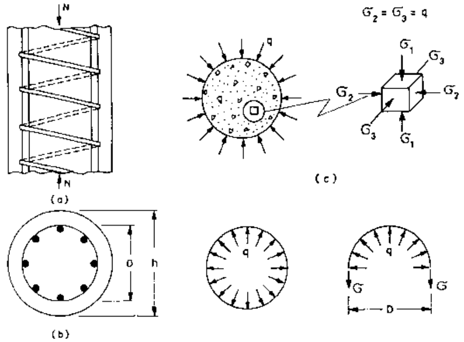
During Lehigh and Illinois tests, it was observed that up to the maximum strength of the column, tied and spiral columns behaved almost identically and the spirals added nothing to the strength. Therefore tied and spiral columns having the same cross-sections and longitudinal steel areas have equal maximum strengths.

The responses of these two columns, however, differ from each other once the cover concrete starts to crush!



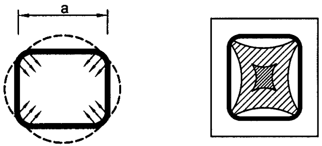
Point A: Both concrete and steel reach their limiting strengths, i.e. $\sigma_c = f_c$, $\sigma_s = f_y$.

Beyond Point A: Crushing of cover concrete starts. Under increasing axial strains developing within the column, Poisson's ratio increases considerably and the core concrete tries to expand in the lateral direction. This tendency toward lateral expansion is prevented by the ties or spiral at varying extents. Ties cannot provide a very strong resistance against this action, while spirals do.

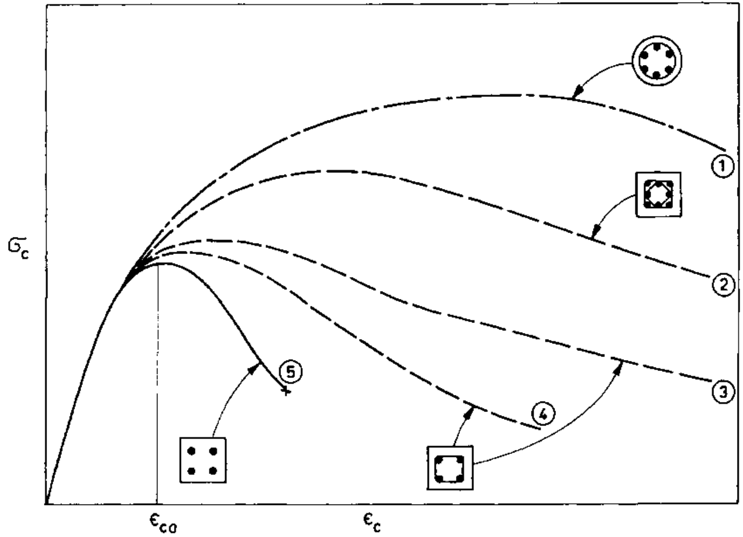


In case of spiral columns, this interaction between the steel and concrete generates **axial tensile** stresses in the spiral and radial compressive stresses in concrete (confinement).

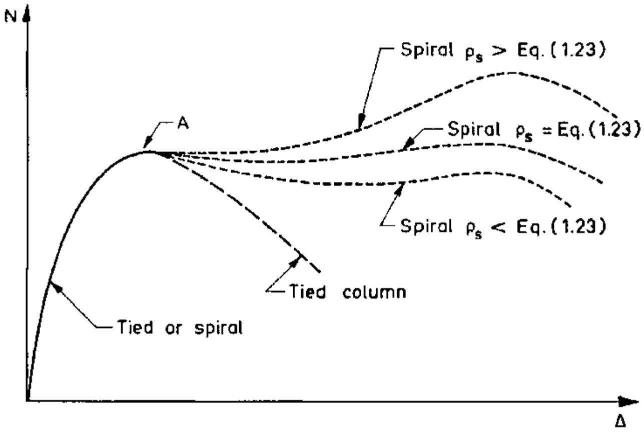
In this case, the lateral expansion of the core concrete (i.e. the change in its diameter) is proportional to the total elongation of the spiral steel, which is obviously proportional to the axial rigidity of the spiral steel.



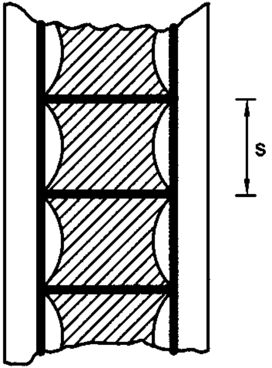
In case of tied columns, the interaction between the steel and concrete also generates **tensile stresses** in the tie rod. This time, however, the lateral expansion of the core is opposed by the flexural action of the tie steel. The tie steel between two longitudinal bars acts like a simply supported beam subjected to transverse loading. Evidently the resulting deformations will be proportional with the flexural rigidity of the tie bar. Which is fairly small compared to its axial rigidity. For this reason, the confinement provided by the rectangular hoop is not as effective as the circular hoops or the spiral.



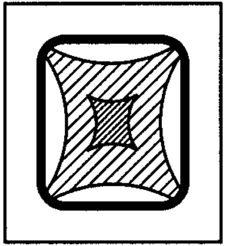
σ_c
ϵ_{co}
ϵ_c
1
2
3
4
5



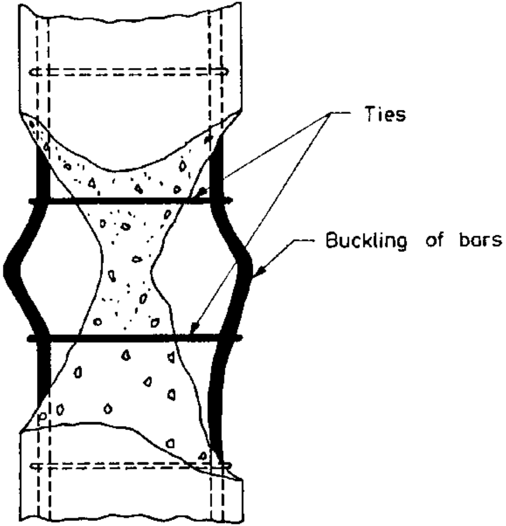
As the column loses strength due to the spalling of the cover, it starts to gain strength due to the increased strength of core concrete, caused by the confinement. Therefore, at the transition state some load drop beyond point A due to spalling of cover concrete is possible. However, it starts to regain strength as the spirals start to become effective. As a result of this, a second peak is reached in the N-Δ curve of spiral columns, as shown in Fig. 4.2.



(b)



(c)



4.3 STRENGTH OF UNCONFINED CONCRETE COLUMNS UNDER UNIAXIAL COMPRESSION

We said that the ultimate strength of a uniaxially loaded column can be expressed as the sum of strengths of concrete and the longitudinal reinforcement. We also see that the strength of tied and spiral columns is same at the first peak. The strength at the first peak, for both tied and spiral columns is the concrete area multiplied by the compressive strength of concrete, plus yield strength of steel times the total area of the longitudinal bars. Concrete strength is multiplied by 0.85 to reflect the difference between the strengths of the column and 150×300 standard cylinder.

$$N_{or} = 0.85\, f_{ck}\, (A_c - A_{st}) + A_{st}\, f_{yk} \quad \text{or simply } N_{or} = 0.85\, f_{ck}\, A_c + A_{st}\, f_{yk}$$

4.4 STRENGTH OF CONFINED CONCRETE COLUMNS UNDER UNIAXIAL COMPRESSION

Remember, based on the tests made by Richart at Illinois University back in 1928, Zia and Cowan suggested the following expression for the confined strength of concrete.

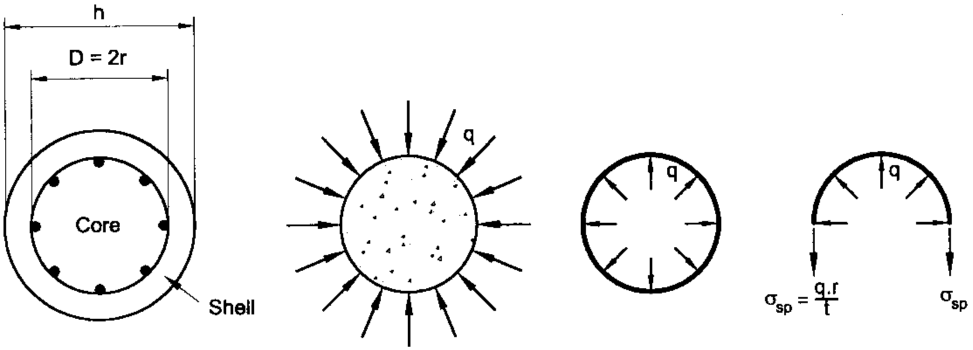
$$f_{c1} = f_{cc} = f_{ck} + 4\sigma_2$$

f_{cc} or f_{c1} strength of concrete under triaxial compression ($\sigma_2=\sigma_3$)

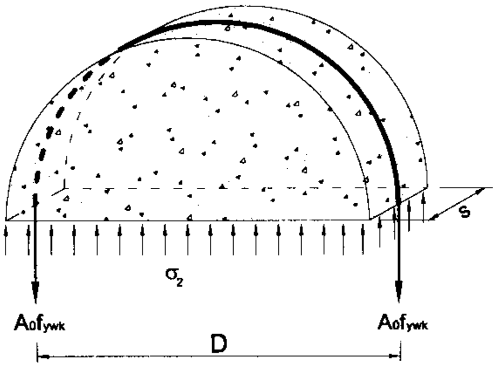
f_{ck} strength of concrete under uniaxial compression

σ_2 uniform lateral pressure ($\sigma_2=\sigma_3$)

Let us drive an expression for the strength of concrete in the core of a spiral column



a) Section b) Core Concrete c) Spiral d) Spiral Stress



Using the thin tube analogy with the followings in mind we have:

$$\sigma_{sp} = \frac{qr}{t}$$

If "r" is replaced by D/2, where "D" is the core diameter, "q" by "σ_2" and "t" by A_0/s, Eq. (4.4) can be rewritten for the spiral column.

$$\sigma_{sp} = \frac{\sigma_2 \times D \times s}{2A_o}$$

In this equation A_o is the cross-sectional area of the spiral and "s" is the spacing. At the limit state the stress in the spiral should reach the yield strength (σ_{sp}=f_{ywk}). Substituting f_{ywk} for σ_{sp} and solving for σ_2 , the following relationship is obtained.

$$\sigma_2 = \frac{2A_o f_{ywk}}{D(s)}$$

Consequently, the second peak strength of the spiral column can be determined as:

$N_{or2} = (0.85\ f_{ck} + 4\sigma_2)\ A_{ck} + A_{st}\ f_{yk}$

- the gross area of concrete (A_{ck}) by the core area A_{ck} (since cover is lost)
- the unconfined concrete strength term ($0.85f_{ck}$) by the strength of confined concrete $f_{cc} = 0.85f_{ck} + 4\sigma_2$.

2ND PEAK EXPRESSION FOR IDEALLY REINFORCED SPIRAL COLUMN

Going back to Fig. 4.2, it was stated that the strength at the second peak depends on the amount and yield strength of the spiral used. The ratio of the spiral is expressed as the volumetric ratio and is defined by the volume of spiral over a height of “s”, divided by the volume of core concrete bounded by “s”.

$$\rho_s = \frac{A_o \pi D}{\left(\pi D^2 / 4 \right) \times s} = \frac{4A_o}{Ds}$$

It is logical to assume that the strength at the second peak, shown in Fig. 4.2, should not be less than the strength at the first peak, marked as A. Then the minimum ratio of spiral reinforcement which will make the strength at the two peaks equal, can be found by equating the strength lost past the first peak, to the strength gained due to the confinement. The strength lost is the strength of the concrete cover lost.

$$-\Delta F = 0.85 f_{ck} \left(A_c - A_{ck} \right)$$

Concrete strength is multiplied by 0.85 since the strength in the column is less than that of the test cylinder as discussed previously. A_c is the gross concrete area and A_{ck} is the core area.

Strength gained due to confinement in the core concrete can be written as.

$$+\Delta F = (f_{cc} - f_{ck})\, A_{ck}$$

Substituting f_{cc} given in Eq. (4.3) into the above equation,

$$+\Delta F = 4\sigma_2\, A_{ck}$$

Taking σ_2 from previously derived expression;

$$+\Delta F = \frac{8 A_o f_{ywk}}{D \times s} A_{ck}$$

The following relationship is obtained when Eq. (4.13) and (4.14) are equated

$$0.85f_{ck}\left(A_c - A_{ck}\right) = \frac{8A_o}{D \times s} A_{ck} f_{ywk}$$

Substituting ρ_s for $4A_0/(Ds)$ from Eq. (4.12) results in the following equation for the minimum spiral volumetric ratio which will make the strength at the second peak equal to the strength at the first peak

$$\min \rho_s = 0.425 \frac{f_{ck}}{f_{ywk}} \left(\frac{A_c}{A_{ck}} - 1 \right)$$

In the TS500-2000 and the ACI-318-95, Eq. (4.16) is given as the minimum spiral reinforcement; however the constant is taken as 0.45 instead of 0.425

$$\min \rho_s = 0.45 \frac{f_{ck}}{f_{ywk}} \left(\frac{A_c}{A_{ck}} - 1 \right)$$

In the codes a second equation is given for the minimum spiral, because as A_c/A_{ck} ratio approaches 1.0, minimum spiral ratio in above expression will approach zero! The second equation is given below.

$$\min \rho_s = 0.12\left(\frac{f_{ck}}{f_{ywk}}\right) \tag{4.18}$$

Minimum spiral ratio will be the greater value obtained from Eqs. (4.17) and (4.18).