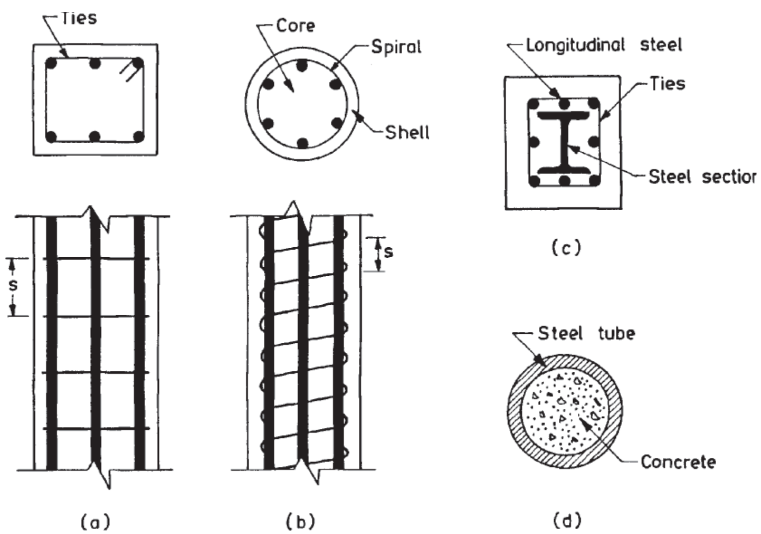
CE 382 Reinforced Concrete Fundamentals

Uniaxial Loading

Introduction

- Columns
  - Vertical members which support floor loads transferred either directly or through beams
  - Uniaxial compression is rarely possible (+bending moment, torsion etc.)
  - Use steel reinforcement for
    - Axial forces, bending
    - Time dependent deformations
    - Settlement
    - Confinement

Axial Load Bearing Members



Elastic Theory

- Transformed Area Concept for Reinforced Concrete member (concrete + steel)
- Assume both steel and concrete are linearly elastic
- Transform total steel area, A_{st}, into equivalent concrete area by multiplying modular ratio, $n = E_s/E_c$
- Net concrete area, $A_{cn} = A_c - A_{st}$
- Total longitudinal steel area, A_{st}

Elastic Theory

- Equilibrium: $N = \sigma_c(A_c - A_{st}) + \sigma_s A_{st}$
- Compatibility: $\varepsilon_c = \varepsilon_s = \varepsilon$
- Force deformation: $\varepsilon_c = \frac{\sigma_c}{E_c}$ & $\varepsilon_s = \frac{\sigma_s}{E_s}$

N: axial load

σ_c: stress in concrete

σ_s: stress in steel

A_c: cross-sectional area of concrete (gross)

A_{st}: cross-sectional area of longitudinal steel (total)

E_c, E_s: modulus of elasticity of concrete and steel

$\varepsilon_c, \varepsilon_s$: strain in concrete and steel

Elastic Theory

- Force deformation into compatibility

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c} \rightarrow \sigma_s = \frac{E_s}{E_c}\sigma_c = n\sigma_c$$

- From equilibrium

$$\sigma_c = \frac{N}{(A_c - A_{st}) + nA_{st}} \quad \& \quad \sigma_s = \frac{N \cdot n}{(A_c - A_{st}) + nA_{st}}$$

$$\rho_t = \frac{A_{st}}{A_c}$$

$$\sigma_c = \frac{N/A_c}{1 + \rho_t(n-1)} \quad \& \quad \sigma_s = n\frac{N/A_c}{1 + \rho_t(n-1)}$$

  - Valid only in the elastic range
  - Do not reflect time dependent deformations or ultimate strength

Redistribution (material-to-material)

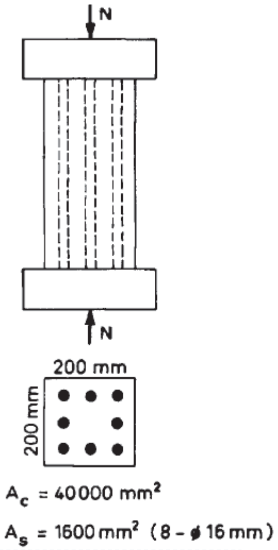
- Uniaxially loaded member
- Composite member (steel and concrete)

- Equilibrium:

$$N = N_c + N_s = A_c\sigma_c + A_s\sigma_s$$

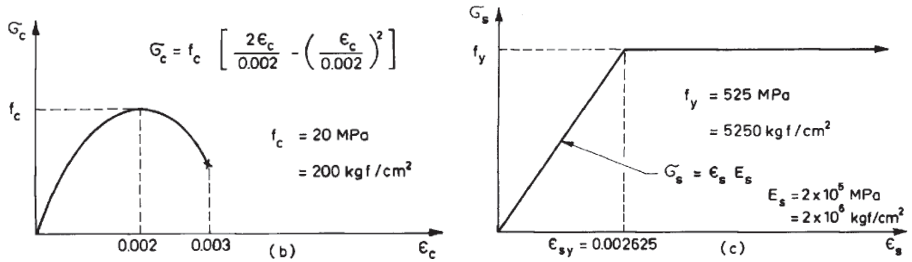
- Compatibility:

$$\varepsilon_c = \varepsilon_s = \varepsilon$$



Redistribution

- Force-deformation relations
  - If both materials were linearly elastic: $\sigma_c = \varepsilon_c E_c$ & $\sigma_s = \varepsilon_s E_s$
  - More realistic stress-strain relations:

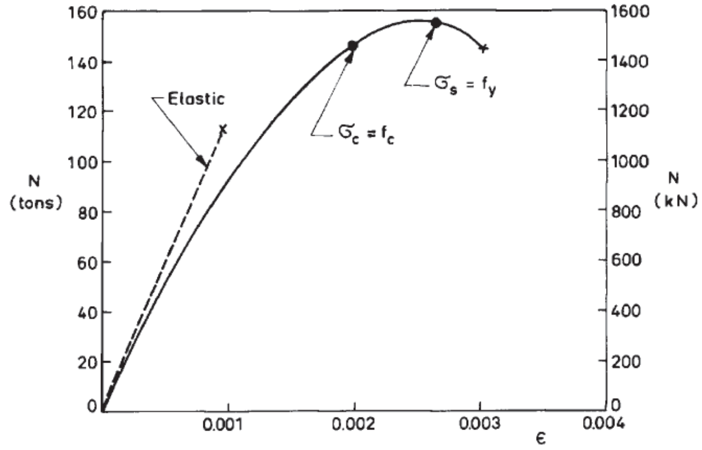


Redistribution

- $\varepsilon = 0.001 \;\rightarrow\; \sigma_c = 20\left\{\frac{2\times 0.001}{0.002} - \left(\frac{0.001}{0.002}\right)^2\right\} = 15$ MPa
- $N_c = \sigma_c A_c = 15 \times 40000 = 600$ kN
- $N_s = \sigma_s A_s = 0.001 \times 2 \times 10^5 \times 1600 = 320$ kN

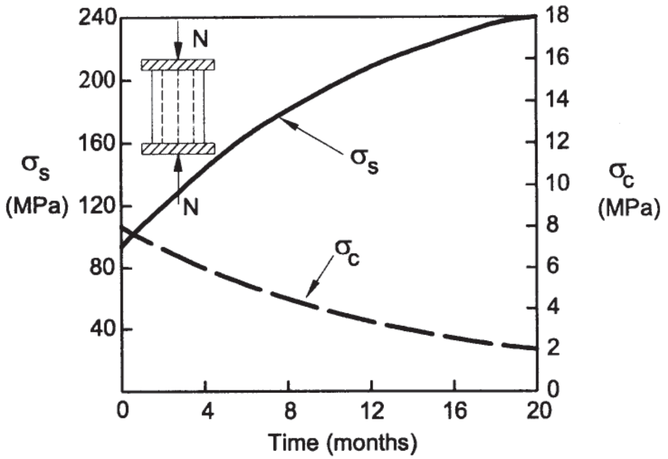
ε	N (kN)	N_c (kN)	N_s (kN)	N_c/N_s
0.001	920	600	320	1.87
0.002	1440	800	640	1.25
0.00262	1562	722	840	0.86
0.003	1440	600	840	0.71

Redistribution



- Reaching ultimate strength for concrete does not mean failure
- Load sharing between concrete and steel
- Calculations exclude time dependent deformations

Redistribution due to time dependent deformations



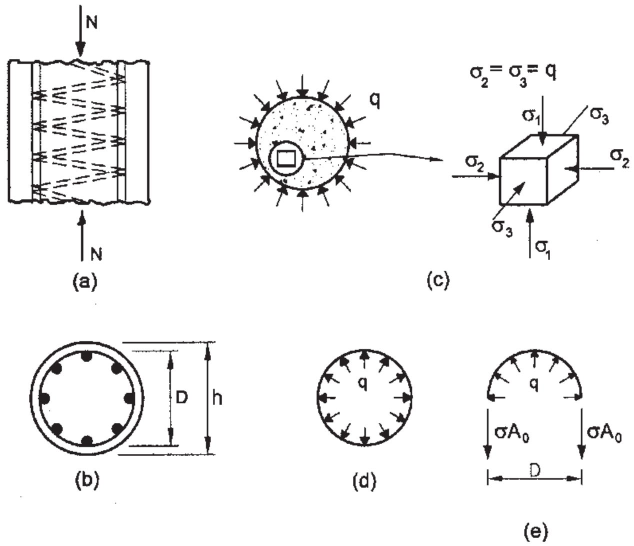
Behavior of Axially Loaded Members

- Stress and deformation computations are difficult due to unknowns associated with time dependency
- Ultimate strength can be computed with reasonable accuracy
- Under uniaxial compression:
  - Steel may yield prior to concrete crushing
  - Concrete may crush prior to steel yielding
  - Column fails if both material reach their limiting values

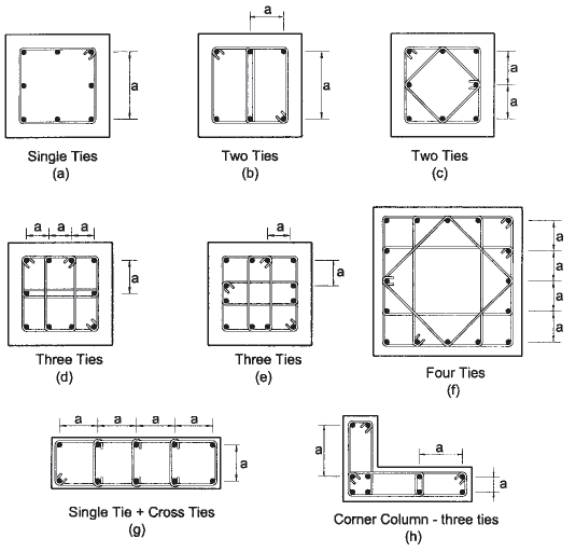
Behavior of Axially Loaded Members

- Strength of RC column ≈ 0.85 × cylinder compressive strength
  - Slower application of load in real life
  - Size effect (specimen size / aggregate size)
  - Shape effect
  - Compaction difference
  - Curing conditions
- When computing strength of a column under uniaxial loading → apply **0.85** factor

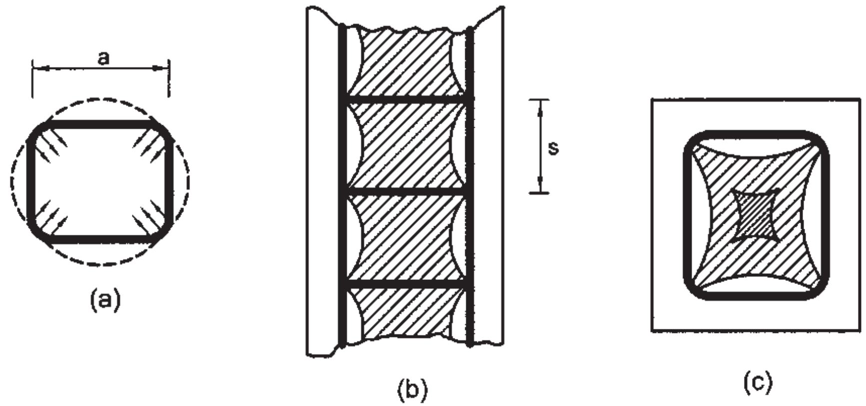
Confinement – continuous spirals



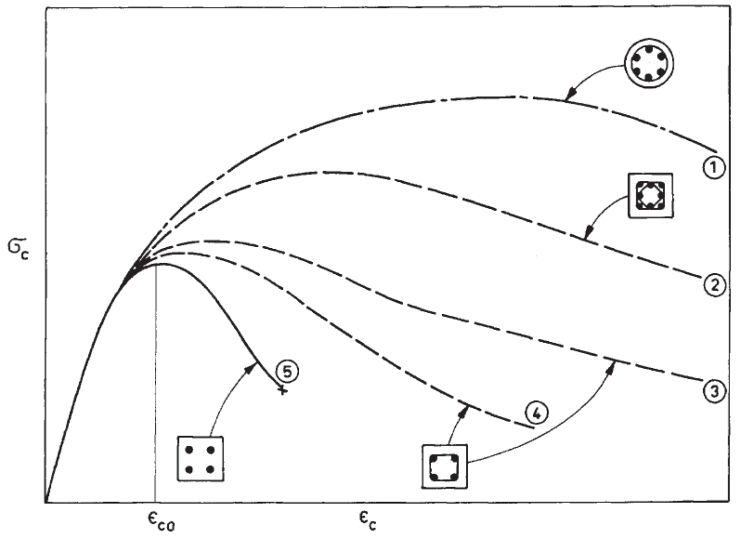
Confinement – rectangular hoops



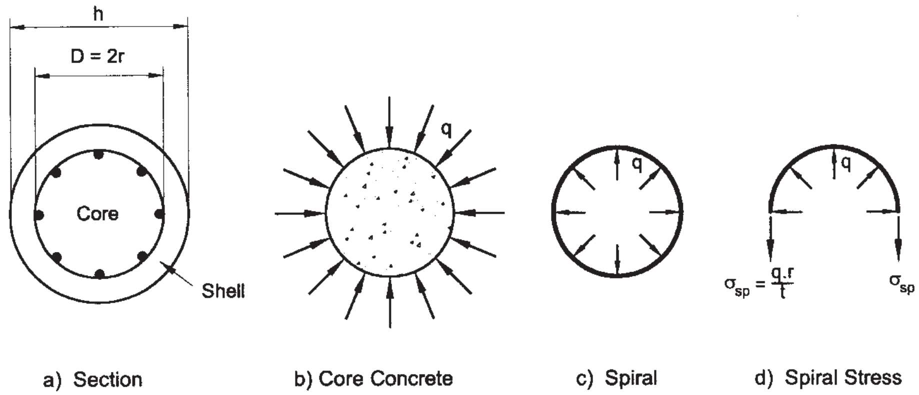
Confinement effect



Comparison of confinement efficiency

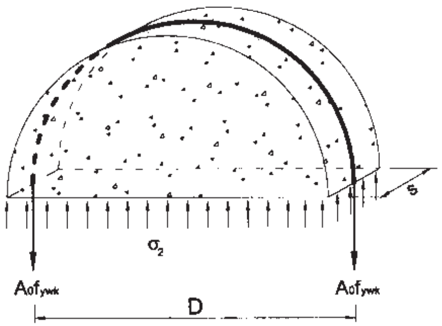


Confinement by spirals



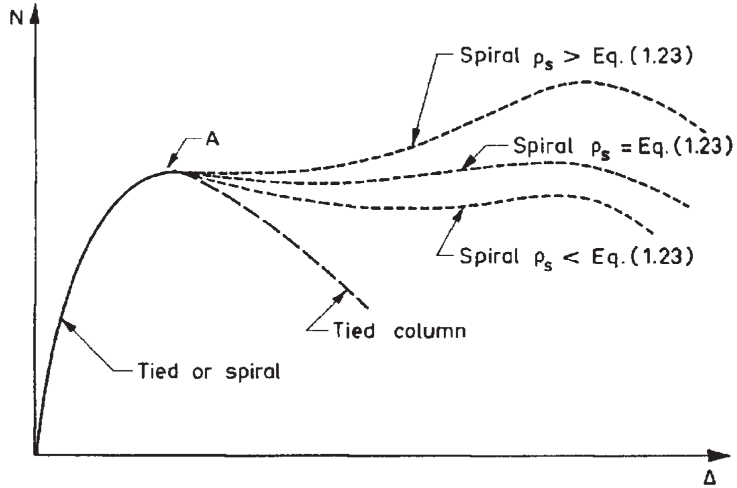
Confinement by spirals

- Pipe analogy
- $\sigma_{sp} = \frac{qr}{t} = \frac{\sigma_2 \frac{D}{2}}{\frac{A_o}{s}} = \frac{\sigma_2 Ds}{2A_o}$
- A_o: spiral area
- s: spiral spacing
- spiral yields $\rightarrow \sigma_{sp} = f_{ywk}$
- $\sigma_2 = \frac{2A_o f_{ywk}}{Ds} = \frac{\rho_v}{2} f_{ywk}$
- $\rho_v = \frac{volume\ of\ spiral}{volume\ of\ concrete} = \frac{A_o \pi D}{\frac{\pi D^2}{4} s} = \frac{4A_o}{Ds}$



Confinement by spirals

- First peak & second peak



Confinement by spirals

- $f_{cc} = f_{ck} + 4\sigma_2 = f_{ck} + 4\frac{2A_o f_{ywk}}{Ds} = f_{ck} + 2\rho_v f_{ywk}$
- Strength loss due to cover spall as a result of concrete crushing:
$$SL = 0.85 f_{ck}(A_c - A_{ck})$$
- Strength gain in core concrete due to spiral confinement
$$SG = (f_{cc} - f_{ck})A_{ck} = 4\sigma_2 A_{ck} = 2\rho_v f_{ywk} A_{ck}$$
- first peak = second peak, $SL = SG$
$$2\rho_v f_{ywk} A_{ck} = 0.85 f_{ck}(A_c - A_{ck})$$

$$\rho_v = \frac{0.85 f_{ck}(A_c - A_{ck})}{2 f_{ywk} A_{ck}} = 0.425 \frac{f_{ck}}{f_{ywk}}\left(\frac{A_c}{A_{ck}} - 1\right)$$

Confinement by spirals

- TS500-2000
- $\min \rho_s = 0.45 \frac{f_{ck}}{f_{ywk}}\left(\frac{A_c}{A_{ck}} - 1\right)$
- $\frac{A_c}{A_{ck}} \to 1 \quad \Rightarrow \quad \rho_s \to 0$
- $\min \rho_s = 0.12\left(\frac{f_{ck}}{f_{ywk}}\right)$
- Use greater of the two equations

Axial strength of columns

- $N_{or} = 0.85 f_{ck}(A_c - A_{st}) + A_{st} f_{yk}$
- $N_{or} \cong 0.85 f_{ck} A_c + A_{st} f_{yk}$
- For a column with spirals:
- $N_{or2} = (0.85 f_{ck} + 4\sigma_2)A_{ck} + A_{st} f_{yk}$
- $N_{or2} = \left(0.85 f_{ck} + 2\rho_s f_{ywk}\right)A_{ck} + A_{st} f_{yk}$

Column design minimum requirements

- Minimum cross-sectional dimensions
  - Minimum stiffness against lateral loads
  - Convenience for casting the concrete
  - Provide adequate space for placing the reinforcement
- Minimum ratio of the longitudinal steel
  - Take care of accidental eccentricities (bending)
  - Time dependent deformations
- Minimum diameter of the longitudinal steel
  - To have a minimum stiffness against buckling of bars
- Maximum ratio of longitudinal steel
  - Convenience of casting the concrete

Column design minimum requirements

- Minimum diameter & maximum spacing for the lateral reinforcement
  - Hold the longitudinal bars in place
  - Prevent the buckling of longitudinal bars
  - Provide confinement & ductility

- Column longitudinal bars should be braced by closely spaced lateral reinforcement: cross-ties or closed ties

Axial tensile strength of columns

- Symmetrically reinforced prismatic member under uniaxial tensile force N

$$N = A_c\sigma_c + A_{st}\sigma_s \quad \text{(valid up to cracking)}$$

- Approximate cracking load

$$N_{cr} = A_c f_{ctd} \qquad f_{ctd} = \frac{0.35}{1.5}\sqrt{f_{ck}}$$

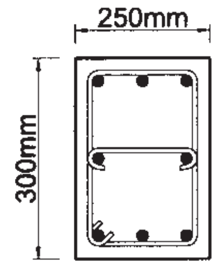
- Cracks due to uniaxial tension extend around the perimeter of the member with almost constant width
- After cracking, resistance provided by concrete diminishes and total load is resisted by steel, $N = A_{st}\sigma_s$
- Ultimate tensile load: $N_r = A_{st}f_{yd}$

Minimum tension reinforcement

- Minimum reinforcement to prevent sudden brittle failure
- Plain concrete: $N = A_c f_{ctk}$
- Reinforced concrete: $N = A_{st} f_{yk}$
- Equating above equations to ensure yielding after cracking
- $\min A_{st} = \frac{f_{ctk}}{f_{yk}} A_c$
- $\min \rho_t = \frac{f_{ctk}}{f_{yk}} = \frac{1.5 f_{ctd}}{1.15 f_{yd}} = 1.3 \frac{f_{ctd}}{f_{yd}}$
- TS500-2000: $\min \rho_t = 1.5 \frac{f_{ctd}}{f_{yd}}$

Example 1

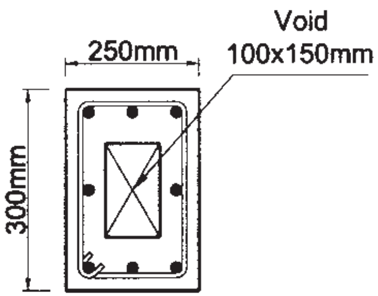
- Compute the ultimate axial load capacities of the column sections shown below. Materials C20, S420
- Case (a)
  - Longitudinal steel: 8ϕ16
  - Ties: ϕ8/200



- $N_{or} = 0.85 f_{cd} A_c + A_{st} f_{yd}$
- $A_c = 300 \times 250 = 75000$ mm²
- $A_{st} = 8\frac{\pi 16^2}{4} = 1600$ mm²
- $N_{or} = 0.85\frac{20}{1.5}75000 + 1600 \times \frac{420}{1.15} = 1413$ kN

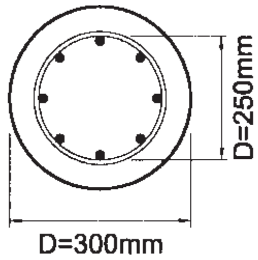
Example 1

- Case (b)
  - Longitudinal steel: 8ϕ16
  - Ties: ϕ8/200
- $N_{or} = 0.85 f_{cd} A_c + A_{st} f_{yd}$
- $A_c = 300 \times 250 - 100 \times 150$
- $A_c = 60000 \text{ mm}^2$
- $N_{or} = 0.85 \times 13 \times 60000 + 1600 \times 365$
- $N_{or} = 1247 \text{ kN}$



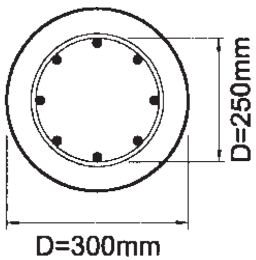
Example 1

- Case (c)
  - Longitudinal steel: 8ϕ16
  - Spiral: ϕ10/80
- $N_{or} = 0.85 f_{cd} A_c + A_{st} f_{yd}$
- $A_c = \frac{\pi 300^2}{4} = 70650 \text{ mm}^2$
- $N_{or} = 0.85 \times 13 \times 70650 + 1600 \times 365$
- $N_{or} = 1365 \text{ kN}$



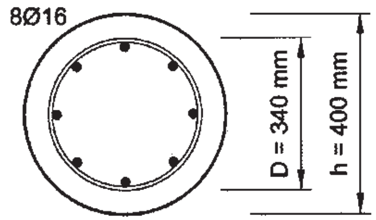
Example 1

- Case (d); second peak capacity
- $\min \rho_s = 0.45 \frac{20}{420} \left(\frac{300^2}{250^2} - 1 \right) = 0.00943$
- $\min \rho_s = 0.12 \frac{20}{420} = 0.00571$
- $A_{ck} = \frac{\pi 250^2}{4} = 49087 \text{ mm}^2$
- $A_c = \frac{\pi 300^2}{4} = 70686 \text{ mm}^2$
- $\rho_s = \frac{4A_o}{Ds} = \frac{4 \times 50}{250 \times 80} = 0.01 > \rho_{s,min}$
- $f_{ccd} = \frac{f_{cc}}{1.5} = \frac{0.85 f_{ck} + 2\rho_s f_{ywk}}{1.5} = \frac{0.85 \times 20 + 2 \times 0.01 \times 420}{1.5} = 16.9 \text{ MPa}$
- $N_{or2} = f_{ccd} A_{ck} + A_{st} f_{yd} = 16.9 \times 49087 + 1600 \times 365$
- $N_{or2} = 1414 \text{ kN}$



Example 2

- Calculate the minimum spiral steel for the given column
  - C20, S420, 8ϕ16
- $A_c = \frac{\pi 400^2}{4} = 125664 \text{ mm}^2$
- $A_{ck} = \frac{\pi 340^2}{4} = 90792 \text{ mm}^2$
- $\min \rho_s = 0.12 \frac{20}{420} = 0.00571$
- $\min \rho_s = 0.45 \frac{20}{420} \left(\frac{125664}{90792} - 1 \right) = 0.00823$

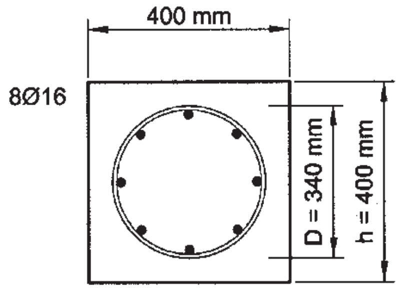


Example 2

- $\rho_s = \frac{4A_o}{Ds} \quad \rightarrow \quad s = \frac{4A_o}{D\rho_s}$
- Use ϕ10 spiral, $A_o = 78.5$ mm^2
- $s = \frac{4 \times 78.5}{340 \times 0.00823} = 112$ mm
- Use ϕ10/110

Example 3

- Calculate the minimum spiral steel for the given column
  - C20, S420, 8ϕ16



- $A_c = 400 \times 400 = 160000$ mm^2
- $A_{ck} = \frac{\pi 340^2}{4} = 90792$ mm^2
- $\min \rho_s = 0.12 \frac{20}{420} = 0.00571$
- $\min \rho_s = 0.45 \frac{20}{420} \left(\frac{160000}{90792} - 1 \right) = 0.01633$

Example 3

- $\rho_s = \frac{4A_o}{Ds} \quad \rightarrow \quad s = \frac{4A_o}{D\rho_s}$
- Use ϕ10 spiral, $A_o = 78.5$ mm^2
- $s = \frac{4 \times 78.5}{340 \times 0.01633} = 56.6$ mm
- Use ϕ10/55