CE 382 Reinforced Concrete Fundamentals

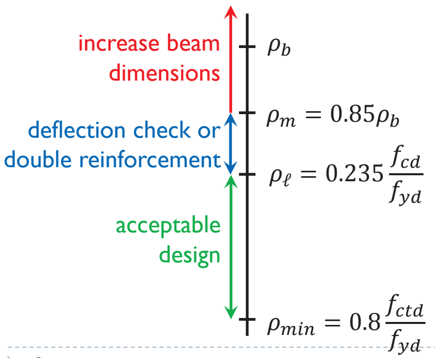
Pure Bending – Design of Beams

Design of Beams

- Preliminary design
  - Establish structural system; locations of columns and structural walls, selection of the floor system…
  - Establish sizes of the members; using experience & intuition and some simple calculations
  - Reinforcement is not calculated in the preliminary design!
- Final Design
  - Maximum ratio of reinforcement $\boxed{\rho_m = 0.85\rho_b}$
  - ρ_m: ensures ductile behavior; but may result in small cross-sectional dimensions which may cause excessive deformations
  - Limiting ratio of reinforcement $\boxed{\rho_\ell = 0.235\dfrac{f_{cd}}{f_{yd}}}$

Design of Beams

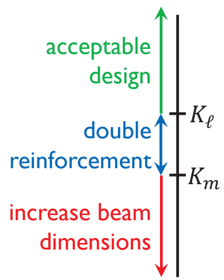
- Ideal ρ or $(\rho - \rho') \leq \rho_\ell$
- Upper limit ρ or $(\rho - \rho') \leq \rho_m$ Requires deflection check very time consuming & complex
- $\rho \leq 0.02$
- $\rho_{\min} = 0.8\dfrac{f_{ctd}}{f_{yd}}$



read the derivation of ρ_{min}

Single reinforced rectangular beams

- Given: M_d, b_w, d
- Calculate: $K = \dfrac{b_w d^2}{M_d}$
- Compare with K_ℓ & K_m
- If $K > K_\ell$ proceed next step
  - If $K > K_m$ OK, but use compression steel
  - If $K < K_m$ change size of the beam
- $A_s = \dfrac{M_d}{f_{yd} j d}$ take j from table
  - to be on the safe side use $j_\ell = 0.86$ if $K > K_\ell$
- Compute shear reinforcement

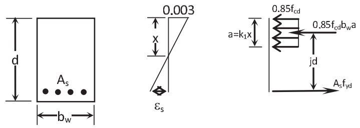


Limiting values for beams

Steel Grade	Concrete Grade	J_m	For ρ_m ρ_m	K_m(*) mm²/kN	j_l	For ρ_l ρ_l	K_l(*) mm²/kN
S220	C16	0.727	0.0267	269	0.86	0.0135	449
S220	C18	0.727	0.0292	247	0.86	0.0148	412
S220	C20	0.727	0.0316	228	0.86	0.0160	380
S220	C25	0.727	0.0413	174	0.86	0.0209	291
S420	C16	0.776	0.0115	308	0.86	0.0071	449
S420	C18	0.776	0.0125	382	0.86	0.0077	412
S420	C20	0.776	0.0136	260	0.86	0.0084	380
S420	C25	0.776	0.0177	199	0.86	0.0109	291
S420	C30	0.784	0.0201	174	0.86	0.0129	247
S420	C35	0.792	0.0223	155	0.86	0.0148	215
S420	C40	0.800	0.0252	136	0.86	0.0174	183
S420	C45	0.809	0.0269	126	0.86	0.0193	165
S420	C50	0.816	0.0283	119	0.86	0.0212	150
S500	C16	0.791	0.0090	324	0.86	0.0059	449
S500	C18	0.791	0.0098	297	0.86	0.0065	412
S500	C20	0.791	0.0106	274	0.86	0.0070	380
S500	C25	0.791	0.0139	210	0.86	0.0092	291
S500	C30	0.799	0.0157	183	0.86	0.0108	247
S500	C35	0.806	0.0174	164	0.86	0.0124	215
S500	C40	0.813	0.0197	144	0.86	0.0146	183
S500	C45	0.821	0.0210	133	0.86	0.0162	165
S500	C50	0.828	0.0222	125	0.86	0.0178	150

Single Reinforced Rectangular Sections

S420 (f_{yd}=365 MPa)								
C14 (f_{cd}=9 MPa)			C16 (f_{cd}=11 MPa)			C20 (f_{cd}=13 MPa)		
K (mm²/kN)	j	ρ	K (mm²/kN)	j	ρ	K (mm²/kN)	j	ρ
1435	0.954	0.0020	1426	0.961	0.0020	1417	0.967	0.0020
1162	0.942	0.0025	1152	0.951	0.0025	1143	0.959	0.0025
980	0.931	0.0030	970	0.941	0.0030	961	0.950	0.0030
851	0.919	0.0035	840	0.932	0.0035	831	0.942	0.0035
754	0.903	0.0040	743	0.922	0.0040	734	0.934	0.0040
679	0.896	0.0045	668	0.912	0.0045	658	0.926	0.0045
619	0.885	0.0050	607	0.902	0.0050	597	0.917	0.0050
570	0.873	0.0055	558	0.893	0.0055	548	0.909	0.0055
530	0.862	0.0060	517	0.883	0.0060	507	0.901	0.0060
493	0.850	0.0065	483	0.873	0.0065	472	0.893	0.0065
466	0.839	0.0070	454	0.864	0.0070	443	0.884	0.0070
441	0.827	0.0075	428	0.854	0.0075	417	0.876	0.0075
420	0.816	0.0080	406	0.844	0.0080	395	0.868	0.0080
401	0.804	0.0085	387	0.834	0.0085	375	0.860	0.0085
384	0.793	0.0090	370	0.825	0.0090	358	0.852	0.0090
369	0.781	0.0095	354	0.815	0.0095	342	0.843	0.0095
			341	0.805	0.0100	328	0.835	0.0100
			328	0.795	0.0105	316	0.827	0.0105
			317	0.786	0.0110	305	0.819	0.0110
						294	0.810	0.0115
						285	0.802	0.0120
						276	0.794	0.0125
						269	0.786	0.0130
						261	0.777	0.0135

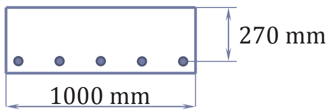


$$\rho = \frac{A_s}{b_w d}$$

$$b_w d^2 = K M_d$$

$$A_s = \frac{M_d}{f_{yd} j d}$$

Example 1



- C20 → $f_{cd} = 13$ MPa
- S420 → $f_{yd} = 365$ MPa
- $K_\ell = 380 \dfrac{mm^2}{kN}$ $\quad j_\ell = 0.861$
- $K_m = 260 \dfrac{mm^2}{kN}$ $\quad j_\ell = 0.776$
- $M_d = 150\ kNm$
- $A_s = ?$
- $K = \dfrac{b_w d^2}{M_d} = \dfrac{1000 \times 270^2}{150000} = 486 \dfrac{mm^2}{kN} > K_\ell$ use single reinforcement
- $A_s = \dfrac{M_d}{f_{yd} j d} = \dfrac{150000000}{365 \times 0.861 \times 270} = 1768\ mm^2 \rightarrow 5\emptyset 22$ (1900 mm²)
- or from table $j = 0.8962 \quad \rightarrow \quad A_s = 1698\ mm^2$

Example 1

- Design the beam using analysis:
  - $M_r = 0.85 f_{cd} k_1 c b_w \left(d - \frac{k_1 c}{2}\right)$

    $150 \times 10^6 = 0.85 \times 13 \times 0.85c \times 1000 \left(270 - \dfrac{0.85c}{2}\right)$

    $3992c^2 - 2535975c + 150 \times 10^6 = 0$

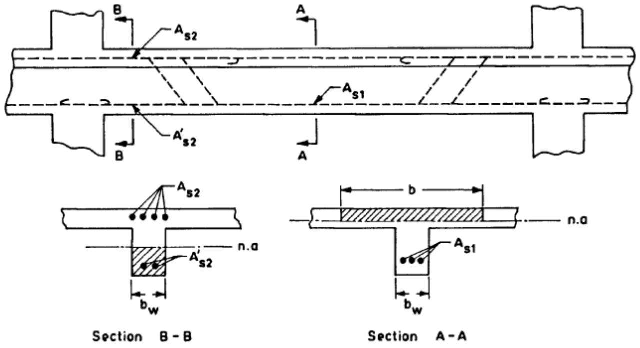
    $c_1 = 66\ mm$ ✓ $\qquad c_2 = 569\ mm$ ✗
  - $F_c = F_s$

    $0.85 f_{cd} k_1 c b_w = A_s f_{yd}$

    $0.85 \times 13 \times 0.85 \times 66 \times 1000 = A_s \times 365$

    $A_s = 1673\ mm^2$

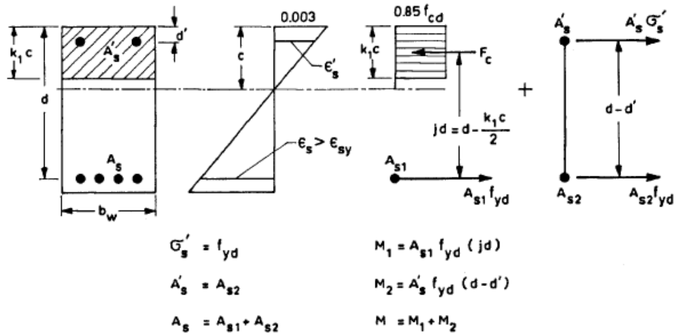
Double reinforced rectangular beams



- @ span → usually positive moment → T-Section
- @ support → usually negative moment → rectangular & double reinforced

Double reinforced rectangular beams

- Given: materials, M_d, b_w & d Find: A_s & A_s'
- Compute $K = \frac{b_w d^2}{M_d}$
- If $K > K_\ell$ → $A_s = \frac{M_d}{f_{yd} j_\ell d}$
- If $\mathrm{K_m} < K < K_\ell$ → compression steel needed



Double reinforced rectangular beams

- Assume $\sigma_s' = f_{yd}$
- $M_1 = M_\ell = \frac{b_w d^2}{K_\ell}$ $\quad j = j_\ell \quad$ $A_{s1} = A_{sl} = \frac{M_1}{f_{yd} j_\ell d}$
- $M_2 = M_d - M_1$ $\quad A_s' = A_{s2} = \frac{M_2}{f_{yd}(d-d')}$
- $A_s = A_{s1} + A_{s2}$
- Check assumption: $c = \frac{A_{s1} f_{yd}}{0.85 f_{cd} k_1 b_w}$ $\quad \varepsilon_s' = 0.003\frac{c-d'}{c}$

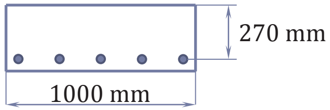
  $\sigma_s' = E_s \varepsilon_s'$ $\quad A_s' = A_{s2}\frac{f_{yd}}{\sigma_s'}$
- generally compression steel yields

Double reinforced rectangular beams

- Given: materials, M_d, b_w, d, A_s'
- Assume $\sigma_s' = f_{yd}$ & compute $M_2 = A_s' f_{yd}(d-d')$
- $M_1 = M_d - M_2$
- $A_{s1} = \frac{M_1}{f_{yd} jd}$ ($j = j_\ell$ can be used)
- Check assumption: $c = \frac{A_{s1} f_{yd}}{0.85 f_{cd} k_1 b_w}$ $\quad \varepsilon_s' = 0.003\frac{c-d'}{c}$
- If $\varepsilon_s' > \varepsilon_{sy}$ → $A_s = A_s' + A_{s1}$
- If $\varepsilon_s' < \varepsilon_{sy}$ → go to general solution

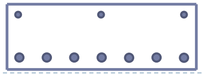
Example 2

- C20 → $f_{cd} = 13$ MPa
- S420 → $f_{yd} = 365$ MPa
- $K_\ell = 380 \dfrac{mm^2}{kN}$ $\quad j_\ell = 0.861$
- $K_m = 260 \dfrac{mm^2}{kN}$ $\quad j_\ell = 0.776$
- $M_d = 220\ kNm$
- $A_s = ?$
- $K = \dfrac{b_w d^2}{M_d} = \dfrac{1000 \times 270^2}{220000} = 331 \dfrac{mm^2}{kN} < K_\ell, > K_m$ use double reinforcement
- $M_1 = \dfrac{b_w d^2}{K_\ell} = \dfrac{1000 \times 270^2}{380} = 191.8\ kNm$
- $A_{s1} = \dfrac{M_1}{f_{yd} j_\ell d} = \dfrac{191800000}{365 \times 0.861 \times 270} = 2260\ mm^2$



Example 2

- $M_2 = M_d - M_1 = 220 - 191.8 = 28.2\ kNm$
- Assume $\sigma_s' = f_{yd}$
- $A_s' = A_{s2} = \dfrac{M_2}{f_{yd}(d-d')} = \dfrac{28200000}{365(270-30)} = 322\ mm^2$
- Check assumption
  - $0.85 \times 13 \times 0.85c \times 1000 = 2260 \times 365$
  - $c = 87.8\ mm$
  - $\varepsilon_s' = 0.003 \dfrac{c-d'}{c} = 0.003 \dfrac{87.8-30}{87.8} = 0.001975 > \varepsilon_{sy} = 0.001825$
- $A_s = A_{s1} + A_{s2} = 2260 + 322 = 2582\ mm^2 \rightarrow 7\phi22$ (2660 mm^2)
- $A_s' = 322\ mm^2 \rightarrow 3\phi14$ (462 mm^2)



Flanged Sections (T-Beams)

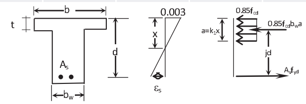
- Select proper chart by b/b_w value
- j values are given as a function of $\overline{K} f_{cd}$ & t/d
- $\overline{K} f_{cd} = \dfrac{bd^2}{M_d} f_{cd}$ $\qquad b$: flange width instead of b_w
- $A_s = \dfrac{M_d}{f_{yd} j d}$

- If tables are not available, approximate solution:
- $jd = 0.9d$
- $jd = d - \dfrac{t}{2}$

take greater value

Flanged Sections (T-Beams)

$b/b_w=6.0$	j Values										
$\overline{K}f_{cd}$	t/d										
	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.30
33.3	0.982	0.982	0.982	0.982	0.982	0.982	0.982	0.982	0.982	0.982	0.982
26.7	0.977	0.977	0.977	0.977	0.977	0.977	0.977	0.977	0.977	0.977	0.977
23.3	0.974	0.974	0.974	0.974	0.974	0.974	0.974	0.974	0.974	0.974	0.974
20.0	0.970	0.970	0.970	0.970	0.970	0.970	0.970	0.970	0.970	0.970	0.970
18.7	0.967	0.967	0.967	0.967	0.967	0.967	0.967	0.967	0.967	0.967	0.967
17.3	0.965	0.965	0.965	0.965	0.965	0.965	0.965	0.965	0.965	0.965	0.965
16.0	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.962
14.7	0.958	0.958	0.958	0.958	0.958	0.958	0.958	0.958	0.958	0.958	0.958
13.3	0.949	0.954	0.954	0.954	0.954	0.954	0.954	0.954	0.954	0.954	0.954
12.7	0.942	0.951	0.951	0.951	0.951	0.951	0.951	0.951	0.951	0.951	0.951
12.0	0.932	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.948
11.3	0.919	0.942	0.945	0.945	0.945	0.945	0.945	0.945	0.945	0.945	0.945
10.7	0.902	0.934	0.941	0.941	0.941	0.941	0.941	0.941	0.941	0.941	0.941
10.0	0.878	0.921	0.936	0.937	0.937	0.937	0.937	0.937	0.937	0.937	0.937
9.7	0.863	0.912	0.932	0.935	0.935	0.935	0.935	0.935	0.935	0.935	0.935
9.3	0.845	0.901	0.927	0.932	0.932	0.932	0.932	0.932	0.932	0.932	0.932
9.0	0.822	0.883	0.920	0.930	0.930	0.930	0.930	0.930	0.930	0.930	0.930
8.7	0.792	0.872	0.911	0.926	0.927	0.927	0.927	0.927	0.927	0.927	0.927
8.3	0.749	0.851	0.899	0.920	0.923	0.923	0.923	0.923	0.923	0.923	0.923
8.0	0.647	0.824	0.884	0.912	0.920	0.920	0.920	0.920	0.920	0.920	0.920
7.7	—	0.785	0.864	0.901	0.915	0.916	0.916	0.916	0.916	0.916	0.916
7.3	—	0.714	0.836	0.886	0.907	0.912	0.912	0.912	0.912	0.912	0.912
7.0	—	—	0.795	0.864	0.896	0.907	0.907	0.907	0.907	0.907	0.907
6.7	—	—	0.701	0.832	0.878	0.898	0.902	0.902	0.902	0.902	0.902
6.3	—	—	—	0.778	0.852	0.884	0.895	0.896	0.896	0.896	0.896
6.0	—	—	—	—	0.807	0.861	0.883	0.890	0.890	0.890	0.890
5.7	—	—	—	—	—	0.820	0.862	0.879	0.872	0.882	0.882
5.3	—	—	—	—	—	—	0.822	0.858	0.862	0.874	0.874
5.0	—	—	—	—	—	—	—	0.813	0.848	0.862	0.864
4.7	—	—	—	—	—	—	—	—	0.792	0.833	0.852
4.3	—	—	—	—	—	—	—	—	—	0.714	0.831



$$\left(\overline{K}f_{cd}\right) = \frac{bd^2}{M_d} f_{cd} \qquad A_s = \frac{M_d}{f_{yd}(j)d}$$

Bending of Bars

- The required area of reinforcement is calculated for the maximum moment.
- Can be reduced where smaller moment exists.
  - Cut-off
  - Bent-up
- $A_s = \frac{M_d}{f_{yd} j d}$

In practice when adjacent spans are not too different from each other, bars can be bent
- $\ell_n/7$ at the exterior supports
- $\ell_n/5$ at the interior supports

- variation in j insignificant $\rightarrow A_s$ proportional M_d
- stress concentration @ cut-off or bent $\rightarrow$ extend bars
  - Cut-off $\rightarrow 20\emptyset$ or d
  - Bent $\rightarrow 8\emptyset$ or $d/3$

Study Example 5.7

Minimum requirements for Beams

- Clear cover
  - 20 mm TS 500-2000
  - 25 mm for beams subject to outside atmosphere
  - 40 mm recommended (for better fire protection)
- Minimum diameter of bar for tension bar: ϕ12
- Minimum spacing between adjacent rows of reinforcement: 20 mm or bar diameter
- Spacing between bars: 20mm, bar diameter or 4/3 of the diameter of the largest aggregate

Minimum requirements for Beams

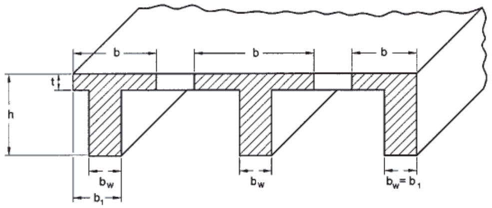
- Reinforcement ratio
  - $\rho = \frac{A_s}{b_w d} \geq 0.8 \frac{f_{ctd}}{f_{yd}}$
  - $(\rho - \rho') \leq \rho_m = 0.85\rho_b$
  - $\rho \leq 0.02$
  - $(\rho - \rho') \leq \rho_\ell = 0.235 \frac{f_{cd}}{f_{yd}}$ authors recommendation
  - For beams deeper than 600 mm
    - two web bars at the mid-depth of the section; minimum $1‰\ (0.001 b_w d)$
    - For each additional 300 mm, an extra row of web longitudinal steel

Minimum requirements for Beams

- Simply supported beam
  - ½ of the tension steel @ span → compression steel @ support
- Continuous beam
  - ⅓ of the mid-span steel → compression steel @ support
- Turkish Seismic Code
  - Seismic zone 1 & 2: 50% of tension steel @support → compression steel @ support
  - Seismic zone 3 & 4: 30% of tension steel @support → compression steel @ support

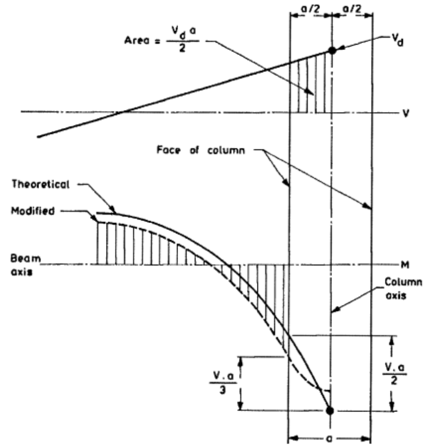
Effective flange width; TS 500-2000

- Symmetrical flange on two sides $b = b_w + \frac{1}{5}\ell_p$
- Unsymmetrical flange on two sides $b = b_1 + \frac{1}{10}\ell_p$
- ℓ_p: distance between the point of inflections
  - $\ell_p \approx 0.8\ell_n$ exterior span of continuous beam
  - $\ell_p \approx 0.6\ell_n$ interior span
  - $\ell_p \approx 1.0\ell_n$ simply supported
  - $\ell_p \approx 1.5\ell_n$ cantilever
  - ℓ_n: clear span



Design Moments

- Members are represented by their centerlines
- Moments at the faces of supports should be used
- $\Delta M = \frac{V_d a}{3}$
- $M_{df} = M_{dc} - \Delta M$



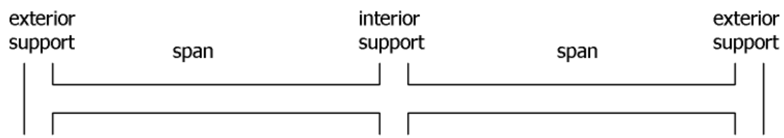
Effect of Material Strength on the Moment Capacity

- Moment capacity is approximately proportional to steel strength
- $concrete\ strength \xrightarrow{influences} c \xrightarrow{influences}$ j
- But this influence can be neglected
- For example
  - for C16 & $\rho = 0.001$
    - for S220: if Δf_c 60% ↘ $\Longrightarrow$ ΔM_r 13% ↘
    - for S420: if Δf_c 60% ↘ $\Longrightarrow$ ΔM_r 20% ↘
- This outcomes are valid only for pure flexure
- Concrete strength affect significantly the moment capacity of a column depending on the level of axial load

Example 3

- Two span continuous beam
- Exterior supports
  - $M_{ext} = -45\ kNm;\ \ V_{ext} = 100\ kN$
- Interior support
  - $M_{int} = -125\ kNm;\ \ V_{int} = 110\ kN$
- Spans
  - $M_{span} = 72\ kNm$

C20 & S420
Columns 400x400mm
Beams 250x400 mm
$t = 120\ mm$
$b = 1000\ mm$
$d = 360\ mm;\ \ d' = 40\ mm$
$K_\ell = 380 \frac{mm^2}{kN}$ $\quad j_\ell = 0.86$
$K_m = 260 \frac{mm^2}{kN}$ $\quad j_m = 0.776$



Example 3

- Modify support moments
  - Exterior supports
    - $M_{d,face} = M_d - \frac{Va}{3} = 45 - \frac{100\times0.4}{3} = 32\ kNm$
  - Interior support
    - $M_{d,face} = M_d - \frac{Va}{3} = 125 - \frac{110\times0.4}{3} = 110\ kNm$
- Calculate the span first $\longrightarrow$ T-beam
  - $jd = 0.9d = 0.9 \times 360 = 324\ mm$ ✓
  - $jd = d - \frac{t}{2} = 360 - \frac{120}{2} = 300\ mm$
  - $A_s = \frac{M_d}{f_{yd} jd} = \frac{72\times10^6}{365\times324} = 609\ mm^2$
  - Use 4ϕ14 (615 mm²) $\longrightarrow$ 2ϕ14 bent + 2ϕ14 straight

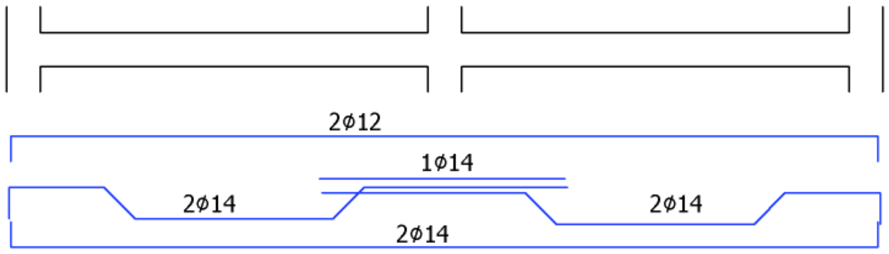
Example 3

- Exterior supports $\longrightarrow$ rectangular beam
  - $K = \frac{b_w d^2}{M_d} = \frac{250\times360^2}{32000} = 1013 > K_\ell$ $\longrightarrow$ use single reinf.
  - $A_s = \frac{M_d}{f_{yd} jd} = \frac{32000000}{365\times0.86\times360} = 283\ mm^2$
  - Available: bent-up + hanger = 2ϕ14+2ϕ12 (534 mm²)
  - $\min A_s = 0.8\frac{f_{ctd}}{f_{yd}} b_w d = 0.8\frac{1.1}{365} 250 \times 360 = 217\ mm^2$
- Interior support $\longrightarrow$ rectangular beam
  - $K = \frac{b_w d^2}{M_d} = \frac{250\times360^2}{110000} = 295 < K_\ell; > K_m$ $\longrightarrow$ double reinf.
  - $M_1 = \frac{b_w d^2}{K_\ell} = \frac{250\times360^2}{380} = 85.3\ kNm$

Example 3

- $A_{s1} = \frac{85300000}{365\times0.86\times360} = 755\ mm^2$
- $M_2 = M_d - M_1 = 110 - 85.3 = 24.7\ kNm$
- assume $\sigma_s' = f_{yd}$
- $A_s' = A_{s2} = \frac{24.7\times10^6}{365(360-40)} = 211\ mm^2$
- Check assumption: $c = \frac{755\times365}{0.85\times13\times0.85\times250} = 117.4\ mm$
- $\varepsilon_s' = 0.003\frac{117.4-40}{117.4} = 0.001978 > \varepsilon_{sy} = 0.001825$
- $A_s = A_{s1} + A_{s2} = 755 + 211 = 966\ mm^2$
- Available: 2ϕ14 + 2ϕ14 + 2ϕ12 (842 mm²)
- add 1ϕ14 (154 mm²)

Example 3



Serviceability – TS 500

- For serviceability limit state calculations → Material Factors = 1.0 ($\gamma_{mc} = 1.0$ & $\gamma_{ms} = 1.0$)
- Beams and slabs do not require deflection calculation if depth to span length ratio is greater than:

Member	Simple support	Edge span	Interior Span	Cantilever
One way slab	1/20	1/25	1/30	1/10
Two way slab (with short side span)	1/25	1/30	1/35	-
Joist slab	1/15	1/18	1/20	1/8
Beam	1/10	1/12	1/15	1/5

Instantaneous Deflection

- For uncracked members ($M_{max} \leq M_{cr}$) → use gross sectional moment of inertia
- $M_{cr} = 2.5 f_{ctd} \frac{I_{gross}}{y}$ & $E_c = 3250\sqrt{f_{ck}} + 14000$
- For cracked members ($M_{max} > M_{cr}$) → use effective moment of inertia
- $I_{ef} = \left(\frac{M_{cr}}{M_{max}}\right)^3 I_{gross} + \left[1 - \left(\frac{M_{cr}}{M_{max}}\right)^3\right] I_{cracked}$
- Calculate deflection in accordance with structural mechanics principles and by considering support conditions
- For continuous beams, take average of the two supports and one span effective moment of inertias.