Basics

$$\frac{d}{dx}e^{kx} = k \cdot e^{kx} \qquad \frac{d}{dx}\arctan(kx) = \frac{k}{1+(kx)^2}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-cb} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Error

$$\varepsilon_{RA} = \left| \frac{x_{k+1} - x_k}{x_{k+1}} \right| \qquad \varepsilon_{RA\%} = \left| \frac{x_{k+1} - x_k}{x_{k+1}} \right| \times 100$$

$$\varepsilon_{AA} = |x_{k+1} - x_k| \qquad RMS = \sqrt{\frac{\sum_{i=1}^{n} \left(f(x_i) - y_i \right)^2}{n}}$$

Taylor Series

$$f(x+h) = f(x) + hf'(x) + \frac{h^2 f''(x)}{2!} + \frac{h^3 f^{(3)}(x)}{3!} + \ldots \frac{h^n f^{(n)}(x)}{n!} + R_n \quad \text{where } R_n = h^{n+1}\frac{f^{(n+1)}(\xi)}{(n+1)!},$$

ξ lies in the interval $[x, x+h]$

Newton-Raphson

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

Secant Formula

$$x_{k+1} = x_k - \frac{f(x_k)(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})}$$

Fixed Point Iteration

$x_{i+1} = g(x_i)$ such that $f(x) = 0$

Newton-Jacobi

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad F = \begin{bmatrix} f_1(x_1, x_2, \cdots, x_n) \\ f_2(x_1, x_2, \cdots, x_n) \\ \vdots \\ f_n(x_1, x_2, \cdots, x_n) \end{bmatrix} \qquad J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \qquad X_{k+1} = X_k - J_k^{-1} \cdot F_k$$

Lagrange's Polynomial

$$f_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i) \text{ where}$$

$$L_i(x) = \prod_{j=0; i \neq j}^{n} \frac{(x - x_j)}{(x_i - x_j)}$$

$$E_n(x) = \prod_{i=0}^{n} (x - x_i) \frac{f^{(n+1)}(\xi)}{(n+1)!}; \; x_0 < \xi < x_n$$

Least Squares Fit

For fitting an n[th] degree polynomial of the form

$$y = C_0 x^0 + C_1 x^1 + C_2 x^2 + \ldots + C_n x^n,$$

$$\begin{bmatrix} \sum x^0 & \sum x^1 & \cdots & \sum x^n \\ \sum x^1 & \sum x^2 & \cdots & \sum x^{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ \sum x^n & \sum x^{n+1} & \cdots & \sum x^{2n} \end{bmatrix} \cdot \begin{bmatrix} C_0 \\ C_1 \\ \vdots \\ C_n \end{bmatrix} = \begin{bmatrix} \sum x^0 y \\ \sum x^1 y \\ \vdots \\ \sum x^n y \end{bmatrix}$$

Numerical Integration

Trapezoidal Rule

$$I = \int_a^b f(x)dx = (b-a)\frac{f(a) + f(b)}{2}$$

Simpson's Rule

$$I = \int_a^b f(x)dx = (b-a)\frac{f(x_0) + 4f(x_1) + f(x_2)}{6}$$

Composite Trapezoidal Rule

$$I = \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n) \right]$$

Truncation Error:

$$E_t = -\frac{(b-a)^3}{12n^3} \sum_{i=1}^{n} f''(\xi_i) \text{ where } \xi_i \in (x_{i-1}, x_i), \text{ i.e.}$$

ξ_i is any value within the segment.

Composite Simpson's Rule

$$I = \frac{h}{3}\left[f(x_0) + 4\sum_{i=1,3,\cdots}^{n-1} f(x_i) + 2\sum_{j=2,4,\cdots}^{n-2} f(x_j) + f(x_n) \right]$$

Truncation Error:

$$E_t = -\frac{(b-a)^5}{180n^5} \sum_{i=1}^{n} f^{(4)}(\xi_i) \text{ where } \xi_i \in (x_{i-1}, x_i), \text{ i.e.}$$

ξ_i is any value within the segment.

Gauss Quadrature

$$\int_a^b g(t)dt = \int_{-1}^{1} f(x)dx,\ \ t = \frac{b+a}{2} + \frac{b-a}{2}x,\ \ dt = \frac{b-a}{2}dx,\ \ \int_{-1}^{1} f(x)dx \approx \sum_{i=0}^{n} w_i f_i$$

N	**Abscissas, $x_{N,k}$ (x_i)**	**Weights, $w_{N,k}$ (w_i)**	**Truncation Error**
2	-0.5773502692 0.5773502692	1.0000000000 1.0000000000	$\frac{f^{(4)}(\xi)}{135}$
3	±0.7745966692 0.0000000000	0.5555555556 0.8888888888	$\frac{f^{(6)}(\xi)}{15,750}$
4	±0.8611363116 ±0.3399810436	0.3478548451 0.6521451549	$\frac{f^{(8)}(\xi)}{3,472,875}$
5	±0.9061798459 ±0.5384693101 0.0000000000	0.2369268851 0.4786286705 0.5688888888	$\frac{f^{(10)}(\xi)}{1,237,732,650}$

Finite Difference

	Forward	**Central**	**Backward**
$\frac{dy}{dx}$	$\frac{(y_{i+1}-y_i)}{\Delta x}$	$\frac{(y_{i+1}-y_{i-1})}{2\Delta x}$	$\frac{(y_i-y_{i-1})}{\Delta x}$
$\frac{d^2y}{dx^2}$	$\frac{(y_{i+2}-2y_{i+1}+y_i)}{(\Delta x)^2}$	$\frac{(y_{i+1}-2y_i+y_{i-1})}{(\Delta x)^2}$	$\frac{(y_i-2y_{i-1}+y_{i-2})}{(\Delta x)^2}$
Error	$O(h)$	$O(h^2)$	$O(h)$

Ordinary Differential Equations

$\frac{dy}{dx} = f(x,y)$	***Euler's Method***	$y(x_{i+1}) = y(x_i) + hf(x_i, y_i)$
	Heun's Method	$y(x_{i+1}) = y(x_i) + h\left(\frac{f(x_i,y_i)+f(x_{i+1},y^*_{i+1})}{2}\right)$ where $y^*_{i+1} = y(x_i) + h\,f(x_i,y_i)$
	Runge-Kutta 4 Method	$y(x_{i+1}) = y(x_i) + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ where $k_1 = f(x_i, y_i)$ $k_2 = f(x_i + \frac{h}{2}, y_i + \frac{h}{2}k_1)$ $k_3 = f(x_i + \frac{h}{2}, y_i + \frac{h}{2}k_2)$ $k_4 = f(x_i + h, y_i + hk_3)$