CE 366-FE 1

(www.ce.metu.edu.tr/~ce366)
Spring' 2010-2011
Homework 2-Solutions

Release Date : **April 26, 2011, Thursday**
Version : **1.0**
Due On : **May 06, 2011, Friday - 5:00 PM**

Rules for Homework:

1. **You may several exams in the first week of May. It is strongly recommended to start doing your homework early, as there will be no extensions given.**
2. Please submit your homework on the boxes labeled "**CE 366**" and located in the **Soil Mechanics Laboratory**. The deadline is strict and **NO EXTENSIONS** will be given.
3. Make sure that you check our website regularly. All announcements and corrections (if necessary) will be made available through our website.
4. Try to be clean, precise when you present your work. State your assumptions if you make any.
5. Discussion with your friends is **strongly encouraged**, however, homework needs to be solved and submitted **individually**.
6. Whenever you have a question about the homework, please contact your teaching assistant first. Remember that all TAs have office hours during the week. If you need further help, you can also contact your sections' instructor.

Version History:

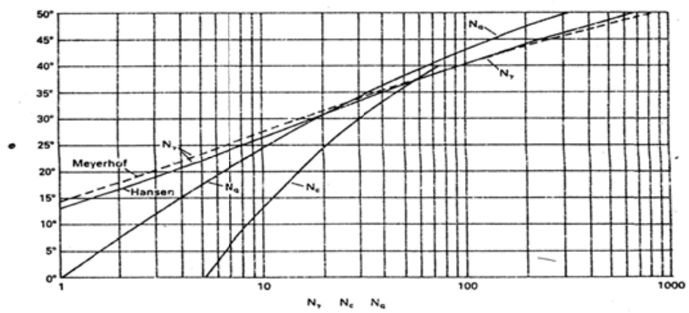
1.0. *(April 26, 2011)* Homework is released.

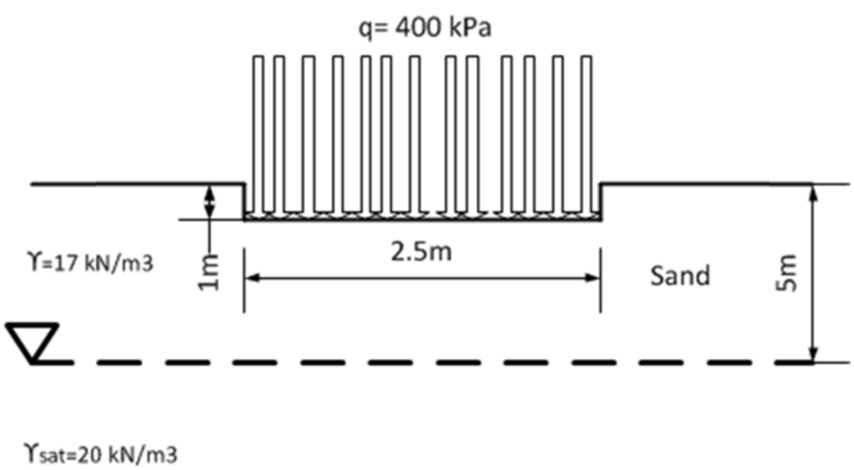
Question	Grade
1 (20%)	
2 (20%)	
3 (20%)	
4 (20%)	
5 (20%)	
Total (100%)	

Surname, Name	
Signature	
Section You Are Registered For	

Question 1 (20%)

A footing 2.5x2.5m carries a pressure of 400 kN/m^2 at a depth of 1m in sand. The saturated unit weight of the sand is 20 kN/m^3 and the unit weight above the water table is 17 kN/m^3. The design shear strength parameters are $c^1 = 0$ and $ø^1 = 40^o$. Determine the factor of safety with respect to shear failure if the water table is 5m below ground level.





For $Ø^1 = 40^0$ ➔ $N_Ɣ = 95$, $N_q = 65$

$q_{nf} = 0.4ƔBN_Ɣ + cN_c + ƔD(N_q\text{-}1)$

$q_{nf} = 0.4*17*2.5*95 + 0 + 17*1*64$

$q_{nf} = 1615 + 1088 = 2703\ kN/m^2$

$q_n = 400 – 1*17 = 383\ kN/m^2$

FS = 2703 / 383 = 7.06

Question 2 (20%)

6.0 m long trapezoidal footing with dimensions, A and B, are shown on the Figure 1. In order for this footing to apply a uniform pressure of 200 kPa, what would be values of A and B? The thickness of the footing is 0.8 m.

(Hint: You can make any assumptions about the weight of the footing ($\gamma_{concrete}$ = 24 kN/m^3). The center of gravity of a trapezoidal footing with length L and widths A and B is given as $\tilde{x} = \frac{L}{3}\left(\frac{2A+B}{A+B}\right)$, x is measured from the side with length B)

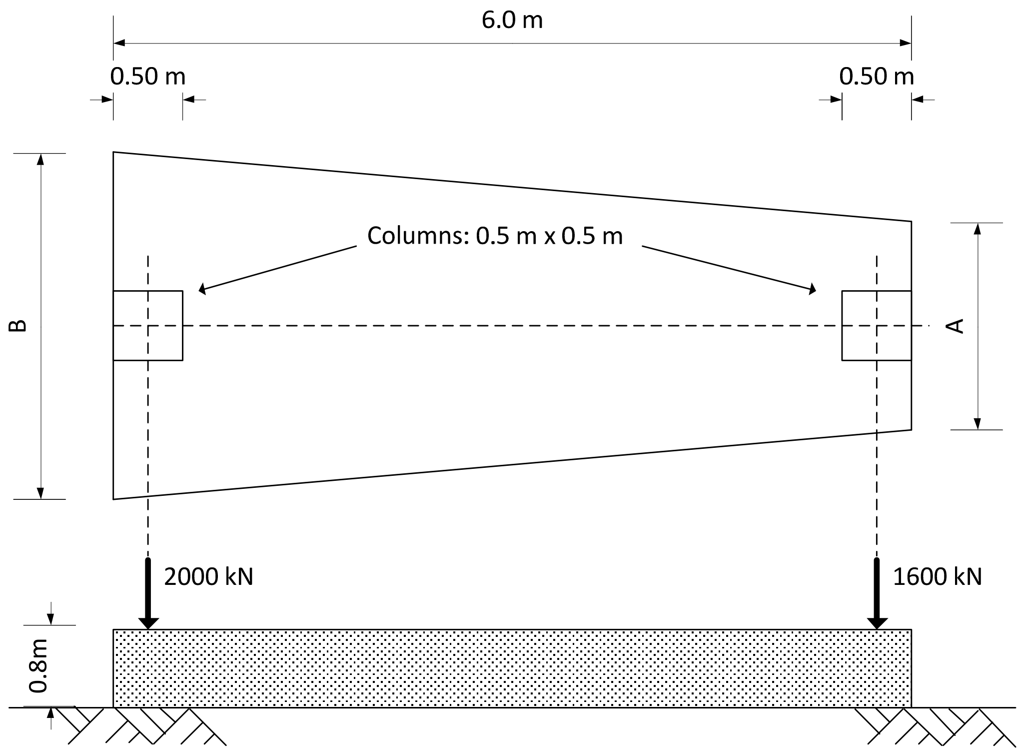


Figure: Overview of Forces Acting on Trapezoidal Footing

Answer: (the weight of the footing is ignored)

Moment can be taken about the left edge of the footing, where B lies, the point of application of soil reaction can be found (x):

2000 x 0.25 + 1600 x 5.75 = (2000 + 1600) x

=> $x := \frac{9700}{3600}$ $\qquad x = 2.7$

In order to have uniform pressure, the loads should be applied at the center of gravity of the footing:

$$\tilde{x} = \frac{L}{3}\left(\frac{2A+B}{A+B}\right) = \frac{6}{3}\left(\frac{2A+B}{A+B}\right) = 2.7$$

This will lead to Equation 1:

$$\left(\frac{B}{A}\right) = \frac{1.3}{0.7}$$ .... Equation 1

Since the foundation pressure is uniform, then considering vertical equilibrium of the forces,

$$\left(\frac{A+B}{2}\right) \times L \times (200) = 3600$$ can be written.

Then Equation 2 can be obtained:

$$A + B = 6$$ .... Equation 2

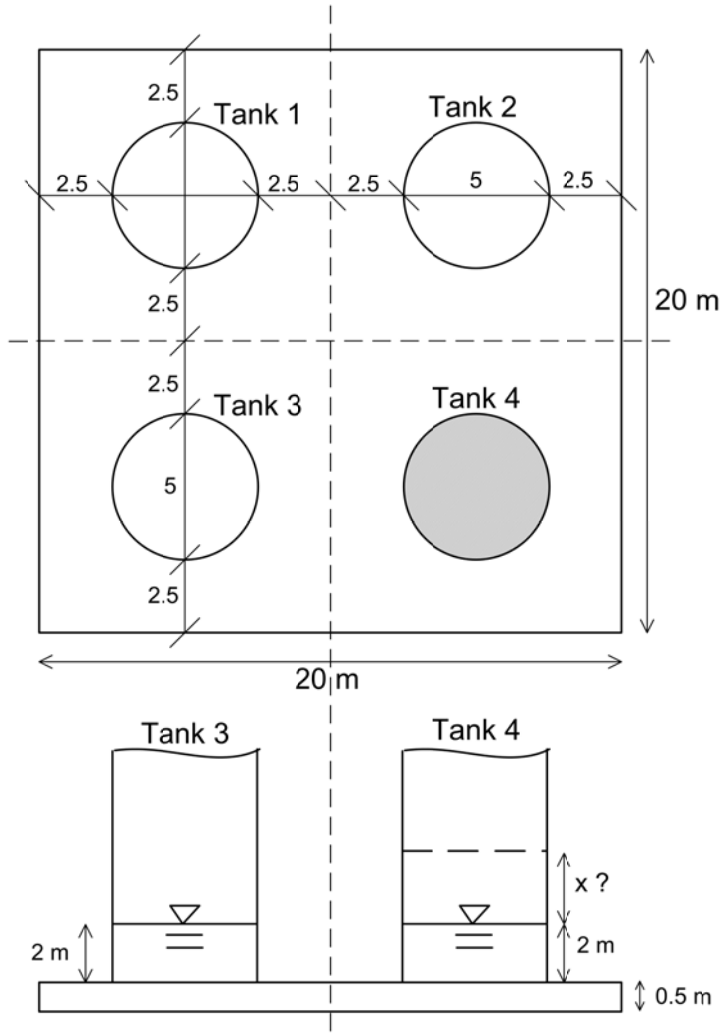
Solving Equations 1 and 2, A and B can be found as 2.1 m and 3.9 m, respectively.

Question 3 (20%)

Foundation of four cylindrical-shaped liquid storage tanks is shown below. The tanks are made of steel, and they are 5 m in diameter. Each of these tanks is full to a height of 2 m by a liquid which has a unit weight of 12 kN/m^3. The owner of this facility would like to increase the liquid level in Tank 4 only. What should be the **allowable increase in liquid level in Tank 4** (x? in the figure) so that the allowable bearing capacity of the soil (which is 25 kPa) is not exceeded.

Hints: Ignore the weight of the steel tanks, and consider the weight of the concrete foundation ($\gamma_{concrete}$ = 24 kN/m^3).

For $\frac{6 \cdot e_1}{B} + \frac{6 \cdot e_2}{L} < 1 \qquad q_{max} = \frac{\Sigma Q}{B \cdot L}\left(1 + \frac{6 \cdot e_1}{B} + \frac{6 \cdot e_2}{L}\right)$



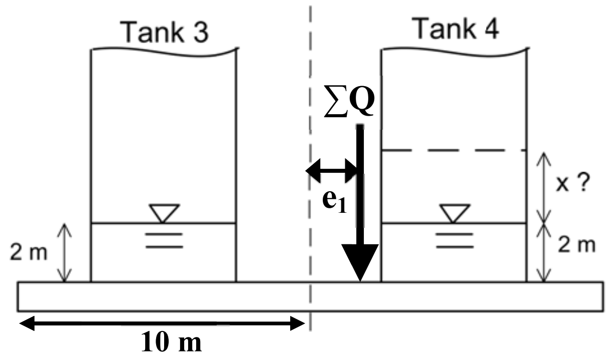
Weight of foundation W = 20 · 20 · 0.5 · 24 = 4800 kN

Weight of liquid in each of tanks 1, 2, 3: $W_1 = W_2 = W_3 = 2 \cdot 12 \cdot \pi D^2/4 = 471.2$ kN

Weight of liquid in Tank 4, $W_4 = (2+x) \cdot 12 \cdot \pi D^2/4 = 471.2+235.6\,x$ kN

$\sum Q = W + W_1 + W_2 + W_3 + W_4 = 4800 + 4 \cdot 471.2 + 235.6x = 6684.8 + 235.6x$

Find eccentricity in both directions:



Take moments about left side of the foundation: $\sum M = \sum Q \cdot (10+e_1)$

$$\sum M = 2 \cdot 471.2 \cdot 5 + 471.2 \cdot 15 + (471.2+235.6\,x) \cdot 15 + 4800 \cdot 10 = 66848 + 3534x$$

$$66848 + 3534x = (6684.8 + 235.6x) \cdot (10+e_1)$$

$$e_1 = \frac{66848+3534x}{6684.8+235.6x} - 10$$

Take moments about the bottom side of the foundation: $\sum M = \sum Q \cdot (10-e_2)$

$$\sum M = 2 \cdot 471.2 \cdot 15 + 471.2 \cdot 5 + (471.2+235.6\,x) \cdot 5 + 4800 \cdot 10 = 66848 + 1178x$$

$$66848 + 1178x = (6684.8 + 235.6x) \cdot (10-e_2)$$

$$e_2 = 10 - \frac{66848+1178x}{6684.8+235.6x}$$

$$q_{\max} = \frac{\Sigma Q}{B \cdot L}\left(1 + \frac{6 \cdot e_1}{B} + \frac{6 \cdot e_2}{L}\right) \le q_{all}$$

$$q_{\max} = \frac{6684.8+235.6x}{20 \cdot 20}\left(1 + \frac{6}{20} \cdot \left(\frac{66848+3534x}{6684.8+235.6x} - 10\right) + \frac{6}{20}\left(10 - \frac{66848+1178x}{6684.8+235.6x}\right)\right)$$

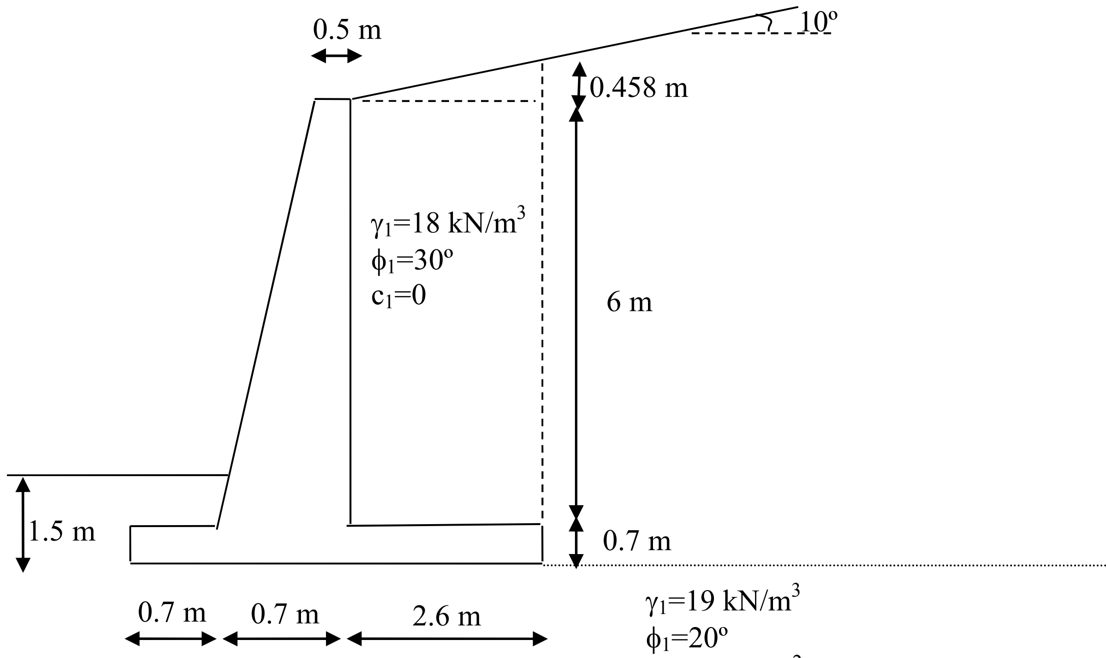
$$q_{\max} = \frac{6684.8+235.6x}{20 \cdot 20}\left(1 + \frac{6}{20} \cdot \frac{66848+3534x}{6684.8+235.6x} - \frac{6}{20} \cdot 10 + \frac{6}{20} \cdot 10 - \frac{6}{20} \cdot \frac{66848+1178x}{6684.8+235.6x}\right)$$

$$q_{\max} = \frac{6684.8+235.6x}{20 \cdot 20}\left(1 + \frac{6}{20} \cdot \frac{(3534x - 1178x)}{6684.8+235.6x}\right)$$

$$q_{\max} = \frac{20 \cdot (6684.8+235.6x) + 6 \cdot (3534x - 1178x)}{20 \cdot 20 \cdot 20} = \frac{133696+18848x}{8000} \le 25$$

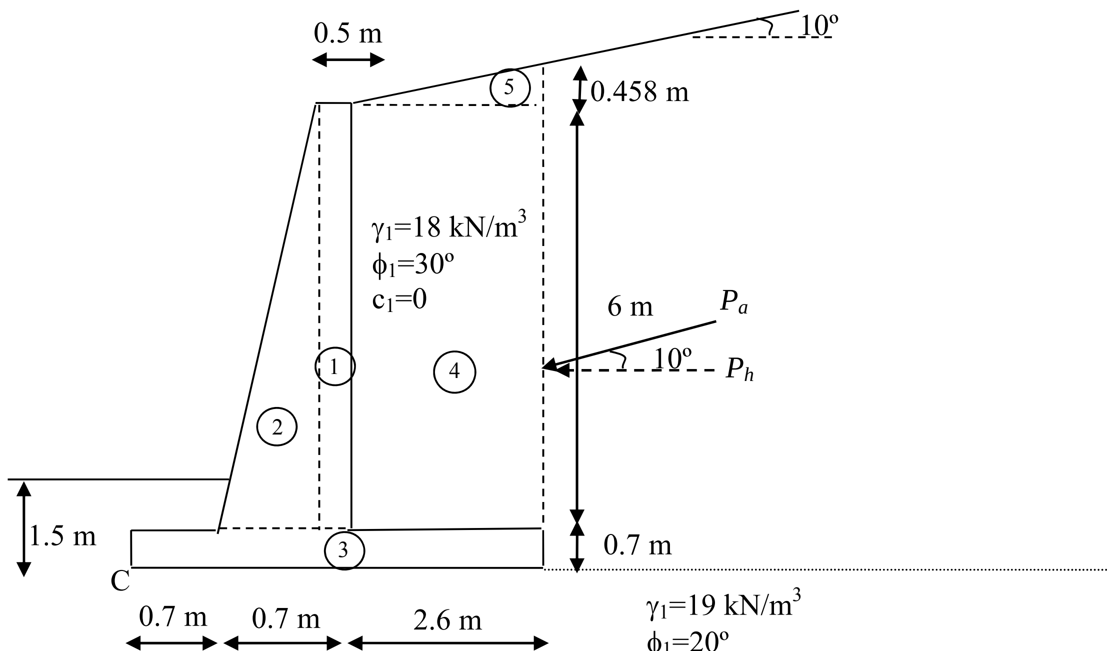
x ≤ 3.52 m

Question 4 (20%)



(Note: Depth of water table is at least 4 m below the base of the retaining wall)

The cross section of a cantilever retaining wall is shown above. Calculate the factors of safety with respect to (i) overturning and (ii) sliding (γ_{conc}=24 kN/m^3)



(Note: Depth of water table is at least 4 m below the base of the retaining wall)

$$P_a = \frac{1}{2}\gamma_1 H^2 K_a$$

$$H = 0.458 + 6 + 0.7 = 7.158m$$

for ϕ_1=30° and α=10° $K_a = 0.35$

$$P_a = \frac{1}{2}18 * (7.158)^2 * 0.35 = 161.4kN/m$$

$$P_v = P_a \sin 10^o = 28.03kN/m$$

$$P_h = P_a \cos 10^o = 158.95kN/m$$

(i) FS against overturning:

Section No	Area (m^2)	Weight/Unit length (kN/m)	Moment arm from point C (m)	Moment (kN-m)
1	6*0.5=3	70.74	1.15	81.35
2	½*0.2*6=0.6	14.15	0.833	11.79
3	4*0.7=2.8	66.02	2	132.04
4	6*2.6=15.6	280.80	2.7	758.16
5	½*2.6*0.458=0.595	10.71	3.13	33.52
		P_v=28.03	4	112.12
		$\sum V$=470.45		$\sum M_R$=1128.98

$$M_o = P_h \frac{H}{3} = 379.25$$

$$FS_{overturning} = \frac{\Sigma M_R}{M_o} = \frac{1128.98}{379.25} = 2.98 > 2$$

(ii) FS against sliding

$$FS_{sliding} = \frac{\Sigma V \tan(k_1\phi_2) + Bk_2c_2 + P_p}{P_a \cos\alpha}$$

$$k_1 = k_2 = \frac{2}{3}$$

$$P_p = \frac{1}{2}\gamma_2 D^2 K_p + 2c_2\sqrt{K_p}D$$

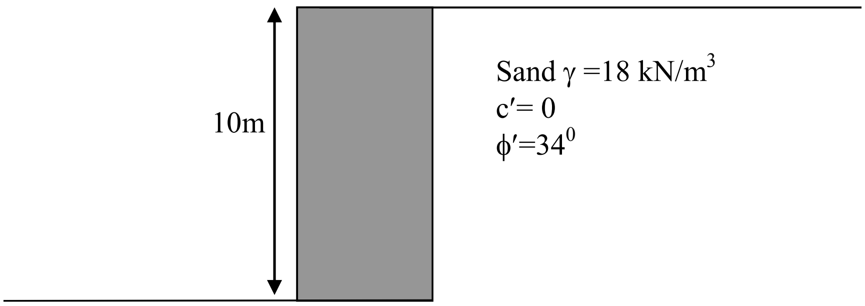
$$K_p = \tan^2(45 + \frac{\phi_2}{2}) = \tan^2(45+10) = 2.04$$

$$P_p = 215kN/m$$

$$FS_{sliding} = \frac{470.45\tan(2/3*20) + 4*2/3*40 + 215}{158.95} = 2.73 > 1.5$$

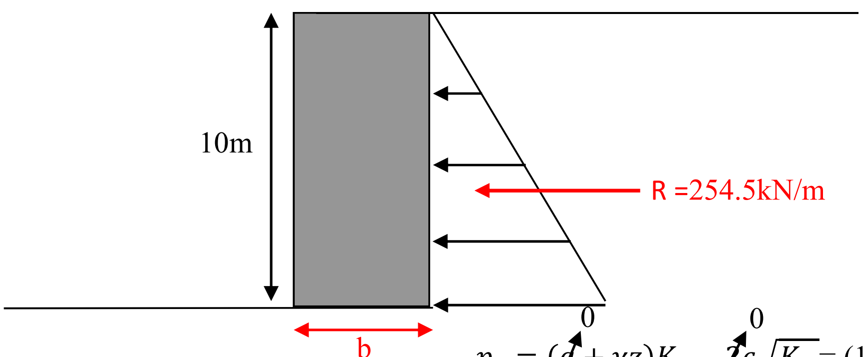
Question 5. ((a): 6%; (b): 7%, (c): 7%)

For the gravity wall shown in the figure given below:



Hint : $K_a = \dfrac{1-\sin\phi'}{1+\sin\phi'}$; $K_p = \dfrac{1+\sin\phi'}{1-\sin\phi'}$; $p_a = K_a \cdot \sigma'_v - 2\cdot c \cdot \sqrt{K_a}$; $p_p = K_p \cdot \sigma'_v + 2\cdot c\cdot \sqrt{K_p}$

a) Determine the **active pressure** acting on the wall by using Rankine's theory and **plot your results** on a similar figure given below. Determine the total resulting force (R) per meter length acting on the wall.



$$p_a = (\cancel{q}^{\,0} + \gamma z)K_a - \cancel{2c}^{\,0}\sqrt{K_a} = (18x10) \text{ x } 0.283 = 50.9 \text{ kPa}$$

$$K_a = \frac{1 - sin34}{1 + sin34} = 0.283$$

$$R = \frac{50.9x10}{2} = 254.5 \text{kN/m}$$

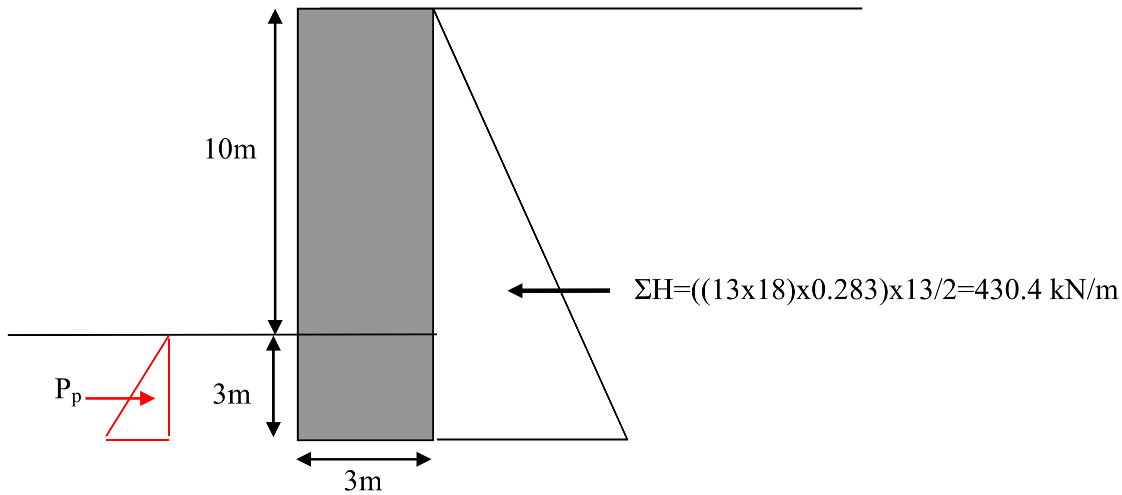
Resultant Rankine Active Force=........254.5......... kN/m.

b) What should be the **width of the concrete gravity wall** to obtain a factor of against sliding as 1.5, if the friction angle between concrete and foundation sand layer is assumed to be 22º (Note: Unit weight of concrete can be assumed as 24 kN/m^3).

$$FS_{sliding} = \frac{\Sigma V tan\delta}{\Sigma H} = \frac{10xbx24xtan(22)}{254.5} = 1.5 \rightarrow \; b = 3.9\,m \cong 4.0\,m$$

Width of Concrete=......4.0......... m.

c) If gravity wall is embedded 3 m into the foundation soil, **estimate the factor of against sliding**, if the friction angle between concrete and foundation sand layer is assumed to be 22º (Note: Apply an additional safety factor of 2.0 on the passive resistance; unit weight of concrete can be assumed as 24 kN/m^3).



$$K_p = \frac{1 + sin34}{1 - sin34} = 3.54$$

$$At\ 3\ m \rightarrow\ p_p = (q + \gamma z)K_p + 2c\sqrt{K_p} = (18x3)\ x3.54 = 191.2\ kPa$$

$$P_p = \frac{191.2x3}{2} = 286.8\ kN/m$$

FS=2 for passive resistance

$$FS_{sliding} = \frac{\Sigma V tan\delta + P_p}{\Sigma H} = \frac{(13x3x24)x\tan(22) + 286.8}{430.4} = \frac{378.2 + \frac{1}{2}x286.8}{430.4}$$

$$FS_{sliding} \cong 1.21$$

Factor of Safety against Sliding=........1.21.................