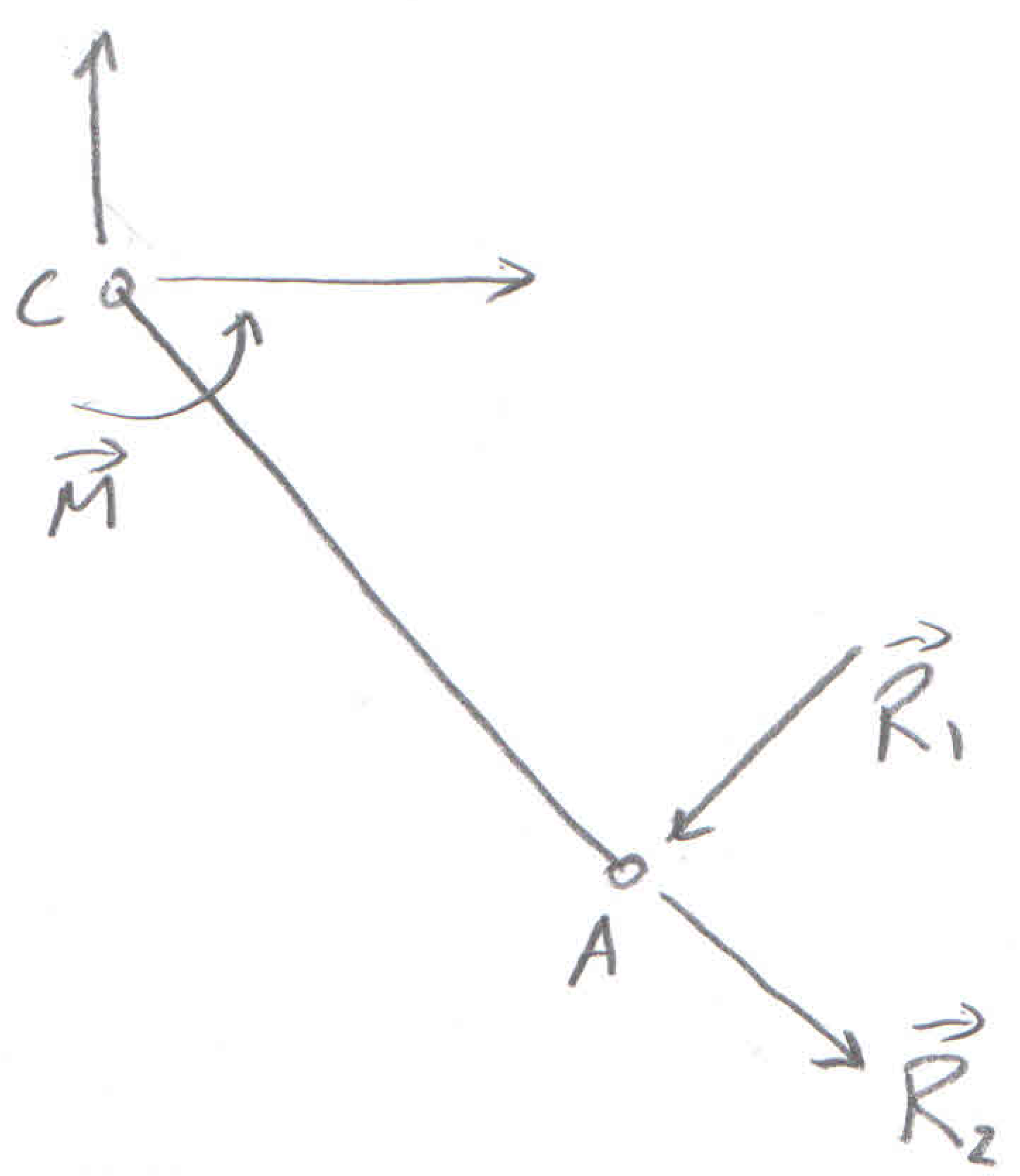
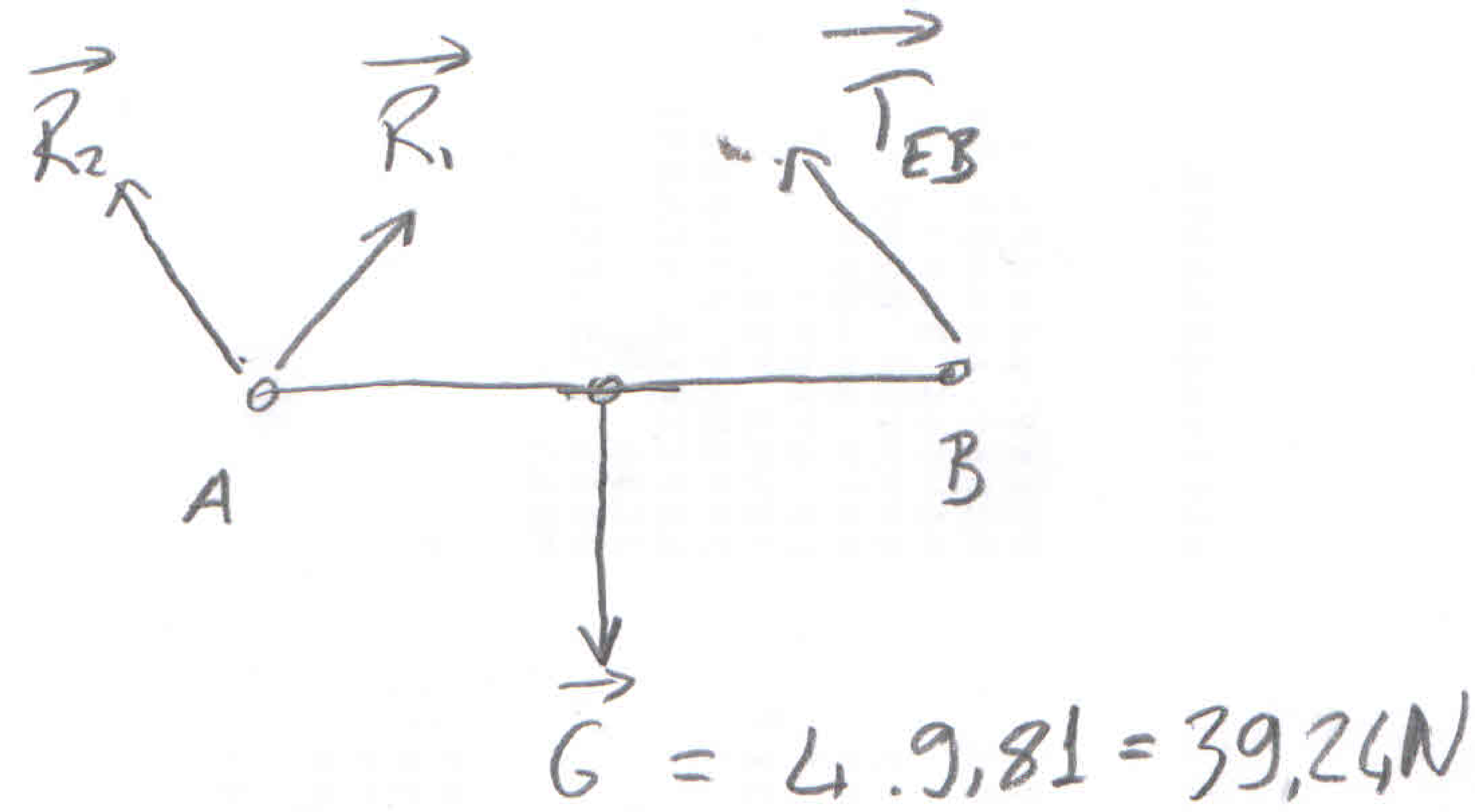
SOLUTION OF HW4

Question 1-)

Bar AC

Bar AB



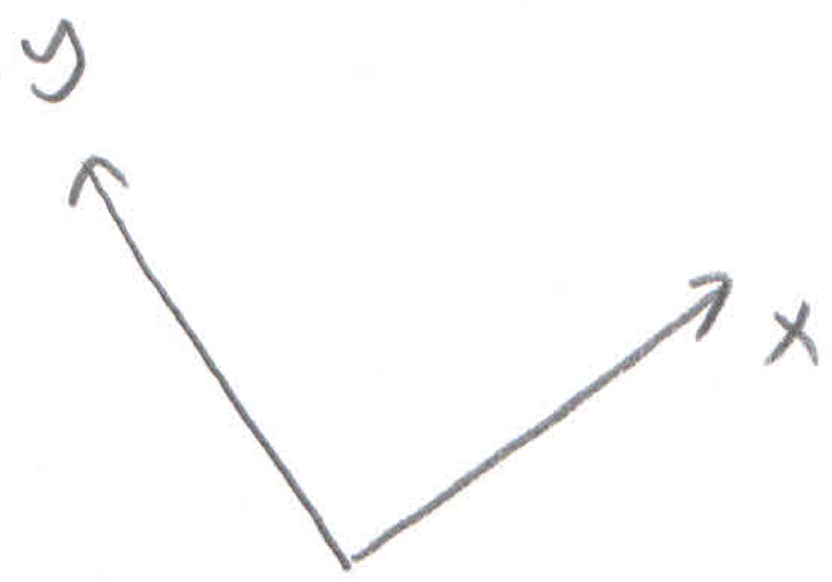


for Bar AC;

$$\overset{+}{\circlearrowright} M_c = 0 \Rightarrow 6\,N_m - R_1(0.45\,m) = 0$$

$$R_1 = \frac{6}{0{,}45} = \frac{40}{3}\,N$$

for Bar AB



$$\sum F_x = \frac{40}{3}\,N - 4 \cdot 9{,}81 \cdot \sin 30^\circ = -4a$$

$$\vec{a} = 1{,}572\ m/s^2 \quad \alpha = 30^\circ$$

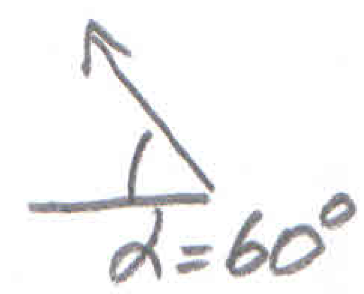
part b.)

$$\overset{+}{\circlearrowright} \sum M_A = 39{,}24 \cdot 0{,}3 - T_{EB} \cdot \cos 30° \cdot 0{,}6$$

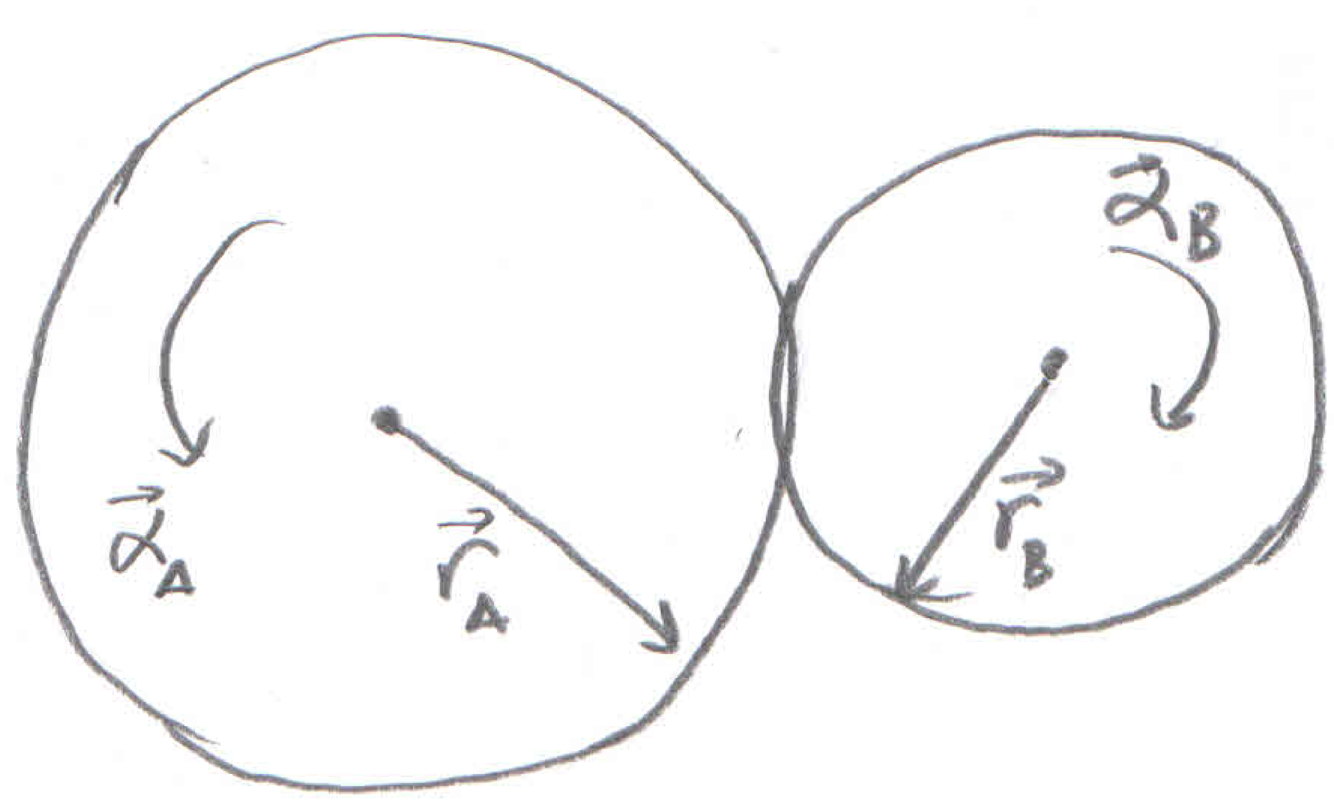
$$= ma\,(\sin 30°) \cdot 0{,}3)$$

$$\Rightarrow T_{EB} = \frac{-4 \cdot 1{,}572 \cdot 0{,}15 + 11{,}772}{0{,}866 \cdot 0{,}6}$$

$$= 20{,}841\,N$$



Question 2.)

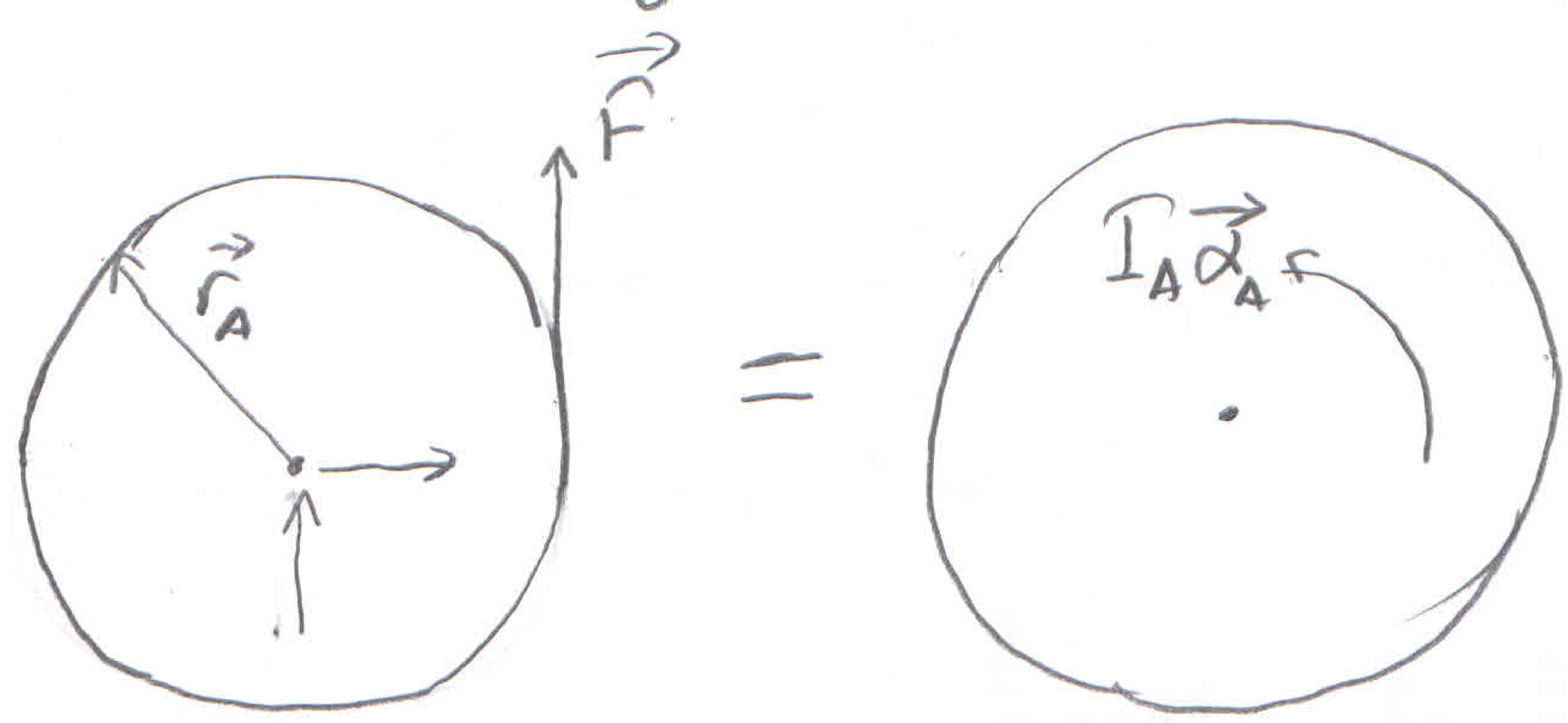


Tangential accelerations of the disks are equal

$$r_A \alpha_A = r_B \alpha_B$$

$$\Rightarrow \alpha_A = \frac{r_B}{r_A} \alpha_B$$

Considering disk A only;

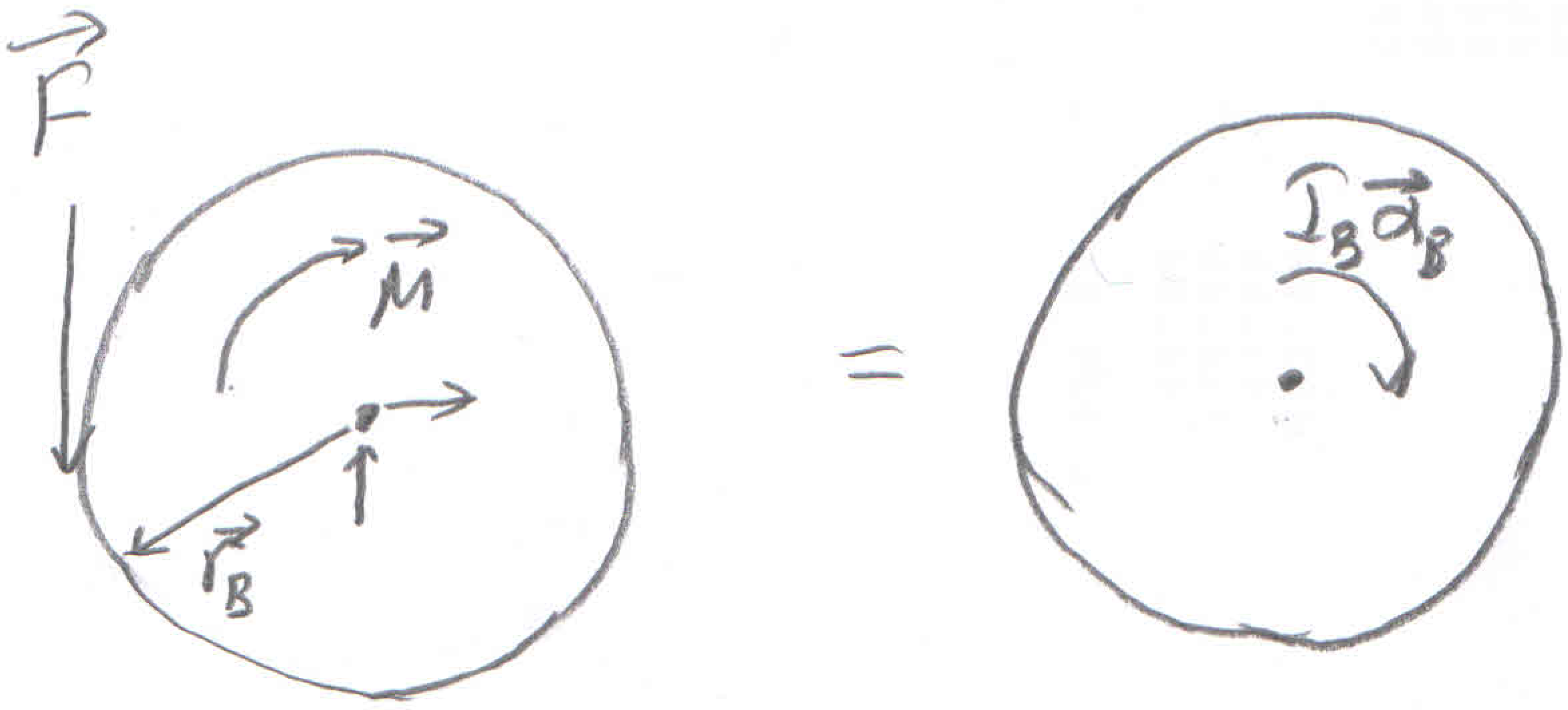


$$I_A = \frac{1}{2} m_A r_A^2$$

$$\overset{+}{\curvearrowleft} \sum M_A = F r_A = I_A \alpha_A$$

$$\Rightarrow \quad F r_A = \frac{1}{2} m_A r_A^2 \alpha_A \quad \Rightarrow F = \frac{1}{2} m_A r_A \alpha_A \qquad (1)$$

Considering disk B;



$$\overset{+}{\curvearrowright} \sum M_B = M - F r_B = I_B \alpha_B \qquad (2)$$

Substituting (1) into (2)

$$M - \left(\frac{1}{2} m_A r_A \alpha_A \right) r_B = I_B \alpha_B$$

Also ; $\quad I_B = \frac{1}{2} m_B r_B^2$

$$M - \frac{1}{2} m_A r_A r_B \left(\frac{r_B}{r_A} \alpha_B \right) = \frac{1}{2} m_B r_B^2 \alpha_B$$

$$\Rightarrow \quad \alpha_B = \frac{2M}{(m_A+m_B)\, r_B^2} = \frac{2 \cdot 0{,}847 \text{ Nm}}{(5{,}4+2{,}7)(0{,}102\text{m})^2}$$

$$= 20{,}10148 \text{ rad/s}^2$$

$\hookrightarrow$ check the dimensions

$$\frac{Nm}{kg \cdot m^2} \Rightarrow \frac{N}{kgm} \Rightarrow \frac{kg \cdot m/s^2}{kgm} = \frac{1}{s^2} \checkmark$$

$$\alpha_A = \frac{r_B}{r_A} \alpha_B = \frac{0{,}102}{0{,}152} \cdot 20{,}10148$$

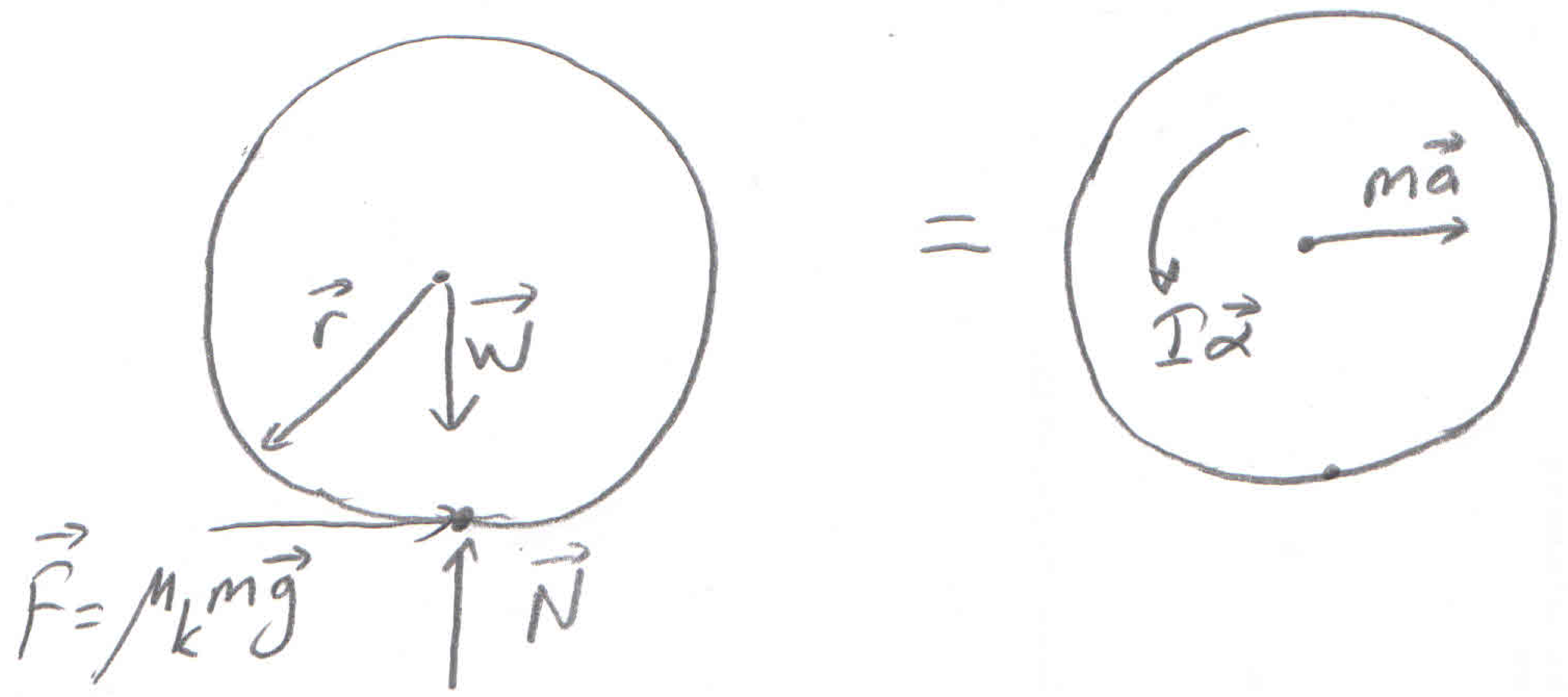
$$= 13{,}491 \text{ rad/s}^2$$

$$F = \frac{1}{2} m_A r_A \alpha_A = \frac{1}{2} \cdot 5{,}4 \cdot 0{,}152 \cdot 13{,}491$$

$$= 5{,}53 \text{ N}$$

$$\vec{F} = 5{,}53 \text{ N} \downarrow$$

Question 3-)



$$\xrightarrow{+} \sum F_x = ma \quad \Rightarrow \mu_k mg = ma$$

$$a = \mu_k g \longrightarrow$$

$$\overset{\curvearrowleft}{+} \sum M \Rightarrow \quad Fr = I\alpha$$

$$(\mu_k mg) r = \frac{2}{5} mr^2 \alpha$$

$$\alpha = \frac{5}{2} \frac{\mu_k g}{r} \curvearrowleft$$

$$\xleftarrow{+} \quad v = v_0 - at = v_0 - \mu_k g t$$

$$\overset{\curvearrowleft}{+} \quad w = \alpha t = \frac{5}{2} \frac{\mu_k g}{r} t$$

considering point of contact with the belt;

$$\xrightarrow{+} \quad v_c = -v + rw$$

$$v_c = -v + r\frac{5}{2} \frac{\mu_k g}{r} t$$

$$v_c = -v + \frac{5\mu_k g}{2} t$$

we know that at $t=t_1$, $v=0$, $v_c=v_1$

$$v_1 = \frac{5\mu_k g}{2} t_1 \Rightarrow t_1 = \frac{2v_1}{5\mu_k g}$$

$$0 = v_0 - \mu_k g\left(\frac{2v_1}{5\mu_k g}\right) \qquad v_0 = \frac{2}{5} v_1$$

@ $t = t_1$

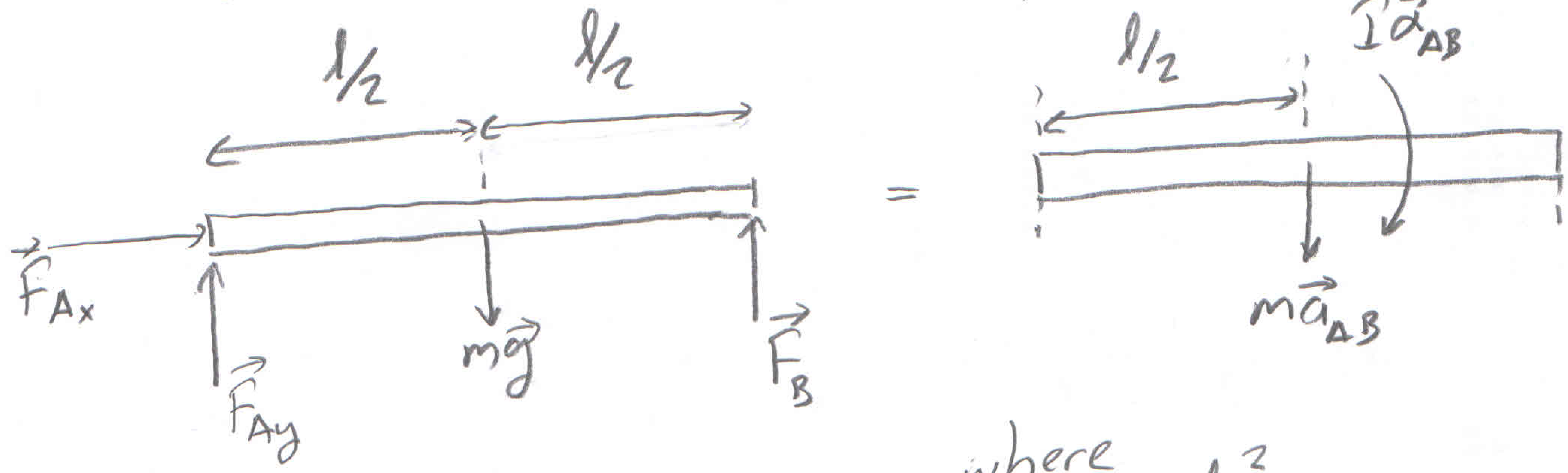
$$\overset{+}{\leftarrow} \quad s = v_0 t_1 - \frac{1}{2} a t_1^2$$

$$s = \left(\frac{2}{5}v_1\right)\left(\frac{2v_1}{5\mu_k g}\right) - \frac{1}{2}(\mu_k g)\left(\frac{2v_1}{5\mu_k g}\right)^2$$

$$\Rightarrow s = \frac{v_1^2}{\mu_k g}\left(\frac{4}{25} - \frac{2}{25}\right) \Rightarrow \vec{s} = \frac{2}{25}\frac{v_1^2}{\mu_k g}(\leftarrow)$$

Question 4-)

If we consider rod AB



where

$$I = \frac{1}{12} m l^2$$

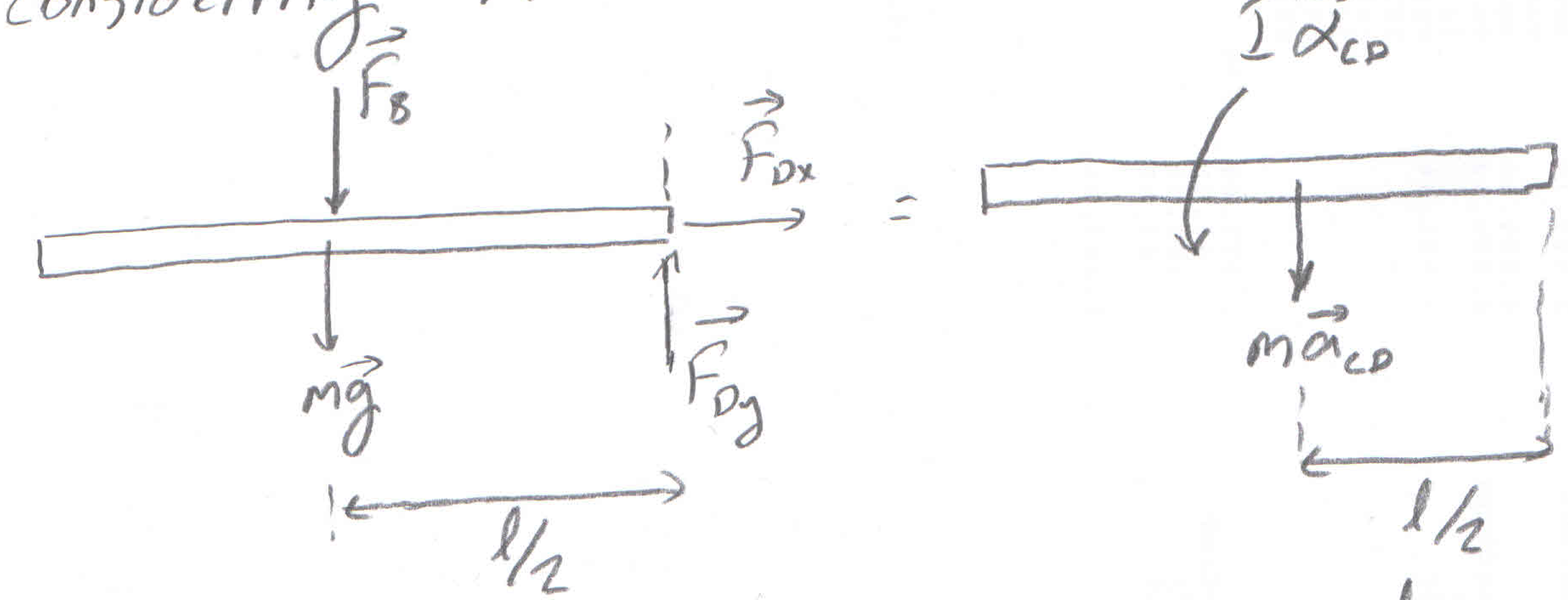
$$m a_{AB} = m \frac{l}{2} \alpha_{AB}$$

$$\overset{+}{\curvearrowright} \sum M_A = mg\left(\frac{l}{2}\right) - F_B l = I\alpha_{AB} + ma_{AB}\left(\frac{l}{2}\right)$$

$$\frac{1}{2}mgl - F_B l = \frac{1}{2}ml^2\alpha_{AB} + \left(m\frac{l}{2}\alpha_{AB}\right)\frac{l}{2}$$

$$\frac{1}{2}mgl - F_B l = \frac{1}{3}ml^2\alpha_{AB} \qquad *1$$

considering Rod CD



where $ma_{CD} = \frac{l}{2}m\alpha_{CD}$

$$\overset{+}{\curvearrowright} \sum M_D = mg\left(\frac{l}{2}\right) + F_B\left(\frac{l}{2}\right) = I\alpha_{CD} + ma_{CD}\left(\frac{l}{2}\right)$$

$$\frac{1}{2}mgl + \frac{1}{2}F_B l = \frac{1}{2}ml^2\alpha_{CD} + \left(m\frac{l}{2}\alpha_{CD}\right)\frac{l}{2}$$

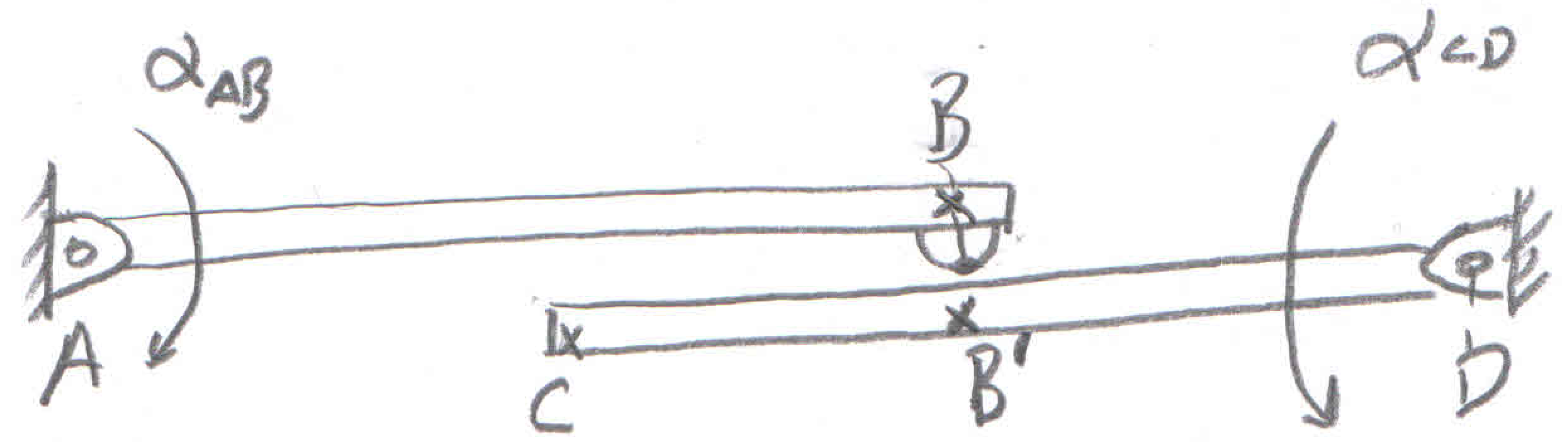
$$mgl + F_B l = \frac{2}{3}ml^2\alpha_{CD} \qquad *2$$

Summing up $*1$ and 2 times $*2$

$$\frac{3}{2}mgl = mgl^2\left(\frac{1}{3}\alpha_{AB} + \frac{2}{3}\alpha_{CD}\right)$$

rearranging gives

$$\alpha_{AB} + 2\alpha_{CD} = \frac{9}{2}(g/l)$$



$\vec{a}_B$ at point B has the direction ↓
$\vec{a}_{B'}$ at point B' " " ↓

$$l\alpha_{AB} = \frac{l}{2}\alpha_{CD} \Rightarrow \alpha_{AB} = \frac{l}{2}\alpha_{CD}$$

$$\Rightarrow \frac{1}{2}\alpha_{CD} + 2\alpha_{CD} = \frac{9}{2}\left(\frac{g}{l}\right)$$

$$\frac{5}{2}\alpha_{CD} = \frac{9}{2}\left(\frac{g}{l}\right) \Rightarrow \alpha_{CD} = 1{,}8\, g/l$$

$$\Rightarrow \vec{a}_C = l\vec{\alpha}_{CD} = 1{,}8g \downarrow$$

we know that $\alpha_{CD} = 1{,}8\ g/l$

and also $mgl + F_B l = \frac{2}{3}ml^2\alpha_{CD}$

$$\Rightarrow mgl + F_B l = \frac{2}{3}ml^2(1{,}8)\left(\frac{g}{l}\right)$$

$$\Rightarrow F_B = 0{,}2\,mg$$

$$\vec{F}_B = 0{,}2\,mg \uparrow$$