CE 388 – FUNDAMENTALS OF STEEL DESIGN

CHAPTER 4: BEAMS

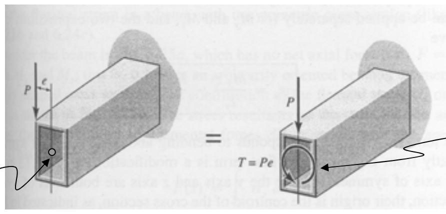
Beams

- A beam is a structural member that carries transverse load and transfers it to its support points through bending moments and shear
- Examples of beams in structures:
  - Girders
  - Stringers
  - Sprandels
  - Purlins
  - Girts, etc.

Shear Center

- The shear center is defined as the point on the cross-section of a beam such that
  - if resultants of the transverse loads pass through the shear center, no twisting of the section takes place
  - Otherwise, twisting will occur in the beam

Shear center

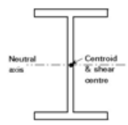


Torsion due to ineccentric load

Twisting of a rectangular tube

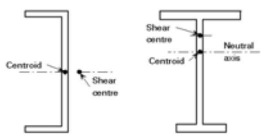
Shear Center

- For doubly symmetric section, the shear center coincides with the centroid, which is the intersection of two axes



(a)
Equal flanged I

- If a beam has only one axis of symmetry, the shear center is on that axis, but it may not coincide with the centroid

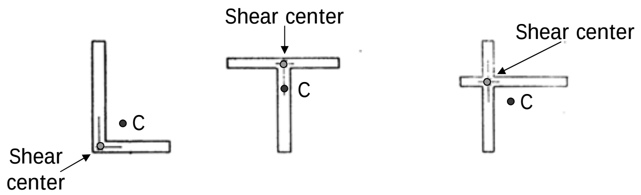


(b) Channel

(c) Unequal flanged I

Shear Center

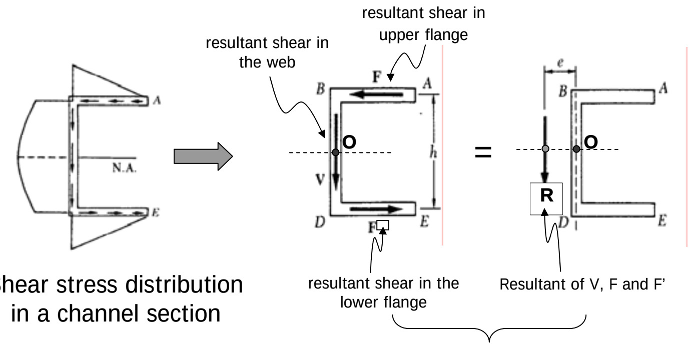
- If the section is made of two intersecting flanges, the shear center is at the point of intersection of flanges



- The shear center of other cross-sections can be located from the fact that the shear center is the resultant of the shear stresses on the cross-section

Shear Center

❑ Example (a channel section):

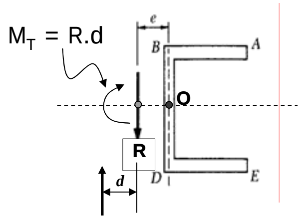


Shear stress distribution in a channel section

$$\sum(F_y)_1 = \sum(F_y)_2 \Rightarrow R = V$$

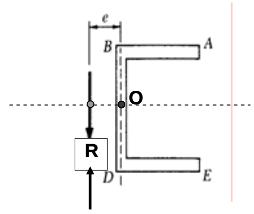
$$\sum(M_o)_1 = \sum(M_o)_2 \Rightarrow F.h = V.e \Rightarrow e = \frac{Fh}{V}$$

Shear Center



applied force (R)

The resultant load does not pass through the shear center - torsion



applied force (R)

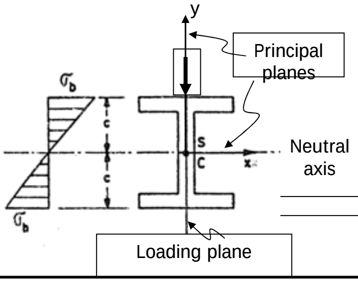
The resultant load does pass through the shear center – no torsion

States of Bending

- A beam may be subjected to three states of bending:
  - Simple Bending
  - Biaxial Bending
  - Bending with Torsion

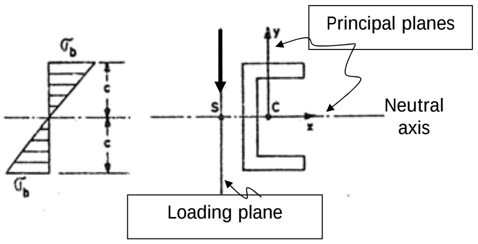
States of Bending

- Simple Bending:
  - The resultant load passes through the shear center (no torsion)
  - Bending takes place about one principal axes only
  - For beams with two axes of symmetry (such as I-sections), the plane of loading coincides with one of the principal planes



States of Bending

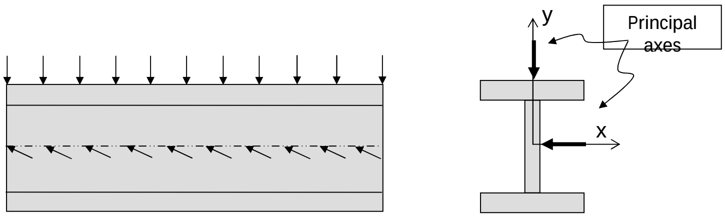
- For beams with one axis of symmetry (such as C-sections), the plane of loading is parallel to one of the principal planes



Single bending for channel section

States of Bending

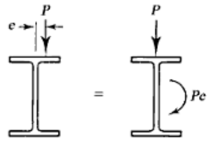
- Biaxial Bending:
  - The resultant load passes through the shear center (no torsion)
  - Bending takes place about both principal axes



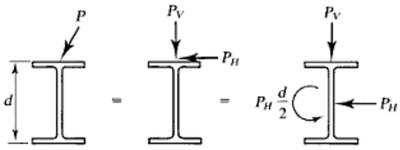
Biaxial bending for I section

States of Bending

- Bending with Torsion:
  - The resultant load ***does not*** pass through the shear center (torsion)
  - Bending takes place about one or both principal axes with torsion



Single bending + torsion



Biaxial bending + torsion

Simple Bending

- Topics Covered in Simple Bending:
  - Flexural analysis
  - Lateral buckling phenomena
  - Lateral supports
  - Allowable bending stress according to TS648
  - Shear analysis
  - Deflection check
  - Failure modes of a web

Flexural Analysis

- Bending stresses varies linearly across the cross-section and are assumed to exist in the longitudional direction
- The flexural formula is used to determine the extreme fiber stress:

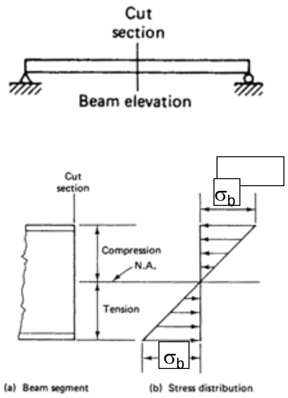
Distance between neutral axis to extreme fiber

Bending moment at the section

$$\sigma_b = \frac{Mc}{I} = \frac{M}{W}$$

Section modulus (I/c)

Extreme fiber stress



Stresses in beams in simple bending

Flexural Analysis

- If a beam is to be designed, the section modulus required to provide sufficient bending strength can be obtained from

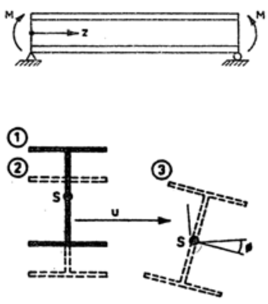
Required section modulus → $$W_{req} = \frac{M}{\sigma_B}$$

M ← Bending moment at the section

σ_B ← Allowable compressive stress due to bending

Lateral Buckling Phenomenon

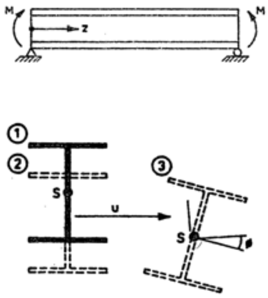
- Conf. 1: Initial configuration of the beam before the load is applied
- Conf. 2: When some moment is applied, the beam deflects due to bending and some portion of it is subjected to compression
- Conf. 3: If moment is increased, at some point the compression zone undergoes displacement normal to the plane of loading



Lateral buckling of beams

Lateral Buckling Phenomenon

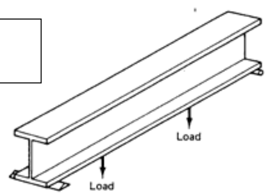
- Beam buckling is called ***lateral torsional buckling*** because buckling displacements are in the lateral direction and also twisting of the section occurs
- Twisting is the result of the fact that tension zone of the beam does not buckle, and moreover it tries to stabilize the compression zone.
- The result is that tension zone remains straight and compression zone deforms laterally



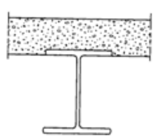
Lateral buckling of beams

Lateral Supports

- If the compression flange of a beam is restrained, lateral buckling problem does not occur
- There are various ways to provide lateral supports to prevent lateral buckling problem
  - A beam which is wholly encased in concrete or which has its compresion flange in a concrete slab is certainly well supported laterally (full lateral support)



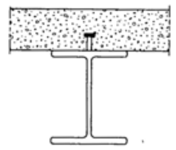
Laterally unsupported beam



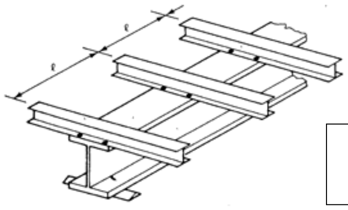
Full lateral support

Lateral Supports

- If the concrete slab rest on the beam, the beam should be anchored to the concrete slab by shear studs (connecters) (full lateral support)
- If there are other beams connected to the compression flange of the beam, they are assumed to provide lateral support at the connections (intermittent lateral support)



Full lateral support



Intermittent lateral support

Allowable Bending Stress according to TS648

- The steel design codes divide the beams into three categories for determining the allowable bending stress
  - Compact beams
  - Laterally supported non-compact beams
  - Laterally unsupported or intermittenly supported beams

Allowable Bending Stress according to TS648

- Compact Beams:
  - They are the beams with stocky components with adequate lateral supports so that local and overall instability will not occur
  - In ASD method, elastic section capacity of a section is considered as the basis (the moment at which outermost fibers yield the reach stress)
  - The true bending strength of a beam is larger. Once the outermost fibers reach the yield stress, the stress in the inner fibers will increase till the whole section is plastified

Allowable Bending Stress according to TS648

- Compact beams are capable of developing their moment capacity before failure
- The allowable compressive stress (σ_B) in bending for compact beams is taken as

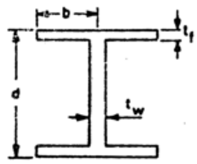
$$\sigma_B = 0.66\sigma_a$$

Allowable Bending Stress according to TS648

- In order for a section to qualify as compact, the following requirements must be met
  - The flanges shall be continuously connected to the webs
  - The depth-thickness ratio will satisfy

$$\frac{d}{t_w} \leq \frac{109.2}{\sqrt{\sigma_a}}\left(1 - \frac{2.33\sigma_w}{\sigma_a}\right)$$

maximum normal stress in the web



A compact section

Allowable Bending Stress according to TS648

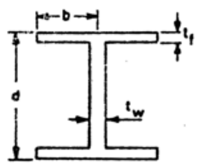
- For unstiffened compression beam flanges (hot rolled sections, such as W-sections), the width-thickness ratio will satisfy

$$\frac{b}{t_f} \leq \frac{13.83}{\sqrt{\sigma_a}}$$

- Compression flange shall be supported laterally at invervals (s) not to exceed

$$s \leq \frac{40b}{\sqrt{\sigma_a}} \quad \text{and} \quad s \leq \frac{1400}{\sigma_a (d / F_b)}$$

Area of compression flange



A compact section

Allowable Bending Stress according to TS648

- Laterally Fully Supported Non-compact Beams:
  - For beams, which are laterally fully supported yet non-compact, it is assumed that the lateral buckling does not occur
  - However, since these beams are non-compact, they may fail before entire section is plastified
  - Hence the allowable compressive stress in bending (allowable bending stress) is taken as

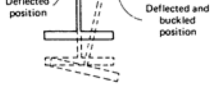
$$\sigma_B = 0.60\sigma_a$$

Allowable Bending Stress according to TS648

- Laterally Unsupported or Intermittently Supported Beams:
  - Lateral buckling may occur, and has to be considered
  - The allowable bending stress is

$$\sigma_B \le 0.60\sigma_a$$

  - It is known that when the compression flange buckles, twisting of the section takes places



Allowable Bending Stress according to TS648

- The two general resistances of a beam to lateral buckling are
  - Lateral bending resistance of the compression flange
  - Torsional resistance of the beam cross-section
- Normally, the total resistance of a section to lateral buckling will be the summation of the two resistances
- However, TS648 conservatively considers only the larger of the two in determination of the allowable bending stress

Allowable Bending Stress according to TS648

- When only lateral bending resistance of the compression flange is considered

$$\text{if } \frac{s}{i_y} \leq \sqrt{\frac{30x10^6 c_b}{\sigma_a}} \Rightarrow \sigma_{B_1} = \left[\frac{2}{3} - \frac{\sigma_a (s/i_y)^2}{90x10^6 c_b}\right]\sigma_a$$

$$\text{if } \frac{s}{i_y} > \sqrt{\frac{30x10^6 c_b}{\sigma_a}} \Rightarrow \sigma_{B_1} = \frac{10x10^6 c_b}{(s/i_y)^2}$$

- When only torsional resistance of the beam is considered

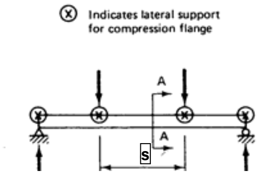
$$\sigma_{B2} = \frac{840x10^3 c_b}{sd/F_b}$$

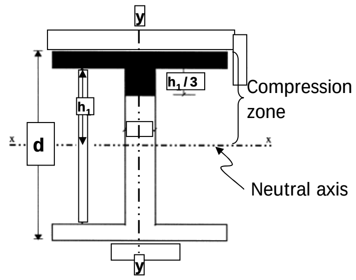
- The allowable compressive stress in bending is larger of σ_{B1} and σ_{B2} and it cannot be larger than $0.60\sigma_a$

$$\sigma_B = \text{larger of } (\sigma_{B_1}, \sigma_{B_2}) \leq 0.60\sigma_a$$

Allowable Bending Stress according to TS648

- In the above equations,
  - s : maximum laterally unsupported length of compression flange (cm)
  - i_y : radious of gyration of the area consisting of compression flange and 1/3 of the compression zone of the web about y-axis
  - d : depth of the beam
  - F_b: area of compression flange (cm^2)
  - c_b: a modifier that takes into account the end restrained conditions and loading conditions because these factors have appreciable effects on lateral buckling resistance of the beam

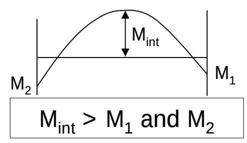




Allowable Bending Stress according to TS648

- If internal moment (M_{int}) at any point between the lateral supports of the beam is larger than end moments M_1 and M_2, then

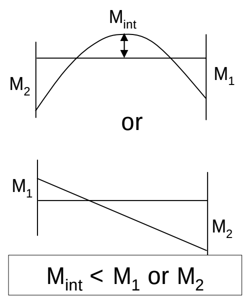
$$c_b = 1.0$$



- If $M_{int} < M_1$ or $M_{int} < M_2$

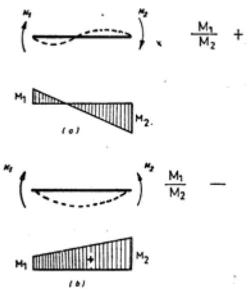
$$c_b = 1.75 + 1.05\left(\frac{M_1}{M_2}\right) + 0.3\left(\frac{M_1}{M_2}\right)^2 \leq 2.3$$

- M_1 : the smaller of the two end moments at lateral supports of the beam
- M_2 : the larger of the two end moments at lateral supports of the beam



Allowable Bending Stress according to TS648

- The ratio (M_1/M_2) is positive if there is a double curvature in the beam. It is negative if there is a single curvature



The sign of ratio M_1 or M_2

- The above formulas are applicable to
  - (a) Doubly symmetric I-beams
  - (b) Channles loaded in a plane paralel to the web
  - (c) I-beams with single symmetry provided that compression flange area is larger than tension flange
  - (d) T-sections loaded in the plane of the web
- For other sections, more rigorous analysis method is necessary

Allowable Bending Stress according to TS648

Example Problems

Shear Analysis

- All beams are subject to shear as well as moment
- Shear is usually not the governing factor in design. However, it may be critical in the following cases
  - Large concentrated loads near supports
  - Very heavily loaded short beams
- The shear stress (τ_v) at a point can be determined from general shear formula:

Vertical shear force at a particular section

Statical moment of area of the portion lying outside the line on which shear stress is computed, taken about neutral axis

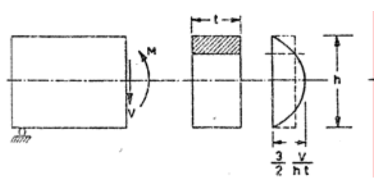
$$\tau_v = \frac{VQ}{It}$$

Moment of inertia about the centroidal axis

Width of the section where shear is computed

Shear Analysis

- Distribution of shear stress for a rectangular section:

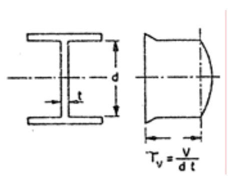


$$(\tau_v)_{\max} = \frac{3}{2}\frac{V}{ht} = 1.5(\tau_v)_{ave}$$

Maximum shear stress (at the mid-point)

Average shear stress

- Distribution of shear stress for a wide-flange beam:



$$(\tau_v)_{\max} >\cong (\tau_v)_{ave}$$

$$(\tau_v)_{ave} = \frac{V}{dt} \quad or \quad (\tau_v)_{ave} = \frac{V}{ht}$$

Shear Analysis

- For design it is required that

$$\text{Calculated shear stress} \longrightarrow \tau \le \tau_{all} \; or \; \tau_{em} \longleftarrow \text{Allowable shear stress}$$

- In AISC, the allowable shear stress is

$$\tau_{all} = 0.4F_y$$

- In TS648, the allowable shear stress is

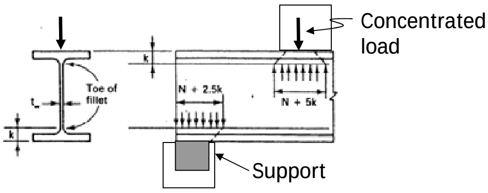
$$\tau_{em} = \frac{\sigma_{çem}}{\sqrt{3}} = 0.346\sigma_a$$

  - For St 37, T_{em} = 831 kgf/cm^2, and for St 52, T_{em} = 1247 kgf/cm^2. The allowable shear stresses for various steels are given in Table 11 of TS648.

Deflection Check

- Deflection of beams are sometimes limited in order to satisfy aesthetic or comfort requirements and/or to prevent damage to non-structural members
- In TS648:
  - For purlins which are longer than 5 m, the maximum allowable deflection is 1/300th of the purlin length
  - For cantilever beams, the max. tip deflection is 1/250th of the beam length
  - The limitations of beams in buildings are not mentioned
- In AISC:
  - The maximum deflection of a beam due to live loads is limited to 1/360 of the span length

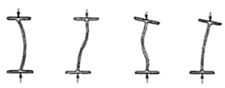
Failure Modes of Web



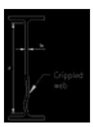
- At supports and at points of concentrated loads, large forces are transmitted from wide flange of a beam to the narrow web (web acts like a column)
- Three possible failure modes are possible
  - Vertical buckling of web
  - Web yielding
  - Web crippling (local web buckling)

Failure Modes of Web

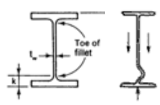
- Vertical web buckling:
  - It is the tendency of a web to buckle as a whole similar to that in columns



- Web crippling:
  - It is the local (not overall) buckling of the web



- Web yielding:
  - It is the yielding of the steel at the junction of the flange and web



  - When loads are transmitted from the flange to the web, the toe of the fillet is most dangerous, because the resisting area has its smallest value there
  - If the load transmitted is excessive, the steel in this area will yield

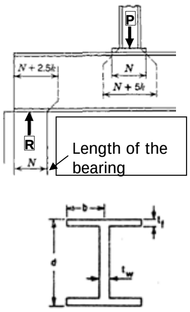
Failure Modes of Web

- Generally, if the web is safe from yielding and crippling, it is also safe from buckling. Therefore, it is required to investigate only yielding and crippling
- Web crippling equations:
  - To prevent web crippling, AISC places the upper limits of concentrated loads
  - For interior loads:

$$P = 67.5 t_w^2 \left[1 + 3 \left(\frac{N}{d} \right) \left(\frac{t_w}{t_f} \right)^{1.5} \right] \sqrt{F_{yw} t_f / t_w}$$

  - For end reactions:

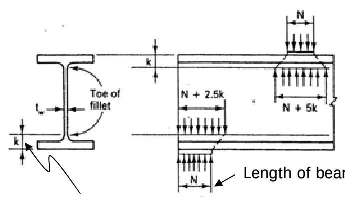
$$R = 34 t_w^2 \left[1 + 3 \left(\frac{N}{d} \right) \left(\frac{t_w}{t_f} \right)^{1.5} \right] \sqrt{F_{yw} t_f / t_w}$$



Yield stress of the beam web

Failure Modes of Web

- Web yielding equations:
  - It is assumed that the load is distributed from place of its application such that the critical area has a length of (N+2.5k) for end reactions and (N+5.0k) for interior loads



Distance between toe of the fillet to the outside of the flane

- For interior loads:

$$\underbrace{\frac{R}{(N+5k).t_w}}_{\text{Compressive stress at the toe of the fillet}} \le \underbrace{0.66\sigma_a}_{\text{Allowable compressive stress for yielding}}$$

- For end reactions:

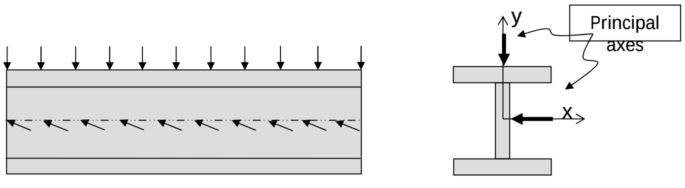
$$\frac{P}{(N+2.5k).t_w} \le 0.66\sigma_a$$

Failure Modes of Web

Example Problem (?)

Biaxial Bending

- From mechanics of material, it is remembered that each beam cross section has a pair of mutually perpendicular axes known as principal axes, for which product of inertia is zero
- Biaxial bending consists of simple bending about two principal axes in the absence of torsion



Biaxial bending for I section

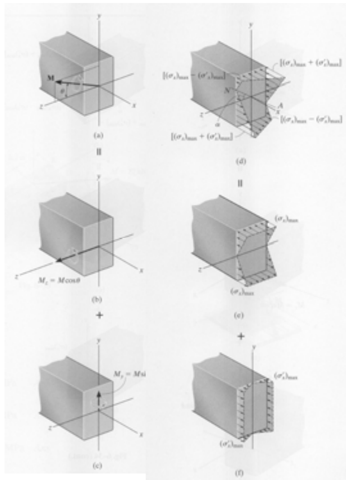
Biaxial Bending

- Combined Bending Stresses:
  - The maximum stresses are caused by a combination of two moment components (M_x and M_y)
  - A beam with an axis of symmetry:
    - Symmetry axis is also a principal axis. The stress at a point (x,y) can be computed as:

$$\sigma_b = \pm\frac{M_x y}{I_x} \pm \frac{M_y x}{I_y}$$

    - The extreme bending stresses are:

$$\sigma_b = \pm\frac{M_x}{W_x} \pm \frac{M_y}{W_y}$$



Combined bending stresses

Biaxial Bending

- A beam with an arbitrary shape:
  - Using principal moment of inertia
    - Determine the direction α of principal axes

$$\tan 2\alpha = -\frac{2I_{xy}}{I_x - I_y}$$

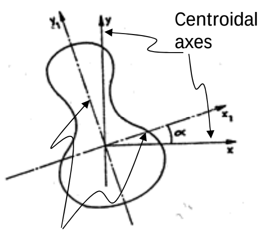
    - Calculate principal moment of inertia I_{x1} and I_{x2}

$$I_{x1} = I_x Cos^2\alpha + I_y Sin^2\alpha - 2I_{xy} Sin\alpha Cos\alpha$$
$$I_{y1} = I_x Sin^2\alpha + I_y Cos^2\alpha + 2I_{xy} Sin\alpha Cos\alpha$$

    - The extreme stresses are obtained from principal moment of inertia

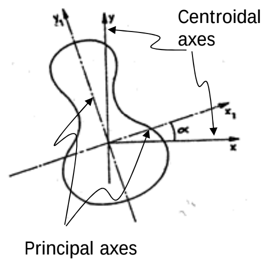
$$\sigma_b = \pm\frac{M_{x1}}{W_{x1}} \pm \frac{M_{y1}}{W_{y1}}$$

M_{x1}, M_{y1}, W_{x1} and W_{y1} are moment components and section modulus about x_1 and y_1 axes



Biaxial Bending

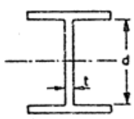
- Using direct formula
  - To avoid tedious calculations in the previous method, one can also use the following direct formula



$$\sigma_b = \frac{M_y I_x - M_x I_{xy}}{I_x I_y - I_{xy}^2} x + \frac{M_x I_y - M_y I_{xy}}{I_x I_y - I_{xy}^2} y$$

Biaxial Bending

- Combined Shear Stresses:
  - Shear stresses are also superimposed in a manner similar to the superposition of the bending moment
  - Open sections:



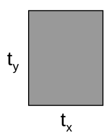
Shear forces paralel to x and y axes

$$\tau_v = \pm \frac{V_y Q_X}{I_x t} \pm \frac{V_x Q_y}{I_y t}$$

Q_x,Q_y: Statics moments about x and y axis

thickness of the section at which shear stress is calculated

  - Solid sections:



$$\tau_{xz} = \frac{V_x Q_y}{I_y t_y} \ (\leftarrow) \qquad \tau_{yz} = \frac{V_y Q_x}{I_x t_x} \ (\downarrow)$$

$$\tau_v = \sqrt{\tau_{xz}^2 + \tau_{yz}^2}$$

Biaxial Bending

- A beam with an arbitrary shape (open sections):

$$\tau_v = \frac{V_y I_y - V_x I_{xy}}{I_x I_y - I_{xy}^2} \cdot \frac{Q_x}{t} + \frac{V_x I_x - V_y I_{xy}}{I_x I_y - I_{xy}^2} \cdot \frac{Q_y}{t}$$

Biaxial Bending

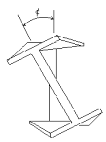
Example Problem

Bending with Torsion

- Bending occuring simultaneously with torsion is called ***bending with torsion***
- Topics covered :
  - Deformations due to torsion
    - Twisting
    - Warping
  - Stresses due to torsion
    - Torsional shear stresses
    - Warping shear stresses
    - Warping normal stresses
    - Approximate method for warping shear and normal stresses
  - Types of torsion
    - Uniform torsion
    - Non-uniform torsion

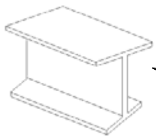
Deformations Due to Torsion

- The members subjected to torsion show two types of deformations:
  - Twisting of the cross-section



The rotation of a cross-section about its shear center through an angle ϕ

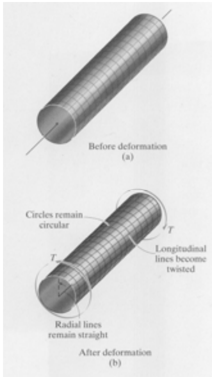
  - Warping of the cross-section



The out of plane deformation of the cross-section. For an I-beam, one corner of the upper flange warps out of the plane, while the other corner warps into the plane

Deformations Due to Torsion

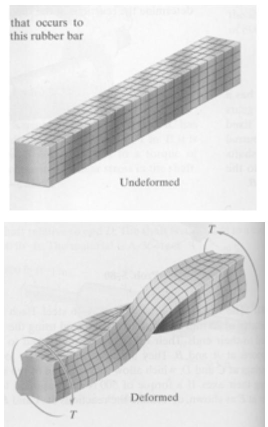
- Circular cross-sections:
  - Each cross-section rotates in its own plane without warping. Hence a plane section before twisting remains a plane section after twisting



Twisting of the circular cross sections

Deformations Due to Torsion

- Non-circular cross-sections:
  - Individual cross-sections along the member will not only rotate but also will deform in a non-uniform manner in the longidutional plane so that plane sections do not remain plane after twisting



Twisting and warping of non-circular cross sections

Stresses Due to Torsion

- The stresses induced on a member as a result of torsion can be classified into three categories:
  - Torsional shear stresses
  - Warping shear stresses
  - Warping normal stresses

Torsional Shear Stresses

- Circular sections:
  - Torsional shear stresses can be computed from the following relationship

Torque acting on the section

Radious of circle

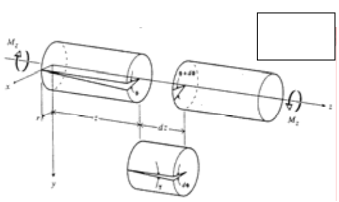
$$\tau = \frac{Tr}{J}$$

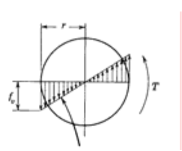
Polar moment of inertia

$$\left\{ \frac{d\phi}{dz} = \frac{T}{GJ} \right.$$

The angle of twist per unit length

Shear modulus



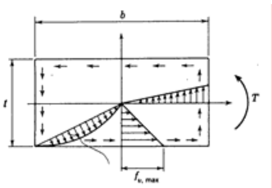


Torsional shear stresses for circular sections

Torsional Shear Stresses

- Solid closed sections:
  - The following relationship exist between the applied torque (T) and angle of twist per unit length ($d\phi/dz$)

$$T = GK_t \frac{d\phi}{dz}$$



Torsional shear stresses for solid closed sections

  - The maximum shear stress (τ_{max}) can be obtained from the following relationship

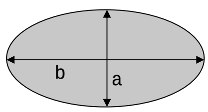
$$\tau_{max} = \frac{Tt_{max}}{K_t}$$

Distance from the center to the furthest point

Torsion coefficient

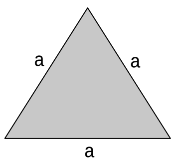
Torsional Shear Stresses

- Torsion Coefficients (K_t):
  - Ellipse:



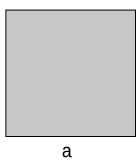
$$K_t = \frac{\pi a^3 b^3}{a^2 + b^2}$$

  - Equilateral triangle



$$K_t = 0.02165a^4$$

  - Square:



$$K_t = 0.141a^4$$

  - Other sections:

Area of the cross-section

$$K_t = 0.025\frac{A^4}{J_g}$$

Polar moment of inertia about center of gravity

Torsional Shear Stresses

- Thin walled closed sections:
  - The relationship T and dϕ/dz

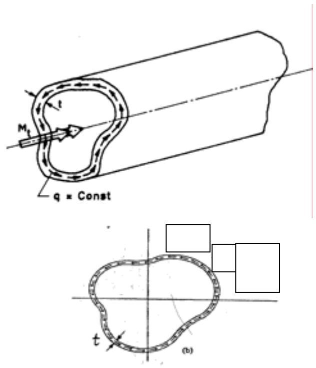
$$T = GK_t \frac{d\phi}{dz}$$

  - The shear stress is assumed to be constant along the thickness and is calculated from:

$$\tau = \frac{T}{2At}$$

Wall thickness

Area enclosed by imaginary curve which traces the tube centerline



Torsional shear stresses for thin walled closed sections

Torsional Shear Stresses

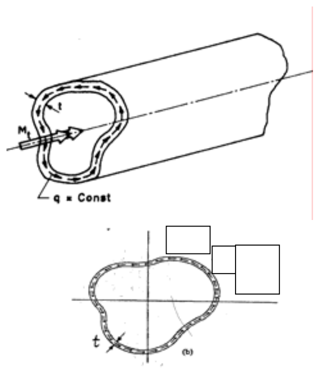
- Torsional coefficient (K_t):

$$K_t = \frac{4A^2t}{S}$$

Perimeter of the tube centerline

$$T = GK_t \frac{d\phi}{dz}$$

$$\frac{d\phi}{dz} = \frac{TS}{4A^2Gt}$$



Torsional shear stresses for thin walled closed sections

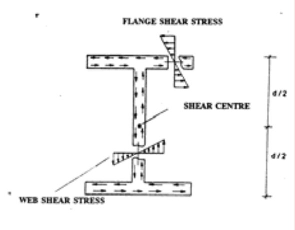
Torsional Shear Stresses

- Thin walled open sections:
  - Shear stress varies linearly over the thickness, zero at the center line and maximum and opposite in sign at the two edges
  - Maximum shear stress can be computed from:

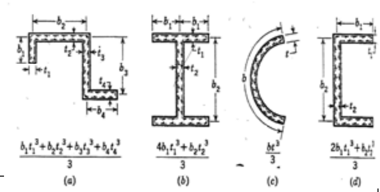
$$\tau_{\max} = \frac{Tt}{K_t}$$

  - Torsional coefficient (K_t):

$$K_t = \frac{1}{3}\sum b_i t_i^3$$



Torsional shear stresses for thin walled open sections



Torsional coefficient

Warping Shear Stresses

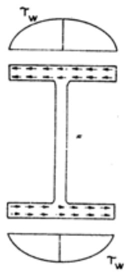
- Warping shear stresses are constant over the thickness of the element and vary along the length of the element and they are parallel to the sides
- The equation for calculating warping shear stress is

Elasticity modulus

Warping statical moment at the section considered

$$\tau_w = \frac{-ES_{ws}\dfrac{d^3\phi}{dz^3}}{t}$$

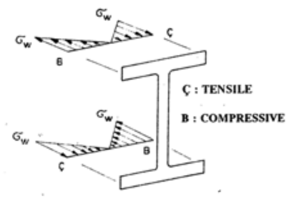
Thickness of the cross-section at the point considered



Warping shear stresses

Warping Normal Stresses

- Warping normal stresses are direct tension and compression stresses resulting from bending of the element due to torsion
- They are constant across the thickness of the element, but vary in magnitude along the length of the element
- These stresses are determined from:



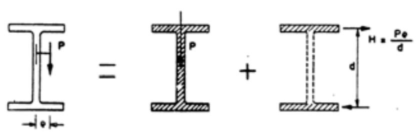
Warping normal stresses

$$\sigma_w = EW_n \frac{d^2\phi}{dz^2}$$

Normalized warping constant

Approximate method for warping shear and normal stresses

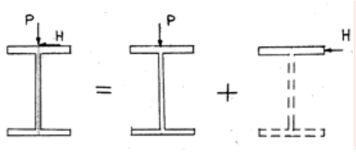
- Some codes including TS648 present conservative methods for calculating stresses due to warping torsion
- Case 1: A vertical eccentric load P



  - The 1st subcase: the force P passes through the shear center
  - The 2nd subcase: the torsional moment Pe induced due to eccentricity is accounted for a force couple H acting on the flanges

Approximate method for warping shear and normal stresses

- Case 2: A horizontal eccentric load H



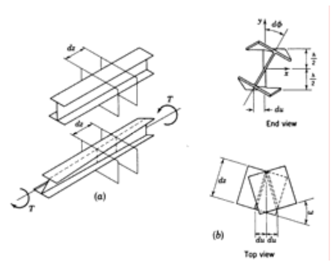
  - The 1st subcase: the force P passes through the shear center
  - The 2nd subcase: the horizontal force H is applied on the flange it acts

Types of Torsion

- Types of torsion:
  - Uniform torsion
  - Non-uniform torsion

Types of Torsion

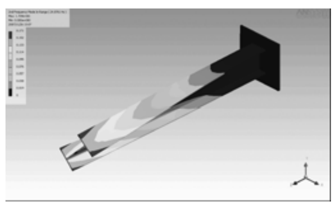
- Uniform torsion:
  - The angle of twist (dϕ/dz) is constant
  - Uniform torsion occurs in
    - Circular sections
    - Unrestrained non-circular beams
  - Stresses developed are
    - Only torsional shear stresses

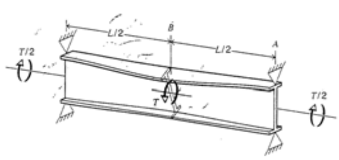


Uniform torsion

Types of Torsion

- Non-uniform torsion:
  - The angle of twist (dϕ/dz) is not constant
  - Uniform torsion occurs in
    - Restrained non-circular sections
  - Stresses developed are
    - Torsional shear stresses
    - Warping shear stresses
    - Warping normal stresses





Non-uniform torsion

Bending with Torsion

Example Problem