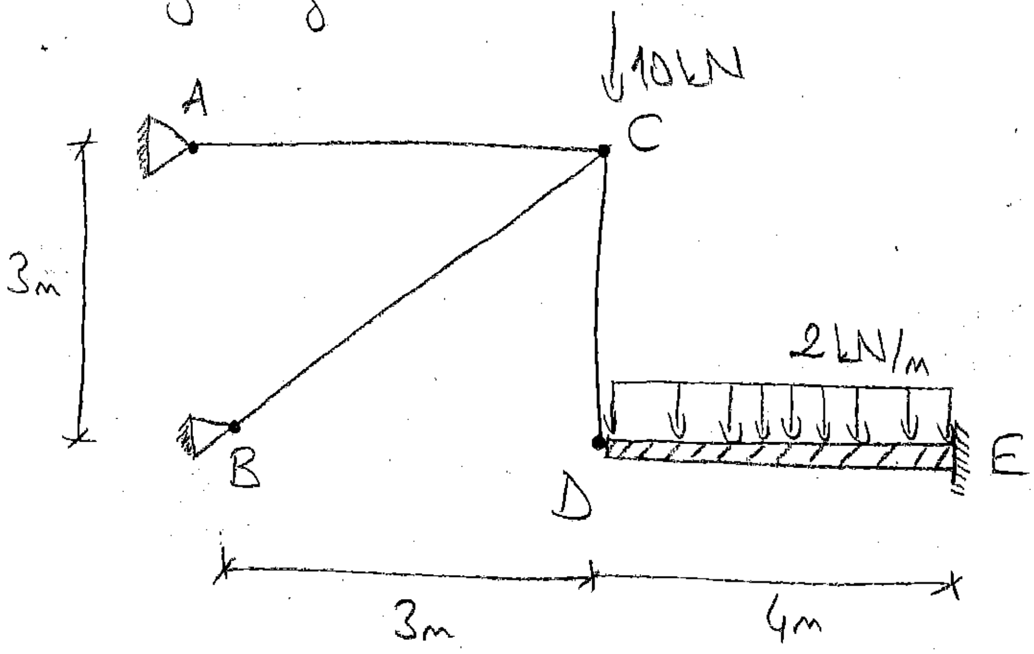
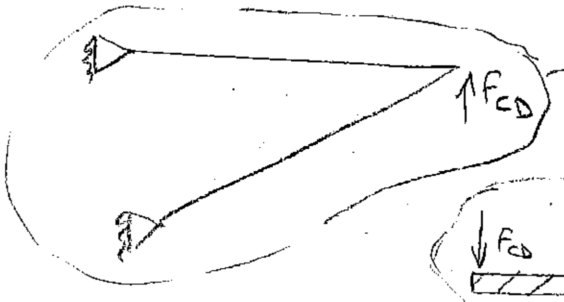
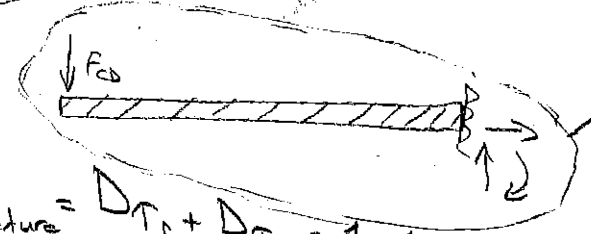
1) Find the force in bar CD and the moment at E for the following system using the force method of analysis. Take $EI_{beam} = 2000\ kN.m^2$ and $EA_{truss} = 200\ kN$. Assume the beam is axially rigid.



First, determine the degree of determinacy.



$D_{T_{tr}} = n + r - 2j = 2 + 4 - 6 = 0$

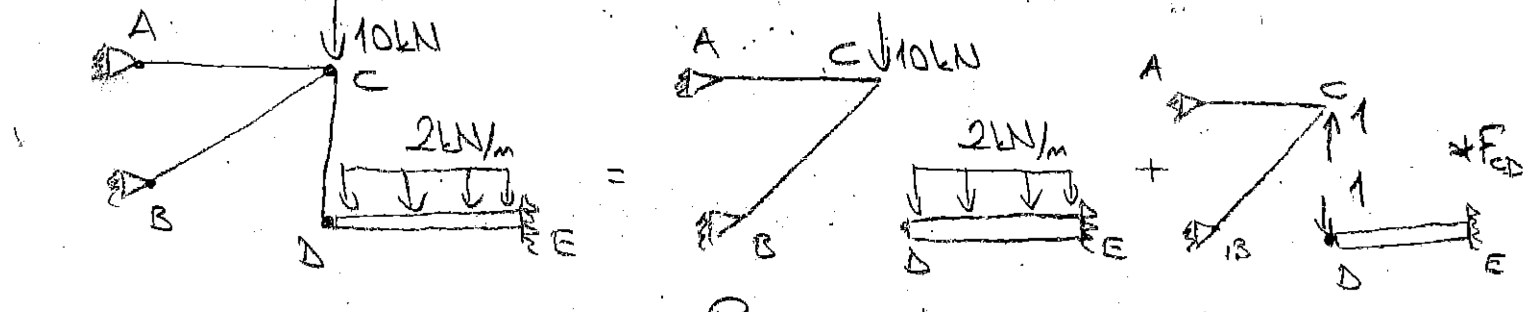


$D_{T_f} = 3n + r - e - x$

$D_{T_f} = 0 + 4 - 3 - 0 = 1$

$D_{T_{structure}} = D_{T_f} + D_{T_{tr}} = 1$

Choose member CD as redundant.



Basic structure Fictitious structure

Write compatibility equation.

$$\Delta_{CD} = \Delta_{CD_0} + f \cdot F_{CD} = 0$$ (Integrity of member CD is satisfied.)

Δ_{CD_0} = The relative displacement due to real forces on basic structure

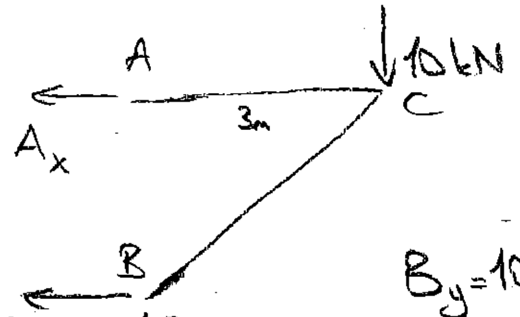
$$\Delta_{CD_0} = \int \frac{M.m.dx}{EI} + \Sigma \frac{N.n.L}{EA}$$

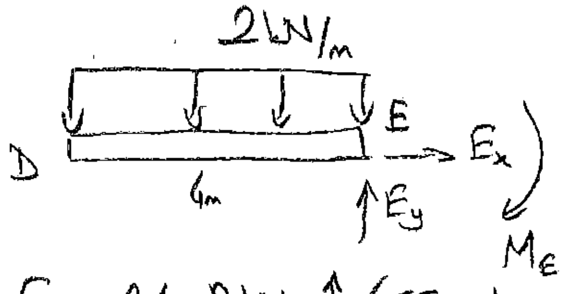
f = The relative displacement due to fictitious forces $\left(\begin{smallmatrix}\uparrow 1\\ \downarrow 1\end{smallmatrix}\right)$

$$f = \int \frac{m.m.dx}{EI} + \Sigma \frac{n.n.L}{EA}$$

Therefore, we have to determine internal forces.

Basic structure:





$B_y = 10\,kN \uparrow$ $(\Sigma F_y = 0)$

$B_x = \frac{-10.3}{3} = -10\,kN \rightarrow$ $(\Sigma M_A = 0)$

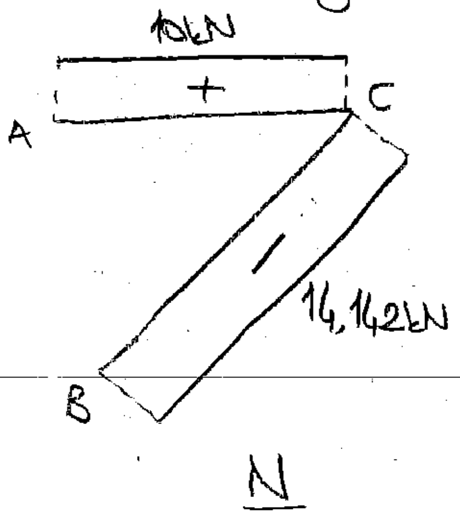
$A_x = -B_x = 10\,kN \leftarrow$ $(\Sigma F_x = 0)$

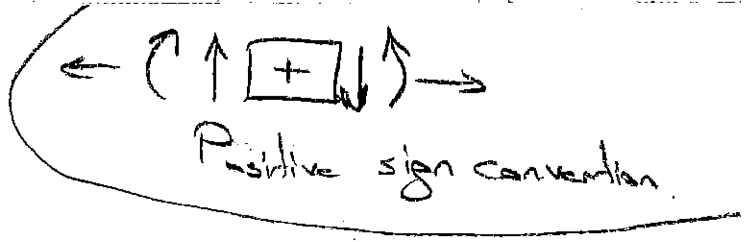
$E_y = 2.4 = 8\,kN \uparrow$ $(\Sigma F_y = 0)$

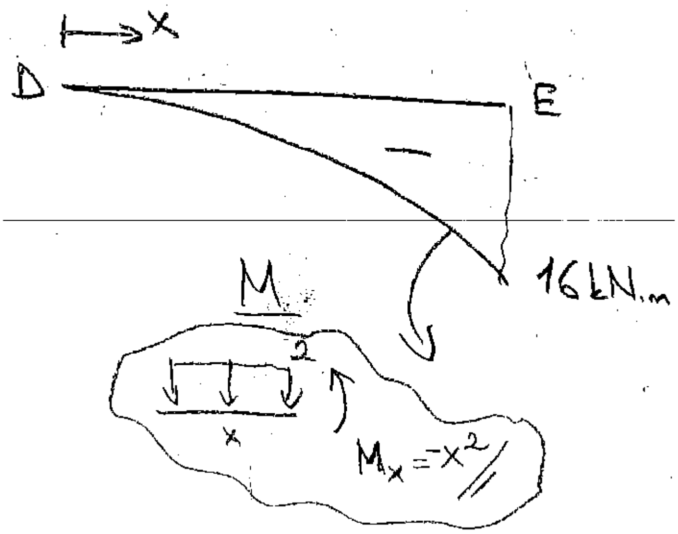
$E_x = 0\,N$ $(\Sigma F_x = 0)$

$M_E = 2.4.2 = 16\,kN.m$ $(\Sigma M_E = 0)$

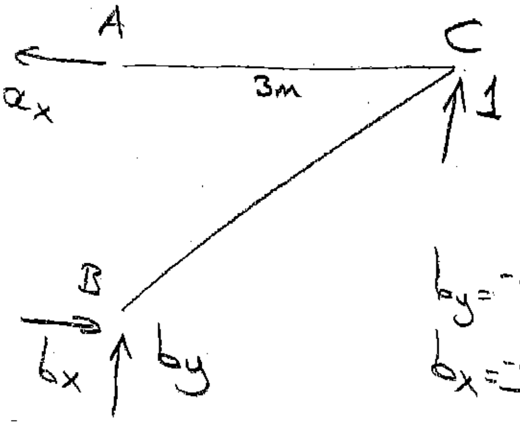
The internal forces are







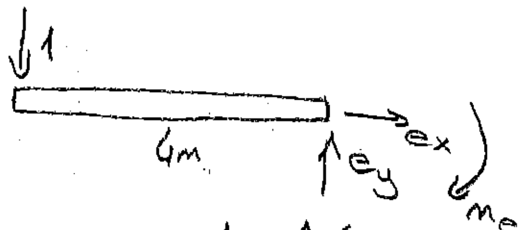
Fictitious structure:



$b_y = -1$ ↓ ($\Sigma F_y = 0$)

$b_x = \frac{-1 \cdot 3}{3} = -1$ ← ($\Sigma M_A = 0$)

$a_x = b_x = -1$ → ($\Sigma F_x = 0$)

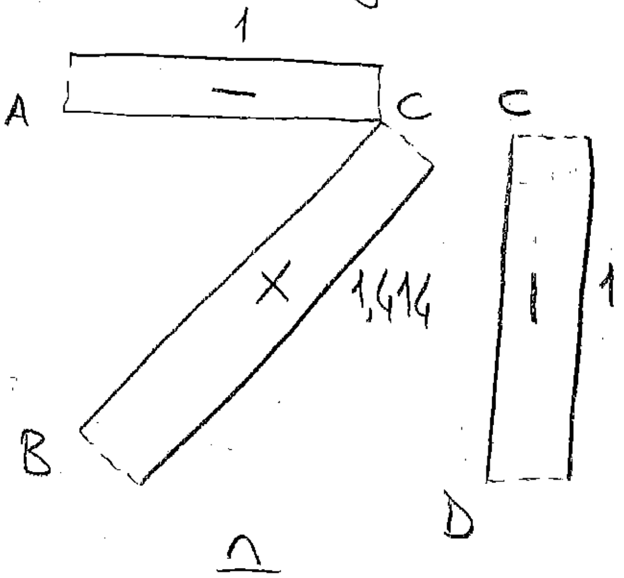


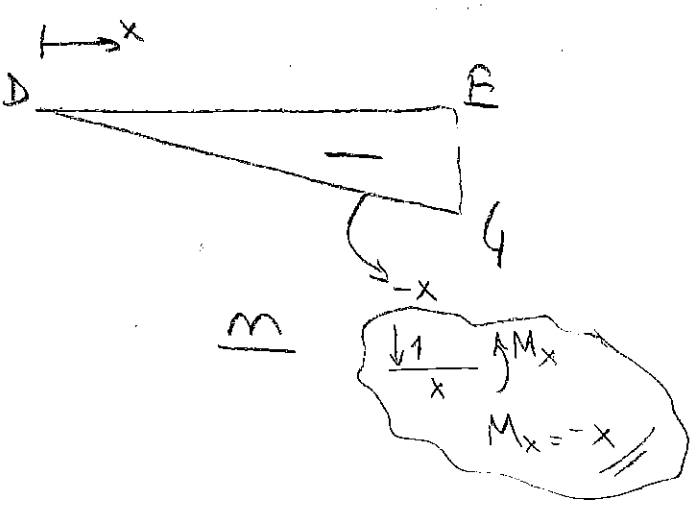
$e_y = 1$ ↑ ($\Sigma F_y = 0$)

$e_x = 0$ ($\Sigma F_x = 0$)

$m_e = 1 \cdot 4 = 4$ ↺ ($\Sigma M_E = 0$)

The internal forces are





$$\Delta_{CD_0} = \underbrace{\int \frac{M \cdot m \cdot dx}{EI}}_{\triangleleft * \triangleleft} + \sum \frac{N \cdot n \cdot L}{EA}$$

$$= \underbrace{\frac{1}{4}\cdot\frac{4\cdot -16\cdot -4}{2000}}_{0{,}032} + \underbrace{\frac{10\cdot -1\cdot 3}{200}}_{-0{,}15} + \underbrace{\frac{-14{,}142\cdot 1{,}414\cdot 4{,}243}{200}}_{-0{,}424} = -0{,}542$$

$$f = \underbrace{\int \frac{m\cdot m\cdot dx}{EI}}_{\triangleleft * \triangleleft} + \sum \frac{n\cdot n\cdot L}{EA}$$

Contribution of member CD

$$= \underbrace{\frac{1}{3}\cdot\frac{4\cdot -4\cdot -4}{2000}}_{0{,}0107} + \underbrace{\frac{-1\cdot -1\cdot 3}{200}}_{0{,}015} + \underbrace{\frac{1{,}414\cdot 1{,}414\cdot 4{,}243}{200}}_{0{,}0424} + \underbrace{\frac{-1\cdot -1\cdot 3}{200}}_{0{,}015} = 0{,}083$$

$$\Delta_{CD_0} + f\cdot F_{CD} = 0 \Rightarrow F_{CD} = \frac{0{,}542}{0{,}083} = 6{,}53\ kN \quad (C)$$

$$M_E = M_{E_0} + m_e \cdot F_{CD} = -16 - 4\cdot 6{,}53 = -42{,}12\ kN.m$$