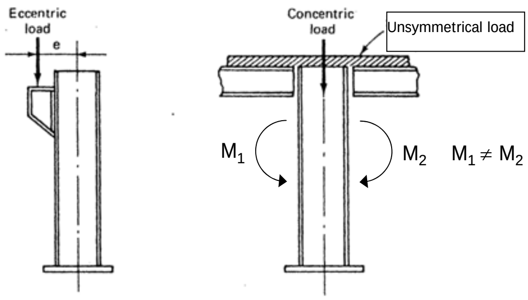
CE 388 – FUNDAMENTALS OF STEEL DESIGN

CHAPTER 5: BEAM-COLUMNS

Introduction

- Generally, axially (concentrically) loaded compression members are non-existent in actual structures
- All compression members are subjected to some amount of bending moment
- The bending moment may be induced by
  - An eccentric load
  - Unsymmetrical loadings
  - Continuous frame action

Introduction

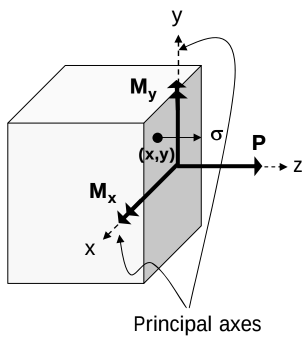


Eccentric loading Unsymmetrical loading

- A structural member that is subjected to both axial compression and bending moment is termed as a ***beam-column***

Stresses in Beam-Columns

- For a member subjected to an axial load P and bending moments M_x and M_y, the normal stress at any point (x,y) can be computed from



$$\sigma = \frac{P}{A} \pm \frac{M_x y}{I_x} \pm \frac{M_y x}{I_y}$$

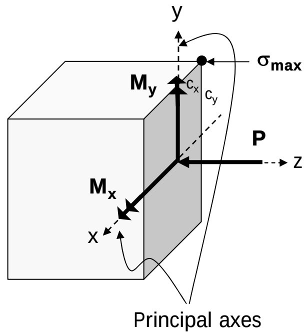
Cross-sectional area Moment of inertia about x-axis Moment of inertia about y-axis

Stresses in Beam-Columns

- Maximum compressive stress

$$\sigma_{\max} = \frac{P}{A} + \frac{M_x c_y}{I_x} + \frac{M_y c_x}{I_y}$$

$$\sigma_{\max} = \underbrace{\sigma_{eb}}_{\text{Compressive stress due to axial force P}} + \underbrace{\sigma_{bx}}_{\text{Maximum comp. stress due to bending moment } M_x} + \underbrace{\sigma_{by}}_{\text{Maximum comp. stress due to bending moment } M_y}$$



Stresses in Beam-Columns

- For a safe design, $\sigma_{max} \le \sigma_{all}$

$$\sigma_{\max} = \sigma_{eb} + \sigma_{bx} + \sigma_{by} \le \sigma_{all}$$

- Dividing the above equation by σ_{all},

$$\frac{\sigma_{eb}}{\sigma_{all}} + \frac{\sigma_{bx}}{\sigma_{all}} + \frac{\sigma_{by}}{\sigma_{all}} \le 1$$

$\sigma_{all} = \sigma_{bem}$ (allowable axial compressive stress)

$\sigma_{all} = \sigma_{Bx}$ (allowable bending stress about x-axis)

$\sigma_{all} = \sigma_{By}$ (allowable bending stress about y-axis)

- Replacing σ_{all} terms by applicable allowable stresses,

$$\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_{bx}}{\sigma_{Bx}} + \frac{\sigma_{by}}{\sigma_{By}} \le 1$$

Interaction equation without second order effects

Second Order (P-Δ) Effects

- In common elastic methods of structural analysis, it is assumed that all deformations are small and thus equilibrium equations are based on undeformed geometry of the structure
- The results of that type of analysis are referred to as first order effects; i.e, first order forces, first-order moments, first order displacements, etc.
- Second order effects consider the changes in member forces and moments as a direct result of structural deformations

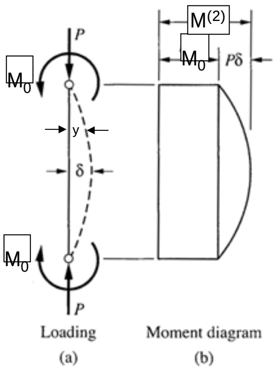
Second Order (P-Δ) Effects

- Consider a column subject to equal and opposite end moments M_0
- The first order moment at a point:

First order moment $\longrightarrow M^{(1)} = M_0$

- At each point, an additional moment P.y is created due to deflection of the column. The second order moment at a point can be computed from

Second order moment $\longrightarrow M^{(2)} = M_0 + P.y$



Second order effect

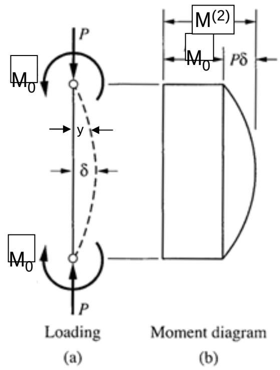
Second Order (P-Δ) Effects

- Maximum second order moment occurs at mid-height where deflection is maximum (δ)

Maximum second order moment → $M_{\max}^{(2)} = M_0 + P.\delta$

- Defining an amplification factor (AF) as the ratio of maximum second order moment to the first order moment,

$$AF = \frac{M_{\max}^{(2)}}{M^{(1)}} = \frac{M_0 + P\delta}{M_0}$$



Second order effect

Second Order (P-Δ) Effects

- Rearranging the terms,

$$AF = \frac{1}{1 - \frac{P\delta}{M_0 + P\delta}} \quad \text{................(1)}$$

Assume δ is very small

$$\frac{\delta}{M_0 + P\delta} = \frac{\delta}{M_0} \quad \text{................(2)}$$

From beam deflection theory

$$\delta = \frac{M_0 L^2}{8EI} \Rightarrow \frac{\delta}{M_0} = \frac{L^2}{8EI} \approx \frac{L^2}{\pi^2 EI} = \frac{1}{P_{cr}} \text{ or } \frac{1}{P_e} \quad \text{................(3)}$$

Elastic (Euler)
Buckling strength

Second Order (P-Δ) Effects

- Eqns. (3) and (2) → Eqn. (1),

$$AF = \frac{1}{1-\frac{P}{P_e}} = \frac{1}{1-\frac{\sigma}{\sigma_e}} \quad \text{................(4)}$$

  - Eqn. (4) is applicable to columns with end moments equal and opposite to each other
  - The second order forces and moments can be determined by modifying the results of the first order analysis using amplification factor

$$M_{\max}^{(2)} = AF.M^{(1)}$$

Design of Beam-Columns According to TS648

- Members subject to axial compression and flexure must be sized to meet the following requirements
  - If $\sigma_{eb} / \sigma_{bem} \leq 0.15$

$$\boxed{\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_{bx}}{\sigma_{Bx}} + \frac{\sigma_{by}}{\sigma_{By}} \leq 1.0}$$

  - If $\sigma_{eb} / \sigma_{bem} > 0.15$

$$\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{c_{mx}\sigma_{bx}}{(1-\frac{\sigma_{eb}}{\sigma_{ex'}})\sigma_{Bx}} + \frac{c_{my}\sigma_{by}}{(1-\frac{\sigma_{eb}}{\sigma_{ey'}})\sigma_{By}} \leq 1.0 \quad \text{(Stability criteria)}$$

$$\frac{\sigma_{eb}}{0.60\sigma_a} + \frac{\sigma_{bx}}{\sigma_{Bx}} + \frac{\sigma_{by}}{\sigma_{By}} \leq 1.0 \quad \text{(Yield criteria)}$$

Design of Beam-Columns According to TS648

- If σ_{eb} / σ_{bem} ≤ 0.15
  - The column is subjected to a small compression force
  - Second order effects are disregarded and a direct interaction equation is used

computed compressive stress due to axial force
computed compressive stress due to bending about x-axis
computed compressive stress due to bending about y-axis

$$\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_{bx}}{\sigma_{Bx}} + \frac{\sigma_{by}}{\sigma_{By}} \leq 1.0$$

allowable compressive stress due to axial force
allowable compressive stress due to bending about x-axis
allowable compressive stress due to bending about y-axis

Design of Beam-Columns According to TS648

- If σ_{eb} / σ_{bem} > 0.15
  - The second order effects may be important, and are taken into account by multiplying computed compressive stresses about x and y axes by amplification factors AF_x and AF_y

$$\frac{\sigma_{eb}}{\sigma_{bem}} + c_{mx}\underbrace{\frac{1}{(1-\frac{\sigma_{eb}}{\sigma_{ex'}})}}_{AF_x}\frac{\sigma_{bx}}{\sigma_{Bx}} + c_{my}\underbrace{\frac{1}{(1-\frac{\sigma_{eb}}{\sigma_{ey'}})}}_{AF_y}\frac{\sigma_{by}}{\sigma_{By}} \leq 1.0$$

$$\left.\begin{aligned}\sigma_{ex'} &= \frac{\pi^2 E}{(\lambda_x)^2}.\frac{1}{2.5} = \frac{8290000}{(\lambda_x)^2}\\ \sigma_{ey'} &= \frac{\pi^2 E}{(\lambda_y)^2}.\frac{1}{2.5} = \frac{8290000}{(\lambda_y)^2}\end{aligned}\right\}$$

Euler buckling stresses about x and y-axes divided by a factor of safety 2.5

Design of Beam-Columns According to TS648

- The factors cm_x and cm_y (≤ 1.0) are known as modification factors or reduction factors

Modification or reduction factors

$$\frac{\sigma_{eb}}{\sigma_{bem}} + c_{mx}\frac{1}{(1-\frac{\sigma_{eb}}{\sigma_{ex'}})}\frac{\sigma_{bx}}{\sigma_{Bx}} + c_{my}\frac{1}{(1-\frac{\sigma_{eb}}{\sigma_{ey'}})}\frac{\sigma_{by}}{\sigma_{By}} \leq 1.0$$

- AF_x and AF_y are derived for the case of equal and opposite end moments, which is the most severe case for a beam-column
- They overestimate the second order effects for other load conditions and end moments
- cm_x and cm_y are introduced to modify (or to recude) the second order effects for other load conditions and end moments

Design of Beam-Columns According to TS648

- To find value of c_m, TS648 divides the beam-columns into three categories,
  - (i) A compression member in a frame where sidesway is permitted

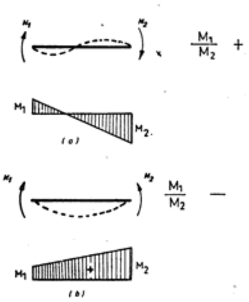
$$C_m = 0.85$$

  - (ii) A compression member in a frame where sidesway is prevented and member not subjected to transverse loading between their ends

$$C_m = 0.6 - 0.4(M_1 / M_2) \geq 0.4$$

Design of Beam-Columns According to TS648

- M_1/M_2 is the ratio of smaller end moment to the larger end moment
- The ratio M_1/M_2 is positive when the end moments M_1 and M_2 are in the same direction (reverse curvature), and is negative otherwise (single curvature)
- Note that when the two end moments have the same magnitude and opposite direction (M_1 = -M_2), c_m = 1, which is the maximum



The sign of ratio M_1 or M_2

Design of Beam-Columns According to TS648

(iii) A compression member in a frame where sidesway is prevented and member is subjected to transverse loading between their ends

$$C_m = 1 + \frac{\sigma_{eb}}{\sigma_{e'}}\psi$$

dimensionless constant

- For some end restraints and load conditions, ψ can be obtained from table
- For other conditions,

Max. deflection

$$\psi = \frac{\pi^2 \delta_o EI}{M_0 s^2} - 1$$

Max. moment

Table 8-1

Case	ψ	C_m
(a)	0	1.0
(b)	−0.4	$1-0.4\frac{f_a}{F_e'}$
(c)	−0.4	$1-0.4\frac{f_a}{F_e'}$
(d)	−0.2	$1-0.2\frac{f_a}{F_e'}$
(e) $L/2$	−0.3	$1-0.3\frac{f_a}{F_e'}$
(f)	−0.2	$1-0.2\frac{f_a}{F_e'}$

ψ values for some end restraints and load conditions

Design of Beam-Columns According to TS648

- If a member is subjected to a combination of axial tension and bending, the following expression must be satisfied:

$$\frac{\sigma_{eb}}{0.60\sigma_a} + \frac{\sigma_{bx}}{\sigma_{Bx}} + \frac{\sigma_{by}}{\sigma_{By}} \leq 1.0$$

Design of Beam-Columns According to TS648

Example Problems