ULTIMATE STRENGTH OF MEMBERS SUBJECTED TO FLEXURE

ULTIMATE STRENGTH OF MEMBERS SUBJECTED TO FLEXURE

5.4 Design of Beams

5.4.1 General

5.4.2 Rectangular Beam Reinforced or Tension only

5.4.3 Double Reinforced Rectangular Beams

5.4.4 Flanged Sections (T-Beams)

5.4.5 Bending of Bars (for flexure only)

5.4.6 Minimum Requirements for Beams

5.4.7 Design Moments

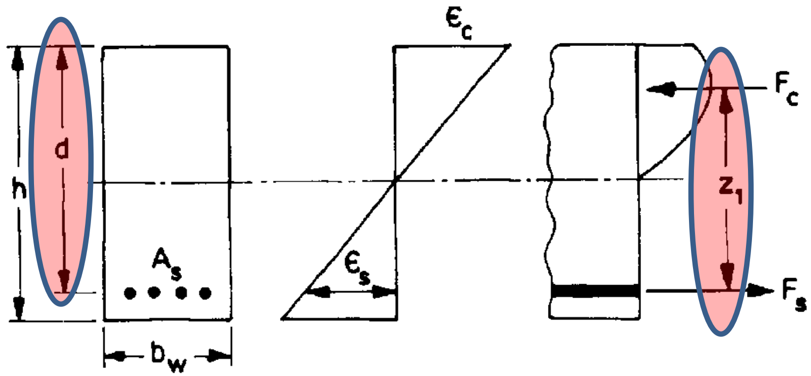
5.5 Effect of Material Strength on the Moment Capacity

INTRODUCTION

Plain concrete is not used in members subjected to flexure, because the flexural strength of such members are governed by the low tensile strength of concrete.

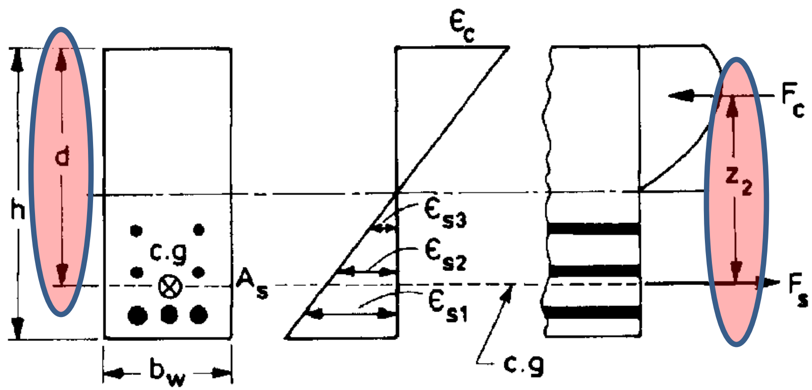
In plain concrete flexural members when the failure load is reached (which corresponds to the cracking load) concrete in compression is still understressed. Therefore, in such beams high capacity of concrete in compression cannot be employed. The only way to make use of this reserved strength in compression is to increase the strength of the beam in the tension zone. This can be accomplished by providing steel bars in the tension zone to take the resulting tension force.

Steel concentrated at one layer: Higher moment capacity compared to (b)



(a)

Steel distributed over the tension zone: Lower moment capacity compared to (a) but smaller crack width

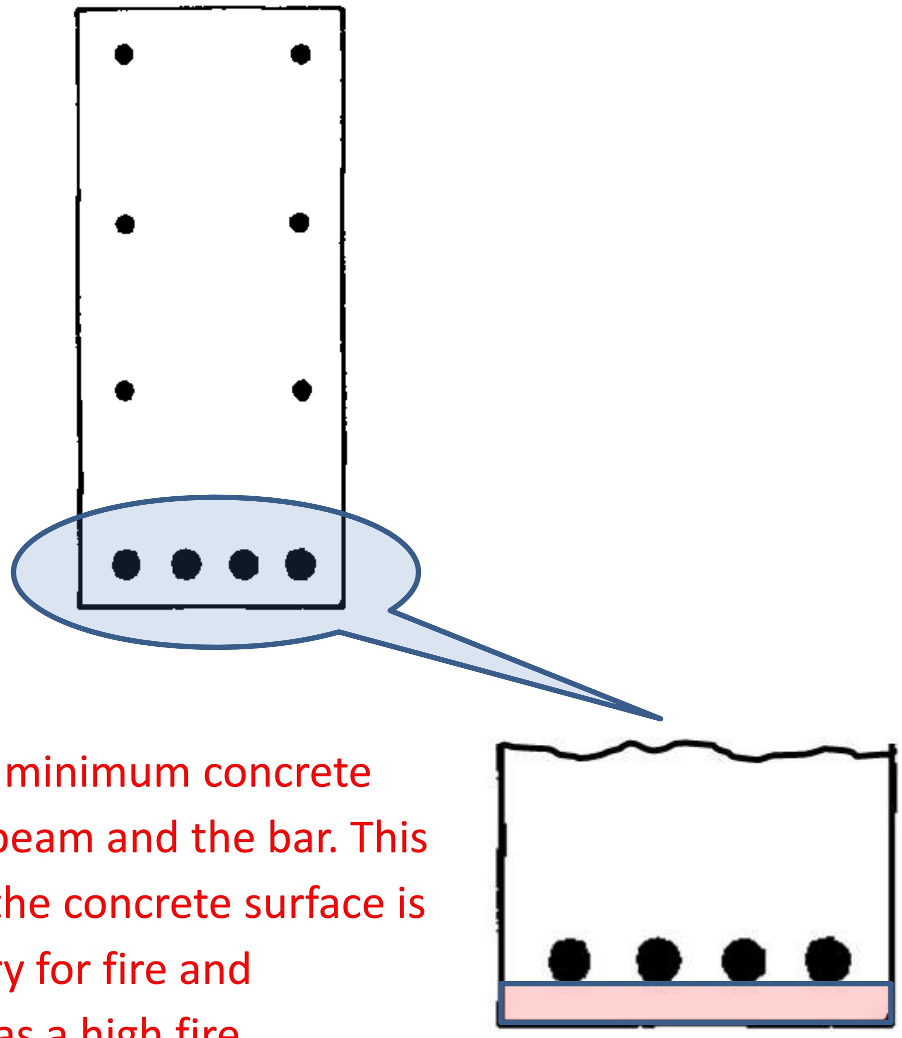


(b)

In practice, steel for tension is not usually distributed in tension zone, but is placed as near to the tension face as possible. The main objective in doing so is to increase the moment arm, thus increasing the moment capacity.

The distance from the centroid of tensile steel to the extreme fiber in compression is more important than the total depth, "h." The distance from the centroid of the tension steel to the compression face is called "effective depth" and is denoted by "d."

In deep members (h > 600 mm) extra steel bars are placed in the web to control the width of cracks caused by flexure and shrinkage,



When steel is placed in the tension zone, a minimum concrete cover is provided between the face of the beam and the bar. This distance from the lower face of the bar to the concrete surface is called "clear cover". Clear cover is necessary for fire and corrosion protection and bond. Concrete has a high fire resistance as compared to steel. Usually a clear cover of 40 to 50 mm will protect the steel from fire effects. Such a clear cover is more than what is needed for corrosion protection and bond.

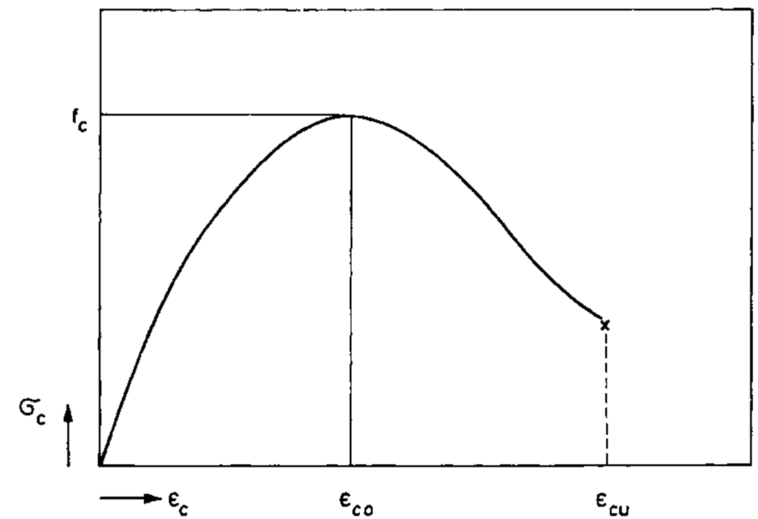
FLEXURAL BEHAVIOR

The behavior of reinforced concrete is ***non-linear, inelastic, time dependent and extremely complex***. However, basic approaches of *"Mechanics of Deformable Bodies"* still hold true. The analysis of deformable bodies are carried out in three basic steps:

1) The forces are studied and equilibrium requirements are established
2) The deformations and conditions of geometric fit are studied and requirements of geometric compatibility are established
3) The relationship between force and deformation (or stress and strain) for the material or materials are established.

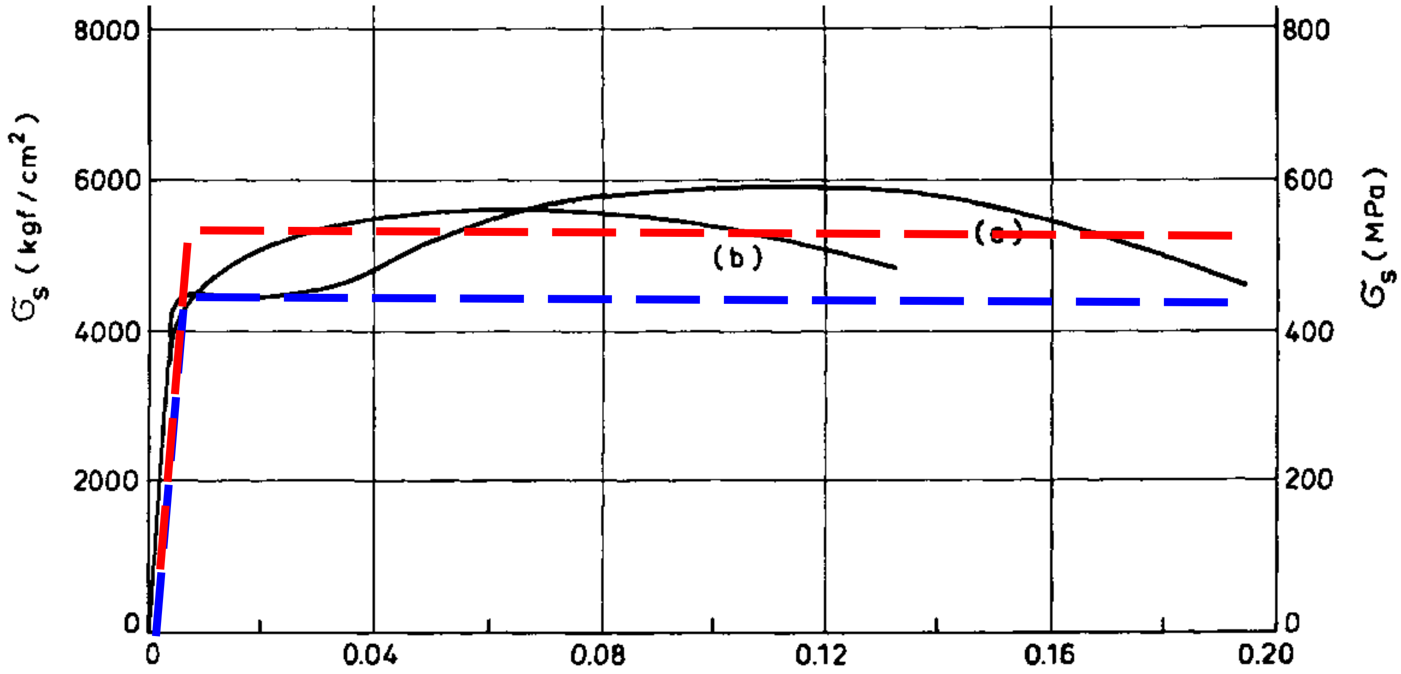
The basic assumptions that may be employed in the behavioral study of flexural members are listed below

1. Plane section remains plane after bending.
2. There is perfect bond between steel and concrete, i.e. stress in steel and concrete, located at the same distance from the neutral axis will have identical strains.
3. The stress block in the compression zones of the members can be assumed to be same as the σ–ε diagrams obtained from the tests on uniaxially loaded specimens. Idealized diagrams such as the one shown in the figure can be used for this purpose.

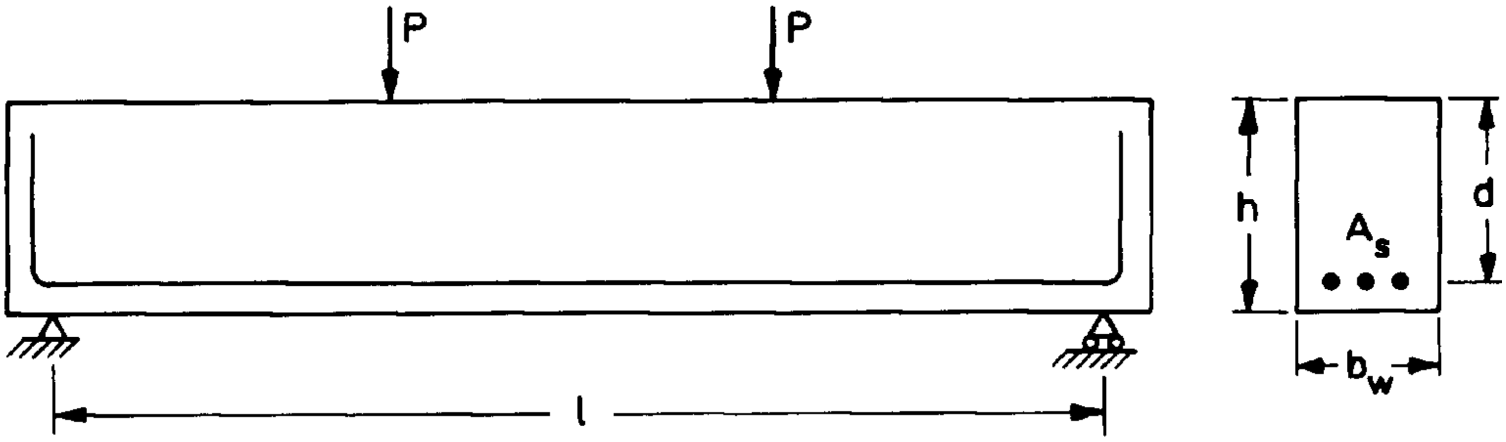


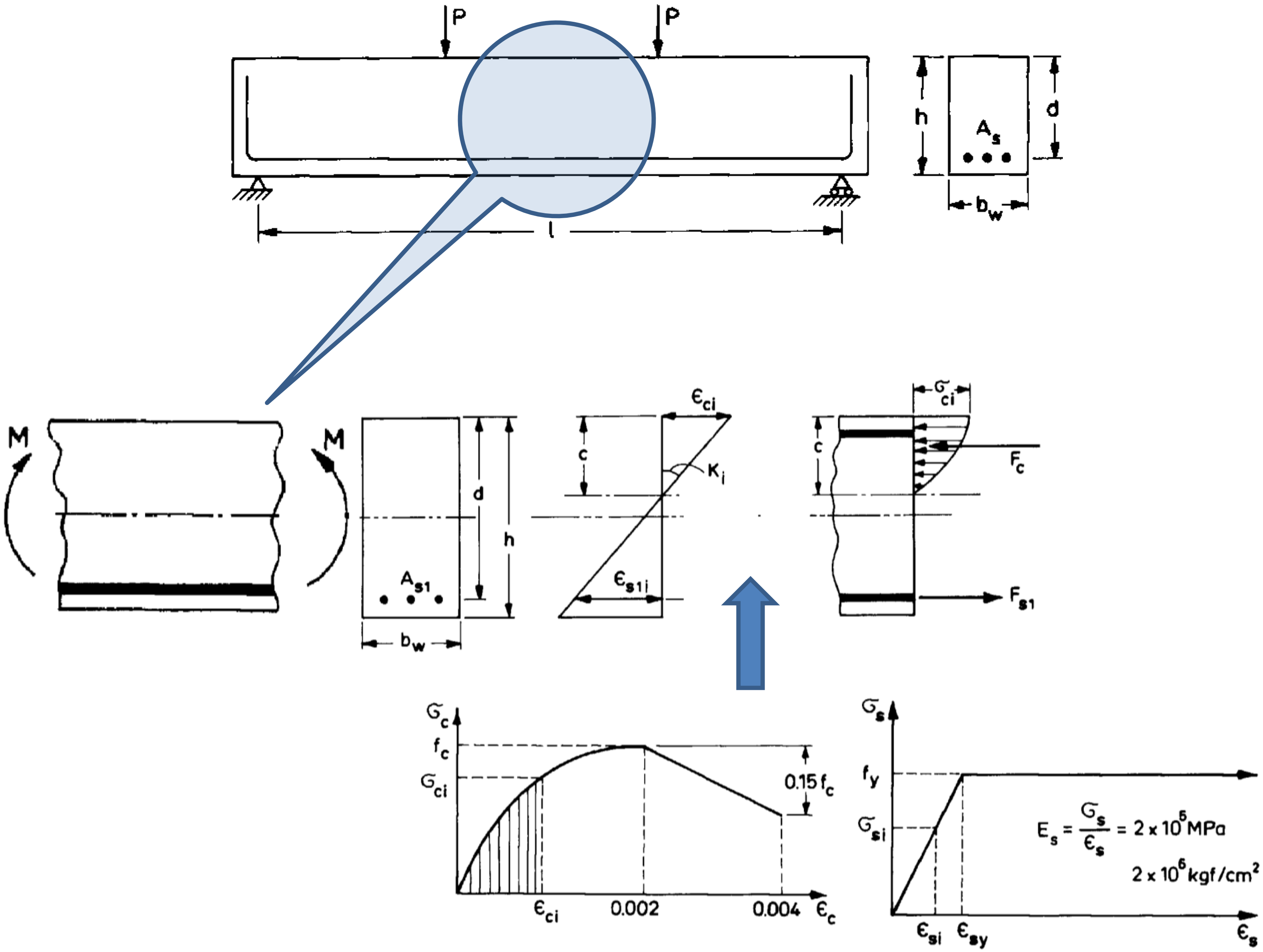
In the usual analysis, the tensile strength of concrete is neglected after cracking. This does not introduce a serious error, since the tensile strength of concrete is very low.

Typical stress-strain diagram for reinforcing steel bars may be idealized by approximate bi-linear curves (elasto-plastic) and these idealized curves may be used in the analysis.

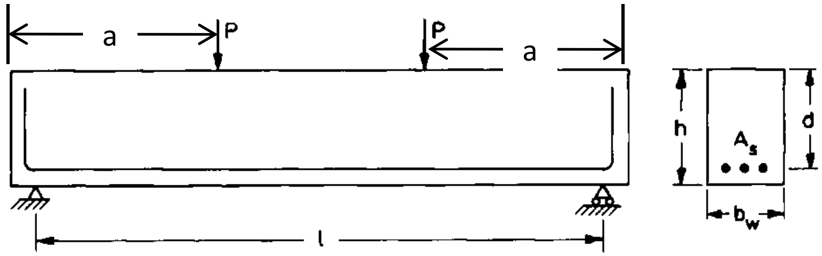


To study the behavior of reinforced concrete members subjected to flexure, the beam shown in figure below will be considered. The beam has a rectangular cross-section and is reinforced only in the tension zone. **The amount of reinforcement is assumed to be small.**

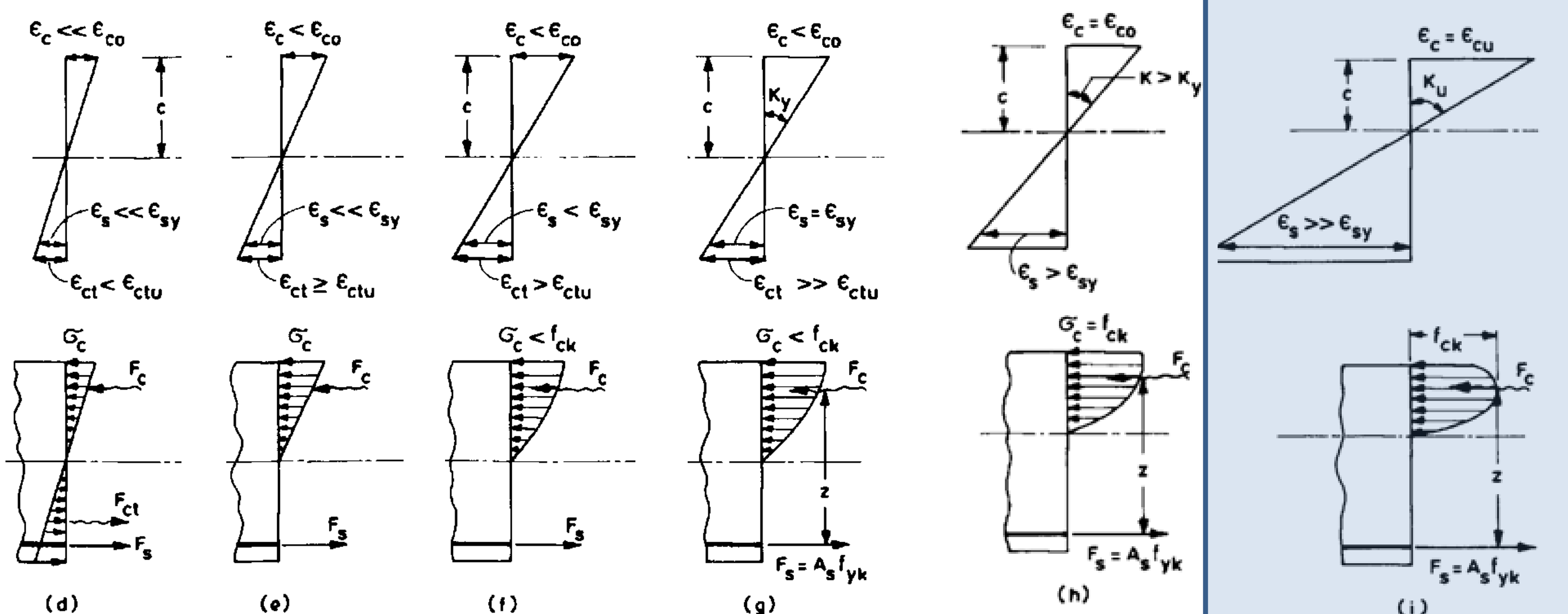




P
P
h
d
A_s
b_w
l
M
M
d
h
A_{s1}
b_w
ϵ_{ci}
c
K_i
ϵ_{s1i}
σ_{ci}
c
F_c
F_{s1}
σ_c
f_c
σ_{ci}
0.15 f_c
ϵ_{ci}
0.002
0.004
ϵ_c
σ_s
f_y
σ_{si}
$E_s = \frac{\sigma_s}{\epsilon_s} = 2 \times 10^6$ MPa
2×10^6 kgf/cm^2
ϵ_{si}
ϵ_{sy}
ϵ_s



$P_1 < P_{cr}$	$P_2 > P_{cr}$	$P_3 > P_2$	$P_y > P_3$	$P_4 > P_y$	$P_u > P_4$
	$K_2 > K_{cr}$	$K_3 > K_2$	$K_y > K_3$	$K_4 > K_y$	$K_u >> K_y$
			$F_s = F_{sy}$	$F_s = F_{sy}$	$F_s = F_{sy}$
			$F_c = F_s = F_{sy}$	$F_{c4} = F_{c5} = F_{sy}$	$F_{c6} = F_{c5}$

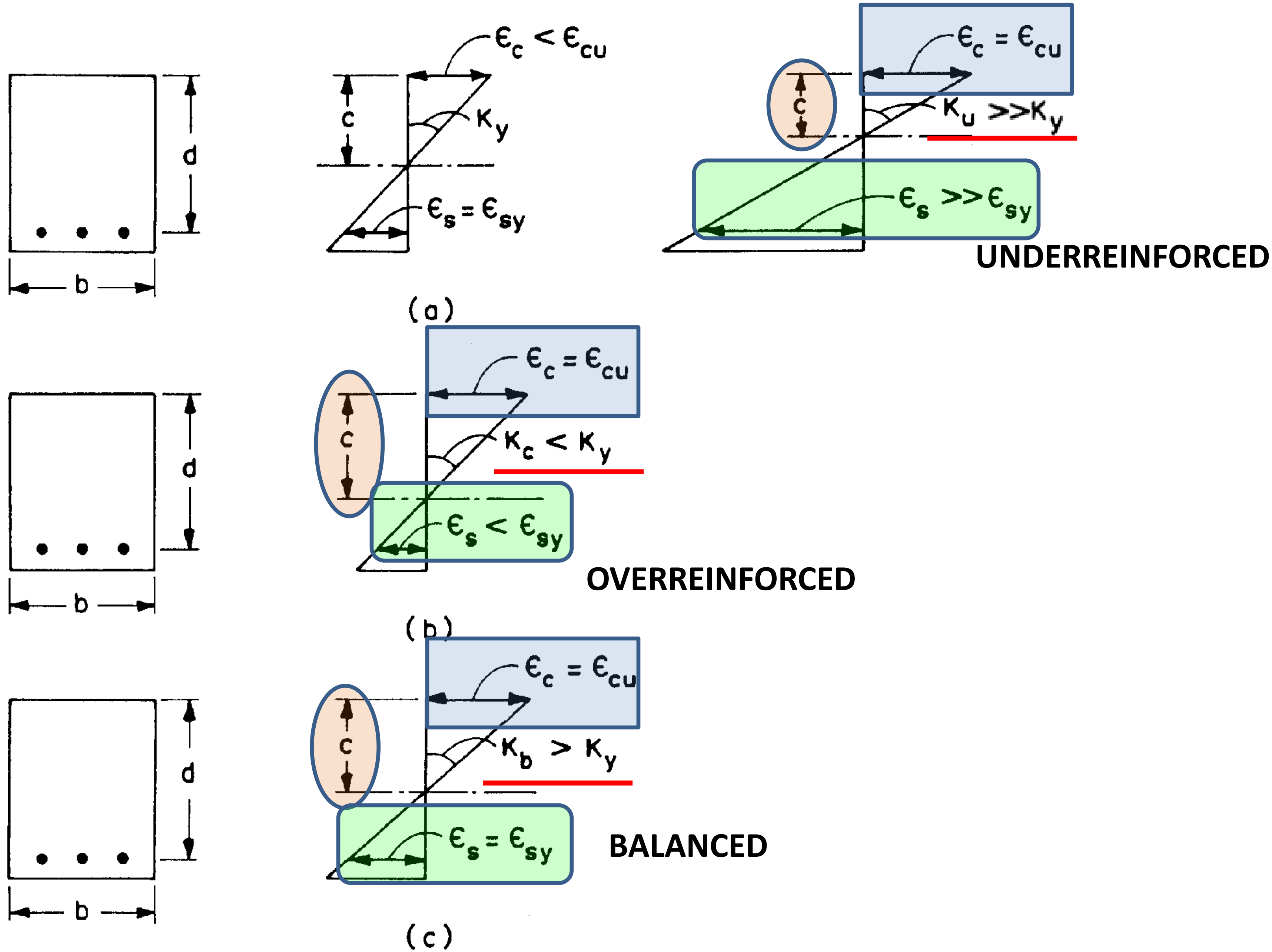


$M = P \times a = M_r = F_s\,(z) = F_c\,(z)$ where z is the distance b/w F_c and F_s

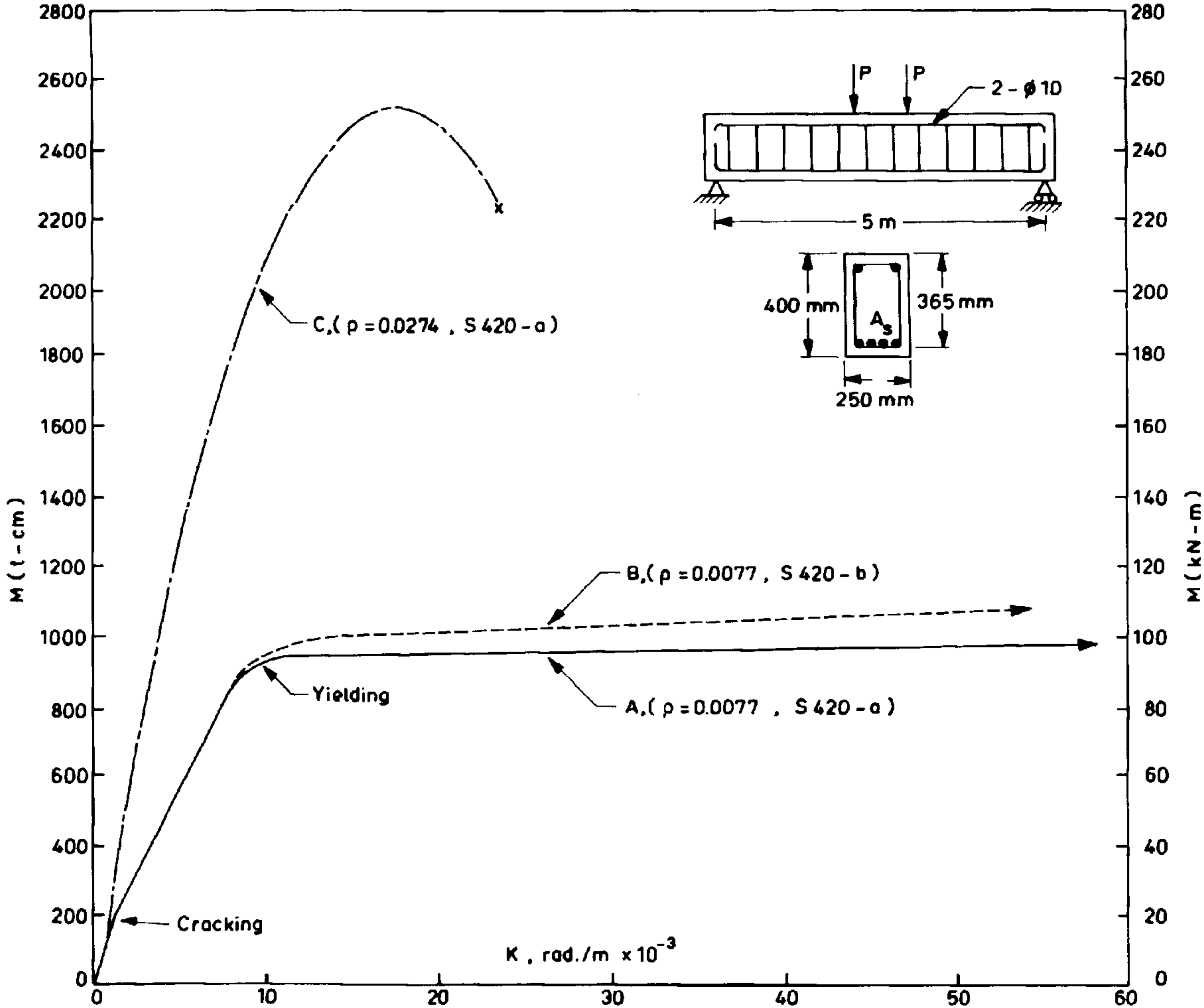
In beams, the type of failure is a function of the percentage of tension steel , $\rho = \frac{A_s}{b_w d}$.

The behavior of beams having large steel percentages (overreinforced beams) is quite different from the behavior described for the underreinforced beams.

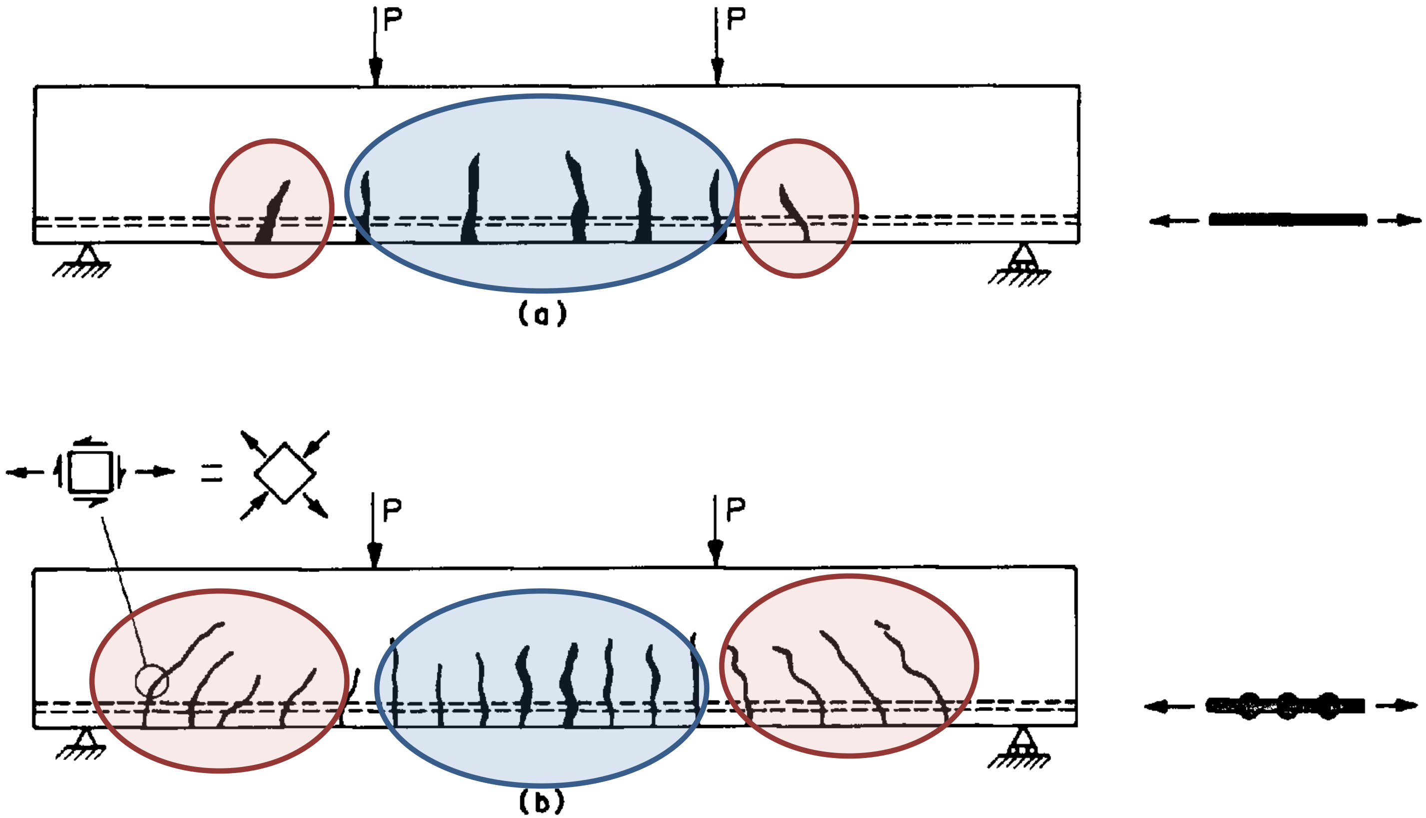
In overreinforced beams, the extreme concrete fiber in compression reaches the strain capacity, ε_{cu} prior to the yielding of tension steel. Therefore, the failure is sudden and brittle. Such failures are very undesirable and can be classified as "compression failure". Codes prohibit overreinforced beams by limiting the percentage of tension steel.



d
b
c
$\epsilon_c < \epsilon_{cu}$
K_y
$\epsilon_s = \epsilon_{sy}$
$\epsilon_c = \epsilon_{cu}$
$K_u >> K_y$
$\epsilon_s >> \epsilon_{sy}$
UNDERREINFORCED
(a)
$\epsilon_c = \epsilon_{cu}$
$K_c < K_y$
$\epsilon_s < \epsilon_{sy}$
OVERREINFORCED
(b)
$\epsilon_c = \epsilon_{cu}$
$K_b > K_y$
$\epsilon_s = \epsilon_{sy}$
BALANCED
(c)



2800
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
0
M (t - cm)
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
M (kN - m)
0
10
20
30
40
50
60
K , rad./m x 10⁻³
P
P
2 - ø 10
5 m
400 mm
365 mm
250 mm
As
C,(ρ = 0.0274 , S 420 - a)
B,(ρ = 0.0077 , S 420 - b)
A,(ρ = 0.0077 , S 420 - a)
Yielding
Cracking


P
P
(a)
P
P
(b)

ANALYSIS OF BEAMS

In the analysis problem, the geometric dimensions of the cross-section, steel area and material strengths are known. The object is to find the resisting or the ultimate moment.

Ultimate Strength Theory:

The ultimate strength theory is based on the actual behavior of steel and concrete.

The aim is to compute the carrying capacity of the section at the ultimate stage.

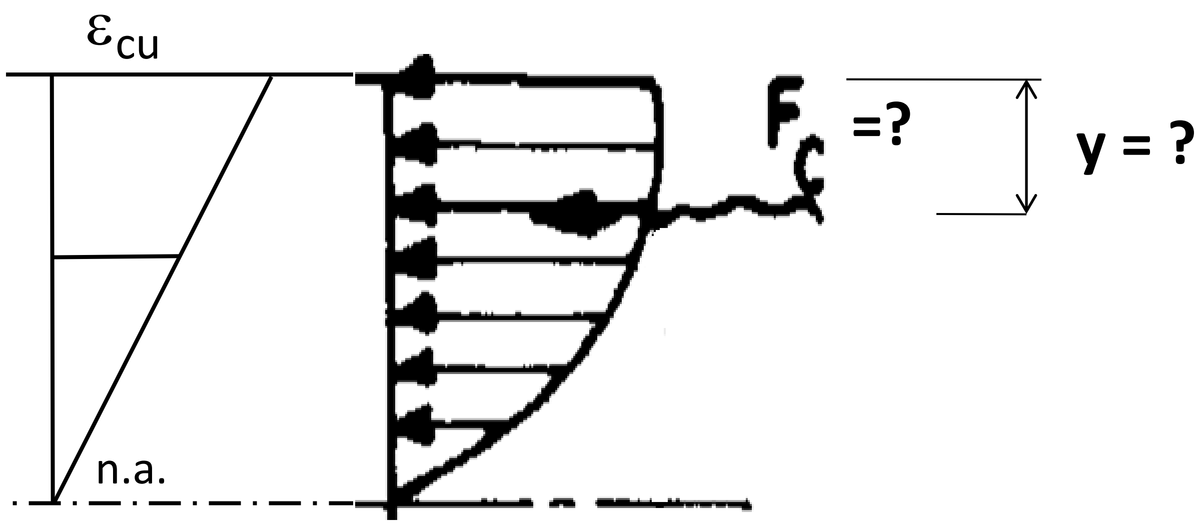
In this method, it is not intended to compute the stresses at any stage. In ***Ultimate Strength Theory***, the equilibrium and the compatibility equations are same as those developed in basic courses like *Mechanics of Deformable Bodies.* The only difference is in the force deformation relations, which try to represent the actual behavior by considering the nonlinear response of steel and concrete. Stress-strain curves for steel and concrete can be assumed to be similar to the ones presented in Chapter 1 or more simplified models can be used.

1. Equilibrium requirements must be satisfied.
2. The geometric compatibility must be established
3. The force and deformation relationships for the materials shall be employed.

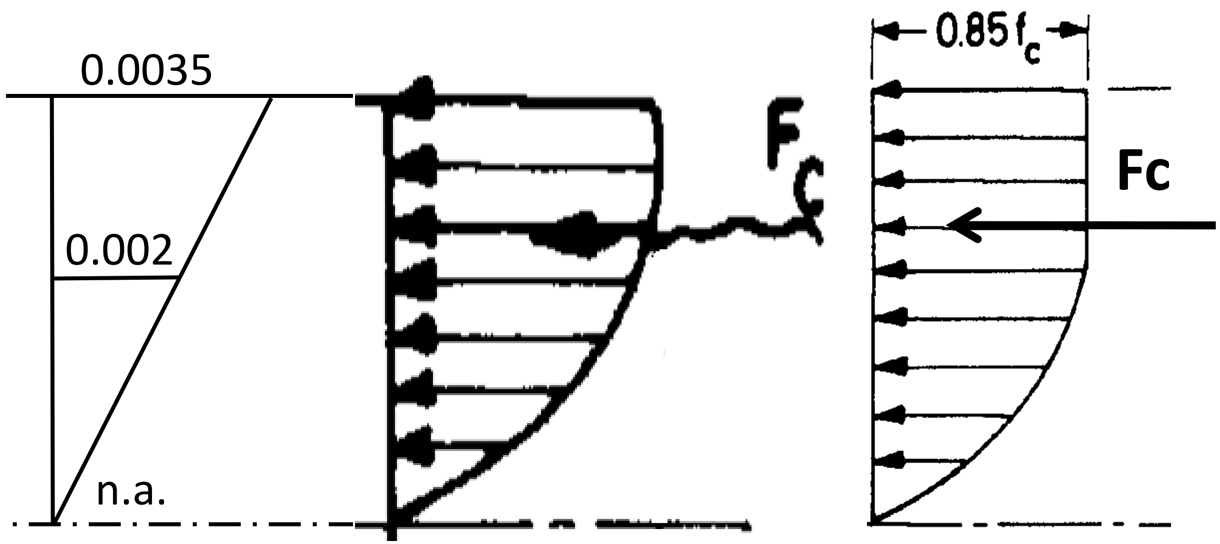
The basic assumptions made in the **Ultimate Strength Theory** for reinforced concrete are listed below:

1. Plane section remains plane after bending
2. Concrete cannot take any tension
3. There is perfect bond between steel and concrete, i.e. stress in steel and concrete, located at the same distance from the neutral axis will have identical strains
4. Stress-strain relation for the reinforcing steel is elasto-plastic, i.e. $\sigma_{si} = \varepsilon_{si} E_s \leq f_y$
5. Stress block of concrete in the compression zone is assumed to be same as the σ-ε curves obtained experimentally from uniaxially loaded specimens
6. Maximum strain in the extreme fiber of concrete in compression is $\varepsilon_{cu} \approx 0.003$

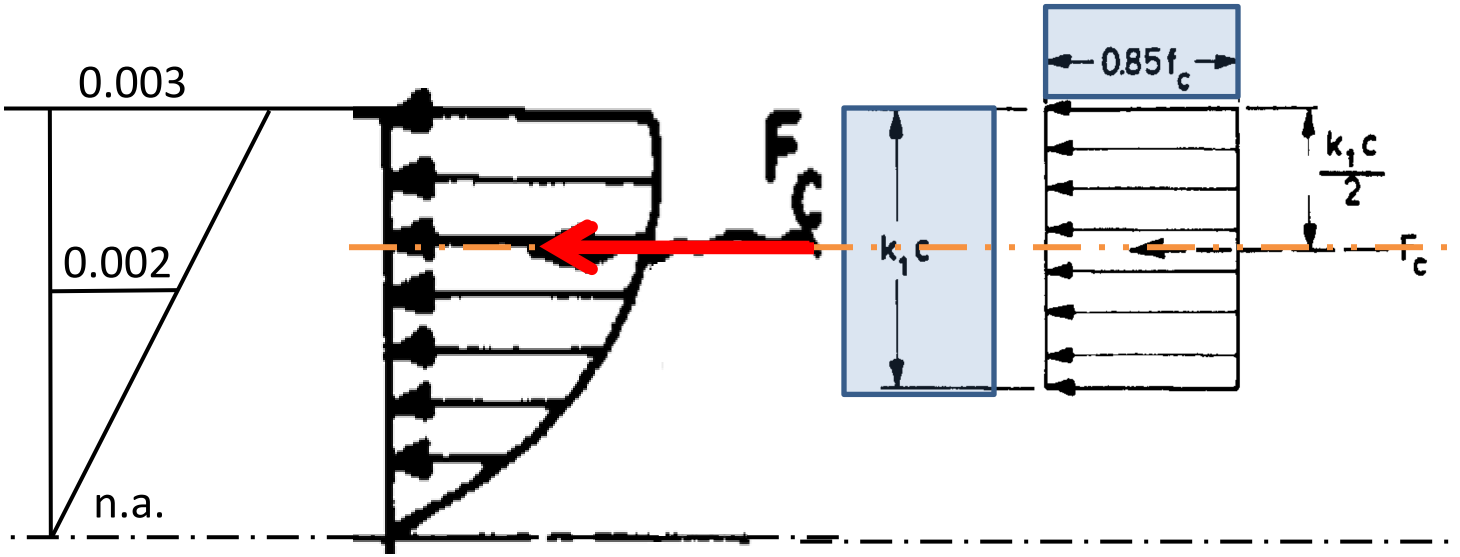
MAIN DIFFICULTY IN THE ANALYSIS



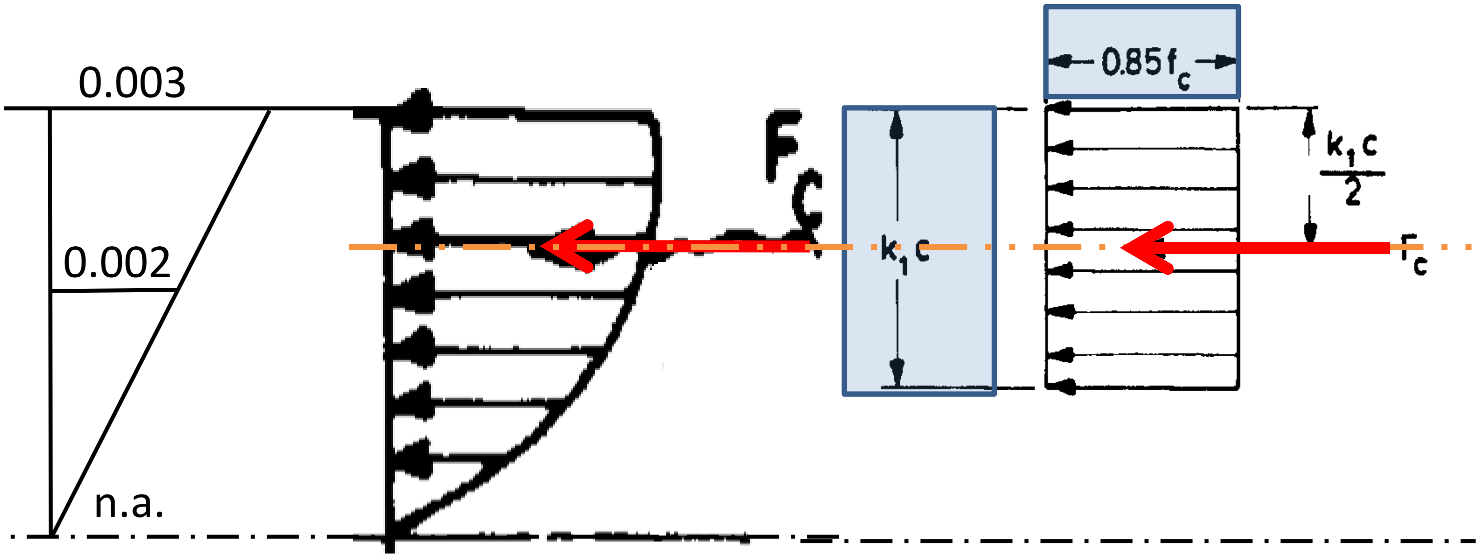
Parabolic Stress Block: EuroCode 2



Rectangular Stress Block: ACI-08 & TS-500



Rectangular Stress Block: ACI-08 & TS-500



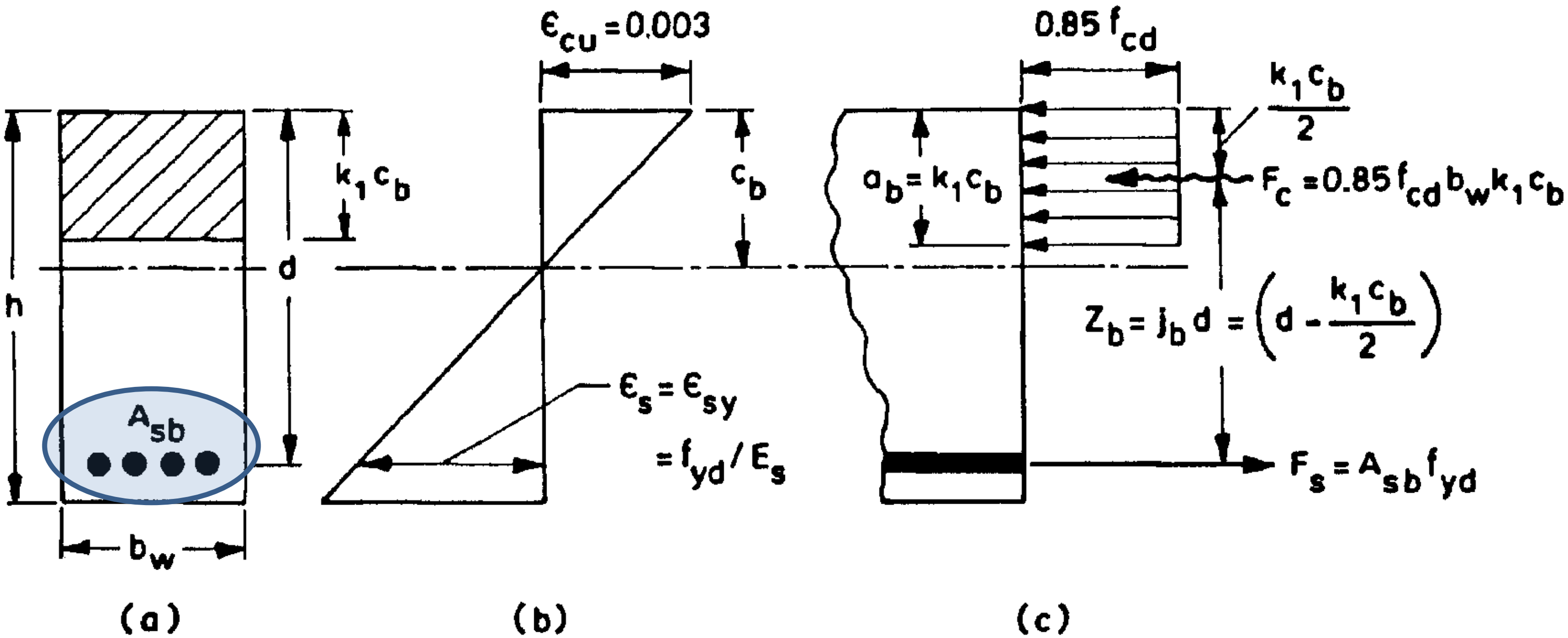
Rectangular Stress Block: k_1 Parameters

Concrete Class	Concrete Strength (MPa)	k_1
C12, C16, C20, C25	≤ 25	0.85
C30	30	0.82
C35	35	0.79
C40	40	0.76
C45	45	0.73
C50	50	0.70

It is always desirable to have a member whose failure is governed by tension rather than compression, because such failures are more ductile. Therefore, in flexural members the tension steel should yield before the crushing of concrete in the extreme compression fiber.

To determine whether a beam is overreinforced (compression failure) or underreinforced (tension failure), the borderline case, i.e. **balanced case** should be known. In the coming slides, the balanced case will be discussed. Next, the necessary equations for the underreinforced beam will be developed. <u>The overreinforced beam will not be treated since such beams are not permitted in practice.</u>

BALANCED CASE

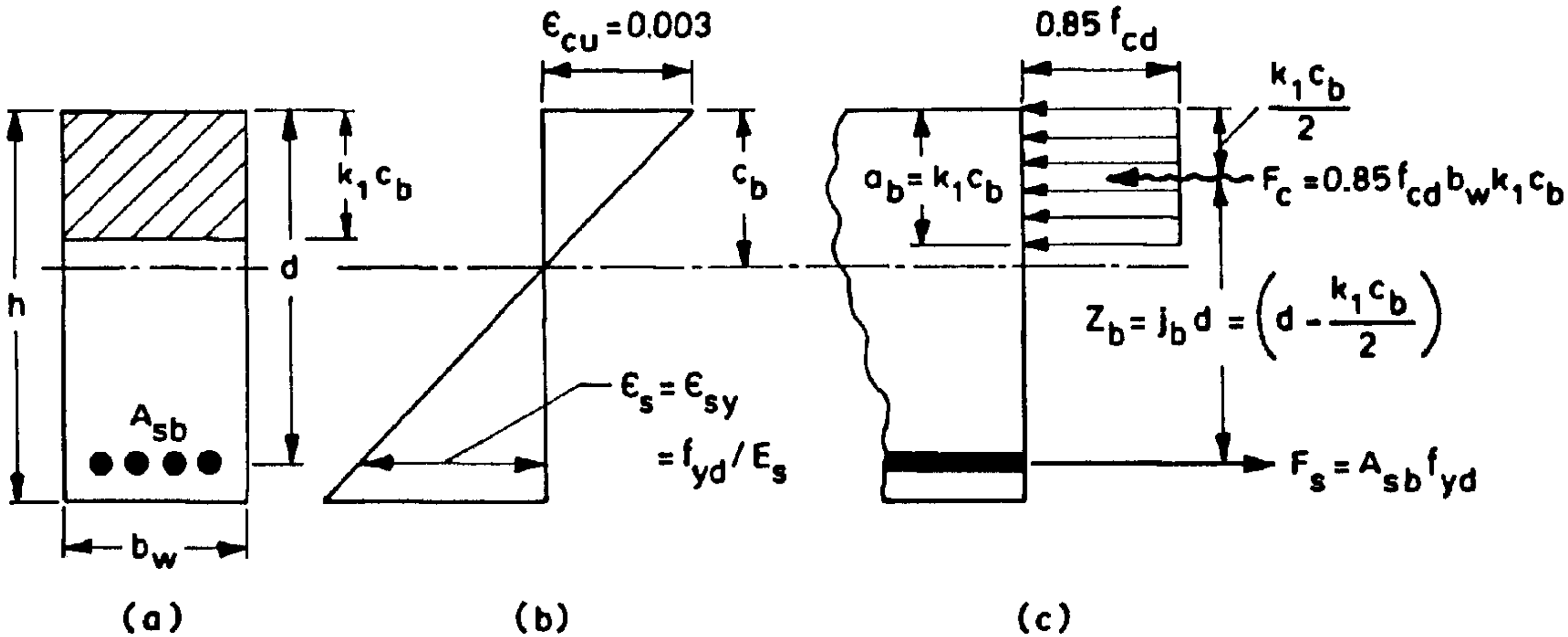


Equilibrium:

$$\Sigma F = 0 \qquad A_{sb} f_{yd} - 0.85 f_{cd} b_w k_1 c_b = 0$$

Compatibility: $$\frac{\varepsilon_{sy}}{d - c_b} = \frac{0.003}{c_b} \qquad \text{or,} \qquad \frac{c_b}{d} = \frac{0.003}{0.003 + \varepsilon_{sy}}$$

BALANCED CASE



Equilibrium:

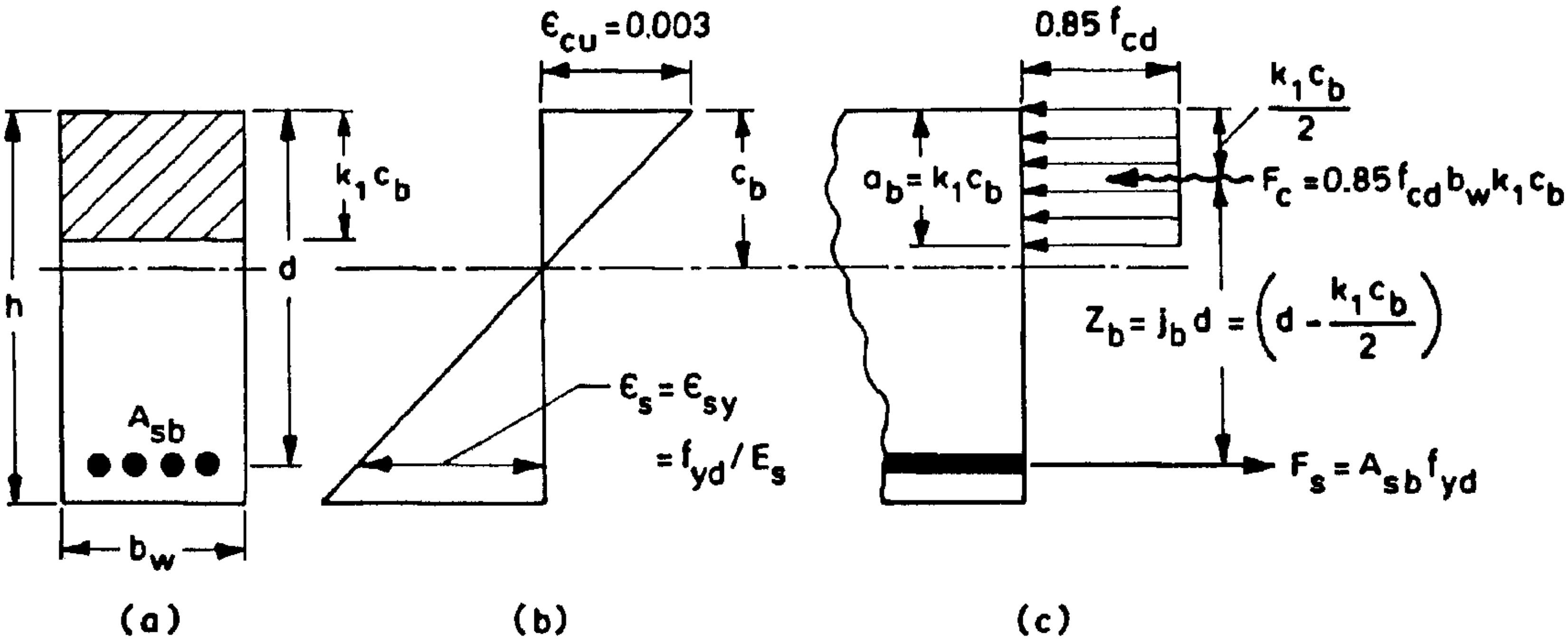
$$\Sigma F = 0 \quad A_{sb} f_{yd} - 0.85 f_{cd} b_w k_1 c_b = 0$$

Compatibility:

$$\frac{c_b}{d} = \frac{0.003}{0.003 + \varepsilon_{sy}}$$

$$\Sigma M = 0 \quad M_b = F_c(z_b) = F_c(z_b) = A_{sb} f_{yd}(j_b d) = A_{sb} f_{yd}\left(d - \frac{k_1 c_b}{2}\right)$$

BALANCED CASE



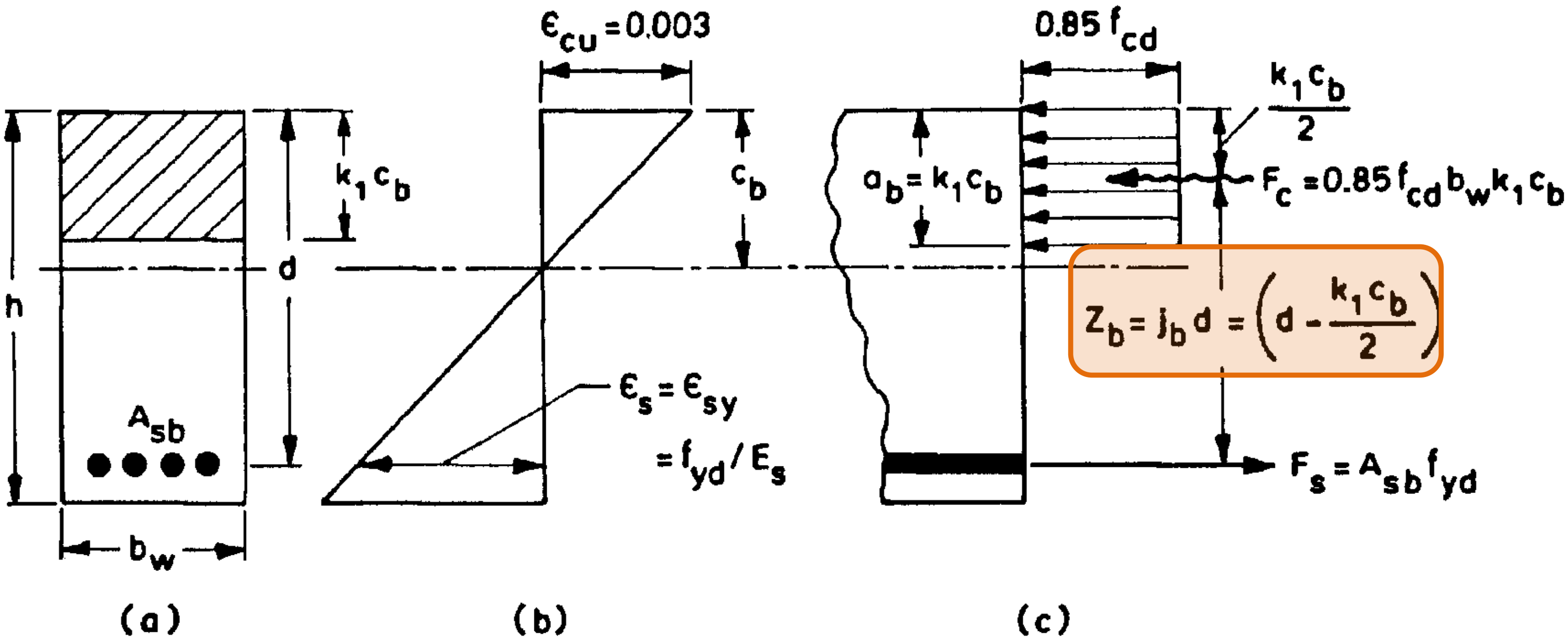
Solving for A_{sb}

$$A_{sb} = \frac{0.85\, f_{cd}}{f_{yd}} k_1 c_b b_w$$

dividing both sides by b_w d

$$\rho_b = \frac{0.85 f_{cd}}{f_{yd}} k_1 \frac{c_b}{d}$$

Balanced Case



$$\rho_b = \frac{0.85 f_{cd}}{f_{yd}} k_1 \frac{c_b}{d}$$

dividing both sides by $b_w d^2$ and rearranging

$$\frac{b_w d^2}{M_b} = K_b = \frac{1}{\rho_b f_{yd} j_b}$$

To assure tension failure (*underreinforced beam*) the percentage of steel should be less than the ρ_b.

Since K_b is inverse of the balanced moment, it can also be said that the beam will be *underreinforced* if the $K = b_w d^2/M$ computed is greater than K_b.

Note that c_b , ρ_b and K_b are all material dependent parameters. Hence;

B A L A N C E D   V A L U E S

Class of Steel	Class of Concrete	K_b $\left(\frac{cm^2}{t}\right)$	K_b $\left(\frac{mm^2}{kN}\right)$	$\frac{c_b}{d}$	j_b	ρ_b
S220	C14	29.0	(290)	0.758	0.678	0.0266
″	C16	24.5	(245)	″	″	0.0316
″	C20	20.7	(207)	″	″	0.0373
″	C25	15.8	(158)	″	″	0.0488
S420	C14	32.6	(326)	0.622	0.736	0.0144
″	C16	27.6	(276)	″	″	0.0135
″	C20	23.3	(233)	″	″	0.0160
″	C25	17.8	(178)	″	″	0.0209
″	C30	15.7	(157)	″	″	0.0237
″	C35	14.2	(142)	″	″	0.0263
″	C40	12.5	(125)	″	″	0.0297
″	C45	11.7	(117)	″	″	0.0317
″	C50	11.1	(111)	″	″	0.0334
S500	C14	34.0	(340)	0.580	0.754	0.0089
″	C16	28.8	(288)	″	″	0.0106
″	C20	24.4	(244)	″	″	0.0125
″	C25	18.6	(186)	″	″	0.0164
″	C30	16.4	(164)	″	″	0.0186
″	C35	14.8	(148)	″	″	0.0206
″	C40	13.1	(131)	″	″	0.0232
″	C45	12.3	(123)	″	″	0.0248
″	C50	11.6	(116)	″	″	0.0262