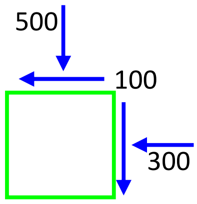
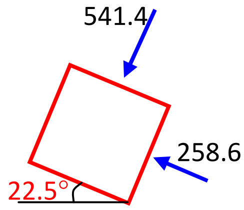
Hw5- Q1. Solution

i) Assuming counter-clockwise shear is positive and τ_{xy} is in the direction illustrated on the right. See the following solution steps marked with the same colors on the Mohr circle given below.

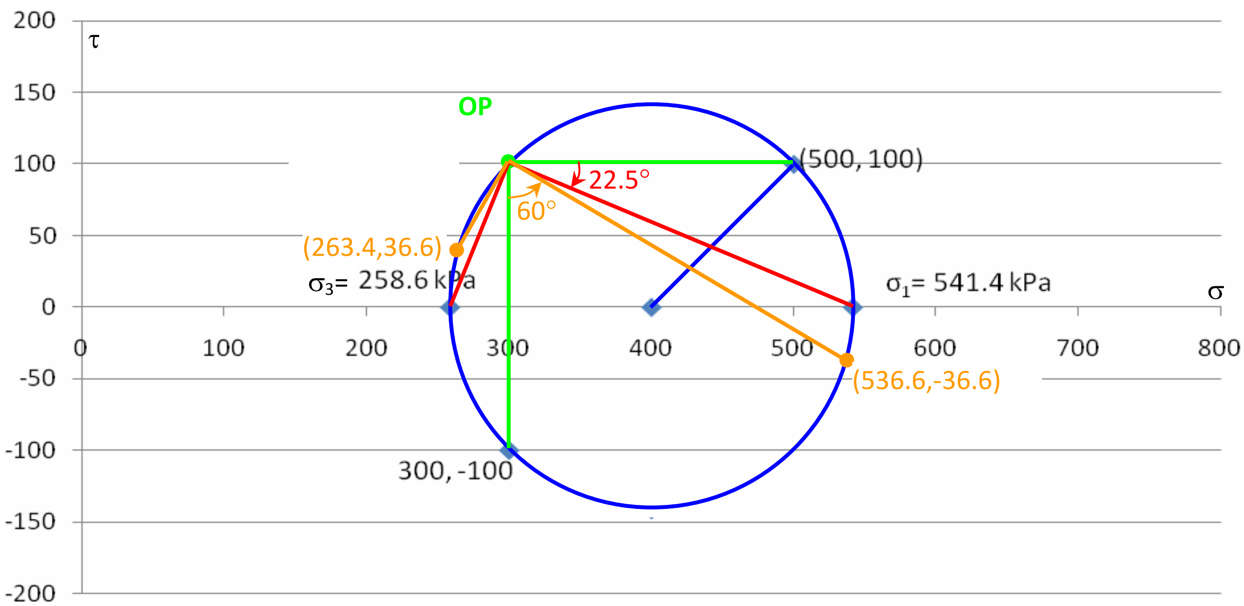


Step 1: Mark the given stress state of pairs of normal and shear stress. Stresses on the horizontal plane are given as (σ,τ) = (500,100) and stresses on the vertical plane are (300,-100). A unique Mohr circle can be drawn through these two points, given that the center of the circle must be on the σ-axis. The center of the circle is (300+500)/2 = 400. Radius can be calculated as $\sqrt{(500\text{-}400)^2+100^2}$ = 141.4. The principal stresses can be calculated as **400±141.4**.

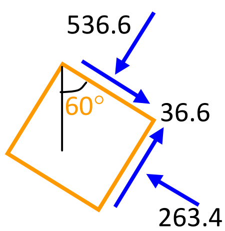
Step 2: Origin of Planes can be determined as (300,100), by drawing a horizontal line from the point that represents the stresses on the horizontal plane, or a vertical line from the point that represents the stresses on the vertical plane.



Step 3: Principal stresses act on the planes parallel to the lines drawn from the OP to the principle stress points on the Mohr circle. From geometry (the arc between (500,100) and σ_1 measures 45°), the principal stress planes make 22.5° angle with the horizontal and vertical, to the clockwise direction.



ii) **Step 4:** Drawing from OP, lines that are 60° counter-clockwise from the horizontal and vertical directions intersect the circle at two points. These mark the stresses on the planes in question. Their coordinates can be found from geometry: the arc between each orange point and nearest principal stress measures 15°. Drawing radii to the orange points, their distance from the center of the circle can be determined by trigonometry, and their coordinates can be calculated as $\sigma = 400 \pm R\cos 15$ and $\tau = \mp R\sin 15$, which result in (263.4,36.6) and (536.6,-36.6).



<u>Note</u> that the shear direction was not given (as assumed at the beginning of the solution) in Question 1 by mistake. If you assume shears in the other direction, the principal stresses won't change in magnitude, principal stress planes will be 22.5° from horizontal and vertical in the counterclockwise direction. The stresses in part (ii) will be $\sigma = 400 \pm R\sin 15$ and $\tau = \pm R\cos 15$, which result in (363.4,-136.6) and (436.6,136.6).

<u>Q2. Solution</u>

a)

Test number	1	2	3
Confining Pressure (kPa)	40	80	120
Deviatoric stress (kPa)	110	190	240
Pore water pressure at failure (kPa)	20	15	35
σ'_3	20	65	85
σ'_1	130	255	325
$p' = (\sigma'_1 + \sigma'_3)/2$	75	160	205
$q = (\sigma'_1 - \sigma'_3)/2$	55	95	120

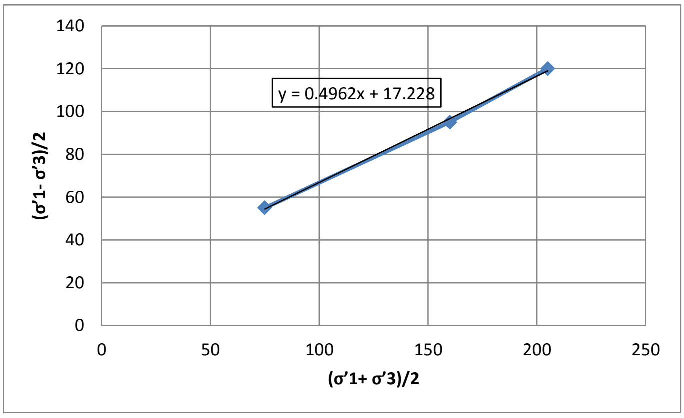
Fitting the best line in p'- q space

$\tan(\alpha')=0.4962 = \sin\phi'$ -> $\phi'=29.75°$

$a' = 17.228$

$c' = a'/\cos\phi' = 17.228/\cos 29.75 = 19.84$

c' = 20 kPa **ϕ' = 30°**



b)

$$T = \pi \times c_u \times \left(\frac{D^2 \times L}{2} + \frac{D^3}{6}\right)$$

$$0.065 = \pi \times c_u \times \left(\frac{0.060^2 \times 0.14}{2} + \frac{0.060^3}{6}\right)$$

$$0.065 = 0.905 \times 10^{-3} \times c_u$$

$$c_u = 71.84 kN/m^2$$

Solution of Q3.

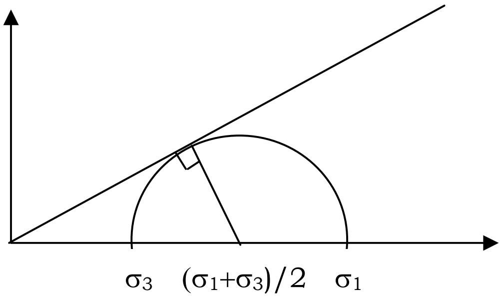
a) $\sigma_1 - \sigma_3 = 1046 kPa$

$\sigma_3 = 420 kPa$

$\Rightarrow \sigma_1 = 1466 kPa$

$$c = 0 \Rightarrow \sin\phi = \frac{\frac{\sigma_1 - \sigma_3}{2}}{\frac{\sigma_1 + \sigma_3}{2}}$$

$\Rightarrow \phi = 33.68^o$



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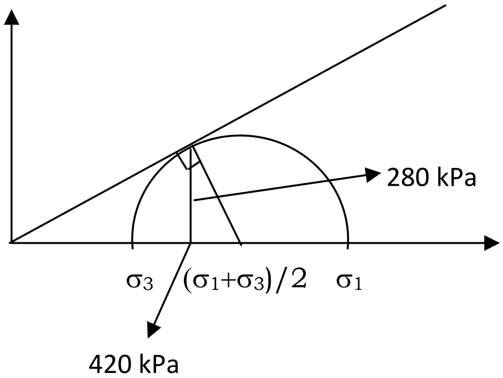
b)



$c = 0$

$\tau_f = c' + \sigma'_f \tan\phi$

$\tau_f = 280 kPa$

$\sigma_f = 420 kPa$

$$\Rightarrow \tan\phi = \frac{280}{420}$$

$\Rightarrow \phi = 33.68^o$

$$\sin\phi = \frac{\frac{\sigma_1 - \sigma_3}{2}}{\frac{\sigma_1 + \sigma_3}{2}} = 0.554$$

$\Rightarrow \sigma_1 = 3.45\sigma_3$

$x = \sqrt{280^2 + 420^2}$

$\Rightarrow x = 504.78$

$$\tan\phi = \frac{\frac{\sigma_1 - \sigma_3}{2}}{x} = \frac{1.225\sigma_3}{504.78} = 0.67$$

$\Rightarrow \sigma_3 = 274.6 kPa$

$\Rightarrow \sigma_1 = 947.4 kPa$

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c) The friction angle for a soil type is unique, it does not vary with testing methodology or strength sustained by the soil. However, the location of mohr circle can move depending on the strength or resistance of the corresponding specimen.