M E T U Department of Mathematics

	Ba	ra	
		Final	
Code:	Math~260	Last Name:	
Acad.Year	r: 2006-2007	Name:	Student No.:
Semester:	Spring	Department:	Section:
Date:	29.5.2007	Signature:	
Time:	16:40	4 QUESTIONS ON 4 PAGES	
Duration:	120 minutes	TOTAL 80 POINTS	
1 2	3 4		

Please carefully write the logical steps leading to your answers. Correct answers without any correct reasoning will not get any points.

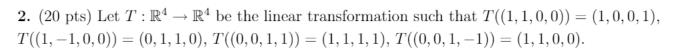
1. (20 pts) Let V be the vector space of polynomials in x of degree less than or equal to 3. Let

$$(f|g) = \int_0^1 x^2 f(x)g(x)dx$$

(a) Show that (|) is an inner product.

(b) Find an orthogonal basis of the subspace S of V spanned by $1, x^2$ and x^3 .

(c) Find the orthogonal projection of x to the subspace S.



(a) Find $T((x_1, x_2, x_3, x_4))$.

(b) Find a basis for Ker(T).

(c) Find a basis for Range(T).

(d) Find the matrix of T with respect to the standard bases $\{(1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1)\}$ in the domain and range.

3. (20 pts) A 3×3 matrix A has eigenvalues 1, -1 and 2 with corresponding eigenvectors

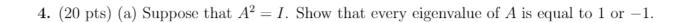
$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \text{ and } v_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

(a) Is A invertible?

(b) Is A diagonalizable? If it is, determine an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

(c) Find A.

(d) Find Tr(A) and det(A).



(b) Let $V = \mathbb{R}^3$ and W be the z-axis in \mathbb{R}^3 . Let $T: V \to W$ be orthogonal projection. Choose bases for V, W, and find the matrix of T with respect to these bases.

(c) Show that for any real numbers x_1, x_2, y_1, y_2 ,

$$2x_1y_1 + 2x_1y_2 + 3x_2y_2 \le \sqrt{2x_1^2 + 2x_1x_2 + 3x_2^2}\sqrt{2y_1^2 + 2y_1y_2 + 3y_2^2}$$

(Hint: first define an appropriate inner product on \mathbb{R}^2)