RULES

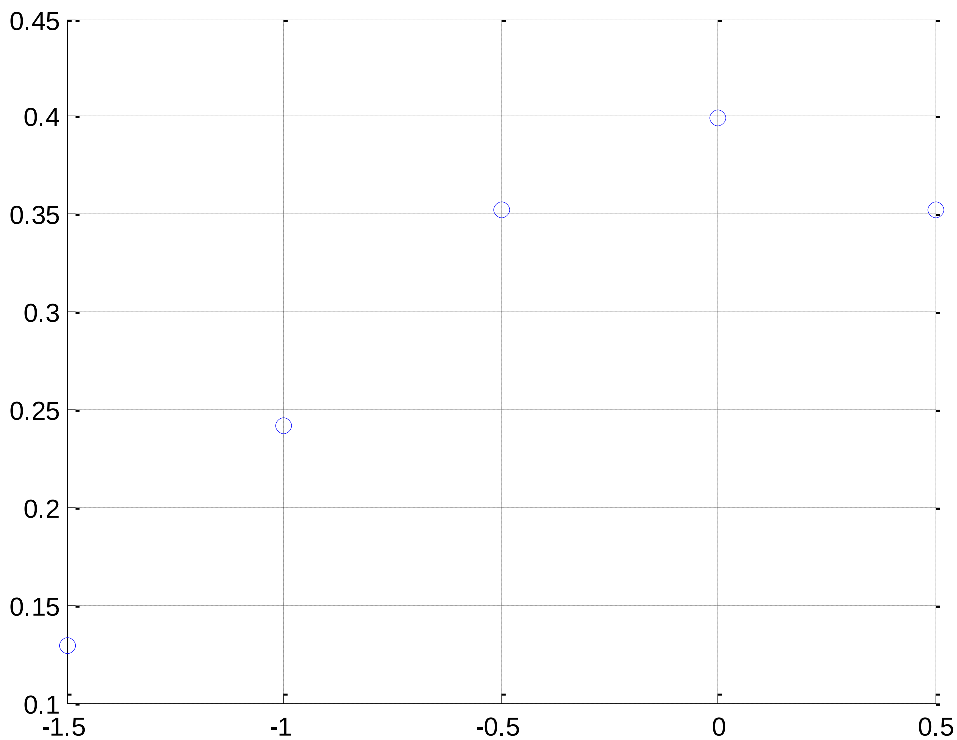
1. This is the **version 5.0**. In case there are any corrections for the solutions of Exercise 5, we will post an updated version on our website. You can follow the changes in the exercises by the **Version History** section below.

Version History

V5.0 Solutions of Exercise 5 are released.

1.

a) The plot is given below:



b)

i) By Trapezoidal Rule:

$$I = \frac{f(b)+f(a)}{2}*(b-a) = \frac{f(0.5)+f(-1.5)}{2}*(0.5+1.5)$$

$$I = \frac{0.352065+0.129518}{2}*2 = 0.481583$$

ii) By Simpson's Rule:

$$I = \frac{1}{3}*(\frac{2}{2})*[f(0.5)+4f(-0.5)+f(-1.5)]$$

$$I = \frac{1}{3}*(0.352065+4*0.352065+0.129518) = 0.629948$$

iii) By Composite Trapezoidal Rule:

$$h = \frac{b-a}{4} = 0.5$$

$$I = 0.5*(\frac{f(-1.5)+2f(-1)}{2}+\frac{f(-1)+2f(-0.5)}{2}+\frac{f(-0.5)+2f(0)}{2}+\frac{f(0)+f(0.5)}{2})$$

$$I = 0.5*(\frac{0.129518+2*0.241971+2*0.352065+2*0.398942+0.352065}{2})$$

$$I = 0.616885$$

iv) By Simpson's Rule:

$$I = (2*0.5)*(\frac{f(-1.5)+4f(-1.0)+f(-0.5)}{6})+(2*0.5)*(f(-0.5)+4f(0)+f(0.5))$$

$$I = \frac{0.129518+4*0.241971+2*0.352065+4*0.398942+0.352065}{6}$$

$$I = 0.624894$$

c) Obviously Simpson's Rule is more accurate than Trapezoidal Rule by the nature of the method. Trapezoidal Rule is exact for first order polynomials where Simpson's Rule is exact for second and third order polynomials. For composite ones, the accuracy increases with increasing n. For the same n, Composite Simpson's Rule is more accurate than Composite Trapezoidal Rule.

2. The arc length can be found as:

$$y = x^3 \Rightarrow \frac{dy}{dx} = 3x^2$$

$$arclength = \int_0^1 \sqrt{(1+9x^4)} * dx$$

$$f(x) = \sqrt{(1+9x^4)}$$

a) By Simple Trapezoidal Rule:

$$I = \frac{f(0)+f(1)}{2} * 1 = \frac{1+\sqrt{10}}{2} = 2.081139$$

b) By Composite Trapezoidal Rule:

$$h = \frac{b-a}{N} = \frac{1-0}{4} = 0.25$$

$$I = 0.25 * \frac{f(0)+f(0.25)}{2} + 0.25 * \frac{f(0.25)+f(0.5)}{2} + 0.25 * \frac{f(0.5)+f(0.75)}{2} + \frac{f(0.75)+f(1)}{2} * 0.25$$

$$I = 0.25 * \frac{f(0)+2f(0.25)+2f(0.5)+2f(0.75)+f(1)}{2}$$

$$I = 0.25 * \frac{1+2*1.017426+2*1.25+2*1.961544+3.162278}{2} = 1.577527$$

c) By Composite Simpson's Rule

$$h = 0.25$$

$$I = 2*0.25*\left(\frac{f(0)+4f(0.25)+f(0.5)}{6}\right) + 2*0.25*\left(\frac{f(0.5)+4f(0.75)+f(1)}{6}\right)$$

$$I = 0.5*\left(\frac{1+4*1.017426+1.25}{6}\right) + 0.5*\left(\frac{1.25+4*1.961544+3.162278}{6}\right)$$

$$I = 1.548180$$

d) This code can be used to define the function:

```
function y=funct(x)
y=sqrt(1+9*x^4);
end
```

This code can be used to perform numerical integration by both rules:

```
clear all
clc

% input parameters after defining the function at funct.m
a=0;           % lower limit of the integral
b=1;           % upper limit of the integral
N=4;           % number of intervals

h=(b-a)/N;          % step size
n_trap=(b-a)/h;     % number of times you need to reach b (trapezoidal
rule)
n_simp=(b-a)/(2*h);     % number of times you need to reach b
(Simpson's rule)

%%%% Trapezoidal Rule

trapezoidal=0;  % initializing sum
x=a;
for i=1:n_trap
   trapezoidal=trapezoidal+(funct(x)+funct(x+h))*h/2; % updating sum
   x=x+h; %updating x
end
trapezoidal

%%%% Simpson's Rule

simpson=0;  % initializing sum
x=a;
for i=1:n_simp
   simpson=simpson+(funct(x)+4*funct(x+h)+funct(x+2*h))*h/3; %
updating sum
   x=x+2*h; %updating x
end
simpson
```

The results can be tabulated as follows:

Interval Number	Trapezoidal Rule	Simpson's Rule
4	1.577527367662456	1.548180018535910
6	1.561044728169447	1.547848743099227
8	1.555278165908198	1.547861765323446

As you can see, increasing interval number increases accuracy.