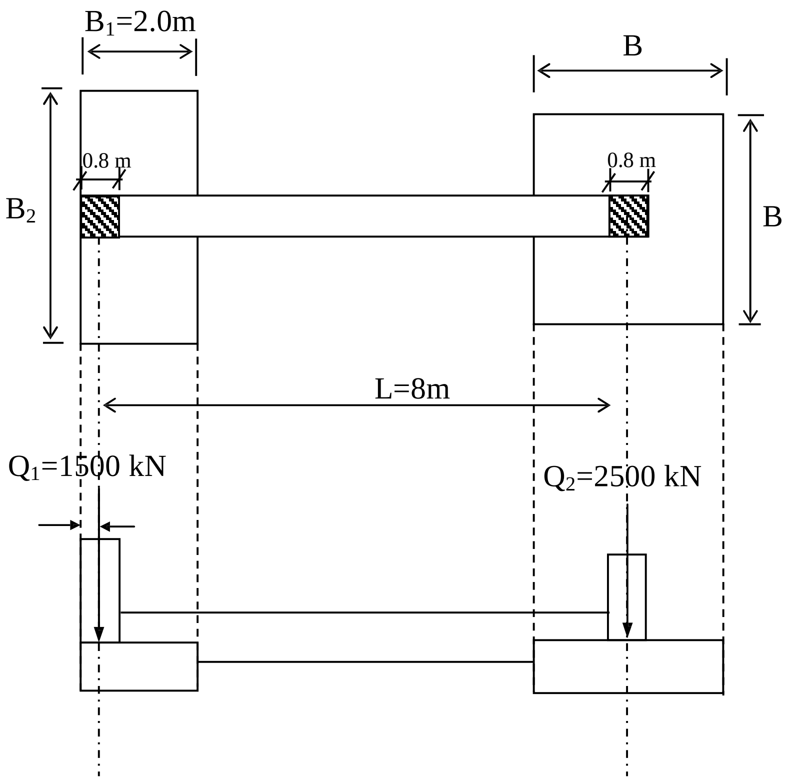
CE 366 – SHALLOW FOUNDATIONS

P.1) CANTILEVER FOOTING

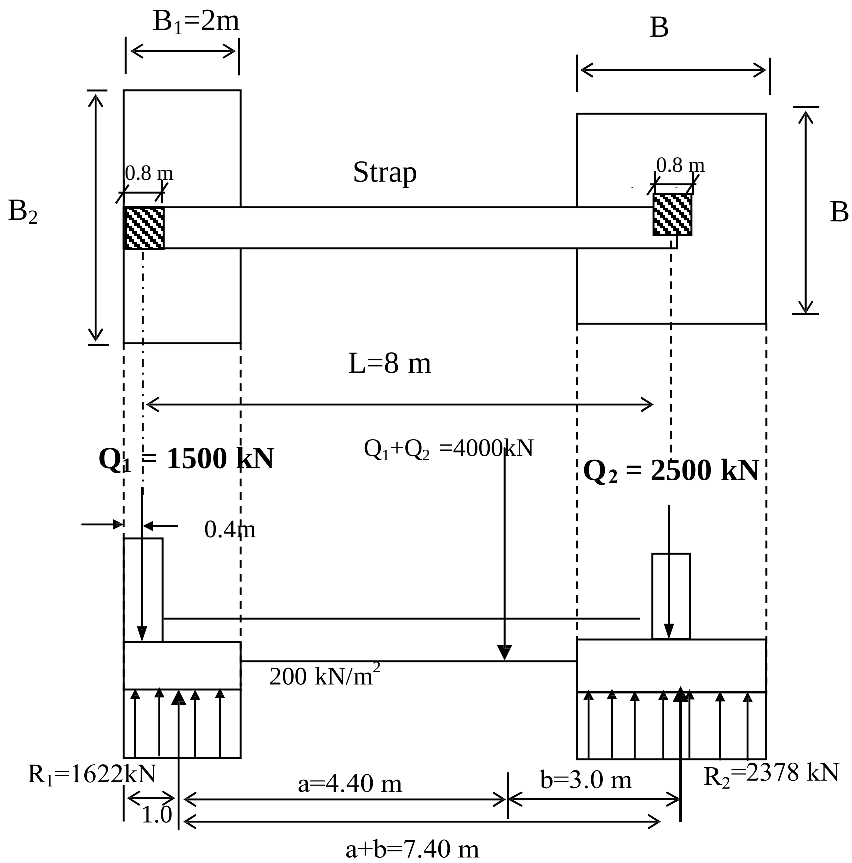
Question:

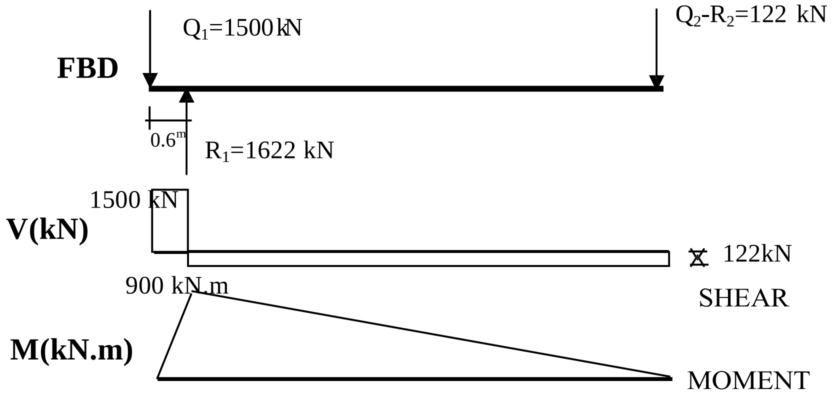
Given: Q_1 = 1500 kN, Q_2 = 2500 kN, q_{all} = 200 kN/m^2

Ignore the weight of footings and find dimensions B and B_2 of a cantilever footing for a uniform soil pressure distribution. Draw shear and bending moment distributions.



Solution:





Locate $\Sigma Q = Q_1 + Q_2$

$b = 1500 \times 8 / 4000 = 3$

For B_1=2m

$a = 8.0 + 0.4 - 1.0 - 3.0$

$\rightarrow a = 4.40$m

$R_1 = (4000 \times 3) / 7.4 = 1622$kN

$R_2 = 4000 - 1622 = 2378$ kN

Determine B_2,

$q_{all} = Q / (2 \times B_2)$

$200 = 1620 / (2 \times B_2) \rightarrow B_2 = 4$ m

OR

Without considering resultant ($Q_1 + Q_2$)

Moment w.r.t Q_2 or (R_2);

$Q_1 x8 - R_1 x7.4 = 0 \rightarrow R_1 = 1622$ kN

From force equilibrium;

$\Sigma F_{vertical} = 0$

$1500 + 25000 - 1622 - R_2 = 0$

$\rightarrow R_2 = 2378$ kN

Determine B_2,

$q_{all} = Q / (2 \times B_2)$

$200 = 1622 / (2 \times B_2) \rightarrow$ **$B_2 \approx 4$ m**

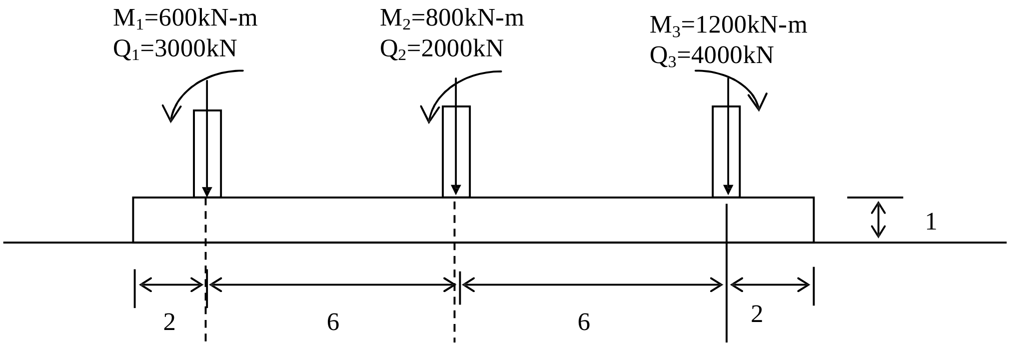
Similary,

$200 = 2378 / B^2 \rightarrow$ **$B \approx 3.45$ m**

P.2) TRAPEZOIDAL FOOTING

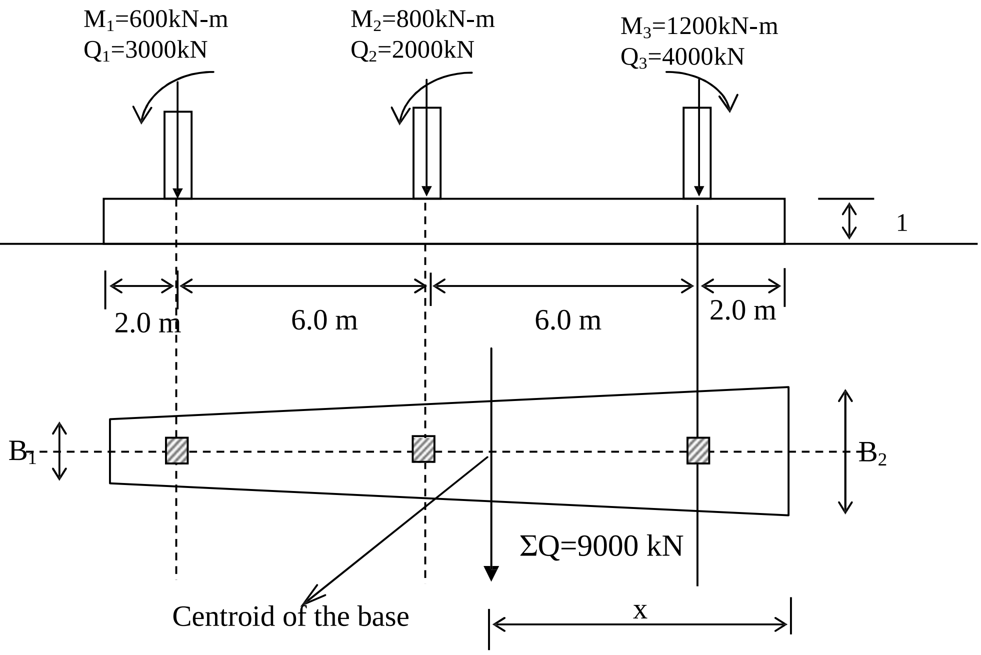
Question:

Determine B_1 and B_2 of a trapezoidal footing for a uniform soil pressure of 300 kN/m^2. (γ_{conc} = 24kN/m^3)



Solution:

After finding the location of resultant force, you can decide whether the wider side of the trapezoid will be at the left or right side.



General formulation for finding coordinates of a centroid;

$\bar{x} = \frac{\overline{x_1}A_1+\overline{x_2}A_2}{A_1+A_2}$ $\bar{y} = \frac{\overline{y_1}A_1+\overline{y_2}A_2}{A_1+A_2}$

ΣQ=9000kN

Weight of footing= 16x24x(B_1+B_2)/2 = 192(B_1+B_2)

Area of footing = 8(B_1+B_2)

ΣF$_y$=0 9000+192(B_1+B_2)=8(B_1+B_2)x300

B_1+B_2 = 4.07m ………(1)

ΣM=0 (moment about centroid of the base)

3000(14-x)+600+2000(8-x)+800-4000(x-2)-1200+(wght of ftg)x0=(base pressure)x0

x = 7.36m

x=7.36=$\frac{1}{3}x16x\,\frac{2B_1+B_2}{B_1+B_2}$ $B_2 = 1.63B_1$ ………..(2)

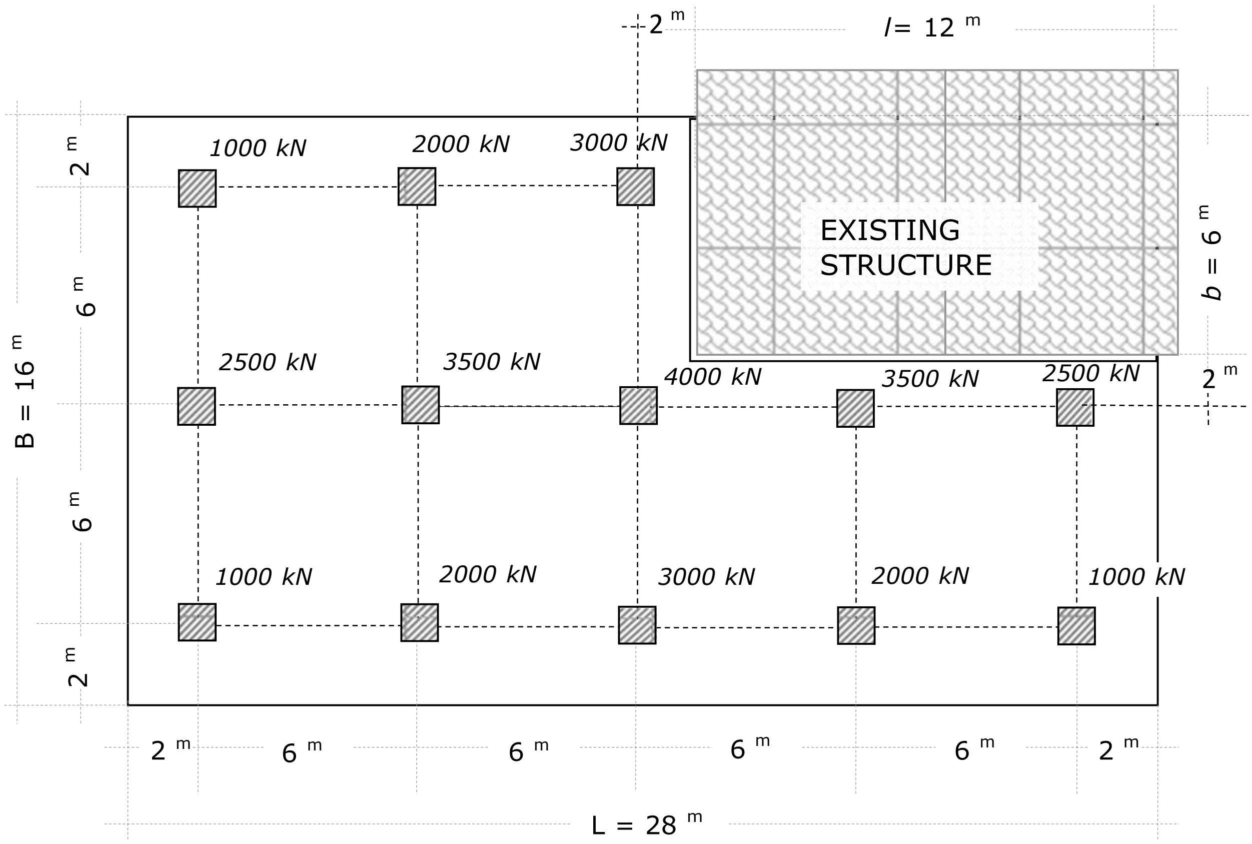
From (1) and (2) B_1= 1.55m ; B_2 = 2.52m

Note: You can also take moment about the left or right side but keep in mind that weight and base pressure will also have moment about left or right side.

P.3) MAT FOUNDATION

Question:

A mat foundation rests on a sand deposit whose allowable bearing value is 150 kN/m^2. Column loads are given in the figure. The thickness of the mat is 2.0 m ($\gamma_{concrete}$ = 24 kN/m^3). Calculate base pressures assuming that the lines passing through the centroid of the mat and parallel to the sides are principal axes. Find the base pressure distribution beneath the base and check whether the mat foundation given is safe?



Solution:

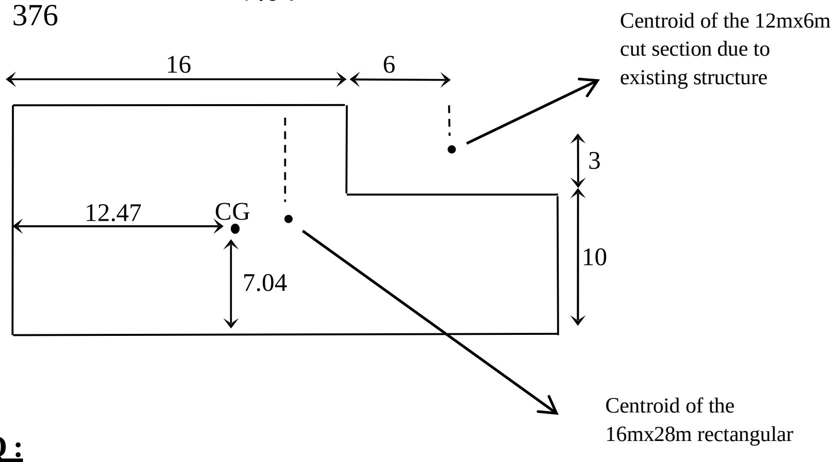
Area of foundation = 28 × 16 – 12 × 6 = 376 m^2

Total vertical load = Σ V = Column loads + Weight of mat=31000+(376) × 24 × 2 = 49048 kN

* **<u>Center of gravity (CG) of mat:</u>**

$$\frac{28 \times 16 \times 14 - 6 \times 12 \times (16+6)}{376} = 12.47 \quad \text{m from left}$$

$$\frac{28 \times 16 \times 8 - 6 \times 12 \times (3+10)}{376} = 7.04 \quad \text{m from bottom}$$



* **<u>Location of ΣQ :</u>**

→ Take moment about the left side:

= (1 / 49048) . [2 × (1000+2500+1000) + 8 × (2000+3500+2000) + 14 × (3000 +

+ 4000 + 3000) + 20 × (3500+2000) + 26 × (2500+1000) + 376 × 2 × 24 × 12.47]

= 12.95 m from left

→ Take moment about bottom side :

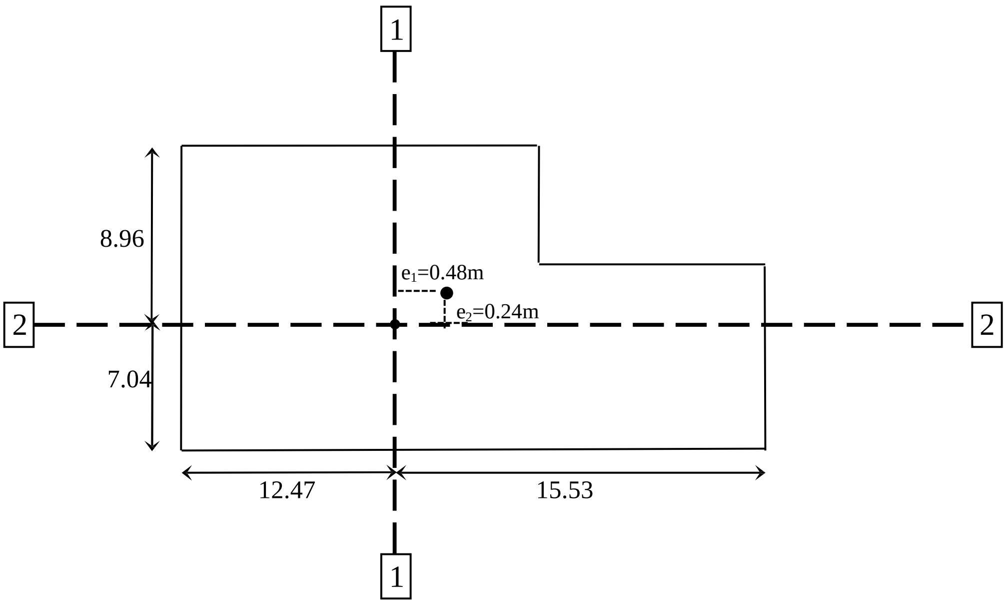
= (1 / 49048) . [2 × (1000+2000+3000+2000+1000) + 8 × (2500 + 3500 + 4000 +

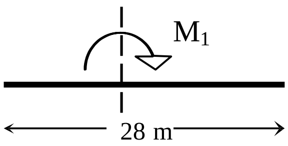
+ 3500 + 2500) + 14 × (1000+2000+3000) + 376 × 2 × 24 × 7.04]

= 7.28 m from bottom

Eccentricity :

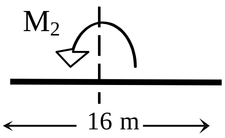
$e_1 = 12.95 - 12.47 = 0.48$ m
$e_2 = 7.28 - 7.04 = 0.24$ m





M_1 about 1-1 axis:

$M_1 = \Sigma Q \cdot e_1 = 49048 \cdot (0.48) = 23543$ kN.m



M_2 about 2-2 axis:

$M_2 = \Sigma Q \cdot e_2 = 49048 \cdot (0.24) = 11772$ kN.m

$$I_{1-1} = \left[\frac{B \cdot L^3}{12} + B \cdot L \cdot (D_1)^2\right] - \left[\frac{b \cdot l^3}{12} + b \cdot l \cdot (d_1)^2\right] =$$

$$= \left[\frac{16 \times 28^3}{12} + 16 \times 28 \times (14 - 12.47)^2\right] - \left[\frac{6 \times 12^3}{12} + 6 \times 12 \times (22 - 12.47)^2\right] = 22915 \quad m^4$$

$$I_{2-2} = \left[\frac{L \cdot B^3}{12} + B \cdot L \cdot (D_2)^2\right] - \left[\frac{l \cdot b^3}{12} + b \cdot l \cdot (d_2)^2\right] =$$

$$= \left[\frac{28 \times 16^3}{12} + 16 \times 28 \times (8 - 7.04)^2\right] - \left[\frac{12 \times 6^3}{12} + 12 \times 6 \times (13 - 7.04)^2\right] = 7197 \quad m^4$$

Note: In soil mechanics compression is taken as positive (+)

$$q = \frac{\Sigma Q}{Area} \pm \frac{M_1 \cdot y_1}{I_{1-1}} \pm \frac{M_2 \cdot y_2}{I_{2-2}}$$

$$q = \frac{49048}{376} \pm \frac{23543 \cdot y_1}{22915} \pm \frac{11772 \cdot y_2}{7197} = 130.4 \pm 1.03 y_1 \pm 1.64 y_2$$

$q_A = 130.4 \pm 1.03 y_1 \pm 1.64 y_2 = 130.4 + 1.03 \cdot (3.53) + 1.64 \cdot (2.96) = 138.9$ kN/m^2

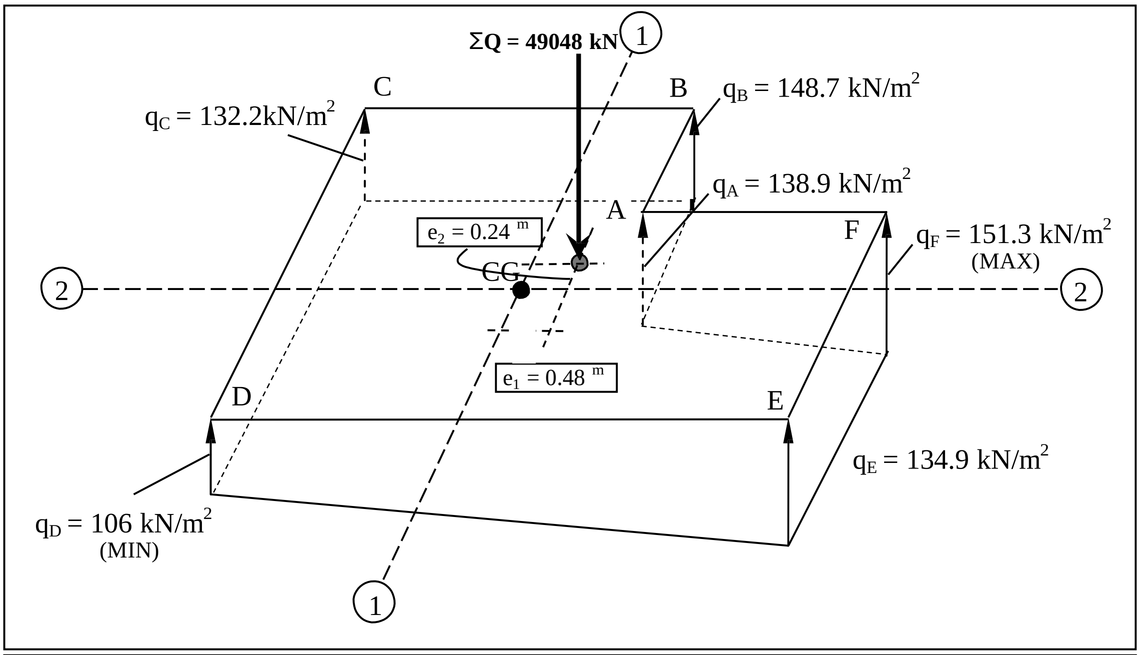
$q_B = 130.4 + 1.03 \cdot (3.53) + 1.64 \cdot (8.96) = 148.7$ kN/m^2

$q_C = 130.4 - 1.03 \cdot (12.47) + 1.64 \cdot (8.96) = 132.2$ kN/m^2

$q_D = 130.4 - 1.03 \cdot (12.47) - 1.64 \cdot (7.04) = 106$ kN/m^2

$q_E = 134.9$ kN/m^2

$q_F = 151.3$ kN/m^2

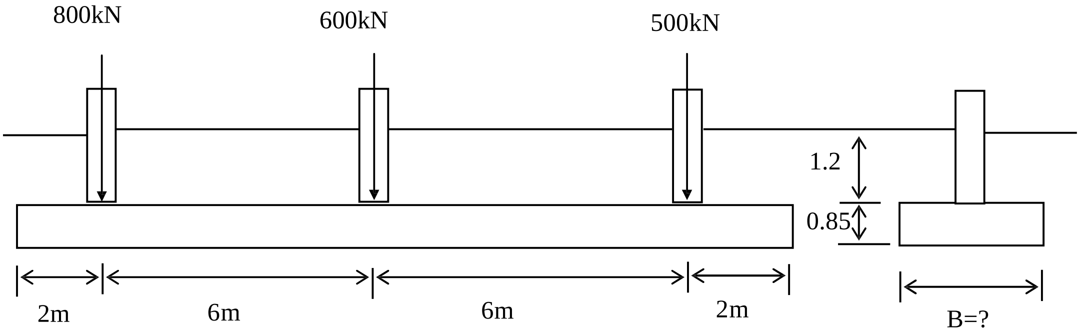


Since at all critical points stress values are almost ≈< $q_{all} = 150$ kN/m^2 given mat foundation is safe.

P.4) COMBINED FOOTING ANALYZED BY RIGID METHOD

Question:

A rectangular combined footing which supports three columns is to be constructed on a sandy clay layer *whose allowable bearing value (base pressure) is 84 kN/m^2*.The thickness of the concrete footing is 0.85m. There is a 1.20m thick soil fill having same unit weight with sandy clay on the footing. Unit weight of the sandy clay and the concrete are 20 kN/m^3 and 24 kN/m^3 respectively. Analyze the footing by rigid method and plot shear and moment diagrams.



Solution:

Neglecting column weights;

ΣQ_{net}=800+600+500+(24-20)x16xBx0.85 = 1900+54.4B

$$e = \frac{\Sigma M}{\Sigma V} = \frac{800x6 - 500x6}{1900 + 54.4B} = \frac{1800}{1900 + 54.4B}$$

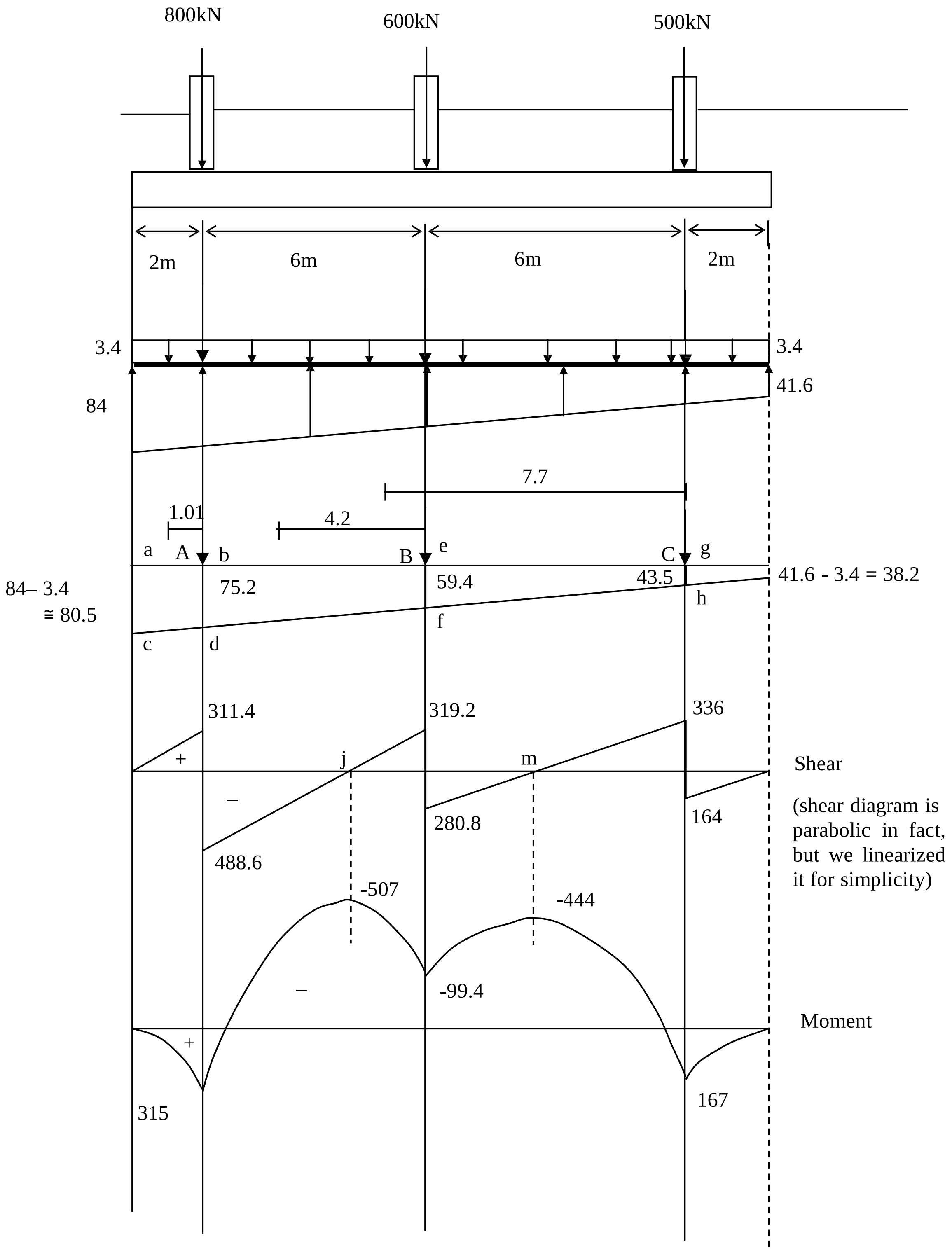
$$q_{\max} = \frac{\Sigma V}{BxL}(1 + \frac{6e}{L}) = 84kN/m^2$$

$$84 = \frac{1900 + 54.4B}{Bx16}(1 + \frac{6x1800}{(1900 + 54.4B)x16}) \quad \Rightarrow B = 2.0m$$

Use B=2.0m q_{max} = 84 kN/m^2 ; q_{min} = 41.6 kN/m^2

Downward uniform pressure = 0.85 × (24-20) = 3.4 kN/m^2

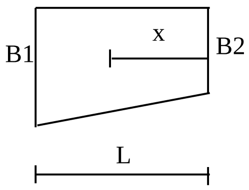
These are the diagrams related to forces and moments acting on the foundation. Explanations are at the next page;



$$V_A = \underbrace{\frac{80.5+75.2}{2}}_{\text{area (abcd)}} \text{ x } 2 \text{ x } \underset{B\,=\,2}{2} = 311.4\,\text{kN}$$

$V_{A'}$ = 311.4 – 800 = – 488.6 kN

To find the centroid of a trapezoid on the horizontal axis;



$$x = \frac{L}{3}\,\frac{2B1+B2}{B1+B2}$$ x: distance from shorter dimension

$$x\,(\text{abcd}) = \frac{2}{3}\,\frac{2\text{x}80.5+75.2}{80.5+75.2} = 1.01\,\text{m}$$

M_A = 311.4 x 1.01 = 315 kN.m

$$V_{BA} = \frac{80.5+59.4}{2} \text{ x } 8 \text{ x } 2 - 800 = 319.2\,\text{kN}$$

V_{BC} = 319.2 – 600 = -280.8 kN

$$x\,(\text{aefc}) = \frac{8}{3}\,\frac{2\text{x}80.5+59.4}{80.5+59.4} = 4.2\,\text{m}$$

$$M_B = \left[\frac{80.5+59.4}{2} \text{ x } 8 \text{ x } 2\right] \text{ x } 4.2 - 800 \text{ x } 6 = -99.4\,\text{kN.m}$$

$$V_{CB} = \frac{80.5+43.5}{2} \text{ x } 14 \text{ x } 2 - 800 - 600 = 336\,\text{kN}$$

V_C = 336 – 500 = -164 kN

Check the end point; $\frac{80.5+38.2}{2} \text{ x } 16 \text{ x } 2 - 800 - 600 - 500 = 0 \longrightarrow$ OK

$$x\,(aghc) = \frac{14}{3}\,\frac{2x80.5 + 43.5}{80.5 + 43.5} = 7.7\,m$$

$$M_C = \left[\frac{80.5 + 43.5}{2}\,x\,14\,x\,2\right]x\,7.7 - 800\,x\,12 - 600\,x\,6 = 167\,kN.m$$

Slope of V b/w x = 2 and x = 8 m Æ (488.6+319.2)/6=134.6 kN/m

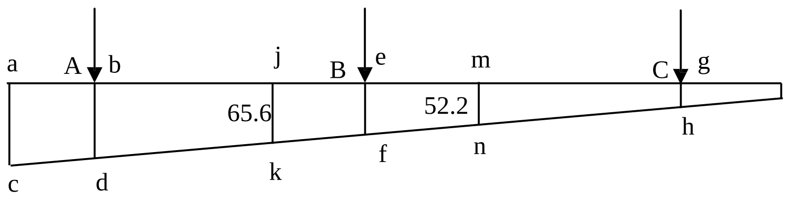
Equation of V b/w x = 2 and x = 8 m Æ V(x) = -488.6 + 134.6 (x) for V(x)=0 Æ x=3.6 m

Base pressure @ x = 2 + 3.6 = 5.6 m Æ 65.6 kN/m^2

Similarly @ x = 10.7 m V = 0

Base pressure @ x =10.7 m Æ 52.2 kN/m^2

maximum points of the moment diagram;



j and m are the points where shear forces are equal to zero (i.e. moment is max)

<u>the distance between point b and point j</u>

$$x\,(ajkc) = \frac{(2 + 3.6)}{3}\,\frac{2x80.5 + 65.6}{80.5 + 65.6} = 2.9\,m$$

$$M_{AB} = \left[\frac{80.5 + 65.6}{2}\,x\,(2 + 3.6)\,x\,2\right]x\,2.9 - 800\,x\,3.6 = -\,507\,kN.m$$

$$x\,(amnc) = 5.73\,m$$

M_{BC} = -444 kN.m