CE 366 Foundation Engineering – I

2011-2012 Spring Semester

Homework 2

Due on: 11.05.2012

Question 1 (20%)

A vertical concrete column is to carry a total load of 520 kN, inclusive of self-weight above ground level. The column is to be supported by a square concrete footing B x B founded at a depth of 1.5 m in a 14 m thick deposit of firm boulder clay. The clay is fully saturated and overlies Bunter sandstone. The properties of the clay are E = 10500 kPa, pore pressure constant, A = 0.4 and m_v = 0.00012 m^2/kN. Neglect the difference in density between the concrete and the clay.

(a) If B = 2.0 m, calculate the **total** settlement at the center of the footing. You can use superposition for calculating settlement.

(b)

For immediate settlement use:

$$S_i = \frac{qB(1-\nu^2)}{E} I_s$$

Where;

$$I_s = F_1 + \left[\frac{1-2\nu}{1-\nu}\right] F_2$$

(c) Calculate the size of footing required to provide a factor of safety of 3 against an undrained shear failure of the foundation soil.

$$N_c = 5\left(1 + 0.2\frac{D}{B}\right)\left(1 + 0.2\frac{B}{L}\right)$$

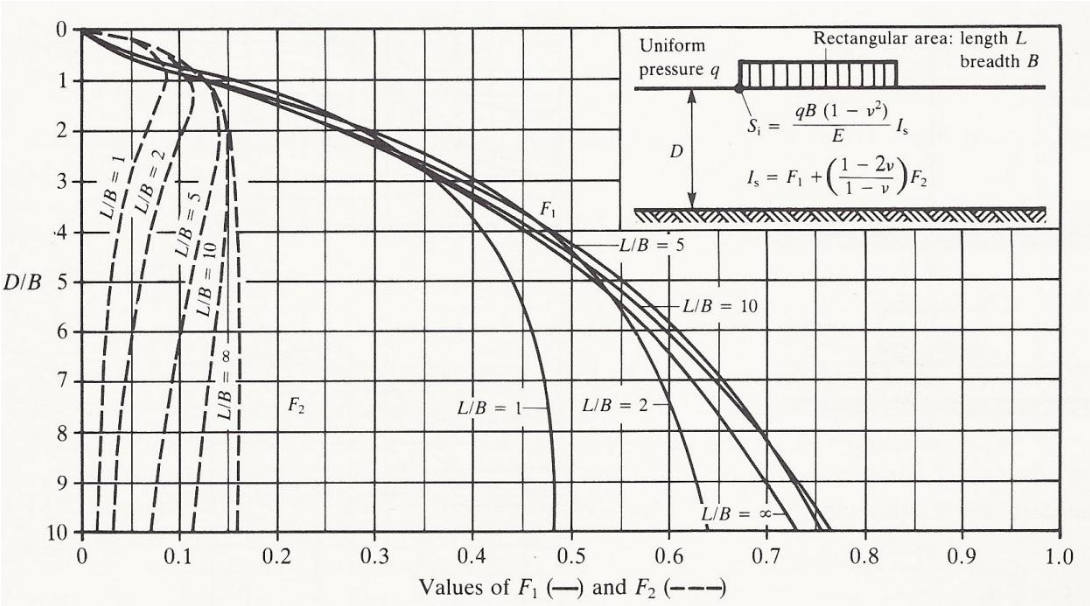


Figure 1. Values of influence factor F_1 and F_2 for calculating the immediate surface settlement S_i under the corner of a uniformly loaded flexible rectangular area on a soil layer of finite thickness (After Steinbrenner)

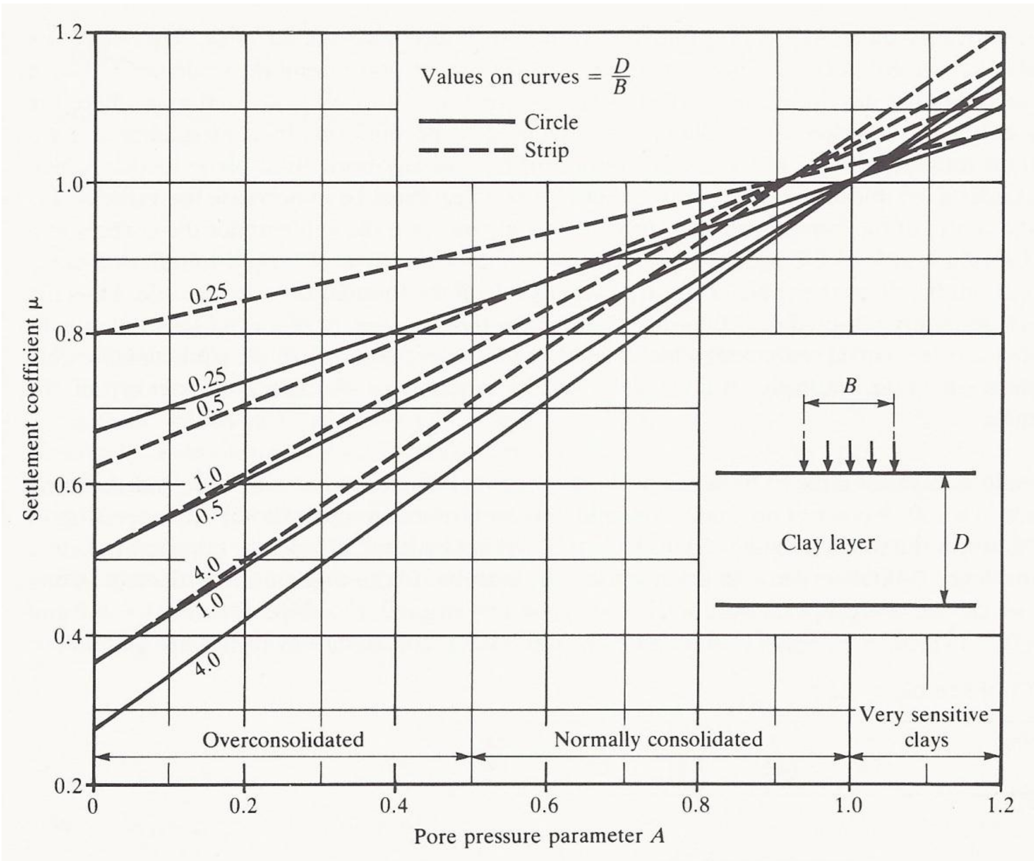


Figure 2. Values of settlement coefficient μ (From Scott, 1963. Copyright Addison-Wesley Inc. Reprinted with permission)

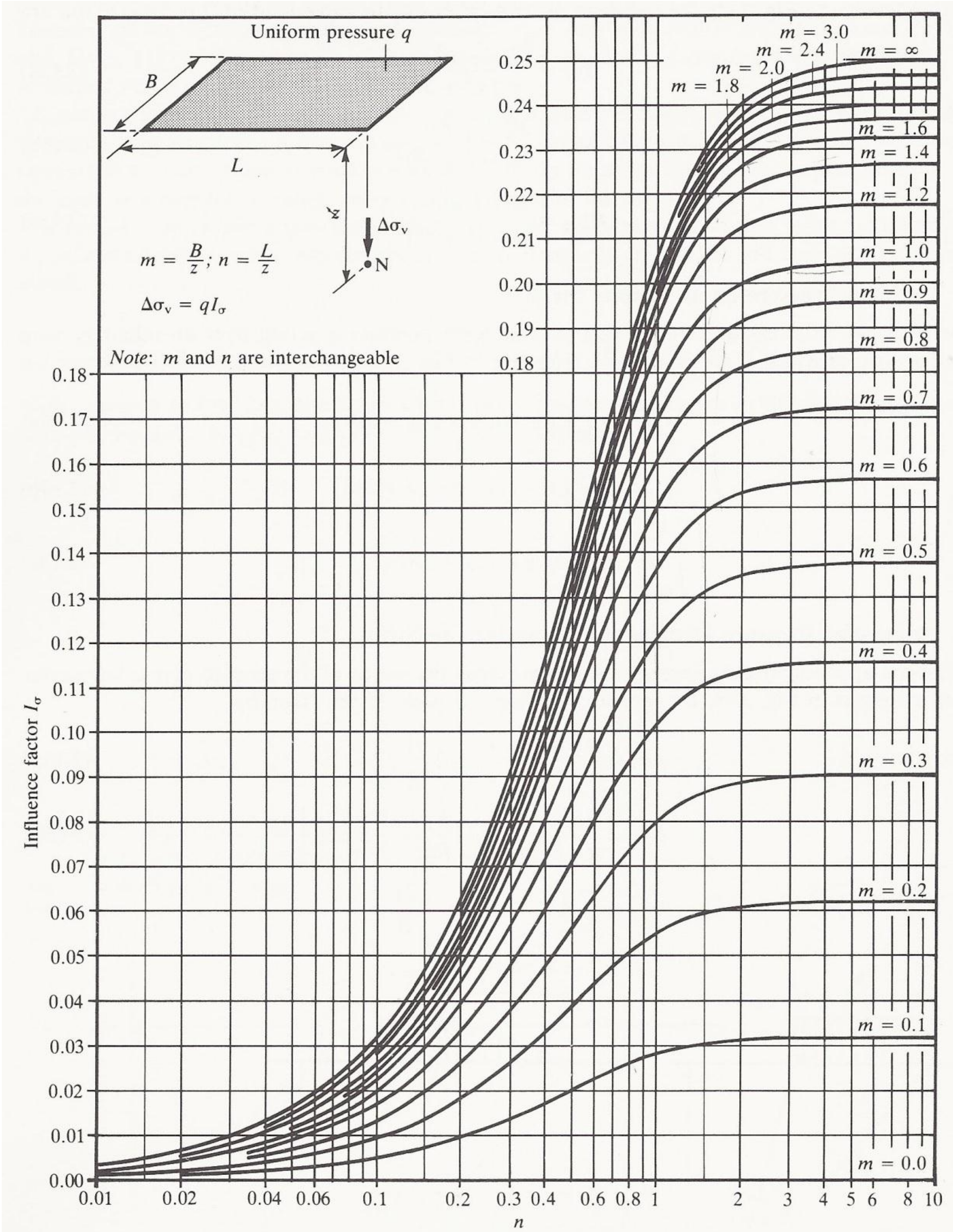
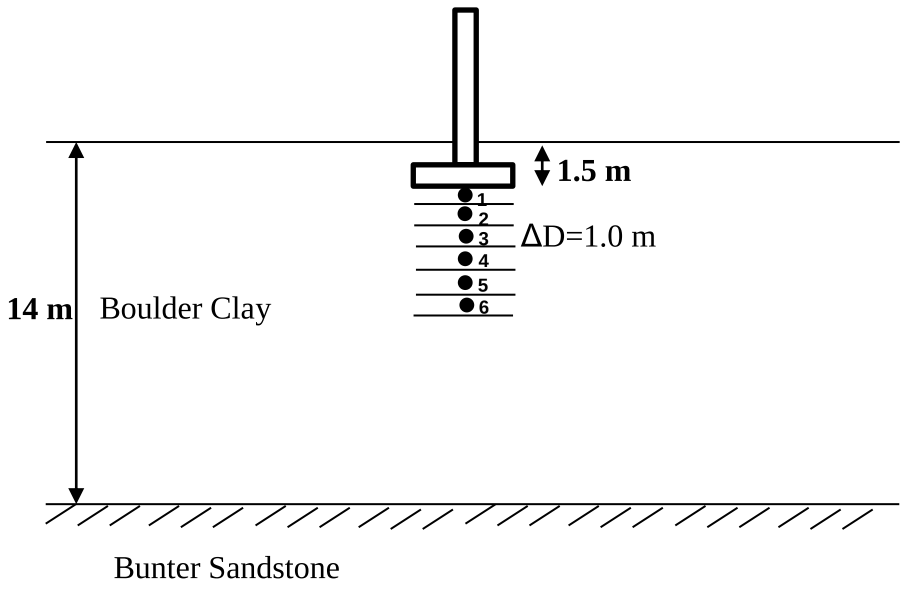


Figure 3. Values of I_σ for calculating increase in total vertical stress $\Delta\sigma_v$ under the corner of a unifromly loaded rectangular area (After Fadum, 1948 and reproduced with permission of Professor Fadum)

Solution

(a) As a concrete footing is a rigid foundation, there can be no differential settlement. As discussed earlier, we proceed by calculating the central settlement for the equivalent flexible foundation and then multiply by 0.8 to account for the actual rigidity of the base.



Immediate Settlement at the center of footing: The immediate surface settlement, S_i at the corner of a rectangular foundation is given by

$$S_i = \frac{qB(1-\nu^2)}{E} I_s$$

Here q = 520 / (2 x 2) = 130 kPa, B = 2/2 = 1 m, ν = 0.5 and E = 10500 kPa. The value of I_s for ν = 0.5 reduces to $I_s = F_1$. The parameter F_1 is obtained from the Figure 1 where for D/B = 12.5 and L/B = 1 we find F_1= 0.49. Hence, I_s= 0.49 and thus

$$S_i = \frac{130 \times 1 \times 0.75 \times 0.49}{10500} = 0.0046\, m$$

Therefore, the immediate surface settlement at the center of the footing is

$$0.0046 \times 4 = 0.018\, m = 18\, mm$$

Consolidation Settlement at the center of footing: The consolidation settlement of a soil element is given by the following equation

$$\Delta S_{cons} = \mu \, m_v \, \Delta D \, \Delta\sigma_v$$

The value of μ is determined from the Figure 2. Thus, for D/B = 12.5 / 2 = 6.25 and pore pressure parameter, A = 0.4 we have μ = 0.60 for a strip and μ = 0.53 for a circle. For a square foundation the value for μ will lie closer to the value for the circle than for the strip. Therefore take μ = 0.53.

As the foundation is square, take ΔD = ½ B = 1 m.

6 sublayers is take into consideration, as shown in the previous figure. The value of $\Delta\sigma_v$ at the mid-depth of each sublayer is determined by the use of Fadum 's influence chart (Figure 3). We thus obtain the following results:

Point	**z (m)**	**B (m)**	**L (m)**	**m=B/z**	**n=L/z**	I_σ	$\Delta\sigma_v$ **(kPa)**	ΔS_{cons} **(m)**
1	0.5	1	1	2.00	2.00	0.229	119.08	0.0304
2	1.5	1	1	0.67	0.67	0.123	63.96	0.0164
3	2.5	1	1	0.40	0.40	0.060	31.20	0.008
4	3.5	1	1	0.29	0.29	0.037	19.24	0.0048
5	4.5	1	1	0.22	0.22	0.024	12.48	0.0032
6	5.5	1	1	0.18	0.18	0.016	8.32	0.002
								Σ = 0.0648 m ≅ 65 mm

Hence central settlement of equivalent flexible footing = 18 + 65 = 83 mm.

Therefore settlement of actual footing = 0.8 x 65 = 66.4 mm.

This should be satisfactory.

(b) The net ultimate bearing pressure for the footing is given by

$$q_{net,ult} = c_u N_c$$

Wherein;

$$N_c = 5\left(1 + 0.2\frac{D}{B}\right)\left(1 + 0.2\frac{B}{L}\right)$$

Thus,

$$q_{net,ult} = 56 \times 5\left(1 + 0.2\frac{1.5}{B}\right)(1 + 0.2 \times 1) = 336\left(1 + \frac{0.3}{B}\right)$$

Neglecting the difference in density between the soil and the footing, the net applied pressure is given by

$$q_{net} = \frac{520}{B^2}$$

For a specified factor of safety of 3 against undrained shear failure

$$3 = \frac{336\left(1 + \frac{0.3}{B}\right)}{\frac{520}{B^2}}$$

From which;

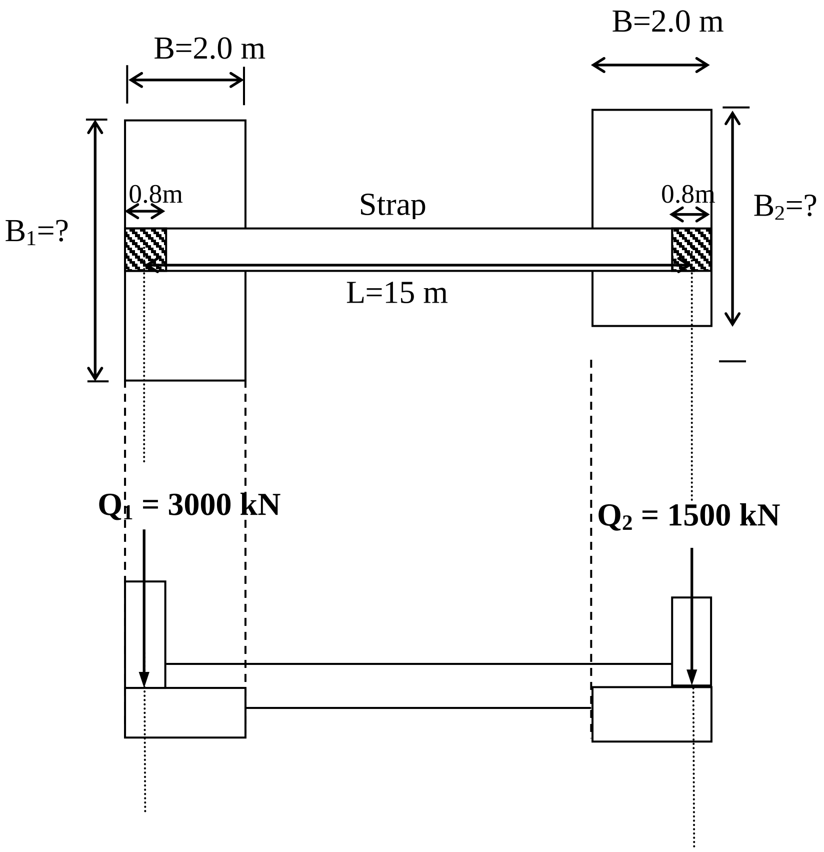
$$B^2 + 0.3B - 4.643 = 0$$

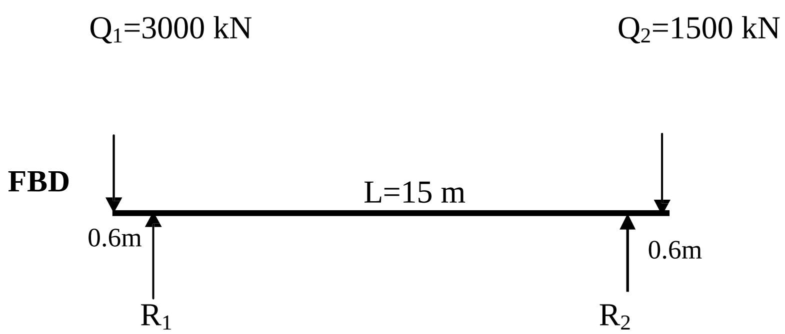
Hence;

$$B = 2.0\ m.$$

Question 2 (10%)

The figure given below shows the foundation plan view and cross-section of a residential building. To achieve uniform pressure distribution beneath footings, two footings were combined by a strap. The net allowable bearing capacity for the foundation system is estimated as 220 kN/m^2. Neglect the weight of footing and estimate the minimum footing dimensions B_1 and B_2. Draw shear and moment diagrams.





Moment w.r.t location of R_2;

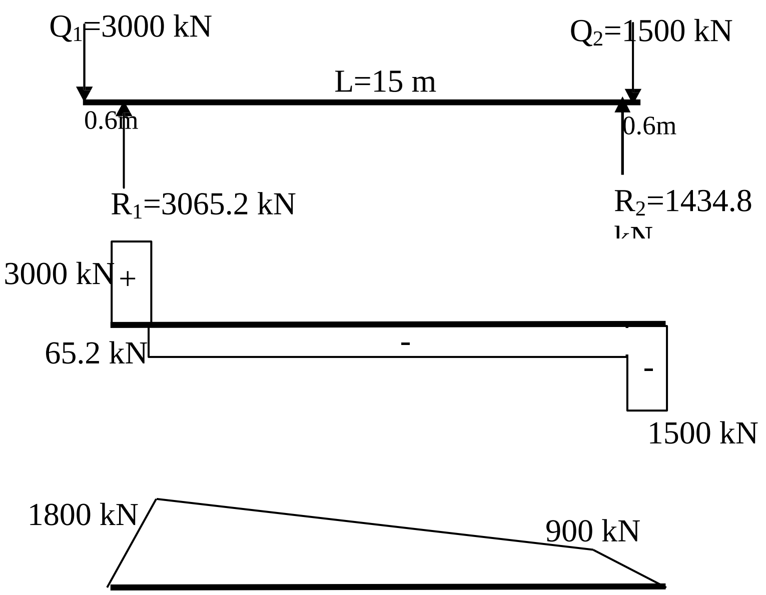
$\Sigma M = 0$; $3000x14.4 - R_1x13.8 - 1500x0.6 = 0$ → $R_1 \approx 3065.2$ kN

From vertical force equilibrium;

$\Sigma F_v = 0$; $3000+1500-3065.2-R_2=0$ → $R_2 = 1434.8$ kN

$$q_{all} = 220kpa = \frac{3065.2}{2xB_1} \rightarrow B_1 = 6.96m \approx 7.0\ m$$

$$q_{all} = 220kpa = \frac{1434.8}{2xB_2} \rightarrow B_2 = 3.26m \approx 3.3\ m$$



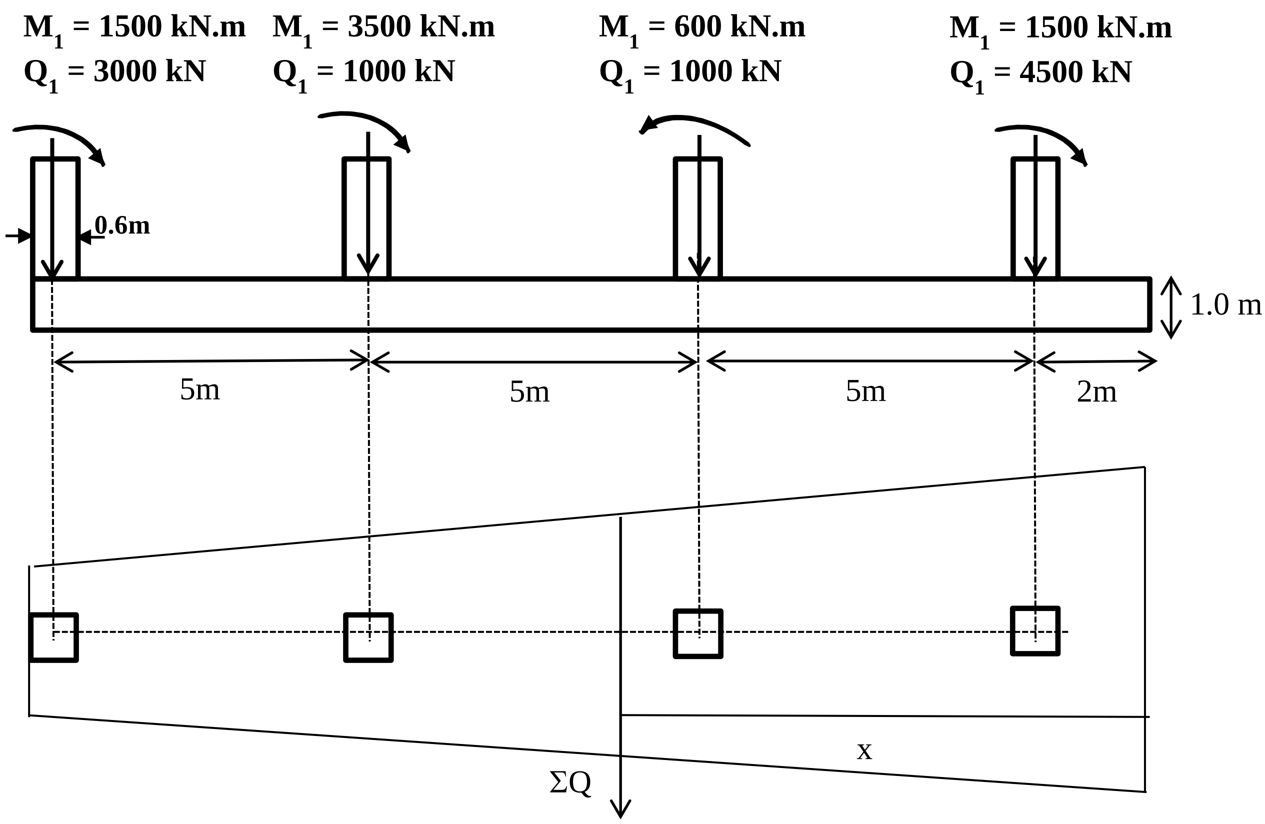
Free Body Diagram

Shear Force (kN) Diagram

Bending Moment (kN.m) Diagram

Question 3 (10%)

Determine B_1 and B_2 of a trapezoidal footing for a uniform soil pressure of 200 kN/m². (γ_{conc} = 24kN/m³)



$$Weight\ of\ footing = \frac{17.3x1x24x(B_1 + B_2)}{2} = 207.6x(B_1 + B_2)$$

$$Area\ of\ trapezoidal\ footing, A = \frac{17.3x(B_1 + B_2)}{2} = 8.65x(B_1 + B_2)$$

$\Sigma F_V = 0$;

3000+1000+1000+4500+ (207.6 x (B_1+B_2)) – (8.65x(B_1+B_2)x200)=0 → B_1+B_2=6.24 m ----(1)

$\Sigma M = 0$;

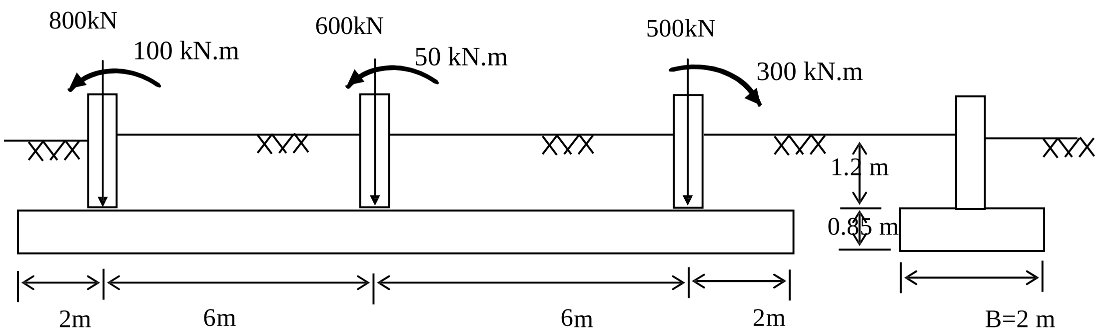
3000.(17-x)-1500+1000.(12-x)-3500-1000.(x-7)+600-4500.(x-2)-1500=0 → x=7.69 m

$x = \frac{1}{3}L\frac{2B_1+B_2}{B_1+B_2}$ → 7.69= $\frac{1}{3}17.3\frac{2B_1+B_2}{B_1+B_2} \rightarrow 2.03B_1 = B_2 - - - -(2)$

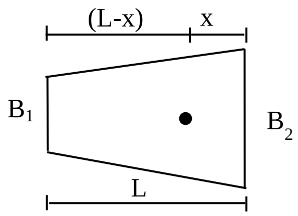
From equation 1 and 2; $\boldsymbol{B_1 = 2.05\ m \approx 2.1\ \ and\ \ B_2 = 4.19\ m \approx 4.2m}$

Question 4 (20%)

A rectangular combined footing which supports three columns is to be constructed on a sandy clay layer. The thickness of the concrete footing is 0.85m. Unit weights of the soil and the concrete are 20 kN/m^3 and 24 kN/m^3 respectively. Analyze the footing by rigid method and plot base pressure distribution, shear and moment diagrams.



Hint:



$$x = \frac{1}{3}L\frac{2B_1 + B_2}{B_1 + B_2}$$

$$A = L\frac{B_1 + B_2}{2}$$

Note: Weight of columns and weight of soil above footing are neglected (i.e: Assume that ΣV=2008.8 kN)

∑V =800+600+500+(24-20)x16x2x0.85 = 2008.8 kN

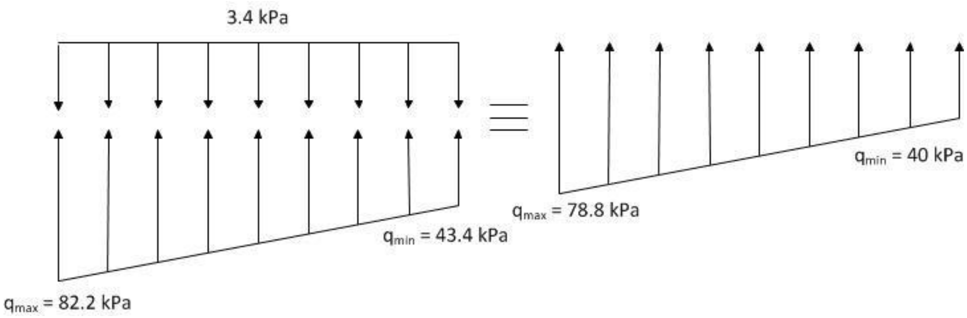
∑M = 100 + 800x6 + 50 – 300 – 500x6 = 1650 kNm (Moment w.r.t. point B)

e = ∑M / ∑V = 1650 / 2008.8 ≈ 0.822 m

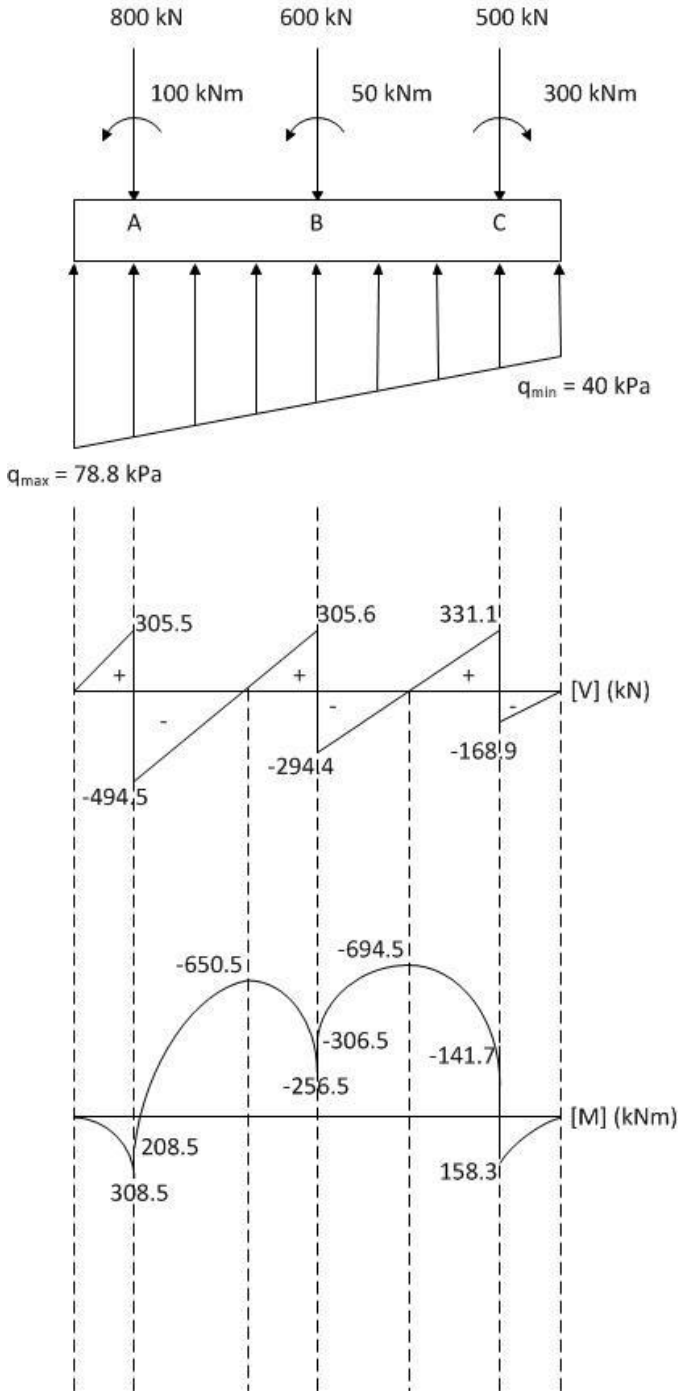
$q_{min,max} = \frac{\sum V}{BxL}(1 \mp \frac{6e}{L})$ $q_{min,max} = \frac{2008.8}{2x16}(1 \mp \frac{6x0.822}{16})$

➔ q_{min} = 43.4 kPa ; q_{max} = 82.2 kPa

Due to the difference between the unit weight of concrete and soil, a uniformly distributed pressure of 3.4 kPa (Uniform base pressure: [2008.8-(800+600+500)]/16x2 = 3.4 kPa) can be considered in downward direction



3.4 kPa
q_{max} = 82.2 kPa
q_{min} = 43.4 kPa
q_{max} = 78.8 kPa
q_{min} = 40 kPa



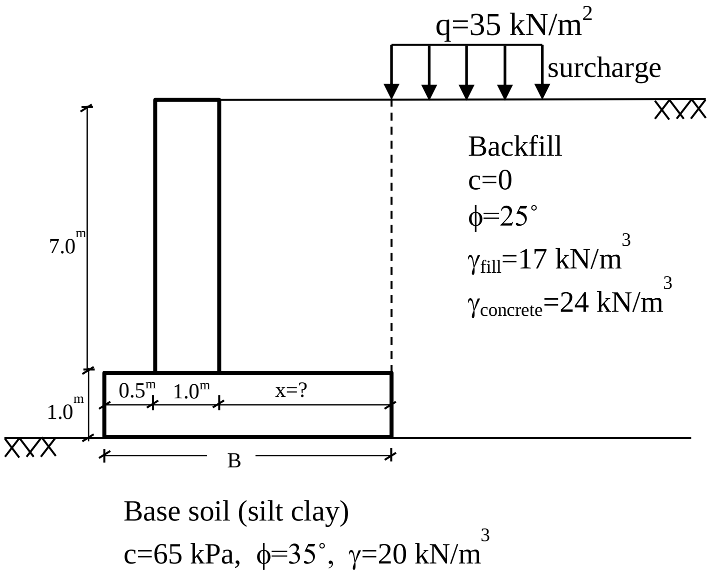
800 kN
600 kN
500 kN
100 kNm
50 kNm
300 kNm
A
B
C
q_{min} = 40 kPa
q_{max} = 78.8 kPa
305.5
305.6
331.1
[V] (kN)
-494.5
-294.4
-168.9
-650.5
-694.5
-306.5
-141.7
-256.5
[M] (kNm)
208.5
308.5
158.3

Question 5 (20%)

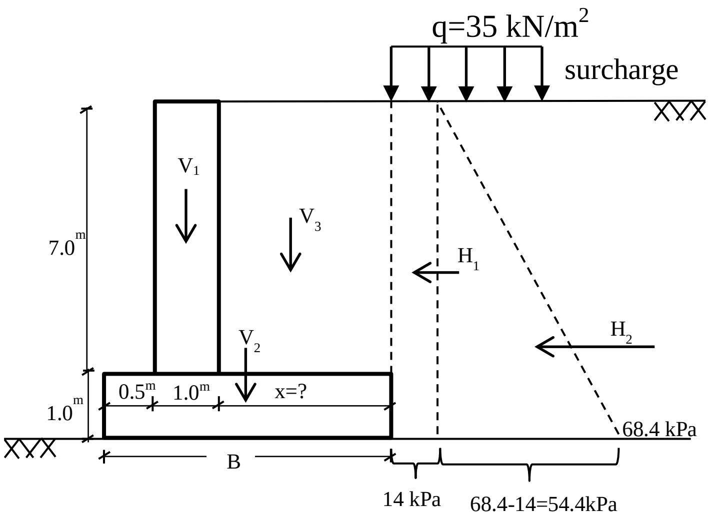
For the RC retaining wall given in the figure below

i) Calculate the required extent of the base slab forward the back of the wall such that a factor of safety of 2.0 against overturning is met.

ii) Subsequently check if the required minimum factor of safety of 1.5 is met against base sliding. If not, design a base key. Calculate the passive pressure starting from the bottom level olf the base slab. Also use 2/3 of the shear strenght parameters of the base soil in calculations and apply a FS 2.0 for the passive resistance.



Solution:



i) $K_A = \frac{1-sin\phi}{1+sin\phi} = \frac{1-sin25}{1+sin25} = 0.4$

$P_A = K_A(\gamma z + q) - \cancel{2c\sqrt{K_A}}$ c=0

$P_A(@z = 0m) = 0.4x(0 + 35) = 14\ kPa$

$P_A(@z = 8m) = 0.4x(17x8 + 35) = 68.4\ kPa$

Force (kN/m)	Moment Arm(m)	Moment (kN.m/m)
V_1 =7*1*24=168	0.5+0.5=1	168
V_2 =(0.5+1+x)*1*24 =(1.5+x)*24=36+24x	(0.5+1+x)/2=0.75+0.5x	27+18x+18x+12x^2 =27+36x+12x^2
V_3 =x*7*17=119x	1.5+x/2	178.5x+59.5x^2
ΣV=204+143x		ΣM_R=195+214.5x+71.5x^2
H_1 =14*8=112	8/2=4	448
H_2 =54.4*8*(1/2)=217.6	8/3=2.67	580.3
ΣH=329.6		ΣM_D =1028.3

$$FS_{ov} = \frac{\Sigma M_R}{\Sigma M_D} = \frac{195 + 214x + 71.5x^2}{1028.3} = 2.0$$

$$71.5x^2 + 214x - 1861.6 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-214 \pm \sqrt{214^2 - 4 * 71.5 * 1861.6}}{2 * 71.5} \rightarrow x \cong 3.8m$$

$$B = 0.5 + 1 + 3.8 = 5.3m$$

ii) Check for base sliding;

Resisting Force:

$$F_R = C_{base} * B + (\Sigma V)tan\phi_{base}$$

$$= \left(\frac{2}{3} * 65\right) * 5.3 + (204 + 143 * 3.8)tan\left(\frac{2}{3} * 35\right) = 229 + 322$$

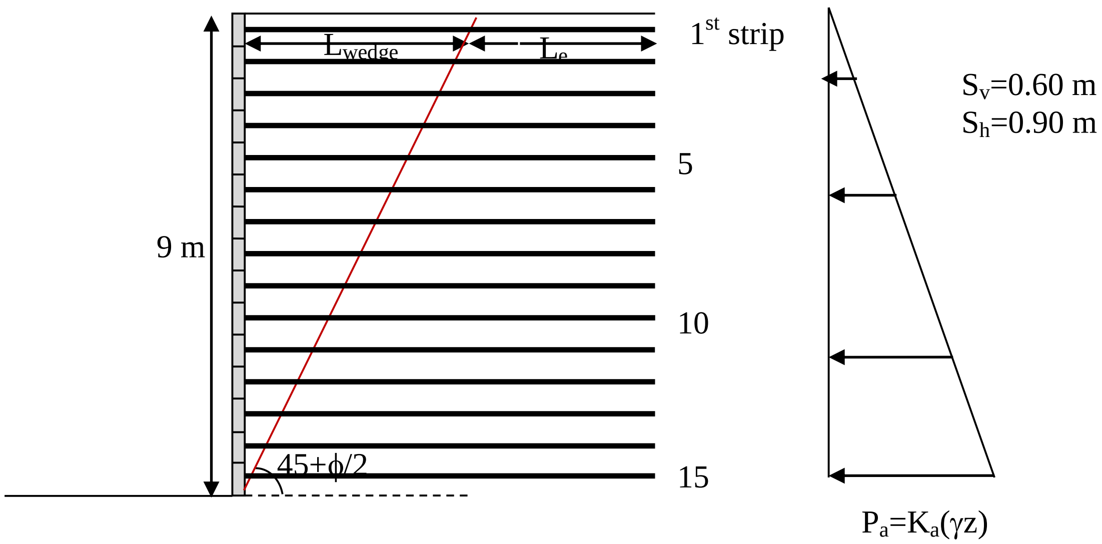
$$F_R = 552kPa$$

$$FS_{sliding} = \frac{F_R}{F_D} = \frac{552}{329.6} = 1.67 > 1.5\ OK$$

Question 6 (20%)

A 9-m high mechanically stabilized earth retaining wall with galvanized steel strip reinforcement in a granular backfill was constructed. Granular backfill has unit weight of 20 kN/m^3 and c'=0 ϕ'=36º. Information about galvanized steel reinforcement: width of the strips 7.5 cm, vertical spacing 60 cm, horizontal spacing 90 cm, yield strength of strip 240 MPa, and soil-strip friction angle 20º, strip thickness 5 mm, strip length 12 m (constant strip length is used). Corrosion rate of galvanized steel strip is 0.025 mm/year and the life span of the structure is 50 years. The first strip is placed at 30 cm below the top of the wall, the lowest strip is placed at 30 cm above the base of the wall.

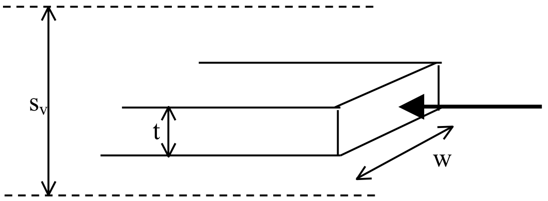
a) Check whether the thickness and length of the strips are sufficient for satisfying factor of safety of 3.0 against tie break and pullout. If they are not sufficient, determine the new thickness and length to satisfy required F.S values.
b) If you were to use different lengths of strips at different depths, calculate the lengths you would use for the 1st, 5th, 10th and 15th strips from the top of the wall.
c) Comment on how you would incorporate in calculating factor of safety against tie break and pull out, the effect of a surcharge load placed at the ground surface a few meters away from the face of the wall. Sketch the problem, write the equations for F.S. and comment in a few sentences about where in the equations you will incorporate some changes.



a)

- As far as the tie breaking is concerned,

Bottom reinforcement (number 15) is the most critical one since the vertical and therefore lateral pressure is maximum at that level.



$P_a=(\gamma z+q)K_a-2c(K_a)^{0.5}$
q=0 and c=0
$P_a=(\gamma z)K_a$

$$(FS)_{breaking}=\frac{w.t.f_y}{T_{max}}=3.0$$

$T= S_v.S_h.(\gamma z)K_a$

$\phi=36^0 \Rightarrow K_a=0.26$

T_{max}=0.60x0.90x(20x9)x0.26=25.27 kN

Corrosion rate $\Rightarrow$ 0.025mm/yr. x 50 =1.25mm

$$(FS)_{breaking}=\frac{0.075x(5-1.25)x10^{-3}x2.4x10^5}{25.27}=2.67<3.0$$

NOT OK, change the thickness of the strip

$$(FS)_{breaking}=\frac{0.075xtx2.4x10^5}{25.27}=3.0$$

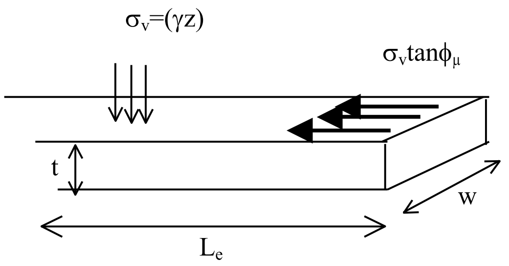
t=4.21 mm

Corrosion rate $\Rightarrow$ 0.025mm/yr. x 50 =1.25mm

t=4.21+1.25 = 5.46 mm

USE t_{design} =6 mm (t =5 mm is not sufficient)

- As far as tie pull-out is concerned,



Frictional resistance is available on both surface (top and bottom)

Friction angle between soil and reinforcement

$$(\text{F.S.})_{\text{pull-out}} = \frac{2\cancel{(\gamma z)}\tan\phi_\mu L_e w}{\cancel{(\gamma z)}K_a S_v S_h} = \frac{2\tan\phi_\mu L_e w}{K_a S_v S_h} = 3.0$$

Since length of strip is constant at each elevation, and L_r is the largest at the uppermost strip, L_e will be the smallest for the uppermost strip. We can check the uppermost strip for F.S. pullout, and decide the L_e used in the design is safe or not. If that uppermost strip satisfies F.S. pullout, all other strips (with less L_r, and more L_e than required) will be safer.

$$\tan(45-\phi/2) = \frac{L_{wedge}}{9-0.30} \Rightarrow \tan(27) = \frac{L_{wedge}}{8.70} \Rightarrow L_{wedge} = 4.43m$$

$$L_e = 12 - 4.43 = 7.57$$

$$(FS)_{pull-out} = \frac{2x0.075x7.57\tan 20}{0.26x0.60x0.90} = 2.94 \approx 3.0 \quad \text{OK}$$

If (FS)$_{pull\ out}$=2.9 <3.0 is considered;

$$(FS)_{pull-out} = \frac{2x0.075xL_e\tan 20}{0.26x0.60x0.90} = 3.0$$

L_e=7.71 m

- Total tie length L=L_{wedge}+L_e=4.43+7.71=12.14 m USE L=13 m

b)

Length in the wedge changes for different strip layers;

L_{wedge}= H*tan(45-ϕ/2)

Reinforcement Number	H (m)	L_{wedge} (m) (H*tan27)	L_e (m)	L (m)	L_{design} (m)
1	8.70	4.43	7.71	12.14	13.00
2	8.10	4.13	7.71	11.84	12.00
3	7.50	3.82	7.71	11.53	12.00
4	6.90	3.52	7.71	11.23	12.00
5	6.30	3.21	7.71	10.92	11.00
6	5.70	2.90	7.71	10.61	11.00
7	5.10	2.60	7.71	10.31	11.00
8	4.50	2.29	7.71	10.00	10.00
9	3.90	1.99	7.71	9.70	10.00
10	3.30	1.68	7.71	9.39	10.00
11	2.70	1.38	7.71	9.09	10.00
12	2.10	1.07	7.71	8.78	9.00
13	1.50	0.76	7.71	8.47	9.00
14	0.90	0.46	7.71	8.17	9.00
15	0.30	0.15	7.71	7.86	8.00

1st strip
→ L_{wedge}= H*tan(45-ϕ/2)=8.7xtan27=4.43m
→ L_e= 7.71 m
→ L= L_{wedge} +L_e=4.43+7.71 =12.14 m Use L=13 m

2nd strip
→ L_{wedge}= H*tan(45-ϕ/2)=8.1xtan27=4.13m
→ L_e= 7.71 m
→ L= L_{wedge} +L_e=4.13+7.71 =11.84 Use L=12 m

3rd strip
→ L_{wedge}= H*tan(45-ϕ/2)=7.5xtan27=3.82m
→ L_e= 7.71 m
→ L= L_{wedge} +L_e=3.82+7.71 =11.53 Use L=12 m

4th strip
→ L_{wedge}= H*tan(45-ϕ/2)=6.9xtan27=3.52m
→ L_e= 7.71 m
→ L= L_{wedge} +L_e=3.52+7.71 =11.23 Use L=12 m

5^{th} strip
→ L_{wedge}= H*tan(45-ϕ/2)=6.3xtan27=3.21m
→ L_e= 7.71 m
→ L= L_{wedge} +L_e=3.21+7.71=10.92 Use L=11 m

6^{th} strip
→ L_{wedge}= H*tan(45-ϕ/2)=5.7xtan27=2.90m
→ L_e= 7.71 m
→ L= L_{wedge} +L_e=2.90+7.71 =10.61 Use L=11 m

7^{th} strip
→ L_{wedge}= H*tan(45-ϕ/2)=5.1xtan27=2.60m
→ L_e= 7.71 m
→ L= L_{wedge} +L_e=2.60+7.71 =10.31 Use L=11 m

8^{th} strip
→ L_{wedge}= H*tan(45-ϕ/2)=4.5xtan27=2.29m
→ L_e= 7.71 m
→ L= L_{wedge} +L_e=2.29+7.71 =10.00 Use L=10 m

9^{th} strip
→ L_{wedge}= H*tan(45-ϕ/2)=3.9xtan27=1.99m
→ L_e= 7.71 m
→ L= L_{wedge} +L_e=1.99+7.71 =9.70 Use L=10 m

10^{th} strip
→ L_{wedge}= H*tan(45-ϕ/2)=3.3xtan27=1.68m
→ L_e= 7.72 m
→ L= L_{wedge} +L_e=1.68+7.71 =9.39 Use L=10 m

11^{th} strip
→ L_{wedge}= H*tan(45-ϕ/2)=2.7xtan27=1.38m
→ L_e= 7.71 m
→ L= L_{wedge} +L_e=1.38+7.71 =9.09 Use L=10 m

12^{th} strip
→ L_{wedge}= H*tan(45-ϕ/2)=2.1xtan27=1.07m
→ L_e= 7.71 m
→ L= L_{wedge} +L_e=1.07+7.71 =8.78 Use L=9 m

13^{th} strip
→ L_{wedge}= H*tan(45-ϕ/2)=1.5xtan27=0.76 m
→ L_e= 7.71 m
→ L= L_{wedge} +L_e=0.76+7.71 =8.47 Use L=9 m

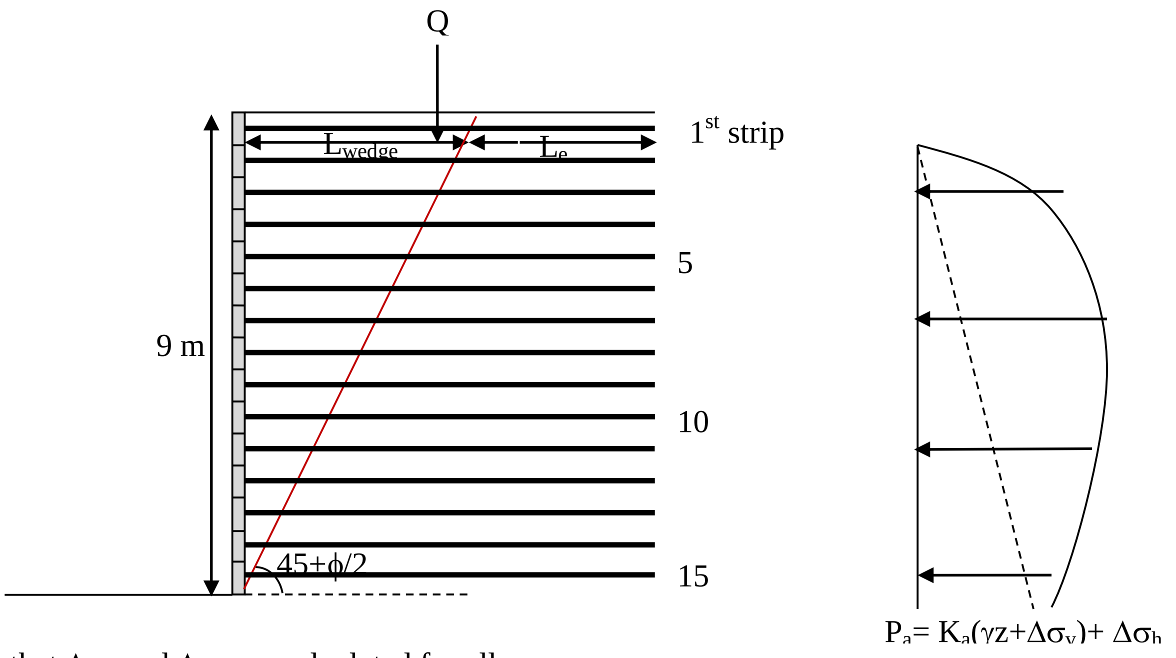
14^{th} strip
→ L_{wedge}= H*tan(45-ϕ/2)=0.9xtan27=0.46m
→ L_e= 7.71 m
→ L= L_{wedge} +L_e=0.46+7.71 =8.17 Use L=9 m

15^{th} strip
→ L_{wedge}= H*tan(45-ϕ/2)=0.3xtan27=0.15m
→ L_e= 7.71 m
→ L= L_{wedge} +L_e=0.15+7.71=7.86 Use L=8 m

c)

Additional vertical and horizontal stresses due to surcharge will be developed due to the surcharge placed at the ground surface a few meters away from the face of the wall. The additional vertical stress is not same throughout the depth. Same is valid for the additional horizontal stress. Therefore, additional stresses (both vertical and horizontal) should be calculated separately for all depths.

Boussinesq or 2V:1H assumption can be used to calculate additional vertical stresses. Boussinesq can be used to calculate additional horizontal stresses. (See Figure 1.2 at page 7 in Lecture Notes)



Note that $\Delta\sigma_v$ and $\Delta\sigma_h$ are calculated for all

$\Delta\sigma_V$: Additional vertical stress due to surcharge
$\Delta\sigma_h$: Additional horizontal stress due to surcharge

F.S. for breaking will be;

$$(FS)_{breaking} = \frac{w.t.f_y}{T_{\max}} = \frac{w.t.f_y}{[\Delta\sigma_h + K_a(\Delta\sigma_v + \gamma z)].S_v.S_h}$$

Note that in the formula above, $\Delta\sigma_v$ and $\Delta\sigma_h$ are the additional vertical and horizontal stresses calculated at the depth where the critical strip will be placed. Additional vertical stresses due to surcharge will be more at the shallower depths, and less at deeper depths. Therefore after including the additonal vertical and horizontal stresses in calculations, one can see which one of the strips will be the most crticial against tie break (i.e. when surcharge is applied, the bottom strip may not always be the most critical against tie break).

F.S. for pull-out will be;

$$(FS)_{pull-out} = \frac{2.\mathrm{w}.\mathrm{L_e}.\sigma_{\mathrm{v}} \tan\phi_{\mu}}{\sigma_{\mathrm{h}}.\mathrm{S_v}.\mathrm{S_h}} = \frac{2.\mathrm{w}.\mathrm{L_e}.(\Delta\sigma_{\mathrm{v}} + \gamma \mathrm{z})\tan\phi_{\mu}}{[\Delta\sigma_{\mathrm{h}} + \mathrm{K_a}\,\mathrm{x}\,(\gamma\,\mathrm{z})].\mathrm{S_v}.\mathrm{S_h}}$$

Note that in the formula above, $\Delta\sigma_v$ and $\Delta\sigma_h$ are the additional vertical and horizontal stresses at the depth where the strips will be placed.