[redacted]

CE 382 - Reinforced Concrete Fundamentals

HOMEWORK 3

1) For C25 & S420 $\quad f_{cd} = 17$ MPa, $f_{yd} = 365$ MPa.

$j'_m = 0.776 \quad K_m = 199\ mm^2/kN$

$j'_\ell = 0.86 \quad K_\ell = 291\ mm^2/kN$.

K101 (Flanged Section)

$$K = \frac{bd^2}{M_d} = \frac{1000 \times 460^2}{275\,000} = 769 > K_\ell \quad \text{O.K.}$$

$j'd = 0.9d = 414\ mm \longrightarrow$ Use this

$j'd = d - \frac{t}{2} = 330\ mm$

$$A_s = \frac{M_d}{f_{yd}\, j'd} = \frac{275\,000}{0.365 \times 414} = 1820\ mm^2 \longrightarrow$$

$4\phi20$ Bent $= 1256\ mm^2$

$4\phi14$ Straight $= 616\ mm^2$

$+$

$1872\ mm^2$ ✓

Support 1 (Rectangular)

$$M_d = 245 - \frac{250 \times 0.4}{3} = 211.7\ kNm$$

$$K = \frac{300 \times 460^2}{211700} = 300 > K_\ell \quad \text{O.K.}$$

$$A_s = \frac{211700}{0.365 \times 0.86 \times 460} = 1466\ mm^2 \longrightarrow$$

Available K101- $4\phi20 = 1256\ mm^2$

Hanger- $2\phi12 = 226\ mm^2$

$+$

$1482\ mm^2$ ✓

K102 (Flange)

$K = \frac{1000 \times 460^2}{100\,000} = 2116\ mm^2/kN > K_\ell$ O.K.

$A_s = \frac{100\,000}{0.365 \times 414} = 662\ mm^2 \longrightarrow$ 2Ø14 Straight $= 308\ mm^2$

2Ø20 Bent $= 628\ mm^2$

+ ―――――

936 mm^2 ✓

Support 2 (Rect.)

$M_d = 330 - \frac{270 \times 0.4}{3} = 294\ kNm$

$K = \frac{300 \times 460^2}{294\,000} = 216 < K_\ell$ Need Double Reinforcement

$M_1 = \frac{300 \times 460^2}{291} = 218.14\ kNm \quad M_2 = M_d - M_1 = 75.86\ kNm$

$A_{s1} = \frac{218\,140}{0.365 \times 0.86 \times 460} = 1511\ mm^2 \quad A_{s2} = A_s' = \frac{M_2}{f_{yd}(d-d')} = 495\ mm^2$

$A_s = 1511 + 495 = 2006\ mm^2$

Top $\longrightarrow$ K101 – 4Ø20 Bent $= 1256\ mm^2$

K102 – 2Ø20 Bent $= 628\ mm^2$

Hanger – 2Ø12 $= 226\ mm^2$

+ ―――――

2110 mm^2 ✓

Bottom $\longrightarrow$ K101 – 4Ø14 Straight $= 616\ mm^2$

K102 – 2Ø14 Straight $= 308\ mm^2$

+ ―――――

924 $mm^2 > A_s'$ ✓

K103 (Flange)

$K = \frac{1000 \times 460^2}{200\,000} = 1058 > K_\ell$ O.K.

$A_s = \frac{200\,000}{0.365 \times 414} = 1324\ mm^2 \longrightarrow$ 4Ø20 Bent $= 1256\ mm^2$

2Ø14 Straight $= 308\ mm^2$

+ ―――――

1564 mm^2 ✓

Support 3

$$M_d = 300 - \frac{195 \times 0.4}{3} = 276 \ kNm$$

$$K = \frac{300 \times 460^2}{276000} = 230 < K_\ell \quad \text{Double Reinf.}$$

$$M_1 = \frac{300 \times 460^2}{291} = 218.14 \ kNm \qquad M_2 = M_d - M_1 = 57.86 \ kNm$$

$$A_{s1} = \frac{218140}{0.365 \times 0.86 \times 460} = 1511 \ mm^2 \qquad A_{s2} = A_s' = \frac{57860}{0.365 \times 420} = 378 \ mm^2$$

$$A_s = A_{s1} + A_{s2} = 1889 \ mm^2$$

Top $\longrightarrow$ K102 - 2Ø20 Bent = 628 mm^2
K103 - 4Ø20 Bent = 1256 mm^2
Hanger - 2Ø12 = 226 mm^2 (+)
2110 mm^2 ✓

Bottom $\longrightarrow$ K102 - 2Ø14 Str = 308 mm^2
K103 - 2Ø14 Str = 308 mm^2 (+)
616 mm^2 ✓

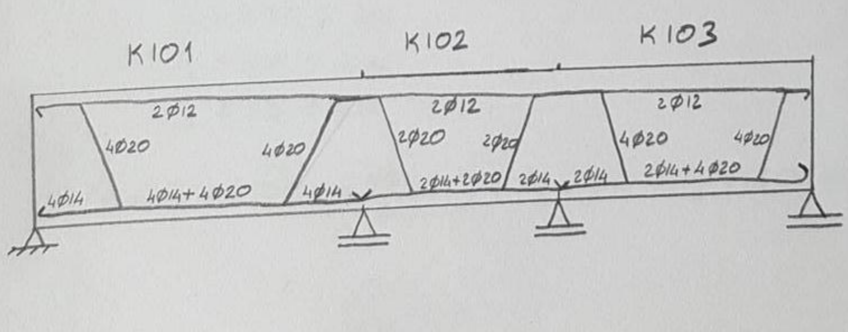
Support 4

$$M_d = 177 - \frac{170 \times 0.4}{3} = 154.3 \ kNm$$

$$K = \frac{300 \times 460^2}{154300} = 411 > K_\ell \quad \text{O.K.}$$

$$A_s = \frac{154300}{0.365 \times 0.86 \times 460} = 1069 \ mm^2$$

$\longrightarrow$ K103 - 4Ø20 Bent = 1256 mm^2
Hanger - 2Ø12 = 226 mm^2 (+)
1482 mm^2 ✓



2) a. $N_{or} = 0.85 f_{cd} A_c + A_s f_{yd} = 0.85 \times 17 \times (400 \times 400) + 4 \times \frac{\pi 24^2}{4} \times 365$

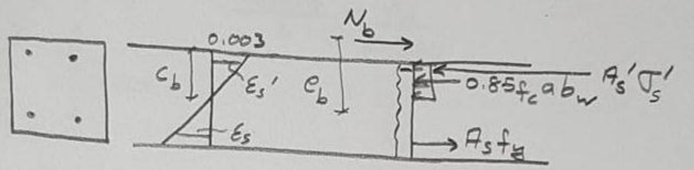
$N_{or} = 2972.5 \text{ kN}$

b. Tension Capacity

Direct Tensile $\longrightarrow$ $f_{ctd} = 0.35\sqrt{17} = 1.44 \text{ MPa}$

$N_t = f_{ctd} \times A_c + A_s f_{yd} = 891 \text{ kN}$

c.



At balance case $\varepsilon_s = \varepsilon_y = \frac{365}{200000} = 0.00183$

$\frac{0.003}{c_b} = \frac{0.00183}{360 - c_b} \longrightarrow c_b = 223.6 \text{ mm}$

$\frac{0.003}{c_b} = \frac{\varepsilon_s'}{c_b - 40} \longrightarrow \varepsilon_s' = 2.5 \times 10^{-3} > \varepsilon_y$ Comp. steel yields

$N_b + \cancel{A_s f_y} = 0.85 f_c a b_w + \cancel{A_s' \sigma_s'}$

$N_b = 1218 \text{ kN}$

$M_b = 2 \times (A_s f_y) \times (160) + (0.85 f_c a b_w) \times (200 - \frac{a}{2}) = \boxed{221 \text{ kNm}}$

d. $F_c = 0.85 \times 17 \times 0.85c \times 400 = 4913c \text{ N}$

$F_s = 2 \times \frac{\pi 24^2}{4} \times 365 = 330244 \text{ N}$

$F_s' = 2 \times \frac{\pi 24^2}{4} \times \sigma_s' = 904.8 \times 200000 \times \left(0.003 - \frac{0.12}{c}\right) = 542880 - \frac{21715200}{c}$

$N = 0 = F_c + F_s' - F_s \longrightarrow c = 48.35 \text{ mm}$

$F_c = 237.5 \text{ kN}$

$F_s = 330.2 \text{ kN}$

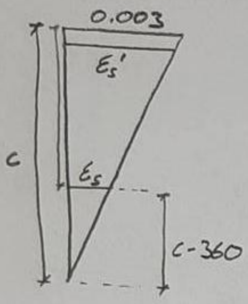
$F_s' = 33.7 \text{ kN}$

$M = F_c \times (200 - \frac{0.85 \times c}{2}) + F_s \times 160 + F_s' \times 160 = \boxed{110.4 \text{ kNm}}$

e. $N = 0.85 N_{or} = 0.85 \times 2972.5 = 2527$ kN

$N > N_b \rightarrow$ Compression Failure

Assume all section is in compression.



$\varepsilon_s' > \varepsilon_y$ by definition.

$$\frac{0.003}{c} = \frac{\varepsilon_s}{c-360}$$

$F_c = 0.85 \times 17 \times 0.85c \times 400 = 4913\,c$

$F_s' = 365 \times 2 \times \frac{\pi 24^2}{4} = 330252$

$F_s = 2 \times \frac{\pi 24^2}{4} \times 200000 \times \left(0.003 - \frac{1.08}{c}\right) = 542\,880 - \frac{195436800}{c}$

$N = 2527000 = F_c + F_s' + F_s \longrightarrow c = 429.3$ mm

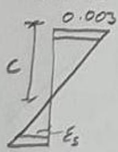
$c > h$ Assumption is true.

$$\left.\begin{array}{l} F_c = 2109 \text{ kN} \\ F_s' = 330.2 \text{ kN} \\ F_s = 87.7 \text{ kN} \end{array}\right\} M = F_c \times \left(200 - \frac{a}{2}\right) + F_s' \times 160 + F_s \times 160 = \boxed{103.9 \text{ kNm}}$$

f. $N = 0.45 N_{or} = 1337.6$ kN

$N > N_b \longrightarrow$ Compression Failure

Assume bottom steel is under tension.



$$\frac{0.003}{c} = \frac{\varepsilon_s}{360-c}$$

$$\left.\begin{array}{l} F_c = 4913\,c \\ F_s' = 330252 \\ F_s = \frac{195\,436\,800}{c} - 542\,880 \end{array}\right\} N = F_c + F_s' - F_s$$

$\hookrightarrow c = 252.2$ mm

$c < h$ Assumption is true

$F_c = 1239$ kN

$F_s = 330.2$ kN

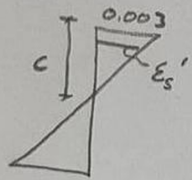
$F_s' = 232$ kN

$$M = F_c \times \left(200 - \frac{a}{2}\right) + 160 \times [F_s + F_s'] = \boxed{205 \text{ kNm}}$$

g. $N = 0.15 N_{or} = 0.15 \times 2372.5 = 446\ kN$

$N < N_b \rightarrow$ Tension Failure

Assume $\sigma_s = f_y$ and $\sigma_s' < f_y$



$$\frac{0.003}{c} = \frac{\varepsilon_s'}{c-40} \qquad \varepsilon_s' = 0.003 - \frac{0.12}{c}$$

$F_c = 4913\,c$

$F_s = 330\,244$

$F_s' = A_s' \times 200000 \times \varepsilon_s' = 542\,880 - \dfrac{21\,715\,200}{c}$

$N = F_c + F_s - F_s' \longrightarrow c = 75.7\ mm$

From compatibility $\varepsilon_s' = 0.0014 < \varepsilon_y$

$\varepsilon_s = 0.011 > \varepsilon_y$ Assumption is true.

$$\left.\begin{array}{l} F_c = 371.9\ kN \\ F_s = 330.2\ kN \\ F_s' = 256\ kN \end{array}\right\} \quad M = F_c \times \left(200 - \frac{0.85c}{2}\right) + 160(F_s + F_s') = \boxed{156.2\ kNm}$$