Handout #7

## **Integration Formulas**

## a) Trapezoidal Rule:

$$I \approx \frac{h}{2} \left[ f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right]$$
or
$$I \approx \frac{(b-a)}{2n} \left[ f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right],$$

$$E = -\frac{(b-a)^3}{12n^3} \sum_{i=1}^n f^{(2)}(\varepsilon_i) \quad \text{or} \quad E = -\frac{(b-a)^3}{12n^2} \overline{f}^n \quad \text{for n segments (or intervals)}.$$

## **b)** Simpson's 1/3 Rule:

$$I \approx \frac{h}{3} \left[ f(x_0) + 4 \sum_{i=1,3,5,\dots}^{n-1} f(x_i) + 2 \sum_{i=2,4,6,\dots}^{n-2} f(x_i) + f(x_n) \right]$$
or
$$I \approx \frac{b-a}{3n} \left[ f(x_0) + 4 \sum_{i=1,3,5,\dots}^{n-1} f(x_i) + 2 \sum_{i=2,4,6,\dots}^{n-2} f(x_i) + f(x_n) \right]$$

$$E = -\frac{(b-a)^5}{90n^5} \sum_{i=1}^{n/2} f^{(4)}(\varepsilon_{2i}) = -\frac{(b-a)^5}{180n^4} \quad \text{for n segments (or intervals)}.$$

## c) Gauss-Legendre Formulas:

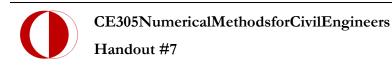
Let your original integral be of the form:

$$\int_{a}^{b} g(t)dt \tag{1}$$

Gauss Legendre Formulation has the variable x for  $\int_{1}^{1} f(x)dx$ 

Now, change (transform) t to x by;

$$t = \frac{b+a}{2} + \frac{b-a}{2}x$$
 (2)



and dt to dx by;

$$dt = \frac{b - a}{2} dx \tag{3}$$

Now substitute equations (2) and (3) into equation (1) and get ready to use Gauss-Legendre formula  $\int_{-1}^{1} f(x)dx \approx \sum_{i=0}^{n} w_i f_i$  with

N	Abscissas, x <sub>N, k</sub>	Weights, $w_{N, k}$ $(W_i)$	Truncation Error
2	-0.5773502692 0.5773502692	1.0000000000 1.0000000000	$\frac{f^{(4)}(\varepsilon)}{135}$
3	±0.7745966692 0.0000000000	0.555555556 0.8888888888	$\frac{f^{(6)}(\varepsilon)}{15,750}$
4	±0.8611363116 ±0.3399810436	0.3478548451 0.6521451549	$\frac{f^{(8)}(\varepsilon)}{3,472,875}$
5	±0.9061798459 ±0.5384693101 0.00000000000	0.2369268851 0.4786286705 0.5688888888	$\frac{f^{(10)}(\varepsilon)}{1,237,732,650}$