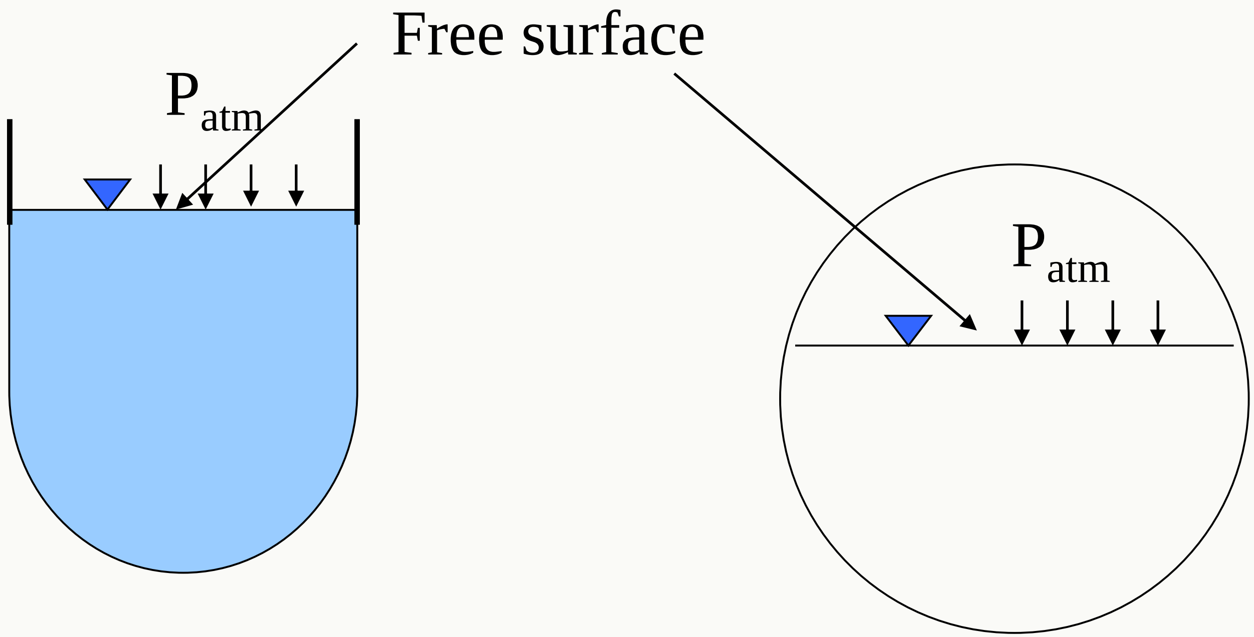
Flow in Open Channels

OPEN-CHANNEL FLOW

- Open-channel flow is a flow of liquid (basically water) in a conduit with a free surface. That is a surface on which pressure is equal to local atmospheric pressure.

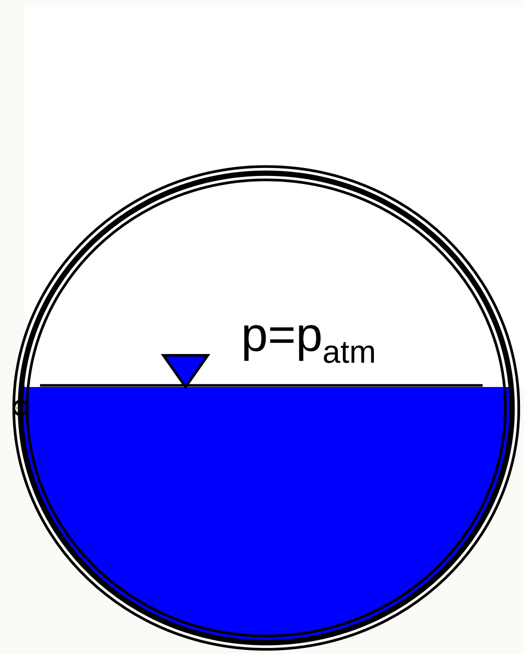


Classification of Open-Channel Flows

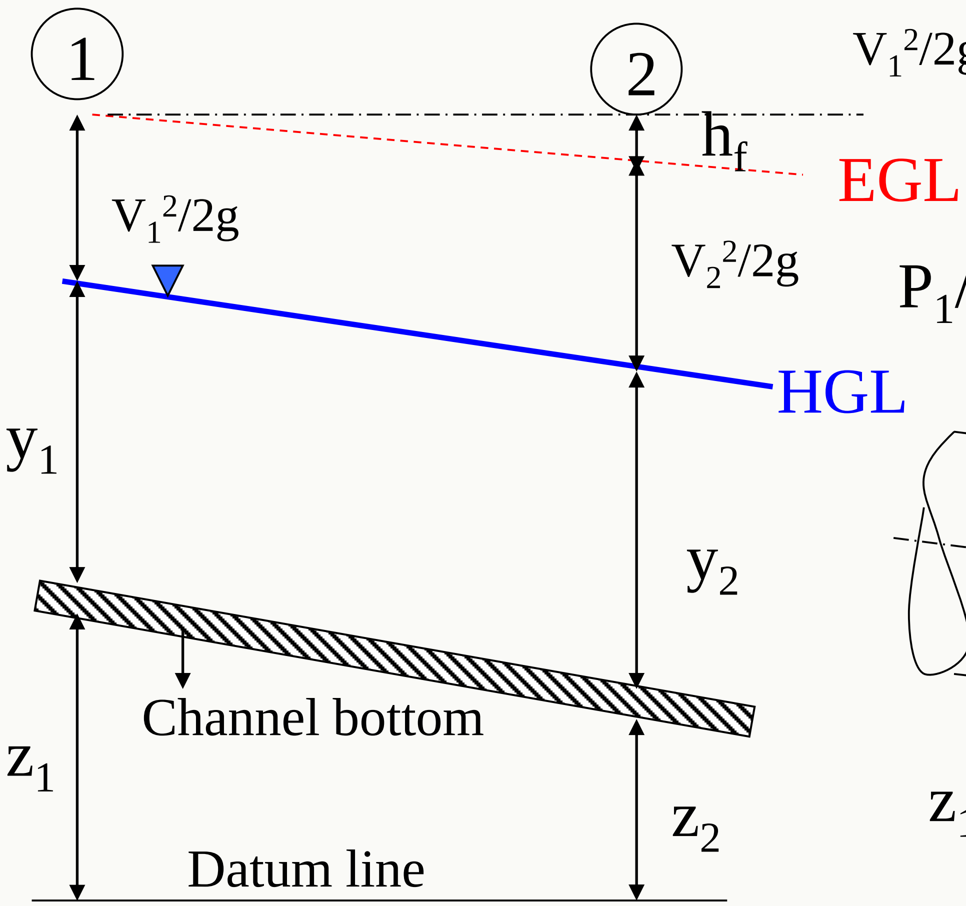
Open-channel flows are characterized by the presence of a liquid-gas interface called the *free surface.*

- Natural flows: rivers, creeks, floods, etc.

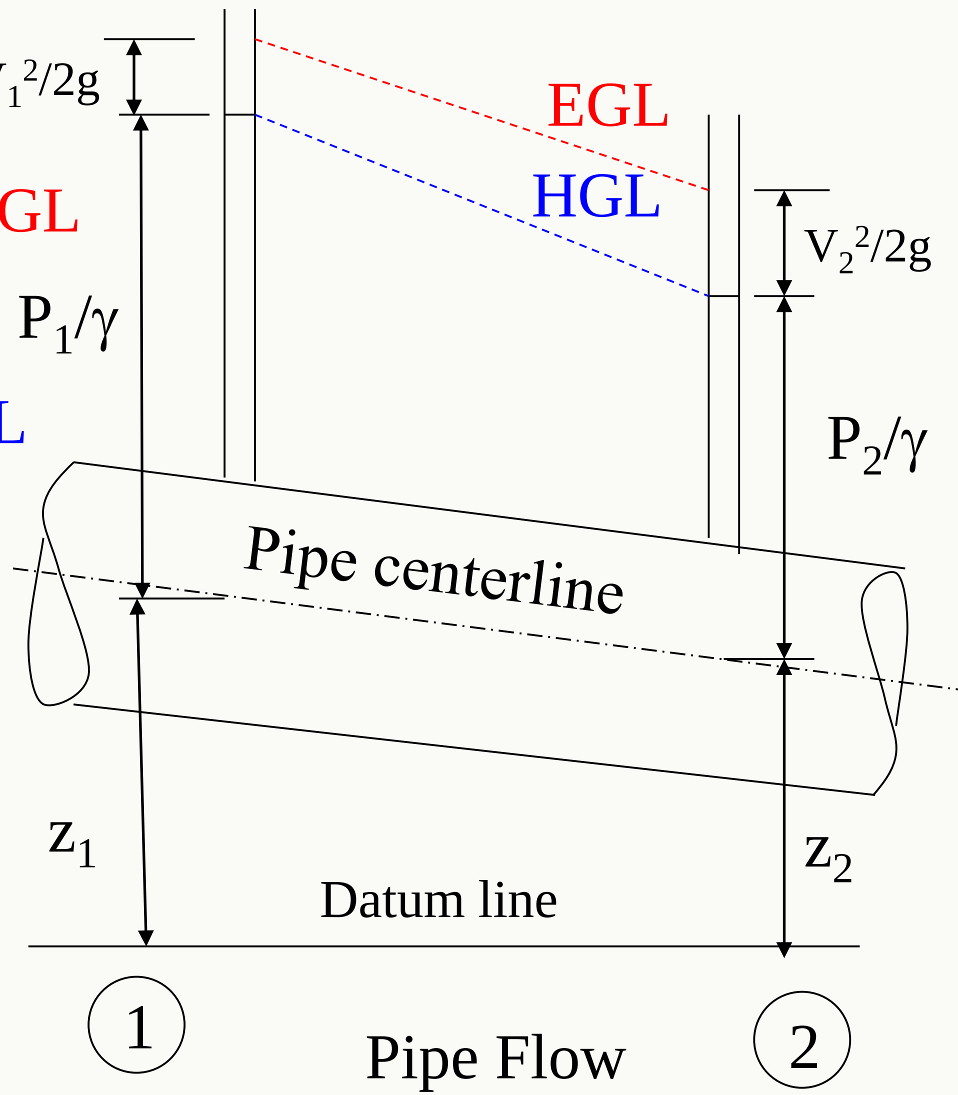
- Human-made systems: fresh-water aquaducts, irrigation, sewers, drainage ditches, etc.



Comparison of OCF and Pipe Flow



Open-Channel Flow



Pipe Flow

Comparison of OCF and Pipe Flow

1) OCF must have a free surface

2) A free surface is subject to atmospheric pressure

3) The driving force is mainly the component of gravity along the flow direction.

4) HGL is coincident with the free surface.

5) Flow area is determined by the geometry of the channel plus the level of free surface, which is likely to change along the flow direction and with as well as time.

1) No free surface in pipe flow

2) No direct atmospheric pressure, hydraulic pressure only.

3) The driving force is mainly the pressure force along the flow direction.

4) HGL is (usually) above the conduit

5) Flow area is fixed by the pipe dimensions The cross section of a pipe is usually circular..

Comparision of OCF and Pipe Flow

6) The cross section may be of any from circular to irregular forms of natural streams, which may change along the flow direction and as well as with time.

7) Relative roughness changes with the level of free surface

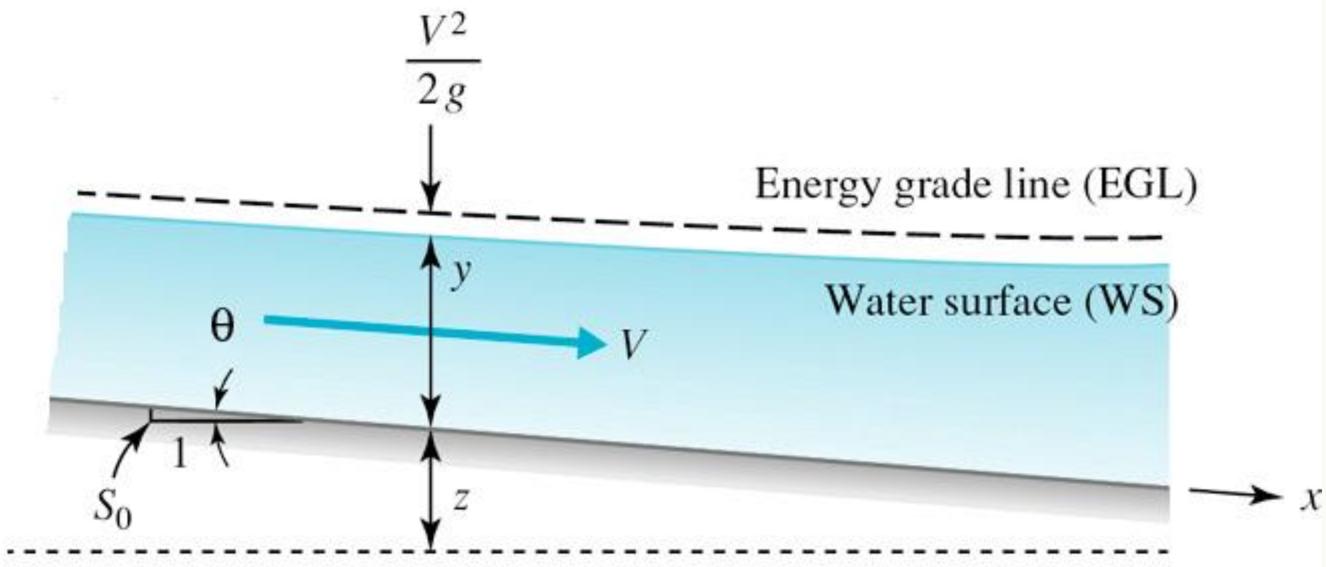
8) The depth of flow, discharge and the slopes of channel bottom and of the free surface are interdependent.

6) The cross section of a pipe is usually circular

7) The relative roughness is a fixed quantity.

8) No such dependence.

Energy grade line and hydraulic grade line



Kinds of Open Channel

- Canal
- Flume
- Chute
- Drop
- Culvert
- Open-Flow Tunnel

Kinds of Open Channel

- CANAL is usually a long and mild-sloped channel built in the ground.

Kinds of Open Channel

- FLUME is a channel usually supported on or above the surface of the ground to carry water across a depression.

Kinds of Open Channel

- CHUTE is a channel having steep slopes.

Kinds of Open Channel

- DROP is similar to a chute, but the change in elevation is affected in a short distance.

Kinds of Open Channel

- CULVERT is a covered channel flowing partly full, which is installed to drain water through highway and railroad embankments.

Kinds of Open Channel

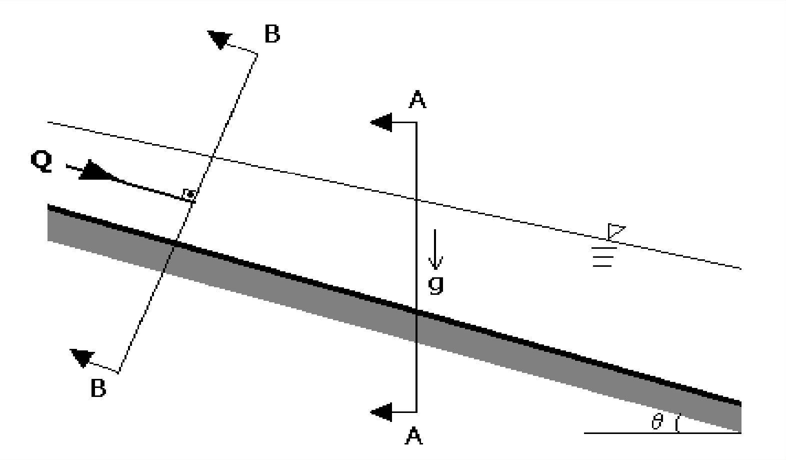
- OPEN-FLOW TUNNEL is a comparatively long covered channel used to carry water through a hill or any obstruction on the ground.

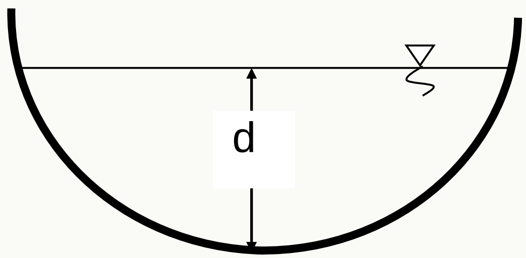
Channel Geometry

- A channel built with unvarying cross section and constant bottom slope is called a **PRISMATIC CHANNEL.**

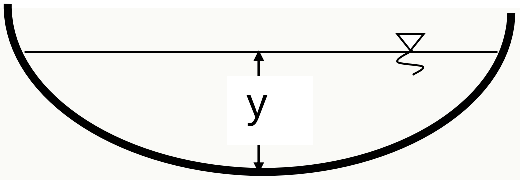
- Otherwise, the channel is **NONPRISMATIC.**

- **THE CHANNEL SECTION** is the cross section of a channel taken normal to the direction of the flow.



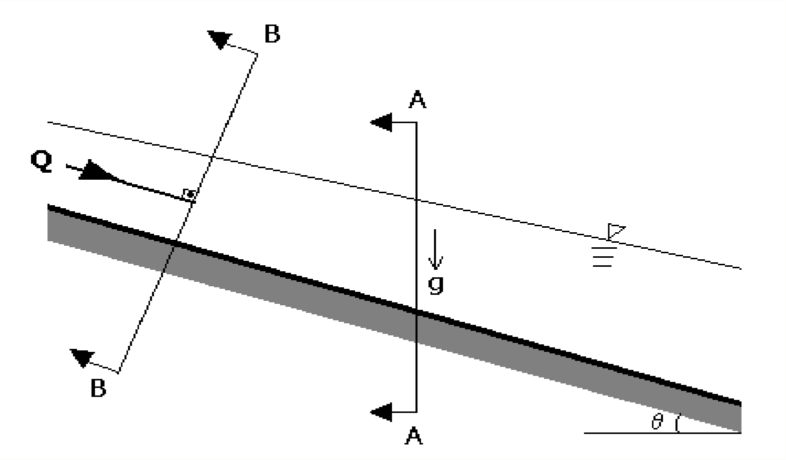


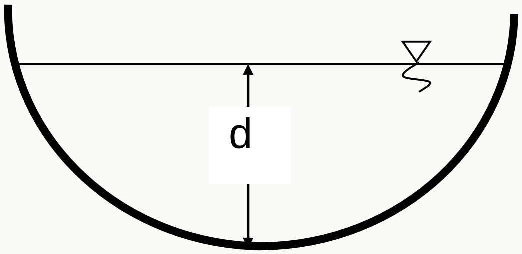
The channel section (B-B)



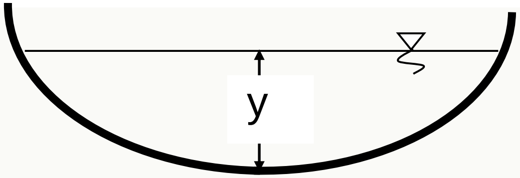
The vertical channel section (A-A)

- **THE VERTICAL CHANNEL SECTION** is the vertical section passing through the lowest or bottom point of the channel section.





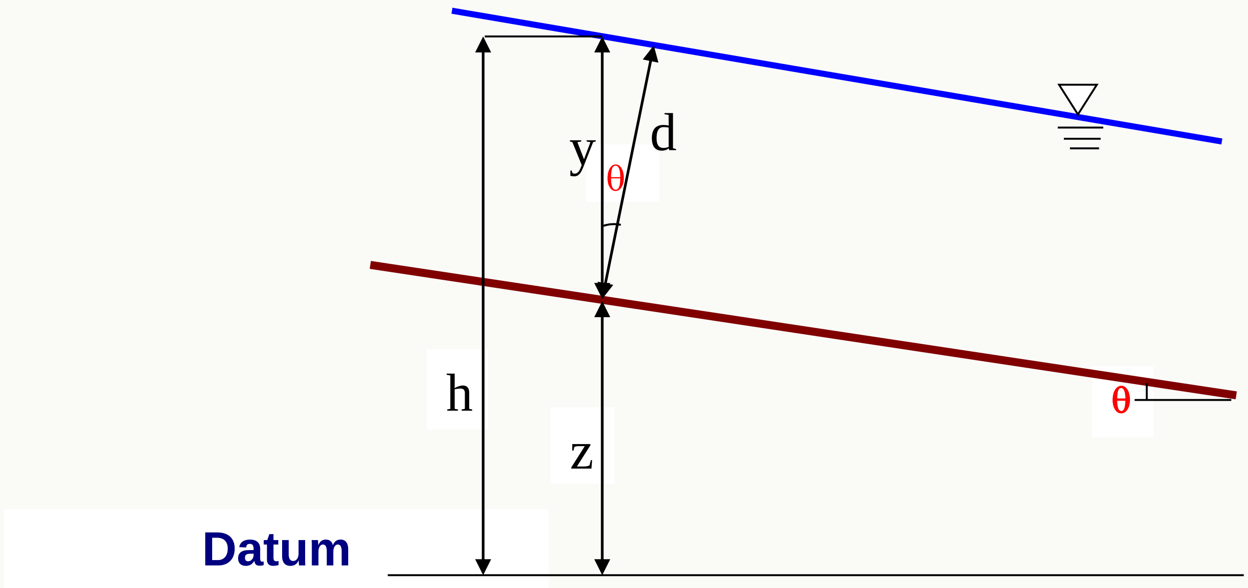
The channel section (B-B)



The vertical channel section (A-A)

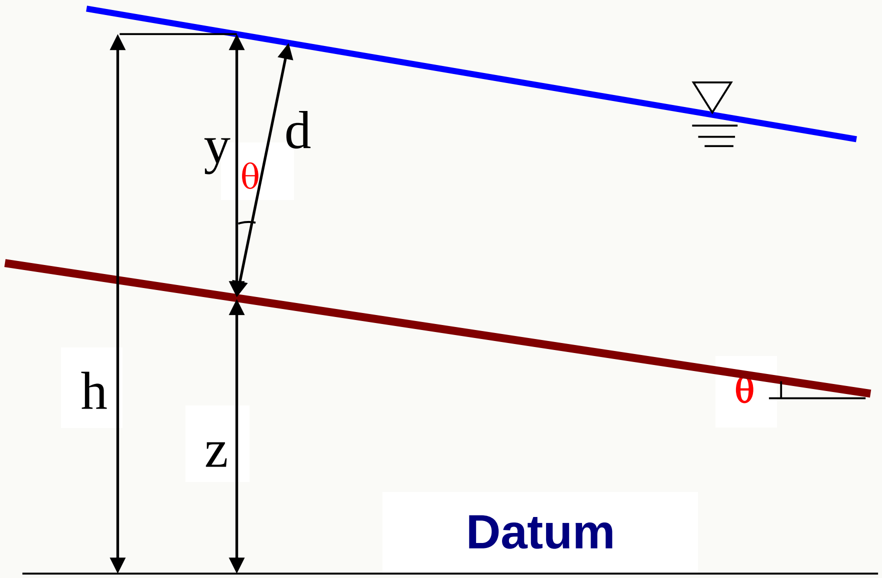
Geometric Elements of Channel Section

- **THE DEPTH OF FLOW, y,** is the vertical distance of the lowest point of a channel section from the free surface.



Geometric Elements of Channel Section

- **THE DEPTH OF FLOW SECTION**, d, is the depth of flow normal to the direction of flow.



θ is the channel bottom slope

$d = y\cos\theta$.

For mild-sloped channels $y \approx d$.

Geometric Elements of Channel Section

- **THE TOP WIDTH, T,**

is the width of the channel section at the free surface.

- **THE WATER AREA, A,**

is the cross-sectional area of the flow normal to the direction of flow.

- **THE WETTED PERIMETER, P,**

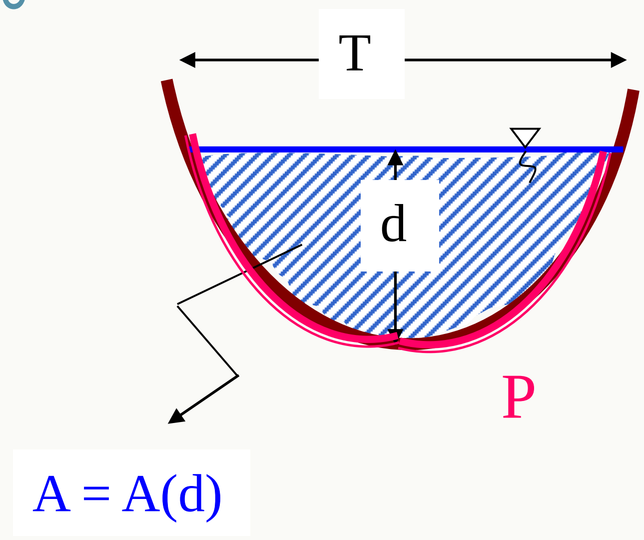
is the length of the line of intersection of the channel wetted surface with a cross-sectional plane normal to the direction of flow.

- **THE HYDRAULIC RADIUS, R = A/P,**

is the ratio of the water area to its wetted perimeter.

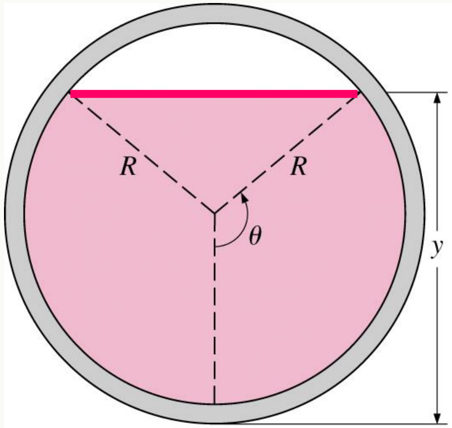
- **THE HYDRAULIC DEPTH, D = A/T,**

is the ratio of the water area to the top width.



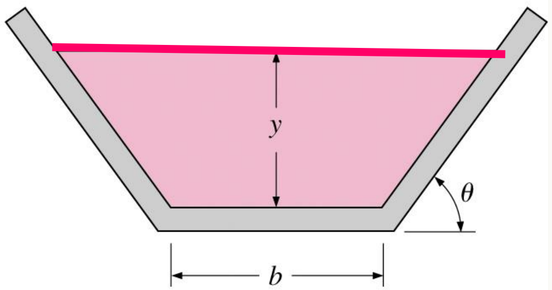
Channel Geometry

- The wetted perimeter ***does not*** include the free surface.
- Examples of R for common geometries shown in Figure at the left.



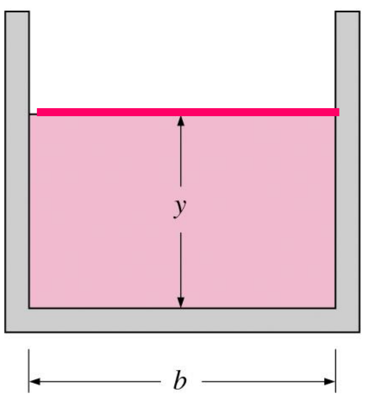
$$A_c = R^2(\theta - \sin\theta\cos\theta)$$
$$p = 2R\theta$$
$$R_h = \frac{A_c}{p} = \frac{\theta - \sin\theta\cos\theta}{2\theta}R$$

(*a*) Circular channel (θ in rad)



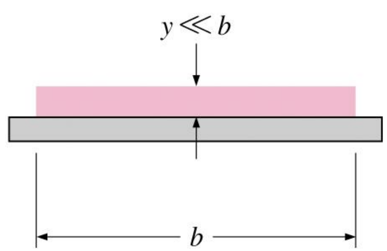
$$R_h = \frac{A_c}{p} = \frac{y(b + y/\tan\theta)}{b + 2y/\sin\theta}$$

(*b*) Trapezoidal channel



$$R_h = \frac{A_c}{p} = \frac{yb}{b + 2y} = \frac{y}{1 + 2y/b}$$

(*c*) Rectangular channel

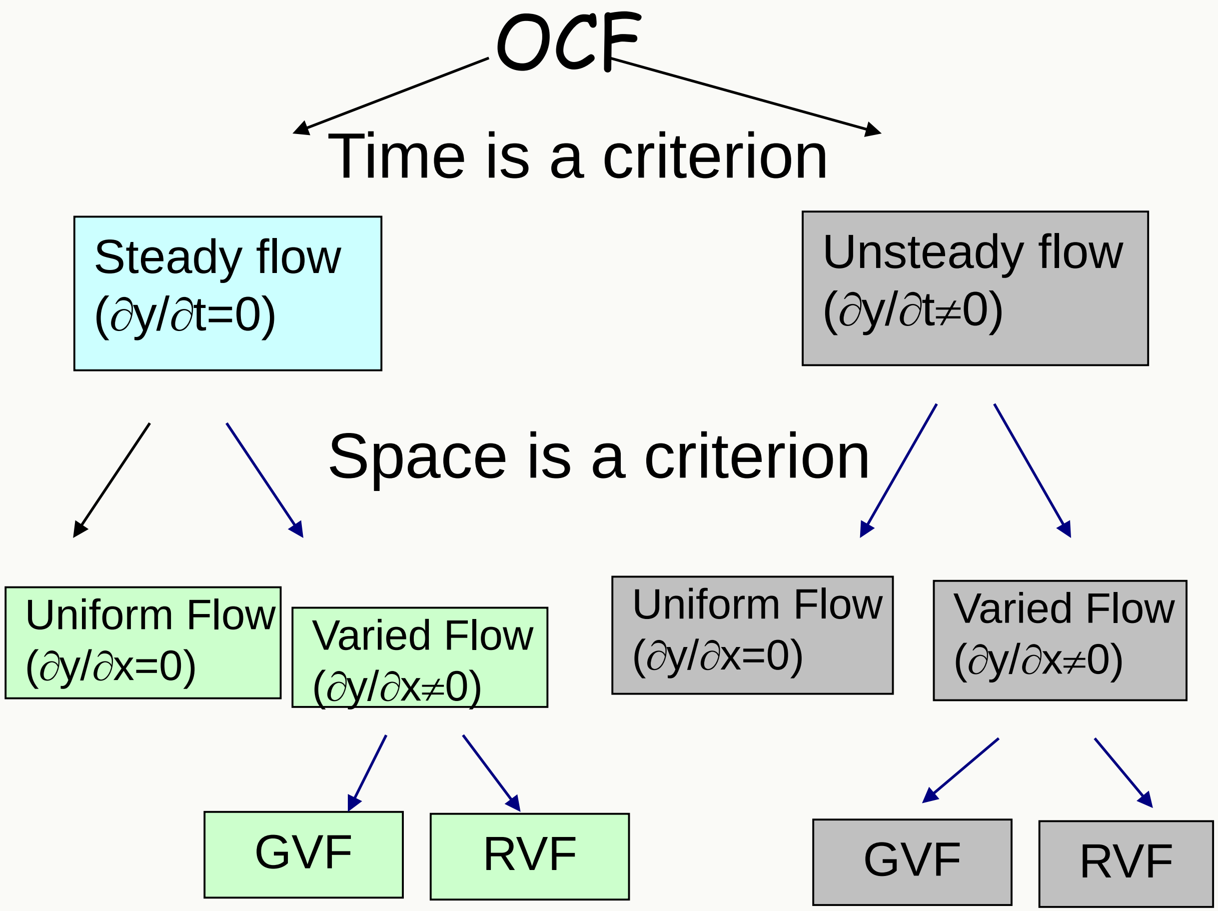


$$R_h = \frac{A_c}{p} = \frac{yb}{b + 2y} \cong \frac{yb}{b} \cong y$$

(*d*) Liquid film of thickness y

Types of Flow

- Criterion: Change in flow depth with respect to time and space



Types of Flow

- Criterion: Change in discharge with respect to time and space

OCF

Time is a criterion

Steady flow ($\partial Q/\partial t=0$)

Unsteady flow ($\partial Q/\partial t\neq 0$)

Space is a criterion

Continuous Flow ($\partial Q/\partial x=0$)

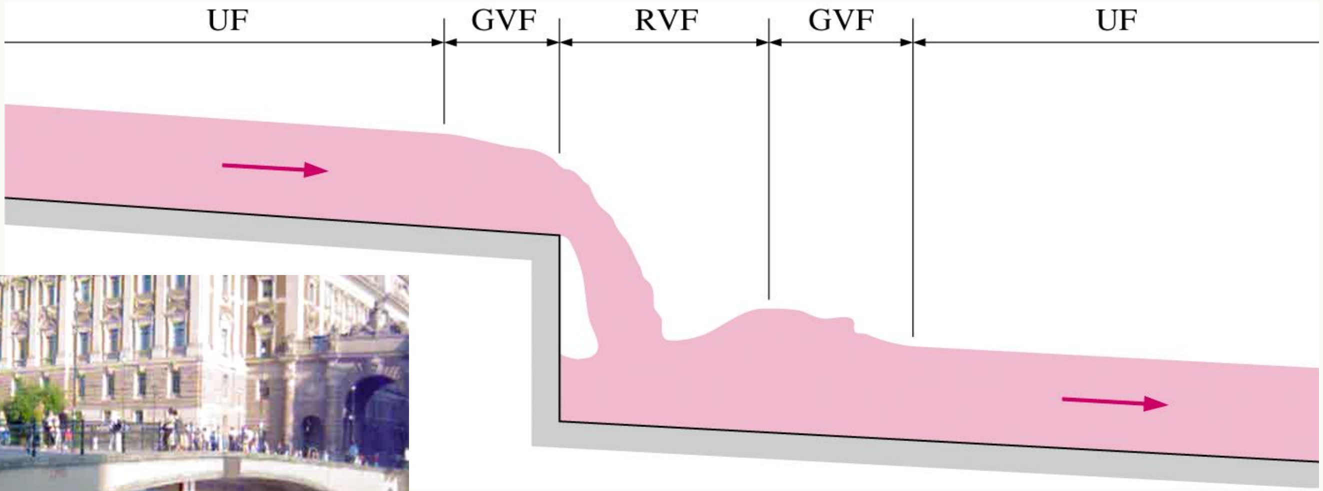
Spatially-Varied Flow ($\partial Q/\partial x\neq 0$)

Continuous Flow ($\partial Q/\partial x=0$)

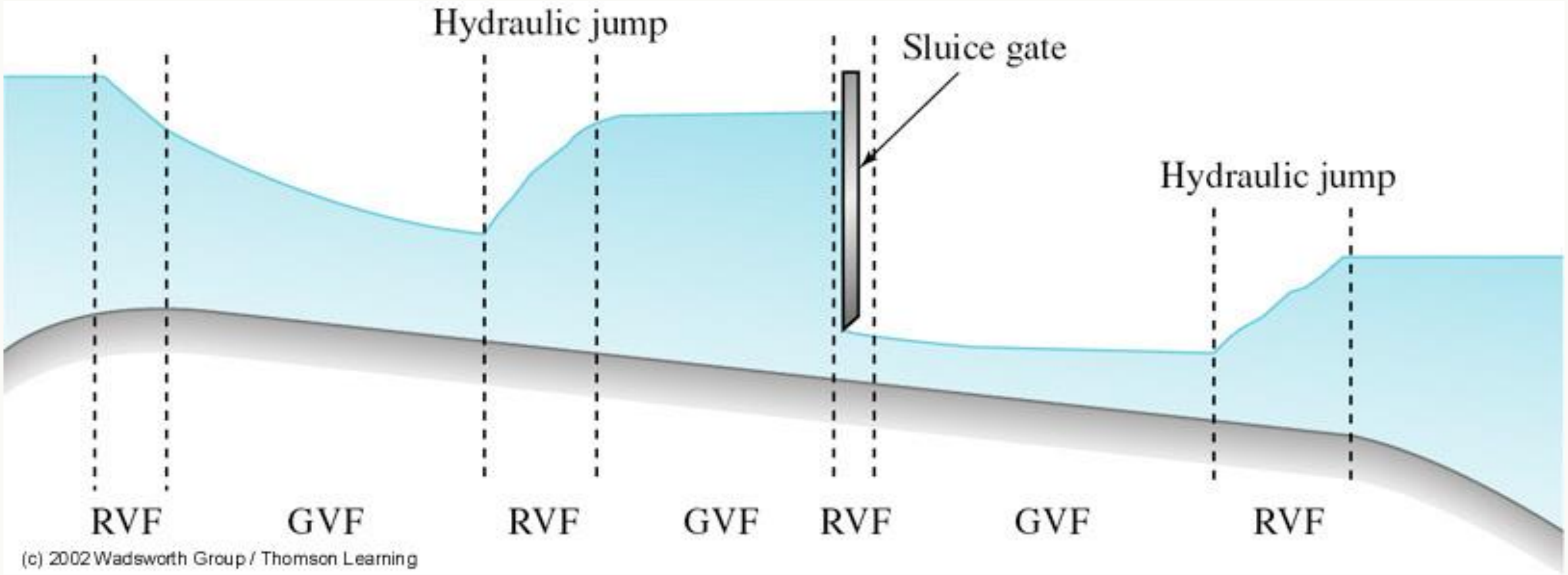
Spatially-Varied Flow ($\partial Q/\partial x\neq 0$)

Classification of Open-Channel Flows

- Obstructions cause the flow depth to vary.
- Rapidly varied flow (RVF) occurs over a short distance near the obstacle.
- Gradually varied flow (GVF) occurs over larger distances and usually connects UF and RVF.



Steady non-uniform flow in a channel.



State of Flow

- Effect of viscosity:

$$Re = \frac{VR}{\upsilon}$$

OCF

Laminar OCF, Re < 500

Transitional OCF, 500 < Re < 1000

Turbulent OCF, Re > 1000

Note That R in Reynold number is Hydraulic Radius

Effect of Gravity

- In open-channel flow the driving force, that is the force causing the motion is the component of gravity along the channel bottom. Therefore, it is clear that, the effect of gravity is very important in open-channel flow.
- In an open-channel flow Froude number is defined as:

$$F_r = \frac{\text{Inertia Force}}{\text{Gravity Force}}, \quad \text{and} \quad F_r^2 == \frac{V^2}{gD} \quad \text{or} \quad F_r = \frac{V}{\sqrt{gD}}$$

- In an open-channel flow, there are three types of flow depending on the value of Froude number:

$F_r>1 \longrightarrow$ Supercritical Flow

$Fr=1 \longrightarrow$ Critical Flow

$F_r<1 \longrightarrow$ Subcritical Flow

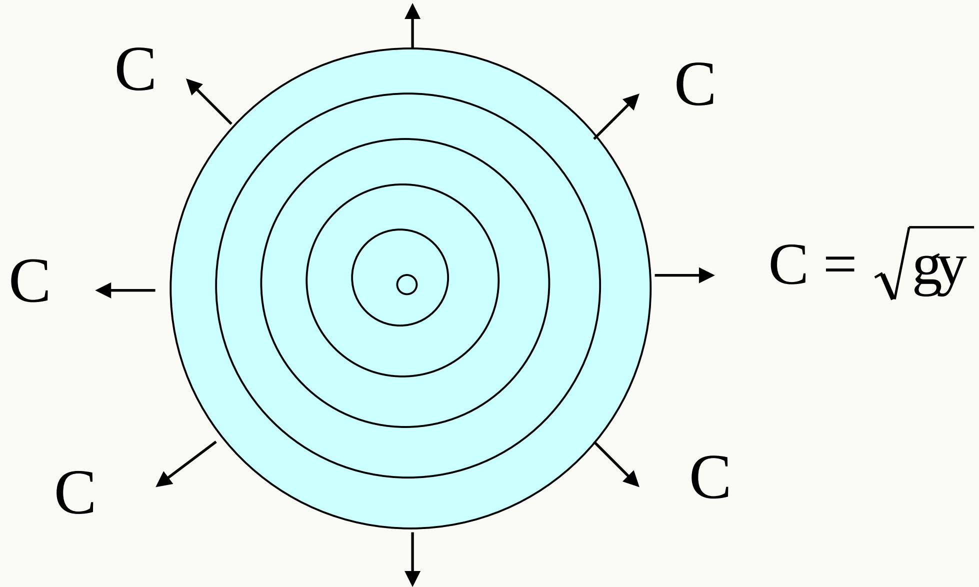
In wave mechanics, the speed of propagation of a small amplitude wave is called the celerity, C.

The celerity, C is equal to: $C = \sqrt{gy}$

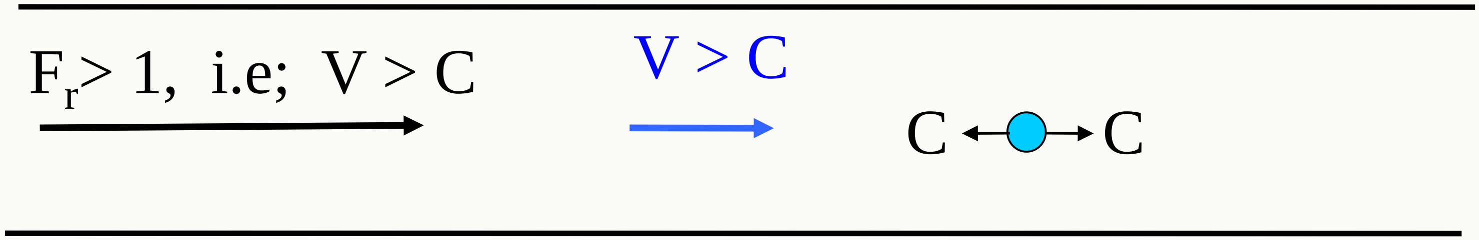
For a rectangular channel, the hydraulic depth, D=y. Therefore, Froude number becomes:

$$F_r = \frac{V}{\sqrt{gy}} = \frac{V}{C}$$

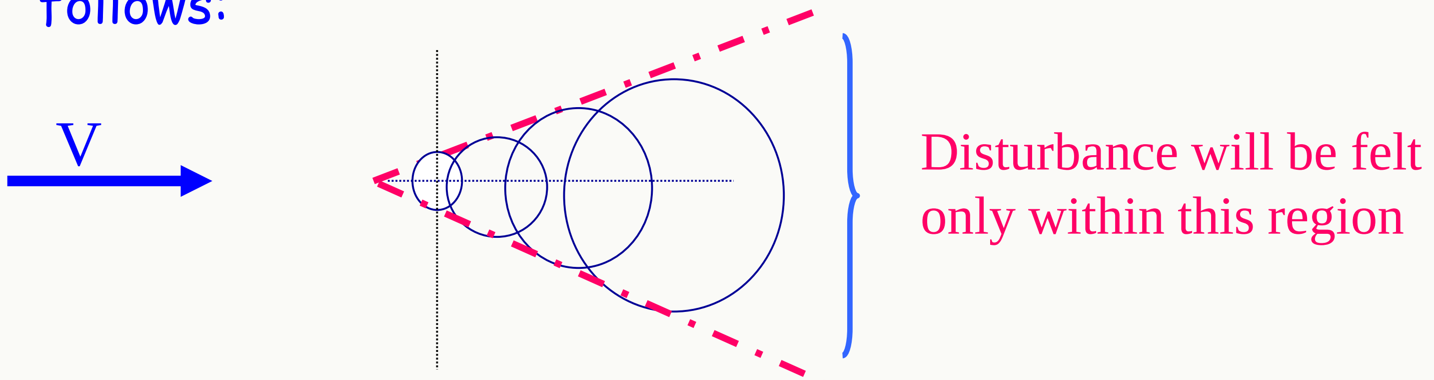
If we disturb water, which is not moving, a disturbance wave occur, and it propagates in all directions with a celerity, C, as:



- Now let us consider propagation of a small amplitude wave in a supercritical open channel flow:



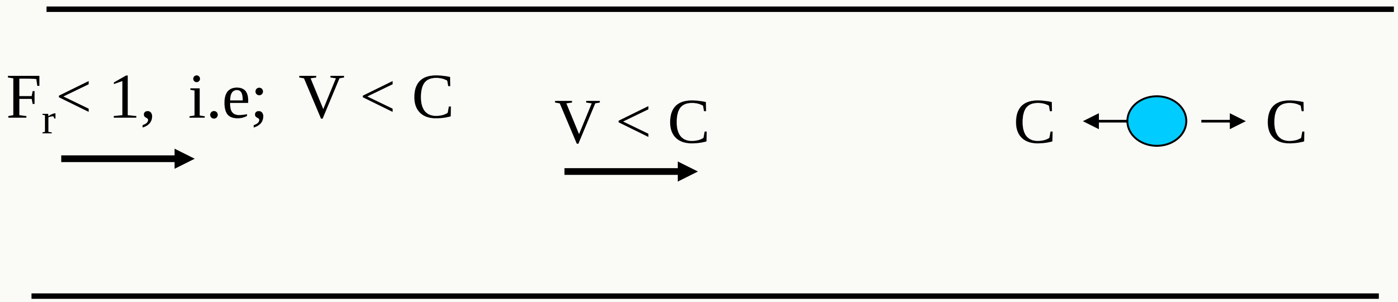
- Since $V > C$, it CANNOT propagate upstream it can propagate only towards downstream with a pattern as follows:



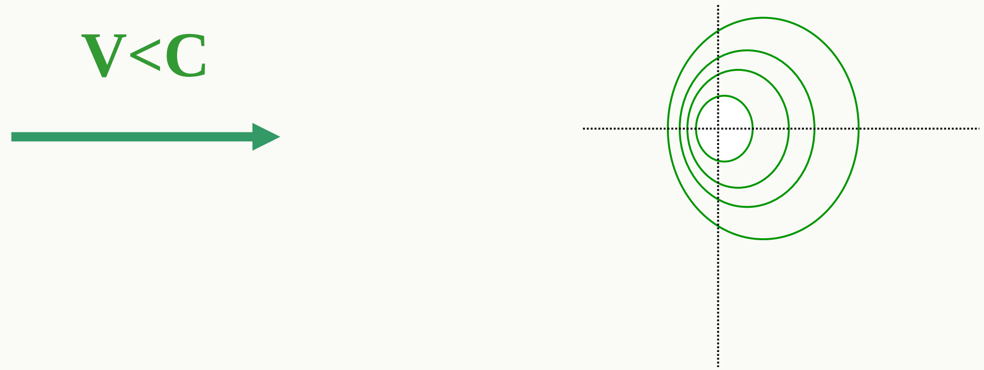
- This means the flow at upstream will not be affected.

  In other words, there is no hydraulic communication between upstream and downstream flow.

- Now let us consider propagation of a small amplitude wave in a subcritical open channel flow:

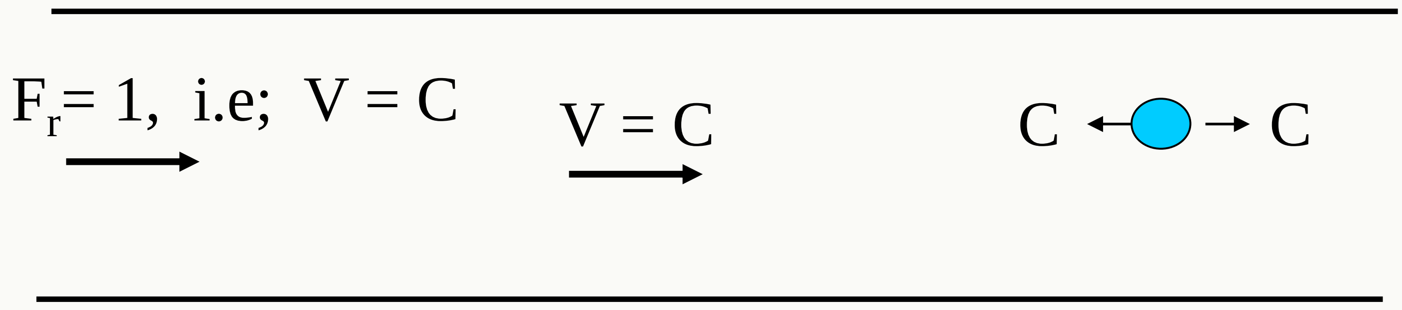


- Since V < C, it CAN propagate both upstream and downstream with a pattern as follows:

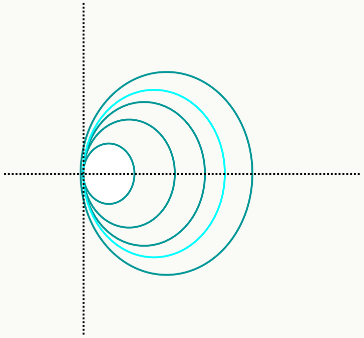


- This means the flow at upstream and downstream will both be affected.
- In other words, there is hydraulic communication between upstream and downstream flow.

Now let us consider propagation of a small amplitude wave in a critical open channel flow:



Since V = C, it can propagate only downstream with a pattern as follows:



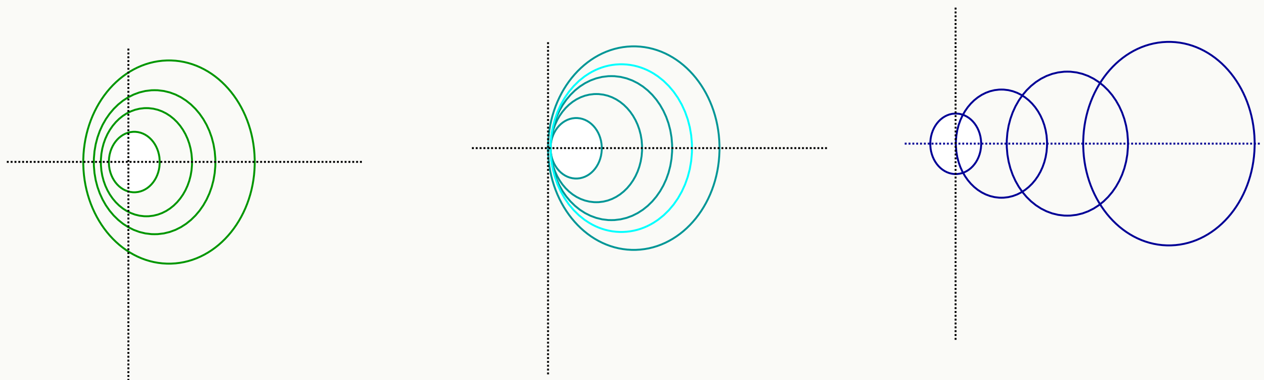
This means the flow at downstream will be affected.

- In an open-channel flow, a control is any local feature which fixes a depth-discharge relationship.

State of Flow

- Effect of gravity:

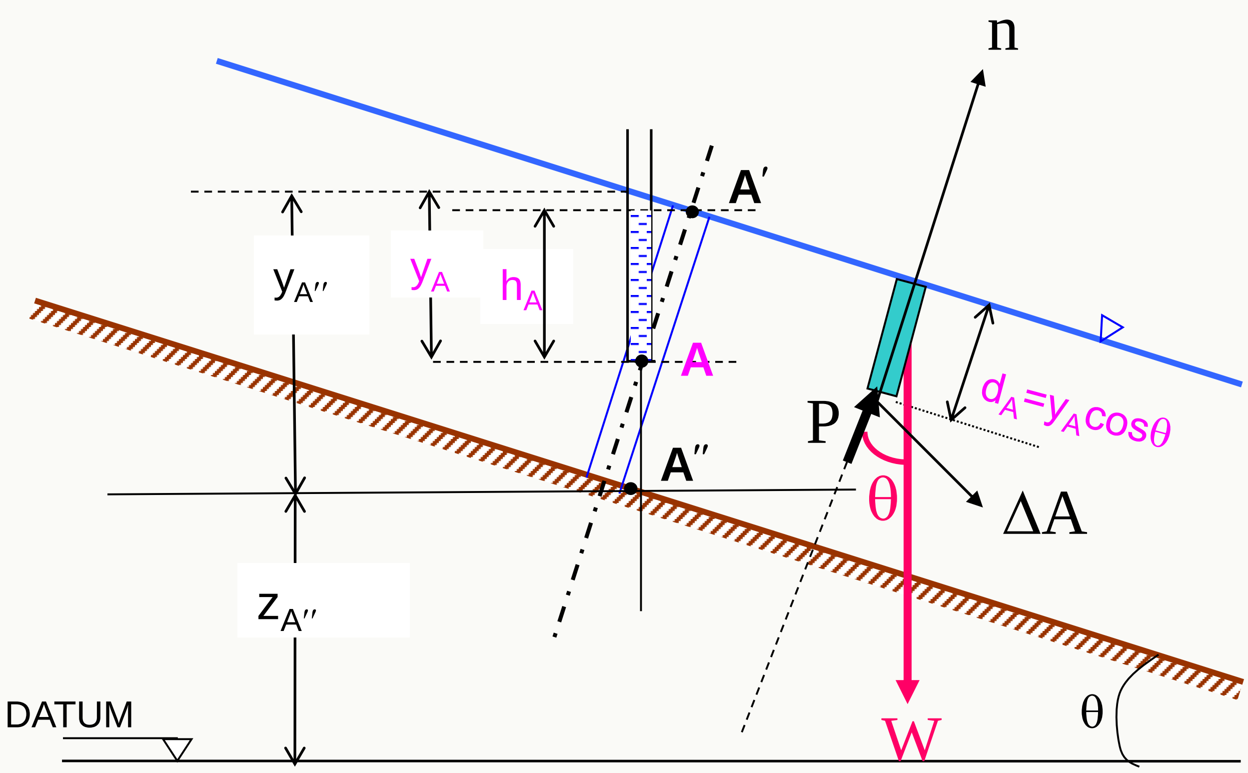
$$Fr = \frac{V}{\sqrt{gD}}$$



$V < \sqrt{gD}$ $\qquad$ $V = \sqrt{gD}$ $\qquad$ $V > \sqrt{gD}$

D in Froude Number is Hydraulic Depth

Pressure Distribution



Let us consider a rectangular element, which has a height of d_A, and base area of ΔA.

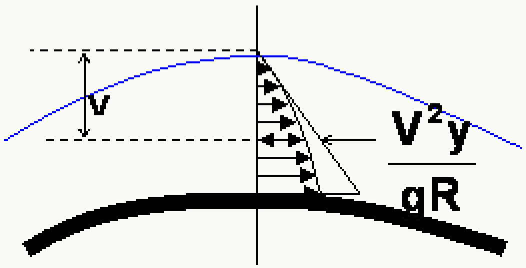
$\Sigma F_n=0 \rightarrow P\Delta A - W\cos\theta = 0$

$P\Delta A = W\cos\theta$, $W=\gamma V=\gamma \Delta A . d_A \Rightarrow$ $P=\gamma d_A \cos\theta = \gamma y_A \cos^2\theta$

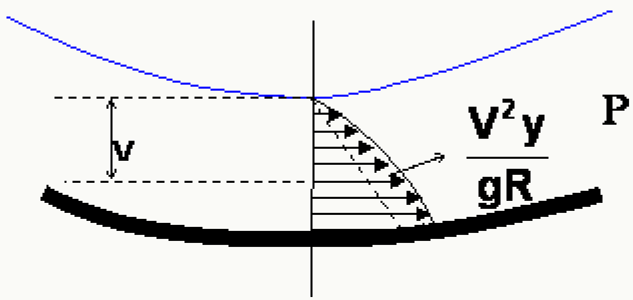
If θ is small $\Rightarrow$ $P=\gamma\, y$ $\Rightarrow$ **Hydrostatic Pressure Distribution**

Pressure Distribution

Convex and concave geometries



$$P = \gamma y\left(1 - \frac{V^2}{gR}\right)$$ Convex Flow



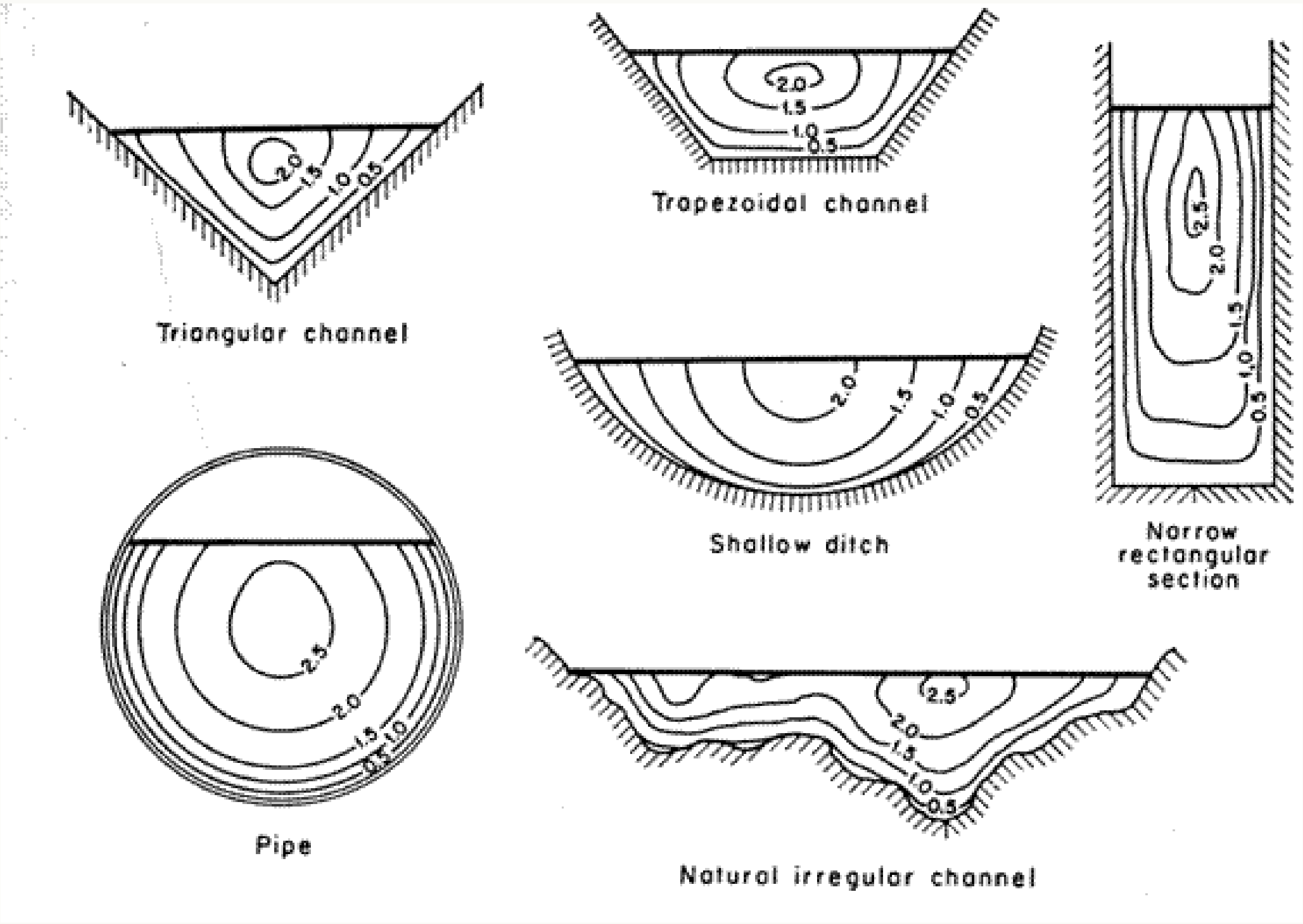
$$P = \gamma y\left(1 + \frac{V^2}{gR}\right)$$ Concave Flow

Velocity Distribution

The velocity distribution in an open-channel flow is quite nonuniform because of :

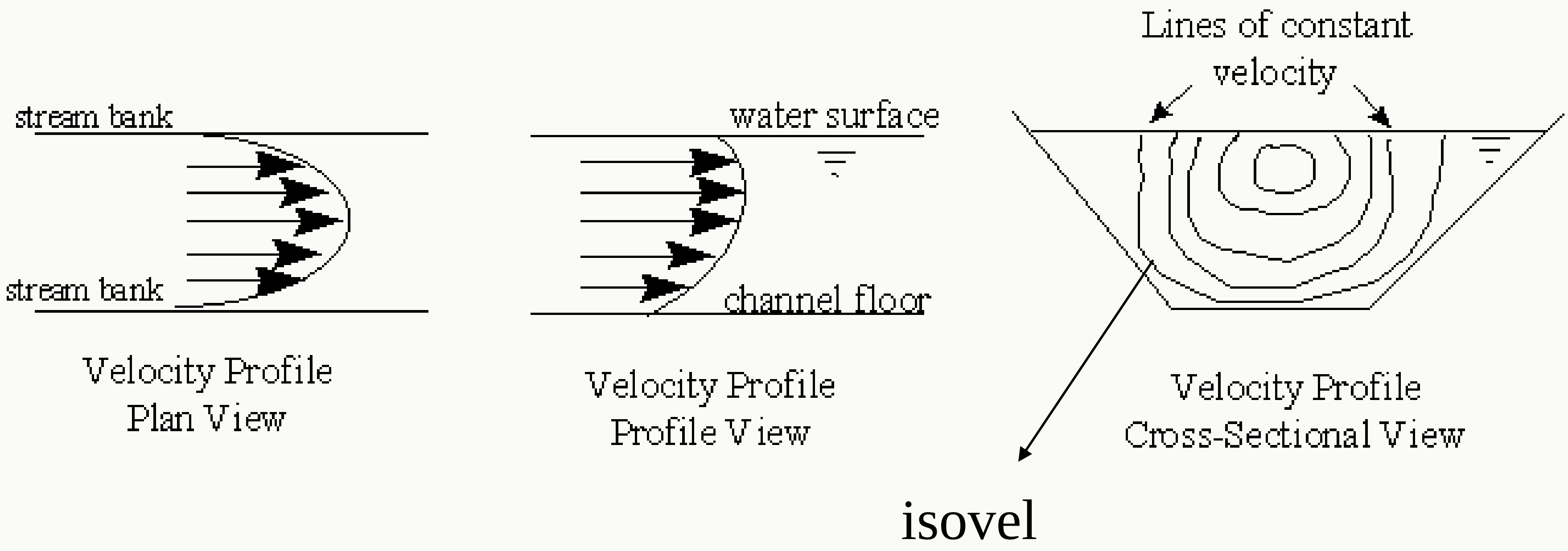
•Nonuniform shear stress along the wetted perimeter,

•Presence of free surface on which the shear stress is zero.

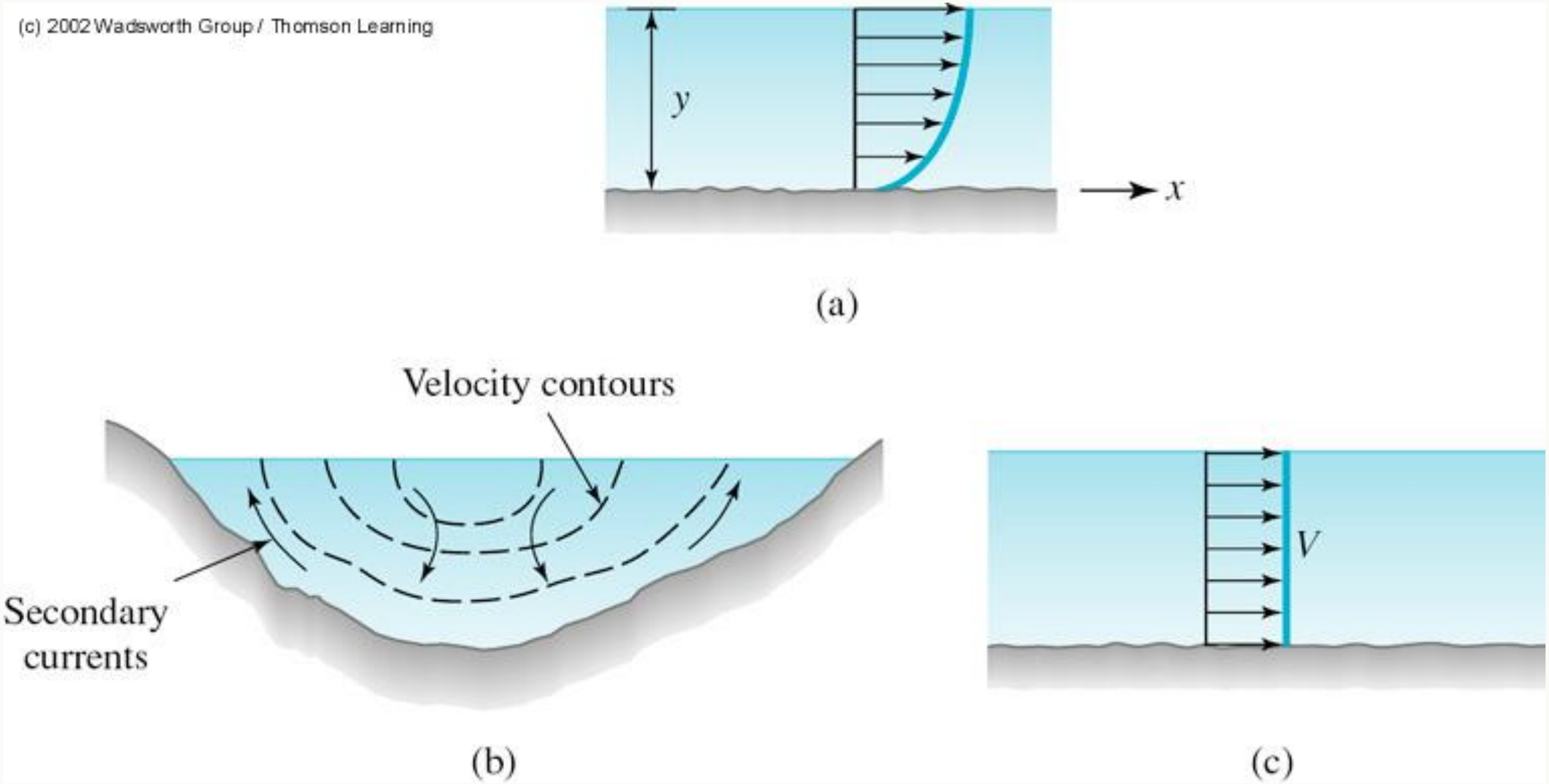


Velocity Profiles

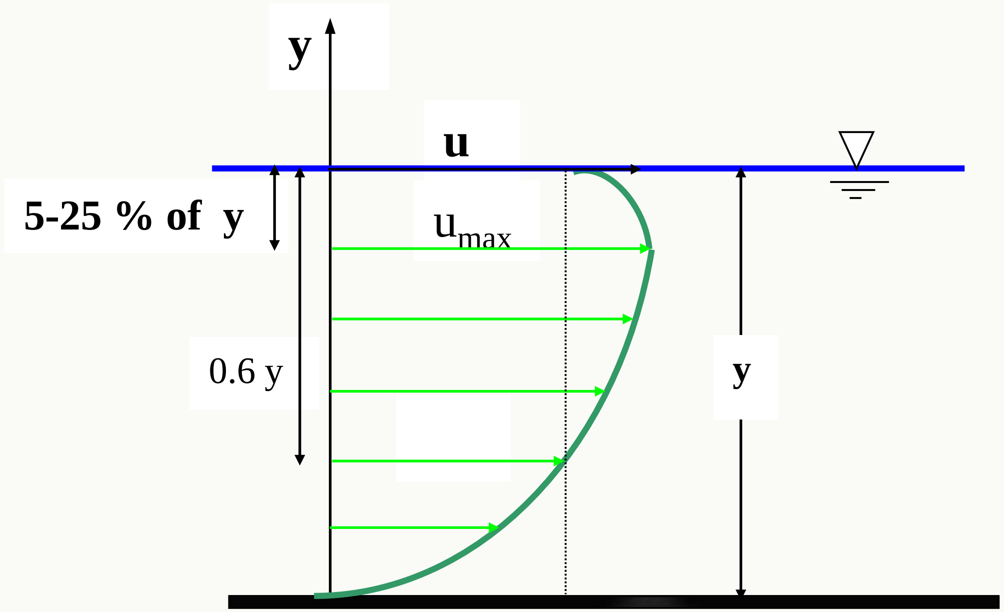
- In order to understand the velocity distribution, it is customary to plot the isovels, which are the equal velocity lines at a cross section.



- Velocity is zero on bottom and sides of channel due to no-slip condition
- the maximum velocity is usually below the free surface.
- It is usually three-dimensional flow.
- However, 1D flow approximation is usually made with good success for many practical problems.



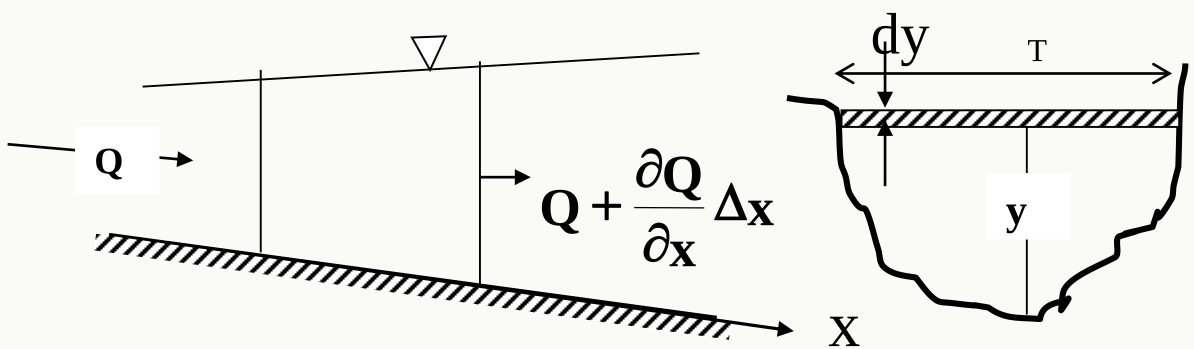
Velocity Distribution across a vertical line:



$$\alpha = \frac{\int_A u^3 dA}{u^3 A} \approx \frac{\sum u^3 \Delta A}{u^3 A}, \qquad \beta = \frac{\int_A u^2 dA}{u^2 A} \approx \frac{\sum u^2 \Delta A}{u^2 A}$$

Equation Of Continuity

$$\int_{cs} \rho \vec{u} \cdot d\vec{A} = -\frac{\partial}{\partial t} \int_{cv} \rho \, d\forall$$



- i) Unsteady Flow

$$\rho \int_{cs} \vec{u} \cdot d\vec{A} = -\rho \frac{\partial}{\partial t} \int_{cv} d\forall \quad \Rightarrow \int_{cs} \vec{u} \cdot d\vec{A} = -\frac{\partial}{\partial t} \forall_{cv}$$

$$-Q + \left(Q + \frac{\partial Q}{\partial x} \Delta x\right) = -\frac{\partial}{\partial t}(A \cdot \Delta x) \quad \Rightarrow \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

$$A = A(y),\, y = y(x, t)$$

$$\frac{\partial A}{\partial t} = \frac{\partial A}{\partial y}\frac{\partial y}{\partial t} \qquad dA = T dy \qquad \frac{dA}{dy} = T$$

$$\frac{\partial A}{\partial t} = T\frac{\partial y}{\partial t} \qquad \Rightarrow T\frac{\partial y}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

- ii) Steady Flow

$$\frac{\partial y}{\partial t} = 0 \quad \Rightarrow \frac{\partial Q}{\partial x} = 0$$

$$Q_1 = Q_2 \;\Rightarrow (U_{av} A)_1 = (U_{av} A)_2$$

Total Head at A Cross Section:

- The total head at a cross section is:

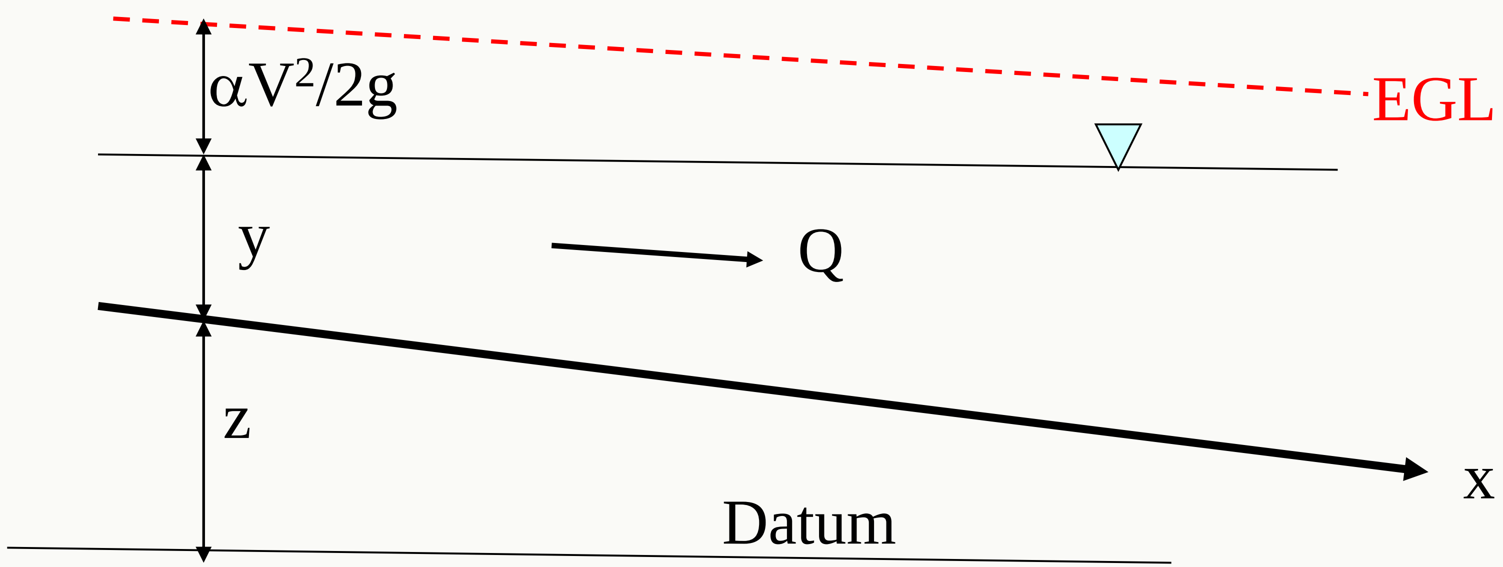
$$H = z + \frac{P}{\gamma} + \alpha \frac{V_{av}^2}{2g}$$

- Where H=total head

  Z=elevation of the channel bottom

  $P/\gamma = y$ = the vertical depth of flow (provided that pressure distribution is hydrostatic)

  $V^2/2g$= velocity head



Energy relationships

$$z_1 + \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + h_\ell$$

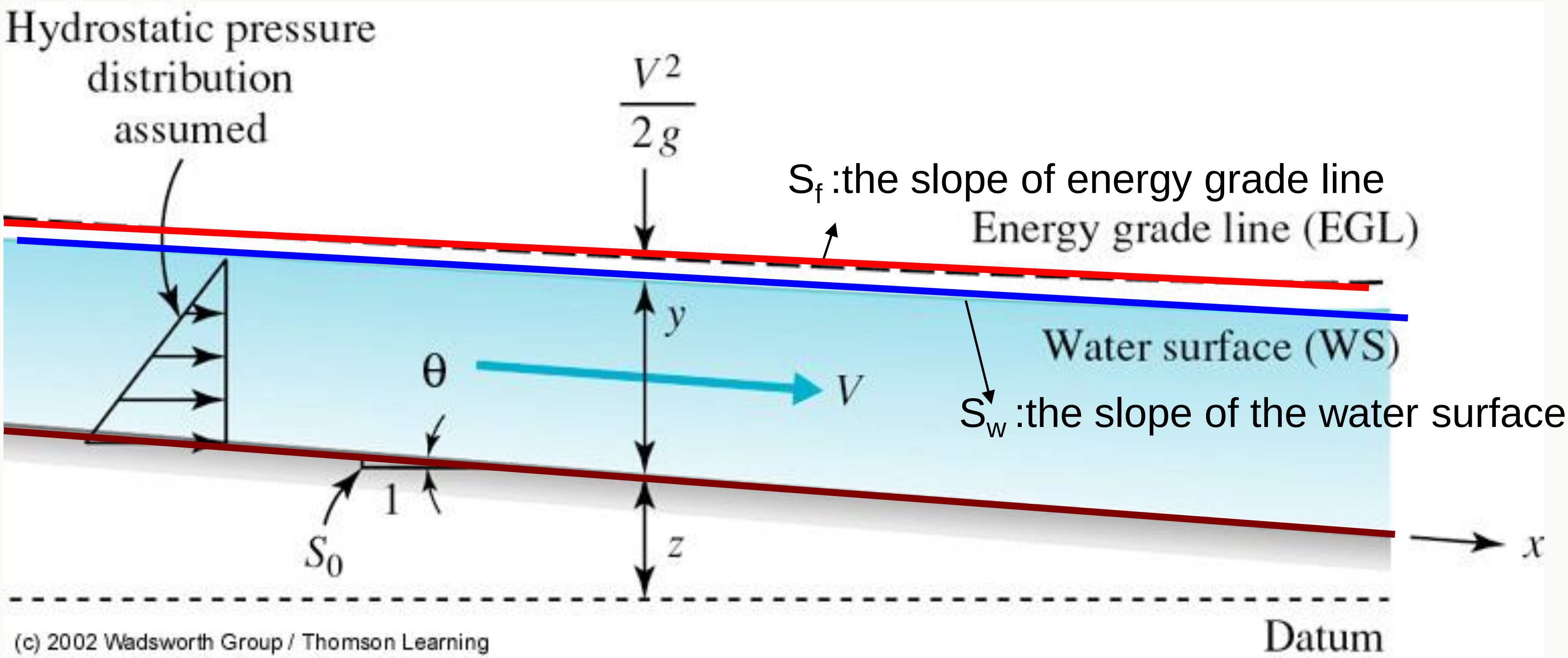
$$z_1 + y_1 + \frac{V_1^2}{2g} = z_2 + y_2 + \frac{V_2^2}{2g} + h_\ell$$

Turbulent flow ($\alpha \cong 1$)

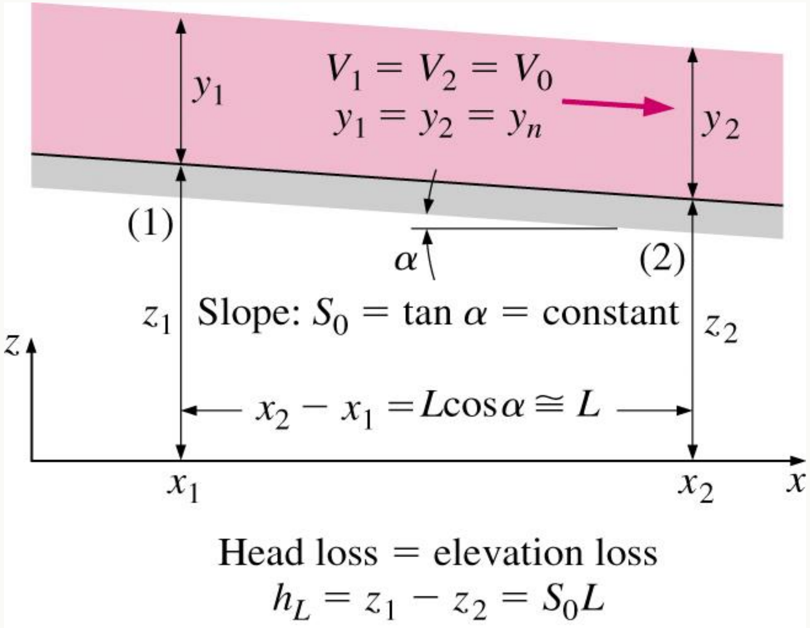
y - depth of flow

Energy Grade Line And Hydraulic Grade Line In Open Channel Flow

Sf :the slope of energy grade line
Sw :the slope of the water surface
So :the slope of the bottom



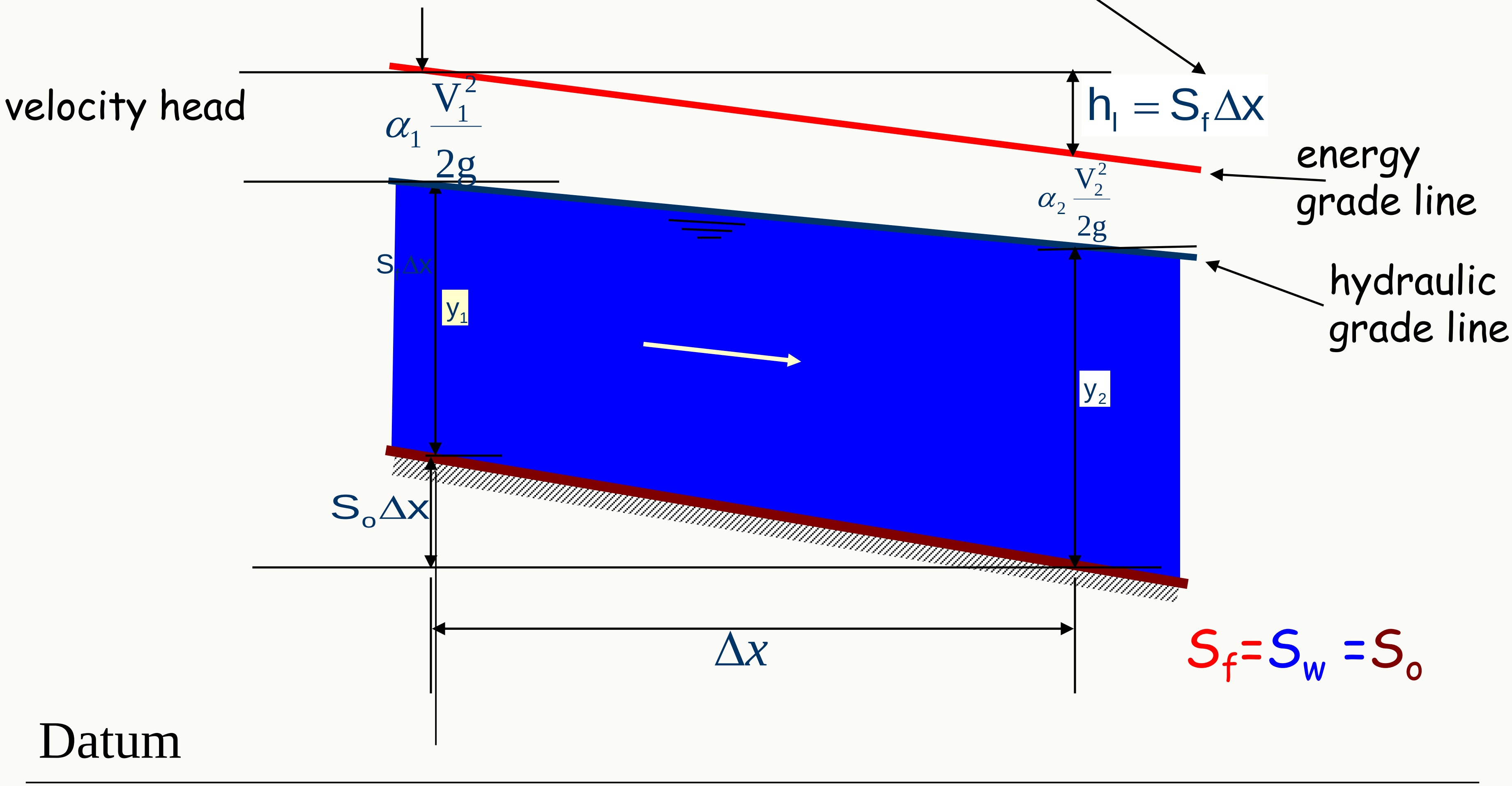
Uniform Flow in Channels



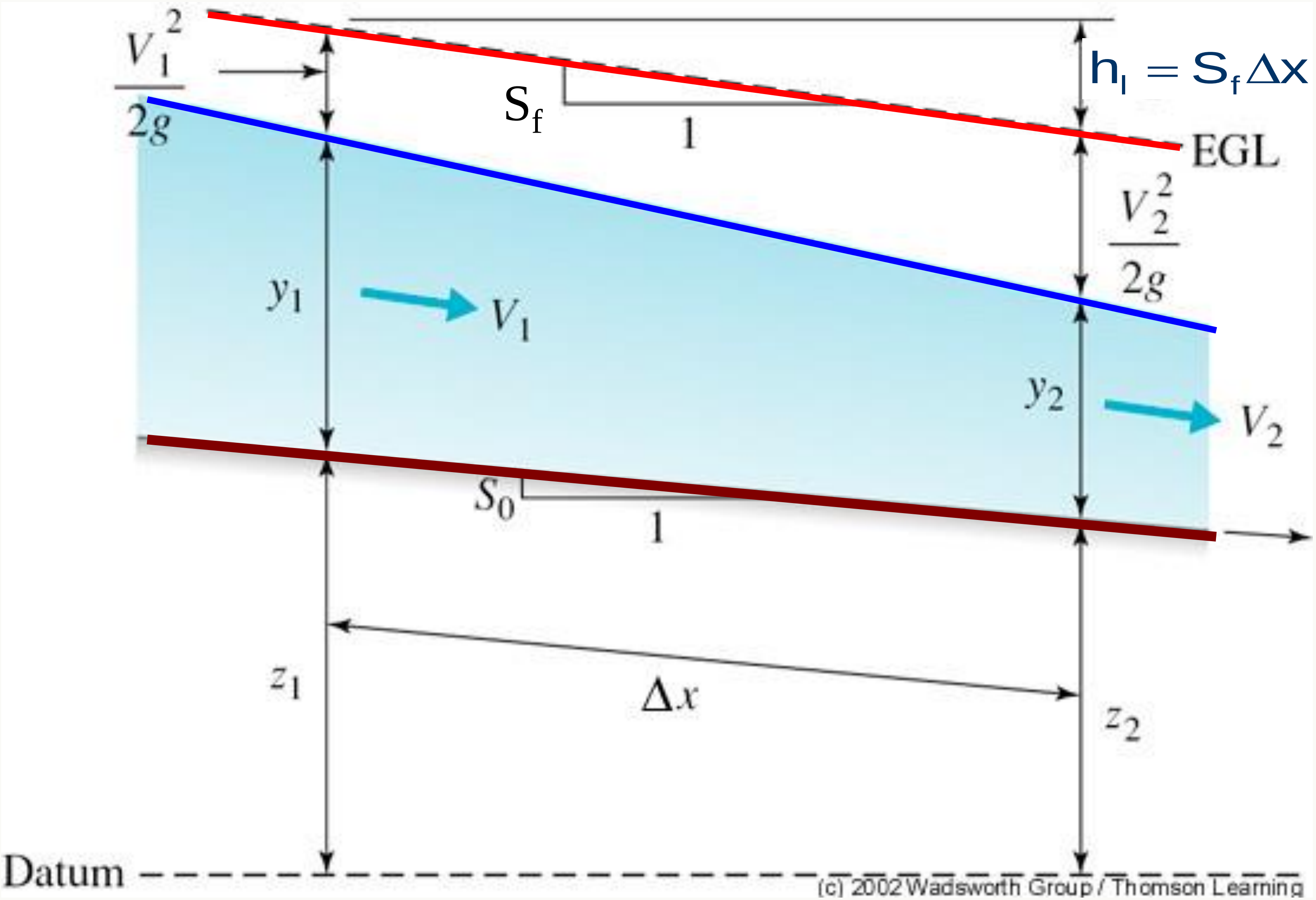
- Flow in open channels is classified as being *uniform* or *nonuniform*, depending upon the depth *y*.
- Depth in Uniform Flow is called **normal depth y_n**
- *Uniform depth* occurs when the flow depth (and thus the average flow velocity) remains constant
- Common in long straight runs
- Average flow velocity is called **uniform-flow velocity V_0**
- *Uniform depth* is maintained as long as the slope, cross-section, and surface roughness of the channel remain unchanged.
- During uniform flow, the terminal velocity reached, and the head loss equals the elevation drop

Uniform Flow in Channels

$$z_1 + y_1 + \frac{V_1^2}{2g} = z_2 + y_2 + \frac{V_2^2}{2g} + h_\ell$$



Non-uniform gradually varied flow. $S_f \neq S_w \neq S_o$



Steady-Uniform Flow: Force Balance

Shear force = $\tau_o P \Delta x$

Wetted perimeter = P

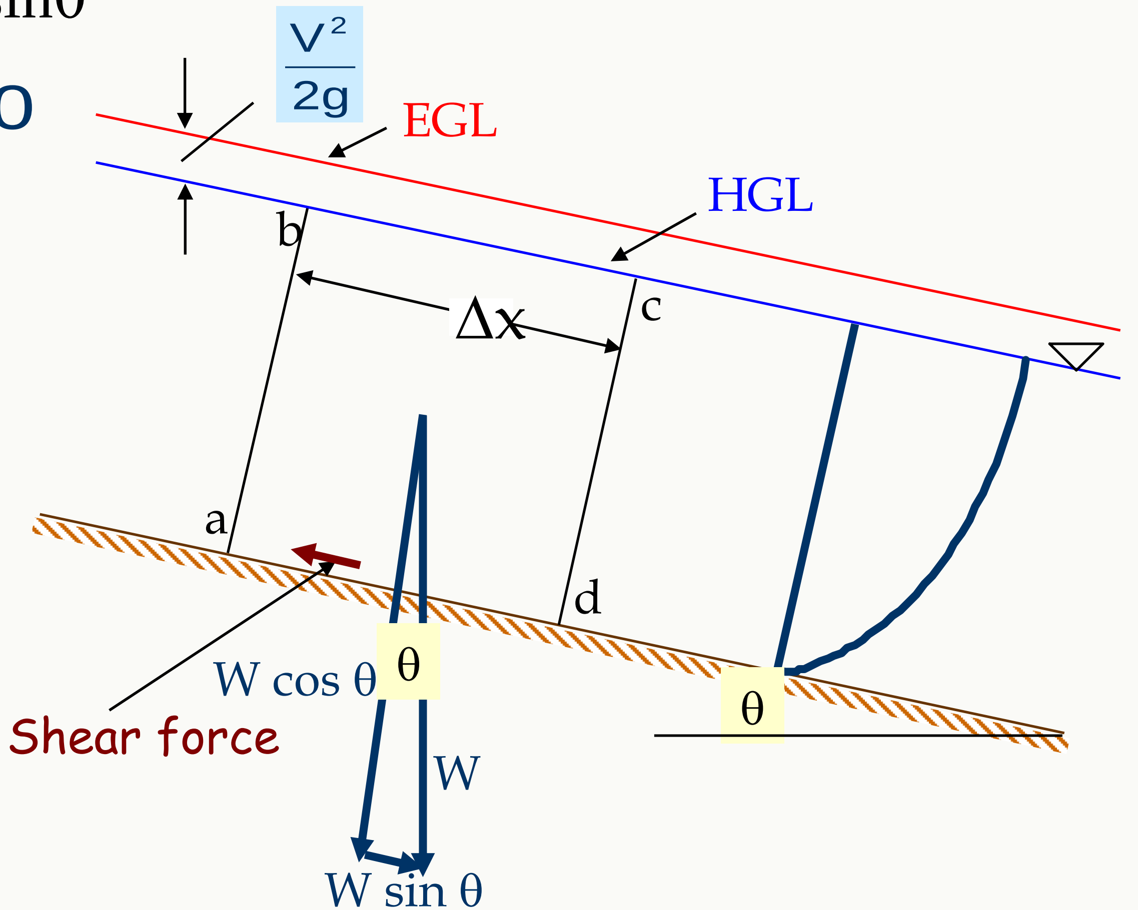
Gravitational force = $\gamma A \Delta x \sin\theta$

$$\gamma A \Delta x \sin\theta - \tau_o P \Delta x = 0$$

$$\tau_o = \gamma \frac{A}{P} \sin\theta$$

$$S_o = \sin\theta$$

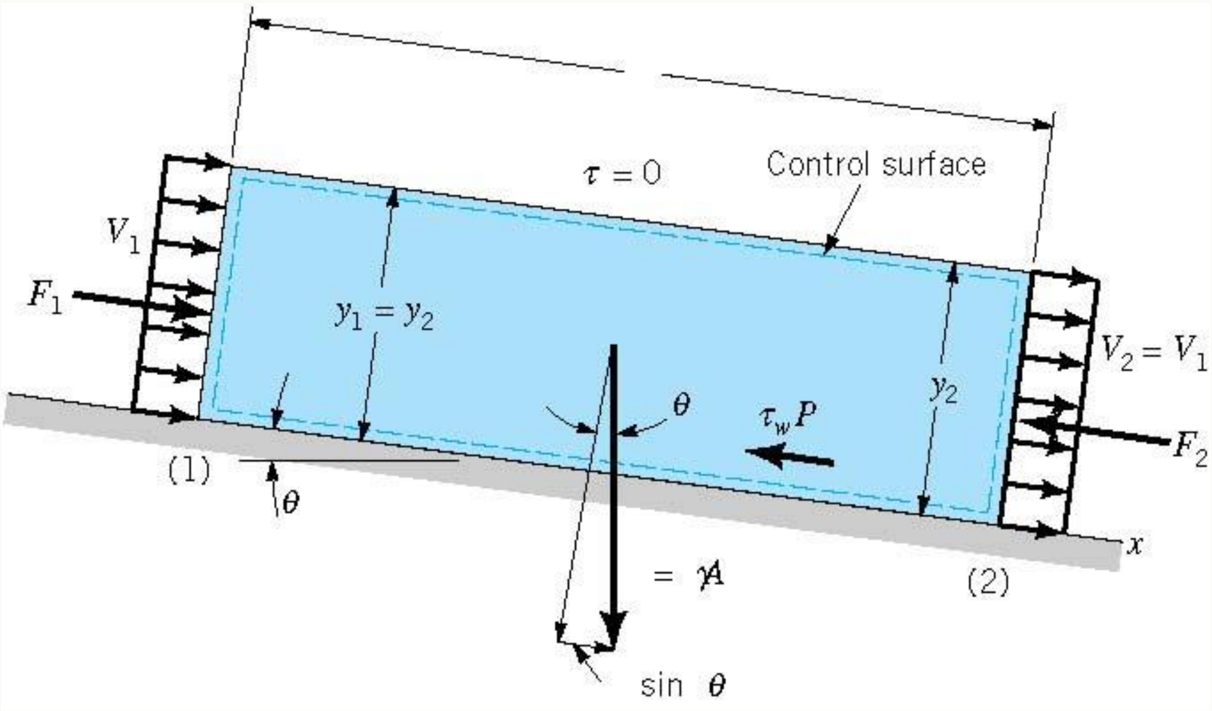
$$\tau_o = \gamma R S_o$$



Hydraulic radius $R = \frac{A}{P}$

Relationship between shear and velocity?

Steady-Uniform Flow: Force Balance



Relationship between shear and velocity?

- Resistance Equation

$$\tau_0 = kV^2$$

$$V = \sqrt{\frac{\gamma}{k}}\sqrt{RS_o}$$

- Wall-Shear Stress

$$\tau_0 = \gamma R S_o$$

$$V = C\sqrt{RS_o}$$

$$C = \sqrt{\frac{\gamma}{k}}$$

Manning Equation for Uniform Flow

$$V = \frac{1}{n} R^{2/3} S_o^{1/2}$$ (SI System)

Discharge: $Q = VA$

$$Q = \frac{1}{n} A R^{2/3} S_o^{1/2}$$

Chezy equation (1768)

- Introduced by the French engineer Antoine Chezy in 1768 while designing a canal for the water-supply system of Paris

$$V = C\sqrt{R_h S_f}$$

C = Chezy coefficient

$$60\frac{\sqrt{m}}{s} < C < 150\frac{\sqrt{m}}{s}$$

where 60 is for rough and 150 is for smooth

also a function of **R** (like *f* in Darcy-Weisbach)

Darcy-Weisbach equation (1840)

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{4R_h}\frac{V^2}{2g}$$

$$LS_f = f\frac{L}{4R_h}\frac{V^2}{2g}$$

$$R_h S_f = f\frac{V^2}{8g} \quad \Rightarrow \quad V = \sqrt{\frac{8g}{f}}\sqrt{R_h S_f}$$

IMPORTANT:

In Uniform Flow $S_f=S_o$

Manning Equation (1891)

$$V = \frac{1}{n} R_h^{2/3} S_f^{1/2}$$ (SI System)

Notes: The Manning Equation

1) is dimensionally nonhomogeneous

2) is very sensitive to *n*

Is *n* only a function of roughness? NO!

Dimensions of *n*? $T / L^{1/3}$

$$V = \frac{1.49}{n} R_h^{2/3} S_f^{1/2}$$ (English system)

Values of Manning *n*

Values of Manning's Roughness Coefficient *n*

Glass, plastic, machined metal	0.010
Dressed timber, joints flush	0.011
Sawn timber, joints uneven	0.014
Cement plaster	0.011
Concrete, steel troweled	0.012
Concrete, timber forms, unfinished	0.014
Untreated gunite	0.015–0.017
Brickwork or dressed masonry	0.014
Rubble set in cement	0.017
Earth, smooth, no weeds	0.020
Earth, some stones and weeds	0.025
Natural river channels:	
Clean and straight	0.025–0.030
Winding, with pools and shoals	0.033–0.040
Very weedy, winding and overgrown	0.075–0.150
Clean straight alluvial channels	$0.031d^{1/6}$ (d=D-75 size in ft.)

$n = 0.031d^{1/6}$ d in ft

$n = 0.038d^{1/6}$ d in m

d = median size of bed material

Relation between Resistance Coefficient

For uniform free surface and pipe flows: $\tau_0 = \gamma R S$

$+$

Darcy's friction factor:

$$f = 8\frac{\tau_0}{\rho V^2}$$

$\Downarrow$

Chézy Equation:

$$V = C\sqrt{RS}$$

$\Rightarrow$

$$C = \sqrt{\frac{8g}{f}}$$

$\Updownarrow$

$\Rightarrow$

$$C = \frac{R^{1/6}}{n}$$

Manning Equation:

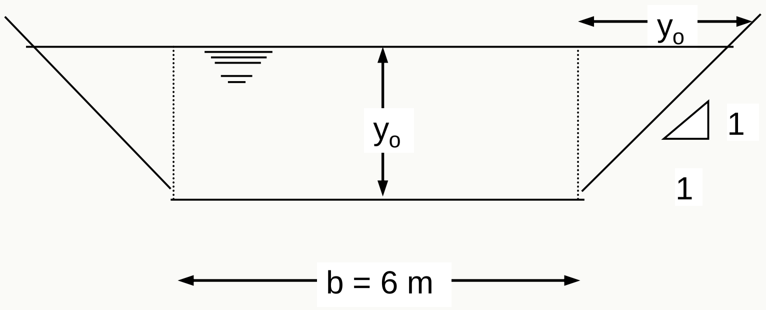
$$V = \frac{1}{n}R^{2/3}\sqrt{S}$$

$$\boxed{n = \frac{R^{1/6}}{2\sqrt{2g}}\sqrt{f}}$$

Example 3.1

A trapezoidal channel has a base width b = 6 m and side slopes 1H:1V. The channel bottom slope is So = 0.0002 and the Manning roughness coefficient is n = 0.014. compute

a) the depth of uniform flow if Q = 12.1 m3/s
b) the state of flow
c) the average wall-shear stress along the wetted perimeter.



Solution of Exp.3.1

a) Manning's equation is used for uniform flow;

$$Q = \frac{A}{n} R^{2/3} \sqrt{S_o}$$

$$A = b.y_o + 2.(y_o{}^2/2) = y_o(b + y_o)$$

$$P = b + 2\sqrt{2}\, y_o = 6 + 2\sqrt{2}\, y_o$$

$$AR^{2/3} = \frac{Qn}{\sqrt{S_o}} = 11.978$$

$$11.978 = y_o(6 + y_o)\left(\frac{y_o(6 + y_o)}{6 + 2\sqrt{2}y_o}\right)^{2/3}$$

Y(m)	A(m²)	P(m)	R(m)	$AR^{2/3}$
1	7	8.28	0.84	6.23
1.2	8.64	9.39	0.92	8.17
1.4	10.36	9.96	1.04	10.63
1.5	11.25	10.24	1.098	11.976

by trial & error   yo=1.5 m

Solution of Exp.3.1

b) The state of flow

$$Fr = \frac{V_{ave}}{\sqrt{gD}} \quad , D = \frac{A}{T} \quad , \quad T = b + 2y_o$$

$A = 1.5\ (6+1.5) = 11.25\ m^2$

$T = 6+2 \times 1.5 = 9\ m$

$D = 11.25 / 9 = 1.25\ m$

$$V_{ave} = \frac{Q}{A} = \frac{12.1}{11.25} = 1.076\ m/s$$

$$Fr = \frac{1.076}{\sqrt{9.81x1.25}} = 0.307 < 1 \ \underline{\text{ Subcritical}}$$

c) $\tau_\omega = \gamma.R.S_o$

$R = A/P$,

$P = 6+2\sqrt{2} \times 1.5 = 10.24\ m$

$R = 11.25 / 10.24 = 1.098\ m$

$$\tau_\omega = 9810x1.098x0.0002 = 2.15\ Pa$$

Graphical Solution

Input diameter, Manning coefficient, and channel slope:

$d := 5 \qquad n := 0.013 \qquad S_0 := 0.0005 \qquad c_1 := 1.0$

Define geometric functions:

$$\alpha(y) := \operatorname{acos}\left(1 - 2\cdot\frac{y}{d}\right)$$

$$A(y) := \frac{d^2}{4}\cdot\left(\alpha(y) - \sin(\alpha(y))\cdot\cos(\alpha(y))\right)$$

$$P(y) := \alpha(y)\cdot d$$

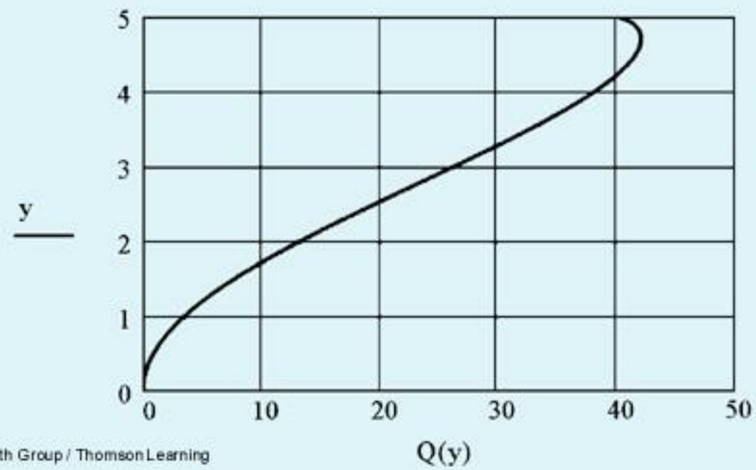
$$R(y) := \frac{A(y)}{P(y)}$$

Define discharge function, (i.e., Manning's equation):

$$Q(y) := \frac{c_1}{n}\cdot A(y)\cdot R(y)^{\frac{2}{3}}\cdot\sqrt{S_0}$$

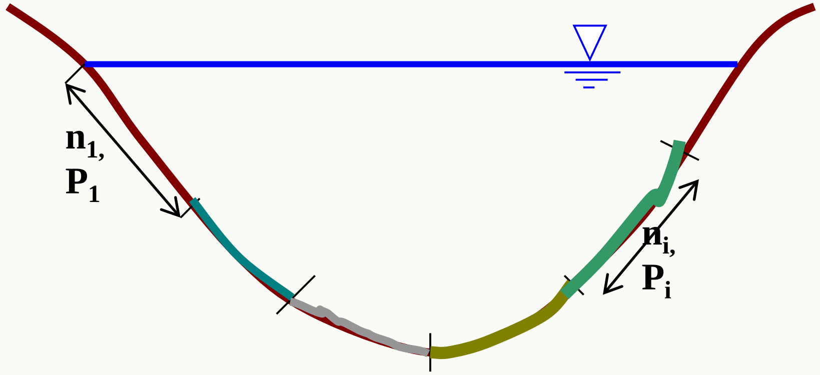
Plot depth versus discharge:

$y := 0, 0.01 .. d$



Composite Section

- A channel section, which is composed, of different roughness along the wetted perimeter is called composite section. For such sections an equivalent Manning roughness can be defined as



$$n_{eq} = \sqrt{\frac{\sum n_i^2 P_i}{\sum P_i}}$$

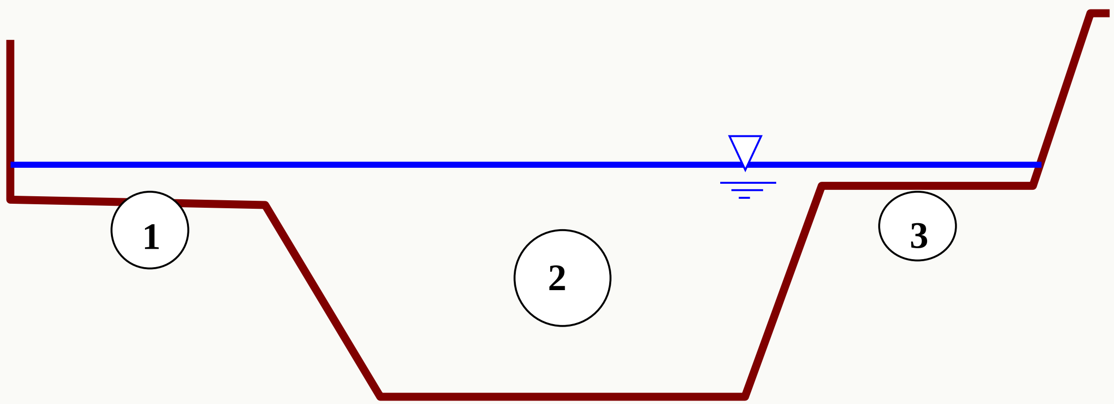
(Pavlovski's eq.

$$F = \sum_{i=1}^{n} F_i$$
)

$$Q = \frac{A}{n_{eq}} R^{2/3} \sqrt{S_f}$$

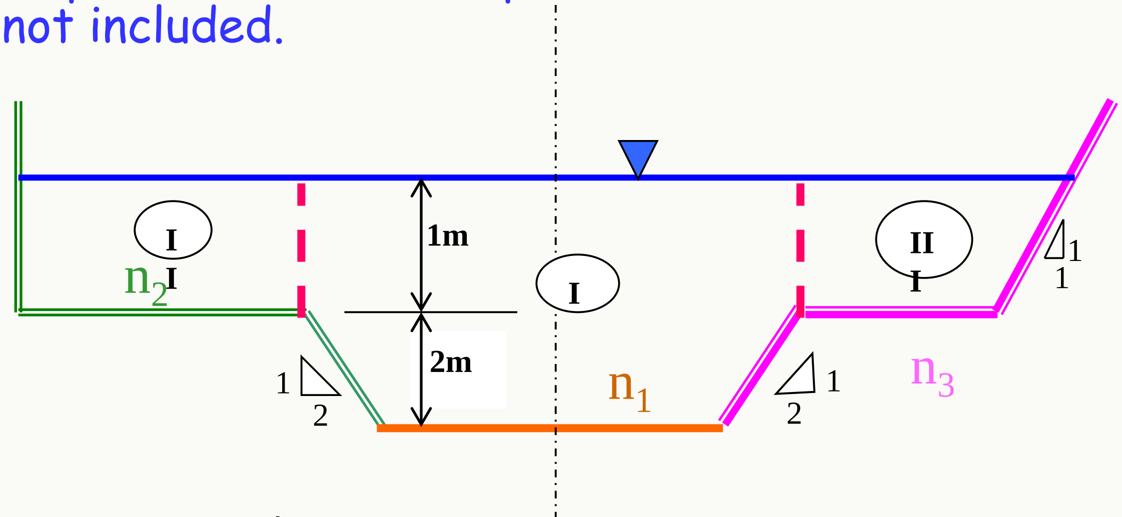
Compound Channel

- is the channel for which the cross section is composed of several distinct subsections



Discharge computation in Compound Channels

- To compute the discharge, the channel is divided into 3 subsections by using vertical interfaces as shown in the figure:
- Then the discharge in each subsection is computed separately by using the Manning equation.
- In computation of wetted perimeter, water-to-water contact surfaces are not included.

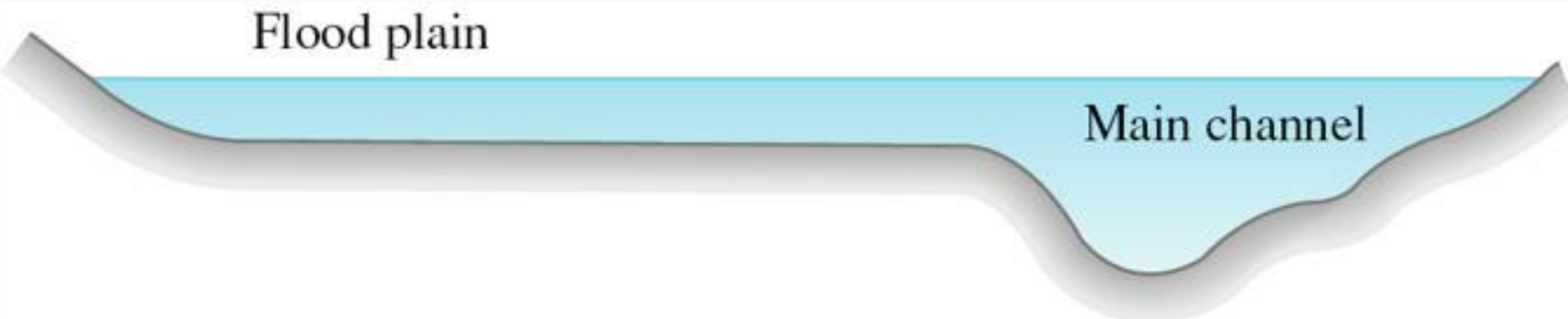


$$Q_i = \frac{A_i}{n_i}\left(\frac{A_i}{P_i}\right)^{2/3}\sqrt{S_o} \qquad i = 1,2,3$$

$$Q_{total} = \sum_{i=1}^{3} Q_i$$

Compound Channel

Generalized section representation

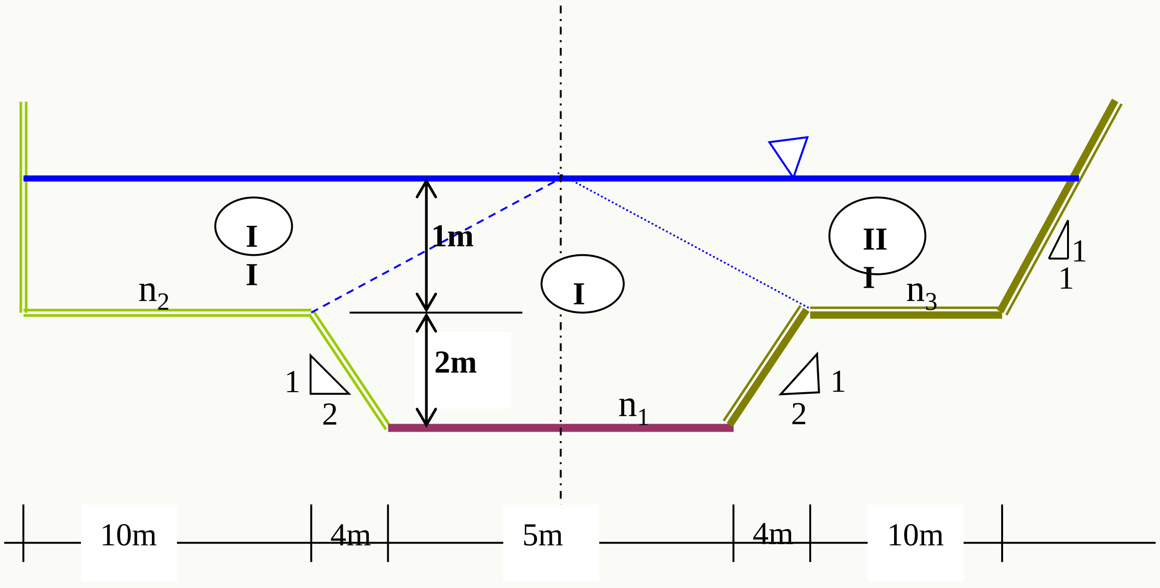


actual cross section

compound-composite cross section.

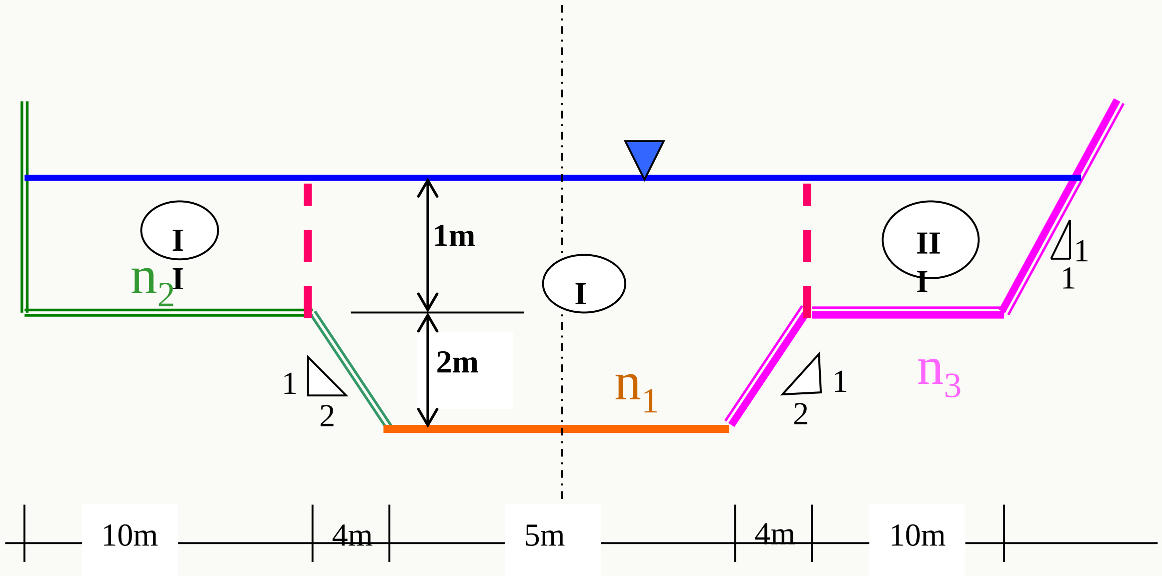
Example 3.2:

Example 3.2: Determine the discharge passing through the cross section of the compound channel shown below. The Manning roughness coefficients are n_1 = 0.02, n_2 = 0.03 and n_3 = 0.04. The channel bed slope for the whole channel is So = 0.008.



Solution of Example 3.2

- Divide the channel into 3 subsections by using vertical interfaces as shown in the figure:



$$Q_i = \frac{A_i}{n_i}\left(\frac{A_i}{P_i}\right)^{2/3}\sqrt{S_o} \qquad i = 1,2,3$$

$$Q_{total} = \sum_{i=1}^{3} Q_i$$

Example 3.2

- For the main channel (subsection I):

The main channel is a composite channel too. Therefore, we need to find an equivalent value of n.

$$n_{eq} = \left(\frac{\sum n_i^{\,2} P_i}{\sum P_i} \right)^{1/2}$$

$$n_{eq} = \left(\frac{n_1^{\,2} 5 + n_2^{\,2} \sqrt{5} * 2 + n_3^{\,2} \sqrt{5} * 2}{5 + 4\sqrt{5}} \right)^{1/2} = \left(\frac{(0.02)^2 5 + 2\sqrt{5}(0.03^2 + 0.04^2)}{5 + 4\sqrt{5}} \right)^{1/2}$$

$$n_{eq} = 0.03074$$

$$A_1 = \frac{1}{2}(5 + 13) * 2 + (13 * 1) = 31\,\mathrm{m}^2$$

$$P_1 = 5 + 2\mathrm{x}2\sqrt{5} = 13.944\,\mathrm{m}$$

$$Q_1 = \frac{31}{0.03074} \left(\frac{31}{13.944} \right)^{2/3} \sqrt{0.008} = 154.05\,\mathrm{m}^3 / s$$

Example 3.2

- **For the subsection II:**

$$A_2 = 10 * 1 = 10 \ \ \mathrm{m}^2$$

$$P_2 = 10 + 1 = 11 \, \mathrm{m}$$

$$Q_2 = \frac{10}{0.030}\left(\frac{10}{11}\right)^{2/3}\sqrt{0.008} = 27.97 \, \mathrm{m}^3 / s$$

For the subsection III:

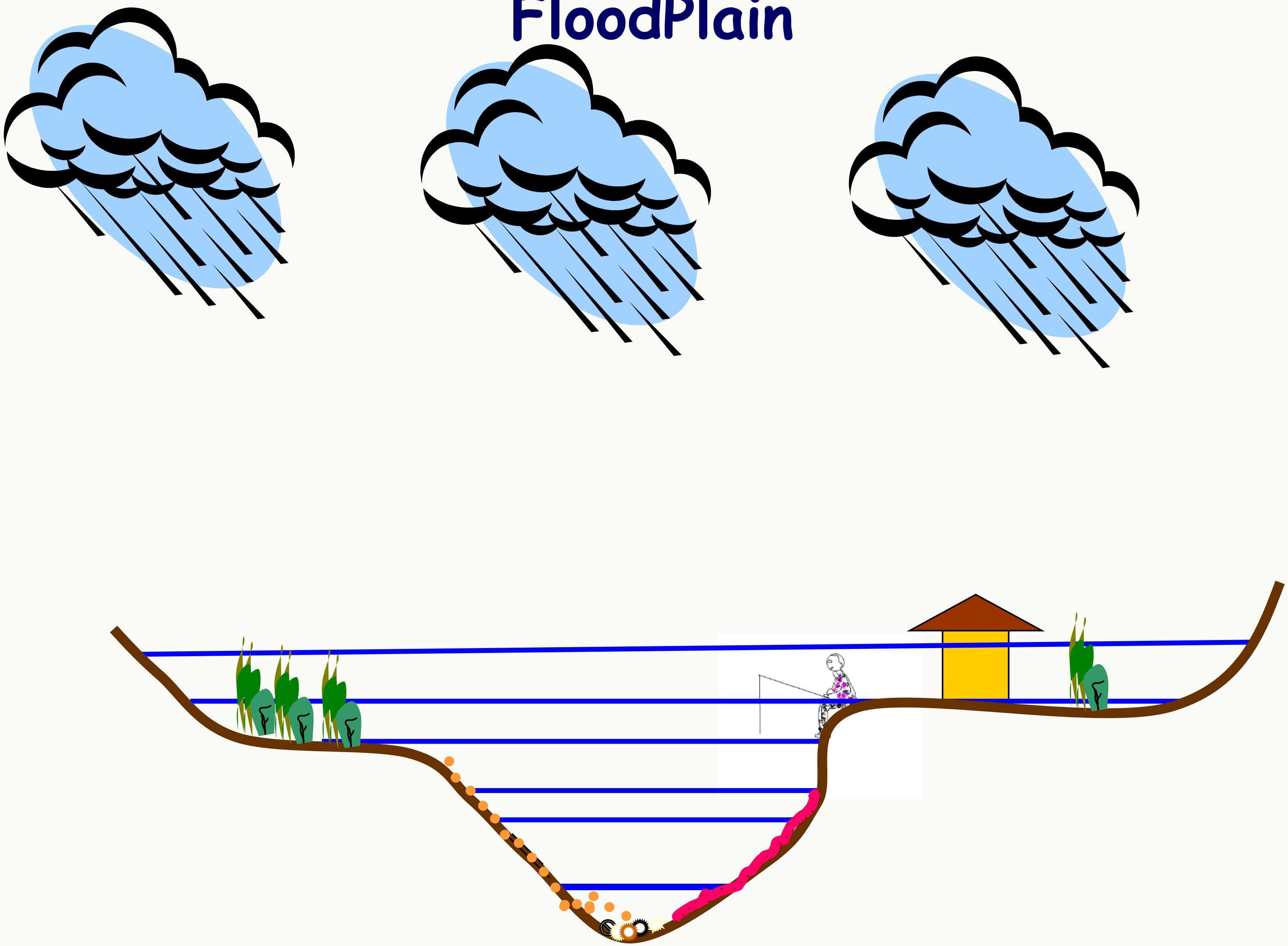
$$A_3 = \frac{1}{2}(10 + 11) * 1 = 10.5 \, \mathrm{m}^2$$

$$P_3 = 10 + \sqrt{2} = 11.41 \, \mathrm{m}$$

$$Q_3 = \frac{10.5}{0.040}\left(\frac{10.5}{11.41}\right)^{2/3}\sqrt{0.008} = 22.21 \, \mathrm{m}^3 / s$$

$$Q_{total} = Q_1 + Q_2 + Q_3 = 154.05 + 27.97 + 22.21 = 204.23 \, \mathrm{m}^3 / s$$

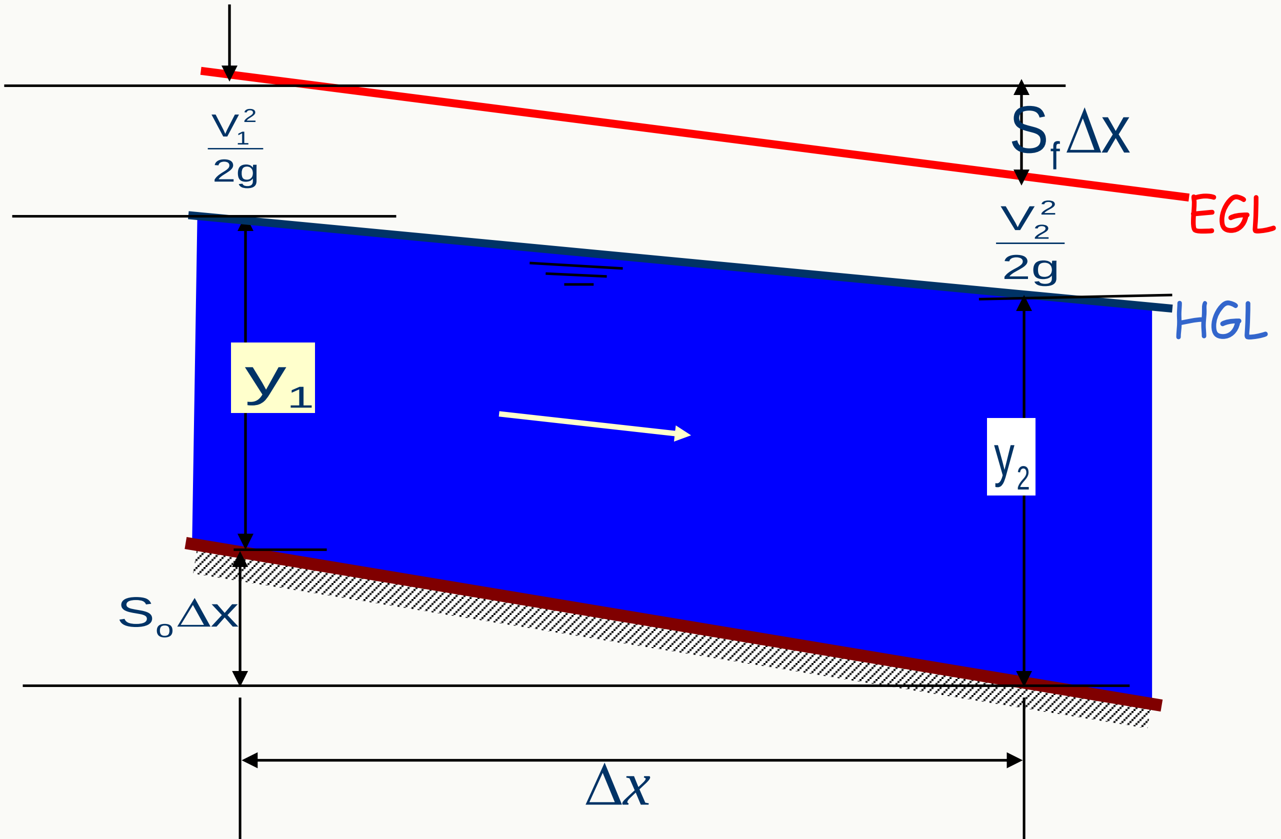
FloodPlain



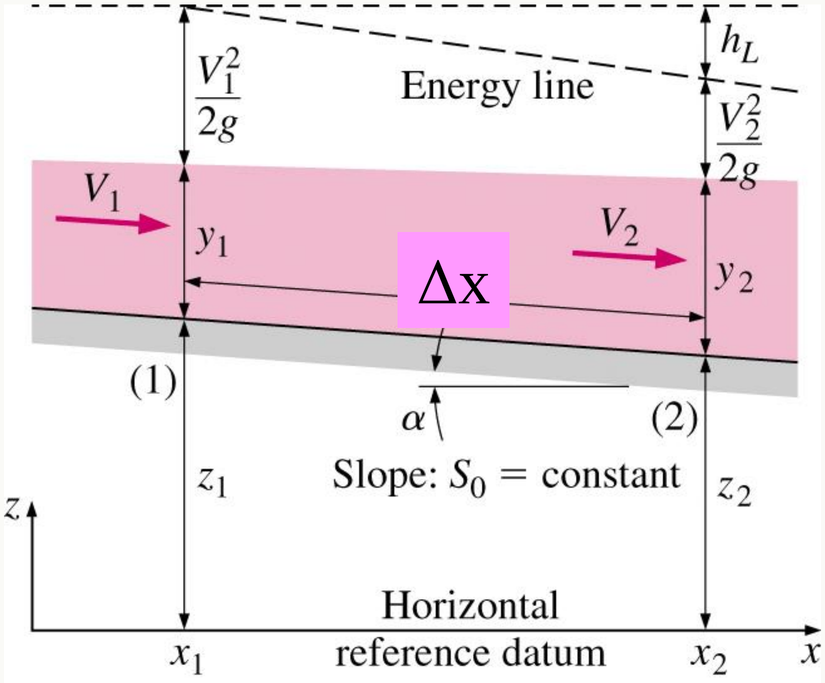
Energy Concept

- Component of energy equation
  1) z is the elevation head
  2) y is the gage pressure head-potential head
  3) $V^2/2g$ is the dynamic head-kinetic head

$$H_1 = z_1 + y_1 + \frac{V_1^2}{2g}$$



Continuity and Energy Equations



- 1D steady continuity equation can be expressed as

$$V_1A_1 = V_2A_2$$

- 1D steady energy equation between two stations

$$z_1 + y_1 + \frac{V_1^2}{2g} = z_2 + y_2 + \frac{V_2^2}{2g} + h_\ell$$

- Head loss h_L

$$h_\ell = S_f \Delta x$$

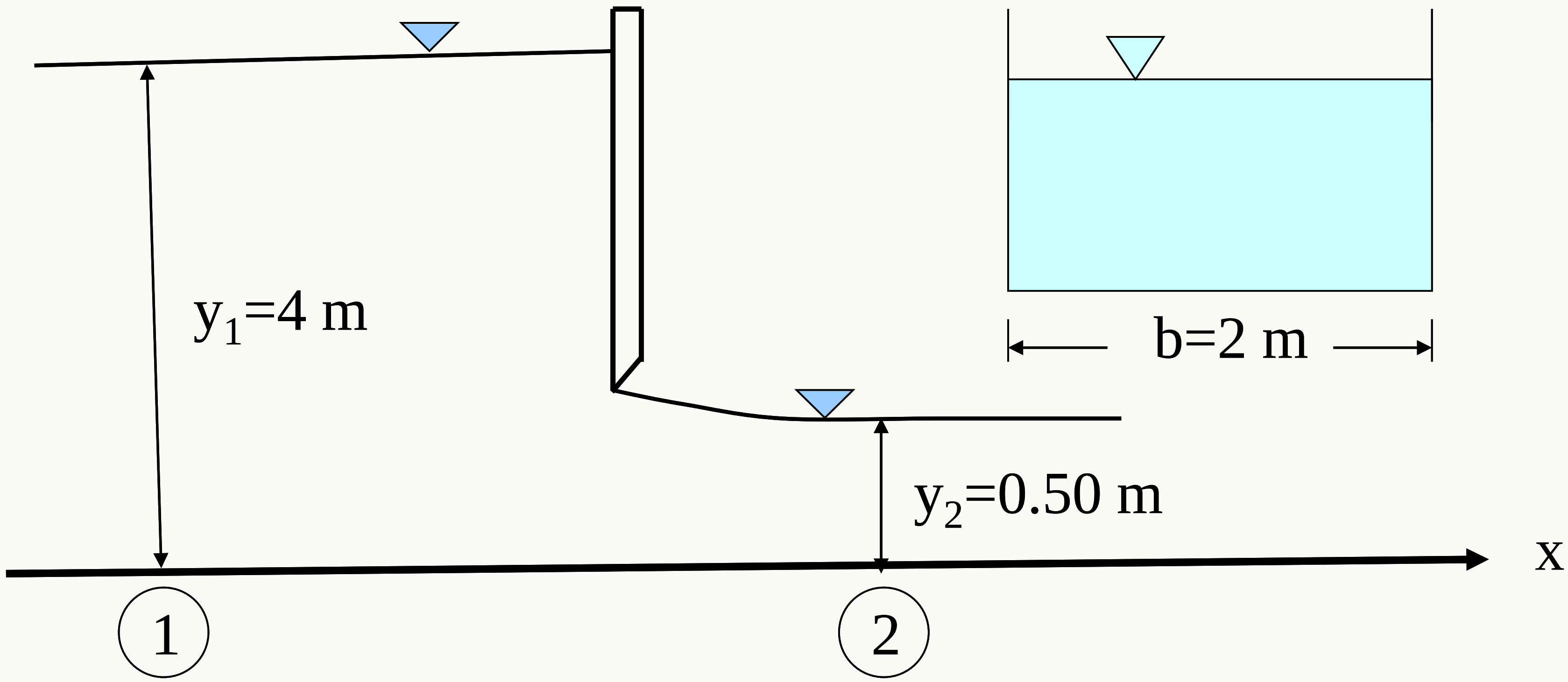
- The change in elevation head can be written in terms of the bed slope θ

$$S_o = \frac{(z_1 - z_2)}{\Delta x}$$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + (S_f - S_o)\Delta x$$

Example 1

- Water flows under a sluice gate in a horizontal rectangular channel of 2 m wide. If the depths of flow before and after the gate are 4 m, and 0.50 m, compute the discharge in the channel.



Solution:

The energy equation between sections (1) and (2) is:

- $H_1=H_2+h_f$
- The head loss between sections (1) and (2) can be neglected.
- Therefore:

$$z_1 + y_1 + \alpha_1 \frac{V_1^2}{2g} = z_2 + y_2 + \alpha_2 \frac{V_2^2}{2g}$$

Choose the channel bottom as datum. Then $z_1=z_2=0$, $\alpha=1$

For rectangular channels, we can define unit discharge, q, as:

$$q = \frac{Q}{b} = \frac{Vby}{b} = Vy \longrightarrow V = \frac{q}{y}$$

Therefore Energy equation between sections (1) and (2) becomes:

$$y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2gy_2^2}$$

- Solving for q:

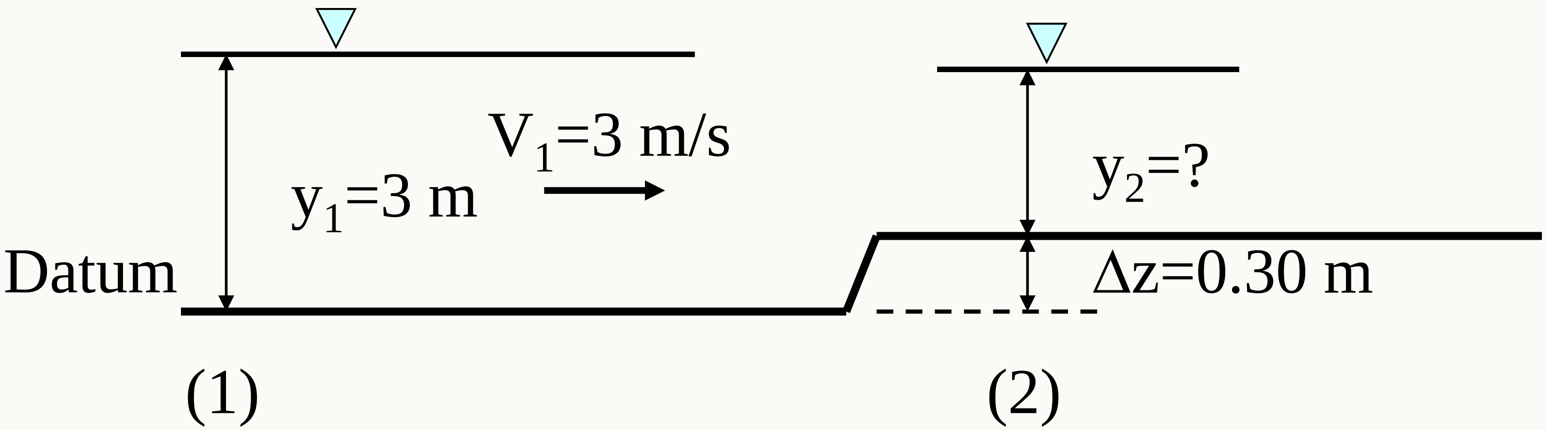
$$\frac{q^2}{2g} \quad \frac{1}{y_2^2} \quad \frac{1}{y_1^2} = y_1 \quad y_2 \quad \text{substituting the values :}$$

$$\frac{q^2}{2g} \quad \frac{1}{0.50^2} \quad \frac{1}{4^2} = 3.5 \ \ \text{solving for } q = 4.176\ m^3/s/m$$

- The total discharge is Q=q.b=4.176*2=8.35 m^3/s

Example 2:

Water flow with a velocity of 3 m/s, and a depth of 3 m in a rectangular channel of 2 m wide. Then there is an upward step of 30 cm as shown in figure below. Compute the depth of flow over the step.



- Energy Eq. Between Sections (1) & (2):

$$z_1 + y_1 + \frac{q^2}{2gy_1^2} = z_2 + y_2 + \frac{q^2}{2gy_2^2} \qquad q = V_1 y_1 = V_2 y_2 = 3.3 = 9\ \mathrm{m^3/s/m}$$

$$3 + \frac{9^2}{2g3^2} = 0.30 + y_2 + \frac{9^2}{2gy_2^2} \Rightarrow \qquad y_2 + \frac{4.1284}{y_2^2} = 3.1587$$

The last equation contains only one unknown: y_2.

However, it is a third degree polynomial of y_2.

- $Y^3-3.1587y^2+4.1284=0$ This polynomial has three possible solutions:
- $Y_{(1)}=2.496 \approx 2.5$ m
- $Y_{(2)}=1.66$ m
- $Y_{(3)}=\cancel{-0.996 \approx -1\ \text{m}}$ Negative depth is not acceptable
- But both 2.5 m and 1.66 m depths are quite possible.
- Which one will occur on the step????
- Nor Energy equation neither continuity equation will help to decide.
- Luckily, in 1912, Bakhmeteff introduced the concept of
- SPECIFIC ENERGY, which is the key to even the most complex open-channel flow phenomena.
- Then let's learn the specific energy concept.

SPECIFIC ENERGY CONCEPT

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + (S_f - S_o)\Delta x$$

$$E_1 = E_2 + (S_f - S_o)\Delta x$$

$$E_1 = y_1 + \frac{V_1^2}{2g}$$

E is called as specific energy

$$E_2 = y_2 + \frac{V_2^2}{2g}$$

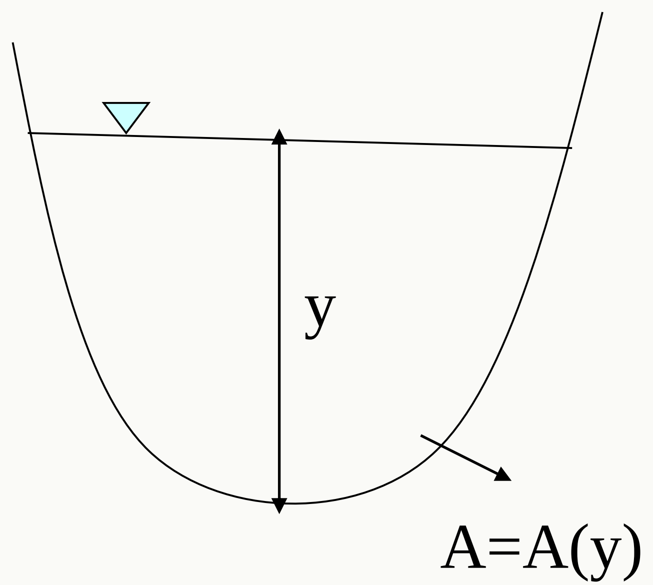
If channel bottom is horizontal and no head loss $E_1 = E_2$

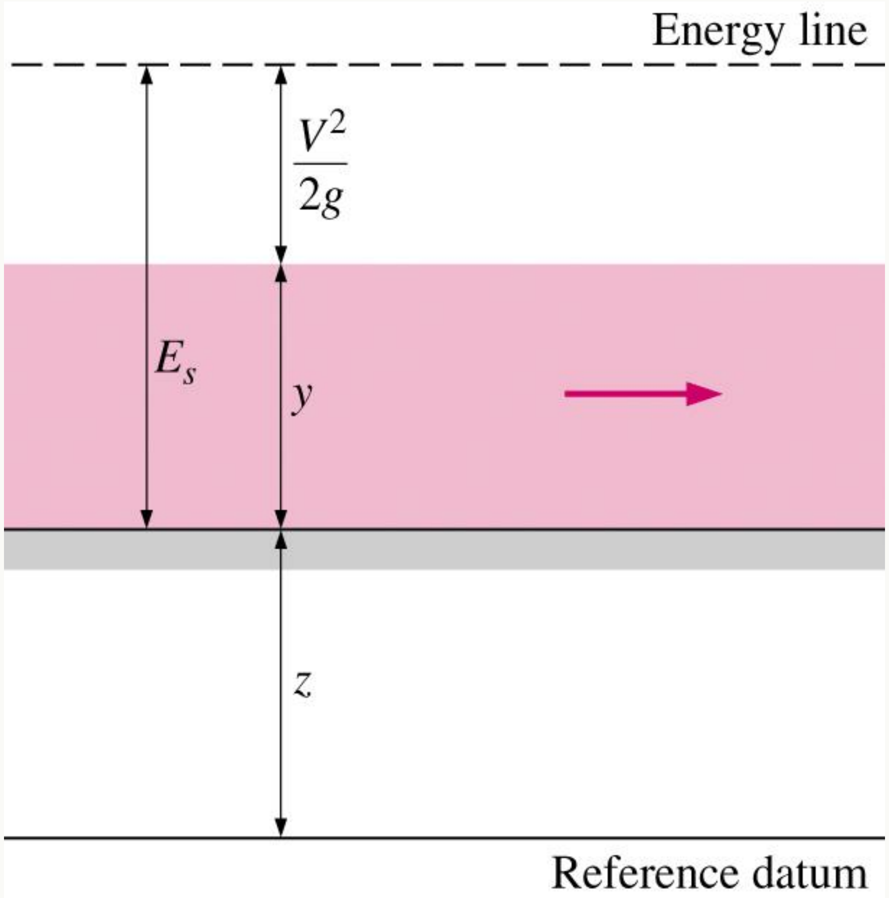
For uniform flow $S_f = S_o$ then $E_1 = E_2$

SPECIFIC ENERGY CONCEPT

- Specific Energy, E, is the energy referred as the channel bed as datum,i.e:
- Taking the datum z=0 as the bottom of the channel, the specific energy E is the sum of the depth of flow and the velocity head is the specific energy.

$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2(y)}$$





Specific-Energy Curve

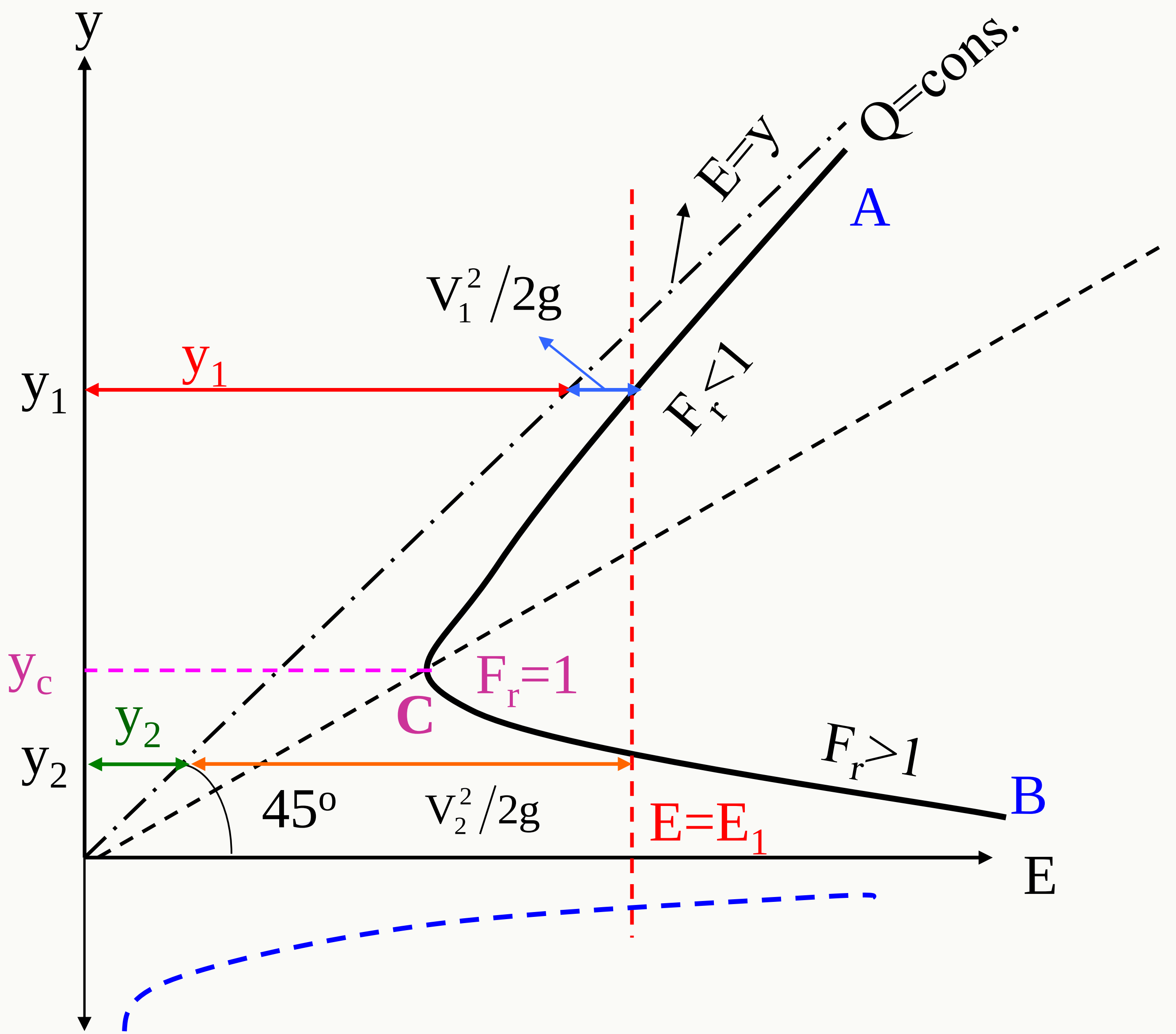
- For a given Q, E is only a function of y, i.e: E=E(y)
- Above equation can be written as:

$$(E \quad y)A(y)^2 = \frac{Q^2}{2g} = \text{cons tan t}$$

The plot of E vs y is called specific-energy curve. Above equation has two asymptotes:

(E-y)=0 and y=0, in fact one section of the curve falls within the 45^o angle between these two asymptotes in the first quadrant as in figure below.

There is another section of the curve shown as broken line, but this is of no practical interest as it yield negative values for y.

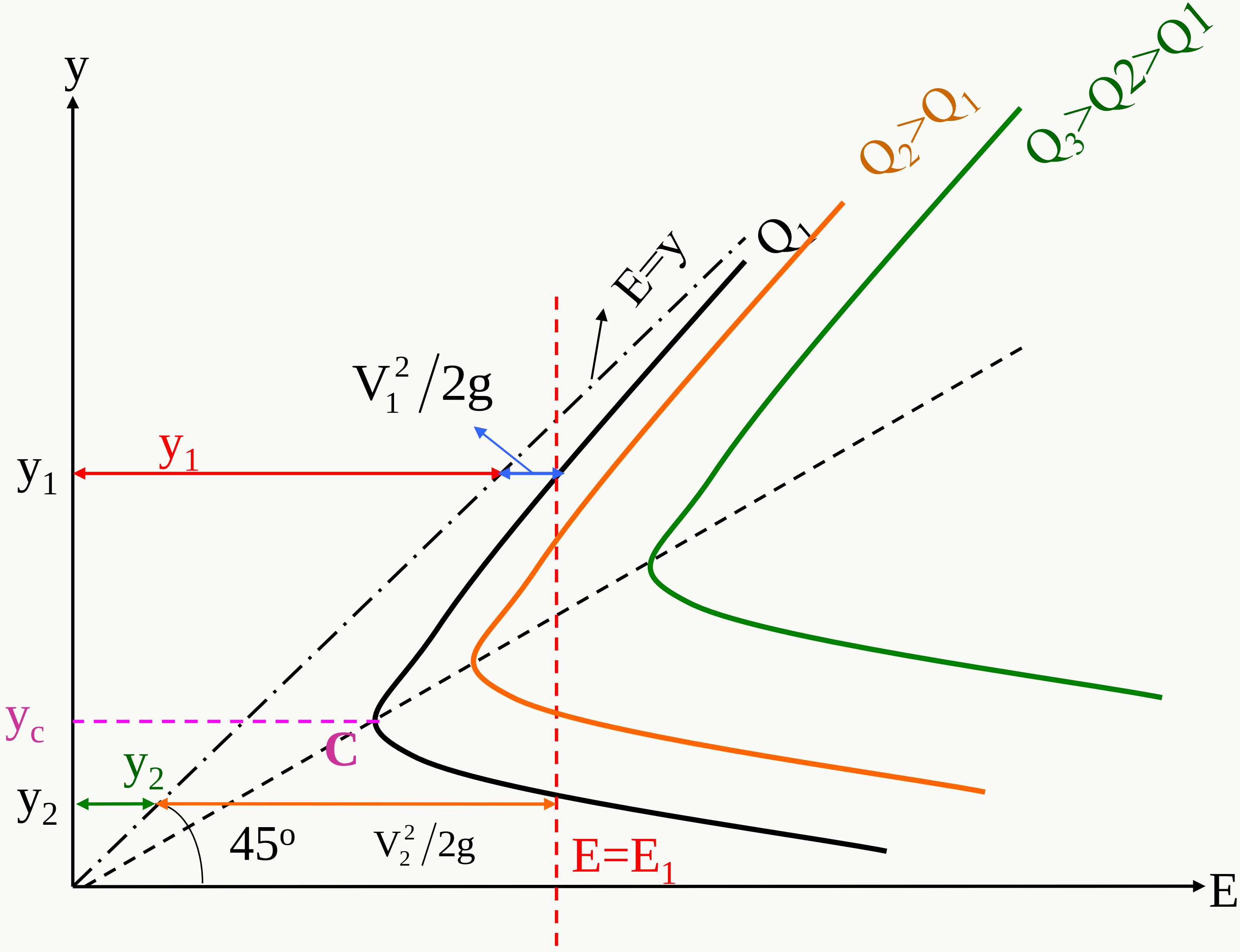


y
Q=cons.
E=y
A
$V_1^2/2g$
y_1
y_1
$F_r<1$
y_c
$F_r=1$
C
y_2
y_2
$F_r>1$
B
45°
$V_2^2/2g$
$E=E_1$
E

- If we regard this curve as a means of solving Eq.:

$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2(y)}$$

- for y, given E and Q, the three solutions of cubic are clearly shown by drawing a vertical line coresponding to the given value of E.
- Only two of them are physically real, so for given values of E and Q, there are two possible depths of flow, unless the vertical line referred misses the curve altogether, a case which will be discussed later.
- These two possible flow depths, for a given E and Q, are referred as alternate depths.
- Alternatively we may say that the curve represents two possible regimes of flow- slow and deep on the upper limb, fast and shallow on the lower limb-meeting at the crest of the curve, C.
- Other curves might be drawn for other values of Q; since, for a given value of y, E increases with Q, curves having higher values of Q will occur inside and to the right of those having lower values of Q.



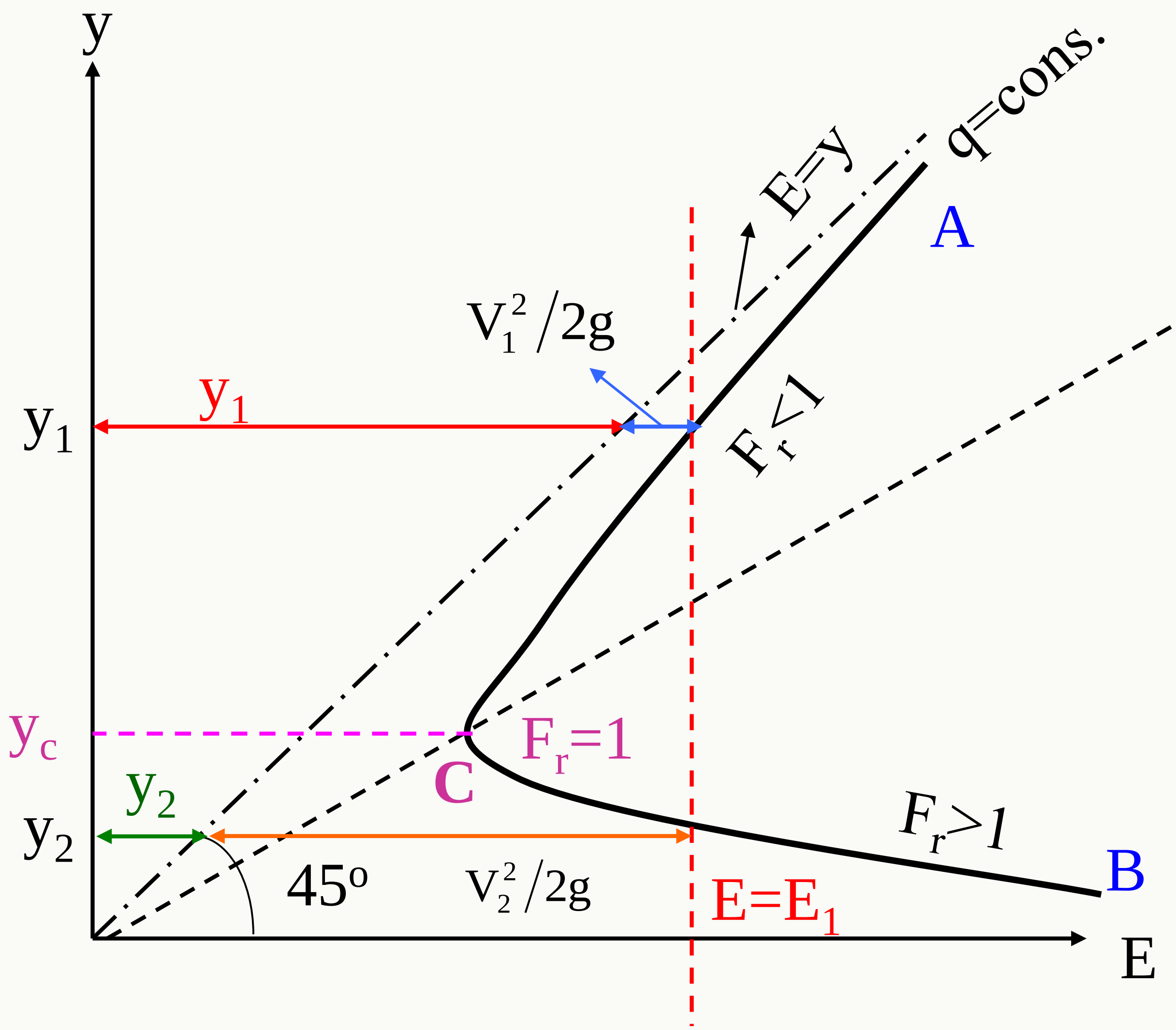
y
E
$Q_3>Q2>Q1$
$Q_2>Q_1$
Q_1
E=y
$V_1^2/2g$
y_1
y_1
y_c
C
y_2
y_2
45o
$V_2^2/2g$
$E=E_1$

Specific Energy for rectangular channels

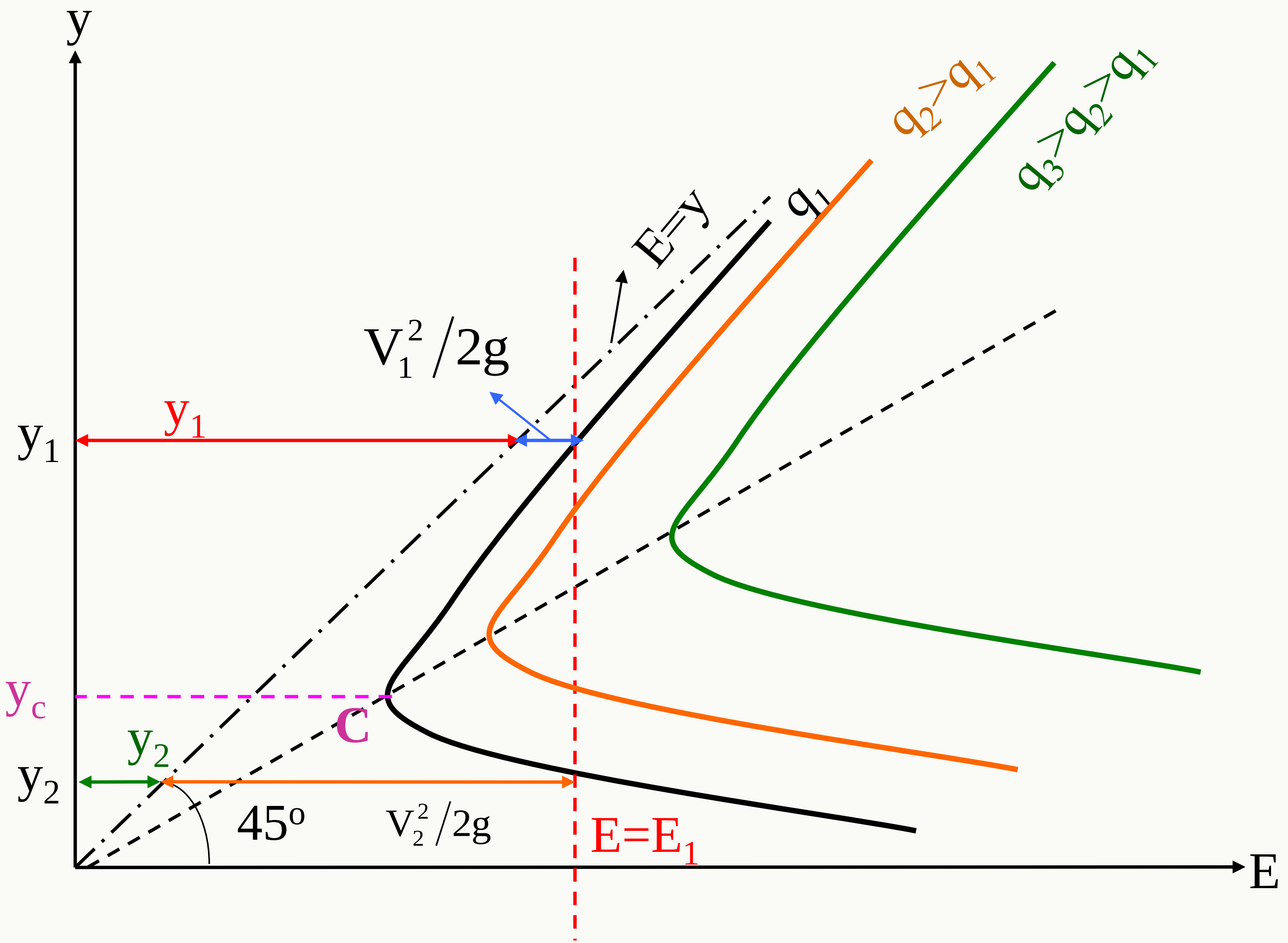
- Specific Energy, E, for rectangular channels is defined in terms of unit discharge q as:

$$E = y + \frac{V^2}{2g} = y + \frac{q^2}{2gy^2}$$

- Therefore specific- energy curve is drawn for a given unit discharge.



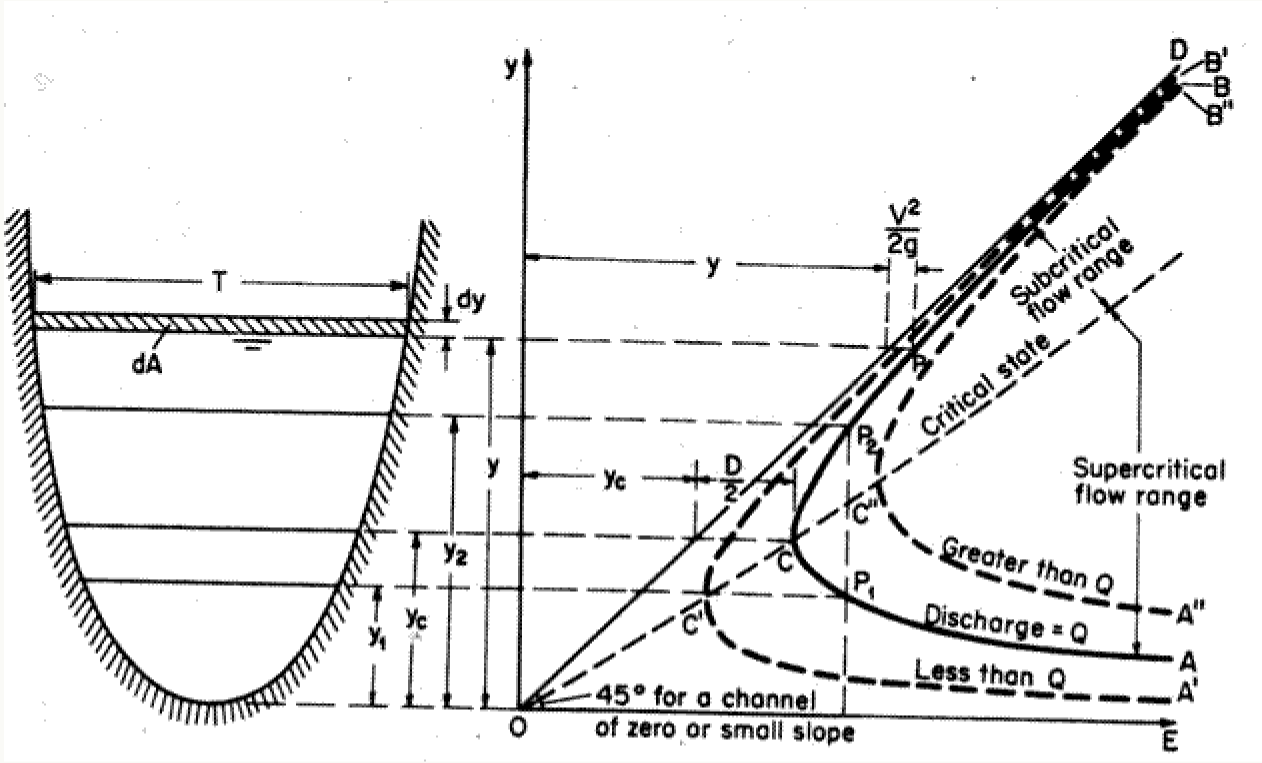
y
q=cons.
E=y
A
$V_1^2/2g$
y_1
y_1
$F_r<1$
y_c
$F_r=1$
C
y_2
y_2
$F_r>1$
B
45o
$V_2^2/2g$
$E=E_1$
E



y
E
$q_2>q_1$
$q_3>q_2>q_1$
q_1
E=y
$V_1^2/2g$
y_1
y_1
y_c
C
y_2
y_2
45°
$V_2^2/2g$
$E=E_1$

SPECIFIC ENERGY CONCEPT

$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2}$$

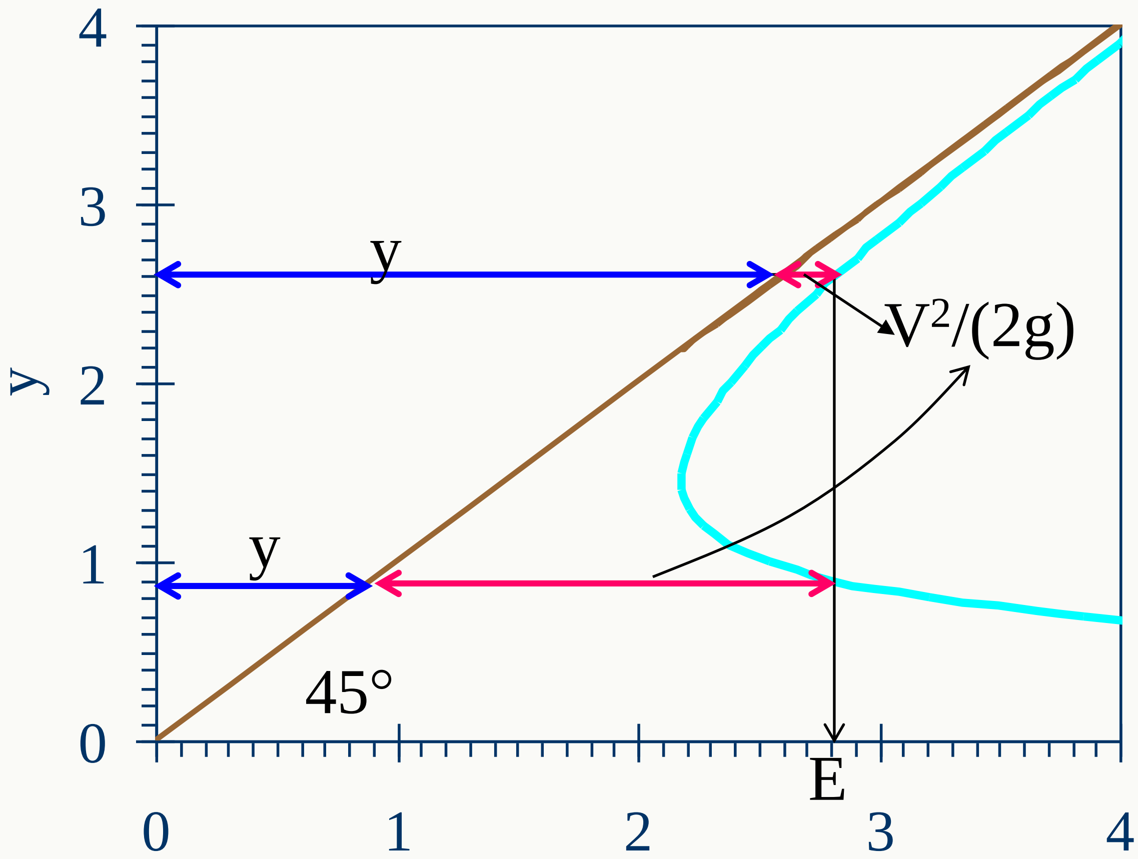


- For a given Q, E = E(y) only. The plot of E vs. y gives Specific Energy Curve.

Specific Energy

- Plot E vs y for constant Q
  - Easy to see breakdown of *E* into pressure (y) and dynamic ($V^2/2g$) head
  - $E \to \infty$ as $y \to 0$
  - $E \to y$ for large y
  - *E* reaches a minimum

What is the physical interpretation of local minimum?

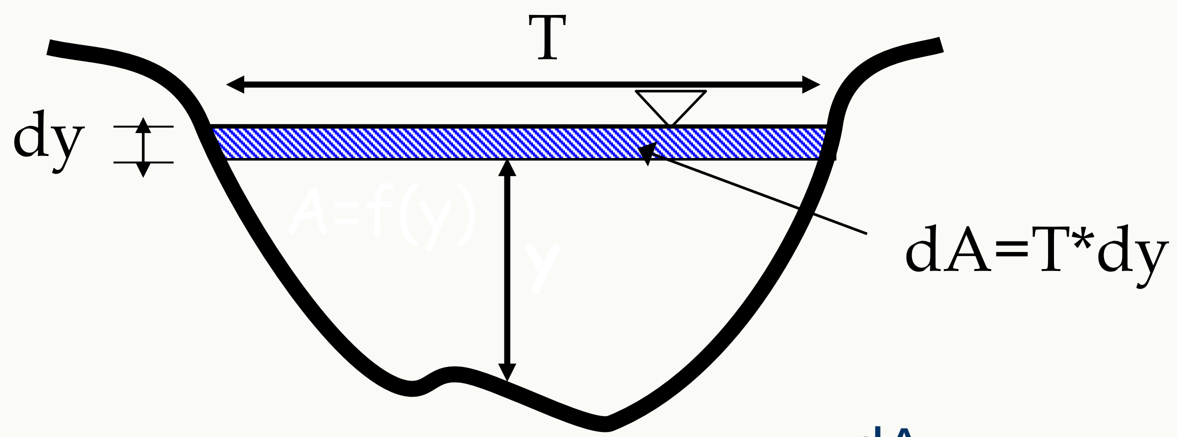


Minimum Specific Energy

Remember that:

$$Fr = \frac{V}{\sqrt{gD}} = \frac{Q/A}{\sqrt{gD}} \Rightarrow Fr^2 = \frac{Q^2/A^2}{gD} = \frac{Q^2}{gDA^2} = \frac{Q^2}{g\frac{A}{T}A} = \frac{Q^2T}{gA^3}$$

$$\therefore F_r^2 = \frac{Q^2T}{gA^3}$$



$$\frac{dA}{dy} = T$$

Minimum Specific Energy

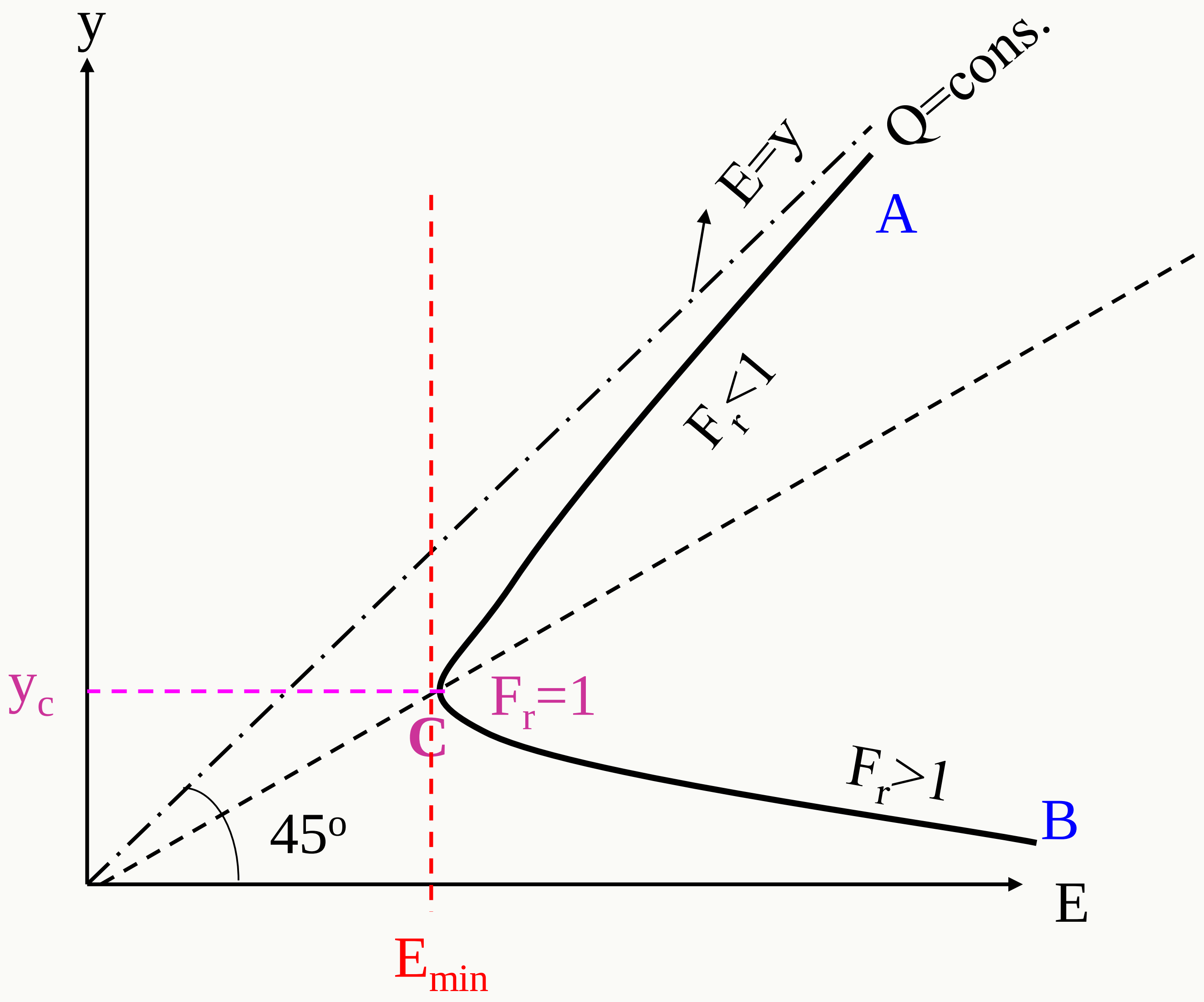
$$E = y + \frac{Q^2}{2gA^2}$$

For a given Q, when specific energy is minimum: $\frac{dE}{dy} = 0$

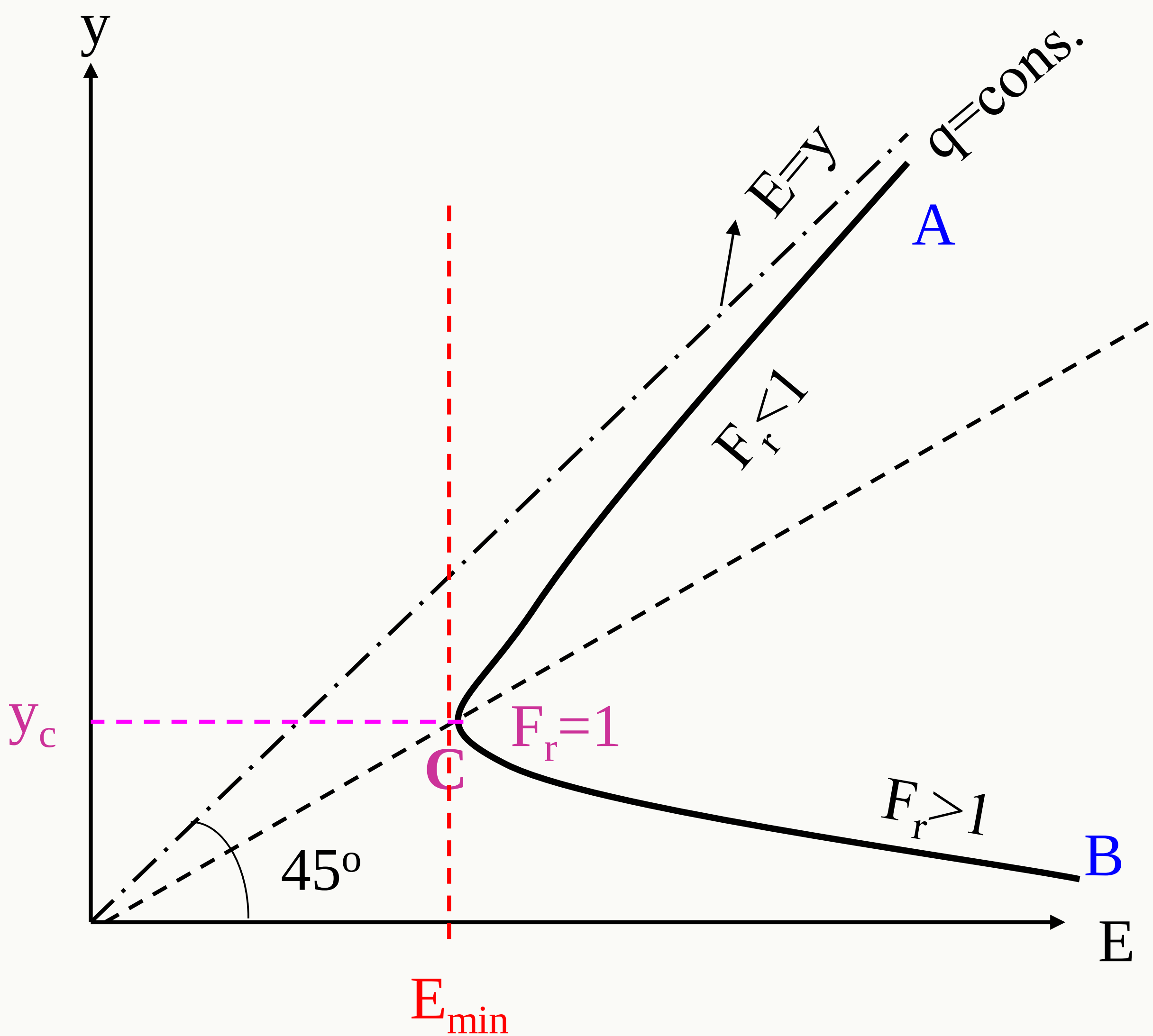
$$\frac{dE}{dy} = 1 \quad \frac{Q^2}{2g} \frac{d}{dA} \quad \frac{1}{A^2} \quad \frac{dA}{dy} = 1 \quad \frac{Q^2}{2g} \frac{2}{A^3} \frac{dA}{dy} = 1 \quad \frac{Q^2 T}{gA} \quad \text{or}$$

$$\frac{dE}{dy} = 1 \quad F_r^2 = 0 \quad \longrightarrow$$

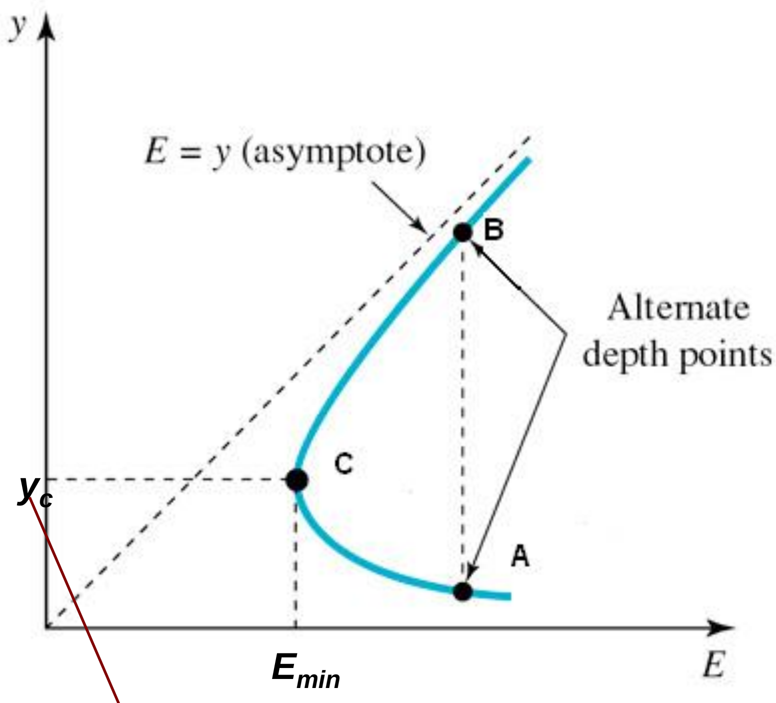
F_r=1, i.e: flow is critical


y
Q=cons.
E=y
A
F_r<1
y_c
F_r=1
C
F_r>1
B
45°
E
E_min

Minimum Specific Energy for Rectangular Channels



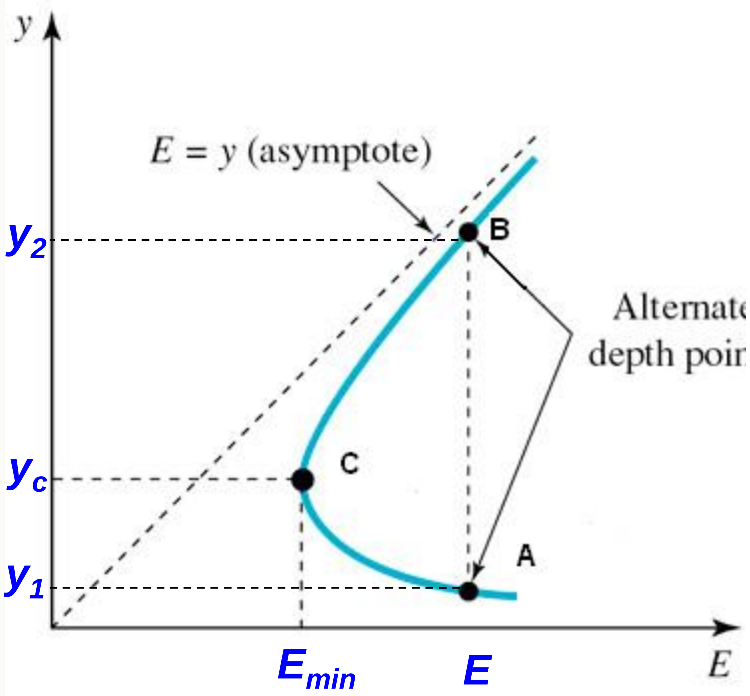
Characteristics Of The Specific Energy Curve



- Curve has 2 asymptotes: E = y and y = 0 lines
- Curve has 2 limbs AC and BC
- Limb AC approaches the horizontal axis as $y \to 0$
- Limb BC approaches to E = y line as $y \to \infty$
- On this curve Q remains constant
- At any point P on this curve, the ordinate represents the depth and the abscissa represents the specific energy.
- For a given specific energy E, there are 2 possible flow depths: y_1 and y_2

Critical depth

Characteristics Of The Specific Energy Curve



$$Fr = \frac{V}{\sqrt{gD}} \Rightarrow Fr^2 == \frac{Q^2T}{gA^3}$$

At point C, the specific energy is minimum

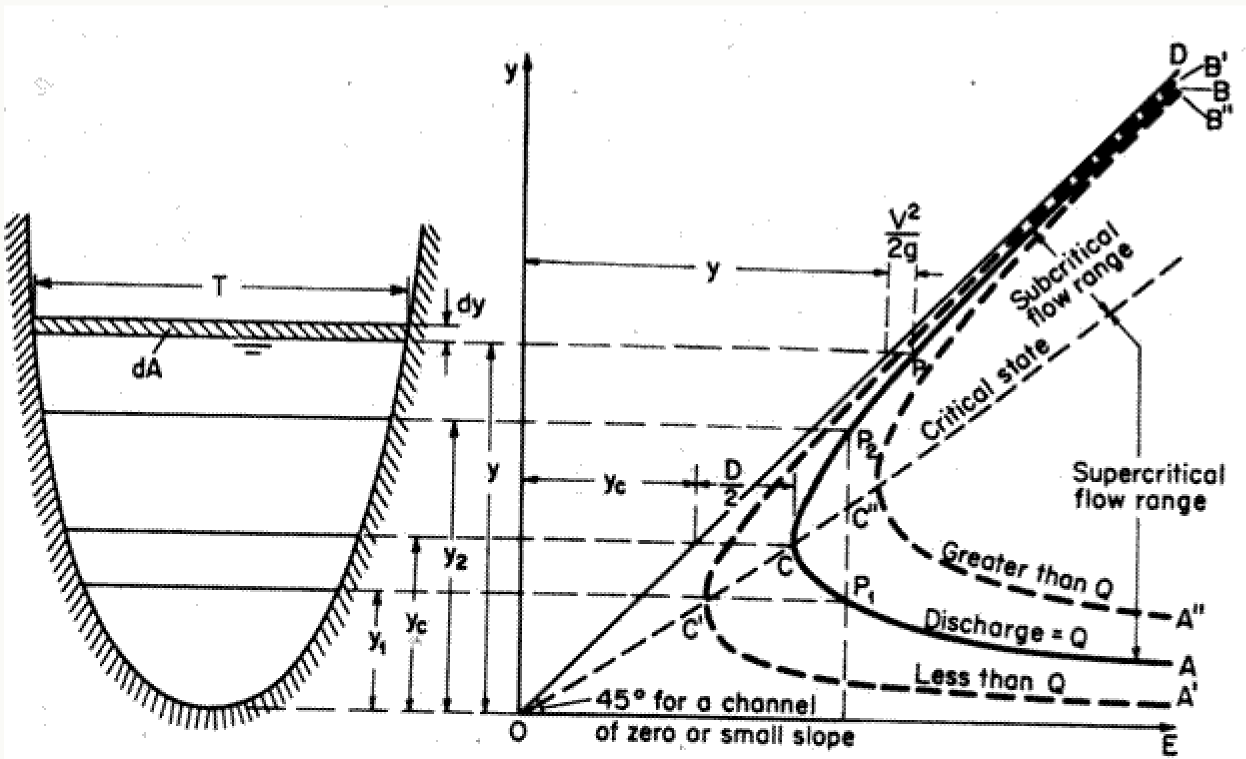
- Minimum specific energy corresponds to critical state of flow, i.e. Fr = 1.
- At the critical state, the two alternate depths become one, which is known as *critical depth*, y_c.

- If $y < y_c$, $V > V_c \rightarrow$ $Fr > 1$
- the limb AC corresponds to supercritical flow and y_2, corresponding to *supercritical depth*

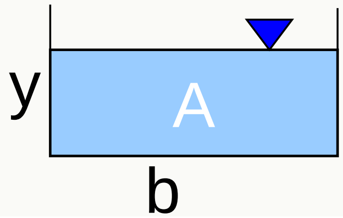
- If $y > y_c$, $V < V_c \rightarrow$ $Fr < 1$
- the limb BC corresponds to subcritical flow and y_1 corresponding to *subcritical depth*

- The depths y_1 and y_2 are called ***alternate depths.***

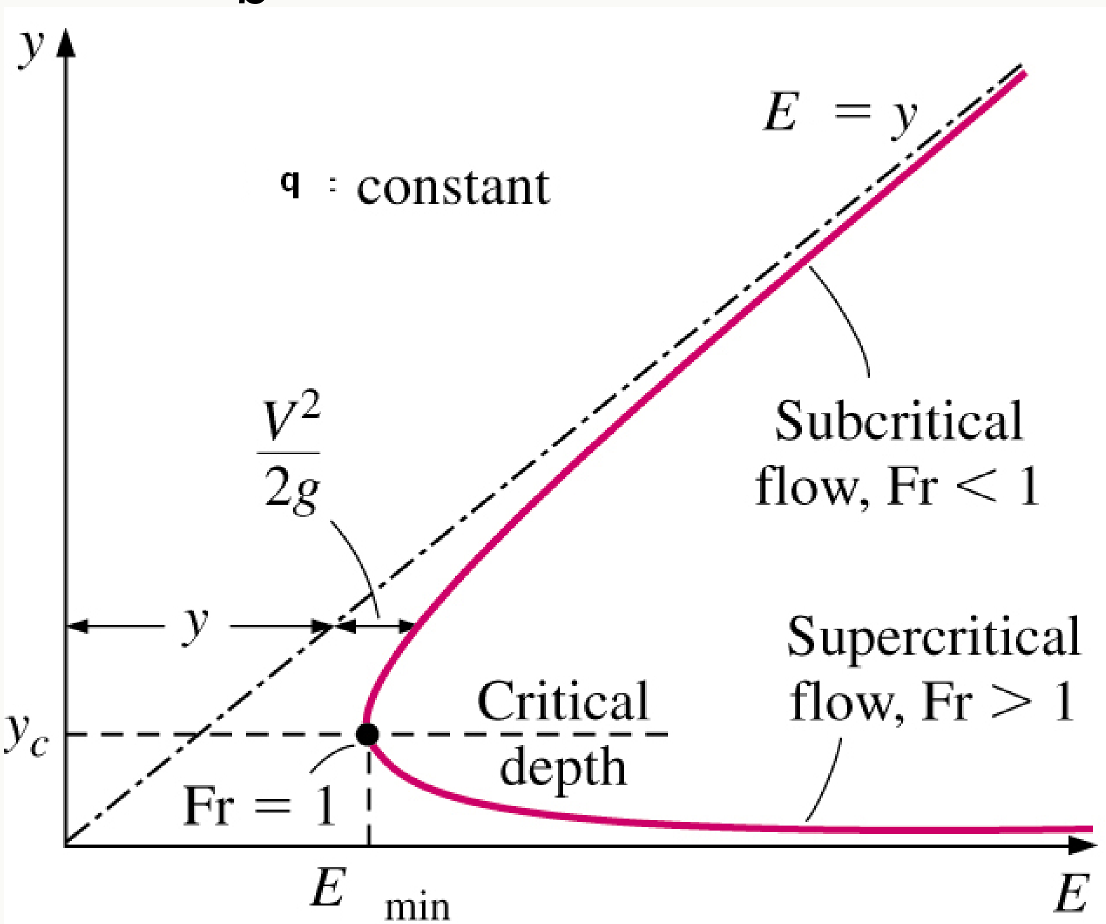
- As Q increase, the curves will move towards right



Specific Energy For rectangular Channel



$$Q = AV = byV \qquad \frac{Q}{b} = q \quad \text{unit discharge}$$



$$E = y + \frac{V^2}{2g}$$

$$E = y + \frac{Q^2}{2gA^2}$$

$$E = y + \frac{q^2}{2gy^2}$$

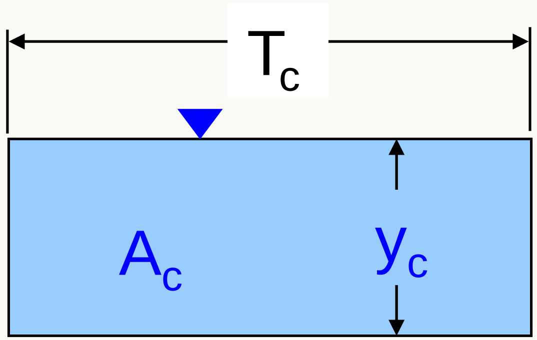
Critical Flow:Rectangular channel

$$1 = \frac{Q^2 T_c}{g A_c^3}$$

$$T_c = b$$

$$Q = qb$$

$$A_c = y_c b$$



$$1 = \frac{q^2 b^2}{g y_c^3 b^3} = \frac{q^2}{g y_c^3}$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

Only for rectangular channels!

$$q = \sqrt{g y_c^3}$$

Given the depth we can find the discharge

$$V_c = \sqrt{g y_c} \qquad q = V_c y_c$$

Critical Flow Relationships: Rectangular Channels

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} \qquad y_c^3 = \left(\frac{V_c^2 y_c^2}{g}\right) \qquad \text{because} \qquad q = V_c y_c$$

$$\frac{V_c}{\sqrt{y_c g}} = 1$$ Froude number for critical flow

$$y_c = \frac{V_c^2}{g} \longrightarrow \frac{y_c}{2} = \frac{V_c^2}{2g}$$ velocity **head** = 0.5 (depth)

$$E = y + \frac{V^2}{2g}$$

Therefore: $E_c = y_c + \dfrac{y_c}{2} = \dfrac{3}{2} y_c$ or $y_c = \dfrac{2}{3} E_c$

Critical Depth

Minimum energy for q

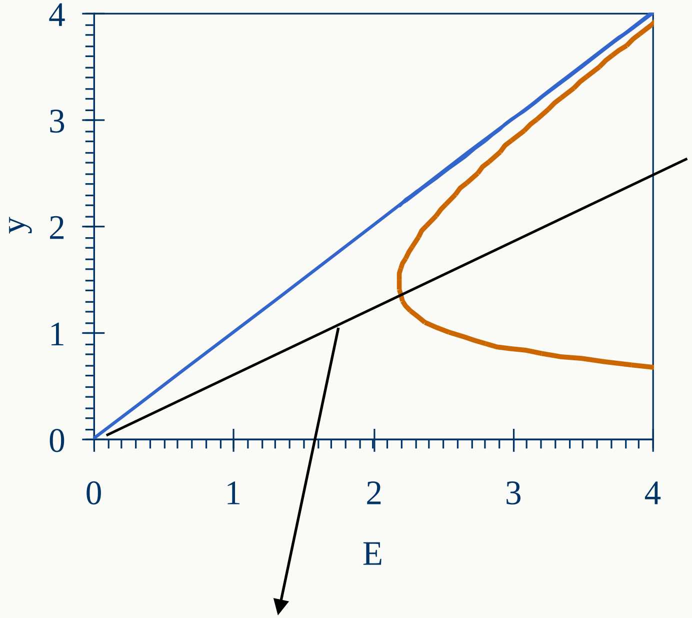
$$\frac{dE}{dy} = 0$$

Fr=1

$$\frac{y_c}{2} = \frac{V_c^2}{2g}$$

Fr>1 = Supercritical

Fr<1 = Subcritical



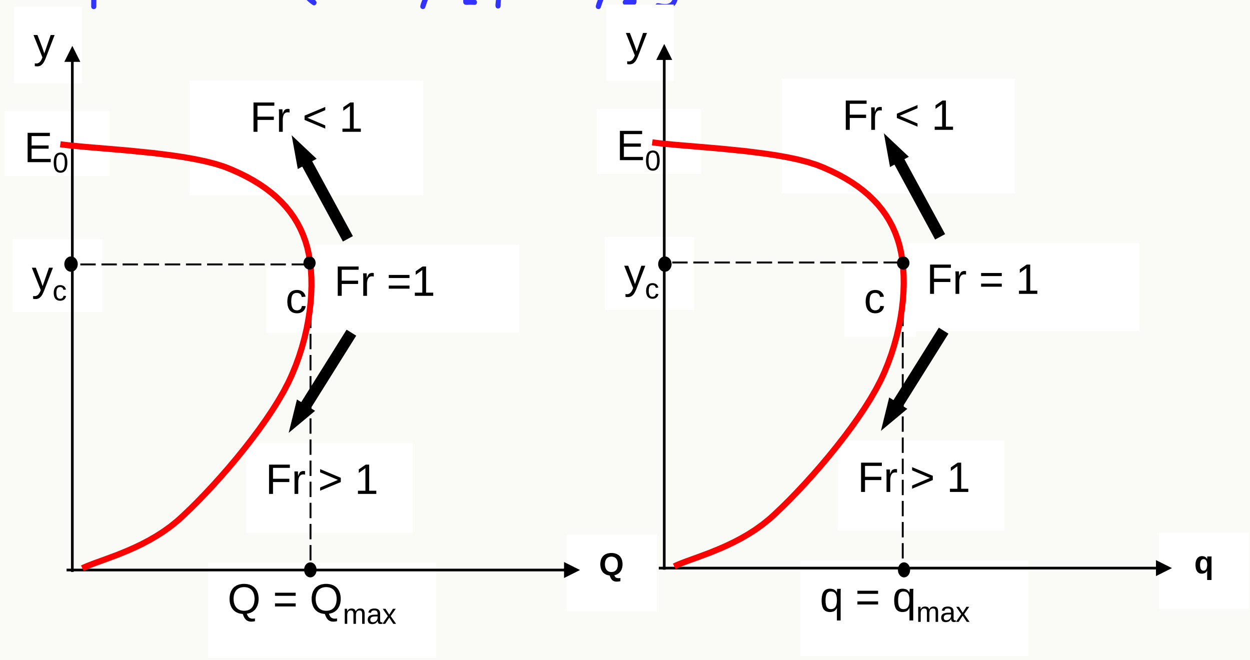
$$y_c = \frac{2}{3} E_c$$

Koch Parabola

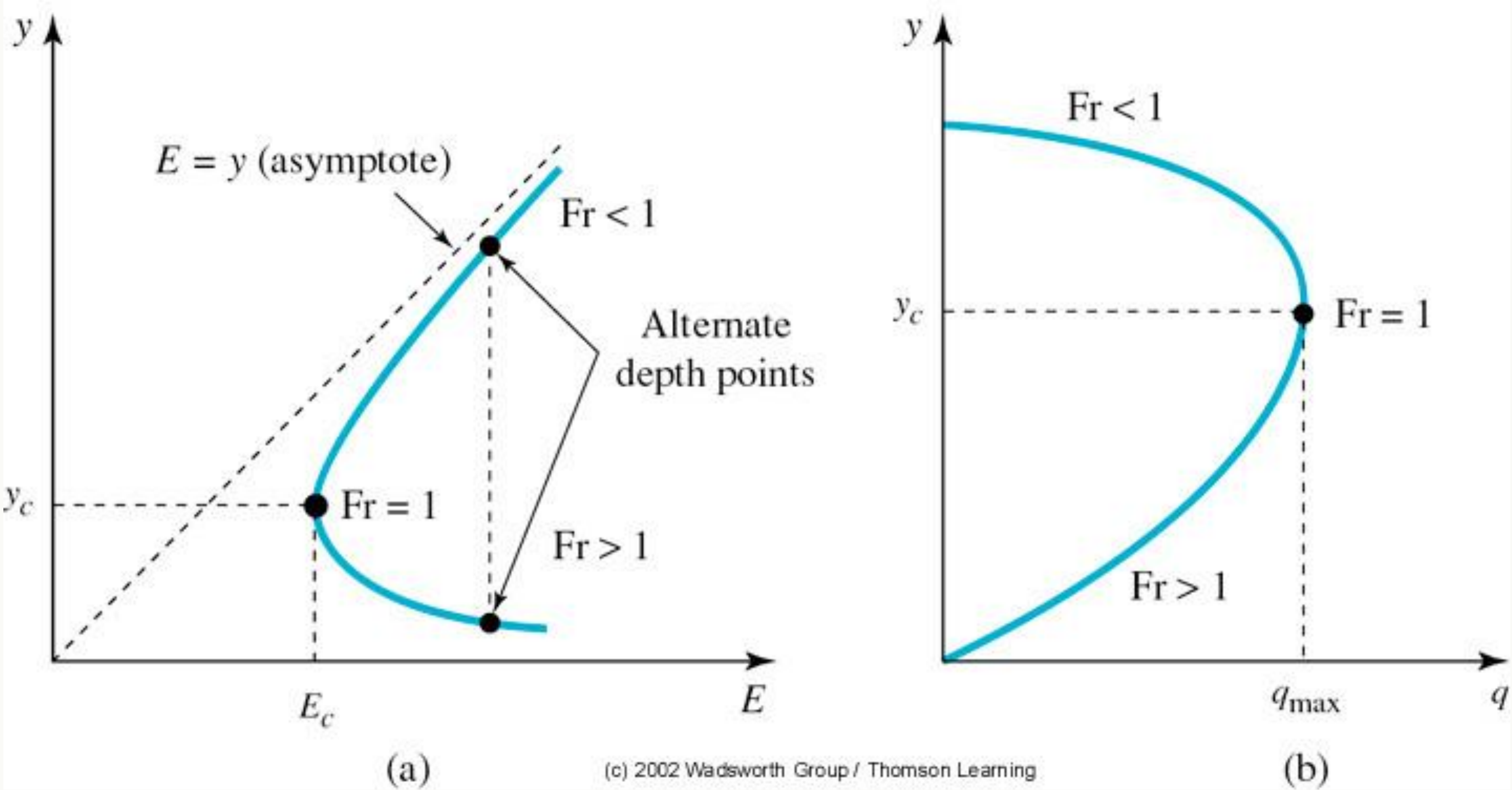
- For a given E,

$$Q = [2gA^2(E - y)]^{1/2} \qquad \text{or} \qquad q = y\sqrt{2g(E \quad y)}$$

- and since A=A(y), then Q = Q(y) [q=q(y) for rectangular channels]
- The plot of Q vs. y [q vs y] gives Koch Parabola.



Therefore, the total discharge Q for arbitrary cross sections, and unit discharge q for rectangular cross sections become maximum for a given E, when the flow is critical.



Variation of specific energy and (unit) discharge with depth: (a) *E* versus *y* for constant *q*; (b) (*q) Q* versus *y* for constant *E*.

Characteristics of Critical Flow

Arbitrary Cross Section	Rectangular Cross Section
• $Fr = 1 \rightarrow \frac{Q^2}{g} = \frac{A_c^3}{T_c}$	• $Fr = 1 \quad q^2 = gy_c^3 \quad y_c = \sqrt[3]{\frac{q^2}{g}}$
• $\frac{V_c^2}{2g} = \frac{D_c}{2} \rightarrow E_c = y_c + \frac{D_c}{2}$	• $\frac{V_c^2}{2g} = \frac{y_c}{2} \rightarrow E_c = \frac{3}{2} y_c$
• For a given Q, $E = E_{min}$	• For a given q, $E = E_{min}$
• For a given specific energy,	• For a given specific energy
E_o, $Q = Q_{max}$	E_o, $q = q_{max}$

Discussion

A long rectangular channel carries water with a flow depth of y_1 on a horizontal channel. If there is a rise on the channel bed:

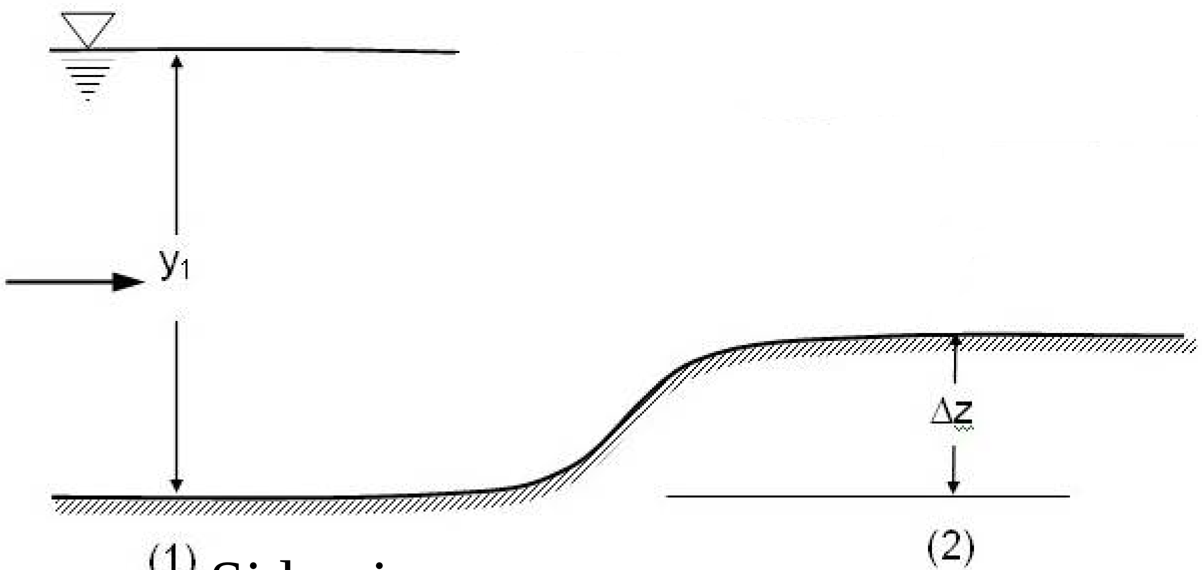
a) What is the relation of total head between section 1 and 2, if head loss assumed to be negligible.

$$H_1 = H_2$$

a) What is the relation of specific energy between section 1 and 2

$$E_1 = E_2 + \Delta z$$

b) How does the water surface profile react to channel bed elevation change



Side view

CHANNEL TRANSITION
for rectangular channels

- Change on the bottom elevation of channel
- Change on the width of the channel
- Change on the bottom elevation and width of the channel

Upward Step-Constant width

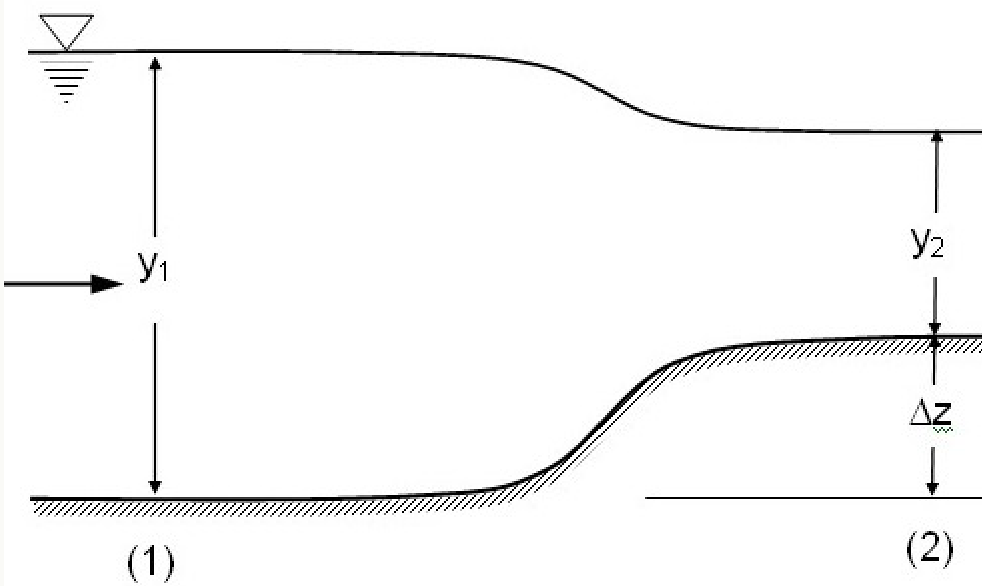
$$Q = VA = V_1(y_1 b) = V_2(y_2 b) \Rightarrow q_1 = q_2$$

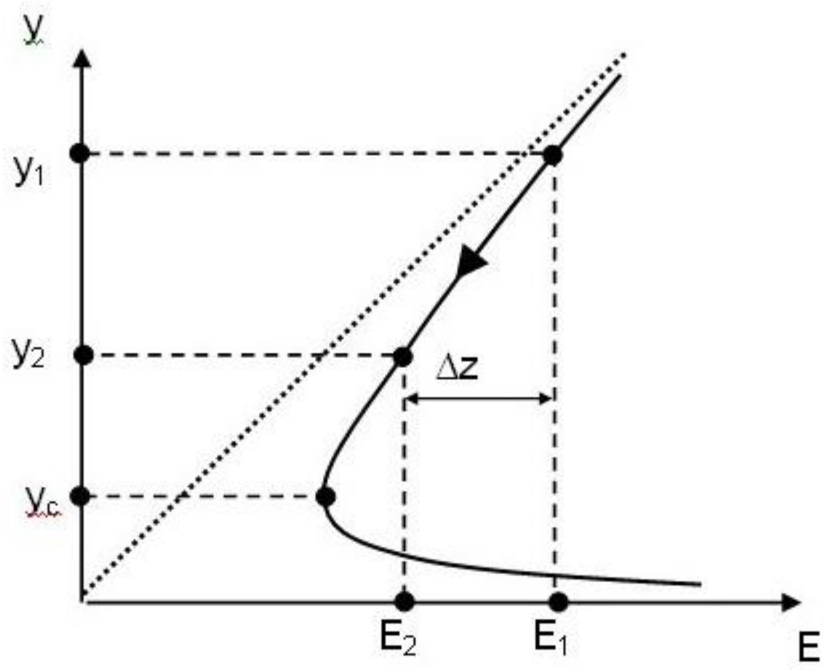
$$H_1 = H_2$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$E_1 = E_2 + \Delta z$$

1) Subcritical flow





Upward Step-Constant width

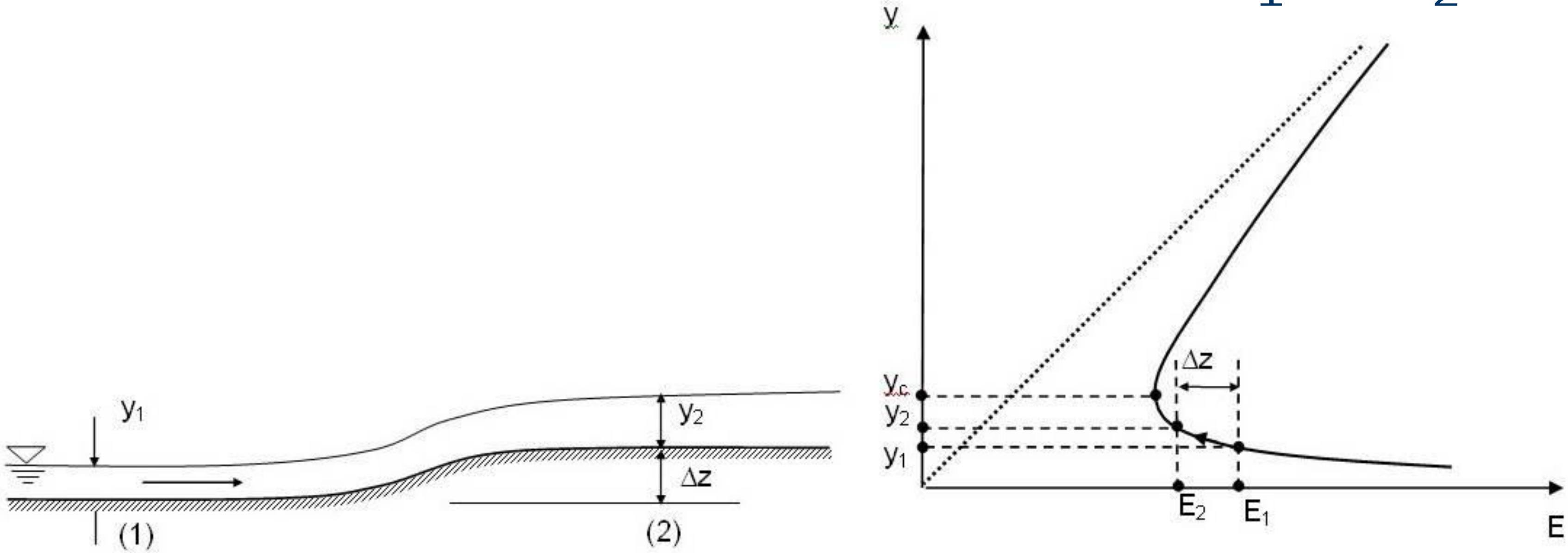
$$q_1 = q_2$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$H_1 = H_2$$

$$E_1 = E_2 + \Delta z$$

2) Supercritical Flow



$E_1 = \Delta z + E_2$

Downward Step-Constant width

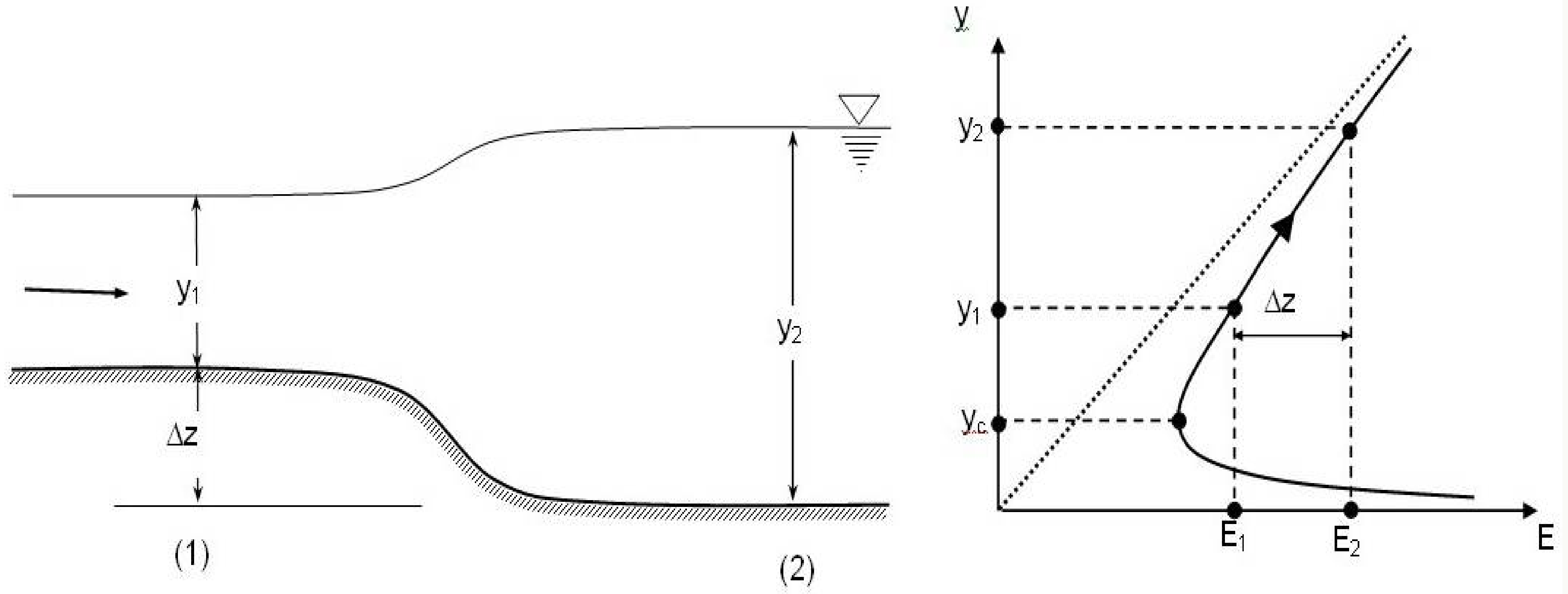
$$Q = VA = V_1(y_1 b) = V_2(y_2 b) \Rightarrow q_1 = q_2$$

$$H_1 = H_2$$

$$E_2 = E_1 + \Delta z$$

1) Subcritical flow

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$



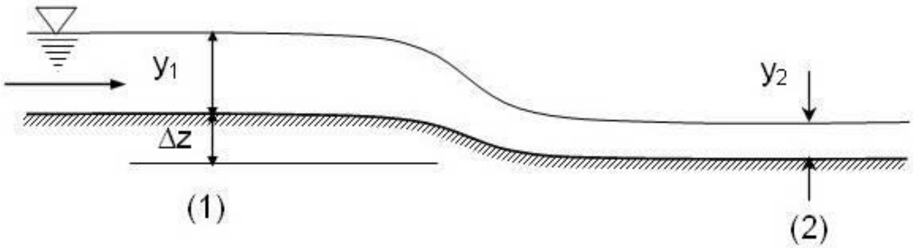
$$q_1 = q_2$$

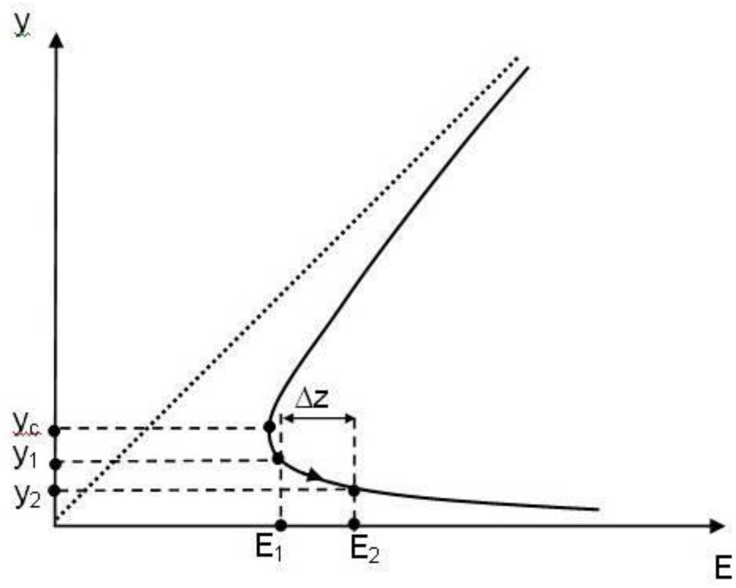
$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$H_1 = H_2$$

$$E_2 = E_1 + \Delta z$$

2) Supercritical Flow

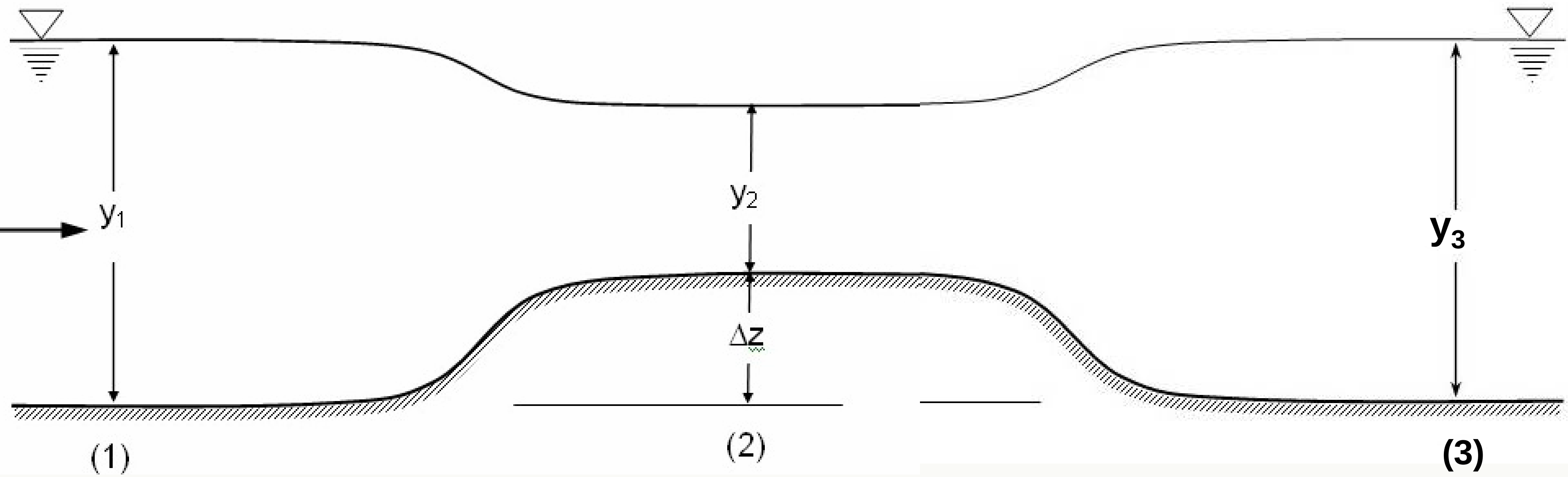




$E_1 + \Delta z = E_2$

Upward Step-Constant width

Subcritical flow



$$E_1 = E_2 + \Delta z$$

$$E_3 = E_2 + \Delta z$$

Specific Energy: Step Up Additional Consideration

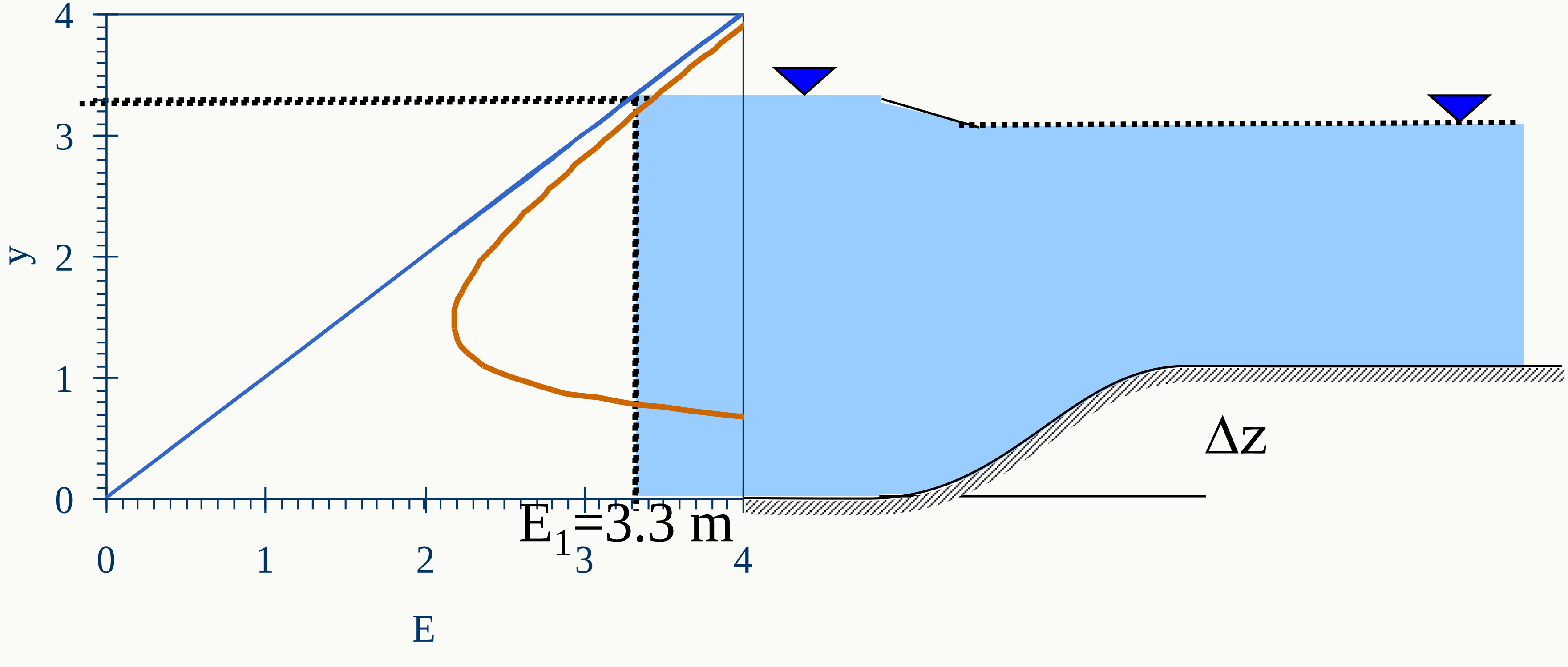
$$q_1 = q_2$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$H_1 = H_2$$

$$E_1 = E_2 + \Delta z$$

$$E_2 = E_1 - \Delta Z$$



Specific Energy: Step Up Additional Consideration

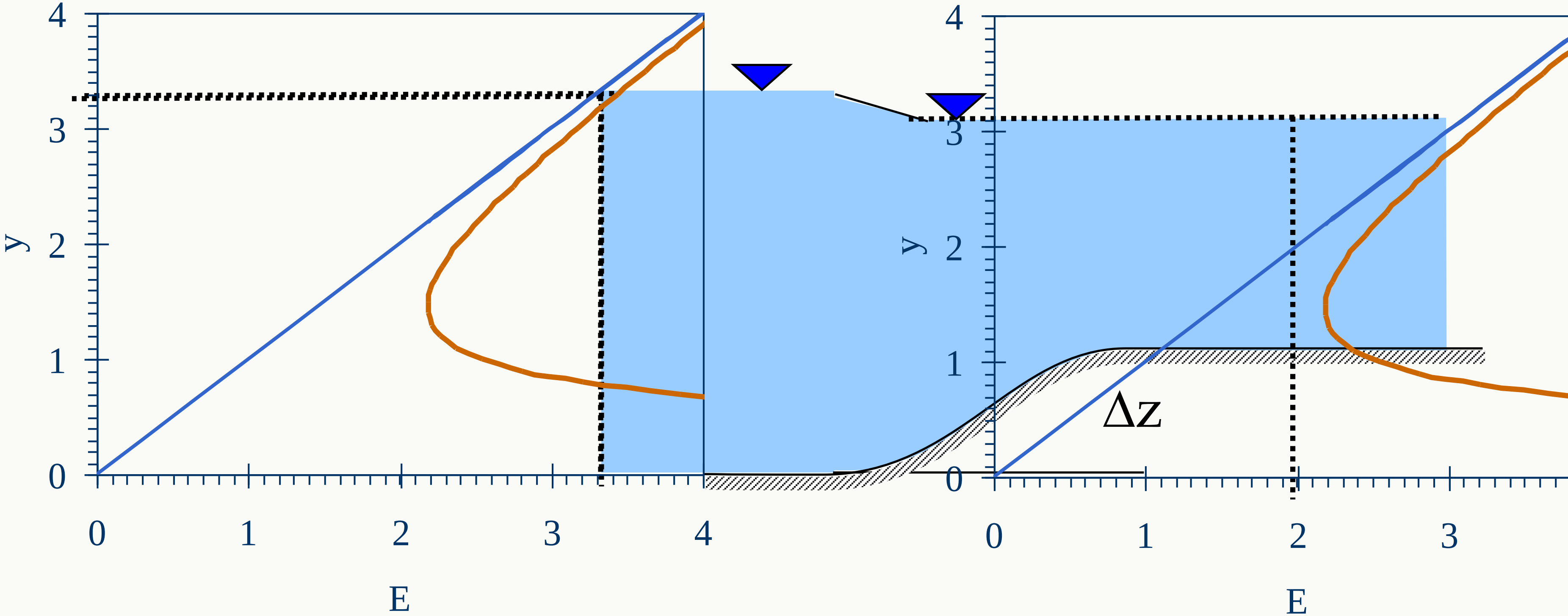
$$q_1 = q_2$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$H_1 = H_2$$

$$E_1 = E_2 + \Delta z$$

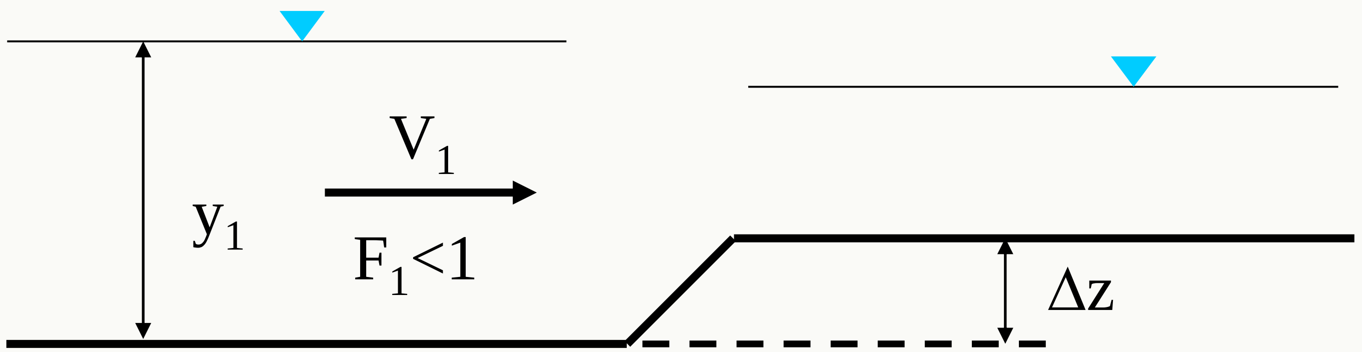
$$E_2 = E_1 - \Delta Z$$



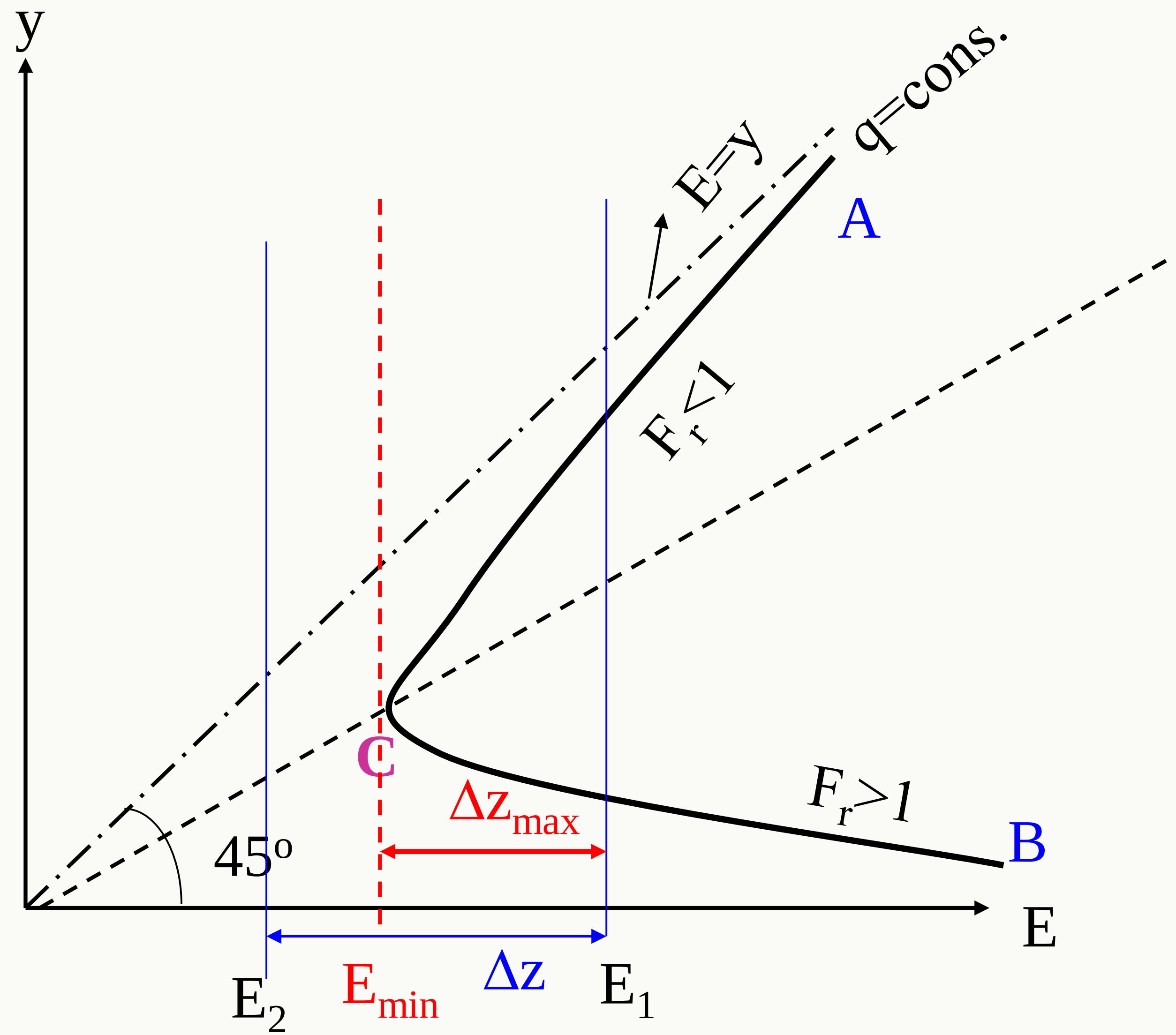
Choking

- In an open-channel flow, whenever there is a contraction in a flow area, there may be a case where the specific energy of the flow may not be enough to pass the given discharge.
- Let us consider different cases where this may happen:

a) Consider the case where there is an upward step in a rectangular channel which has a constant width b:



- The energy equation between sections (1) and (2):
- $E_1=\Delta z+E_2$, or $E_2=E_1-\Delta z$,
- if $E_2<E_{min}$, as shown in figure below,
- What does it mean?



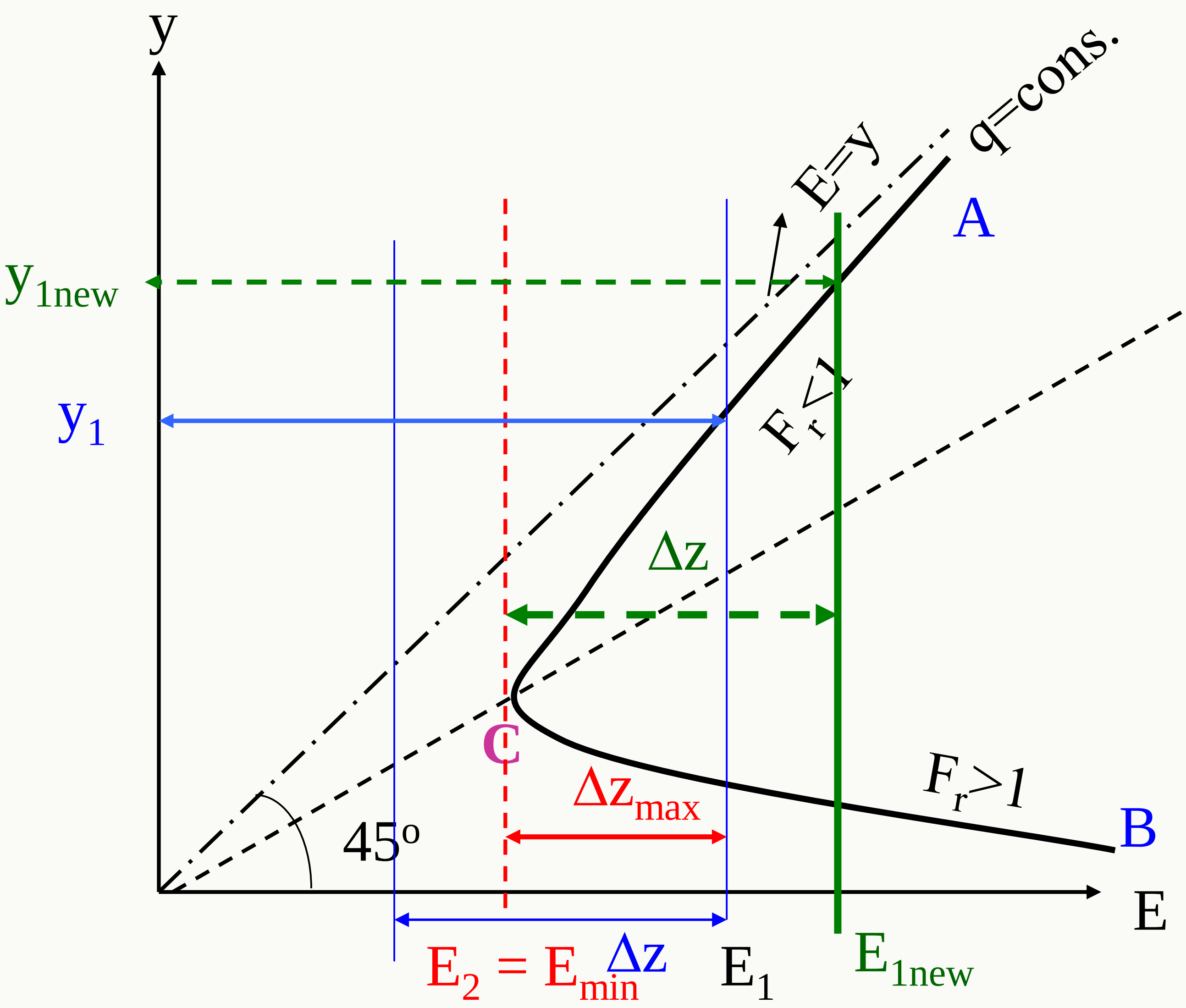
- It means that the available specific energy is not enough to pass the given discharge. Therefore this means that:
- E_1, q, and Δz cannot occur simultaneously in the channel. One of them must change.
- This phenomenon is called choking.
- Which one will change and how will it change?
- In a design problem, we can change Δz easily, such that:

$$\Delta z \leq \Delta z_{max}$$

- However, if it is not a design problem and if Δz is a fixed quantity, then either E_1, or q will change, i.e:
- Either E_1 will increase, or q will decrease.
- Which one will change depends on the controls in the channel.
- Let's see how they will change:

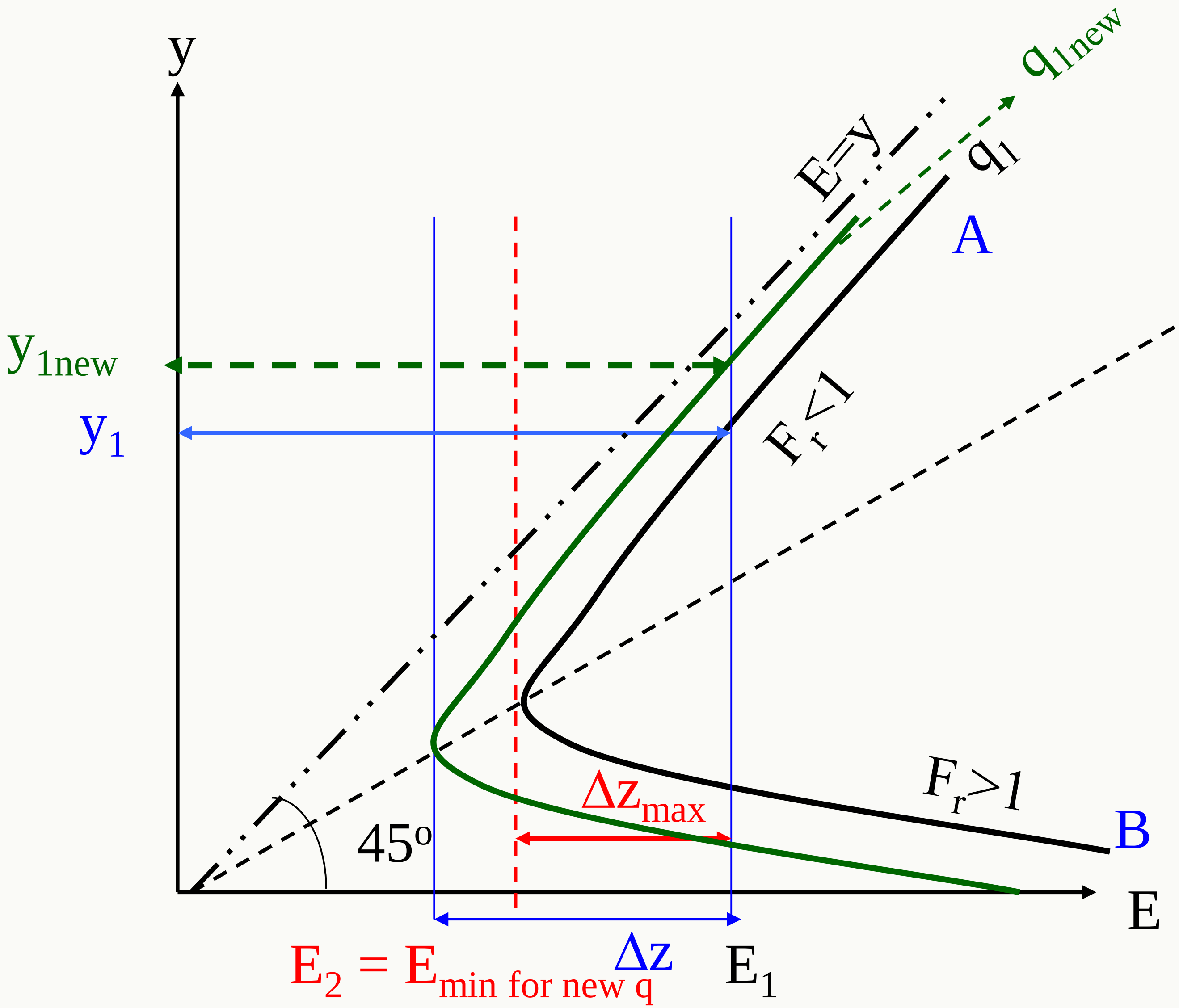
- Case 1) How E_1 will increase?
- E_1 will increase such that $E_2 = E_1 - \Delta z$ have a point of contact with q constant curve. Therefore:
- $E_2 = E_1 - \Delta z = E_{min}$ for the given q value.

Change in E1, keeping q constant

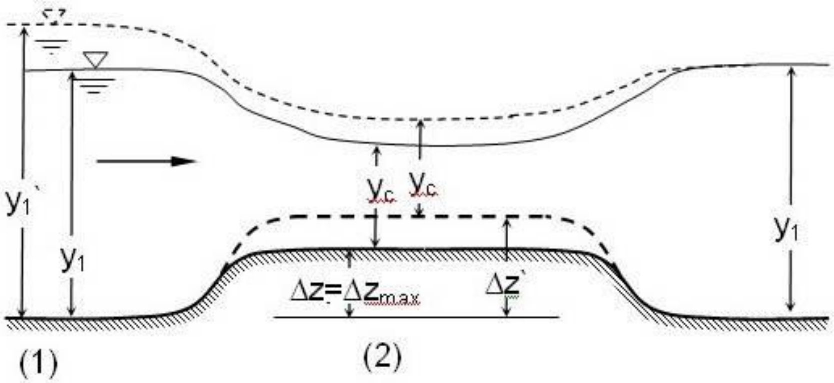


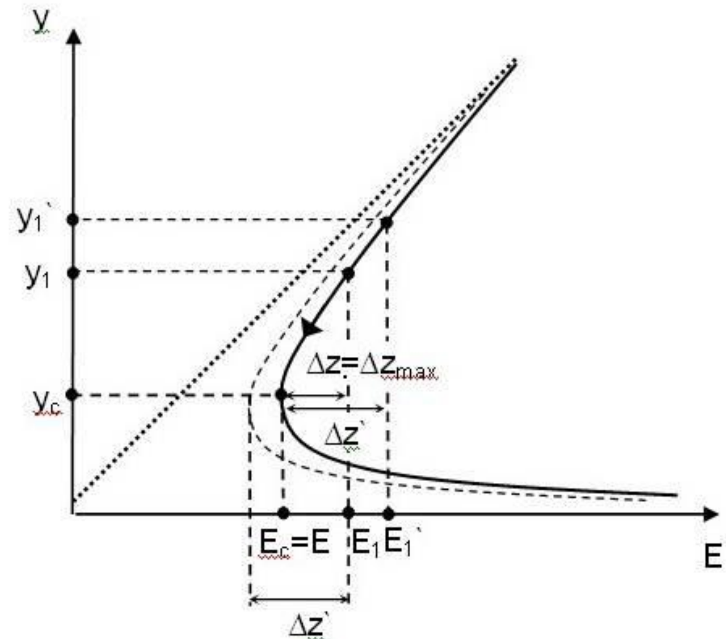
- Case 2) How q will decrease?
- q will decrease such that $E_1 - \Delta z$ have a point of contact with a new q constant curve. Therefore:
- $E_2 = E_1 - \Delta z = E_{min}$ for the new q value.

Change in q, keeping E1 constant



1) Upward step





$$E_1 = \Delta z_{max} + E_c \ , \ \Delta z` > \Delta z_{max}$$

$$E_1` = \Delta z` + E_c$$

$$q_1 = q_2$$

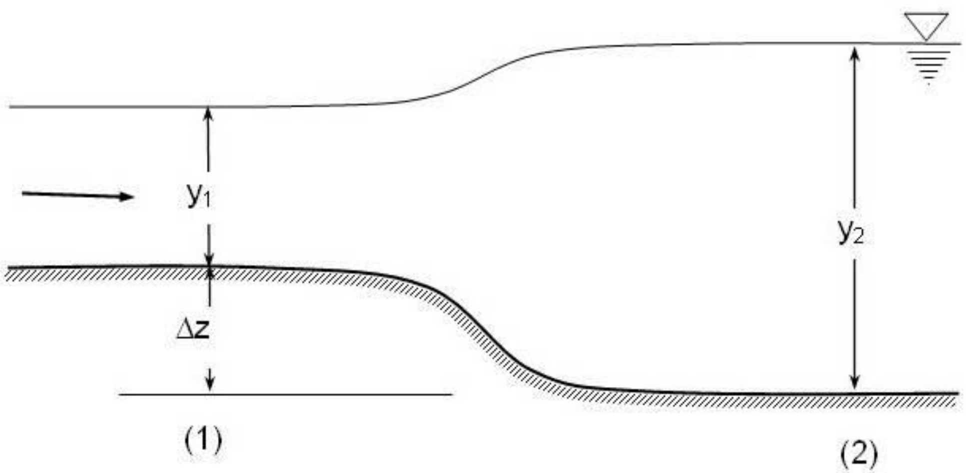
$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

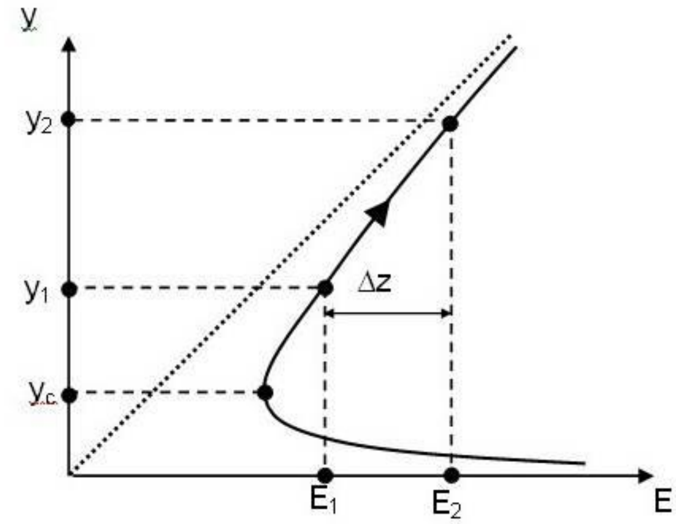
B.) Downward Step (Constant Width)

$$H_1 = H_2$$

$$E_1 = E_2 - \Delta z$$

1) Subcritical Flow





$E_1 + \Delta z = E_2$

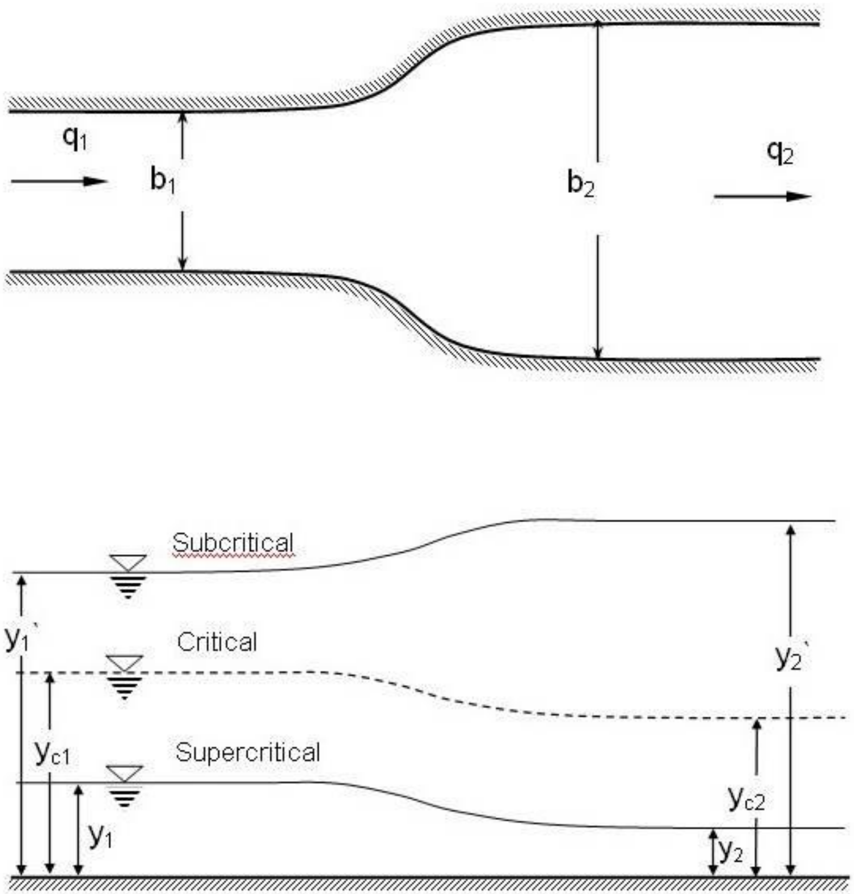
Channel Expansion (constant bed elevation)

$$q_1 \neq q_2$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

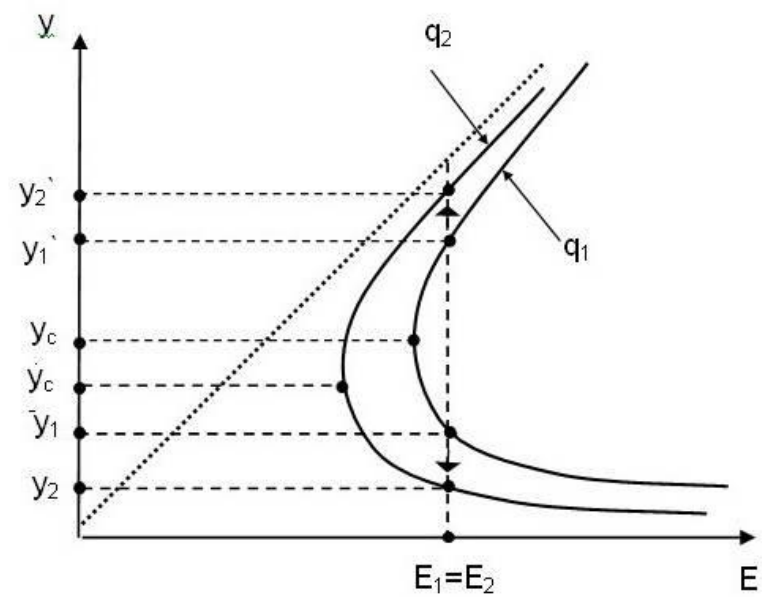
$$H_1 = H_2$$

$$E_1 = E_2$$



$q_1=Q/b_1$

$q_2=Q/b_2$



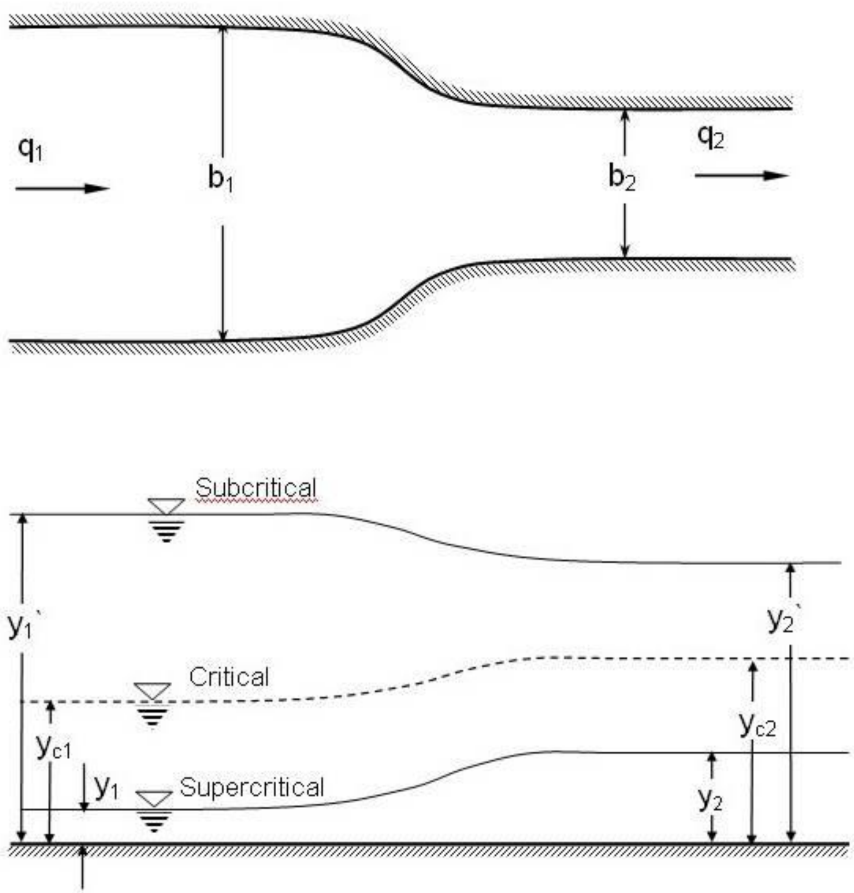
Channel Contraction (constant bed elevation)

$$q_1 \neq q_2$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

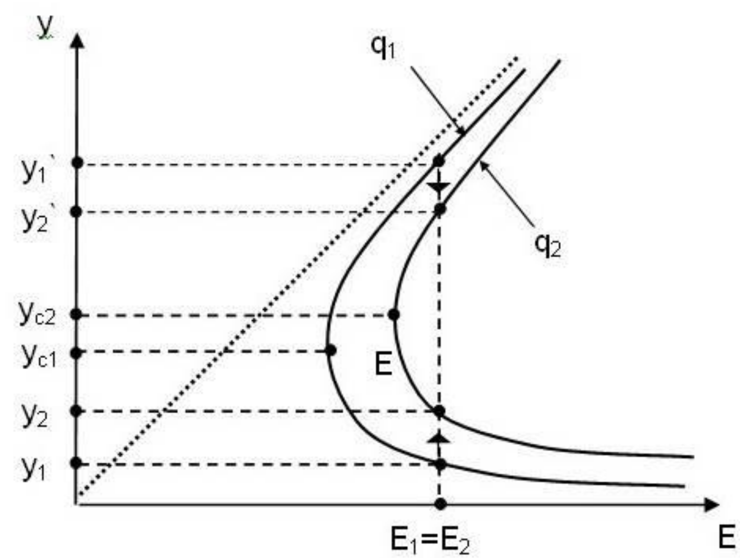
$$H_1 = H_2$$

$$E_1 = E_2$$



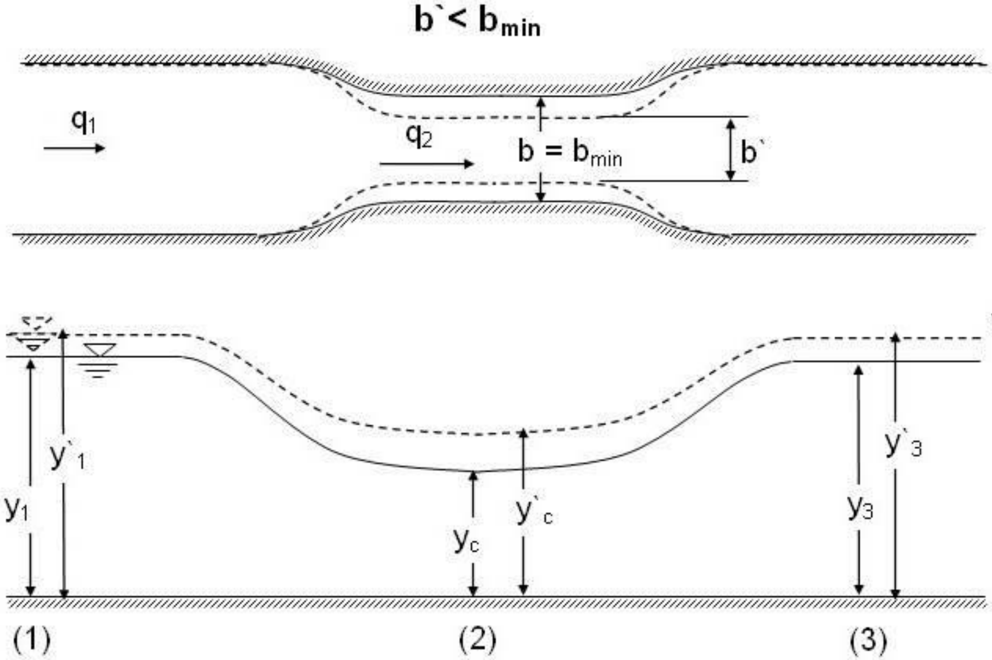
$q_1 = Q/b_1$

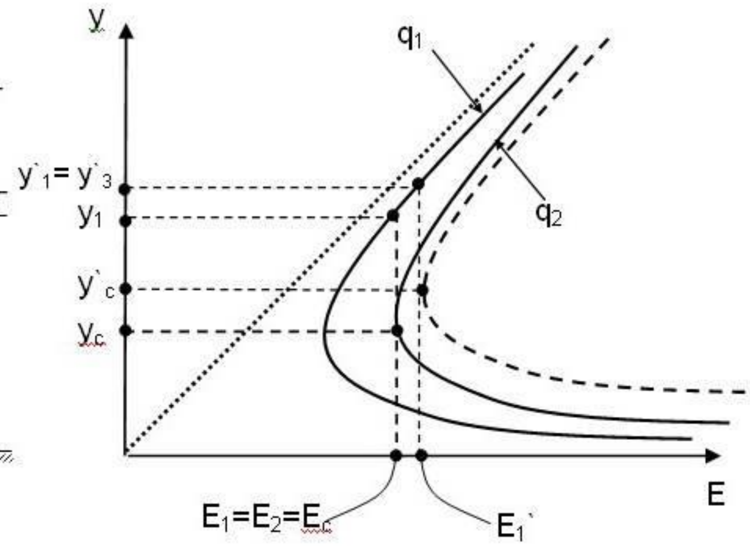
$q_2 = Q/b_2$



CHOKING

2) Contraction





$$E_1 = (E_c)_2 = E_3 \quad \text{if} \quad b = b_{min}, \quad (E_c)_2 = \frac{3}{2} y_c$$

$$E_1' = (E_c)_2' = E_3' \quad \text{if} \quad b < b_{min}, \quad (E_c)_2' = \frac{3}{2} y'$$

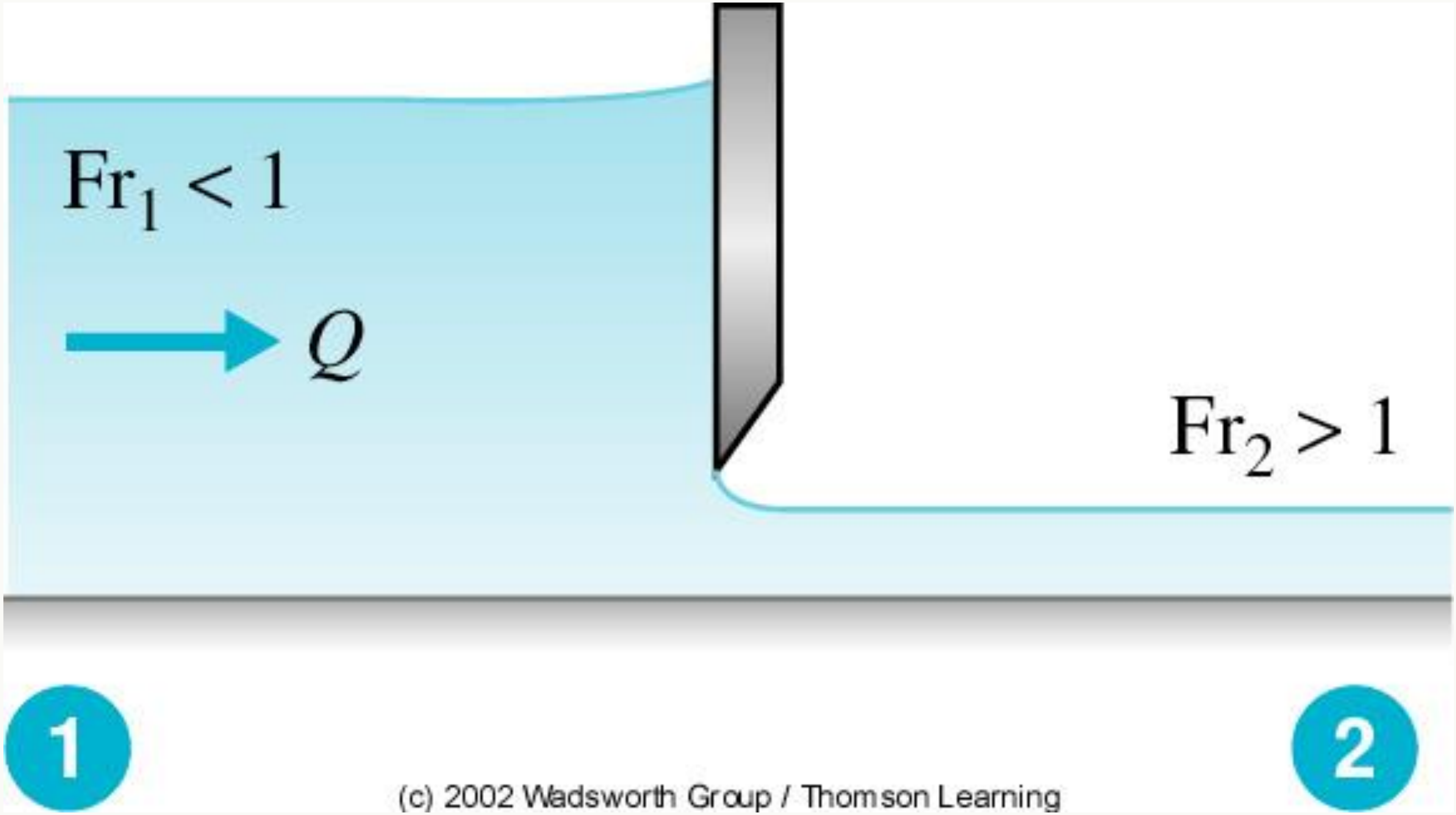


Figure E10.7

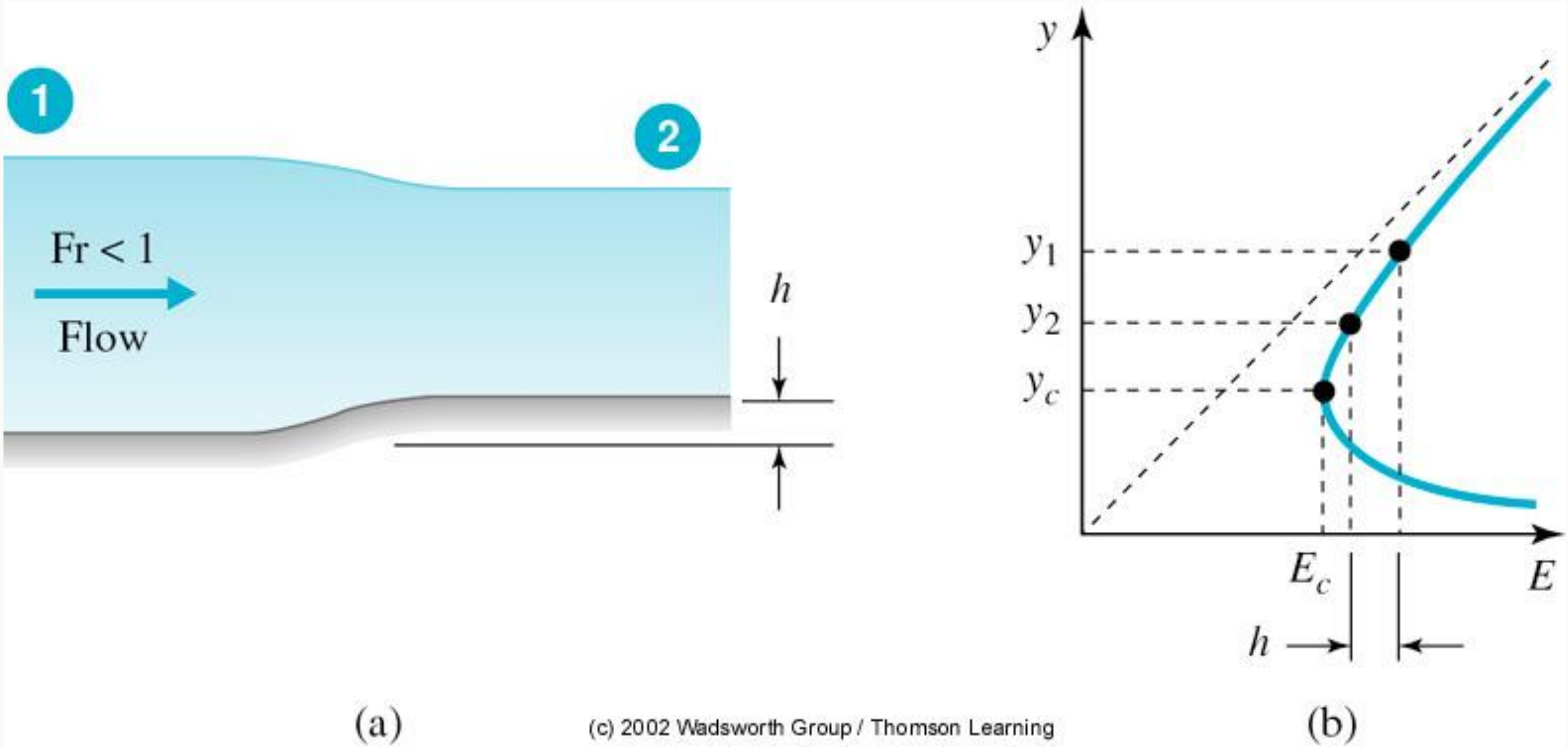


Figure 10.7 – Channel constriction: (a) raised channel bottom; (b) specific energy diagram.

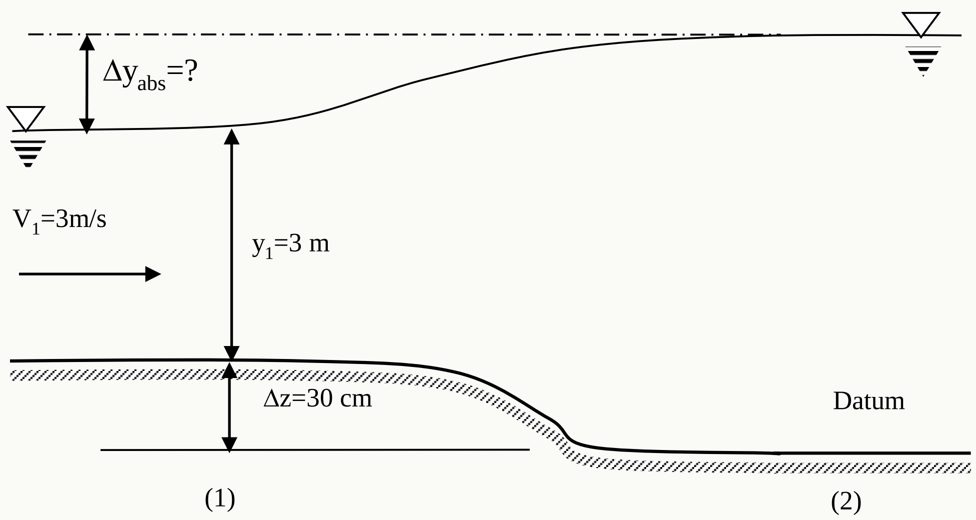
Example 3.3

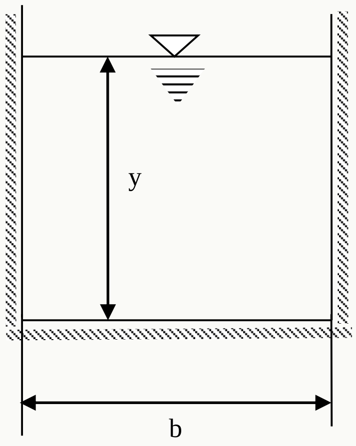
Water is flowing in a rectangular channel. Find the change in depth and in absolute water level produced by a smooth downward step of 0.30 m if the upstream velocity and depth are given as.

a) V_1=3 m/s and y_1=3 m.

b) V_1=5 m/s and y_1=0.60 m.

Draw the water-surface profiles for both cases





Example 3.3 Solution

Energy Eq'n between (1) and (2) :$E_1+\Delta z=E_2+h_l$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 3 + \frac{3^2}{19.62} = 3.46\text{m} \Rightarrow E_2 = 3.46 + 0.30 = 3.76 \text{ m}$$

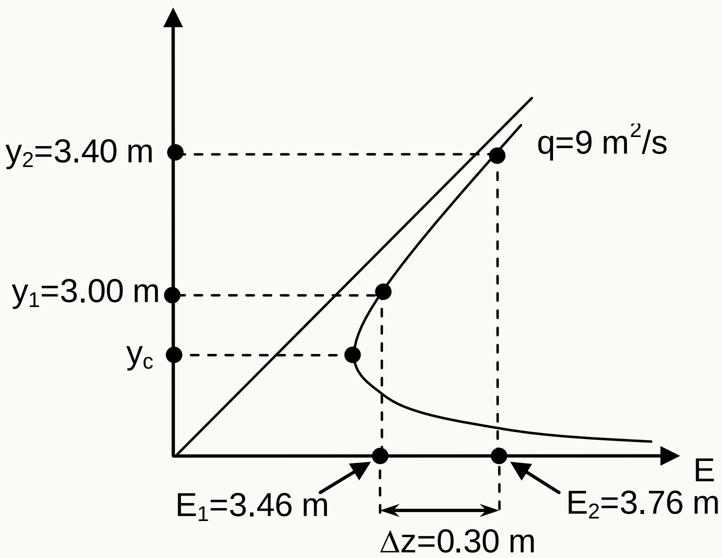
$$E_2 = y_2 + \frac{q^2}{2gy_2^2} \quad ,q = V_1 y_1 = 3 \times 3 = 9\,\text{m}^2/\text{s}$$

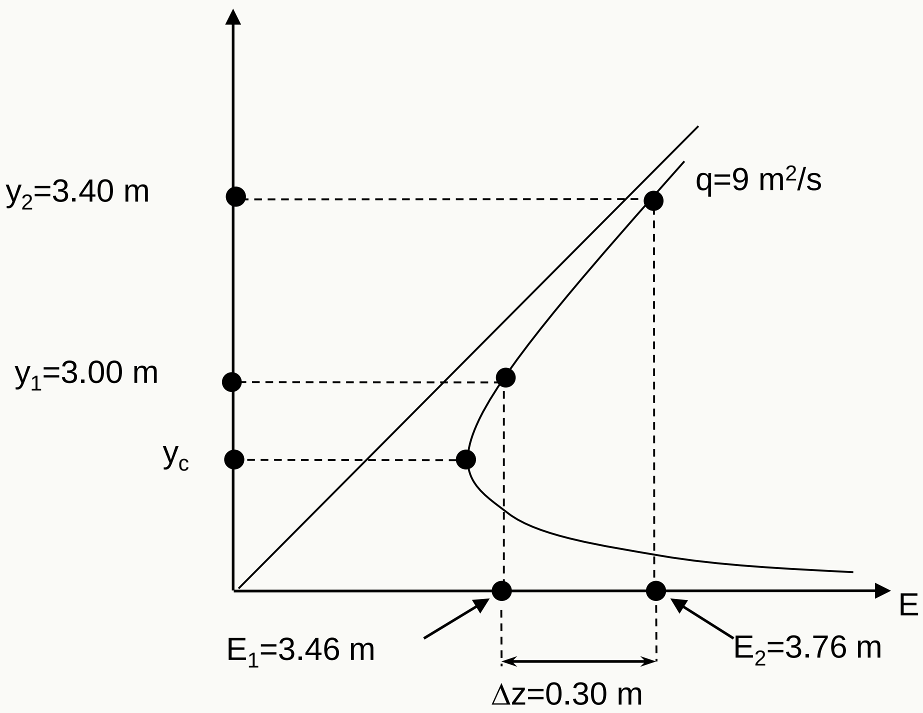
$$3.76 = y_2 + \frac{9^2}{19.62 y_2^2} = y_2 + \frac{4.128}{y_2^2}$$

There are 2 possible solutions. To determine which one occurs we should compute upstream Froude number.

$$F_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{3}{\sqrt{9.81 \times 3}} = 0.553 < 1 \text{ subcritical flow.}$$

Therefore, y_2 will correspond to subcritical flow.





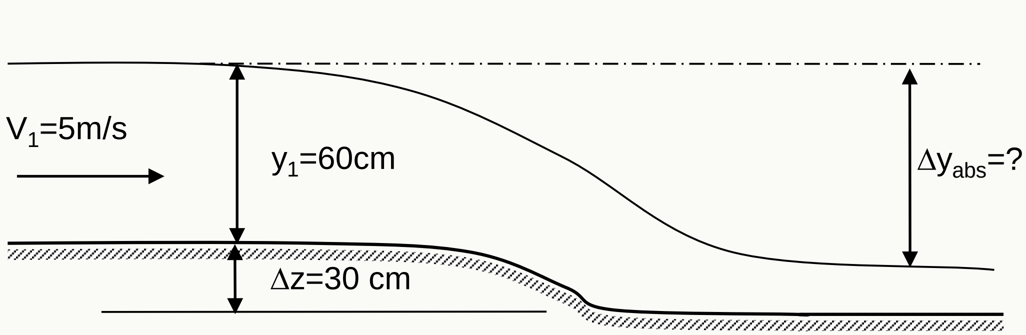
y_2 must be greater than 3 m. The root of greater than 3 m can found by trial and error as;

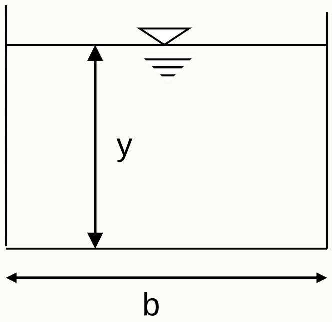
y_2 = 3.40 m

Δy_{abs} = y2 - (Δz + y_1)

= 3.40 - (3.0 + 0.30)

Δy_{abs} = 0.10 m.





Energy equation between section (1) and (2)

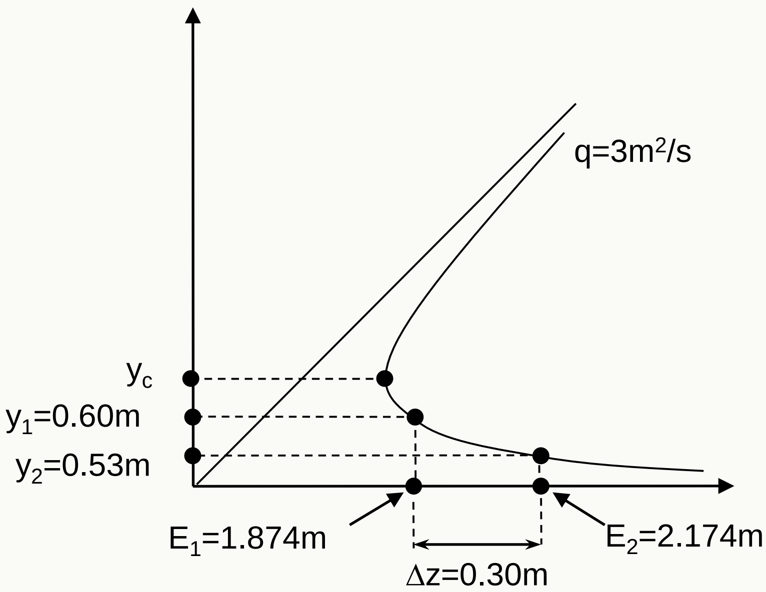
$$\Delta z + E_1 = E_1 + \cancel{h_\ell}^{\cong 0} \qquad E_1 = y_1 + \frac{V_1^2}{2g} = 0.60 + \frac{5^2}{19.62} = 1.874\,\text{m}$$

$$E_2 = 0.30 + 1.874 = 2.174\ \text{m} = y_2 + \frac{q^2}{2gy_2^2}\ ,\ q = V_1 y_1 = 5 \times 0.6 = 3\,\text{m}^2/\text{s}$$

$$2.174\,\text{m} = y_2 + \frac{3^2}{19.62 y_2^2} \Rightarrow 2.174\,\text{m} = y_2 + \frac{0.4587}{y_2^2}$$ **Two possible solutions.**

The upstream Froude number

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{5}{\sqrt{9.81 \times 0.60}} = 2.06 > 1, \text{ supercritical flow}$$



y_2 must be smaller than 0.60 m

can be solved by trial and error to obtain y_2=0.528 m.

Or y_2 = 0.53 m.

$Dy_{abs} = (\Delta z + \square y_1) - y_2$

$= (0.30 + 0.60) - 0.53$

$\Delta y_{abs} = \square 0.37$ m.

Solution of Specific Energy Equation

$$E = y + \frac{q^2}{2gy^2}$$

$$E = y + \frac{q^2}{2gy^2} \Rightarrow E = y + \frac{C}{y^2}$$

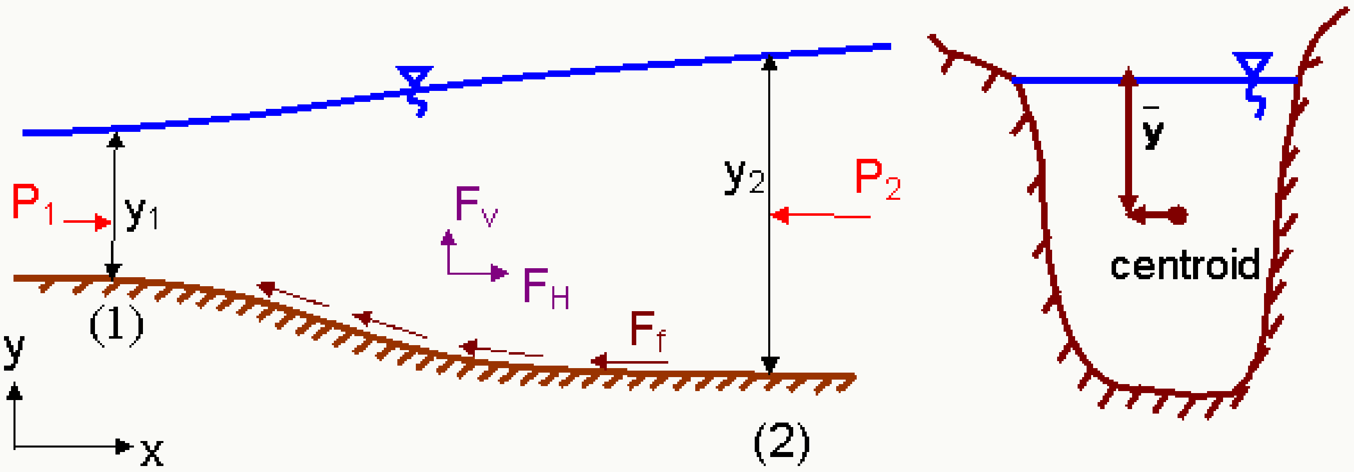
Subcritical Root

$$y^* = E - \frac{C}{y^2}$$

Supercritical root

$$y^* = \sqrt{\frac{E - y}{C}}$$

SPECIFIC FORCE CONCEPT



Momentum Equation in x-direction:

$$F_T = P_1 - P_2 - F_f + F_H = \rho Q V_2 - \rho Q V_1 \quad \text{........} \quad (1)$$

For uniform vel. distn. and horizontal channel,

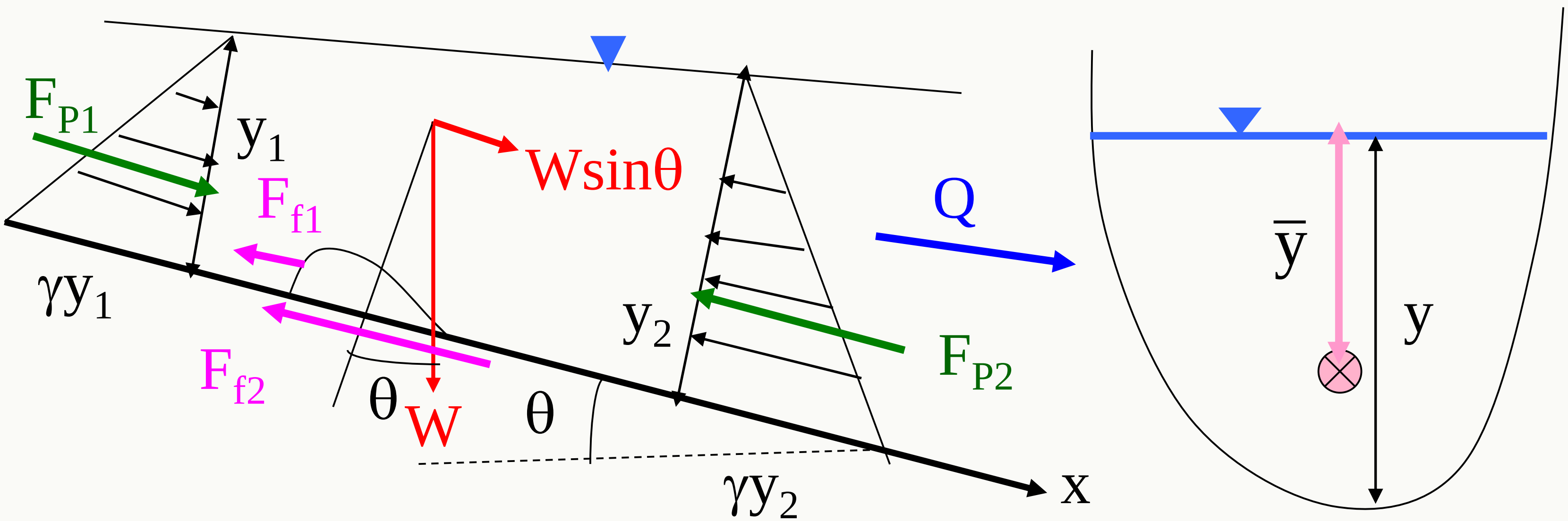
$$F_T = \rho Q (V_2 - V_1)$$

If the vel. distribution is not uniform,

$$F_T = \rho Q (\beta_2 V_2 - \beta_1 V_1)$$ **where β is the momentum correction factor**

Momentum Equation in Open-Channel flow

- Consider a steady flow in an open-channel of arbitrary cross section:



- The momentum Equation along the x-direction is:

- $F_{P1} - F_{P2} - F_{f1} - F_{f2} + W\sin\theta = \rho Q(\beta_2 V_2 - \beta_1 V_1)$

- Where:
- F_{pi}= pressure force at section (i), i=1,2
- F_{f1}=resistance to the flow applied by the obstacle in the channel
- F_{f2}= resistance due to wall shear stress

  Assume that $\beta_1=\beta_2=1$, and $P_f=F_{f1}+F_{f2}$= total resistance to the flow

  note that $V_i=Q/A_i$, i=1,2 and

$$F_{Pi} = \gamma\bar{y}_i A_i \qquad i = 1,2$$

- Where $\bar{y}$ is the depth of the centroid of the cross section
- Substituting these into the momentum equation:

$$\gamma\bar{y}_1 A_1 \quad \gamma\bar{y}_2 A_2 \quad P_f + W\sin\theta = \rho Q^2 \quad \frac{1}{A_2} \quad \frac{1}{A_1}$$

- Now, let's divide by γ, and assume that θ is small, and reaarange the equation as:

$$\frac{P_f}{\gamma} = \bar{y}_1 A_1 + \frac{Q^2}{gA_1} \qquad \bar{y}_2 A_2 + \frac{Q^2}{gA_2}$$

In this equation, the total friction force is in the form of difference of two terms. These terms are in the form as:

$$\underbrace{F}_{\text{Specific Force}} = \underbrace{\bar{y}}_{\substack{\text{pressure force}\\ \text{per unit weight}}} + \underbrace{\frac{Q^2}{gA}}_{\substack{\text{momentum flux}\\ \text{per unit weight}}}$$

The term F is defined as the specific force in open-channel flow. In terms of specific force above equation becomes:

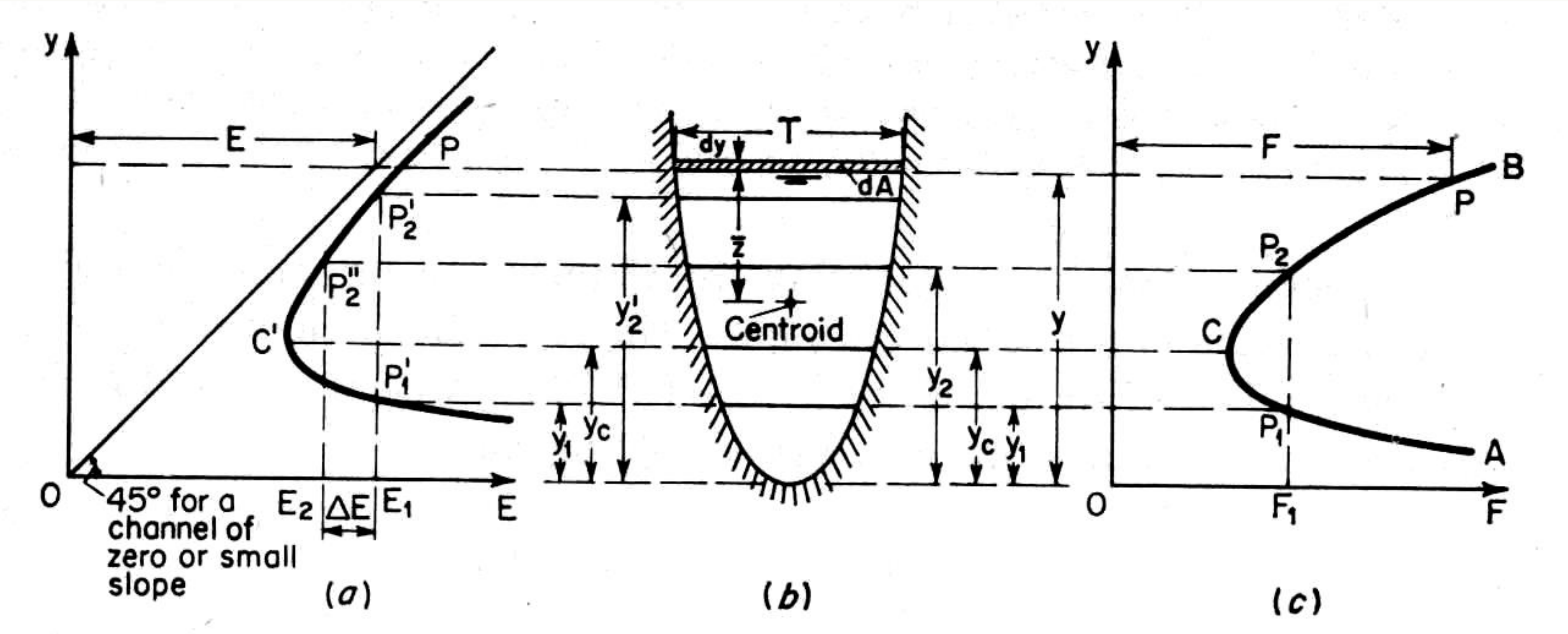
$$\frac{P_f}{\gamma} = F_1 \qquad F_2$$

Hence, momentum equation is reduced to a simple force balance equation in the direction of flow.

Specific force for any channel section

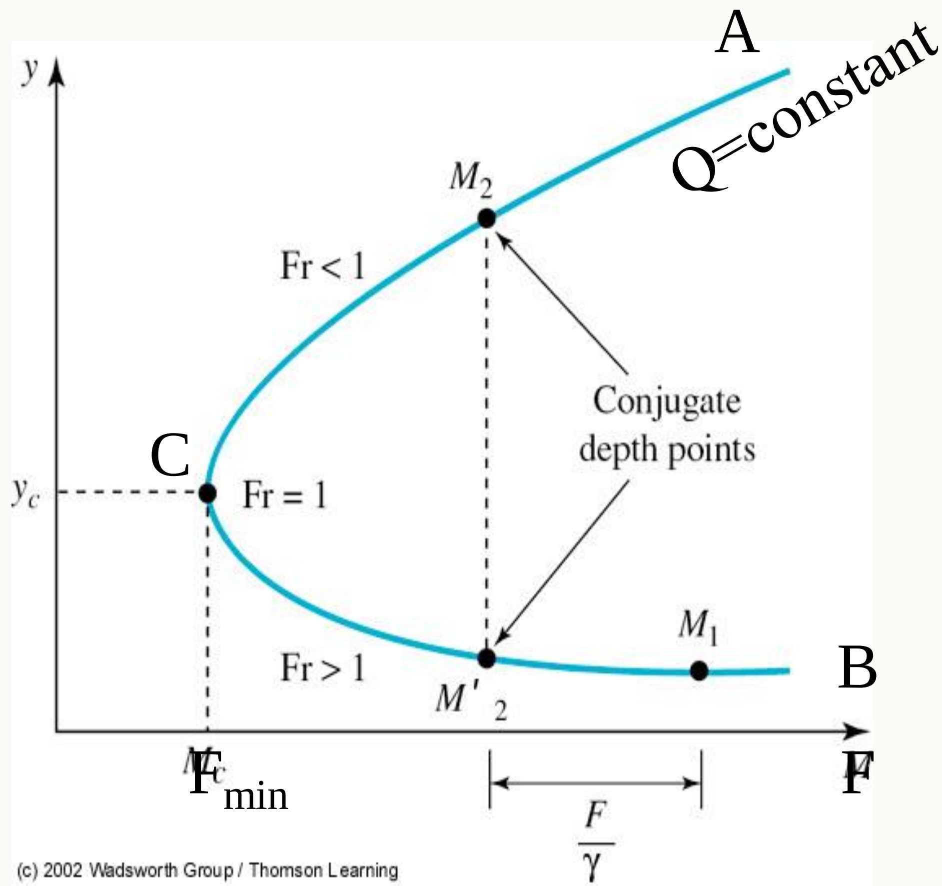
$$F = \bar{y}A + \frac{Q^2}{gA}$$

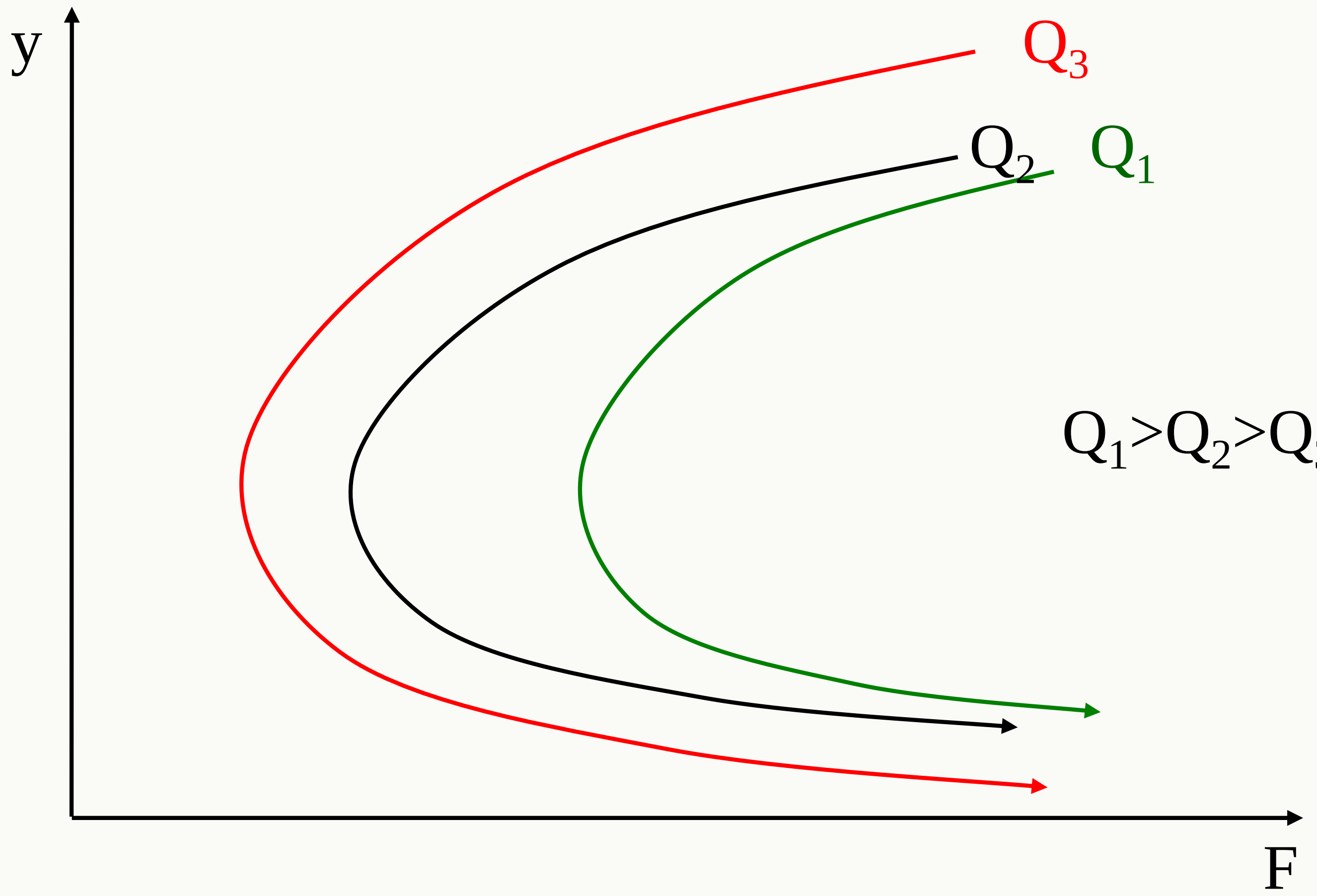
For a given Q, F = F (y)
The plot of F vs y gives specific force curve



Characteristics of the Specific Force Curve:

- Specific force curve has two limps AC and BC
- At point C, flow is critical, and
- Specific force becomes minimum when the flow is critical
- Lower limp AC corresponds to supercritical flow
- Upper limp BC corresponds to subcritical flow.
- For a given F and Q, there are two possible flow regimes represented by the depths y_1 and y_2 which are called ***conjugate or sequent depths***, corresponding to supercritical and subcritical flows, respectively.





y
Q_3
Q_2
Q_1
$Q_1>Q_2>Q_3$
F

Characteristics of the Specific Force Curve:

- **For rectangular channels**, the specific force can be written as specific force per unit width as

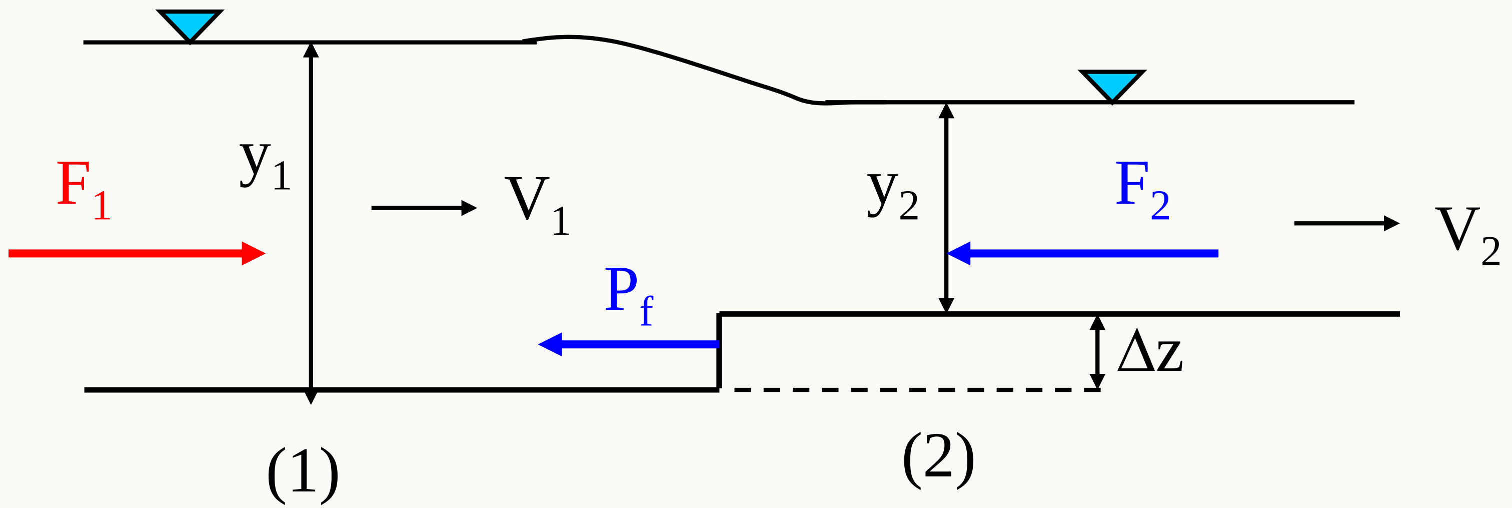
$$\frac{F}{b} = \frac{1}{2}y^2 + \frac{q^2}{gy}$$

Application of Specific Force Concept

- The specific force concept reduces momentum equation to a force balance equation in the direction of flow
- The direction of specific force can be considered as the direction of the pressure force.
- Let us discuss the various flow cases:

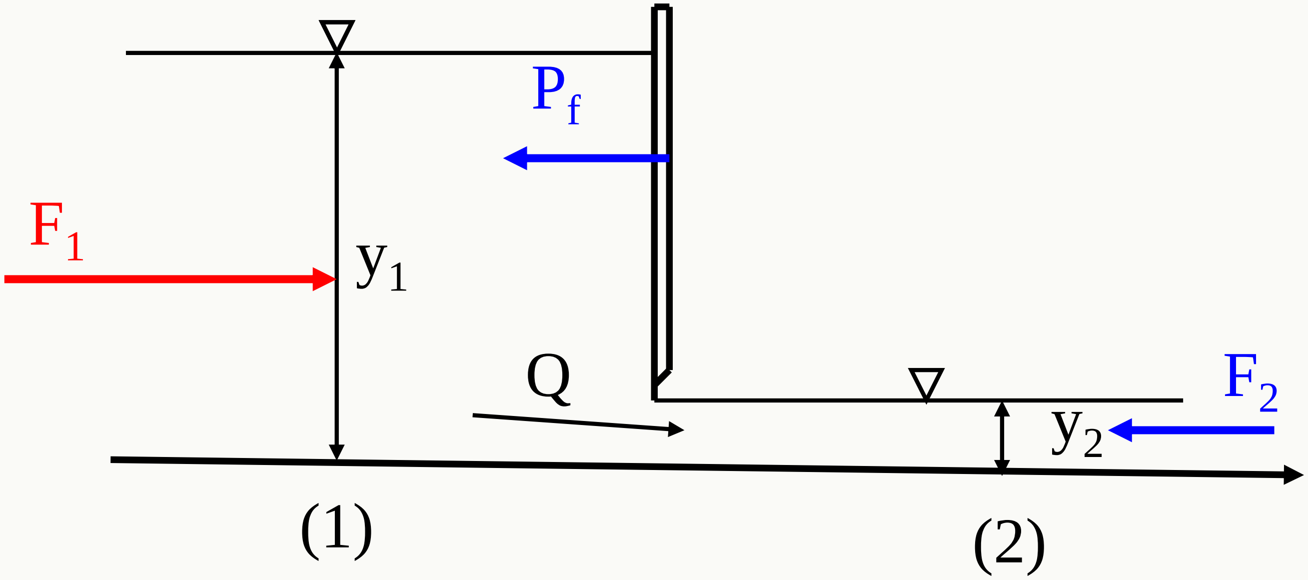
Case A) Head loss, $h_f=0$, $P_f \neq 0$

i) Flow over a smooth step



$$E_1 = \Delta z + E_2, \quad \text{and} \qquad F_1 = F_2 + \frac{P_f}{\gamma}$$

2) Flow under a sluice gate:



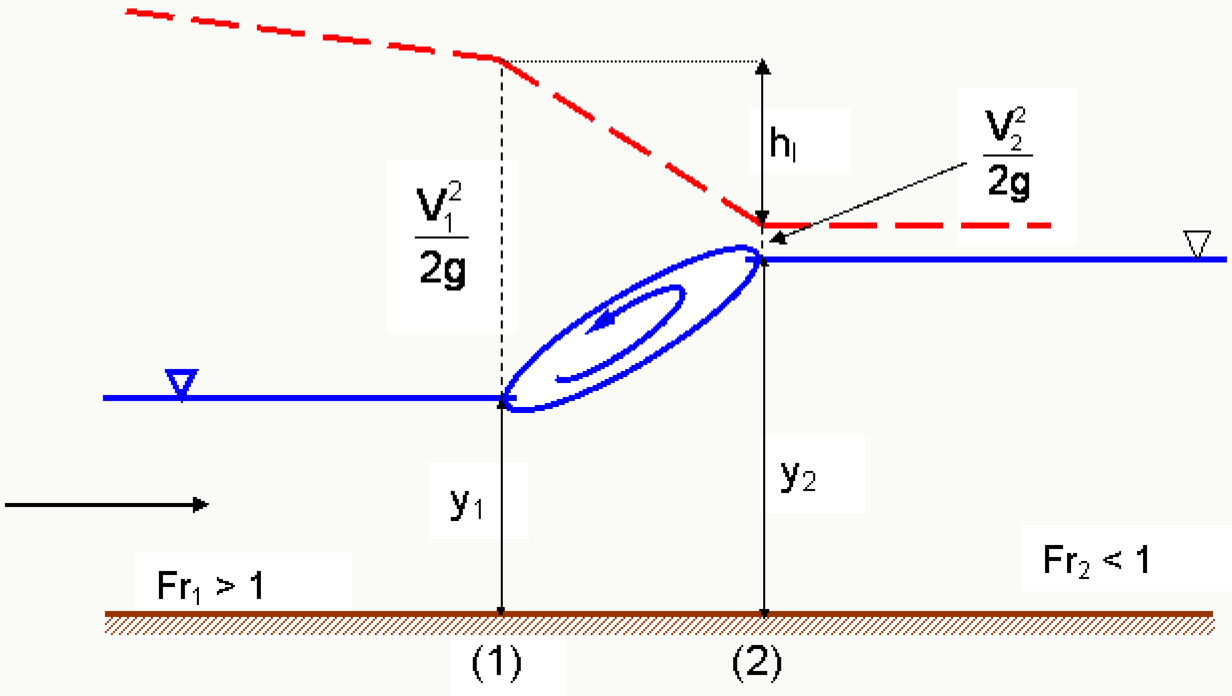
$$E_1 = \Delta z + E_2, \quad \text{and} \qquad F_1 = F_2 + \frac{P_f}{\gamma}$$

Case B) Head loss, hf ≠ 0, Pf=0

This is the case of hydraulic jump. Then let's learn the hydraulic jump.

HYDRAULIC JUMP

- Hydraulic jump is a rapidly varied flow in which flow changes abruptly from supercritical state to a subcritical state accompanied by considerable turbulence and energy loss.



$$F_1 = F_2 \quad \rightarrow \quad \bar{y}_1 A_1 + \frac{Q^2}{gA_1} = \bar{y}_2 A_2 + \frac{Q^2}{gA_2}$$

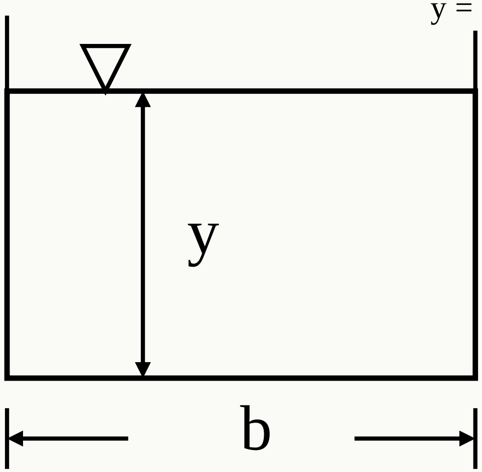
$$E_1 = E_2 + h_\ell$$

Rectangular Channels

$$\mathbf{F_1 = F_2} \qquad \rightarrow \qquad \mathbf{\bar{y}_1 A_1 + \frac{Q^2}{gA_1} = \bar{y}_2 A_2 + \frac{Q^2}{gA_2}}$$

For a rectangular channel, above equation reduces to a simple form:

$\bar{y} = \frac{y}{2}$



A=by, Q=qb

$$\frac{y_1}{2} by_1 + \frac{q^2 b^2}{gby_1} = \frac{y_2}{2} by_2 + \frac{q^2 b^2}{gby_2}$$

Dividing by b, and rearranging:

$$\frac{q^2}{g}\left(\frac{1}{y_1} \quad \frac{1}{y_2}\right) = \frac{1}{2}\left(y_2^2 \quad y_1^2\right) \rightarrow \frac{q^2}{g}\,\frac{y_2 \quad y_1}{y_1 y_2} = \frac{1}{2}\left(y_2 \quad y_1\right)\left(y_2 + y_1\right)$$

Simplifying, and dividing by y_1^2 and reaaranging

$$2\frac{q^2}{g y_1^3} = \frac{y_2}{y_1}\ \frac{y_2}{y_1} + 1 \ = 2F_1^2$$

this equation is a second order polynomial of y_2/y_1:

$$\frac{y_2}{y_1}^{2} + \frac{y_2}{y_1} \quad 2F_1^2 = 0$$

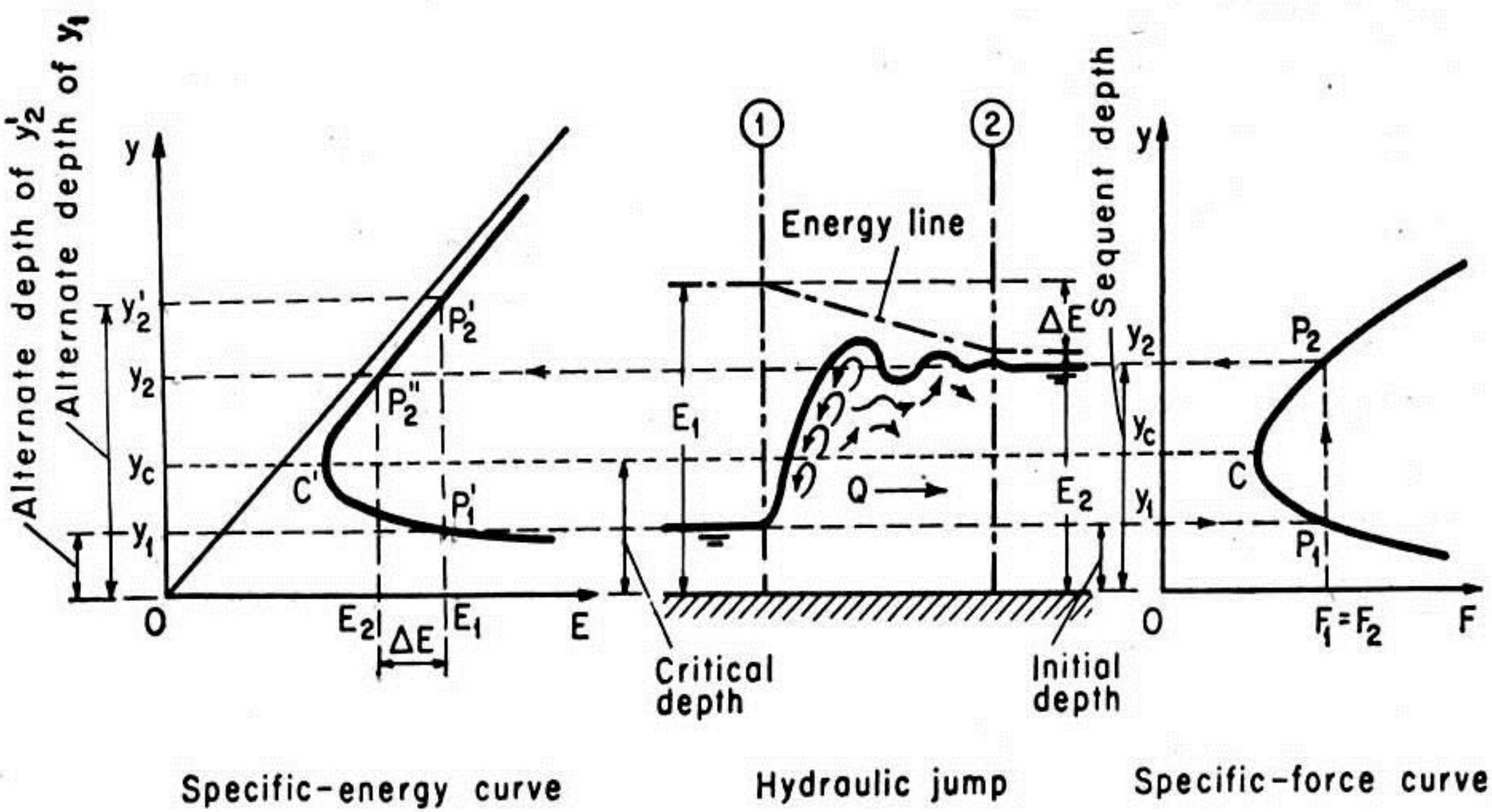
The positive root is the solution:

$$\mathbf{\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1 + 8F_{r1}^2} - 1\right)}$$

The energy equation: $E_1 = E_2 + h_f$, subtituting, E values and after certin manupulations, the head loss can be written as:

$$h_f = \frac{(y_2 \quad y_1)^3}{4 y_1 y_2}$$

For a simple jump, since $F_1 = F_2$, y_1 and y_2 are the conjugate (sequent) depths.



HYDRAULIC JUMP INTERPRETED BY SPECIFIC-ENERGY AND SPECIFIC-FORCE CURVES

PRACTICAL APPLICATIONS OF HYDRAULIC JUMP

- To dissipate energy
- To recover head or raise the water level
- To increase weight on apron
- To mix chemicals used for water purification
- To aerate water

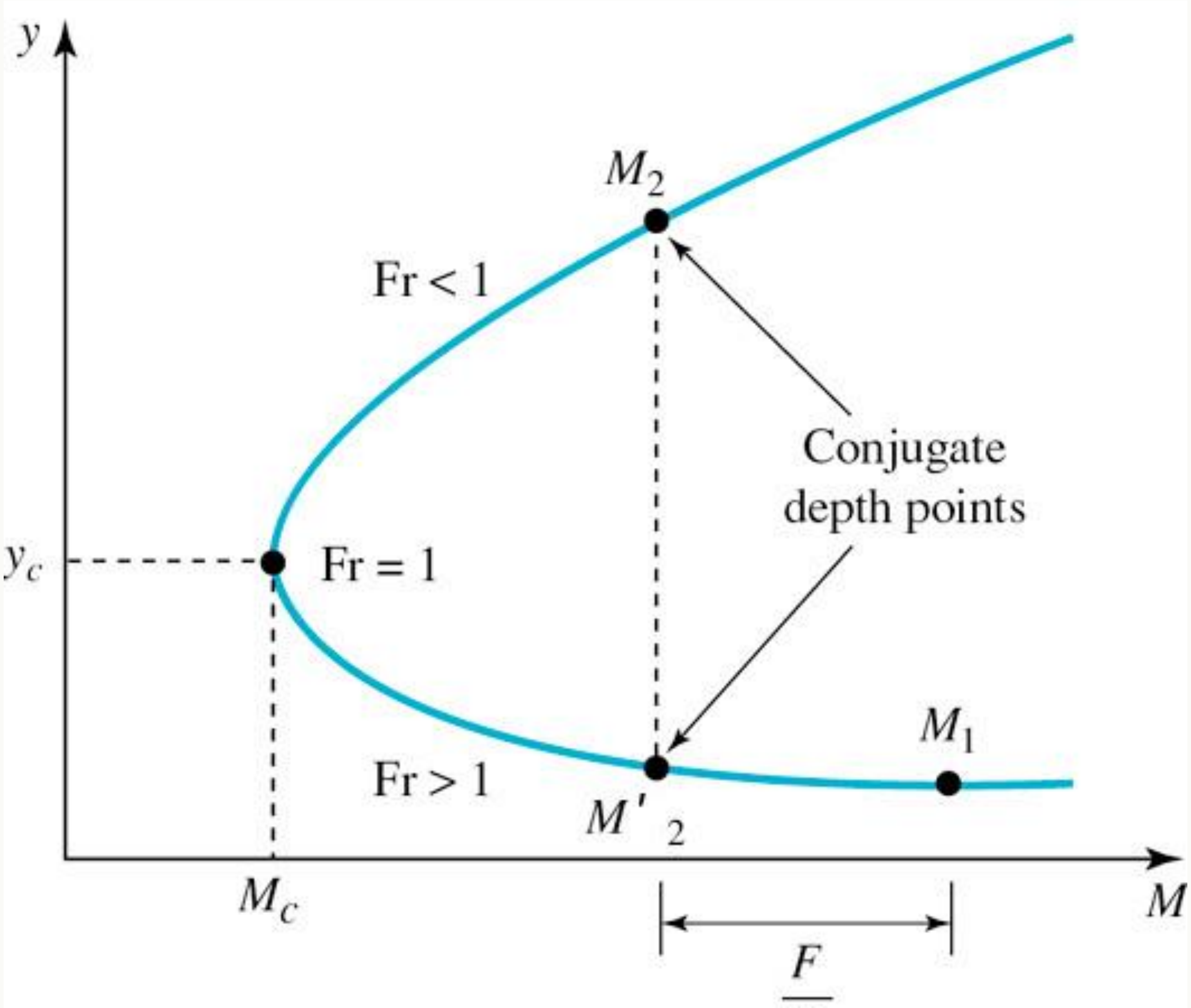
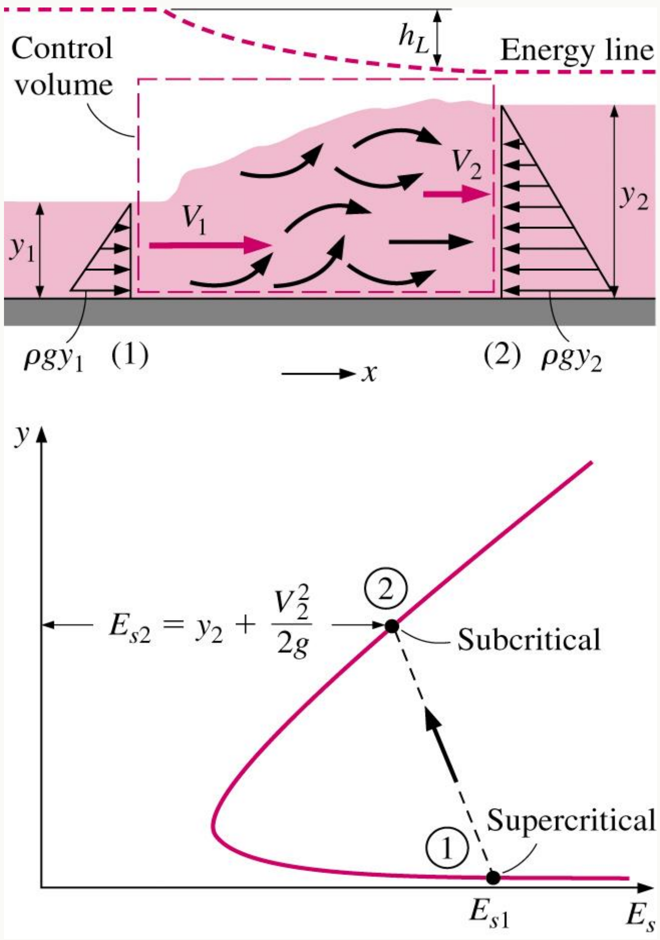


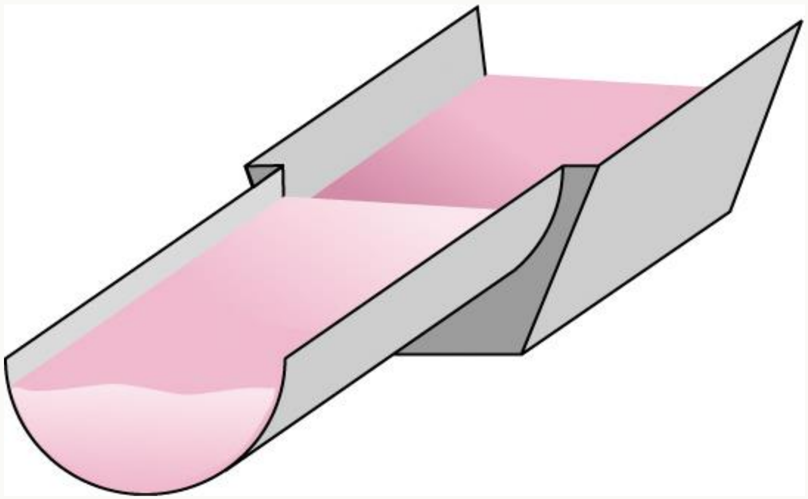
Figure 10.14 – Variation of the momentum function with depth.

Rapidly Varied Flow and Hydraulic Jump



- Consider the CV surrounding the hydraulic jump
- Assumptions
  1. V is constant at sections (1) and (2), and β_1 and $\beta_2 \approx 1$
  2. P = $\rho g y$
  3. τ_w is negligible relative to the losses that occur during the hydraulic jump
  4. Channel is wide and horizontal
  5. No external body forces other than gravity

Rapidly Varied Flow and Hydraulic Jump



- Flow is called **rapidly varied flow (RVF)** if the flow depth has a large change over a short distance
  - Sluice gates
  - Weirs
  - Waterfalls
  - Abrupt changes in cross section
- Often characterized by significant 3D and transient effects
  - Backflows
  - Separations

Rapidly Varied Flow and Hydraulic Jump

- Continuity equation
- X m $\rho y_1 b V_1 = \rho y_2 b V_2 \longrightarrow y_1 V_1 = y_2 V_2$ $\Rightarrow$ $V_2 = (y_1/y_2)V_1$

$$\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$$

$$P_{1,avg} A_1 - P_{2,avg} A_2 = \dot{m}(V_2) - \dot{m}(V_1)$$

$$P_{1,avg} = \frac{\rho g y_1}{2}, \quad P_{2,avg} = \frac{\rho g y_2}{2}, \quad \dot{m} = \dot{m}_2 = \dot{m}_1 = \rho A_1 V_1 = \rho y_1 b V_1$$

$$y_1^2 - y_2^2 = \frac{2 y_1 V_1}{g}(V_2 - V_1) \Rightarrow y_1^2 - y_2^2 = \frac{2 y_1 V_1^2}{g y_2}(y_1 - y_2)$$

Quadratic equation for y_2/y_1

$$\left(\frac{y_2}{y_1}\right)^2 + \frac{y_2}{y_1} - 2Fr_1^2 = 0$$

Rapidly Varied Flow and Hydraulic Jump

- Solving the quadratic equation and keeping only the positive root leads to the **depth ratio**

$$\boxed{\frac{y_2}{y_1} = 0.5\left(-1 + \sqrt{1 + 8Fr_1^2}\right)}$$

- Energy equation for this section can be written as

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_L$$

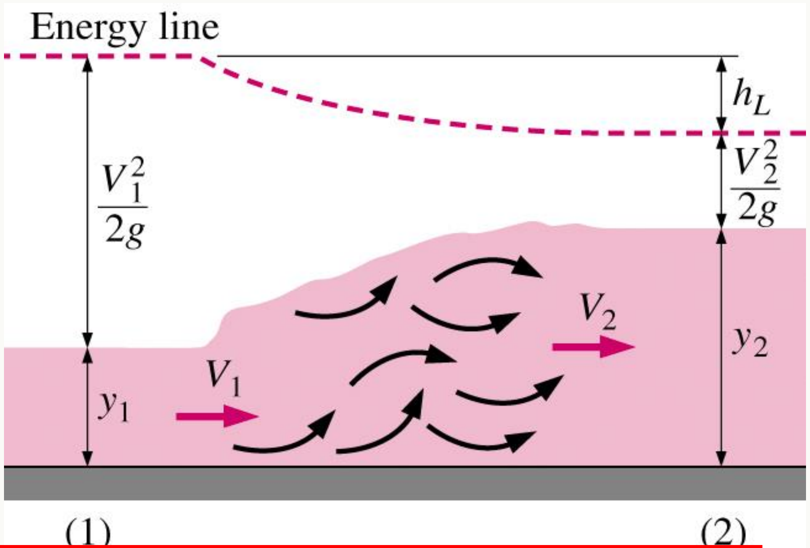
$$V_2 = (y_1/y_2)V_1$$

- Head loss associated with hydraulic jump

$$\boxed{h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} = y_1 - y_2 + \frac{y_1 Fr_1^2}{2}\left(1 - \frac{y_1^2}{y_2^2}\right)}$$

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}}$$

Rapidly Varied Flow and Hydraulic Jump



$$\text{Dissipation ratio} = \frac{h_L}{E_{s1}} = \frac{h_L}{y_1 + V_1^2/2g}$$

- Often, hydraulic jumps are avoided because they dissipate valuable energy
- However, in some cases, the energy must be dissipated so that it doesn't cause damage
- A measure of performance of a hydrualic jump is its fraction of energy dissipation, or **energy dissipation ratio**

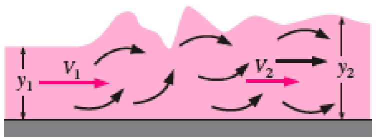
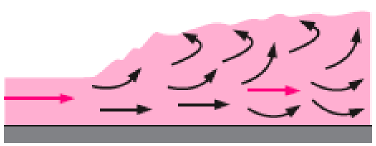
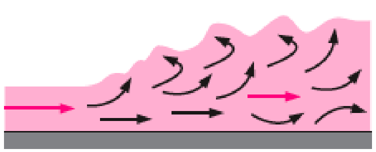
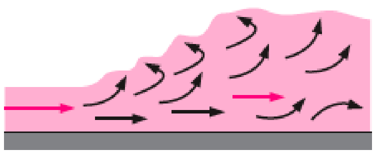
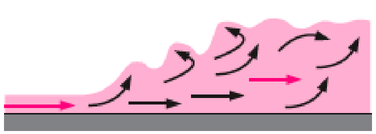
Rapidly Varied Flow and Hydraulic Jump

- Experimental studies indicate that hydraulic jumps can be classified into 5 categories, depending upon the upstream Fr

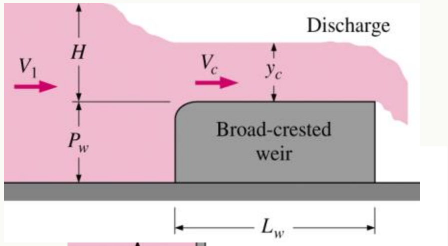
TABLE 13–4

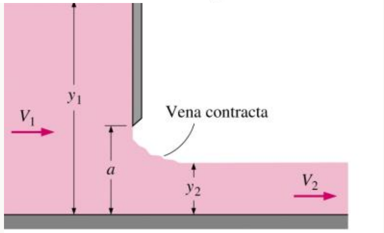
Classification of hydraulic jumps

Source: U.S. Bureau of Reclamation (1955).

Upstream Fr_1	Depth Ratio y_2/y_1	Fraction of Energy Dissipation	Description	Surface Profile
<1	1	0	*Impossible jump*. Would violate the second law of thermodynamics.	
1–1.7	1–2	<5%	*Undular jump* (or *standing wave*). Small rise in surface level. Low energy dissipation. Surface rollers develop near Fr = 1.7.	
1.7–2.5	2–3.1	5–15%	*Weak jump*. Surface rising smoothly, with small rollers. Low energy dissipation.	
2.5–4.5	3.1–5.9	15–45%	*Oscillating jump*. Pulsations caused by entering jets at the bottom generate large waves that can travel for miles and damage earth banks. Should be avoided in the design of stilling basins.	
4.5–9	5.9–12	45–70%	*Steady jump*. Stable, well-balanced, and insensitive to downstream conditions. Intense eddy motion and high level of energy dissipation within the jump. Recommended range for design.	
>9	>12	70–85%	*Strong jump*. Rough and intermittent. Very effective energy dissipation, but may be uneconomical compared to other designs.	

Flow Control and Measurement



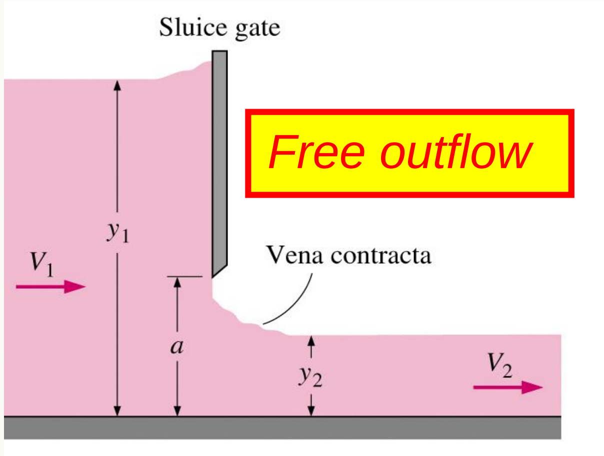


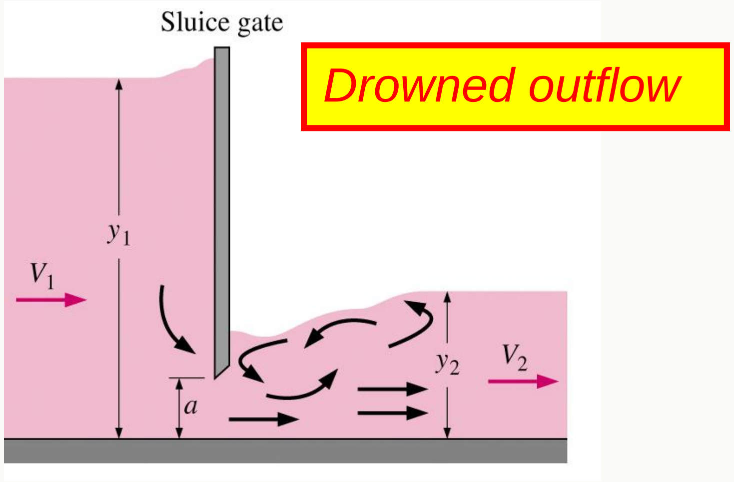
(a) Sluice gate with free outflow

- Flow rate in pipes and ducts is controlled by various kinds of valves
- In OC flows, flow rate is controlled by partially blocking the channel.
  - Weir : liquid flows over device
  - Underflow gate : liquid flows under device
- These devices can be used to control the flow rate, and to measure it.

Flow Control and Measurement

Underflow Gate



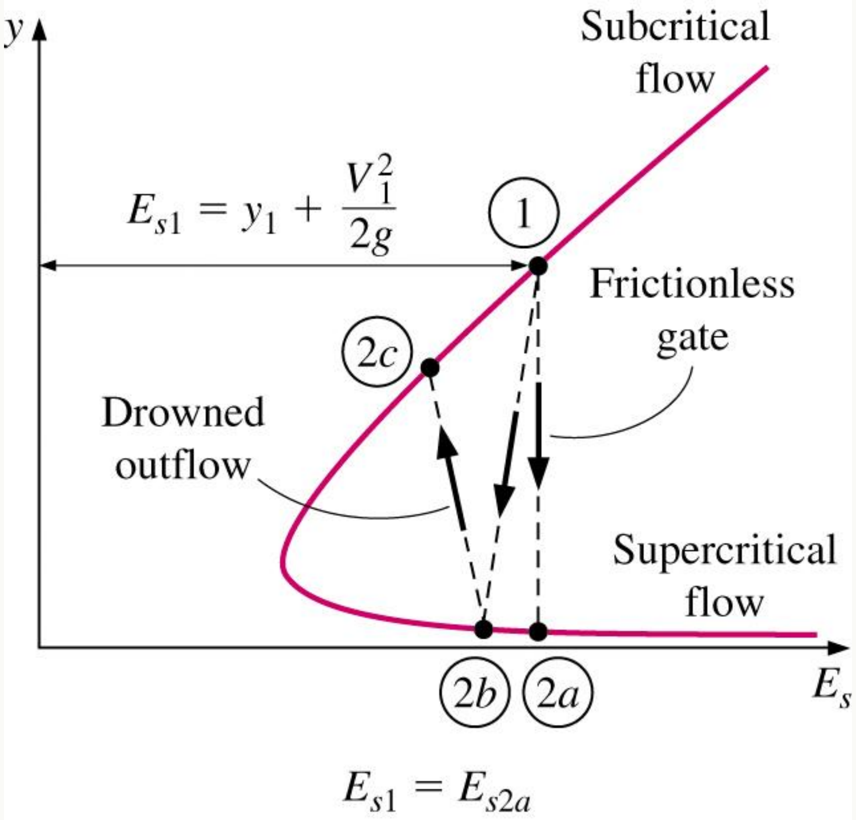


- Underflow gates are located at the bottom of a wall, dam, or open channel
- Outflow can be either free or drowned
- In free outflow, downstream flow is supercritical
- In the drowned outflow, the liquid jet undergoes a hydraulic jump. Downstream flow is subcritical.

Flow Control and Measurement

Underflow Gate

Schematic of flow depth-specific energy diagram for flow through underflow gates

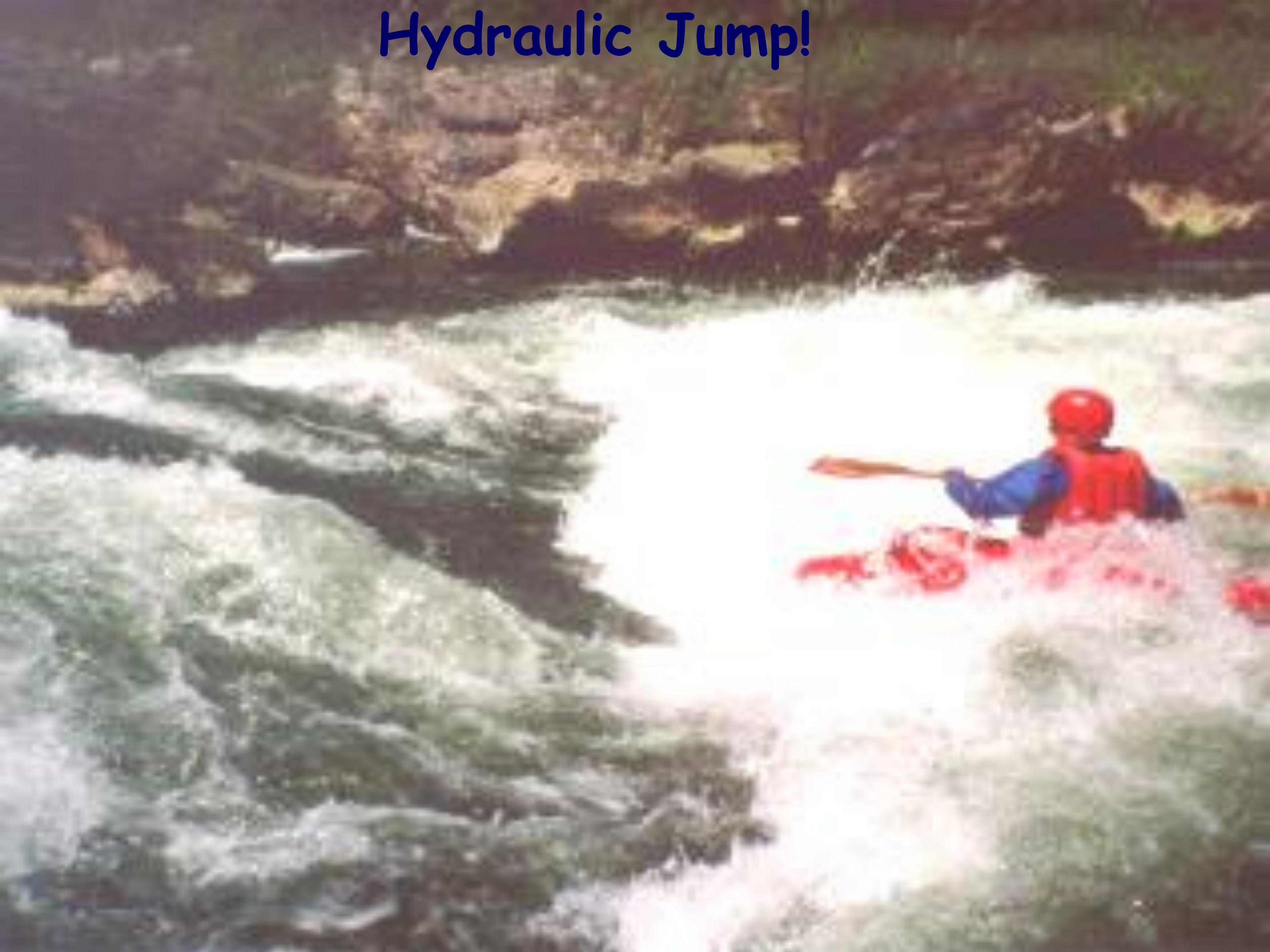


- E_s remains constant for idealized gates with negligible frictional effects
- E_s decreases for real gates
- Downstream is supercritical for free outflow (2b)
- Downstream is subcritical for drowned outflow (2c)

Hydraulic Jump

- Used for energy dissipation
- Occurs when flow transitions from supercritical to subcritical
  - base of spillway
- We would like to know depth of water downstream from jump as well as the location of the jump
- Which equation, Energy or Momentum?

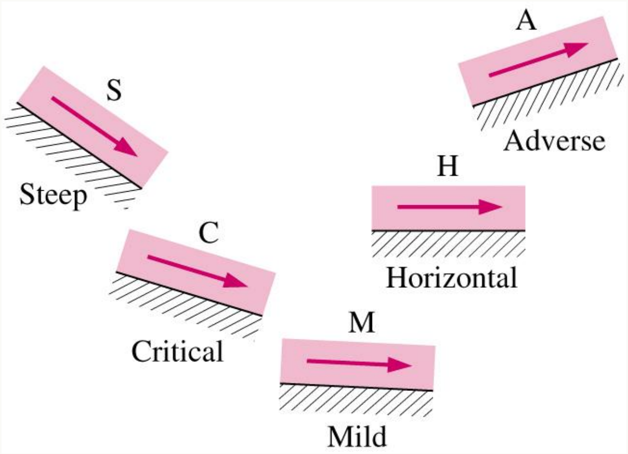
Hydraulic Jump!

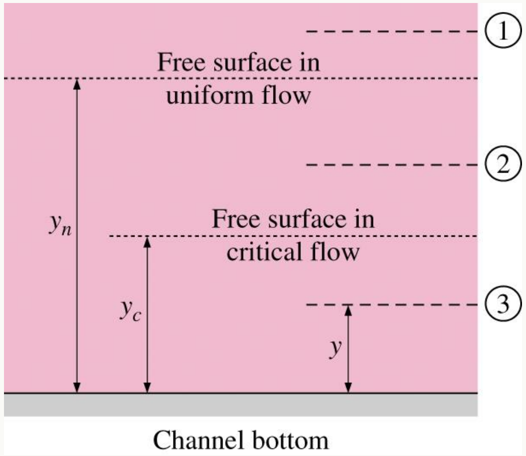


Gradually Varied Flow

$$\boxed{\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2}}$$

Gradually Varied Flow





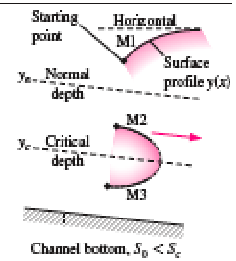
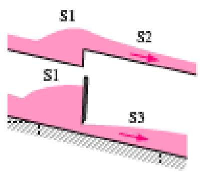
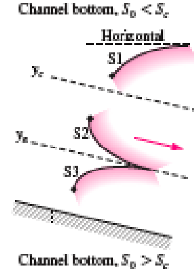
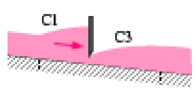
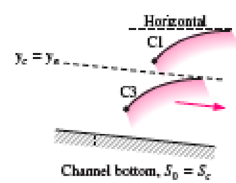
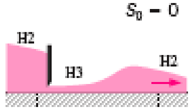
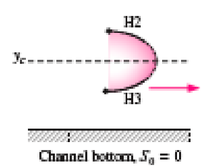
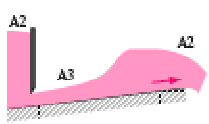
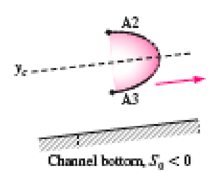
- This result is important. It permits classification of liquid surface profiles as a function of Fr, S_0, S_f, and initial conditions.
- Bed slope S_0 is classifed as
  - Steep : $y_n < y_c$
  - Critical : $y_n = y_c$
  - Mild : $y_n > y_c$
  - Horizontal : $S_0 = 0$
  - Adverse : $S_0 < 0$
- Initial depth is given a number
  - 1 : $y > y_n$
  - 2 : $y_n < y < y_c$
  - 3 : $y < y_c$

Gradually Varied Flow

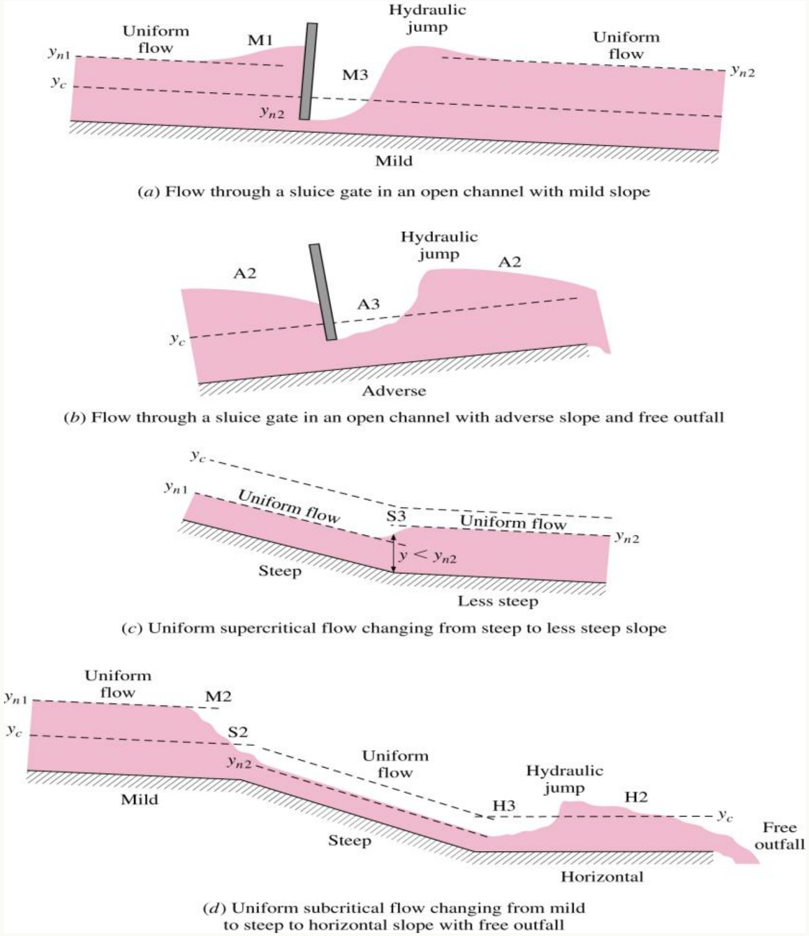
- 12 distinct configurations for surface profiles in GVF.

TABLE 13–3

Classification of surface profiles in gradually varied flow

Channel Slope	Profile Notation	Flow Depth	Froude Number	Profile Slope	Surface Profile
Mild (M) $y_c < y_n$, $S_0 < S_c$ 	M1	$y > y_n$	Fr < 1	$\frac{dy}{dx} > 0$	
	M2	$y_c < y < y_n$	Fr < 1	$\frac{dy}{dx} < 0$	
	M3	$y < y_c$	Fr > 1	$\frac{dy}{dx} > 0$	
Steep (S) $y_c > y_n$, $S_0 < S_c$ 	S1	$y > y_c$	Fr < 1	$\frac{dy}{dx} > 0$	
	S2	$y_n < y < y_c$	Fr > 1	$\frac{dy}{dx} < 0$	
	S3	$y < y_n$	Fr > 1	$\frac{dy}{dx} > 0$	
Critical (C) $y_c = y_n$, $S_0 < S_c$ 	C1	$y > y_c$	Fr < 1	$\frac{dy}{dx} > 0$	
	C3	$y < y_c$	Fr > 1	$\frac{dy}{dx} > 0$	
Horizontal (H) $y_n \to \infty$, $S_0 = 0$ 	H2	$y > y_c$	Fr < 1	$\frac{dy}{dx} < 0$	
	H3	$y < y_c$	Fr > 1	$\frac{dy}{dx} > 0$	
Adverse (A) $S_0 < 0$, y_n: does not exist 	A2	$y > y_c$	Fr < 1	$\frac{dy}{dx} < 0$	
	A3	$y < y_c$	Fr > 1	$\frac{dy}{dx} > 0$	

Gradually Varied Flow



- Typical OC system involves several sections of different slopes, with *transitions*
- Overall surface profile is made up of individual profiles described on previous slides

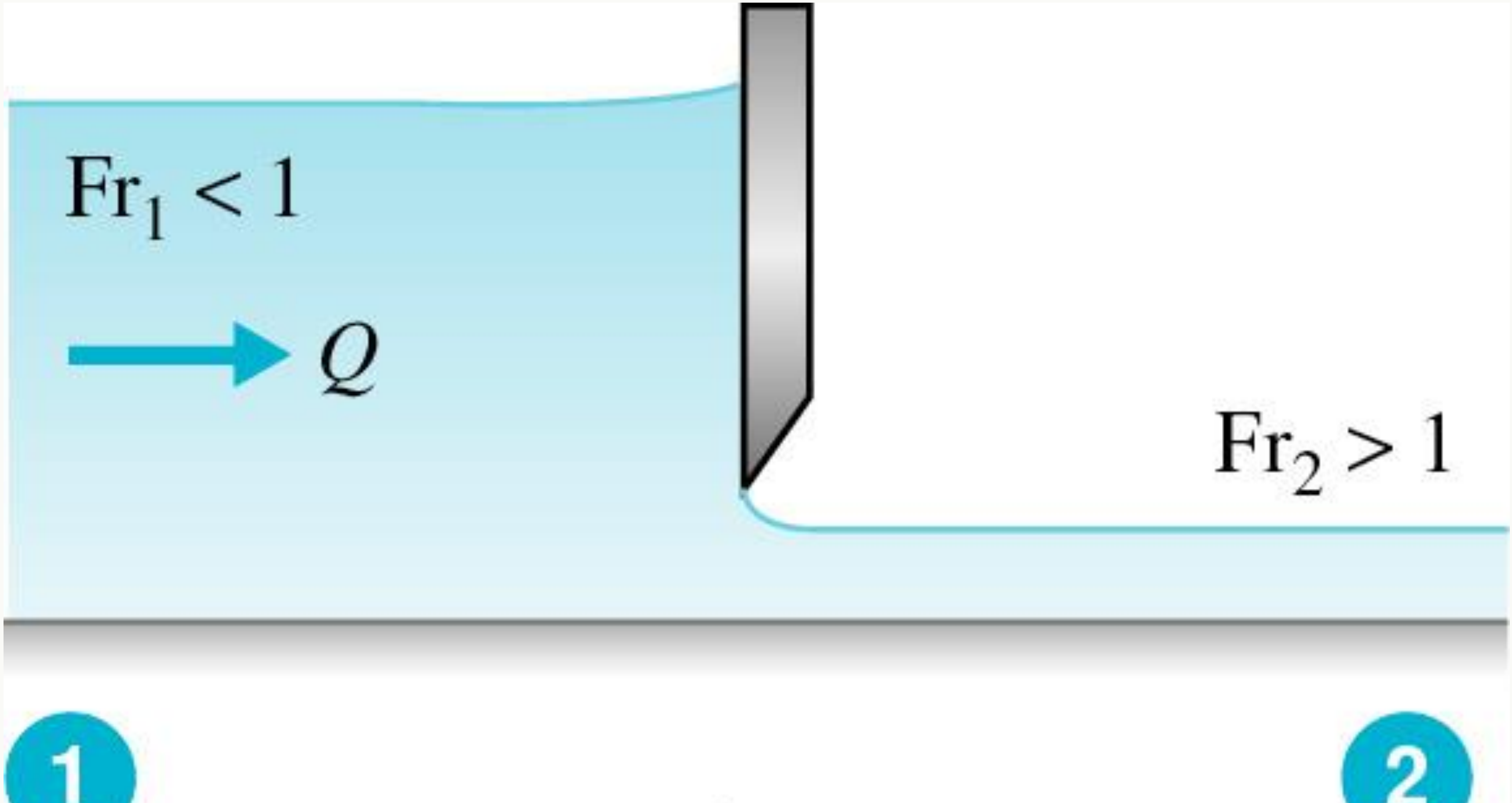


Figure E10.7

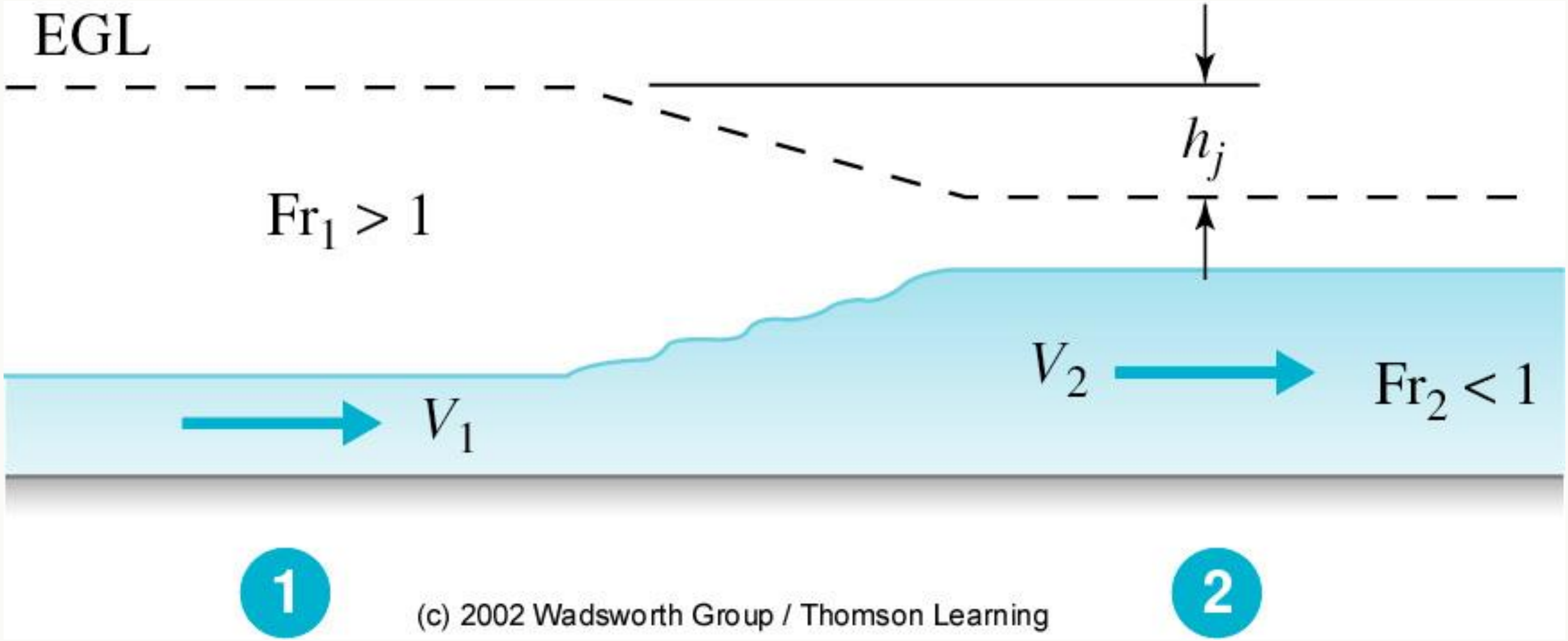


Figure 10.15 – Idealized hydraulic jump.

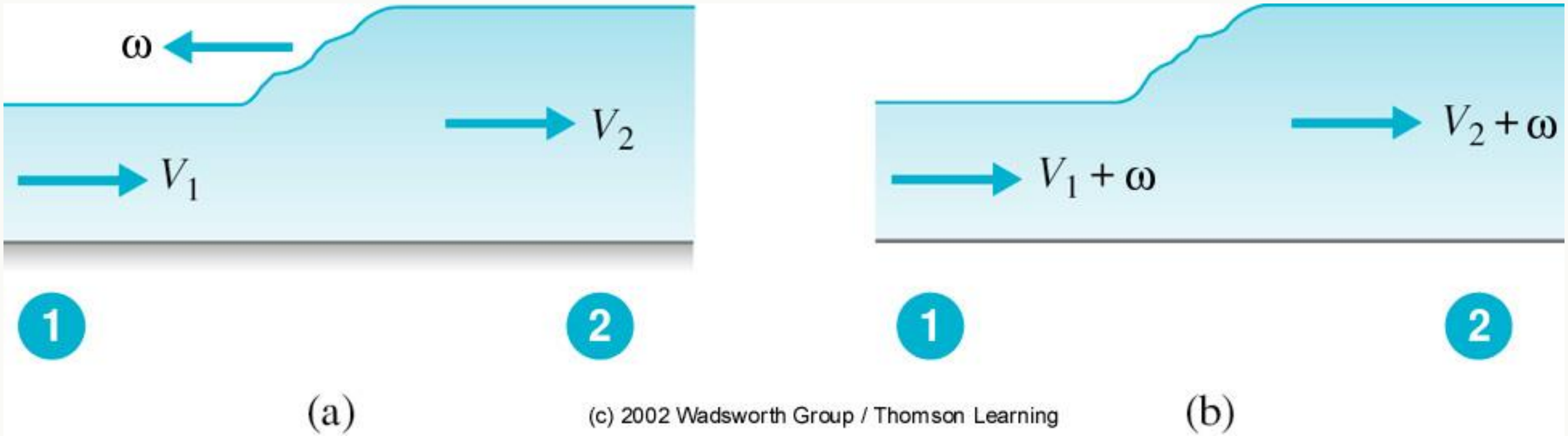


Figure 10.16 – Translating hydraulic jump: (a) front moving upstream; (b) front appears stationary by superposition.

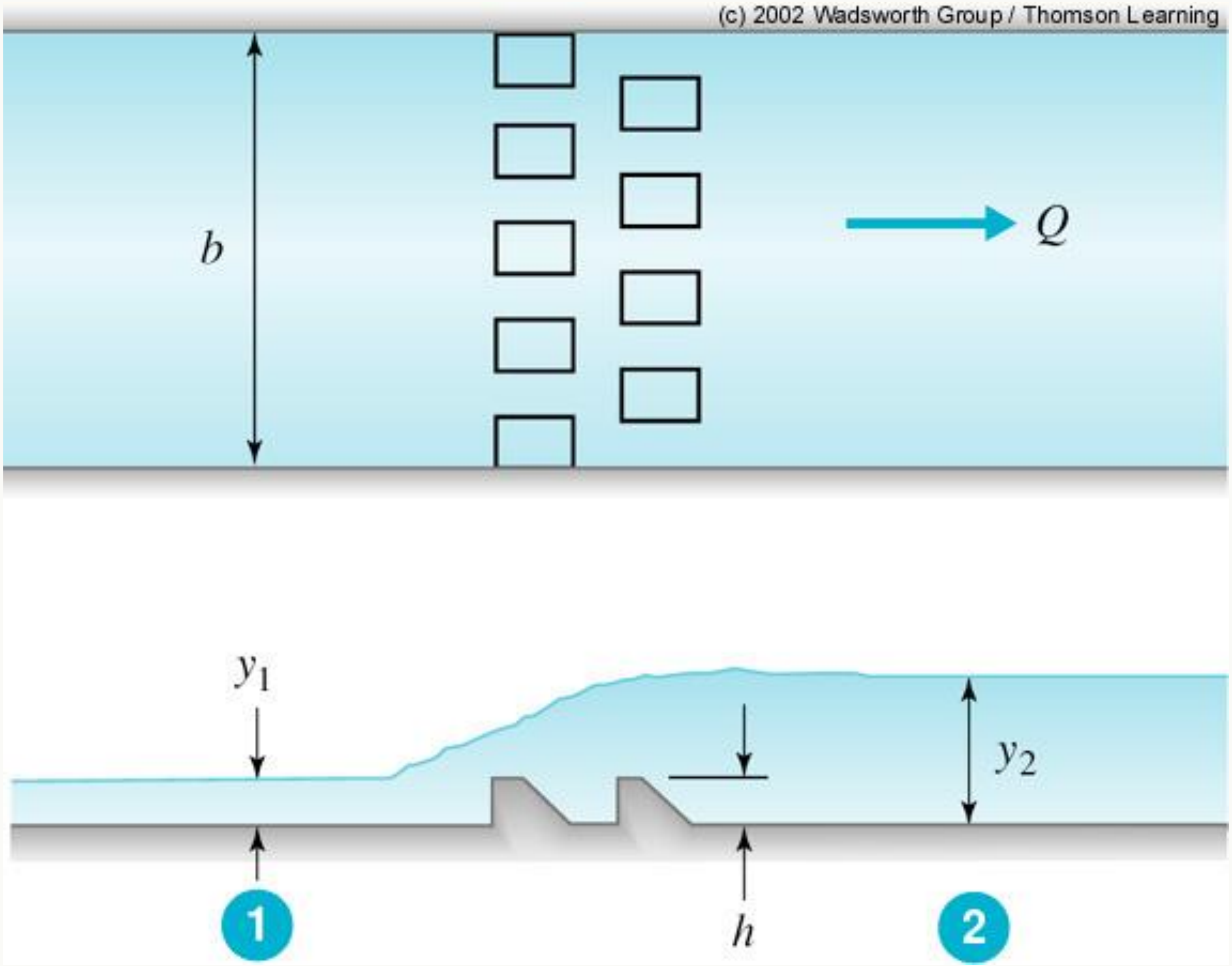


Figure 10.17 – Stilling basin with baffle blocks.

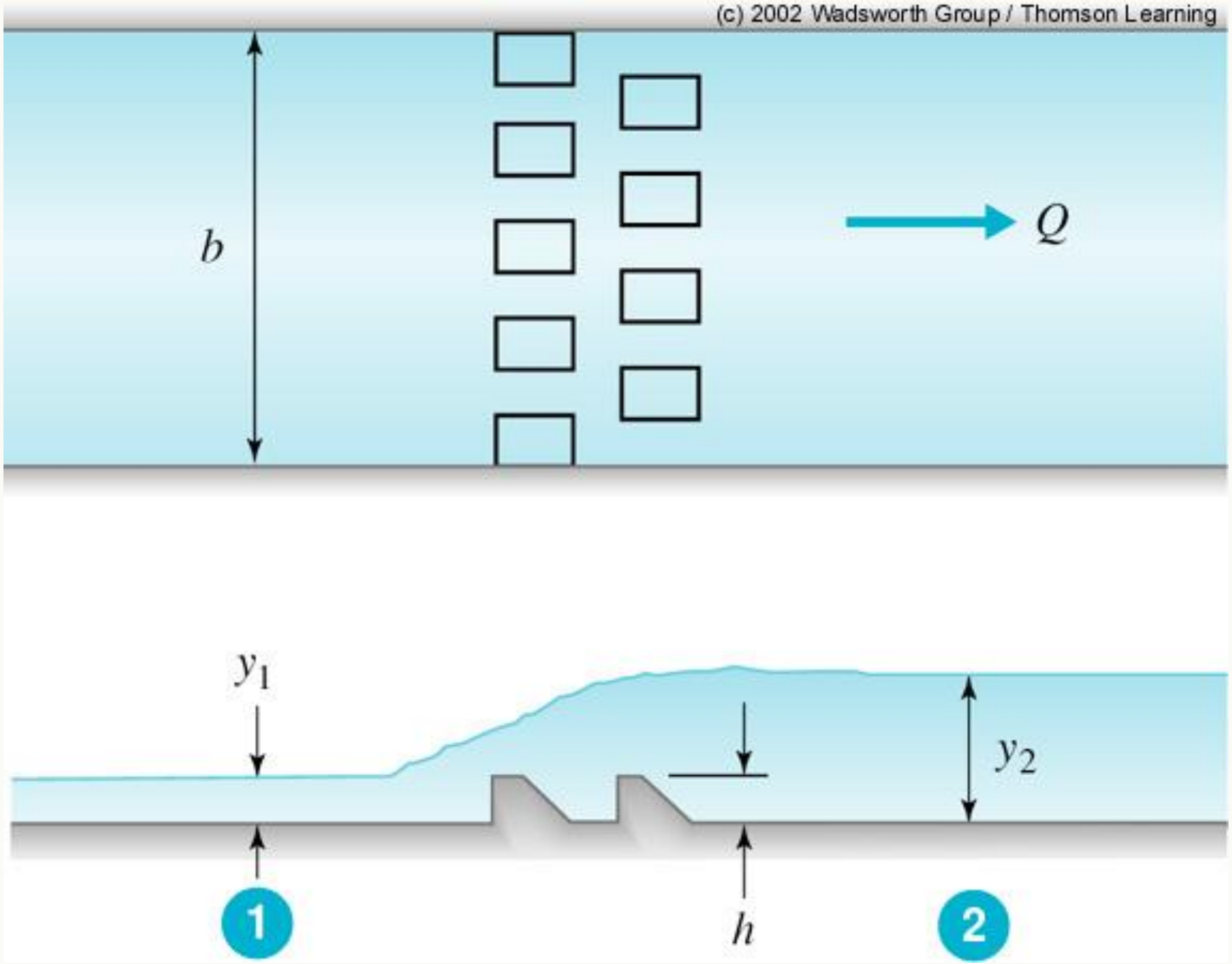


Figure E10.10

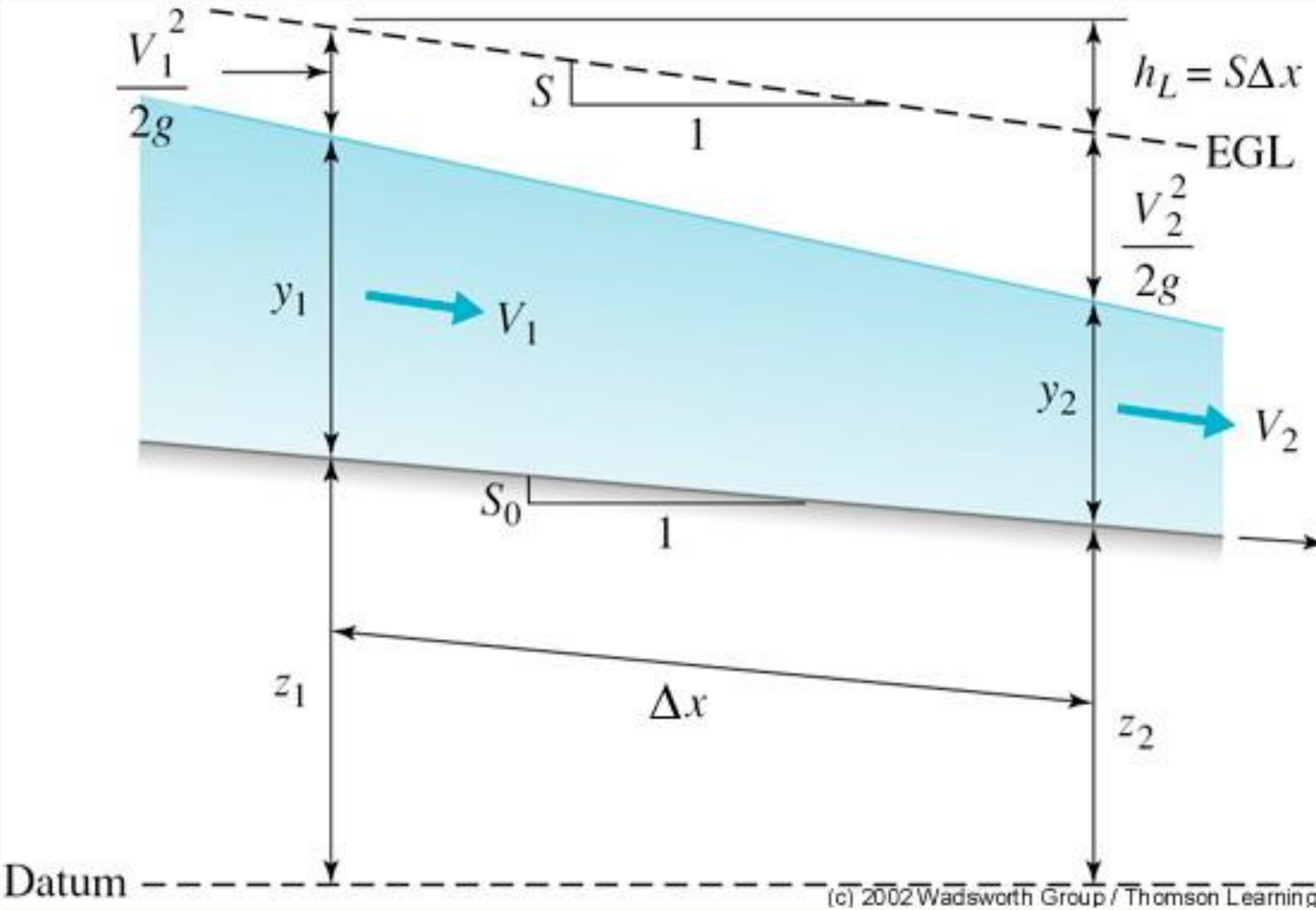


Figure 10.18 – **Non-uniform gradually varied flow.**

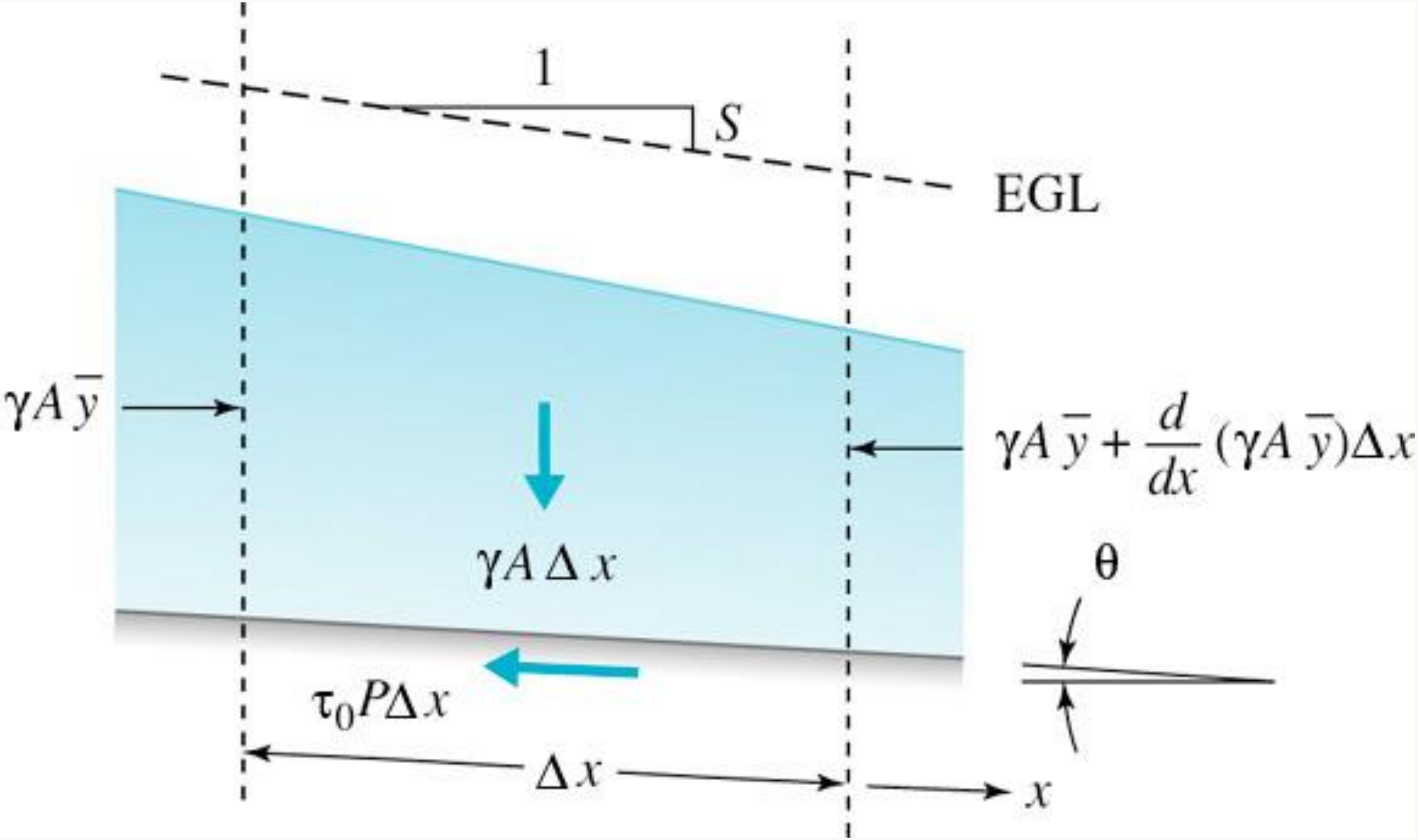


Figure E10.12

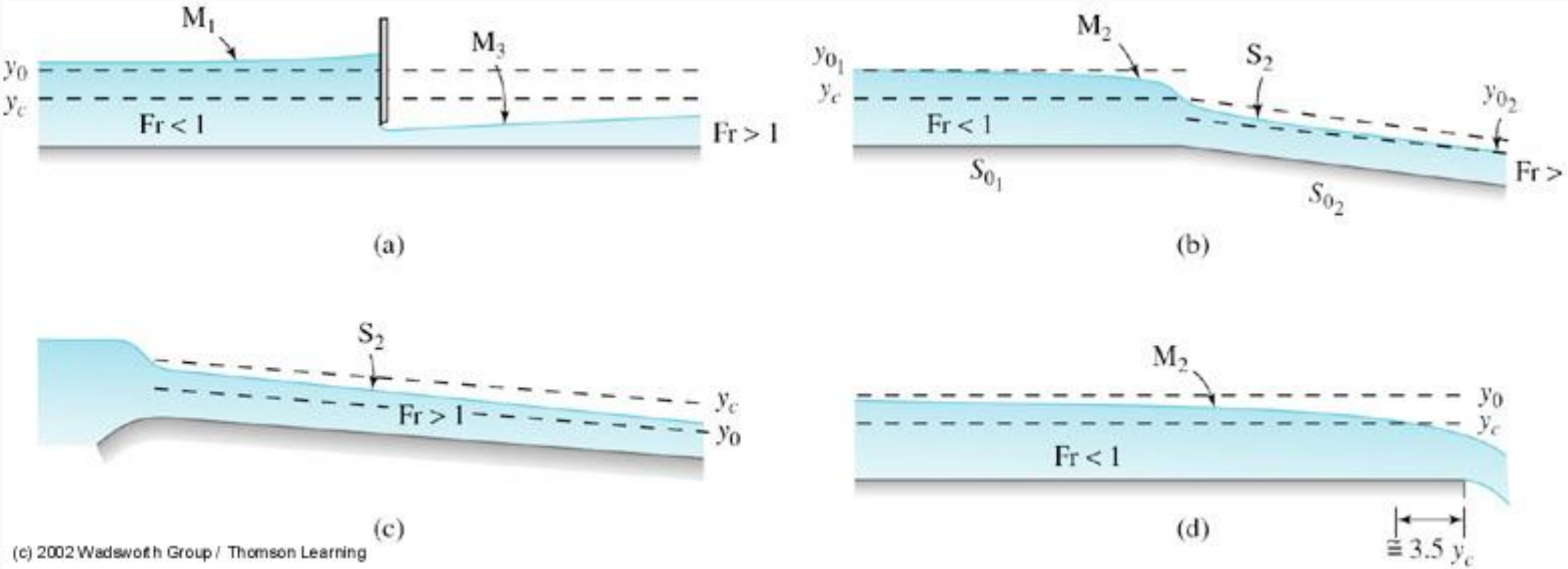


Figure 10.19 – **Representative controls: (a) sluice gate; (b) change in slope from mild (S_{01})to steep (S_{02}); (c) entrance to a steep channel; (d) free outfall.**

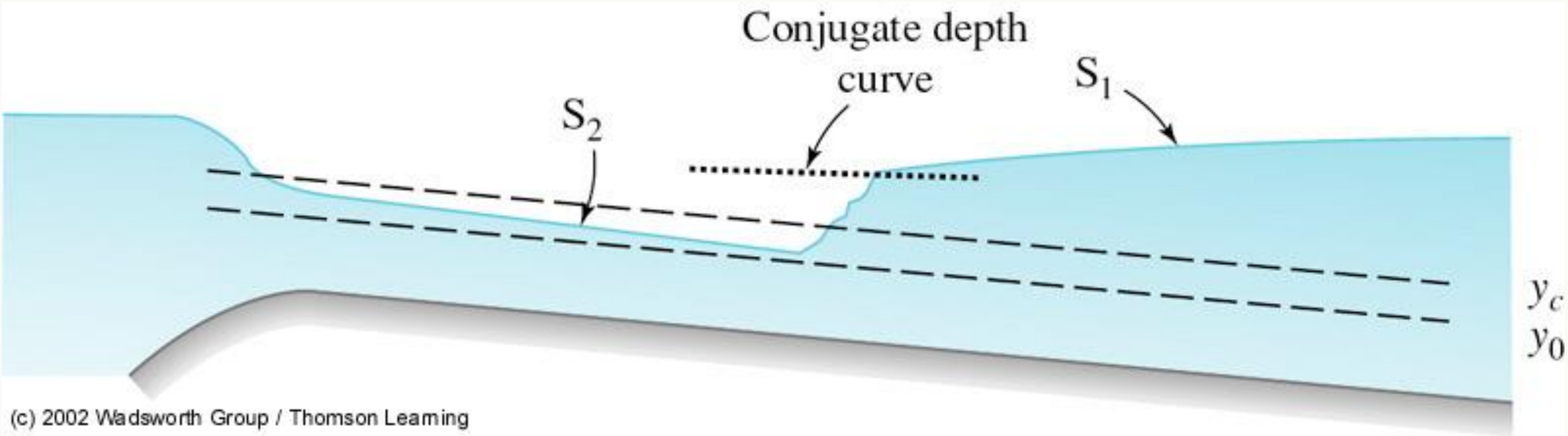


Figure 10.20 – **Example of profile synthesis**

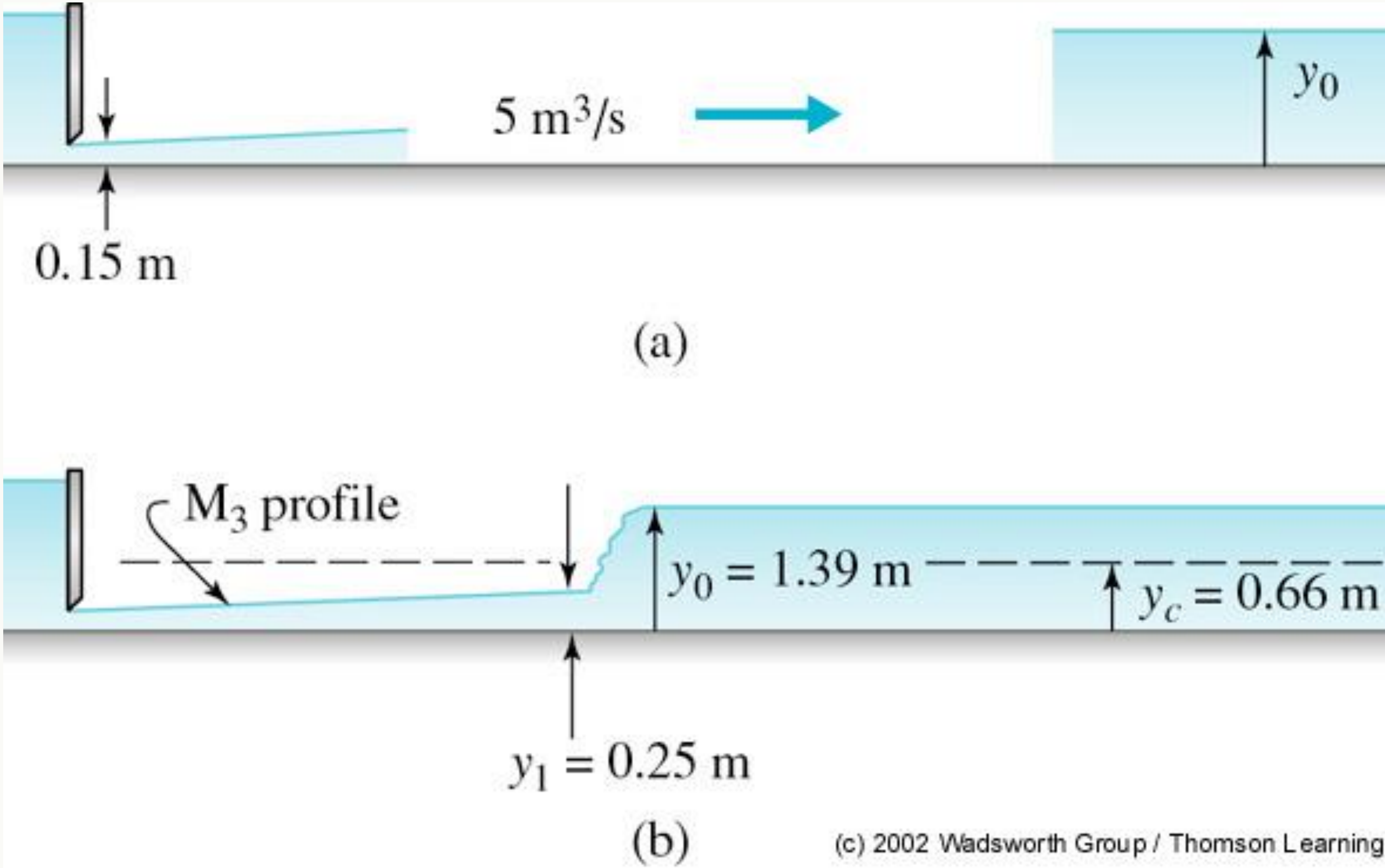


Figure E10.14

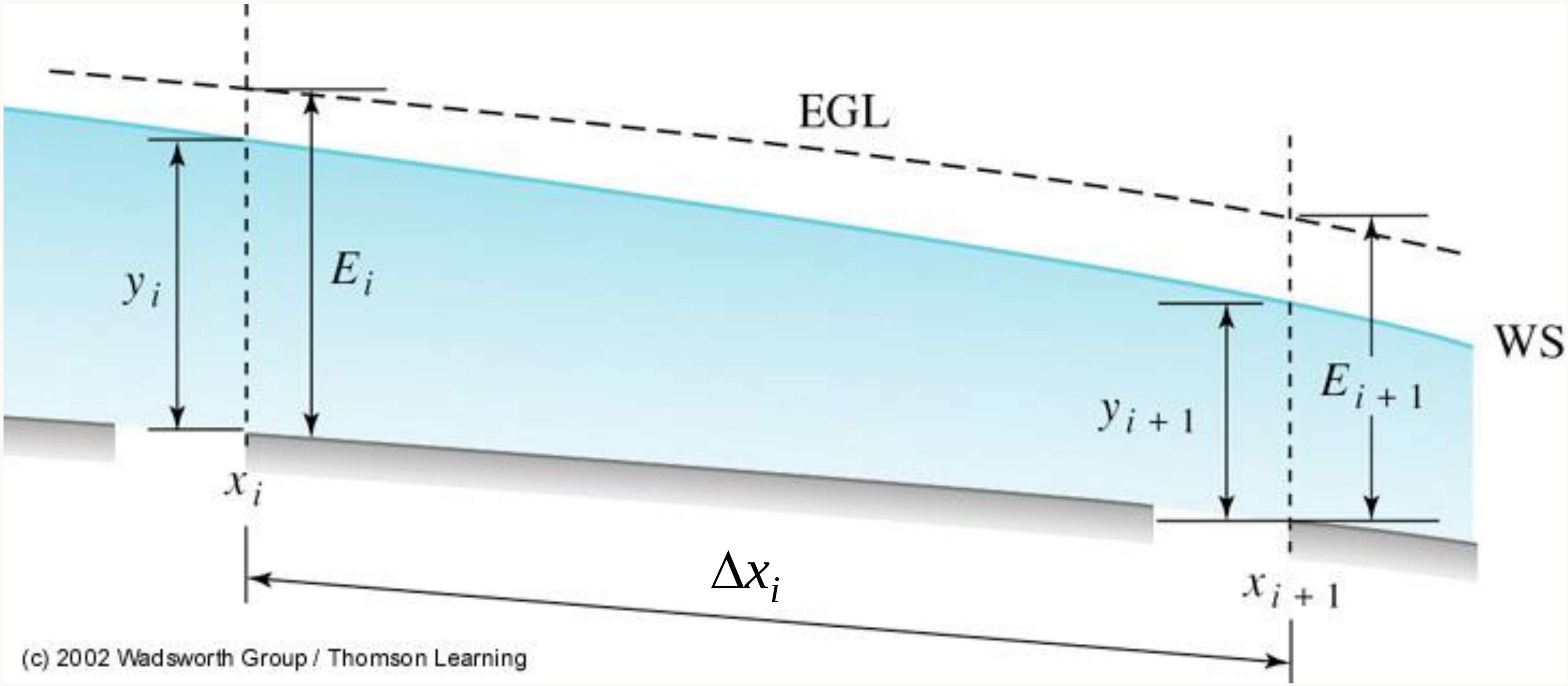


Figure 10.21 – Notation for computing gradually varied flow.

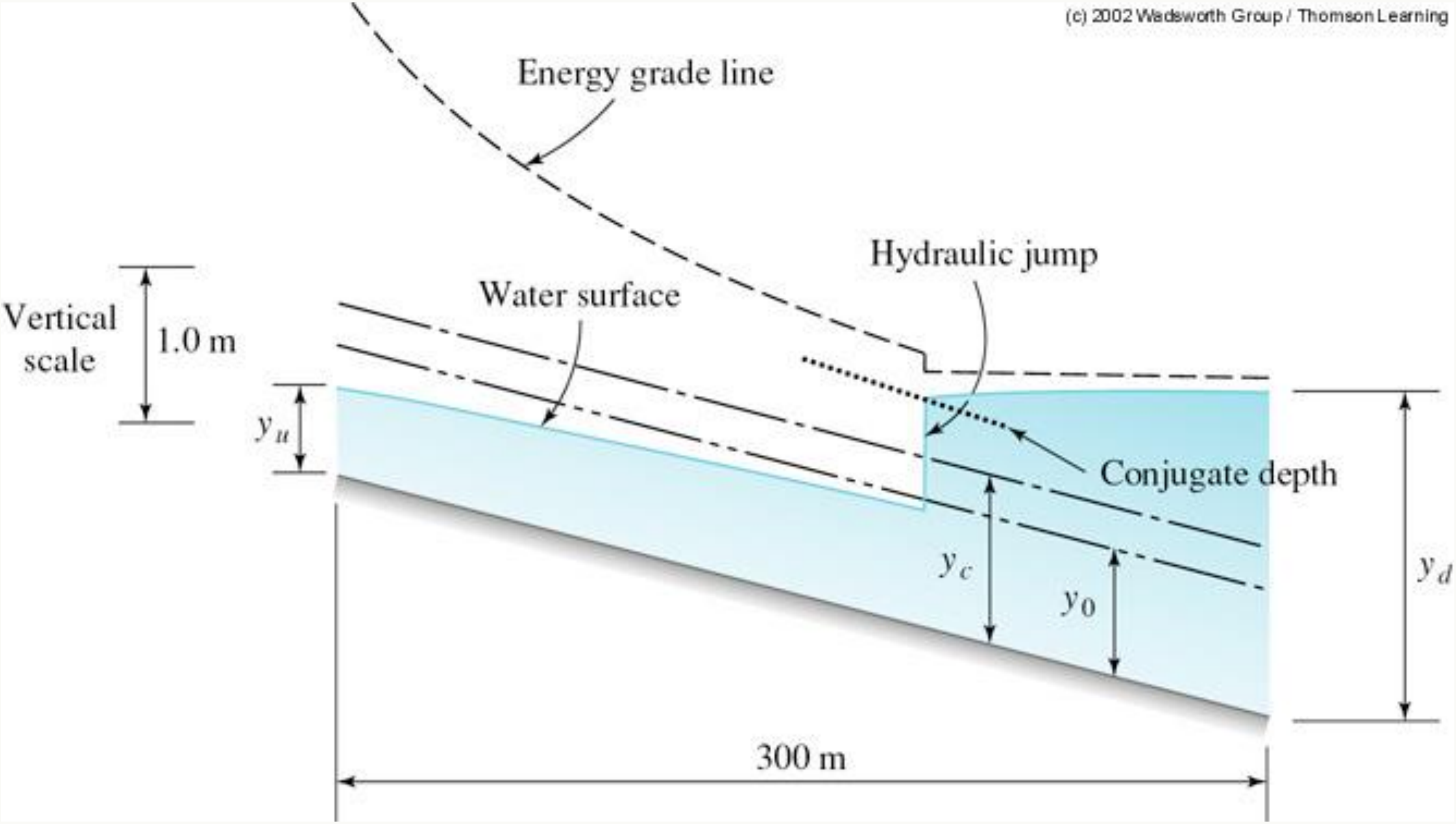
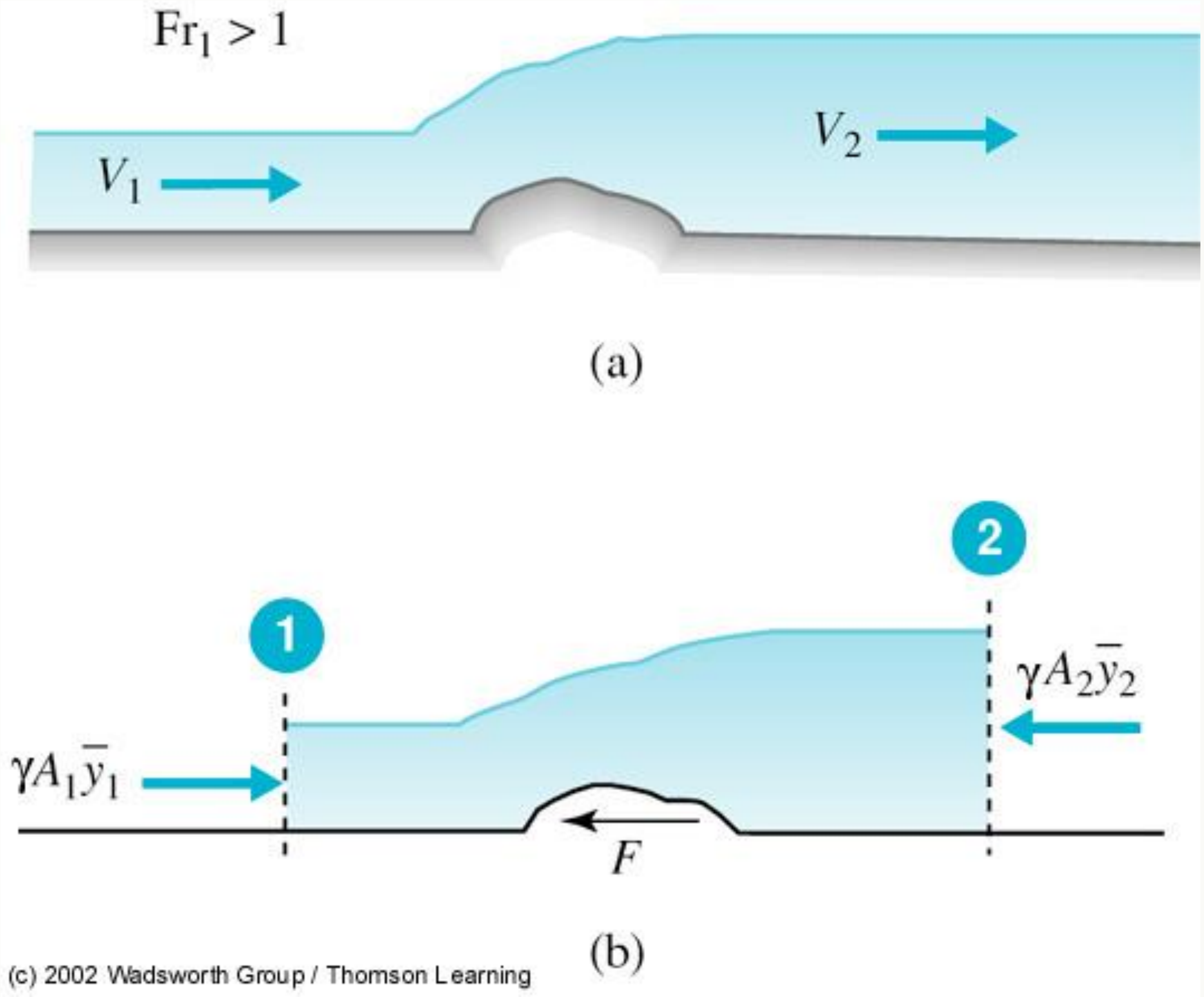
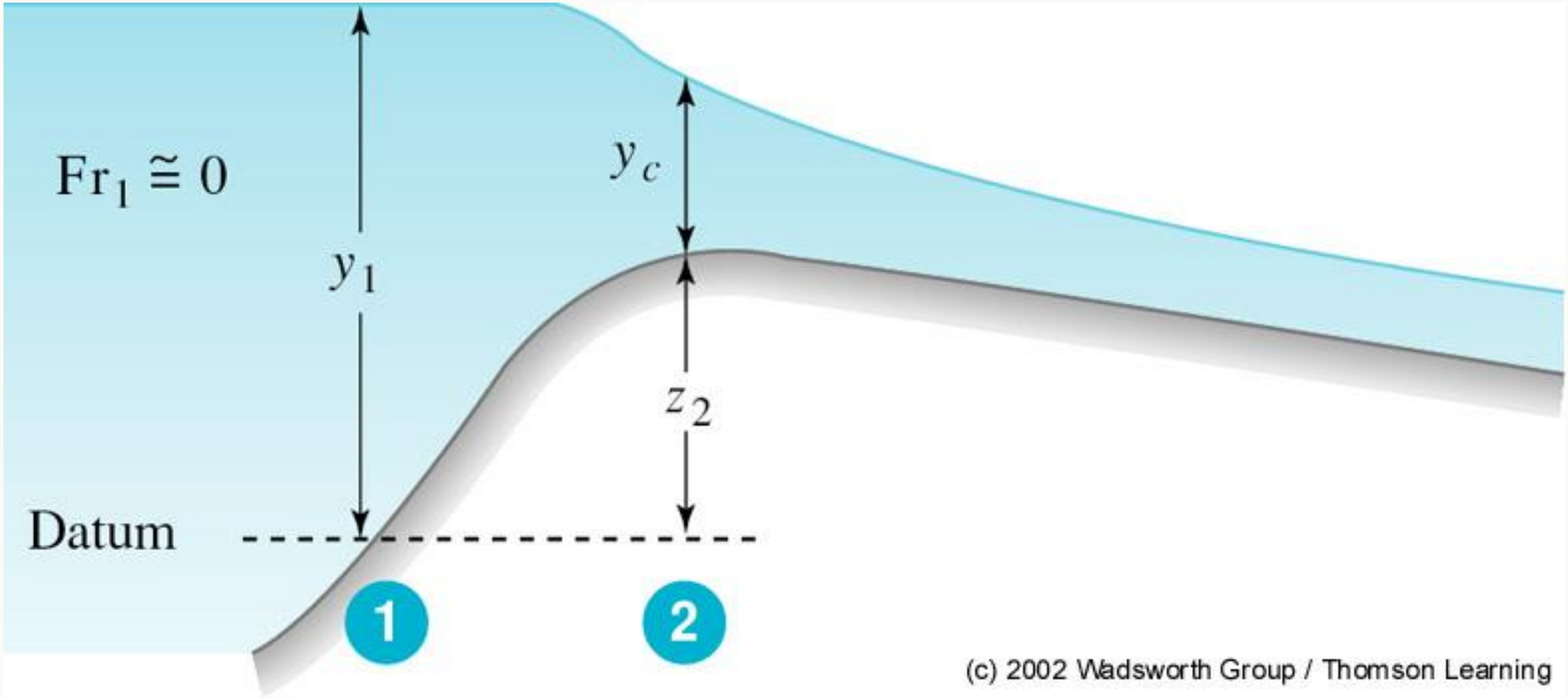


Figure E10.16



Channel flow over an obstacle: (a) idealized flow; (b) control volume.

Outflow from a reservoir with critical flow at the channel entrance.



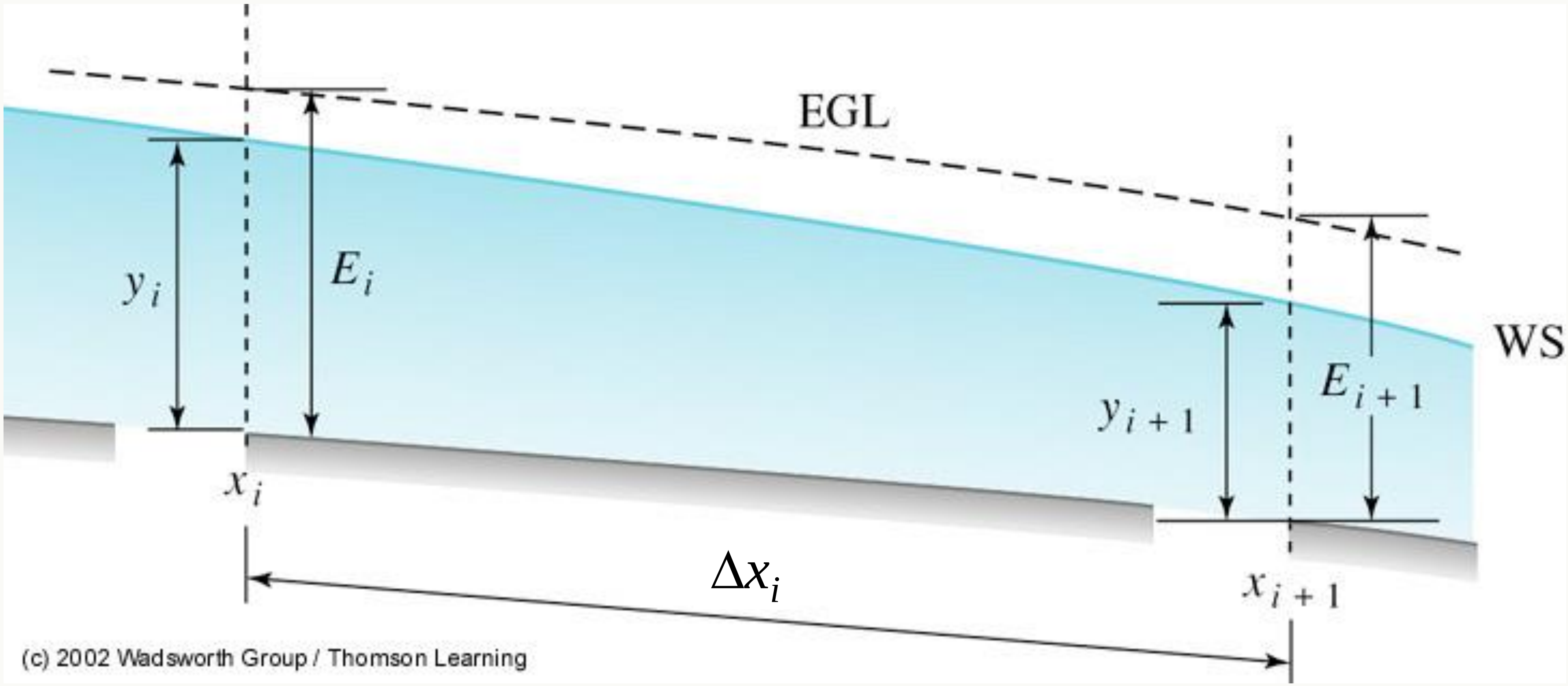


Figure 10.21 – Notation for computing gradually varied flow.

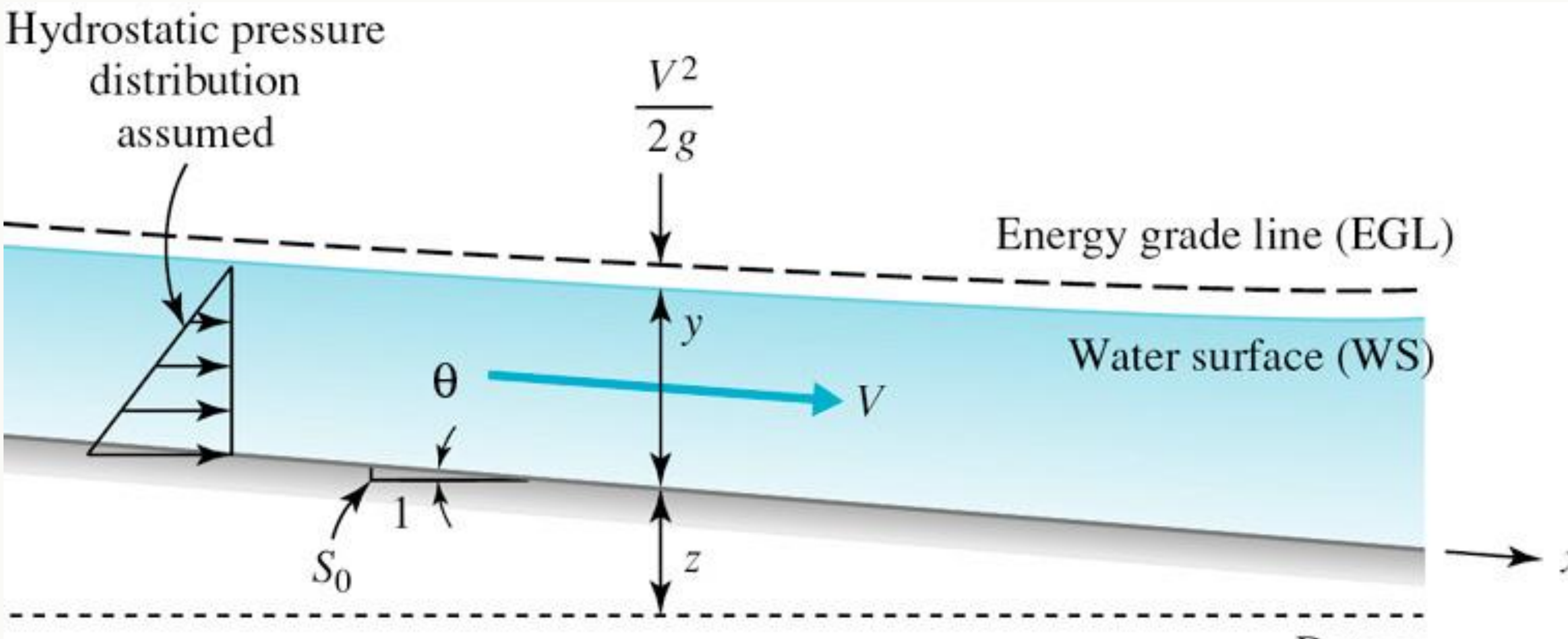


Figure 10.3 – **Reach of open-channel flow.**

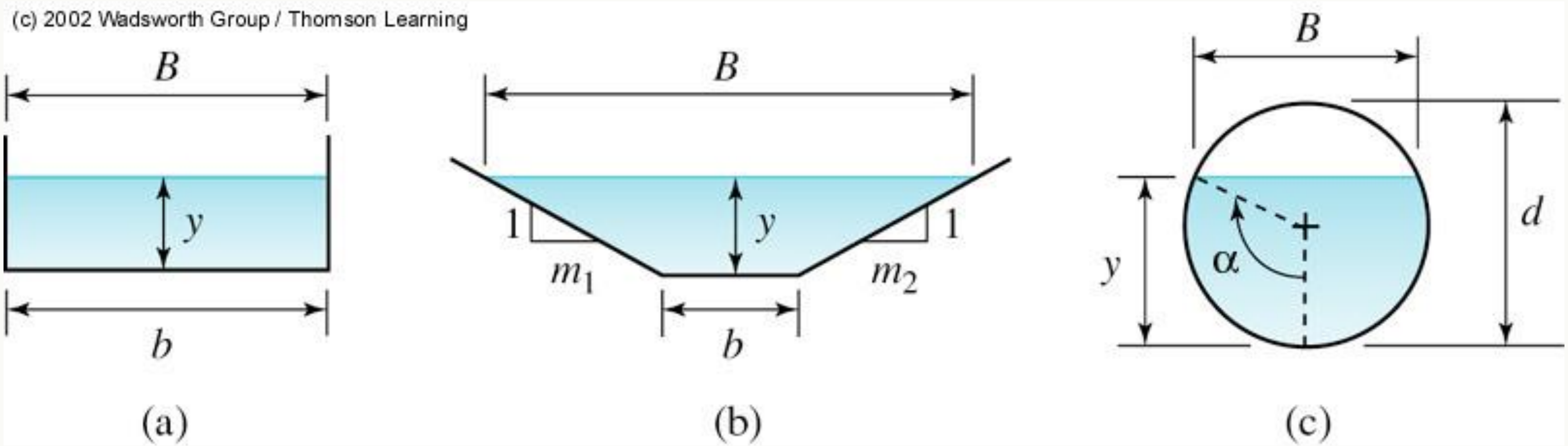


Figure 10.4 – Representative regular cross sections: (a) rectangular; (b) trapezoidal; (c) circular.

The Manning formula

$$Q = \frac{1.49\, A\, R^{2/3}\, S^{1/2}}{n}$$

where: Q = quantity of flow in cubic feet per second

n = Manning coefficient of roughness dependent upon material of conduit

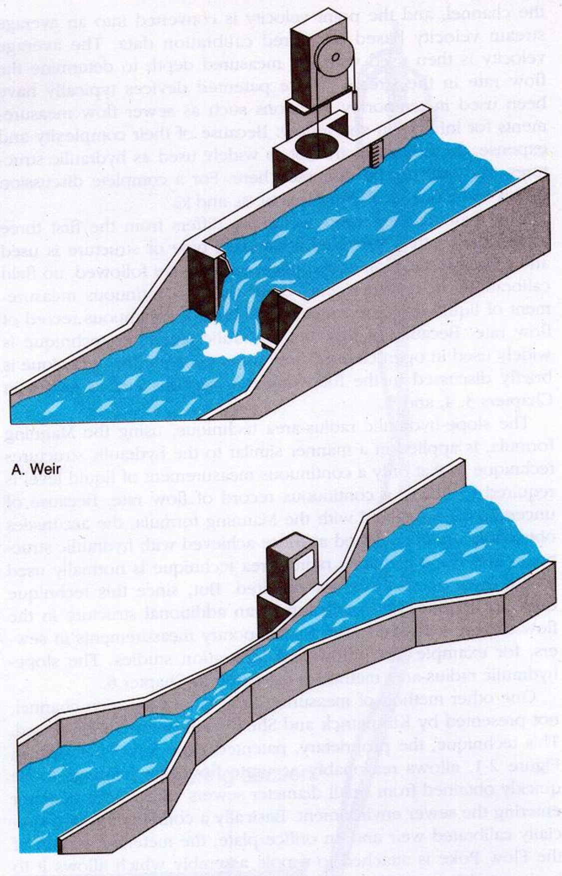
A = cross sectional area of flow in square feet

R = hydraulic radius in feet (cross sectional area divided by the wetted perimeter)

S = slope of the hydraulic gradient

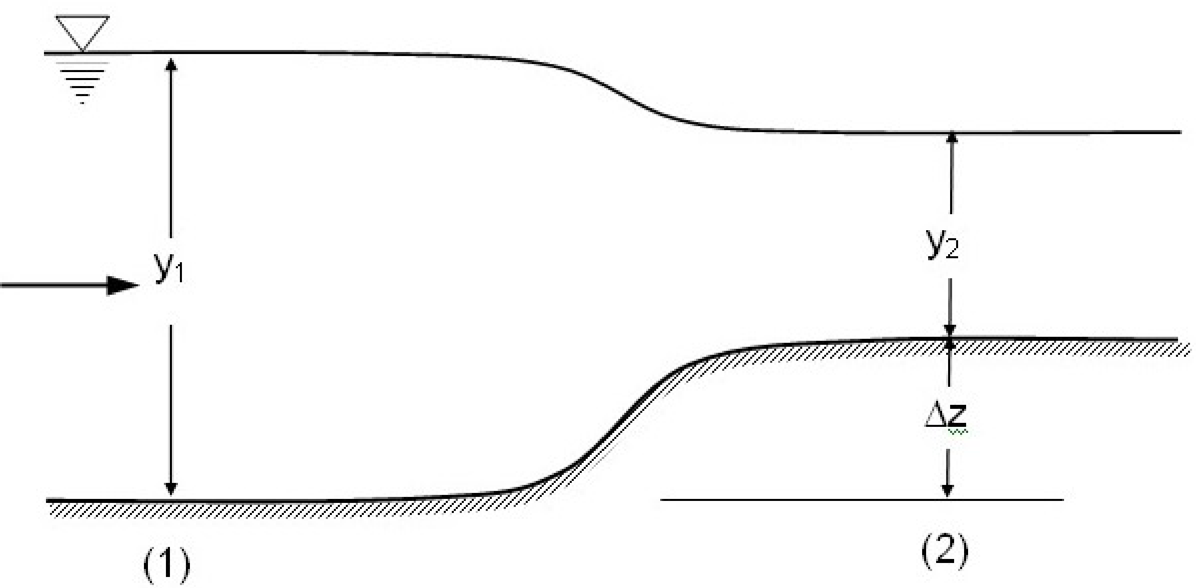
For flow rate in million gallons per day, the Manning formula is

$$Q = \frac{669\, A\, R^{2/3}\, S^{1/2}}{n}$$



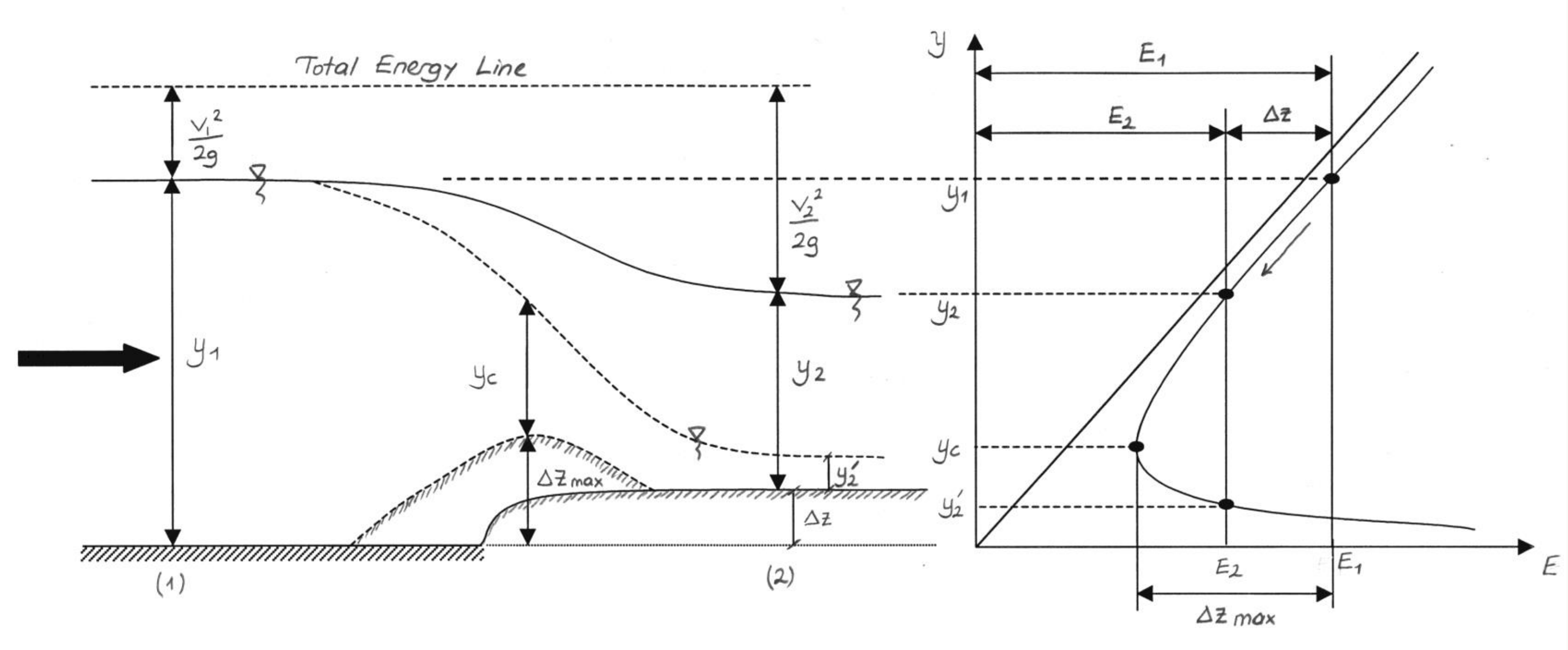
A. Weir

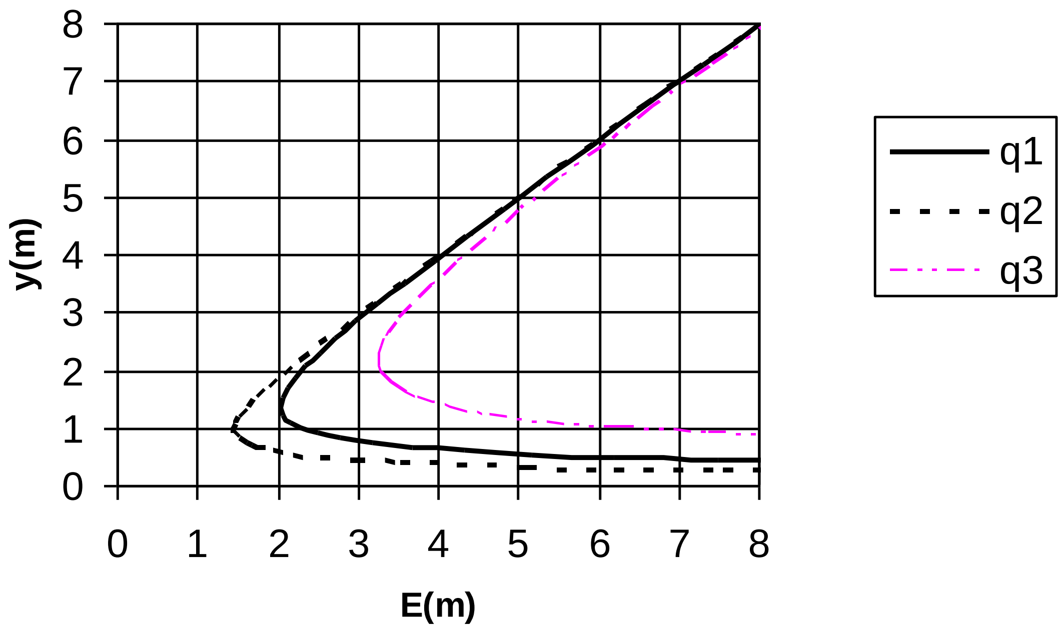
B. Flume



y1
y2
Δz
(1)
(2)

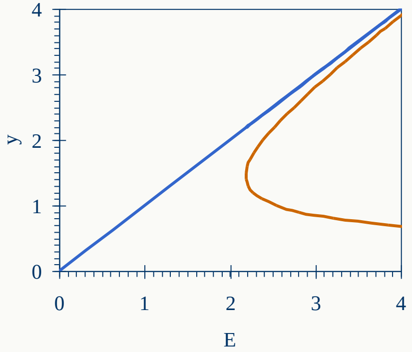
Further for Choking





Specific Energy Depth Curve

Critical Flow



Difficult to measure depth

- Characteristics
  - Unstable surface
  - Series of standing waves
- Occurrence
  - Broad crested weir (and other weirs)
  - Channel Controls (rapid changes in cross-section)
  - Over falls
  - Changes in channel slope from mild to steep
- Used for flow measurements

Unique relationship between depth and discharge

Classification of Open-Channel Flows

- Like pipe flow, OC flow can be laminar, transitional, or turbulent depending upon the value of the Reynolds number

$$Re = \frac{\rho V R_h}{\mu} = \frac{V R_h}{\nu}$$

  – Where
    - ρ = dens … atic viscosity
    - V = average velocity
    - R_h = **Hydraulic Radius = A_c/p**
      – A_c = cross-section area
      – P = wetted perimeter
      – Note that **Hydraulic Diameter** was defined in pipe flows as $D_h = 4A_c/p = 4R_h$ (D_h is not $2R_h$, BE Carefull!)

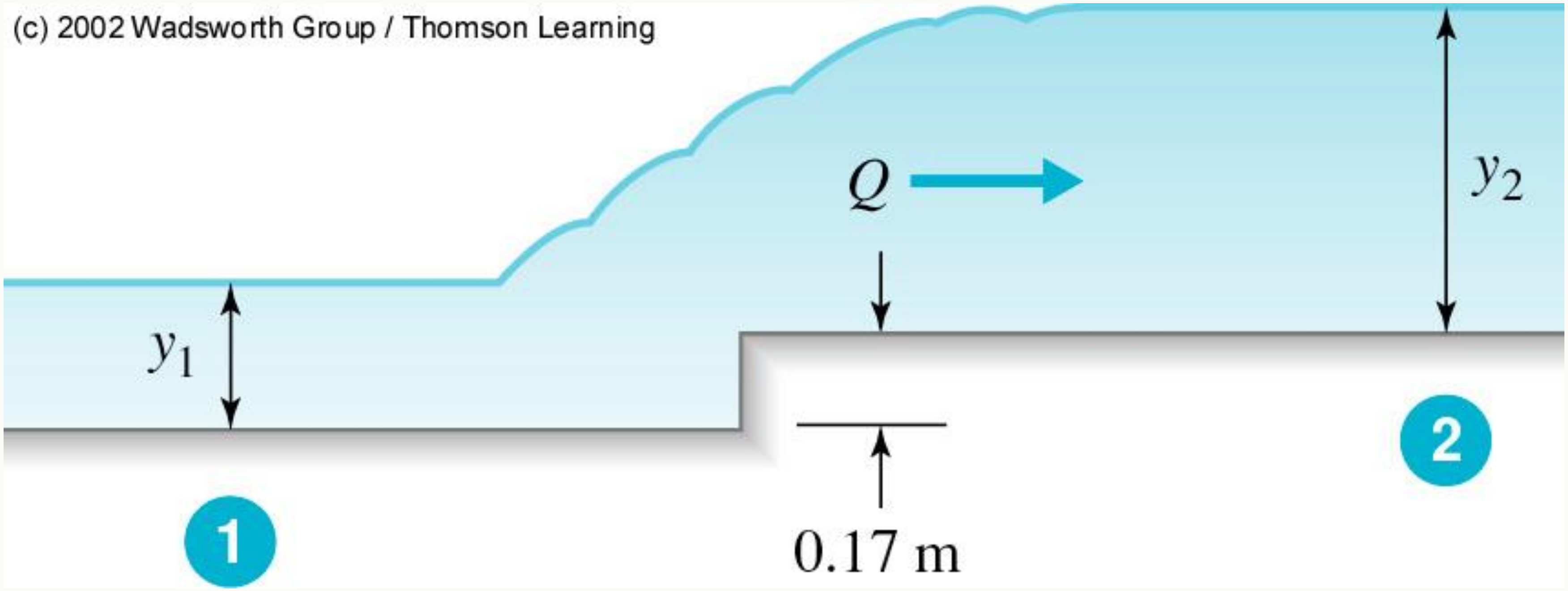


Figure P10.44

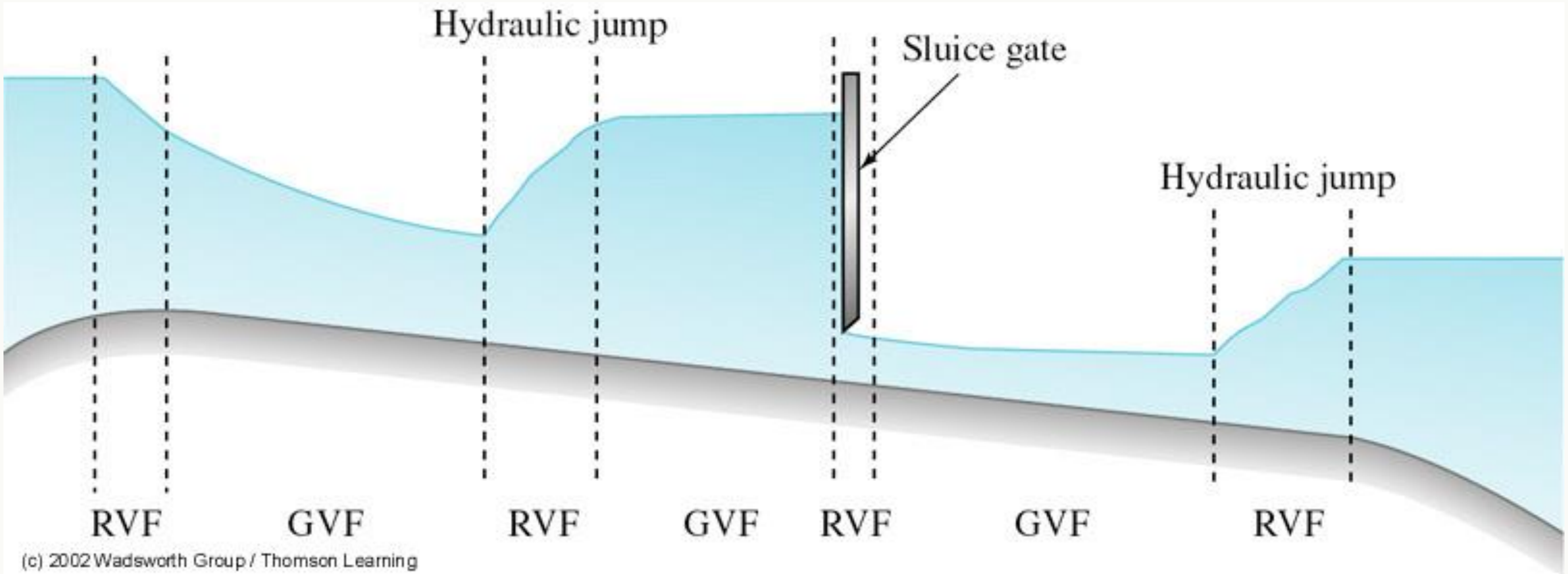


Figure 10.2 – Steady non-uniform flow in a channel.