Q2) ORIGIN := 1

CALCULATION OF 15 X 15 STIFFNESS MATRIX

$$
KG(N, Ndof, ELT, CON) := \left|
\begin{array}{l}
\text{for } i \in 1..Ndof \\
\quad \text{for } j \in 1..Ndof \\
\quad\quad K_{i,j} \leftarrow 0 \\
\text{for } i \in 1..N \\
\quad \left|
\begin{array}{l}
M \leftarrow CON_{i,5} \\
E \leftarrow ELT_{M,1} \\
I \leftarrow ELT_{M,2} \\
L \leftarrow ELT_{M,3} \\
kel \leftarrow \begin{pmatrix}
\frac{12 \cdot E \cdot I}{L^3} & \frac{6 \cdot E \cdot I}{L^2} & -\frac{12 \cdot E \cdot I}{L^3} & \frac{6 \cdot E \cdot I}{L^2} \\
\frac{6 \cdot E \cdot I}{L^2} & \frac{4 \cdot E \cdot I}{L} & -\frac{6 \cdot E \cdot I}{L^2} & \frac{2 \cdot E \cdot I}{L} \\
-\frac{12 \cdot E \cdot I}{L^3} & -\frac{6 \cdot E \cdot I}{L^2} & \frac{12 \cdot E \cdot I}{L^3} & -\frac{6 \cdot E \cdot I}{L^2} \\
\frac{6 \cdot E \cdot I}{L^2} & \frac{2 \cdot E \cdot I}{L} & -\frac{6 \cdot E \cdot I}{L^2} & \frac{4 \cdot E \cdot I}{L}
\end{pmatrix} \\
\text{for } j \in 1..4 \\
\quad a_j \leftarrow CON_{i,j} \\
\text{for } j \in 1..4 \\
\quad \text{for } q \in 1..4 \\
\quad\quad K_{a_j, a_q} \leftarrow K_{a_j, a_q} + kel_{j,q} \quad \text{if } a_j \neq 0 \wedge a_q \neq 0
\end{array}
\right. \\
K
\end{array}
\right.
$$

KG(N,Ndof,ELT,CON) : The program that calculates the local element stiffnesses (ignoring axial deformations) and makes the assembly to construct the global structural stiffness matrix.

N : Total number of elements.

Ndof : Total number of degrees of freedom.

ELT : The array that is used to input stiffness properties of elements. The first column is the E value, the second column is the I value and the third column is the length of the element.

CON : The array that is used to map the local degrees of freedom to the global degrees of freedom. Using CON, the local stiffness matrices are filled into the global structural stiffness matrix. The first four columns indicate the global degrees of freedom that correspond to the local degrees of freedom. If a local degree of freedom does not match with any of the global degrees of freedom, then 0 is input for that local degree of freedom. The last column is used for input of the material type. The numbering of local degrees of freedom that were used in the program for columns and beams are shown in the word file (Numbering.pdf) (Figure 1). In addition the numbering of global degrees of freedom is also shown (Figure 2).

INPUTS

$\mathrm{Ndof} := 15 \qquad \mathrm{N} := 15$

$$\mathrm{CON} := \begin{pmatrix} 1 & 6 & 0 & 0 & 1 \\ 2 & 8 & 1 & 6 & 1 \\ 3 & 10 & 2 & 8 & 1 \\ 4 & 12 & 3 & 10 & 1 \\ 5 & 14 & 4 & 12 & 1 \\ 1 & 7 & 0 & 0 & 1 \\ 2 & 9 & 1 & 7 & 1 \\ 3 & 11 & 2 & 9 & 1 \\ 4 & 13 & 3 & 11 & 1 \\ 5 & 15 & 4 & 13 & 1 \\ 0 & 6 & 0 & 7 & 2 \\ 0 & 8 & 0 & 9 & 2 \\ 0 & 10 & 0 & 11 & 2 \\ 0 & 12 & 0 & 13 & 2 \\ 0 & 14 & 0 & 15 & 2 \end{pmatrix}$$

$$\mathrm{ELT} := \begin{pmatrix} 30\cdot10^{6} & 2\cdot10^{-3} & 3 \\ 30\cdot10^{6} & 1.6\cdot10^{-3} & 5 \end{pmatrix}$$

Note : KN,m and ton are used for units.

$\mathrm{KG(N, Ndof, ELT, CON)} =$

	1	2	3	4	5	6
1	$1.0667\cdot10^{5}$	$-5.3333\cdot10^{4}$	0	0	0	0
2	$-5.3333\cdot10^{4}$	$1.0667\cdot10^{5}$	$-5.3333\cdot10^{4}$	0	0	$4\cdot10^{4}$
3	0	$-5.3333\cdot10^{4}$	$1.0667\cdot10^{5}$	$-5.3333\cdot10^{4}$	0	0
4	0	0	$-5.3333\cdot10^{4}$	$1.0667\cdot10^{5}$	$-5.3333\cdot10^{4}$	0
5	0	0	0	$-5.3333\cdot10^{4}$	$5.3333\cdot10^{4}$	0
6	0	$4\cdot10^{4}$	0	0	0	$1.984\cdot10^{5}$
7	0	$4\cdot10^{4}$	0	0	0	$1.92\cdot10^{4}$
8	$-4\cdot10^{4}$	0	$4\cdot10^{4}$	0	0	$4\cdot10^{4}$
9	$-4\cdot10^{4}$	0	$4\cdot10^{4}$	0	0	0
10	0	$-4\cdot10^{4}$	0	$4\cdot10^{4}$	0	0
11	0	$-4\cdot10^{4}$	0	$4\cdot10^{4}$	0	0
12	0	0	$-4\cdot10^{4}$	0	$4\cdot10^{4}$	0
13	0	0	$-4\cdot10^{4}$	0	$4\cdot10^{4}$	0
14	0	0	0	$-4\cdot10^{4}$	$4\cdot10^{4}$	0
15	0	0	0	$-4\cdot10^{4}$	$4\cdot10^{4}$	0

15 X 15 STIFFNESS MATRIX

1.07E+05	-5.33E+04	0	0	0	0	0	-4.00E+04	-4.00E+04	0	0	0	0	0	0
-5.33E+04	1.07E+05	-5.33E+04	0	0	4.00E+04	4.00E+04	0	0	-4.00E+04	-4.00E+04	0	0	0	0
0	-5.33E+04	1.07E+05	-5.33E+04	0	0	0	4.00E+04	4.00E+04	0	0	-4.00E+04	-4.00E+04	0	0
0	0	-5.33E+04	1.07E+05	-5.33E+04	0	0	0	0	4.00E+04	4.00E+04	0	0	-4.00E+04	-4.00E+04
0	0	0	-5.33E+04	5.33E+04	0	0	0	0	0	0	4.00E+04	4.00E+04	4.00E+04	4.00E+04
0	4.00E+04	0	0	0	1.98E+05	1.92E+04	4.00E+04	0	0	0	0	0	0	0
0	4.00E+04	0	0	0	1.92E+04	1.98E+05	0	4.00E+04	0	0	0	0	0	0
-4.00E+04	0	4.00E+04	0	0	4.00E+04	0	1.98E+05	1.92E+04	4.00E+04	0	0	0	0	0
-4.00E+04	0	4.00E+04	0	0	0	4.00E+04	1.92E+04	1.98E+05	0	4.00E+04	0	0	0	0
0	-4.00E+04	0	4.00E+04	0	0	0	4.00E+04	0	1.98E+05	1.92E+04	4.00E+04	0	0	0
0	-4.00E+04	0	4.00E+04	0	0	0	0	4.00E+04	1.92E+04	1.98E+05	0	4.00E+04	0	0
0	0	-4.00E+04	0	4.00E+04	0	0	0	0	4.00E+04	0	1.98E+05	1.92E+04	4.00E+04	0
0	0	-4.00E+04	0	4.00E+04	0	0	0	0	0	4.00E+04	1.92E+04	1.98E+05	0	4.00E+04
0	0	0	-4.00E+04	4.00E+04	0	0	0	0	0	0	4.00E+04	0	1.18E+05	1.92E+04
0	0	0	-4.00E+04	4.00E+04	0	0	0	0	0	0	0	4.00E+04	1.92E+04	1.18E+05

CONDENSATION

The system is condensed to the translational degrees of freedom.

$$
K := \left|\begin{array}{l}
K \leftarrow KG(N, Ndof, ELT, CON) \\
Ktt \leftarrow submatrix(K, 1, 5, 1, 5) \\
Ktr \leftarrow submatrix(K, 1, 5, 6, 15) \\
Krt \leftarrow submatrix(K, 6, 15, 1, 5) \\
Krr \leftarrow submatrix(K, 6, 15, 6, 15) \\
kcon \leftarrow Ktt - Ktr \cdot Krr^{-1} \cdot Krt
\end{array}\right.
$$

CONDENSED STIFFNESS MATRIX

$$
K = \begin{pmatrix}
9.0874 \times 10^{4} & -5.3226 \times 10^{4} & 1.5208 \times 10^{4} & -2.8406 \times 10^{3} & 414.7009 \\
-5.3226 \times 10^{4} & 7.671 \times 10^{4} & -5.0475 \times 10^{4} & 1.4409 \times 10^{4} & -2.1035 \times 10^{3} \\
1.5208 \times 10^{4} & -5.0475 \times 10^{4} & 7.591 \times 10^{4} & -4.8876 \times 10^{4} & 1.1028 \times 10^{4} \\
-2.8406 \times 10^{3} & 1.4409 \times 10^{4} & -4.8876 \times 10^{4} & 6.801 \times 10^{4} & -3.1224 \times 10^{4} \\
414.7009 & -2.1035 \times 10^{3} & 1.1028 \times 10^{4} & -3.1224 \times 10^{4} & 2.1961 \times 10^{4}
\end{pmatrix}
$$

MASS MATRIX

$$M := \begin{pmatrix} 25 & 0 & 0 & 0 & 0 \\ 0 & 25 & 0 & 0 & 0 \\ 0 & 0 & 25 & 0 & 0 \\ 0 & 0 & 0 & 25 & 0 \\ 0 & 0 & 0 & 0 & 25 \end{pmatrix}$$

Equation of Motion

$$\underline{M}\,\ddot{\underline{U}} + \underline{K}\,\underline{U} = \underline{P}$$

where

$$\underline{P} = \begin{bmatrix} P_1 & P_2 & P_3 & P_4 & P_5 \end{bmatrix}^T \qquad \underline{U} = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 & u_5 \end{bmatrix}^T$$

$$\ddot{\underline{U}} = \begin{bmatrix} \ddot{u}_1 & \ddot{u}_2 & \ddot{u}_3 & \ddot{u}_4 & \ddot{u}_5 \end{bmatrix}^T$$