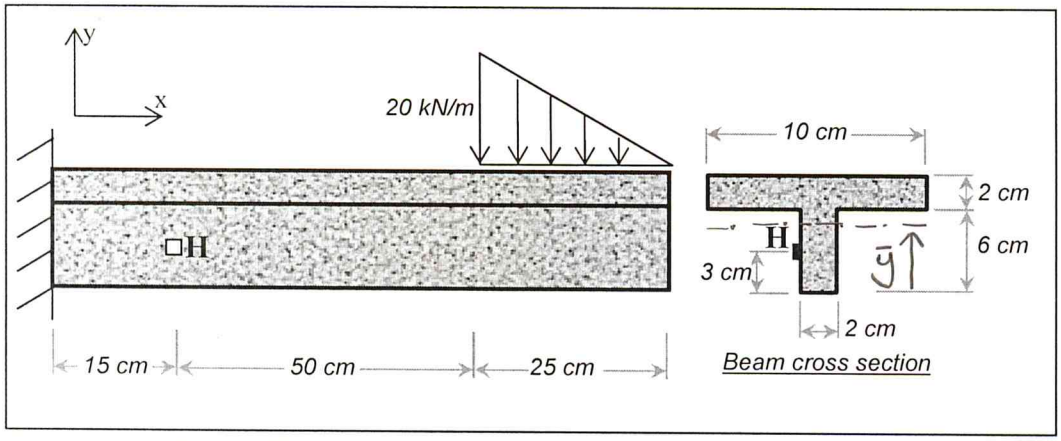
METU Civil Engineering Department	CE 224 Mechanics of Materials Summer 2014	Quiz #5
Name: SOLUTION	Signature:	Time: 30 mins

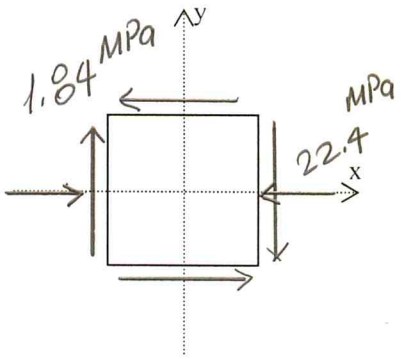
For the beam and loading shown,

i. determine the stress state at point H and show these stresses on the square element given
ii. draw the corresponding Mohr's circle
iii. using the Mohr's circle drawn in part-ii, determine the principal stresses and show them on a properly oriented square element

Please show all your calculations clearly!



Beam cross section



Stress state at point H

$$\bar{y} = \frac{10\times2\times7 + 6\times2\times3}{10\times2 + 6\times2} = 5.5\ cm$$

$$I = \tfrac{1}{12}\times10\times2^3 + 10\times2\times1.5^2 + \tfrac{1}{12}\times2\times6^3 + 2\times6\times2.5^2 = 162.7\ cm^4$$

$$V_H = 20\times0.25\times\tfrac{1}{2} = 2.5\ kN$$

$$M_H = (20\times0.25\times\tfrac{1}{2})(0.5+0.25\times\tfrac{1}{3}) = 1.46\ kN\text{-}m$$

$$\sigma_y = 0$$

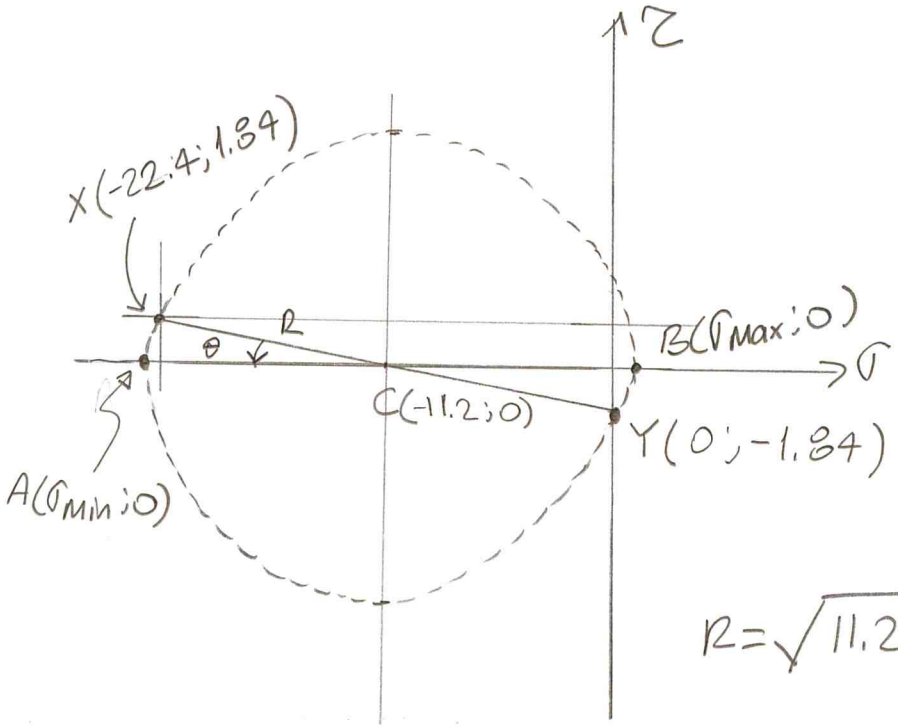
$$\sigma_x = \frac{M_H \times c}{I} = \frac{1.46\times10^6\times25}{162.7\times10^4} = 22.4\ MPa$$

$$Q = 3\times2\times4 = 24\ cm^3$$

$$\tau_{xy} = \frac{V\times Q}{I\times t} = \frac{2500\times24\times10^3}{162.7\times10^4\times20} = 1.84\ MPa$$

$$\sigma_{ave} = \frac{-22.4+0}{2} = -11.2\ MPa \Rightarrow C(-11.2;0)$$

$X(-22.4;1.84)$ $Y(0;-1.84)$



$$R = \sqrt{11.2^2 + 1.84^2} = 11.35$$

$$\sigma_{Max} = -11.2 + 11.35 = 0.15\ MPa$$

$$\sigma_{Min} = -11.2 - 11.35 = -22.55\ MPa$$

$$\tan\theta = \frac{1.84}{11.2} \Rightarrow \theta = 9.3°$$