Karaesmen

MIDDLE EAST TECHNICAL UNIVERSITY
FACULTY OF ENGINEERING

CE 204 UNCERTAINTY and DATA ANALYSIS
Spring Semester 2012-2013

Homework 2- Date Due: April 2nd, 2013 Tuesday till 16.00

IMPORTANT NOTICE:

- *You are allowed to collaborate with other students (or ask questions to your assistants/ instructors) on homework provided that you stay away from plagiarizing (according to dictionaries "to plagiarize" means to steal and pass off ideas and/or words/ solutions of another as one's own without citing the source). That is, collaboration is accepted if you write and give your own solutions. If you are caught on plagiarizing or cheating by handing in "too similar" homework, you will be graded by zero on this homework.*

1. **(Exam1, 2011-12 Spring)** The compressive strength of concrete to be used for a mass dam is modeled by a normal distribution with a mean of 40 MPa and coefficient of variation of 0.15 by the design engineer.
   a) If the specifications require the compressive strength to exceed 50 MPa, what is the probability of satisfying the specification limits?
   b) Is it reasonable to use normal distribution model for the compressive strength of concrete? Why yes/why no?
   c) What must be the specification limit of concrete so that the probability of exceeding it, is 0.95?
   d) If another engineer proposes to model the concrete by a lognormal distribution with the same mean and coefficient of variation as given above, what is the probability of the strength to exceed 50 MPa? Compare your results in parts "a" and "d" and comment on them.

X: compressive strength, $\delta = 0.15$, $\sigma_X = \mu \cdot \delta = 0.15 \times 40 = 6$ MPa

$N_X(40\text{MPa}, 6\text{ MPa})$

a) $P(X > 50) = P\left(Z > \frac{50-40}{6}\right) = P(Z > 1.667) \approx 1 - 0.952$
≈ 0.048

b) $P(X < 0) = P\left(Z < \frac{0-40}{6}\right) = P(Z < -6.667) \approx 0.0$ so it is reasonable to use normal distribution.

c) $P(X_{max} < X) = 0.95 \Rightarrow P\left(Z > \frac{X_{max}-40}{6}\right) = 0.95$

$\frac{x_{max}-40}{6} = -1.65 \Rightarrow x_{max} = 40 - 6 \times 1.65 = 30.1$ MPa

d) $\xi^2 = \ln(1+\delta^2) = \ln(1+0.15^2) = 0.02225 \Rightarrow \xi = 0.1492$ (or ≈ 0.15)

$\lambda = \ln(40) - \frac{1}{2}\xi^2 = 3.6778$

$P(x > 50) = P\left(Z > \frac{\ln 50 - 3.6778}{0.1492}\right) = P(Z > 1.5705)$
$\approx 1 - 0.942 \cong 0.058.$

With the given information, results of "a" & "b"

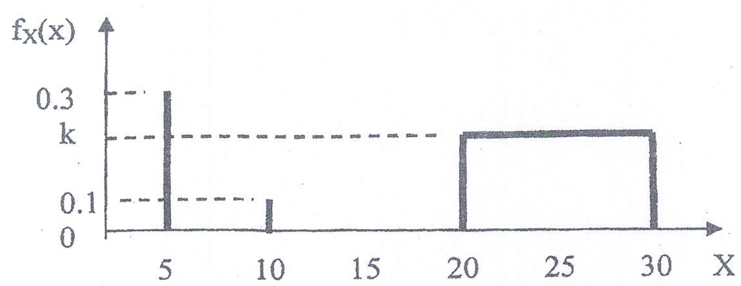
2. **(Exam1, 2011-12 Spring)** The random variable, X takes the values 5 and 10 with probabilities 0.3 and 0.1, respectively and is uniformly distributed between 20 and 30 as shown in the figure below.

a) Compute the appropriate value of k so that X has a proper probability function. Write down the expressions for the probability and cumulative probability distributions of the random variable X. Plot the cumulative probability distribution function.

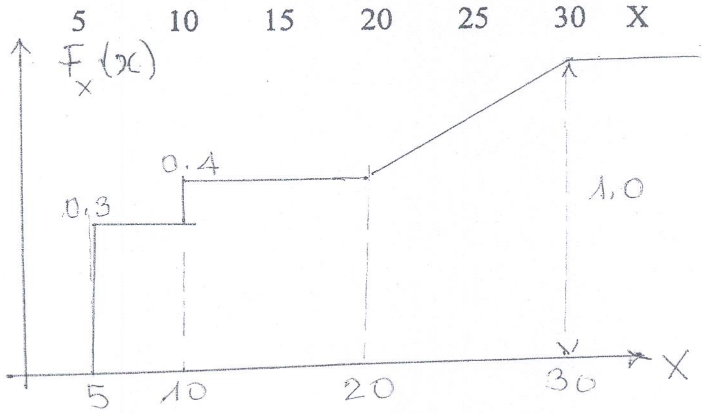
b) Find the mean, median and mode of the random variable X.

c) Find the variance, standard deviation and coefficient of variation of X.

d) What is the probability that X is greater than 21, if it is known that X is less than 25?



a)



$\Sigma P_X(x) = 0.3 + 0.1 + k(30-20) = 1.0 \Rightarrow k = 0.06$

$P_X(x) = 0.3$ for $x = 5$

$P_X(x) = 0.1$ 〃 $x = 10$

$f_X(x) = \frac{0.6}{30-20} = 0.06$ for $20 \leq x \leq 30$.

$= 0$, elsewhere.

$F_X(x) = 0, \ x < 5; \quad F_X(x) = 0.3, \ x < 10; \quad F_X(x) = 0.4, \ x < 20;$

$F_X(x) = 0.4 + \int_{20}^{x} 0.06\,dx = 0.4 + 0.06\Big|_{20}^{x} = 0.06x - 0.8, \ 20 \leq x \leq 3$

b) $E(x) = 5 \times 0.3 + 10 \times 0.1 + \int_{20}^{30} x \cdot 0.06\,dx = 2.5 + \frac{0.06x^2}{2}\Big|_{20}^{30} = 2.5 + 15 = 17.5$

$F_X(x_{med}) = 0.5$ so $x > 20 \Rightarrow 0.4 + \int_{20}^{x_{med}} 0.06\,dx = 0.5 \Rightarrow 0.06x_{med} = 1.3$

$x_{med} = 23.3$

$x_{mode} = 5.$

c) $V(x) = E(x^2) - \overline{E(x)}^2 = 25 \times 0.3 + 100 \times 0.1 + \int_{20}^{30} x^2 (0.06)\,dx - (17.5)^2 = 91.25$

$\sigma_X = \sqrt{91.25} = 9.552, \quad \delta = 9.552/17.5 = 0.546$

d) $P_X(x > 21) = 0.3 + 0.1 + \int_{20}^{21} 0.06\,dx = 0.4 + 0.06(21-20) = 0.46.$

$P_X(X > 21 \mid \ldots) \ldots = \frac{P(x<25) - P(x>21)}{\ldots} = \frac{0.7 - 0.46}{\ldots} \approx 0.3\ldots$

3. **(3.14)** A structure with a design life of 50 years is planned for a site where high-intensity earthquakes may occur with a return period of 100 years. The structure is designed to have a 0.99 probability of not suffering damage within its design life. Damage effects between earthquakes are statistically independent.

   a) If the occurrence of high-intensity earthquakes at the site is modeled by a Bernoulli sequence, what is the probability of damage to the structure under a single earthquake?

   b) Using the damage probability to a single earthquake from Part (a), what would be the probability of damage to the structure in the next 20 years, assuming that the occurrences of earthquakes constitute a Poisson process?

Let D be the design life of the structure $\Rightarrow D = 50$ years

Mean rate of high intensity earthquake : $1/100$ per year,

a) Probability of damage within 50 years : $1/100$

" " no damage within 50 years : $1 - 1/100 = 99/100$

Probability of earthquake each year : θ

" " of damage each year : $\frac{1}{100} \cdot \theta = 0.01\theta$.

Probability of no damage in 50 years :

$$P_X(x=0) = \binom{50}{0} (0.01\theta)^0 (1-0.01\theta)^{50-0}$$

For $P_X(x=0) = 0.99 = \frac{50!}{(50-0)!\,0!} \cdot 1 \, (1-0.01\theta)^{50}$

$(1-0.01\theta) = 0.99^{1/50} \Rightarrow 1-0.01\theta = 0.9998 \Rightarrow \theta = 0.020099$

b) Assuming Poisson Process :

Mean number of damaging earthquakes : $0.01\theta = 0.0002/\text{year}$

Probability of damage in 20 years : $\sum_{x=1}^{\infty} \frac{e^{-\lambda^*} \lambda^{*x}}{x!} = \sum_{1}^{\infty} \frac{e^{-(20 \times 0.002)} \lambda^*}{x!}$

λ^* for 20 years : $20 \times 0.0002 = 0.004$ $= 1 - Pr(\text{no dam}$

Probability of damage in 20 years : $1 - \frac{e^{-0.004} (0.004)^0}{0!}$

$= 1 - 0.99608 \approx 0.004$

4. **(3.27)** One of the hazards to an existing underground pipeline is due to improperly conducted excavations. Consider a system consisting of 150 km of pipeline. Suppose the number of excavations along this pipeline over the next year follows a Poisson process with a mean rate of 1 per 75 km. Forty percent of the excavations are expected to result in damage to the pipeline. Assume the events of damage between excavations are statistically independent.

   a) What is the probability that there will be at least two excavations along the pipeline next year?

   b) Suppose that two excavations will be performed. What is the probability that the pipeline will be damaged?

   c) What is the probability that the pipeline will not be damaged from excavations next year?

Let X be the total number of excavations along the pile for the next year.

X has Poisson distribution with mean, $\lambda = \frac{1}{75km} 150km = 2$

a) Probability of $X \geq 2 = 1 - \sum_{x=0}^{1} \frac{e^{-2} 2^x}{x!}$

$$= 1 - \underbrace{\frac{e^{-2} 2^0}{0!}}_{0.1353} - \underbrace{\frac{e^{-2} 2}{1!}}_{0.2707} \cong 0.594.$$

b) Probability of getting damaged in each excavation is 0.4.

Probability of no damage in each excavation is $(1-0.4) = 0.6$

Probability of no damage in two excavations; $(0.6 \times 0.6) = 0.36$

Probability of damage: $1 - 0.36 = 0.64$.

c) Prob. of no damage in x excavations;

$P\left(\text{no damage}/x \text{ excavations}\right) \cdot P(x \text{ excavations})$

$$= \sum_{x=0}^{\infty} 0.6^x \left(\frac{e^{-2} 2^x}{x!}\right) = e^{-2} \sum_{x=0}^{\infty} \frac{0.6^x 2^x}{x!}$$

$$= e^{-2} \left[0.6 + 0.6 \times 2 + \frac{0.6^2 \times 4}{2!} + \frac{0.6^3 2^3}{3!} + \dots\right] \cong 0.4493$$

(OR 40% damaging earthquakes will have $\lambda = 0.4 \times 1/75 \times 150 = 0.8$,

no damage in x excavations; $\frac{e^{-0.8} 0.8^0}{0!} = e^{-0.8} = 0.4493$

5. **(3.45)** The time between severe earthquakes at a given region follows a lognormal distribution with a coefficient of variation of 40%. The expected time between severe earthquakes is 80 years.

a) Determine the parameters of this lognormally distributed recurrence time T. (Ans. 4.3078, 0.3853)

b) Determine the probability that a severe earthquake will occur within 20 years from the previous one.

c) Suppose the last severe earthquake in the region took place 100 years ago. What is the probability that severe earthquake will occur over the next year?

a) $\xi^2 = \ln(1+\delta^2) = \ln(1+0.4^2) = 0.14842 \Rightarrow \xi \approx 0.3853$

$\lambda = \ln\mu - \frac{1}{2}\xi^2 = \underbrace{\ln 80}_{2.99573} - \frac{1}{2} \times 0.14842 = 4.3078$

b) $P(x \leq 20) = P\left(z \leq \underbrace{\frac{\ln 20 - 4.3078}{0.3853}}_{-3.4054}\right) \approx 0.0003$

c) $P\left(100 < x < 101 / x > 100\right) = \frac{P(100 < x < 101)}{P(x > 100)}$

$= \frac{P\left((\ln 100 - 4.3078)/0.3853 < z < (\ln 101 - 4.3078)/0.3853\right)}{P\left(z > \frac{\ln 100 - 4.3078}{0.3853}\right)}$

$= P(0.7719 < z < 0.788) / P(z > 0.7719)$

$= \frac{0.009}{1 - 0.7719} \approx 0.0395$