CE 382 - Reinforced Concrete Fundamentals

HOMEWORK 2

1) a) $\sigma_c = 19200\,\varepsilon_c$

$N_c + N_{st} = 1500\,kN$, From compatibility $\varepsilon_c = \varepsilon_s = \varepsilon$

$N_c = \sigma_c \times A_c$ $\qquad \sigma_c = 19200\,\varepsilon$

$N_{st} = \sigma_{st} \times A_{st}$ $\qquad \sigma_{st} = 200000\,\varepsilon$

$$\left.\begin{aligned} N_c &= (19200\varepsilon) \times (300 \times 300) = 1728\,000\,\varepsilon \;kN \\ N_{st} &= (200000\varepsilon) \times \left(6 \times \frac{\pi 20^2}{4}\right) = 377000\,\varepsilon \;kN \end{aligned}\right\} \varepsilon = 7.126 \times 10^{-4}$$

check $\sigma_{st} = 200\,000\,\varepsilon \leq f_y$ ✓

$$\boxed{\begin{aligned} N_c &= 1231.4\;kN \\ N_{st} &= 268.6\;kN \end{aligned}}$$

b) $\sigma_c = 26480\,\varepsilon_c\,[1 - 221\,\varepsilon_c]$, $\varepsilon_c = \varepsilon_{st} = \varepsilon$

$N_c = (26480\,\varepsilon_c - 3642080\,\varepsilon_c^2) \times (300 \times 300) = 2\,383\,200\,\varepsilon - 327\,787\,200\,\varepsilon^2\;kN$

$N_{st} = 377\,000\,\varepsilon\;kN$

$$-327\,787\,200\,\varepsilon^2 + 2760200\,\varepsilon - 1500 = 0 \xrightarrow{solve} \varepsilon_{1,2} = \frac{-b \mp \sqrt{\Delta}}{2a}$$

$\varepsilon_1 = 7.84 \times 10^{-3}$ $\longrightarrow$ ε_1 is not possible because it creates the maximum stress in steel which is f_y and the equality is not checked $N_{st} + N_c = 1500$.

$\varepsilon_2 = 5.84 \times 10^{-4}$

$\varepsilon = 5.84 \times 10^{-4}$

$$\boxed{\begin{aligned} N_c &= 1280\;kN \\ N_{st} &= 220\;kN \end{aligned}}$$

c) Assuming linear-elastic behaviour for concrete makes the stress in concrete smaller. As can be seen from the graph on the question sheet, for $f_c = 30\,MPa$ ε reaches higher value for non-linear curve, which increases its capacity.

2) a) $N_{or} = 0.85 f_{ck} A_c + A_{st} f_{yk}$

$$N_{or} = 0.85 \times 20 \times (300 \times 300) + 8 \times \frac{\pi 12^2}{4} \times 420 = 1910 \text{ kN}$$

b) $N_{or} = 0.85 \times 40 \times (300 \times 300) + 8 \times \frac{\pi 12^2}{4} \times 420 = 3440 \text{ kN}$

c) $N_{or} = 0.85 \times 20 \times (300 \times 300) + 8 \times \frac{\pi 24^2}{4} \times 420 = 3050 \text{ kN}$

d) $N_{or} = 0.85 \times 40 \times (300 \times 300) + 8 \times \frac{\pi 24^2}{4} \times 420 = 4580 \text{ kN}$

e) Higher concrete strength and larger steel area causes higher axial load capacity.

3) a) $f_{cc} = 0.85 f_c + 6 \sigma_2$, $\sigma_2 = 2 f_{yw} \frac{A_o}{D.s}$

$N_{or} = N_{or2}$

$$N_{or} = 0.85 f_c A_c + A_{st} f_y = 0.85 \times 35 \times (550 \times 300) + 12 \times \frac{\pi 20^2}{4} \times 420$$
$$= 6492 \text{ kN}$$

$$f_{cc} = 0.85 \times 35 + 6 \times 2 \times 420 \times \frac{\pi 12^2}{4} \times \frac{1}{250 \times s} = 29.75 + \frac{2280}{s}$$

$$N_{or2} = (f_{cc} \times \underbrace{2 A_{ck}}^{2\text{ cores}}) + A_{st} f_y = \left(29.75 + \frac{2280}{s}\right) \times 2 \times \frac{\pi 250^2}{4} + 12 \times \frac{\pi 20^2}{4} \times 420$$

$$N_{or2} = 4504 + \frac{223\,838.5}{s} \text{ kN}$$

$$N_{or} = 6492 = N_{or2} \longrightarrow \boxed{s = 112.6 \text{ mm}}$$

b) $N_{or} = N_{or2} = 6492 \text{ kN}$

4) a) * For 1st cross section,

$N_{or} = 0.85 \times 20 \times \frac{\pi 500^2}{4} + 8 \times \frac{\pi 20^2}{4} \times 420 = \boxed{4393.5\ kN}$

$f_{cc} = 0.85 f_c + 4\sigma_2 = 0.85 f_c + 8 \frac{A_0 f_{yw}}{D \times S} = 0.85 \times 20 + 8 \times \frac{\pi 10^2}{4} \times \frac{220}{300 \times 40} = 28.5\ MPa$

$N_{or2} = 28.5 \times \frac{\pi 300^2}{4} + 8 \times \frac{\pi 20^2}{4} \times 420 = \boxed{3070\ kN}$

* For 2nd cross-section,

$N_{or} = 0.85 \times 20 \times \frac{\pi 350^2}{4} + 8 \times \frac{\pi 20^2}{4} \times 420 = \boxed{2691\ kN}$

$f_{cc} = 28.5\ MPa$ (Same with the first one)

$N_{or2} = 28.5 \times \frac{\pi 300^2}{4} + 8 \times \frac{\pi 20^2}{4} \times 420 = \boxed{3070\ kN}$

* For 3rd cross-section,

$N_{or} = 0.85 \times 20 \times (350 \times 350) + 8 \times \frac{\pi 20^2}{4} \times 420 = \boxed{3138\ kN}$

$N_{or2} = 28.5 \times \frac{\pi 300^2}{4} + 8 \times \frac{\pi 20^2}{4} \times 420 = \boxed{3070\ kN}$

Since N_{or2} directly related with core area, stirrup diameter and spacing; it does not change with the changes in the shell area.

b) For 1st section,

$N_{or2} = N_{or} + 0.2 \times N_{or} = 5272.2\ kN$

$5272.2 = f_{cc} \times \frac{\pi 300^2}{4} \times 10^{-3} + 1055.6 \rightarrow f_{cc} = 59.6\ MPa$

$59.6 = 0.85 \times 20 + 8 \times \frac{\pi 16^2}{4} \times \frac{220}{300 \times S} \rightarrow \boxed{S = 10.8\ mm}$

For 2nd,

$N_{or2} = 3229.2\ kN = f_{cc} \times \frac{\pi 300^2}{4} \times 10^{-3} + 1055.6 \quad f_{cc} = 30.75\ MPa$

$30.75 = 0.85 \times 20 + 8 \times \frac{\pi 10^2}{4} \times \frac{220}{300 \times S} \rightarrow \boxed{S = 33.5\ mm}$

For 3rd,

$3765.6 = 70.68 f_{cc} + 1055.6 \quad f_{cc} = 38.3\ MPa$

$38.3 = 17 + \frac{460.8}{S} \rightarrow \boxed{S = 21.6\ mm}$

5) a) Compatibility $\varepsilon_{st} = \varepsilon_c = \varepsilon$

$\sigma_{st} = 200000\,\varepsilon$ MPa

$\sigma_c = 8$ MPa

Hognestad model $\rightarrow E_c = 12680 + 460 f_c = 28780$ MPa

$$\varepsilon_{co} = \frac{2f_c}{E_c} = 2.43 \times 10^{-3}$$

$$\sigma_c = f_c \left[2\frac{\varepsilon_c}{\varepsilon_{co}} - \left(\frac{\varepsilon_c}{\varepsilon_{co}}\right)^2 \right]$$

$$8 = 35 \left[\frac{2\varepsilon}{2.43\times10^{-3}} - \left(\frac{\varepsilon}{2.43\times10^{-3}}\right)^2 \right] \longrightarrow \varepsilon = 3\times10^{-4}$$

$\sigma_{st} = 60$ MPa, $\sigma_c = 8$ MPa

$$N = 8\times(400\times600) + 60\times10\,\frac{\pi 20^2}{4} = \boxed{2108.5\ kN}$$

b) $\sigma_c = 28$ MPa, $E_c = 28780$ MPa, $\varepsilon_{co} = 2.43\times10^{-3}$

$$28 = 35\left[\frac{2\varepsilon}{0.00243} - \left(\frac{\varepsilon}{0.00243}\right)^2\right] \longrightarrow \varepsilon = 1.34\times10^{-3}$$

$\sigma_{st} = 268$ MPa, $\sigma_c = 28$ MPa

$$N = 28\times(400\times600) + 268\times10\,\frac{\pi 20^2}{4} = \boxed{7562\ kN}$$

c) $\sigma_{st} = 420$ MPa

$420 = \varepsilon\times 200\,000 \longrightarrow \varepsilon = 2.1\times10^{-3}$

$$\sigma_c = 35\left[\frac{2\times2.1}{2.43} - \left(\frac{2.1}{2.43}\right)^2\right] = 34.35\ MPa$$

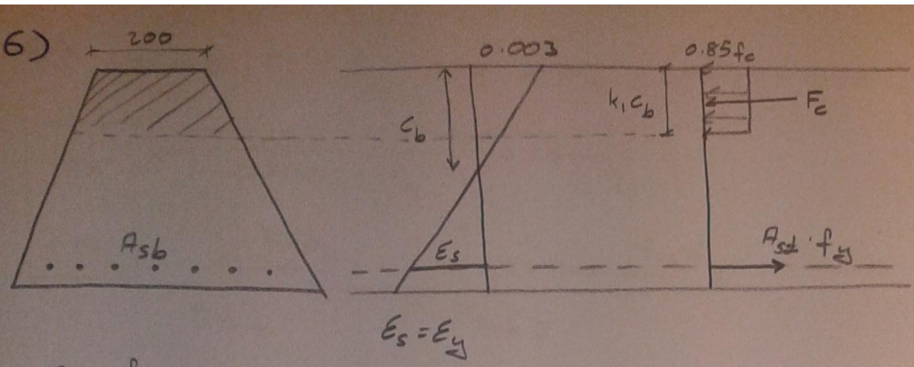
$$N = 34.35\times(400\times600) + 420\times10\times\frac{\pi 20^2}{4} = \boxed{9563.5\ kN}$$

d) $$N_{or} = 35\times(400\times600) + 420\times10\times\frac{\pi 20^2}{4} = \boxed{9719.5\ kN}$$

e) Direct tensile $\rightarrow f_{ct} = 0.35\sqrt{f_c} = 2.1$ MPa

$$N_{cr} = 2.1\times(400\times600) = \boxed{504\ kN}$$

6)



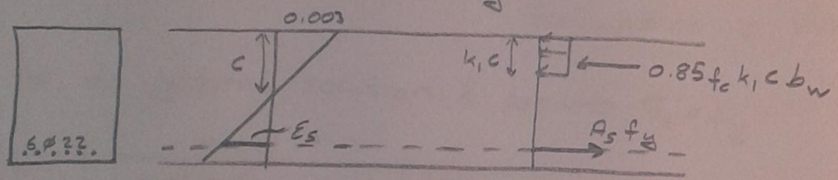
$\varepsilon_y = \frac{f_y}{200000} = 2.1 \times 10^{-3}$ I take clear length as 40 mm.

Compatibility $\longrightarrow$ $\frac{0.003}{c_b} = \frac{0.0021}{560 - c_b}$ $c_b = 329.4$ mm

$F_c = 0.85 \times 25 \times \left(\frac{200 + 386.7}{2} \times 230 \right) = 1745.4$ kN

$1745.4 \times 10^3 = A_{sb} \times 420 \longrightarrow \boxed{A_{sb} = 4156 \text{ mm}^2}$

7) Assume underreinforced $\sigma_s = f_y$



$\frac{c}{660} = \frac{0.003}{0.003 + \varepsilon_s}$ and $0.85 f_c k_1 c b_w = A_s f_y$

$0.85 \times 30 \times 0.85 \times c \times 400 = 6 \times \frac{\pi 22^2}{4} \times 420$

$c = 110.5$ mm

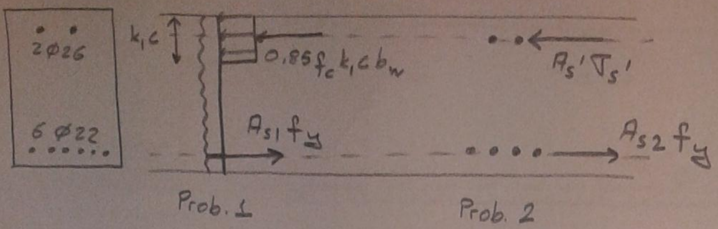
$\varepsilon_s > \varepsilon_y = 0.0021$ $\varepsilon_s = 0.015$

O.K ✓ Assumption is true.

$M_r = A_{st} \times f_y \times \left(d - \frac{k_1 c}{2} \right) = 6 \times \frac{\pi 22^2}{4} \times 420 \times \left(660 - \frac{0.85 \times 110.5}{2} \right)$

$\boxed{M_r = 587.3 \text{ kNm}}$

8) Assume $\sigma_s' = f_y$ and apply superposition



$$A_s' \cancel{\sigma_s'} = A_{s2} \cancel{f_y} \rightarrow A_s' = A_{s2} \qquad A_s' = 2 \times \frac{\pi 26^2}{4} = 1062 \text{ mm}^2$$

$$A_s = A_{s1} + A_{s2} \rightarrow A_{s1} = 6 \times \frac{\pi 22^2}{4} - 1062 = 1219 \text{ mm}^2$$

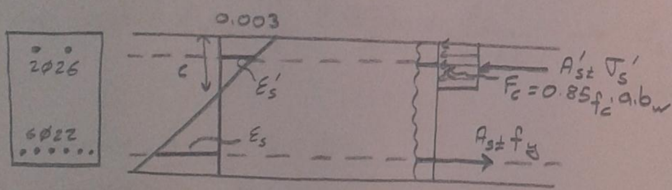
$$0.85 \times 30 \times 0.85 \times c \times 400 = 1219 \times 420 \qquad c = 59 \text{ mm}$$

From similarity of triangles,

$$\frac{0.003}{c} = \frac{\varepsilon_s'}{c-40} \qquad \varepsilon_s' = 0.001 < 0.021$$

Assumption is WRONG!

So, do general solution & assume $\sigma_s = f_y$



$$\sigma_s' = 200000\, \varepsilon_s'$$

$$\frac{0.003}{c} = \frac{0.003 + \varepsilon_s}{660} \quad \text{and} \quad \frac{0.003}{c} = \frac{\varepsilon_s'}{c-40}$$

$$(0.85 \times 30 \times 0.85 \times c \times 400) + \left[2 \times \frac{\pi 26^2}{4} \times 200\,000 \times \left(\frac{0.003 \times (c-40)}{c}\right)\right] = 6 \times \frac{\pi 22^2}{4} \times 420$$

$$8670c + 637\,115 - \frac{25\,484\,600}{c} = 957\,934 \rightarrow c = 75.79 \text{ mm}$$

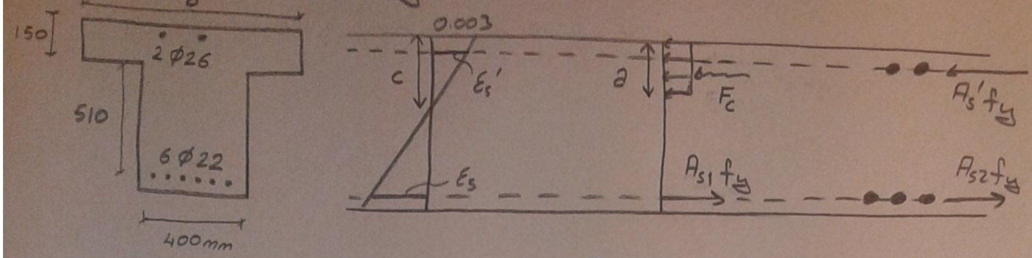
$$\varepsilon_s' = 1.42 \times 10^{-3}$$

$\varepsilon_s = 0.023 > 0.021$ Assumption is TRUE!

$$M_r = \left[(0.85 \times 30 \times 0.85 \times 75.79 \times 400) \times \left(660 - \frac{0.85 \times 75.79}{2}\right)\right] + \left[2 \times \frac{\pi 26^2}{2} \times 284 \times (660 - 40)\right]$$

$$\boxed{M_r = 599.5 \text{ kNm}}$$

9) Assume $\sigma_s' = f_y$ and $a < t$



$$A_s = A_{s1} + A_{s2}$$

$$A_s' f_y = A_{s2} f_y \longrightarrow A_s' = A_{s2} = 2 \times \frac{\pi 26^2}{4} = 1062 \text{ mm}^2$$

$$A_{s1} = 6 \times \frac{\pi 22^2}{4} - 1062 = 1219 \text{ mm}^2$$

$$\Sigma F = 0 \rightarrow \quad F_c = 1219 \times 420 \times 10^{-3} = 512 \text{ kN}$$

From question 8 $\rightarrow M_r = 599.5$ kNm

$$M_r = F_c \times \left(660 - \frac{a}{2}\right) + 1062 \times 420 \times (660 - 40)$$

$$599.5 \times 10^6 = 512 \times 10^3 \times \left(660 - \frac{a}{2}\right) + 276544800 \longrightarrow a = 58.4 \text{ mm}$$

$$a = k_1 c \rightarrow c = 68.7 \text{ mm}$$

Compatibility: $\dfrac{0.003}{c} = \dfrac{\varepsilon_s'}{c-40} \rightarrow \varepsilon_s' = 1.25 \times 10^{-3} \ngtr \varepsilon_y$

Assumption is wrong!!!

* Assume only $a < t$

$$\frac{0.003}{c} = \frac{\varepsilon_s'}{c-40} \qquad \varepsilon_s' = 0.003 - \frac{0.12}{c}$$

$$\sigma_s' = 200000 \times \left(0.003 - \frac{0.12}{c}\right) = 600 - \frac{24000}{c}$$

$$(0.85 \times 30 \times 0.85c \times b) + \left[1062 \times \left(600 - \frac{24000}{c}\right)\right] = 2281 \times 420 \quad (\text{Eq } 1)\ \Sigma F = 0.$$

$$\left[(0.85 \times 30 \times 0.85c \times b) \times \left(660 - \frac{0.85c}{2}\right)\right] + \left[\left(1062 \times \left(600 - \frac{24000}{c}\right)\right) \times (660 - 40)\right] = 599.5 \times 10^6$$

(Eq 2)

2 equations, 2 unknowns (c & b). Solve:

c = 73.51 mm (Assumption is TRUE)

$$\boxed{b = 419 \text{ mm}}$$