CE 382 Reinforced Concrete Fundamentals

Combined Flexure & Axial Load – Design of RC Columns

Design of Columns

- Consider all possible load combinations.
- Design for most critical N_d, M_d' combination.
- Combination with a smaller axial load may be more critical.
- TS 500-2000 minimum eccentricity:

$$\boxed{e_{min} = 15mm + 0.03h}$$

- h: dimension in the direction of eccentricity

Design of Columns

- Maximum axial load:
  - TS 500-2000: $N_d \leq 0.6 f_{ck} A_c$ or $N_d \leq 0.9 f_{cd} A_c$
  - TEC2007: $N_d \leq 0.5 f_{ck} A_c$ or $N_d \leq 0.75 f_{cd} A_c$

$$\boxed{\min A_c = \frac{N_d}{0.75 f_{cd}} \geq 75000\ mm^2}$$

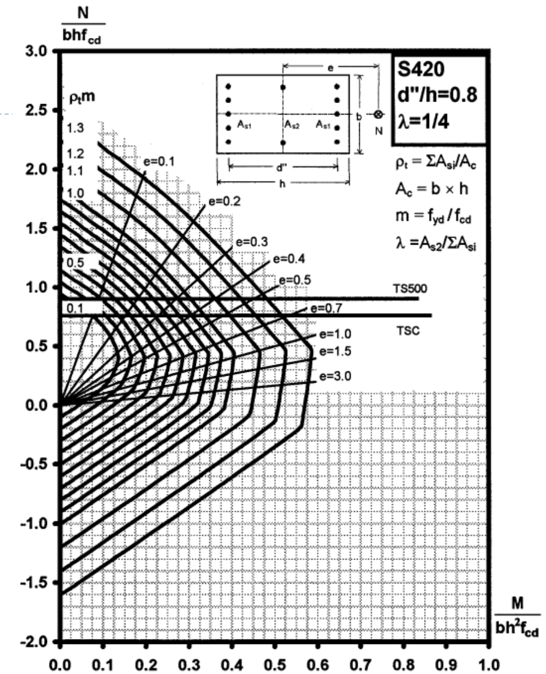
- In the final design $\rightarrow$ design moment should include second order moments
- $P - \delta$ effect is calculated by:
  - Non-linear structural analysis
  - Approximate methods

Design Charts

- Non-dimensional form

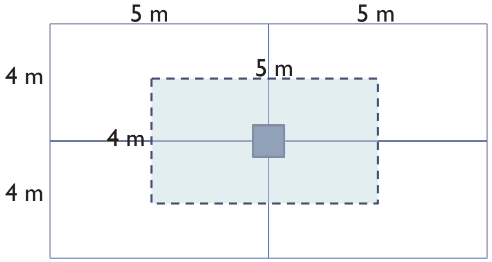
$\bar{n} = \frac{N}{bhf_{cd}}$ & $\bar{m} = \frac{M}{bh^2 f_{cd}}$

- variables:
  - Geometry
  - $\frac{d''}{h}$
  - ρ_t: ratio of longitudinal steel
  - Arrangement of steel
  - Steel grade
- $A_{st} = bh\rho_t$
- $m = \frac{f_{yd}}{f_{cd}}$
- $(\rho_t m)\frac{f_{cd}}{f_{yd}} = \rho_t$



Example 1

- $n = 8\ story$
- Assume $10\ kN/m^2$
- From tributary area:
- $N = 8 \times 4 \times 5 \times 10$
- $N = 1600\ kN$
- Non-sway frame
- C25 $(f_{cd} = 17\ MPa)$, S420 $(365\ MPa)$
- $A_c = \frac{1600000}{0.75 \times 17} = 125490\ mm^2 > 75000\ mm^2$
- Use $350 \times 400\ mm$ column $(140000\ mm^2)$



Example 2

- C25 & S420
- 350×400 mm

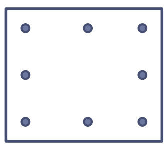
$N_d\ (kN)$	$M'_d\ (kNm)$
1800	180
1600	230
1200	200

→ less critical (both N & M is less)

- Solution:
  - estimate $d' = 35\ mm$, $d'' = h - 2d' = 330\ mm$
  - $d''/h = 0.9$ $\lambda = 1/4$ (assumed)
  - $m = \frac{f_{yd}}{f_{cd}} = \frac{365}{17} = 21.5$

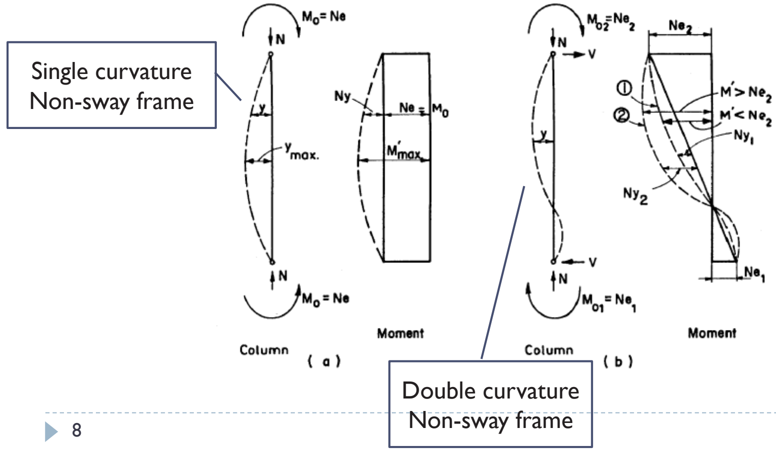
Example 2

- $\frac{1800000}{350 \times 400 \times 17} = 0.76$ $\frac{180000000}{350 \times 400^2 \times 17} = 0.19$ $\rho_t m = 0.5$
- $\frac{1600000}{350 \times 400 \times 17} = 0.67$ $\frac{230000000}{350 \times 400^2 \times 17} = 0.24$ $\rho_t m = 0.6$
- $\rho_t m = 0.6 \quad \rightarrow \quad \rho_t = \frac{0.6}{21.5} = 0.028$
- $A_{st} = \rho_t bh = 0.028 \times 350 \times 400 = 3920\ mm^2$
- Use 8Ø26 $(4247\ mm^2)$



Slenderness Effect

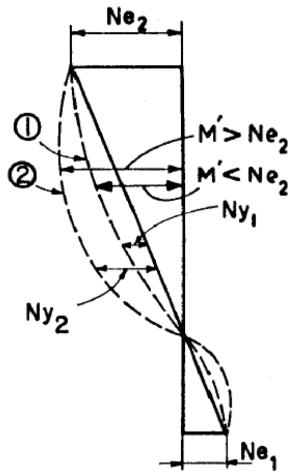
- Second order moments → estimate the amount of displacement
- In real structures → boundary conditions are complex



Slenderness Effect

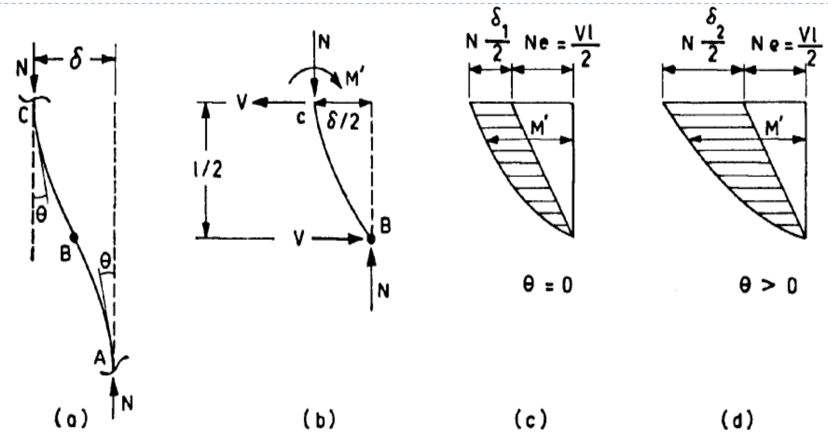
- $M'_d = M_o + Ny = Ne + Ny = N(e + y)$
- «y» depends on the slenderness of the column
- ℓ/h column length / cross-sectional dimension in moment direction
- More general:
- ℓ/i column length / radius of gyration
- Short column → y is small → second order moment can be neglected

Slenderness Effect



- Curve 1
  - Medium slenderness
  - Total moment including the second order moments remains less than the end moments
  - $M' < N \times e_2$
- Curve 2
  - Very slender
  - Second order moments are greatly magnified
  - $M' > N \times e_2$

Sway frame (unbraced frame)



- Shear due to lateral forces (earthquake, wind)
- Symmetrical case; equal end moments; inflection point @ midheight

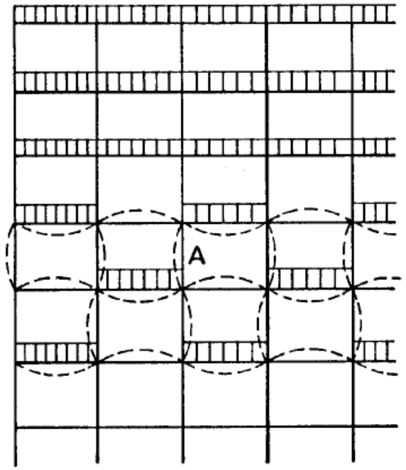
Sway frame (unbraced frame)

- $\theta = 0$ → infinitely rigid floor
- Usual case $\theta > 0$
  - displacements and second order moments are magnified
- Flat plate & block-joist floor (shallow beams, same depth as joist)
  - relative displacement between the two column end are very high
  - second order moments > first order moments
- Magnitude of second order moments depends on θ
- θ depends on the stiffness of columns relative to that of the floor (usually beams), α

Sway frame (unbraced frame)

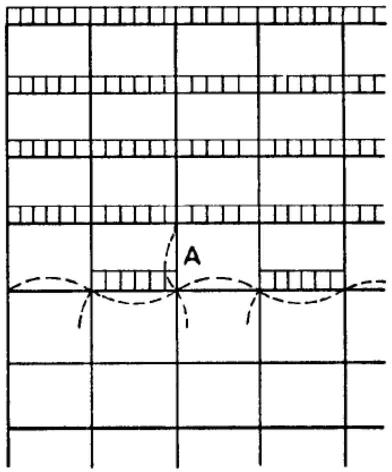
- $\alpha = \dfrac{\sum \frac{I}{\ell}(columns)}{\sum \frac{I}{\ell}(floor\ members)}$
- Beam-column type of structure → floor members: beams
- Flat type od floor system → floor members: slabs (very flexible)
- Use structural wall to take care of lateral loads.
- Deflected shape of the column depends on boundary conditions & arrangement of live load.

Sidesway prevented



- Column A in single curvature
- Live load arrangement to yield maximum moment for the column
- Second order moments definitely increase the design moments.

Sidesway prevented

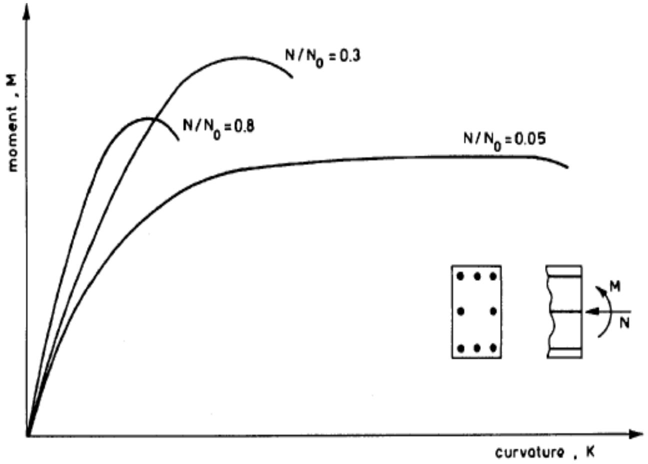


- Live load arrangement for maximum axial load @ column A
- Double curvature
- Second order moments may or may not affect the design moment, depending on the slenderness of the column

Slenderness

- If sidesway is not prevented
  - Design moment is magnified by second order moments regardless of the slenderness ratio
  - All columns have to be considered in a story
- If sidesway is prevented
  - Just individual columns should be checked

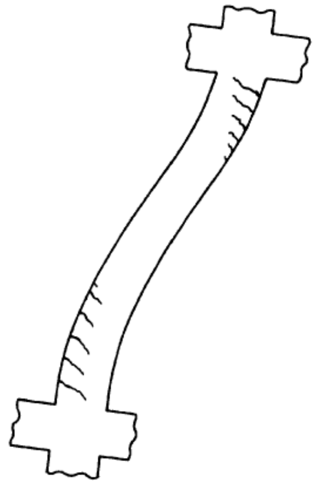
Flexural Rigidity



- Slope of the moment-curvature curve, EI
- Non-linear
- Depends on the level of axial load
- There is no unique $M - \mathcal{K}$ curve for a given cross-section

Flexural Rigidity

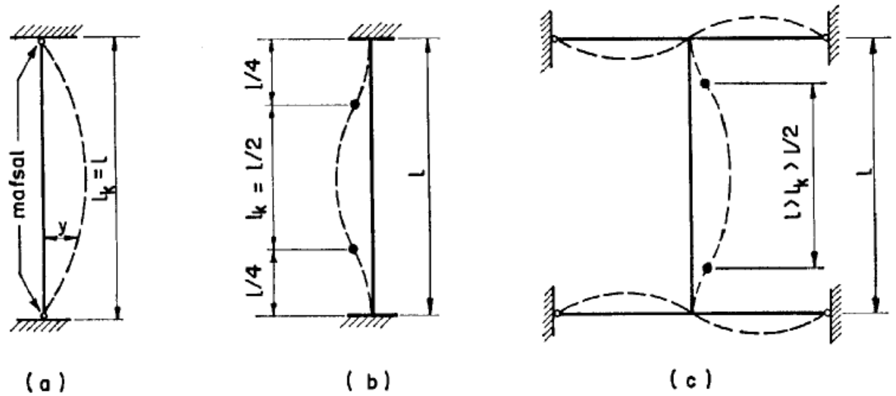
- Elastic Modulus, E, influenced by creep → change in flexural rigidity
- Concrete cracks under flexural moments → reduce moment of inertia, I
- I is not constant along the length of column
- Also cracking can occur due to shrinkage
- Severe seismic action → reversed cyclic loading → stiffness ↘



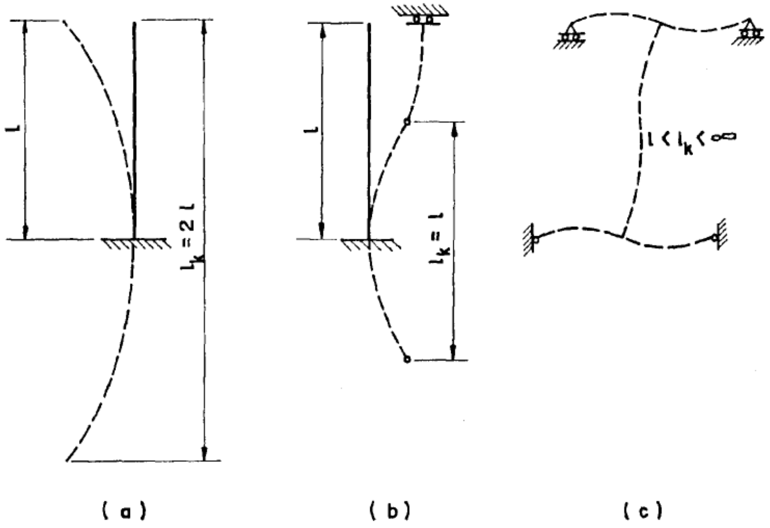
Effective Length of Column

- The deflection of an elastic beam-column depends on the ratio of the axial load to the buckling load
- $\dfrac{N}{N_{cr}} = \dfrac{N}{\frac{\pi^2 EI}{l_k^2}}$
- l_k: effective length; distance between the point of inflections; depends mainly on boundary conditions

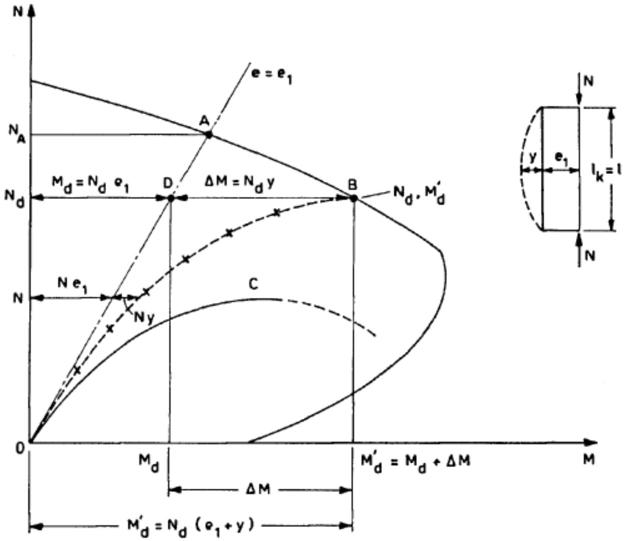
Effective Length of Column – sidesway prevented



Effective Length of Column – unbraced frame



Design Methods for calculating Second Order Moments



- If there is no slenderness effect & with constant eccentricity, e_1 $\rightarrow$ failure at point A with N_A
- For very slender column $\rightarrow$ instability $\rightarrow$ path C
- Realistic path OB for RC columns
- @ $N_d \rightarrow M_d = N_d e_1$ & $\Delta M = N_d y$
- $M_d' = M_d + \Delta M = N_d(e_1 + y)$
- From non-linear analysis
- Add ΔM to M_d (complimentary moment method, CEB)
- Multiply M_d by a magnification factor (moment magnifier method, ACI, TS500)

Moment Magnification Method

- Find the moment magnifier β (for $\frac{\ell_k}{i} \leq 100$)
- Check whether the frame is braced or unbraced
  - $\varphi = 1.5\Delta_i \frac{\Sigma\left(N_{di}/\ell_i\right)}{V_{fi}}$
  - If $\varphi \leq 0.05$ non-sway (braced) frame
  - If $\varphi > 0.05$ sway (unbraced) frame
  - If ℓ_i of each member are identical
  - $\varphi = 1.5 \frac{\sum N_{di} \times \Delta_i}{V_{fi} \times \ell_i}$

    Second order moment

    First order moment
  - This equation requires structural analysis to find Δ_i, V_{fi} & N_{di}

Δ_i: lateral displacement of i^{th} floor, relative to the floor below
N_{di}: axial force on each column & structural wall at i^{th} floor
ℓ_i: length of each column & structural wall from center-to-center of joints
V_{fi}: sum of horizontal shear forces in i^{th} floor
Load Combinations:

$$F_d = 1.0G + 1.0Q + 1.0E$$
$$F_d = 1.0G + 1.3Q + 1.3W$$

Moment Magnification Method

- Approximate method
  - The frame is braced if
  - $H\sqrt{\frac{\sum N_d}{\Sigma(E_c I_c)_r}} \leq 0.6$ for $n \geq 4$
  - $H\sqrt{\frac{\sum N_d}{\Sigma(E_c I_c)_r}} \leq 0.2 + 0.1n$ for $n < 4$
  - H: Height of the building
  - $(E_c I_c)_r$: gross uncracked flexural rigidity of rigid vertical members (walls or bays with cross-bracing; columns not included)
  - n: # of stories

Moment Magnification Method

- Second order moments can be neglected if
  - $\left(\frac{\ell_k}{i}\right) \leq 34 - 12\frac{M_{d1}}{M_{d2}} \leq 40$ for braced frame
  - $\left(\frac{\ell_k}{i}\right) \leq 22$ for unbraced frame
  - ℓ_k: effective length, between the point of inflection
  - i: radius of gyration
  - M_{d1} & M_{d2}: design end moments, first order moments from analysis; $M_{d2} \geq M_{d1}$
  - Single curvature $\frac{M_{d1}}{M_{d2}} \rightarrow (+)$
  - Double curvature $\frac{M_{d1}}{M_{d2}} \rightarrow (-)$

Moment Magnification Method

- For rectangular sections: $i \approx 0.30h$
- For circular sections: $i \approx 0.25h$
- h: dimension in the direction of bending
- $\ell_k = k\ell_n$
- ℓ_n: clear distance; from the top of the slab to the bottom of the beam above
- Relative stiffness: $\alpha = \frac{\Sigma(I/\ell)_{column}}{\Sigma(I/\ell)_{beam}}$
- Calculate for top & bottom joints of the column
- Bigger one α_2
- Smaller one α_1

$$\alpha_2 > \alpha_1$$

Moment Magnification Method

- Braced frame:
  - $k = 0.7 + 0.05(\alpha_1 + \alpha_2)$
  - $k = 0.85 + 0.05\alpha_1 \leq 1.0$

  Use smaller of the two equations
- Unbraced frame:
  - $\alpha_m = 0.5(\alpha_1 + \alpha_2)$
  - $k = \frac{20-\alpha_m}{20}\sqrt{1+\alpha_m}$ if $\alpha_m < 2.0$
  - $k = 0.9\sqrt{1+\alpha_m}$ if $\alpha_m \geq 2.0$
- Column with hinge at one end & unbraced
  - $k = 2.0 + 0.3\alpha$
  - α: relative stiffness at the joint without hinge

Moment Magnification Method

- In the calculation of $\alpha = \frac{\Sigma(I/\ell)_{column}}{\Sigma(I/\ell)_{beam}}$
  - I_{column} $\rightarrow$ gross concrete section
  - I_{beam} $\rightarrow$ cracked section
  - approximately:
    - for flanged sections: $I_{cr} \approx \frac{1}{12}b_w h^3$
    - for rectangular sections: $I_{cr} \approx \frac{1}{24}b_w h^3$
    - for flat plate: 50% of the gross moment of inertia of column strip

Moment Magnification Method

- For braced frames:
  - $M'_d = \beta M_{d2}$
  - $\beta = \frac{C_m}{1-1.3\left(\frac{N_d}{N_{cr}}\right)} \geq 1.0$
  - N_{cr}: Euler buckling load
  - $C_m = \left(0.6 + 0.4\frac{M_{d1}}{M_{d2}}\right) \geq 0.4$

Moment Magnification Method

- For unbraced frames:
  - $M'_d = \beta_s M_{d2}$
  - β_s is calculated for the whole floor
  - $\beta_s = \frac{1}{1-1.3\left(\frac{\sum N_d}{\sum N_{cr}}\right)} \geq 1.0$
  - If $\frac{\sum N_d}{\sum N_{cr}} > 0.45 \rightarrow$ increase the column dimensions
  - Also compute β for individual columns taking $C_m = 1.0$
  - The bigger of β and β_s should be used for unbraced frames.
  - If $\left(\frac{\ell_k}{i}\right) > \frac{35}{\sqrt{\frac{N_d}{f_{ck}A_c}}} \quad \rightarrow \quad M_d = \beta\beta_s M_2$

Moment Magnification Method

- $N_{cr} = \frac{\pi^2 EI}{\ell_k^2}$
- $\ell_k = k\ell_n$
- EI: Effective flexural rigidity
- $EI = \frac{0.4E_cI_c}{1+R_m}$
- or
- $EI = \frac{0.2E_cI_c+E_sI_s}{1+R_m}$

E_c: modulus of elasticity of concrete

I_c: moment of inertia of gross concrete section

E_s: modulus of elasticity of steel

I_s: moment of inertia of longitudinal reinforcement about the centroid of the gross concrete section

R_m: coefficient for the effect of creep

Moment Magnification Method

- Braced:
  - $R_m = \frac{N_{gd}}{N_d}$

    Design sustained axial load = load factor $\times N_g$

    Total design axial load
- Unbraced:
  - $R_m = \frac{\sum V_{gd}}{\sum V_d}$

    Sum of the design sustained shear forces in the floor

    Sum of the design shear forces
- Normally, $V_{gd} = 0 \rightarrow R_m = 0$ unless there is earth pressure
- Recommendation: $R_m \geq 0.5$ when $V_{gd} = 0$

Minimum requirements for columns

- Minimum cross-sectional dimension:
  - 250 mm (TSC & TS500); 300 mm for circular columns
- Minimum cross-sectional area:
  - $A_c \geq \frac{N_d}{0.5 f_{ck}} = \frac{N_d}{0.75 f_{cd}} \geq 75000\ mm^2$ (TSC)
  - $A_c \geq \frac{N_d}{0.6 f_{ck}} = \frac{N_d}{0.9 f_{cd}} \geq 75000\ mm^2$ (TS500-2000)
- Minimum & maximum reinforcement ratio:
  - $0.01 \leq \rho_t \leq 0.04$ (TS 500 & TSC)
  - TS500 $\rho_t \geq 0.005$ if more than 30% of required is used

Minimum requirements for columns

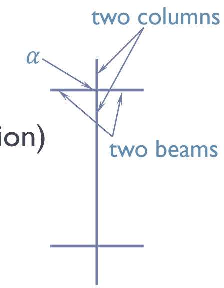
- Minimum bar diameters:
  - Longitudinal: 14 mm (TS500 & TSC)
  - Transverse: 8 mm (TS500 & TSC)
- Length of confined regions at column ends
  - $h,\ \ell_n/6$ & 500 mm (TSC)
- Maximum spacing of lateral reinforcement
  - $b/3,$ 100 mm confined region (TSC)
  - $b/2,$ 200 mm mid region (TSC)
  - $12\emptyset_\ell$, 200 mm (TS500) $\emptyset_\ell$: diameter of long. bars
- Minimum eccentricity: $e_{min} = 15mm + 0.03h$ (TS500)

Example 3

- Braced frame
- Interior column (500×500 mm, ℓ =6 m, single curvature)
- Beams (flanged, 250×500 mm, ℓ =5 m)
- C20 & S420
- $N_d = 2500\ kN,\ N_{gd} = 1800\ kN$
- $M_{d1} = 200\ kNm, M_{d2} = 250\ kNm$
- Find the required reinforcement.

Example 3

- $i = 0.3h = 0.3 \times 500 = 150\ mm$
- Columns:
  - $I = \frac{500^4}{12} = 5.21 \times 10^9\ mm^4$
  - $\ell = 6\ m \rightarrow \frac{I}{\ell} = 0.87 \times 10^6\ mm^3$
- Beams:
  - $I_{cr} = \frac{250 \times 500^3}{12} = 2.6 \times 10^9\ mm^2$ (cracked section)
  - $\ell = 5\ m \rightarrow \frac{I}{\ell} = 0.52 \times 10^6\ mm^3$
- $\alpha_1 = \alpha_2 = \frac{\Sigma(I/\ell)_{column}}{\Sigma(I/\ell)_{beam}} = \frac{2 \times 0.87 \times 10^6}{2 \times 0.52 \times 10^6} = 1.7$



Example 3

- $k = 0.7 + 0.05(\alpha_1 + \alpha_2) = 0.87$
- $k = 0.85 + 0.05\alpha_1 = 0.935$
- $\rightarrow \; k = 0.87$

Use smaller of the two equations

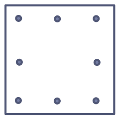
- Clear height of the column
  - $\ell_n = \ell - 0.5 = 6.0 - 0.5 = 5.5\ m$ (beam depth 500 mm)
  - $\ell_k = k\ell_n = 0.87 \times 5.5 = 4.8\ m$
  - $\frac{\ell_k}{i} = \frac{4.8}{0.15} = 32$
  - $\frac{\ell_k}{i} = 32 \overset{?}{\leq} 34 - 12\frac{M_{d1}}{M_{d2}} = 34 - 12\frac{200}{250} = 24.4$
  - Second order moments should be calculated ($\beta > 1$)

Example 3

- $R_m = \frac{N_{gd}}{N_d} = \frac{1800}{2500} = 0.72$
- $E_c = 28000\ MPa$ (for C20)
- $EI = \frac{0.4E_cI_c}{1+R_m} = \frac{0.4\times28000\times5.21\times10^9}{1+0.72} = 33.9 \times 10^3\ kNm^2$
- $C_m = 0.6 + 0.4\frac{M_{d1}}{M_{d2}} = 0.92$
- $N_{cr} = \frac{\pi^2 EI}{\ell_k^2} = \frac{\pi^2\times33.9\times10^3}{4.8^2} = 14507\ kN$
- $\beta = \frac{C_m}{1-1.3\frac{N_d}{N_{cr}}} = \frac{0.92}{1-1.3\frac{2500}{14507}} = 1.18 > 1.0$
- $\min M_{d2} = N_d(15 + 0.03h) = 2500(15 + 0.03 \times 500)$
- $\min M_{d2} = 75\ kNm$

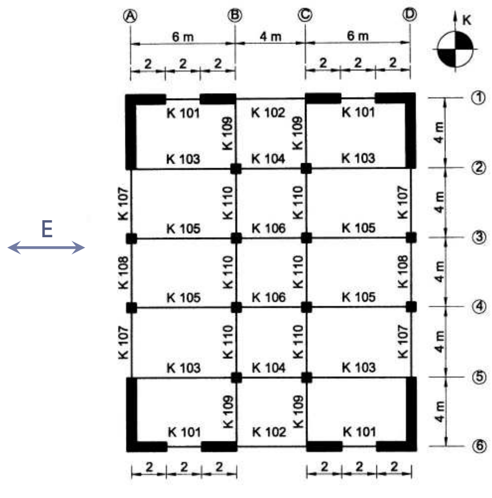
Example 3

- $M_{d2} = 250\ kNm > \min M_d$
- $M_d' = \beta M_{d2} = 1.18 \times 250 = 295\ kNm$
- Use design charts; assume $\lambda = 1/4, d''/h = 0.8$
  - $\frac{N_d}{bhf_{cd}} = \frac{2500000}{500\times500\times13} = 0.77 \qquad \frac{M_d'}{bh^2 f_{cd}} = \frac{295\times10^6}{500^3\times13} = 0.18$
  - $\rightarrow \; \rho_t m = 0.45 \qquad m = \frac{f_{yd}}{f_{cd}} = \frac{365}{13} = 28 \quad \rightarrow \; \rho_t = 0.016$
  - $\max \rho_t = 0.04 > \rho_t = 0.016 > \min \rho_t = 0.01$ ✓
  - $A_{st} = \rho_t bh = 0.016 \times 500 \times 500 = 4000\ mm^2$
  - Use 8ϕ26 (4245 mm²)

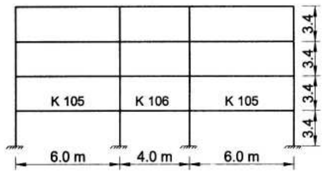


$\lambda = \frac{1}{4}$

Example 4



(A) Typical Floor Plan



(b) Frames 3 and 4

- 4 story building
- C30 & S420

Example 4

- Preliminary design of interior columns:
  - Tributary area $A_{tr} = \frac{(6+4)}{2}\frac{(4+4)}{2} = 20\ m^2$
  - Assume dead weight + live load $= 15\ kN/m^2$
  - Column load $N_d = 4 \times 20 \times 15 = 1200\ kN$
  - Structural walls → assume braced frame
  - $A_c \geq \frac{N_d}{0.75 f_{cd}} = \frac{1200000}{0.75 \times 20} = 80000\ mm^2 > 75000\ mm^2$
  - Member dimensions:
    - Columns: 300×300 mm
    - Beams: 250×500 mm
    - Slabs: 120 mm
    - RC walls: 200 mm (thickness)

Preliminary design not given

Example 4

- Dead load on all spans
- 5 different live load arrangements (1.4G+1.6Q)
- Seismic load (B: 1.0G+1.0Q+1.0E & C: 0.9G+1.0E)

Load Comb.	N_d (kN)	M_{d1} (kNm)	M_{d2} (kNm)	min. M_d (kNm)	M_d (kNm)	N_{gd} (kN)	R_m	Remarks
A1	1240	13.6	27.2	29.8	29.8	960	0.77	Dbl. Crv.
A2	1250	5.5	11.0	30.0	30.0	960	0.77	Dbl. Crv.
A3	1320	12.3	23.3	31.7	31.7	960	0.73	Dbl. Crv.
A4	1330	11.3	21.2	31.9	31.9	960	0.73	Dbl. Crv.
A5	1295	13.6	27.2	31.0	31.0	960	0.74	Dbl. Crv.
B(seis)	900	20.5	24.6	21.6	24.6	-	-	Dbl. Crv.
C(seis)	608	18.6	20.6	14.6	20.6	-	-	Dbl. Crv.

Example 4

- For load combination B:
  - Total axial load at base $\sum N_{di} = 13000\ kN$
  - Total shear of column and walls at base $V_{fi} = 2287\ kN$
  - First story drift (lateral displacement of the first floor relative to the base) $\Delta_i = 3.9\ mm$
  - check whether the frame is braced or not:
  - $\varphi = 1.5 \frac{(\sum N_{di})\Delta_i}{V_{fi}\ell_i} = 1.5 \frac{13000 \times 0.0039}{2287 \times 3.4} = 0.0098 < 0.05$ braced
  - Columns:
  - $I_c = \frac{0.3^4}{12} = 0.000675\ m^4$
  - $\left(\frac{I}{\ell}\right)_c = 0.0002$

Example 4

- Beams: (T-Section)
  - $I_b = \frac{0.25 \times 0.5^3}{12} = 0.0026\ m^4$ (cracked value)
  - K106: $\left(\frac{I}{\ell}\right)_b = 0.00065$ & K105: $\left(\frac{I}{\ell}\right)_b = 0.00043$
- Joints at the first floor level:
  - $\alpha = \frac{\Sigma(I/\ell)_c}{\Sigma(I/\ell)_b} = \frac{2 \times 0.0002}{0.00065 + 0.00043} = 0.37 = \alpha_2$
- Joint at the base: $\alpha = \frac{1 \times 0.0002}{\infty} = 0 = \alpha_1$
- Effective length of columns:
  - $k = 0.7 + 0.05(0.37 + 0) = 0.72$
  - $k = 0.85 + 0.05(0) = 0.85 < 1.0$

Example 4

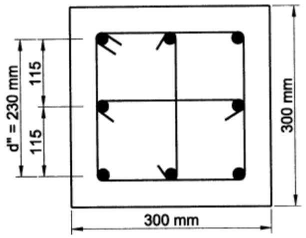
- $\ell_n = 3.4 - 0.5 = 2.9\ m$ (beam depth is 0.5 m)
- $\ell_k = 0.72 \times 2.9 = 2.1\ m$
- Slenderness ratio:
  - $i = 0.3h = 0.3 \times 0.3 = 0.09\ m$
  - $\frac{\ell_k}{i} = \frac{2.1}{0.09} = 23.3$
- Limiting value:
  - double curvature $\rightarrow \frac{M_{d1}}{M_{d2}} < 0$
  - $34 - 12\frac{M_{d1}}{M_{d2}} > 34$
  - $23.3 < 34$ no slenderness effect $\beta = 1.0$

Example 4

- To illustrate the procedure to be followed lets calculate β
  - $E_c = 32000\ MPa$ (for C30) $\rightarrow E_cI_c = 21600\ kNm^2$
  - $EI = \frac{0.4E_cI_c}{1+R_m} = \frac{0.4\times21600}{1+0.77} = 4881\ kNm^2$ biggest R_m from table
  - $N_{cr} = \frac{\pi^2 EI}{\ell_k^2} = \frac{\pi^2 4881}{2.1^2} = 10910\ kN$
  - $C_m = 0.6 + 0.4\frac{M_{d1}}{M_{d2}} = 0.6 + 0.4(-0.83) = 0.27 \geq 0.4$
  - $C_m = 0.4$
  - $\beta = \frac{C_m}{1-1.3\frac{N_d}{N_{cr}}} = \frac{0.4}{1-1.3\frac{900}{10910}} = 0.47 \rightarrow \beta = 1.0$

Example 4

- $N_d = 900\ kN$ & $M_d' = \beta M_{d2} = 1.0 \times 24.6kNm$
- $\frac{N_d}{bhf_{cd}} = \frac{900000}{300\times300\times20} = 0.5$ & $\frac{M_d}{bh^2 f_{cd}} = 0.045$
- Assume $\lambda = \frac{1}{4}$ & $\frac{d''}{h} = 0.8$
- From chart $\rho_t m < 0.1 \rightarrow$ use minimum $\rho_t = 0.01$
- $A_{st} = 0.01 \times 300 \times 300 = 900\ mm^2$
- Use 8ϕ14 (1230 mm²)



Study Example 6.14
Study Example 6.15