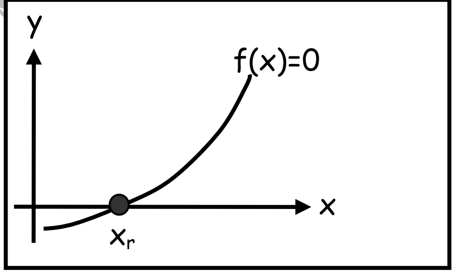
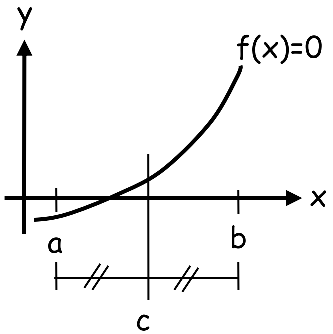
SOLUTION OF NON-LINEAR EQUATIONS

→ Graphical Method



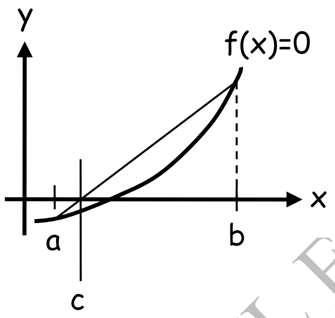
Bracketing Methods

Bisection M.



- must know an interval (i.e. [a, b]) which includes the root
- f(a).f(b)<0 → usually indicated there is a root in [a, b]
- c= mid pt. of [a, b]
- if f(a).f(c)<0, let b=c o/w let a=c

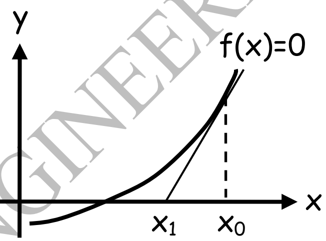
False Position M.



- must know an interval (i.e. [a, b]) which includes the root
- f(a).f(b)<0 → usually indicated there is a root in [a, b]
- $c = b - \dfrac{f(b)(b-a)}{f(b)-f(a)}$
- if f(a).f(c)<0, let b=c o/w let a=c

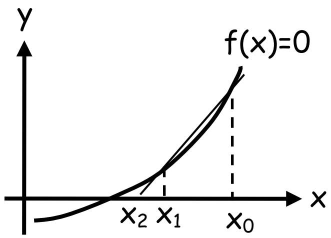
Open Methods

Newton-Raphson M.



- one initial guess (i.e. x_0) is required.
- $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$
- iteration is required

Secant M.



- two initial guess (i.e. x_0 and x_1) are required.
- $x_{i+1} = x_i - \dfrac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$
- iteration is required

Fixed Pt. Iteration

f(x)=0

- reorganize f(x) s.t. $x_{i+1}=g(x_i)$
- converges when $|g'(x_r)| < 1$
- one initial guess is required
- iteration is required

SYSTEM OF LINEAR EQUATIONS

Direct Methods

Naive Gauss Elimination M.

- Using elementary row operations convert $[A \mid b]$ into $[\,U \mid b^*]$
- U is an upper triangular matrix
- Solve

Gauss Elimination M.

- Partial pivoting
- Using elementary row operations convert $[A \mid b]$ into $[\,U \mid b^*]$
- U is an upper triangular matrix
- Solve

Simpler methods

- Graphical (n=2)
- Cramer's (n=3)
- Elimination of unknowns
- Matrix inversion $\underline{X} = A^{-1}.\underline{b}$

Gauss-Jordan Elimination M.

- Partial pivoting
- Using elementary row operations convert $[A \mid b]$ into $[\,I \mid b^*]$
- U is an upper triangular matrix whose diagonal elements are all 1
- Solve

LU Decomposition M.

- Decompose A into LU where L and U are lower and upper triangular matrices, respectively.
- Doolittle's Algorithm for decomposition: diagonal elements of L are all 1
- $A\underline{X} = \underline{b}$

  $A = LU$

  $LU\ \underline{X} = \underline{b}$

  $U\ \underline{X} = \underline{Y}$

  $L\underline{Y} = \underline{b}$

Indirect Methods

Gauss Jacobi M.

- Apply elementary row operations to obtain all diagonal elements of A as 1, call that A*
- $A\underline{X} = \underline{b}$

  $A^* = I + B$

  $(I + B)\underline{X} = \underline{b}^*$

  $I\underline{X} + \underline{BX} = \underline{b}^*$

  $I\underline{X} = -\underline{BX} + \underline{b}^*$

  $\underline{X} = -\underline{BX} + \underline{b}^*$

  $\underline{X}^{i+1} = -\underline{BX}^{i} + \underline{b}^*$
- or

$$x_i^{k+1} = \frac{b_i - \sum_{\substack{j=1 \\ i \neq j}}^{n} a_{ij} x_j^k}{a_{ii}}$$

$i = 1, 2, \ldots, n$

Gauss Seidel M.

- Similar to Gauss Jacobi but use updated values of currunt iteration.

$$x_i^{k+1} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{k+1} - \sum_{j=i+1}^{n} a_{ij} x_j^k}{a_{ii}}$$

$i = 1, 2, \ldots, n$