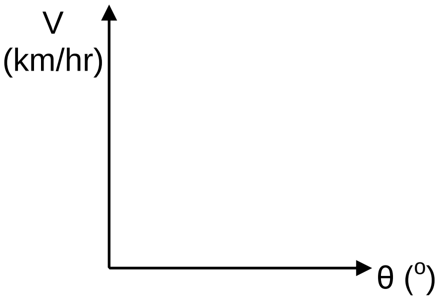
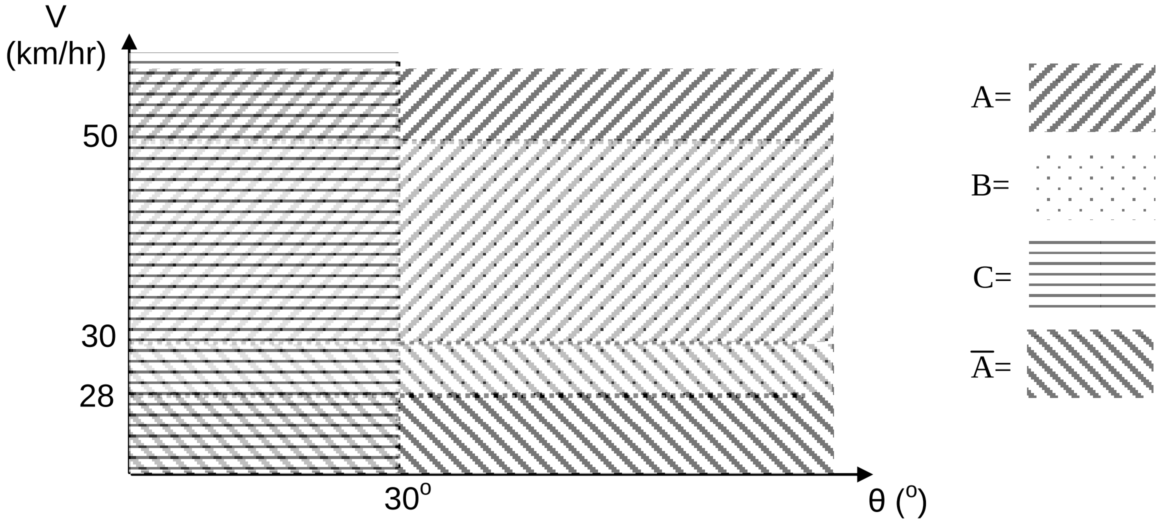
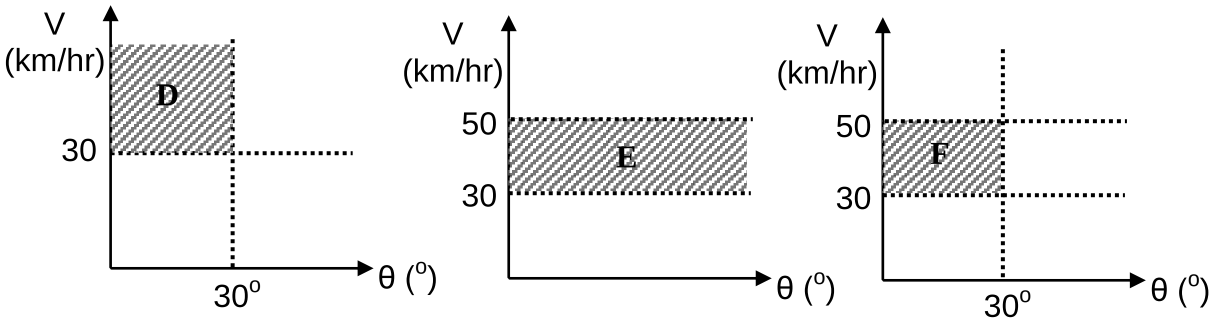
1) a)



b)



c)



d) Two events are mutually exclusive if they can not occur at the same time.

2) a) $\left.\begin{array}{l} P(F/H) = 0.95 \\ P(F/M) = 0.60 \\ P(F/P) = 0.10 \end{array}\right\}$ Ministry of Public Works

$\left.\begin{array}{l} P(H) = 0.40 \\ P(M) = 0.35 \\ P(P) = 0.25 \end{array}\right\}$ Chamber of Civil Engineers

$$\begin{aligned} P(Feasible) &= P(F \cap H) + P(F \cap M) + P(F \cap P) = \\ &= P(F/H).P(H) + P(F/M).P(M) + P(F/P).P(P) \\ &= 0.95 \times 0.40 + 0.60 \times 0.35 + 0.10 \times 0.25 \\ &= 0.615 \end{aligned}$$

b) $P(H/\overline{F}) = \dfrac{P(\overline{F} \cap H)}{P(\overline{F})} = \dfrac{P(\overline{F}/H).P(H)}{P(\overline{F})} = \dfrac{0.05 \times 0.40}{1 - 0.615} = 0.052$

3) a) In bad weather, the erection can not be finished in one day.

P(B)=0.20   P(G)=0.80

3 Companies $\Longrightarrow$ $P(C)=\frac{1}{3}$

P(C/B)=0   P(F/G)=0.70

Note: The events B, G, C and F stand for bad weather, good weather, successful crane renting and finished in one day, respectively.

$$\begin{aligned} P(\text{No crane on Tuesday}) &= P(B\cap\overline{C})+P(G\cap\overline{C}) \\ &= P(\overline{C}/B).P(B)+P(\overline{C}/G).P(G) \\ &= 1\times 0.20+\frac{2}{3}\times 0.80 \\ &= 0.733 \end{aligned}$$

b) $$\begin{aligned} P(\text{Finished on Tuesday}) &= P(G\cap F\cap C) \\ &= P(F/G).P(G).P(C) \\ &= 0.70\times 0.80\times 0.333 \\ &= 0.187 \end{aligned}$$

Note: The probability of the crane renting is statistically independent of the weather conditions and the one-day finish.

c) $$\begin{aligned} P(\text{Finished more than one day}) &= 1-P(\text{Finished on Tuesday}) \\ &= 1-0.187=0.813 \end{aligned}$$

4) a) P(L)=0.6   P(A)=0.3   P(H)=0.1   P(S/L)=1.0   P(S/A)=0.1   P(S/H)=0.5

$$\begin{aligned} P(Shortage) &= P(\overline{S}\cap A)+P(\overline{S}\cap H) \\ &= P(\overline{S}/A).P(A)+P(\overline{S}/H).P(H) \\ &= (1.0-0.1)\times 0.3+(1.0-0.5)\times 0.1 \\ &= 0.32 \end{aligned}$$

b) $$P(A/S)=\frac{P(S\cap A)}{P(S)}=\frac{P(S/A).P(A)}{P(S)}=\frac{0.1\times 0.3}{0.32}=0.094$$

c) $$\begin{aligned} P(Shortage\text{ at least one month}) &= 1-P(No\ Shortage) \\ &= 1-\binom{2}{0}\times 0.32^{0}\times(1-0.32)^{2} \\ &= 0.538 \end{aligned}$$

5) a) P(A)=0.08   P(B)=0.12

$$\begin{aligned} P(Function) &= 1-P(A\cap B)=1-0.08\times 0.12=1-0.0096 \\ &= 0.9904 \end{aligned}$$

b) $P(Function) = 1 - P(A \cap B) = 1 - p^2 = 0.99 \quad \Longrightarrow \quad p = 0.1$

c) $P(Function) = 1 - P(A_1 \cap A_2 \cap A_3 \cap .... \cap A_n) = 1 - 0.5^n = 0.99$

$$\begin{aligned} 0.5^n &= 0.01 \\ \log(0.5^n) &= \log(0.01) \\ n \times \log(0.5) &= -2 \\ n &= 6.644 \end{aligned}$$

Therefore, take n=7.

d) $P(Function) = 1 - [P(A \cap \overline{B} \cap B \cap A) + P(A \cap B \cap B \cap B) + P(\overline{A} \cap B \cap A \cap B)]$

$$\begin{aligned} &= 1 - [0.08 \times (1 - 0.12) \times 0.12 \times 0.08 + 0.08 \times 0.12 \times 0.12 \times 0.12 + \\ &(1 - 0.08) \times 0.12 \times 0.08 \times 0.12] \\ &= 1 - [0.08^2 \times 0.12 \times (1 - 0.12 + 0.12) + 0.12^2 \times 0.08 \times 0.92] \\ &= 0.998 \end{aligned}$$

II. way :

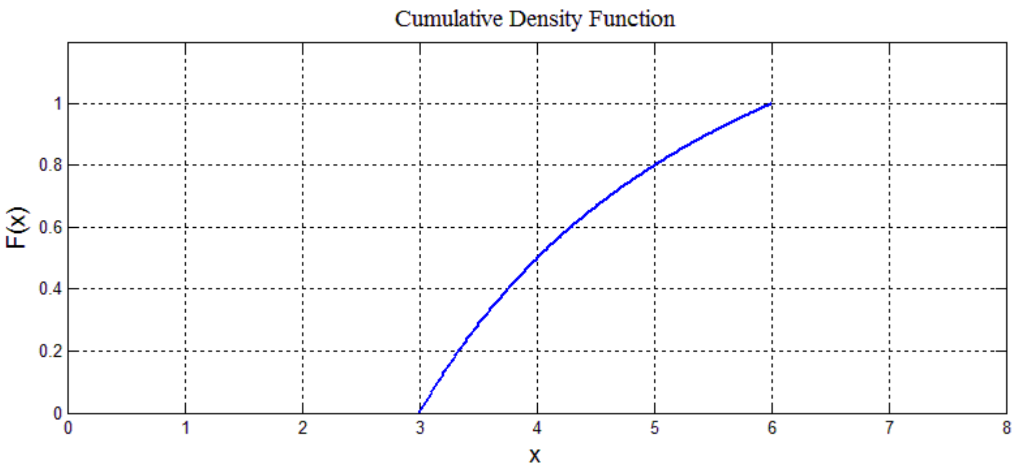
$$\begin{aligned} P(Function) &= 1 - P(A \cup B) \times P(A \cap B) \\ &= 1 - (0.08 + 0.12 - 0.08 \times 0.12) \times (0.08 \times 0.12) \\ &= 0.998 \end{aligned}$$

6)

a) $F_x(x) = \int_3^6 \frac{c}{x^2} * dx = \frac{-c}{x}\Big|_3^6 = 1 \;\Rightarrow\; \frac{-c}{6} + \frac{c}{3} = 1$

$$c = 6$$

b) $F_x(x) = \int_3^x \frac{6}{x^2} * dx = \frac{-6}{x}\Big|_3^x = \frac{-6}{x} + \frac{6}{3} = \frac{-6}{x} + 2 \quad for\ 3 \le x \le 6$



c) $E(x) = \int_3^x x * f_x(x) * dx = \int_3^x x * \frac{6}{x^2} * dx = 6 * \ln(x)|_3^6 = 4.159$

d) $F_x(x_{median}) = 0.5 \;\Rightarrow\; \frac{-6}{x_{median}} + 2 = 0.5$

$$\frac{6}{x_{median}} = 1.5$$
$$x_{median} = 4$$

The probability density function (pdf) attained its largest at x=3. Therefore, the mode is 3.

e) $V(x) = \int_3^x (x-\mu)^2 * f_x(x) * dx = \int_3^x (x-4.159)^2 * \frac{6}{x^2} * dx = 0.704$

$\sigma_x = \sqrt{V(x)} = \sqrt{0.704} = 0.839$

$\delta = \frac{\sigma_x}{\mu} = \frac{0.839}{4.159} = 0.202$

f) $P(Collapse) = P(x > 5.5) = \int_{5.5}^6 f_x(x) * dx = \int_{5.5}^6 \frac{6}{x^2} * dx = 0.091$

7)

a) If the cumulative distribution function is differentiated with respect to s, the probability density function is obtained.

$$f_s(s) = \begin{cases} 0 & for\ s \le 0 \\ \dfrac{-s^2}{288} + \dfrac{s}{24} & for\ 0 < s \le 12 \\ 0 & for\ s > 12 \end{cases}$$

$$E(s) = \int_0^{12} s * f_s(s) * ds = \int_0^{12} s * \left(\frac{-s^2}{288} + \frac{s}{24}\right) * ds = \left(\frac{-s^4}{1152} + \frac{s^3}{72}\right)\Bigg|_0^{12} = 6$$
$$\frac{df_s(s)}{ds} = \frac{-s}{144} + \frac{1}{24} = 0 \Rightarrow s_{mode} = 6$$

b) $\sum P_R(R) = 1 \Rightarrow 0.75 + c + 0 = 1 \Rightarrow c = 0.25$

The probability of failure is

$$P(Failure) = P(10 \le s \le 12 \cap R = 10)$$

Assume that the maximum load and the resistance are statistically independent variables.

$$P(Failure) = P(10 \le s \le 12/R = 10) * P(R = 10) + P(S > 13/R = 13) * P(R = 13)$$
$$P(Failure) = 0.074 * 0.75 + 0 * 0.25$$
$$P(Failure) = 0.056$$

<u>Note:</u> $P(10 \le s \le 12) = F_s(12) - F_s(10) = 1 - 0.926 = 0.074$

8) $\left.\begin{array}{l} \mu = 12\ min \\ \delta = 0.2 \end{array}\right\}$ σ=12*0.2=2.4 min

a) $P(10 < T < 15) = P\left(\frac{10-12}{2.4} < \frac{T-\mu}{\sigma} < \frac{15-12}{2.4}\right) = P(-0.833 < z < 1.25)$

$$= P(z < 1.25) - (1 - P(z < 0.833))$$
$$= 0.8944 - (1 - 0.7975)$$
$$= 0.6919$$

b) $P(|T-\mu| \leq 3) = P(-3 \leq T-\mu \leq 3) = P\left(\frac{-3}{2.4} \leq \frac{T-\mu}{\sigma} \leq \frac{3}{2.4}\right)$

$$= P(-1.25 \leq z \leq 1.25)$$
$$= 2 * (P(z \leq 1.25) - 0.5)$$
$$= 2 * (0.8944 - 0.5)$$
$$= 0.7888$$

$P(|T-\mu| > 2) = 1 - P(|T-\mu| \leq 2) = 1 - P\left(\frac{-2}{2.4} \leq \frac{T-\mu}{\sigma} \leq \frac{2}{2.4}\right)$

$$= 1 - P(-0.833 \leq z \leq 0.833)$$
$$= 1 - \Big(P(z \leq 0.833) - \big(1 - P(z \leq 0.833)\big)\Big)$$
$$= 1 - \big(0.7975 - (1 - 0.7975)\big)$$
$$= 0.405$$

c) $P(T > 12/8 < T < 16) = \frac{P(T>12 \cap 8<T<16)}{P(8<T<16)} = \frac{P(12<T<16)}{P(8<T<16)}$

$$= \frac{P\left(\frac{12-12}{2.4} \leq \frac{T-\mu}{\sigma} \leq \frac{16-12}{2.4}\right)}{P\left(\frac{8-12}{2.4} \leq \frac{T-\mu}{\sigma} \leq \frac{16-12}{2.4}\right)}$$
$$= \frac{P(0 \leq z \leq 1.667)}{P(-1.667 \leq z \leq 1.667)}$$
$$= \frac{P(z \leq 1.667) - P(z \leq 0)}{2 * \big(P(z \leq 1.667) - P(z \leq 0)\big)}$$
$$= \frac{0.9522 - 0.5}{2 * (0.9522 - 0.5)}$$
$$= 0.5$$

d) $P(T < t) = 0.15 \;\Rightarrow\; P\left(\frac{T-\mu}{\sigma} < \underbrace{\frac{t-12}{2.4}}_{\text{a}}\right) = 0.15$

From normal distribution table, a=-1.036

$$\frac{t-12}{2.4} = -1.036$$
$$t = 9.51\, min$$

<u>Hint:</u> The value of "a" can be obtained by determining the z value corresponding to a probability of 0.15. However, the normal distribution table distributed in this course shows probability values higher than 0.50. Therefore, the above value is found by utilizing the symmetry property of the normal distribution. In other words, the needed "a" value is the negative of the z value corresponding to a probability of 0.85.

e) $P(T < 0) = P\left(\frac{T-\mu}{\sigma} < \frac{0-12}{2.4}\right) = P(z < -5) \cong 0$

Therefore, it is reasonable to use normal distribution.

9)

a) $\theta = \frac{1}{25} = 0.04$

$$P(t < 5) = \int_0^5 \theta * e^{-\theta * t} * dt = \int_0^5 0.04 * e^{-0.04*t} * dt = -e^{-0.04*t}|_0^5 = -0.818 + 1 = 0.182$$

Hint: This part may also be solved by using the binomial distribution.

$$P(Overtopped\ for\ the\ first\ time\ during\ the\ 5-year\ period) = 1 - \binom{5}{0} * (1-0.04)^5 * 0.04^0$$
$$= 0.185$$

b) $P(Overtopped\ twice\ during\ the\ 5-year\ period) = \binom{5}{2} * (1-0.04)^3 * 0.04^2 = 0.0142$

c) $P(First\ time\ overtopping\ in\ the\ fifth\ year) = (1-0.04)^4 * 0.04^1 = 0.034$

$P(No\ overtopping\ in\ five\ years) = (1-0.04)^5 * 0.04^0 = 0.849$

10)

a) $\lambda = 4.60\ kN$

$\xi = 2.38\ kN$

$$P(Failure) = P(L > 150) = P(lnL > \ln(150)) = P\left(\frac{\ln L - \lambda}{\xi} > \frac{\ln(150) - 4.60}{2.38}\right)$$
$$= P(z > 0.173)$$
$$= 1 - P(z \leq 0.173)$$
$$= 1 - 0.5687$$
$$= 0.4313$$

b) $P(Yellow\ Flag) = \binom{5}{1} * (1-0.4313)^4 * 0.4313^1 = 0.226$

c) $P(Red\ Flag) = 1 - \binom{5}{0} * (1-0.4313)^5 * 0.4313^0 - \binom{5}{1} * (1-0.4313)^4 * 0.4313^1$

$$= 1 - 0.059 - 0.226$$
$$= 0.715$$

d) $P(Out\ of\ Service) = P(Red\ Flag) + P(Yellow\ Flag \cap Out\ of\ Service)$

$$= 0.715 + 0.226 * 0.5$$
$$= 0.828$$