CE 382 Homework #1

19/03/2012
Monday

1) $f_c = 50$ MPa

$E_c = 12680 + 460.(50) = \underline{35680 \text{ MPa}}$

$\varepsilon_{co} = 2.50 / 35680 = \underline{2.8 . 10^{-3}}$

for $\sigma_c = 40$ MPa

$$\sigma_c = f_c \left[2\varepsilon_c / \varepsilon_{co} - \left(\frac{\varepsilon_c}{\varepsilon_{co}} \right)^2 \right] \qquad 40 = 50 \left[\frac{2\,\varepsilon_c}{2.8 . 10^{-3}} - \frac{\varepsilon_c^2}{7.86 . 10^{-6}} \right]$$

$$\Rightarrow \varepsilon_c^2 - 5.6 . 10^{-3} . \varepsilon_c + 6.272 . 10^{-6} = 0$$

2 Solutions -,

$\varepsilon_c = 4.05 . 10^{-3}$

$\underline{\varepsilon_c = 1.55 . 10^{-3}}$

We choose the smaller one due to $\underline{\underline{\sigma_c < f_c}}$

for 40 MPa to 15 MPa

$\sigma = E . \varepsilon$ (Hooke's Law)

$(40 - 15) = \varepsilon . (35680) \qquad \varepsilon_r = 7.00 . 10^{-4}$

$\varepsilon = \varepsilon_c - \varepsilon_r = \underline{\underline{8.49 . 10^{-4}}}$

2) for 45 MPa :

$$45 = 50 \left[\frac{2\,\varepsilon_c}{\varepsilon_{co}} - \left(\varepsilon_c / \varepsilon_{co} \right)^2 \right]$$

$$\varepsilon_c^2 - 5{,}6.10^{-3}\,\varepsilon_c + 7{,}056.10^{-6} = 0$$

2 Solutions :

$\varepsilon_c = 1{,}91\ 10^{-3}$

$\varepsilon_c = 3{,}68\ 10^{-3}$

We choose the smaller ε due to the strain should be lower than $2{,}8.10^{-3}$ ($\varepsilon < \varepsilon_{co}$)

3) Concrete should relax on an elastic curve so that ;

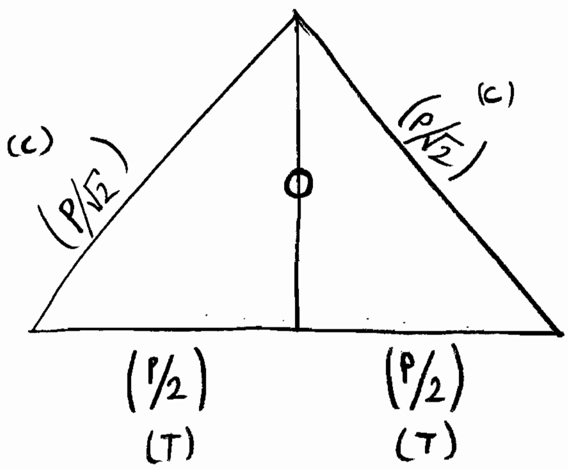
$$\sigma = E.\,\varepsilon_r$$

$$(45-0) = 35680 .\ \varepsilon \qquad \varepsilon_r = 1{,}26 .10^{-3}$$

Remaining ε will be ; $\varepsilon = \varepsilon_c - \varepsilon_r$

$$\varepsilon = 6{,}5.10^{-4}$$

1,2) a)



$A_c = 150^2 = 22500\ mm^2$

$f_c = 20$ MPa

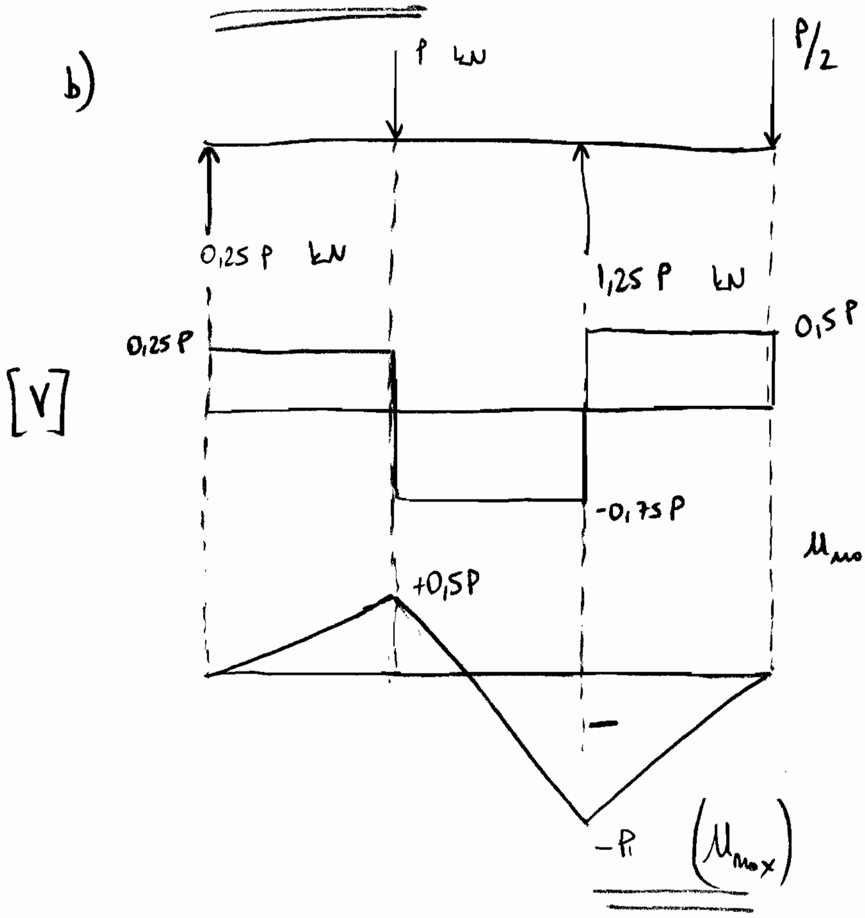
$f_{cT} = 0{,}35 \cdot \sqrt{f_c} = \underline{\underline{1{,}6 \text{ MPa}}}$

$\frac{P/\sqrt{2}}{22500} \leqslant 20$ MPa $\Rightarrow$ $P \leqslant 636$ kN (In Compression)

$\frac{(P/2)}{22500} \leqslant 1{,}6$ MPa $\rightarrow$ $P \leqslant 72$ kN (In Tension)

$\underline{\underline{P_{max} = 72 \text{ kN}}}$

b)



$f_{CTF} \approx 0{,}7\sqrt{f_c} = 3{,}1$ MPa

$\sigma_{Max} = \frac{M_{max}\, c}{I}$

$c = 200mm$

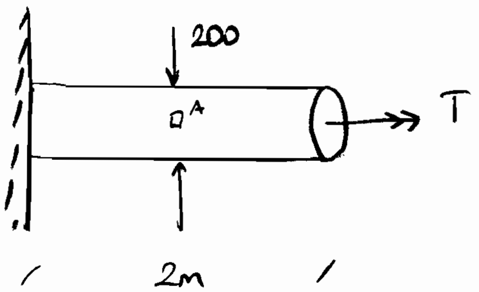
$I = 200 \cdot (400)^3/12 = 1{,}07 \cdot 10^9\ mm^4$

$M_{max} = (3.1) \cdot (1{,}07 \cdot 10^9)/200$

$= 16{,}54$ kN.m

$M_{max} = P = \underline{\underline{16.54 \text{ kN}}}$

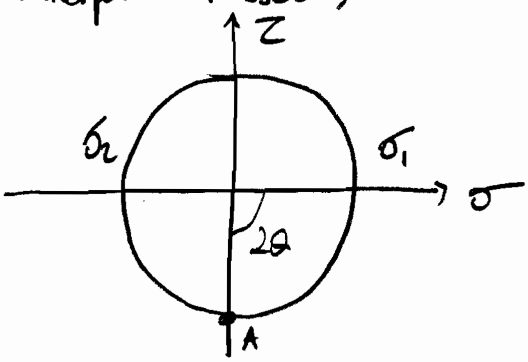
c)



$$\tau_{max} = \frac{Tc}{J} = \frac{(P \cdot 10^6\ Nmm)(100)}{(\pi/2 \cdot 100^4\ mm^4)}$$

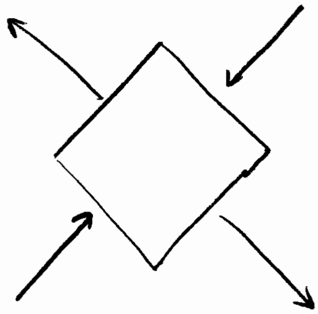
$$= (0{,}637\ P\ MPa)$$

Principal Stresses :



At A $\quad \sigma_x = 0 \quad \underline{\underline{\tau_{xy} = -0{,}637 P}}$

$\sigma_y = 0$



$\underline{\sigma_2 = -0{,}637 P}$

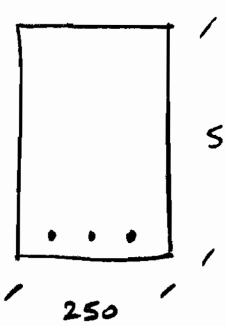
$\underline{\sigma_1 = 0{,}637 P}$

$f_{cts} = 0{,}5 \sqrt{f_c} = \underline{2{,}2\ MPa}$

$\sigma_1 = 0{,}637 P = 2{,}2 \Rightarrow \quad \underline{\underline{P_{max} = 3{,}45\ kN}}$

44) Concrete will crack due to the excessive shear stress, so that it must be checked.

$$\tau = \frac{VQ}{It}$$



$t = 250$ mm

$V_{max} = P/2$

$Q = 250 \cdot 250 \cdot 125$

$= 7812500\ mm^3$

$I = \frac{1}{12} bh^3 = \frac{1}{12} \cdot 250 \cdot 500^3$

$= 2604166667\ mm^4$

$\tau = P \cdot 6 \cdot 10^{-6}\ N/mm^2$

for max ; $\theta = 45°$

$f_{ctf} = \sqrt{f_c} \cdot 0{,}7 = 3{,}13\ MPa$

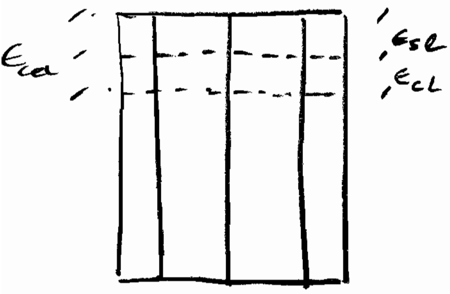
$3{,}13 = 6 \cdot 10^{-6} \cdot P$

$\underline{\underline{P = 521{,}7\ kN}}$ / $\underline{\theta = 45°}$

Tension crack angle

1,b) $\varepsilon_{cE} = \frac{\sigma_{co}}{E_c} \phi_{cE} = \frac{8}{20.000} (2) = 8 . 10^{-4}$

Stresses in steel and concrete due to creep (only)



① Equilibrium:

$$F_s - F_c = 0$$

$$\sigma_s . A_s - \sigma_c A_c = 0$$

$$\epsilon_s . E_s . A_s - \epsilon_c E_c A_c = 0$$

$$\epsilon_s . (200.000) . (8 . 531) - \epsilon_c (20.000) . (400^2 - 8.531) = 0$$

$$\underline{\underline{\epsilon_s = 3{,}76 \ \epsilon_c}}$$

② Displacement Compatibility

$$\epsilon_{sl} + \epsilon_{cl} = \epsilon_{cel}$$

$$\epsilon_s + \epsilon_c = 0{,}0008 \quad \rightarrow \quad \epsilon_s = 6.32 . 10^{-4}$$

$$\epsilon_c = 1{,}68 . 10^{-4}$$

$\sigma_s = E_s . \epsilon_s = 126.35$ MPa (in Compression)

$\sigma_c = E_c . \epsilon_c = 3{,}36$ MPa (in Tension)

Superposition of Creep Effect:

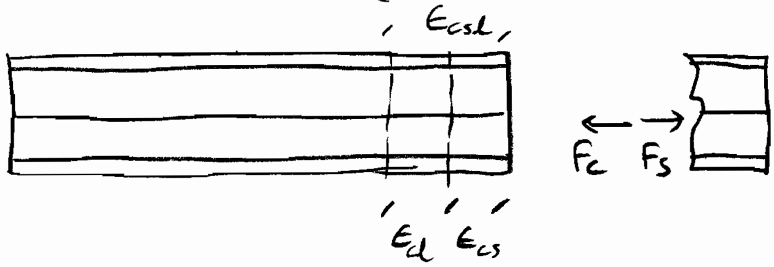
$\sigma_c = 8 - 3{,}36 = (4{,}64$ MPa Comp)

$\sigma_s = 120 + 126.35 = (246.35$ MPa Comp)

1,7) $h_0 = \frac{2 A_c}{u} = \frac{2(250)(700)}{2(250)(700)} = 184\,mm$

from (Table 1,3) pg. 48 ; Assuming Dry Climate ;

$\epsilon_{cs} = 0{,}00059$ (Interpolated Result)



① Equilibrium ;

$$-F_c + F_s = 0$$

$$\Rightarrow -\epsilon_c E_c A_c + \epsilon_s . E_s . A_s = 0$$

$$\Rightarrow +\epsilon_c \underbrace{(28{,}500)}_{\text{from the book}} . \overbrace{(250.700 - 10 \times 201)}^{A_c} = \epsilon_s . \overbrace{(200\,000)}^{E_s} . \underbrace{(10.201)}_{A_s}$$

$$\epsilon_c\,(4{,}93\ 10^9) = \epsilon_s . (4{,}02 . 10^8)$$

$$\Rightarrow \epsilon_c\,(12{,}26) = \epsilon_s$$

② Disp. Comp

$$\epsilon_{cl} + \epsilon_{sl} = 0{,}00059$$

$$\epsilon_{cl} = 4{,}45 . 10^{-5}$$

$$\epsilon_{sl} = 5{,}45 . 10^{-4}$$

$\sigma_s = E_s . \epsilon_s = (200.000)(5{,}45 . 10^{-4}) = (109{,}07\ MPa)$ Compression

$\sigma_c = E_c . \epsilon_c = (28.500)(4{,}45 . 10^{-5}) = (1{,}27\ MPa)$ Tension