RULES

1. This is the **version 6.0**. In case there are any corrections for the solutions of Exercise 6, we will post an updated version on our website. You can follow the changes in the exercises by the **Version History** section below.

Version History

V6.0 Solutions of Exercise 6 are released.

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1.

a) First of all the integral should be transformed into the form such that Gaussian Quadrature is applicable:

$$t=\frac{b+a}{2}+\frac{b-a}{2}x=\frac{1.5+0.5}{2}+\frac{1.5-0.5}{2}x$$
$$t=1+0.5x$$
$$dt=\frac{b-a}{2}dx=\frac{1.5-0.5}{2}dx=0.5dx$$

Thus,

$$I=\int_{0.5}^{1.5} e^{x}\cos(x)\,dx=\int_{-1}^{1} e^{1+0.5x}\cos(1+0.5x)0.5\,dx$$

Now, we can put the 2 point GQ numbers into the function:

$$I=w_1 f_1+w_2 f_2=1*f(\frac{\sqrt{3}}{3})+1*f(-\frac{\sqrt{3}}{3})$$
$$I=e^{1+0.5*\frac{\sqrt{3}}{3}}*\cos(1+0.5*\frac{\sqrt{3}}{3})*0.5+e^{1+0.5*\frac{-\sqrt{3}}{3}}*\cos(1+0.5*\frac{-\sqrt{3}}{3})*0.5$$
$$I=1.276395857886311$$

b) and c)

```
clear all
clc
format long
x2=[sqrt(3)/3;-sqrt(3)/3]; % Points of 2-point GQ
w2=[1;1]; % The weights of 2-point GQ
x3=[0.7745966692;-0.7745966692;0]; % Points of 3-point GQ
w3=[0.555555555;0.555555555;0.8888888888]; % The weights of 3-point
GQ
integral2=0; %Initializing the 2-point GQ integral
integral3=0; %Initializing the 3-point GQ integral
%2-point GQ
for i=1:2
    t=1+0.5*x2(i,1);
    integral2=integral2+w2(i,1)*exp(t)*cos(t)*0.5;
end
integral2 %visualizing the resultant integral of 2-point GQ
%3-point GQ
for i=1:3
    t=1+0.5*x3(i,1);
    integral3=integral3+w3(i,1)*exp(t)*cos(t)*0.5;
end
integral3 %visualizing the resultant integral of 3-point GQ
```

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The output of the code given above is:

```
Command Window

integral2 =

   1.276395857886311


integral3 =

   1.275069035909317
```

d) The true error can be calculated as:

$$\varepsilon_{abs.true,2-point} = |1.27640 - 1.27508| = 0.00132$$

$$\varepsilon_{abs.true,3-point} = |1.27507 - 1.27508| = 0.00001$$

2.

The general formulation for central difference formula is given as:

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2\Delta x} \quad \text{where } \Delta x = 0.1$$

According to this formulation, the value of $f'(x)$ at x=0.2, 0.4 and 0.5 can be found as:

$$f'(0.2) = \frac{f(0.3) - f(0.1)}{2*0.1} = \frac{0.400 - 0.425}{0.2} = -0.1250$$

$$f'(0.4) = \frac{f(0.5) - f(0.3)}{2*0.1} = \frac{0.525 - 0.400}{0.2} = 0.6250$$

$$f'(0.5) = \frac{f(0.6) - f(0.4)}{2*0.1} = \frac{0.675 - 0.450}{0.2} = 1.1250$$

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3.

a) $m\ddot{u} + ku = P(t)$

$$\frac{du}{dt} = v \qquad (1)$$

$$m\frac{dv}{dt} + ku = P_0\left(1 - \frac{t}{T}\right)$$

$$\frac{dv}{dt} = \frac{P_0}{m}\left(1 - \frac{t}{T}\right) - \frac{k}{m}u \qquad (2)$$

b)

h=0.125

at t=0

$u(0) = 0$

$v(0) = 0$

at t=0.125

$$u(0.125) = u(0) + v(0)\cdot\Delta t = 0 + 0\cdot 0.125 = 0$$

$$v(0.125) = v(0) + \left[\frac{P_0}{m}\left(1 - \frac{t}{T}\right) - \frac{k}{m}u(0)\right]\cdot\Delta t$$

$$v(0.125) = 0 + \left[\frac{500000}{10000}\left(1 - \frac{0}{0.5}\right) - \frac{10000000}{10000}0\right]\cdot 0.125 = 6.250000 \text{ m/s}$$

at t=0.250

$$u(0.250) = u(0.125) + v(0.125)\cdot 0.125 = 0.781250 \text{ m}$$

$$v(0.250) = v(0.125) + \left[\frac{P_0}{m}\left(1 - \frac{t}{T}\right) - \frac{k}{m}u(0.125)\right]\cdot\Delta t$$

$$v(0.250) = 6.250000 + \left[\frac{500000}{10000}\left(1 - \frac{0.125}{0.5}\right) - \frac{10000000}{10000}0\right]\cdot 0.125 = 10.937500 \text{ m/s}$$

at t=0.375

$$u(0.375) = u(0.250) + v(0.250)\cdot 0.125 = 2.1484375 \text{ m}$$

$$v(0.375) = v(0.250) + \left[\frac{P_0}{m}\left(1 - \frac{t}{T}\right) - \frac{k}{m}u(0.250)\right]\cdot\Delta t$$

$$v(0.375) = 10.937500 + \left[\frac{500000}{10000}\left(1 - \frac{0.25}{0.5}\right) - \frac{10000000}{10000}0.585938\right]\cdot 0.125 = -83.593750 \text{ m/s}$$

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at t=0.5

$$u(0.5) = u(0.375) + v(0.375)\cdot 0.125 = -8.300781 \text{ m}$$

$$v(0.5) = v(0.375) + \left[\frac{P_0}{m}\left(1-\frac{t}{T}\right) - \frac{k}{m}u(0.375)\right]\cdot \Delta t$$

$$v(0.5) = -83.59375 + \left[\frac{500000}{10000}\left(1-\frac{0.375}{0.5}\right) - \frac{10000000}{10000}(2.1484375)\right]\cdot 0.125 = -350.585938 \text{ m/s}$$

$$F = k\cdot u(0.5) = 10000\cdot(-8.300781) = -83007.81 \text{ kN}$$

c) The code is added as a zip file. Please download from the link given in the website.

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