Flow in Open Channels

DESIGN OF OPEN CHANNELS FOR UNIFORM FLOW

Hydraulic Efficiency of Cross-sections

- Conveyance of the channel section

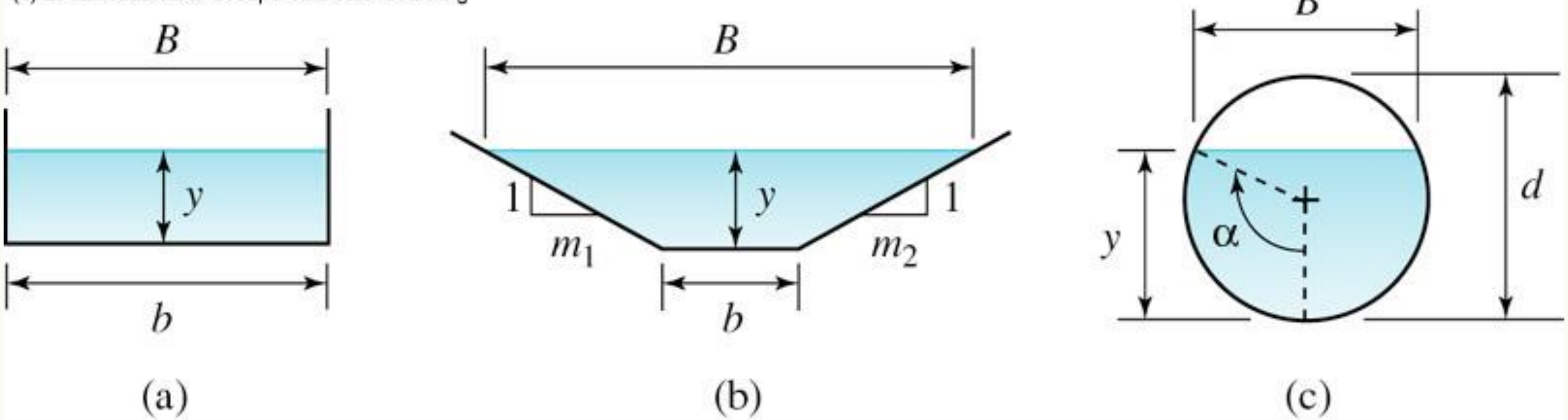
$$Q = \frac{A}{n} R^{2/3} S_o^{1/2}$$

$$\frac{Q}{\sqrt{S_o}} = \frac{A}{n} R^{2/3} = \frac{A^{5/3}}{nP^{2/3}} = K_o$$

Where

K_0 = Conveyance of the channel section

It is a measure of carrying capacity of a channel section.



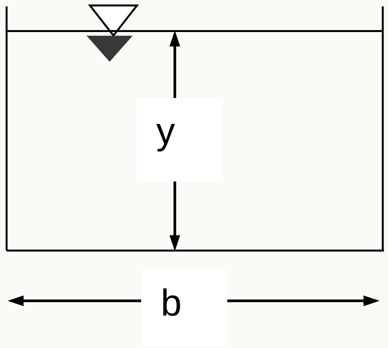
B
y
b
(a)
B
1
y
1
m_1
m_2
b
(b)
B
d
y
α
(c)

Best Hydraulic Section

- A channel section having the least wetted perimeter for a given area has the maximum conveyance; such a section is known as **the best hydraulic section**

Example 3.7

- What are the most efficient dimensions (the best hydraulic section) for a concrete (n=0.012) rectangular channel to carry 3.5 m^3/s at So=0.0006?



Given:
n=0.012
Q=3.5 m^3/s
S_o=0.0006

Find b and y.

Solution:

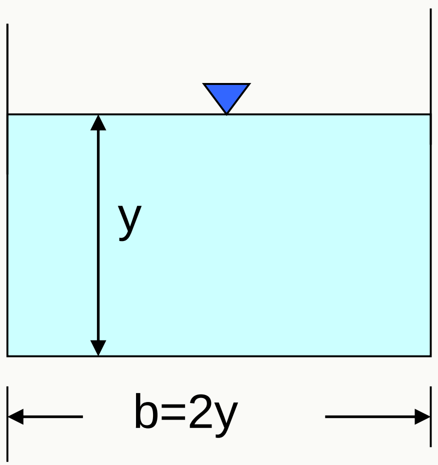
The best hydraulic section for a rectangular channel is needed

$$\left.\begin{aligned} A &= by \\ P &= b + 2y \end{aligned}\right\} \text{P must be minimum for a given A :} \quad b = \frac{A}{y}$$

$$\therefore P = \frac{A}{y} + 2y \Rightarrow \frac{dP}{dy} = 0 \qquad \frac{dP}{dy} = -\frac{A}{y^2} + 2 = 0 \Rightarrow A = 2y^2$$

$$b = \frac{A}{y} = \frac{2y^2}{y} = 2y \quad \therefore \quad b = 2y$$

Therefore, the best hydraulic section for a rectangular channel is



Example 3.7

When $Q = 3.5\ m^3/s$, $n = 0.012$ and $S_o = 0.0006$

$$Q = \frac{A}{n}\left(\frac{A}{P}\right)^{2/3}\sqrt{S_o}$$

$$\left.\begin{array}{l} A = by = 2y^2 \\ P = b + 2y = 2y + 2y = 4y \end{array}\right\} R = \frac{2y^2}{4y} = \frac{y}{2}$$

$$3.5 = \frac{2y^2}{0.012}\left(\frac{y}{2}\right)^{2/3}\sqrt{0.0006}$$

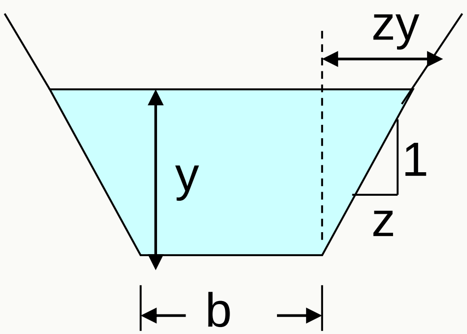
$$1.36 = y^2 y^{2/3} = y^{8/3} \qquad y = (1.36)^{3/8} = 1.123\ m.$$

$$b = 2y = 2.245\ m.$$

$$\therefore y = 1.123\ m, \qquad b = 2.245\ m$$

Example: The best hydraulic section for a trapezoidal channel

- Consider the trapezoidal section shown below:



$$A = by + zy^2 \quad \text{and} \quad P = b + 2y\sqrt{1+z^2}$$

$$by = A \quad zy^2 \quad \text{or} \quad b = \frac{A}{y} \quad zy$$

hence wetted perimeter becomes :

$$P = \frac{A}{y} \quad zy + 2y\sqrt{1+z^2} \quad \text{For a given } A,\ P = P(z, y)$$

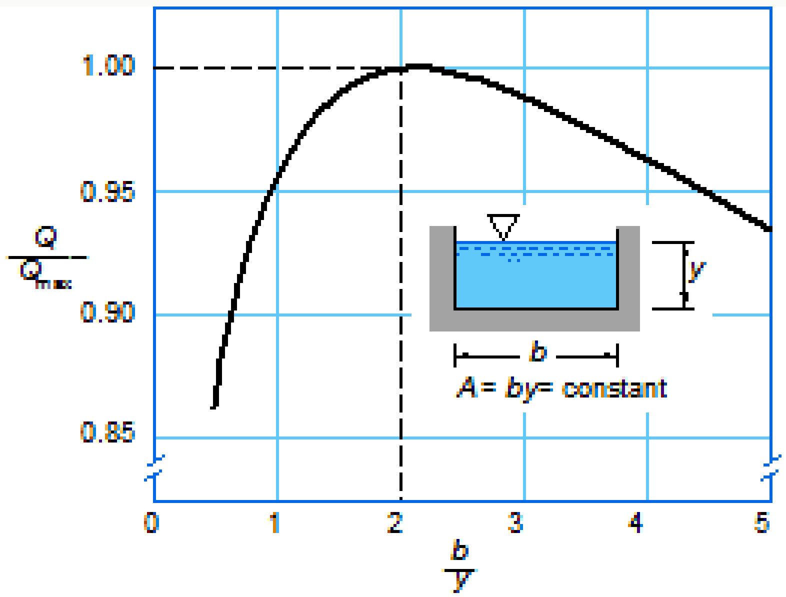
$$\text{Therefore when } P = P_{min} \quad \frac{\partial P}{\partial z} = 0, \ \text{and} \ \frac{\partial P}{\partial y} = 0$$

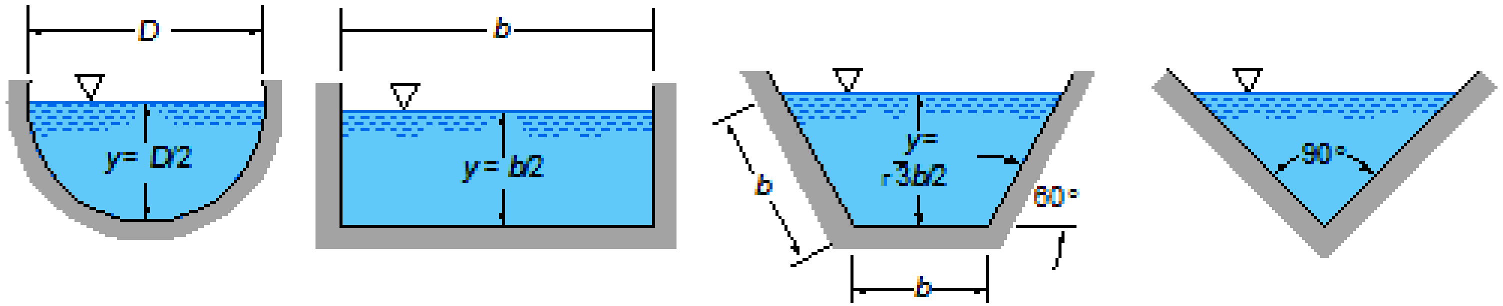
$$\frac{\partial P}{\partial z} = \quad y + 2y\frac{z}{\sqrt{1+z^2}} = 0 \ \text{solving for } z, \quad z = \frac{1}{\sqrt{3}}$$

substituting the value of z : $P = \dfrac{A}{y} + \sqrt{3}y,\ \text{and}\ \dfrac{\partial P}{\partial y} = \quad \dfrac{A}{y^2} + \sqrt{3} = 0$

$$A = \sqrt{3}y^2 \qquad b = \frac{2y}{\sqrt{3}}$$

Best Hydraulic Sections





Design of Open Channels

- Nonerodible channels
(channels with fixed boundaries)

Erodible channels
(channels with movable boundaries)

Precautions

- Steep slopes cause high velocities which may create erosion in erodible (unlined) channels
- Very mild slopes may result in low velocities which will cause silting in channels.
- The proper channel cross-section must have adequate hydraulic capacity for a minimum cost of construction and maintenance.

Typical Cross Sections

- The cross-sections of unlined channels are recommended as trapezoidal in shape with side slopes depending mainly on the kind of foundation material

(considering construction techniques and equipment, and stability of side inclination, the United States Bureau of Reclamation (USBR) and the Turkish State Hydraulic Works (DSİ) use standard 1.5H:1V side slopes for trapezoidal channels)

Recommended side slopes

Table 1 Recommended side slopes

Material	Side Slope (H:V)
Rock	Nearly vertical
Muck and peat soils	¼: 1
Stiff clay or earth with concrete lining	½:1 to 1:1
Earth with stone lining or earth for large channels	1:1
Firm clay or earth for small ditches	1.5:1
Loose sandy earth	2:1
Sandy loam or porous clay	3:1

Recommended side slopes

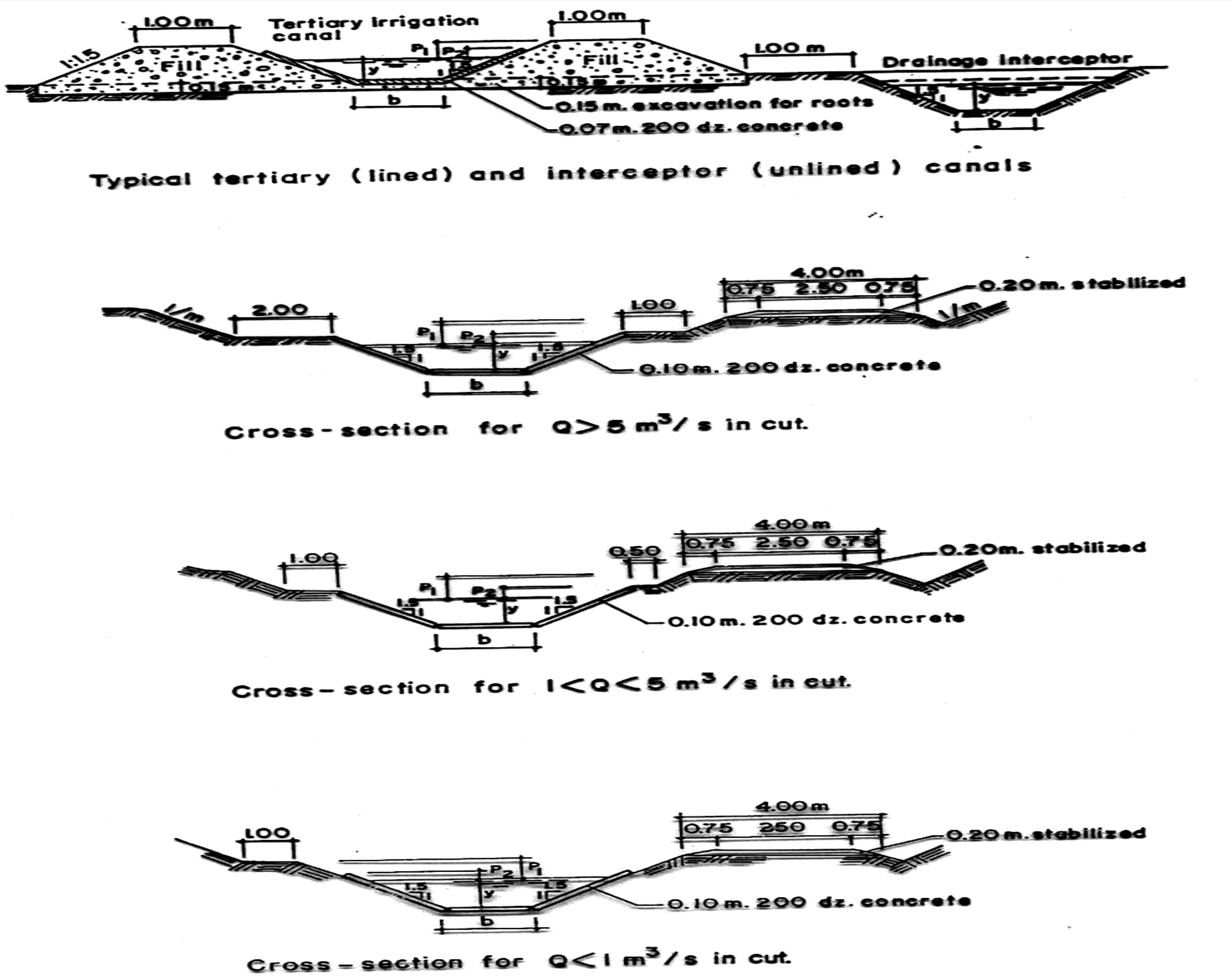


Fig. 1 Recommended channel cross-sections (Kızılkaya, 1988).

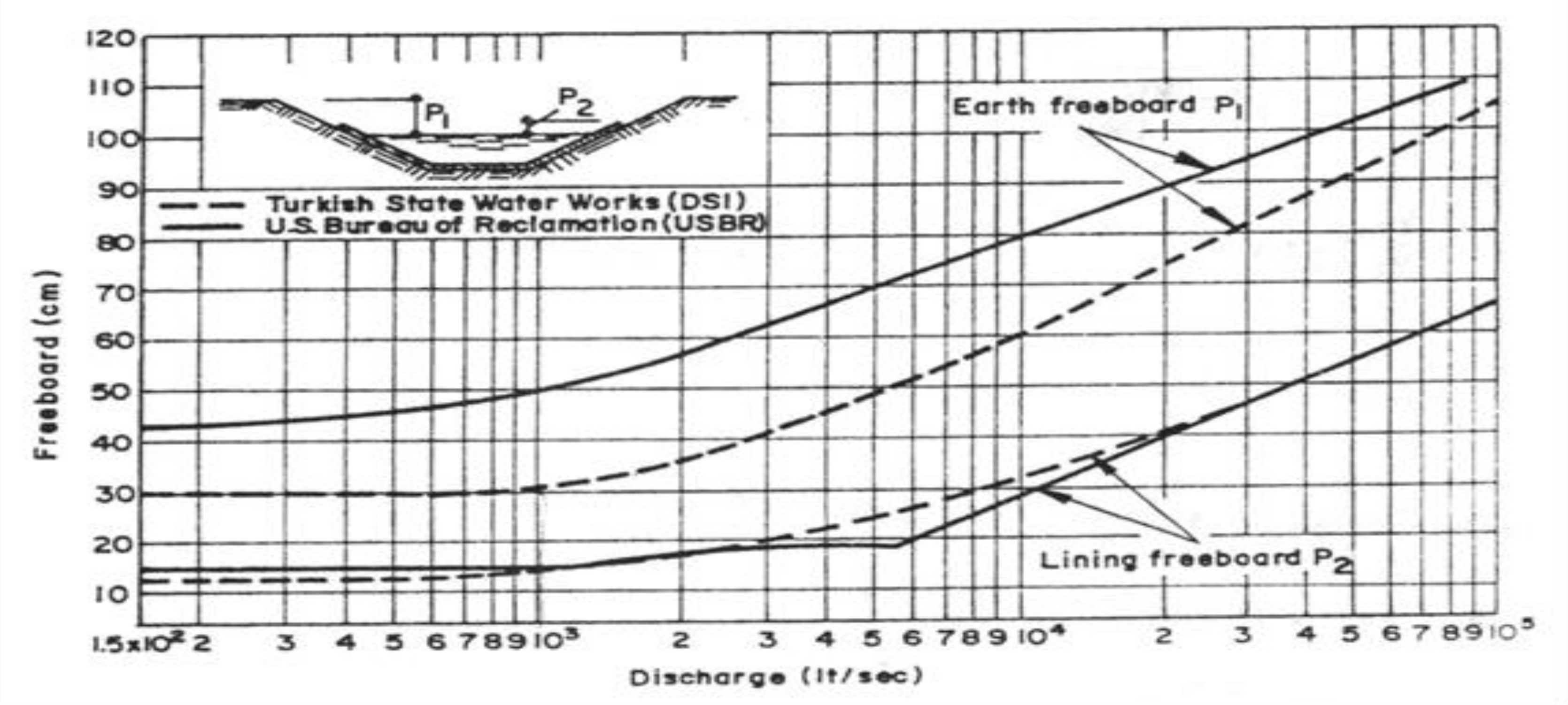
Freeboard

The freeboard, f, is determined either by

1. an empirical equation $f = 0.2\,(1+y)$

   where f is the freeboard in m, and y is the water depth in m, or

2. by the curves given in Figure 1 for irrigation canals for the USBR and DSI practices.



DESIGN OF NONERODIBLE CHANNELS

- For nonerodible channels the designer simply computes the dimensions of the channel by a uniform-flow formula and then finalize the dimensions on the basis of hydraulic efficiency, practicability, and economy.

Minimum Permissible velocity

- In the design of lined channels the minimum permissible velocity is considered to avoid deposition if water carries silt or debris
- V_{min} = 0.75 m/s (non-silting velocity)

The determination of section dimensions for nonerodible channels, includes the following steps:

- All necessary information, i.e. the design discharge, the Manning roughness coefficient and the bed slope are determined.

$$Q, n, S_o$$

- Compute the section factor, Z, from the Manning equation

$$A = (b + zy)y$$

$$P = b + 2y\sqrt{1 + z^2}$$

- If the expressions for A and R for the selected shape are substituted in the above equation, one obtains 3 unknowns (b, y, z) for trapezoidal sections, and 2 unknowns (b,y) for rectangular sections.

$$Z = AR^{2/3} = \frac{[(b + zy)y]^{5/3}}{\left[b + 2y\sqrt{1 + z^2}\right]^{2/3}}$$

Various combinations of b, y and z can be found to satisfy the above section factor Z.

The final dimensions are decided on the basis of hydraulic efficiency, practicability and economy.

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Various combinations of b, y and z can be found to satisfy the above section factor Z.
The final dimensions are decided on the basis of hydraulic efficiency, practicability and economy.

Methods and Procedures

- 1) Assume side slope z
- 2) get the value of b (or y) from the experience curve, Fig. 2,
- 3) solve for y (or b).

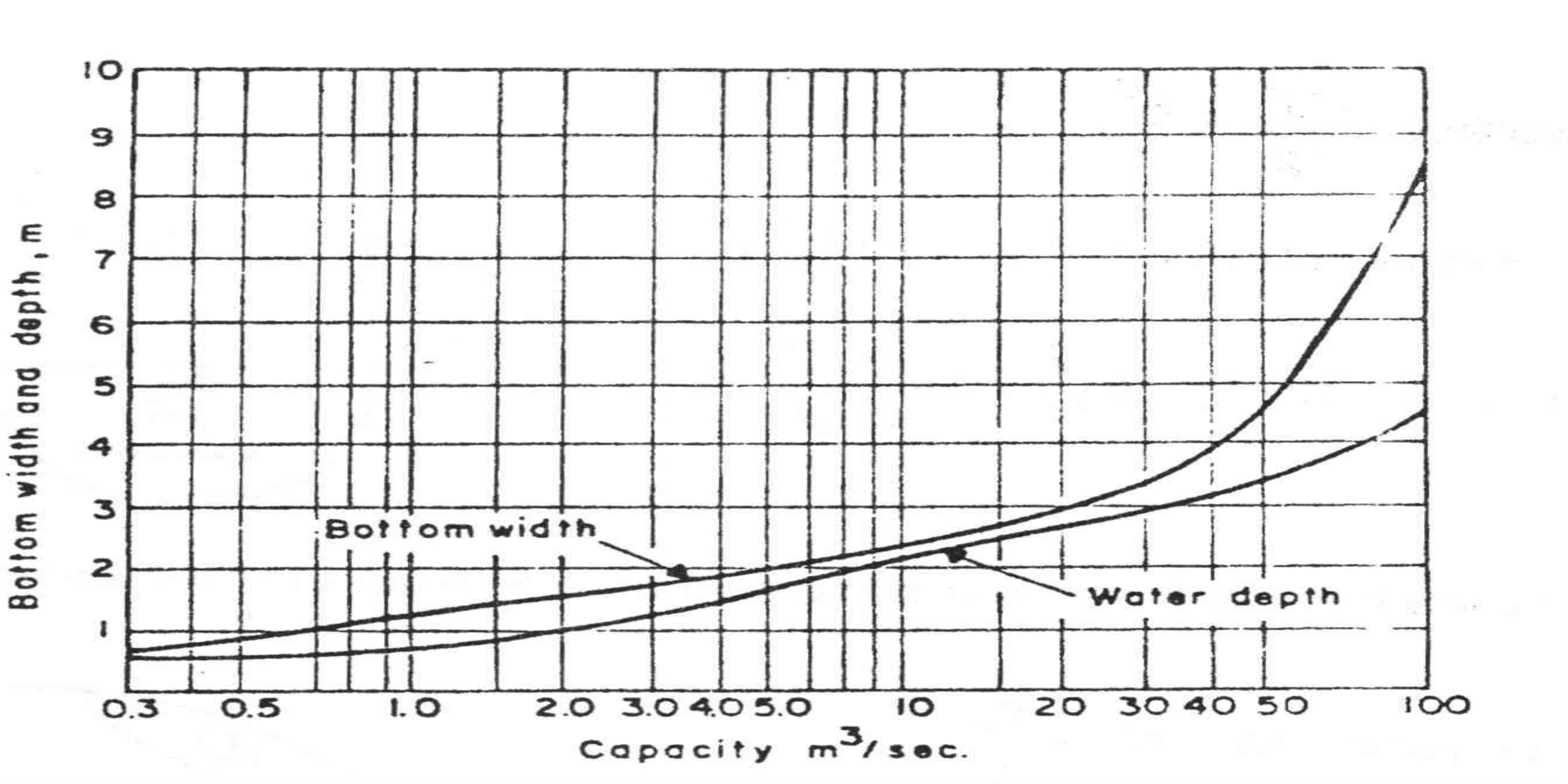


Fig 2: Experience Curves showing bottom width and water depth of lined channels

- If the best hydraulic section is required, then:
- 1) substitute A and R for best hydraulic section in

$$Z = AR^{2/3} = \frac{\left[(b+zy)y\right]^{5/3}}{\left[b+2y\sqrt{1+z^2}\right]^{2/3}}$$

- 2)Solve for y

For example in trapezoidal sections we have:

$$z = \frac{1}{\sqrt{3}} \qquad A = \sqrt{3}y^2 \qquad b = \frac{2}{3}\sqrt{3}y$$

$$R = \frac{y}{2} \qquad T = \frac{4}{3}\sqrt{3}y$$

Checks

1) In the proximity of critical depth, flow becomes unstable with excessive wave action, hence it is recommended that:

for subcritical flows: $y > 1.1y_c$ (or $Fr < 0.86$)

for supercritical flows: $y < 0.9y_c$ (or $Fr > 1.13$)

2) 2) Check the minimum permissible velocity if the water carries silt.

$V_{av} > V_{min} = 0.75$ m/s (non-silting velocity)

Finalization

1) Modify the dimensions for practicability
2) Add a proper freeboard to the depth of the channel section. Recommended freeboard for canals is given in Fig. 1
3) Draw channel cross section and show dimensions and the given parameters, Q, S_o,n.

Example 3.11

OPEN CHANNEL DESIGN EXAMPLE

A trapezoidal channel carrying 11.5m³/s clear water is built with nonerodible (concrete) having a slope of 0.0016 and n = 0.025. Proportion the section dimensions.

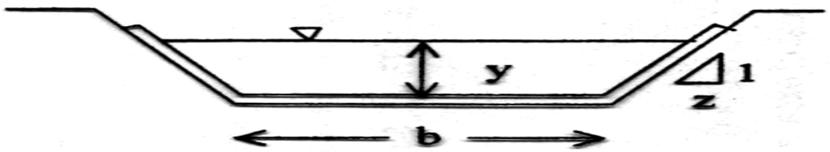


Figure 1.

Solution:

$Q = 11.5\ m^3/s,\ S_o = 0.0016,\ n = 0.025$

$$Z = \frac{nQ}{\sqrt{S_o}} = \frac{0.025*11.5}{0.04} = 7.1875$$

T=b+2zy

A=(b+zy)y

$$P = b + 2y\sqrt{1+z^2}$$

$$R = \frac{A}{P} = \frac{(b+zy)y}{b+2y\sqrt{1+z^2}}$$

Assume b=6 m and z = 2

$$Z = AR^{2/3} = \frac{[(6+2y)y]^{5/3}}{[6+2y\sqrt{5}]^{2/3}} = 7.1875$$

By trial and error y=1.04 m

For Q = 11.5 m^3/s from Fig. 2 (DSİ's curve):

Height of lining above water surface = 0.33 m

Height of bank above water surface = 0.63 m

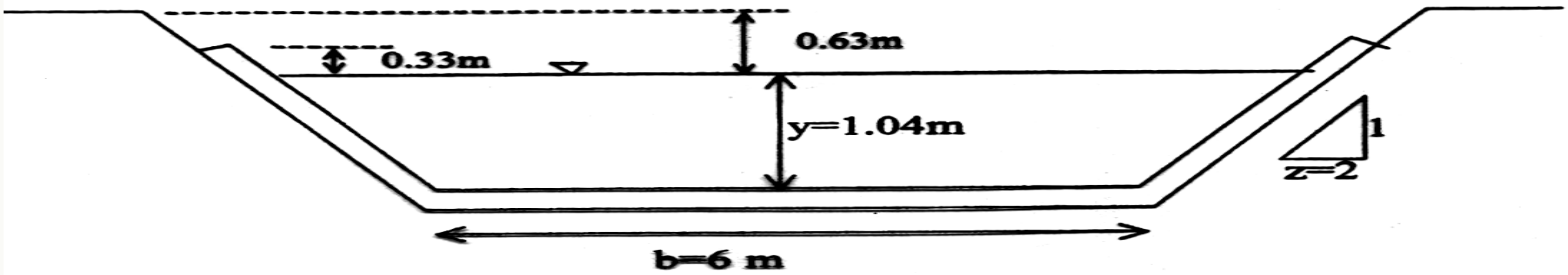


Figure 2.

Check stability:

At critical flow $\frac{Q^2}{g} = \frac{A^3}{T}$, y_c=0.692 m is solved by trial and error.

$$Fr = \frac{V}{\sqrt{gD}}$$

D=A/T=0.82 and Fr=0.48 (subcritical flow)

y > 1.1y_c or Fr< 0.86 (Stable flow)

Check silting velocity:

V=Q/A=1.37 m/s > 0.75 m/s (no silting)

Design the channel given in Example 1 by using experience curve and take z=1.5.

For Q = 11.5 m^3/s from Fig. 3 $\rightarrow$ b = 2.50 m.

$$AR^{2/3} \frac{[(2.50+1.5y)y]^{5/3}}{\left[2.50+2y\sqrt{1+(1.5)^2}\right]^{2/3}} = 7.1875$$ is solved by iteration then y = 1.56 m

$Fr = 0.47 < 0.86$ (Stable flow)

$V = \frac{Q}{A} = 1.52\ m/s > 0.75\ m/s$ O.K. (no silting)

The channel dimensions become:

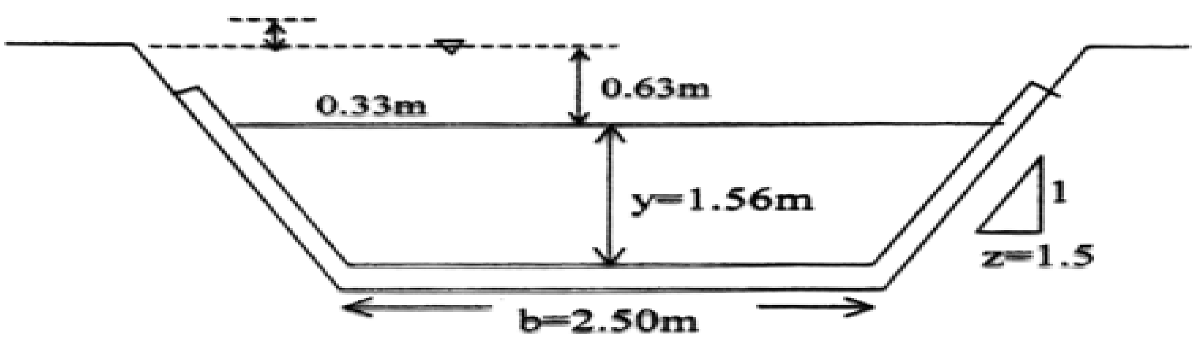


Figure 3.

Example 3.13

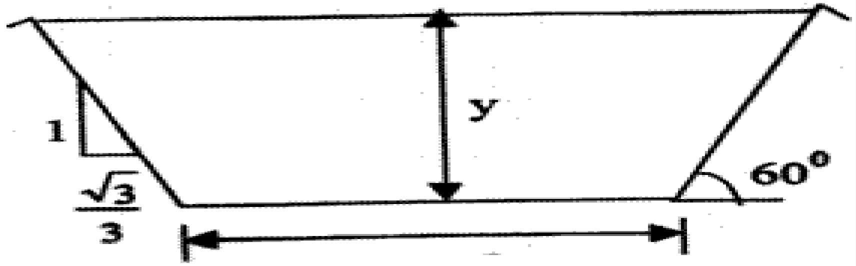
Example 3.10:

Proportion the canal section of Example 1 using best hydraulics section approach.

Solution:

The best trapezoidal section is half of hexagonal. The characteristics of such a section are

$$A = \sqrt{3}y^2$$
$$P = 2\sqrt{3}y$$
$$R = y/2$$
$$T = (4/3)\sqrt{3}\,y$$



given below,

$$AR^{2/3} = \frac{nQ}{\sqrt{S}} = \frac{0.025 \times 11.5}{\sqrt{0.0016}} = 7.1875$$

$$AR^{2/3} = (\sqrt{3}y^2)(y/2)^{2/3} = 7.1875$$

$$y = 2.03\text{m}$$

$$b = y/\cos 30^\circ = 2.34$$

Check silting:

$$A = \sqrt{3}y^2 = 7.14\ \text{m}^2$$

$V = Q/A = 1.61\ \text{m/s} > 0.75\ \text{m/s}$ Silting is not expected

Check stability:

$Fr = 0.42 < 0.86$ (Stable flow)

Example 3.14

The section becomes:

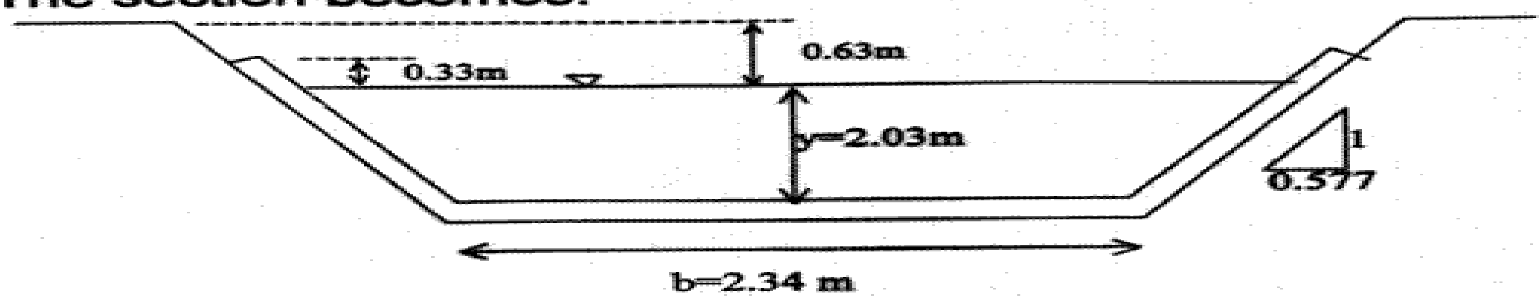


Figure 4.

Example 3.11

Determine the optimum section dimensions of the canal of example 1. Unit cost of the excavation, lining and land purchasing are \$18.75/m^3, \$4.75/m^2, and \$11.25/m^2, respectively.

Solution:

The total cost construction, $\Sigma C = C_1 + C_2 + C_3$ should be minimized. Here, C_1 is the cost of excavation, C_2 is the cost of lining, C_3 is the cost of purchasing the land. The total cost can be expressed in terms of canal geometric elements and unit cost as:

$$\Sigma C = \left(by_* + 1.5_2 y_*^2\right)C_1' + \left(b + 2y_* \sqrt{1 + (1.5)^2}\right)C_2' + (b + 2 \times 1.5 y_*)C_3'$$

where C_1', C_2' and C_3' are the respective unit cost and $y_* = y + f$ with $f = 0.2(1 + y)$.

In the computations, <u>a value is assumed for b</u> and corresponding water depth is computed from Manning's equation. The relation between b and $\sum C$ is given in Figure 5. As it can be seen from the Figure 5., optimum bottom width of channel is 2.10 m with $\sum C = \$362.77/m$ and the optimum water depth is determined as y=1.65 m.

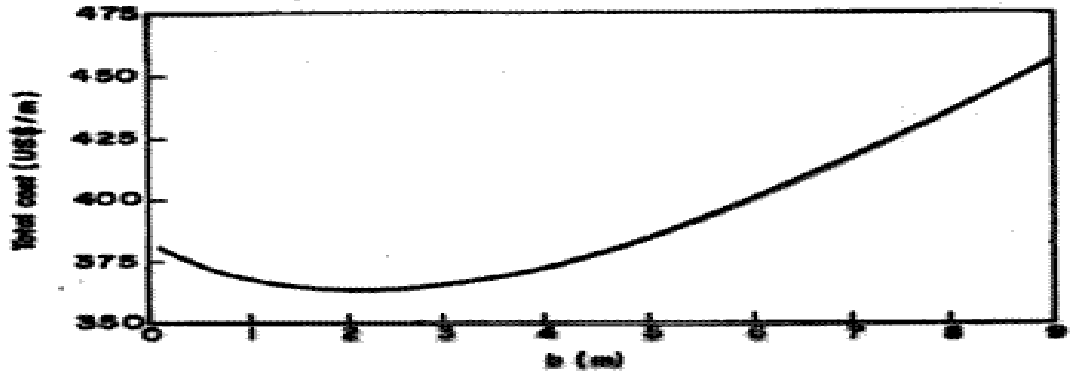


Figure 5.

The optimum section dimensions are shown in Figure.6 (check silting and stability)

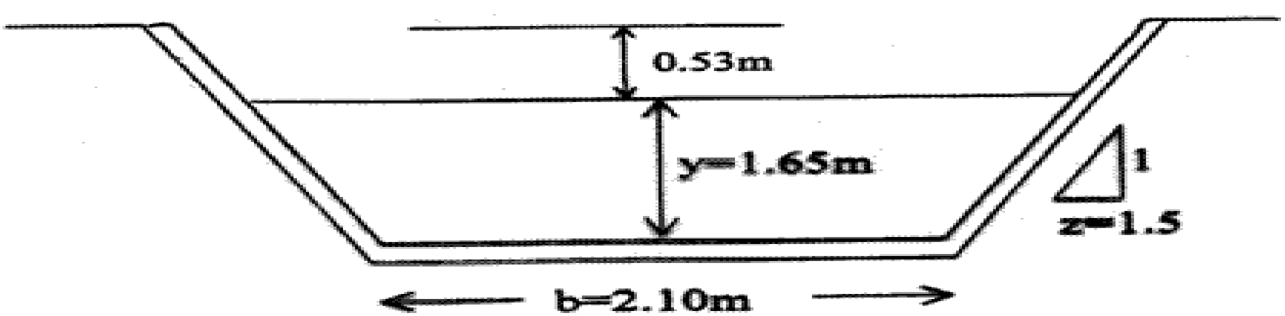


Figure 6

DESIGN OF ERODIBLE CHANNEL

- Channels used as irrigation and drainage canals, are unlined for cost reasons and they must be so proportioned as to permit neither silting nor scouring in objectionable quantity.
- There are two methods of approach to the proper design of erodible channels:
- Method of maximum permissible velocity
- Method of permissible tractive

Method of Permissible Tractive Force

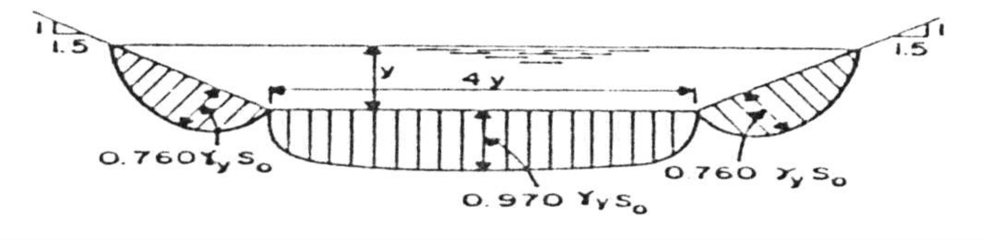
- Average unit tractive force

  For a uniform flow the average unit tractive force acting on the wetted perimeter of the channel section:

  $\tau_o = \gamma R S_o$

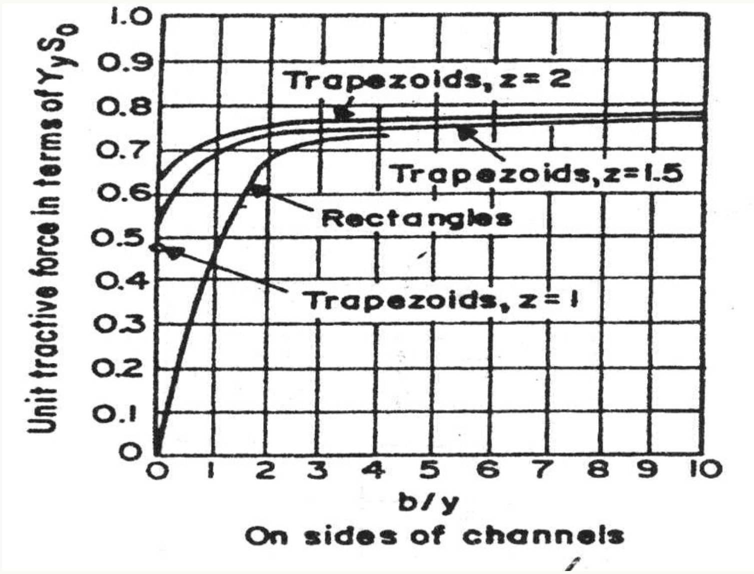
Unit tractive force

A typical distribution of unit tractive force in a trapezoidal channel is shown below.



Distribution of unit tractive force in a trapezoidal channel with b=4y

Maximum unit tractive force



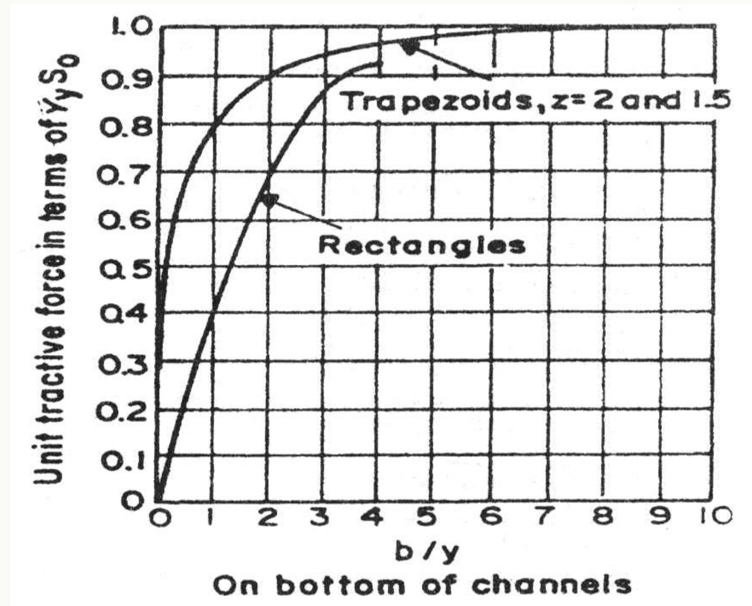
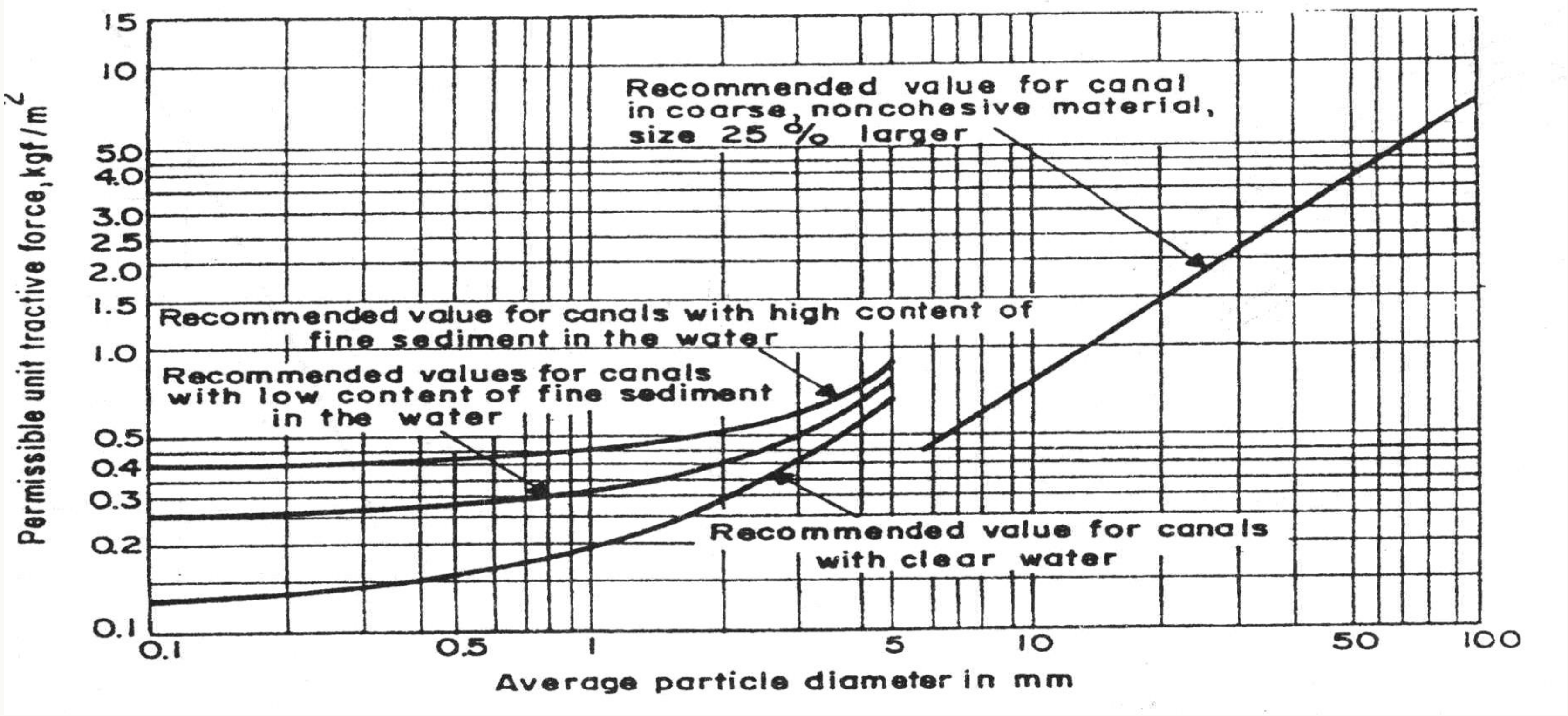


Fig 6: Maximum unit tractive forces in terms of □$\gamma y S_0$

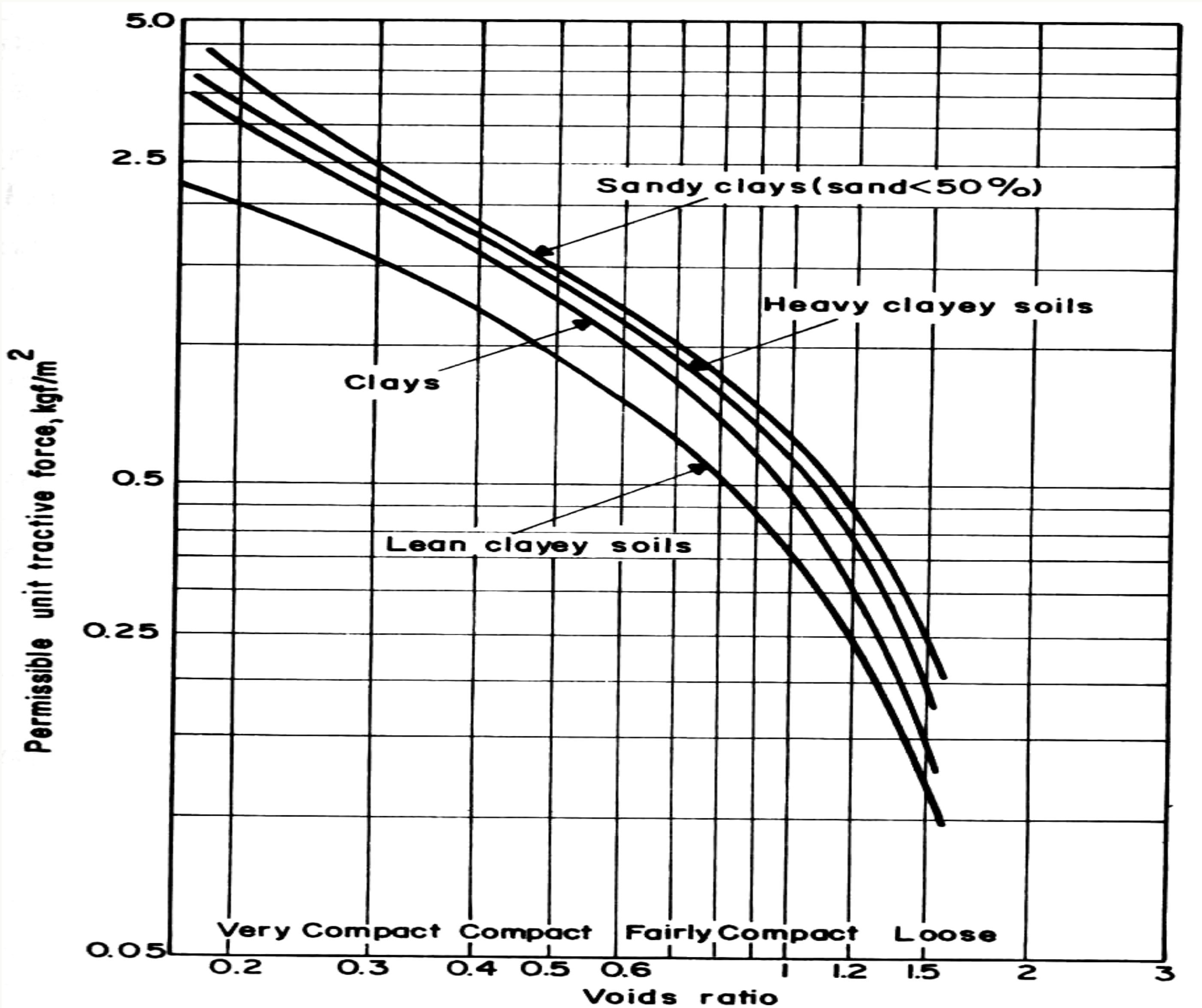
Permissible tractive force

- The permissible tractive force is the maximum tractive force that will not cause serious erosion of the material forming the channel bed on a level surface. This unit tractive force is determined by laboratory experiments and the value thus obtained is known as the critical tractive force

- For noncohesive materials the permissible tractive force is a function of the average particle diameter (Fig. 4)



For cohesive materials it is function of the void ratio (Fig. 8)

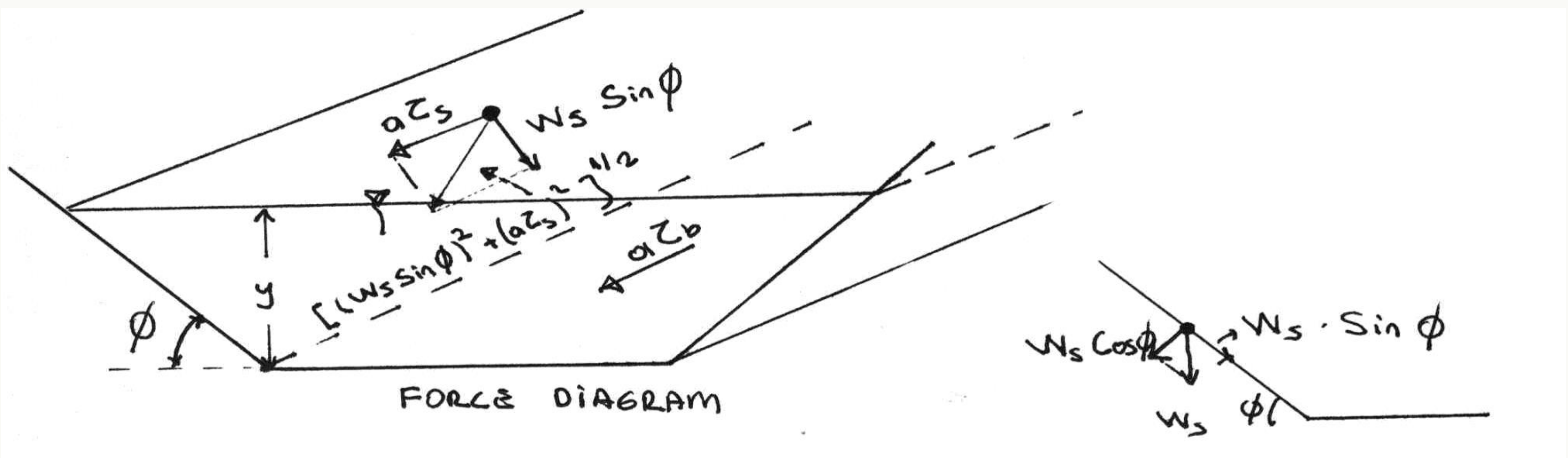


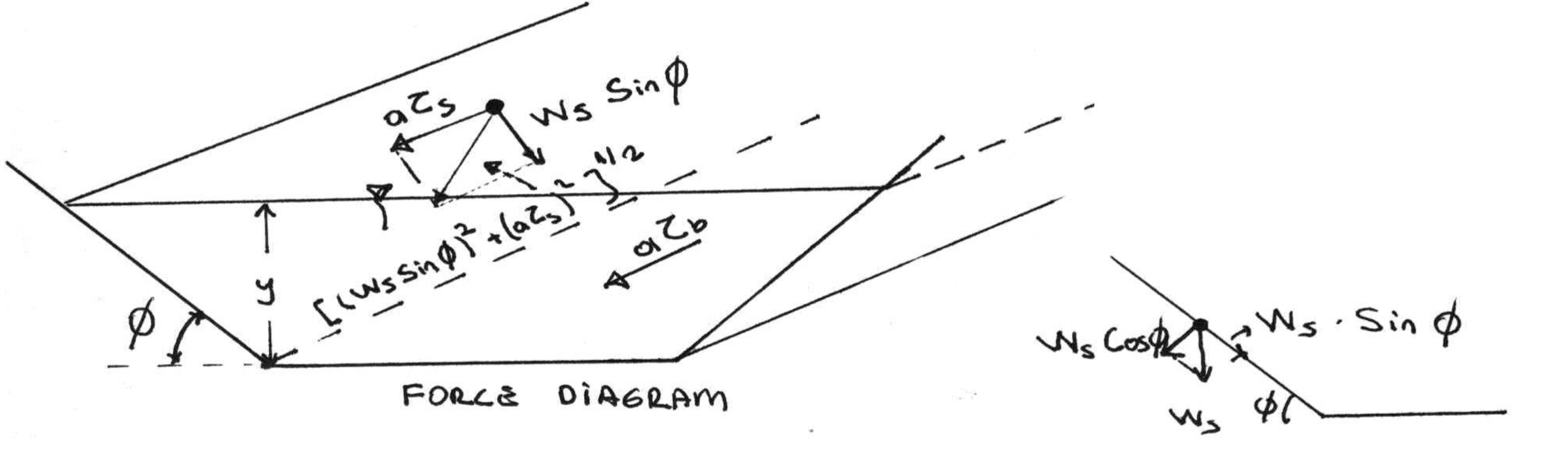
Motion of the particle on the side

- $a\tau_s$ =the tractive force on the side slope
- Ws.sinϕ=the gravity force component

where

- a is the effective area of the particle,
- τ is the unit tractive force on the side of the channel, Ws is the submerged weight of the particle.
- the resultant force
- Ws cosϕ tanθ =the resistance to motion of the particle
- tanθ =the coefficient of friction (θ is the angle of repose)





- $a\tau s$=the tractive force on the side slope
- Ws.sinϕ=the gravity force component

1) a is the effective area of the particle,
2) τ is the unit tractive force on the side of the channel, Ws is the submerged weight of the particle.
3) the resultant force
4) Ws cosϕ tanθ =the resistance to motion of the particle
5) tanθ =the coefficient of friction (θ is the angle of repose)

- The resultant force $\left[\left(W_s \sin\phi\right)^2 + \left(a\tau_s\right)^2\right]^{1/2} =$

- The resistance to motion of the particle $W_s \cos\phi \tan\theta$

tanθ =the coefficient of friction (θ is the angle of repose)

$$W_s \cos\phi \tan\theta = \sqrt{W_s^2 \sin^2\phi + a^2\tau_s^2}$$

$$W_s \cos\phi \tan\theta = \sqrt{W_s^2 \sin^2\phi + a^2\tau_s^2}$$

$$\tau_s = \frac{W_s}{a} \cos\phi \tan\theta \sqrt{1 - \frac{\tan^2\phi}{\tan^2\theta}}$$

$$\tau_b = \frac{W_s}{a} \tan\theta$$

$$W_s \tan\theta = a\tau_b$$

$$K = \frac{\tau_s}{\tau_b} = \mathrm{Cos}\phi \sqrt{1 - \frac{\tan^2\phi}{\tan^2\theta}}$$

$$K = \sqrt{1 - \frac{\sin^2\phi}{\sin^2\theta}}$$

Design Procedure

- Given: Design discharge Q, type of soil and channel bed slope So;
- Side slope z, roughness coefficient n, and angle of repose θ are selected
- The permissible tractive force, τ_p, is determined either from Fig. 4 (Noncohesive) or Fig. 5 (Cohesive)
- Any b/y ratio is assumed and tractive force of water;
- On the channel bed, $C_1\gamma y S_o$, and
- On the sides of the channel, $C_2\gamma y S_o$ are determined from Fig. 3.

- The value of y is determined from the following inequalities and its smaller value is accepted
-

$$C_1 \gamma y S_o \leq \tau_p = (\tau_b)_{permissible}$$

$$C_2 \gamma y S_o \leq \left[(\tau_b)_{permissible} = \tau_p . K = \tau_p \sqrt{1 - \frac{\sin^2 \phi}{\sin^2 \theta}} \right]$$

- Using the assumed b/y ratio and the computed value of y the channel capacity (discharge) is determined by the Manning equation

$$Q = \frac{A}{n} R^{2/3} S_o^{1/2}$$

- If the computed channel capacity is different from the design discharge, a new value for b/y ratio is assumed and the procedure is repeated until the computed discharge is equal to the given design discharge.
- Critical flow conditions are checked
- A freeboard is added on the water depth.

Notes

- For cohesive soils, only the tractive force of bottom is critical
- For noncohesive soils, one must compute the value of K from its equation. On the basis of stability criteria, if:
- $K < 0.78$ side shear controls the required depth
- $K > 0.78$ bed shear controls the required depth

Example 3.15

- Design an unlined trapezoidal canal using the tractive force approach to carry Q=4.6 m3/s. Given are z=2.0, n=0.02 and So=0.0016. The canal material is moderately rounded noncohesive soil that 25 % material by weight is larger than 25 mm.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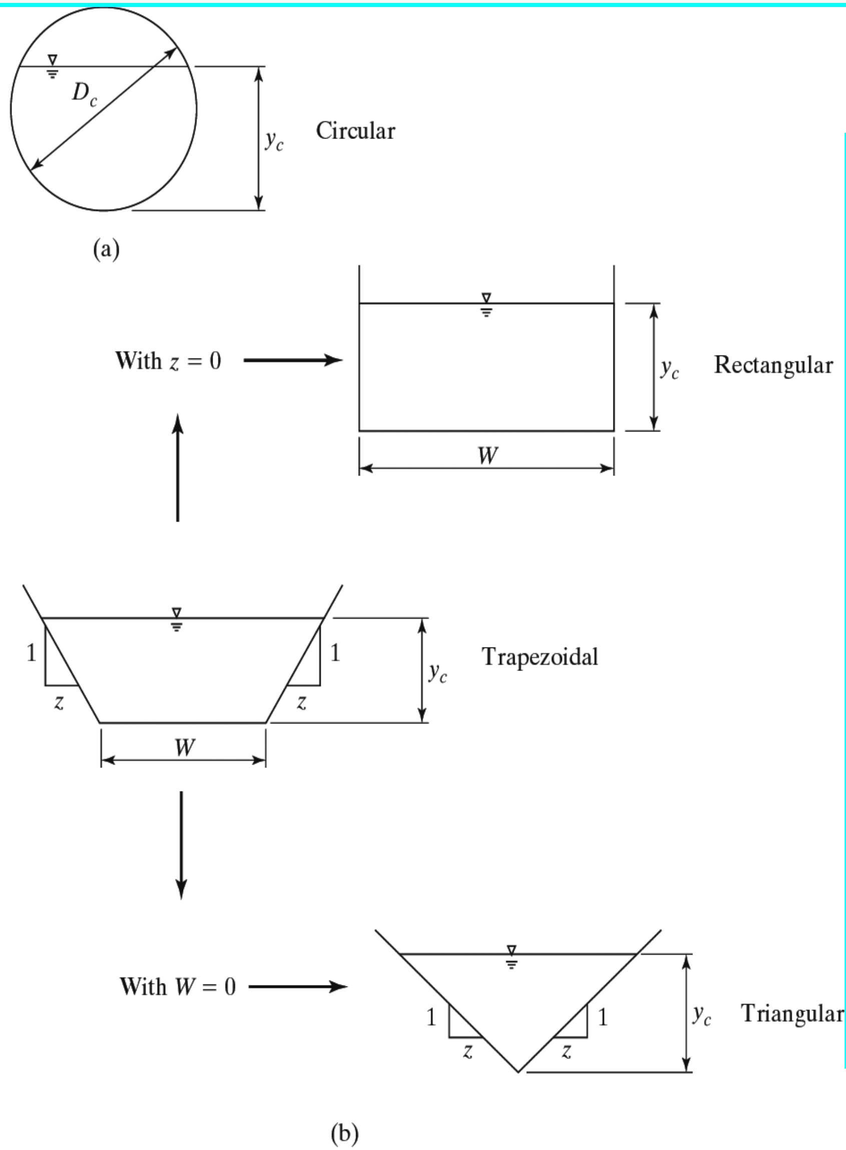

Figure 4.12

Basic channel shapes and their variations used by the HEC-1 flood hydrograph package for kinematic wave stream routing.

- Circular pipe diameter D
- Rectangular culvert
- Trapezoidal channel
- Triangular channel

Hydraulic Shapes

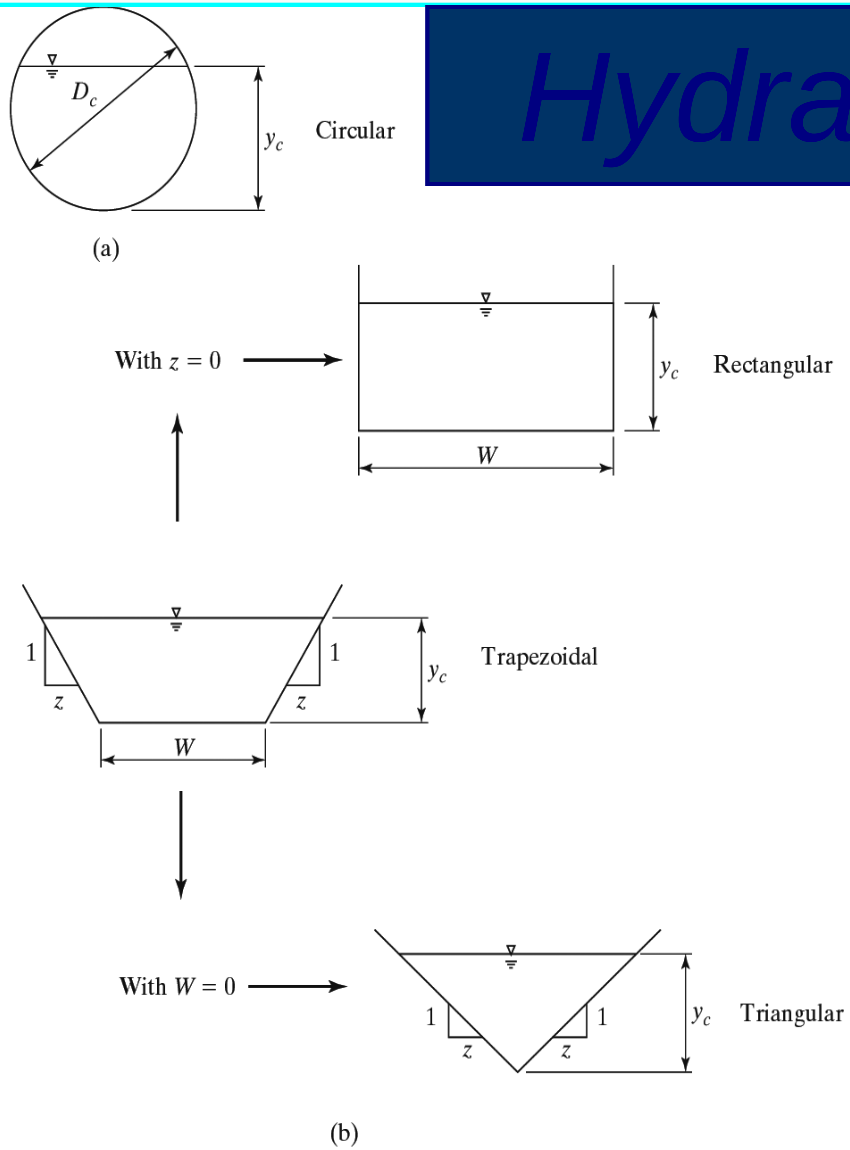

Figure 4.12

Basic channel shapes and their variations used by the HEC-1 flood hydrograph package for kinematic wave stream routing.

Manning's Equation used to estimate flow rates

$$Q = 1.49/n \; A \; R^{2/3} \; S^{1/2}$$

Where **Q = flow rate**

n = roughness

A = cross sect A

R = A / P

S = Slope

Furrow Irrigation