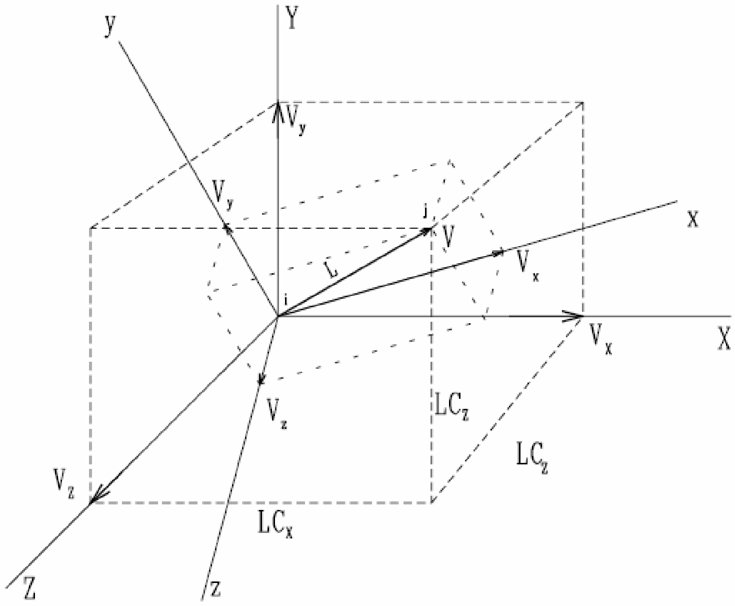
CE483 ADVANCED STRUCTURAL ANALYSIS
Coordinate Transformations in 3D

Vector transformation from (X,Y,Z) coordinate system to (x,y,z) coordinate system

$$\begin{Bmatrix} V_x \\ V_y \\ V_z \end{Bmatrix} = \underbrace{\begin{bmatrix} l_{xX} & l_{xY} & l_{xZ} \\ l_{yX} & l_{yY} & l_{yZ} \\ l_{zX} & l_{zY} & l_{zZ} \end{bmatrix}}_{[\gamma]} \begin{Bmatrix} V_X \\ V_Y \\ V_Z \end{Bmatrix}$$



Where l_{ij} is the direction cosines of rotated axis *i* with respect to original axis *j*

For instance, l_{xj} (l_{xX}, l_{xY}, l_{xZ}) is the direction cosines of *x* with respect to *X, Y, Z*.

γ is the vector transformation matrix
γ is an orthogonal matrix with the property
$\gamma^{-1}=\gamma^{T}$

T is the force transformation matrix for 12 degree-of-freedom 3D frame element

$$\left\{\begin{array}{c} F_{x1} \\ F_{y1} \\ F_{z1} \\ \hline M_{x1} \\ M_{y1} \\ M_{z1} \\ \hline F_{x2} \\ F_{y2} \\ F_{z2} \\ \hline M_{x2} \\ M_{y2} \\ M_{z2} \end{array}\right\} = \underbrace{\left[\begin{array}{c|c|c|c} [\gamma] & & & \\ \hline & [\gamma] & & \\ \hline & & [\gamma] & \\ \hline & & & [\gamma] \end{array}\right]}_{\textbf{[T]}} \left\{\begin{array}{c} F_{X1} \\ F_{Y1} \\ F_{Z1} \\ \hline M_{X1} \\ M_{Y1} \\ M_{Z1} \\ \hline F_{X2} \\ F_{Y2} \\ F_{Z2} \\ \hline M_{X2} \\ M_{Y2} \\ M_{Z2} \end{array}\right\}$$