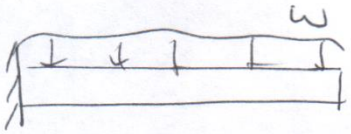
CE425
HW#5
SOLUTIONS

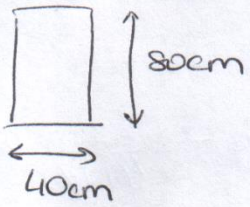
Q1) Plot the tip displacement of the cantilever (flexural and shear deformations seperately) versus:

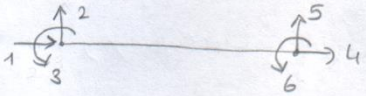
a) E/G ratio (use 40cm x 80cm section and L=4m)

b) I/A ratio (use E=200GPa, ν = 0.3 and L=4m)

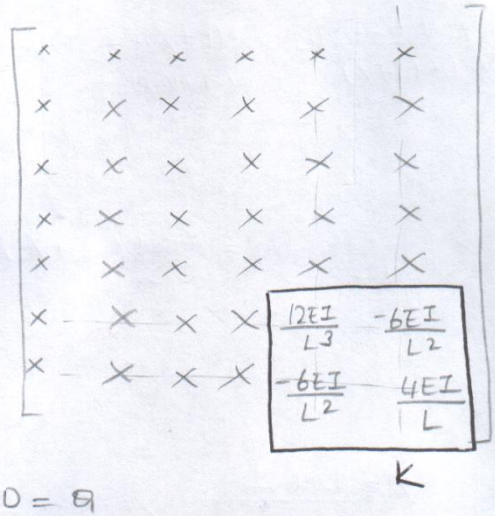
c) L (use E=200GPa, ν=0.3 and 40cm x 80cm section)



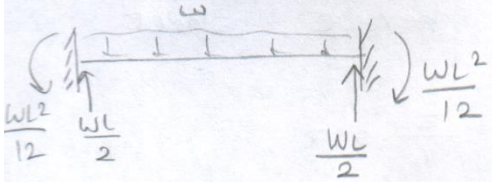




* Bernoulli Euler Beam Theory, K = (considering only flexure)



For FEM;



$$\underset{\sim}{K}\cdot\underset{\sim}{D}=\underset{\sim}{Q}$$

$$Q = Q_a - Q_F = \begin{bmatrix} -WL/2 \\ WL^2/12 \end{bmatrix} \begin{bmatrix} \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ \frac{-6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} D_3 \\ D_4 \end{bmatrix} = \begin{bmatrix} -WL/2 \\ WL^2/12 \end{bmatrix}$$

(1) ⇒

$$\frac{12EI}{L^3} D_3 - \frac{6EI}{L^2} D_4 = \frac{-WL}{2}$$

$$\left(\frac{3}{2L}\right) \Big/ \frac{-6EI}{L^2} D_3 + \frac{4EI}{L} D_4 = \frac{WL^2}{12} \quad \Rightarrow D_3 = \frac{-WL^4}{8EI}$$

* Timoshenko Beam Theory, (considering shear and flexure):

$$K = \begin{bmatrix} \frac{12EI}{L^3(1+\phi)} & \frac{-6EI}{L^2(1+\phi)} \\ \frac{-6EI}{L^2(1+\phi)} & \frac{(4+\phi)EI}{(1+\phi)L} \end{bmatrix}, \quad \underset{\sim}{K} \cdot \underset{\sim}{D} = \underset{\sim}{Q}$$

(Q_1 is same)

$$(4+\phi) \Big/ \frac{12EI}{L^3(1+\phi)} D_3 - \frac{6EI}{L^2(1+\phi)} D_4 = \frac{-WL}{2}$$

$$\left(\frac{6}{L}\right) \Big/ \frac{-6EI}{L^2(1+\phi)} D_3 + \frac{(4+\phi)EI}{(1+\phi)L} D_4 = \frac{WL^2}{12}$$

$48EI - 36EI + 12EI\phi = 12EI(1+\phi)$

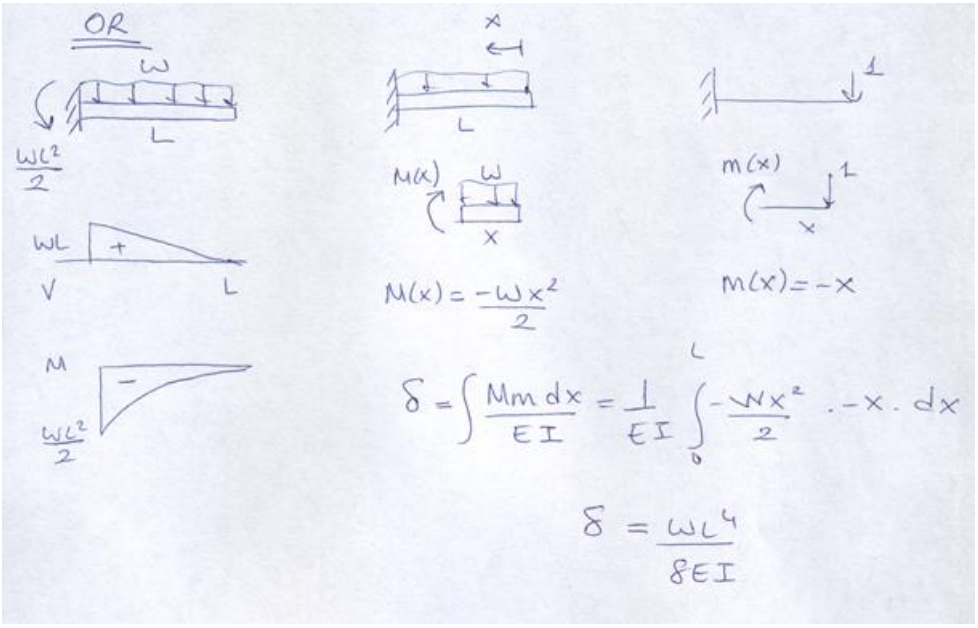
$$\Rightarrow \frac{12EI(4+\phi) - 36EI}{L^3(1+\phi)} \cdot D_3 = \frac{-WL(4+\phi)}{2} + \frac{WL}{2}$$

$$\Rightarrow \frac{12EI}{L^3} \cdot D_3 = \frac{-WL(3+\phi)}{2} \Rightarrow D_3 = \underbrace{\frac{-WL^4}{8EI}}_{\text{only for flexure}} \underbrace{- \frac{WL^4\phi}{24EI}}_{\text{only for shear}}$$

$$\phi = \frac{12EI}{A_s \cdot G \cdot L^2}, \quad A_s = \frac{5}{6} \cdot A = \frac{5}{6} \cdot b \cdot h$$

$$I = \frac{1}{12} \cdot b \cdot h^3$$

Using the variables given, for different cases tip displacements are graphed using MS Excel. Any value can be assumed for W. (2)

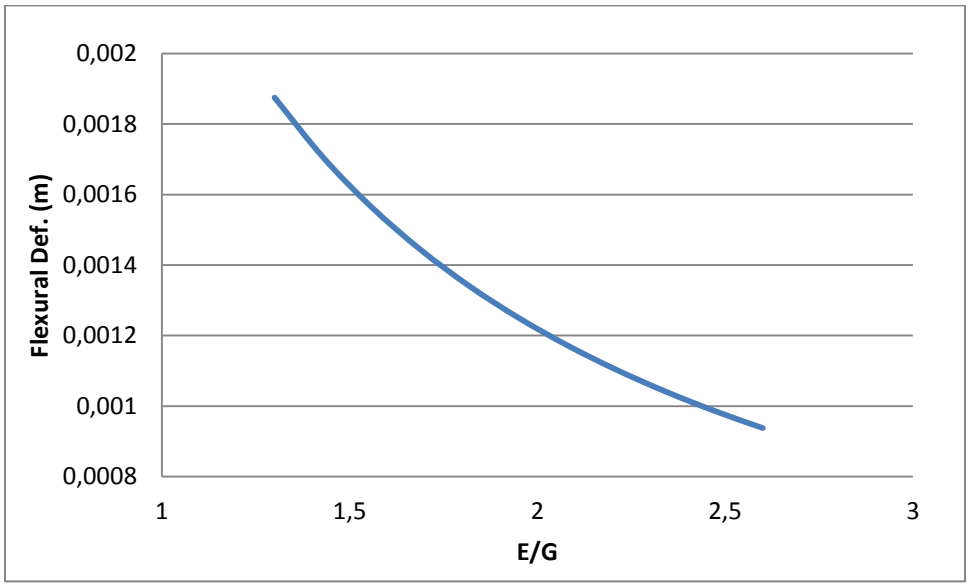

OR
W
L
WL²/2
WL
+
V
L
M
−
WL²/2
x
L
M(x)
W
x
1
m(x)
1
x
M(x) = −Wx²/2
m(x) = −x
δ = ∫ Mm dx/EI = 1/EI ∫₀ᴸ −Wx²/2 . −x . dx
δ = WL⁴/8EI

a)

W (N/m) (assumed)	E (Pa)	G (Pa)	ø
100000	2E+11	76923076923	0,1248
L (m)	I (m^4)	Poisson's Ratio	A (m^2)
4	0,017067	0,3	0,32

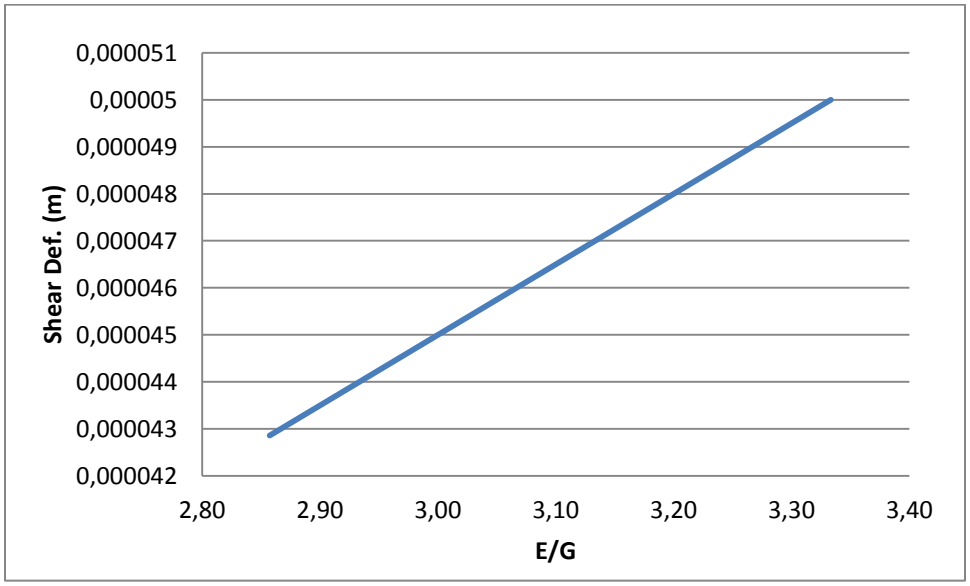
For Flexural Deformation G is kept constant, E is increasing;

E (Pa)	E/G	Flexural Def. (m)
2E+11	2,6	0,0009375
1,9E+11	2,47	0,000986842
1,8E+11	2,34	0,001041667
1,7E+11	2,21	0,001102941
1,6E+11	2,08	0,001171875
1,5E+11	1,95	0,00125
1,4E+11	1,82	0,001339286
1,3E+11	1,69	0,001442308
1,2E+11	1,56	0,0015625
1,1E+11	1,43	0,001704545
1E+11	1,3	0,001875



For ShearDeformation E is kept constant, G is decreasing;

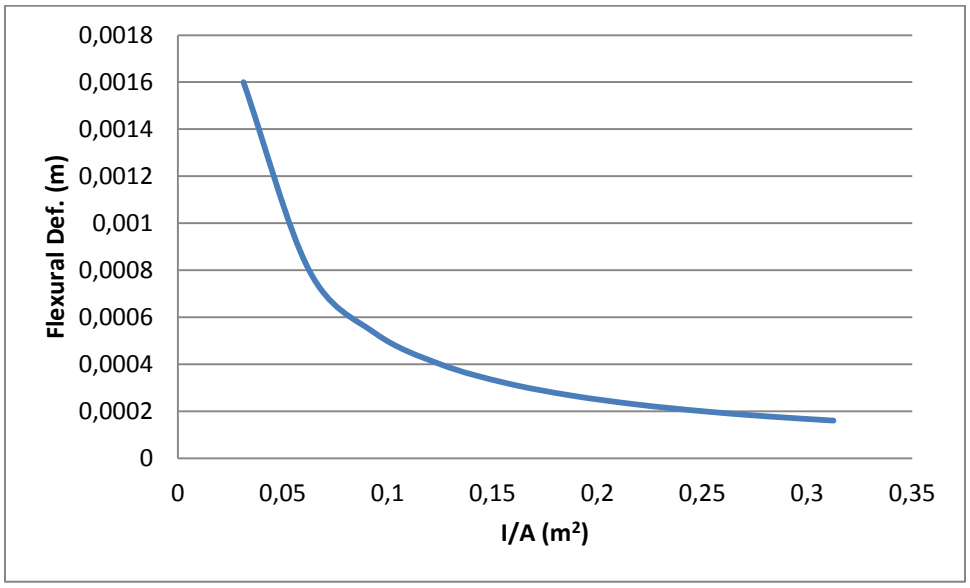
G(Pa)	E/G	Shear Def. (m)
70000000000	2,86	4,28571E-05
69000000000	2,90	4,34783E-05
68000000000	2,94	4,41176E-05
67000000000	2,99	4,47761E-05
66000000000	3,03	4,54545E-05
65000000000	3,08	4,61538E-05
64000000000	3,13	0,000046875
63000000000	3,17	4,7619E-05
62000000000	3,23	4,83871E-05
61000000000	3,28	4,91803E-05
60000000000	3,33	0,00005



b)

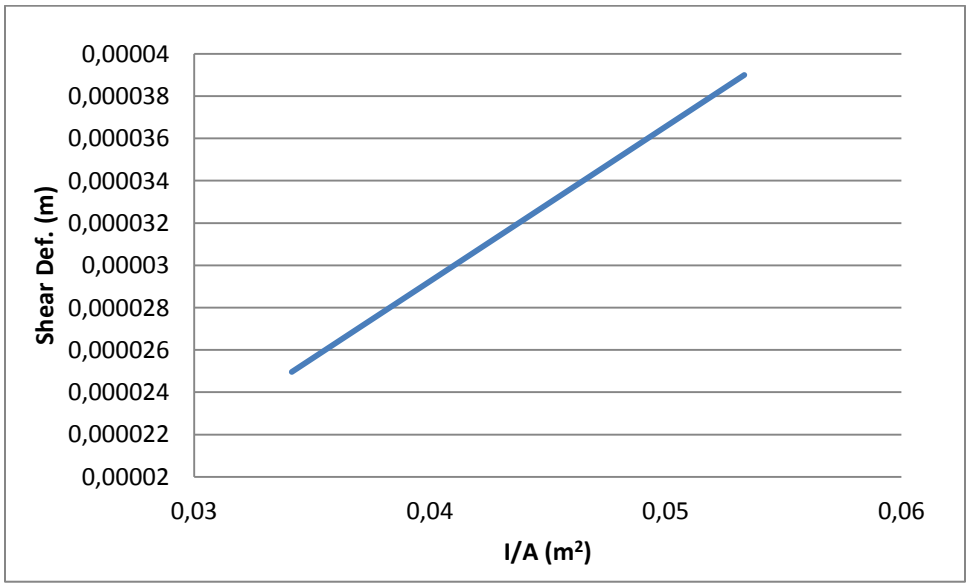
For Flexural Deformation Ais kept constant, I is increasing;

I (m^4)	I/A (m^2)	Flexural def. (m)
0,01	0,03125	0,0016
0,02	0,0625	0,0008
0,03	0,09375	0,000533333
0,04	0,125	0,0004
0,05	0,15625	0,00032
0,06	0,1875	0,000266667
0,07	0,21875	0,000228571
0,08	0,25	0,0002
0,09	0,28125	0,000177778
0,1	0,3125	0,00016



For ShearDeformation I is kept constant, A is decreasing;

A (m^2)	I/A (m^2)	Shear def. (m)
0,5	0,034133	0,00002496
0,48	0,035556	0,000026
0,46	0,037101	2,71304E-05
0,44	0,038788	2,83636E-05
0,42	0,040635	2,97143E-05
0,4	0,042667	0,0000312
0,38	0,044912	3,28421E-05
0,36	0,047407	3,46667E-05
0,34	0,050196	3,67059E-05
0,32	0,053333	0,000039



c)

L (m)	Flexural def. (m)	ø	Shear def. (m)
1	3,66211E-06	1,9968	2,4375E-06
2	5,85938E-05	0,4992	0,00000975
3	0,000296631	0,221866667	2,19375E-05
4	0,0009375	0,1248	0,000039
5	0,002288818	0,079872	6,09375E-05
6	0,004746094	0,055466667	0,00008775