In[•]:=
```
Quit[]
```
detén núcleo del sistema

In[1]:=
```
constants = {kB → 1.380649 × 10^-23 (*in joules*),
    hbar → 1.054 × 10^-34 (*J s*), h → 1.054 × 10^-34 * 2 π, c → 299 792 458 (*m/s*)};
```

In[2]:=
$$\alpha = \frac{\pi^2\, kB^4}{15\, hbar^3\, c^3}\ (*j\ \ m^{-3}K^{-4}*);$$

$$\alpha = 2\,\frac{\pi^2\, kB^4}{30\, hbar^3\, c^3}\ (*j\ \ m^{-3}K^{-4}*);$$

$$\beta = \frac{2.4041}{\pi^2}\ \frac{kB^3}{hbar^3\, c^3}\ (*m^{-3}K^{-3}*);$$

$$\beta = \frac{2\, Zeta[3.0]}{\pi^2}\ \frac{kB^3}{hbar^3\, c^3}\ (*m^{-3}K^{-3}*);$$

In[6]:=
$$dndf\gamma[f\_, T\_] := \frac{8\,\pi}{c^3}\ \frac{f^2}{Exp\left[\frac{h\,f}{kB\,T}\right] - 1}$$

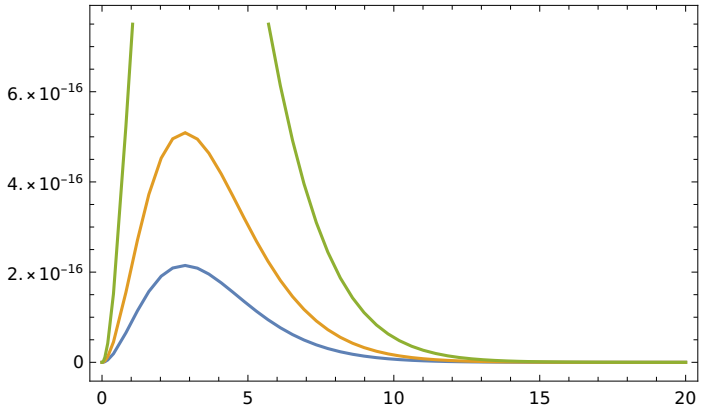
In[7]:=
$$d\epsilon df\gamma[f\_, T\_] := \frac{8\,\pi\, h}{c^3}\ \frac{f^3}{Exp\left[\frac{h\,f}{kB\,T}\right] - 1}$$

$$d\epsilon dfx[x\_, T\_] := \frac{8\,\pi\, h}{c^3}\left(\frac{kB\,T}{h}\right)^3\ \frac{x^3}{Exp[x] - 1}$$

In[9]:=
```
dϵdfx[x, T] /. constants /. T → 3000
```
Out[9]=
$$\frac{1.5113 \times 10^{-16}\, x^3}{-1 + e^x}$$

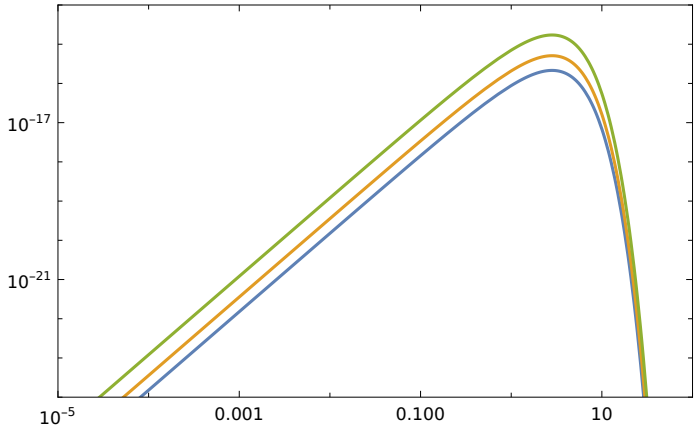
In[10]:=
```
Plot[{dϵdfx[x, T] /. constants /. T → 3000, dϵdfx[x, T] /. constants /. T → 4000,
  dϵdfx[x, T] /. constants /. T → 6000}, {x, 0, 20}, Frame → True]
```
representación gráfica · marco · verdadero

Out[10]=




In[11]:=
```
LogLogPlot[{dϵdfx[x, T] /. constants /. T → 3000,
  dϵdfx[x, T] /. constants /. T → 4000, dϵdfx[x, T] /. constants /. T → 6000},
 {x, 10^-5, 100}, Frame → True, PlotRange → {{10^-5, 100}, {10^-24, 10^-14}}]
```
representación log log · marco · ver··· · rango de representación

Out[11]=




In[12]:=
$$\frac{8\,\pi\,h}{c^3}\left(\frac{kB\,T}{h}\right)^3\frac{c^3\,h^2}{8\,\pi\,kB^3\,T^3}$$

Out[12]= 1

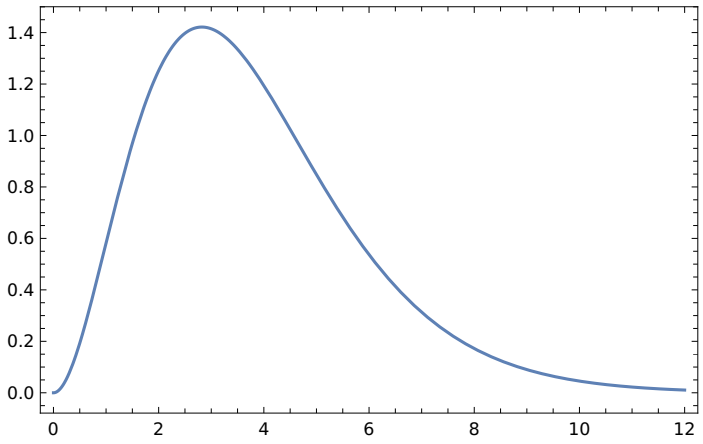
In[13]:=
```
Ndϵdfx[x_] := c^3 h^2/(8 π kB^3 T^3) 8 π h/c^3 (kB T/h)^3 x^3/(Exp[x] - 1) (*normalized and independent of T*)
```

In[14]:=
```
Ndϵdfx[x]
```

Out[14]=
$$\frac{x^3}{-1+e^x}$$

In[15]:=
```
Plot[NdϵdfX[x], {x, 0, 12}, Frame → True]
```
representación gráfica  marco  verdac

Out[15]=




In[16]:=
```
Integrate[dϵdfγ[f, T], {f, 0, Infinity}, Assumptions → {h > 0, kB > 0, T > 0}]
```
integra  infinito  asunciones

$$\frac{1}{(2\pi)^3}\ \texttt{Integrate[dndfγ[f, T], \{f, 0, Infinity\}, Assumptions → \{h > 0, kB > 0, T > 0\}]}$$
integra  infinito  asunciones

Out[16]=
$$\frac{8\ kB^4\ \pi^5\ T^4}{15\ c^3\ h^3}$$

Out[17]=
$$\frac{2\ kB^3\ T^3\ Zeta[3]}{c^3\ h^3\ \pi^2}$$

In[18]:=
$$\frac{\frac{1}{(2\pi)^3}\,\text{Integrate}\left[\text{dndf}\gamma[f, T], \{f, 10.0\,\text{kB}\,T/h, \text{Infinity}\}, \text{Assumptions} \to \{h > 0, \text{kB} > 0, T > 0\}\right]}{\frac{2\,\text{kB}^3\,T^3\,\text{Zeta}[3]}{c^3\,h^3\,\pi^2}}$$

$$\frac{\frac{1}{(2\pi)^3}\,\text{Integrate}\left[\text{dndf}\gamma[f, T], \{f, 20.0\,\text{kB}\,T/h, \text{Infinity}\}, \text{Assumptions} \to \{h > 0, \text{kB} > 0, T > 0\}\right]}{\frac{2\,\text{kB}^3\,T^3\,\text{Zeta}[3]}{c^3\,h^3\,\pi^2}}$$

$$\frac{\frac{1}{(2\pi)^3}\,\text{Integrate}\left[\text{dndf}\gamma[f, T], \{f, 30.0\,\text{kB}\,T/h, \text{Infinity}\}, \text{Assumptions} \to \{h > 0, \text{kB} > 0, T > 0\}\right]}{\frac{2\,\text{kB}^3\,T^3\,\text{Zeta}[3]}{c^3\,h^3\,\pi^2}}$$

Out[18]= 0.00230393

Out[19]= $3.78946 \times 10^{-7}$

Out[20]= $3.74085 \times 10^{-11}$

In[22]:=
```
gγ = 2;
ϵγ[T_] := π^2/30 gγ kB^4/(hbar^3 c^3) T^4
nγ[T_] := Zeta[3]/π^2 gγ kB^3/(hbar^3 c^3) T^3
```

In[25]:=
```
Integrate[nγ[T], {}]
integra
```

Integrate: Invalid integration variable or limit(s) in {}.

Out[25]= $\text{Integrate}\left[\frac{2\,\text{kB}^3\,T^3\,\text{Zeta}[3]}{c^3\,\text{hbar}^3\,\pi^2}, \{\}\right]$

In[26]:=
```
Eγ0 = 6.242 * 10^18 ϵγ[T]/nγ[T] /. constants /. T → 2.725(*in eV*)
EB0 = 939 × 10^6(*in eV*);
EB0 / Eγ0
```

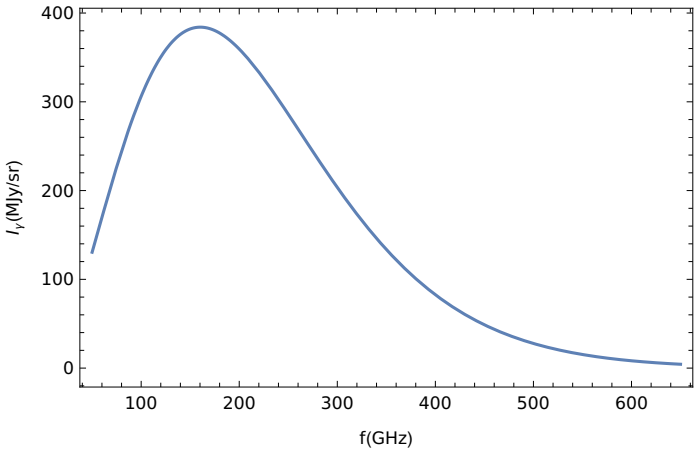
Out[26]= 0.000634347

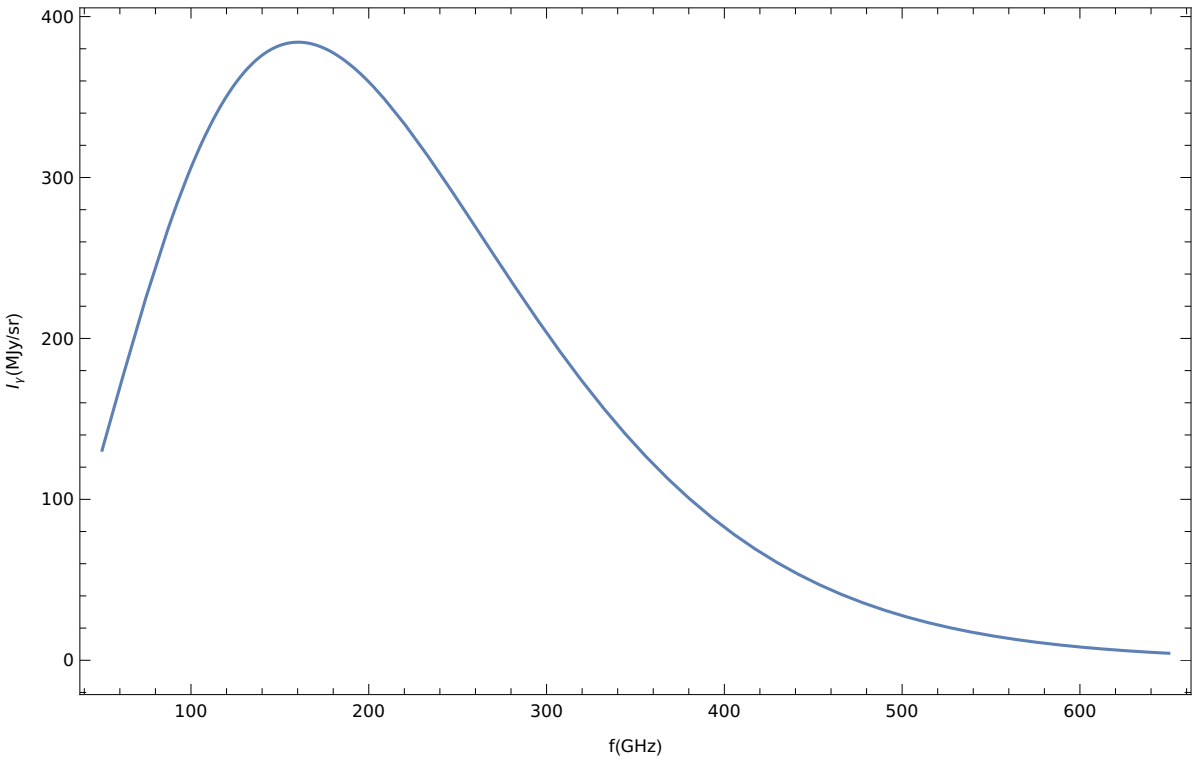
Out[28]= $1.48026 \times 10^{12}$

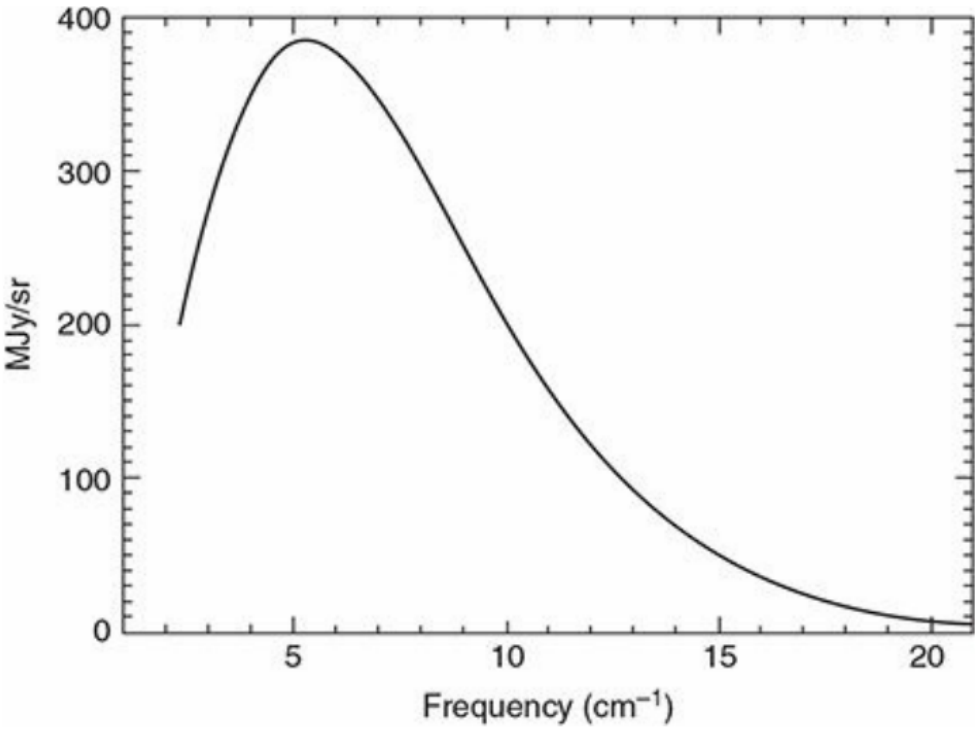
In[29]:=
```
(*Intensity: what is measured by experiments*)

Iγ[f_, T_] := 2 h / c^2 f^3 / (Exp[h f / (kB T)] - 1) (*j s s^2/m^2/sr/s^3 = j/m^2/sr =

    Watts/sr = j/s m^2/sr/Hz = W/m2/sr/Hz  = 10^26 My = 10^26 10^-23 erg/s/cm^2/sr/Hz*)
```

In[30]:=
```
Plot[10^-6 × 10^26 Iγ[10^9 f, T] /. constants /. T → 2.725,
 representación gráfica
 {f, 50 * 10^0, 650 * 10^0}, Frame → True, FrameLabel → {"f(GHz)", "I_γ(MJy/sr)"}]
                              marco  ver···  etiqueta de marco
```

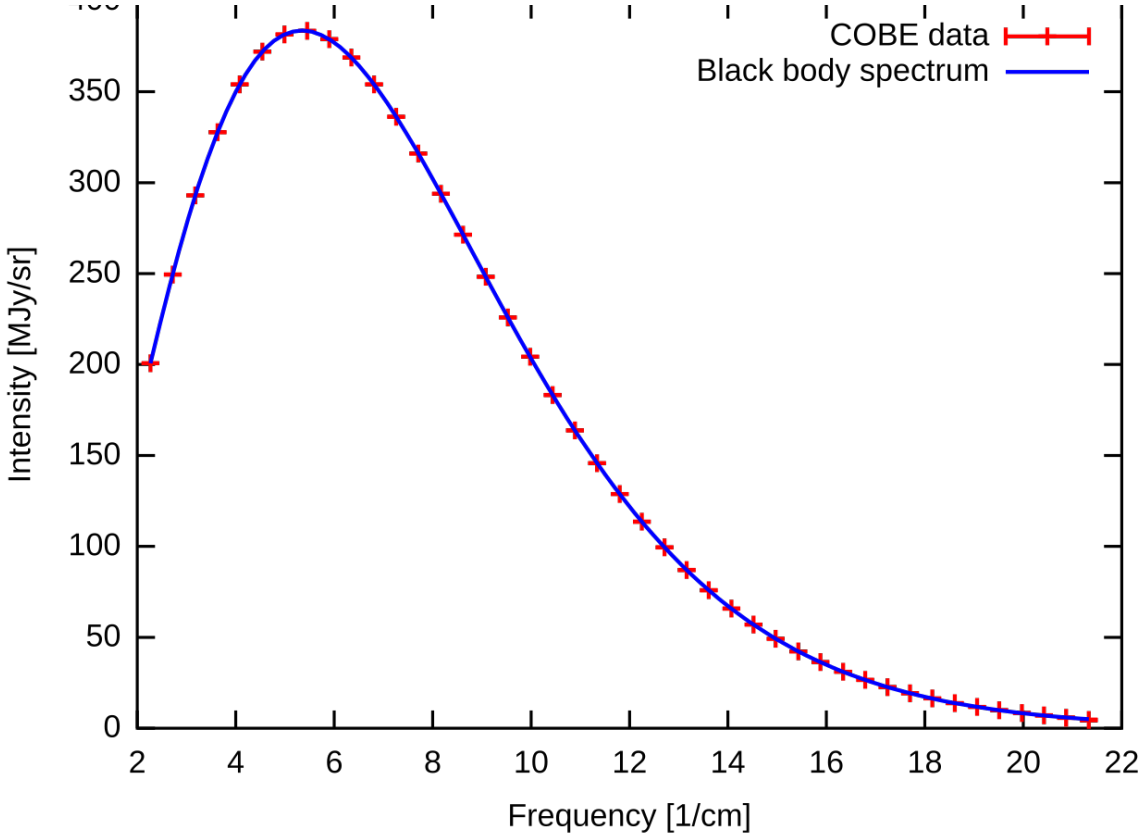
Out[30]=

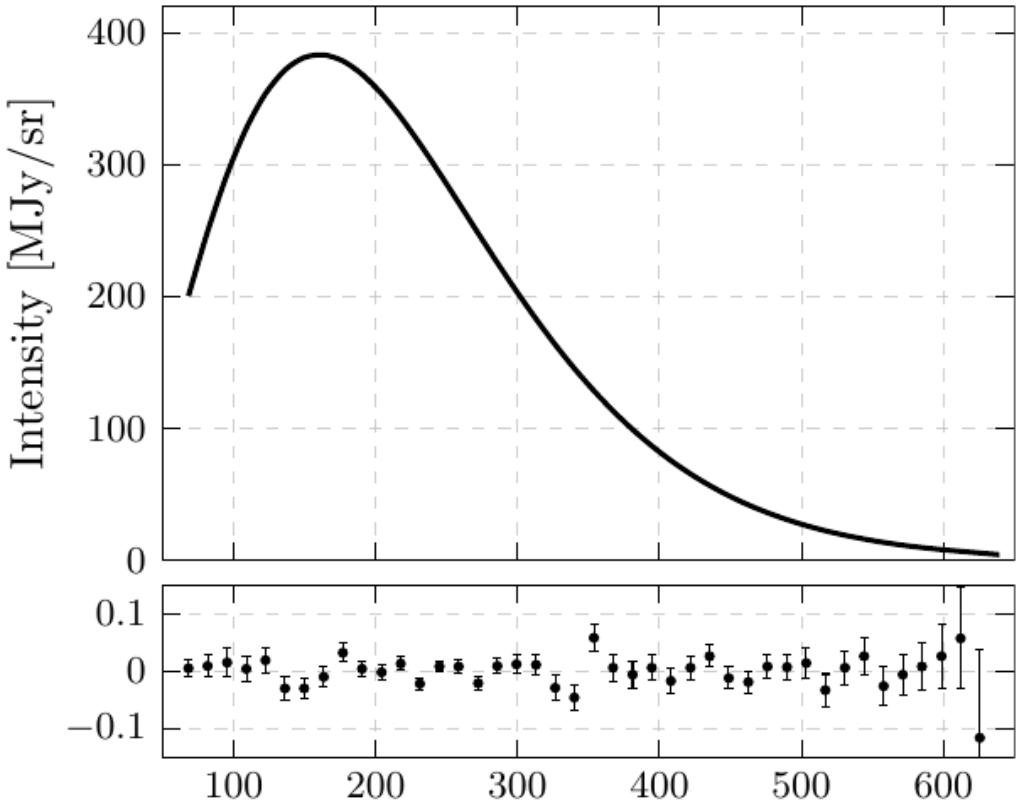





Cosmic microwave background spectrum (from COBE)

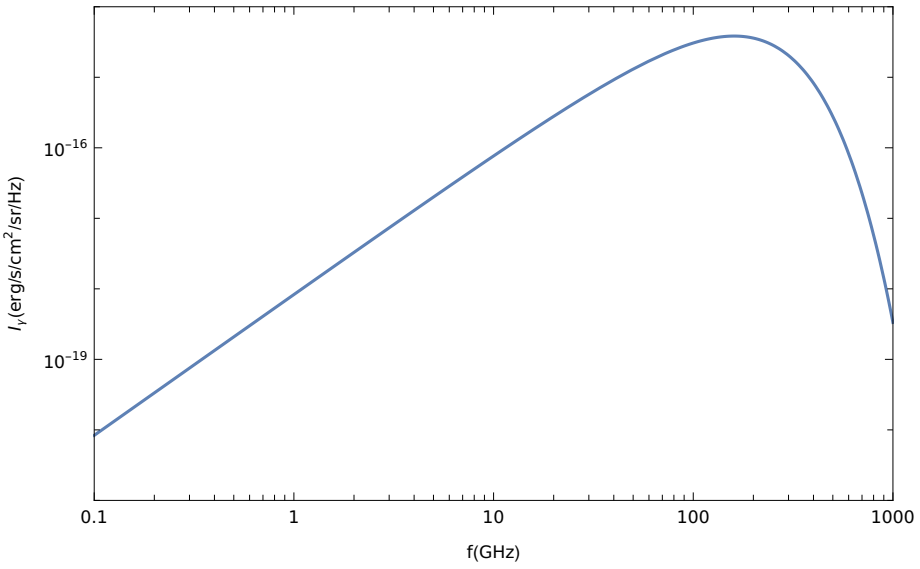

COBE data
Black body spectrum
Intensity [MJy/sr]
Frequency [1/cm]
0
50
100
150
200
250
300
350
2
4
6
8
10
12
14
16
18
20
22


Intensity [MJy/sr]
400
300
200
100
0
0.1
0
−0.1
100
200
300
400
500
600

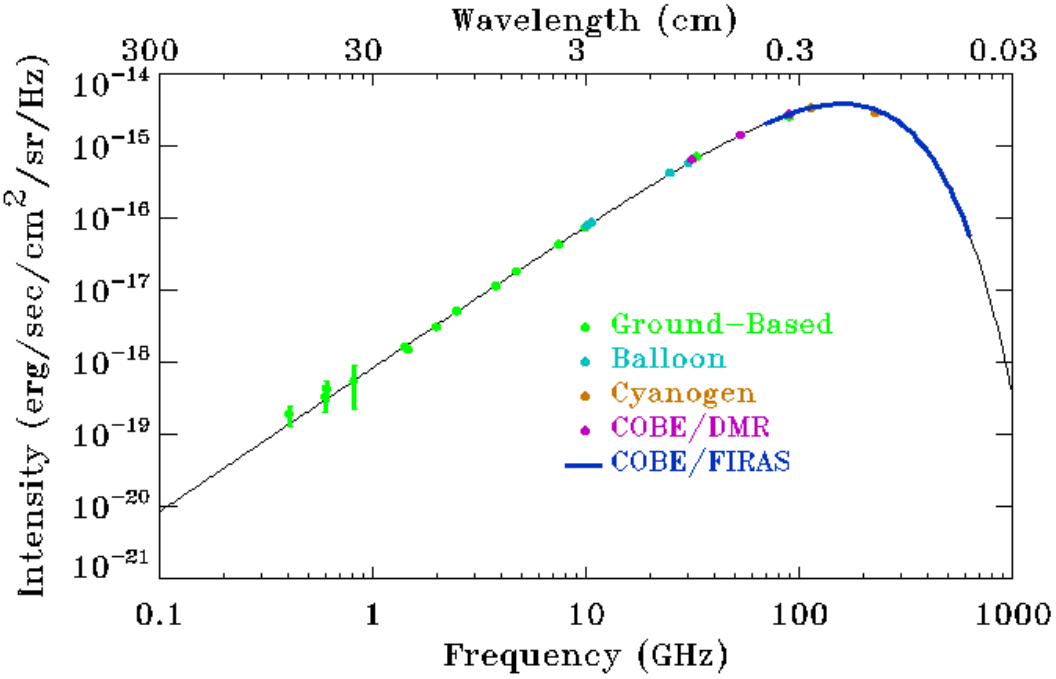
Frequency [GHz]

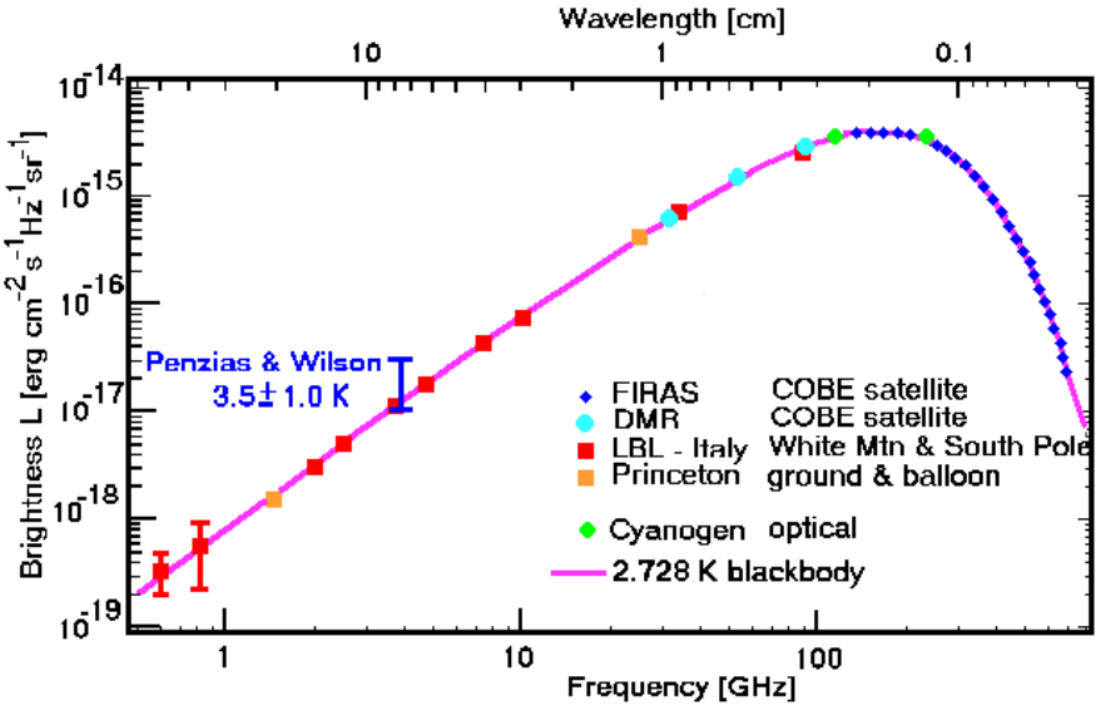
In[31]:=
```
LogLogPlot[10^3 Iγ[10^9 f, T] /. constants /. T → 2.725, {f, 0.1 * 10^0, 1000 * 10^0},
 representación log log
  Frame → True, PlotRange → {{0.1 * 10^0, 1000 * 10^0}, {10^-21, 10^-14}},
  marco  ver···  rango de representación
  FrameLabel → {"f(GHz)", "I_γ(erg/s/cm^2/sr/Hz)"}]
  etiqueta de marco
```

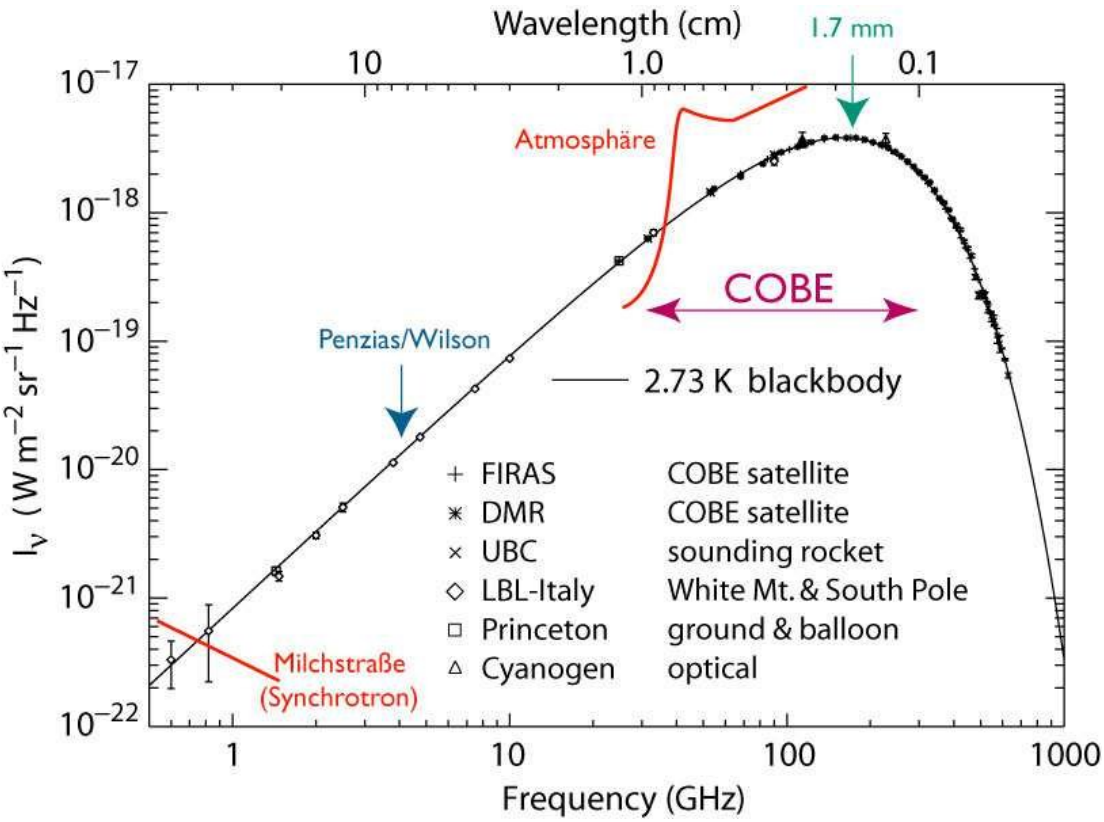
Out[31]=




```
(*1Jy= 10^-23 erg/s/cm^2/Hz*)
```


Wavelength (cm)
300
30
3
0.3
0.03
Intensity (erg/sec/cm^2/sr/Hz)
10^-14
10^-15
10^-16
10^-17
10^-18
10^-19
10^-20
10^-21
0.1
1
10
100
1000
Frequency (GHz)
Ground−Based
Balloon
Cyanogen
COBE/DMR
COBE/FIRAS


Wavelength [cm]
10
1
0.1
Brightness L [erg cm^-2 s^-1 Hz^-1 sr^-1]
10^-14
10^-15
10^-16
10^-17
10^-18
10^-19
Penzias & Wilson
3.5± 1.0 K
FIRAS COBE satellite
DMR COBE satellite
LBL - Italy White Mtn & South Pole
Princeton ground & balloon
Cyanogen optical
2.728 K blackbody
1
10
100
Frequency [GHz]




In[•]:=
```
0.25 / 19 000
```

Out[•]=
```
0.0000131579
```

In[32]:=
```
H0 = 70 km / (10^6 × 3.0857 * 10^13 km) (*1/s*)
ΩR0 = 9.0 * 10^-5
σe = 6.65 * 10^-29 (*cm^2*)
nB0 = 0.25
vc = 3 * 10^8 (*m/s*)
```

Out[32]=
```
2.26853 × 10^-18
```

Out[33]=
```
0.00009
```

Out[34]=
```
6.65 × 10^-29
```

Out[35]=
```
0.25
```

Out[36]=
```
300 000 000
```

In[38]:=
$$HR[a\_] := H0 \frac{(\Omega R0)^{1/2}}{a^2}$$
$$\Gamma e[a\_] := \frac{nB0}{a^3} \sigma e\, vc$$

In[40]:=
$$\frac{\Gamma e[a]}{HR[a]}$$

Out[40]=
$$\frac{0.231749}{a}$$

In[41]:=
$$\Gamma e\left[10^{-5}\right]$$
$$HR\left[10^{-5}\right]$$

Out[41]= $4.9875 \times 10^{-6}$

Out[42]= $2.15212 \times 10^{-10}$

In[46]:=
$$\Omega M0 = 0.31;$$
$$\Omega \Lambda 0 = 0.69;$$

In[49]:=
$$Hcdm[a\_] := H0 \left(\frac{\Omega R0}{a^4} + \frac{\Omega M0}{a^3} + \Omega \Lambda 0\right)^{1/2}$$

In[72]:=
$$across = Solve[Hcdm[a] == \Gamma e[a], a, Reals]$$
resuelve número
$$zcross = \frac{1}{a} - 1 /. across[[1, 1]]$$
$$\frac{2.725}{a} /. across[[1, 1]]$$

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

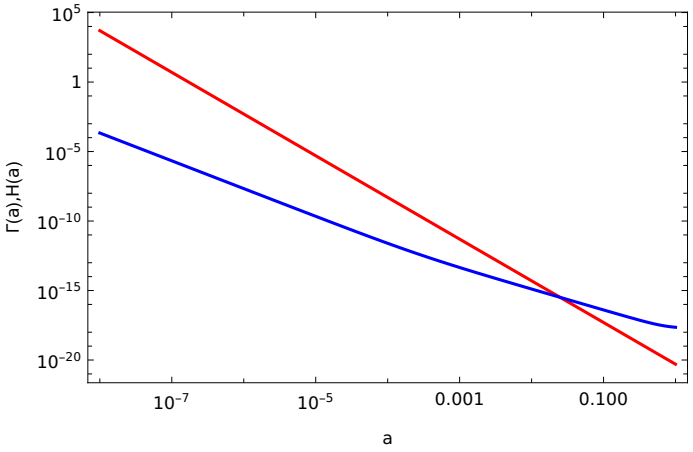
Out[72]= {{a → 0.024886}}

Out[73]= 39.1833

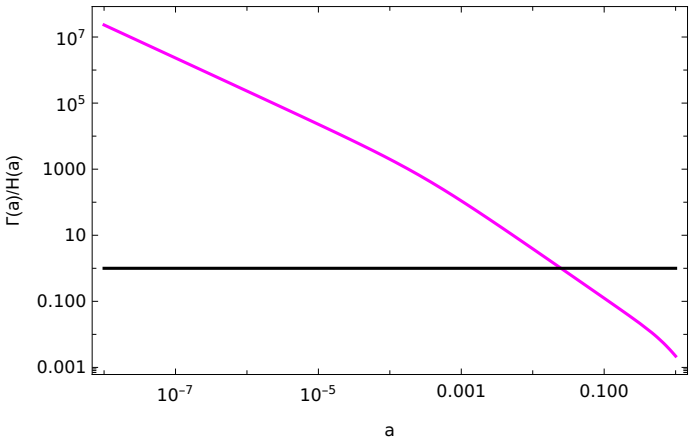
Out[74]= 109.499

In[70]:=
```
LogLogPlot[{Γe[a], Hcdm[a]}, {a, 10^-8, 1}, Frame → True,
 PlotStyle → {Red, Blue}, FrameLabel → {"a", "Γ(a),H(a)"}]
LogLogPlot[{Γe[a] / Hcdm[a], 1}, {a, 10^-8, 1}, Frame → True,
 PlotStyle → {Magenta, Black}, FrameLabel → {"a", "Γ(a)/H(a)"}]
```

Out[70]=




Out[71]=



In[109]:=
```
me = 0.511 * 10^6;
Δ = 13.6;
η_B = Ω_B * h^2 * 2.68 * 10^-8;
η_B /. h → 0.7 /. Ω_B → 5
ηη_B = 0.75 * η_B /. h → 0.7 /. Ω_B → 0.5
(*η_B=4.6*10^-10*)
eq1 = Xp + 2 * N[Zeta[3]] / Pi^2 ηη_B (2 Pi T / me)^(3/2) Xp^2 Exp[Δ / T]
```
función zeta · exponenci

Out[112]=
$$6.566 \times 10^{-8}$$

Out[113]=
$$4.9245 \times 10^{-9}$$

Out[114]=
$$Xp + 5.17197 \times 10^{-17}\, e^{13.6/T}\, T^{3/2}\, Xp^2$$

In[115]:=
```
NSolve[eq1 == 1. /. Xp → 0.1 /. T → (1 + z) * 2.75 * 8.617 * 10^-5, z]
```
resuelve numéricamente
```
NSolve[eq1 == 1. /. Xp → 0.5 /. T → (1 + z) * 2.75 * 8.617 * 10^-5, z]
```
resuelve numéricamente

NSolve: Inverse functions are being used by NSolve, so some solutions may not be found; use Reduce for complete solution information.

Out[115]=
$$\{\{z \to 1310.72\}, \{z \to 6.10523 \times 10^{15}\}\}$$

NSolve: Inverse functions are being used by NSolve, so some solutions may not be found; use Reduce for complete solution information.

Out[116]=
$$\{\{z \to 1440.84\}, \{z \to 4.8257 \times 10^{14}\}\}$$

In[117]:=
```
list1 = {};
For[i = 1000, i < 2000,
```
para cada
```
  zz = 1.0 i;
  tt = (1 + zz) * 2.75 * 8.617 * 10^-5;
  eq2 = Last[NSolve[eq1 == 1. /. T → tt, Xp]];
```
últ··· resuelve numéricamente
```
  XXp = Xp /. eq2;
  list1 = AppendTo[list1, {zz, XXp}];
```
añade al final
```
  i++]
list13 = list1;
```

In[123]:=
```
Pic1 = ListLogLinearPlot[{list11, list12, list13},
  representación log lineal de lista
 Frame → True, FrameLabel → {"z", "Xe^eq"}](*ΩB0=0.005,0.05,0.5*)
  marco  ver···  etiqueta de marco
```

Out[123]=