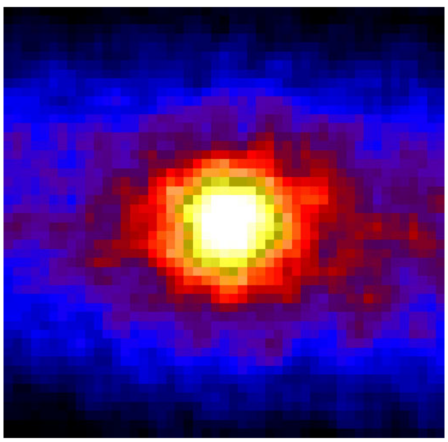
Neutrinos in Cosmology

We have seen that neutrinos could provide an important contribution to the total energy density of the Universe if they have a mass in the 10 eV range.

$$\Omega_\nu h^2 \approx \frac{\sum m_{\nu,i}}{94\,\mathrm{eV}}\,.$$

Because neutrinos only interact weakly with matter, they are very important in astrophysics. Where other particles become trapped or can only propagate through very slow diffusive processes, neutrinos are able to escape. Neutrinos can thus connect regions of matter that would otherwise be isolated from each other. Because they are massless (or almost massless), they move at the speed of light, which makes the energy transfer very efficient.

For example, neutrinos produced near the centre of the Sun can be detected at the Earth after a time of flight of around 8 minutes, and permit the study of the nuclear reactions that take place in the core of 'our star'.
The photons generated by the energy-producing nuclear reactions at the centre, however, diffuse slowly to the surface with an average diffusion time of around a million years!



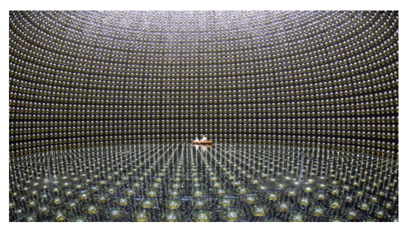


Image of the Sun taken through the Earth, in "neutrino light", at the Super-Kamiokande detector (Japan). The image has been obtained with a 503 days exposure, by registering neutrinos emitted from the solar core and detected in a 50 000-ton water pool located 1 km underground. At night, neutrinos were transparently traversing the whole earth before being registered in this image.

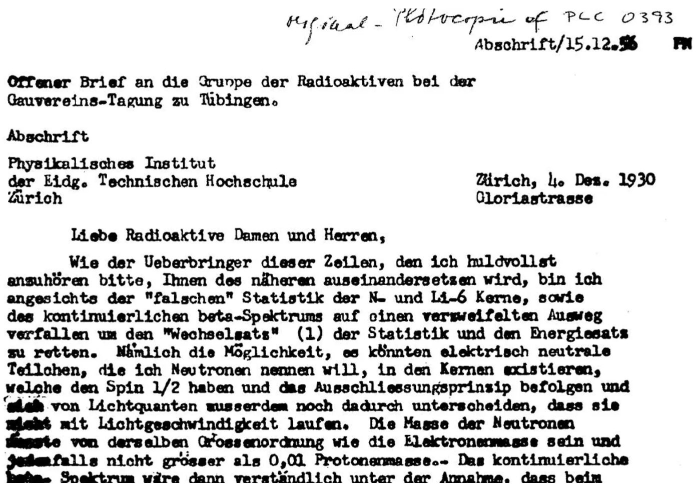
"Estimados y radiactivos damas y caballeros. He encontrado una medida desesperada para salvar la ley de conservación de la energía suponiendo que en el núcleo existen partículas sin carga eléctrica a los que llamaré neutrones. Las observaciones de la desintegración beta tienen sentido si además del electrón, un neutrón es emitido de tal manera que la suma de sus energías es constante...."

"Por ahora no me atrevo a publicar los detalles de esta idea, les confío a ustedes mi querida gente radiactiva la pregunta de cuán probable sería encontrar evidencia experimental de tal neutrón".

"Debo admitir que mi solución puede parecer casi imposible ya que si existiera ya deberíamos haber visto estos neutrones. Pero si no nos arriesgamos no avanzaremos.
Querida gente radiactiva, examinen y juzguen».

He supposedly stated: “I have done a terrible thing, I have postulated a particle that cannot be detected”.

Soon the Italian Enrico Fermi (Nobel Prize laureate in 1938) was able to demonstrate an elegant theory that included Pauli’s lightweight, neutral particle. It was called the neutrino.



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Abschrift/15.12.56 FN

Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung zu Tübingen.

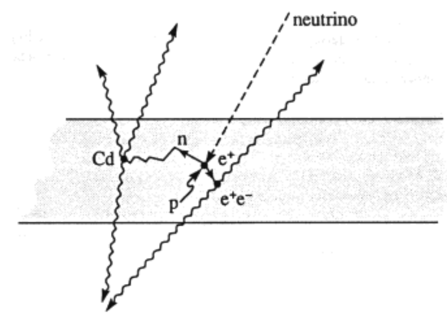
Abschrift

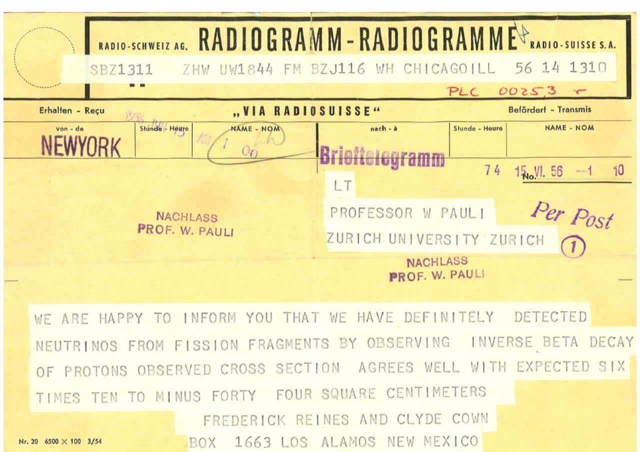
Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim





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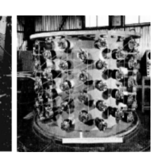
PROFESSOR W PAULI
ZURICH UNIVERSITY ZURICH
Per Post
NACHLASS PROF. W. PAULI
WE ARE HAPPY TO INFORM YOU THAT WE HAVE DEFINITELY DETECTED NEUTRINOS FROM FISSION FRAGMENTS BY OBSERVING INVERSE BETA DECAY OF PROTONS OBSERVED CROSS SECTION AGREES WELL WITH EXPECTED SIX TIMES TEN TO MINUS FORTY FOUR SQUARE CENTIMETERS
FREDERICK REINES AND CLYDE COWN
BOX 1663 LOS ALAMOS NEW MEXICO

Nos alegra informale que hemos definitivamente detectado neutrinos de fragmentos de fisión nuclear al observar la desintegración beta inversa de protones. [...]

Frederick Reines y Clyde Cowan
Los Alamos, New Mexico
14 Junio 1956



Al día siguiente Pauli respondió a Reines y Cowan:

Gracias por el mensaje. Todo llega a quien sabe esperar.

Pauli.

The History of Neutrinos

Neutrinos were postulated in 1930 by Wolfgang Pauli to explain a supposed anomaly in the energy spectrum of β-decay. While at the time only two emerging particles from the nuclear decay could be detected:

$$^A_Z X \rightarrow \ ^A_{Z+1}Y \ + \ e^-$$

the energy of the produced electron was not monochromatic, as it should be if the original nucleus decays at rest into only two bodies. To save the principle of energy conservation, Pauli suggested that a third particle was always produced in β-decay, but that it was not detectable. The third particle had to be neutral (or else the conservation of charge would have been violated!) and have very small mass, since the total energy of the detected particles accounted for almost all the mass of the parent nuclei. For these reasons, Fermi called the 'invisible' particle the *neutrino*.[1]

The proper description of β^--decay is thus

$$^A_Z X \rightarrow \ ^A_{Z+1}Y \ + \ e^- + \bar{\nu}_e \tag{14.1}$$

If energetically allowed, a proton within a nucleus can transform into a neutron under the emission of a positron:

$$^A_Z X \rightarrow \ ^A_{Z-1}Y \ + \ e^+ + \nu_e \tag{14.2}$$

The $\bar{\nu}_e$ in (14.1) is the antineutrino. The production of an antiparticle in β^- decay is necessary for the conservation of lepton number, as discussed in Section 6.3. The reverse happens in β^+ decay, where a positron (anti-electron) is emitted, thus a neutrino is needed to reset the lepton number to what it was prior to the decay.

The existence of neutrinos was finally demonstrated by the observation of the reaction:

$$p \ + \ \bar{\nu}_e \rightarrow n \ + \ e^+ \tag{14.3}$$

The original experiment was performed by Clyde Cowan and Fred Reines in 1955. For this fundamental discovery Reines was awarded the 1995 Nobel prize in physics.[2] The experiment was performed in an underground laboratory, 11 metres below one of the nuclear reactors at Savannah River. The signal of an antineutrino capture was the simultaneous detection of the positron, as it annihilated with an electron in the target,[3] and a recoiling neutron.

[1] Pauli was quite uneasy about predicting a particle whose existence nobody, including himself, thought could ever be proven. Only some 25 years later, when the first powerful nuclear reactors were put into operation, was the time ripe for its experimental discovery.

[2] The prize was shared between Reines (Cowan had died many years earlier) and Martin Perl for his discovery of the τ lepton.

[3] The result of the annihilation is two 511 keV photons in opposite directions.

Muon neutrinos were first observed in 1962 in an experiment led by Leon Lederman, Melvin Schwartz and Jack Steinberger. They received the Nobel prize in physics in 1988.

Neutrino Interactions with Matter

Neutrino interactions with matter are divided into two kinds. *Neutral current* (NC) interactions are mediated by the neutral Z bosons. *Charge current* (CC) interactions, on the other hand, involve the exchange of W^+ and W^- bosons.

NC interactions are responsible for annihilation reactions involving neutrinos,

$$e^+ + e^- \rightarrow \nu_\mu + \bar{\nu}_\mu$$

for example, and interactions such as

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$

where neutrinos scatter with matter and thereby gain or lose energy from the collision partner without any additional matter being created or destroyed. Such collisions are called *elastic scatterings*. Both types of interaction are shown in Fig. 14.1.

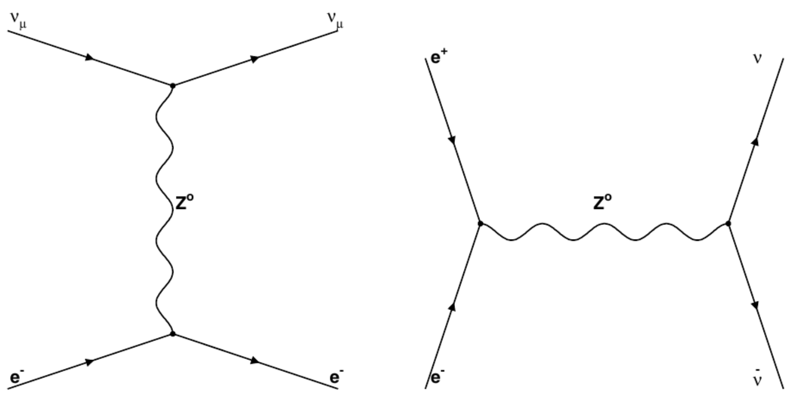


Fig. 14.1. Feynman graphs for NC interactions: elastic $e-\nu$ scattering and electron positron annihilation into neutrinos.

In CC interactions there is an exchange of lepton partners. For example, an antineutrino can be absorbed by a proton, producing a neutron and a positron in the final state as shown in Fig. 14.2.

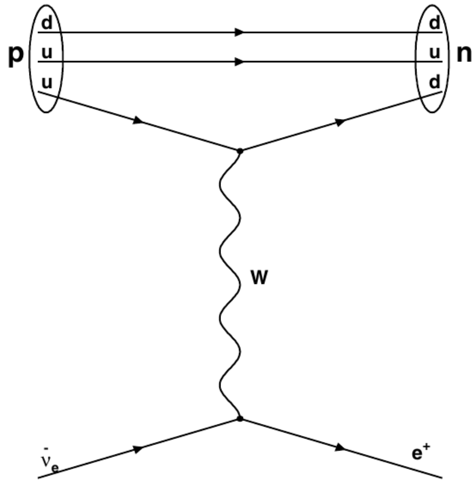


Fig. 14.2. Feynman diagram for antineutrino absorption. One of the quarks inside the proton changes flavour thereby transforming the nucleon into a neutron.

The approximate expression for high-energy neutrino interactions with matter can thus be written

$$\sigma \sim \frac{\alpha^2 s}{m_W^4} \sim G_F^2 E_{cm}^2$$

where E_{cm} is the centre of momentum energy of the incoming neutrino.

$$\begin{aligned}\sigma_{weak} &= [2 \cdot 10^{-11} \times 1.1664 \cdot 10^{-11} \times E_{cm}]^2 \\ &\sim 5 \cdot 10^{-44} \left(\frac{E_{cm}}{1\ \mathrm{MeV}}\right)^2 \ \mathrm{cm}^2\end{aligned}$$

Thus, we find that in the *laboratory* frame: $\sigma(\nu e) \approx 2G_F^2 m_e E_\nu$

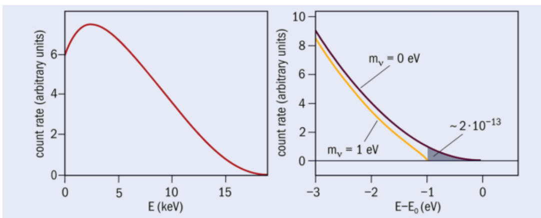
In other words: *the cross-section for neutrino interactions rises linearly with neutrino energy.*

Neutrino Masses

Laboratory experiments have, so far, not succeeded in measuring the mass of any neutrino. Instead, the negative results have been expressed in the form of upper limits, due to the finite resolution of the experiments. The best (lowest) upper limits on the mass of the electron neutrino come from the studies of the electron energy spectrum in tritium decay:

$$^{3}\mathrm{H} \rightarrow ^{3}\mathrm{He} + e^{-} + \bar{\nu}_e$$

As the minimum amount of energy taken by the ν_e is its mass, the end-point energy of the emitted electron is a measurement of m_{ν_e}. According to these experiments the mass of the electron neutrino is lower than 3 eV at the 95 per cent confidence level.



The beta spectrum of tritium (left), showing in detail the effect of different neutrino masses on the endpoint (right). Credit: CERN

Stellar Neutrinos

Neutrinos are very efficient in the process of the cooling of stars, in spite of their low probability of interaction with matter. Next we compute the mean free path of neutrinos in stellar environments.

As seen from (14.10), neutrinos in the MeV range have a cross-section of $\sigma_\nu \approx 10^{-44}$ cm^2 for matter interactions. The mean free path for neutrinos is related to the density of matter and the cross-section for interaction as (see (12.14)):

$$\lambda_\nu = \frac{1}{n\sigma_\nu} \tag{14.15}$$

where n is the number of nucleons per cubic centimetre.

For a hydrogen star with mass density ρ, the number density is given by

$$n = \frac{\rho}{m_H} = \left(\frac{\rho}{1 \text{ g cm}^3}\right) \cdot 6 \cdot 10^{23} \text{ cm}^{-3} \tag{14.16}$$

Inserting (14.16) in (14.15) we find

$$\lambda_\nu \approx \frac{2 \cdot 10^{20}}{\rho/(1 \text{ g cm}^3)} \text{ cm} \tag{14.17}$$

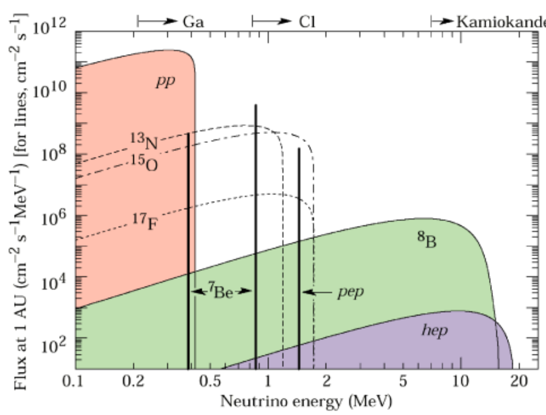
For normal stellar matter, $\rho \approx 1$ g/cm^3, the mean free path becomes 100 pc, and even at $\rho \approx 10^6$ g/cm^3 λ_ν is still 3000 solar radii! In other words, neutrinos with MeV-scale energies escape from the stellar interior without scattering even for extremely high densities. This is what makes neutrinos so special in astrophysical environments.

Solar Neutrinos

The low cross-sections of weak interactions allow stars to burn their fuel slowly instead of exploding soon after formation. Our Sun, for example, is believed to be 4.5 billion years old, and is predicted to continue in its present luminous state for at least as long. The main source of energy in hydrogen-burning stars (as the Sun) is through the pp-fusion reaction:

$$4\text{p} + 2e^- \rightarrow {}^4\text{He} + 2\nu_e + 26.731 \text{ MeV}$$

where, on average, only about 2 per cent (0.6 MeV) of the energy is carried by the neutrinos according to the ‘standard solar model’, SSM. The details of the nuclear reaction – what is called the *pp* or *proton proton* chain – and the spectrum of the produced neutrinos are shown in Table 14.1 and Fig. 14.4.[6]



Neutrino Absorption Experiments

The field of observational neutrino astronomy started with the pioneering experiment of Ray Davis and collaborators in the early 1960s. A tank filled with 615 tons of a cleaning fluid, C_2Cl_4, was installed at the Homestake Mine in South Dakota, USA, about 1.5 km below ground. The nuclear reaction that makes neutrino detection possible is:

$$\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^-$$

The high threshold for the reaction, 0.814 MeV, permits the observation of only a small fraction of the solar spectrum, as shown in Fig. 14.4. From the accumulation of argon in the tank, the flux of electron neutrinos can be calculated. The argon is chemically extracted, and *single atoms* are counted through their subsequent decay.[7]

The experimental and theoretical rates for absorption of solar neutrinos *per target atom* are expressed in *Solar Neutrino Units*, SNU. For convenience:

$$1 \text{ SNU} = 10^{-36} \text{ s}^{-1}$$

While the predicted rate for neutrino capture at the chlorine experiment is 7.9 ± 2.6 SNU, the observed rate averaged over more than 20 years is only 2.6 ± 0.2 SNU, as shown in Fig. 14.5. This long-standing disagreement has been called the 'solar neutrino problem'. Several other experimental techniques have been pursued to verify the nature of the disagreement.

The most efficient solar neutrino absorption experiments to date have been performed with the gallium detectors, SAGE and GALLEX. The nuclear reaction that takes place in these detectors is:

$$\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^-$$

The energy threshold for the reaction is only 0.233 MeV – significantly lower than for the ^{37}Cl experiment, as shown in Fig. 14.4. The gallium experiments are therefore sensitive to a much larger fraction of the total spectrum of solar neutrinos. In particular, they probe the main reaction in the chain: $p + p \rightarrow^2 \text{H} + e^+ + \nu_e$, which produces neutrinos with $E \leq 0.420$ MeV.

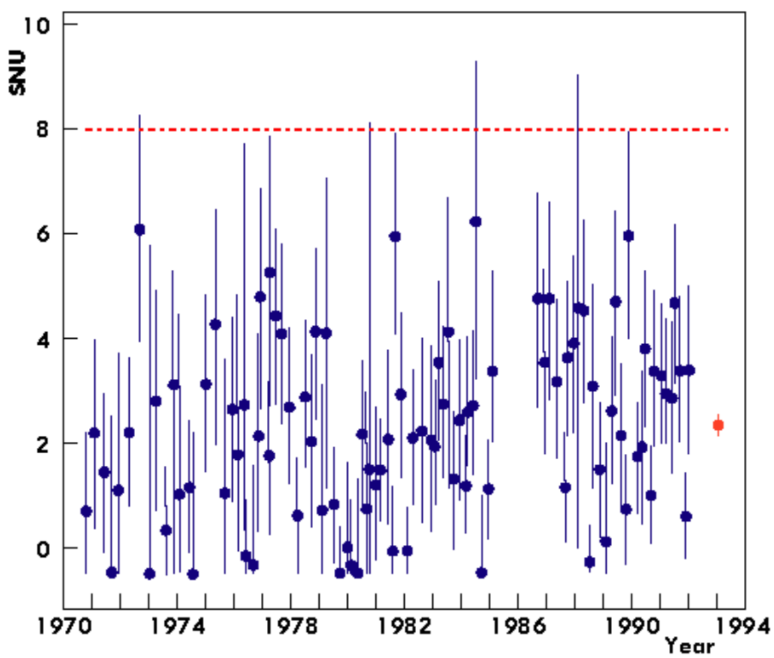


Fig. 14.5. Observational results from the Homestake neutrino experiment as a function of time. The line at 7.9 SNU shows the prediction of the 'standard solar model'.

Neutrino Scattering Experiments

In the Japanese mine at Kamioka a large water tank was fitted with thousands of photomultipliers (PMs). This enabled the detection of solar neutrinos through the elastic scattering process:

$$\nu + e^- \rightarrow \nu + e^-$$

The recoiling electrons emit Cherenkov light, detectable with the PMs, and can thus be counted. (In Section 14.10 the Cherenkov process will be explained in detail.) The method is very attractive because of its relative simplicity (water is cheap) and because the scattered electrons, on average, follow the direction of the incoming neutrinos. That allows the experimenters to verify that the signal really comes from the Sun, as demonstrated in Fig. 14.6. One disadvantage is that there is an experimental threshold for the detectability of the recoiling electron corresponding to $E_\nu > 7 - 9$ MeV.[8] Therefore, only the highest-energy neutrinos emitted from the Sun can be measured, thereby reducing the detectable flux by about 10^4. Another difficulty is the indistinguishable detector signal from gamma-rays generated by radioactive impurities in the water or induced by cosmic rays, implying a high rate of background events, as shown in Fig. 14.6.

On the other hand, one clearly superior aspect of the scattering experiments is that they are sensitive to *all* neutrino species, although with different cross-sections (see equation (14.10)).

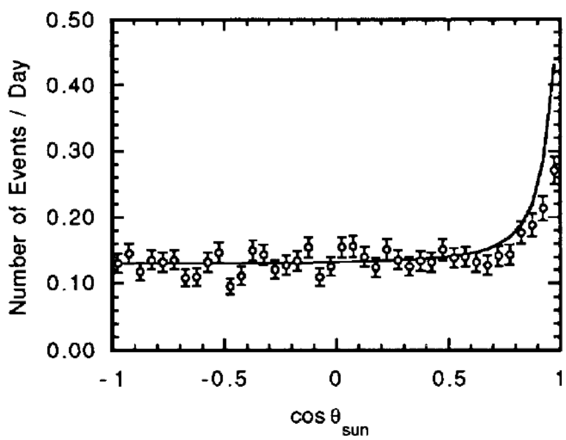


Figure 2: Angular distribution of events with respect to the Sun, Kamiokande [29].

Neutrino Oscillations

If neutrinos are massless they are by definition stable: that is, they cannot decay into any other *lighter* particle. There is, however, no compelling reason why they cannot have finite masses. For ν masses less than an MeV or so, the radiative decay of a heavier ν_a to a lighter ν_b through $\nu_a \rightarrow \nu_b + \gamma$ is kinematically possible, but the estimated lifetime in the Standard Model is much longer than the age of the Universe.

In such a scenario, mixing of neutrino species may occur if the *weak-interaction eigenstates*, ν_e, ν_μ and ν_τ, are not *mass eigenstates*: that is, the states in which neutrinos propagate in vacuum.

In general, any flavour or weak-interaction neutrino eigenstate, ν_f, can be expressed as a linear superposition of orthogonal mass eigenstates, ν_m:

$$|\nu_f> = \sum_m c_{fm} |\nu_m>$$

For example, let us consider the situation where there are two neutrino mass eigenstates associated with two flavour eigenstates.

The unitary transformation matrix connecting the mass eigenstates with the flavour eigenstates can be described with one parameter, the mixing angle θ.

$$\begin{pmatrix} \nu_\mu \\ \nu_e \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \tag{14.27}$$

Although the states $|\nu_e>$ and $|\nu_\mu>$ (and their antiparticles) are produced in weak interactions, such as in the decay $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$, the *physical* states: that is, the eigenstates of the Hamiltonian with definite masses, are ν_1 and ν_2.

Therefore, the time evolution of a muon neutrino wave function with momentum p is

$$|\nu_e(t)> = -\sin\theta e^{-iE_1 t}|\nu_1> + \cos\theta e^{-iE_2 t}|\nu_2> \tag{14.28}$$

where E_1 and E_2 are the energies of the two mass eigenstates. Two energy levels arise if ν_1 and ν_2 have different masses, as they must have the same momentum, p. Then, for very small neutrino masses: that is, $m_i \ll E_i$,

$$E_i = p + \frac{m_i^2}{2p} \tag{14.29}$$

The probability $P(\nu_e \rightarrow \nu_e) = |<\nu_e|\nu_e>|^2$, that an electron neutrino *remains* a ν_e after travelling a time t is (Problem 14.7):

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2(2\theta)\sin^2[\frac{1}{2}(E_2 - E_1)t] \tag{14.30}$$

For very small neutrino masses, inserting (14.29) we get

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2(2\theta)\sin^2\left[\left(\frac{m_2^2 - m_1^2}{4E}\right)t\right] \tag{14.31}$$

where E is the energy of ν_e.

It thus follows that the probability that the electron neutrino becomes a muon neutrino at a time t is

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2(2\theta)\sin^2\left[\frac{\Delta m^2}{4E}t\right] \tag{14.32}$$

where $\Delta m^2 = |m_2^2 - m_1^2|$.

From (14.32) and Fig. 14.8 it is seen that the probability function for flavour change *oscillates*, with an amplitude given by $\sin^2(2\theta)$ and oscillation frequency $\sim \Delta m^2/E$. Therefore, for suitable neutrino masses and mixing angles, the presumed deficit of solar electron neutrinos can be explained by the oscillation phenomenon.

To summarize, the amplitude and oscillation length of the flavour oscillation are (reinserting factors of $\hbar$ and c):

$$\begin{cases} A &= \sin^2(2\theta) \\ L_\nu &= \frac{4\pi E\hbar}{\Delta m^2 c^3} \end{cases} \tag{14.33}$$

Numerically, the oscillation length becomes

$$L_\nu = 1.27 \left(\frac{E}{1\,\mathrm{MeV}}\right)\left(\frac{1\ \mathrm{eV}^2}{\Delta m^2}\right) \ \text{metres.} \tag{14.34}$$

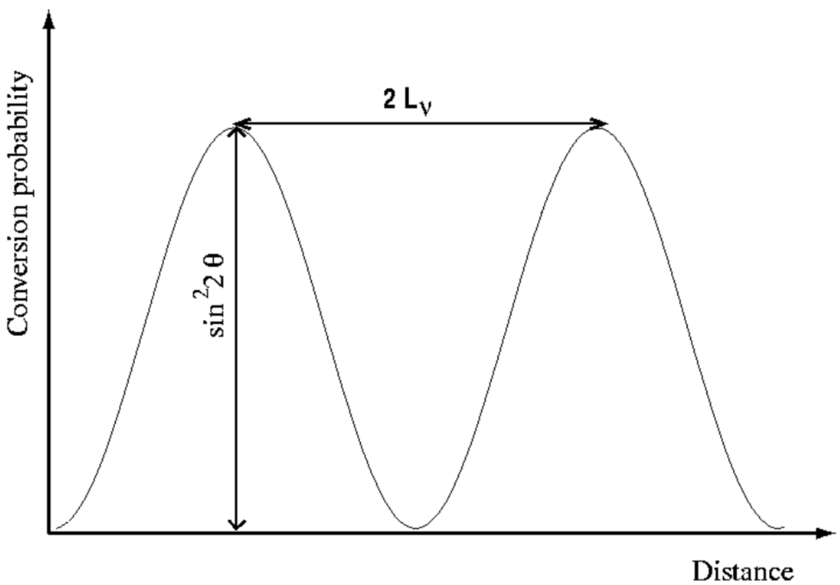


Fig. 14.8. Probability distribution for $\nu_e \to \nu_\mu$.