Early Universe: Introduction

Disclaimer

Discussions taken from Barbara Ryden [1], Daniel Baumann [2], Kolb & Turner [3], Rubakov & Gorbunov [4] books.

1 Units used in Cosmology

- Astronomical unit (AU), equal to the mean distance between the Earth and Sun:

$$1\,\text{AU} = 1.5 \times 10^{11}\,\text{m}. \tag{1}$$

  - Average distance from the Sun to Neptune: 30.1 A.U.
  - Distance from the Sun to Voyager 1 in October 2024: 165 A.U.
  - Distance light travels in one day: 173 A.U.
  - Distance light travels in one Julian year (365.25 days): 63241 A.U.
  - Distance of the outer limit of Oort cloud from the Sun: 75000 A.U.
  - Distance to Proxima Centauri (Sun's nearest neighboring star): 268000 A.U.

- Parsec (pc), equal to the distance at which 1 AU subtends an angle of 1 arcsecond:

$$1\,\text{pc} = 3.09 \times 10^{16}\,\text{m}. \tag{2}$$

  - Distance to Proxima Centauri: 1.3 pc. The distance to Arcturus and Capella is more than 10 pc, the distances to Canopus and Betelgeuse are about 100 pc and 200 pc, respectively; Crab Nebula - the remnant of supernova seen by naked eye - is 2 kpc away from us.
  - Distance to the center of our galaxy: 8.5 kpc. Our Galaxy is of spiral type, the diameter of its disc is about 30 kpc and the thickness of the disc is about 250 pc.
  - The distance to the nearest dwarf galaxies, satellites of our Galaxy, is about 30 kpc. Fourteen of these satellites are known; the largest of them — Large and Small Magellanic Clouds — are 50 kpc away. Search for new, dimmer satellite dwarfs is underway; we note in this regard that only eight of Milky Way satellites were known by 1994.
  - The nearest "usual" galaxy –the spiral galaxy M31 in Andromeda constellation — is 760 kpc away from the Milky Way. Despite the large distance, it occupies a sizeable area on the celestial sphere: its angular size is larger than that of the Moon! Another nearby galaxy is in the Triangulum constellation. Our Galaxy together with the Andromeda and Triangulum galaxies, their satellites and other 35 smaller galaxies constitute the Local Group, the gravitationally bound object consisting of more than 50 galaxies.
  - Distance to the Virgo cluster (the nearest big cluster of galaxies): 16.5 Mpc. Its angular size is about 5 degrees. Clusters of galaxies are the largest gravitationally bound systems in the Universe.

- About 20 superclusters are known by now. The Local Group belongs to a supercluster with the center in the direction of Virgo constellation. The size of this supercluster is about 30 Mpc, and besides the Virgo cluster and Local Group it contains about a hundred groups and clusters of galaxies. The nearest to Virgo is the supercluster in the Hydra and Centaurus constellations; its distance to the Virgo supercluster is about half a hundred megaparsec. Distance to the Coma cluster: 99 Mpc.
- Distance to QSO J0313-1806 (the most distant, and hence also the oldest known quasar at $z = 7.64$): 900 Mpc.
- Radius of observable Universe: $\sim$14000 Mpc.

• Solar Mass ($\mathrm{M}_\odot$):

$$1\,\mathrm{M}_\odot = 1.99 \times 10^{30}\,\mathrm{kg}. \tag{3}$$

- The most massive confirmed exoplanet is Iota Draconis b, which has 16.4 times the mass of Jupiter, that is $16.4 \times 0.00095\,\mathrm{M}_\odot \sim 0.01558\,\mathrm{M}_\odot$.
- The largest black hole merger detected by LIGO to date was GW190521, which created a black hole with a mass of 142 solar masses.
- R136a1 is one of the most massive and luminous stars known: $\sim 200\,\mathrm{M}_\odot$.
- The Milky Way's central black hole, Sagittarius A*, has a mass of $4.3 \times 10^6\,\mathrm{M}_\odot$.
- The M87* supermassive black hole has a mass of $6.5 \times 10^9\,\mathrm{M}_\odot$.
- Mass of the Milky Way galaxy: $M_{\mathrm{MW}} \sim 10^{12}\,\mathrm{M}_\odot$.
- Mass of the Coma cluster: $M_{\mathrm{Coma}} \sim 7 \times 10^{14}\,\mathrm{M}_\odot$.
- The Laniakea Supercluster encompasses approximately 100,000 galaxies stretched out over 160 Mpc. It has the approximate mass of $10^{17}\,\mathrm{M}_\odot$.

• Sun's luminosity ($\mathrm{L}_\odot$):

$$1\,\mathrm{L}_\odot = 3.83 \times 10^{26}\,\mathrm{watts}. \tag{4}$$

- The peak luminosity of SN 1987A, the famous supernova observed in the Large Magellanic Cloud in 1987, was approximately $10^9\,\mathrm{L}_\odot$.
- Approximately luminosity of the Milky Way galaxy: $L_{\mathrm{MW}} \sim 3 \times 10^{10}\,\mathrm{L}_\odot$.
- The luminosity of QSO J0313–1806 is approximately $3.6 \times 10^{13}\,\mathrm{L}_\odot$.

• Age of the Universe (t_U):

$$t_U = 13.8\,\mathrm{Gyr}. \tag{5}$$

- Matter-dark energy equality: 5.1 Gy ago, that is, $t_{M\Lambda} = 8.8 \times 10^9$ y.
- Time of formation of CMB (recombination): 380 ky.
- Radiation-Matter equality: $t_{RM} = 50 \times 10^3$ y.
- Time of formation of light nuclei (Big Bang Nucleosynthesis): 1 s.
- Time of electroweak symmetry breaking: 10^{-12} s.

2 Elementary particles

Elementary particles are classified according their spin s: bosons have integer s while fermions have half-integer s.

2.1 Bosons

The bosons with spin $s = 1$ are dubbed as vector particles and are associated with the mediators of the fundamental interactions: photon with electromagnetism, gluon with strong force and $Z, W^{\pm}$ with weak force. The Higgs particle is associated with the field responsible for giving mass to all known particles. The properties of the known bosonic particles are displayed in Table 1.

Boson	Spin	Charge	Mass (GeV/c^2)
Photon	1	0	0
Gluon	1	0	0
Z^0	1	0	91.2
$W^{\pm}$	1	± 1	80.4
Higgs	0	0	125

Table 1: Bosonic elementary particles.

2.2 Fermions

Fermions are classified accordingly to leptons and quarks, and both are presented in families or generation for a total of 3 of those. Leptons do not undergo the strong interaction while quarks do (see Tables 2 and 3).

Lepton	Spin	Charge (e)	Mass (GeV/c^2)
$e^{\pm}$	1/2	± 1	0.000511
ν_e	1/2	0	$\lesssim 10^{-15}$
$\mu^{\pm}$	1/2	± 1	0.105
ν_μ	1/2	0	$\sim 10^{-12}$
$\tau^{\pm}$	1/2	± 1	1.777
ν_τ	1	0	$\sim 10^{-11}$

Table 2: Leptons.

Quark	Spin	Charge (e)	Mass (GeV/c^2)
up	1/2	2/3	0.0023
down	1/2	-1/3	0.0048
charm	1/2	2/3	1.275
strange	1/2	-1/3	0.095
top	1/2	2/3	173
bottom	1/2	-1/3	4.18

Table 3: Quarks.

2.2.1 Composite baryons

Particles made up from quarks are called baryons, and are classified as mesons or hadrons if they are formed by a pair quark-antiquark or a ternary of quarks (anti-quarks), respectively.

Meson	Spin	Charge (e)	Mass (GeV/c^2)
$\pi^0\,(u\bar{u}-d\bar{d})$	0	0	0.135
$\pi^+\,(u\bar{d})$	0	1	0.140
$\pi^-\,(d\bar{u})$	0	-1	0.140
Hadron	Spin	Charge (e)	Mass (GeV/c^2)
$p\,(uud)$	1/2	1	0.938
$n\,(udd)$	1/2	0	0.939
$\Lambda^0\,(uds)$	1/2	0	1.116

Table 4: Baryons.

3 Planck Units

The Planck system is based on the combination of four universal constants:

$$G_N = 6.67\times 10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kg\ sec}^2}, \tag{6}$$

$$c = 299792458\,\frac{\mathrm{m}}{\mathrm{sec}}, \tag{7}$$

$$\hbar = 1.055\times 10^{-34}\,\mathrm{J\,s} = 6.582\times 10^{-25}\,\mathrm{GeV\,s}, \tag{8}$$

$$k_B = 1.38\times 10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}} = 8.617\times 10^{-14}\,\frac{\mathrm{GeV}}{\mathrm{K}}, \tag{9}$$

with $1\,\mathrm{eV} = 1.602\times 10^{-19}$ J. The Planck length, mass and time are defined as

$$\ell_P \equiv \left(\frac{G_N\hbar}{c^3}\right)^{1/2} = 1.62\times 10^{-35}\,\mathrm{m}, \tag{10}$$

$$M_P \equiv \left(\frac{\hbar c}{G_N}\right)^{1/2} = 2.18\times 10^{-8}\,\mathrm{kg}, \tag{11}$$

$$t_P \equiv \left(\frac{G_N\hbar}{c^5}\right)^{1/2} = 5.39\times 10^{-44}\ \mathrm{sec}\,. \tag{12}$$

The Planck energy is defined as

$$E_P \equiv M_P c^2 = 1.96\times 10^9\,\mathrm{J} = 1.22\times 10^{19}\,\mathrm{GeV}, \tag{13}$$

while the Planck temperature becomes

$$T_P \equiv E_P/k_B = 1.42\times 10^{32}\,\mathrm{K}. \tag{14}$$

4 Natural units

When distance, mass, time, and temperature are measured in the appropriate Planck units, then $c = k_B = \hbar = G_N = 1$. For instance, the Einstein's equation for energy, $E = mc^2$ becomes $E = m$.

From $\hbar = 1$ it follows that

$$1\,\mathrm{GeV} = \frac{1}{6.6 \times 10^{-25}}\,\mathrm{s}^{-1}. \tag{15}$$

From $c = 1$ we obtain

$$1\,\mathrm{s} = 3 \times 10^{8}\,\mathrm{m}. \tag{16}$$

From this two expressions

$$10^{-15}\,\mathrm{m} \approx 5\,\mathrm{GeV}^{-1}. \tag{17}$$

Hence

$$1\,\mathrm{mb} \equiv 10^{-31}\,\mathrm{m}^2 \approx 2.56\,\mathrm{GeV}^{-2}, \tag{18}$$

or

$$1\,\mathrm{GeV}^2 \approx 0.39 \times 10^{-31}\,\mathrm{m}^{-2}. \tag{19}$$

The mass of the electron in natural units can be expressed as

$$m_e \approx 0.5\,\mathrm{GeV} = 0.25 \times 10^{13}\,\mathrm{m}^{-1} = 0.77 \times 10^{21}\,\mathrm{s}^{-1}. \tag{20}$$

5 Review questions

1. Fill in the Table 5 with the year of discovery or first detection and lifetime.

2. Which of these particles were firstly predicted and then detected? Which were detected without being expected?

3. Plot the quark and charged lepton masses in some way which suggest regularities, i.e, family number versus mass. If there us a fourth family, what masses might you predict for the members?. What could you infer about the neutrino masses in comparison with the rest of masses?

4. Plot the masses of the unstable particles as function of their lifetime. Is there any pattern? Could you get some physical insight about the relation between type of interactions and lifetime?

6 Problems

1. Convert the following quantities by inserting the appropriate factors of $c, \hbar$, G_N and k_B:

   a) $T_0 = 2.725$ K $\rightarrow$ eV.

Particle	Date	lifetime (s)	Particle	Date	lifetime (s)
γ			u		
g			d		
$W^\pm$			c		
Z^0			s		
H			t		
e^-			b		
ν_e			p		
μ^-			n		
ν_μ			Λ^0		
τ^-			π^0		
ν_τ			$\pi^\pm$		

Table 5: Year of first detection and lifetime of some relevant particles.

b) $\rho_\gamma = \pi^2 T_0^4/15 \to \mathrm{eV}^4$ and g/cm^3.

c) $M_P \equiv 1.2 \times 10^{19}$ GeV $\to$ K.

2. The range of length scales involved in cosmology are hard to grasp. The best we can do is to consider relative distances and compare them to something more familiar. In this exercise, we will make some attempt at obtaining a more intuitive understanding of the vastness of the cosmos.

   a) Consider shrinking the Earth to the size of a basketball. What would then be the size of the Moon and its orbit around the Earth?

   b) Now imagine scaling the Earth down to the size of a peppercorn. What would then be the size of the Sun and the Earth's orbit? How far away would the most distant planet in the Solar System be?

   c) The "Solar Neighborhood" is a collection of about fifty nearby stars, spread across about 65 light-years, that travel together with the Sun. Scaling this region down, so that it fits inside a basketball court, what would be the size of the Solar System?.

   d) Shrinking our Galaxy to the size of the basketball court, what would now be the size of the Solar Neighbourhood?

   e) The "Local Group" comprises about fifty nearby galaxies, spread across about 10 million light-years. If we squeezed this region into the size of the basketball court, what would be the size of our Milky Way galaxy?

   f) The largest structures in the universe, like our "Local Supercluster," are about 500 million light-years across. Scaling these superclusters down to the dimensions of the basketball court, what would be the size of our Local Group?

   g) This radius of the observable universe is 46.5 billion light-years. Compressing the observable universe to the size of the basketball court, what would be the size of the largest superclusters?

3. A key parameter in cosmology is the Hubble constant

$$H_0 = 70 \text{ km/s/Mpc}.$$

In the following, you will use the measured value of the Hubble constant to estimate a few fundamental scales of our universe.

a) What is the Hubble time $t_{H_0} \equiv H_0^{-1}$ in years? This is a rough estimate of the age of the universe.

b) What is the Hubble distance $d_{H_0} \equiv cH_0^{-1}$ in meters? This is a rough estimate of the size of the observable universe.

c) The average density of the universe today is

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G_N},$$

where G_N is the Nweton's constant. What is $\rho_{c,0}$ in g/cm^{-3}? How does this compare to the density of water.

d) Let us assume that the universe is filled with only hydrogen atoms. What is then the total number of atoms in the universe? How does this compare to the number of hydrogen atoms in your brain?
Hint: Assume that the brain is mostly water. Use $m_{\mathrm{H}} = 2 \times 10^{-24}$ g and $m_{\mathrm{H_2O}} = 3 \times 10^{-23}$ g.

e) The maximal energy scale probed by the Large Hadron Collider (LHC) is $E_{\mathrm{max}} \sim 1$ TeV. What length scale ℓ_{min} does this correspond to? How does this compare to the size of the universe d_{H_0}?

4. The typical mass of a galaxy is given by $10^{12}\,\mathrm{M}_{\odot}$. Assuming that the matter in form of nucleons (that, is in protons and neutrons) contribute 1/5 to the mass of a galaxy, estimate the typical number of nucleons in a galaxy.

5. When a charged π^- meson with very low velocity reacts with a proton, a neutron and a neutral π^0 meson are produced, $\pi^- + p \to \pi^0 + n$. Suppose that the proton, neutron and π^- masses are known: $m_p = 938.3$ MeV, $m_n = 939.6$ MeV, $m_{\pi^-} = 139.6$ MeV. Determine the mass of the π^0, if the neutron kinetic energy is measured to be $T_n \equiv E_n - m_n = 0.4$ MeV.

6. Show that a free electron and a free positron, both with mass m_e, cannot annihilate into a single freely propagating photon.

7. The cosmic microwave background radiation (CMBR) consists of photons of typical energy 3×10^{-4} eV. How high an energy must a cosmic gamma photon γ_c have if pair production on this background $\gamma_c + \gamma_{\mathrm{CMB}} \to e^+ + e^-$ is to be kinematically allowed?

8. Suppose that the Milky Way galaxy is a typical size, containing say 10^{11} stars, and that galaxies are typically separated by a distance of one Mpc. Estimate the density of the Universe in SI units. How does this compare with the density of the Earth?

References

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[4] V. A. Rubakov and D. S. Gorbunov, *Introduction to the Theory of the Early Universe: Hot big bang theory.* World Scientific, Singapore, 2017.