Early Universe
Homework #1

1. Solve the following exercises from the book "Introduction to cosmology", 2nd edition, by Barbara Ryden: 2.5, 3.4, 4.2, 5.1, 5.4, 5.5, 5.7, 5.10, 6.1, 6.3, 6.9.

2. In the real Universe the expansion is not completely uniform. Rather, galaxies exhibit some random motion relative to the overall Hubble expansion, known as their peculiar velocity and caused by the gravitational pull of their near neighbours. Supposing that a typical (e.g. root mean square) galaxy peculiar velocity is 600 km/s, how far away would a galaxy have to be before it could be used to determine the Hubble constant to ten per cent accuracy, supposing $H_0 = 70$ km/s/Mpc. Assume in your calculation that the galaxy distance and redshift could be measured exactly. Unfortunately, that is not true of real observations.

3. The curvature of the two-dimensional surface is characterized by what is known as the Gaussian curvature, which may be defined as the limit

$$K = \frac{3}{\pi} \lim_{\mathfrak{s} \to 0} \left(\frac{2\pi\, \mathfrak{s} - C(\mathfrak{s})}{\mathfrak{s}^3} \right) ,$$

   where $\mathfrak{s}$ is the measured radius of a circle on the surface and $C(\mathfrak{s})$ the length of the circumference. Suppose a circle is drawn around the North Pole on a sphere with radius a.

   a) Show that the curvature of the sphere is equal to $K = 1/a^2$.

   b) We may introduce coordinates (r', ϕ) on the sphere with the property that the circumference of a circle around the North Pole has the value exactly equal to $2\pi r'$. With the auxiliary angle $\theta = \mathfrak{s}/a$, we obtain that $r' = a \sin\theta$, so $\mathfrak{s} = a \arcsin(r'/a)$. Show that the line element can be expressed as

$$ds^2 = \frac{1}{1 - K\, r'^2} dr'^2 + r'^2 d\phi^2 \, .$$

4. Consider a FLRW Universe dominated by a perfect fluid with pressure $p = \omega\epsilon$, with $\omega = const$. Taking into account that the time-dependent density parameter is defined as $\Omega(t) = \epsilon(t)/\epsilon_c(t)$, where the time-dependent critical density is $\epsilon_c(t) = 3H^2/(8\pi G_N)$. Show that

$$\frac{d\Omega}{d \ln a} = (1 + 3w)\Omega(\Omega - 1).$$

   Discuss the evolution of $\Omega(a)$.