The Cosmic Microwave Background

8.1 Observing the CMB

Dicke, Peebles, Roll, and Wilkinson in a companion letter in this issue." The companion paper by Dicke and his collaborators points out that the radiation could be a relic of an early, hot, dense, and opaque state of the universe.

Measuring the spectrum of the CMB, and confirming that it is indeed a blackbody, is not a simple task, even with modern technology. The mean energy per CMB photon (6.34×10^{-4} eV) is tiny compared to the energy required to break up an atomic nucleus (~ 2 MeV) or even the energy required to ionize an atom (~ 10 eV). However, the mean photon energy is comparable to the rotational energy of a small molecule such as H_2O. Thus, CMB photons can zip along for more than 13 billion years through tenuous intergalactic gas, then be absorbed a microsecond away from the Earth's surface by a water molecule in the atmosphere. Microwaves with wavelengths shorter than λ ~ 3 cm are strongly absorbed by water molecules. Penzias and Wilson observed the CMB at a wavelength λ = 7.35 cm, corresponding to a photon energy $E = 1.7\times10^{-5}$ eV, because that was the wavelength of the signals that Bell Labs had been bouncing off orbiting satellites. Thus, Penzias and Wilson were detecting CMB photons far on the low-energy tail of the blackbody spectrum (Figure 2.7), with an energy just 0.027 times the mean photon energy.

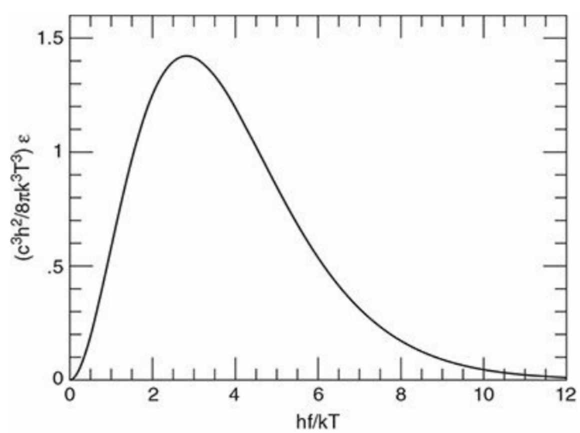
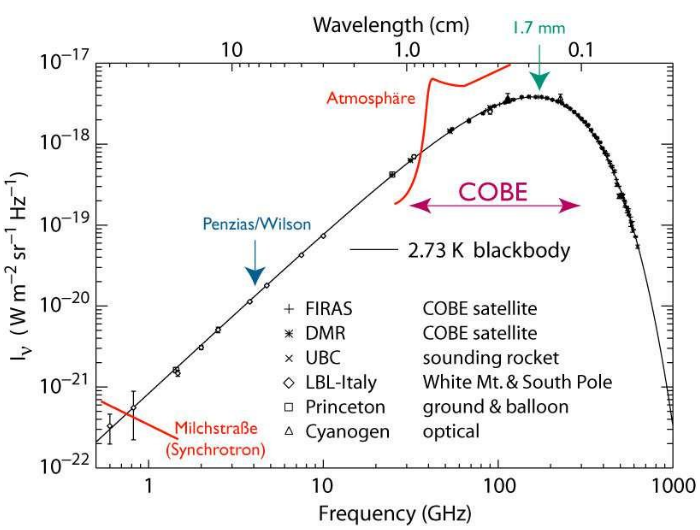
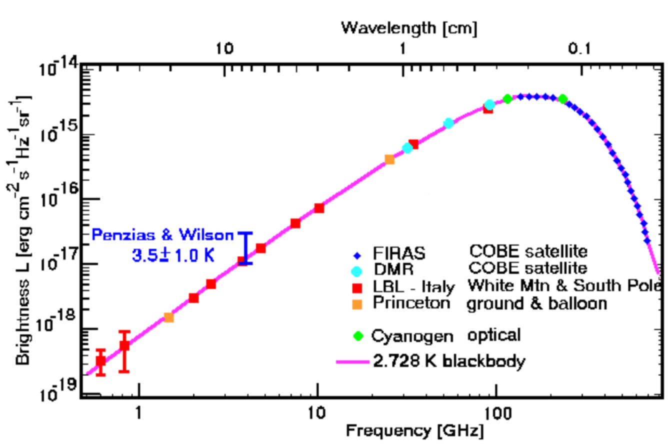
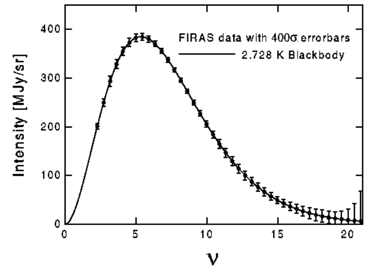


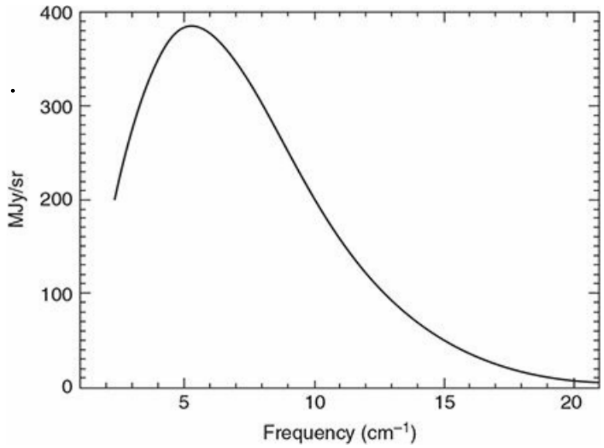
Figure 2.7 The energy density of blackbody radiation, expressed as a function of frequency *f*.





Result number one: At any angular position (θ, ϕ) on the sky, the spectrum of the cosmic microwave background is very close to that of an ideal blackbody, as illustrated in Figure 8.1. How close is very close? *COBE* could have detected fluctuations in the spectrum as small as one part in 10^4. No deviations were found at this level within the wavelength range investigated by *COBE*.





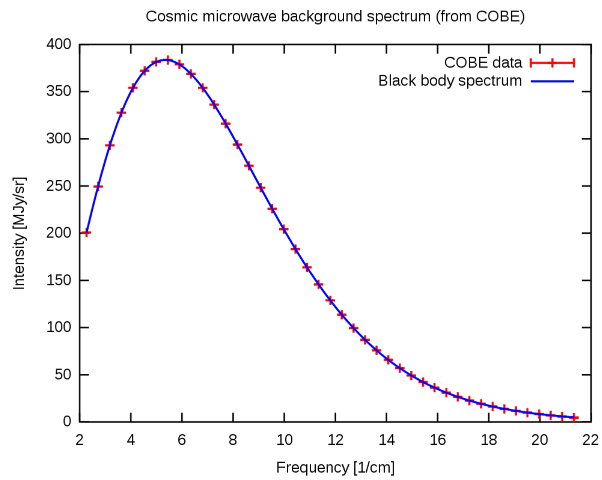
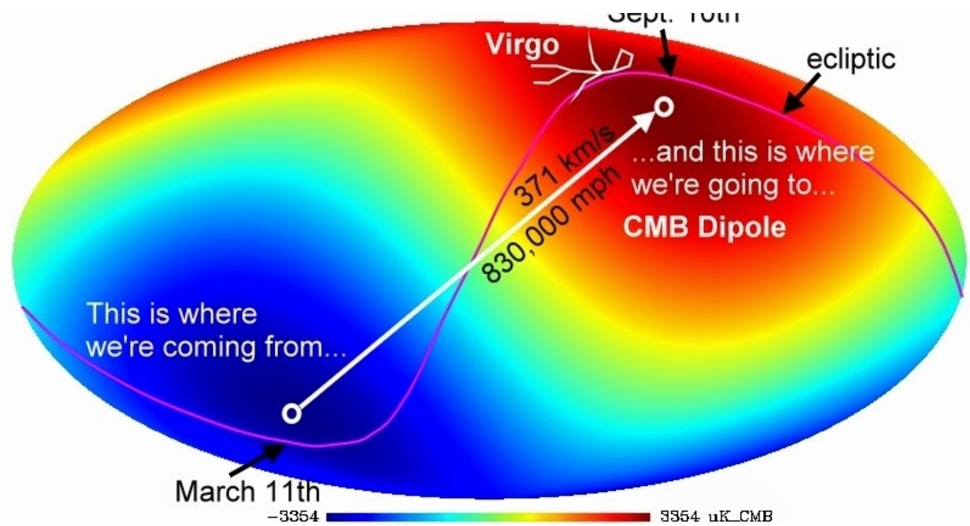


Figure 8.1 The spectrum of the cosmic microwave background, as measured by *COBE*. The uncertainties in the measurement are smaller than the thickness of the line. [Fixsen *et al.* 1996 *ApJ*, **473**, 576]

Result number two: The CMB has the dipole distortion in temperature shown in Figure 8.2. That is, although each point on the sky has a blackbody spectrum, in one half of the sky the spectrum is slightly blueshifted to higher temperatures, and in the other half the spectrum is slightly redshifted to lower temperatures. [4] This dipole distortion is a simple Doppler shift, caused by the net motion of *WMAP* relative to a frame of reference in which the CMB is isotropic. After correcting for the orbital motion of *WMAP* around the Sun ($v \sim 30\ \mathrm{km\ s^{-1}}$), for the orbital motion of the Sun around the galactic center ($v \sim 235\ \mathrm{km\ s^{-1}}$), and for the orbital motion of our galaxy relative to the center of mass of the Local Group ($v \sim 80\ \mathrm{km\ s^{-1}}$), it is found that the Local Group is moving in the general direction of the constellation Hydra, with a speed $v_{\mathrm{LG}} = 630 \pm 20\ \mathrm{km\ s^{-1}} = 0.0021c$. This peculiar velocity for the Local Group is what you'd expect as the result of gravitational acceleration by the largest lumps of matter in the vicinity of the Local Group. The Local Group is being accelerated toward the Virgo cluster, the nearest big cluster to us. In addition, the Virgo cluster is being accelerated toward the Hydra-Centaurus supercluster, the nearest supercluster to us. The combination of these two accelerations, working over the age of the universe, has launched the Local Group in the direction of Hydra, at 0.2% the speed of light.



Result number three: After the dipole distortion of the CMB is subtracted away, the remaining temperature fluctuations, shown in Figure 8.3, are small in amplitude. Let the temperature of the CMB, at a given point on the sky, be $T(\theta, \phi)$. The mean temperature, averaging over all locations, is

$$\langle T \rangle = \frac{1}{4\pi} \int T(\theta, \phi) \sin\theta \, d\theta \, d\phi = 2.7255\,\mathrm{K}.$$

The dimensionless temperature fluctuation at a given point (θ, ϕ) on the sky is

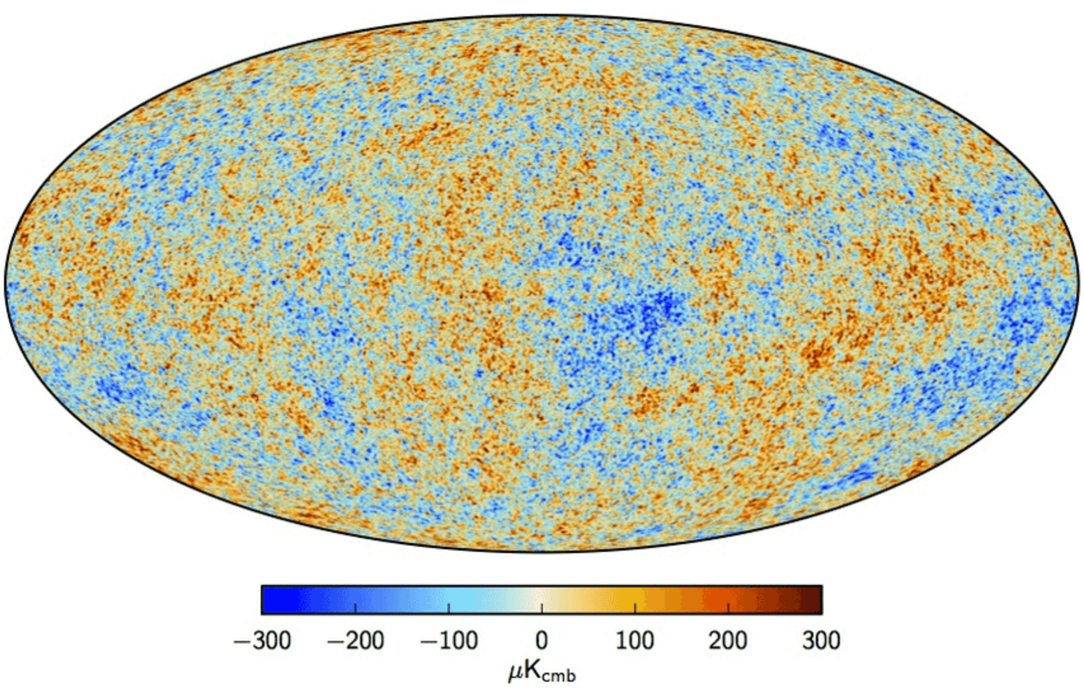
$$\frac{\delta T}{T}(\theta, \phi) \equiv \frac{T(\theta, \phi) - \langle T \rangle}{\langle T \rangle}. \tag{8.7}$$

After subtraction of the Doppler dipole, the root mean square temperature fluctuation found by *COBE* was

$$\left\langle \left(\frac{\delta T}{T} \right)^2 \right\rangle^{1/2} = 1.1 \times 10^{-5}. \tag{8.8}$$

(Given the limited angular resolution of the *COBE* satellite, this excludes the temperature fluctuations on an angular scale < 10°.) Even taking into account the blurring from *COBE*'s low resolution, the fact that the CMB temperature varies by only 30 microKelvin across the sky represents a remarkably close approach to isotropy.

The observations that the CMB has a nearly perfect blackbody spectrum and that it is nearly isotropic (once the Doppler dipole is removed) provide strong support for the Hot Big Bang model of the universe. A background of nearly isotropic blackbody radiation is natural if the universe was once hot, dense, opaque, and nearly homogeneous, as it was in the Hot Big Bang scenario. If the universe did not go through such a phase, then any explanation of the cosmic microwave background will have to be much more contrived.



8.2 Recombination and Decoupling

Table 3.3 Key events in the formation of the cosmic microwave background.

Event	redshift	temp (eV)	temp (K)	time (yrs)
Matter-radiation equality	3400	0.81	9390	50 000
Recombination	1300	0.30	3480	250 000
Photon decoupling	1100	0.25	2940	380 000
Last-scattering	1100	0.25	2940	380 000

$$X \equiv \frac{n_p}{n_p + n_{\rm H}} = \frac{n_p}{n_{\rm bary}} = \frac{n_e}{n_{\rm bary}}.$$

$$\mathrm{H} + \gamma \rightarrow p + e^-.$$

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$$I_\nu = \frac{2h\nu^3/c^2}{e^{h\nu/kT}-1}$$

$$I_\nu \propto \nu^2 T \qquad \text{when } h\nu \ll kT$$

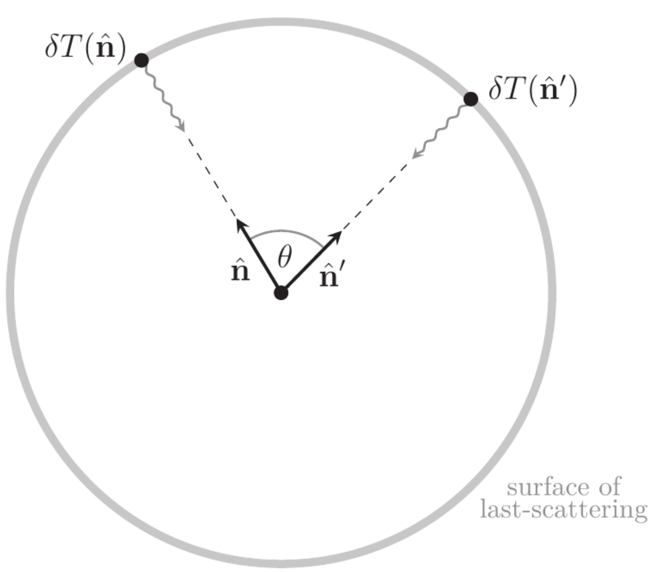
$$\frac{\delta I_\nu}{I} = \frac{\delta T}{T}$$

$$\frac{\delta T}{T}(\theta,\phi) = \frac{T(\theta,\phi) - \langle T\rangle}{\langle T\rangle}\,, \qquad \frac{\delta T}{T}(\theta,\phi) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} a_\ell^m\, Y_\ell^m(\theta,\phi)$$

where $Y_\ell^m(\theta,\phi)$ are the usual spherical harmonic functions:

$$\begin{aligned}
Y_0^0(\theta,\phi) &= \frac{1}{2}\sqrt{\frac{1}{\pi}} \\
Y_1^{-1}(\theta,\phi) &= \frac{1}{2}\sqrt{\frac{3}{2\pi}}\,\sin\theta\, e^{-i\phi} \\
Y_1^{0}(\theta,\phi) &= \frac{1}{2}\sqrt{\frac{3}{\pi}}\,\cos\theta \\
Y_1^{1}(\theta,\phi) &= -\frac{1}{2}\sqrt{\frac{3}{2\pi}}\,\sin\theta\, e^{i\phi}
\end{aligned}$$

and so on.



A statistical measure of the temperature fluctuations is the correlation function, $C(\theta)$. Consider two points on the surface of last scattering, in directions represented by the vectors $\mathbf{r}$ and $\mathbf{r}'$, separated by the angle θ such that $\cos\theta = \mathbf{r}\cdot\mathbf{r}'$. The correlation function $C(\theta)$ is found by multiplying together the values of $\delta T/T$ at the two points, and averaging the product over all pairs of points separated by the angle θ:

$$C(\theta) = \left\langle \frac{\delta T}{T}(\mathbf{r}) \frac{\delta T}{T}(\mathbf{r}') \right\rangle_{\mathbf{r}\cdot\mathbf{r}'=\cos\theta} \tag{10.6}$$

If we knew the value of $C(\theta)$ for all angles θ we would have a complete statistical description of the temperature fluctuations over the entire sky (all angular scales). In practice, however, this information is available only over a limited range of scales. Using the expansion of $\delta T/T$ in spherical harmonics (eq 10.5), the correlation function can be written in the form:

$$C(\theta) = \frac{1}{4\pi} \sum_{\ell=0}^{\infty} (2\ell+1)\, C_\ell\, P_\ell(\cos\theta) \tag{10.7}$$

where P_ℓ are the Legendre polynomials:

$$\begin{aligned} P_0(x) &= 1 \\ P_1(x) &= x \\ P_2(x) &= \frac{1}{2}(3x^2 - 1) \end{aligned}$$

and so on, and

$$C_\ell = \langle |a_\ell^m|^2 \rangle = \frac{1}{2\ell+1} \sum_m |a_\ell^m|^2 \,. \tag{10.8}$$

In this way, the correlation function $C(\theta)$ can be broken down into its multipole components C_ℓ.

For a given CMB set of data (either an all sky map obtained by a satellite mission, or an observation over a more restricted portion of the sky from one of the ground-based experiments), it is possible to measure C_ℓ for angular scales larger than the resolution of the data and smaller than the patch of sky examined. Generally speaking, a term C_ℓ is a measure of $\delta T/T$ on the angular scale $\theta \sim 180^\circ/\ell$.

The $\ell = 0$ term of the correlation function (the monopole) vanishes if the mean temperature has been defined correctly. The $\ell = 1$ term (the dipole) reflects the motion of the Earth through space, as we shall see in a moment. The moments with $\ell \geq 2$ are the ones of interest here, as they represent the fluctuations present at the time of last scattering.

The *power spectrum* of temperature fluctuations of the CMB is usually plotted as:

$$\Delta_{\mathrm{T}}^2 \equiv \frac{\ell(\ell+1)}{2\pi} C_\ell \; \langle T \rangle^2 \tag{10.9}$$

as a function of multipole moment ℓ ; the units of Δ_{T}^2 are $\mu\mathrm{K}^2$. The most precise determination of Δ_{T} over the largest range of scales has been provided by the Planck mission and is reproduced in Figure 10.2.

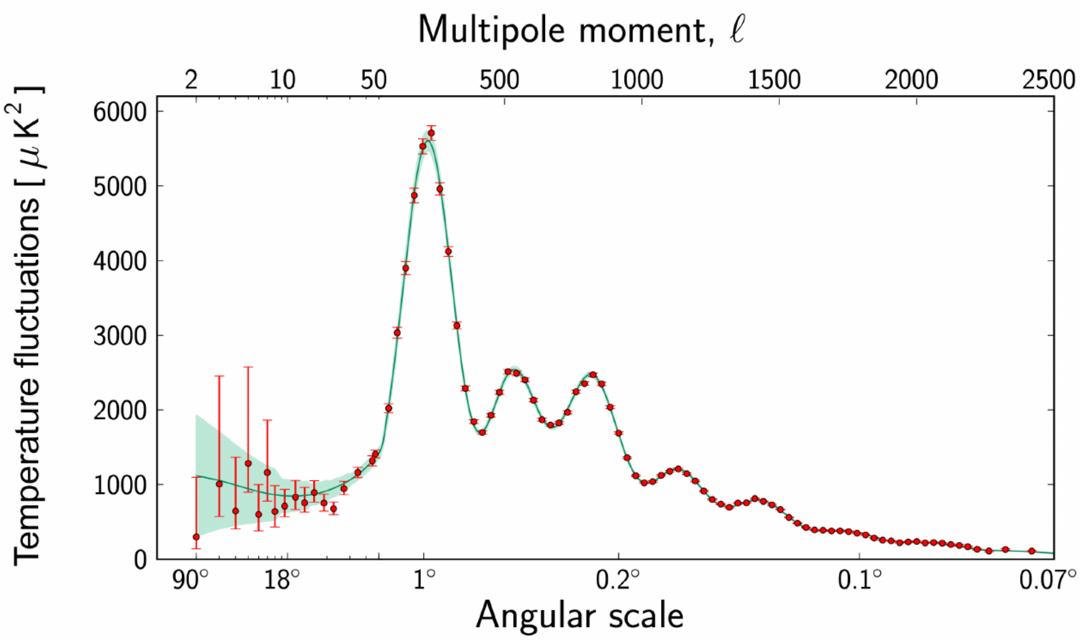
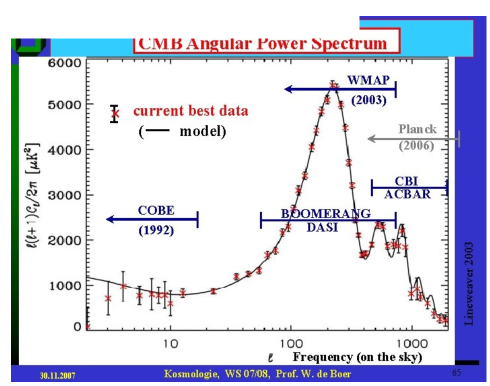


Figure 10.2: Power spectrum of the temperature fluctuations of the Cosmic Microwave Background measured by the Planck satellite. The red points are the measurements of Δ_{T}^2 as defined in eq. 10.9, with error bars that become larger at the largest scales because there are fewer pairs of directions, $\mathbf{r}$ and $\mathbf{r}'$, separated by such large values of θ on the sky. This is what is often referred to as 'cosmic variance'. The green curve is the best fit to the data obtained with the parameters that the define the 'standard model' of cosmology. The pale green area around the curve illustrates the predictions from the range of values of these parameters consistent with the Planck data.



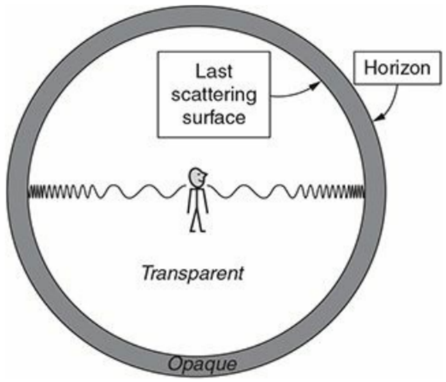


Figure 8.4 An observer is surrounded by a spherical last scattering surface. The photons of the CMB travel straight to us from the last scattering surface, being continuously redshifted.

If hydrogen had remained ionized (and note the qualifying *if*), then photons would have remained coupled to the electrons and protons until a relatively recent time. Taking into account the transition from a radiation-dominated to a matter-dominated universe, and the resulting change in the expansion rate, we can compute that *if* hydrogen had remained fully ionized, then decoupling would have taken place at a scale factor $a \approx 0.0254$, corresponding to a redshift $z \approx 38$ and a CMB temperature $T \approx 110$ K. However, at such a low temperature, the CMB photons are too low in energy to keep the hydrogen ionized. Thus, the decoupling of photons is not a gradual process, caused by the continuous lowering of free electron density as the universe expands. Rather, it is a relatively sudden process, caused by the plummeting of free electron density during the epoch of recombination, as electrons combined with protons to form hydrogen atoms.

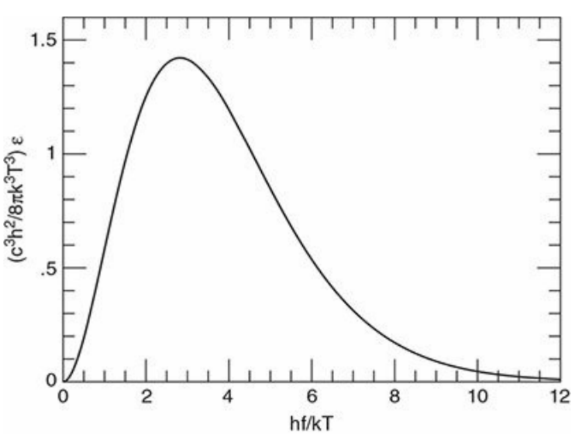


Figure 2.7 The energy density of blackbody radiation, expressed as a function of frequency *f*.

$$T_{rec} \sim \frac{Q}{2.7k} \sim \frac{13.6\,\mathrm{eV}}{2.7(8.6 \times 10^{-5}\,\mathrm{eV\,K^{-1}})} \sim 60\,000\,\mathrm{K}.$$

Alas, this crude approximation is a little *too* crude to be useful. It doesn't take into account the fact that CMB photons are not of uniform energy – a blackbody spectrum has an exponential tail (see Figure 2.7) trailing off to high energies. Although the mean photon energy is $2.7kT$, about one photon in 500 will have $E > 10kT$, one in 3 million will have $E > 20kT$, and one in 30 billion will have $E > 30kT$. Although extremely high energy photons make up only a tiny fraction of the CMB photons, the total number of CMB photons is enormous, with 1.6 billion photons for every baryon. The vast swarms of photons that surround every newly formed hydrogen atom greatly increase the probability that the atom will collide with a photon from the high-energy tail of the blackbody spectrum, and be photoionized.

Recombination was not an instantaneous process; it happened sufficiently gradually that at any given instant, the assumption of kinetic and chemical equilibrium is a reasonable approximation. However, as shown in Figure 8.5, it did proceed fairly rapidly by cosmological standards. The fractional ionization went from $X = 0.9$ at a redshift $z = 1480$ to $X = 0.1$ at a redshift $z = 1260$. In the Benchmark Model, the time that elapses from $X = 0.9$ to $X = 0.1$ is $\Delta t \approx 70\,000$ yr $\approx 0.28 t_{rec}$.

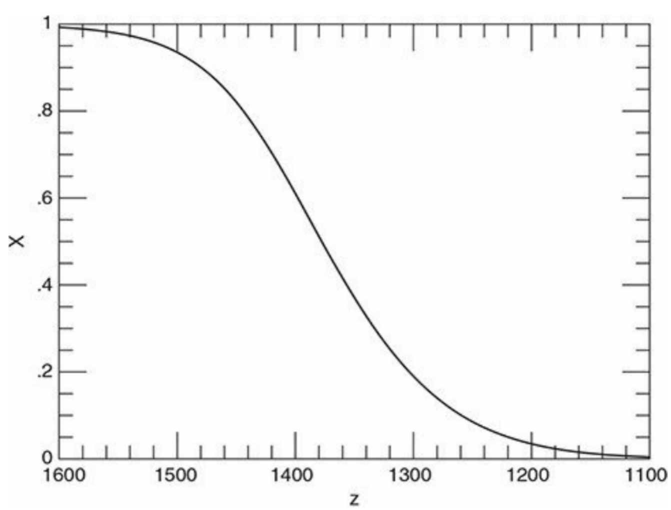
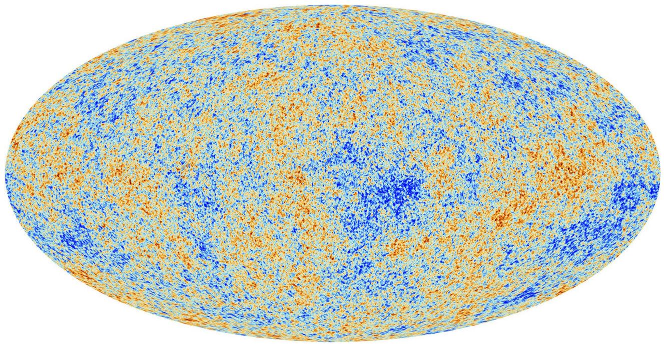


Figure 8.5 Fractional ionization X as a function of redshift during the epoch of recombination. A baryon-to-photon ratio $\eta = 6.1\times10^{-10}$ is assumed. Redshift decreases, and thus time increases, from left to right.

Table 8.1 Events in the early universe.

Event	*Redshift*	*Temperature (K)*	*Time (Myr)*
Radiation–matter equality	3440	9390	0.050
Recombination	1380	3760	0.25
Photon decoupling	1090	2970	0.37
Last scattering	1090	2970	0.37

8.4 Temperature Fluctuations



$$\frac{\delta T}{T}(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi), \tag{8.50}$$

where $Y_{lm}(\theta, \phi)$ are the usual spherical harmonic functions. What concerns cosmologists is not the exact pattern of hot spots and cold spots on the sky, but their statistical properties. The most important statistical property of $\delta T/T$ is the correlation function $C(\theta)$. Consider two points on the last scattering surface. Relative to an observer, they are in the directions $\hat{n}$ and $\hat{n}'$, and are separated by an angle θ given by the relation $\cos\theta = \hat{n} \cdot \hat{n}'$. To find the correlation function $C(\theta)$, multiply together the values of $\delta T/T$ at the two points, then average the product over all points separated by the angle θ:

$$C(\theta) = \left\langle \frac{\delta T}{T}(\hat{n}) \frac{\delta T}{T}(\hat{n}') \right\rangle_{\hat{n}\cdot\hat{n}'=\cos\theta}.$$

$$C(\theta) = \frac{1}{4\pi} \sum_{l=0}^{\infty} (2l+1) C_l P_l(\cos\theta),$$

and so forth. In this way, a measured correlation function $C(\theta)$ can be broken down into its multipole moments C_l. The l = 0 (monopole) term of the correlation function vanishes if we've defined the mean temperature correctly. The l = 1 (dipole) term results primarily from the Doppler shift due to our motion through space. For larger values of l, the termC_l is a measure of temperature fluctuations on an angular scale $\theta \sim 180^\circ/l$. Thus, the multipole l is interchangeable, for all practical purposes, with the angular scale θ. The moments with $l \geq 2$ are of the most interest to astronomers, since they tell us about the fluctuations present at the time of last scattering.

In presenting the results of CMB observations, it is customary to plot the function

$$\Delta_T \equiv \left(\frac{l(l+1)}{2\pi} C_l \right)^{1/2} \langle T \rangle \,, \tag{8.54}$$

since this function tells us the contribution per logarithmic interval in l to the total temperature fluctuation δT of the cosmic microwave background. Figure 8.6, which shows results from the *Planck* satellite, is a plot of Δ_T as a function of l. The detailed shape of the Δ_T versus l curve contains a wealth of information about the universe at the time of photon decoupling. In the next section we will examine, very briefly, the physics behind the temperature fluctuations, and how we can extract cosmological information from the temperature anisotropy of the cosmic microwave background.

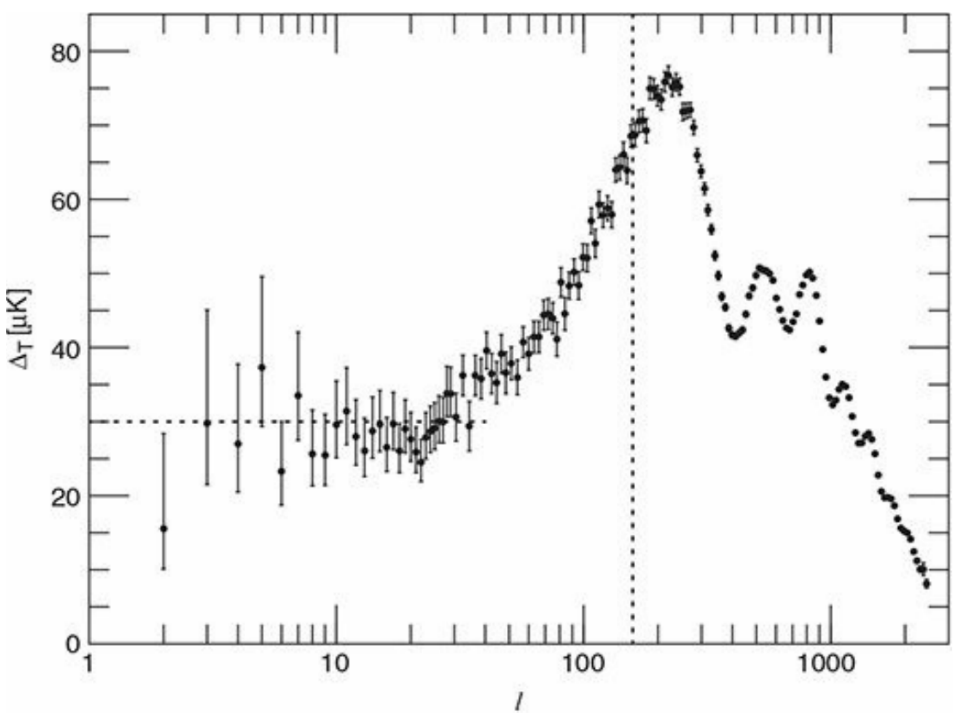


Figure 8.6 Temperature fluctuations Δ_T of the CMB, as observed by *Planck*, expressed as a function of the multipole l. The vertical dotted line shows l_{hor}, the multipole corresponding to the horizon size at last scattering. The horizontal dotted line shows the value $\Delta_T \approx 30\ \mu$K at which Δ_T levels off at small l. [data courtesy of *Planck*/ESA]

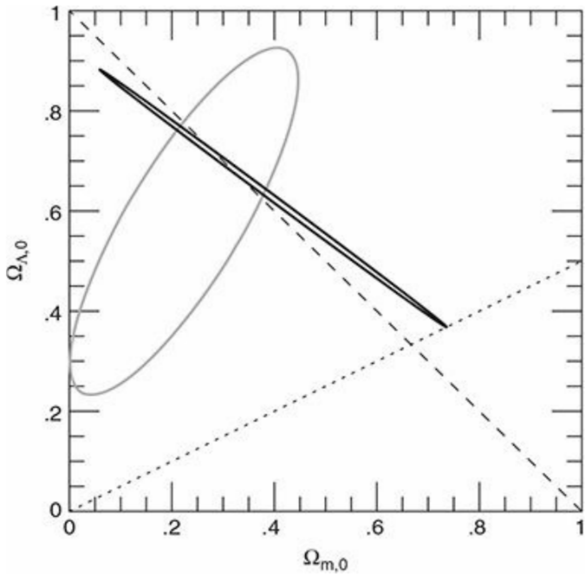


Figure 8.7 The elongated black ellipse represents the 95% confidence interval for the values of $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$ that best fit the *Planck* CMB data. For comparison, the gray ellipse shows the 95% confidence interval from the supernova data, repeated from Figure 6.6. [Anže Slosar & José Alberto Vázquez, BNL]