Early Universe: Introduction

Disclaimer

Discussions taken from Barbara Ryden [1], Daniel Baumann [2] and Kolb & Turner [3] books

1 Units used in Cosmology

- Astronomical unit (AU), equal to the mean distance between the Earth and Sun:

$$1\,\mathrm{AU} = 1.5 \times 10^{11}\,\mathrm{m}. \tag{1}$$

- Parsec (pc), equal to the distance at which 1 AU subtends an angle of 1 arcsecond:

$$1\,\mathrm{pc} = 3.09 \times 10^{16}\,\mathrm{m}. \tag{2}$$

  - Distance to Proxima Centauri (Sun's nearest neighboring star): 1.3 pc.
  - Distance to the center of our galaxy: 8.5 kpc.
  - Distance to the Andromeda galaxy (M31): 0.76 Mpc.
  - Distance to the Virgo cluster (the nearest big cluster of galaxies): 16.5 Mpc.
  - Distance to the Coma cluster: 99 Mpc.
  - Distance to QSO J0313-1806 (the most distant, and hence also the oldest known quasar at $z = 7.64$): 900 Mpc.

- Solar Mass ($\mathrm{M}_\odot$):

$$1\,\mathrm{M}_\odot = 1.99 \times 10^{30}\,\mathrm{kg}. \tag{3}$$

  - Mass of the Milky Way galaxy: $M_{\mathrm{MW}} \sim 10^{12}\,\mathrm{M}_\odot$.
  - Mass of the Coma cluster: $M_{\mathrm{Coma}} \sim 7 \times 10^{14}\,\mathrm{M}_\odot$.

- Sun's luminosity ($\mathrm{L}_\odot$):

$$1\,\mathrm{L}_\odot = 3.83 \times 10^{26}\,\mathrm{watts}. \tag{4}$$

  - Approximately luminosity of the Milky Way galaxy: $L_{\mathrm{MW}} \sim 3 \times 10^{10}\,\mathrm{L}_\odot$.

- Age of the Universe (t_U):

$$t_U = 13.8\,\mathrm{Gyr}. \tag{5}$$

  - Time of formation of CMB (recombination): 380 ky.
  - Time of formation of light nuclei (Big Bang Nucleosynthesis): 1 s.
  - Time of electroweak symmetry breaking: 10^{-12} s.

2 Elementary particles

Elementary particles are classified according their spin s: bosons have integer s while fermions have half-integer s.

2.1 Bosons

The bosons with spin $s = 1$ are dubbed as vector particles and are associated with the mediators of the fundamental interactions: photon with electromagnetism, gluon with strong force and $Z, W^{\pm}$ with weak force. The Higgs particle is associated with the field responsible for giving mass to all known particles. The properties of the known bosonic particles are displayed in Table 1.

Boson	Spin	Charge	Mass (GeV/c^2)
Photon	1	0	0
Gluon	1	0	0
Z^0	1	0	91.2
$W^{\pm}$	1	± 1	80.4
Higgs	0	0	125

Table 1: Bosonic elementary particles.

2.2 Fermions

Fermions are classified accordingly to leptons and qarks. Leptons do not undergo the strong interaction while quarks do (see Tables 2 and 3).

Lepton	Spin	Charge	Mass (GeV/c^2)
$e^{\pm}$	1/2	± 1	0.000511
ν_e	1/2	0	$\lesssim 10^{-15}$
$\mu^{\pm}$	1/2	± 1	0.105
ν_μ	1/2	0	$\sim 10^{-12}$
$\tau^{\pm}$	1/2	± 1	1.777
ν_τ	1	0	$\sim 10^{-11}$

Table 2: Leptons.

Quark	Spin	Charge	Mass (GeV/c^2)
up	1/2	2/3	0.0023
down	1/2	-1/3	0.0048
charm	1/2	2/3	1.275
strange	1/2	-1/3	0.095
top	1/2	2/3	173
bottom	1/2	-1/3	4.18

Table 3: Quarks.

2.2.1 Composite baryons

Particles made up from quarks are called baryons, and are classified as mesons or hadrons if they are formed by a pair quark-antiquark or a ternary of quarks (anti-quarks), respectively.

Meson	Spin	Charge	Mass (GeV/c^2)
$\pi^0\,(u\bar{u} - d\bar{d})$	0	0	0.135
$\pi^+\,(u\bar{d})$	0	1	0.140
$\pi^-\,(d\bar{u})$	0	-1	0.140
Hadron	Spin	Charge	Mass (GeV/c^2)
$p\,(uud)$	1	1	0.938
$n\,(udd)$	1	0	0.939
$\Lambda^0\,(uds)$	1	0	1.116

Table 4: Baryons.

3 Natural (Planck) Units

The Planck system is based on the combination of four universal constants:

$$G_N = 6.67 \times 10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kg\ sec}^2}, \tag{6}$$

$$c = 299792458\,\frac{\mathrm{m}}{\mathrm{sec}}, \tag{7}$$

$$\hbar = 1.055 \times 10^{-34}\,\mathrm{J\,s} = 6.582 \times 10^{-25}\,\mathrm{GeV\,s}, \tag{8}$$

$$k_B = 1.38 \times 10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}} = 8.617 \times 10^{-14}\,\frac{\mathrm{GeV}}{\mathrm{K}}, \tag{9}$$

with $1\,\mathrm{eV} = 1.602 \times 10^{-19}$ J. The Planck length, mass and time are defined as

$$\ell_P \equiv \left(\frac{G_N \hbar}{c^3}\right)^{1/2} = 1.62 \times 10^{-35}\,\mathrm{m}, \tag{10}$$

$$M_P \equiv \left(\frac{\hbar c}{G_N}\right)^{1/2} = 2.18 \times 10^{-8}\,\mathrm{kg}, \tag{11}$$

$$t_P \equiv \left(\frac{G_N \hbar}{c^5}\right)^{1/2} = 5.39 \times 10^{-44}\,\mathrm{sec}\,. \tag{12}$$

The Planck energy is defined as

$$E_P \equiv M_P c^2 = 1.96 \times 10^9\,\mathrm{J} = 1.22 \times 10^{19}\,\mathrm{GeV}, \tag{13}$$

while the Planck temperature becomes

$$T_P \equiv E_P / k_B = 1.42 \times 10^{32}\,\mathrm{K}. \tag{14}$$

When distance, mass, time, and temperature are measured in the appropriate Planck units, then $c = k_B = \hbar = G_N = 1$. For instance, the Einstein's equation for energy, $E = mc^2$ becomes $E = m$.

4 Problems

1. Check the relations for the Planck length, time and mass.

2. Convert the following quantities by inserting the appropriate factors of $c, \hbar$, G_N and k_B:

   a) $T_0 = 2.725$ K $\rightarrow$ eV.

   b) $\rho_\gamma = \pi^2 T_0^4/15 \rightarrow$ eV^4 and g/cm^3.

   c) $M_P \equiv 1.2 \times 10^{19}$ GeV $\rightarrow$ K.

3. The range of length scales involved in cosmology are hard to grasp. The best we can do is to consider relative distances and compare them to something more familiar. In this exercise, we will make some attempt at obtaining a more intuitive understanding of the vastness of the cosmos.

   a) Consider shrinking the Earth to the size of a basketball. What would then be the size of the Moon and its orbit around the Earth?

   b) Now imagine scaling the Earth down to the size of a peppercorn. What would then be the size of the Sun and the Earth's orbit? How far away would the most distant planet in the Solar System be?

   c) The "Solar Neighborhood" is a collection of about fifty nearby stars, spread across about 65 light-years, that travel together with the Sun. Scaling this region down, so that it fits inside a basketball court, what would be the size of the Solar System?.

   d) Shrinking our Galaxy to the size of the basketball court, what would now be the size of the Solar Neighbourhood?

   e) The "Local Group" comprises about fifty nearby galaxies, spread across about 10 million light-years. If we squeezed this region into the size of the basketball court, what would be the size of our Milky Way galaxy?

   f) The largest structures in the universe, like our "Local Supercluster," are about 500 million light-years across. Scaling these superclusters down to the dimensions of the basketball court, what would be the size of our Local Group?

   g) This radius of the observable universe is 46.5 billion light-years. Compressing the observable universe to the size of the basketball court, what would be the size of the largest superclusters?

4. A key parameter in cosmology is the Hubble constant

$$H_0 = 70 \text{ km/s/Mpc}.$$

   In the following, you will use the measured value of the Hubble constant to estimate a few fundamental scales of our universe.

   a) What is the Hubble time $t_{H_0} \equiv H_0^{-1}$ in years? This is a rough estimate of the age of the universe.

   b) What is the Hubble distance $d_{H_0} \equiv cH_0^{-1}$ in meters? This is a rough estimate of the size of the observable universe.

c) The average density of the universe today is

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G_N},$$

where G_N is the Nweton's constant. What is $\rho_{c,0}$ in g/cm^{-3}? How does this compare to the density of water.

d) Let us assume that the universe is filled with only hydrogen atoms. What is then the total number of atoms in the universe? How does this compare to the number of hydrogen atoms in your brain?
Hint: Assume that the brain is mostly water. Use $m_{\mathrm{H}} = 2\times10^{-24}$ g and $m_{\mathrm{H_2O}} = 3\times10^{-23}$ g.

e) The maximal energy scale probed by the Large Hadron Collider (LHC) is $E_{\mathrm{max}} \sim 1$ TeV. What length scale ℓ_{min} does this correspond to? How does this compare to the size of the universe d_{H_0}?

5. The typical mass of a galaxy is given by $10^{12}\,\mathrm{M}_\odot$. Assuming that the matter in form of nucleons (that, is in protons and neutrons) contribute 1/5 to the mass of a galaxy, estimate the typical number of nucleons in a galaxy.

6. When a charged π^- meson with very low velocity reacts with a proton, a neutron and a neutral π^0 meson are produced, $\pi^- + p \to \pi^0 + n$. Suppose that the proton, neutron and π^- masses are known: $m_p = 938.3$ MeV, $m_n = 939.6$ MeV, $m_{\pi^-} = 139.6$ MeV. Determine the mass of the π^0, if the neutron kinetic energy is measured to be $T_n \equiv E_n - m_n = 0.4$ MeV.

7. Show that a free electron and a free positron, both with mass m_e, cannot annihilate into a single freely propagating photon.

8. The cosmic microwave background radiation (CMBR) consists of photons of typical energy 3×10^{-4} eV. How high an energy must a cosmic gamma photon γ_c have if pair production on this background $\gamma_c + \gamma_{\mathrm{CMB}} \to e^+ + e^-$ is to be kinematically allowed?

9. Suppose that the Milky Way galaxy is a typical size, containing say 10^{11} stars, and that galaxies are typically separated by a distance of one Mpc. Estimate the density of the Universe in SI units. How does this compare with the density of the Earth?

References

[1] B. Ryden, *Introduction to cosmology*. Cambridge University Press, 1970.

[2] D. Baumann, *Cosmology*. Cambridge University Press, 7, 2022.

[3] E. W. Kolb, *The Early Universe*, vol. 69. Taylor and Francis, 5, 2019.