Cosmological Parameters

Deceleration parameter

$$q_0 \equiv -\left(\frac{\ddot{a}a}{\dot{a}^2}\right)_{t=t_0} = -\left(\frac{\ddot{a}}{aH^2}\right)_{t=t_0}.$$

$$q_0 = \Omega_{r,0} + \frac{1}{2}\Omega_{m,0} - \Omega_{\Lambda,0}.$$

$$a(t) \approx 1 + H_0(t-t_0) - \frac{1}{2}q_0 H_0^2 (t-t_0)^2.$$

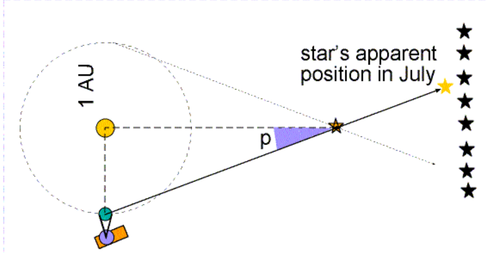
Proper distance

$$d_p(t_0) = c\int_{t_e}^{t_0} \frac{dt}{a(t)}.$$

$$d_p(t_0) \approx \frac{c}{H_0}\left[z - \left(1+\frac{q_0}{2}\right)z^2\right] + \frac{cH_0}{2}\frac{z^2}{H_0^2} = \frac{c}{H_0}z\left[1 - \frac{1+q_0}{2}z\right].$$

Parallax distance

$$d_\pi \equiv \frac{b}{\tan\theta} \simeq \frac{b}{\theta} = 1\,\mathrm{pc}\,\frac{b}{1\,\mathrm{AU}}\,\frac{1\,\mathrm{arcsec}}{\theta},$$



Stellar parallaxes need largest possible baseline

GAIA satellite's error: $\Delta\theta \sim 10\,\mu\mathrm{arcsec}$

Luminosity Distance

The luminosity distance is a way of expressing the amount of light received from a distant object.

A luminous astronomical source with a known *absolute luminosity* L, the total emitted power, is called *standard candle*. Assuming that a simple inverse-square law for the reduction of the light intensity with distance holds, the power received by a detector of area A located at a distance d_L is given by

$$P = L \frac{A}{4\,\pi\, d_L^2}\,.$$

Knowing L and measuring P, one can derive the *luminosity distance* defined as

$$d_L \equiv \left(\frac{L}{4\,\pi\, F}\right)^{1/2}\,,$$

where $F \equiv P/A$ is the measured flux, i.e., the power received per unit area.

This distance is what the proper distance to the standard candle would be if the universe were static and Euclidean.

Geometric effects: FLRW metric:

$$ds^2 = -c^2 dt^2 + a(t)^2 [dr^2 + S_\kappa(r)^2 d\Omega^2],$$

$$S_\kappa(r) = \begin{cases} R_0 \sin(r/R_0) & (\kappa = +1) \\ r & (\kappa = 0) \\ R_0 \sinh(r/R_0) & (\kappa = -1). \end{cases} \longrightarrow \qquad A_p(t_0) = 4\pi S_\kappa(r)^2.$$

Expansion of the universe effects:

There are actually two effects, which looks like double counting but is not:

- The individual photons lose energy $\propto (1+z)$, so have less energy when they arrive.
- The photons arrive less frequently $\propto (1+z)$.

The net result is that in an expanding, spatially curved universe, the relation between the observed flux f and the luminosity L of a distant light source is

$$f = \frac{L}{4\pi S_\kappa(r)^2(1+z)^2},$$

and the luminosity distance is

$$d_L = S_\kappa(r)(1+z).$$

For flat a flat Universe: $$d_L = r(1+z) = d_p(t_0)(1+z) \qquad [\kappa = 0].$$

When $z \ll 1$, $$d_P(t_0) \approx \frac{c}{H_0} z\left(1 - \frac{1+q_0}{2} z\right).$$

$$d_L \approx \frac{c}{H_0} z\left(1 - \frac{1+q_0}{2} z\right)(1+z) \approx \frac{c}{H_0} z\left(1 + \frac{1-q_0}{2} z\right). \qquad [\kappa = 0].$$

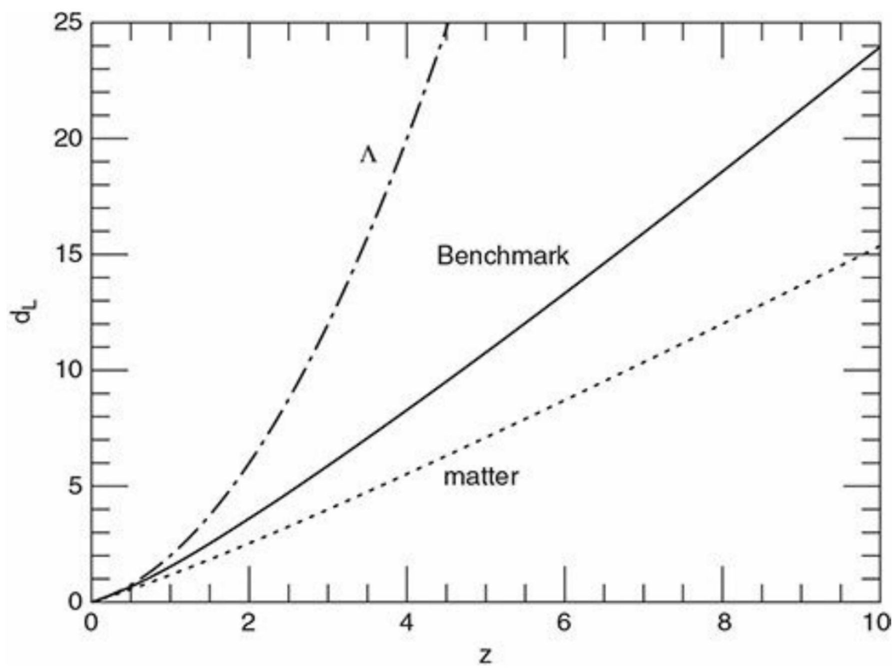
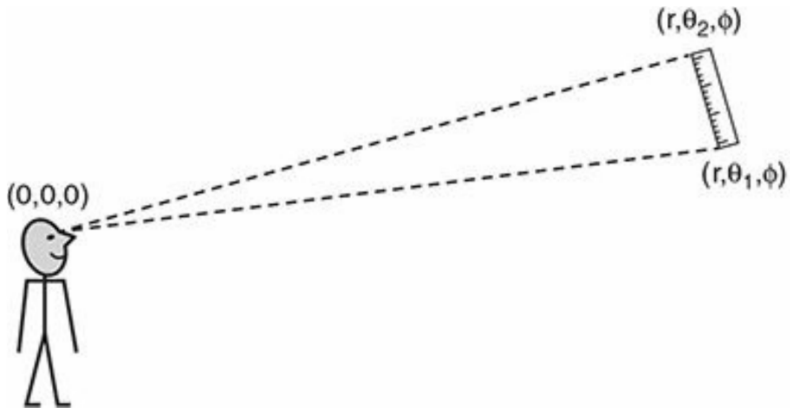


Figure 6.2 Luminosity distance of a standard candle with observed redshift z, in units of the Hubble distance, c/H_0. The bold solid line gives the result for the Benchmark Model. For comparison, the dot-dash line indicates a flat, lambda-only universe, and the dotted line a flat, matter-only universe.

Angular-diameter Distance

The angular diameter distance is a measure of how large objects appear to be.

An observer at the origin observes a standard yardstick,
of known proper length l, at comoving coordinate distance r.



angular-diameter distance

$$d_A \equiv \frac{\ell}{\delta\theta}.$$

The distance ds between the two ends of the yardstick, measured at the time te when the light was emitted, can be found from the FRLW metric:

$$ds = a(t_e)S_\kappa(r)\delta\theta.$$

$$\ell = a(t_e)S_\kappa(r)\delta\theta = \frac{S_\kappa(r)\delta\theta}{1+z}.$$

Thus, the angular-diameter distance d_A to a standard yardstick is

$$d_A \equiv \frac{\ell}{\delta\theta} = \frac{S_\kappa(r)}{1+z}.$$

The relation between the angular-diameter distance and the luminosity distance is

$$d_A = \frac{d_L}{(1+z)^2}.$$

Thus, if you observe a redshifted object that is both a standard candle and a standard yardstick, the angular-diameter distance that you compute for the object will be smaller than the luminosity distance. Moreover, if the universe is spatially flat,

$$d_A(1+z) = d_p(t_0) = \frac{d_L}{1+z} \qquad [\kappa = 0].$$

In a flat universe, therefore, if you compute the angular-diameter distance d_A of a standard yardstick, it isn't equal to the current proper distance $d_p(t_0)$; rather, it is equal to the proper distance at the time the light from the object was emitted: $d_A = d_p(t_0)/(1+z) = d_p(t_e)$.

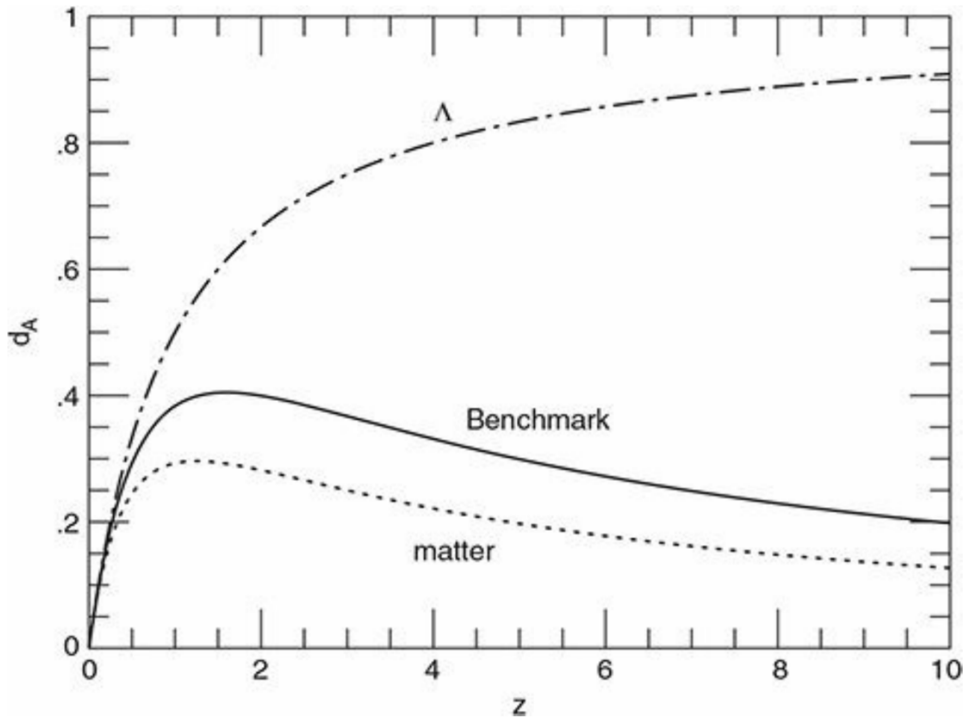


Figure 6.4 Angular-diameter distance of a standard yardstick with observed redshift z, in units of the Hubble distance, c/H_0. The bold solid line gives the result for the Benchmark Model. For comparison, the dot-dash line indicates a flat, lambda-only universe, and the dotted line a flat, matter-only universe.

When $z \ll 1$, $$d_A \approx \frac{c}{H_0} z \left(1 - \frac{3+q_0}{2} z\right).$$

In model universes other than the lambda-only model, the angular-diameter distance d_A has a maximum for standard yardsticks at some critical redshift z_c. For instance, the Benchmark Model has a critical redshift $z_c = 1.6$, where $d_A(\mathrm{max}) = 0.405 c/H_0 = 1770$ Mpc. If the universe were full of glow-in-the-dark yardsticks, all of the same size ℓ, their angular size $\delta\theta$ would decrease with redshift out to $z = z_c$, but then would increase at larger redshifts. The sky would be full of big, faint, redshifted yardsticks.

Unfortunately, we can thus not learn anything about the cosmological parameters by just comparing the distance measures d_L and d_A to a single object. However, if we have a number of sources at different redshifts, we can use the dependence on z of either d_L, d_A or both, to determine the cosmological parameters.

Standard Candles and H_0

- Identify a population of standard candles with luminosity L.
- Measure the redshift z and flux f for each standard candle.
- Compute $d_L = (L/4\pi f)^{1/2}$ for each standard candle.
- Plot cz versus d_L.
- Measure the slope of the cz versus d_L relation when $z \ll 1$; this gives H_0.

A very important class of standard candles is given by variable stars for which there is an empirical evidence of a relation between their *luminosity curves* and in particular the period of variability and their absolute luminosity. There are three famous kinds of such variables:

- *RR* Lyrae, with a period between a few hours to one day. They allow to determine distances up to $\sim 1\,\text{Mpc}$;

- *Cepheids*, with a period ranging from 2 to 40 days; they allow to determine distances up to $\sim 10\,\text{Mpc}$. In particular in 1924 the discovery of a Cepheid made by Hubble in the Andromeda Galaxy made it possible to determine the distance of the Andromeda galaxy hosting the Cepheid (about $0.7\,\text{Mpc}$). This distance is much bigger than the size of our galaxy. In this way it became possible to give a solution to quite a strong scientific controversy, whether the Milky Way Galaxy should be considered encompassing the whole universe or just one among billions of other galaxies. Cepheids also played a crucial role in the discovery of the universe expansion made by Hubble in 1929. In 1994 the Hubble Space Telescope managed for the first time to determine the distance of a Cepheid in the Virgo Cluster about $\sim 17\,\text{Mpc}$ from us. This made possible a precise calibration of the next rung in the cosmic distance ladder and the most precise measurement of the Hubble constant, the parameter that describes the expansion of the universe at present, before 2013, when the *Planck* satellite provided the most precise measurement of the Hubble constant from CMB observations.

- *Supernovae type Ia.* They have a very well determined maximum luminosity in their light curve, the change of luminosity with time after the explosion. They are useful in determining the distances of the furthest galaxies, up to a few hundred Mpc. In 1998 they surprisingly provided the first strong observational evidence of an accelerated expansion of the universe at present.

Another important class of standard candles is represented by *spiral galaxies*, since their luminosity is strongly correlated with their maximum rotation velocity (*Tully–Fisher relation*). This method works for distances up to $\sim 100\,\text{Mpc}$.

$$H_0 = 68 \pm 2 \, \mathrm{km \, s^{-1} \, Mpc^{-1}}.$$

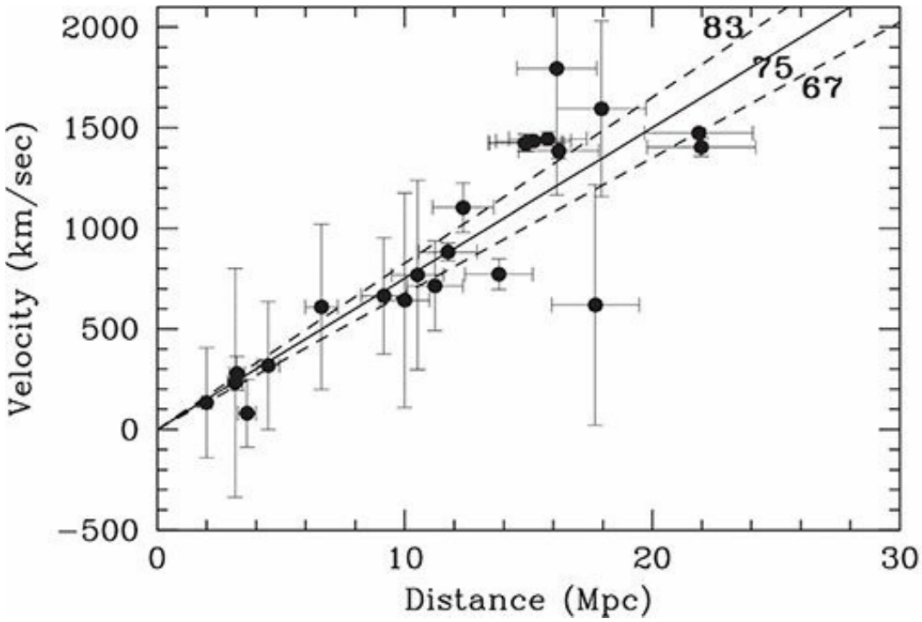


Figure 2.5 A more recent version of Hubble's plot, showing *cz* versus distance. In this case, the galaxy distances have been determined using Cepheid variable stars as standard

There is a hidden difficulty involved in using Cepheid stars to determine H_0. Cepheids can take you out only to a distance d_L ~ 30 Mpc; on this scale, the universe cannot be assumed to be homogeneous and isotropic. In fact, the Local Group is gravitationally attracted toward the Virgo cluster, causing it to have a peculiar motion in that direction. It is estimated, from dynamical models, that the recession velocity *cz* that we measure for the Virgo cluster is 250 km s^{-1} less than it would be if the universe were perfectly homogeneous. The plot of *cz* versus d_L given in Figure 2.5 uses recession velocities that are corrected for this "Virgocentric flow," as it is called.

Standard Candles and Acceleration

To determine the value of H_0 without having to worry about Virgocentric flow and other peculiar velocities, we need to determine the luminosity distance to standard candles with $d_L > 100$ Mpc, or $z > 0.02$.

For a standard candle to be seen at $d_L > 1000$Mpc, it must be very luminous.

Nowadays, the bolometric *apparent magnitude* of a light source is defined in terms of the source's bolometric flux as

$$m \equiv -2.5 \log_{10}(f/f_x),$$

where the reference flux f_x is set at the value $f_x = 2.53\times10^{-8}$ watt m^{-2}.

The bolometric *absolute magnitude* of a light source is defined as the apparent magnitude that it would have if it were at a luminosity distance of $d_L = 10$ pc. Thus, a light source with luminosity L has a bolometric absolute magnitude

$$M \equiv -2.5 \log_{10}(L/L_x),$$

where the reference luminosity is $L_x = 78.7\ L_\odot$, since that is the luminosity of an object that produces a flux $f_x = 2.53 \times 10^{-8}$ watt m^{-2} when viewed from a distance of 10 parsecs. The bolometric absolute magnitude of the Sun is thus $M = 4.74$. Although the system of apparent and absolute magnitudes seems strange to the uninitiated, the apparent magnitude is really nothing more than a logarithmic measure of the flux, and the absolute magnitude is a logarithmic measure of the luminosity.

Given the definitions of apparent and absolute magnitude, the relation between an object's apparent magnitude and its absolute magnitude can be written in the form

$$M = m - 5\log_{10}\left(\frac{d_L}{10\,\mathrm{pc}}\right),$$

where d_L is the luminosity distance to the light source. If the luminosity distance is given in units of megaparsecs, this relation becomes

$$M = m - 5\log_{10}\left(\frac{d_L}{1\,\mathrm{Mpc}}\right) - 25.$$

distance modulus to a light source

$$m - M = 5\log_{10}\left(\frac{d_L}{1\,\mathrm{Mpc}}\right) + 25.$$

The distance modulus of the Large Magellanic Cloud, for instance, at d_L = 0.050 Mpc, is $m - M = 18.5$. The distance modulus of the Virgo cluster, at d_L = 15Mpc, is $m - M = 30.9$. When $z \ll 1$, the luminosity distance to a light source is

$$d_L \approx \frac{c}{H_0} z \left(1 + \frac{1 - q_0}{2} z\right).$$

$$m - M \approx 43.23 - 5 \log_{10}\left(\frac{H_0}{68\,\mathrm{km\,s^{-1}\,Mpc^{-1}}}\right) + 5 \log_{10} z + 1.086(1 - q_0) z.$$

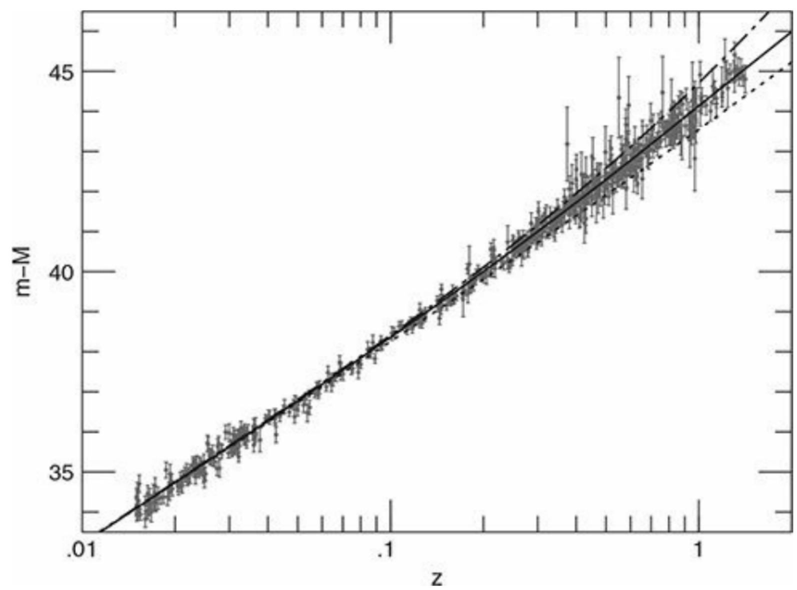


Figure 6.5 Distance modulus versus redshift for a set of 580 type Ia supernovae. The bold solid line gives the expected relation for the Benchmark Model. For comparison, the dot-dash line indicates a flat, lambda-only universe, and the dotted line a flat, matter-only universe. [data from Suzuki *et al.* 2012, *ApJ*, **716**, 85]

Figure 6.5 shows the plot of distance modulus versus redshift for a compilation of actual supernova observations from a variety of sources. The solid line running through the data is the result expected for the Benchmark Model. At a redshift $z \approx 1$, supernovae in the Benchmark Model are about 0.6 magnitudes fainter than they would be in a flat, matter-only universe; it was the observed faintness of Type Ia supernovae at $z > 0.3$ that led to the conclusion that the universe is accelerating. However, at $z \approx 1$, supernovae in the Benchmark Model are about 0.6 magnitudes brighter than they would be in a flat, lambda-only universe. Thus, the observations of Type Ia supernovae that tell us that the universe is accelerating also place useful upper limits on the magnitude of the acceleration.

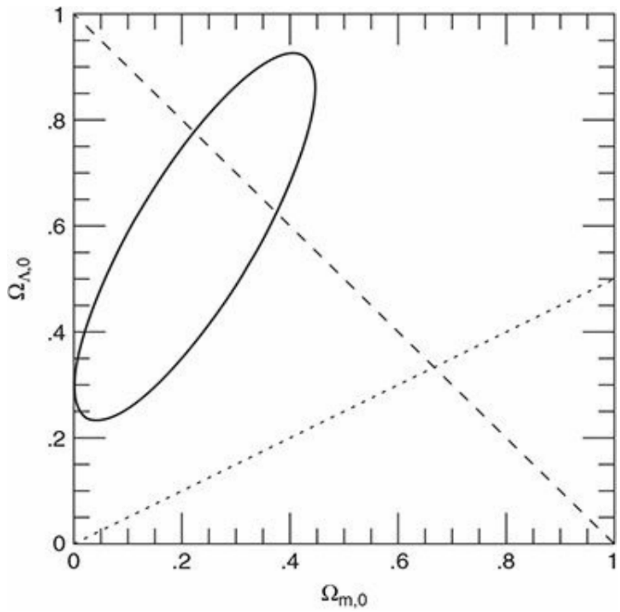


Figure 6.6 The values of $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$ that best fit the supernova data. The bold elliptical contour represents the 95% confidence interval. For reference, the dashed line represents flat universes, and the dotted line represents coasting ($q_0 = 0$) universes: compare to Figure 5.6. [Anže Slosar & José Alberto Vázquez, Brookhaven National Laboratory]

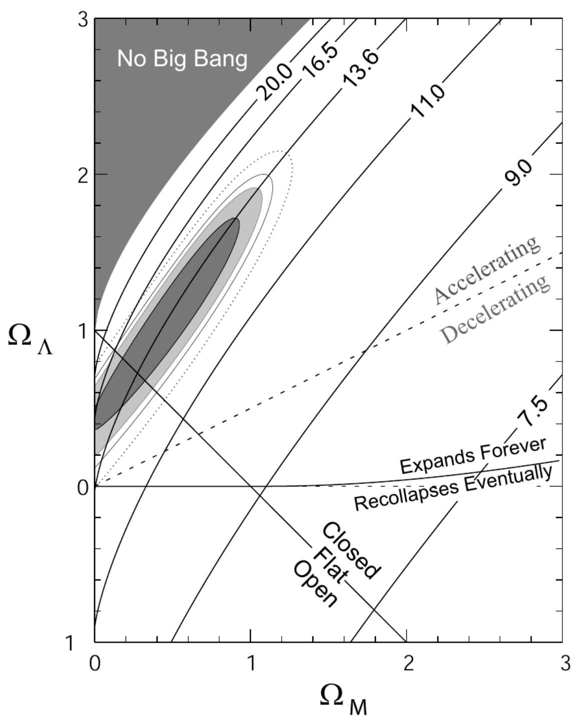


Fig. 4.6. Best-fit coincidence regions in the Ω_M–Ω_Λ plane from the analysis of the *Supernova Cosmology Project* supernova Hubble diagram shown in Fig. 4.5. The dark and light ellipses show the 68 per cent and 90 per cent confidence regions. The outer ellipses show the 95 and 99 per cent confidence levels. A flat Universe ($\Omega_K = 0$) would fall on top of the diagonal solid line passing through the Einstein de Sitter solution. To the right of that line the Universe is closed, and to the left it is open. The dashed line shows the division between acceleration and deceleration for the Universe. Also shown are isochrones of constant age of the Universe in units of billion of years (h=0.71 was assumed). The data suggests that the rate of expansion of the Universe is currently accelerating. Courtesy of Robert Knop.

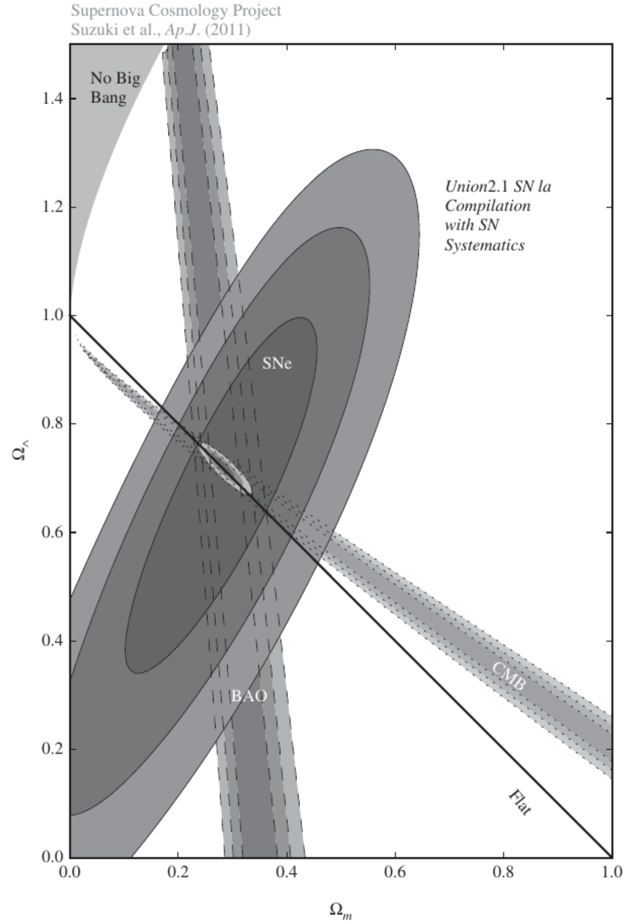
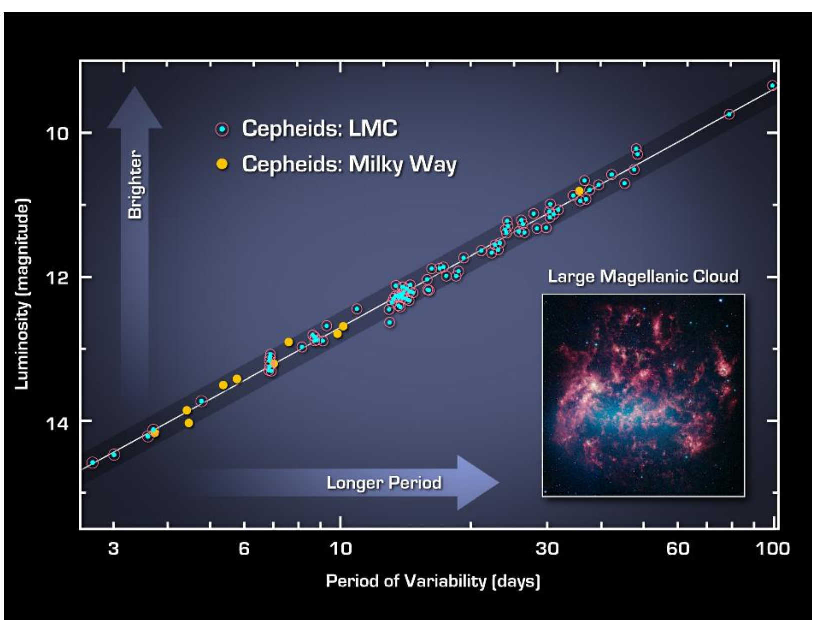
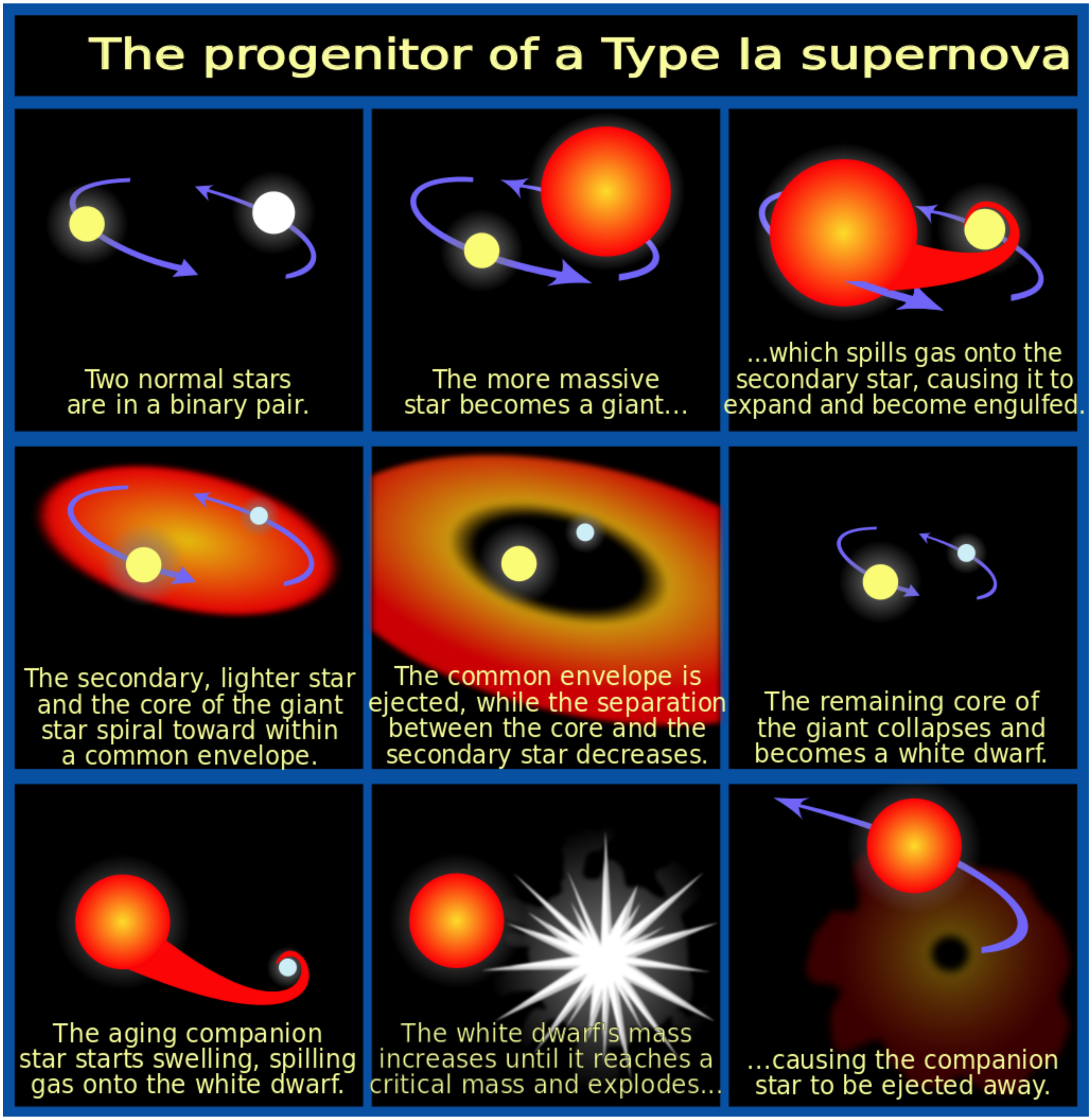
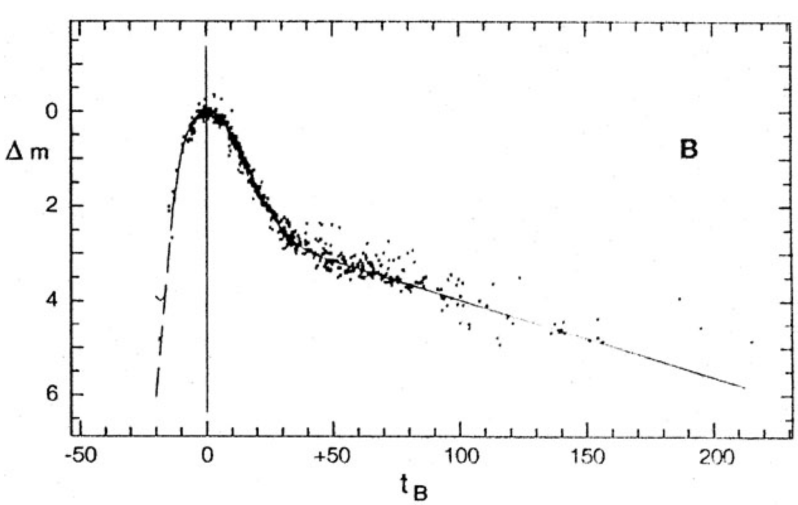


Figure A2.4 The contours marked 'SNe' show observational constraints from the supernova luminosity–redshift relation from the Union2.1 data set compiled the Supernova Cosmology Project. They are displayed in the Ω_0–Ω_Λ plane as introduced in Section 7.3, alongside constraints from the cosmic microwave background (CMB) and from a technique known as baryon acoustic oscillations (BAO). Only a very small region, with $\Omega_0 \simeq 0.3$ and $\Omega_\Lambda \simeq 0.7$, matches all three data sets. (From Suzuki et al., Astrophys. J. **746**, 85 (2012), courtesy Supernova Cosmology Project.)

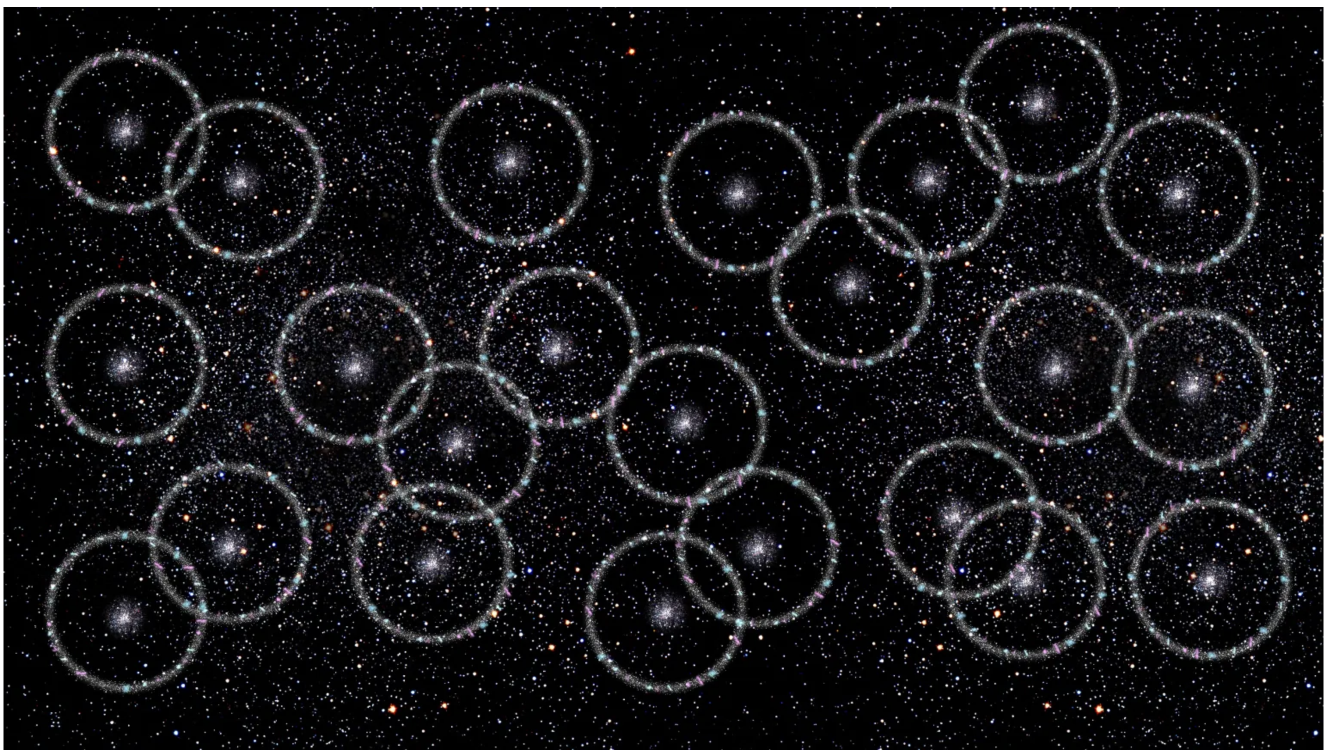



The progenitor of a Type Ia supernova
Two normal stars are in a binary pair.
The more massive star becomes a giant...
...which spills gas onto the secondary star, causing it to expand and become engulfed.
The secondary, lighter star and the core of the giant star spiral toward within a common envelope.
The common envelope is ejected, while the separation between the core and the secondary star decreases.
The remaining core of the giant collapses and becomes a white dwarf.
The aging companion star starts swelling, spilling gas onto the white dwarf.
The white dwarf's mass increases until it reaches a critical mass and explodes...
...causing the companion star to be ejected away.


0
Δ m
2
4
6
B
-50
0
+50
100
150
200
t B

An example of *standard yardstick* is given by *eclipsing binaries* stars. One can reconstruct the orbital parameters of these binary systems and in particular the size of the orbit. With this technique it is possible to measure distances up to $3\,\mathrm{Mpc}$. Therefore they are useful extra-galactic standard yardsticks.

Other example: Bryon acoustic oscillations



Standard sirens

The discovery of gravitational waves paves the way to a completely new method to measure the distance of an object that does not rely on the cosmic distance ladder and, therefore, offers a completely independent cross check. The gravitational wave frequency modulation, the so-called *chirp*, generated by the merging of a compact binary system, also contains information on the mass of the emitting object and from this one can calculate the absolute amplitude of the signal. When this is compared with the measured amplitude, since one knows that this decays inversely proportionally with the distance of the observer from the emitting source, one can immediately extract the distance of the source. Such objects have been dubbed *standard sirens* [4].