General relativity: gravitation.

Newtonian gravitation: $\vec{F} = -\frac{G_N M m}{r^2}\hat{U}_r$. "Phenomenological law".

The phenomenological character of the gravitational force last for more than 200 years.

With the appearance of the Special Relativity the Newtonian approach to gravitation suffered of additional drawbacks:

- the gravitational force is not covariant.
- The gravitational force linking two bodies should act instantly.

The formulation of a theory of gravitation (by Einstein) is deeply related with three basic principles:

Principles of $\begin{cases} \text{Equivalence.} \\ \text{Covariance,} \\ \text{Consistency.} \end{cases}$

* Special relativity is inconsistent with Newtonian gravitation.

$$\vec{F} = m_G \vec{g} = -m_G \vec{\nabla}\Phi \quad ; \nabla^2\Phi = 4\pi G_N \rho.$$

* Einstein realized that a theory of general relative motion (involving accelerated observers) would also shed light on the problem of gravitation.

* Geometry: Minkowski space-time and curved spaces.

Mass in Newtonian theory

The gravitational force $\vec{f}$ on a test particle of gravitational mass m_G at some point is $\vec{f} = m_G \vec{g} = -m_G \vec{\nabla}\phi$.

with $\nabla^2 \phi = 4\pi G_N \rho$.

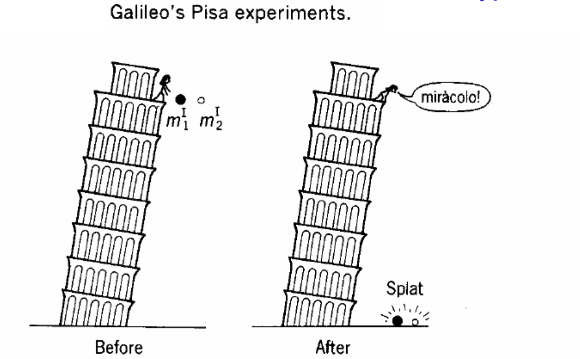
This eq. clearly is not consistent with special relativity.
If ∇^2 is promoted to $\Box^2$ then the resulting expression is also not relativistic $\Box^2 \phi = \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \phi = -4\pi G_N \rho$ because ρ does not transform as a Lorentz scalar.

The eq. of motion of a particle of inertial mass m_I in a gravitational field is

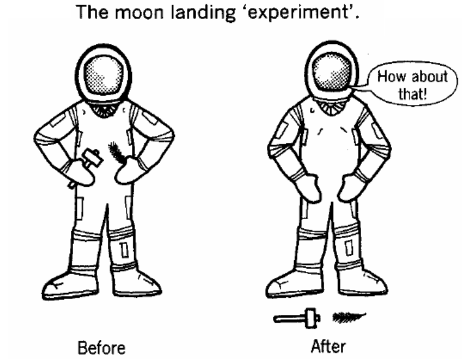
$$\frac{d^2 \vec{x}}{dt^2} = -\frac{m_G}{m_I} \vec{\nabla}\phi .$$

It is a well-established experimental fact that m_G/m_I is the same for all the particles: $\frac{m_{Ga}}{m_{Ia}} = \frac{m_{Gb}}{m_{Ib}} = \alpha$.

Galileo's Pisa experiments.



The moon landing 'experiment'.



The motion of a gravitational test particles in a gravitational field is independent of its mass an composition.

By a suitable choice of units we can take $\alpha = 1$, from which we obtain
Inertial mass = gravitational mass.

This equality has been tested to 1 part in 10^{11}.

In Newtonian theory, this is simply an extraordinary coincidence with no explanation.

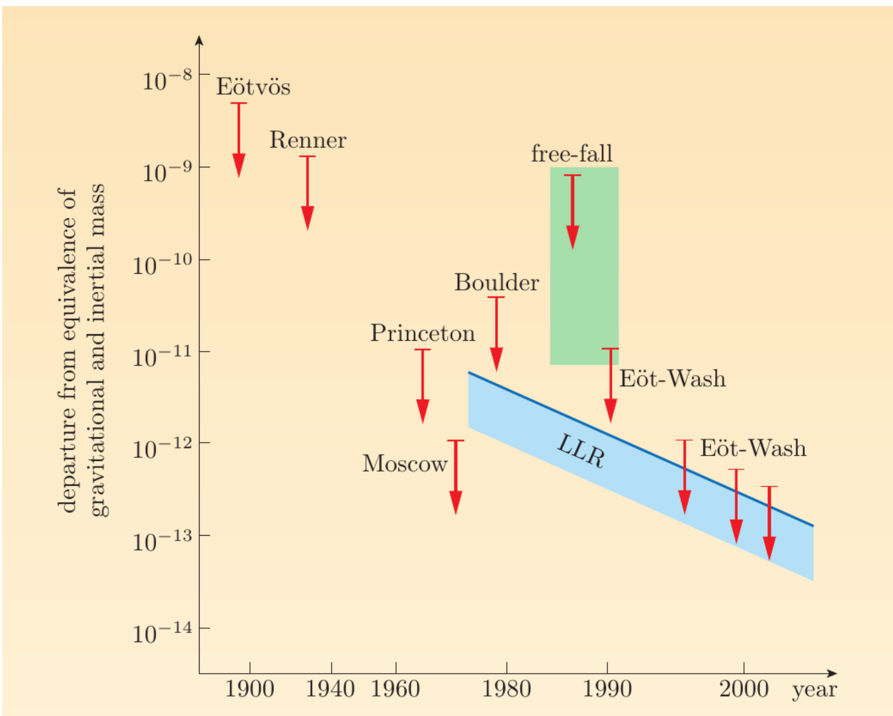
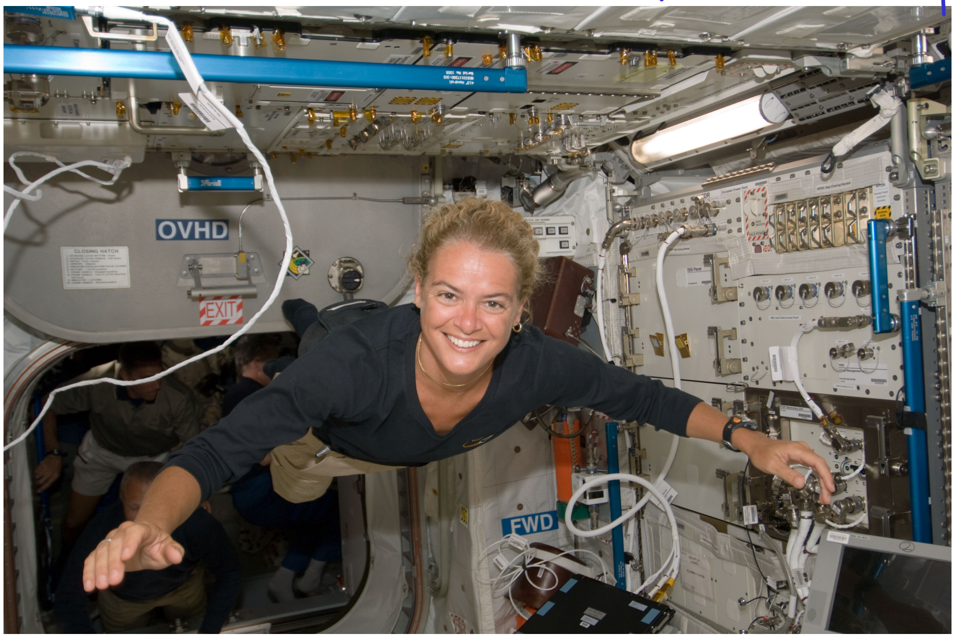


Figure 4.6 Tests of the weak equivalence principle. Most use torsion balances to seek tiny differences in the gravitational and inertial mass of a body, but the green region represents the results of experiments in drop towers, and LLR indicates lunar ranging experiments that compare the acceleration of the Earth and the Moon in the gravitational field of the Sun.

Experimental tests of the strong equivalence principle are much less clear-cut, but most theories that violate it predict that the locally measured value of the gravitational constant, G, may vary with time. Current constraints on the rate of change of G are approaching one part in 10^{13} year^{-1}. Einstein's theory of general relativity is thought to be the only theory of gravity that is consistent with the strong equivalence principle.

Principle of equivalence.

Despite $M_G/M_I = 1$, they seem to suggest of a deeper fundamental fact.



Julie Payette.

Cups, saucers, cannon balls, feathers, and any other objects moving freely within the shuttle remain at rest or in uniform motion with respect to them (neglecting air resistance, etc., as Einstein did). From a study of the motion of such objects over a short period of time the astronauts cannot tell whether they are falling freely in the gravitational field of the Earth or at rest in empty space far from any source of gravitation. In effect, the gravitational field has vanished in the freely falling frame of the space shuttle.

The gravitational field has vanished in the freely falling frame of the space shuttle.

The equality of gravitational and inertial mass is essential to reach this conclusion. If a cannonball and feather fell toward the Earth with different accelerations, they would not remain at rest or in uniform motion with respect to each other in the shuttle's interior. The detection of a small difference in acceleration would suffice to distinguish the presence of a gravitational attraction.

Freely falling frames are locally inertial frames

The equality of gravitational and inertial mass not only implies that a gravitational field can be eliminated by falling freely, but also that one can be created by acceleration.

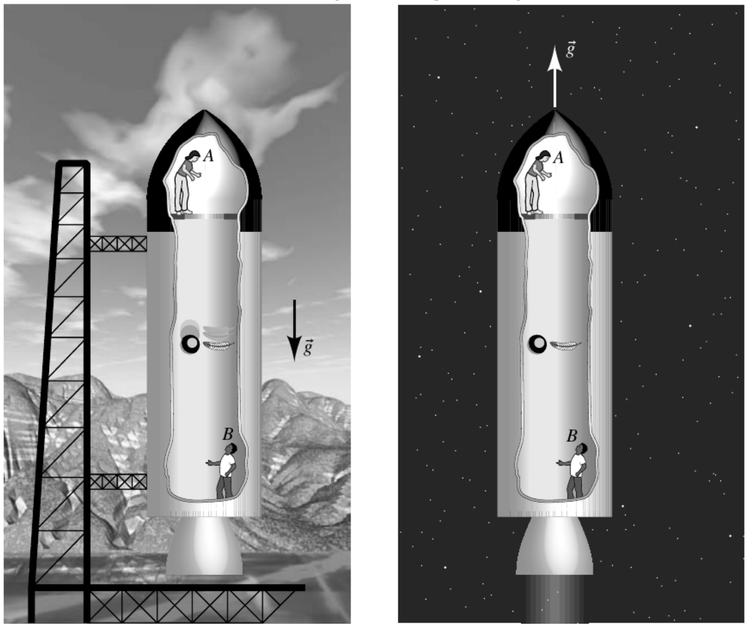


FIGURE 6.4 The equivalence of a uniform acceleration and a uniform gravitational field. On the left is a laboratory at rest on the surface of the Earth. An observer inside lets go of a cannonball and feather. If the gravitational and inertial masses are equal, both fall to the floor with an acceleration g. On the right is a closed laboratory deep in space, far from any sources of gravitational force. The laboratory is being accelerated upward with an acceleration g. An observer inside the laboratory lets go of a cannonball and feather at the same time. Both drop to the floor with acceleration g. An observer inside a closed laboratory cannot distinguish whether they are in one situation or the other.

An experimenter inside who drops a cannonball and a feather will observe that they fall to the floor of the laboratory with equal accelerations—the same result as for the laboratory at rest in a gravitational field.
By this, or any other mechanical experiment with particles, the experimenter inside cannot tell whether the laboratory is unaccelerated in a uniform gravitational field or accelerated in empty space. The two laboratories are equivalent as far as these experiments are concerned.

A reference frame fixed in a freely-falling lift is a locally inertial frame.
Thus, we cannot distinguish between gravity and acceleration.

Weak equivalence principle

Within a sufficiently localized region of spacetime adjacent to a concentration of mass, the motion of bodies subject to gravitational effects alone cannot be distinguished by any experiment from the motion of bodies within a region of appropriate uniform acceleration.

The weak equivalence principle is a direct consequence of the fact that the acceleration of freely falling objects does not depend on their composition, and it is therefore sometimes referred to as the **principle of universality of free fall**. Note that this does not apply to very massive objects that would substantially change the gravitational field in their vicinity. Moreover, it only relates to gravitational forces, so experiments involving electromagnetic forces or nuclear interactions are specifically excluded.

The absence of local experiments that distinguish between uniform acceleration and uniform gravitational fields follows immediately from the equality of gravitational and inertial mass as long as those experiments concern the motion of bodies such as cannonballs and feathers. But what about experiments with photons or neutrinos? What about electromagnetic fields or the fields of quantum chromodynamics? Could the two laboratories be distinguished by these effects? Einstein's equivalence principle is the idea that there is no experiment that can distinguish a uniform acceleration from a uniform gravitational field. The two are fully "equivalent."

Strong equivalence principle

Within a sufficiently localized region of spacetime adjacent to a concentration of mass, the physical behaviour of bodies cannot be distinguished by *any* experiment from the physical behaviour of bodies within a region of appropriate uniform acceleration.

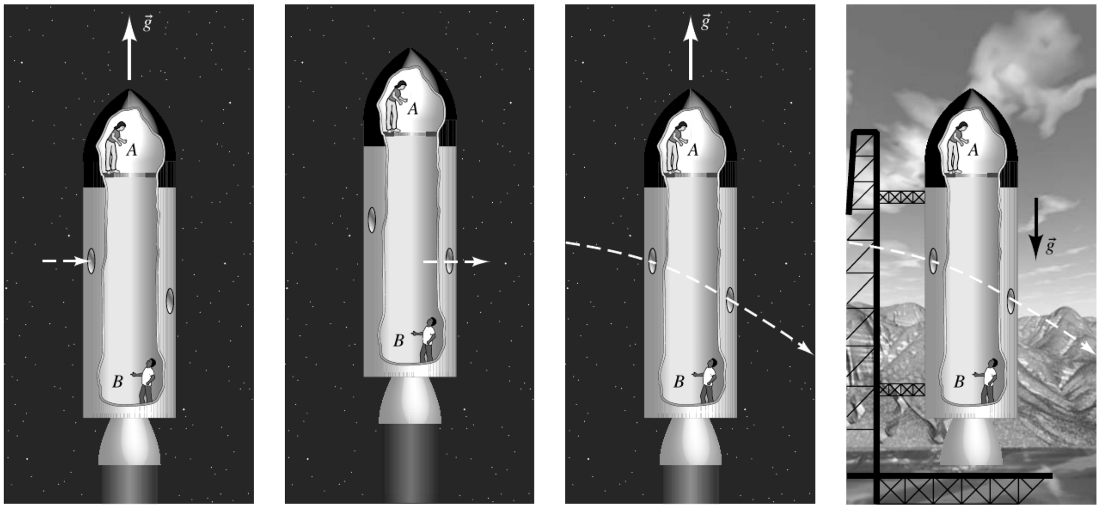
In a freely falling (non-rotating) laboratory occupying a small region of spacetime, the laws of physics are those of special relativity.

All these observations would hold exactly if the gravitational field of the Earth were truly uniform. Of course, the gravitational field of the Earth is not uniform but acts radially inwards towards its centre of mass, with a strength proportional to $1/r^2$. Thus, if the elevator were left to free-fall for a long time or if it were very large (i.e. a significant fraction of the Earth's radius), two particles released from rest near the walls of the elevator would gradually drift inwards, since they would both be falling along radial lines towards the centre of the Earth.
What the observer in the elevator would be experiencing would be the tidal forces resulting from the residual inhomogeneity in the strength and direction of the gravitational field once the main acceleration has been subtracted.
It should always be remembered that these tidal forces can never be completely abolished in an elevator (laboratory) of finite, i.e. non-zero, size.

Predicted effects by Einstein:

• Gravitational deflection of light.

The light rays are deflected towards concentrations of mass!



inertial frame when light ray enters | inertial frame when light ray exits | rocket frame | gravitational field

FIGURE 6.5 A light ray traverses a laboratory accelerating upward in empty space. The first two pictures are views from the inertial frame. The path of the light ray is straight. However, the laboratory moves upward in the time the light ray takes to cross. The exit point of the light ray is, therefore, lower in the laboratory than the entry point. Thus, for an observer in the accelerating laboratory the light ray falls with an acceleration g. The equivalence principle implies that the same observation is made in a laboratory at rest in a uniform gravitational field. A light ray in a gravitational field must fall with the same acceleration as other objects. Gravity attracts light.

• Gravitational redshift of light.

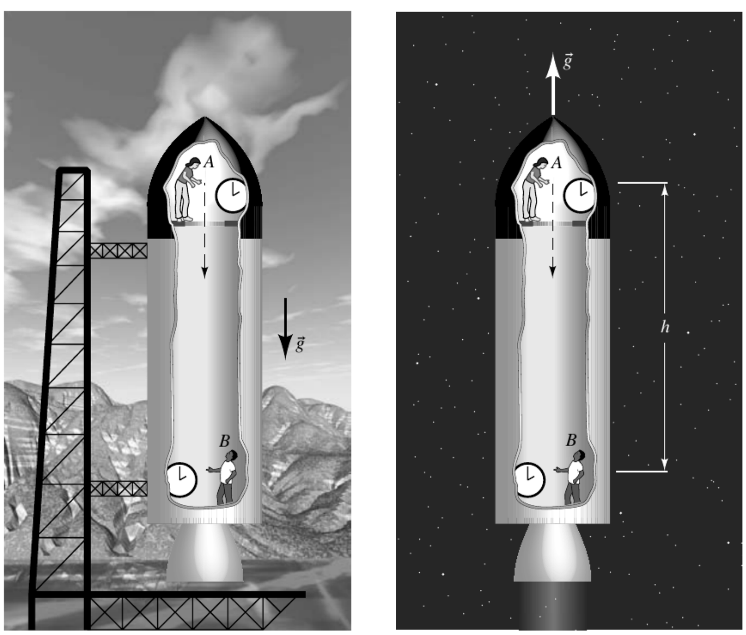


FIGURE 6.6 On the left is a rocket at rest in a uniform gravitational field. Alice, in the nose of the rocket, emits signals at equal intervals on a clock at her height. Bob, in the tail, measures the time interval between receipt of the signals on an identical clock at his location. The equivalence principle implies that the relation between the intervals of emission and reception must be the same as if the rocket ship were accelerating vertically upward far from any source of gravitational attraction, as shown at right. There signals are received at shorter intervals than they are emitted because the accelerating tail is catching up with the signals. The equivalence principle implies that in the gravitational field, the signals are received at a faster rate than they are emitted.

Freely falling (accelerated) observer is equivalent to a locally inertial observer is deeply related with coordinate transformations, much broader than the Lorentz transformations.

Principle of covariance.

According to the principle of relativity, the laws of physics should take the same form in all inertial frames.

The principle of covariance extends the principle of relativity by requiring the physical equivalence of all frames, including non-inertial ones.

Principle of consistency.

A new theory that aims to replace or supersede earlier theories should account for the successful predictions of those earlier theories.
For GR, it should predict in the limit of zero curvature the results of

1. Special relativity.

The special relativity limit is guaranteed by using a spacetime that is locally equivalent to Minkowski spacetime.

2. Newtonian gravitation.

This limit provides a useful constraint on the kinds of tensor equations that can be used in the formulation of G.R.