Planck units

$$\ell_P \equiv \left(\frac{G\hbar}{c^3}\right)^{1/2} = 1.62 \times 10^{-35}\,\text{m.} \qquad M_P \equiv \left(\frac{\hbar c}{G}\right)^{1/2} = 2.18 \times 10^{-8}\,\text{kg,} \qquad t_P \equiv \left(\frac{G\hbar}{c^5}\right)^{1/2} = 5.39 \times 10^{-44}\,\text{s.}$$

$$E_P = M_P c^2 = 1.96 \times 10^9\,\text{J} = 1.22 \times 10^{28}\,\text{eV.} \qquad T_P = E_P/k = 1.42 \times 10^{32}\,\text{K.}$$

When distance, mass, time, and temperature are measured in the appropriate Planck units, then $c = k = \hbar = G = 1$. This is convenient for individuals who have difficulty in remembering the numerical values of physical constants.

Fundamental Observations

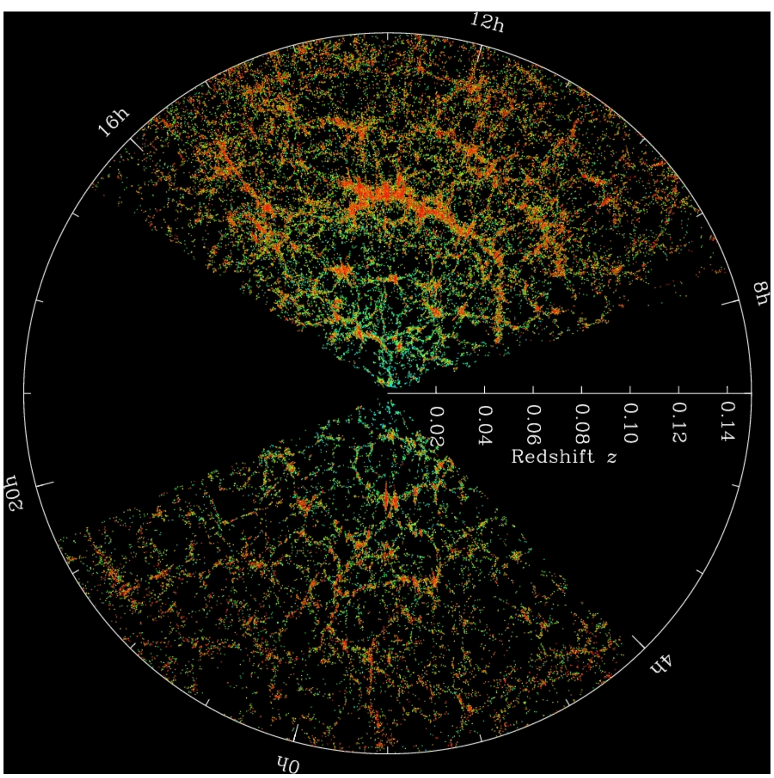
The Universe is Isotropic and Homogeneous

What does it mean to state that the universe is isotropic and homogeneous? Saying that the universe is *isotropic* means that there are no preferred directions in the universe; it looks the same no matter which way you point your telescope. Saying that the universe is *homogeneous* means that there are no preferred locations in the universe; it looks the same no matter where you set up your telescope. Note the very important qualifier: the universe is isotropic and homogeneous *on large scales*. In this context, "large scales" means that the universe is only isotropic and homogeneous on scales of roughly 100 Mpc or more.

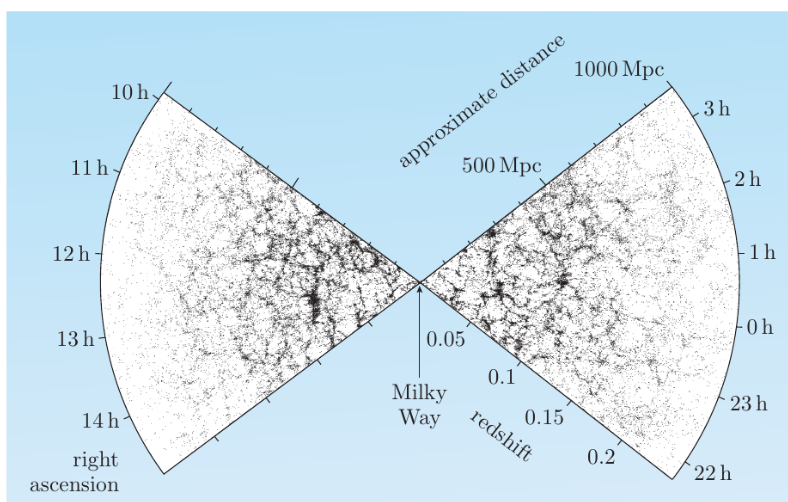
On small scales, the universe is obviously inhomogeneous, or lumpy, in addition to being anisotropic. For instance, a sphere 3 m in diameter, centered on your navel, will have an average density of ~ 100 kg m^{-3}, in round numbers. However, the average matter density of the universe as a whole is $\rho_0 \approx 2.7 \times 10^{-27}$ kg m^{-3}. Thus, on a scale d ~ 3 m, the patch of the universe surrounding you is more than 28 orders of magnitude denser than average.

Figure 2.3 Left: a pattern that is anisotropic, but is homogeneous on scales larger than the stripe width. Right: a pattern that is isotropic about the origin, but is inhomogeneous.

A map of galaxy positions in a narrow slice of the Universe, as measured by the Sloan Digital Sky Survey. Our galaxy is located at the centre, and the survey radius is around 600 Mpc. The galaxy positions were obtained by measurement of the shift of spectral lines.



The distribution of galaxies reported by the 2dF survey.



Redshift is Proportional to Distance

$$z \equiv \frac{\lambda_{ob} - \lambda_{em}}{\lambda_{em}}.$$

Strictly speaking, when $z < 0$, this quantity is called a blueshift, rather than a redshift. However, the vast majority of galaxies have $z > 0$.

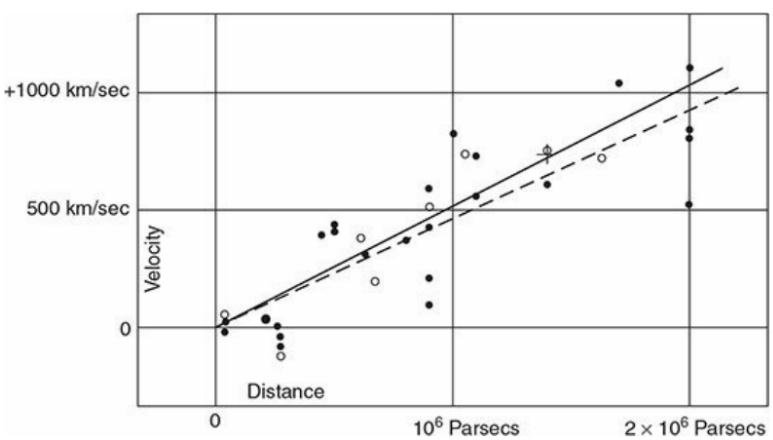
The astronomer Edwin Hubble invested a great deal of effort into measuring the distances to galaxies. By 1929, he had estimated distances for a sample of 20 galaxies whose value of z had been measured. Figure shows Hubble's plot of redshift (z) versus distance (r) for these galaxies. He noted that the more distant galaxies had higher redshifts, and fitted the data with the famous linear relation now known as Hubble's law:

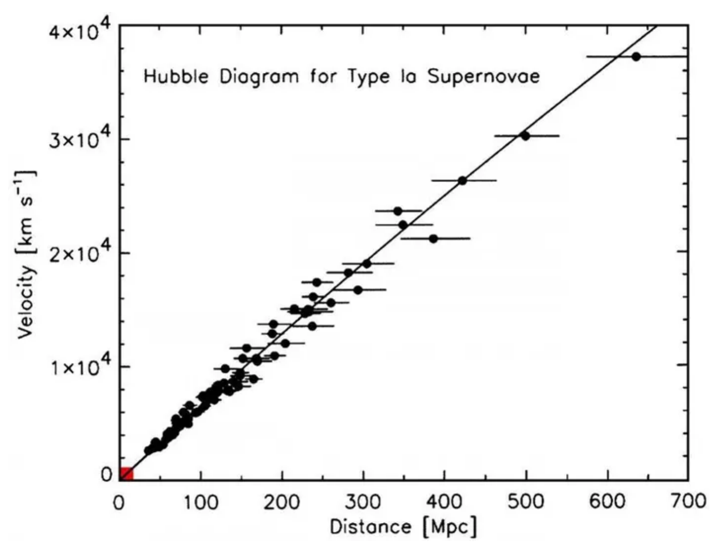
$$z = \frac{H_0}{c} r,$$

where H0 is a constant (now called the Hubble constant). Interpreting the redshifts as Doppler shifts, Hubble's law takes the form

$$\frac{\lambda_{obs}}{\lambda_{em}} = \frac{(1+\beta)^{1/2}}{(1-\beta)^{1/2}} = z + 1 \simeq (1 + \beta/2)^2 \approx (1+\beta) \ ; \quad \beta \ll 1 .$$

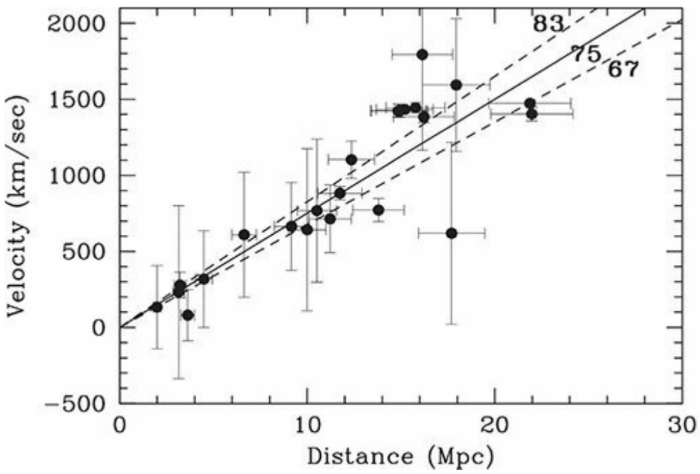
$$\Rightarrow \quad z = \frac{H_0}{c} r = \frac{v}{c} \quad \Rightarrow \quad \boxed{v = H_0 r.}$$

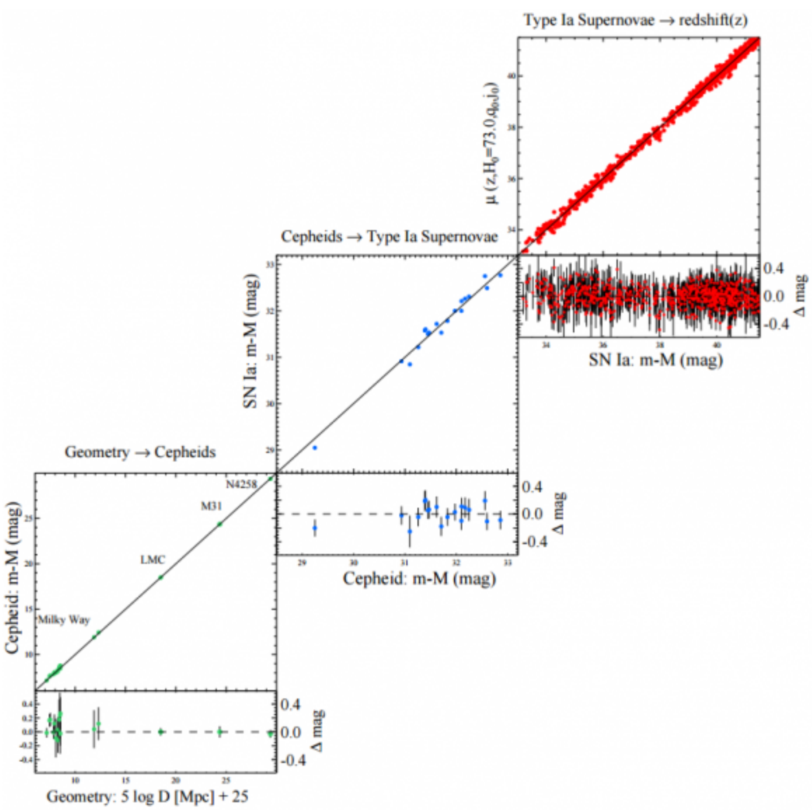


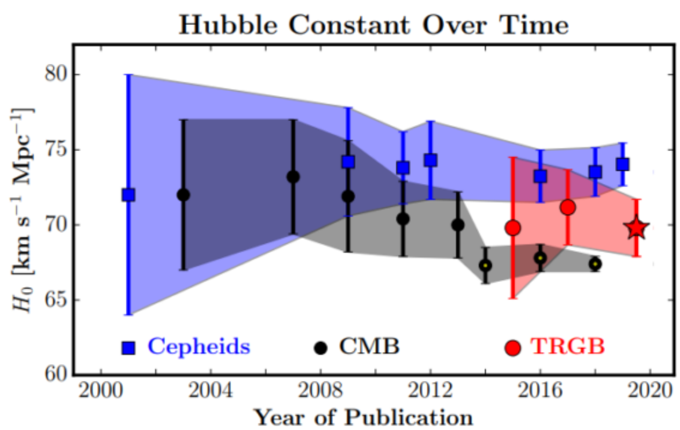


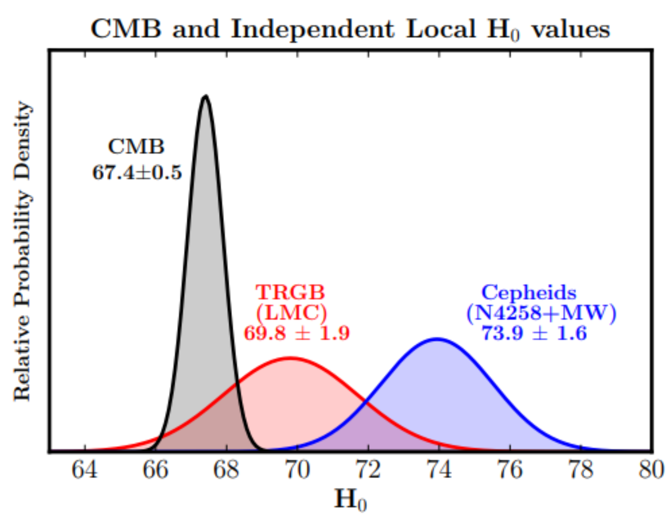
This figure shows a more recent determination of the Hubble constant from nearby galaxies, using data obtained using the Hubble Space Telescope. Notice that galaxies with a radial velocity v = cz ≈ 1000 km/s, which Hubble thought were at a distance r ≈ 2 Mpc, are now more accurately placed at a distance r ≈ 15 Mpc. The best current estimate of the Hubble constant, combining the results from various research techniques, is

$$H_0 = 68 \pm 2\,\mathrm{km\,s^{-1}\,Mpc^{-1}}.$$







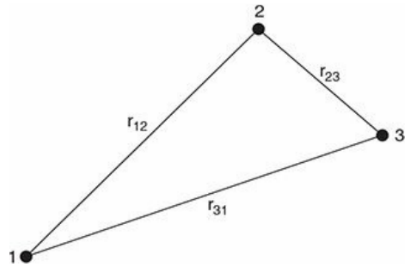


To see on a more mathematical level what we mean by homogeneous, isotropic expansion, consider three galaxies at positior $\vec{r}_1$, $\vec{r}_2$, and $\vec{r}_3$.

$$r_{12} \equiv |\vec{r}_1 - \vec{r}_2|$$

$$r_{23} \equiv |\vec{r}_2 - \vec{r}_3|$$

$$r_{31} \equiv |\vec{r}_3 - \vec{r}_1| \,.$$



Homogeneous and uniform expansion means that the shape of the triangle is preserved as the galaxies move away from each other. Maintaining the correct relative lengths for the sides of the triangle requires an expansion law of the form

$$r_{12}(t) = a(t) r_{12}(t_0)$$

$$r_{23}(t) = a(t) r_{23}(t_0)$$

$$r_{31}(t) = a(t) r_{31}(t_0).$$

Here the function a(t) is a scale factor, equal to one at the present moment (t= t0) and totally independent of location or direction. The scale factor a(t) tells us how the expansion (or possibly contraction) of the universe depends on time. At any time t, an observer in galaxy 1 will see the other galaxies receding with a speed

$$v_{12}(t) = \frac{dr_{12}}{dt} = \dot{a} r_{12}(t_0) = \frac{\dot{a}}{a} r_{12}(t)$$

$$v_{31}(t) = \frac{dr_{31}}{dt} = \dot{a} r_{31}(t_0) = \frac{\dot{a}}{a} r_{31}(t).$$

An observer in galaxy 2 or galaxy 3 will find the same linear relation between observed recession speed and distance, with (1/a) da/dt playing the role of the Hubble constant. Since this argument can be applied to any trio of galaxies, it implies that in any universe where the distribution of galaxies is undergoing homogeneous, isotropic expansion, the velocity–distance relation takes the linear form v = Hr, with

$$H = \dot{a}/a.$$

If galaxies are currently moving away from each other, then it implies they were closer together in the past. Consider a pair of galaxies currently separated by a distance r, with a velocity v = H0 r relative to each other. If there are no forces acting to accelerate or decelerate their relative motion, then their velocity is constant, and the time that has elapsed since they were in contact is

$$t_0 = \frac{r}{v} = \frac{r}{H_0 r} = H_0^{-1};$$

Hubble time

$$t_0 = 14.38 \pm 0.42 \text{ Gys.}$$

A Big Bang model may be broadly defined as a model in which the universe expands from an initially highly dense state to its current low-density state.

The Hubble time of ~ 14.4 Gyr is comparable to the ages computed for the oldest known stars in the universe. This rough equivalence is reassuring. However, the age of the universe – that is, the time elapsed since its original highly dense state – is not necessarily exactly equal to the Hubble time. We know that gravity exists, and that galaxies contain matter.

If gravity working on matter is the only force at work on large scales, then the attractive force of gravity will act to slow the expansion. In this case, the universe was expanding more rapidly in the past than it is now, and the universe is somewhat younger than 1/H0.

On the other hand, if the energy density of the universe is dominated by a cosmological constant, then the dominant gravitational force is repulsive, and the universe may be older than 1/H0.

Hubble distance:

- $d_H = H_0^{-1} = 4380 \pm 130$ Mpc.
- $d_{light} = c t_0 = H_0^{-1}$.

In a Big Bang model, the properties of the universe evolve with time; the average density decreases, the mean distance between galaxies increases, and so forth. However, Hubble's law can also be explained by a Steady State model. The Steady State model was first proposed in the 1940s by Hermann Bondi, Thomas Gold, and Fred Hoyle, who were proponents of the perfect cosmological principle, which states that not only are there no privileged locations in space, there are no privileged moments in time. Thus, a Steady State universe is one in which the global properties of the universe, such as the mean density ρ0 and the Hubble constant H0, remain constant with time.

Si $H_0 = \text{cte} \Rightarrow \frac{dr}{dt} = H_0 r \Rightarrow r(t) \sim e^{H_0 t}$

$r \to 0$ pour $t \to -\infty$: a Steady State universe is infinitely old

$dM/dt/V = 3 H_0 \rho_0 = 5.6 \times 10^{-28}\ \mathrm{kg\,m^{-3}\,Gyr^{-1}}$

This corresponds to creating roughly one hydrogen atom per cubic kilometer per year.

matter density $\rho_0 \approx 2.7 \times 10^{-27}$ kg m^{-3}

Photons

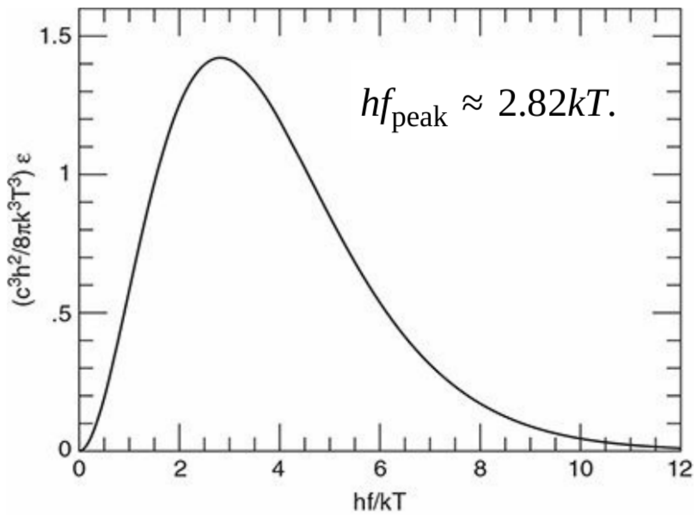
$$\lambda = c/f. \qquad E_\gamma = hf, \qquad h = 2\pi\,\hbar$$

Photons, in general, are easily created. One way to make photons is to take a dense, opaque object – such as the filament of an incandescent lightbulb – and heat it up. If an object is opaque, then the protons, neutrons, electrons, and photons that it contains frequently interact, and attain thermal equilibrium; that is, they all have the same temperature T. The density of photons in the object, as a function of photon energy, will depend only on T. It doesn't matter whether the system is a tungsten filament, an ingot of steel, or a sphere of ionized hydrogen and helium. The energy density of photons in the frequency range f → f + df is given by the blackbody function

$$\varepsilon(f)df = \frac{8\pi h}{c^3}\frac{f^3\,df}{\exp(hf/kT)-1},$$

$$\varepsilon_\gamma = \alpha T^4,$$

$$\alpha = \frac{\pi^2}{15}\frac{k^4}{\hbar^3 c^3} = 7.566\times 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}.$$



Since the energy of a photon is E γ = hf, the number density of photons in the frequency range f → f + df is,

$$n(f)df = \frac{\varepsilon(f)df}{hf} = \frac{8\pi}{c^3}\frac{f^2\,df}{\exp(hf/kT)-1}.$$

Integrated over all frequencies, the number density of photons in blackbody radiation is

$$n_\gamma = \beta T^3, \qquad \beta = \frac{2.4041}{\pi^2}\frac{k^3}{\hbar^3 c^3} = 2.029\times 10^7\,\mathrm{m^{-3}\,K^{-3}}.$$

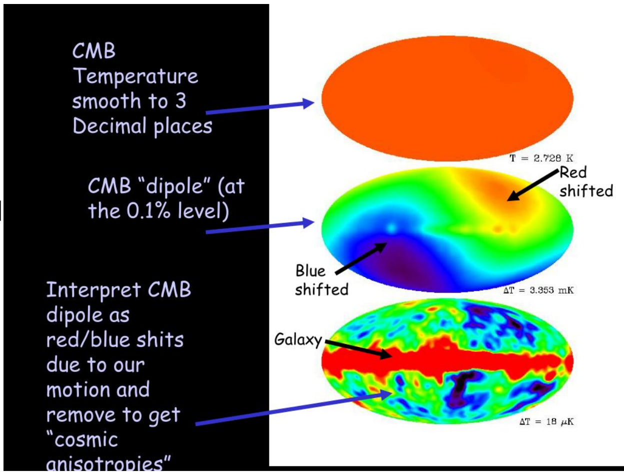
$\rho_\gamma(T) = \frac{\pi^2}{30} g_\gamma T^4$; $n_\gamma(T) = \frac{\zeta(3)}{\pi^2} g_\gamma T^3$; $n_\nu(T) = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_\nu T^3$.

$\bar{E}_\gamma = \frac{\rho_\gamma(T)}{n_\gamma(T)} = h\bar{f}_\gamma = 2.7\,k_B T$

For $T = 310$ K, $\bar{E}_\gamma = 0.072$ eV $\Rightarrow \lambda = 1.7\times 10^{-5}$ m mid-infrared.

For $T_{sun} = 5800$ K ; $\bar{E}_\gamma = 1.3$ eV $\Rightarrow \lambda = 9\times 10^{-7}$ m $= 900$ nm near infrared.

Cosmic Microwave Background

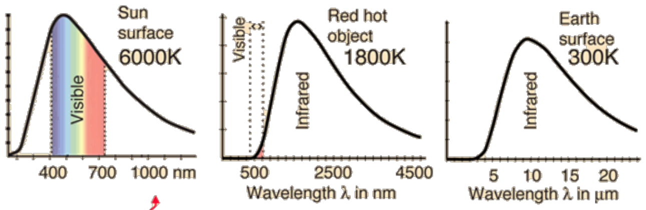


The discovery of the cosmic microwave background (CMB) by Arno Penzias and Robert Wilson in 1965 has entered cosmological folklore. Using a microwave antenna at Bell Labs, they found an isotropic background of microwave radiation. More recently, space-based experiments have revealed that the cosmic microwave background is exquisitely well fitted by a blackbody spectrum with a temperature

$$T_0 = 2.7255 \pm 0.0006\,\mathrm{K}.$$

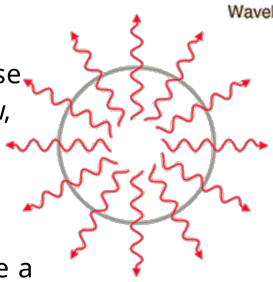
The energy density of the CMB is,

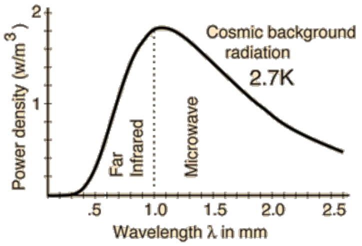
$$\varepsilon_\gamma = 4.175 \times 10^{-14}\,\mathrm{J\,m^{-3}} = 0.2606\,\mathrm{MeV\,m^{-3}}.$$



The number density of CMB photons is,

$$n_\gamma = 4.107 \times 10^{8}\,\mathrm{m^{-3}}.$$



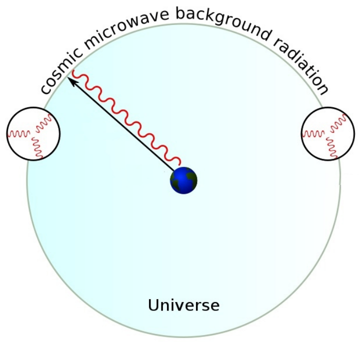


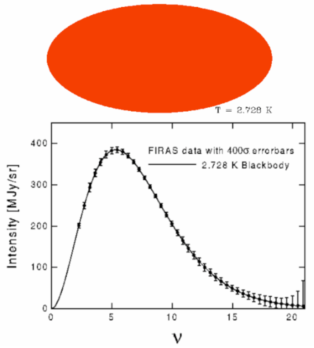
Thus, there are about 411 CMB photons per cubic centimeter of the universe at the present day. The mean energy of CMB photons, however, is quite low,

$$E_{\mathrm{mean}} = 6.344 \times 10^{-4}\,\mathrm{eV}.$$

This is too low in energy to photoionize an atom, much less photodissociate a nucleus. The mean CMB photon energy corresponds to a wavelength of 2 millimeters, in the microwave region of the electromagnetic spectrum – hence the name “cosmic microwave background.”

In a Big Bang universe, however, a cosmic background radiation arises naturally if the universe was initially very hot as well as very dense. If mass is conserved in an expanding universe, then in the past the universe was denser than it is now. Assume that the early dense universe was very hot (T ≫ 104 K, or kT ≫ 1 eV). At such high temperatures, the baryonic matter in the universe was completely ionized, and the free electrons rendered the universe opaque. A dense, hot, opaque body, produces blackbody radiation. So, the early hot dense universe was full of photons, banging off the electrons like balls in a pinball machine, with a spectrum typical of a blackbody. However, as the universe expanded, it cooled. Eventually, the temperature became sufficiently low that ions and electrons combined to form neutral atoms. When the universe no longer contained a significant number of free electrons, the blackbody photons started streaming freely through the universe, without further scattering off free electrons.





$$E_{\text{mean}} = 6.344 \times 10^{-4}\,\text{eV}.$$ $\lambda \approx 2\,mm$ microwave.

$T_{CMB} = 2970\,K \Rightarrow T_{CMB}/T^{0}_{CMB} = 1090$

$$dQ = dE + PdV,$$

where dQ is the amount of heat flowing into or out of the photon gas in the volume V, dE is the change in the internal energy, P is the pressure, and dV is the change in volume of the box. Since, in a homogeneous universe, there is no net flow of heat (everything is the same temperature, after all), dQ = 0. Thus, the first law of thermodynamics, applied to an expanding homogeneous universe, is

$$\frac{dE}{dt} = -P(t)\frac{dV}{dt}.$$

Since, for the photons of the CMB, $E = \varepsilon_\gamma V = \alpha T^4 V$ and $P = P_\gamma = \alpha T^4/3$,

$$\alpha\left(4T^3\frac{dT}{dt}V + T^4\frac{dV}{dt}\right) = -\frac{1}{3}\alpha T^4\frac{dV}{dt},$$

$$\frac{1}{T}\frac{dT}{dt} = -\frac{1}{3V}\frac{dV}{dt}.$$ $V \propto a^3$ $$\frac{d}{dt}(\ln T) = -\frac{d}{dt}(\ln a).$$

$$T(t) \propto a(t)^{-1};$$

The temperature of the cosmic background radiation has dropped by a factor of 1090 since the universe became transparent, because the scale factor a(t) has increased by a factor of 1090 since then.
What we now see as a cosmic microwave background was once, at the time the universe became transparent, a cosmic near infrared background, with a temperature comparable to that of a relatively cool star like Proxima Centauri.

$$T(t) \approx 10^{10}\,\text{K}\left(\frac{t}{1\,\text{s}}\right)^{-1/2}, \qquad kT(t) \approx 1\,\text{MeV}\left(\frac{t}{1\,\text{s}}\right)^{-1/2}.$$

One second after the Big Bang the temperature of the universe was therefore about 1 MeV (or 10^11 K). While there was very little time available in the early universe, the rates of reactions were extremely high, so that many things happened in a short amount of time.

Different Types of Particles

FERMIONS
matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2			Quarks spin = 1/2		
Flavor	Mass GeV/c^2	Electric charge	Flavor	Approx. Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0	u up	0.003	2/3
e electron	0.000511	–1	d down	0.006	–1/3
ν_μ muon neutrino	<0.0002	0	c charm	1.3	2/3
μ muon	0.106	–1	s strange	0.1	–1/3
ν_τ tau neutrino	<0.02	0	t top	175	2/3
τ tau	1.7771	–1	b bottom	4.3	–1/3

BOSONS
force carriers
spin = 0, 1, 2, ...

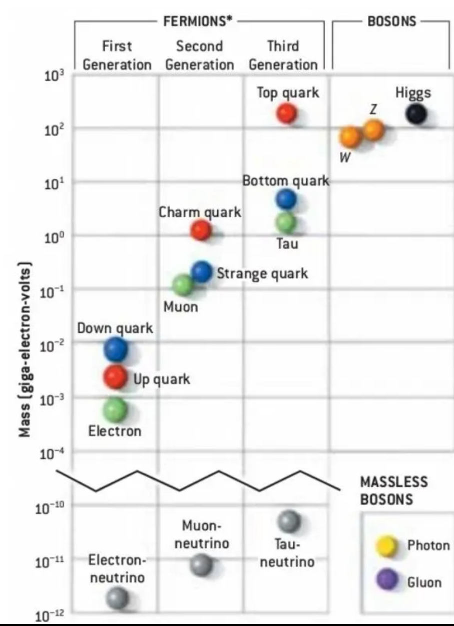
Unified Electroweak spin = 1			Strong (color) spin = 1		
Name	Mass GeV/c^2	Electric charge	Name	Mass GeV/c^2	Electric charge
γ photon	0	0	g gluon	0	0
W^-	80.4	–1	H	125	0
W^+	80.4	+1	Higgs	spin	0
Z^0	91.187	0			

Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$
Baryons are fermionic hadrons.
There are about 120 types of baryons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	–1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	–1	1.672	3/2

Mesons $q\bar{q}$
Mesons are bosonic hadrons.
There are about 140 types of mesons.

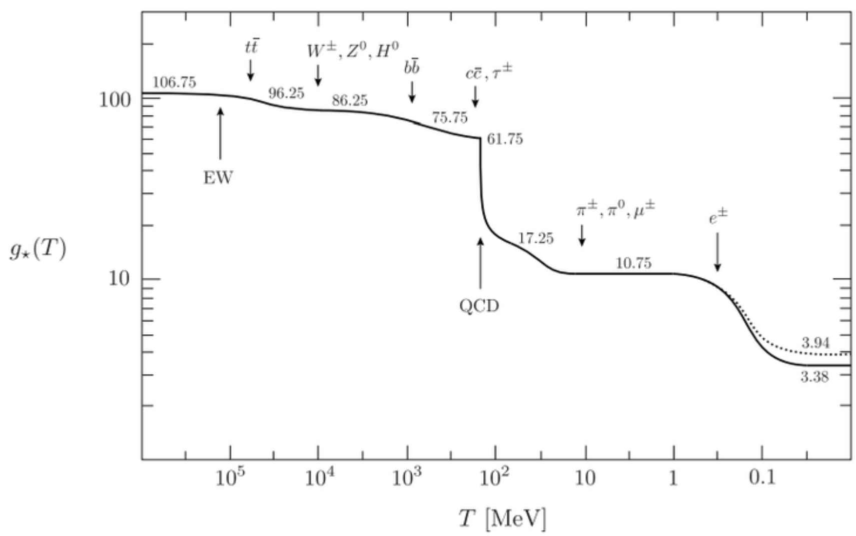
Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	–1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2 .980	0

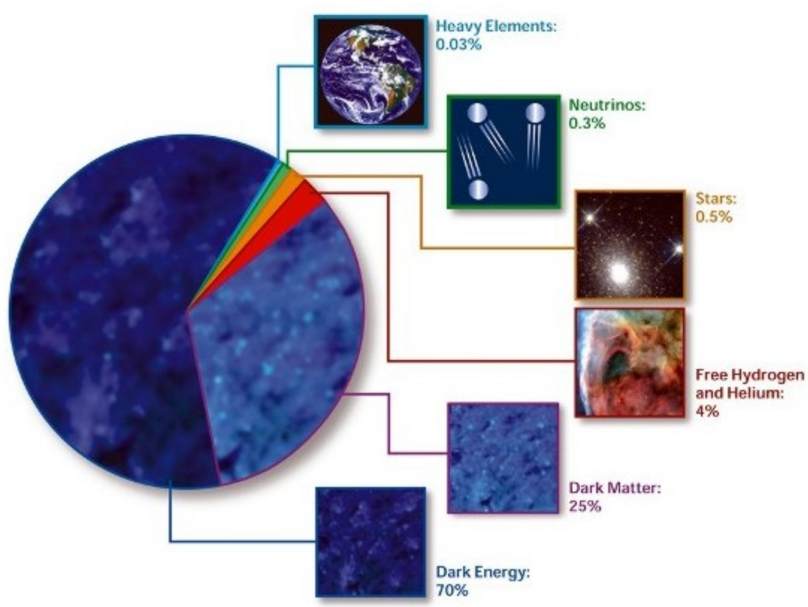


PROPERTIES OF THE INTERACTIONS

Property \ Interaction	Gravitational	Weak (Electroweak)	Electromagnetic (Electroweak)	Strong: Fundamental	Strong: Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons	Mesons
Strength relative to electromag for two u quarks at: 10^{-18} m	10^{-41}	0.8	1	25	Not applicable to quarks
Strength relative to electromag for two u quarks at: 3×10^{-17} m	10^{-41}	10^{-4}	1	60	
for two protons in nucleus	10^{-36}	10^{-7}	1	Not applicable to hadrons	20

Relativistic degrees of freedom





A grandes escalas $n_{e^-} = n_{p^+}$.

Como $m_p/m_e = 1836 \Rightarrow$ materia ordinaria = bariónica.

$\tau_n = 880s$

ρ_B: H, He $\quad M_{He}/M_H \simeq 0.24$. $\quad \tau_p > 10^{29} H_0^{-1}$.

Dark matter

Any massive component of the universe which doesn't emit, absorb, or scatter light at all.

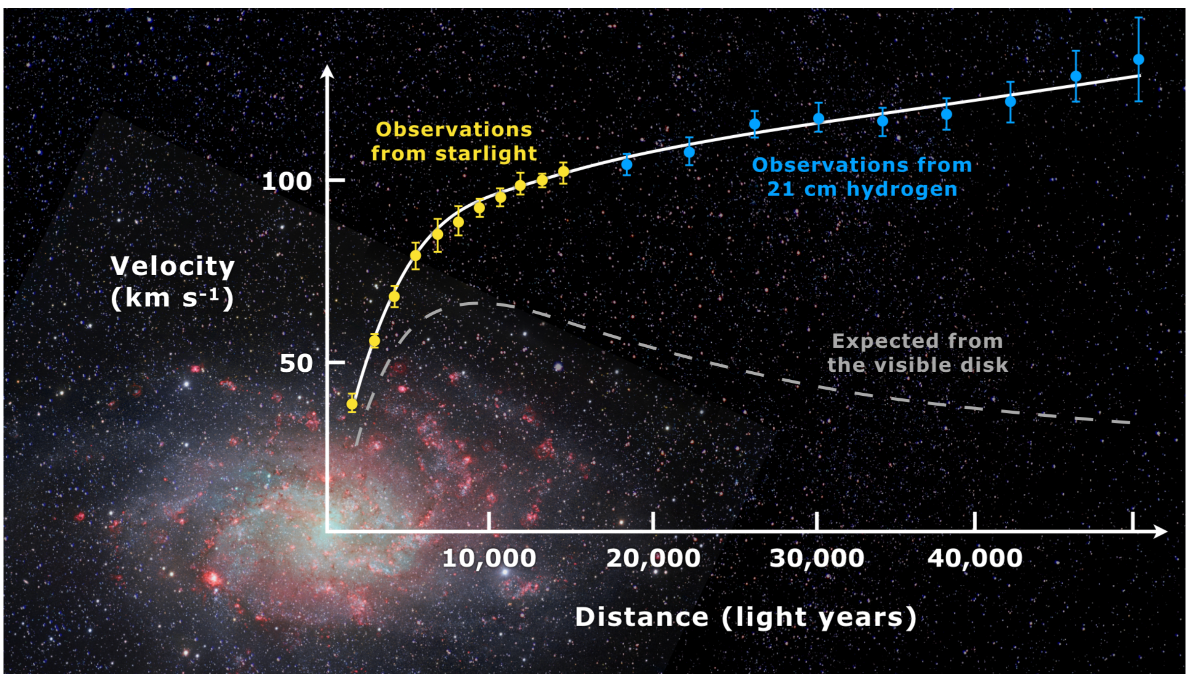
In some extensions to the Standard Model of particle physics, there exist massive particles that interact, like neutrinos, only through the weak nuclear force and through gravity. These particles, which have not yet been detected in the laboratory, are generically referred to as weakly interacting massive particles, or WIMPs.

There is a large amount of astronomical evidence indicating that there is more matter than what can be associated with the luminous parts of the galaxies.
For example, the orbital velocity of stars and gas clouds as a function of their distance from the galaxy core in spiral galaxies contradicts Kepler's third law unless the distribution of mass extends far beyond the visible galaxy core.
For distances beyond the visible galaxy disk, the orbital velocities should decrease with the distance as

$$v_{orb} \propto r^{-\frac{1}{2}}$$

Instead, the observations indicate that the rotation curves flatten out, that is

$$v_{orb} \approx \text{const}$$



According to Kepler's third law: $GM(r) = v_{orb}^2 \cdot r \longrightarrow v_{orb} = \sqrt{\frac{GM}{r}}$.

In fact, the amount of dark matter needed to explain the rotation curves may be in excess of what is consistent with the limits on baryonic matter from primordial nucleosynthesis. Although some fraction of the 'invisible' matter is certainly baryonic – for example, faint stars and black holes – other dark matter candidates have been proposed, for example exotic putative particles produced in connection with the Big Bang such as axions, heavy neutrinos, dark photons, dark higgses, monopoles or primordial black holes.

Key events in the history of the universe.

Event	temperature	energy	time
Inflation	$< 10^{28}$ K	$< 10^{16}$ GeV	$> 10^{-34}$ s
Dark matter decouples	?	?	?
Baryons form	?	?	?
EW phase transition	10^{15} K	100 GeV	10^{-11} s
Hadrons form	10^{12} K	150 MeV	10^{-5} s
Neutrinos decouple	10^{10} K	1 MeV	1 s
Nuclei form	10^{9} K	100 keV	200 s
Atoms form	3400 K	0.30 eV	260 000 yrs
Photons decouple	2900 K	0.25 eV	380 000 yrs
First stars	50 K	4 meV	100 million yrs
First galaxies	20 K	1.7 meV	1 billion yrs
Dark energy	3.8 K	0.33 meV	9 billion yrs
Einstein born	2.7 K	0.24 meV	13.8 billion yrs