Early Universe: Out of Equilibrium dynamics

Disclaimer

Discussions taken from Barbara Ryden [1], Daniel Baumann [2] and Kolb & Turner [3] books

1 Temperature in radiation domination

For $a < a_{RM} \approx 3400$, the scale factor varies with time as

$$a(t) \propto t^{1/2}, \tag{1}$$

such that

$$H_R(t) = \frac{1}{2t}. \tag{2}$$

On the other hand,

$$H_R(T) = \sqrt{\frac{\epsilon_R}{3M_p^2}} = \sqrt{\frac{\pi^2 g_*}{90}}\frac{T^2}{M_P}. \tag{3}$$

Therefore

$$T \approx 1.31 \times 10^{10}\,\mathrm{K}\left(\frac{1\,\mathrm{s}}{t}\right)^{1/2} \tag{4}$$

$$= 1.1341\,\mathrm{MeV}\left(\frac{1\,\mathrm{s}}{t}\right)^{1/2}. \tag{5}$$

The mean energy per photon is

$$\overline{E_\gamma} \approx 2.7\,k_B\,T \approx 3\,\mathrm{MeV}\left(\frac{1\,\mathrm{s}}{t}\right)^{1/2}. \tag{6}$$

Typical energies in atomic physics:

$$E_I(\mathrm{H}) = 13.6\,\mathrm{eV};\quad E_I(\mathrm{He}) = 24.6\,\mathrm{eV};\quad E_I(\mathrm{Cs}) \approx 4\,\mathrm{eV}. \tag{7}$$

Nuclear physics deals with the much higher energy processes of fission and fusion (splitting or merging atomic nuclei). The binding energy B of a nucleus is the energy required to pull it apart into its component protons and neutrons. Equivalently, it is the energy released when a nucleus is fused together from individual protons and neutrons. For instance, when a neutron and a proton are bound together to form a deuteron, an energy of $B_D = 2.22$ MeV is released

$$p^+ + n \leftrightarrow D + 2.22\,\mathrm{MeV}. \tag{8}$$

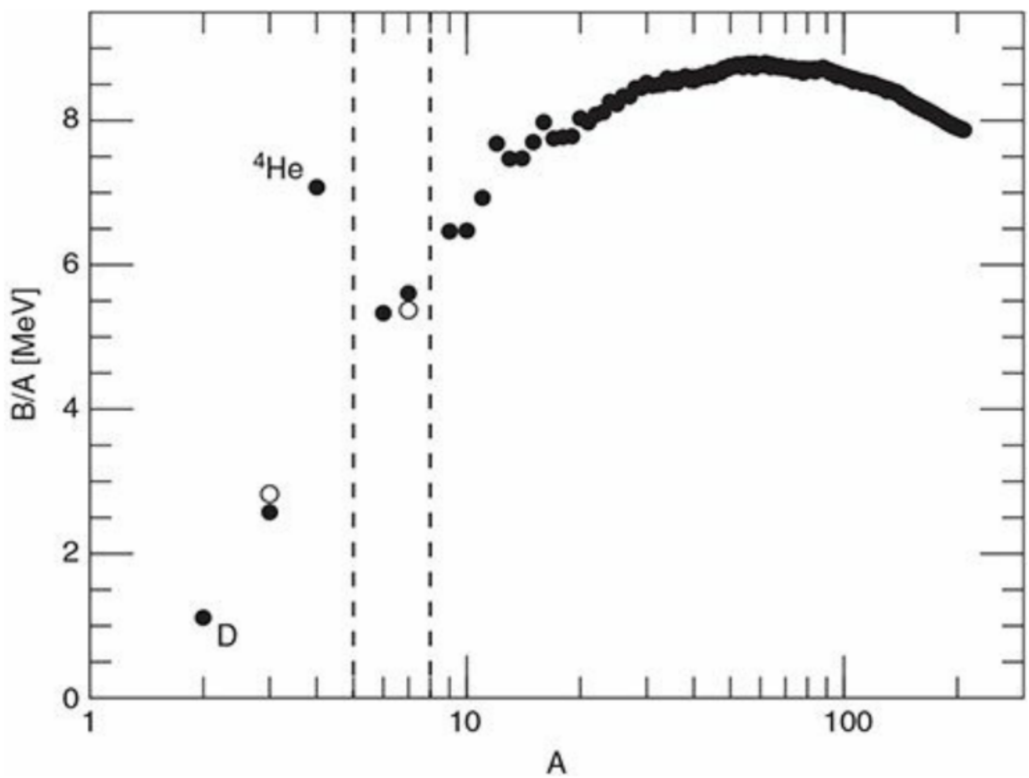

Figure 1: Binding energy per nucleon (B/A) as a function of the number of nucleons (A). Stable isotopes are shown as solid dots; the open dots represent the isotopes ^{3}H and ^{7}Be.

2 Neutron-to-proton ratio

2.1 Equilibrium at $t = 0.1$ s

At early times, the baryonic matter in the universe was mostly in the form of protons and neutrons, which were coupled to each other by processes mediated by the weak interaction, such as β-decay and inverse β-decay:

$$n + \nu_e \leftrightarrow p^+ + e^-, \tag{9}$$
$$n + e^+ \leftrightarrow p^+ + \bar{\nu}_e. \tag{10}$$

Above $T = 0.1$ MeV, only free protons and neutrons existed, while other light nuclei hadn't been formed yet. We can therefore first solve for the neutron-to-proton ratio and then use this abundance as an input for the synthesis of deuterium, helium, etc.

At $t = 0.1$ s the temperature was $T \approx 10^{10}$ K and $\overline{E_\gamma} \approx 10$ MeV $> m_e$. Hence the pair production and annihilation was in chemical equlibrium:

$$\gamma + \gamma \leftrightarrow e^- + e^+. \tag{11}$$

In summary, at this early time, all particles, including protons and neutrons, were in kinetic equilibrium. Hence

$$\mu_n + \mu_\nu = \mu_p + \mu_e. \tag{12}$$

Assuming that $\mu_\nu = \mu_e \approx 0$ we have that $\mu_n = \mu_p$. Thus, the ratio of the number densities of neutrons and protons is given by

$$\left(\frac{n_n}{n_p}\right)_{\text{eq}} = \left(\frac{m_n}{m_p}\right)^{3/2} e^{-(m_n - m_p)/T} \approx e^{-\mathcal{Q}_n/T}. \tag{13}$$

2.2 Boltzmann equation and neutron freeze-out

Let the neutron fraction

$$X_n \equiv \frac{n_n}{n_n + n_p}. \tag{14}$$

In equilibrium

$$X_n^{\text{eq}} = \frac{e^{-\mathcal{Q}_n/T}}{1 + e^{-\mathcal{Q}_n/T}}. \tag{15}$$

Neutrons follow this equilibrium abundance until weak interaction processes such as β-decay effectively shut off at a temperature of around 1 MeV (roughly at the same time as neutrino decoupling).

To track the evolution let's solve the Boltzmann equation identifying $n_1 = n_n, n_3 = n_p$ and $n_2 = n_4 = n_\ell = n_\ell^{\text{eq}}$:

$$\frac{1}{a^3}\frac{d(n_n a^3)}{dt} = -\Gamma_n \left[n_n - \left(\frac{n_n}{n_n}\right)_{\text{eq}} n_p \right], \tag{16}$$

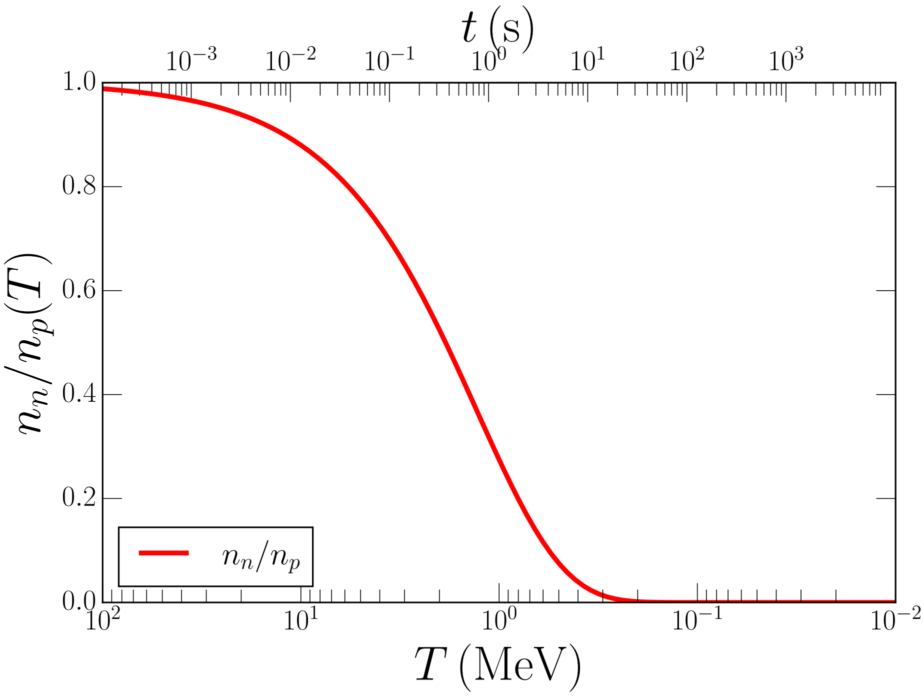

Figure 2: n_n/n_p ratio during equilibrium.

where

$$\Gamma_n = n_\ell \langle \sigma v \rangle_{\text{weak}} = \frac{255}{\tau_n} \frac{12 + 6x + x^2}{x^5}, \qquad x = \mathcal{Q}_n/T. \tag{17}$$

Here τ_n is the neutron lifetime

$$\tau_n = 886.7 \pm 0.8\,\text{s}. \tag{18}$$

Expressing n_n in terms of X_n leads to

$$\frac{dX_n}{dt} = -\Gamma_n \left[X_n - (1 - X_n) e^{-\mathcal{Q}_n/T} \right], \tag{19}$$

and trading t for x

$$\frac{dX_n}{dx} = -\frac{\Gamma_n(x)}{H(\mathcal{Q}_n)}\, x \left[e^{-x} - X_n(1 + e^{-x}) \right]. \tag{20}$$

Here

$$H(x) = \sqrt{\frac{\epsilon_R}{3M_p^2}} = \sqrt{\frac{\pi^2 g_*}{90}} \frac{T^2}{M_P} = \sqrt{\frac{\pi^2 g_*}{90}} \frac{\mathcal{Q}_n^2}{M_P} x^{-2} = H(\mathcal{Q}_n)\, x^{-2}. \tag{21}$$

The solution gives

$$X_n^\infty \equiv X_n(x \to \infty) \approx 0.145. \tag{22}$$

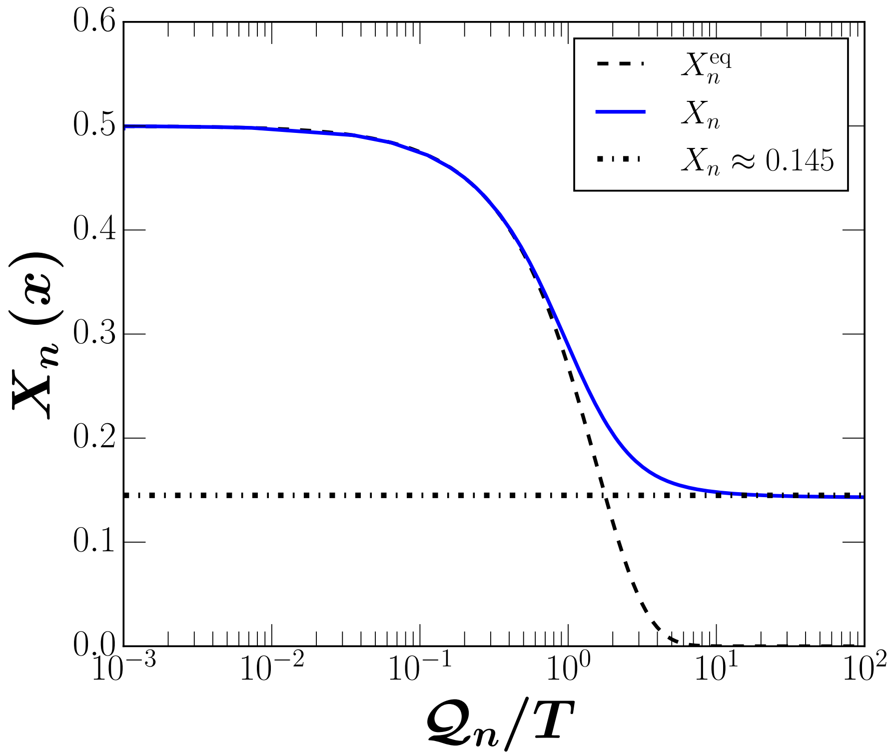

Figure 3: Fractional neutron abundance as a function of x.

2.3 Neutron decay

Notice that the mass difference between neutrons and protons is of just 1 MeV:

$$\mathcal{Q}_n = m_n c^2 - m_p c^2 = 1.29\,\mathrm{MeV}. \tag{23}$$

This in turn has profund implications in what concerns to the neutron decay,

$$n \to p^+ + e^- + \bar{\nu}_e. \tag{24}$$

At temperatures below 0.2 MeV (or $t \gtrsim 100$ s) the finite lifetime of the neutron becomes important, that is,

$$N_n(t) = N_0\, e^{-\frac{t}{\tau_n}} = N_0\, e^{-\frac{t}{887\,\mathrm{s}}}. \tag{25}$$

It is now a race against time. If the onset of nucleosynthesis takes significantly longer than the neutron lifetime, then the number of free neutrons becomes exponentially small and not much fusion will occur. Fortunately, this is not the case in our universe. The neutron bound into a stable atomic nucleus is preserved against decay. Neutrons are still around today because they've been tied up in deuterium, helium, and other atoms.

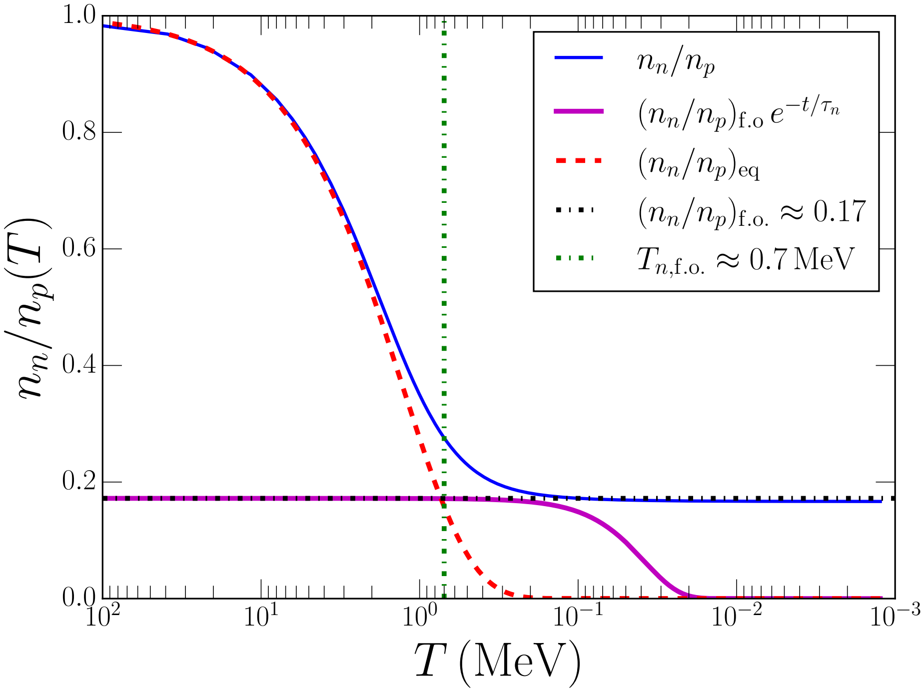

Figure 4: n_n/n_p ratio as a function of T.

3 Deuterium synthesis

4 Beyond deuterium

5 Baryon asymmetry of the Universe

References

[1] B. Ryden, *Introduction to cosmology*. Cambridge University Press, 1970.

[2] D. Baumann, *Cosmology*. Cambridge University Press, 7, 2022.

[3] E. W. Kolb, *The Early Universe*, vol. 69. Taylor and Francis, 5, 2019.