Early Universe: Out of Equilibrium dynamics

Disclaimer

Discussions taken from Barbara Ryden [1], Daniel Baumann [2] and Kolb & Turner [3] books

1 Temperature in radiation domination

For $a < a_{RM} \approx 3400$, the scale factor varies with time as

$$a(t) \propto t^{1/2}, \tag{1}$$

such that

$$H_R(t) = \frac{1}{2t}. \tag{2}$$

On the other hand,

$$H_R(T) = \sqrt{\frac{\epsilon_R}{3M_p^2}} = \sqrt{\frac{\pi^2 g_*}{90}}\frac{T^2}{M_P}. \tag{3}$$

Therefore

$$T \approx 1.31 \times 10^{10}\,\mathrm{K}\left(\frac{1\,\mathrm{s}}{t}\right)^{1/2} \tag{4}$$

$$= 1.1341\,\mathrm{MeV}\left(\frac{1\,\mathrm{s}}{t}\right)^{1/2}. \tag{5}$$

The mean energy per photon is

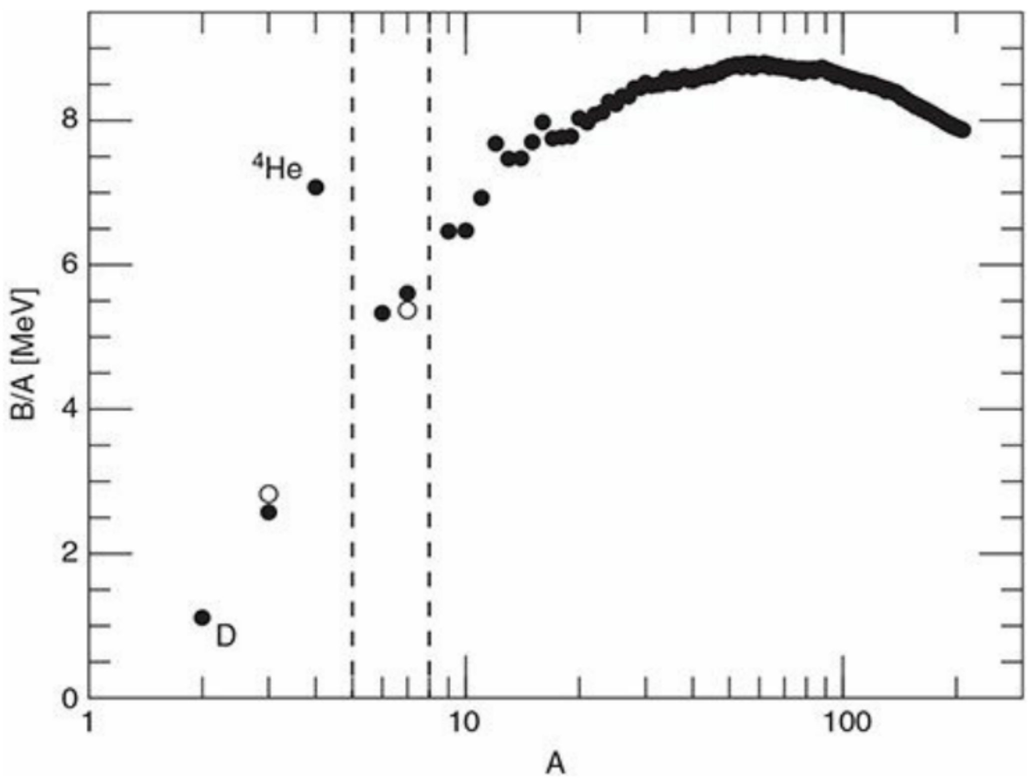
$$\overline{E_\gamma} \approx 2.7\,k_B\,T \approx 3\,\mathrm{MeV}\left(\frac{1\,\mathrm{s}}{t}\right)^{1/2}. \tag{6}$$

Typical energies in atomic physics:

$$E_I(\mathrm{H}) = 13.6\,\mathrm{eV};\quad E_I(\mathrm{He}) = 24.6\,\mathrm{eV};\quad E_I(\mathrm{Cs}) \approx 4\,\mathrm{eV}. \tag{7}$$

Nuclear physics deals with the much higher energy processes of fission and fusion (splitting or merging atomic nuclei). The binding energy B of a nucleus is the energy required to pull it apart into its component protons and neutrons. Equivalently, it is the energy released when a nucleus is fused together from individual protons and neutrons. For instance, when a neutron and a proton are bound together to form a deuteron, an energy of $B_D = 2.22$ MeV is released

$$p^+ + n \leftrightarrow D + 2.22\,\mathrm{MeV}. \tag{8}$$



rm BB

Figure 1: Binding energy per nucleon (B/A) as a function of the number of nucleons (A). Stable isotopes are shown as solid dots; the open dots represent the isotopes ^{3}H and ^{7}Be.

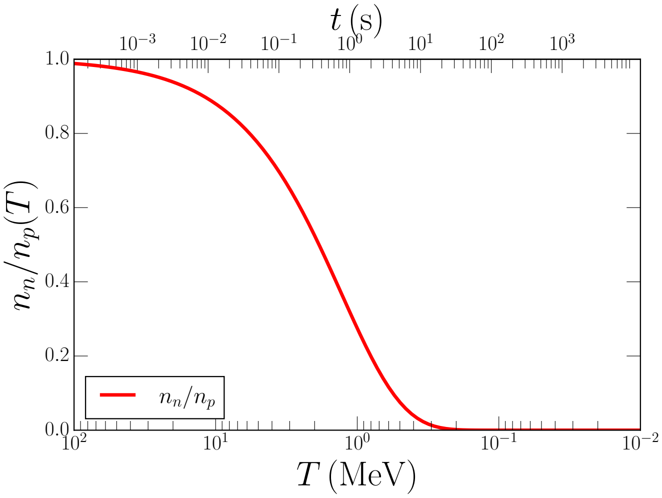


Figure 2: n_n/n_p ratio during equilibrium.

2 Neutron-to-proton ratio

2.1 Equilibrium at $t = 0.1$ s

At early times, the baryonic matter in the universe was mostly in the form of protons and neutrons, which were coupled to each other by processes mediated by the weak interaction, such as β-decay and inverse β-decay:

$$n + \nu_e \leftrightarrow p^+ + e^-, \tag{9}$$
$$n + e^+ \leftrightarrow p^+ + \bar{\nu}_e. \tag{10}$$

Above $T = 0.1$ MeV, only free protons and neutrons existed, while other light nuclei hadn't been formed yet. We can therefore first solve for the neutron-to-proton ratio and then use this abundance as an input for the synthesis of deuterium, helium, etc.

At $t = 0.1$ s the temperature was $T \approx 10^{10}$ K and $\overline{E_\gamma} \approx 10$ MeV $> m_e$. Hence the pair production and annihilation was in chemical equlibrium:

$$\gamma + \gamma \leftrightarrow e^- + e^+. \tag{11}$$

In summary, at this early time, all particles, including protons and neutrons, were in kinetic equilibrium. Hence

$$\mu_n + \mu_\nu = \mu_p + \mu_e. \tag{12}$$

Assuming that $\mu_\nu = \mu_e \approx 0$ we have that $\mu_n = \mu_p$. Thus, the ratio of the number densities of neutrons and protons is given by

$$\left(\frac{n_n}{n_p}\right)_{\rm eq} = \left(\frac{m_n}{m_p}\right)^{3/2} e^{-(m_n - m_p)/T} \approx e^{-Q_n/T}. \tag{13}$$

2.2 Boltzmann equation and neutron freeze-out

Let the neutron fraction

$$X_n \equiv \frac{n_n}{n_n + n_p}. \tag{14}$$

In equilibrium

$$X_n^{\rm eq} = \frac{e^{-\mathcal{Q}_n/T}}{1 + e^{-\mathcal{Q}_n/T}}. \tag{15}$$

Neutrons follow this equilibrium abundance until weak interaction processes such as β-decay effectively shut off at a temperature of around 1 MeV (roughly at the same time as neutrino decoupling).

To track the evolution let's solve the Boltzmann equation identifying $n_1 = n_n, n_3 = n_p$ and $n_2 = n_4 = n_\ell = n_\ell^{\rm eq}$:

$$\frac{1}{a^3}\frac{d(n_n a^3)}{dt} = -\Gamma_n \left[n_n - \left(\frac{n_n}{n_n}\right)_{\rm eq} n_p \right], \tag{16}$$

where

$$\Gamma_n = n_\ell \left\langle \sigma v \right\rangle_{\rm weak} = \frac{255}{\tau_n}\frac{12 + 6x + x^2}{x^5}, \qquad x = \mathcal{Q}_n/T. \tag{17}$$

Here τ_n is the neutron lifetime

$$\tau_n = 886.7 \pm 0.8\,{\rm s}. \tag{18}$$

Expressing n_n in terms of X_n leads to

$$\frac{dX_n}{dt} = -\Gamma_n \left[X_n - (1 - X_n) e^{-\mathcal{Q}_n/T} \right], \tag{19}$$

and trading t for x

$$\frac{dX_n}{dx} = -\frac{\Gamma_n(x)}{H(\mathcal{Q}_n)}\, x \, \left[e^{-x} - X_n(1 + e^{-x}) \right]. \tag{20}$$

Here

$$H(x) = \sqrt{\frac{\epsilon_R}{3M_p^2}} = \sqrt{\frac{\pi^2 g_*}{90}}\frac{T^2}{M_P} = \sqrt{\frac{\pi^2 g_*}{90}}\frac{\mathcal{Q}_n^2}{M_P} x^{-2} = H(\mathcal{Q}_n)\, x^{-2}. \tag{21}$$

The solution gives

$$X_n^\infty \equiv X_n(x \to \infty) \approx 0.145. \tag{22}$$

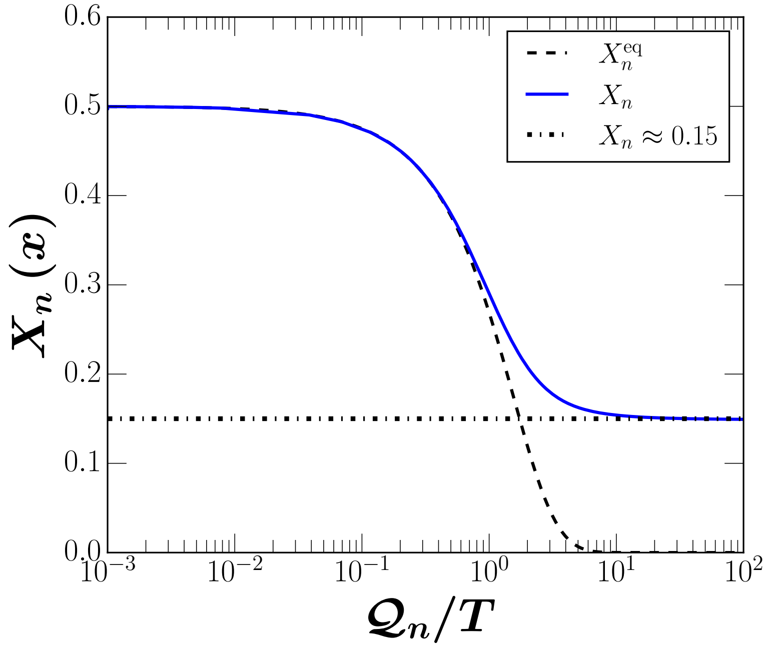


Figure 3: Fractional neutron abundance as a function of x.

2.3 Neutron decay

Notice that the mass difference between neutrons and protons is of just 1 MeV:

$$\mathcal{Q}_n = m_n c^2 - m_p c^2 = 1.29\,\mathrm{MeV}. \tag{23}$$

This in turn has profund implications in what concerns to the neutron decay,

$$n \to p^+ + e^- + \bar{\nu}_e. \tag{24}$$

At temperatures below 0.2 MeV (or $t \gtrsim 100$ s) the finite lifetime of the neutron becomes important, that is,

$$N_n(t) = N_0\, e^{-\frac{t}{\tau_n}} = N_0\, e^{-\frac{t}{887\,\mathrm{s}}}. \tag{25}$$

It is now a race against time. If the onset of nucleosynthesis takes significantly longer than the neutron lifetime, then the number of free neutrons becomes exponentially small and not much fusion will occur. Fortunately, this is not the case in our universe. The neutron bound into a stable atomic nucleus is preserved against decay. Neutrons are still around today because they've been tied up in deuterium, helium, and other atoms.

3 Deuterium synthesis

At this point the Universe contains protons and free neutrons. These free neutrons have to be combined with protons to form heavier nuclei and thus avoid the decreasing in its number density due to decay.

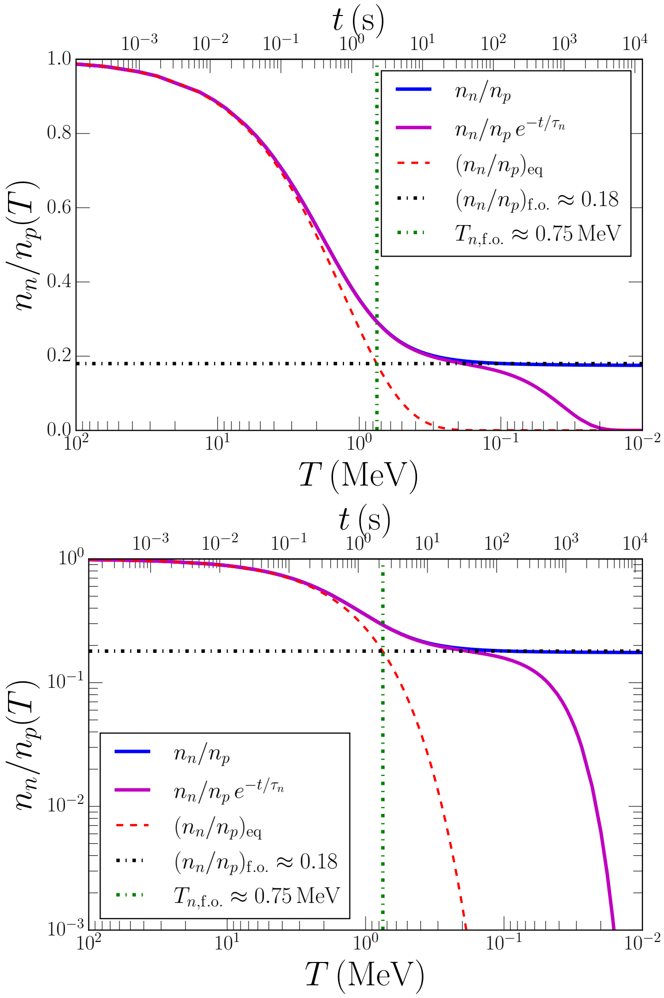


Figure 4: n_n/n_p ratio as a function of T.

Deuterium can be produced through the following reactions

$$n + p^+ \leftrightarrow \mathrm{D} + \gamma, \qquad\qquad (T \sim 0.1\,\mathrm{MeV}), \tag{26}$$

which leads to

$$\mu_n + \mu_p = \mu_{\mathrm{D}}. \tag{27}$$

These chemical potentials become absent

$$\begin{aligned}\left(\frac{n_{\mathrm{D}}}{n_n n_p}\right)_{\rm eq} &= \frac{g_{\mathrm{D}}}{g_n g_p}\left(\frac{m_D}{m_n m_p}\frac{2\pi}{T}\right)^{3/2} e^{-(m_{\mathrm{D}}-m_n-m_p)/T} \\ &\approx \frac{3}{4}\left(\frac{4\pi}{m_p T}\right)^{3/2} e^{B_{\mathrm{D}}/T},\end{aligned} \tag{28}$$

with

$$B_{\mathrm{D}} = m_n + m_p - m_{\mathrm{D}} = 2.22\,\mathrm{MeV}. \tag{29}$$

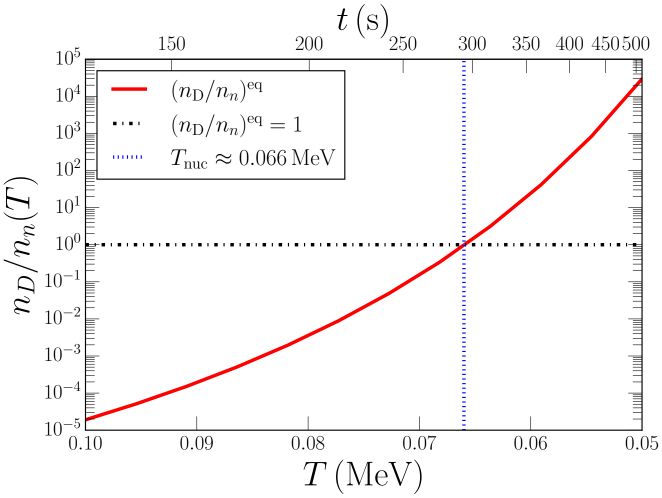


Figure 5: Deuteron-to-neutron ratio during equilibrium.

The nucleosynthesis temperature is defined as

$$\frac{n_{\rm D}}{n_n}(T_{\rm nuc}) = 1. \tag{30}$$

The deuteron-to-proton ratio can be written as

$$\left(\frac{n_{\rm D}}{n_n}\right)_{\rm eq} = 6 n_p^{\rm eq} \left(\frac{\pi}{m_p T}\right)^{3/2} e^{B_{\rm D}/T}. \tag{31}$$

Expressing

$$n_p^{\rm eq} \sim n_B = \eta_B\, n_\gamma(T) = \eta_B\, \frac{\zeta(3)}{\pi^2} g_\gamma T^3, \tag{32}$$

we can obtain the estimate

$$\left(\frac{n_{\rm D}}{n_n}\right)_{\rm eq} = 6\eta_B\, \frac{\zeta(3)}{\pi^2} g_\gamma \left(\frac{\pi\, T}{m_p}\right)^{3/2} e^{B_{\rm D}/T}, \tag{33}$$

leading to

$$T_{\rm nuc} \approx 0.66\,{\rm MeV} \to t_{\rm nuc} \approx 300\,{\rm s}. \tag{34}$$

Moreover

$$X_n(t_{\rm nuc}) \simeq 0.15 e^{-t_{\rm nuc}/\tau_n} \approx 0.11. \tag{35}$$

4 Beyond deuterium

4.1 Helium

Since the binding energy of helium is larger than that of deuterium, the Boltzmann factor $e^{B/T}$ favours helium over deuterium. Indeed, helium is produced almost immediately after deuterium. The reaction proceeds in two steps:

- First step: formation of ^{3}He and ^{3}H=T:

$$
\begin{aligned}
\mathrm{D} + p^+ &\leftrightarrow {}^3\mathrm{He} + \gamma, && (36)\\
\mathrm{D} + n &\leftrightarrow {}^3\mathrm{H} + \gamma, && (37)\\
\mathrm{D} + \mathrm{D} &\leftrightarrow {}^3\mathrm{H} + p^+, && (38)\\
\mathrm{D} + \mathrm{D} &\leftrightarrow {}^3\mathrm{He} + n, && (39)\\
\mathrm{D} + \mathrm{D} &\leftrightarrow {}^4\mathrm{He} + \gamma, \quad \text{(unlikely.)} && (40)
\end{aligned}
$$

  This means that basically at the beginning of the nucleosynthesis there are no large amounts of ^{3}H and ^{3}He.

- Second step: formation of ^{4}He:

$$
\begin{aligned}
{}^3\mathrm{H} + p^+ &\leftrightarrow {}^3\mathrm{He} + n, && (41)\\
{}^3\mathrm{H} + p^+ &\leftrightarrow {}^4\mathrm{He} + \gamma, && (42)\\
{}^3\mathrm{H} + \mathrm{D} &\leftrightarrow {}^4\mathrm{He} + n, && (43)\\
{}^3\mathrm{He} + \mathrm{D} &\leftrightarrow {}^4\mathrm{He} + p^+, && (44)\\
{}^3\mathrm{H} + \mathrm{D} &\leftrightarrow {}^4\mathrm{He} + \gamma. && (45)
\end{aligned}
$$

All these reactions involve the strong nuclear force, and have large cross-sections and fast reaction rates. Thus, once nucleosynthesis begins, D, ^{3}H, and ^{3}He are all efficiently converted to ^{4}He.

Since two neutrons go into one nucleus of ^{4}He, the final helium abundance is equal to half of the neutron abundance at $t_{\rm nuc}$, so that $n_{\rm He} = \frac{1}{2} n_n(t_{\rm nuc})$. Hence, the mass fraction of helium is

$$
Y_{^4\mathrm{He}} = \frac{4n_{\rm He}}{n_{\rm H}} = \frac{4n_{\rm He}}{n_p} \simeq \frac{2X_n(t_{\rm nuc})}{1 - X_n(t_{\rm nuc})} \sim 0.25. \qquad (46)
$$

To compute the maximum possible value of $Y_{^4\mathrm{He}}$, suppose that every neutron present after the proton–neutron freezeout is incorporated into a ^{4}He nucleus. Given a neutron-to-proton ratio of $n_n/n_p = 1/5$, we can consider a representative group of 2 neutrons and 10 protons. The 2 neutrons can fuse with 2 of the protons to form a single ^{4}He nucleus. The remaining 8 protons, though, will remain unfused. The mass fraction of ^{4}He will then be

$$
Y^{\rm max}_{^4\mathrm{He}} = \frac{4}{12} = \frac{1}{3}. \qquad (47)
$$

4.2 Heavier nuclei

Small amounts of beryllium and lithium are created by the reactions

$$
\begin{aligned}
{}^3\mathrm{He} +{}^4\mathrm{He} &\leftrightarrow {}^7\mathrm{Be} + \gamma, && (48)\\
{}^3\mathrm{He} + \mathrm{D} &\leftrightarrow {}^6\mathrm{Li} + \gamma, && (49)\\
{}^3\mathrm{H} +{}^4\mathrm{He} &\leftrightarrow {}^7\mathrm{Li} + \gamma, && (50)
\end{aligned}
$$

along with

$$
\begin{aligned}
{}^7\mathrm{Be} + n &\leftrightarrow {}^7\mathrm{Li} + p^+, \quad \text{(conversion)}, && (51)\\
{}^7\mathrm{Li} + p^+ &\leftrightarrow {}^4\mathrm{He} +{}^4\mathrm{He}, \quad \text{(Li destruction)}, && (52)\\
{}^7\mathrm{Be} + e^- &\to {}^7\mathrm{Li} + \nu_e, \quad \text{(Be decay)}. && (53)
\end{aligned}
$$

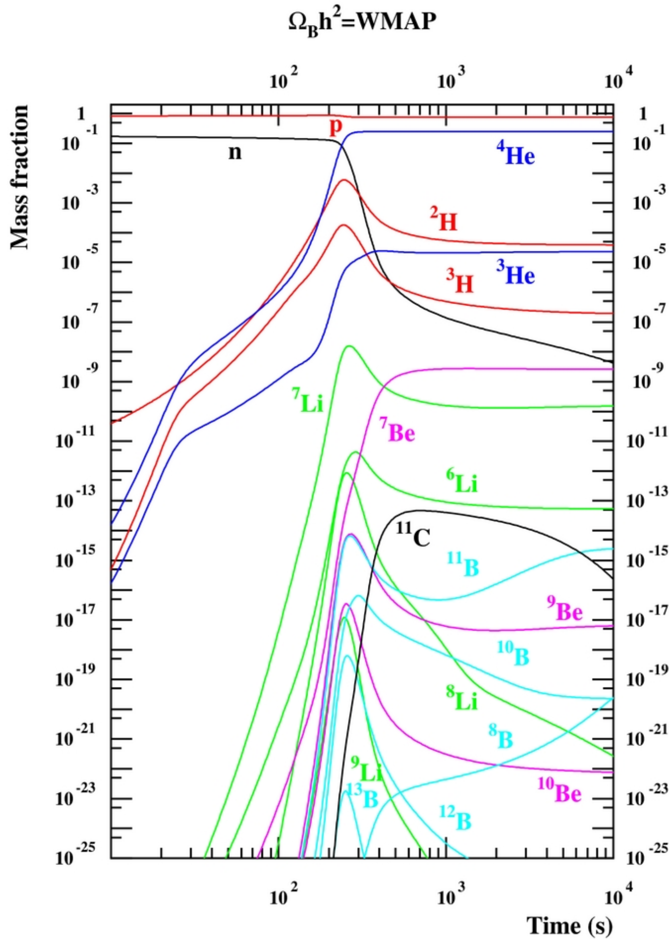


Figure 6: Mass fraction of nuclei as a function of time during the epoch of nucleosynthesis.

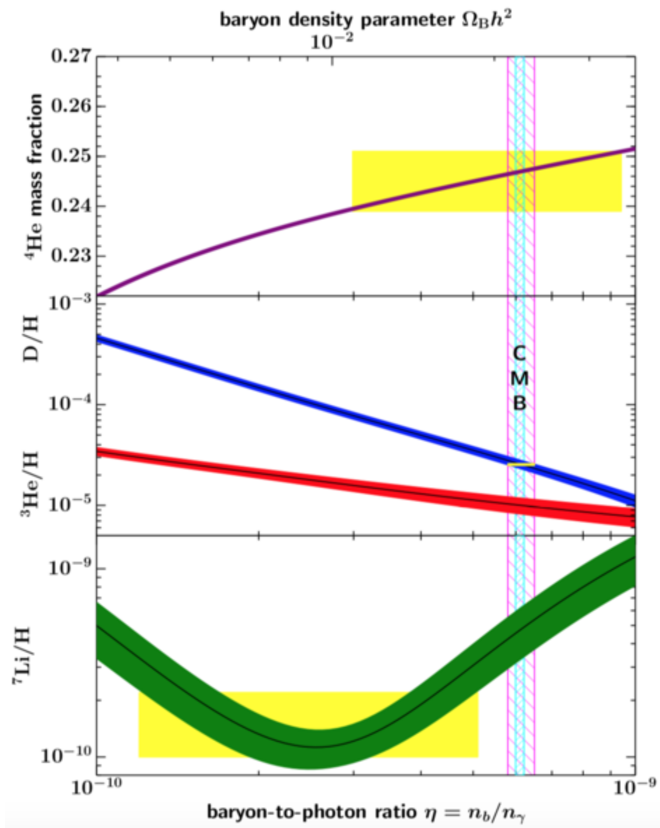


Figure 7: The mass fraction of ^{4}He, and the number densities of D, ^{3}He, and ^{7}Li expressed as a fraction of the H number density.

- Helium-4 can be measured from the light of ionized gas clouds, because the strength of some emission lines depends on the amount of helium. We have to correct for the fact that ^{4}He is also produced in stars. One way to do this, is to correlate the measured helium abundance with the abundances of heavier elements, such as nitrogen and oxygen. The larger the amount of oxygen, the more primordial helium has been destroyed. Extrapolating the measurements to zero oxygen gives an improved estimate of the primordial helium abundance.

- Deuterium is very weakly bound and therefore easily destroyed in the late universe. BBN is the only source of significant deuterium in the universe. The best way to determine the primordial (un-processed) value of the deuterium abundance is to measure the spectra of high-redshift, low-metallicity quasars. In particular, the Lyman-α line of these spectra is a sensitive probe of the amount of deuterium.

- Helium-3 can be created and destroyed in stars. Its abundance is therefore hard to measure and interpret. Given these uncertainties, helium-3 is usually not used as a cosmological probe.

- Lithium is mostly destroyed by stellar nucleosynthesis. The best estimate of its primordial abundance follows from the measurement of metal-poor stars in the Galactic halo. Rhe measured lithium abundance is significantly smaller than the predicted value. This is called the lithium problem. It is unclear whether this problem is due to systematic errors in the measurements or signals the need for new physics during BBN.

This allows a powerful test of the Hot Big Bang model, encompassing ten orders of magnitude in abundance. There turn out to be only two important input parameters which affect the abundances.

- The number of massless neutrino species in the Universe, which affects the expan-ion temperature–time relation and hence the way in which nuclear reactions go out of thermal equilibrium. So far we have assumed there are three neutrino types as in the Standard Model of particle interactions, but other numbers are possible in principle.

- The density of baryonic matter in the Universe, from which the nuclei are composed. If the density of baryons were changed, it is reasonable to imagine that the details of how they form nuclei are changed. The absolute density of baryons, ϵ_B , is what matters.

5 Baryon asymmetry of the Universe

Baryon-to-photon ratio:

$$\eta_B = \frac{n_B}{n_\gamma} = 6.1 \times 10^{-10}. \tag{54}$$

At earlier times $T \gg 300$ MeV the following reaction was present:

$$\bar{q} + q \leftrightarrow \gamma + \gamma. \tag{55}$$

If $n_{\bar{q}} = n_q$ from the beginning and no interactions exist producing only quarks or antiquarks, there would not be a sizeable amount of baryonic matter today. To avoid this fate, there must be a small excess of quarks over antiquarks:

$$\delta n_q = \frac{n_q - n_{\bar{q}}}{n_q + n_{\bar{q}}} \ll 1, \tag{56}$$

in such a way after $\bar{q} - q$ annihilation some quark number density remained as a relic.

As an illustrative example let's consider:

$$n_q = 800.000.003, \tag{57}$$
$$n_{\bar{q}} = 800.000.000, \tag{58}$$
$$\delta n_q = \frac{3}{1.6 \times 10^9}. \tag{59}$$

Since $n_\gamma \sim n_q$, then

$$\frac{n_B}{n_\gamma} \approx \frac{\frac{1}{3}\delta n_q}{n_q} \approx 6.1 \times 10^{-10}. \tag{60}$$

5.1 Freeze-out of protons?

Let's assume that protons remained in chemical equilibrium thanks to the interaction with photons

$$p^+ + p^- \leftrightarrow \gamma + \gamma. \tag{61}$$

The instantaneous freeze-out approximation reads

$$\Omega_B^{\text{f.o}} \simeq 0.1 \frac{x_f}{g_*(m_p)^{1/2}} \frac{10^{-8}\,\text{GeV}^{-2}}{\langle\sigma v\rangle_{\bar{p}p\to\gamma\gamma}}, \tag{62}$$

where $x_f = 20 - 30$ and $\langle\sigma v\rangle_{\bar{p}p\to\gamma\gamma} \sim m_\pi^{-2}$. It follows that

$$\Omega_B^{\text{f.o}} \sim 10^{-10} \ll \Omega_B. \tag{63}$$

An easy solution to this dead end would be to invoke a small initial difference between the number of protons and antiprotons. In this way there would not be dynamical solution to the BAU.

5.2 Sakharov conditions

In 1967, Sakharov identified three necessary conditions that any successful theory of baryogenesis must satisify.

1. Bayon number violation.
   - In Grand Unified Theories:
     $$p \to e^+ + \pi^0. \tag{64}$$
   - Non-perturbative effects known as sphalerons.
2. CP violation.
   - In meson decays due to weak interactions.
   - New interactions violating CP.
3. Out of equilibrium.
   - Reactions rates that depart form equilibrium.
   - Cosmological phase transitions.

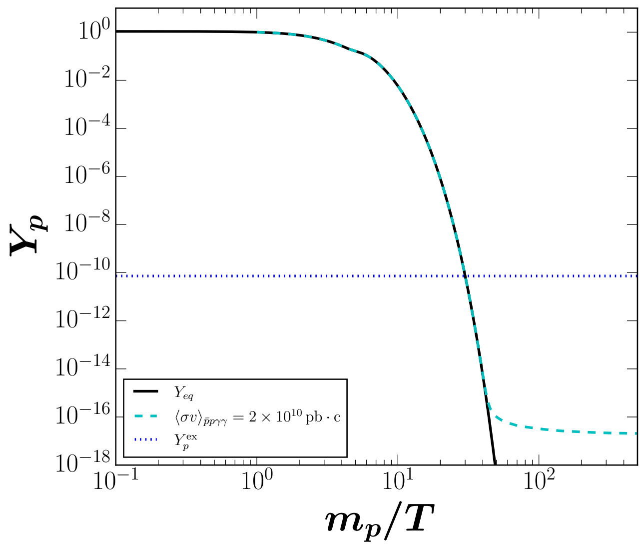


Figure 8: Caption

References

[1] B. Ryden, *Introduction to cosmology*. Cambridge University Press, 1970.

[2] D. Baumann, *Cosmology*. Cambridge University Press, 7, 2022.

[3] E. W. Kolb, *The Early Universe*, vol. 69. Taylor and Francis, 5, 2019.