Universidad de Antioquia - Instituto de Física

Universo Temprano

First exam (20%) 16/10/2024

REMARK: All mathematical procedures must be done and the answers must be simplified and justified.

1. (20%) The number density of photons in the frequency range f to $f + df$ is

$$n(f) = \frac{8\pi}{c^3} \frac{f^2\, df}{e^{hf/kT} - 1}. \tag{1}$$

   a) Show that for an energy threshold $E_0 \ll kT$, the fraction of blackbody photons that have energy $hf > E_0$ is

$$\frac{n(hf < E_0)}{n_\gamma} \approx 0.21 \left(\frac{E_0}{kT}\right)^2. \tag{2}$$

   b) Microwave (and far infrared) photons with a wavelength $\lambda < 3$ cm are strongly absorbed by H_2O and O_2 molecules. What fraction of the photons in today's cosmic microwave background have $\lambda > 3$ cm, and thus are capable of passing through the Earth's atmosphere and being detected on the ground?

   c) Estimate the corresponding total number of today's CMB photons with $\lambda > 3$ cm, knowing that the distance to the horizon is about 14000 Mpc.

2. (20%) Suppose you are in a flat, matter-only universe that has a Hubble constant $H_0 = 68$ km/s/Mpc. You observe a galaxy with $z = 1$. How long will you have to keep observing the galaxy to see its redshift change by one part in 10^6.

3. (20%) The most likely cosmology describing our own current Universe has a flat geometry with a matter density of $\Omega_{M,0} = 0.3$ and a cosmological constant with $\Omega_{\Lambda,0} = 0.7$. What will the values of Ω and Ω_Λ be when the Universe has expanded to be five times its present size? Use an approximation suggested by this result to find the late-time solution to the Friedmann equation for our Universe.

4. (20%) At what z does the transition from deceleration to acceleration occur for dark energy with equation of state $p = \omega\epsilon$, $\omega =$ const? For what value of the parameter ω this transition would occur now? Give numerical estimate using the values given in class.

5. (20%) **To be solved at home and presented on friday 18/10 at 11:59 pm.**
Consider the Figure 1 which shows the properties of universes containing matter and a cosmological constant, in the $\Omega_{M,0} - \Omega_{\Lambda,0}$ plane. A flat Universe ($k = 0$) would fall on top of the diagonal dashed line passing through the Einstein de Sitter solution. To the right of that line the Universe is closed, and to the left it is open. The dotted line shows the division between acceleration and deceleration for the Universe. Also shown are isochrones of constant age of the Universe in units of billion of years ($h = 0.68$ was assumed).

   a) Make a computational program with the aim of reproducing ALL the lines shown in the figure.

   b) Add to the plot the best-fit coincidence regions in the $\Omega_M - \Omega_\Lambda$ plane from the analysis of the Supernova Cosmology Project data (the ellipses at the 68 per cent and 90 per cent confidence regions). What is suggesting the supernova data?

   BONUS (20%): For those who answer these questions in an article format, addressing the following:

   - Title: A concise and informative title that accurately reflects the content.
   - Abstract: A brief overview of the article, including the model's key points and findings.
   - Introduction: A comprehensive introduction to the topic, providing necessary background information and context.
   - Description of the Model: A detailed explanation of the model, including its underlying principles, assumptions, and mathematical framework.
   - Analysis of Results: A thorough examination of the results obtained from the model, supported by data, calculations, and visualizations.
   - Conclusions: A summary of the key conclusions drawn from the analysis, highlighting the significance of the findings and potential implications.

   Additional Considerations:

   - Clarity and Coherence: Ensure that the article is well-organized, easy to follow, and logically structured.
   - Relevance: Relate the model to current research trends or unanswered questions in particle physics.
   - Depth: Provide a sufficient level of detail to demonstrate a deep understanding of the subject matter.
   - Citations: Properly cite any sources used to support your arguments or provide additional information.

   By following these guidelines, you can produce a high-quality article that effectively communicates your understanding of a cosmological model and its implications.

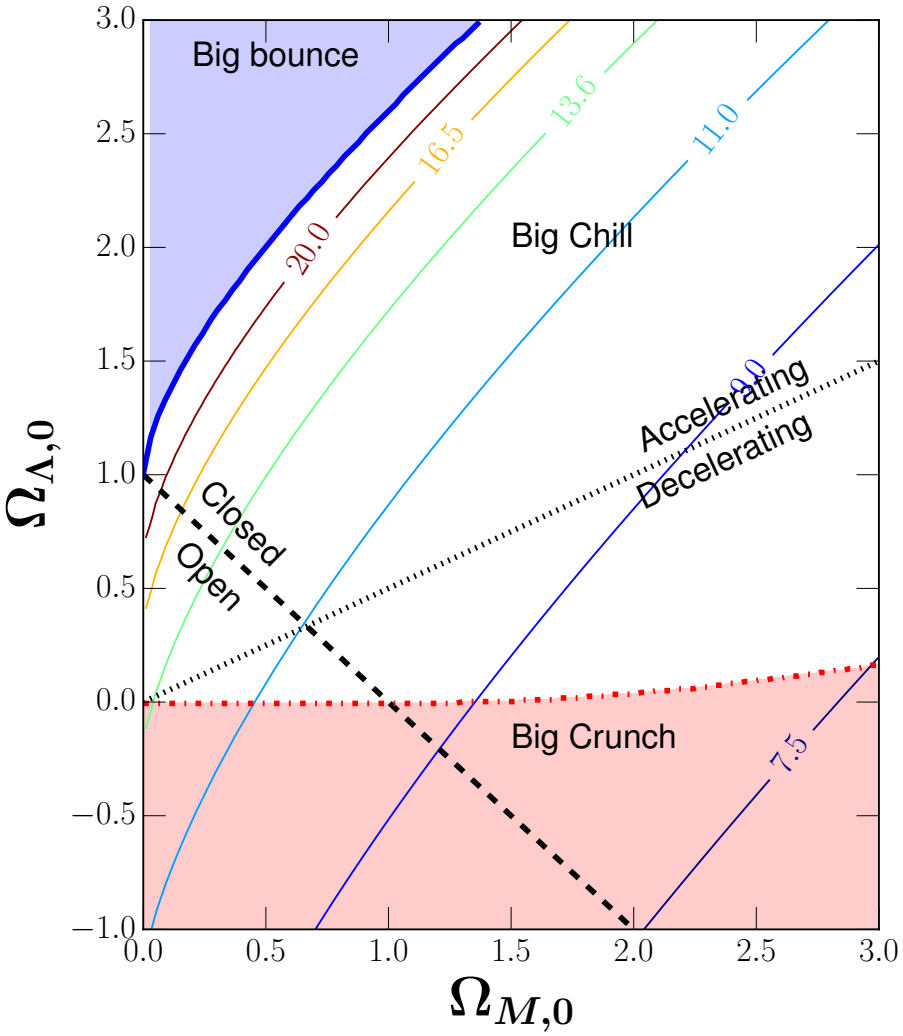

Figure 1: Properties of universes containing matter and a cosmological constant.