Universidad de Antioquia - Instituto de Física

Universo Temprano

Second exam (20%) 19/02/2025

REMARK: All mathematical procedures must be done and the answers must be simplified and justified.

The exam consists of two sets of problems: one to be completed in class, contributing either 60% or 80% of the total grade, and another to be completed at home, contributing either 20% or 40%, ensuring that the total adds up to 100%.

1. (20%) Consider a Universe with just one energy density component characterized by an equation of state parameter ω. Identify a necessary and sufficient condition that ω must satisfy for the deceleration parameter q_0 to be negative.

2. (20%) Consider a galaxy of physical (visible) size 5 kpc. What angle would this galaxy subtend if situated at redshift 0.1? Redshift 1? Do the calculation in a flat universe and matter-dominated.

3. (20%) How many neutrino species would be needed, to make the epoch of matter-radiation equality coincide with the epoch of last scattering?

4. Assume that neutrinos have a mass given by $m_\nu c^2 = 10$ eV.

   a) (20%) The transition from the relativistic to the non-relativistic regime occurs when the thermal energy of a neutrino, approximately given by $3k_B T$, becomes comparable to its mass-energy. Derive an approximate expression for the redshift $z_{\rm nr}$ at which this transition occurs.

   b) (20%) Estimate the comoving distance (in Mpc) that such neutrinos travel while still relativistic.

5. (20%) Compute the Hubble radius at $T = 0.1$ MeV. To what comoving scale does this correspond?

6. (20%) Determine η_B in terms of $\Omega_B h^2$.

7. (20%) We calculated the epoch of radiation-matter equality (a_{RM}) assuming all three neutrinos are massless. Now, suppose instead that two neutrinos remain massless while the third has a mass of $m = 10$ eV. What is a_{RM} is this scenario?

8. (20%) Argue and describe the effect on the recombination temperature in the early Universe when the baryon-to-photon ratio, η_B, is changed to a lower value, such as $\eta_B = 4 \times 10^{-10}$, and to a higher value, such as $\eta_B = 8 \times 10^{-10}$.

9. Suppose there exists a very light, stable fermion f ($g_f = 2$) that interacted with Standard Model particles in the early Universe exclusively via the exchange of a very heavy mediator—a gauge boson BB with mass $m_B = 10$ TeV. The cross section associated with the reaction that maintained chemical equilibrium is given by $\sigma_f = 0.01T^2/m_B^4$, where T is the plasma temperature

   a) (20%) Estimate the decoupling temperature of the fermion.

b) (20%) Assuming the fermion decoupled while still relativistic, determine the maximum possible mass of f such that it does not overclose the Universe (that is, ensuring $\Omega_f h^2 < 1$).

10. **To be solved at home and presented on friday 21/2 at 11:59 pm.**
Consider Figure 1, which shows photon decoupling during recombination.

a) (20%) Write a Python program to reproduce the entire figure.

b) (20%) Using the same program, generate a figure that displays the mean free path of photons and the age of the Universe as functions of $(1+z)$ are displayed. Assume different values for $\Omega_{M,0}$ and $\Omega_{B,0}$ (see figure 3.10 in the Kolb and Turner's book).

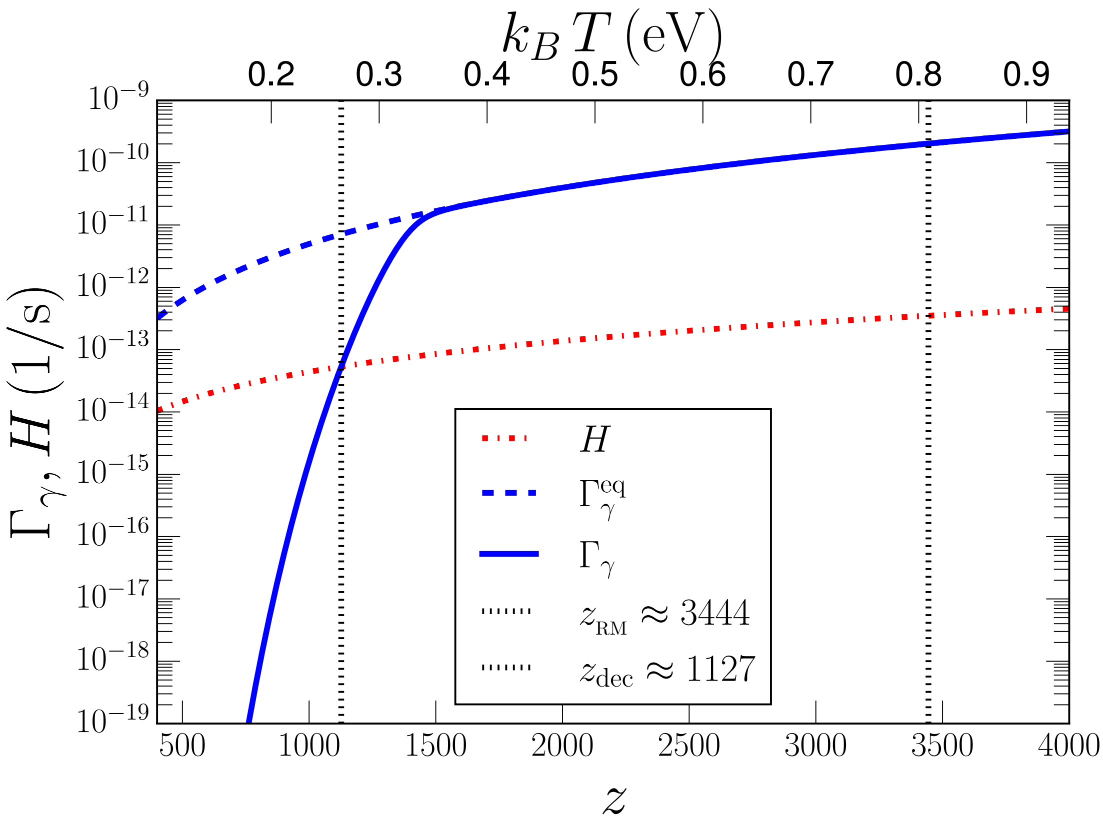

Figure 1: Photon decoupling during recombination