Model Universes

In a spatially homogeneous and isotropic universe, the relation between the energy density $\varepsilon(t)$, the pressure $P(t)$, and the scale factor $a(t)$ is given by the Friedmann equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2 a^2},$$

the fluid equation,

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0,$$

and the equation of state,

$$P = w\varepsilon.$$

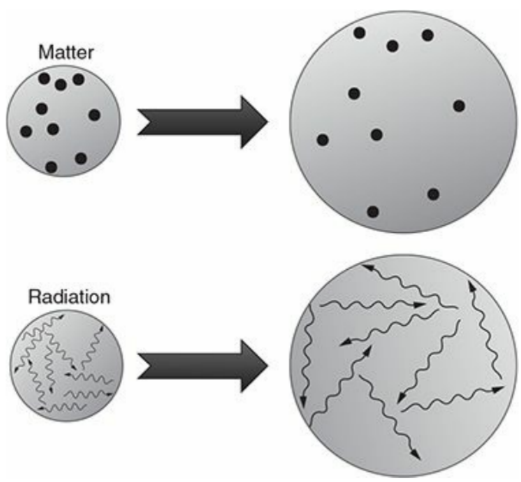
Evolution of Energy Density

$$\varepsilon = \sum_i \varepsilon_i. \qquad P = \sum_i w_i \varepsilon_i. \longrightarrow \dot{\varepsilon}_i + 3\frac{\dot{a}}{a}(\varepsilon_i + P_i) = 0$$

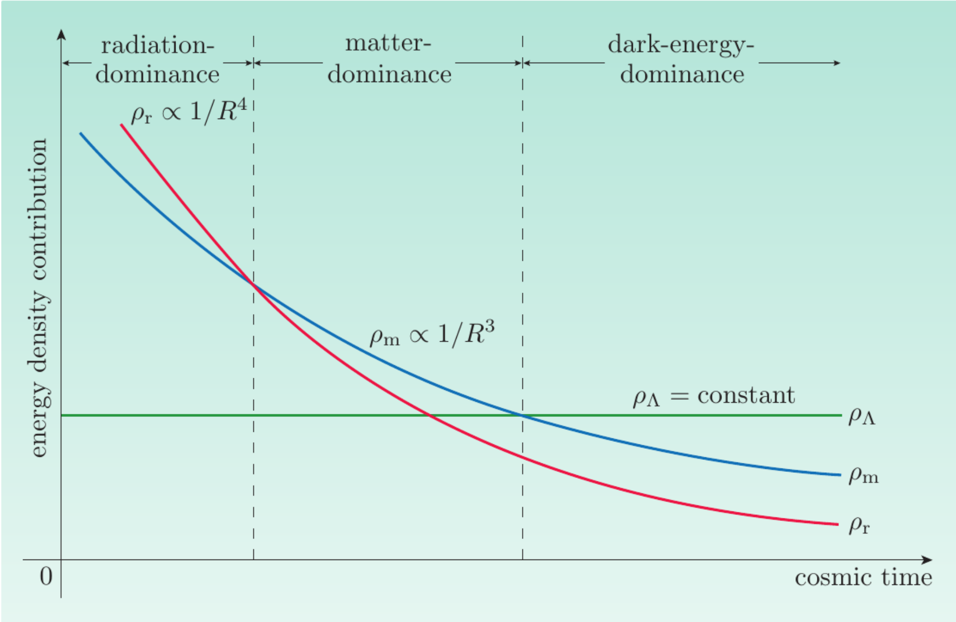
$$\varepsilon_i(a) = \varepsilon_{i,0} a^{-3(1+w_i)}.$$

$$\varepsilon_m(a) = \varepsilon_{m,0}/a^3.$$

$$\varepsilon_r(a) = \varepsilon_{r,0}/a^4.$$



The possible evolution of the density of radiation, matter and dark energy over cosmic time in our Universe.



Cosmic composition

At cosmic time $t = t_0$, the sources of cosmic gravitation are specified by just three values: $\rho_{m,0}$, $\rho_{r,0}$ and ρ_Λ. Given these three values, the cosmic density and pressure at any other cosmic time can be determined, provided that the cosmic scale factor $R(t)$ is known as an explicit function of cosmic time.

Table 5.2 Properties of the Benchmark Model.

List of ingredients		
Photons:	$\Omega_{\gamma,0} = 5.35 \times 10^{-5}$	
Neutrinos:	$\Omega_{\nu,0} = 3.65 \times 10^{-5}$	
Total radiation:	$\Omega_{r,0} = 9.0 \times 10^{-5}$	
Baryonic matter:	$\Omega_{\text{bary},0} = 0.048$	
Nonbaryonic dark matter:	$\Omega_{\text{dm},0} = 0.262$	
Total matter:	$\Omega_{m,0} = 0.31$	
Cosmological constant:	$\Omega_{\Lambda,0} \approx 0.69$	
Important epochs		
Radiation–matter equality:	$a_{rm} = 2.9 \times 10^{-4}$	$t_{rm} = 0.050$ Myr
Matter–lambda equality:	$a_{m\Lambda} = 0.77$	$t_{m\Lambda} = 10.2$ Gyr
Now:	$a_0 = 1$	$t_0 = 13.7$ Gyr

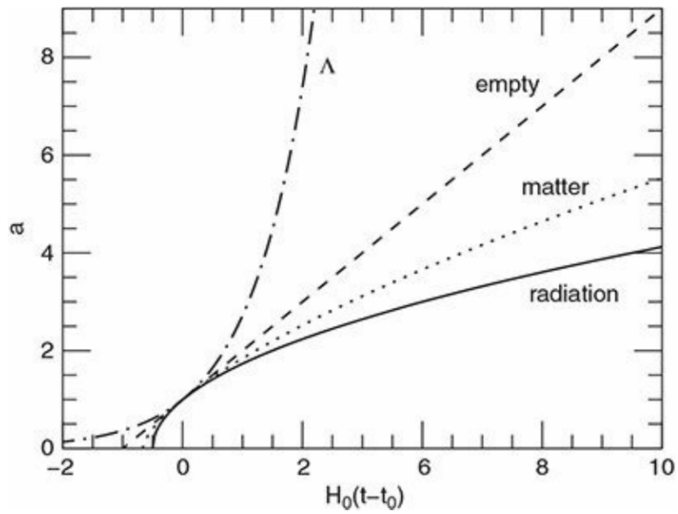


Figure 5.2 Scale factor versus time for an expanding, empty universe (dashed), a flat, matter-dominated universe (dotted), a flat, radiation-dominated universe (solid), and a flat, Λ-dominated universe (dot-dash).

Empty Universes

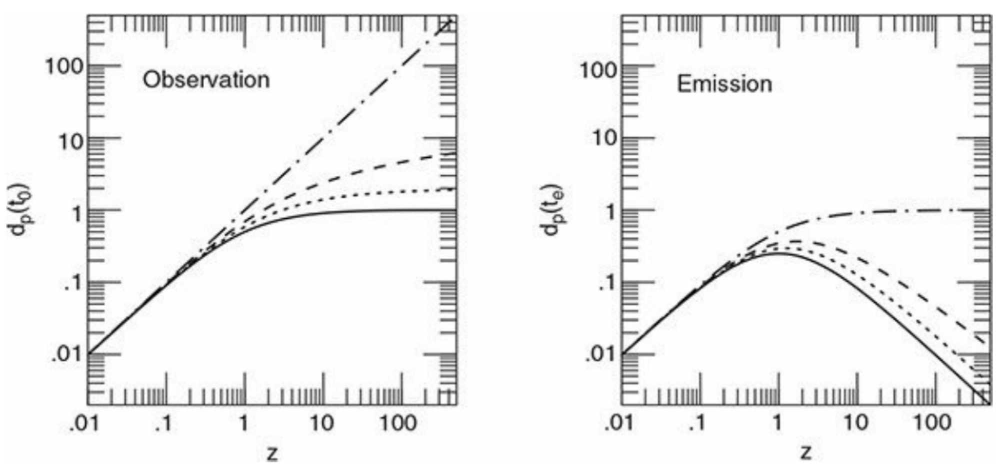


Figure 5.3 The proper distance to an object with observed redshift z, measured in units of the Hubble distance, c/H_0. Left panel: the proper distance at the time the light is observed. Right panel: proper distance at the time the light was emitted. Line types are the same as those of Figure 5.2.

Single-component Universes

$$\dot{a}^2 = \frac{8\pi G\varepsilon_0}{3c^2} a^{-(1+3w)}.$$

$$a(t) = \left(\frac{t}{t_0}\right)^{2/(3+3w)}.$$

$$t_0 = \frac{1}{1+w}\left(\frac{c^2}{6\pi G\varepsilon_0}\right)^{1/2}.$$

$$H_0 \equiv \left(\frac{\dot{a}}{a}\right)_{t=t_0} = \frac{2}{3(1+w)} t_0^{-1}.$$

$$t_0 = \frac{2}{3(1+w)} H_0^{-1}.$$

In a spatially flat universe, if $w > -1/3$, the universe is *younger* than the Hubble time. If $w < -1/3$, the universe is *older* than the Hubble time.

$$\varepsilon(a) = \varepsilon_0 a^{-3(1+w)},$$

$$\varepsilon(t) = \varepsilon_0 \left(\frac{t}{t_0}\right)^{-2},$$

$$\varepsilon_0 = \varepsilon_{c,0} = \frac{3c^2}{8\pi G} H_0^2 = \frac{c^2}{6\pi(1+w)^2} t_0^{-2},$$

$$1 + z = \frac{a(t_0)}{a(t_e)} = \left(\frac{t_0}{t_e}\right)^{2/(3+3w)}$$

$$t_e = \frac{t_0}{(1+z)^{3(1+w)/2}} = \frac{2}{3(1+w)H_0} \frac{1}{(1+z)^{3(1+w)/2}}.$$

The current proper distance to the galaxy is

$$d_p(t_0) = c\int_{t_e}^{t_0}\frac{dt}{a(t)} = ct_0\frac{3(1+w)}{1+3w}[1-(t_e/t_0)^{(1+3w)/(3+3w)}],$$

$$d_p(t_0) = \frac{c}{H_0}\frac{2}{1+3w}[1-(1+z)^{-(1+3w)/2}].$$

$$d_{\rm hor}(t_0) = c\int_0^{t_0}\frac{dt}{a(t)}.$$

$$d_{\rm hor}(t_0) = ct_0\frac{3(1+w)}{1+3w} = \frac{c}{H_0}\frac{2}{1+3w}.$$

Lambda only

$$\dot{a}^2 = \frac{8\pi G\varepsilon_\Lambda}{3c^2}a^2,$$

$$\dot{a} = H_0 a,$$

$$H_0 = \left(\frac{8\pi G\varepsilon_\Lambda}{3c^2}\right)^{1/2}.$$

$$a(t) = e^{H_0(t-t_0)}.$$

$$d_p(t_0) = c\int_{t_e}^{t_0}e^{H_0(t_0-t)}dt = \frac{c}{H_0}[e^{H_0(t_0-t_e)}-1] = \frac{c}{H_0}z,$$

$$d_p(t_e) = \frac{c}{H_0}\frac{z}{1+z},$$