Early Universe: Out of Equilibrium dynamics

Disclaimer

Discussions taken from Barbara Ryden [1], Daniel Baumann [2] and Kolb & Turner [3] books

1 Successes and drawbacks of the Hot Big Bang Model

	z	t
Distant quasars	< 10	> 0.5 Gyr
LSS	1090	0.37 Gyr
Nuc	3×10^8	3 min
Neutron decoupling	4×10^9	1 s

Table 1: Successes of the Hot Big Bang Model

The universe is nearly flat today, and was even flatter in the past
The universe is nearly isotropic and homogeneous today, and was even more so in the past
The universe is apparently free of magnetic monopoles

Table 2: Drawbacks of the Hot Big Bang Model

2 The flatness problem

From the Friedmann equation

$$\frac{H^2}{H_0^2} = \Omega - \frac{k\,c^2}{R_0^2 H_0^2\,a^2}, \tag{1}$$

we have that

$$1 - \Omega_0 = -\frac{k\,c^2}{R_0^2 H_0^2\,a_0^2}. \tag{2}$$

Hence

$$1 - \Omega = \frac{H_0^2(1-\Omega_0)}{H(t)^2\,a(t)^2} \propto \begin{cases} a \propto t, & \text{RD;} \\ a \propto t^{2/3}, & \text{MD.} \end{cases} \tag{3}$$

Experimentally

$$|1 - \Omega_0| \leq 0.005. \tag{4}$$

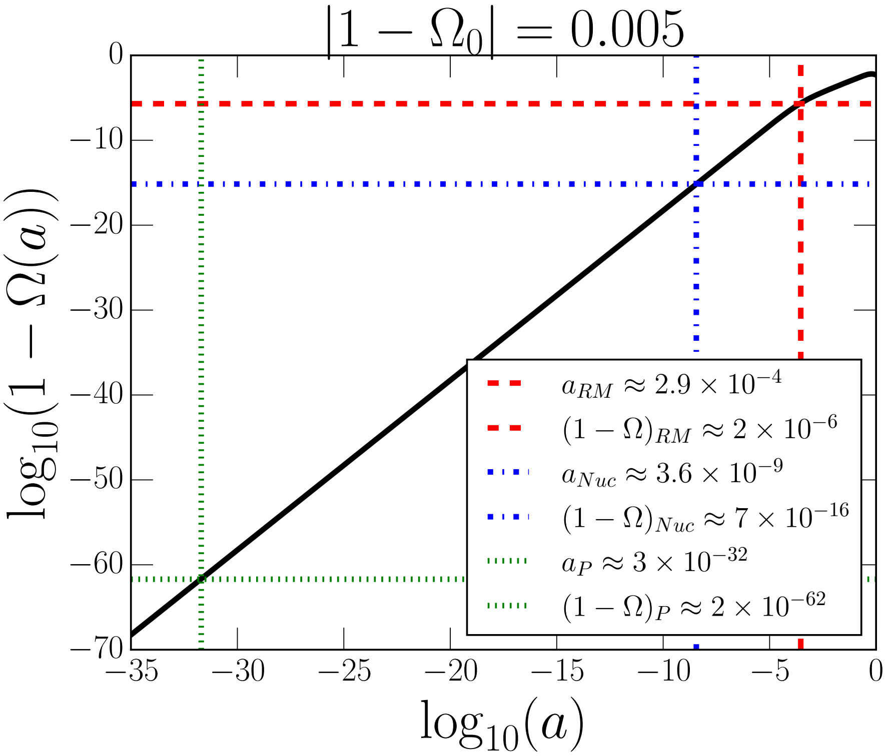

Figure 1: $1 - \Omega(a)$ as a function of a.

3 The horizon problem

3.1 Current proper distances

The horizon distance is given by

$$d_{\rm hor}(t_0) = c\, a(t_0) \int_0^{t_0} \frac{dt}{a(t)} = c \int_0^1 \frac{da}{a^2\, H(a)} \tag{5}$$

$$= c \int_0^{a_0} \frac{da}{a^2\, H_0 \sqrt{\Omega_{R,0}/a^4 + \Omega_{M,0}/a^3 + \Omega_{\Lambda,0}}} \approx 14118\,\text{Mpc}. \tag{6}$$

The current proper distance to the last scattering surface is

$$d_p(t_0) = c\, a(t_0) \int_{t_{ls}}^{t_0} \frac{dt}{a(t)} = c \int_{a_{ls}}^{a_0} \frac{da}{a^2\, H(a)} \approx 0.98\, d_{\rm hor} \tag{7}$$

Next let's consider two points in opposite sites of the last scattering surface. The are separated by a proper distance of

$$d_p \approx 1.96\, d_{\rm hor}(t_0), \tag{8}$$

as seen by an observed on Earth.

Since the two points are farther apart than the horizon distance, they are causally disconnected. That is, they haven't had time to send messages to each other, and in particular, haven't had time to come into thermal equilibrium with each other. Nevertheless, the two points have the same temperature to within one part in 10^5*. Why? How can two points that haven't had time to swap information be so nearly identical in their properties?*

3.2 Proper distances at the time of last scattering

The horizon distance at the time of last scattering was

$$d_{\rm hor}(t_{ls}) = c\, a(t_{ls}) \int_0^{t_{ls}} \frac{dt}{a(t)} = c\, a(t_{ls}) \int_0^{a_{ls}} \frac{da}{a^2\, H(a)} \approx 0.257\,\text{Mpc}. \tag{9}$$

3.3 How many patches?

In a spatially flat Universe the angular-diameter distance is

$$d_A = \frac{d_p(t_0)}{1+z} \overset{z\to\infty}{\approx} \frac{d_{\rm hor}(t_0)}{z}. \tag{10}$$

Hence the angular-diameter distance to the last scattering surface is

$$d_A = \frac{14118\,\text{Mpc}}{1090} \approx 12.952\,\text{Mpc}. \tag{11}$$

Thus, points on the last scattering surface that were separated by a horizon distance will have an angular separation equal to

$$\theta_{\rm hor} = \frac{d_{\rm hor}(t_{ls})}{d_A} = \frac{0.257\,\text{Mpc}}{12.95\,\text{Mpc}} \approx 0.02\,\text{rad} \approx 1.1^\circ \tag{12}$$

as seen from the Earth today.

Points that are separated by more than 2° on the sky seem never to have been in causal contact, since their past light cones don't overlap.

The size of a causally-connected patch of space is determined by the maximal distance from which light can be received. The surface of last scattering can be divided into some 40 000 patches, each 1.1° across:

$$\#\,\text{of patches} \sim \frac{4\pi}{0.02^2} \approx 40000. \tag{13}$$

4 The Monopole problem

Monopoles may be originated in the early universe within so called grand unified theories (GUTs). The typical mass-energy of these particles is

$$M_{\rm M} = E_{\rm GUT} \sim 10^{15}\,\text{GeV}. \tag{14}$$

Since one monopole per horizon volume, the monopole number density at their formation time, $t_{\rm GUT} \sim 10^{-36}$ s, is

$$n_{\rm M}(t_{\rm GUT}) = \frac{1}{(2\,c\,t_{\rm GUT})^3} \sim 10^{82}\,\text{m}^{-3}. \tag{15}$$

The corresponding energy density is

$$\epsilon_{\rm M}(t_{\rm GUT}) = M_{\rm M}\, n_{\rm M}(t_{\rm GUT}) \sim 10^{97}\,\text{GeV/m}^3. \tag{16}$$

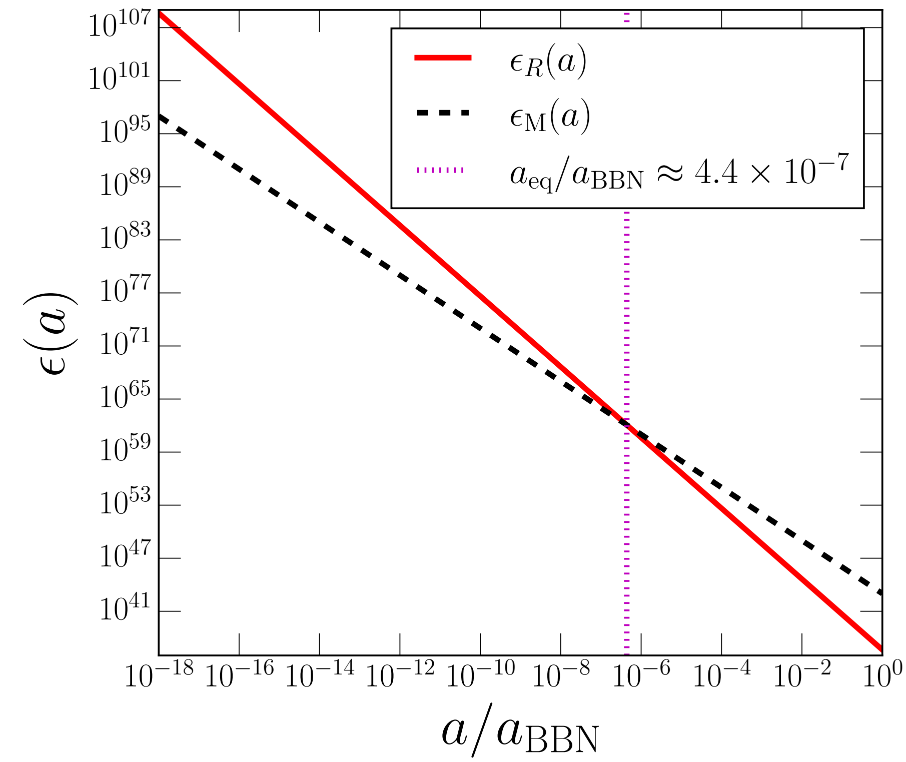

Figure 2: Energy density of radiation and monopole as a funtion of $a/a_{\rm BBN}$.

For comparison purposes, the energy density of radiation at $t_{\rm GUT}$ is

$$\epsilon_R(t_{\rm GUT}) = \frac{\pi^2}{30} g_* T_{\rm GUT}^4 \approx 10^{57}\,{\rm GeV}^4 \tag{17}$$

$$\sim 10^{108}\,{\rm GeV/cm}^3, \tag{18}$$

where

$$T_{\rm GUT} \approx 1.31 \times 10^{10}\,{\rm K} \left(\frac{1\,{\rm s}}{t_{\rm GUT}}\right)^{1/2} = 1.1341\,{\rm MeV} \left(\frac{1\,{\rm s}}{t_{\rm GUT}}\right)^{1/2} \tag{19}$$

$$\approx 10^{15}\,{\rm GeV}. \tag{20}$$

5 Inflation to rescue

6 Field theory of inflation

References

[1] B. Ryden, *Introduction to cosmology*. Cambridge University Press, 1970.

[2] D. Baumann, *Cosmology*. Cambridge University Press, 7, 2022.

[3] E. W. Kolb, *The Early Universe*, vol. 69. Taylor and Francis, 5, 2019.