Student Information

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Answer 1

a)

To find the size of our Monte Carlo simulation using normal approximation, we can use the formula;

$$N \geq p^*(1-p^*)(\frac{z_{\alpha/2}}{\varepsilon})^2$$

So that it satisfies;

$$P\{|\hat{p}-p| > \varepsilon\} \leq \alpha$$

In our case, since we don't have a preliminary estimator of probability $\hat{p}$, we think of the worst case where $\hat{p}(1-\hat{p}) \leq 0.25$ so we take 0.25. Also, since we want to be %99 confident with our estimation, α = 1 - 0.99 = 0.01. Lastly, ε is our precision which is 0.02. So the formula becomes;

$$N \geq 0.25(\frac{z_{0.005}}{0.02})^2$$

where $z_{0.005} = 2.576$ (from table A4).

$$N \geq 0.25(\frac{2.576}{0.02})^2$$
$$N \geq 4147.36$$

So in order to estimate the probability %99 confidence and 0.02 precision, the minimum size of our Monte Carlo simulation must be 4148.

b)

i) The weight of each individual automobile is a random variable that follows gamma distribution with parameters $\alpha = 120$ $\lambda = 0.1$. We can represent this random variable with W_A. E(W_A) $= \frac{\alpha}{\lambda} = \frac{120}{0.1} = 1200$kg.

ii) Similar to automobile, the weight of a truck also follows gamma distribution with parameters $\alpha = 14$ and $\lambda = 0.001$. So the expected value E(W_T) $= \frac{\alpha}{\lambda} = \frac{14}{0.001} = 14000$kg.

iii) Expected value for the total weights of all automobiles that pass over the bridge in a day is the product of expected value of number of automobiles that uses bridge in a day and the expected value of weight of an automobile which is 1200kg (from part **i**).

$$E(T_A) = E(N_A) \cdot E(W_A)$$

The number of automobiles that use bridge in a day is Poisson random variable with the parameters $\lambda_A = 60$. So the expected value is E(N_A) $= \lambda_A = 60$.

$$E(T_A) = E(N_A) \cdot E(W_A) = 60 \cdot 1200 = 72000kg$$

iv) Similar to automobile, in order to find the expected value for the total weights of all trucks that pass over the bridge in a day is the product of expected value of number of trucks that uses bridge in a day and the expected value of weight of an truck which is 14000kg (from part **ii**).

$$E(T_T) = E(N_T) \cdot E(W_T)$$

The number of trucks that use bridge in a day is Poisson random variable with the parameters λ_A $= 12$. So the expected value is E(N_T) $= \lambda_T = 12$.

$$E(T_T) = E(N_T) \cdot E(W_T) = 12 \cdot 14000 = 168000kg$$

Answer 2

i) By using the template given us and utilizing the code segments given in examples 5.9 and 5.11, I have written the following code;

```
N = 4148;
lambdaA = 60;  % number of automobiles
lambdaT = 12;  % number of trucks

TotalWeight = zeros(N,1); % a vector that keeps the total weight for each Monte Carlo run
for k = 1:N;
  % first generate the number of passed vehicles for each type from Poisson
  numA = 0;
  numT = 0;

  % number of automobiles
  U = rand;
  i = 0;
  F = exp(-lambdaA);
  while(U >= F);
      i = i + 1;
      F = F + exp(-lambdaA) * (lambdaA^i)/gamma(i + 1);
  end;
  numA = i;
  % number of trucks
  U = rand;
  i = 0;
  F = exp(-lambdaT);
  while(U >= F);
      i = i + 1;
      F = F + exp(-lambdaT) * (lambdaT^i)/gamma(i + 1);
  end;
  numT = i;
    weight = 0; % total weight of vehicles for this run
  % calculate the total weight of automobiles
    for f=1:numA;
        weight = weight +  sum( -1/0.1 * log(rand(120,1)) );
    end;
  % calculate the total weight of trucks
    for f=1:numT;
        weight = weight +  sum( -1/0.001 * log(rand(14,1)) );
    end;

    TotalWeight(k) = weight;
end;

p_est = mean(TotalWeight > 250000);
expectedWeight = mean(TotalWeight);
stdWeight = std(TotalWeight);

fprintf('Estimated probability = %f\n',p_est);
fprintf('Expected weight = %f\n',expectedWeight);
fprintf('Standard deviation = %f\n',stdWeight);
```

When we run this code in MATLAB, we have to following results;

```
>> hw4Temp
Estimated probability = 0.410077
Expected weight = 240886.716597
Standard deviation = 50675.338839
```

From this result, we can conclude that the estimation of the probability with %99 confidence and with 0.02 precision is 0.410. Also we expect that, the total weight of automobiles and trucks that passes the bridge in a day is 240886.716kg and lastly the standard deviation of this weight is 50675.338.

ii) In order to limit the number of trucks that passes the bridge in a day, we can decrement the λ_T one by one and see if the new probability is below 0.1. When we have the probability below 0.1, we can stop decrementing λ_T. When we start this process, we see that when we make $\lambda_T = 8$, we get the following results;

```
>> hw4Temp
Estimated probability = 0.065815
Expected weight = 184766.949119
Standard deviation = 41507.484498
```