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Answer 1

a-)

BASIS: For n=1 $2^3 - 3^1 = 5$ (5 is divisible by 5).

INDUCTIVE STEP

INDUCTIVE HYPOTHESIS: For a $k > 1$, $k \in N$, assume $2^{3k} - 3^k$ is divisible by 5.

Then for $k+1$, we get $2^{3k+3} - 3^{k+1}$ which is equal to $8 * 2^k - 3 * 3^k$. We can rewrite this equation as $3 * (2^{3k} - 3^k) + 5 * 2^{3k}$. From the Inductive Hypothesis, we know that $2^{3k} - 3^k$ is divisible by 5, hence $3 * (2^{3k} - 3^k)$ is also divisible by 5. Also we know that $5 * 2^{3k}$ is divisible by 5 since it is a multiple of 5. Hence $3*(2^{3k} - 3^k) + 5*2^{3k}$ is divisible by 5 since both of the terms are divisible by 5.

b-)

BASIS: For n=2 $4^2 - 7 * 2 - 1 = 1$ which is bigger than 0.

INDUCTIVE STEP

INDUCTIVE HYPOTHESIS: For a $k > 1$, $k \in N$, assume $4^k - 7k - 1 > 0$. So $4^k > 7k + 1$ and we can derive $4 * 4^k > 28k + 4$.

Then for k+1, we get $4^{k+1} - 7*(k+1) - 1$ which is equal to $4*4^k - 7k - 8$. We know from Inductive Hypothesis that $4 * 4^k > 28k + 4$. Also since $k > 2$, $28k + 4 > 7k + 8$. Hence, $4 * 4^k > 7k + 8$.

Answer 2

a-)

For this question we can divide this question into 4 different cases A, B, C, D.

Case A, a string containing seven 1's and three 0's. By using permutation, we get $|A| = 10!/(7! * 3!) = 120$.
Case B, a string containing eight 1's and two 0,s. By using permutation, we get $|B| = 10!/(8!*2!) = 45$.

Case C, a string containing nine 1's and one 0. By using permutation, we get $|C| = 10!/(9! * 1!) = 10$.
Case D, a string containing ten 1's. We get only 1 string, so $|D| = 1$.
Since A, B, C, D are all disjoint, by the sum rule, $|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| = 120 + 45 + 10 + 1 = 176$.

b-)

For this question we can divide this question into 3 different cases A, B, C.

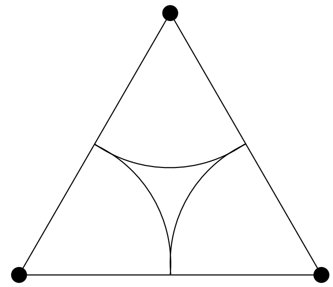
Case A, we have a collection containing 1 Statistical Methods textbook and 3 Discrete Mathematics textbooks. Since Discrete Mathematics textbooks are identical, we can make this collection in only 1 way.
Case B, we have a collection containing 2 Statistical Methods textbooks and 2 Discrete Mathematics textbooks. Again we can make this collection in only 1 way since books are identical.
Case C, we have a collection containing 3 Statistical Methods textbooks and 1 Discrete Mathematics textbook. Again we can make this collection in only 1 way since books are identical.
Since A, B, C are all disjoint, by the sum rule, $|A \cup B \cup C| = |A| + |B| + |C| = 1 + 1 + 1 = 3$.

c-)

For this question, we can find the number of functions that are not onto and subtract it from the all possible functions.
For each element in domain, there are 3 possibilities. So by product rule, all functions possible are $3^5 = 243$.
To find the number of functions that are not onto, we need to leave 1 element from co-domain unassigned. We can use combination to find that element. After that for each element in domain, there will be 2 possibilities and we can use product rule there. However, when we do that we subtract 2 times the case where all elements in domain is assigned to the same element co-domain. So we need to add this case (inclusion-exclusion principle).To find number of this case we can use combination to find the elements that will be unassigned.
$243 - \binom{3}{1} * 2^5 + \binom{3}{2} * 1^5 = 150$

Answer 3

To solve this question, we can use the Pigeonhole Principle. In this case pigeons are kids and we have 5 kids. For the pigeonholes, we have maximum 4 number of areas in the equilateral triangle where we place 3 of the kids on the vertices of the triangle and 1 at the middle of the triangle so that they are at least 250 meters away from each other.



Here if we place kids at the vertices of the triangle there will be 1 are that we can place the fourth kid. Hence we have 4 places and 5 kids. Hence by Pigeonhole Principle, there will be at least two kids within 250 meters of each other.

Answer 4

a-)

We can rewrite the recurrence relation in this form $a_n - 3a_{n-1} = 5^{n-1}$. For the homogeneous solution we need to make this recurrence relation equal to 0. So we get $a_n - 3a_{n-1} = 0$. Now we need to write the characteristic equation to find the characteristic root.
Characteristic Equation $--> \alpha - 3 = 0$, $\alpha = 3$.
$a_n^{(h)} = A * 3^n$

b-)

For the particular solution we need to make a guess for a_n. Here at this recurrence relation we have the term 5^{n-1}. So we can make the guess that $a_n^{(p)} = B * 5^n$. Now, in order to find B here, we need to replace a_n with $B * 5^n$. Now the equation becomes;
$B * 5^n - 3 * B * 5^{n-1} = 5^{n-1}$
$B * 5 * 5^{n-1} - 3 * B * 5^{n-1} = 5^{n-1}$
$5^{n-1} * (5 * B - 3 * B) = 5^{n-1}$
$2 * B = 1$
From this we get that $B = 1/2$. So particular solution becomes $a_n^{(p)} = \frac{5^n}{2}$

c-)

a_n can be written in terms of particular solution and homogeneous solution $a_n = a_n^{(h)} + a_n^{(p)}$. Now we can use the solutions I found at part a and part b. $a_n = A * 3^n + \frac{5^n}{2}$. To find A here, we can use the initial condition $a_1 = 4$. Let's insert $n = 1$.
$a_1 = 4 = A * 3^1 + \frac{5^1}{2}$
$A * 3 = \frac{3}{2}$ So we get $A = \frac{1}{2}$. And $a_n = \frac{3^n + 5^n}{2}$
Now for the proof part, we need Basis and Inductive step to prove by mathematical induction.

BASIS: For n=1 $a_1 = \frac{(3^1+5^1)}{2} = 4$

INDUCTIVE STEP

INDUCTIVE HYPOTHESIS: For a $k > 1$, $k \in N$, assume $a_k = \frac{(3^k+5^k)}{2}$.
For $k+1$, we get $a_{k+1} = 3a_k + 5^k$ from recurrence relation, and we want to know if a_{k+1} is equal to $\frac{(3^{k+1}+5^{k+1})}{2}$. From Inductive Hypothesis we know that $a_k = \frac{(3^k+5^k)}{2}$, so we can replace a_k here. Now we get;

$a_{k+1} = \frac{3*(3^k+5^k)}{2} + 5^k$

$a_{k+1} = \frac{3^{k+1}}{2} + \frac{3*5^k}{2} + \frac{2*5^k}{2}$

$a_{k+1} = \frac{3^{k+1}}{2} + \frac{5*5^k}{2}$

$a_{k+1} = \frac{3^{k+1}}{2} + \frac{5^{k+1}}{2}$

$a_{k+1} = \frac{(3^{k+1}+5^{k+1})}{2}$