Enabling Partial Pivoting in Task Flow LU Factorization

Master Defense

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- Three months at *Innovative Computer Laboratory*

- Three months at *Inria Bordeaux Sud-Ouest*

Summary

Context and Motivations

LU Decomposition Algorithms

LU Decomposition over Task Flow Model

Performance

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Architecture Trends

- Computing platforms are more and more complex
- We classify them into four categories:
  - Shared memory architectures
  - Distributed memory architectures
  - Hierarchical architectures
  - Heterogeneous architectures

Programming Paradigms

Historically, we use:

- Message passing (MPI) for distributed memory architectures
- Threads library (OMP) for shared memory architectures
- Accelerator library (Cuda, OpenCL) for accelerator of heterogeneous architectures

⇒ Several programs for the same algorithm
⇒ Weak portability
⇒ Weak scalability

Solution:

Other paradigm of programming which separate algorithm from architectures used: **Task Flow Model**

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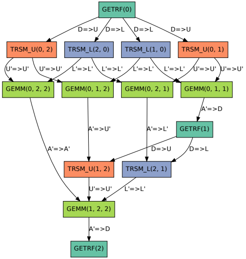
Other paradigm of programming which separate algorithm from architectures used: **Task Flow Model**

Task Flow Model

Programs can be represented by a Direct Acyclic Graph (DAG) where:

- Vertices are tasks
- Edges are data dependencies between tasks



Execution of tasks and data movement between them assured by the **Runtime**

$\Rightarrow$ One single implementation of the DAG in a specific language

Runtimes

Advantages :

- Abstraction of architectures
- Portability of performance
- Good reactivity for load imbalance

Question:

How to cope with parallel program consisting fine grain tasks?

Challenge:

- DAGuE for large hierarchical architectures
- StarPU for heterogeneous architectures

Summary

Why LU decomposition

In order to solve square systems of linear equations:

$$Ax = b$$

Use LU decomposition:

$$A = LU$$

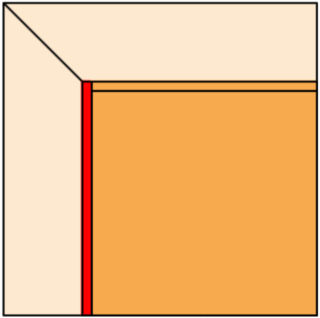
Where L is a lower triangular matrix with the identity diagonal and U an upper triangular matrix.
Then solve:

$$Ly = b \text{ then } Ux = y$$

LU algorithm

Let be A a $n * n$ square matrix
For k from 1 to n

For i from $k+1$ to n
$a_{i,k} = a_{i,k}/a_{k,k}$
For i from $k+1$ to n
For j from $k+1$ to n
$a_{i,j} = a_{i,j} - a_{i,k} *_{k,j}$



LU decomposition problem

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- $a_{k,k}$ may be equal or close to zero
- Numerical value may be deteriorated due to fixed precision used by computers

$\Rightarrow$ LU decomposition is not stable

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LU decomposition problem

Let be A a $n * n$ square matrix
For k from 1 to n
    Search for pivot then swap
    For i from $k+1$ to n
        $a_{i,k} = a_{i,k}/a_{k,k}$
    For i from $k+1$ to n
        For j from $k+1$ to n
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Solution is pivoting

Partial Pivoting Algorithm

The partial pivoting consist to look for the element with the maximal absolute value on the k^{th} column from $a_{k,k}$, then swap its row with the row consisting $a_{k,k}$.

The factorization amount to $PA = LU$

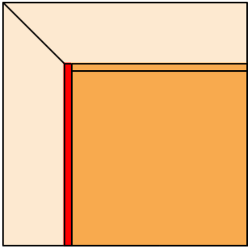
The partial pivoting is:

- Practically stable and accurate
- Commonly used in the scientific community
- Used in the LINPACK benchmark to rank the TOP 500 super-computers

LU implementation ($PA = LU$)

Software/Algorithms follow hardware evolution in time

- 70's - LinPACK, vector operations
- 80's - LAPACK, block, cache-friendly
- 90's - ScaLAPACK, distributed memory



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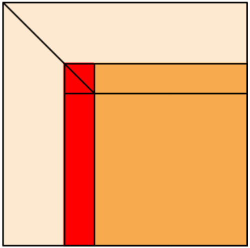

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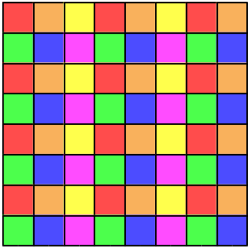

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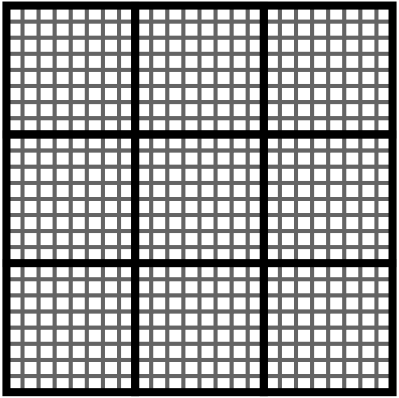
LU Decomposition over Task Flow Model

Performance

Task flow LU ($A = LU$)

In order to cope with the task flow model, linear algebra algorithms are expressed in terms of tasks operating on fine grain squares sub-matrices, also called tiles.



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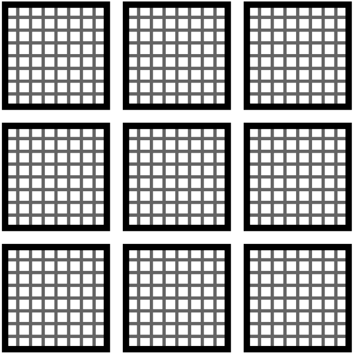

Tiles are not contiguous in memory

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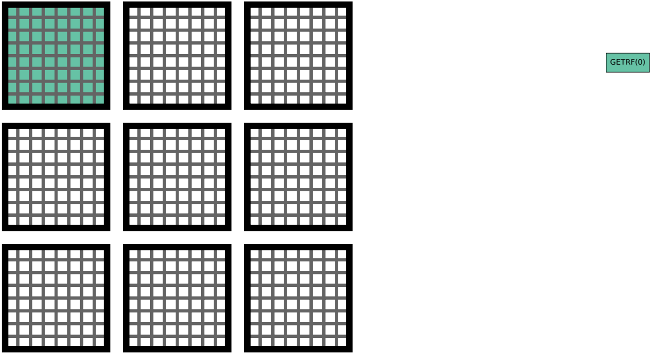

Task to factorize diagonal tiles: GETRF

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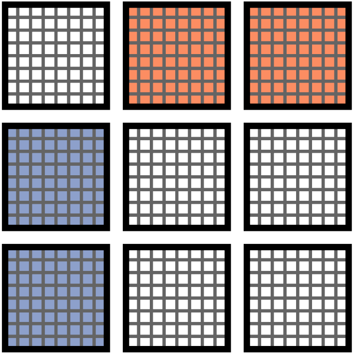

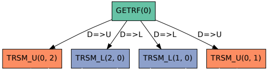

Task to update other panel tiles and block line tiles: TRSM_L and TRSM_U

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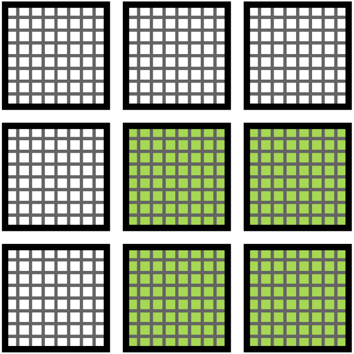

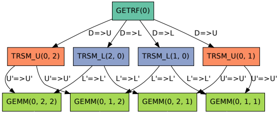

Task to update trailing sub-matrix: GEMM

Task flow LU ($A = LU$)

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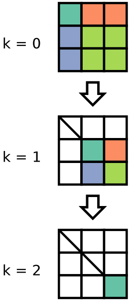
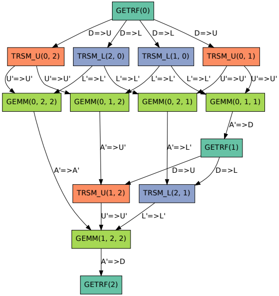
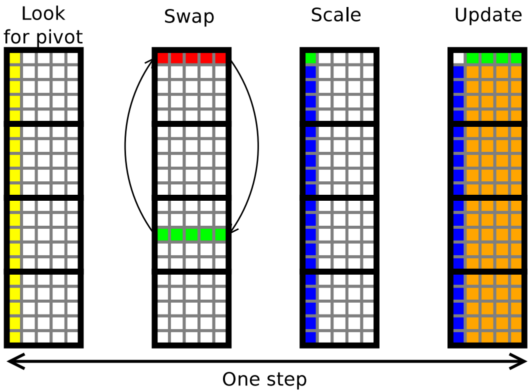




Panel Factorization Problems

Tasks of panel factorization



- Look for a pivot is distributed over several tiles
- Tasks are fine grained

Panel Factorization Problems

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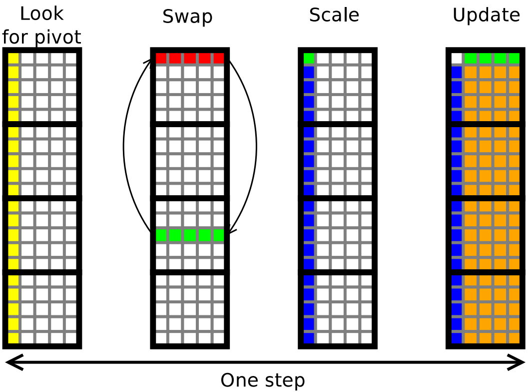

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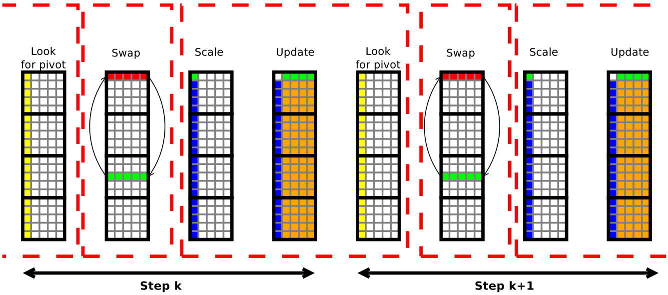
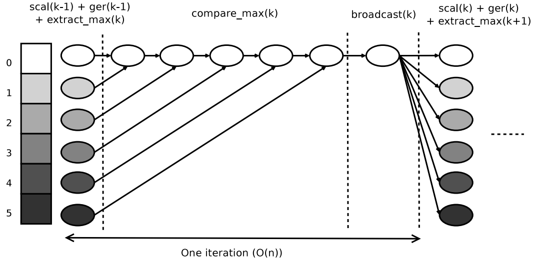
Panel Factorization Problems

Reduce fine grained tasks



Panel Factorization Problems

Natural task flow of panel factorization



$\Rightarrow$ Serialized task flow!

Optimizations:

Use *all_reduce* operation (using Bruck's algorithm)

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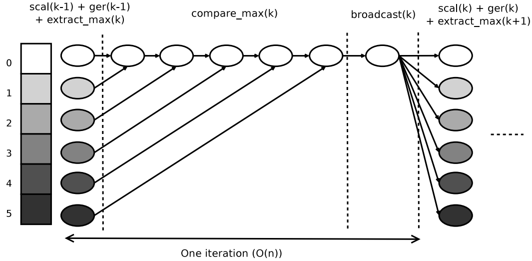

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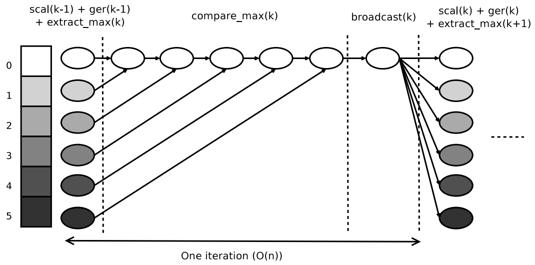

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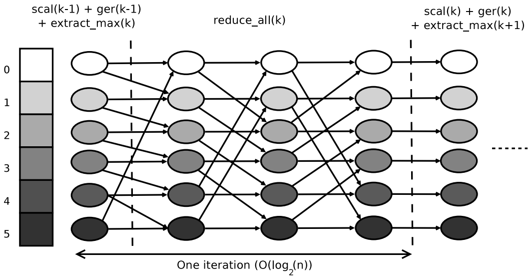
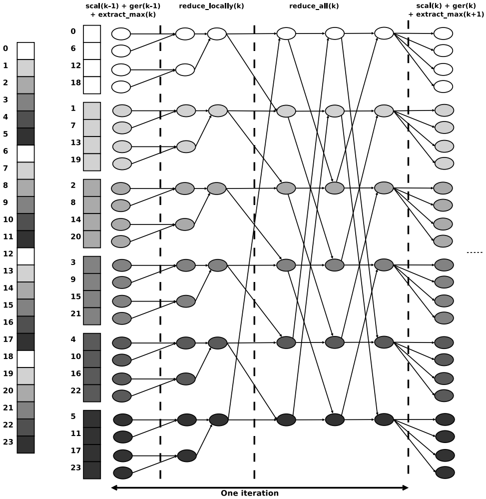
Panel Factorization Problems

Optimized task flow of panel factorization for distributed architectures



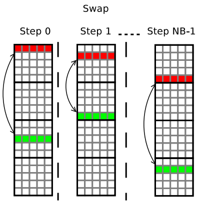
Panel Factorization Problems

Optimized task flow of panel factorization for hierarchical architectures



Update Problems

Update trailing sub-matrix



The upper tile exchange rows with other tile depending on pivots values.

Problem

- Dynamic decision for a static DAG
- Pivots implies swaps in a specific order
- Serialized communications

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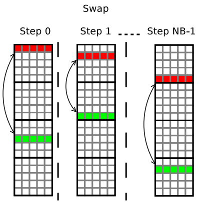

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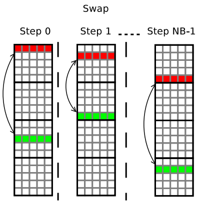

The upper tile exchange rows with other tile depending on pivots values.

Problem → Solutions

- Dynamic decision for a static DAG → Generate tasks for all possible communications?
- Pivots implies swaps in a specific order → Use permutation instead of pivots
- Serialized communications → Separate Swap from/into upper tile

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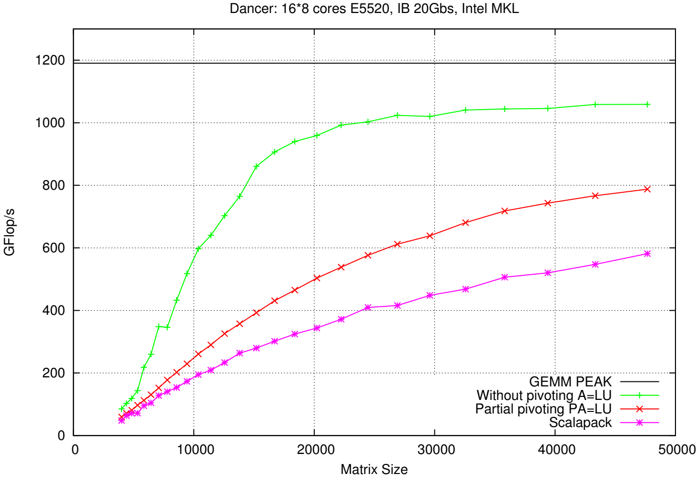

Dancer: 16*8 cores E5520, IB 20Gbs, Intel MKL
GFlop/s
0
200
400
600
800
1000
1200
0
10000
20000
30000
40000
50000
Matrix Size
GEMM PEAK
Without pivoting A=LU
Partial pivoting PA=LU
Scalapack

Conclusion and future work

Software contribution:

- Implemented and validated $A = LU$ on DAGuE and StarPU
- Implemented and validated $PA = LU$ on DAGuE and StarPU

Conclusion:

- Exhibited the feasibility of partial pivoting on task flow model
- Obtained encouraging performance with partial pivoting over DAGuE

Future work:

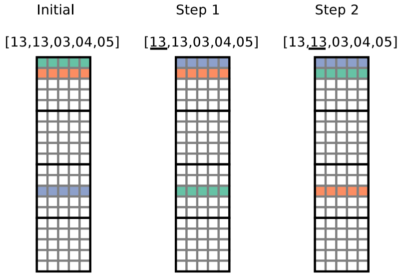
- Integrate GPU pivoting operations to DAGuE and StarPU
- Try other startegies of panel factorizations on several methods
- Build a new benchmark based on the DAGuE partial pivoting

Thank you !

ANNEXE

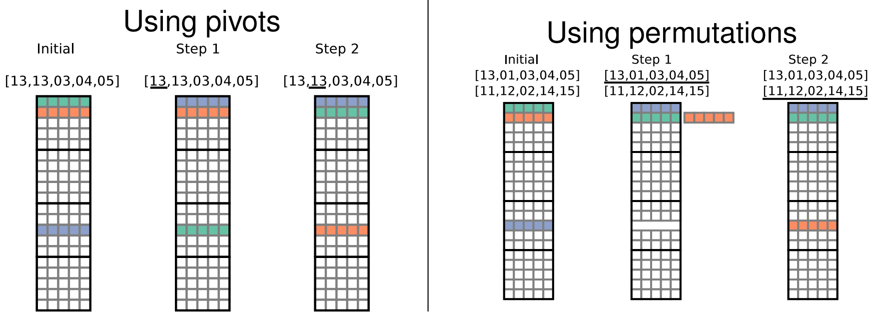
Using permutations instead of pivots

Using pivots



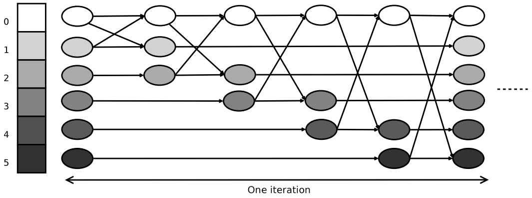
ANNEXE

Using permutations instead of pivots



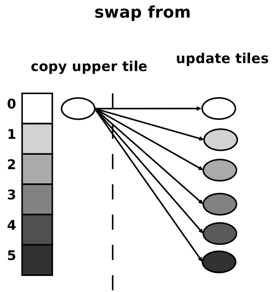
Update Problems

Natural task flow of one swap in update



Update Problems

Optimized task flow of swaps of update for distributed architectures



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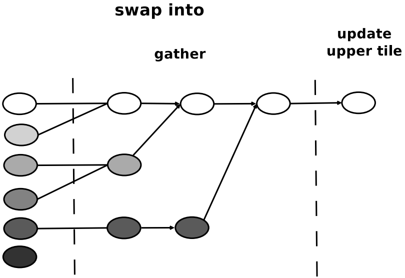

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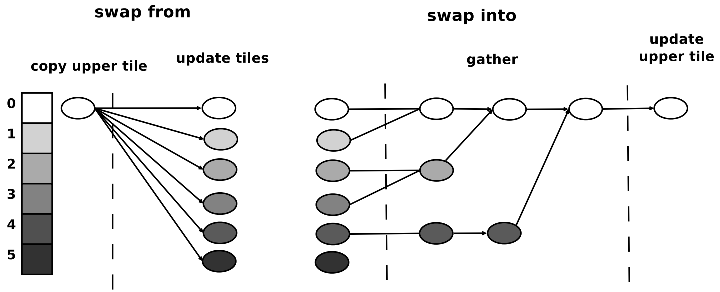

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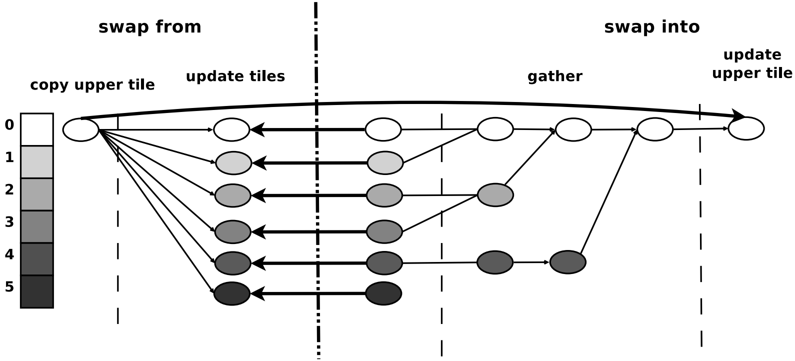

Update Problems

Optimized task flow of swaps of update for hierarchical architectures