LU Decompositions over DAGuE

Friday Lunch Talk

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May 25, 2012

Summary

LU Decomposition Algorithms

DAGuE Runtime System

Static Pivoting

A generic update engine for dynamic pivoting

Partial Pivoting

Performance

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LU Decomposition Algorithms

Introduction

LU decomposition algorithm :

- Used to solve linear equation
- Used in Linpack Benchmark and HPL
- Implemented in most of mathematic library

Why implementing a new LU decomposition ?

- Trend to use Runtime
- Three layer architecture: Algorithm, Runtime and Hardware

$\rightarrow$ performant and portable code

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LU Decomposition Algorithms

The Challenge

DAGuE provide static DAG

The LU algorithm may be static:

- Without pivoting
- Criteria substitution
- Incremental pivoting

Or dynamic:

- Partial pivoting

How to programm dynamic application with static task flow ?

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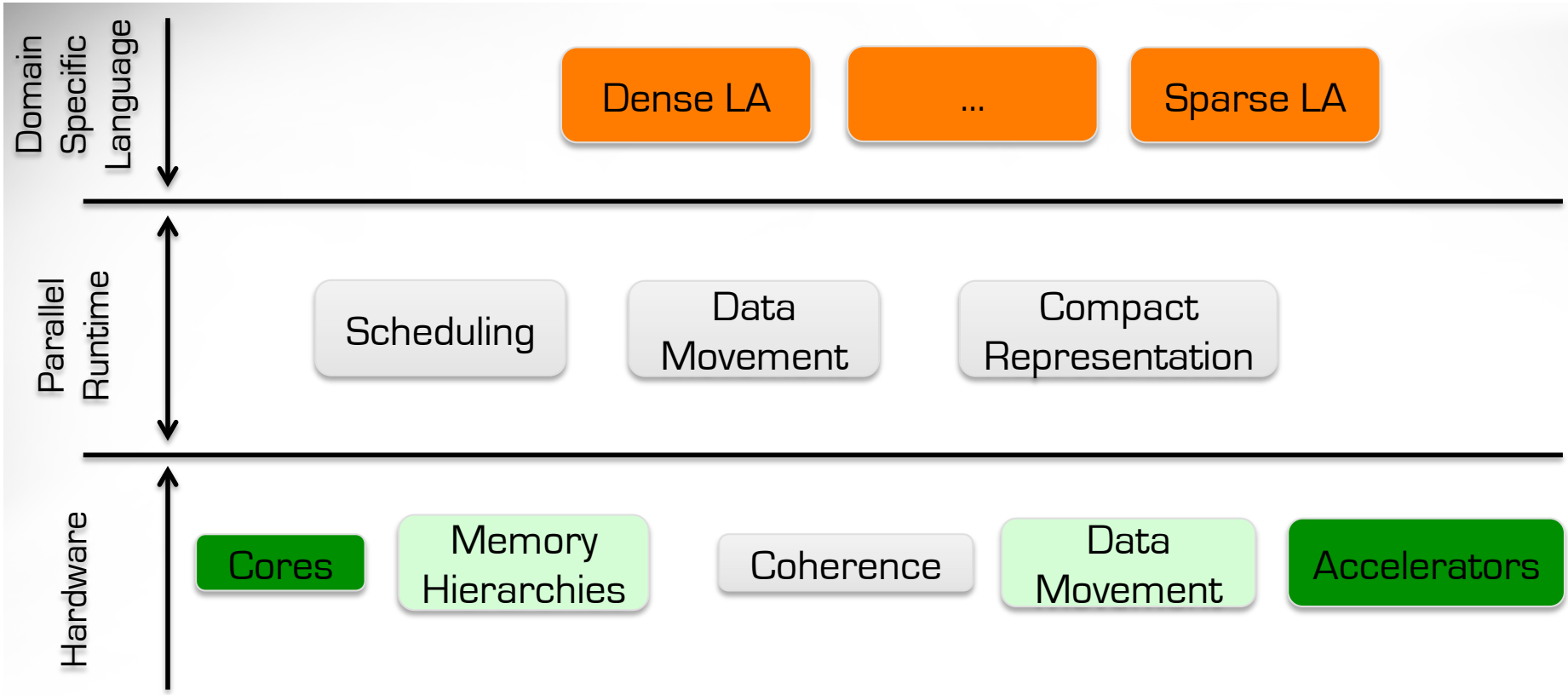
DAGuE

Quick presentation



DAGuE

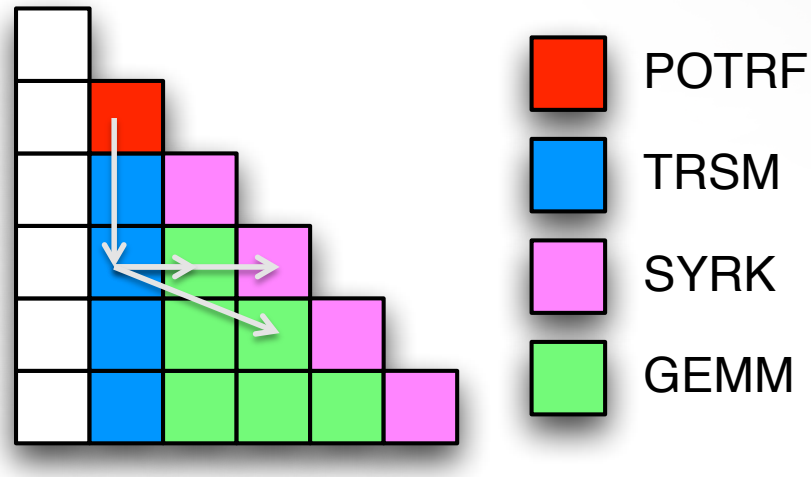
Quick presentation

DAGuE is a Direct Acyclic Graph scheduler Engine based on task flow model where :

- nodes are tasks
- edges are dependancies

```
TRSM(k, n)
// Execution space
k = 0..SIZE-1
n = k+1..SIZE-1
: A(n, k) // Parallel Partitionning
READ  T <- T POTRF(k)
RW    C <- (k == 0) ? A(n, k)
                    : C GEMM(k-1, n, k)
        -> A SYRK(k, n)
        -> A GEMM(k, n+1..SIZE-1, n)
        -> B GEMM(k, n, k+1..n-1)
        -> A(n, k)

BODY*
```



Compared with the serial code this code lost all control flow

DAGuE

Advantages :

- Independence between performances and computers
- Provide multicore parallelism
- Good reactivity for load imbalance
- Natural look ahead

Problems :

- DAG is a static representation of a task flow

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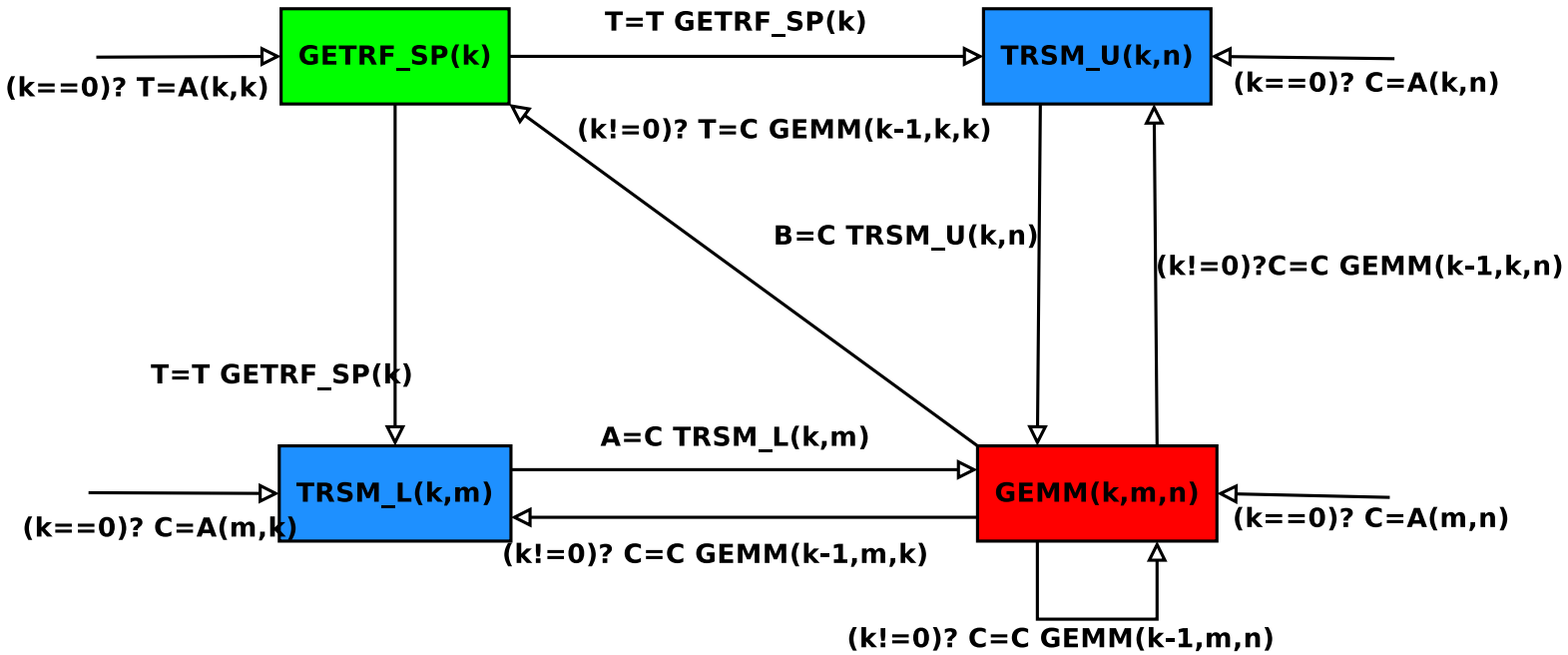
Static Pivoting

Motivation

- Static pivoting matches the the task flow programming model
- Good efficiency
- Stable for several problems
- Preprocess can improve stability
- Good upper bound for the partial pivoting

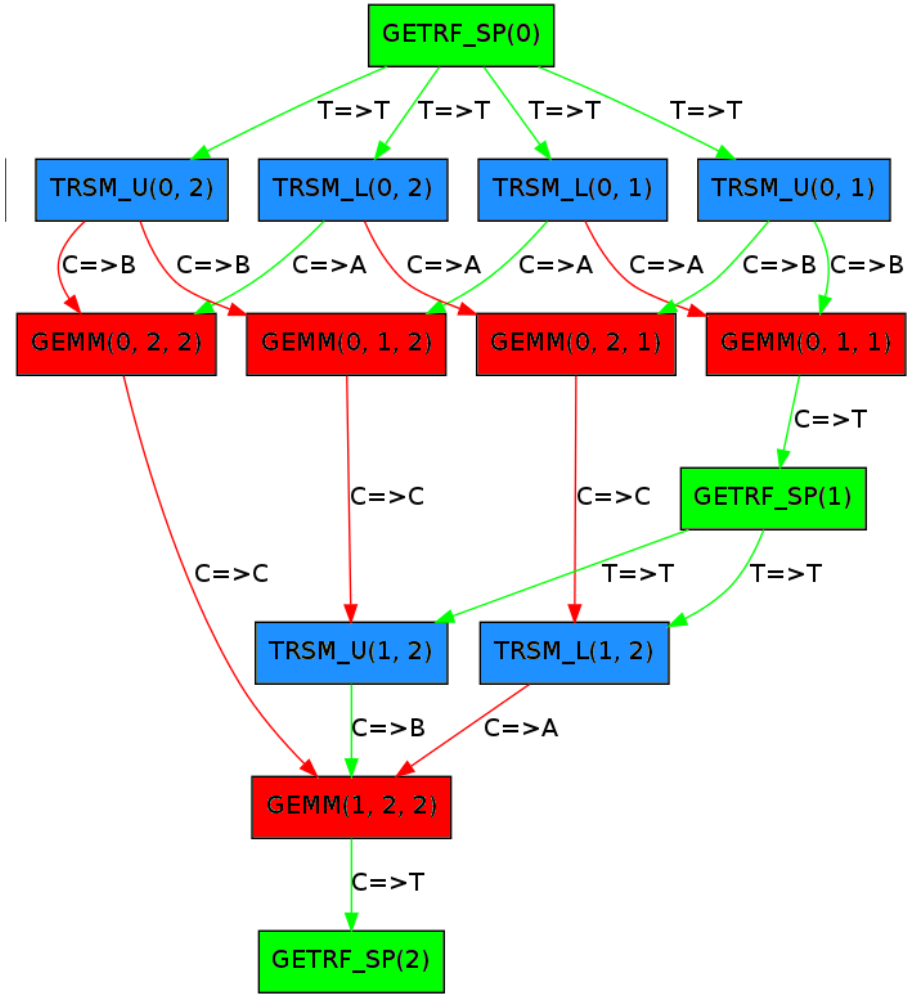
Static Pivoting

Algebraic Representation



Static Pivoting

DAG for a matrix 3*3



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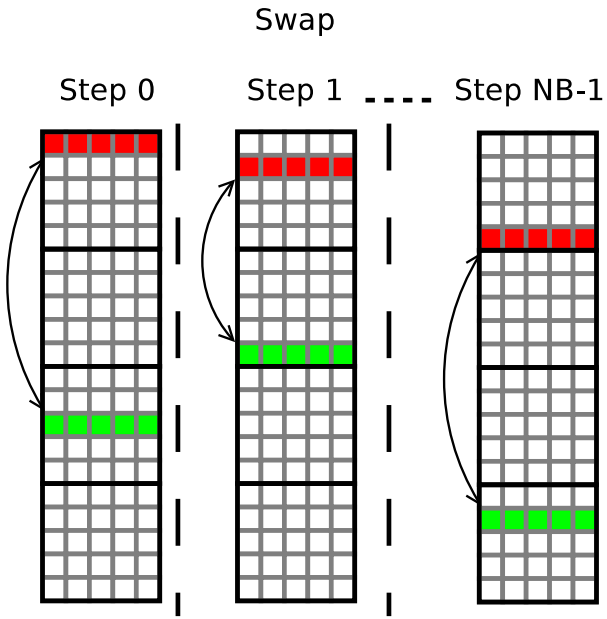
Update Issue



The tile U exchange swap rows with other concerned tile.

Problem

- A dynamic decision for a static DAG
  $\rightarrow$ Prepare tasks for all possible communications?

A generic update engine for dynamic pivoting

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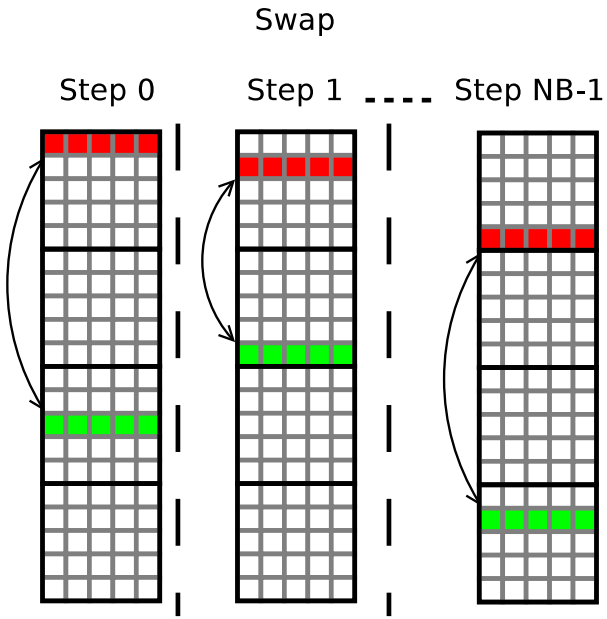

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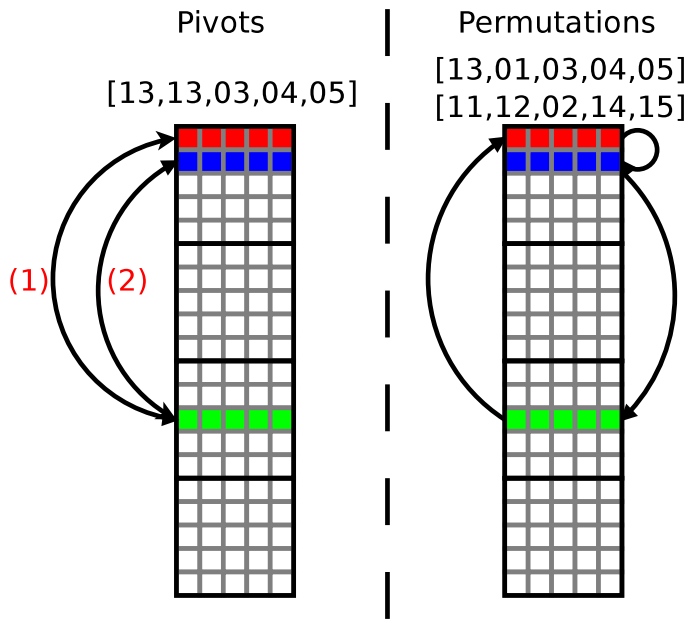
A generic update engine for dynamic pivoting

Solutions

Ideas:

- Avoiding useless swap to increase parallelism
  → Use of permutations instead of pivots indexes
- Updating the main tile is more urgent
  → Parallelize the swap **from** and the swap **into** the tile U
- Limiting the number of communication (not the volume)
  → Gather communications of all rows over two buffers



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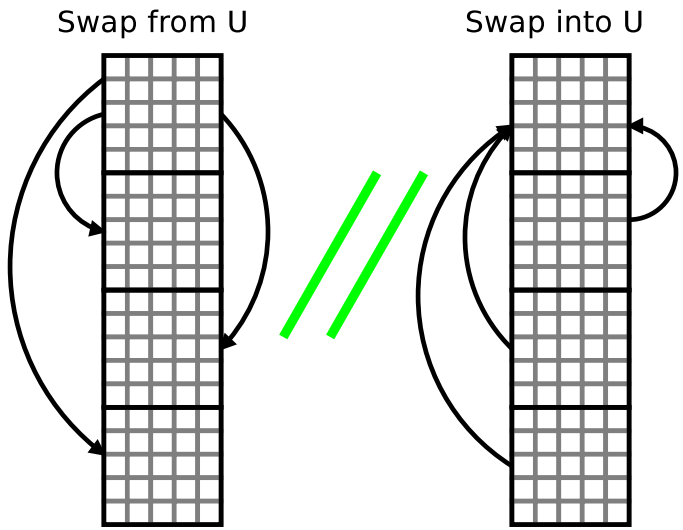

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$\rightarrow$ Five kinds of tasks : COPY, COLLECT, RECEIVE, SEND and PASTE.

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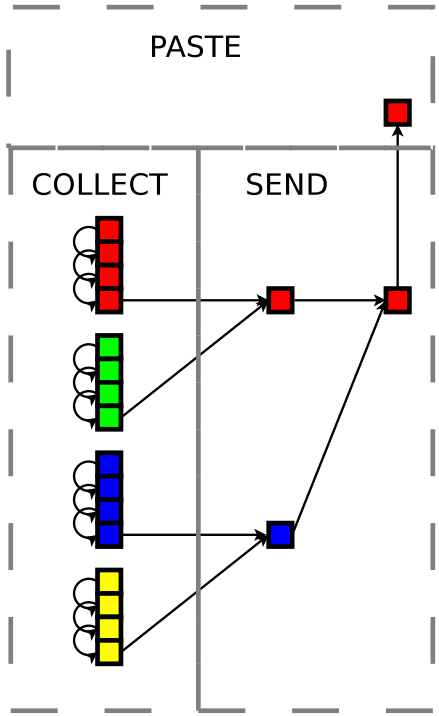
A generic update engine for dynamic pivoting

Swap into U

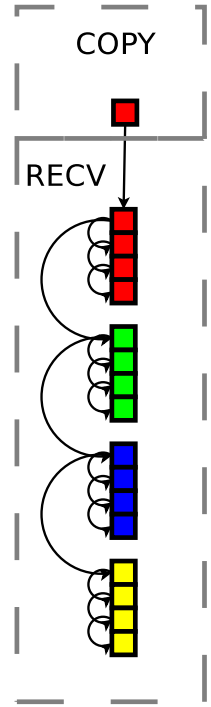
- COLLECT: Collecting the rows needed by the tile U into a buffer.
- SEND: Gather the buffers collected by COLLECT of each node.
- PASTE: Overwrite the tile U with the buffer.
- COPY
- RECEIVE



A generic update engine for dynamic pivoting

Swap from U

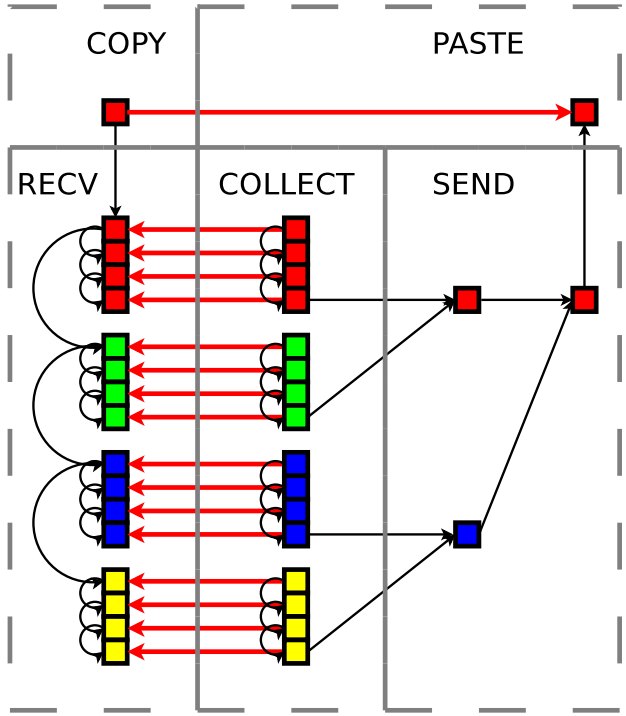
- COLLECT
- SEND
- PASTE
- COPY: Copy tile U into a buffer.
- RECEIVE: Receive the buffer U and make the swap from it.



A generic update engine for dynamic pivoting

Update Tasks Synchronisation

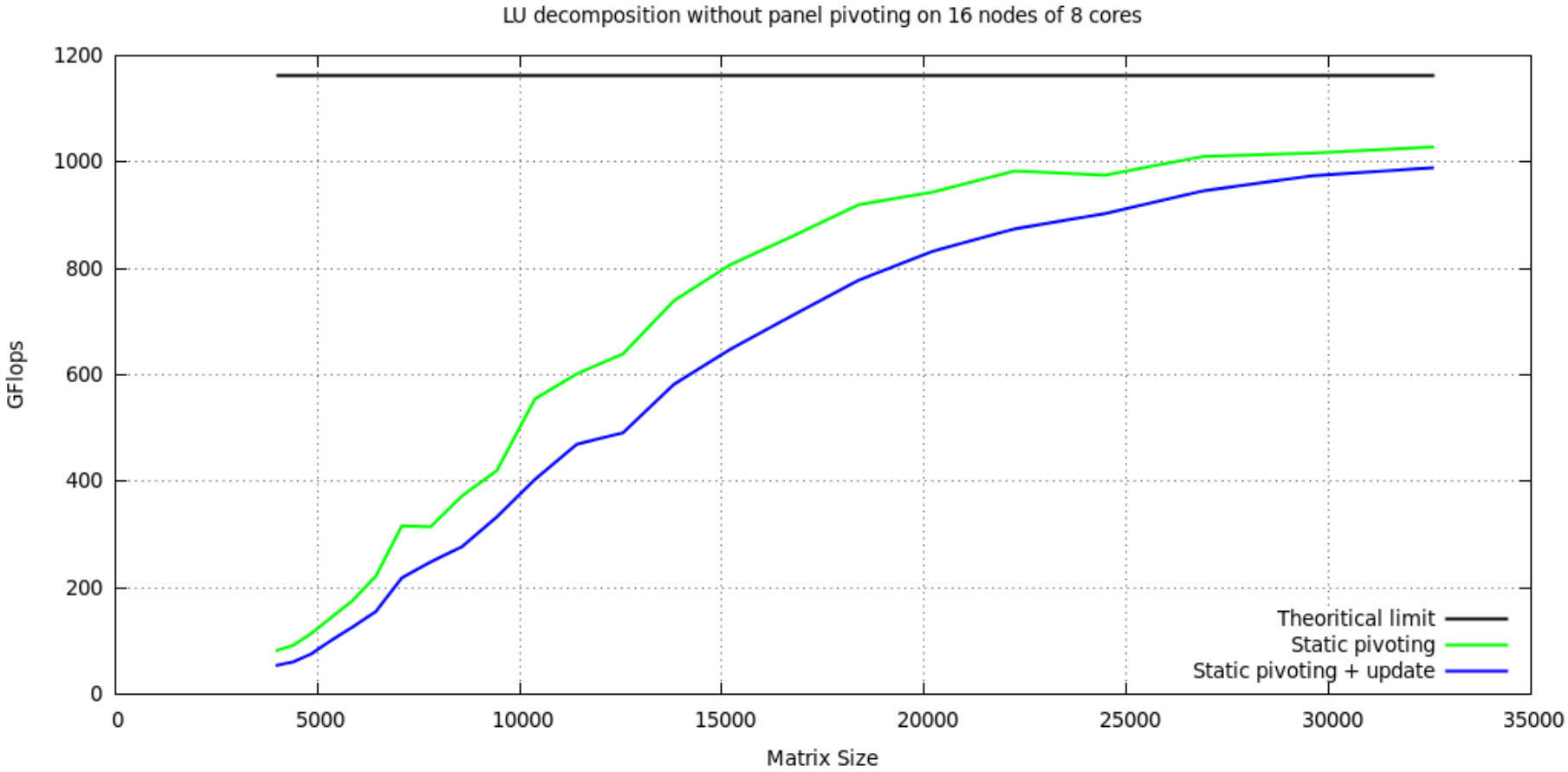
- COLLECT
- SEND
- PASTE
- COPY
- RECEIVE



The red arrows prevent the **READ AFTER WRITE**.

A generic update engine for dynamic pivoting

Update Impact



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Partial Pivoting

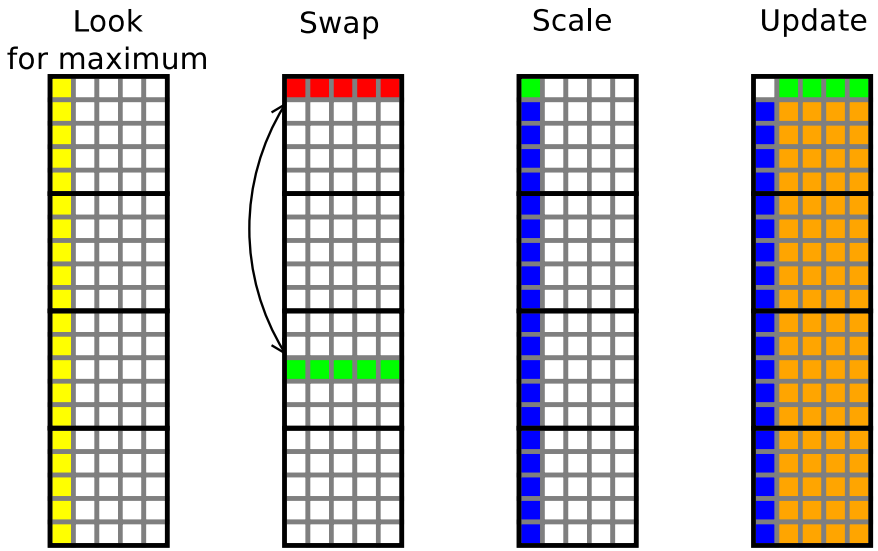
Heuristic Factorization

Several heuristic to factorize the panel:

- Partial pivoting
- Threshold pivoting
- Tournament pivoting
  - Internal partial pivoting
  - Panel rank revealing

Partial Pivoting

Operations of Panel LU Decomposition

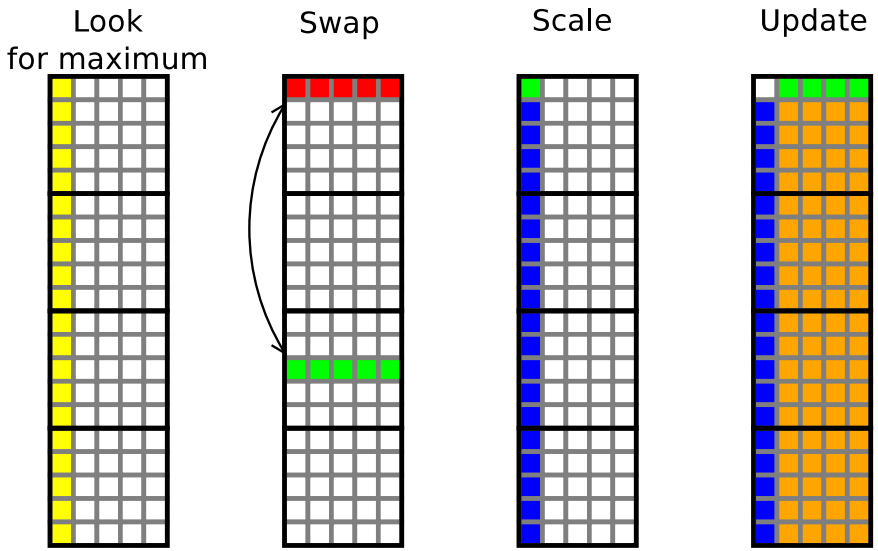


Problem for implementing with task flow model is based on

- Swap line is dynamically decided but the DAG is static
- Minimize latency for the panel

Partial Pivoting

Operations of Panel LU Decomposition



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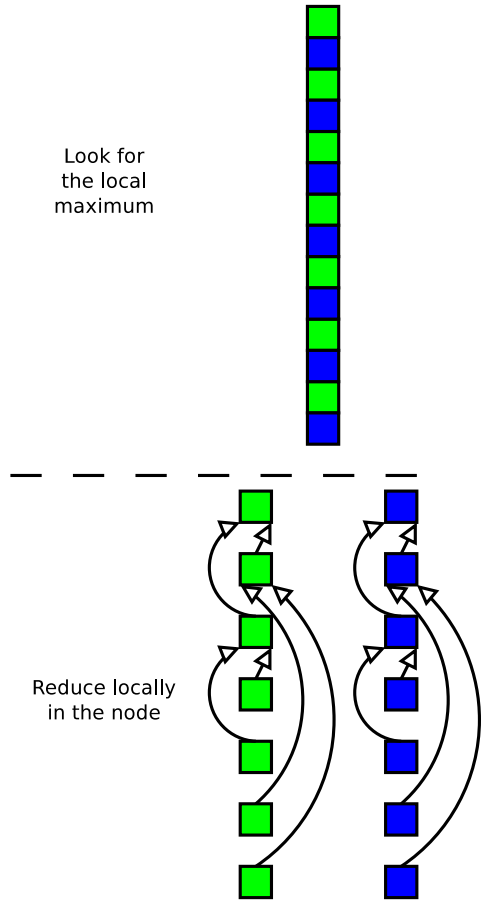
Partial Pivoting

Solutions

Solutions:

- Start looking for the maximum locally then reduce locally the result
- Share the global result by using Bruck's algorithm
- Use internal blocking



Partial Pivoting

Implemented version

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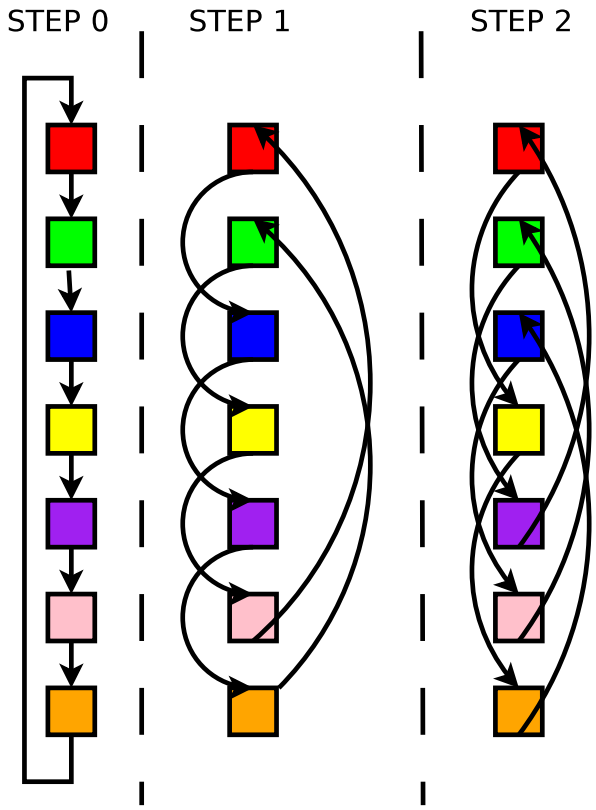

Partial Pivoting

Implemented version

Optimizations:

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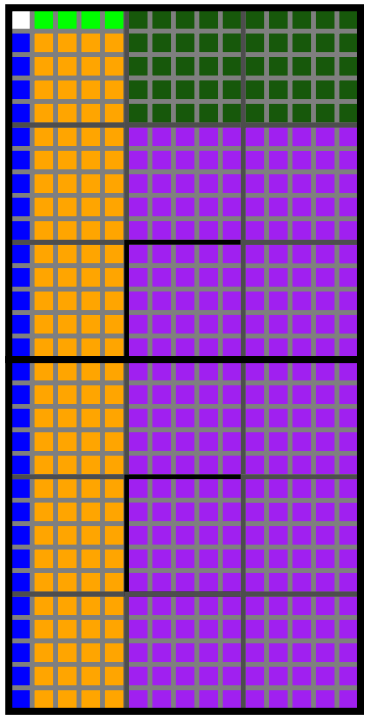
Panel update



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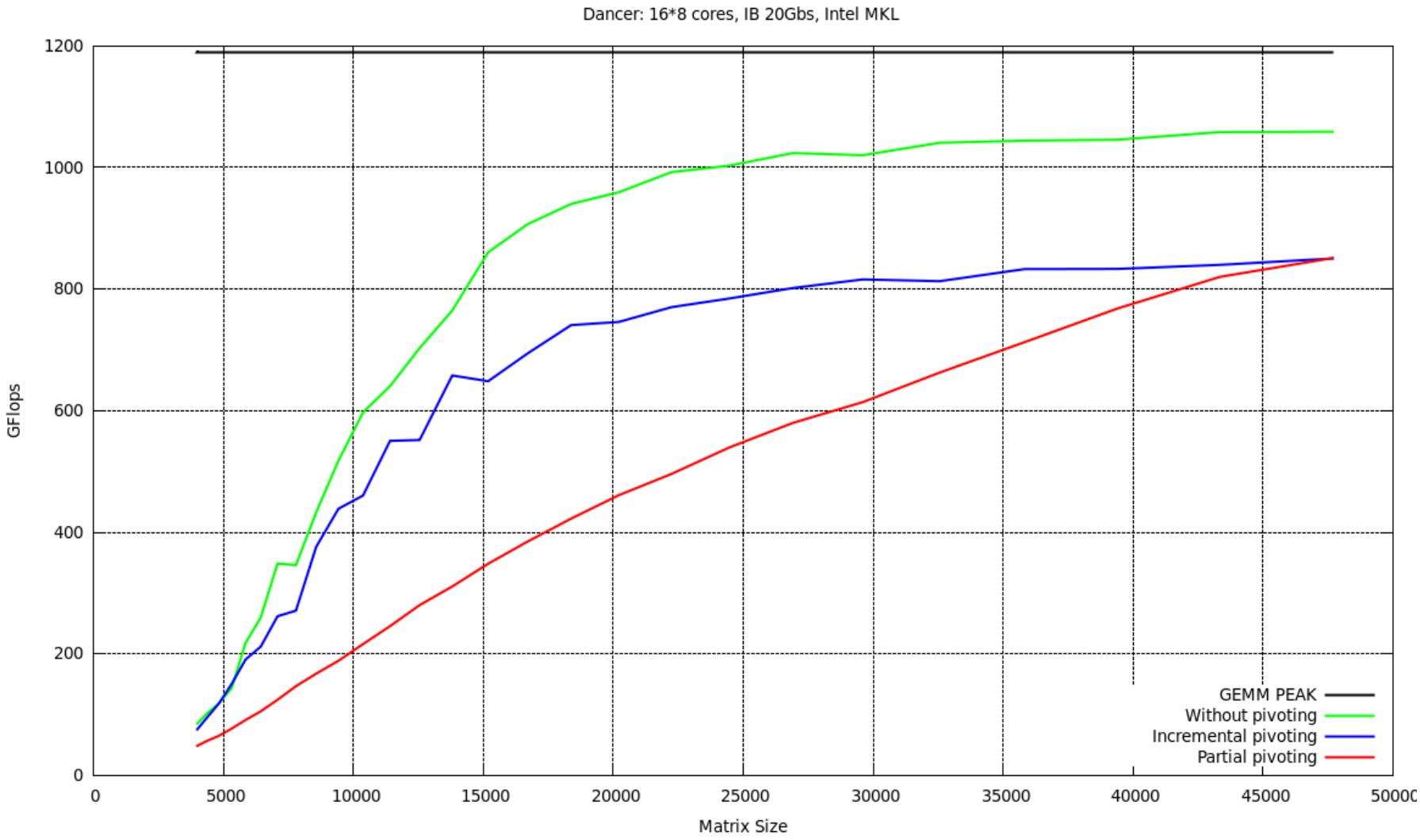

Performance

- Shared memory
- Problem scalability
- Strong scalability

Conclusion and future work