Enabling Partial Pivoting in Task Flow LU Factorization

Master Defense

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- Three months at *Innovative Computer Laboratory*

- Three months at *Inria Bordeaux Sud-Ouest*

Summary

Context and Motivations

- $A = LU$
- $PA = LU$

LU Decomposition Without Pivoting ($A = LU$)

LU Decomposition with Partial Pivoting ($PA = LU$)

- Panel Factorization
- Update of Trailing Sub-matrix

Performance

Summary

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Architecture Trends

- Computing platforms are more and more complex
- We classify them into four categories:
  - Shared memory architectures
  - Distributed memory architectures
  - Hierarchical architectures
  - Heterogeneous architectures

Programming Paradigm

Historically, we use:

- Message passing (MPI) for distributed memory architectures
- Threads library (OMP) for shared memory architectures
- Accelerator library (OpenCL) for accelerator of heterogeneous architectures

$\Rightarrow$ Several programs for the same algorithm
$\Rightarrow$ Weak portability
$\Rightarrow$ Weak scalability

Solution:

Recent paradigm of programming which separate algorithm from architectures used: **Task Flow Model**

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Task Flow Model

Programs can be represented by a Direct Acyclic Graph (DAG) where:

- Vertices are tasks
- Edges are data dependencies between tasks

⇒ One single implementation of the DAG in a specific language

Execution of tasks and data movement between them assured by the **Runtime**

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Runtimes

Advantages :

- Abstraction of architectures
- Portability of performance
- Good reactivity for load imbalance
- Natural look ahead

Challenge:

At the moment, runtimes are efficient for model of architectures

- DAGuE for large hierarchical architectures
- StarPU for heterogeneous architectures

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Why LU decomposition

In order to solve square systems of linear equations:

$$Ax = b$$

Use LU decomposition (Gaussian elimination):

$$A = LU$$

Where L is a lower triangular matrix with the identity diagonal and U an upper triangular matrix.
Then solve:

$$Ly = b \text{ then } Ux = y$$

LU algorithm

Let be A a $n * n$ square matrix

For k from 1 to n

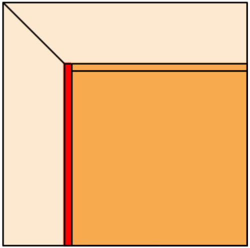
  For i from $k+1$ to n

    $a_{i,k} = a_{i,k}/a_{k,k}$

  For i from $k+1$ to n

    For j from $k+1$ to n

      $a_{i,j} = a_{i,j} - a_{i,k} *_{k,j}$



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LU decomposition problem

Let be A a $n * n$ square matrix
For k from 1 to n

For i from $k + 1$ to n
$a_{i,k} = a_{i,k}/a_{k,k}$
For i from $k + 1$ to n
For j from $k + 1$ to n
$a_{i,j} = a_{i,j} - a_{i,k} *_{k,j}$

- $a_{k,k}$ may be equal or close to zero
- Numerical value may be deteriorated due to fixed precision used by computers

$\Rightarrow$ LU decomposition is not stable

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LU decomposition problem

Let be A a $n * n$ square matrix
For k from 1 to n
  Search for pivot then swap
  For i from $k+1$ to n
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  For i from $k+1$ to n
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Solution is pivoting

Partial Pivoting Algorithm

The partial pivoting consist to look for the element with the maximal absolute value on the k^{th} column from $a_{k,k}$, then swap its row with the row consisting $a_{k,k}$.

The factorization amount to $PA = LU$

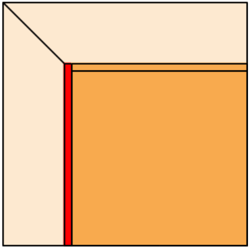
The partial pivoting is:

- Practically stable and accurate
- Commonly used in the scientific community
- Used in the LINPACK benchmark to rank the TOP 500 super-computers

LU implementation ($PA = LU$)

Software/Algorithms follow hardware evolution in time

- 70's - LinPACK, vector operations
- 80's - LAPACK, block, cache-friendly
- 90's - ScaLAPACK, distributed memory



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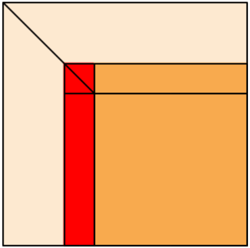

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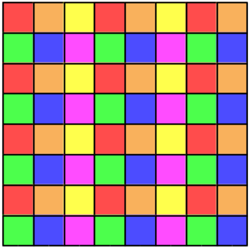

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Task flow LU ($A = LU$)

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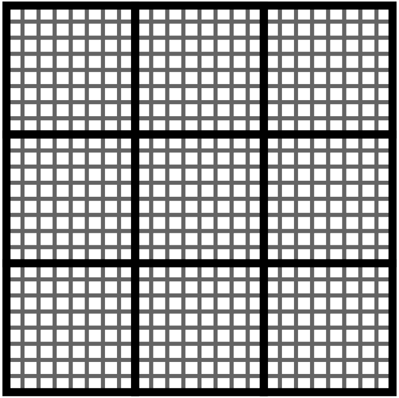

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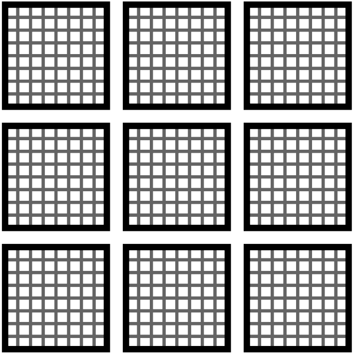

Tiles are not contiguous in memory

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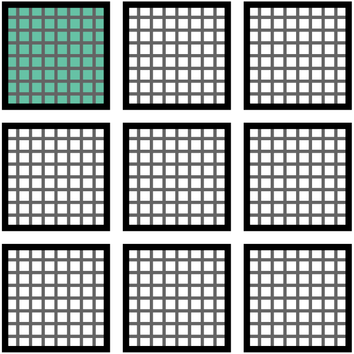

Task to factorize diagonal tiles: GETRF

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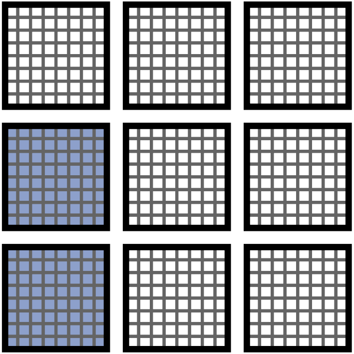

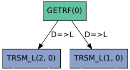

Task to update other panel tiles: TRSM_L

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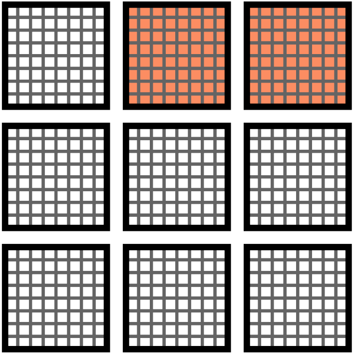

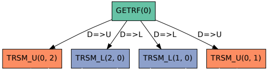

Task to update eliminated rows tiles: TRSM_U

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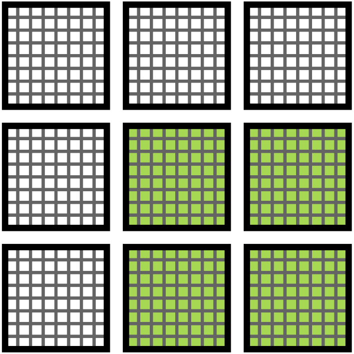

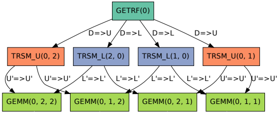

Task to update trailing sub-matrix: GEMM

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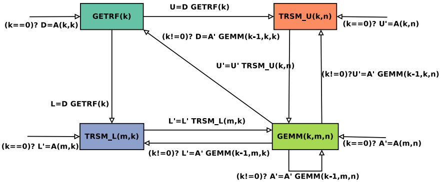

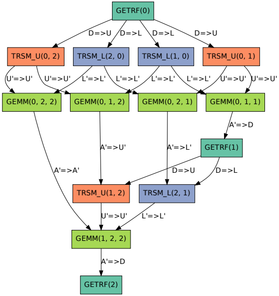

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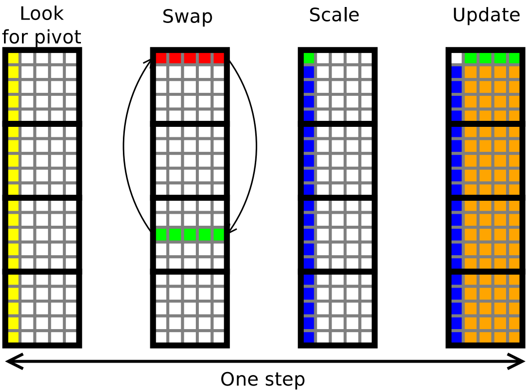
Panel Factorization Problems

Tasks of panel factorization



- Look for a pivot is distributed over several tiles
- Tasks are fine grained

Panel Factorization Problems

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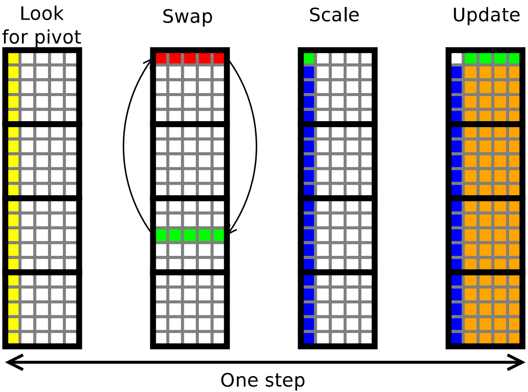

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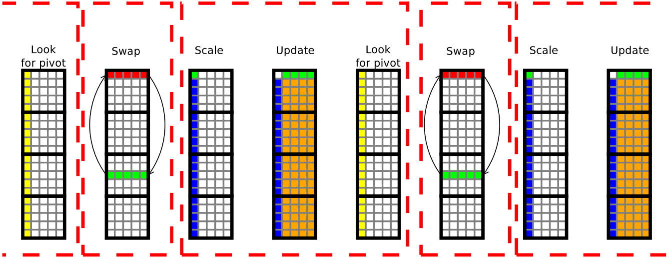
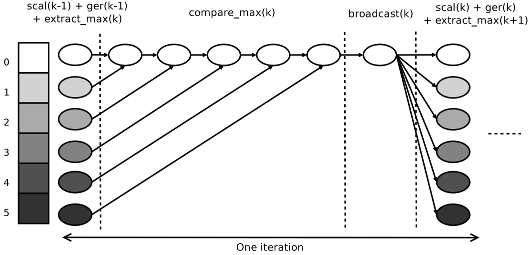
Panel Factorization Problems

Reduce fine grained tasks



Panel Factorization Problems

Natural task flow of panel factorization



⇒ Serialized task flow!

Optimizations:

Use *all_reduce* operation (using Bruck's algorithm)

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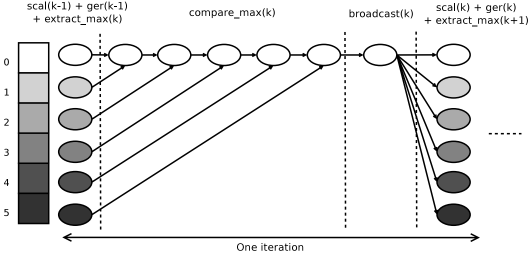

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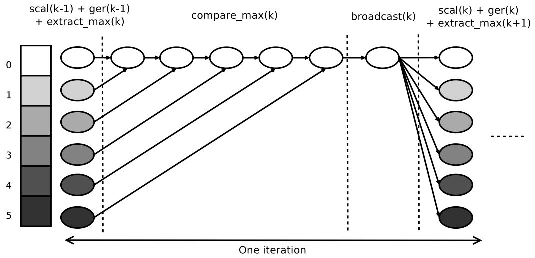

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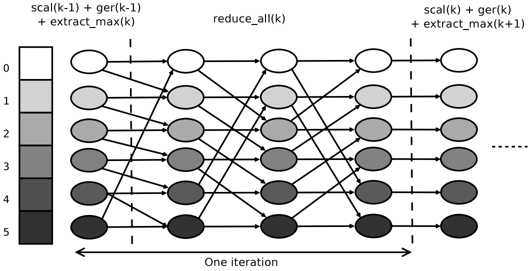
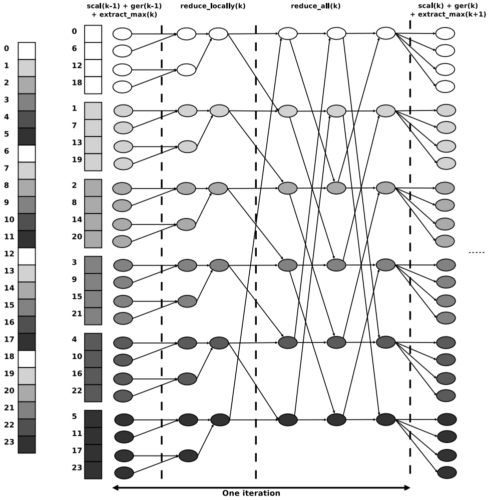
Optimized task flow of panel factorization for distributed architectures



Panel Factorization Problems

Optimized task flow of panel factorization for hierarchical architectures



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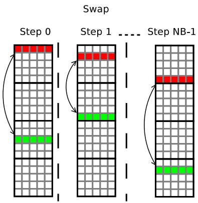
Update trailing sub-matrix



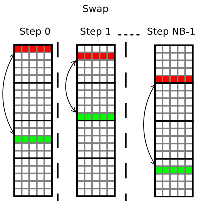
The upper tile exchange rows with other tile depending on pivots values.

Problem

- Dynamic decision for a static DAG $\rightarrow$ Generate tasks for all possible communications?
- Pivots implies swaps in a specific order $\rightarrow$ Use permutation instead of pivots
- Serialized communications $\rightarrow$ Separate Swap from/into upper tile

Update Problems

Update trailing sub-matrix



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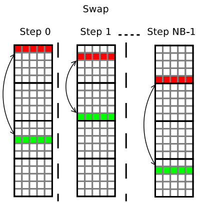

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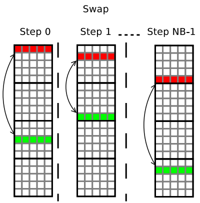

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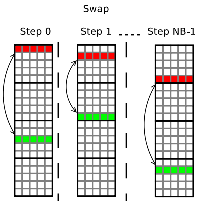

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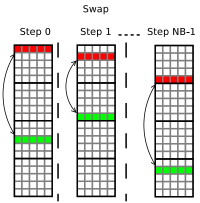

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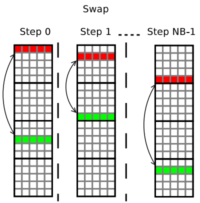

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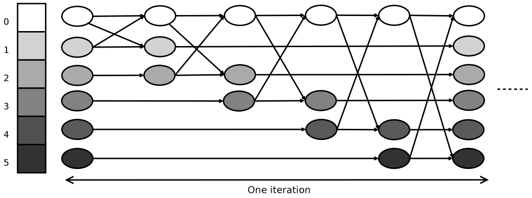
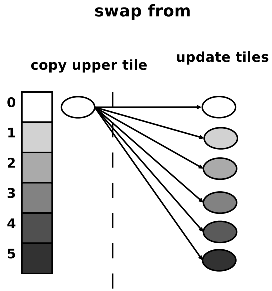
Natural task flow of one swap in update



Update Problems

Optimized task flow of swaps of update for distributed architectures



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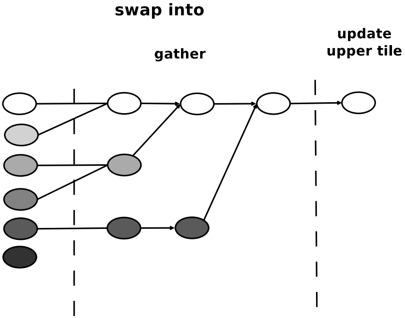

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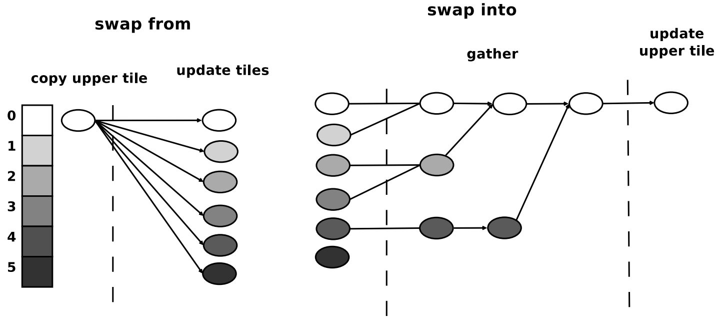

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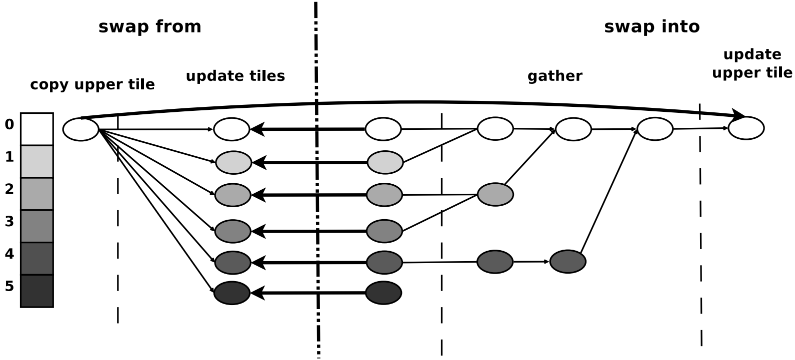
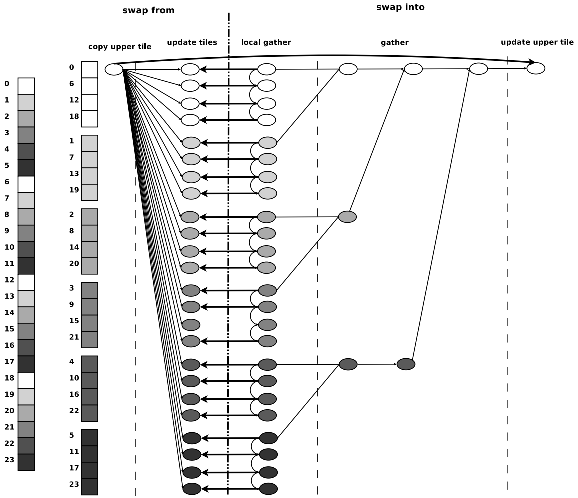


Update Problems

Optimized task flow of swaps of update for hierarchical architectures



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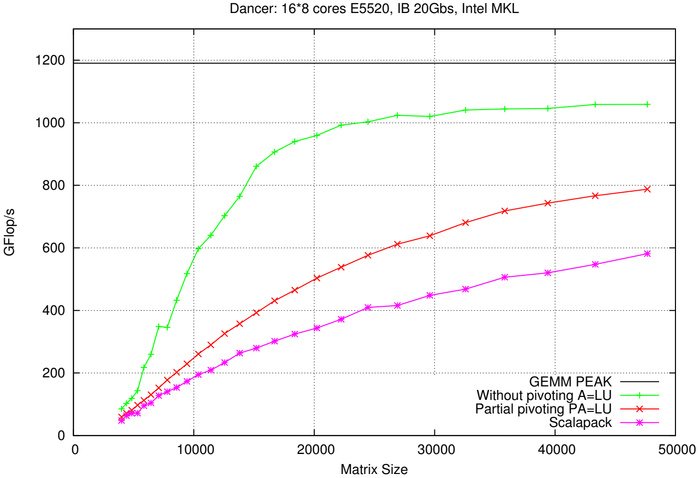

Dancer: 16*8 cores E5520, IB 20Gbs, Intel MKL
GFlop/s
Matrix Size
0
200
400
600
800
1000
1200
0
10000
20000
30000
40000
50000
GEMM PEAK
Without pivoting A=LU
Partial pivoting PA=LU
Scalapack

Conclusion and future work

Conclusion :

- Implemented static pivoting ($A = LU$) on DAGuE and StarPU
- Implemented partial pivoting ($PA = LU$) on DAGuE and StarPU
- Exhibited the feasibility of partial pivoting on task flow model
- Obtained encouraging performances with partial pivoting over DAGuE
- Performances of partial pivoting over StarPU still not exploitable

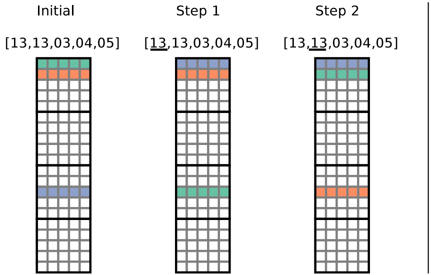
Future work :

- Integrate GPU pivoting operations to DAGuE and StarPU
- Try other startegies of panel factorizations on several methods
- Build a new benchmark based on the DAGuE partial pivoting

Thank you !

ANNEXE

Using permutations instead pivots



ANNEXE

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