Sommersemester 2025 Homework 10

1. Questions

(1) Have you already presented a solution on the board? If not, what are your plans? Note that you need to present at least one to get a positive grade.

(2) Consider
$$\mathbb{Z}[\sqrt{-5}] = \{a + b\sqrt{-5} \colon a, b \in \mathbb{Z}\} \subseteq \mathbb{C}.$$

    (a) Check that $\mathbb{Z}[\sqrt{-5}]$ is a ring.

    (b) Show that $2 \in \mathbb{Z}[\sqrt{-5}]$ divides $(1 + \sqrt{-5})(1 - \sqrt{-5})$.

    (c) Show that 2 does not divide $1 + \sqrt{-5}$ or $1 + \sqrt{-5}$.