Sommersemester 2025 Blatt 1

1. Questions

(1) Show that a Dyhopantine equation $a_1x_1 + \ldots + a_nx_n = c$ has a solution if and only if $\gcd(a_1, \ldots, a_n)|c$.
(2) Consider the equation $ax + by = c$ with $\gcd(a, b)|c$. Determine the solution space.

2. Comments

(1) Before attempting to solve Question 1, first make sure you understand what the question is asking.
  (a) Remember that in a Dyhopantine equation, unlike a course in linear algebra, you are only interested in integer solutions.
  (b) The notation $a|b$ is read "a divides b" and it means that there is a positive integer c such that $b = ac$.
  (c) The notation $\gcd(-)$ refers to greatest common divisor.
  (d) First start with an example:
    (i) Compute $2 \times 3 + 4 \times 2 + 8 \times 1$.
    (ii) Do you know a solution to $2x_1 + 4x_2 + 8x_2 = 22$?
    (iii) What is the greatest common divisor of $2, 4$ and 8? Does it divide 22?
    (iv) Choose any three numbers a, b, c that you want. Compute $2a + 4b + 8c$. Is it even or is it odd? Can you choose other numbers a, b, c to make $2a + 4b + 8c$ an odd number?
    (v) Does the equation $2x_1 + 4x_2 + 8x_2 = 21$ have a solution?
(2) Before attempting to solve Question 2, first make sure you understand what the question is asking.
  (a) Find a solution to the equation $2x + 3y = 5$. This means that find integers a, b such that $2a + 3b = 5$.
  (b) From your answer to Question 1, you know that there is a solution to the homogeneous equation $2x + 3y = 0$. Find at least one solution. Call them c, d. This means that you are asked to find integers c, d such that $2c + 3d = 0$.
  (c) Now, let a, b, c, d as you chose above.
    (i) Compute $2a + 3b + 2c + 3d$.
    (ii) Compute $2(a + c) + 3(b + d)$.
    (iii) What can you conclude?