Sommersemester 2025 Homework 2

1. Questions

(1) What did you learn in class this week? What was one interesting thing? What did you find most difficult?

(2) You saw in the lecture that the positive integers (x, y, z) with $\gcd(x, y, z) = 1$ satisfying the equation

$$x^2 + y^2 = z^2$$

are of the form

$$x = a^2 - b^2 \quad , \quad y = 2ab \quad , \quad z = a^2 + b^2$$

with a, b positive integers, $a > b$, $a \neq b \mod 2$ and $\gcd(a, b) = 1$. Now, consider the map

$$f\colon \{(a,b) \in \mathbb{Z}_{\geq 1} \colon a > b, a \neq b \mod 2, \gcd(a,b) = 1\} \to \{\text{Pythagorean triples}\}$$
$$(a,b) \mapsto \begin{bmatrix} a^2 - b^2 \\ 2ab \\ a^2 + b^2 \end{bmatrix}.$$

Is f injective?