Sommersemester 2025 Blatt 5

1. Questions

(1) Bring the quadratic equation
$$x^2 + xy + y^2 - 1 = 0$$
into normal form. Give a parametrisation of the solution space.
(2) Bring the quadratic equation
$$2x^2 - 5xy - 3y^2 = 0$$
into normal form. Give a parametrisation of the solution space.
(3) Find rational solutions to
$$x^3 + y^3 = x^2 + y^2.$$
(4) For $n \geq 2$, find a parametrisation of the rational solution space of $y^2 = x^n$ by $\mathbb{Q}$. What about integer solutions?
(5) Find a parametrisation of the rational solution space of $y^2 = x^2(x+1)$ by $\mathbb{Q}$.
(6) Show that if F is a degenerate quadratic equation in two variables with integer coefficients, then it describes either a point or a union of two lines.

2. Comments

(1) Before you try to solve question 1 and 2, look at your notes from the lecture and make sure you remember the definition of a *normal form* and a *parametrisation*. Remember that Eva is not enrolled in the lecture, so you should check in with her to see if she needs help.
(2) For Question 3, consider the case $x = 0$ and $x \neq 0$ separately.
   (a) When a number is not zero, we can divide by it! So for the second case, consider $h := \dfrac{y}{x} \in \mathbb{Q}$.
(3) For Question 4, you need to consider the case n is odd and the case n is even separately.
(4) We did not have time to consider Question 6 during Week 4. Martin suggested that we do it this week.