Ausgewählte Kapital der Mathematik 2 **Özgür Esentepe**

Sommersemester 2025 Blatt 11

1. Questions

(1) Prove that if $x^2 - dy^2 = c$ has one solution, then it has infinitely many solutions.
(2) Find all the integer solutions to the equation $x^2 - 2y^2 = 1$.
(3) Find all the integer solutions to the equation $x^2 - 3y^2 = 1$.
(4) Show that there are infinitely many Pythagorean triples $x < y < z$ in which x and y are consecutive integers.

2. Comments

(1) Let (w, v) be a solution of this equation and (r, s) be a solution of the equation $x^2 - cy^2 = 1$. Use a trick similar to 1d from Blatt 10. Explain how this gives us the answer.
(2) For both 2 and 3, you are expected to find a minimal solution (what does a minimal solution mean?) and then use the formula you learned in class.
(3) Rewrite the question: there are infinitely many Pythagorean triples $x, x+1, z$. Now, we have the equation

$$x^2 + (x+1)^2 = z^2$$

for which we need to show that there are infinitely many solutions. Turn this equation into an equation of the form

$$u^2 - 2z^2 = -1$$

and then use Question 1 from Blatt 10 to relate it to the equation

$$u^2 - 2z^2 = 1.$$

How does this finish the solution?