Initiations of Chaotic Regimes of Attitude Dynamics of Multi-Spin Spacecraft and Gyrostat-Satellites Basing on Multiscroll Strange Chaotic Attractors

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Initiations of Chaotic Regimes of Attitude Dynamics of Multi-Spin Spacecraft and Gyrostat-Satellites Basing on Multiscroll Strange Chaotic Attractors

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***Abstract*—In this paper the possibility of chaotic regimes intentional initiations in attitude dynamics of multi-spin spacecraft (MSSC) and gyrostat-satellites (GS) are considered. These considering chaotic regimes correspond to phase trajectories along strange chaotic attractors, which can be presented in the MSSC/GS phase space. Intentional initiations of chaotic regimes can be applied to the attitude reorientation of MSSC/GS in cases of accidents and failures of main attitude control systems, and/or in cases of cancellations of uncontrolled rotations. Also these regimes can be used for the implementation of special tasks of the motion control of space and underwater robots, including the fast random observation of surroundings, fast random search of objects, and chaotic maneuvering.**

Keywords— dynamical chaos, dual-spin spacecraft, multi-spin spacecraft, attitude control, spatial reorientation, chaotic maneuvering, space and underwater robots

I. INTRODUCTION

The study of various regimes of the spacecraft (SC) attitude dynamics still is important task of the modern spaceflight dynamics, especially in the framework of formulations of investigation and using of unconventional and irregular motion regimes, including homo(hetero)clinic chaotizations and implementations of phase trajectories along strange chaotic attractors [1-31].

The chaotic attractors' principal presence in the phase space of MSSC angular (attitude) motion is shown in [27], were also the well-known dynamical systems (Lorenz, Sprott, Wang, Qi, Li, Chen, Lü, Liu, Čelikovský, Burke, Shaw, Arneodo, Coullet, etc.) are observed in the indicated sense [1-28]. In this paper we consider the following development of this dynamical problem. So, here we will show the synthesis of the MSSC dynamical parameters which deliver the implementation of regimes corresponding to strange chaotic attractors.

The MSSC [27-29] represents the multi-body (multi-rotor) constructional scheme with conjugated pairs of rotors placed on the inertia principle axes of the main body (fig.1). General properties of the MSSC attitude dynamics are connected with the internal redistribution of the angular momentum between the system bodies (the main body, and rotors) due to the internal torques action. These properties of dynamics can be applied to the implementation of spatial (attitude/attitude) reorientations of the MSSC and also to the roll-walking motions of multi-rotor walking robots [28, 29].

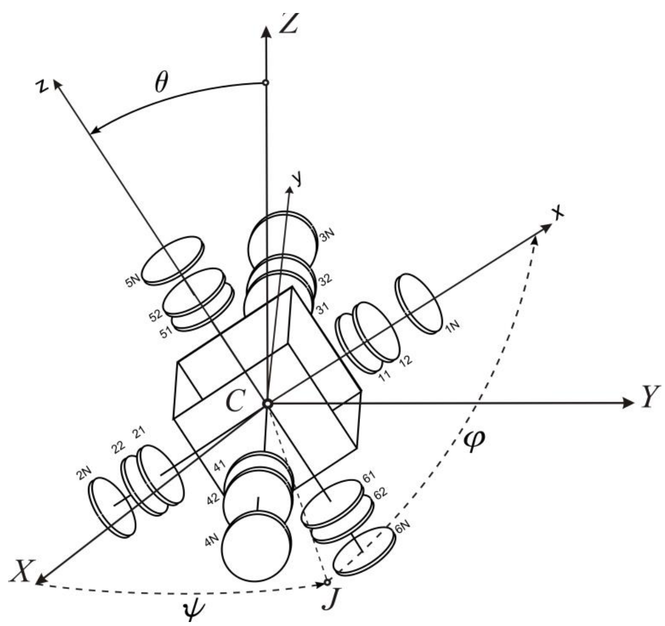

Fig.1. The MSSC and main coordinates systems

II. MATHEMATICAL MODEL OF MSSC

Following to the work [27, 28] we can present the main equations of the MSSC attitude dynamics as the dynamical equations for the multi-rotor system with 6N rotors (Fig.1) contained into N layers on six general directions coinciding with the principle axes of the main body. In these equations he following notations are used: $\boldsymbol{\omega}=[p, q, r]^{\mathrm{T}}$ – the vector of the absolute angular velocity of the main body (in projections on the connected frame $Cxyz$); $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ are the general moments of inertia of the main body; M_x^e, M_y^e M_z^e – are the external torques acting on the system (in general applied to the main body); σ_{kl} is the relative angular velocity of the kl-th rotor (relatively the main body); I_l and J_l are the longitudinal and the equatorial inertia moments of the l-layer-rotor relatively the point *O*; $M^e_{jlx}, M^e_{jly}, M^e_{jlz}$ are external torques acting only on the *jl*-th rotor, and M^i_{jl} is the torque from internal forces acting between the main body and the *jl*-th rotor (the internal engines torques). Then the indicated equations have the form [29, 28]:

$$\begin{cases} A\dot{p} + \sum_{l=1}^{N} I_l\left(\dot{\sigma}_{1l} + \dot{\sigma}_{2l}\right) + \left(C - B\right)qr + \\ \quad + \left[q\sum_{l=1}^{N} I_l\left(\sigma_{5l} + \sigma_{6l}\right) - r\sum_{l=1}^{N} I_l\left(\sigma_{3l} + \sigma_{4l}\right)\right] = M_x^e; \\ B\dot{q} + \sum_{l=1}^{N} I_l\left(\dot{\sigma}_{3l} + \dot{\sigma}_{4l}\right) + \left(A - C\right)pr + \\ \quad + \left[r\sum_{l=1}^{N} I_l\left(\sigma_{1l} + \sigma_{2l}\right) - p\sum_{l=1}^{N} I_l\left(\sigma_{5l} + \sigma_{6l}\right)\right] = M_y^e; \\ C\dot{r} + \sum_{l=1}^{N} I_l\left(\dot{\sigma}_{5l} + \dot{\sigma}_{6l}\right) + \left(A - C\right)qp + \\ \quad + \left[p\sum_{l=1}^{N} I_l\left(\sigma_{3l} + \sigma_{4l}\right) - q\sum_{l=1}^{N} I_l\left(\sigma_{1l} + \sigma_{2l}\right)\right] = M_z^e; \end{cases} \tag{1}$$

with the addition of the relative motion equations of the rotors $\left(l = 1..N\right)$:

$$\begin{cases} I_l\left(\dot{p} + \dot{\sigma}_{1l}\right) = M_{1l}^i + M_{1lx}^e; \; I_l\left(\dot{p} + \dot{\sigma}_{2l}\right) = M_{2l}^i + M_{2lx}^e; \\ I_l\left(\dot{q} + \dot{\sigma}_{3l}\right) = M_{3l}^i + M_{3ly}^e; \; I_l\left(\dot{q} + \dot{\sigma}_{4l}\right) = M_{4l}^i + M_{4ly}^e; \\ I_l\left(\dot{r} + \dot{\sigma}_{5l}\right) = M_{5l}^i + M_{5lz}^e; \; I_l\left(\dot{r} + \dot{\sigma}_{6l}\right) = M_{6l}^i + M_{6lz}^e \end{cases} \tag{2}$$

Also presented equations can be rewritten in the unbalanced-gyrostat-form:

$$\begin{cases} \hat{A}\dot{p} + \dot{D}_{12} + \left(\hat{C} - \hat{B}\right)qr + \left[qD_{56} - rD_{34}\right] = M_x^e; \\ \hat{B}\dot{q} + \dot{D}_{34} + \left(\hat{A} - \hat{C}\right)rp + \left[rD_{12} - pD_{56}\right] = M_y^e; \\ \hat{C}\dot{r} + \dot{D}_{56} + \left(\hat{B} - \hat{A}\right)pq + \left[pD_{34} - qD_{12}\right] = M_z^e; \end{cases} \tag{3}$$

$$\begin{cases} \dot{D}_{12} = M_{12}^i + M_{12}^e; \\ \dot{D}_{34} = M_{34}^i + M_{34}^e; \\ \dot{D}_{56} = M_{56}^i + M_{56}^e, \end{cases} \tag{4}$$

where

$$\hat{A} = A - 2\sum_{j=1}^{N} I_j; \quad \hat{B} = B - 2\sum_{j=1}^{N} I_j; \quad \hat{C} = C - 2\sum_{j=1}^{N} I_j;$$

$$A = \tilde{A} + 4\bar{J} + 2\bar{I}, \quad B = \tilde{B} + 4\bar{J} + 2\bar{I}; \quad C = \tilde{C} + 4\bar{J} + 2\bar{I};$$

$$\bar{J} = \sum_{l=1}^{N} J_l; \; \bar{I} = \sum_{l=1}^{N} I_l;$$

$$\begin{cases} D_{12} = \sum_{j=1}^{N}\left[\Delta_{1j} + \Delta_{2j}\right], \quad D_{34} = \sum_{j=1}^{N}\left[\Delta_{3j} + \Delta_{4j}\right], \\ \qquad D_{56} = \sum_{j=1}^{N}\left[\Delta_{5j} + \Delta_{6j}\right]; \\ \Delta_{1j} = I_j\left(p + \sigma_{1j}\right); \qquad \Delta_{2j} = I_j\left(p + \sigma_{2j}\right); \\ \Delta_{3j} = I_j\left(q + \sigma_{3j}\right); \qquad \Delta_{4j} = I_j\left(q + \sigma_{4j}\right); \\ \Delta_{5j} = I_j\left(r + \sigma_{5j}\right); \qquad \Delta_{6j} = I_j\left(r + \sigma_{6j}\right). \end{cases} \tag{5}$$

The summarized rotors' internal (*i*) and external (*e*) torques are:

$$\begin{cases} M_{12}^i = \sum_{l=1}^{N}\left(M_{1l}^i + M_{2l}^i\right), \; M_{12}^e = \sum_{l=1}^{N}\left(M_{1lx}^e + M_{2lx}^e\right); \\ M_{34}^i = \sum_{l=1}^{N}\left(M_{3l}^i + M_{4l}^i\right), \; M_{34}^e = \sum_{l=1}^{N}\left(M_{3ly}^e + M_{4ly}^e\right); \\ M_{56}^i = \sum_{l=1}^{N}\left(M_{5l}^i + M_{6l}^i\right), \; M_{56}^e = \sum_{l=1}^{N}\left(M_{5lz}^e + M_{6lz}^e\right). \end{cases} \tag{6}$$

Also for the kinematical description of the attitude motion of the MSSC main body we have to add the well-known Euler kinematical equations:

$$\begin{cases} \dot{\theta} = p\cos\varphi - q\sin\varphi; \\ \dot{\psi} = (p\sin\varphi + q\cos\varphi)/\sin\theta; \\ \dot{\varphi} = r - \operatorname{ctg}\theta(p\sin\varphi + q\cos\varphi); \end{cases} \tag{7}$$

where the Euler angles are used (fig.1): θ - the nutation, ψ - the precession, φ - the intrinsic rotation.

Let us in this paper consider the case of the MSSC controlled attitude motion at the creation of the following artificial controlling internal and external torques:

$$M_{12}^i = \alpha_p \dot{p}; \qquad M_{34}^i = \beta_q \dot{q}; \qquad M_{56}^i = \gamma_r \dot{r}; \tag{8}$$

$$M_x^e = m_x + \alpha_1 p; \; M_y^e = m_y + \beta_1 q; \; M_z^e = m_z + \gamma_1 r, \tag{9}$$

with the constant controlling terms/coefficients:

$$\left\{m_x, m_y, m_z, \alpha_1, \beta_1, \gamma_1, \alpha_p, \beta_q, \gamma_r\right\} \sim \text{const}$$

Hear we can note using only linear internal control torques proportional to the angular accelerations of the main MSSC-body(8) (these torques are formed by the internal rotors' engines); and also using linear external control torques applied to the main body (9) which are formed by thrusters (e.g. this torques can be formed by electrically powered propulsion systems – this type of engines is characterized by the extremely low consumption of the working body (usually compressed gases and/or plasma), that corresponds to the practically constant mass system). Also the indicated type of external torques (9) is appropriate for the modeling of the rotational motion of underwater vehicles/robots and MSSC in resistant environments with dissipative/excitative properties.

Then taking into account equations (4) and torques (8) we obtain analytical forms for the summarized rotors' angular momentums:

$$D_{12} = \alpha_p p + \alpha_0;\; D_{34} = \beta_q q + \beta_0;\; D_{56} = \gamma_r r + \gamma_0, \quad (10)$$

where $\alpha_0, \beta_0, \gamma_0$ – are the constants following from initial conditions.

III. The synthesis of the MSSC parameters delivering the dynamics with implemntation of strange chaotic attractors

As it was indicated in the recent work [1] the natural candidates for the construction of dynamical systems with multi-scroll chaotic attractors are 3D quadratic continuous time systems given by equations

$$\begin{cases} \dot{x} = a_0 + a_1 x + a_2 y + a_3 z + a_4 x^2 + a_5 y^2 + \\ \qquad + a_6 z^2 + a_7 xy + a_8 xz + a_9 yz; \\ \dot{y} = b_0 + b_1 x + b_2 y + b_3 z + b_4 x^2 + b_5 y^2 + \\ \qquad + b_6 z^2 + b_7 xy + b_8 xz + b_9 yz; \\ \dot{z} = c_0 + c_1 x + c_2 y + c_3 z + c_4 x^2 + c_5 y^2 + \\ \qquad + c_6 z^2 + c_7 xy + c_8 xz + c_9 yz; \end{cases} \quad (11)$$

where $\{a_i, b_i, c_i\}_{0 \le i \le 9} \in \mathbb{R}^{30}$ are the constant parameters.

Basing on expressions (10) we can solve the linear algebraic equations (3)relatively $\{\dot{p}, \dot{q}, \dot{r}\}$; and implying the redesignation $(p \leftrightarrow x;\, q \leftrightarrow y;\, r \leftrightarrow z)$ it is possible to write the following correspondences for the system (11) coefficients $\{a_i, b_i, c_i\}$ and the MSSC parameters:

$$\begin{cases} a_0 = \dfrac{m_x}{\hat{A} + \alpha_p};\; b_0 = \dfrac{m_y}{\hat{B} + \beta_q};\; c_0 = \dfrac{m_z}{\hat{C} + \gamma_r}; \\ a_1 = \dfrac{\alpha_1}{\hat{A} + \alpha_p};\quad b_1 = \dfrac{\gamma_0}{\hat{B} + \beta_q};\quad c_1 = \dfrac{-\beta_0}{\hat{C} + \gamma_r}; \\ a_2 = \dfrac{-\gamma_0}{\hat{A} + \alpha_p};\quad b_2 = \dfrac{\beta_1}{\hat{B} + \beta_q};\quad c_2 = \dfrac{\alpha_0}{\hat{C} + \gamma_r}; \\ a_3 = \dfrac{\beta_0}{\hat{A} + \alpha_p};\quad b_3 = \dfrac{-\alpha_0}{\hat{B} + \beta_q};\quad c_3 = \dfrac{\gamma_1}{\hat{C} + \gamma_r}; \\ a_4 = a_5 = a_6 = b_4 = b_5 = b_6 = c_4 = c_5 = c_6 \equiv 0; \\ a_7 = 0;\; b_7 = 0;\; c_7 = \left(\hat{A} - \hat{B} - \beta_q + \alpha_p\right) \Big/ \left(\hat{C} + \gamma_r\right); \\ a_8 = 0;\; b_8 = \left(\hat{C} - \hat{A} - \alpha_p + \gamma_r\right) \Big/ \left(\hat{B} + \beta_q\right);\; c_8 = 0; \\ a_9 = \left(\hat{B} - \hat{C} - \gamma_r + \beta_q\right) \Big/ \left(\hat{A} + \alpha_p\right);\; b_9 = 0;\; c_9 = 0. \end{cases} \quad (12)$$

The obtained correspondences (12) can be used for the synthesis of the control constants at the defined MSSC parameters (inertia moments). As it is clear, we have to find such values of constants from the set

$$Control = \left\{\alpha_p, \alpha_0, m_x, \alpha_1, \beta_q, \beta_0, m_y, \beta_1, \gamma_r, \gamma_0, m_z, \gamma_1\right\} \in \mathbb{R}^{12} \quad (13)$$

which deliver appropriate values of coefficients from the set

$$Coeff = \{a_i, b_i, c_i\}_{0 \le i \le 9} \in \mathbb{R}^{30} \quad (14)$$

corresponding to the dynamical systems with strange chaotic attractors. Here we will see a rather limited number of the systems with strange chaotic attractors, to which we can reduce the correspondences(12) due to incompatibility of dimensions of sets *Coeff* and *Control*. By this reason it is impossible to solve the correspondences (12) relatively the set's *Control* constants as a linear algebraic equations.

From the expressions (12) the form of possible dynamical systems with strange chaotic attractors follows: we have to find the concretized cases of systems (11) with null-coefficients from the set

$$Zeros = \begin{Bmatrix} a_4 = a_5 = a_6 = b_4 = b_5 = b_6 = c_4 = c_5 = c_6 = 0; \\ a_7 = 0;\; b_7 = 0;\; a_8 = 0;\; c_8 = 0;\; b_9 = 0;\; c_9 = 0 \end{Bmatrix} \quad (15)$$

Some "natural gyroscopic systems" will belong to the systems (11) with the zeros (15); e.g. well-known systems with the Wang-Sun four-scroll chaotic attractor [8] and with the Chen-Lee two-scroll chaotic attractor [17] are appropriate in this sense; and we confine by these two systems only in this research.

So, we will try to find in this research concrete control parameters *Control* for the MSSC motion, which will demonstrate the regimes along the indicated attractors [8, 17]. Here we should shortly comment our characteristic "natural gyroscopic systems" – these are the systems corresponding to dynamical equations of the attitude/angular/rotational motion of usual rigid-bodies systems including MSSC and GS.

As we can ascertain [8, 17], the following sets of coefficients are actual for the Wang-Sun system (*WS*) and for the Chen-Lee (*CHL*) system (in both systems the requirements(15) are fulfilled):

$$WS = \begin{Bmatrix} a_0 = 0;\; b_0 = 0;\; c_0 = 0; \\ a_1 = 0.2;\quad b_1 = -0.01;\quad c_1 = 0; \\ a_2 = 0;\quad b_2 = -0.4;\quad c_2 = 0; \\ a_3 = 0;\quad b_3 = 0;\quad c_3 = -1; \\ a_4 = a_5 = a_6 = b_4 = b_5 = b_6 = c_4 = c_5 = c_6 \equiv 0; \\ a_7 = 0;\; b_7 = 0;\; c_7 = -1; \\ a_8 = 0;\; b_8 = -1;\; c_8 = 0; \\ a_9 = 1;\; b_9 = 0;\; c_9 = 0. \end{Bmatrix} \quad (16)$$

$$CHL = \left\{ \begin{array}{l} a_0 = 0;\ \ b_0 = 0;\ \ c_0 = 0; \\ a_1 = 5;\ \ \ b_1 = 0;\ \ \ c_1 = 0; \\ a_2 = 0;\ \ \ b_2 = -10;\ \ \ c_2 = 0; \\ a_3 = 0;\ \ \ b_3 = 0;\ \ \ c_3 = -3.8; \\ a_4 = a_5 = a_6 = b_4 = b_5 = b_6 = c_4 = c_5 = c_6 \equiv 0; \\ a_7 = 0;\ \ b_7 = 0;\ \ c_7 = \dfrac{1}{3}; \\ a_8 = 0;\ \ b_8 = 1;\ c_8 = 0; \\ a_9 = -1;\ \ b_9 = 0;\ \ c_9 = 0. \end{array} \right\} \quad (17)$$

Now it is needed to find the set of parameters *Control* which provides the fulfillment of the system (12) at the concrete numerical values of sets *WS* or *CHL*, in addition of numerical values of the MSSC inertial parameters (these are arbitrary parameters, which are defined by the concrete type of spacecraft). Due to the incompatibility (in general) of indicated systems we will use the well-known first-order optimization algorithm to find a local minimum of a connected function using gradient descent procedure.

For solving our task we will use the gradient descent procedure for the following quadratic function:

$$\begin{aligned} &\Psi\left(\alpha_p, \alpha_0, m_x, \alpha_1, \beta_q, \beta_0, m_y, \beta_1, \gamma_r, \gamma_0, m_z, \gamma_1\right) = \\ &= \left(a_0 - \frac{m_x}{\hat{A} + \alpha_p}\right)^2 + \left(b_0 - \frac{m_y}{\hat{B} + \beta_q}\right)^2 + \left(c_0 - \frac{m_z}{\hat{C} + \gamma_r}\right)^2 + \\ &+ \left(a_1 - \frac{\alpha_1}{\hat{A} + \alpha_p}\right)^2 + \left(b_1 - \frac{\gamma_0}{\hat{B} + \beta_q}\right)^2 + \left(c_1 + \frac{\beta_0}{\hat{C} + \gamma_r}\right)^2 + \\ &+ \left(a_2 + \frac{\gamma_0}{\hat{A} + \alpha_p}\right)^2 + \left(b_2 - \frac{\beta_1}{\hat{B} + \beta_q}\right)^2 + \left(c_2 - \frac{\alpha_0}{\hat{C} + \gamma_r}\right)^2 + \\ &+ \left(a_3 - \frac{\beta_0}{\hat{A} + \alpha_p}\right)^2 + \left(b_3 + \frac{\alpha_0}{\hat{B} + \beta_q}\right)^2 + \left(c_3 - \frac{\gamma_1}{\hat{C} + \gamma_r}\right)^2 + \\ &+ \left(c_7 - \left(\hat{A} - \hat{B} - \beta_q + \alpha_p\right) \Big/ \left(\hat{C} + \gamma_r\right)\right)^2 + \\ &+ \left(b_8 - \left(\hat{C} - \hat{A} - \alpha_p + \gamma_r\right) \Big/ \left(\hat{B} + \beta_q\right)\right)^2 + \\ &+ \left(a_9 - \left(\hat{B} - \hat{C} - \gamma_r + \beta_q\right) \Big/ \left(\hat{A} + \alpha_p\right)\right)^2 \end{aligned} \quad (18)$$

where numerical values a_i, b_i, c_i correspond to numbers (16), or (17), or other appropriate coefficients of systems with strange chaotic attractors.

As it quite understandable, if the function (18) will have the local zero-minimum, then the parameters of this zero-minimum provide the fulfill of conditions (12), that corresponds to the transition of the MSSC-equations to the dynamical system with strange attractors at the selection/assignment of found numerical values of parameters from the set *Control*. Also it is worth to note the possibility of searching "approximate" near-zero-minimums of the function (18), instead its exact zero-minimum (with the defined tolerance) – such near-zero-minimums can give quite appropriate results in the sense of obtaining dynamical systems with strange attractors.

So, after the definition of the function (18), the gradient descent procedure is executed basing on following iterations:

$$\textit{while} \ \left|\sqrt{\Psi\left(X_i\right)}\right| > \varepsilon: \ \ X_{i+1} = X_i - h \cdot \nabla \Psi\left(X_i\right); \quad (19)$$

where $X_0 \in \textit{Control}$ is the initial approximation of finding parameters, and ε is the tolerance. It is needed to take into account, we do not guarantee the local convergence of iterations (19) to the zero-minimum.

IV. MODELLING RESULTS

Now we can show some calculations results for the MSSC dynamical system with control parameters (13) in cases of reducing to the Wang-Sun system and the Chen-Lee system.

1). Firstly we consider results for the Wang-Sun system with coefficients (16) and at the following parameters: $\hat{A} = 90,\ \hat{B} = 70,\ \hat{C} = 50\ [\mathrm{kg \cdot m^2}];\ \ \varepsilon = 0.03.$ The point of the iterations (19) convergence, corresponding to the required control parameters is

$$\begin{aligned} &X_{final} = \left(\alpha_p, \alpha_0, m_x, \alpha_1, \beta_q, \beta_0, m_y, \beta_1, \gamma_r, \gamma_0, m_z, \gamma_1\right) = \\ &= (-3.70594,\ 0,\ 0.00776,\ 16.05099,\ 16.31322,\ 0,\ 0.01781, \\ &-32.38210,\ -49.98084,\ -0.42498,\ 0,\ -0.01916). \end{aligned} \quad (20)$$

Then we obtain the four-scroll Wang-Sun attractor in the dynamical space of MSSC (Fig.2): the red line corresponds to the classical Wang-Sun attractor, black line – to the "approximate" attractor in the MSSC dynamical system (3) with the gyrostatic momentum (10), torques (9) and control parameters (20). It is very important to show the numerical time-dependencies for angular velocity components and for the Euler angles calculated by numerical integration (Fig.3, 4, 5).

2). Secondly we consider results for the Chen-Lee system with coefficients (17) and at the following parameters: $\hat{A} = 90,\ \hat{B} = 70,\ \hat{C} = 50\ [\mathrm{kg \cdot m^2}];\ \ \varepsilon = 0.01.$ In the considering case the point of the iterations (19) convergence, corresponding to required control parameters is

$$\begin{aligned} &X_{final} = \left(\alpha_p, \alpha_0, m_x, \alpha_1, \beta_q, \beta_0, m_y, \beta_1, \gamma_r, \gamma_0, m_z, \gamma_1\right) = \\ &= (-76.39886,\ 0,\ 0,\ 67.99629,\ -63.04526,\ 0,\ 0, \\ &-69.55072,\ -29.45241,\ 0,\ 0,\ -78.07454). \end{aligned} \quad (21)$$

Then we obtain the two-scroll Chen-Lee attractor in the dynamical space of MSSC (Fig.6): the red line corresponds to the classical Chen-Lee attractor, black line – to the "approximate" attractor in the MSSC dynamical system (3) with the gyrostatic momentum (10), torques (9) and control parameters (21). The numerical time-dependencies for angular velocity components and for the Euler angles are presented at figures (Fig.7, 8, 9).

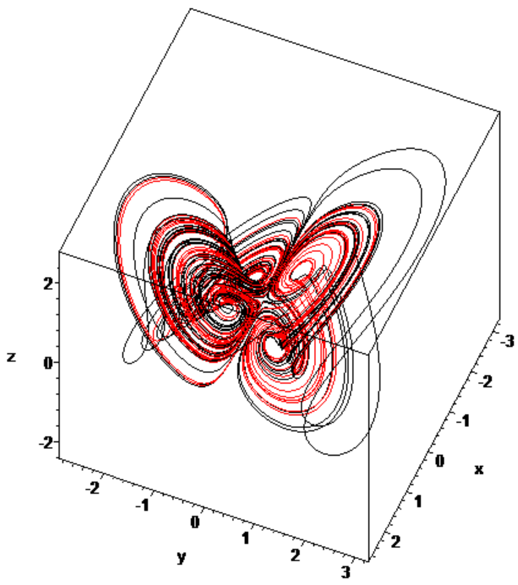

Fig.2. The four-scroll Wang-Sun attractor in the phase space of MSSC at the initial conditions: $x(0)=p(0)=1.05, y(0)=q(0)=1.1$, $z(0)=r(0)=1.5$ [1/s]

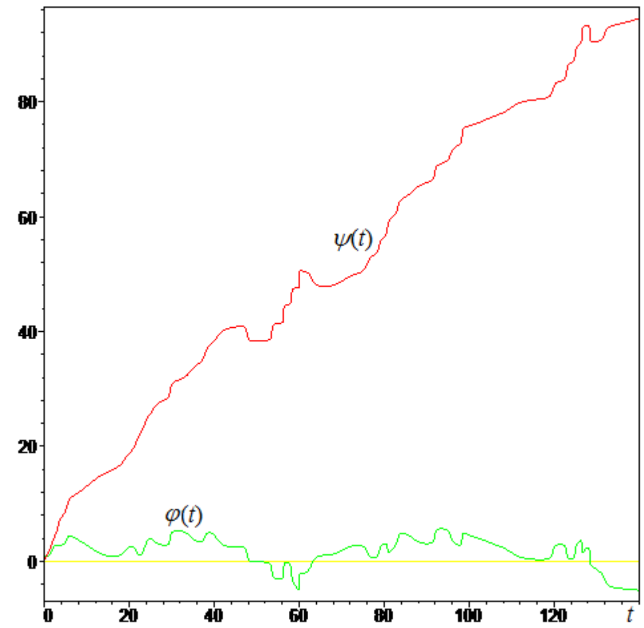

Fig.4. Time-dependencies for the chaotic precession angle and the intrinsic rotation for the main body of MSSC in the Wang-Sun regime

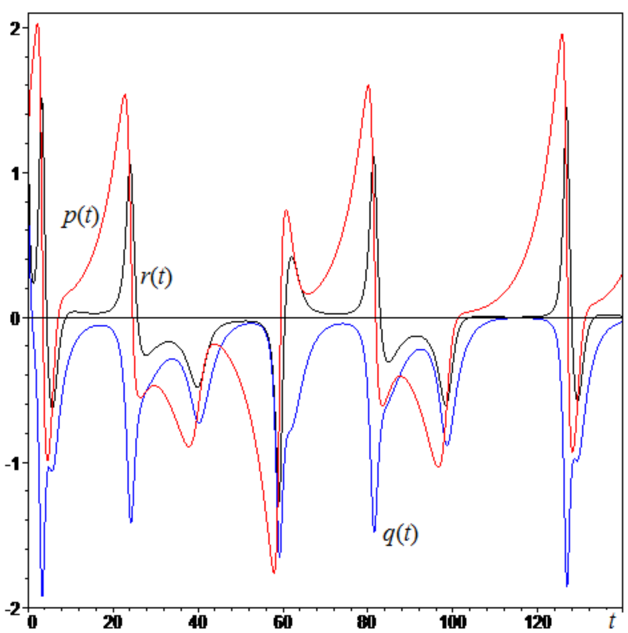

Fig.3. Components of the chaotic angular velocity of the main body of MSSC in the Wang-Sun regime

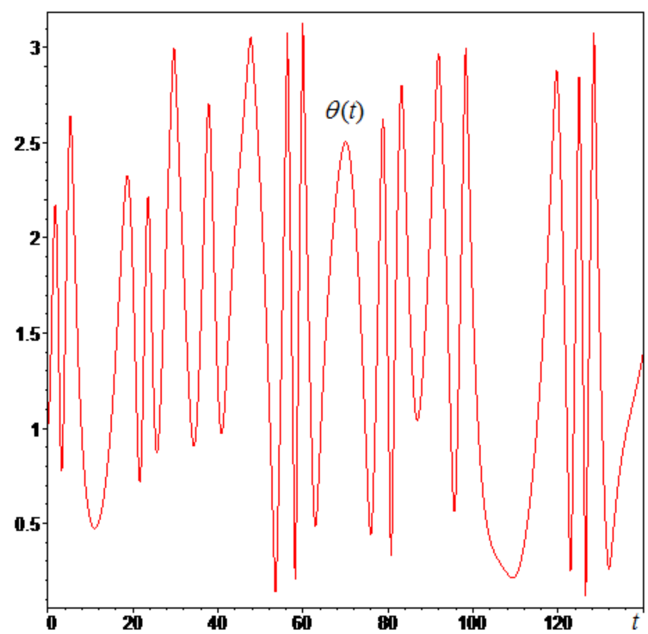

Fig.5. The time-dependency for the chaotic nutation angle of the main body of MSSC in the Wang-Sun regime

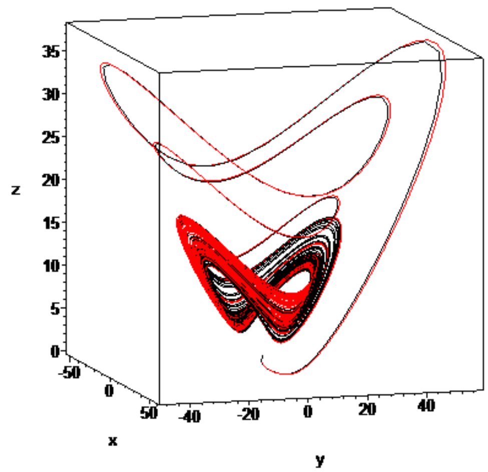

Fig.6. The two-scroll Chen-Lee attractor in the phase space of MSSC at the initial conditions: $x(0)=p(0)=1.05, y(0)=q(0)=1.1$, $z(0)=r(0)=1.5$ [1/s]

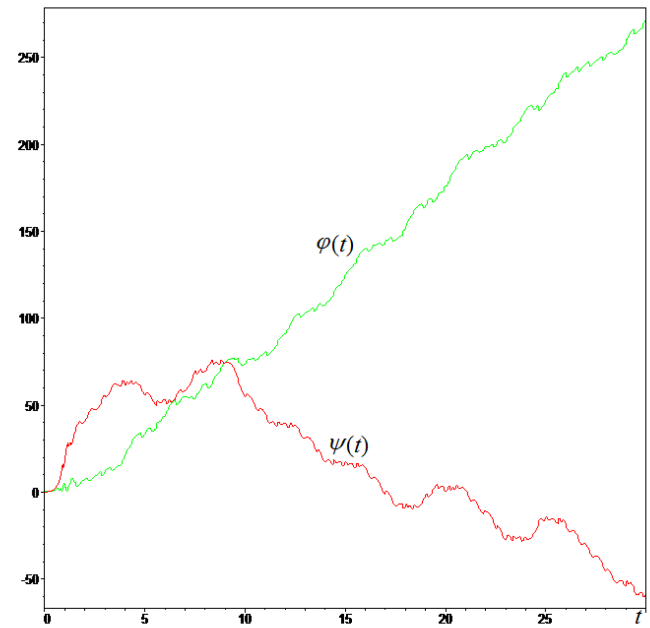

Fig.8. Time-dependencies for the chaotic precession angle and the intrinsic rotation of the main body of MSSC in the Chen-Lee regime

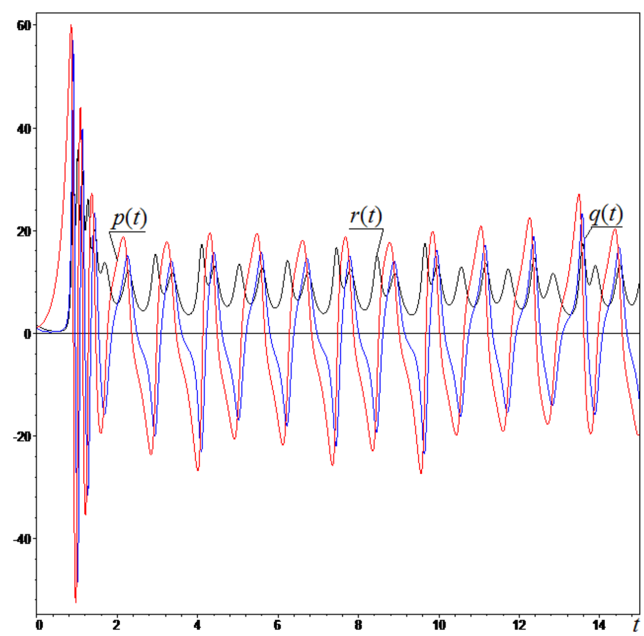

Fig.7. Components of the chaotic angular velocity of the main body of MSSC in the Chen-Lee regime

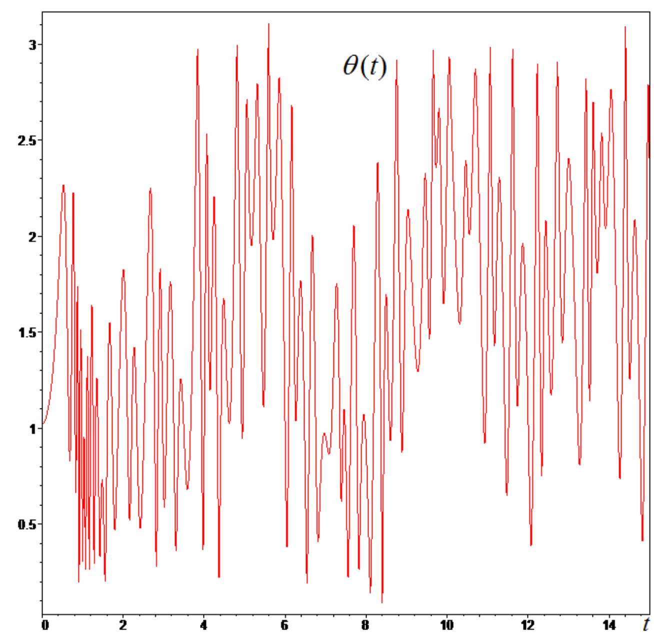

Fig.9. The time-dependency for the chaotic nutation angle of the main body of MSSC in the Chen-Lee regime

CONCLUSION

The possibility of chaotic regimes intentional initiations in attitude dynamics of multi-spin spacecraft (MSSC) and gyrostat-satellites (GS) basing on the activation of strange chaotic attractors was presented. Above considered intentional initiations of chaotic regimes can be applied to the attitude reorientation of MSSC/GS in cases of extreme dynamical situations including accidents and failures of main attitude control systems, and/or in cases of cancellations of uncontrolled rotations.

Also investigated chaotic regimes can be used for the fulfillment of special motion control tasks connected with space and underwater robots, including the fast random observation of surroundings, fast random search of objects, and chaotic maneuvering.

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