2017 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)
September 24–28, 2017, Vancouver, BC, Canada

Variable Topology Truss: Design and Analysis

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***Abstract*— This paper introduces a new class of self-reconfigurable robot: the variable topology truss (VTT). Related to an existing class of robots, the variable geometry truss (VGT), variable topology trusses have the additional capability to change the topology of the truss through self-reconfiguration. The hardware necessary to achieve this is introduced, and the constraints and capabilities of this new type of robot are analyzed by introducing the concept of a topology neighbor graph. Lastly, the minimal reconfigurable VTTs, which require 18 members, are identified and their achievable topologies are enumerated.**

I. Introduction

In many applications, trusses are ideal due to their high structural efficiency, as they consist of members which are only in pure tension or compression, resulting in reduced maximum stress [1]. The class of robots commonly known as variable geometry trusses (VGTs) can be obtained by replacing some or all of the members in a truss with linear actuators [2]. There are many applications for this type of robot, including parallel manipulators, long chain actuators, collapsible structures, and locomotion platforms.

A variable topology truss (VTT) starts from the same truss framework as a VGT: linear actuators form the members (beam elements in the truss) and passive spherical joints connect the members at the nodes in the truss. However, the truss has the additional capability to self-reconfigure, changing its topology by merging or splitting nodes. That is, two separate nodes in the truss can dock to form one single node which connects all of the involved members. Similarly, a single node with a sufficient amount of members can undock into a pair of nodes. In this way, a VTT can be thought of as a chain type self-reconfigurable robot [3] consisting only of linearly actuated elements and unactuated spherical joints.

While VGTs only have control over the shape or geometry of the truss, a VTT can additionally change the topology of the underlying truss. This brings benefits typically associated with reconfigurable robots; the VTT can select the topology which is most suited to the task at hand. For example, a single VTT may have the capability to self-reconfigure for many applications: a collapsible topology for storage, a dynamic rolling gait topology for locomotion, and a topology with a large workspace for manipulation. This task flexibility is not possible with the single fixed topology of a VGT. An example of two structures which might be realizable from one system is shown in Fig. 1.

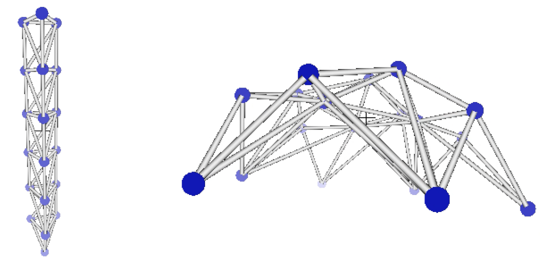

Fig. 1: A tower and dome configuration each with 51 members. We are interested in determining whether it is possible to reconfigure from one to the other.

The additional reconfigurability of a VTT could have applications in space missions as a lightweight adaptable structure, or as versatile structural reinforcement to assist first responders during disaster scenarios. Our previous work [4] explores the challenges of the latter application using an earlier VTT concept.

II. Background

The Stewart platform is one example of a VGT; it can be thought of as an octahedral truss with six actuated members [2]. Rhodes and Mikulas investigated the use of VGTs as a collapsable, controllable beam for space applications [5]. The Tetrobot system is another example of VGT with a variety of configurations, including a six legged walker [6]. Lee and Sanderson presented dynamic rolling locomotion with an icosahedral Tetrobot [7]. The dynamics and control of general VGTs have been studied as well [8].

Arun et. al. claim to enumerate all possible VGTs [2] by constructing them from basic unit cells. Arun specifies that VGTs must be statically determinate and composed of convex polyhedral cells, however, this criteria is too restrictive, as it eliminates many interesting structures. For example, the complete bipartite graph $K_{5,5}$ is generically rigid in 3D space, [9] but is neither statically determinate nor composed of polyhedral cells. By removing a single edge, it can be made statically determinate, yet it is still not composed of polyhedral cells. The only criteria we specify for a VTT in this paper is that the truss is a rigid framework. We consider all suitably rigid bar and joint structures, including statically indeterminate ones. Statically indeterminate structures are necessary to admit reconfiguration of the truss; a statically determinate truss becomes overdetermined after merging two nodes.

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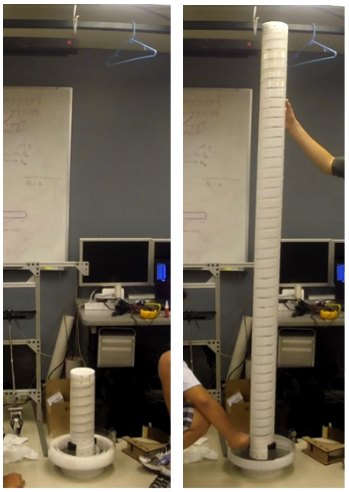


Fig. 2: Spiral Zipper, with an extension ratio of 14:1, arbitrarily large extension ratios are possible.

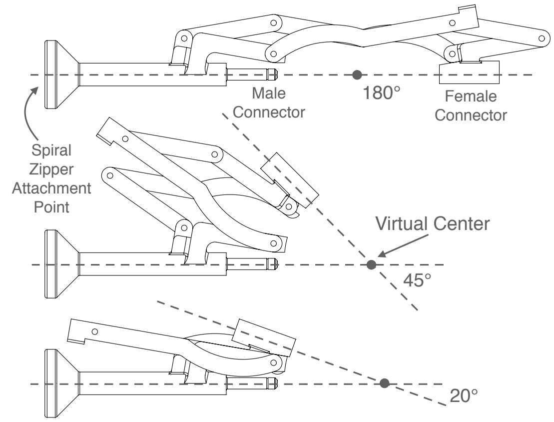


Fig. 3: VTT joint features. Maximum, intermediate, and minimum joint positions are shown. The latching mechanism is not shown.

A critical design parameter in improving the usefulness of VGT type systems is the range of motion of the prismatic joints. In the past work referenced above, all of the actuators were standard single stage linear actuators with an extension/compression ratio of less than 2. Larger ratios will allow the systems to have larger workspaces or smaller storage volumes as well as higher dexterity over these larger workspaces.

III. Mechanical Design and Validation

VGTs and VTTs achieve geometric shape change by actuating member lengths. In a truss with variable member lengths, the nodes must consist of passive revolute joints. In the case of a VTT, where nodes may merge or split, the nodes must also be chainable to support the connection and disconnection of an arbitrary number of members at a single node.

A. Active Prismatic Joint

The ideal prismatic joint is one with large a extension/compression ratio. Collins and Yim have developed such an actuator called the Spiral Zipper shown in Fig. 2 [10].

The Spiral Zipper operates both as a prismatic actuator and a structural component. It extends and retracts a rigid, lightweight tube by nesting the features in the top of a band to the matching features on the bottom. The extension ratio is only limited in principle by the amount of stored band and the material strength. The band forms a circular tube which is the optimal shape for stiffness to weight ratio under buckling loads, the most likely mode of failure for slender truss beams. The zipping feature interface is strongest in compression, although it can support small torsions and moments. To support tension, the actuator will have a reinforcing winched cable on the inside of the tube (not shown). Experimentally, we have shown strength to weight ratios of approximately 10:1 for a prismatic actuator capable of extending $1\,\mathrm{m}$ with a plastic band. Spiral Zipper tubes made of acetal plastic, $1.5\,\mathrm{m}$ long, have been shown to support $530\,\mathrm{N}$. We have also shown the tube can be extended as fast as $0.45\,\mathrm{m/s}$.

B. Passive Chainable Spherical Joints

To maintain the advantageous decoupling of moments and forces, the system needs passive spherical joints at the nodes of the truss. The joints should satisfy the following requirements:

1) The position of the endpoints of the truss members joined at a node should be coincident, but the members should have three degrees of rotational freedom about the node center.
2) The node must be able to constrain an arbitrary number of truss members. The number of members at a node may change during topological reconfiguration.
3) The nodes must minimize interference between members as the angles between members change.

To satisfy the first condition in practice, the truss members themselves do not need to physically occupy the node center as long as they share a common center of rotation. Universal joints and spherical ball joints are typical candidates for a truss node, however they are not chainable to support merging of nodes. Instead, a spherical link chain, made up of links of revolute joints which rotate about the node center, can provide this behavior [11].

A mechanism similar to a concentric multilink spherical (CMS) joint first developed by Hamlin [12] for the Tetrobot Modular Robot satisfies the design, in particular minimizing interference. To provide the ability to attach and detach multiple joints at a single node, which was not part of the CMS joint, development of a male and female connection point and a latch mechanism was implemented. We call this type of spherical linkage a *VVT joint*; it is illustrated in Fig. 3.

The VTT joint is characterized by an offset planar hinge that forms a six-bar linkage. There are six components and seven pin joints that make up the spherical joint. A male connector and Spiral Zipper attachment are integrated into one linkage point, and a female connector and latch are integrated into a second linkage point. The joint is designed such that the links will slide next to each other to allow folding of the linkage. 2D planar movement of the joint is illustrated in Fig. 3. When the joint is fully closed to its

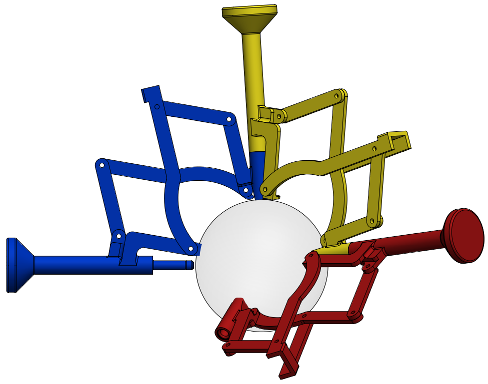


Fig. 4: Chaining of three VTT joints. The truss members are constrained to move over a virtual sphere about the virtual center.

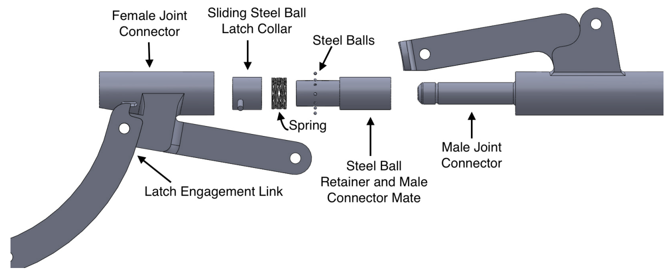


Fig. 5: Exploded view of the female connector latch mechanism

Fig. 6: Full test setup.

minimized position, the angle between the male and female connection points is 20°. When the joint is opened to its maximized position, the angle is 180°.

The center of rotation for the VTT joint linkage is the node center. As more VTT joints are added to the chain, they all share the same center of rotation (Fig. 4).

The latching mechanism is designed to be similar to steering wheel quick connect devices. The female connection point on each joint will incorporate a passive latching mechanism that will hold the male connector from a mating joint into position. Engagement/disengagement of the male connector will occur when the joint containing the female connector is in its fully opened position (180°). In this position, a male connector can be inserted or removed from the female connector of the adjoining joint. Once the joint angle is decreased below 180°, the male connector and female connector will remain latched. An exploded view of the passive latching mechanism is shown in Fig. 5.

Each edge module in the truss consists of an active prismatic joint and two passive VTT joints. When the endpoints of multiple edge modules are brought together, the spherical joints can mate to form a chain of arbitrary length with a free male connector and female connector at either end of the chain. This design allows an arbitrary number of edge modules to be connected at a single point, which corresponds to a single node in the truss. In the same way, two nodes can be brought together and the chains of joints can be appended, forming a single long chain that suitably constrains all of the edge modules.

C. Hardware Prototype

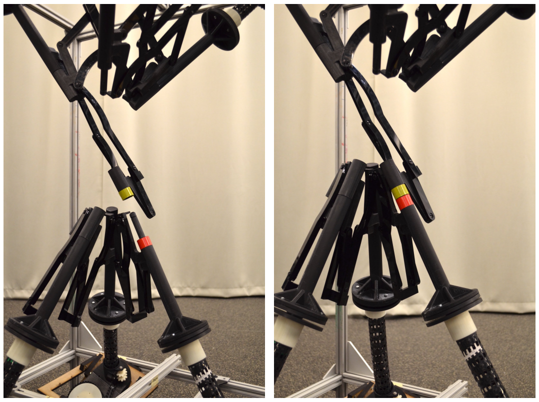
A demonstration of the capability of the spherical joints to connect and disconnect passively for truss reconfiguration was performed (Fig. 6). The demonstration mimics the merging of two nodes corresponding to tetrahedral components of a truss. The exterior of the truss was made from stock 80/20 material, the tetrahedra from Spiral Zippers [10] connected by VTT joints. The upper tetrahedron was fixed, and joystick control was used to maneuver the lower tetrahedron into position and to merge the two nodes into a single node as shown in Fig. 7. Similarly, the splitting capabilities of the VTT joints were also tested.

IV. Robot Topological Analysis

A VTT has the capability to change from one truss topology to another. Yet there are limitations on which topologies can be reached by a given VTT. For example, trusses (a) and (b) shown in Fig. 10 have the same number of nodes and members, but there is no way to reconfigure from one into the other. This section will discuss the rules which govern which reconfiguration actions are possible.

A. Robot Configuration and Topology

We give the term *configuration* the standard definition in robotics: a set of parameters which describes the complete specification of all of the points in a system. A VTT consists of some fixed number of edge modules, so the configuration must encode both the spatial locations of all of the members as well as a description of how the members are connected to one another. We call this description of the connections between the members the *connectivity* of the truss. The natural choice to describe the connectivity is a simple graph



(a) (b)

Fig. 7: (a) Test setup prior to docking. Note the separation of the lower VTT node with the upper VTT node. The male connector is marked in red, and the female connector is marked in yellow. (b) Test setup post docking. Note the two nodes have joined together passively.

with labeled edges, with each graph edge corresponding to a truss member. Then, to fully specify the physical locations of the members, one choice is to provide an *embedding* of this graph, which is a specification of the point in 3D space of each node in the graph. This choice eliminates the ambiguity between multiple solutions that can arise from a specification of member lengths, for example.

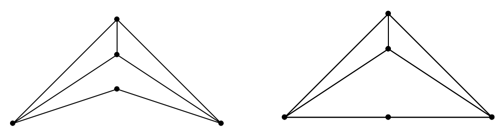
When performing high level planning and design, we often would like to disregard the labels from the labeled graph representation of the connectivity. This yields the *topology* of the truss, which is an isomorphism class of truss connectivity. The truss topology can be represented by an unlabeled graph. In other words, two trusses are said to have the same topology if there is a *graph isomorphism* between their graph representations. The truss topology provides the most general specification that corresponds to the tasks a particular robot configuration is suited to perform.

We call the motion which only moves the position of the nodes a *geometric reconfiguration*, and a motion which merges or splits nodes a *topological reconfiguration*. Topological reconfigurations necessarily change the number of nodes in the graph representation, but the total number of edges must remain the same. The edges correspond to physical actuators; they cannot vanish or reappear, nor can two actuators occupy the same location. Consequently, nodes which are first or second degree neighbors cannot merge. If first degree neighbors in a graph were to combine, the edge between them would be deleted. Similarly, if second degree neighbors were to combine, two edges would collapse to a single edge. Therefore the valid list of node merges for a given connectivity is restricted to the list of third or greater degree neighbors.

B. Rigidity

A VTT is required to be a rigid structure to maintain its shape. In order to maintain controllability of every node on a VTT robot, it is necessary for the truss to satisfy the stronger condition of *infinitesimally rigidity*. Infinitesimal rigidity is the property that no infinitesimal motions can be assigned to the nodes of the truss without violating the distance constraints imposed by the member lengths, aside from the motions that correspond to the six degrees of freedom of a rigid body [13]. However, infinitesimal rigidity is a property that depends on the specific embedding of the graph. A more general property is *generic rigidity*, a property of the topology alone. A generically rigid graph is rigid "almost everywhere," and if it is rigid in one configuration, it is rigid in some neighborhood of that configuration [13]. Similarly, a graph which is generically non-rigid will be flexible "almost everywhere." There may be configurations where a generically rigid graph becomes infinitesimally flexible, and these correspond to singularity configurations of a VTT. Fig. 8 gives one such example in 2D space.

In 2D, there are combinatorial methods for determining whether a graph is generically rigid. Finding a combinatorial method for 3D frameworks is still an open problem in rigidity theory. In this paper, we use the fact that if a graph has at least one infinitesimally rigid configuration, then the graph is generically rigid. An embedding is assigned to a graph, and infinitesimal rigidity is checked. If the graph is not infinitesimally rigid, then the embedding is perturbed to ensure it is not in a singularity position. Since generic (non-singular) configurations occur "almost everywhere," this method works well in practice.



(a) (b)

Fig. 8: An example of a generically rigid graph in 2D. Configuration (a) is infinitesimally rigid, and all nearby configurations are also infinitesimally rigid. Configuration (b) is a singular configuration which is not infinitesimally rigid. Although no nodes can move a finite distance without violating distance constraints, an infinitesimal motion in the vertical direction can be applied to the bottom center node.

Since topologies which are not generically rigid have no infinitesimally rigid configurations, these topologies must be avoided during topological reconfiguration. Any merging or splitting of nodes must preserve the generic rigidity of structure. Additionally, singular configurations of generically rigid configurations must be avoided during geometric reconfiguration.

C. Truss Topology Neighbor Graphs

To describe the reconfiguration capability of a VTT, we introduce the *topology neighbor graph* of a VTT. Each node in the topology neighbor graph represents a unique truss topology. Two nodes are connected if a single topological reconfiguration step takes a configuration in the first topology to a configuration in the second topology. An example of a topology neighbor graph for a truss that can reconfigure to form three topologies is shown in Fig. 9.

The topology neighbor graph for a truss can be generated by starting with some initial topology and successively applying all admissible topological reconfigurations. Admissi-

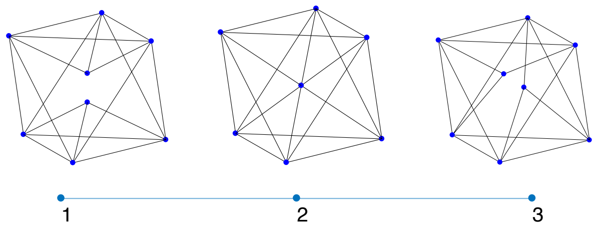


Fig. 9: An example of a topology neighbor graph for an 18 member VTT. The left truss (1), an octahedron with two interior tetrahedral cells, can merge its nodes to form the center truss (2). This truss can split the center node in a topologically distinct way, which is shown on the right (3). The topology neighbor graph shown below the trusses encapsulates this information: topologies 1 and 3 are neighbors to topology 2 but not to each other.

ble node merges must preserve the number of members in the truss, so the possible merges are limited to all pairs of third degree or higher node neighbors in the truss. Admissible merges and splits must also preserve generic rigidity of the truss. After applying a valid topological reconfiguration, the resulting truss connectivity is checked for isomorphism with all of the existing topologies in the neighbor graph. New topologies are added as nodes to the graph, and the graph is fully explored when all reconfiguration options are exhausted. Fig. 10 shows two more complicated examples of VTTs and their associated topology neighbor graphs.

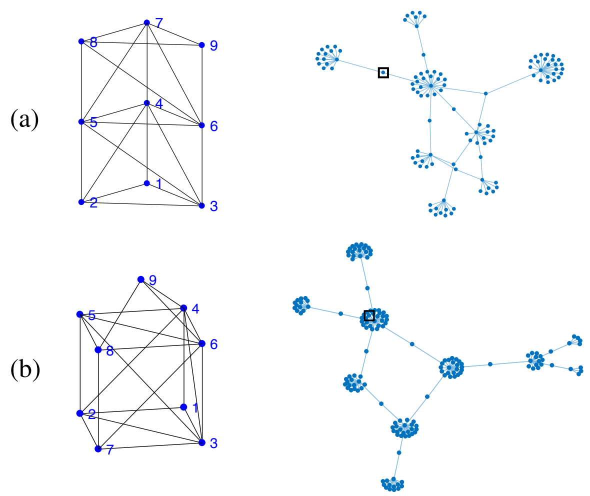


Fig. 10: Two 21 member VTTs and their associated topology neighbor graphs. The black box shows the location of the pictured topology in the neighbor graph. The neighbor graph of truss (a) has 111 topologies, while the neighbor graph of truss (b) has 165 topologies.

The topology neighbor graph can also be thought of as a single connected component in the *topology neighbor supergraph*, which describes the relationship between all truss topologies. Given start and goal truss configurations, the topology neighbor graph can be used to quickly eliminate potentially impossible goal configurations. If the topology of the goal configuration does not lie in the same connected component as the topology of the start configuration, then there is no path through configuration space which connects the start and goal configurations. However, these graphs do not consider self-collisions or actuator constraints, so the existence of a path through the neighbor graph does not guarantee that a path through configuration space exists.

V. Smallest Reconfigurable Trusses

From the restrictions on reconfiguration, we can determine the minimal reconfigurable trusses.

Theorem 1: A minimum of 18 members are necessary to admit reconfiguration in a VTT.

Proof: Since topological reconfiguration steps are reversible, any reconfigurable VTT must have some configuration which is capable of merging two nodes. One necessary condition for generic rigidity in 3D is that the graph representation of the truss must have a minimum vertex degree of three or more (with the exception of the special cases of a single bar and single triangle). Since the two merging nodes must be no closer than third degree neighbors, each of the merging nodes must have three unshared neighbors to satisfy rigidity. Therefore the total number of nodes in this configuration is at least eight. A second necessary condition for generic rigidity in 3D is the following constraint inequality:

$$M \geq 3N - 6 \tag{1}$$

where M is the number of members and N is the number of nodes. This relates the number of position constraints imposed by the members and the degrees of freedom of the nodes. In a minimally rigid structure, the removal of any member results in flexibility. These structures are statically determinate, and the above inequality becomes an equality. With 8 nodes, $M \geq 18$, therefore the minimum number of truss members required to admit reconfiguration is 18. ■

With 18 members, only trusses with 7 or 8 nodes are possible, since the complete graph with 6 nodes only contains 15 members, and a truss of 9 nodes would violate the constraint inequality. There are five non-isomorphic graphs with 7 nodes and 18 edges [14], and they are pictured in Fig. 11. All of them are generically rigid in 3D. One of the five graphs (Fig. 11 (1)) corresponds to the complete graph with 6 nodes with one additional node connected by three edges. This graph is incapable of splitting any node without losing generic rigidity. The remaining four graphs correspond to the four 18 member VTTs which are capable of topological reconfiguration.

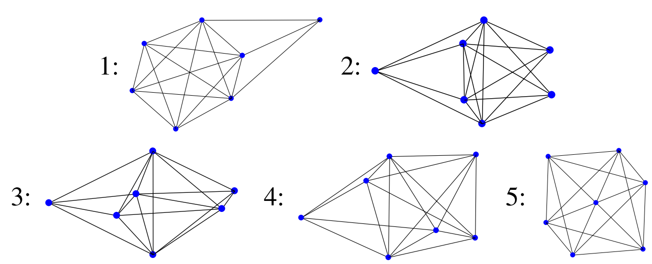


Fig. 11: The five topologies with 7 nodes and 18 members.

Fig. 12 shows the topology neighbor graphs of the five trusses. They form five separate connected components,

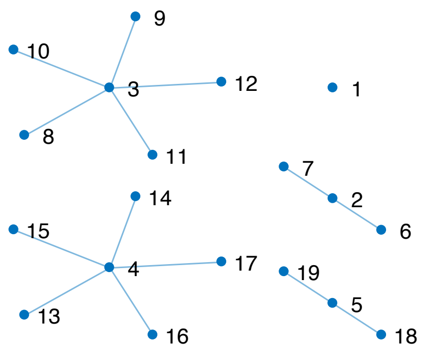


Fig. 12: The topology neighbor graphs of the five topologies with 7 nodes and 18 edges, where numbers 1–5 above correspond to numbers 1–5 in Fig. 11. They form five connected components.

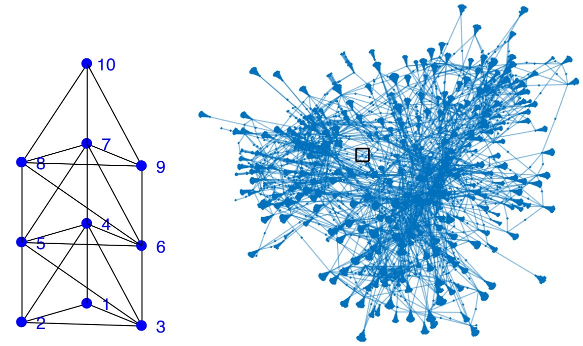


Fig. 13: A 24 member VTT and a subset of the associated topology neighbor graph. The graph generation program was stopped after an hour of execution. The subset contains over 10,000 topologies.

which implies none of these trusses can reconfigure into any of the other four. The fourteen additional nodes (6–19) represent the topologically unique trusses that can be obtained by splitting nodes from the 18 member and 7 node trusses. There are 663 non-isomorphic graphs with 8 nodes and 18 members [14]. All but the fourteen shown must be either non-rigid or isolated topologies incapable of reconfiguration.

VI. Larger Reconfigurable Trusses

A similar process can be applied for 19 and 20 member structures. There are two non-isomorphic 19 member topologies with 7 nodes, each of which neighbor three 8 node topologies. There is only one isomorphism class of 20 member topologies with 7 nodes, and it neighbors two 8 node topologies. With 21 members, rigid 9 node graphs become possible, and the connected components begin to exhibit more interesting organization. Fig. 10 shows two such examples. There is only one graph with 7 nodes and 21 edges: the complete 7 node graph, which neighbors one 8 node graph. This 8 node graph cannot reconfigure further, so all other reconfigurable trusses with 21, 22, or 23 members involve merges and splits between 8 and 9 node graphs.

Rigid 10 node graphs become possible with 24 members, and again there is a jump in complexity of the topology neighbor graphs. The maximum size of the topology neighbor graphs increases rapidly with the maximum number of nodes in the truss. Fig. 13 gives an example of a VTT with 24 members and 10 nodes, here, computing the complete connected component is computationally intractable.

VII. Conclusion

Variable topology trusses are a novel extension to the class of robots known as variable geometry trusses. The ability to topologically reconfigure expands the capabilities of particular trusses. Constraints on reconfiguration and the allowable truss topologies were introduced, leading to the derivation of the minimum number of members in a truss required to admit topological reconfiguration. By exhaustively exploring reconfiguration options, the relationship between all of the topologically distinct reconfigurable structures for 18, 19, and 20 members were characterized.

Future work includes performing a comprehensive study of structures with more members, and identifying promising configurations for common robotics tasks. The motion and reconfiguration planning problems for VTTs also need to be developed. We suspect that topological reconfiguration can be used to move past singular configurations, expanding the workspace of VTTs.

References

[1] A. Ananthanarayanan, M. Azadi, and S. Kim, "Towards a bio-inspired leg design for high-speed running," *Bioinspiration & biomimetics*, vol. 7, no. 4, p. 046005, 2012.

[2] V. A. Reinholtz and L. T. Watson, "Enumeration and analysis of variable geometry truss manipulators," 1990.

[3] M. Yim, D. G. Duff, and K. D. Roufas, "Polybot: a modular reconfigurable robot," in *Robotics and Automation, 2000. Proceedings. ICRA'00. IEEE International Conference on*, vol. 1. IEEE, 2000, pp. 514–520.

[4] A. Spinos and M. Yim, "Towards a variable topology truss for shoring," in *Ubiquitous Robots and Ambient Intelligence (URAI), 2017 14th International Conference on*. IEEE, 2017.

[5] M. D. Rhodes and M. Mikulas Jr, "Deployable controllable geometry truss beam," 1985.

[6] G. J. Hamlin and A. C. Sanderson, *Tetrobot: A modular approach to reconfigurable parallel robotics*. Springer Science & Business Media, 2013, vol. 423.

[7] W. H. Lee and A. C. Sanderson, "Dynamic rolling, locomotion planning, and control of an icosahedral modular robot," in *Intelligent Robots and Systems, 2000.(IROS 2000). Proceedings. 2000 IEEE/RSJ International Conference on*, vol. 3. IEEE, 2000, pp. 2178–2183.

[8] B. Boutin, A. K. Misra, and V. Modi, "Dynamics and control of variable-geometry truss structures," *Acta Astronautica*, vol. 45, no. 12, pp. 717–728, 1999.

[9] E. Bolker and B. Roth, "When is a bipartite graph a rigid framework?" *Pacific Journal of Mathematics*, vol. 90, no. 1, pp. 27–44, 1980.

[10] F. Collins and M. Yim, "Design of a spherical robot arm with the spiral zipper prismatic joint," in *Robotics and Automation (ICRA), 2016 IEEE International Conference on*. IEEE, 2016, pp. 2137–2143.

[11] M. Yim, "Reconfigurable structural member and system," U.S. Patent Provisional Application 62/245,461, Oct. 23, 2015.

[12] G. J. Hamlin and A. C. Sanderson, "Tetrobot modular robotics: Prototype and experiments," in *Intelligent Robots and Systems' 96, IROS 96, Proceedings of the 1996 IEEE/RSJ International Conference on*, vol. 2. IEEE, 1996, pp. 390–395.

[13] J. E. Graver, B. Servatius, and H. Servatius, *Combinatorial rigidity*. American Mathematical Soc., 1993, no. 2.

[14] L. Comtet, "Advanced combinatorics, reidel, dordrecht, 1974," *Riordan, Combinatorical Idendities,(RE Krieger, New York, 1979)*.