6.1

$F_{net} = m.a$

$$F_c - \dot{x}_c B_{eq} - K_s(x_c - x_2) = M_c \cdot \ddot{x}_c \quad , \quad K_s(x_c - x_2) - \dot{x}_2 B_{eq2} = M_{c2} \cdot \ddot{x}_2$$

$$\frac{d^2 x_c}{dt^2} = \frac{dx_c}{dt}\left[\frac{-B_{eq}}{M_c}\right] - \frac{K_s}{M_c}x_c + \frac{K_s x_2}{M_c} + \frac{F_c}{M_c} \quad , \quad \frac{d^2 x_2}{dt^2} = \frac{dx_2}{dt}\left[\frac{-B_{eq2}}{M_{c2}}\right] + \frac{K_s}{M_{c2}}x_c - \frac{K_s}{M_{c2}}x_2$$

6.2

1)

$$\begin{bmatrix} \dot{x}_c \\ \dot{x}_2 \\ \ddot{x}_c \\ \ddot{x}_2 \end{bmatrix} = \underbrace{\begin{array}{c} \begin{array}{cccc} x_c & x_2 & \dot{x}_c & \dot{x}_2 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-K_s}{M_c} & \frac{K_s}{M_c} & \frac{-B_{eq}}{M_c} & 0 \\ \frac{K_s}{M_c} & \frac{-K_s}{M_c} & 0 & \frac{-B_{eq2}}{M_{c2}} \end{bmatrix} \end{array}}_{A} X + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{1}{M_c} \\ 0 \end{bmatrix}}_{B} U$$

2)

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-K_s}{M_c} & \frac{K_s}{M_c} & \frac{-B_{eq}}{M_c} - \frac{\eta_g K_g^2 \eta_g K_t K_m}{M_c R_m r_{mp}^2} & 0 \\ \frac{K_s}{M_c} & \frac{-K_s}{M_c} & 0 & \frac{-B_{eq2}}{M_{c2}} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{\eta_g K_g \eta_m K_t}{M_c R_m r_{mp}} \\ 0 \end{bmatrix}$$

3)

$M_c = 1.1456\ kg \qquad M_{c2} = 0.5425\ kg$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -139.66 & 139.66 & -11.46 & 0 \\ 294.93 & -294.93 & 0 & -2.03 \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ 0 \\ 1.54 \\ 0 \end{bmatrix}$$

$K_s = 160$
$B_{eq_2} = 11$

4) Found eigenvalues on MATLAB

$P_{1,2} = -2.35 \pm j20.27 \qquad P_3 = 0 \quad P_4 = -8.79$

The system is not stable since $\boxed{P_3 = 0}$, there is a pole outside of OLHP

5)
System has 1 integrator (only $P_3 = 0$) so system is Type 1.

6) $C_1 = [1 \quad 0 \quad 0 \quad 0]$ (used matlab in this part)

$$Ob = \begin{bmatrix} C_1 \\ C_1 A \\ C_1 A^2 \\ C_1 A^3 \end{bmatrix}, \quad \det(Ob) \neq 0 \Rightarrow$$ Observability matrix is full rank means that system is fully observable.

$C_2 = [0 \quad 0 \quad 1 \quad 0] \Rightarrow \det(Ob) = 0 \Rightarrow$ Observability matrix is not full rank therefore system is not fully observable

6.3

1) $$\boxed{-K_S x_2 - B_{eq2} \cdot \dot{x}_2 = M_{c2} \cdot \ddot{x}_2} \quad (\star)$$

2) $\mathcal{L}\{\cdot\}$ of $(\star)$

$$-K_S X_2 - B_{eq2} \cdot s X_2 = s^2 M_{c2} X_2$$

$$\Rightarrow X_2 = \frac{1}{s^2 + \frac{B_{eq2}}{M_{c2}} s + \frac{K_s}{M_{c2}}} \qquad \Rightarrow \boxed{w_n^2 = \frac{K_s}{M_{c2}}}$$

$$\Rightarrow 2\zeta w_n = \frac{B_{eq2}}{M_{c2}} = 2\zeta\sqrt{\frac{K_s}{M_{c2}}}$$

$$\Rightarrow \boxed{\xi = \frac{B_{eq2}}{2\sqrt{K_S M_{c2}}}}$$

$$\boxed{w_n = \sqrt{\frac{K_s}{M_{c2}}}} \qquad \boxed{\xi = \frac{B_{eq2}}{2\sqrt{K_S M_{c2}}}}$$

3) $$\boxed{K_S = M_{c2} w_n^2} \qquad \boxed{B_{eq2} = 2\xi\sqrt{K_S M_{c2}}}$$

5) $w_d = w_n\sqrt{1-\xi^2}$ $\quad w_d t_1 = \pi; \; w_d t_2 = 3\pi \quad t_1 = \frac{\pi}{w_d}, \; t_2 = \frac{3\pi}{w_d}$

$$\frac{|x_2(t_1)|}{|x_2(t_2)|} = R_p = \frac{A e^{\left(-\xi w_n \frac{\pi}{w_d}\right)}}{A e^{\left(-\xi w_n \frac{3\pi}{w_d}\right)}} = e^{\xi w_n \frac{\pi}{w_d}} \Rightarrow \boxed{R_p = e^{\frac{\xi\pi}{\sqrt{1-\xi^2}}}}$$

$$\frac{\ln(R_p)}{\pi} = \frac{\xi}{\sqrt{1-\xi^2}} \xRightarrow{(\cdot)^2} \frac{\ln^2(R_p)}{\pi^2} = \frac{\xi^2}{1-\xi^2} \Rightarrow \boxed{\xi = \frac{\ln(R_p)}{\sqrt{\pi^2 + \ln^2(R_p)}}}$$

7.

$t_2 - t_1 = 5.67 - 5.26 = 0.41 \text{ sec} \Rightarrow w_d = \frac{2\pi}{t_2 - t_1} \Rightarrow w_d = 15.32 \ \frac{rad}{sec}$

$R_p = \frac{93.86}{64.04} \Rightarrow \boxed{\xi = 0.1208}$

$w_n = \frac{w_d}{\sqrt{1-\xi^2}} \Rightarrow \boxed{w_n = 15.62 \ \frac{rad}{sec}}$

8. $k_s = w_n^2 M_{c2} \Rightarrow \boxed{k_s = 132.5}$

$B_{eq2} = 2\xi w_n M_{c2} \Rightarrow \boxed{B_{eq2} = 2.05}$