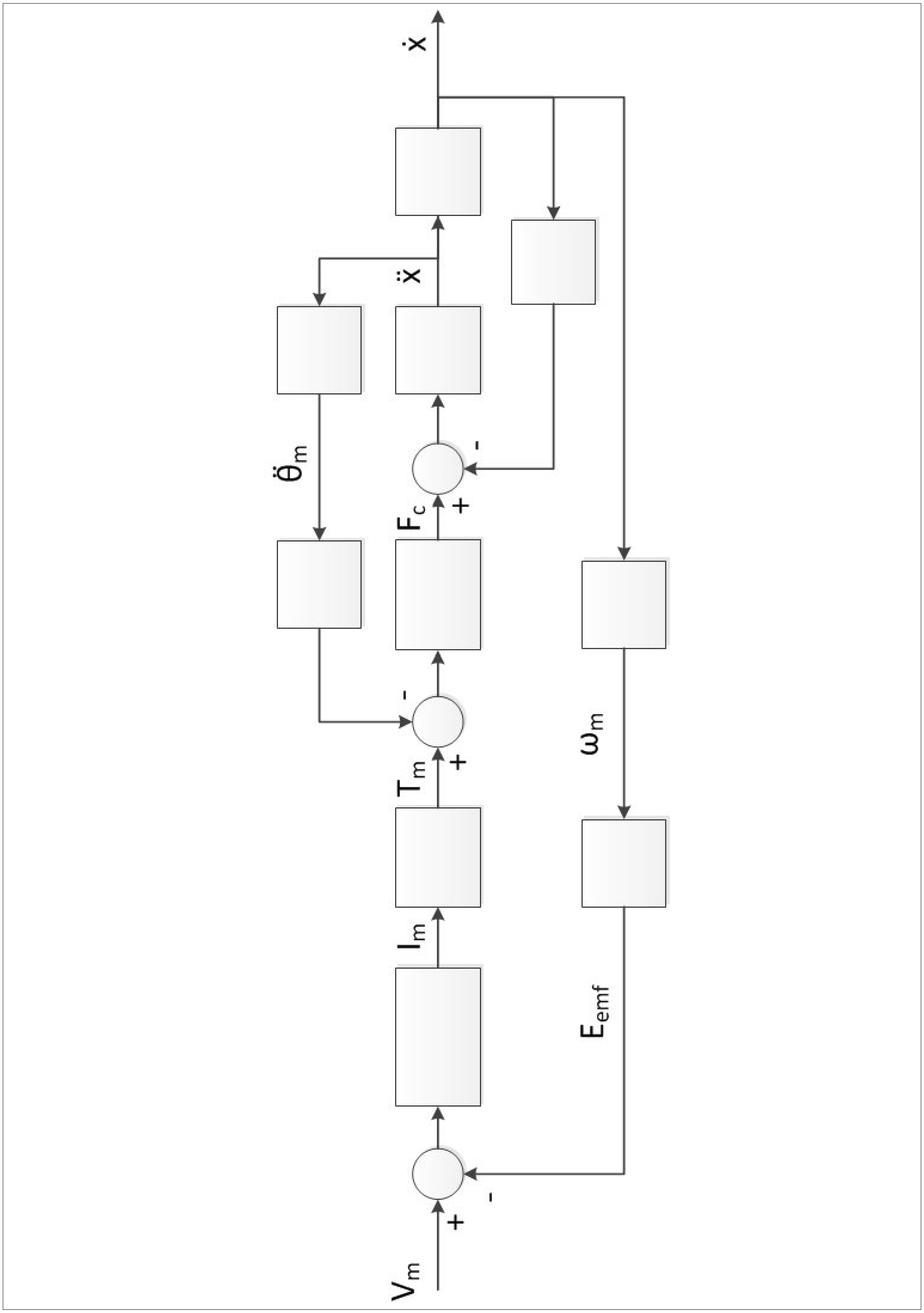


Fig. 4 – The IP02 Servo Plant Detailed Block Diagram

6.2 Assignment #2: Open-Loop Transfer Function

Now that you have a detailed block diagram of the open-loop plant, obtain the overall transfer function from the input to the output. Remember that the transfer function *G(s)* is given by

$$G(s) = \frac{V(s)}{V_m(s)} \tag{1}$$

and is now defined for a velocity output.

(a) Derive and write down this overall open-loop plant transfer function in the box provided below in terms of all the variables defined in Appendix A.

G2(s) =

$$\frac{1}{\mathrm{Mc}_2\, s \left(\frac{\mathrm{Beq}}{\mathrm{Mc}_2\, s} + 1\right)}$$

ans =

$$\frac{1}{\mathrm{Beq} + \mathrm{Mc}_2\, s}$$

G1(s) =

$$\frac{\mathrm{Eff}_g\, \mathrm{Kg}}{\mathrm{Mc}_2\, r_{\mathrm{mp}}\, s \left(\frac{\mathrm{Eff}_g\, \mathrm{Jm}\, \mathrm{Kg}^2}{\mathrm{Mc}_2\, r_{\mathrm{mp}}^2 \left(\frac{\mathrm{Beq}}{\mathrm{Mc}_2\, s} + 1\right)} + 1\right) \left(\frac{\mathrm{Beq}}{\mathrm{Mc}_2\, s} + 1\right)}$$

G1(s) =

$$\frac{\mathrm{Eff}_g\, \mathrm{Kg}\, r_{\mathrm{mp}}}{\mathrm{Beq}\, r_{\mathrm{mp}}^2 + \mathrm{Mc}_2\, r_{\mathrm{mp}}^2\, s + \mathrm{Eff}_g\, \mathrm{Jm}\, \mathrm{Kg}^2\, s}$$

G(s) =

$$\frac{\mathrm{Eff}_g\, \mathrm{Eff}_m\, \mathrm{Kg}\, \mathrm{Kt}\, r_{\mathrm{mp}}}{(\mathrm{Rm} + \mathrm{Lm}\, s) \left(\frac{\mathrm{Eff}_g\, \mathrm{Eff}_m\, \mathrm{Kg}^2\, \mathrm{Km}\, \mathrm{Kt}}{(\mathrm{Rm} + \mathrm{Lm}\, s)\, (\mathrm{Beq}\, r_{\mathrm{mp}}^2 + \mathrm{Mc}_2\, r_{\mathrm{mp}}^2\, s + \mathrm{Eff}_g\, \mathrm{Jm}\, \mathrm{Kg}^2\, s)} + 1\right) (\mathrm{Beq}\, r_{\mathrm{mp}}^2 + \mathrm{Mc}_2\, r_{\mathrm{mp}}^2\, s + \mathrm{Eff}_g\, \mathrm{Jm}\, \mathrm{Kg}^2\, s)}$$

G(s) =

$$\frac{\mathrm{Eff}_g\, \mathrm{Eff}_m\, \mathrm{Kg}\, \mathrm{Kt}\, r_{\mathrm{mp}}}{\mathrm{Beq}\, \mathrm{Rm}\, r_{\mathrm{mp}}^2 + \mathrm{Beq}\, \mathrm{Lm}\, r_{\mathrm{mp}}^2\, s + \mathrm{Mc}_2\, \mathrm{Rm}\, r_{\mathrm{mp}}^2\, s + \mathrm{Lm}\, \mathrm{Mc}_2\, r_{\mathrm{mp}}^2\, s^2 + \mathrm{Eff}_g\, \mathrm{Eff}_m\, \mathrm{Kg}^2\, \mathrm{Km}\, \mathrm{Kt} + \mathrm{Eff}_g\, \mathrm{Jm}\, \mathrm{Kg}^2\, \mathrm{Rm}\, s + \mathrm{Eff}_g\, \mathrm{Jm}\, \mathrm{Kg}^2\, \mathrm{Lm}\, s^2}$$

(b) Now, refer to the IP02 User Manual and use the numerical values for all the parameters for this plant to actually evaluate the open-loop plant transfer function. By noting that the motor inductance is much smaller than motor internal resistance, you may from this point on, assume that L_m=0. This reduces the order of the system by one and will help us with the following analysis. Write down the evaluated G(s) below.

G(s) =

$$\frac{2.4513}{s + 17.1001}$$

(c) Now, determine the open-loop poles, zeros and the DC gain of the plant. Write them down in the box provided below.

Zeros: NONE

Poles: -17.1001

DC Gain: 2.4513/17.1001 = 0.14335

(d) What is the Type of the system? Assuming uncompensated unity-feedback structure (no controller in the forward path) can the steady-state error for this system for a unit-step input be zero? Derive E(s) and find $|e_{ss}|$ for unit step and unit ramp inputs.

System is **Type 0** meaning that there is no integrator on the open-loop transfer function. For a unity-feedback structure steady-state error for unit-step (1/s);

$$E(s) = R(s) - E(s)G(s)H(s), \qquad \text{where H(s) is 1.}$$

$$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)H(s)}$$

$$\frac{F(s)}{E(s)} = G_{OL} = G(s)H(s) \qquad E(s) = R(s)\frac{1}{1+G_{OL}(s)}$$

For unit step response R(s) = 1/s

$$e_{ss} = \lim_{s\to 0}\left[sR(s)\frac{1}{1+G_{OL}(s)}\right]$$
$$= \lim_{s\to 0}\left[s\frac{1}{s}\frac{1}{1+G_{OL}(s)}\right]$$
$$e_{ss} = \frac{1}{1+\lim_{s\to 0} G_{OL}(s)}$$

$$e_{ss} = \frac{1}{1+K_{DC}}$$

$|e_{ss}|$ for unit step: 0.8746

For unit ramp R(s) = $1/s^2$

$$e_{ss} = \lim_{s\to 0}\left[sR(s)\frac{1}{1+G_{OL}(s)}\right]$$
$$= \lim_{s\to 0}\left[s\frac{1}{s^2}\frac{1}{1+G_{OL}(s)}\right]$$
$$= \frac{1}{lim_{s\to 0}s^{-1}G_{OL}(s)}$$

$$e_{ss} \to \infty$$

$|e_{ss}|$ for unit ramp: ∞

6.3 *Assignment #3: Phase-Lead Compensator Design*

This section deals with the design of a phase-lead based closed-loop controller in order to control the velocity of your IP02 cart so as to satisfy the design requirements. We will use the following approach:

i. First, we will try eliminating the system's steady-state error $|e_{ss}|$.
ii. Second, we will find the gain K such that the open-loop bandwidth requirement is met.
iii. Lastly, we will find the resulting phase margin and introduce the phase-lead compensator in order to obtain the desired closed-loop relative stability (desired open-loop phase margin) for the system.

6.3.1 Eliminating Steady-State Error

Suppose we want to control the plant using the unity-feedback structure given in Fig. 5. Find the simplest possible *H*(*s*) using such that we satisfy the steady-state error design specification. Hint: Use *E(s)* and $|e_{ss}|$.

We can eliminate the steady state error by introducing an integrator in H(s). Single integrator is enough since the design requirement is reducing the steady state error to zero only for unit step response.

$H(s) = 1/s$

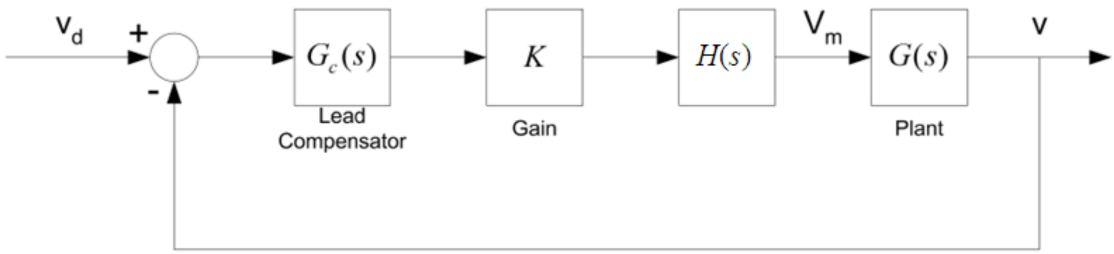


Fig. 5 – Lead-Compensator based Speed Control structure

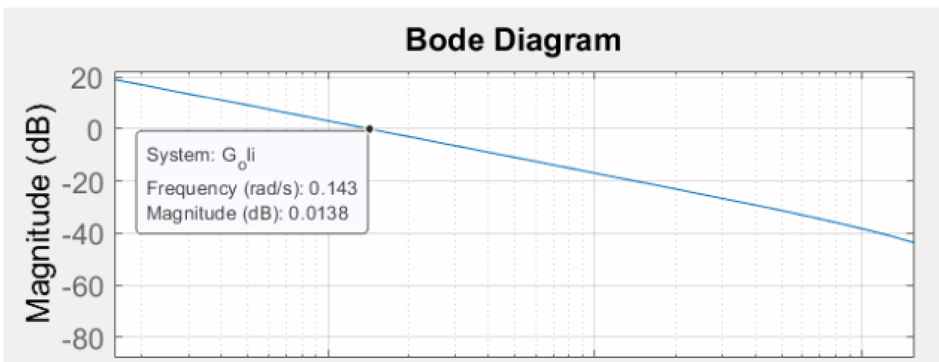
6.3.2 Satisfying Desired Bandwidth

We have found the necessary $H(s)$ in Fig. 5 such that we have zero steady-state error to a unit step input. One design criterion related with the system transient response is given as the open-loop bandwidth of the system. We would like to first find what the present bandwidth (Gain crossover frequency) is, followed by determining the required gain K to set the bandwidth to the desired value.

(a) Sketch the magnitude part of the bode plot of the cascade system $H(s)G(s)$ in the space provided below, indicating all critical points. Determine the gain crossover frequency w_c, hence the present bandwidth of this intermediate system. You can always use MATLAB bode plots for checking your results. But, in this step analytical calculations are expected for bode plots. Especially, calculate gain cross-over frequency and phase at gain crossover frequency.

$$G(s)H(s) = \frac{2.4513}{s\ (s+17.1001)} \quad , |G(j\omega_c)H(j\omega_c)| = 1 \quad \Rightarrow \omega_c = 0.1422 \text{ rad/s}$$

$$\angle G(j\omega_c) + \angle H(j\omega_c) = \pm r \tan^{-1}\left(\frac{\omega}{a}\right) = \text{-arctan}(0.1422/17.1)\text{-}90° = \text{-}90°$$



$w_c =$ 0.1422 rad/s

Phase @ gain cross-over frequency = -90°

(b) Calculate the value of the multiplier Gain, *K,* to bring w_c of $KH(s)G(s)$ to the specified value.

w_c = 80 rad/s from the plot we can see that gain is -68.5 dB, we can shift it up by multipliying it with K,

$$K = 10^{68/20} = 2660.7$$

(e) Using your knowledge from EE302 and assuming that the gain you have found is the combined gain of the open-loop system $KH(s)G(s)$, discuss the effect of changing K (increase/decrease) on the transient behavior of the closed-loop system (rise time t_r and maximum overshoot $M_p\%$). (Note: For this step, ignore the compensator block, i.e., assume $G_c(s)$=1).

$\boldsymbol{K}$	increase / decrease	$\boldsymbol{t_r}$	decrease/increase	$\boldsymbol{M_p\%}$	increase / decrease

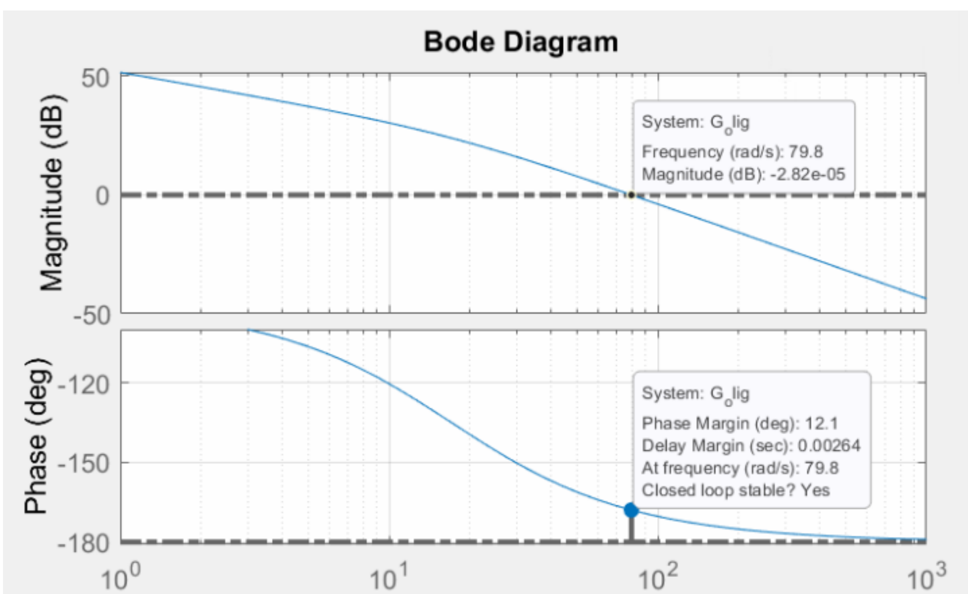
(f) Considering the relation between the gain, K, and the bandwidth, w_c , state the effect of bandwidth on the transient behavior of the closed-loop system.

$\boldsymbol{w_c}$	increase / decrease	$\boldsymbol{t_r}$	decrease/increase	$\boldsymbol{M_p\%}$	increase / decrease

6.3.3 Satisfying Desired Phase Margin

From EE302, the purpose of the *Lead Compensator* block in Fig. 5 is to increase relative stability of the system, by increasing the phase margin of the closed-loop system.

(a) We would like to know what the present phase margin of the system is without the help from the $G_c(s)$ block in Fig. 5. In the space provided below, sketch magnitude and phase of the bode plot of the uncompensated open-loop system $KH(s)G(s)$. In this step you can use MATLAB to plot the desired diagrams. Do not forget to mark all critical points on your plots.



(b) On the plot, indicate the gain crossover frequency w_c and determine the phase margin of the system.

$\Phi_m = 12.1°$

(c) Given the design specification on phase margin, determine the extra phase, Φ_c, required to meet the compensated phase margin of the system. In EE302 you were also adding 5° error margin while calculating required compensator phase. In this step do not add any error margin.

$\Phi_c = 85°\text{-}12.1° = 72.9°$

The phase-lead compensator that we will design has a transfer function $G_c(s)$ of the form

$$G_c(s) = \frac{\alpha\left(s + \frac{w_c^{lead}}{\alpha}\right)}{s + w_c^{lead}\,\alpha} \tag{2}$$

Note that this compensator structure is slightly different from the form we are familiar from our EE302. The main difference of the present form is that it exposes the frequency at which the gain is 1.

(d) Show that $|G_c(jw)| = 1$ at $w = w_c^{lead}$.

$$|G_c(j\omega)| = \frac{\alpha\left(s + \frac{\omega_c^{lead}}{\alpha}\right)}{s + \alpha\,\omega_c^{lead}} \quad \textit{evaluated at } s = j\omega_c^{lead},$$

$$|G_c(j\omega)| = \sqrt{\frac{\alpha^2\left(j\omega_c^{lead} + \frac{\omega_c^{lead}}{\alpha}\right)^2}{\left(j\omega_c^{lead} + \alpha\,\omega_c^{lead}\right)^2}} = \sqrt{\frac{\alpha^2{\omega_c^{lead}}^2\left(j + \frac{1}{\alpha}\right)^2}{{\omega_c^{lead}}^2(j+\alpha)^2}} = \sqrt{\frac{\alpha^2\left(j + \frac{1}{\alpha}\right)^2}{(j+\alpha)^2}} = 1$$

$$= \sqrt{\frac{-\alpha^2 + 2j\alpha + 1}{-\alpha^2 + 2j\alpha + 1}} = 1$$

(e) Show that the maximum phase $\angle G_c(jw)$ is achieved when $w = w_c^{lead}$.

$$\angle G_c(j\omega) = arctan\left(\frac{\omega\alpha}{\omega_c^{lead}}\right) - arctan\left(\frac{\omega}{\omega_c^{lead}\alpha}\right)$$

$$\frac{\partial \angle G_c(j\omega)}{\partial\omega} = \frac{1}{\frac{\omega_c^{lead}}{\alpha} - \frac{\omega^2\alpha}{\omega_c^{lead}}} - \frac{1}{\omega_c^{lead}\alpha - \frac{\omega^2}{\omega_c^{lead}\alpha}} = 0$$

$$\omega = \omega_c^{lead} \; satisfies \; this \; condition \; as \; well \; as \; \omega = -\omega_c^{lead}$$

(f) Determine this maximum phase as a function of the compensator parameters.

$$\angle G_c(j\omega) = arctan(\alpha) - arctan\left(\frac{1}{\alpha}\right)$$

(g) By equating the maximum phase of the compensator to the extra phase required for the design, find the suitable parameter values of the compensator.

w_c^{lead} = 80° α = 6.6514 $G_c(s)$ = (6.6514s+80) / (s+532.1)

(h) You have found values of all blocks in Figure 5. Now, calculate open-loop transfer function of the system shown in Figure 5.

$$\frac{43370s + 521600}{s^3 + 549.2s^2 + 9099s} = \frac{43370\,(s + 12.03)}{s\,(s + 532.1)\,(s + 17.1)}$$

(i) Derive E(s) for the final compensated system. Calculate $|e_{ss}|$ for unit step and unit ramp inputs.

$$E(s) = R(s)\frac{1}{1+G_{OL}(s)}$$

For unit step response R(s) = 1/s

$$e_{ss} = \lim_{s\to 0}\left[sR(s)\frac{1}{1+G_{OL}(s)}\right] = \lim_{s\to 0}\left[s\frac{1}{s}\frac{1}{1+G_{OL}(s)}\right] = \frac{1}{\ }$$

$$E(s) = \frac{s\,(s+532.1)\,(s+17.1)}{(s+430)\,(s+108)\,(s+11.24)}$$

$e_{ss} = 0$

For unit ramp R(s) = 1/s²

$$e_{ss} = \lim_{s\to 0}\left[sR(s)\frac{1}{1+G_{OL}(s)}\right] = \lim_{s\to 0}\left[s\frac{1}{s^2}\frac{1}{1+G_{OL}(s)}\right] = \frac{1}{\ }$$

$$E(s) = \frac{s\,(s+532.1)\,(s+17.1)}{s\,(s+430)\,(s+108)\,(s+11.24)}$$

$e_{ss} = 0.01743$

(j) Comment on the effect of the compensator on the steady-state error performance of the closed-loop system (Hint: recall the type of the open-loop system before compensation)

Compansator reduced the steady state error for unit step to zero and minimized it to 0.01743 for unit ramp, previously it was infinite.

(k) Comment again on the effect of the compensator on the steady-state error of the closed-loop system if the integrator block was not considered in our design.

Steady state error for unit step would be small but not zero, for unit ramp it would be infinite because of the type of the system.

(l) Calculate the closed-loop transfer function of the whole compensated system.

$$T(s) = \frac{43370\,(s + 12.03)}{(s + 430)\,(s + 108)\,(s + 11.24)}$$

(m) Why didn't we add the error margin of 5° to the required compensator phase as we did in EE302? Explain.

Since we used the systems bode plot to calculate the alpha and omega values, it is strictly known that the phase for any given alpha. error margin is unnecessary.