Complex Numbers

Definition: Complex numbers are numbers of the form $a + bi$ where $a, b \in \mathbb{R}$ and i is the imaginary unit such that $i^2 = -1$. The set of complex numbers is denoted by $\mathbb{C}$.

$$\mathbb{C} = \{a + bi \mid a, b \in \mathbb{R}\}$$

addition and multiplication are defined as follows:

$$\begin{aligned}(a+bi)+(c+di) &= (a+c)+(b+d)i \\ (a+bi)(c+di) &= (ac-bd)+(ad+bc)i\end{aligned}$$

The set of real numbers is a subset of the complex numbers, i.e. $\mathbb{R} \subset \mathbb{C}$. The set of imaginary numbers is also a subset of the complex numbers, i.e. $\mathbb{I} \subset \mathbb{C}$.

Euler's formula is a mathematical formula in complex analysis that establishes the fundamental relationship between the trigonometric functions and the complex exponential function. Euler's formula states that for any real number x:

$$e^{ix} = \cos(x) + i\sin(x)$$

where e is Euler's number, the base of natural logarithms, and i is the imaginary unit, which satisfies the equation $i^2 = -1$.

The formula is still valid if x is a complex number. In particular, if $x = \pi$, Euler's formula states that:

$$e^{i\pi} + 1 = 0$$

The complex conjugate of a complex number $z = a + bi$ is given by $\bar{z} = a - bi$. The complex conjugate of a complex number z is denoted by $\bar{z}$.

#EE501 - `Linear Systems Theory` at `METU`