

Function Norms

Definition: Function `norm`s are norm definitions applied to `function space`s. Function spaces are vector spaces whose elements are functions. The most common function spaces are L^p spaces.

$$V = \{f(\cdot) | f : [0, 1] \rightarrow \mathbb{R} \text{ s.t. } \int_0^1 |f(x)|^p dx < \infty \quad 1 \leq p < \infty\}$$

We can define norms on V as follows:

$$\|f\|_p := \left(\int_0^1 |f(x)|^p dx \right)^{1/p}$$

where $p \geq 1$.

Specific Cases

- $\|f\|_1 = \int_0^1 |f(x)| dx$ L_1 - norm
- $\|f\|_2 = \left(\int_0^1 |f(x)|^2 dx \right)^{1/2}$ L_2 - norm
- $\|f\|_\infty = \sup_{x \in [0,1]} |f(x)|$ L_∞ - norm