Convergence

Definition: Let (V, F) be a vector space. A sequence of vectors $\{x_n\}_{n=1}^{\infty}$ in V is said to converge to a vector $x \in V$ if for every $\epsilon > 0$ there exists an integer N such that $\|x_n - x\| < \epsilon$ for all $n \geq N$. In this case we write $x_n \to x$ as $n \to \infty$.

> **Remark**: The sequence is said to be **convergent** if it converges to some vector $x \in V$. Otherwise, it is said to be **divergent**.

Example: Let $V = R \ \ \|v\| = v$, consider the sequence

$$\left\{ (\frac{1}{2})^n \right\}_{n=1}^{\infty}$$

Proove that this sequence converges to 0 as $n \to \infty$.

Proof: DO IT LATER

Definition: Let $(V, F, \|.\|)$ be a normed space. A sequence of vectors $\{x_n\}_{n=1}^{\infty}$ in V is said to be a **Cauchy sequence** if for every $\epsilon > 0$ there exists an integer N such that $\|x_n - x_m\| < \epsilon$ for all $n, m \geq N$.

- **Proof**: $\|x_n - x_m\| < \epsilon$ for all $n, m \geq N$ implies $\|x_n - x\| < \epsilon$ for all $n \geq N$.

#EE501 - `Linear Systems Theory` at `METU`