Fields

Linear Spaces

Definition: A `field` is a set F of `scalar`s with two operations, addition and multiplication, such that the following axioms hold:

Addition

- (A1) $a+b=b+a$ for all $a, b \in F$ (commutativity of addition)
- (A2) $(a+b)+c=a+(b+c)$ for all $a, b, c \in F$ (associativity of addition)
- (A3) There exists an element $0 \in F$ such that $a+0=a$ for all $a \in F$ (existence of additive identity) This element is commonly denoted by 0_F.
- (A4) For every $a \in F$ there exists an element $-a \in F$ such that $a+(-a)=0_F$ (existence of additive inverse) This element is commonly denoted by $(-a)$.

Multiplication

- (M1) $a \cdot b=b \cdot a$ for all $a, b \in F$ (commutativity of multiplication)
- (M2) $(a \cdot b) \cdot c=a \cdot(b \cdot c)$ for all $a, b, c \in F$ (associativity of multiplication)
- (M3) There exists an element in F such that $a \cdot 1_F=a$ for all $a \in F$ (existence of multiplicative identity)
- (M4) For every $a \in F$ except 0_F there exists an element $a^{-1} \in F$ such that $a \cdot a^{-1}=1_F$ (existence of multiplicative inverse)

Distributivity

- (D1) $a \cdot(b+c)=a \cdot b+a \cdot c$ for all $a, b, c \in F$ (distributivity of multiplication over addition)

At least two elements must exist in a field, additive identity 0_F and multiplicative identity 1_F.

Field examples include $\mathbb{R}$, $\mathbb{C}$, $\mathbb{Q}$, $\mathbb{Z}_p$ where p is a prime number. $\mathbb{R}$ are the real numbers, $\mathbb{C}$ are the `complex numbers`, $\mathbb{Q}$ are the rational numbers, $\mathbb{Z}_p$ are the integers modulo p.

In a sense, what I understand from the fields are, mathematical concepts that are used to define the operations on the elements of the vector spaces.

$\mathbb{F}^n$

Linear Spaces

Definition: The set of all `list`s of length n with elements from a `field` $\mathbb{F}$ is denoted by $\mathbb{F}^n$.

$$\mathbb{F}^n=\{(x_1, x_2, \ldots, x_n) \mid x_i \in \mathbb{F}, i=1,2, \ldots, n\}$$

Addition and scalar multiplication are defined as follows:

$$(x_1, x_2, \ldots, x_n)+(y_1, y_2, \ldots, y_n)=(x_1+y_1, x_2+y_2, \ldots, x_n+y_n)$$

$$\alpha(x_1, x_2, \ldots, x_n)=(\alpha x_1, \alpha x_2, \ldots, \alpha x_n)$$

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