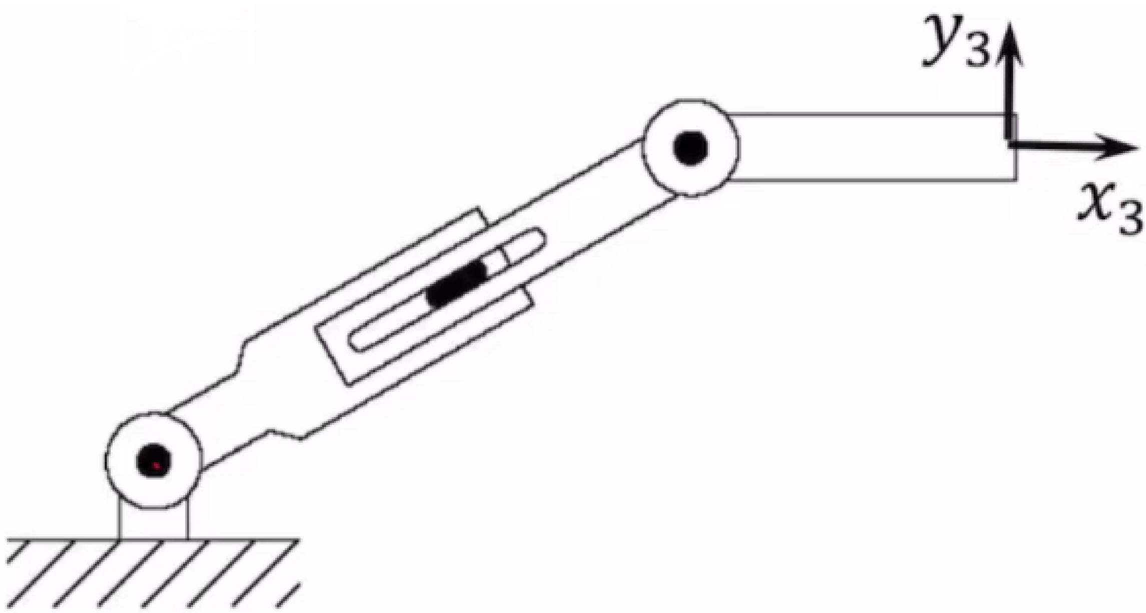
Kinematic DH Parameters for 3-link RPR arm

```
imshow(imread("./figures/RPR.png"))
```



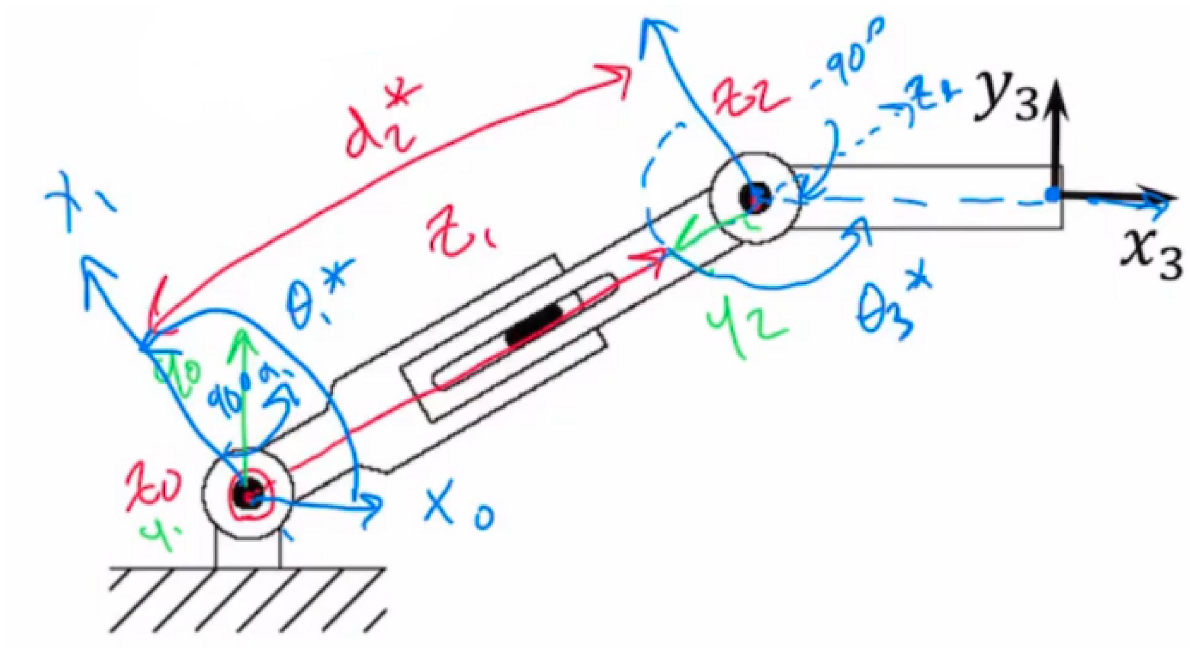
Robot Description

- **{r, p, r} :** r for revolute, p for prismatic
- **{0, 0, L_1} :** radius lenghts
- **{Pi/2, -Pi/2, 0} :** link twists
- **{0, q_2, 0} :** link offset rotation
- **{q1, 0, q3} :** joint rotation

```
imshow(imread("./figures/DH.png"))
```

Joint	Type	r	α	d	θ
1	revolute	0	$\frac{\pi}{2}$	0	θ_1^*
2	prismatic	0	$-\frac{\pi}{2}$	d_2^*	0
3	revolute	r_3	0	0	θ_3^*

```
imshow(imread("./figures/annotated_RPR.png"))
```



$A_i = \mathrm{Rot}_{z,\theta_i}\mathrm{Trans}_{z,d_i}\mathrm{Trans}_{x,\alpha_i}\mathrm{Rot}_{x,\alpha_i}$

$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \alpha_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \sqrt{}0 & 1 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} c_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
imshow(imread("./figures/annotated_RPR.png"))
```

The DH parameters define the geometry of how each rigid body attaches to its parent via a joint. The parameters follow a four transformation convention:

- A — Length of the common normal line between the two *z*-axes, which is perpendicular to both axes
- α — Angle of rotation for the common normal
- d — Offset along the *z*-axis in the normal direction, from parent to child
- θ — Angle of rotation for the *x*-axis along the previous *z*-axis

Specify the parameters for the system as a matrix. Values come from .

```
clc
clear
close all
```

```
arms = [15.0071 24.9199 20 28];
% arms(1) =  distance between lower pole and main pole on ground.
% arms(2) =  lower pole length.
% arms(3) =  distance between higher pole and the main pole on top surface.
% arms(4) =  top pole length.

tops = [50 90];              % top surface dimensions
main = 42;                   % main pole length
% main = 40;                 % main pole length


min_height = 35;
top_pole_sep = 20;
sigma = rad2deg(atan(min_height/(min_height-top_pole_sep/2)));
gamma = rad2deg(atan(min_height/(min_height+top_pole_sep/2)));
```

```
%%

syms a b c alpha beta theta;

fcn = b^2 == a^2 + c^2 - 2*a*c*cos(beta);

S = solve(fcn, c);

S = simplify(S(2))
```

S = $a\cos(\beta) + \sqrt{a^2\cos(\beta)^2 - a^2 + b^2}$

```
%%

fcn2 = c == b/sin(beta)*sin(pi-beta-(asin(sin(beta)*a/b)));

S2 = solve(fcn2,c);

S2 = simplify(S2)
```

S2 =

$$b\sqrt{-\frac{a^2\sin(\beta)^2-b^2}{b^2}}+a\cos(\beta)$$

```
syms t1 d2 Xe Ye ;

dhparams = [0        pi/2        0        t1 ;
            0        -pi/2       d2       0  ;
            10         0            0        pi-t1 ];


H0_1 = DH(dhparams(1,1), dhparams(1,2), dhparams(1,3), dhparams(1,4))
```

H0_1 =

$$\begin{pmatrix} \cos(t_1) & 0 & \sin(t_1) & 0 \\ \sin(t_1) & 0 & -\cos(t_1) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
H1_2 = DH(dhparams(2,1), dhparams(2,2), dhparams(2,3), dhparams(2,4))
```

H1_2 =

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
H2_3 = DH(dhparams(3,1), dhparams(3,2), dhparams(3,3), dhparams(3,4))
```

H2_3 =

$$\begin{pmatrix} -\cos(t_1) & -\sin(t_1) & 0 & -10\cos(t_1) \\ \sin(t_1) & -\cos(t_1) & 0 & 10\sin(t_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
H0_2 = H0_1*H1_2
```

H0_2 =

$$\begin{pmatrix} \cos(t_1) & -\sin(t_1) & 0 & d_2\sin(t_1) \\ \sin(t_1) & \cos(t_1) & 0 & -d_2\cos(t_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
H0_3 = H0_1*H1_2*H2_3
```

H0_3 =

$$\begin{pmatrix} -\cos(t_1)^2 - \sin(t_1)^2 & 0 & 0 & -10\cos(t_1)^2 - 10\sin(t_1)^2 + d_2\sin(t_1) \\ 0 & -\cos(t_1)^2 - \sin(t_1)^2 & 0 & -d_2\cos(t_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
O1_x = simplify(H0_1(1,4))
```

O1_x = 0

```
O1_y = simplify(H0_1(2,4))
```

O1_y = 0

```
O2_x = simplify(H0_2(1,4))
```

O2_x = $d_2\sin(t_1)$

```
O2_y = simplify(H0_2(2,4))
```

O2_y = $-d_2\cos(t_1)$

```
O3_x = simplify(H0_3(1,4))
```

O3_x = $d_2\sin(t_1) - 10$

```
O3_y = simplify(H0_3(2,4))
```

O3_y = $-d_2\cos(t_1)$

```
xeq = Xe == O3_x
```

xeq = $\mathrm{Xe} = d_2\sin(t_1) - 10$

```
yeq = Ye == O3_y
```

yeq = $\mathrm{Ye} = -d_2\cos(t_1)$

```
S = solve([xeq yeq],[t1 d2])
```

```
S = struct with fields:
    t1: [2×1 sym]
    d2: [2×1 sym]
```

```
simplify(S.t1)
```

ans =

$$\begin{pmatrix} 2\,\mathrm{atan}\left(\frac{\mathrm{Ye} - \sqrt{\mathrm{Xe}^2 + 20\,\mathrm{Xe} + \mathrm{Ye}^2 + 100}}{\mathrm{Xe} + 10}\right) \\ 2\,\mathrm{atan}\left(\frac{\mathrm{Ye} + \sqrt{\mathrm{Xe}^2 + 20\,\mathrm{Xe} + \mathrm{Ye}^2 + 100}}{\mathrm{Xe} + 10}\right) \end{pmatrix}$$

```
simplify(S.d2)
```

ans =

$$\begin{pmatrix} -\sqrt{\mathrm{Xe}^2 + 20\,\mathrm{Xe} + \mathrm{Ye}^2 + 100} \\ \sqrt{\mathrm{Xe}^2 + 20\,\mathrm{Xe} + \mathrm{Ye}^2 + 100} \end{pmatrix}$$

```
% these set the relation of x play and the height, to reduce the equation
% to single input


Xe = 0 ;
% Xe = main*sin(deg2rad(sigma)) ;
% Xe = 10 ;
% 38.2743
% -10
% 10
% -33.282

% S=subs(S,Ye,sqrt((sqrt((main*cos(deg2rad(sigma))+10)^2+(main*sin(deg2rad(sigma)))^2))^2-Xe^2)

S=subs(S,Ye,-1/175*Xe^2+42)
```

```
S = struct with fields:
    t1: [2×1 sym]
    d2: [2×1 sym]
```

```
% Ye = 48.47;

t_1 = simplify(S.t1);
d_2 = simplify(S.d2);

t_1(2)
```

ans =

$$2\,\mathrm{atan}\left(\frac{\sqrt{\mathrm{Xe}^2 + 20\,\mathrm{Xe} + 1864} + 42}{\mathrm{Xe} + 10}\right)$$

```
d_2(2)
```

ans = $\sqrt{\mathrm{Xe}^2 + 20\ \mathrm{Xe} + 1864}$

```
t_1 = eval(t_1)
```

```
t_1 = 2×1
   -0.2337
    2.9078
```

```
d_2 = eval(d_2)
```

```
d_2 = 2×1
  -43.1741
   43.1741
```

```
t_3 = -pi + t_1
```

```
t_3 = 2×1
   -3.3753
   -0.2337
```

```
O1_x = simplify(H0_1(1,4));
O1_y = simplify(H0_1(2,4));

O2_x = simplify(H0_2(1,4));
O2_y = simplify(H0_2(2,4));

O3_x = simplify(H0_3(1,4));
O3_y = simplify(H0_3(2,4));

% t1 = deg2rad(180-104.511+90)
% t3 = deg2rad(90-(180-104.511))
% d2 = sqrt(20^2+30^2)


t1 = t_1(2)
```

```
t1 = 2.9078
```

```
d2 = d_2(2)
```

```
d2 = 43.1741
```

```
d1 = triIneq(arms(1),arms(2),(t1-pi/2))
```

```
d1 = 23.6718
```

```
d2
```

```
d2 = 43.1741
```

```
d3 = triIneq(arms(3),arms(4),pi-t1-pi/2)
```

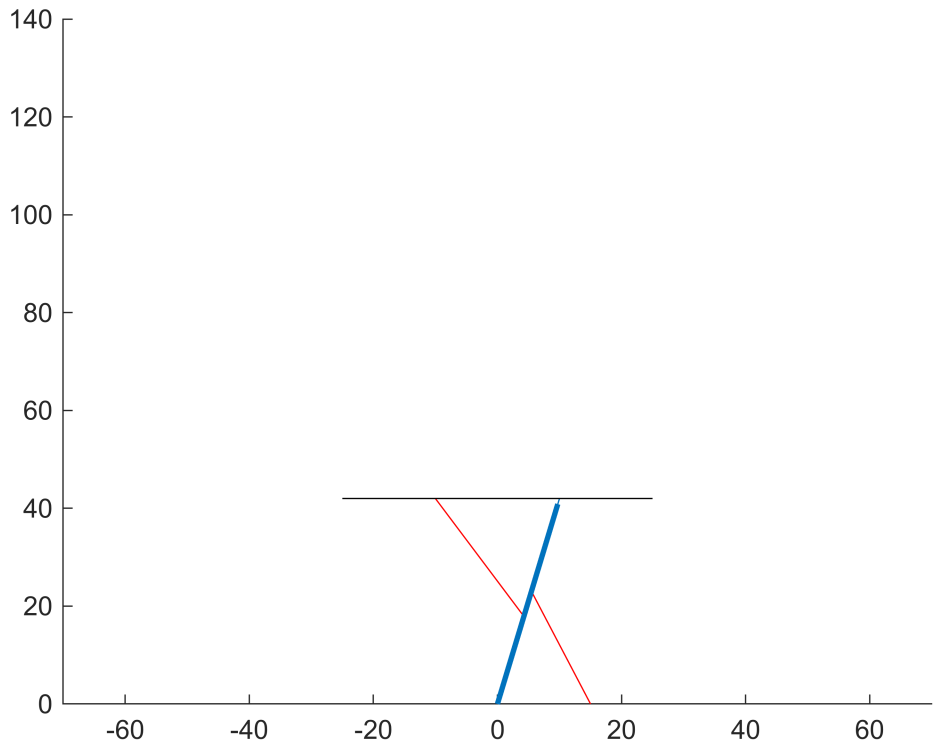
```
d3 = 24.7684
```

```
O2_x = eval(O2_x);
O2_y = eval(O2_y);

O3_x = eval(O3_x);
O3_y = eval(O3_y);

clf
link1 = line([0,O1_x],[0,O1_y]);
link2 = line([O1_x, O2_x],[O1_y, O2_y]);
link3 = line([O2_x, O3_x],[O2_y, O3_y]);
arm1 = line([arms(1),d1*cos(t1-pi/2) ],[0,d1*sin(t1-pi/2) ], 'color','red');
arm2 = line([O2_x-(arms(3)),O2_x-d3*cos(pi-t1-pi/2) ],[O2_y,O2_y+d3*sin(pi-t1-pi/2) ], 'color',
top = line([O2_x,O2_x+15],[O2_y,O2_y ], 'color','black');
top2 = line([ O2_x,O2_x-(tops(1)-(arms(3)))/2-(arms(3))],[O2_y,O2_y ], 'color','black');
middle = line([0, main*cos(t1-pi/2)],[0, main*sin(t1-pi/2)], 'LineWidth', 2);

xlim([-70 70])
ylim([0 140])
```



```
d1 = d1
```

```
d1 = 23.6718
```

```
d3 = d2-d3
```

```
d3 = 18.4056
```

```
d2 = d2-main
```

```
d2 = 1.1741
```

```
function [A] = DH(a,alpha,d,theta)

A = [cos(theta) -sin(theta)*round(cos(alpha))  sin(theta)*round(sin(alpha)) a*cos(theta);
     sin(theta)  cos(theta)*round(cos(alpha)) -cos(theta)*round(sin(alpha)) a*sin(theta);
     0           round(sin(alpha))             round(cos(alpha))            d ;
     0           0                             0                            1];
end
```

```
function [c] = triIneq(a,b,beta)
c = b*(-(a^2*sin(beta)^2 - b^2)/b^2)^(1/2) + a*cos(beta);
end
```