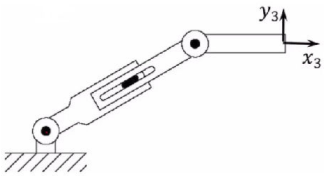
Kinematic DH Parameters for 3-link RPR arm

```
imshow(imread("./figures/RPR.png"))
```



Robot Description

- **{r, p, r} :** r for revolute, p for prismatic
- **{0, 0, L_1} :** radius lenghts
- **{Pi/2, -Pi/2, 0} :** link twists
- **{0, q_2, 0} :** link offset rotation
- **{q1, 0, q3} :** joint rotation

```
imshow(imread("./figures/DH.png"))
```

Joint	Type	r	α	d	θ
1	revolute	0	$\frac{\pi}{2}$	0	θ_1^*
2	prismatic	0	$-\frac{\pi}{2}$	d_2^*	0
3	revolute	r_3	0	0	θ_3^*

```
imshow(imread("./figures/annotated_RPR.png"))
```

$A_i = \text{Rot}_{z,\theta_i}\text{Trans}_{z,d_i}\text{Trans}_{x,\alpha_i}\text{Rot}_{x,\alpha_i}$

$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \alpha_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}c_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E = y \cdot \begin{bmatrix} x \\ y \\ s \end{bmatrix} \begin{bmatrix} e_x & e_y & e_s \end{bmatrix}^{\mathrm{T}}$$

The DH parameters define the geometry of how each rigid body attaches to its parent via a joint. The parameters follow a four transformation convention:

- A — Length of the common normal line between the two *z*-axes, which is perpendicular to both axes
- α — Angle of rotation for the common normal
- d — Offset along the *z*-axis in the normal direction, from parent to child
- θ — Angle of rotation for the *x*-axis along the previous *z*-axis

Specify the parameters for the system as a matrix. Values come from .

```
clc
clear
close all
```

```
% arms = [20 25 10 25];
arms = [15.5959 24.38 10 28];
tops = [50 90];
main = 40;
```

```
%%

syms a b c alpha beta theta;

fcn = b^2 == a^2 + c^2 - 2*a*c*cos(beta);

S = solve(fcn, c);

S = simplify(S(2))
```

S = $a\cos(\beta) + \sqrt{a^2\cos(\beta)^2 - a^2 + b^2}$

```
%%

fcn2 = c == b/sin(beta)*sin(pi-beta-(asin(sin(beta)*a/b)));

S2 = solve(fcn2,c);

S2 = simplify(S2)
```

S2 =

$$b\sqrt{-\frac{a^2\sin(\beta)^2 - b^2}{b^2}} + a\cos(\beta)$$

```
syms t1 d2 Xe Ye ;

dhparams = [0         pi/2        0       t1 ;
            0         -pi/2       d2      0  ;
            10          0            0        pi-t1 ];


H0_1 = DH(dhparams(1,1), dhparams(1,2), dhparams(1,3), dhparams(1,4))
```

H0_1 =

$$\begin{pmatrix} \cos(t_1) & 0 & \sin(t_1) & 0 \\ \sin(t_1) & 0 & -\cos(t_1) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
H1_2 = DH(dhparams(2,1), dhparams(2,2), dhparams(2,3), dhparams(2,4))
```

H1_2 =

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
H2_3 = DH(dhparams(3,1), dhparams(3,2), dhparams(3,3), dhparams(3,4))
```

H2_3 =

$$\begin{pmatrix} -\cos(t_1) & -\sin(t_1) & 0 & -10\cos(t_1) \\ \sin(t_1) & -\cos(t_1) & 0 & 10\sin(t_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
H0_2 = H0_1*H1_2
```

H0_2 =

$$\begin{pmatrix} \cos(t_1) & -\sin(t_1) & 0 & d_2\sin(t_1) \\ \sin(t_1) & \cos(t_1) & 0 & -d_2\cos(t_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
H0_3 = H0_1*H1_2*H2_3
```

H0_3 =

$$\begin{pmatrix} -\cos(t_1)^2 - \sin(t_1)^2 & 0 & 0 & -10\cos(t_1)^2 - 10\sin(t_1)^2 + d_2\sin(t_1) \\ 0 & -\cos(t_1)^2 - \sin(t_1)^2 & 0 & -d_2\cos(t_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
O1_x = simplify(H0_1(1,4))
```

O1_x = 0

```
O1_y = simplify(H0_1(2,4))
```

O1_y = 0

```
O2_x = simplify(H0_2(1,4))
```

O2_x = $d_2\sin(t_1)$

```
O2_y = simplify(H0_2(2,4))
```

O2_y = $-d_2\cos(t_1)$

```
O3_x = simplify(H0_3(1,4))
```

O3_x = $d_2\sin(t_1) - 10$

```
O3_y = simplify(H0_3(2,4))
```

O3_y = $-d_2\cos(t_1)$

```
xeq = Xe == O3_x
```

xeq = $\mathrm{Xe} = d_2\sin(t_1) - 10$

```
yeq = Ye == O3_y
```

yeq = $\mathrm{Ye} = -d_2\cos(t_1)$

```
S = solve([xeq yeq],[t1 d2])
```

```
S = struct with fields:
    t1: [2×1 sym]
    d2: [2×1 sym]
```

```
simplify(S.t1)
```

ans =

$$\begin{pmatrix} 2\,\mathrm{atan}\left(\frac{\mathrm{Ye}-\sqrt{\mathrm{Xe}^2+20\,\mathrm{Xe}+\mathrm{Ye}^2+100}}{\mathrm{Xe}+10}\right) \\ 2\,\mathrm{atan}\left(\frac{\mathrm{Ye}+\sqrt{\mathrm{Xe}^2+20\,\mathrm{Xe}+\mathrm{Ye}^2+100}}{\mathrm{Xe}+10}\right) \end{pmatrix}$$

```
simplify(S.d2)
```

ans =

$$\begin{pmatrix} -\sqrt{\mathrm{Xe}^2+20\,\mathrm{Xe}+\mathrm{Ye}^2+100} \\ \sqrt{\mathrm{Xe}^2+20\,\mathrm{Xe}+\mathrm{Ye}^2+100} \end{pmatrix}$$

```
% S=subs(S,Ye,sqrt((sqrt((22.188+10)^2+33.282^2)-2)^2-Xe^2))
S=subs(S,Ye,sqrt((sqrt((22.188+10)^2+33.282^2))^2-Xe^2))
```

```
S = struct with fields:
    t1: [2×1 sym]
    d2: [2×1 sym]
```

```
t_1 = simplify(S.t1);
d_2 = simplify(S.d2);

t_1(2)
```

ans =

$$2\,\mathrm{atan}\left(\frac{\sqrt{2.1438\mathrm{e}{+}03-\mathrm{Xe}^2}+\sqrt{20\,\mathrm{Xe}+2.2438\mathrm{e}{+}03}}{\mathrm{Xe}+10}\right)$$

```
d_2(2)
```

ans = $\sqrt{20\,\mathrm{Xe}+2.2438\mathrm{e}{+}03}$

```
Xe = -33.282 ;
% 33.282
% -10
% 10
% -33.282


t_1 = eval(t_1)
```

```
t_1 = 2×1
    0.6262
   -2.5154
```

```
d_2 = eval(d_2)
```

```
d_2 = 2×1
  -39.7255
   39.7255
```

```
t_3 = pi - t_1
```

```
t_3 = 2×1
    2.5154
    5.6570
```

```


O1_x = simplify(H0_1(1,4));
O1_y = simplify(H0_1(2,4));

O2_x = simplify(H0_2(1,4));
O2_y = simplify(H0_2(2,4));

O3_x = simplify(H0_3(1,4));
O3_y = simplify(H0_3(2,4));

% t1 = deg2rad(180-104.511+90)
% t3 = deg2rad(90-(180-104.511))
% d2 = sqrt(20^2+30^2)


t1 = t_1(2)
```

```
t1 = -2.5154
```

```
d2 = d_2(2)
```

```
d2 = 39.7255
```

```



d1 = triIneq(arms(1),arms(2),(t1-pi/2))
```

```
d1 = 11.7091
```

```
d2
```

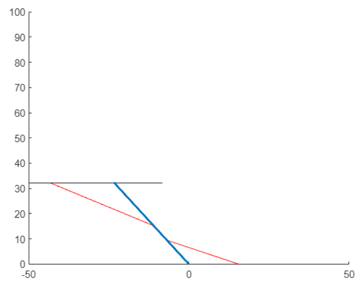
```
d2 = 39.7255
```

```
d3 = triIneq(arms(3),arms(4),pi-t1-pi/2)
```

```
d3 = 20.9413
```

```




O2_x = eval(O2_x);
O2_y = eval(O2_y);

O3_x = eval(O3_x);
O3_y = eval(O3_y);
```

```
clf
link1 = line([0,O1_x],[0,O1_y]);
link2 = line([O1_x, O2_x],[O1_y, O2_y]);
link3 = line([O2_x, O3_x],[O2_y, O3_y]);
arm1 = line([arms(1),d1*cos(t1-pi/2) ],[0,d1*sin(t1-pi/2) ], 'color','red');
arm2 = line([O2_x-20,O2_x-d3*cos(pi-t1-pi/2) ],[O2_y,O2_y+d3*sin(pi-t1-pi/2) ], 'color','red');
top = line([O2_x,O2_x+15],[O2_y,O2_y ], 'color','black');
top2 = line([ O2_x,O2_x-(tops(1)-20)/2-20],[O2_y,O2_y ], 'color','black');
middle = line([0, main*cos(t1-pi/2)],[0, main*sin(t1-pi/2)], 'LineWidth', 2);

xlim([-50 50])
ylim([0 100])
```




```
d1 = d1
```

```
d1 = 11.7091
```

```
d3 = d2-d3
```

```
d3 = 18.7842
```

```
d2 = d2-main
```

```
d2 = -0.2745
```

```
function [A] = DH(a,alpha,d,theta)

A = [cos(theta) -sin(theta)*round(cos(alpha))  sin(theta)*round(sin(alpha)) a*cos(theta);
     sin(theta)  cos(theta)*round(cos(alpha)) -cos(theta)*round(sin(alpha)) a*sin(theta);
     0           round(sin(alpha))             round(cos(alpha))             d ;
     0           0                             0                             1];
end
```

```
function [c] = triIneq(a,b,beta)
c = b*(-(a^2*sin(beta)^2 - b^2)/b^2)^(1/2) + a*cos(beta);
end
```