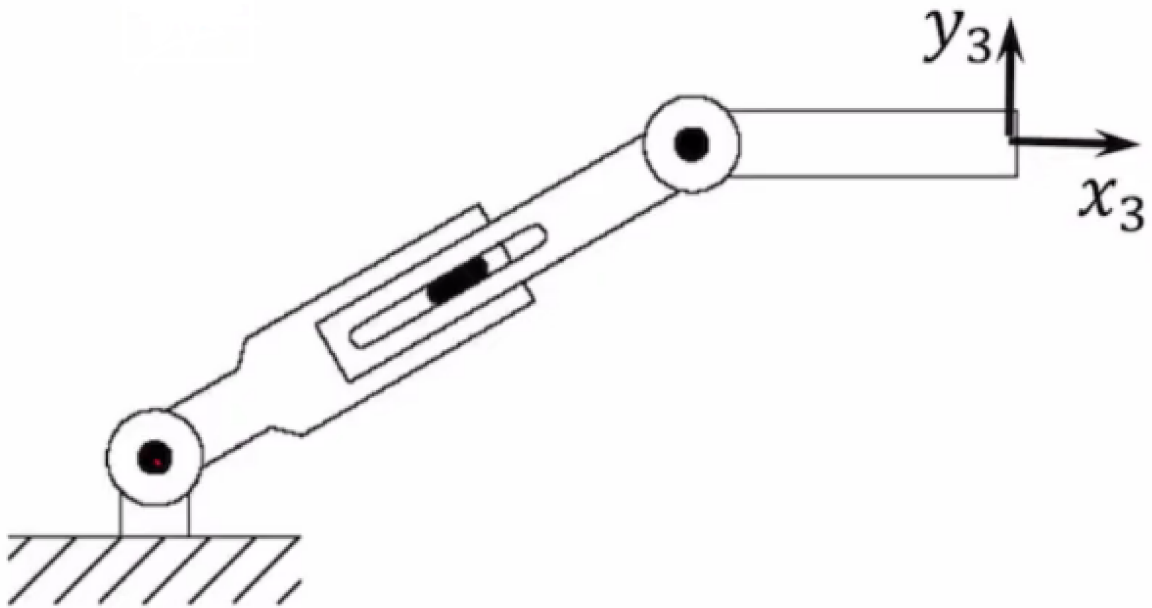


Kinematic DH Parameters for 3-link RPR arm

```
imshow(imread("./figures/RPR.png"))
```



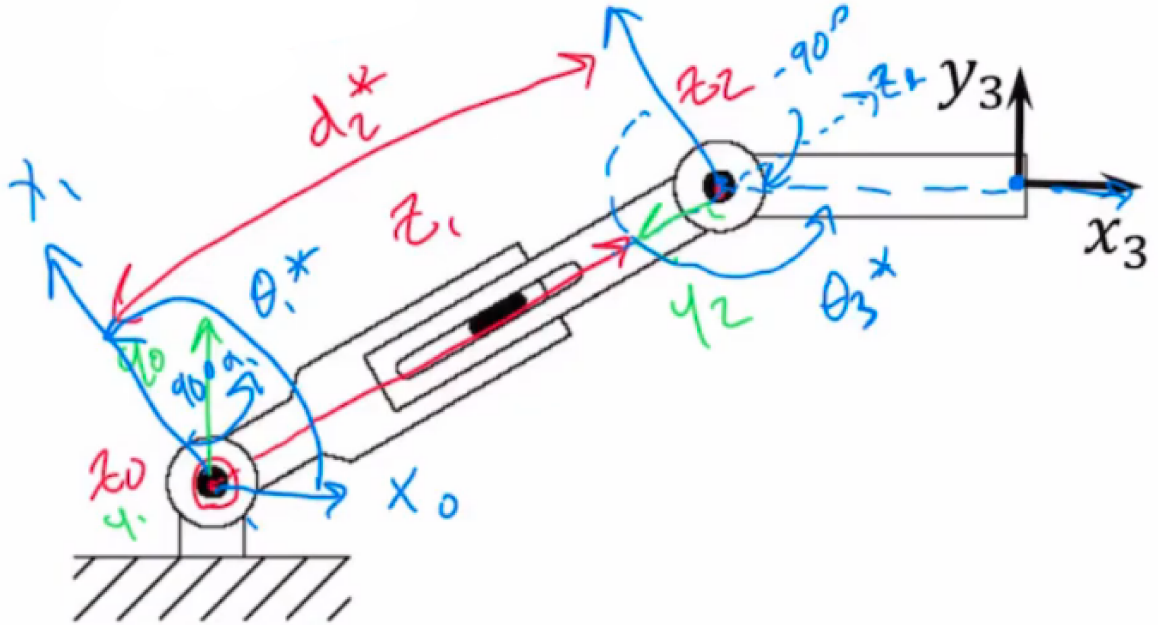
Robot Description

- $\{r, p, r\}$: r for revolute, p for prismatic
- $\{0, 0, L_1\}$: radius lengths
- $\{\pi/2, -\pi/2, 0\}$: link twists
- $\{0, q_2, 0\}$: link offset rotation
- $\{q_1, 0, q_3\}$: joint rotation

```
imshow(imread("./figures/DH.png"))
```

Joint	Type	r	α	d	θ
1	revolute	0	$\frac{\pi}{2}$	0	θ_1^*
2	prismatic	0	$-\frac{\pi}{2}$	d_2^*	0
3	revolute	r_3	0	0	θ_3^*

```
imshow(imread("./figures/annotated_RPR.png"))
```



$$A_i = \text{Rot}_{z,\theta_i} \text{Trans}_{z,d_i} \text{Trans}_{x,\alpha_i} \text{Rot}_{x,\alpha_i}$$

$$A_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \alpha_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \sqrt{0} & 1 & 0 \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i c\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
imshow(imread("./figures/annotated_RPR.png"))
```

The DH parameters define the geometry of how each rigid body attaches to its parent via a joint. The parameters follow a four transformation convention:

- A — Length of the common normal line between the two z-axes, which is perpendicular to both axes
- α — Angle of rotation for the common normal
- d — Offset along the z-axis in the normal direction, from parent to child
- θ — Angle of rotation for the x-axis along the previous z-axis

Specify the parameters for the system as a matrix. Values come from .

```
clc
clear
close all
```

```
arms = [15.0071 24.9199 10 28];
% arms(1) = distance between lower pole and main pole on ground.
% arms(2) = lower pole length.
% arms(3) = distance between higher pole and the main pole on top surface.
% arms(4) = top pole length.

tops = [50 90];           % top surface dimensions
main = 42;                 % main pole length
% main = 40;               % main pole length

min_height = 35;
top_pole_sep = 20;
sigma = rad2deg(atan(min_height/(min_height-top_pole_sep/2)));
gamma = rad2deg(atan(min_height/(min_height+top_pole_sep/2)));
```

```
%%

syms a b c alpha beta theta;

fcn = b^2 == a^2 + c^2 - 2*a*c*cos(beta);

S = solve(fcn, c);

S = simplify(S(2))
```

$$S = a \cos(\beta) + \sqrt{a^2 \cos(\beta)^2 - a^2 + b^2}$$

```
%%

fcn2 = c == b/sin(beta)*sin(pi-beta-(asin(sin(beta)*a/b)));

S2 = solve(fcn2,c);

S2 = simplify(S2)
```

S2 =

$$b \sqrt{-\frac{a^2 \sin(\beta)^2 - b^2}{b^2}} + a \cos(\beta)$$

```
syms t1 d2 Xe Ye ;
```

```
dhparams = [0      pi/2      0      t1 ;
             0      -pi/2     d2      0 ;
             10      0        0      pi-t1 ];
```

```
H0_1 = DH(dhparams(1,1), dhparams(1,2), dhparams(1,3), dhparams(1,4))
```

```
H0_1 =
```

$$\begin{pmatrix} \cos(t_1) & 0 & \sin(t_1) & 0 \\ \sin(t_1) & 0 & -\cos(t_1) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
H1_2 = DH(dhparams(2,1), dhparams(2,2), dhparams(2,3), dhparams(2,4))
```

```
H1_2 =
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
H2_3 = DH(dhparams(3,1), dhparams(3,2), dhparams(3,3), dhparams(3,4))
```

```
H2_3 =
```

$$\begin{pmatrix} -\cos(t_1) & -\sin(t_1) & 0 & -10 \cos(t_1) \\ \sin(t_1) & -\cos(t_1) & 0 & 10 \sin(t_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
H0_2 = H0_1*H1_2
```

```
H0_2 =
```

$$\begin{pmatrix} \cos(t_1) & -\sin(t_1) & 0 & d_2 \sin(t_1) \\ \sin(t_1) & \cos(t_1) & 0 & -d_2 \cos(t_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
H0_3 = H0_1*H1_2*H2_3
```

```
H0_3 =
```

$$\begin{pmatrix} -\cos(t_1)^2 - \sin(t_1)^2 & 0 & 0 & -10 \cos(t_1)^2 - 10 \sin(t_1)^2 + d_2 \sin(t_1) \\ 0 & -\cos(t_1)^2 - \sin(t_1)^2 & 0 & -d_2 \cos(t_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
O1_x = simplify(H0_1(1,4))
```

```
O1_x = 0
```

```
O1_y = simplify(H0_1(2,4))
```

```
O1_y = 0
```

```
O2_x = simplify(H0_2(1,4))
```

```
O2_x = d2 sin(t1)
```

```
O2_y = simplify(H0_2(2,4))
```

```
O2_y = -d2 cos(t1)
```

```
O3_x = simplify(H0_3(1,4))
```

```
O3_x = d2 sin(t1) - 10
```

```
O3_y = simplify(H0_3(2,4))
```

```
O3_y = -d2 cos(t1)
```

```
xeq = Xe == O3_x
```

```
xeq = Xe = d2 sin(t1) - 10
```

```
yeq = Ye == O3_y
```

```
yeq = Ye = -d2 cos(t1)
```

```
S = solve([xeq yeq],[t1 d2])
```

```
S = struct with fields:
```

```
t1: [2x1 sym]
```

```
d2: [2x1 sym]
```

```
simplify(S.t1)
```

ans =

$$\begin{pmatrix} 2 \operatorname{atan}\left(\frac{Y_e - \sqrt{X_e^2 + 20 X_e + Y_e^2 + 100}}{X_e + 10}\right) \\ 2 \operatorname{atan}\left(\frac{Y_e + \sqrt{X_e^2 + 20 X_e + Y_e^2 + 100}}{X_e + 10}\right) \end{pmatrix}$$

```
simplify(S.d2)
```

ans =

$$\begin{pmatrix} -\sqrt{X_e^2 + 20 X_e + Y_e^2 + 100} \\ \sqrt{X_e^2 + 20 X_e + Y_e^2 + 100} \end{pmatrix}$$

```
% these set the relation of x play and the height, to reduce the equation  
% to single input
```

```
Xe = 35;  
% Xe = main*sin(deg2rad(sigma)) ;  
% Xe = 10 ;  
% 38.2743  
% -10  
% 10  
% -33.282
```

```
% S=subs(S,Ye,sqrt((sqrt((main*cos(deg2rad(sigma))+10)^2+(main*sin(deg2rad(sigma)))^2))-Xe^2,
```

```
S=subs(S,Ye,-1/175*Xe^2+42)
```

```
S = struct with fields:  
t1: [2x1 sym]  
d2: [2x1 sym]
```

```
% Ye = 48.47;
```

```
t_1 = simplify(S.t1);  
d_2 = simplify(S.d2);
```

```
t_1(2)
```

ans =

$$2 \operatorname{atan}\left(\frac{\sqrt{X_e^2 + 20 X_e + 1325 + 35}}{X_e + 10}\right)$$

```
d_2(2)
```

$$\text{ans} = \sqrt{Xe^2 + 20 Xe + 1325}$$

```
t_1 = eval(t_1)
```

```
t_1 = 2×1
    -0.9098
     2.2318
```

```
d_2 = eval(d_2)
```

```
d_2 = 2×1
    -57.0088
     57.0088
```

```
t_3 = pi - t_1
```

```
t_3 = 2×1
     4.0513
     0.9098
```

```
O1_x = simplify(H0_1(1,4));
O1_y = simplify(H0_1(2,4));

O2_x = simplify(H0_2(1,4));
O2_y = simplify(H0_2(2,4));

O3_x = simplify(H0_3(1,4));
O3_y = simplify(H0_3(2,4));

% t1 = deg2rad(180-104.511+90)
% t3 = deg2rad(90-(180-104.511))
% d2 = sqrt(20^2+30^2)
```

```
t1 = t_1(2)
```

```
t1 = 2.2318
```

```
d2 = d_2(2)
```

```
d2 = 57.0088
```

```
d1 = triIneq(arms(1),arms(2),(t1-pi/2))
```

```
d1 = 35.0000
```

```
d2
```

```
d2 = 57.0088
```

```
d3 = triIneq(arms(3),arms(4),pi-t1-pi/2)
```

```
d3 = 35.2122
```

```
O2_x = eval(O2_x);
```

```
O2_y = eval(O2_y);
```

```
O3_x = eval(O3_x);
```

```
O3_y = eval(O3_y);
```

```
clf
```

```
link1 = line([0,O1_x],[0,O1_y]);
```

```
link2 = line([O1_x, O2_x],[O1_y, O2_y]);
```

```
link3 = line([O2_x, O3_x],[O2_y, O3_y]);
```

```
arm1 = line([arms(1),d1*cos(t1-pi/2) ],[0,d1*sin(t1-pi/2) ], 'color','red');
```

```
arm2 = line([O2_x-20,O2_x-d3*cos(pi-t1-pi/2) ],[O2_y,O2_y+d3*sin(pi-t1-pi/2) ], 'color','red');
```

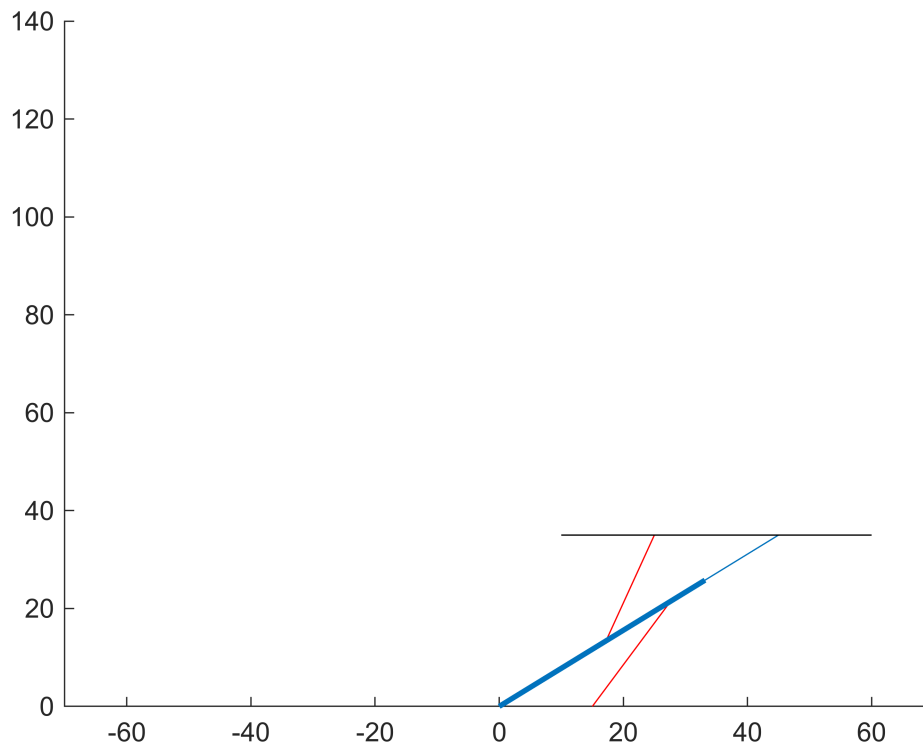
```
top = line([O2_x,O2_x+15],[O2_y,O2_y ], 'color','black');
```

```
top2 = line([ O2_x,O2_x-(tops(1)-20)/2-20],[O2_y,O2_y ], 'color','black');
```

```
middle = line([0, main*cos(t1-pi/2)],[0, main*sin(t1-pi/2)], 'LineWidth', 2);
```

```
xlim([-70 70])
```

```
ylim([0 140])
```




```
d1 = d1
```

```
d1 = 35.0000
```

```
d3 = d2-d3
```

```
d3 = 21.7966
```

```
d2 = d2-main
```

```
d2 = 15.0088
```

```
function [A] = DH(a,alpha,d,theta)
```

```
A = [cos(theta) -sin(theta)*round(cos(alpha)) sin(theta)*round(sin(alpha)) a*cos(theta);  
      sin(theta) cos(theta)*round(cos(alpha)) -cos(theta)*round(sin(alpha)) a*sin(theta);  
      0          round(sin(alpha))          round(cos(alpha))          d ;  
      0          0                          0                      1];
```

```
end
```

```
function [c] = triIneq(a,b,beta)
```

```
c = b*(-(a^2*sin(beta)^2 - b^2)/b^2)^(1/2) + a*cos(beta);
```

```
end
```