1. **[7.5 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE. Each answer should be at most a few sentences.

(a) TRUE FALSE — System is a set of entities that interact in a cohesive manner to accomplish a specific goal

(b) TRUE FALSE — Model is an abstract representation of a system, concentrating on essentials and ignoring the details.

(c) TRUE FALSE — Stochastic model is used for a system whose at least one state variable follows a random distribution.

(d) TRUE FALSE — A discrete RV takes on a countably number of possible outcomes.

(e) TRUE FALSE — A computer simulation is a computer program that mimics the behavior of a physical system.

(f) TRUE FALSE — A system consists of the components: entities, attributes, activities, events and state variables.

(g) TRUE FALSE — The sample (experimental) definition of probability is given by

$$\Pr(\text{outcome}) = \lim_{n\to\infty} \frac{\text{Number of observed outcomes}}{n \text{ repetitions of experiment}}.$$

(h) TRUE FALSE — In the concepts of discrete-event simulation, the event list is a a list of event notices for future events, ordered by time of occurrence, a.k.a. the future event list (FEL).

(i) TRUE FALSE — A histogram is a bar graph showing the probability of samples measured from a system.

(j) TRUE FALSE — The lognormal distribution is commonly used to model the lives of units whose failure modes are of a fatigue-stress nature.

(k) TRUE FALSE — A RV that is modeled with the Weibull distribution is a discrete random variable (RV). Because weibull distribution is a continuous distribution.

(l) TRUE FALSE — Simulations are classified into two main groups: deterministic and stochastic simulations.

(m) TRUE FALSE — The variation of a random variable decreases as its variance decreases. Because the variance of the sample mean is inversely proportional to the sample size.

(m) TRUE FALSE — The main goal of pseudo random number generation is to produce a sequence of numbers in [0,1] that simulates, or imitates, the ideal properties of random numbers (RN).

(o) TRUE FALSE — The random number sequence {8, 7, 17, 3, 6, 8, 7, 17, 3, 6, 8, 7, 17, 3, 6} is generated by a RNG whose period is 8.
Because period of the given sequence is 5

Scanned by TapScanner

2. **[12.5 points]** In order to simulate a system, it is required to determine its components whose definitions are given as

$$\begin{aligned}
\text{Entities} &\triangleq \{\text{entity}_1, \text{entity}_2, \ldots, \text{entity}_K\} \\
\text{Attributes} &\triangleq \{\text{attribute}_1, \text{attribute}_2, \ldots, \text{attribute}_L\} \\
\text{Activities} &\triangleq \{\text{activity}_1, \text{activity}_2, \ldots, \text{activity}_M\} \\
\text{Events} &\triangleq \{\text{event}_1, \text{event}_2, \ldots, \text{event}_N\} \\
\text{State Variables} &\triangleq \{\text{variable}_1, \text{variable}_2, \ldots, \text{variable}_T\}
\end{aligned}$$

(a) Give names to the entities, attributes, activities, events and state variables for system of "Call Center". Briefly explain the effect of the values of K, L, M, N, T numbers on the simulation complexity.

Entities = Customer, personnel, manager

Attributes of a customer = angry, has a problem, impatient

Attributes of a personnel = patient, problem solver, constructive, diction well

Attributes of a manager = fixer, educated, multilingual

Activities = solving the problem, answering calls

Events = customer calls the call center, customer waiting on the phone

State Variable = number of customer waiting for solution, number of busy personnel

System complexity increases if k,L,M,N,T increases. But the accuracy of the system also increases.

(b) Which questions about the system can you answer with simulations by using the definitions you give in option (a). You should write at least 5 questions. Your questions need to match the names you defined above.

1.) The number of people whose problem is solved in one day?

2.) What is the waiting time of the customer on the average phone?

3.) What is the average idle time of the personnel?

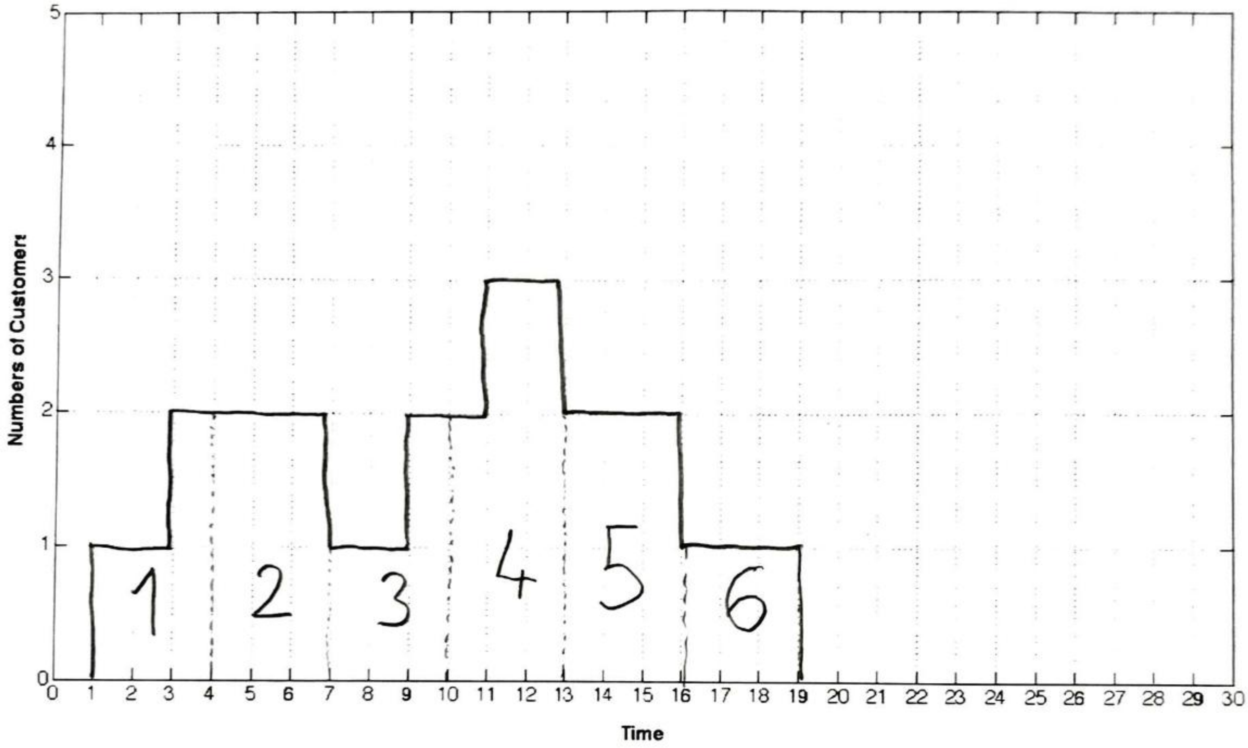
4.) What is the average service time for a customer?

5.) What is the probability that the customers will be happy after the service?

Scanned by TapScanner

3. **[15 points]** Assume a one-teller bank with customer arrivals at times 1, 3, 4, 9, 10 and 11. Each customer takes 3 time units to process. The following questions will be answered by using your play obtained in option (a) below.

(a) Draw a plot that shows number of customers in the bank on the Y-axis. The X-axis shows time going from 0 to 30.



(b) At what time does the last customer leave?

19. minute

(c) What is the maximum waiting time?

5 minute

(d) What is the maximum number of customers waiting?

2 customer

(e) What was the average service time?

$$\frac{\text{Total service time}}{\text{total number of customer}} = \frac{18}{6} = 3 \text{ minute}$$

Scanned by TapScanner

4. **[15 points]** Perform a hand simulation of the scenario developed in our class. A modified version of the inter-arrival and service times are presented in the following table. Simulation starts at time $t = 0$ and the first entity arriving at time $t = 5$ minutes. Simulation stops at $t = 65$ minutes.

(a) Complete the discrete event hand simulation table similar to the one presented in the textbook.

Customer ID	Inter-arrival Time	Service Time	Time Service Begins (clock)	Waiting Time in Queue (minute)	Time Service Ends (clock)	Time Customer spends in system (minute)	Idle time of server (minute)
1	0	5	0	0	5	5	0
2	7	1	7	0	8	1	2
3	3	3	10	0	13	3	2
4	1	7	11	2	20	9	0
5	2	4	13	7	24	11	0
6	2	1	15	9	25	10	0
7	5	1	20	5	26	6	0
8	3	1	23	3	27	4	0
9	4	4	27	0	31	4	0
10	2	4	29	2	35	6	0
11	3	2	32	3	37	5	0
12	4	4	36	1	41	5	0
13	4	4	40	1	45	5	0
14	12	2	52	0	54	2	7
15	8	5	60	0	65	5	6

(b) Compute the average waiting time in the queue? Compute the probability that the customer waits in queue?

$$\text{Average waiting time in the queue} = \frac{\text{total waiting time in queue}}{\text{Number of customer}} = \frac{33}{15} = 2.2 \text{ min}$$

$$\text{Probability that the customer waits in queue} = \frac{\text{Number of customer who wait}}{\text{Number of customer}} = \frac{9}{15} = 0.6$$

(c) Compute the probability that the server is idle?

$$\text{Probability that the server is idle} = \frac{\text{total idle time of server}}{\text{total run time of simulation}} = \frac{17}{65} = 0.26$$

(d) Compute the average service time per customer?

$$\text{Average service time per customer} = \frac{\text{total service time}}{\text{total number of customer}} = \frac{48}{15} = 3.2 \text{ min}$$

Scanned by TapScanner

5. **[15 points]** For the safety of YTÜ students, a speed trap (radar) were put on Yildiz Street, Davutpaşa. The speed trap recorded the speeds of vehicles as

26.5	74.1	80.3	72.3	87.7	60.9	98.3	52.1	40.8
74.3	13.4	92.5	79.2	74.1	84.1	42.6	57.3	65.4
66.5	72.5	55.2	33.2	79.2	33.9	18.4	83.6	10.4
56.3	82.5	99.8	49.9	19.6	68.3	45.4	59.9	16.8

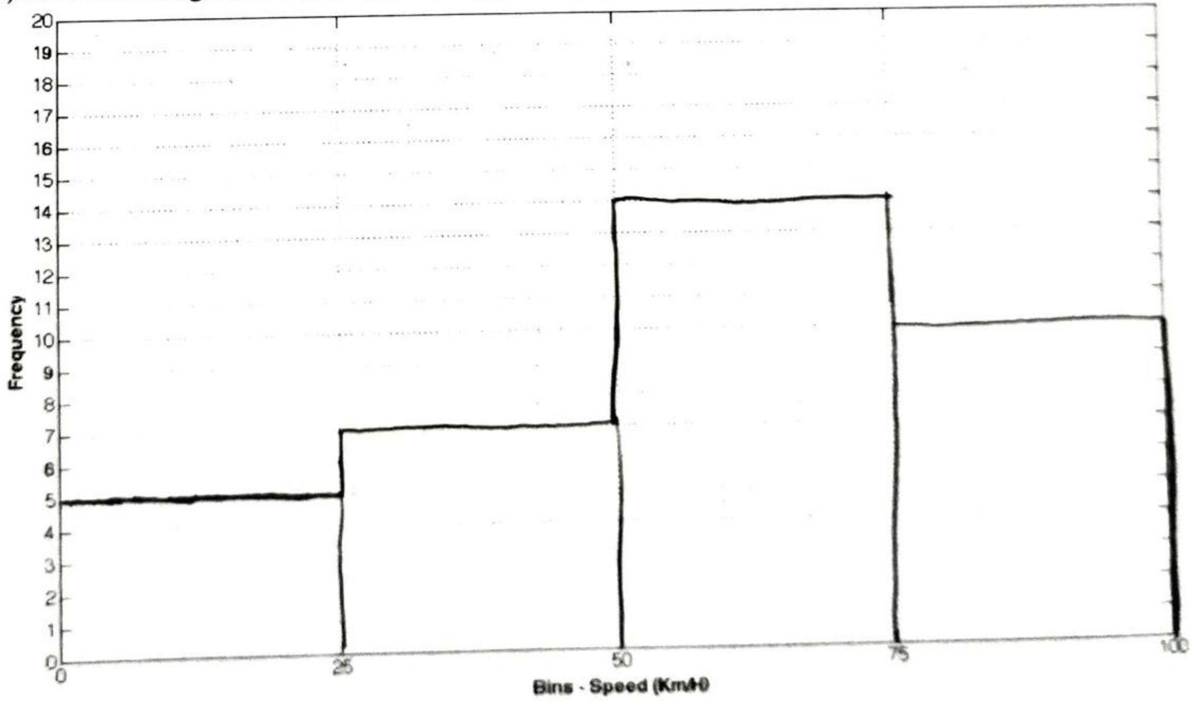
a) Find mean and variance for the speed of these vehicles.

$$\text{mean} = \bar{x} = \frac{\Sigma x}{n} = \frac{2127,3}{36} = 59.091$$

$$\text{variance} = s^2 = \frac{\Sigma (x_i - \bar{x})^2}{n-1} = \frac{22164,16}{35} = 633,26$$

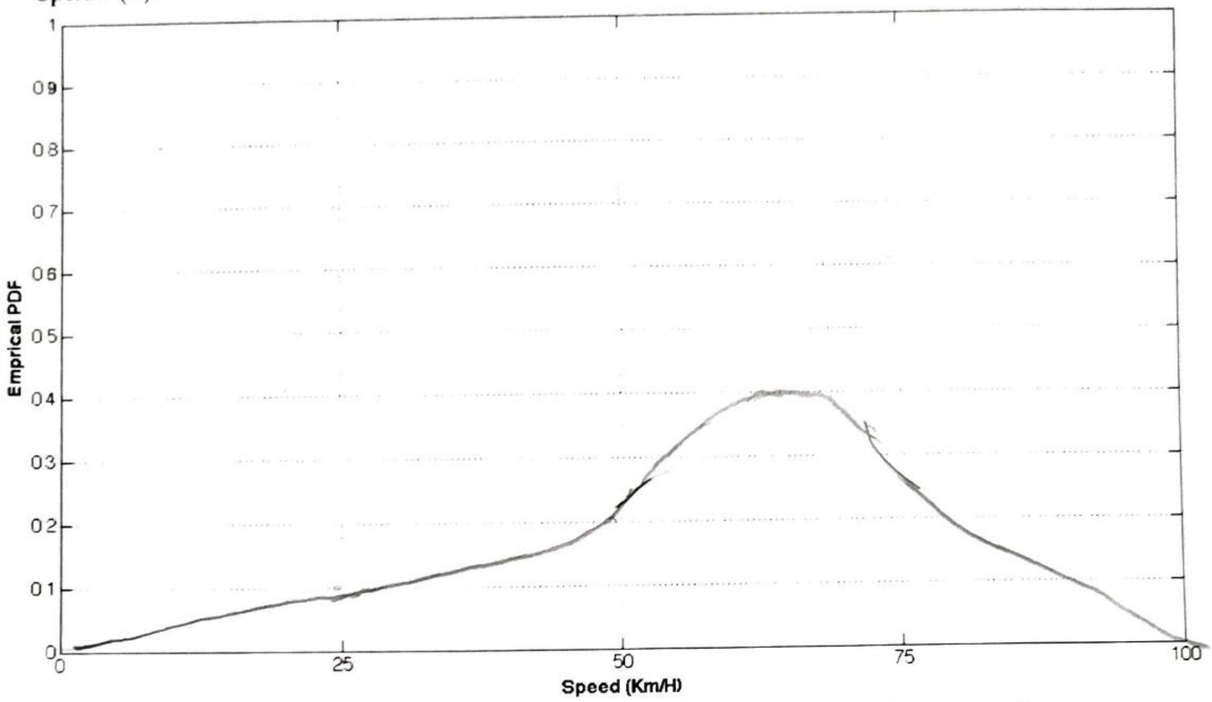
$$\text{standart deviation} = s = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n-1}} \cong 25,15$$

b) Find the histogram for [0,25), [25,50), [50,75), [75,100) ranges of speeds ($\Delta = 25$).



Scanned by TapScanner

c) Find the empirical PDF by using the normalized histogram obtained from the histogram in option (b).



6. **[15 points]** A set of 3 observations $\{0.4, 0.7, 0.9\}$ is collected from a continuous random variable whose distribution follows thee PDF

$$f_X(x) = \begin{cases} \theta x^{\theta-1}, & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Estimate the parameter θ using "method of moments" technique.

$$\mu_1 = E(x) = \int_0^1 x \cdot f(x)\,dx = \int_0^1 x \cdot \theta x^{\theta-1}\,dx = \int_0^1 \theta x^{\theta}\,dx$$

$$= \frac{\theta x^{\theta+1}}{\theta+1}\Bigg|_{x=0}^{x=1} = \frac{\theta}{\theta+1}$$

$$m_1 = \bar{x} = \frac{0.4 + 0.7 + 0.9}{3} = \frac{2}{3}$$

$$\frac{\theta}{\theta+1} = \frac{2}{3} \Rightarrow 3\theta = 2\theta + 2 \Rightarrow \theta = 2$$

Scanned by TapScanner

7. **[20 points]** Let X be a random variable that takes on the value of the number of failed servers for a given period of observation. Over many observation periods, it is obtained that the probability of that 0 servers fail in a given period of observation is 0.5, the probability of that 1 server fails is 0.2, and the probability of that 2 servers fail is 0.3.

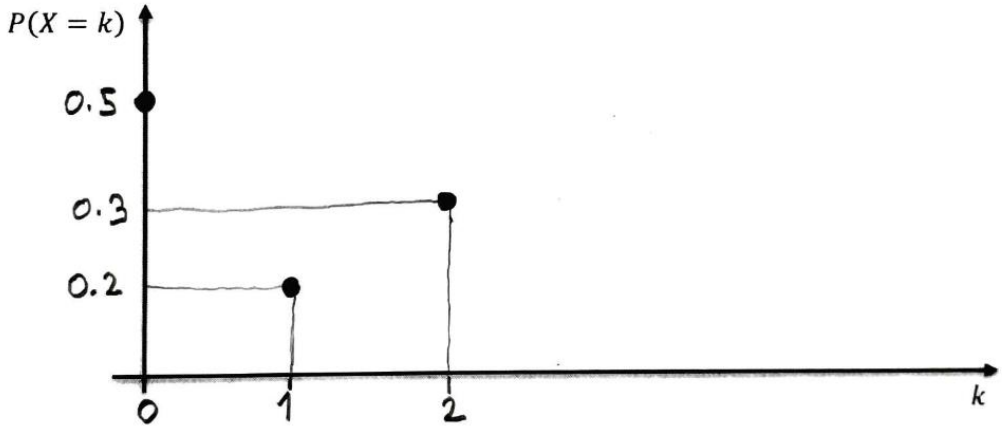
(a) What is the mean value of X?

$$E[x] = \mu_x = (0.5).(0) + (0.2).(1) + (0.3).(2) = 0.8$$

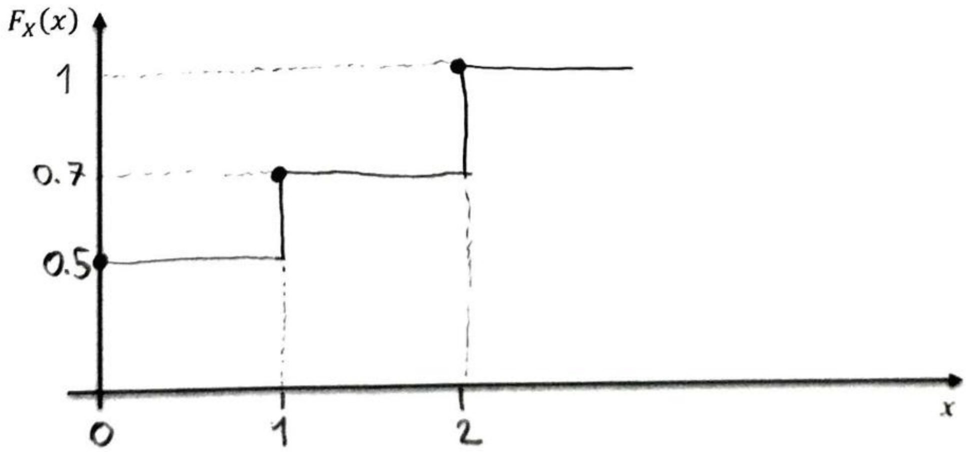
(b) What is the variance of X?

$$\begin{aligned} Var[x] = E[x^2] - E[x]^2 &= ((0.5).0^2 + (0.2).1^2 + (0.3).2^2) - (0.8)^2 \\ &= 0.2 + 1.2 - 0.64 \\ &= 0.76 \end{aligned}$$

(c) Plot the PMF of X.



(d) Plot the CDF of X.



Scanned by TapScanner