Recursividade

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International

Service

Association

International

Service

Association

VISA

International

Service

Association

International

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Association

VISA

International

Service

International

Service

Association

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Service

Association

International

Service

Association

ZERO
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caso **recursivo**

caso **base**

caso **recursivo**

(sub-caso ou caso anterior)

---

caso **base**

caso **recursivo**

(sub-caso ou caso anterior)

---

caso **base**

(caso atômico ou inicial)

caso **recursivo**

(o que fazer a cada nível da recursão)

---

caso **base**

caso **recursivo**

(o que fazer a cada nível da recursão)

---

caso **base**

(o que fazer ao atingir o caso base)

cálculo **fatorial**

cálculo **fatorial**

- 5! = 5 * 4 * 3 * 2 * 1

cálculo fatorial

- 5! = 5 * 4 * 3 * 2 * 1
- 4! = 4 * 3 * 2 * 1

cálculo fatorial

- 5! = 5 * 4 * 3 * 2 * 1
- 4! = 4 * 3 * 2 * 1
- 3! = 3 * 2 * 1

cálculo **fatorial**

- 5! = 5 * 4 * 3 * 2 * 1
- 4! = 4 * 3 * 2 * 1
- 3! = 3 * 2 * 1
- 2! = 2 * 1

cálculo fatorial

- 5! = 5 * 4 * 3 * 2 * 1
- 4! = 4 * 3 * 2 * 1
- 3! = 3 * 2 * 1
- 2! = 2 * 1
- 1! = 1

cálculo fatorial

- 5! = 5 * 4 * 3 * 2 * 1
- 4! = 4 * 3 * 2 * 1
- 3! = 3 * 2 * 1
- 2! = 2 * 1
- 1! = 1
- 0! = 1

cálculo fatorial

```
int fat_iter(int n)
{
   int fat = 1;
   while (n > 1) { fat *= n; n—; }
   return fat;
}
```

- 5! = 5 * 4 * 3 * 2 * 1
- 4! = 4 * 3 * 2 * 1
- 3! = 3 * 2 * 1
- 2! = 2 * 1
- 1! = 1
- 0! = 1

cálculo fatorial

cálculo fatorial

- 5! = 5 * 4 * 3 * 2 * 1

cálculo fatorial

- 5! = 5 * 4 * 3 * 2 * 1
- 4! = 4 * 3 * 2 * 1

cálculo fatorial

- 5! = 5 * 4 * 3 * 2 * 1
- 4! = 4 * 3 * 2 * 1

- 5! = 5 * 4!

cálculo fatorial

- 5! = 5 * 4 * 3 * 2 * 1
- 4! = 4 * 3 * 2 * 1

- 5! = 5 * 4!

- 4! = 4 * 3 * 2 * 1

cálculo fatorial

- 5! = 5 * 4 * 3 * 2 * 1
- 4! = 4 * 3 * 2 * 1

- 5! = 5 * 4!

- 4! = 4 * 3 * 2 * 1
- 3! = 3 * 2 * 1

cálculo fatorial

- 5! = 5 * 4 * 3 * 2 * 1
- 4! = 4 * 3 * 2 * 1

- 5! = 5 * 4!

- 4! = 4 * 3 * 2 * 1
- 3! = 3 * 2 * 1

- 4! = 4 * 3!

cálculo fatorial

caso recursivo

caso base

cálculo fatorial

caso recursivo

(sub-caso ou caso anterior)

caso base

cálculo fatorial

caso recursivo

$n! = n * (n-1)!$

---

caso base

cálculo **fatorial**

caso **recursivo**

n! = n * (n-1)!

caso **base**

(caso atômico ou inicial)

cálculo **fatorial**

caso **recursivo**

$n! = n * (n-1)!$

---

caso **base**

$0! = 1! = 1$

cálculo fatorial

cálculo fatorial

```
int fat_rec(int n)
{
   if (n <= 1) return 1;
   return n * fat_rec(n-1);
}
```

cálculo fatorial

```
int fat_iter(int n)
{
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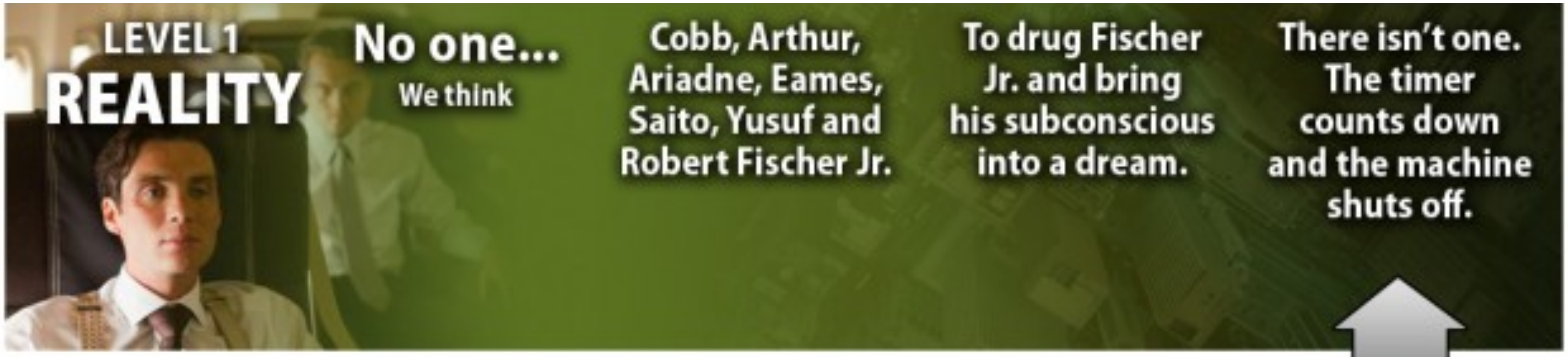


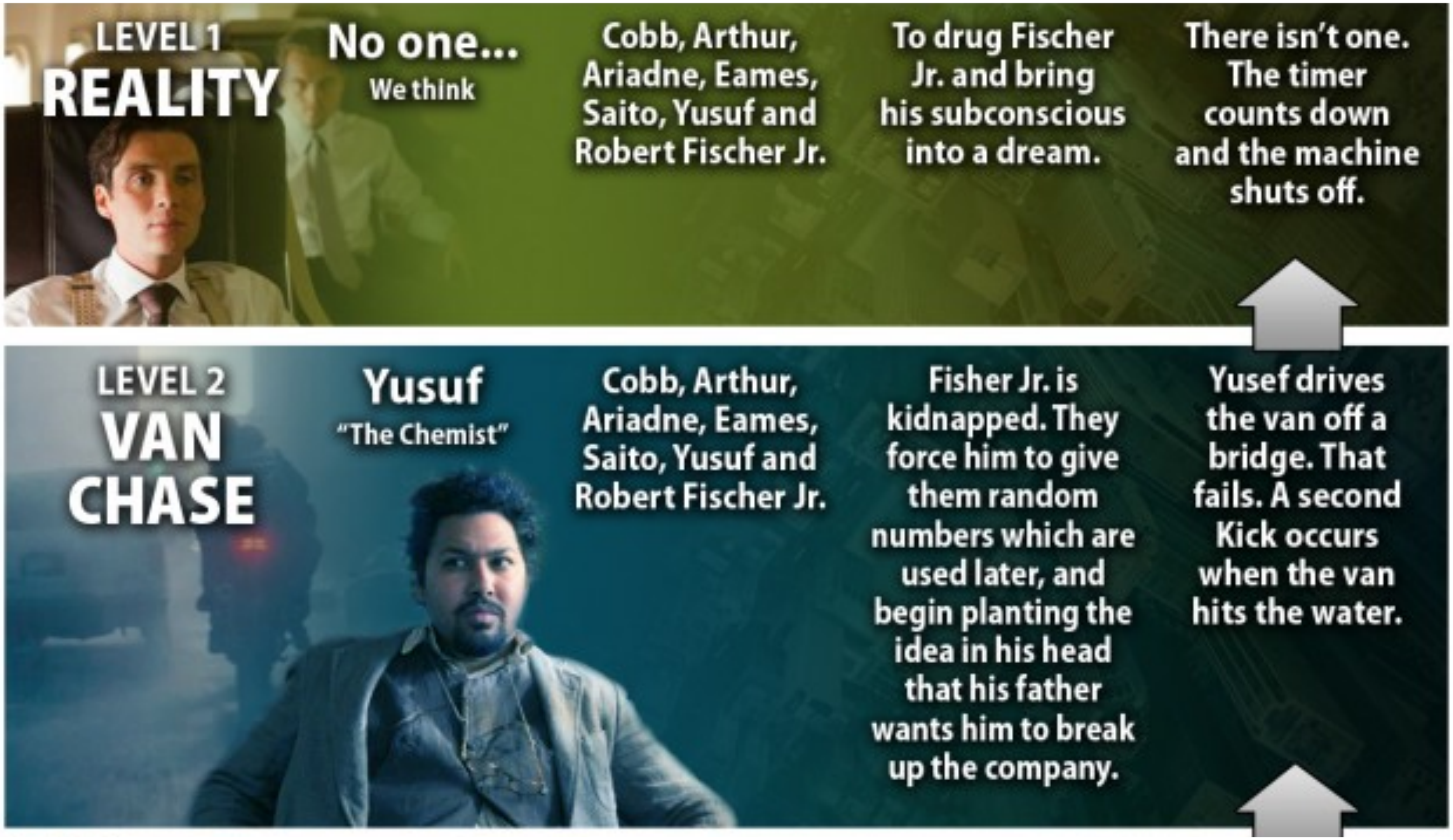
LEVEL 1
REALITY

No one...
We think

Cobb, Arthur, Ariadne, Eames, Saito, Yusuf and Robert Fischer Jr.

To drug Fischer Jr. and bring his subconscious into a dream.

There isn't one. The timer counts down and the machine shuts off.



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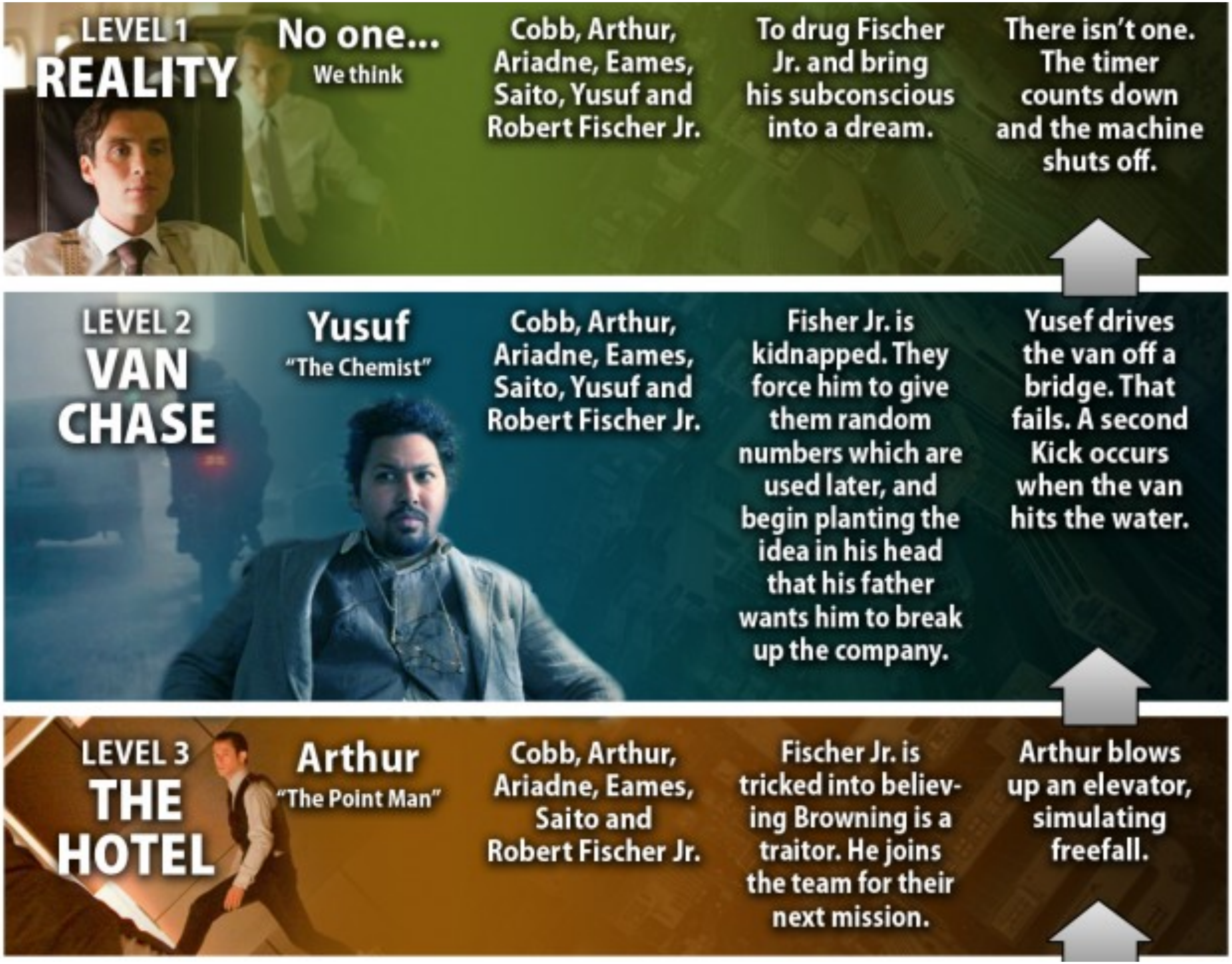
LEVEL 2 VAN CHASE	Yusuf "The Chemist"	Cobb, Arthur, Ariadne, Eames, Saito, Yusuf and Robert Fischer Jr.	Fisher Jr. is kidnapped. They force him to give them random numbers which are used later, and begin planting the idea in his head that his father wants him to break up the company.	Yusef drives the van off a bridge. That fails. A second Kick occurs when the van hits the water.



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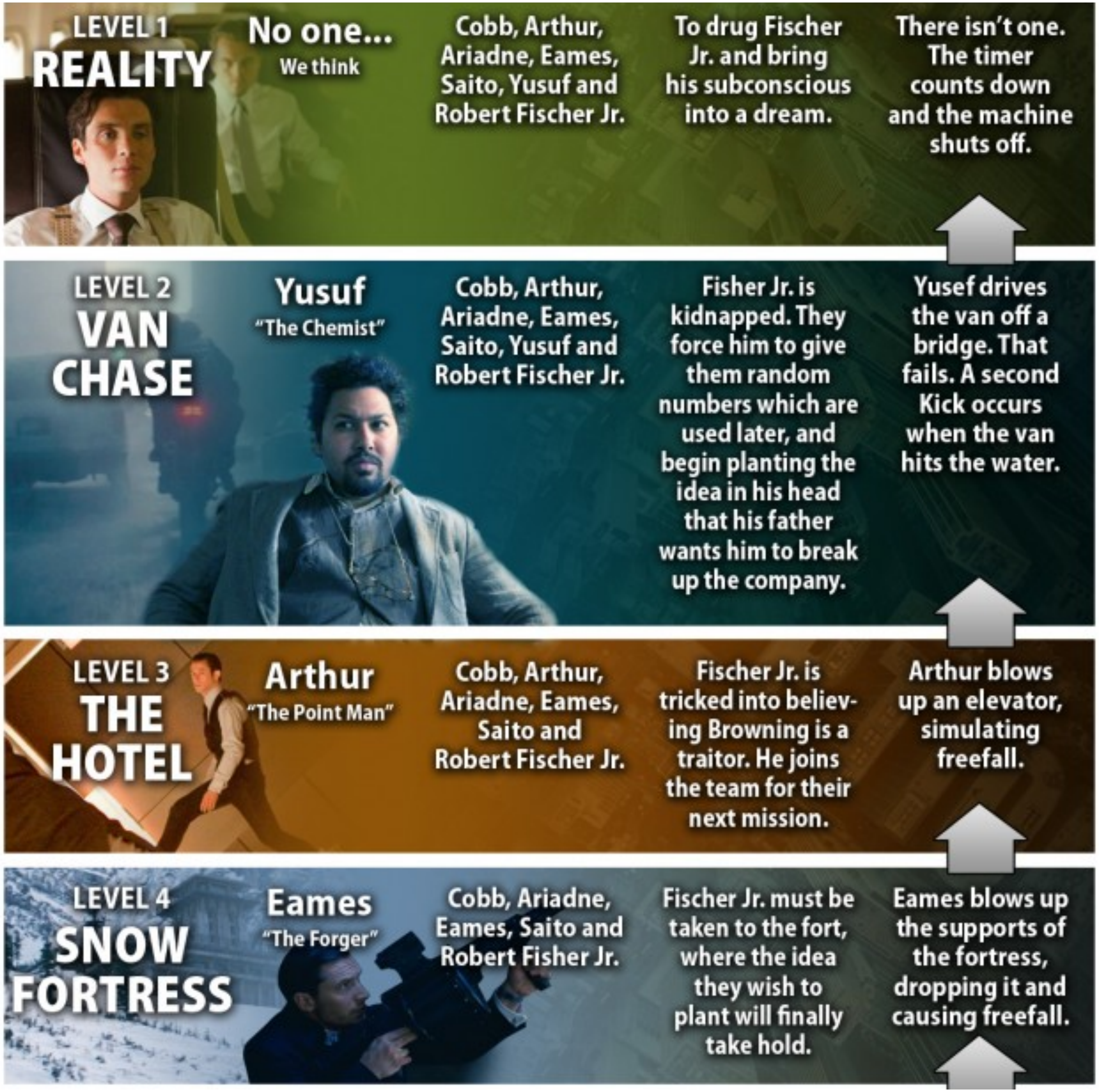

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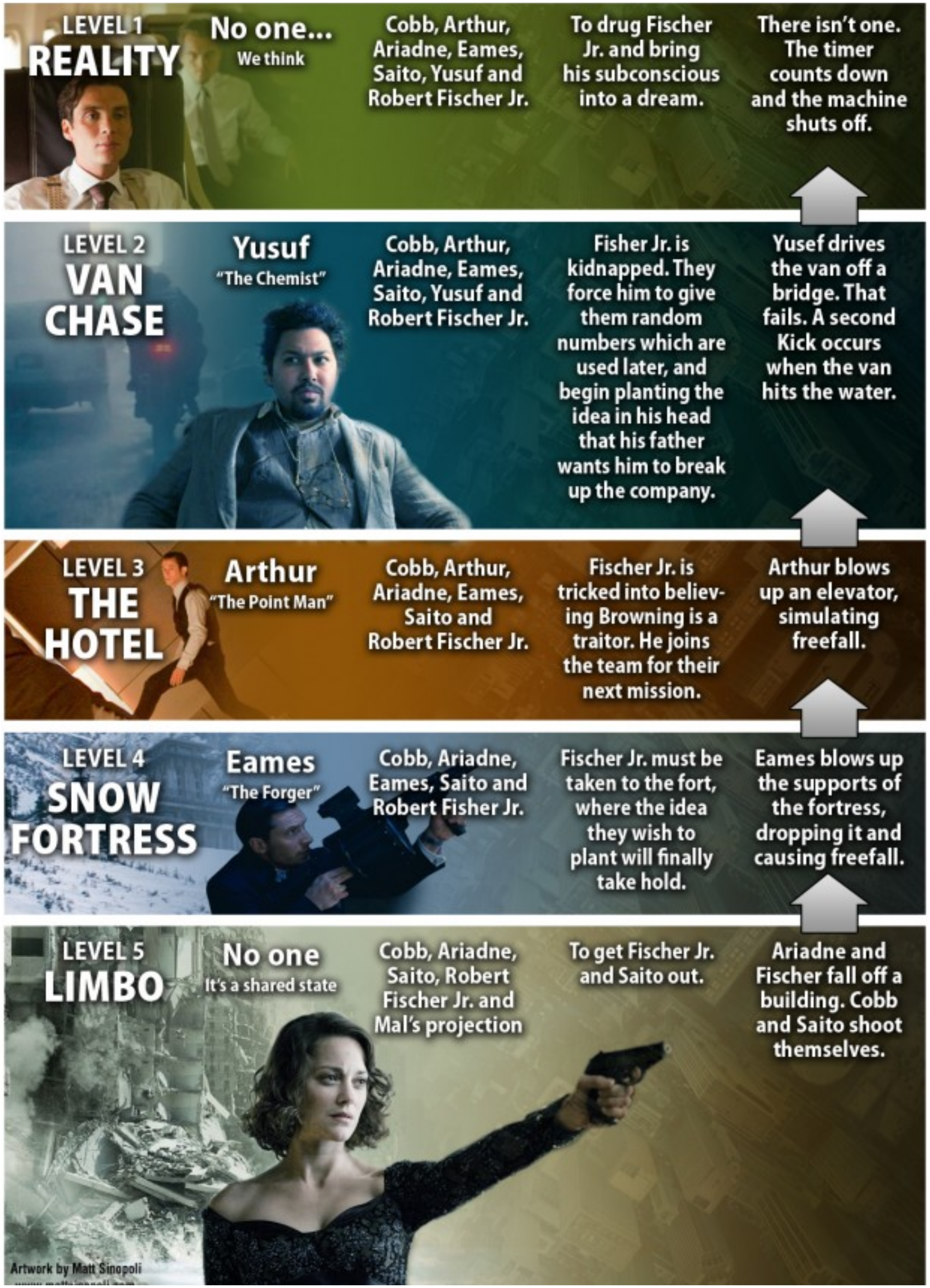

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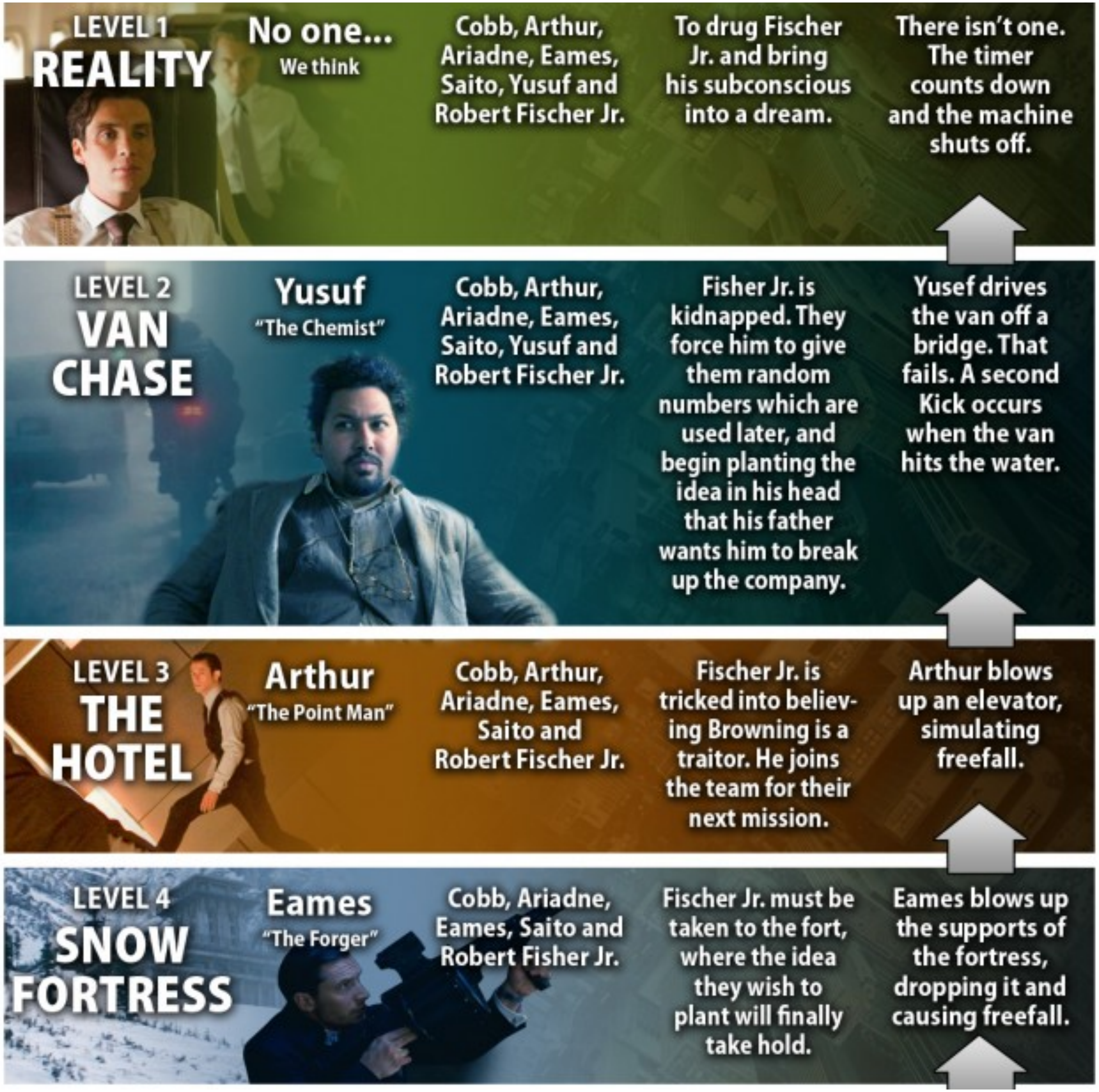
LEVEL 5 LIMBO	No one It's a shared state	Cobb, Ariadne, Saito, Robert Fischer Jr. and Mal's projection	To get Fischer Jr. and Saito out.	Ariadne and Fischer fall off a building. Cobb and Saito shoot themselves.

Artwork by Matt Sinopoli
www.mattsinopoli.com



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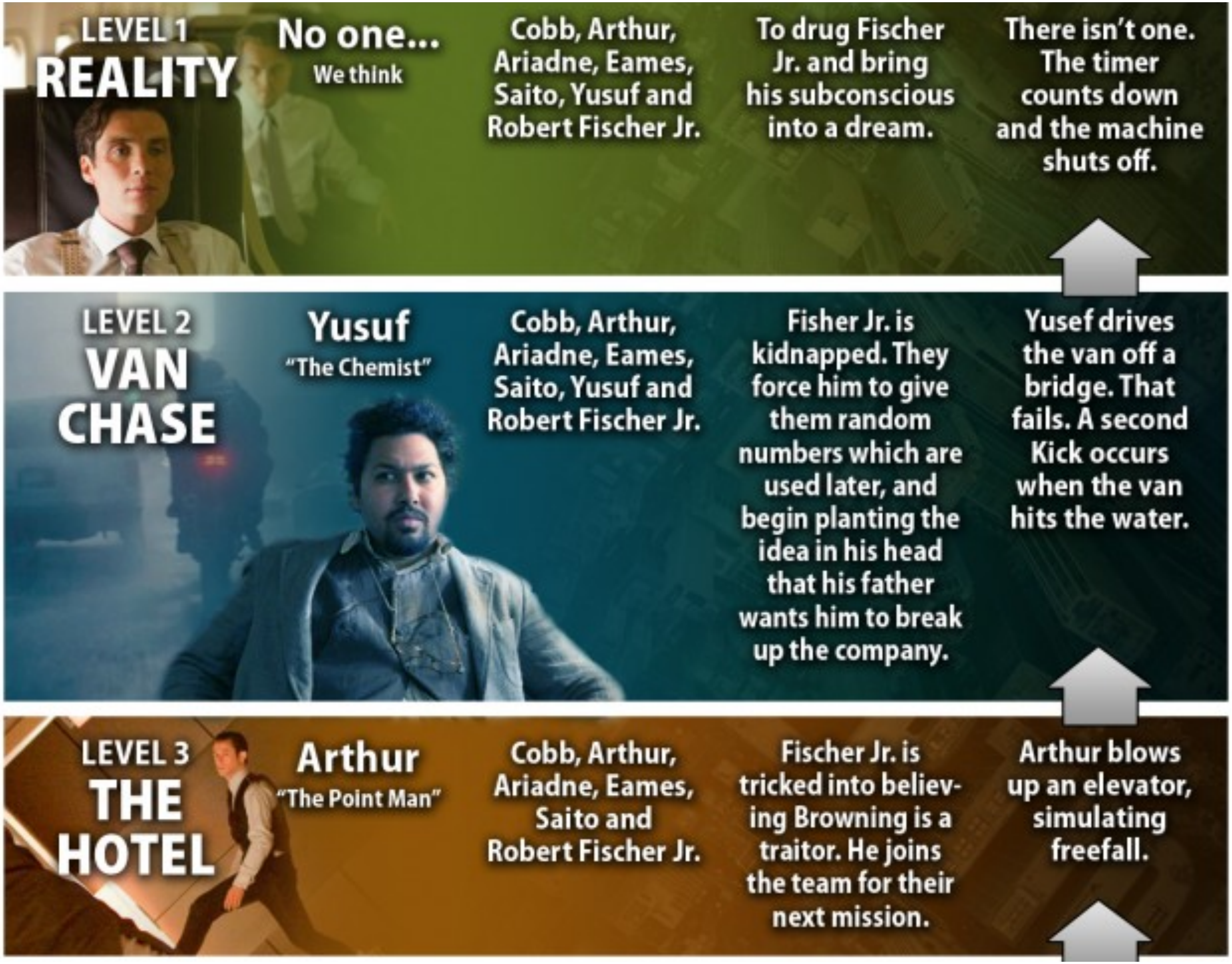

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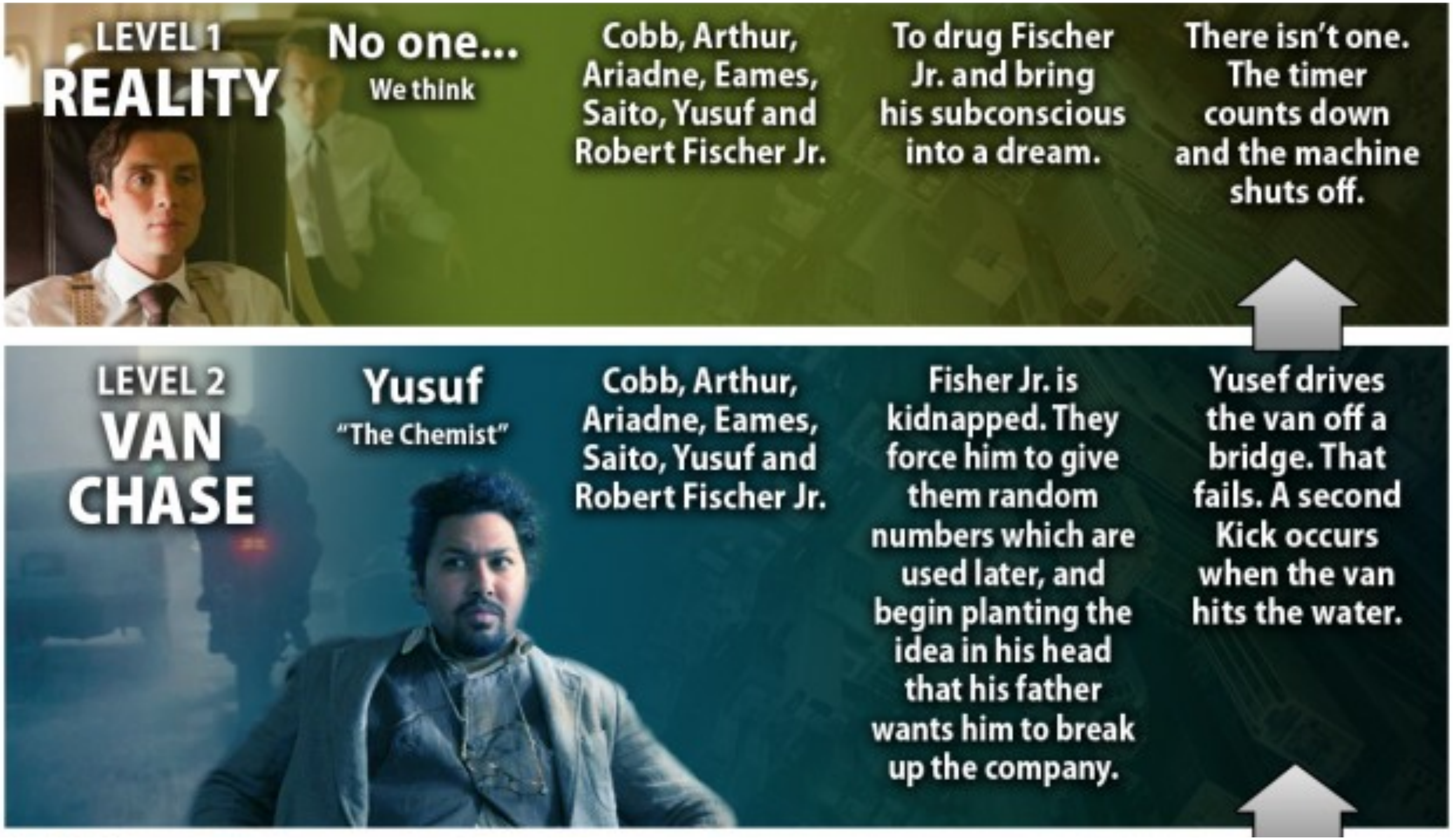

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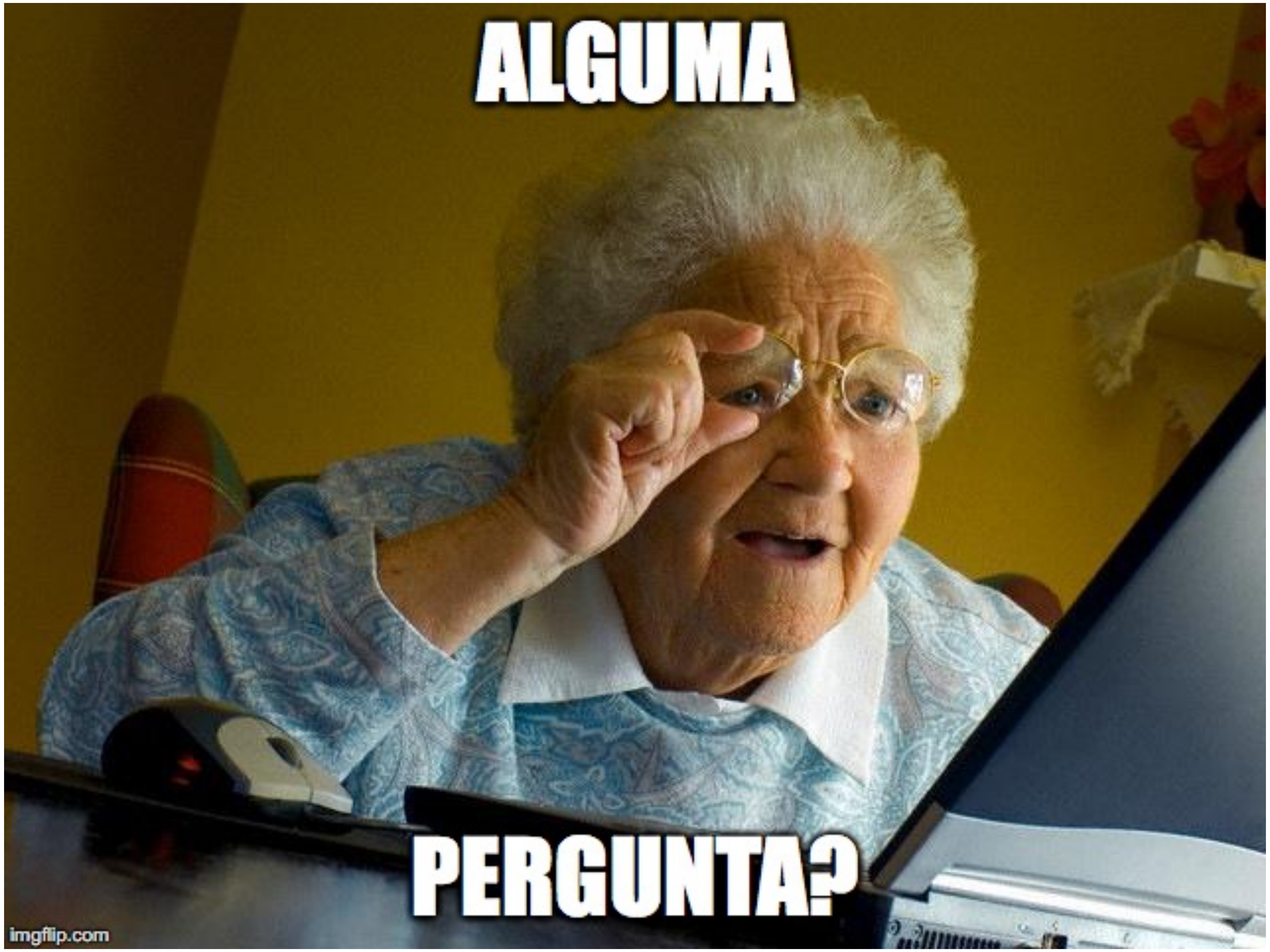
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cálculo fatorial

cálculo fatorial

```
int fat_rec(int n)
{
   if (n <= 1) return 1;
   return n * fat_rec(n-1);
}
```

cálculo fatorial

```
int fat_iter(int n)
{
   int fat = 1;
   while (n > 1) { fat *= n; n—; }
   return fat;
}
```

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cálculo fatorial

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```

cálculo fatorial

```
int fat_iter(int n)
{
   int fat = 1;
   while (n > 1) { fat *= n; n—; }
   return fat;
}
```

Melhor caso

- O(n)

Pior caso

- O(n)

cálculo fatorial

```
int fat_rec(int n)
{
   if (n <= 1) return 1;
   return n * fat_rec(n-1);
}
```

cálculo fatorial

Caso base

- ?

Caso recursivo

- ?

```
int fat_rec(int n)
{
   if (n <= 1) return 1;
   return n * fat_rec(n-1);
}
```

cálculo fatorial

Caso **base**

- 2

Caso **recursivo**

- 2 + T(n-1)

```
int fat_rec(int n)
{
   if (n <= 1) return 1;
   return n * fat_rec(n-1);
}
```

cálculo fatorial

cálculo **fatorial**

$$T(n) = T(n-1) + 2$$

cálculo **fatorial**

$$T(n) = T(n-1) + 2$$

$$T(n) = (T(n-2) + 2) + 2$$

cálculo **fatorial**

$$T(n) = T(n-1) \quad + 2$$

$$T(n) = (T(n-2) + 2) \quad + 2$$

$$T(n) = T(n-2) + 4$$

cálculo **fatorial**

$$T(n) = T(n-1) + 2$$

$$T(n) = (T(n-2) + 2) + 2$$

$$T(n) = T(n-2) + 4$$

$$T(n) = (T(n-3) + 2) + 4$$

cálculo **fatorial**

$$T(n) = T(n-1) + 2$$

$$T(n) = (T(n-2) + 2) + 2$$

$$T(n) = T(n-2) + 4$$

$$T(n) = (T(n-3) + 2) + 4$$

$$T(n) = T(n-3) + 6$$

cálculo fatorial

cálculo **fatorial**

$$T(n) = T(n-1) + 2$$

cálculo **fatorial**

$$T(n) = T(n-1) + 2$$

$$T(n) = T(n-2) + 4$$

cálculo **fatorial**

$$T(n) = T(n-1) + 2$$

$$T(n) = T(n-2) + 4$$

$$T(n) = T(n-3) + 6$$

cálculo **fatorial**

$$T(n) = T(n-1) + 2$$

$$T(n) = T(n-2) + 4$$

$$T(n) = T(n-3) + 6$$

$$\dots$$

cálculo **fatorial**

$$T(n) = T(n-1) + 2$$

$$T(n) = T(n-2) + 4$$

$$T(n) = T(n-3) + 6$$

$$\ldots$$

$$T(n) = T(n-k) + 2k$$

cálculo fatorial

cálculo **fatorial**

$$T(n) = T(n-k) + 2k$$

cálculo **fatorial**

$$T(n) = T(n-k) + 2k$$

Fazendo $k = n$:

cálculo **fatorial**

$$T(n) = T(n-k) + 2k$$

Fazendo $k = n$:

$$T(n) = T(n-n) + 2n$$

cálculo **fatorial**

$$T(n) = T(n-k) + 2k$$

Fazendo $k = n$:

$$T(n) = T(n-n) + 2n$$

$$T(n) = T(0) + 2n$$

cálculo **fatorial**

$$T(n) = T(n-k) + 2k$$

Fazendo $k = n$:

$$T(n) = T(n-n) + 2n$$

$$T(n) = T(0) + 2n$$

$$T(n) = 2 + 2n$$

cálculo fatorial

- $T(n) = 2 + 2n$
  - $O(n)$

```
int fat_rec(int n)
{
   if (n <= 1) return 1;
   return n * fat_rec(n-1);
}
```

cálculo **fatorial**

cálculo fatorial

iterativo	

cálculo fatorial

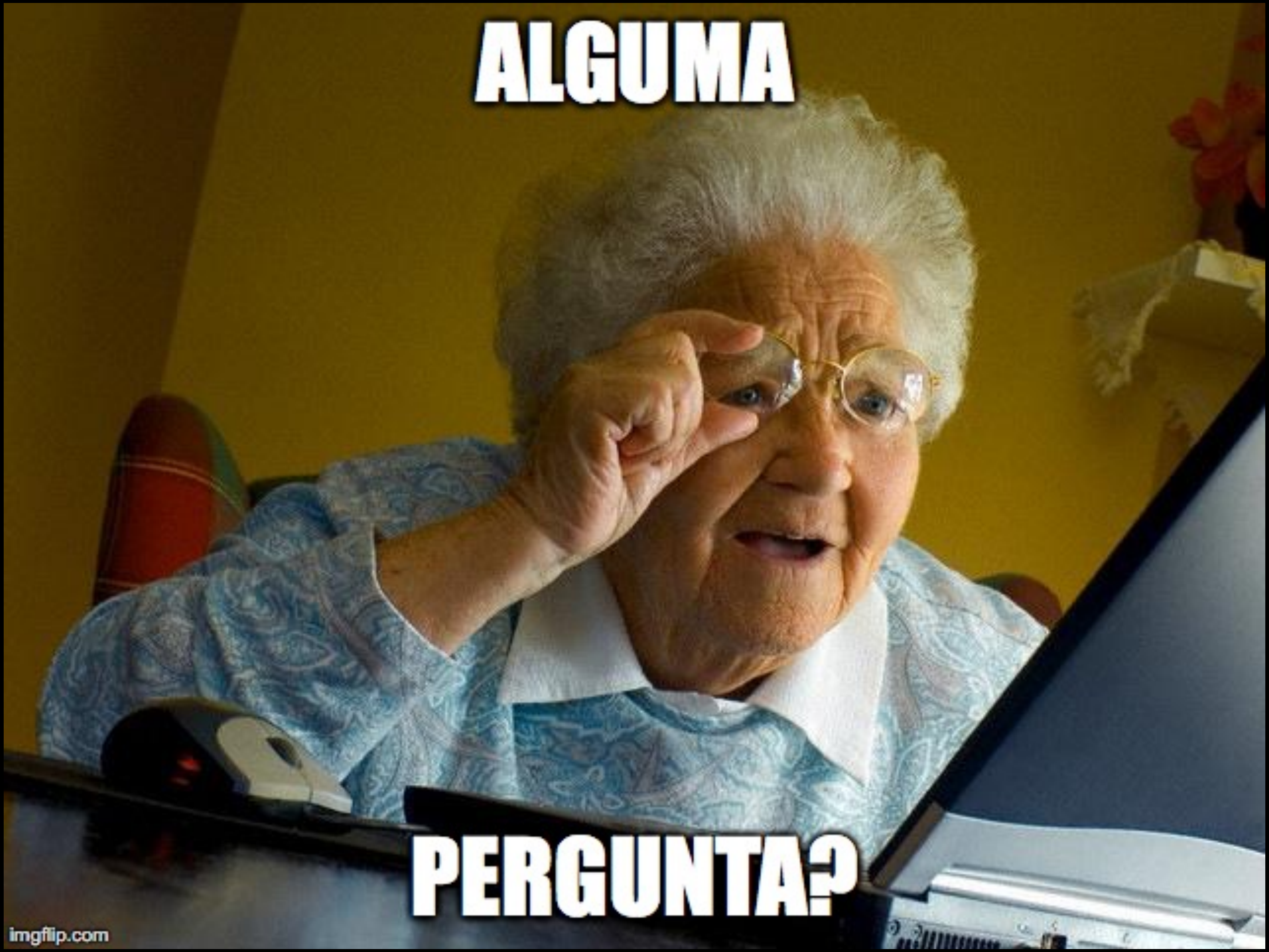
iterativo	recursivo

cálculo **fatorial**

iterativo	recursivo
O(n)	

cálculo **fatorial**

iterativo	recursivo
$O(n)$	$O(n)$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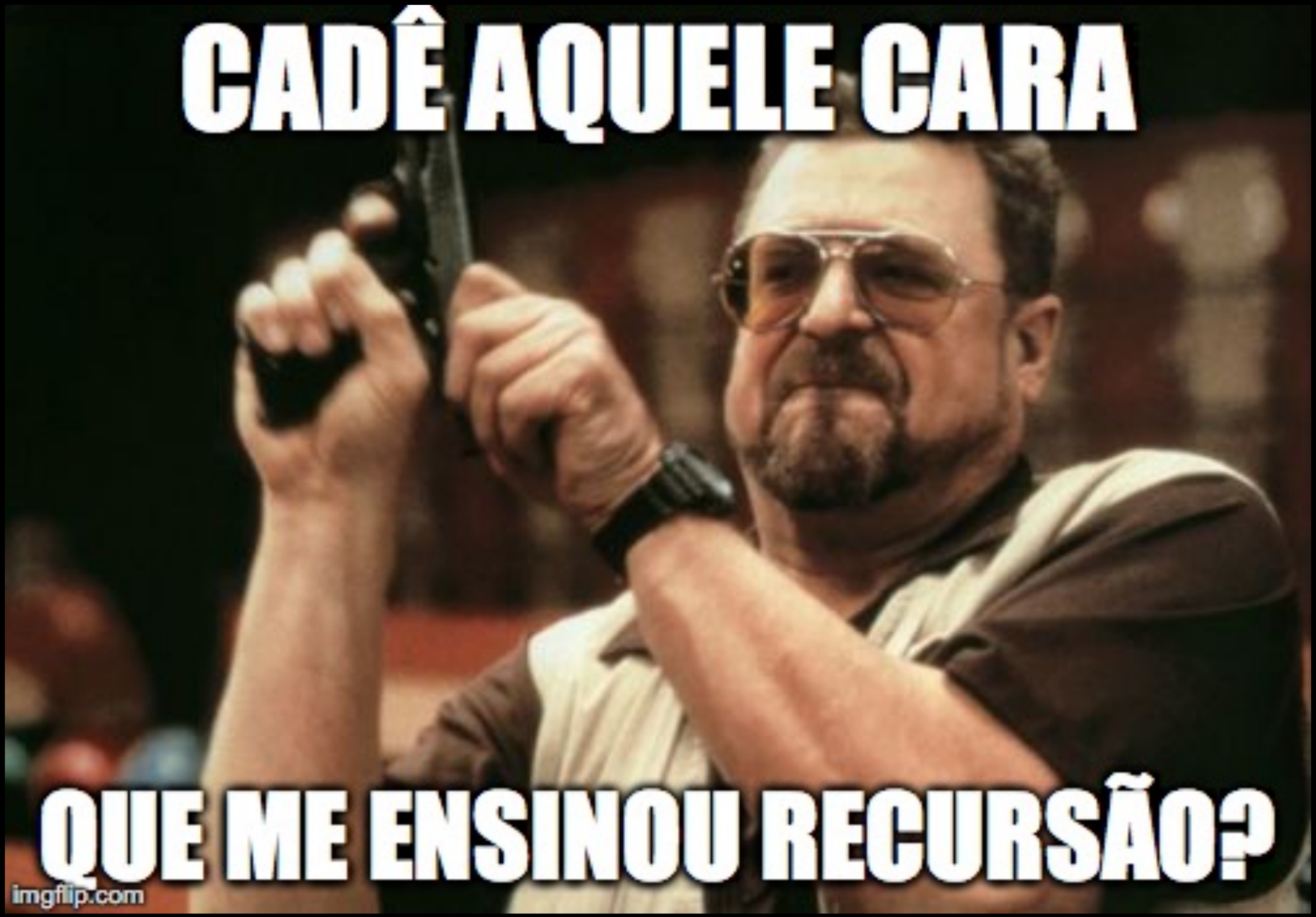
CADÊ AQUELE CARA
QUE ME ENSINOU RECURSÃO?
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dividir para conquistar

dividir para conquistar	intersecções entre subproblemas

dividir para conquistar	intersecções entre subproblemas
recursão de cauda	

dividir para conquistar	intersecções entre subproblemas
recursão de cauda	demais casos

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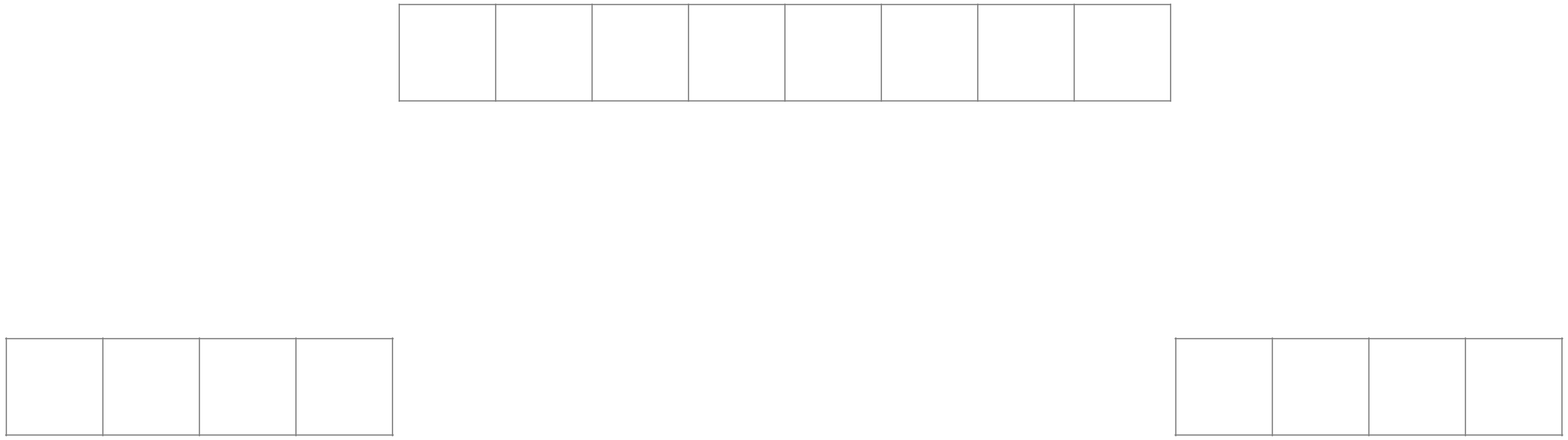
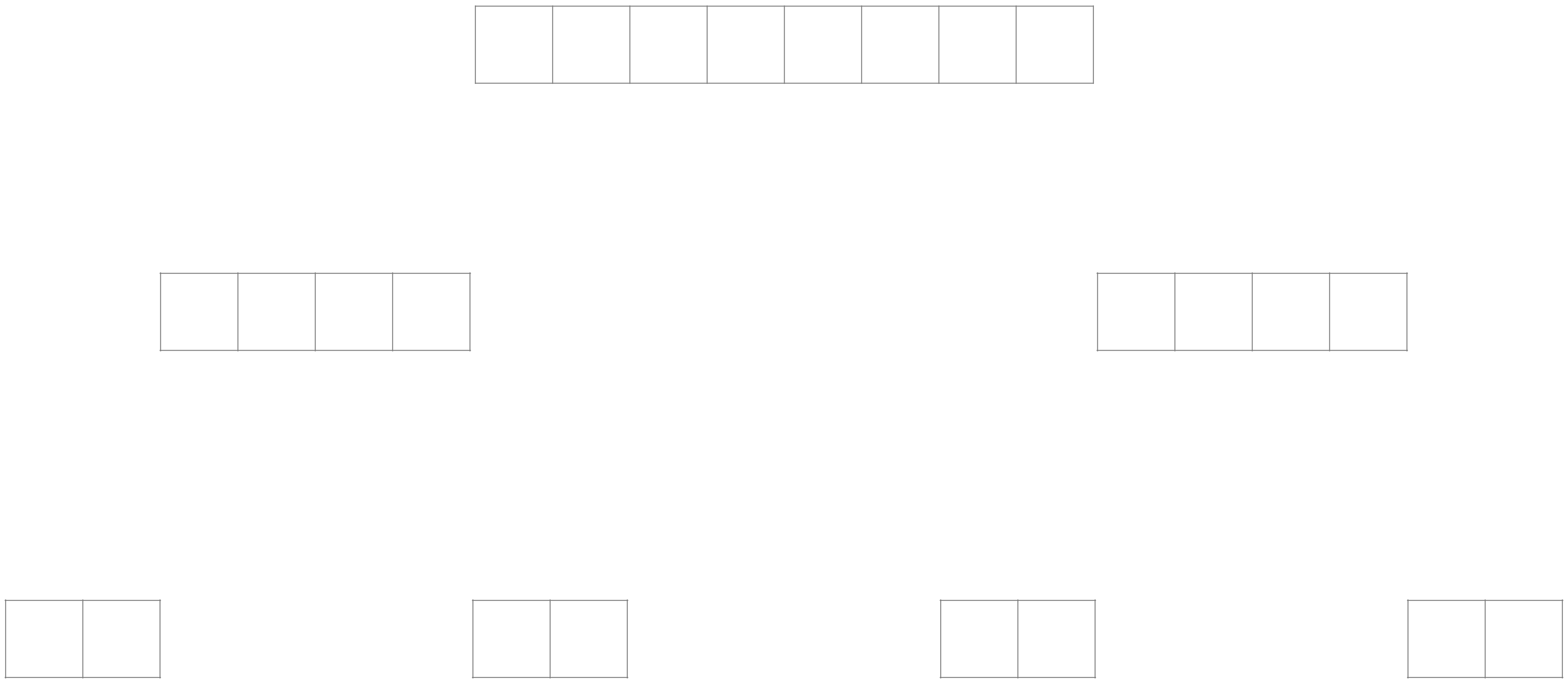
recursão de cauda	demais casos

dividir para conquistar

dividir para conquistar



dividir para conquistar



dividir para conquistar

1	4	7	10	15	16	18	21

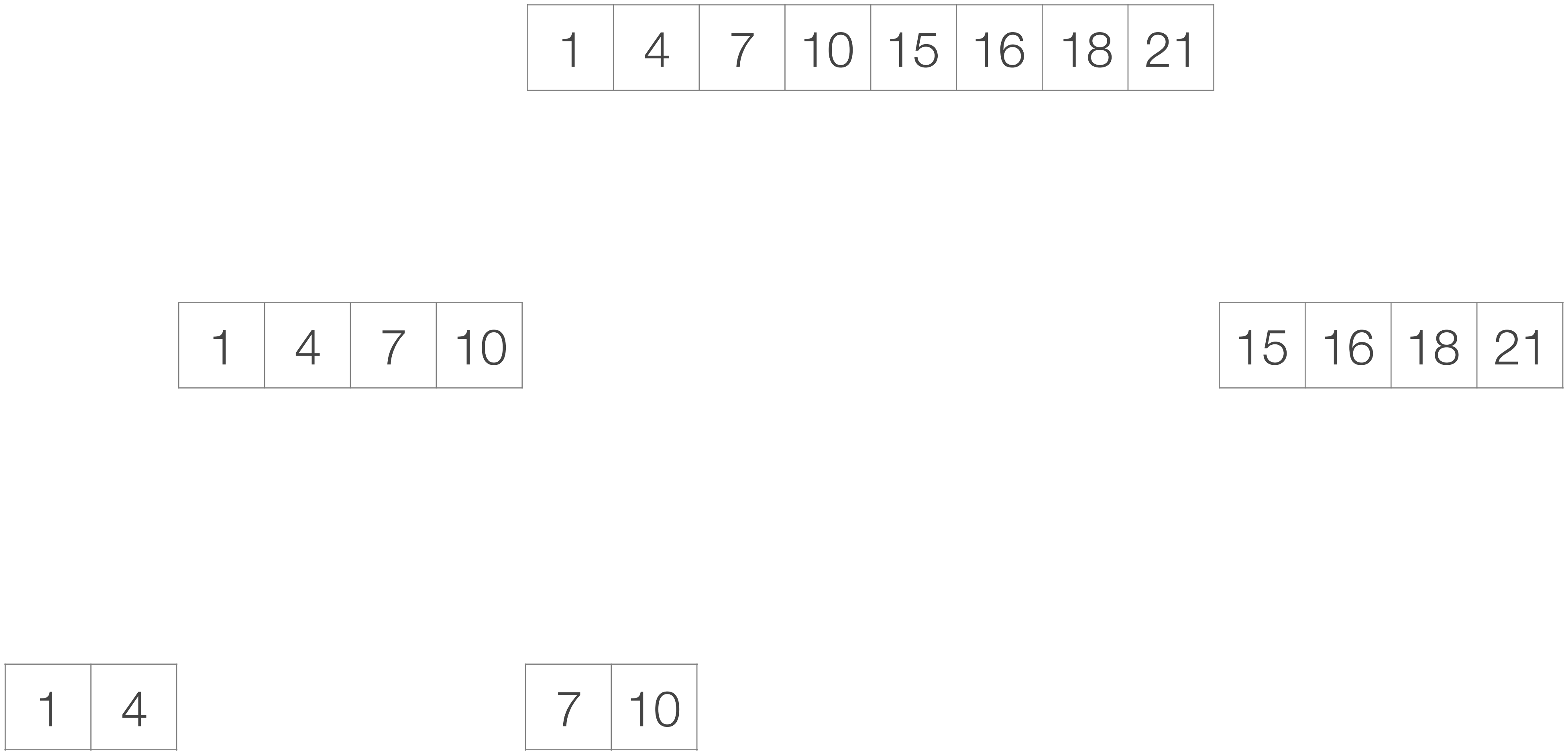
dividir para conquistar

1	4	7	10	15	16	18	21

1	4	7	10

15	16	18	21

dividir para conquistar



1	4	7	10	15	16	18	21

1	4	7	10

15	16	18	21

1	4

7	10

dividir para conquistar

1	4	7	10	15	16	18	21

1	4	7	10

15	16	18	21

1	4

7	10

15	16

18	21

dividir para conquistar

busca(3)

1	4	7	10	15	16	18	21

dividir para conquistar

busca(3)

1	4	7	10	15	16	18	21

1	4	7	10

dividir para conquistar

busca(3)

1	4	7	10	15	16	18	21

1	4	7	10

1	4

dividir para conquistar

busca(3)

O(log n)

1	4	7	10	15	16	18	21

1	4	7	10

1	4

dividir para conquistar

busca(3)

1	7	4	15	10	21	18	16

dividir para conquistar

busca(3)

1	7	4	15	10	21	18	16

1	7	4	15

10	21	18	16

dividir para conquistar

busca(3)

1	7	4	15	10	21	18	16

1	7	4	15

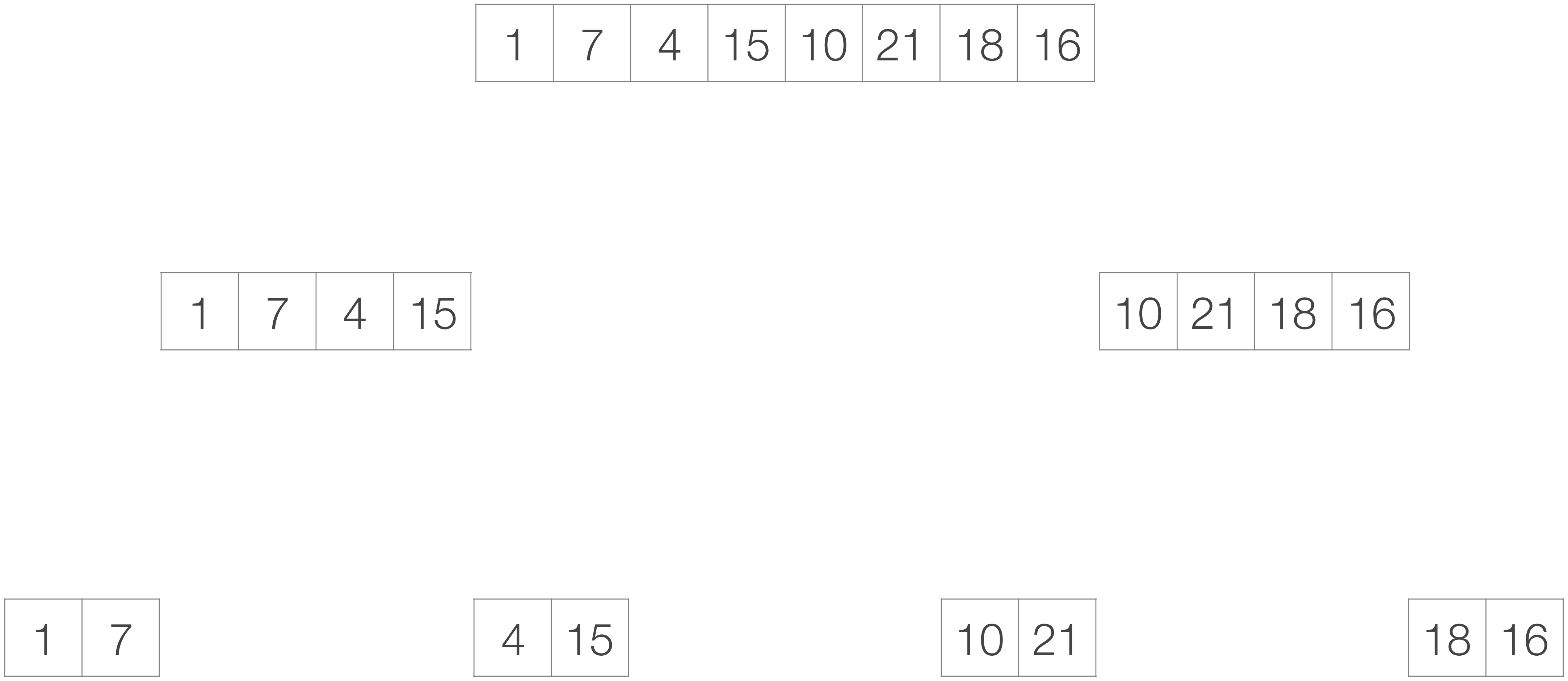
10	21	18	16

1	7

4	15

dividir para conquistar

busca(3)



1	7	4	15	10	21	18	16

1	7	4	15

10	21	18	16

1	7

4	15

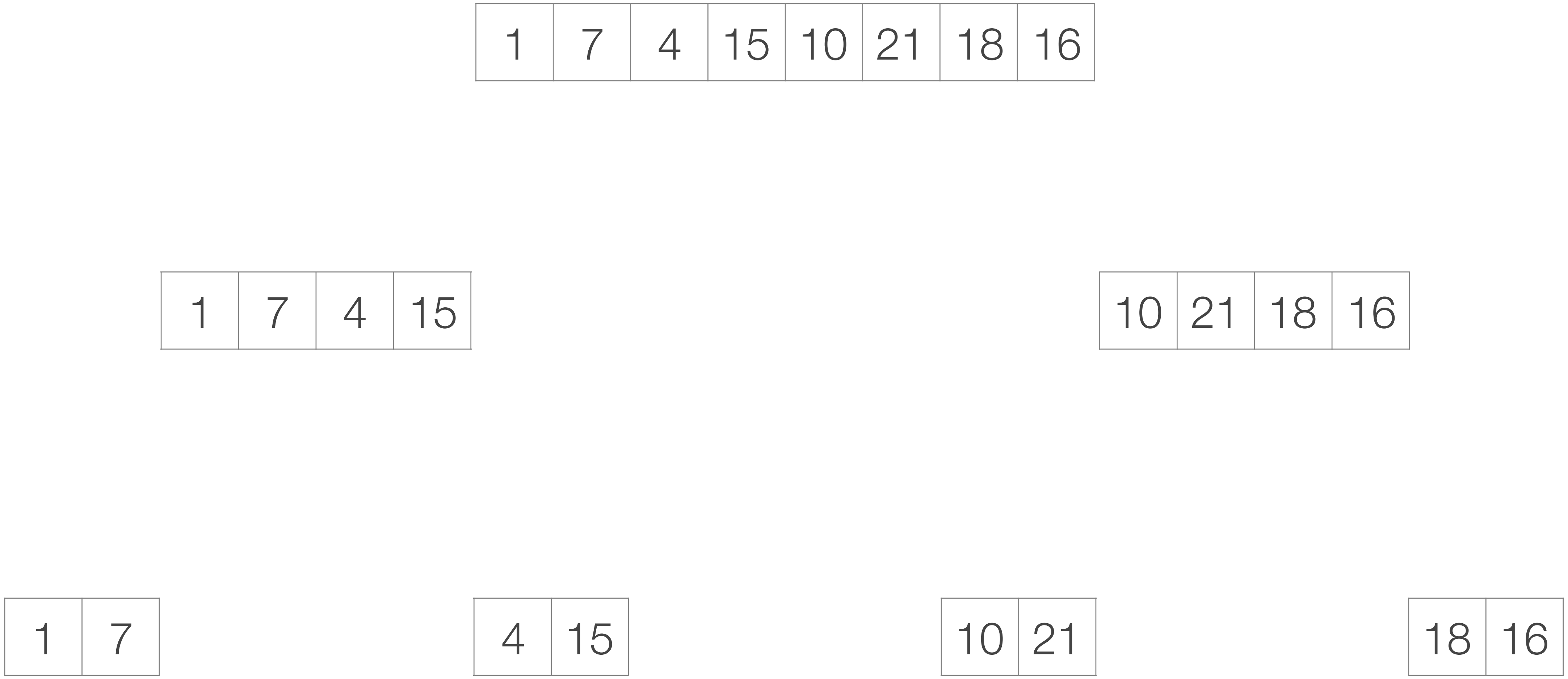
10	21

18	16

dividir para conquistar

busca(3)

O(n)



1	7	4	15	10	21	18	16

1	7	4	15

10	21	18	16

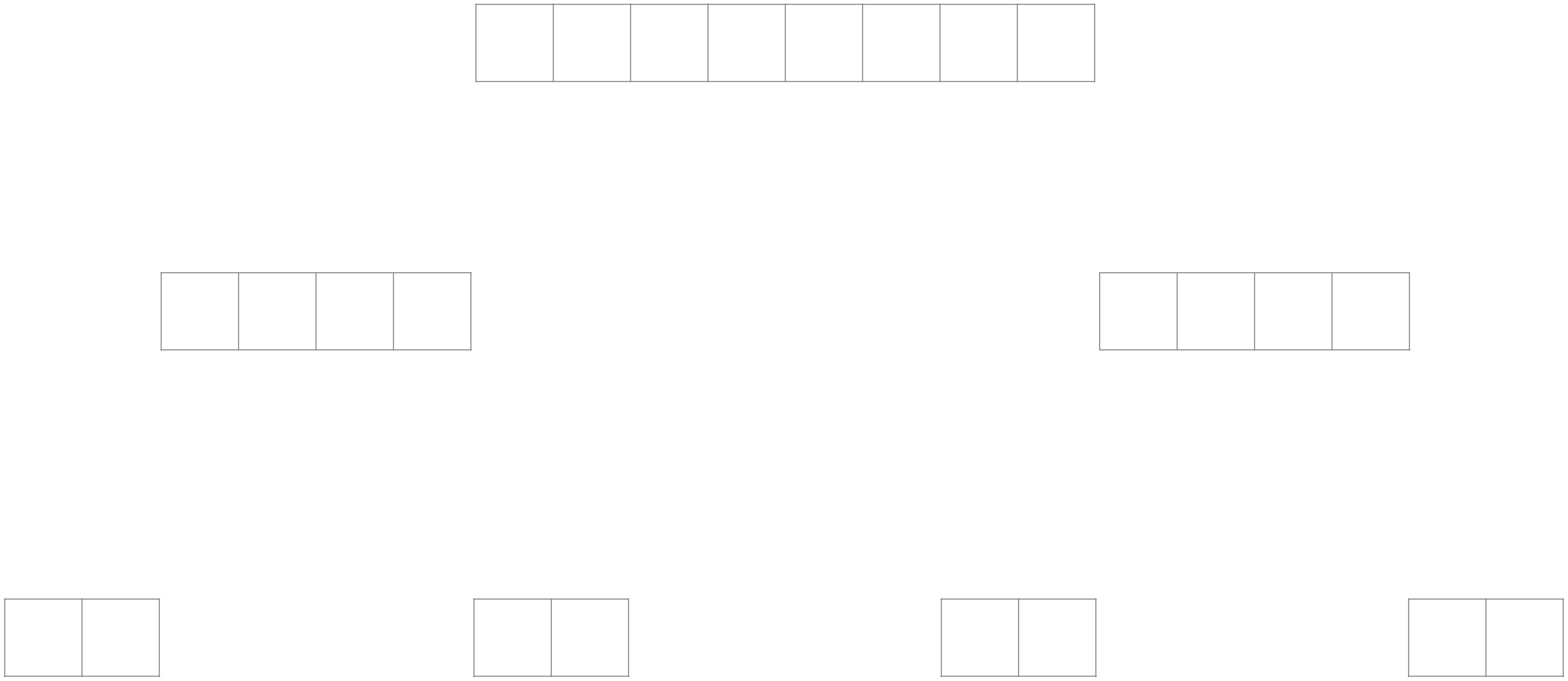
1	7

4	15

10	21

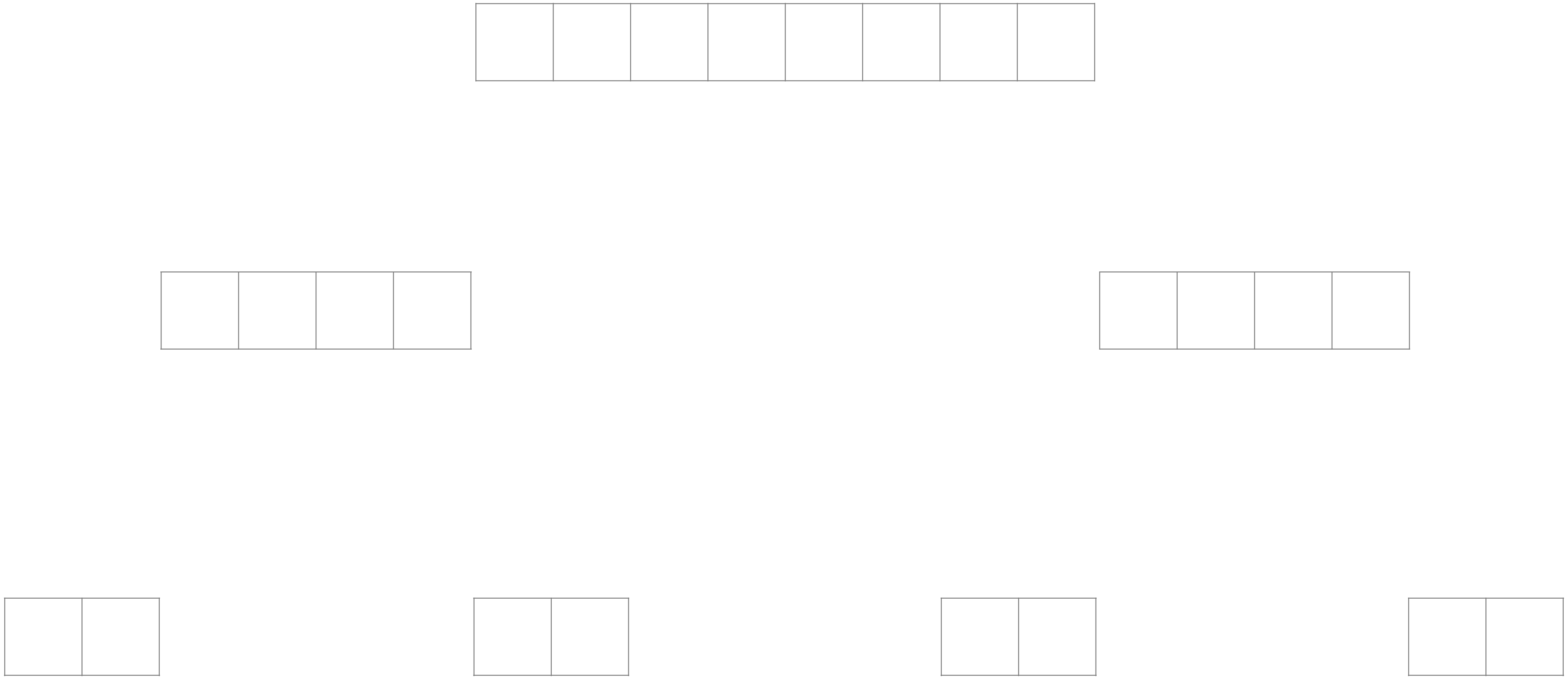
18	16

dividir para conquistar



dividir para conquistar

$aT(n/b)$



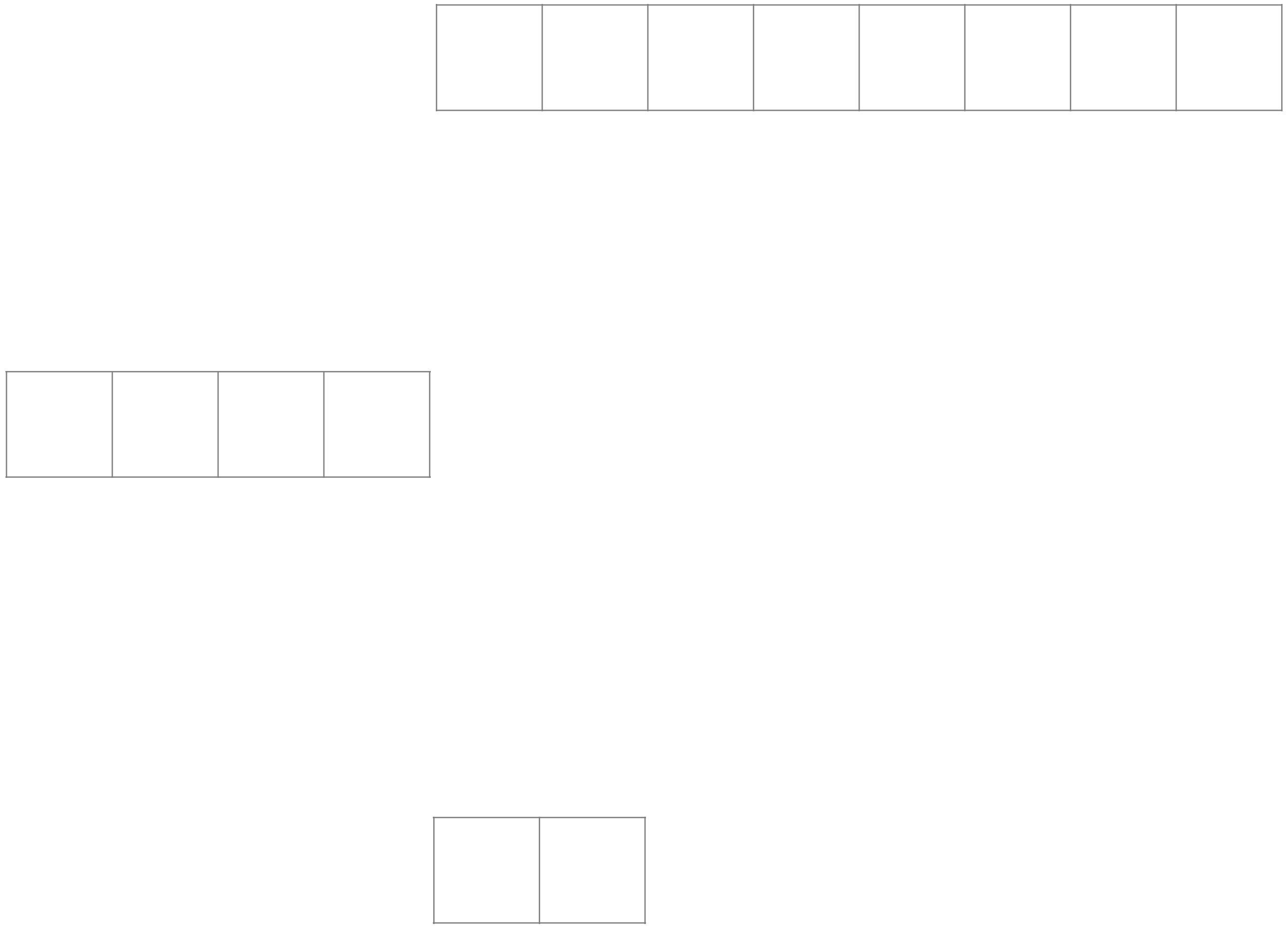
dividir para conquistar

$$aT(n/b)$$

a	fator de arborescência
b	fator de divisão

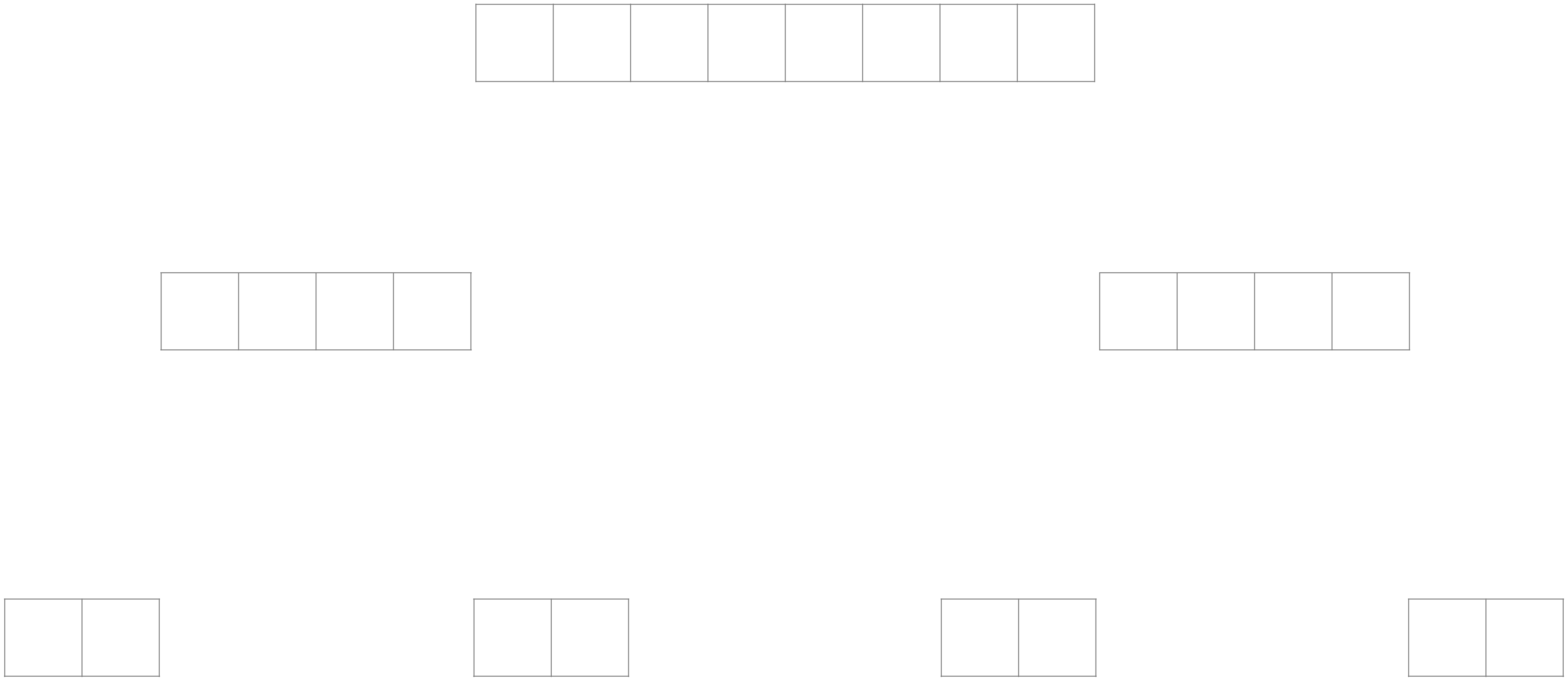
dividir para conquistar

$T(n/2)$



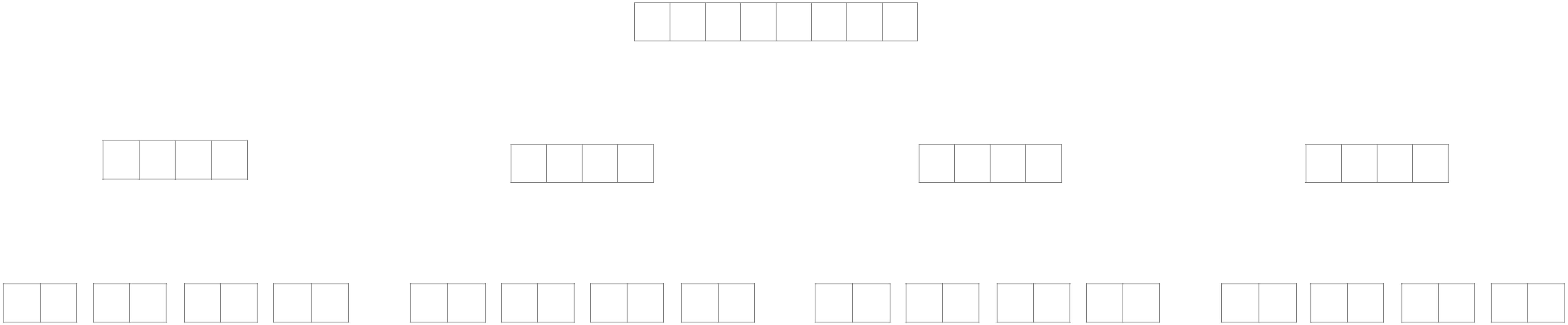
dividir para conquistar

$2T(n/2)$



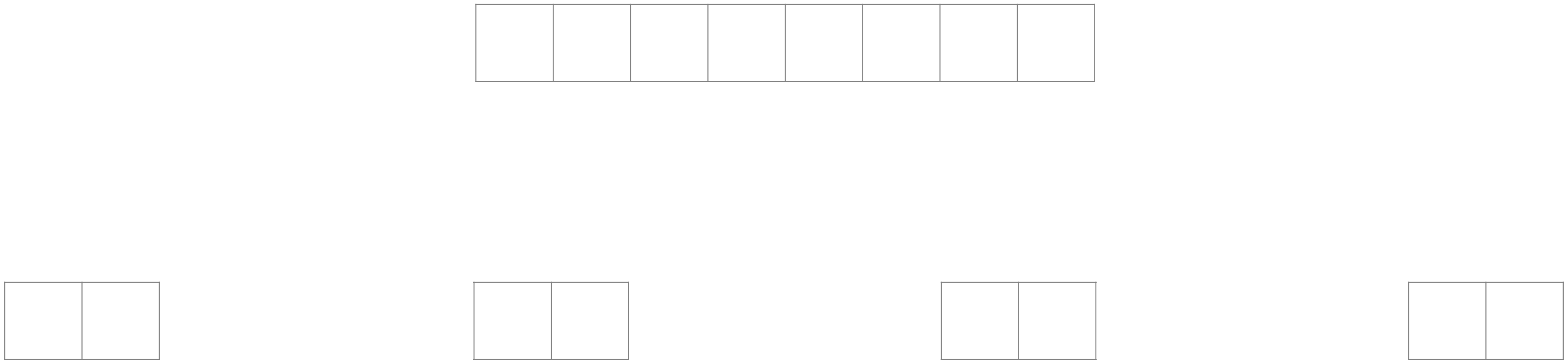
dividir para conquistar

$4T(n/2)$



dividir para conquistar

$4T(n/4)$



dividir para conquistar

dividir para conquistar

$$T(n) = aT(n/b) + f(n)$$

dividir para conquistar

$$T(n) = a\underbrace{T(n/b)}_{\text{dividir}} + f(n)$$

dividir para conquistar

$$T(n) = a\underbrace{T(n/b)}_{\text{dividir}} + \overbrace{f(n)}^{\text{conquistar}}$$

busca **binária**

busca binária

```
int bin_rec(int v[], int chave, int inicio, int fim)
```

busca **binária**

```
int bin_rec(int v[], int chave, int inicio, int fim)
{
```

busca **binária**

```
int bin_rec(int v[], int chave, int inicio, int fim)
{
   int tamanho = fim - inicio;
```

busca **binária**

```
int bin_rec(int v[], int chave, int inicio, int fim)
{
    int tamanho = fim - inicio;
    if (tamanho == 0) { return -1; }
```

busca binária

```
int bin_rec(int v[], int chave, int inicio, int fim)
{
   int tamanho = fim - inicio;
   if (tamanho == 0) { return -1; }
   int meio = inicio + floor(tamanho / 2);
```

busca binária

```
int bin_rec(int v[], int chave, int inicio, int fim)
{
   int tamanho = fim - inicio;
   if (tamanho == 0) { return -1; }
   int meio = inicio + floor(tamanho / 2);
   if (chave == v[meio]) { return meio; }
```

busca **binária**

```
int bin_rec(int v[], int chave, int inicio, int fim)
{
    int tamanho = fim - inicio;
    if (tamanho == 0) { return -1; }
    int meio = inicio + floor(tamanho / 2);
    if (chave == v[meio]) { return meio; }
    else if (chave < v[meio]) {
```

busca **binária**

```
int bin_rec(int v[], int chave, int inicio, int fim)
{
    int tamanho = fim - inicio;
    if (tamanho == 0) { return -1; }
    int meio = inicio + floor(tamanho / 2);
    if (chave == v[meio]) { return meio; }
    else if (chave < v[meio]) {
        return bin_rec(v, chave, inicio, meio);
```

busca **binária**

```
int bin_rec(int v[], int chave, int inicio, int fim)
{
   int tamanho = fim - inicio;
   if (tamanho == 0) { return -1; }
   int meio = inicio + floor(tamanho / 2);
   if (chave == v[meio]) { return meio; }
   else if (chave < v[meio]) {
      return bin_rec(v, chave, inicio, meio);
   }
```

busca **binária**

```
int bin_rec(int v[], int chave, int inicio, int fim)
{
    int tamanho = fim - inicio;
    if (tamanho == 0) { return -1; }
    int meio = inicio + floor(tamanho / 2);
    if (chave == v[meio]) { return meio; }
    else if (chave < v[meio]) {
        return bin_rec(v, chave, inicio, meio);
    }
    return bin_rec(v, chave, meio + 1, fim);
```

busca **binária**

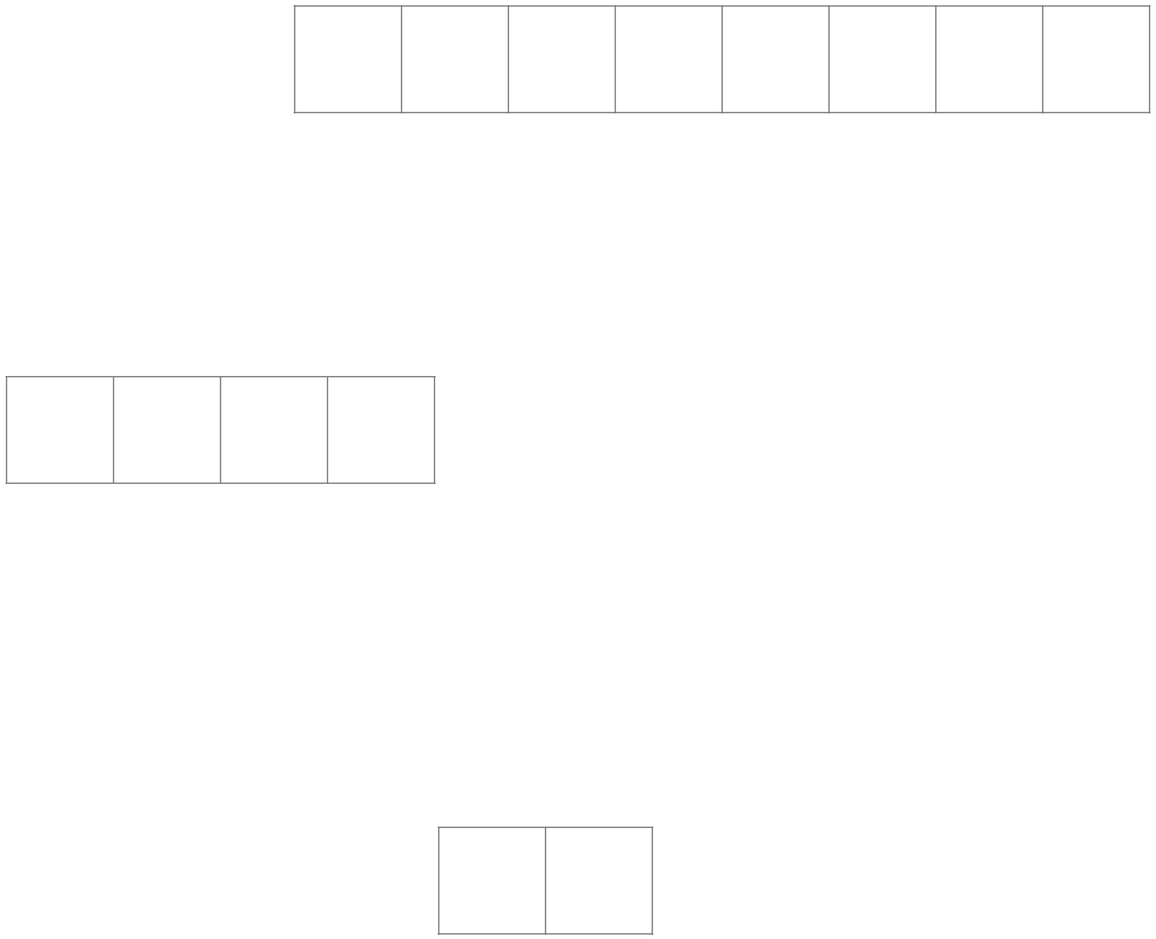
```
int bin_rec(int v[], int chave, int inicio, int fim)
{
    int tamanho = fim - inicio;
    if (tamanho == 0) { return -1; }
    int meio = inicio + floor(tamanho / 2);
    if (chave == v[meio]) { return meio; }
    else if (chave < v[meio]) {
        return bin_rec(v, chave, inicio, meio);
    }
    return bin_rec(v, chave, meio + 1, fim);
}
```

dividir para conquistar

$$T(n) = a\underbrace{T(n/b)}_{\text{dividir}} + \overbrace{f(n)}^{\text{conquistar}}$$

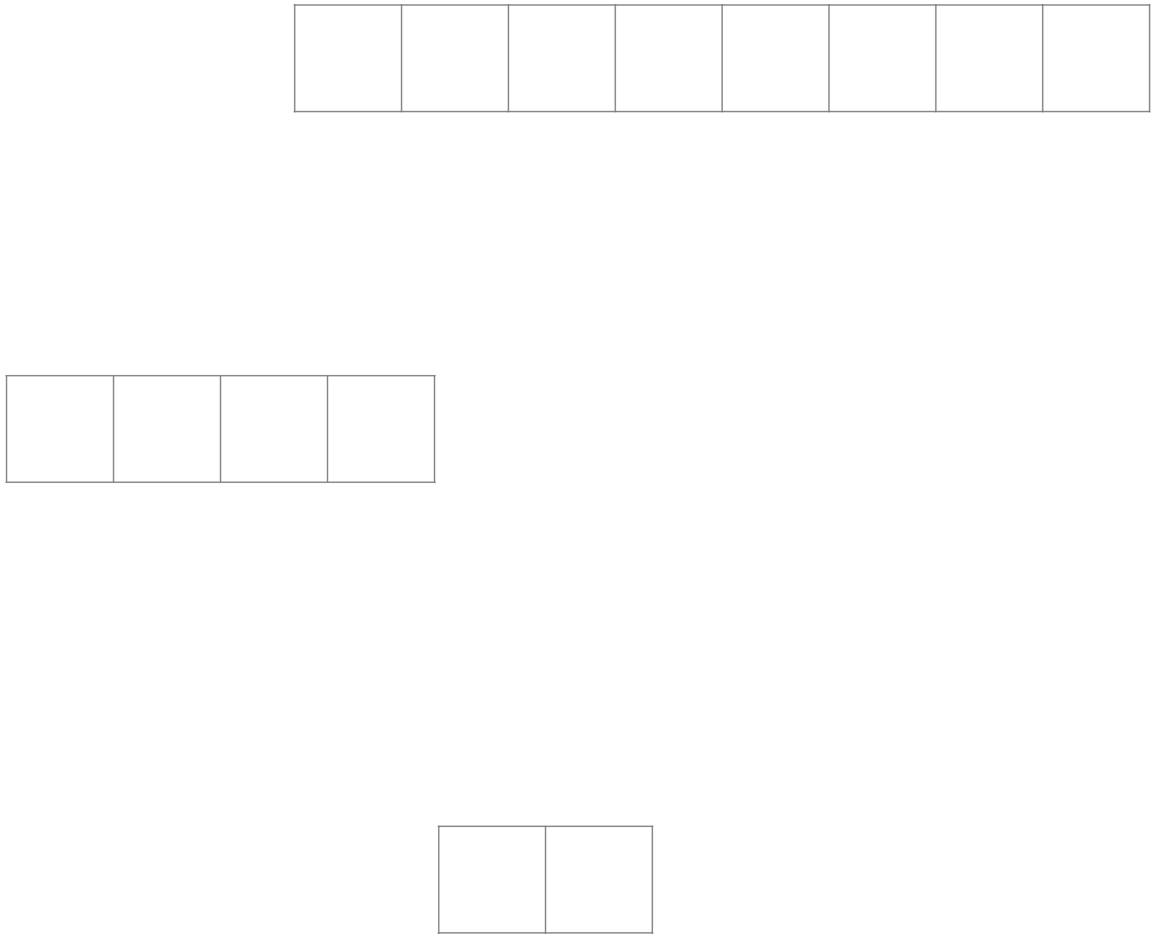
dividir para conquistar

$$T(n/2) + 1$$



dividir para conquistar

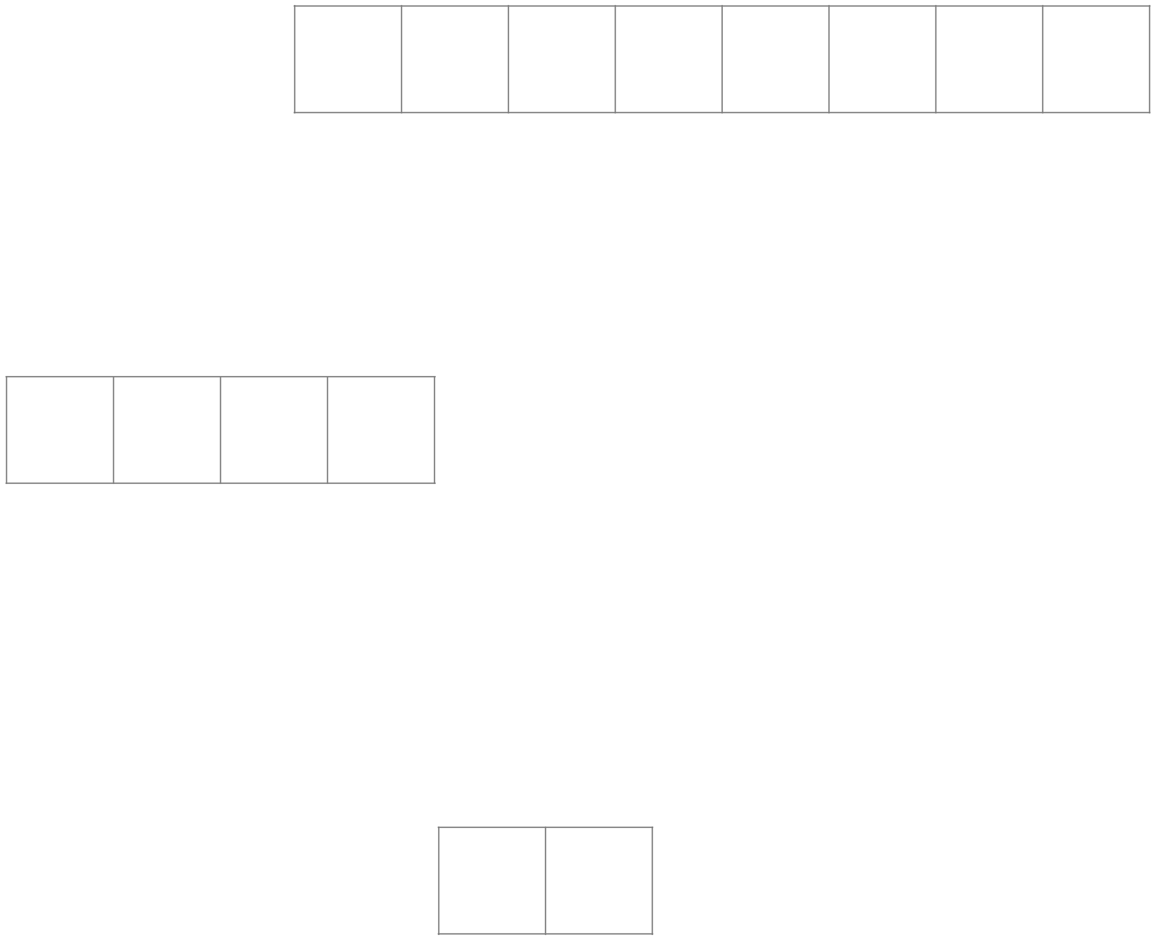
$T(n/2) + 1$



1

dividir para conquistar

$T(n/2) + 1$

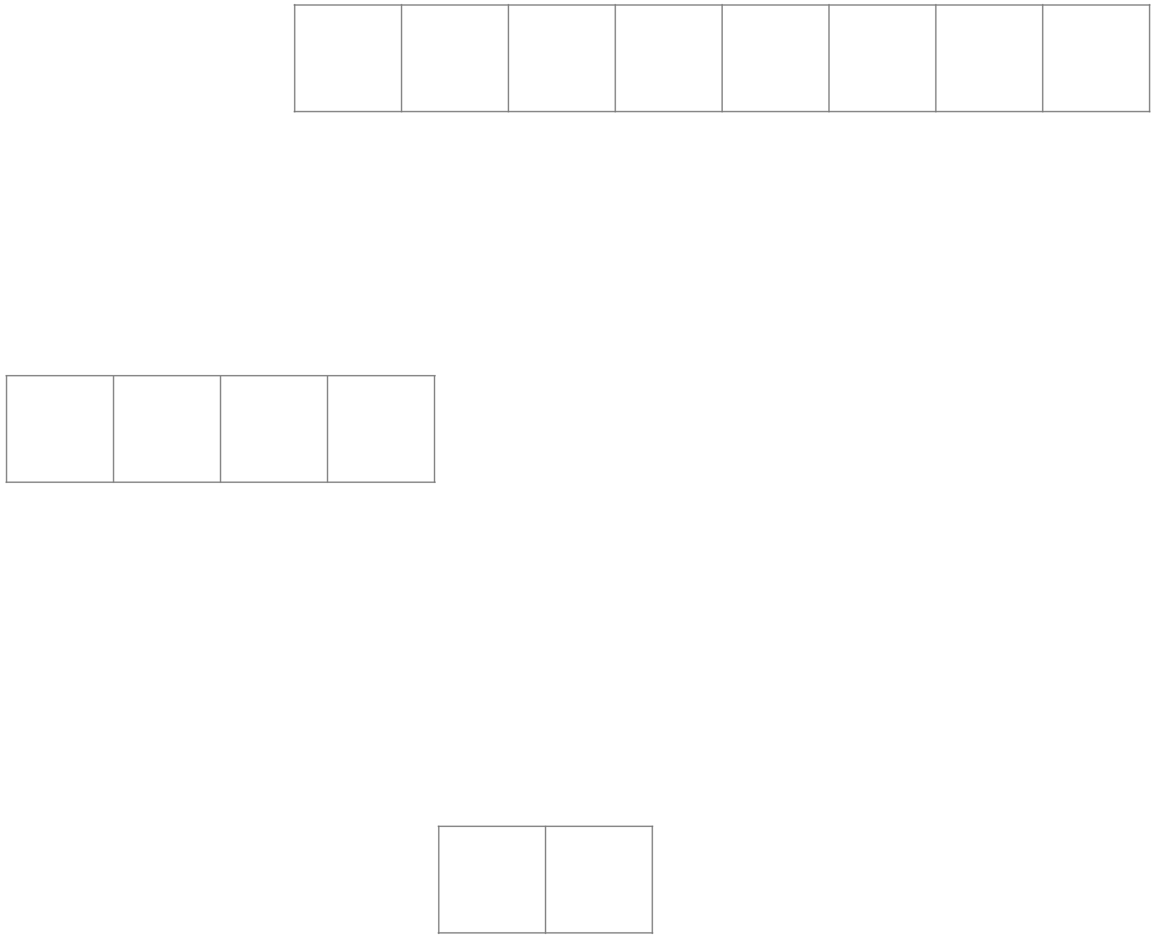


1

1

dividir para conquistar

$T(n/2) + 1$



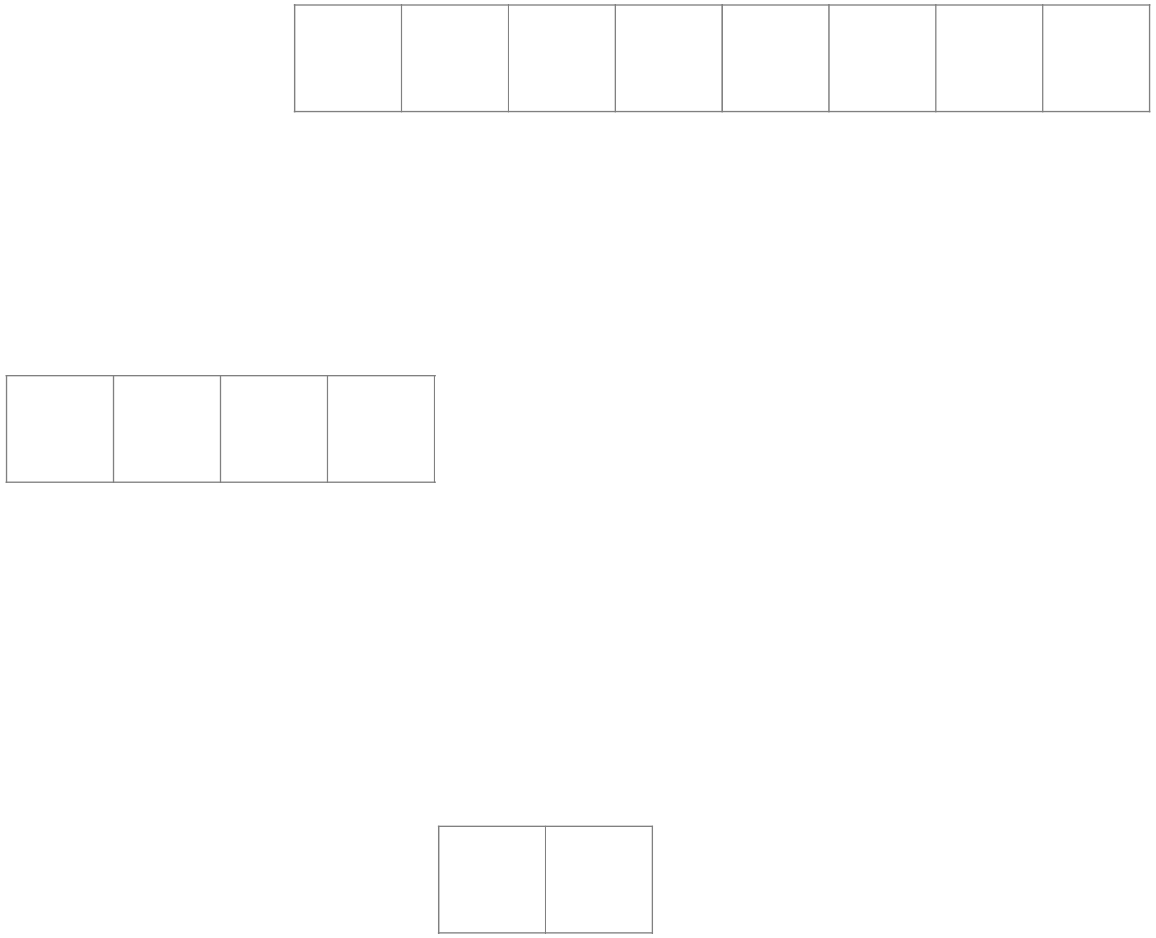
1

1

1

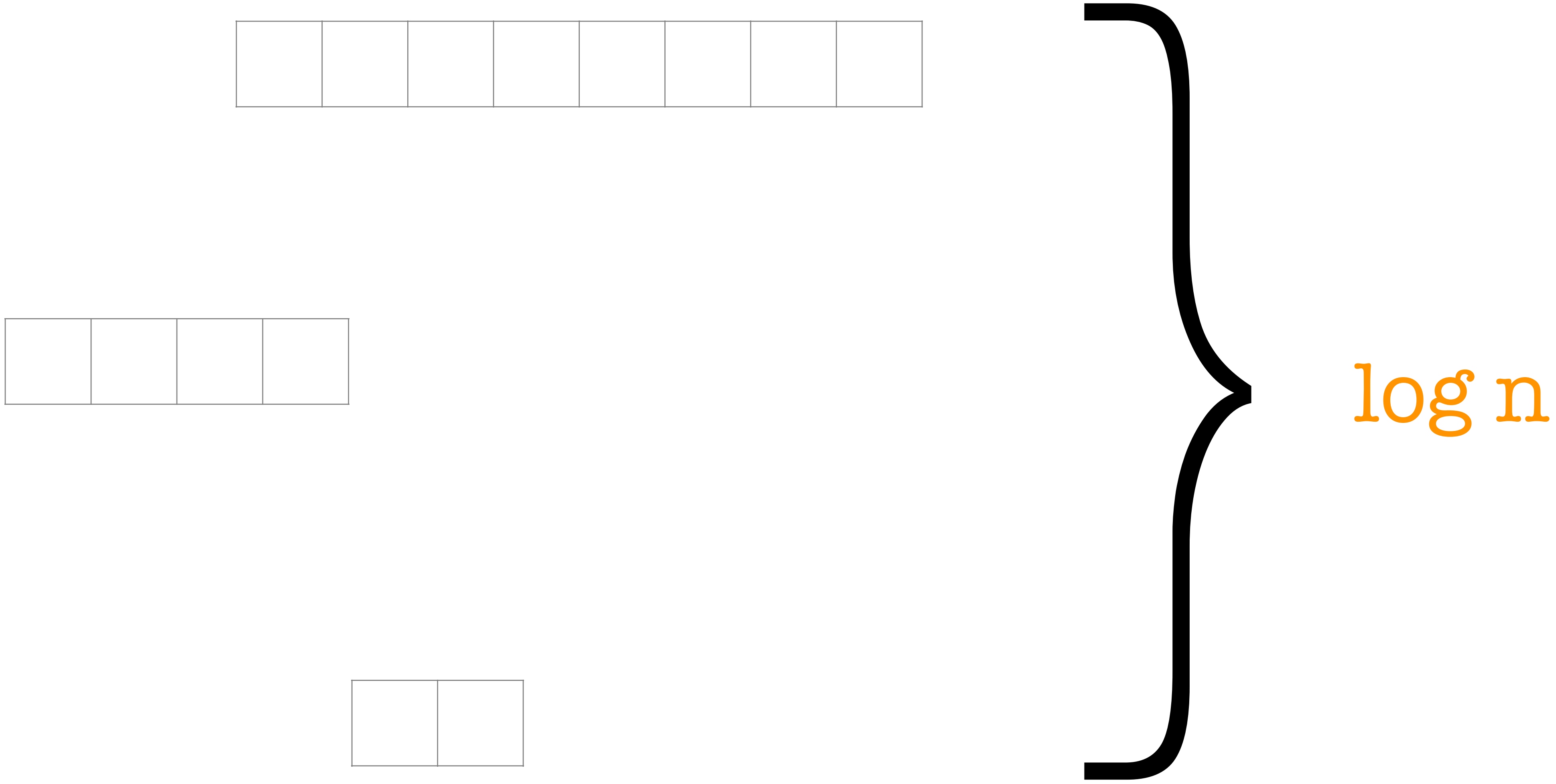
dividir para conquistar

$$T(n/2) + 1$$



dividir para conquistar

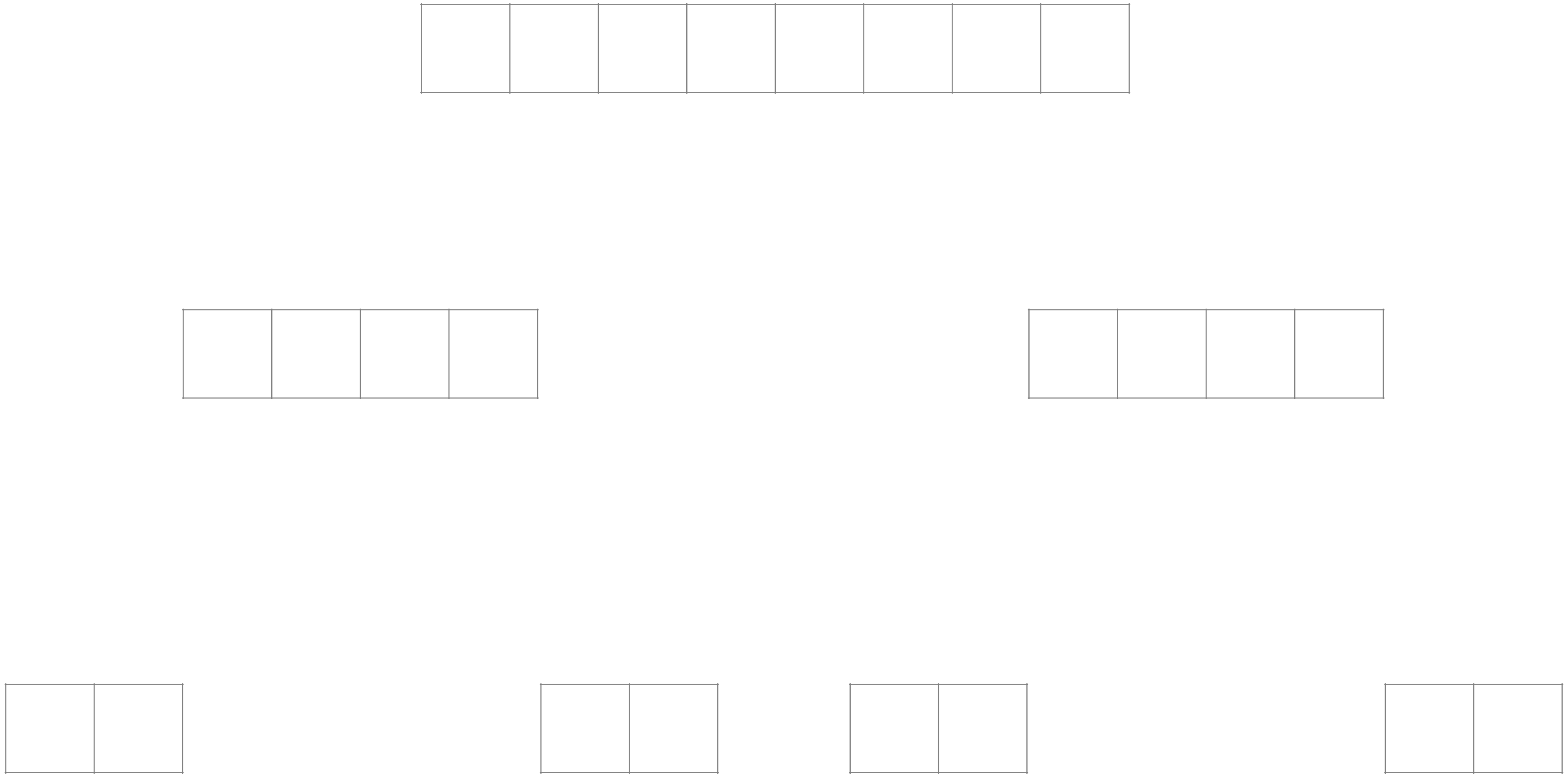
$T(n/2) + 1$



log n

dividir para conquistar

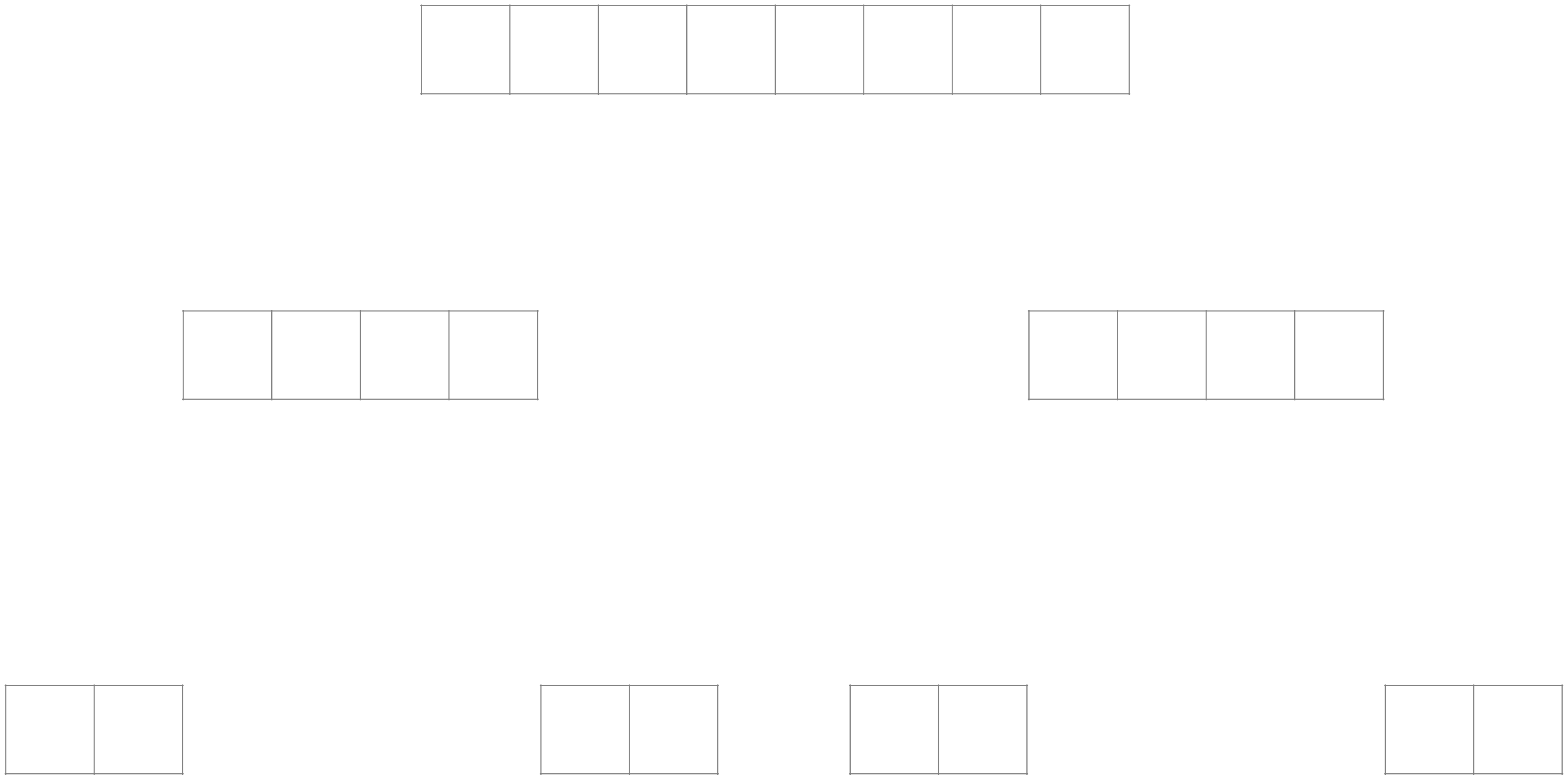
$2T(n/2) + 1$



dividir para conquistar

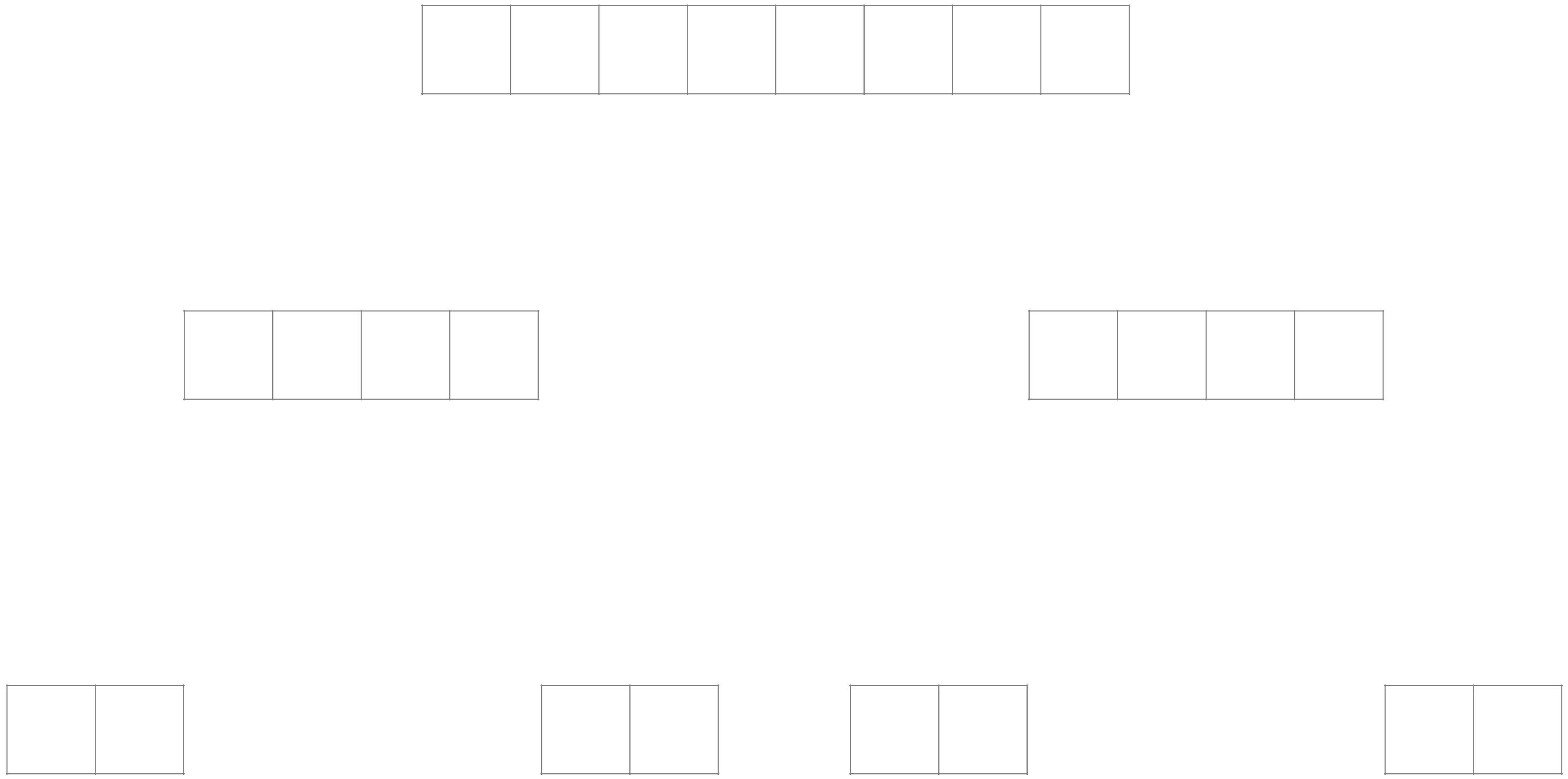
$2T(n/2) + 1$

1



dividir para conquistar

$2T(n/2) + 1$

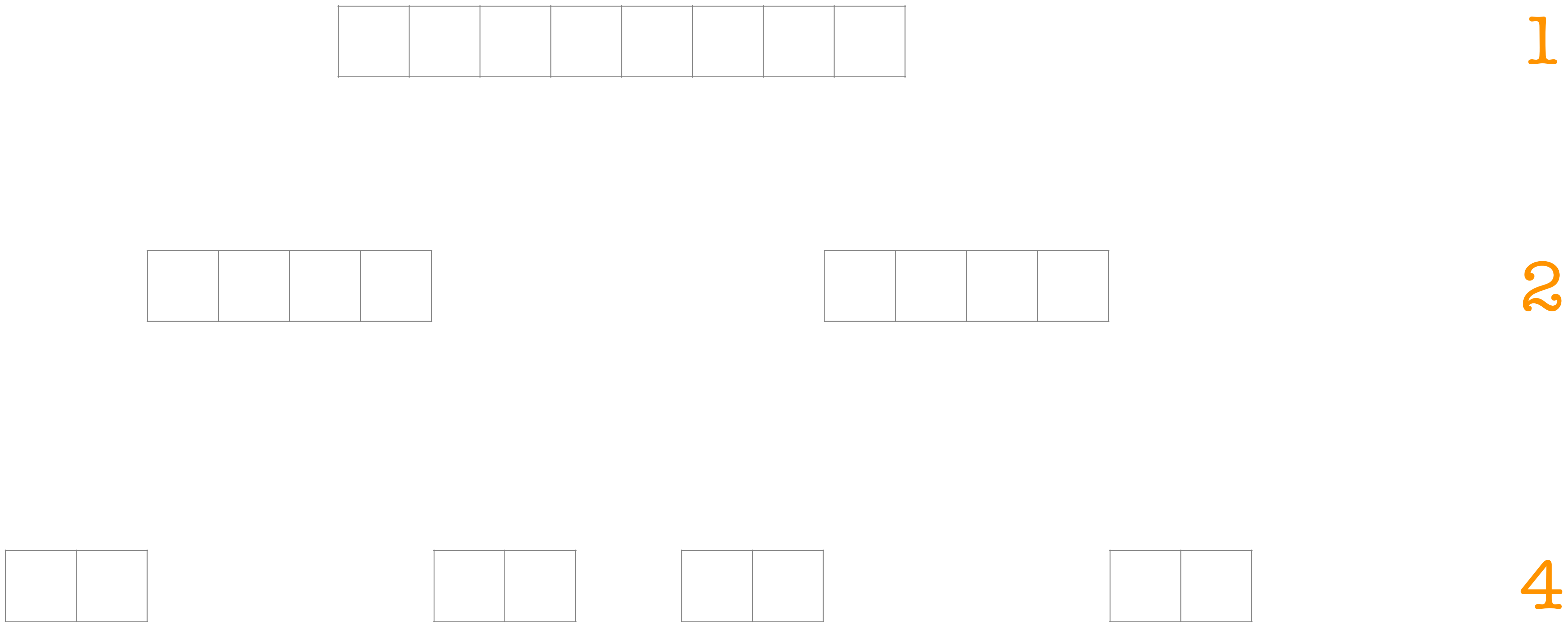


1

2

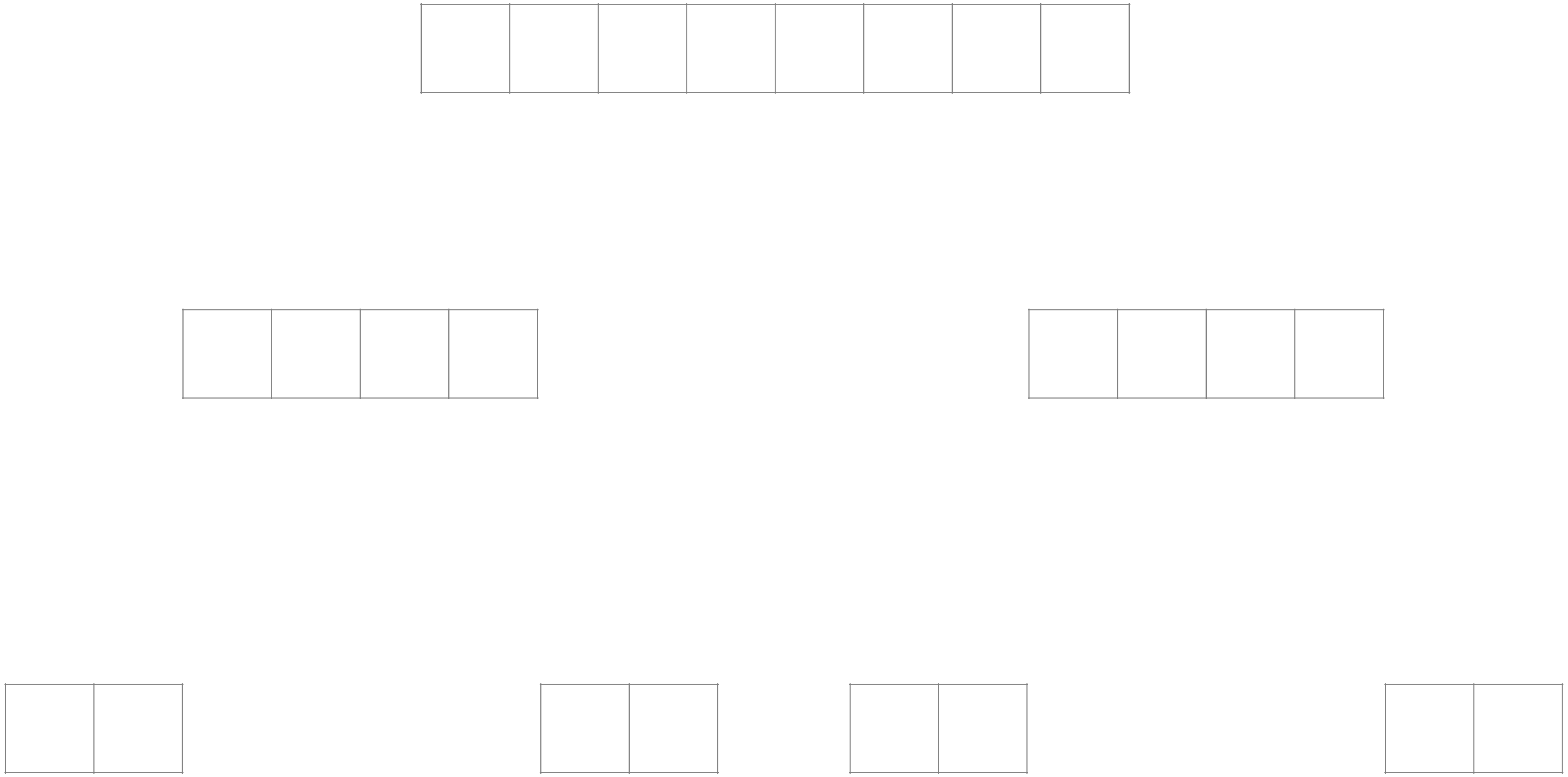
dividir para conquistar

$$2T(n/2) + 1$$



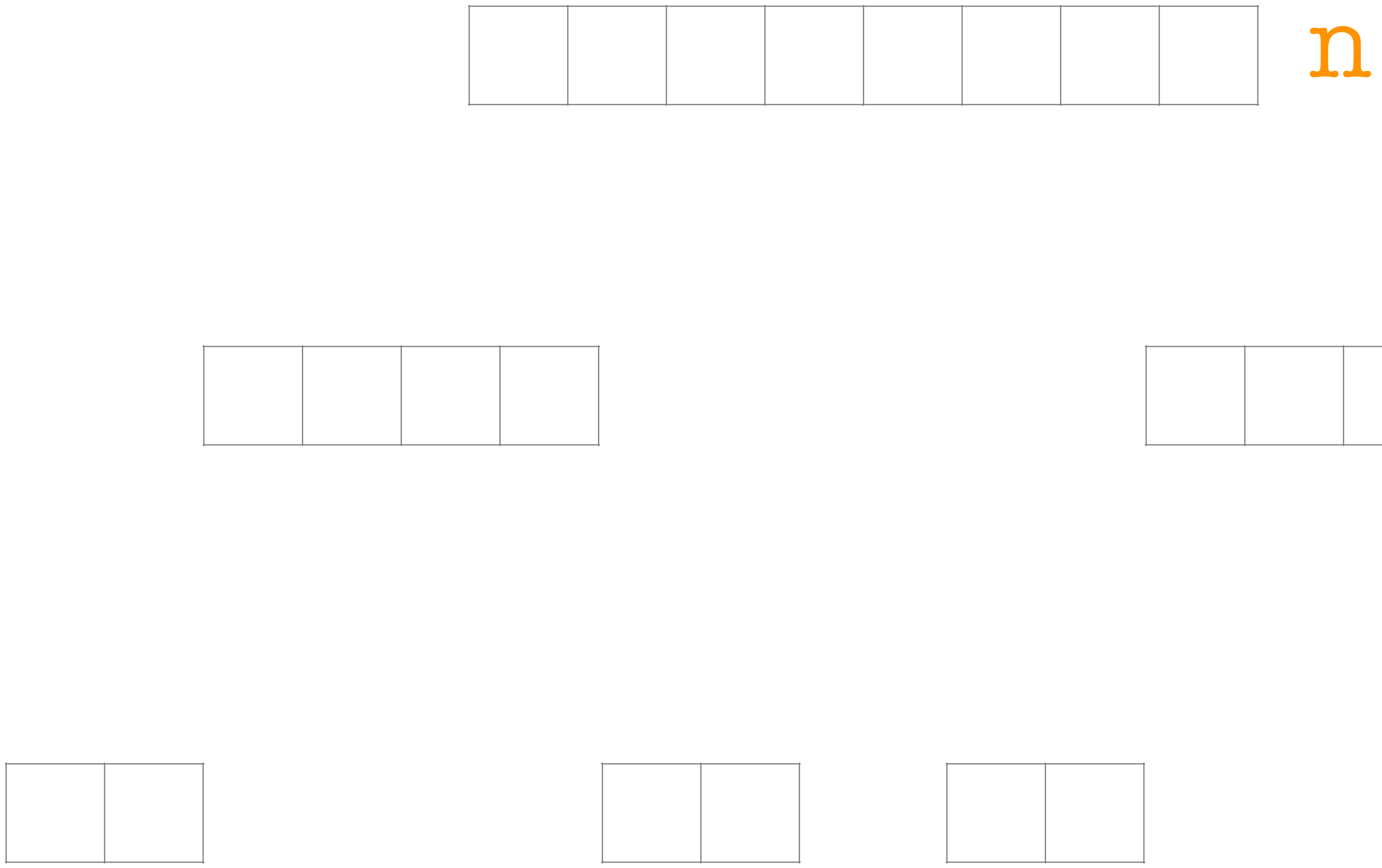
dividir para conquistar

$$2T(n/2) + n$$



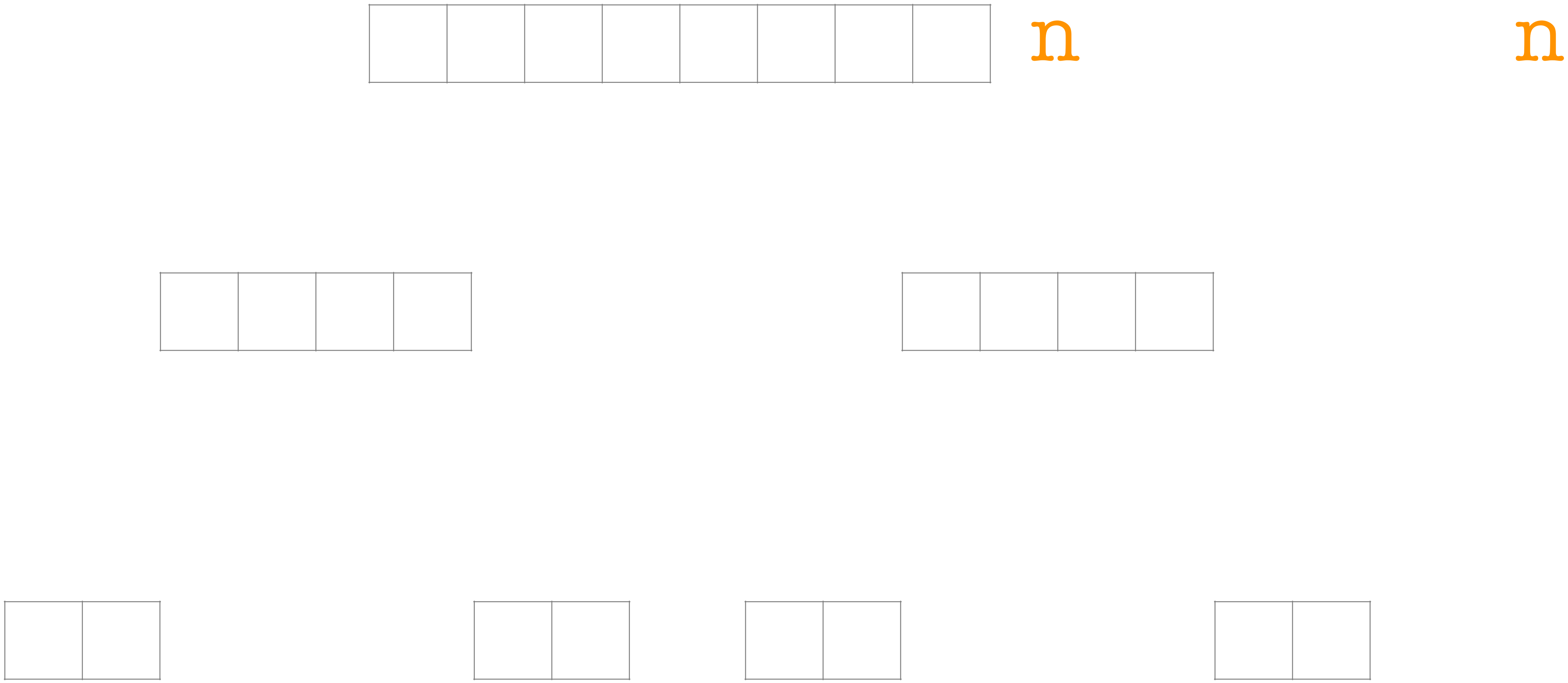
dividir para conquistar

$$2T(n/2) + n$$



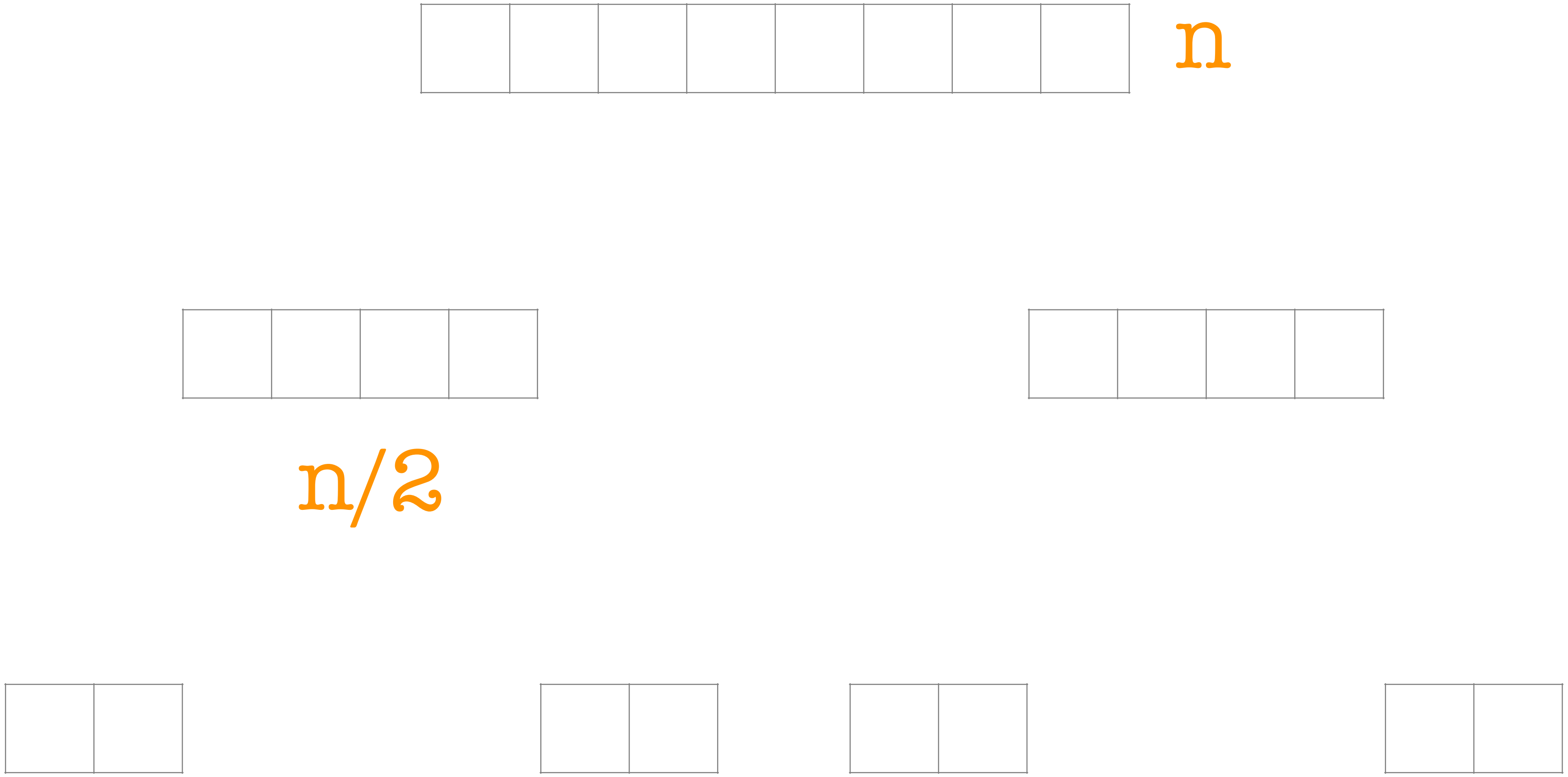
dividir para conquistar

$2T(n/2) + n$



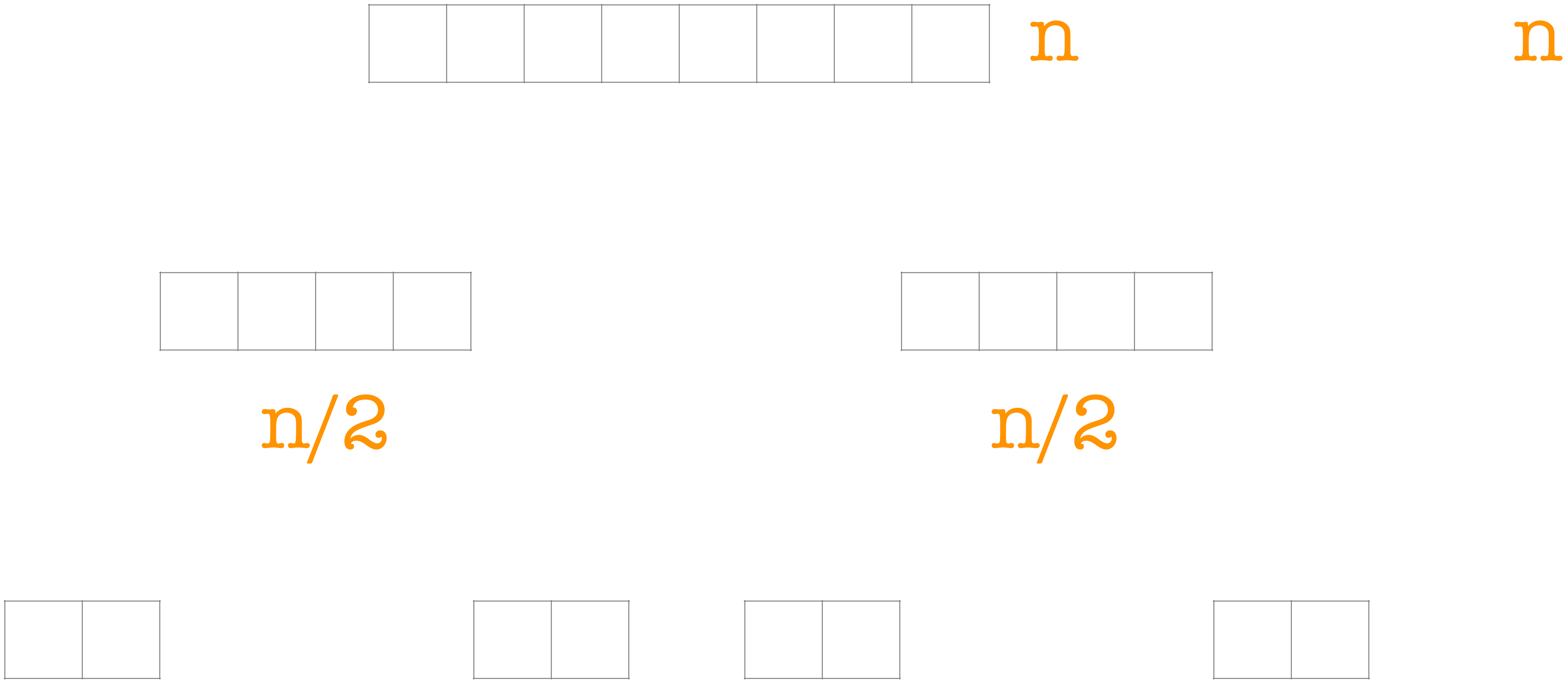
dividir para conquistar

$$2T(n/2) + n$$



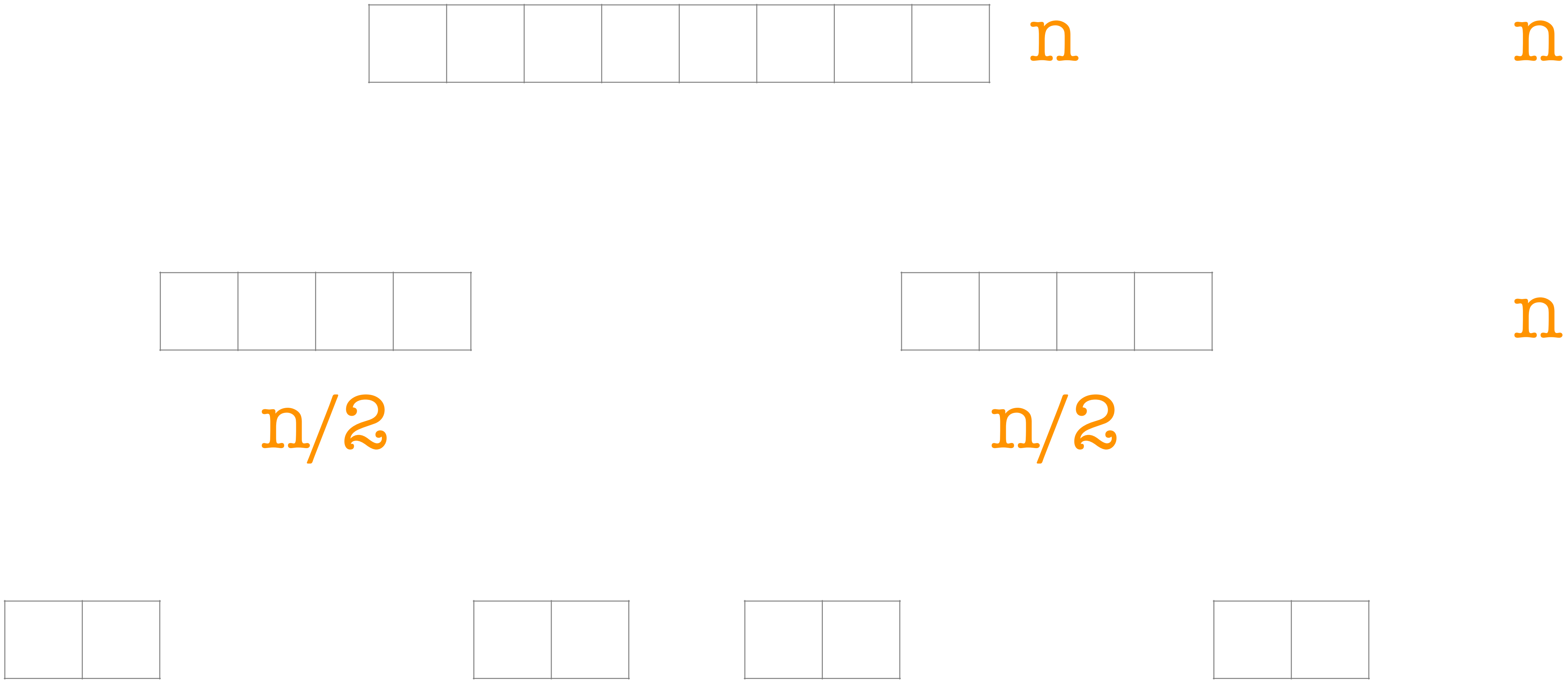
dividir para conquistar

$2T(n/2) + n$



dividir para conquistar

$2T(n/2) + n$


n

n

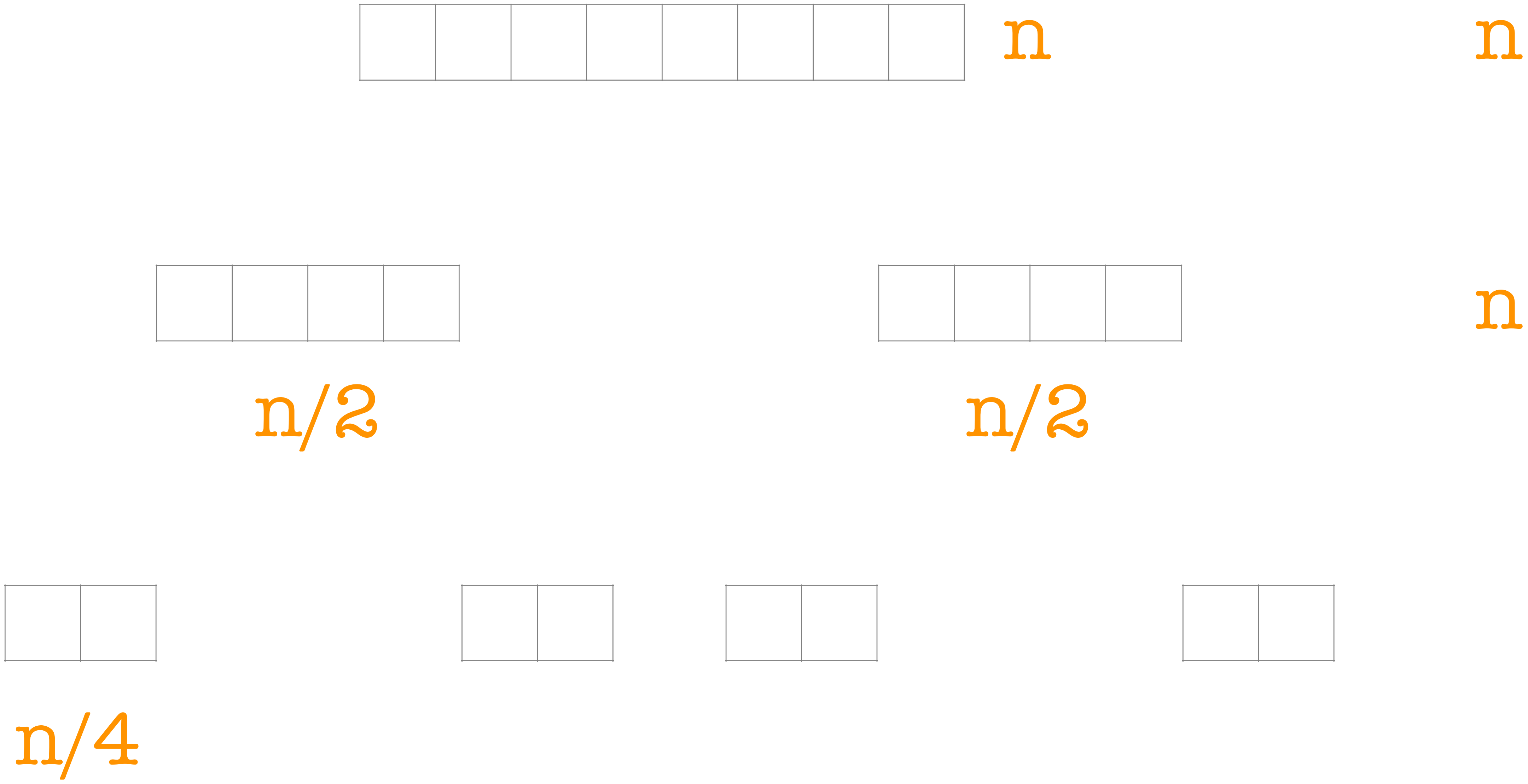
n/2

n/2

n

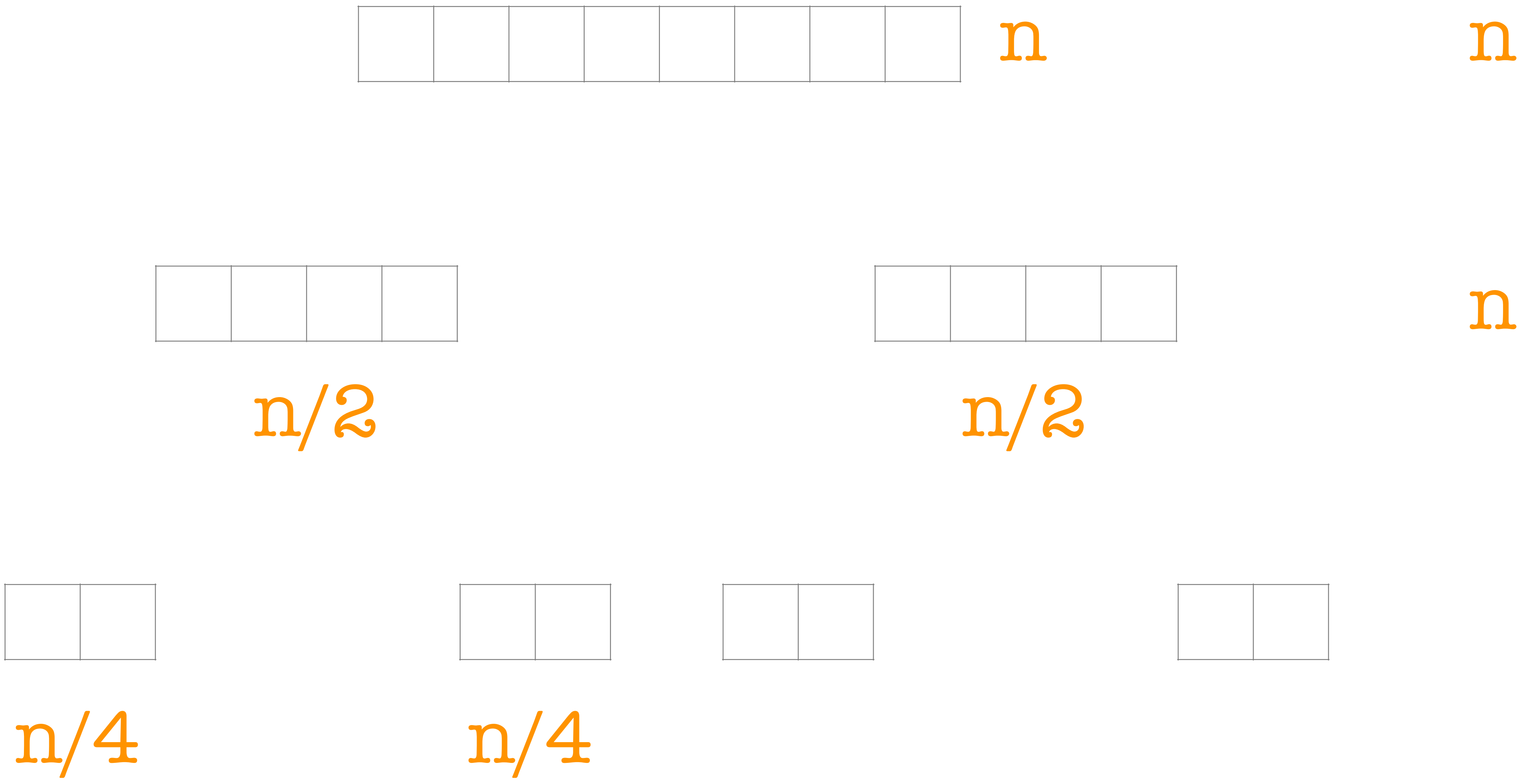
dividir para conquistar

$$2T(n/2) + n$$



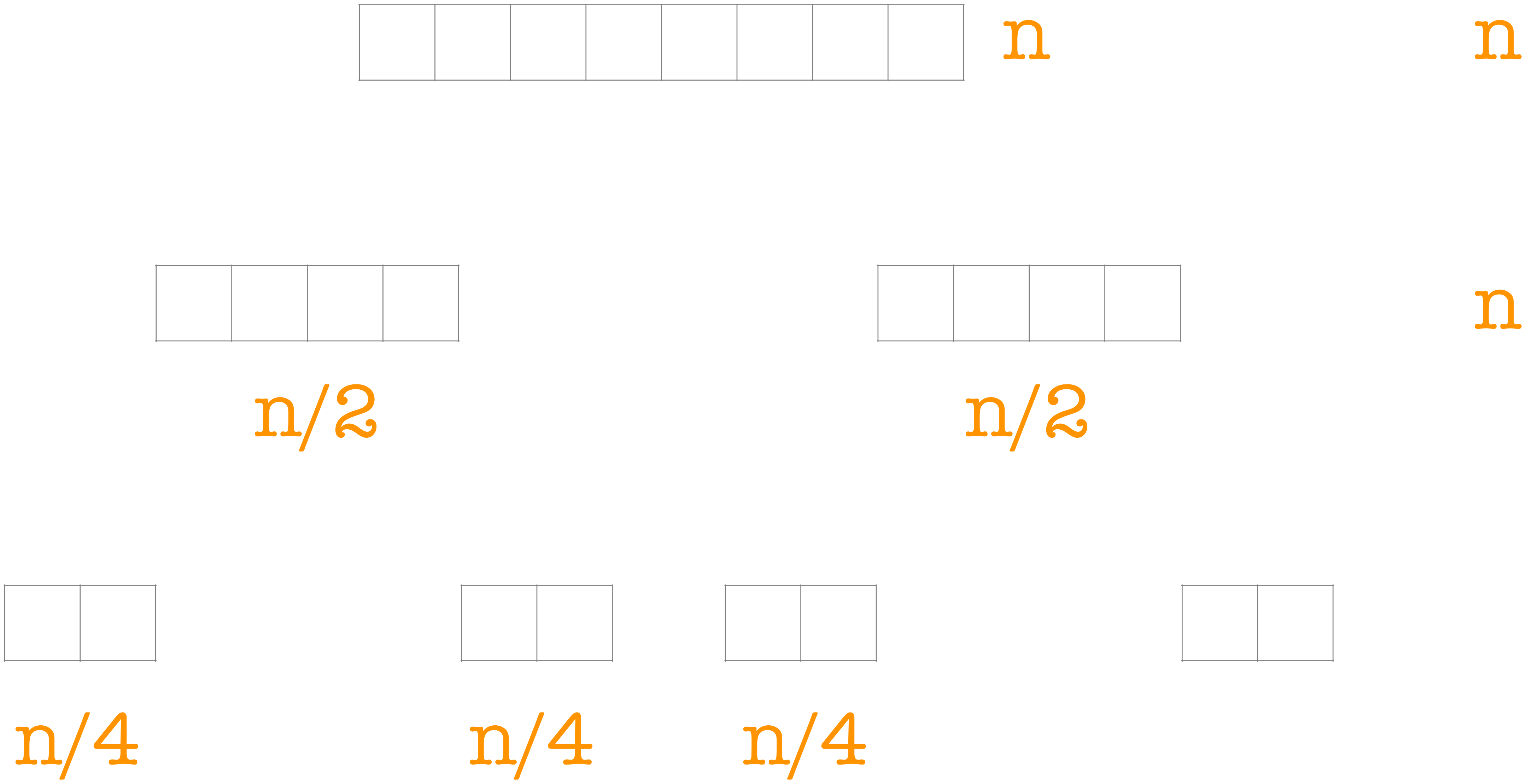
dividir para **conquistar**

$$2T(n/2) + n$$



dividir para conquistar

$2T(n/2) + n$



n

n

n

n/2

n/2

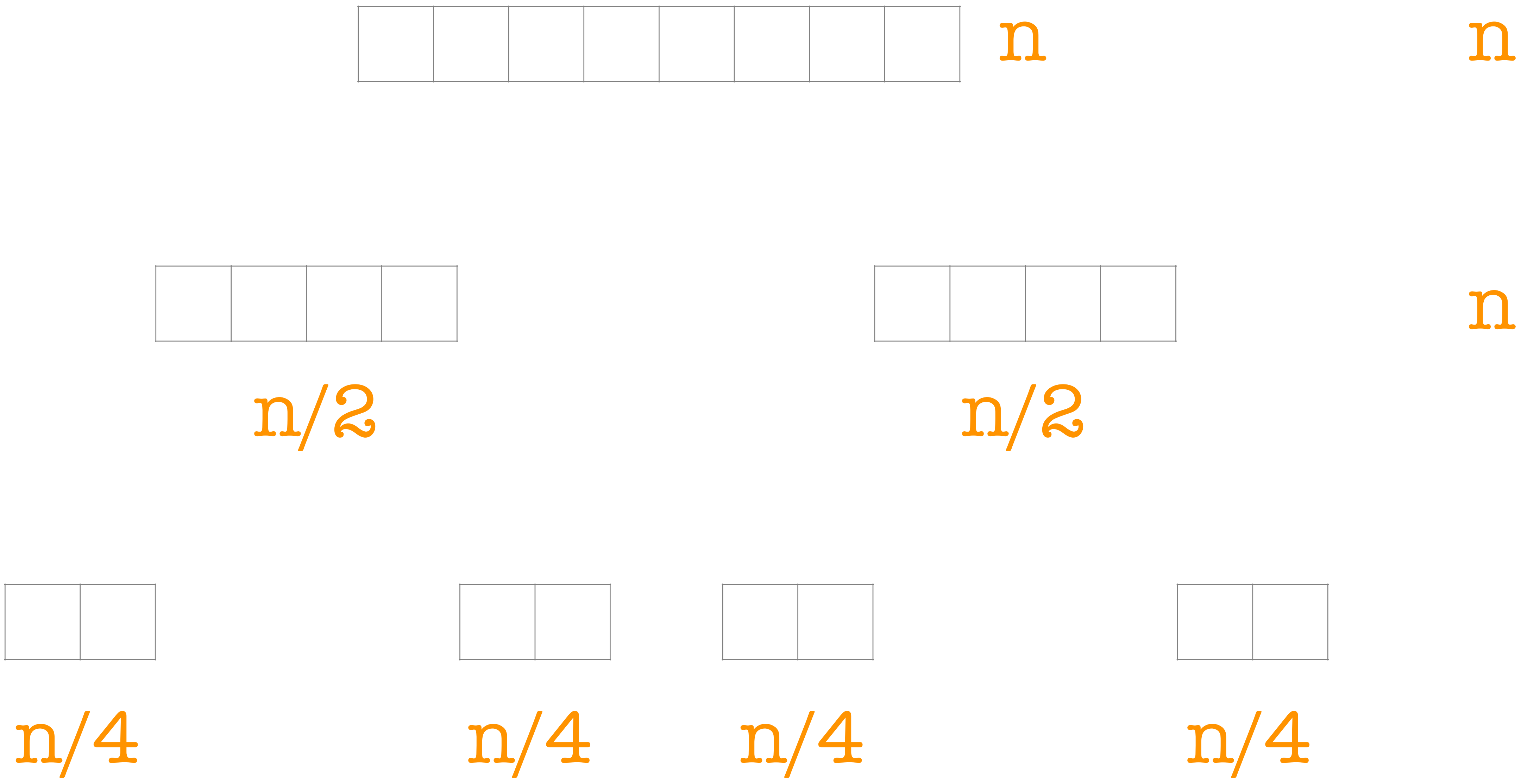
n/4

n/4

n/4

dividir para conquistar

$2T(n/2) + n$



n

n

n

n/2

n/2

n/4

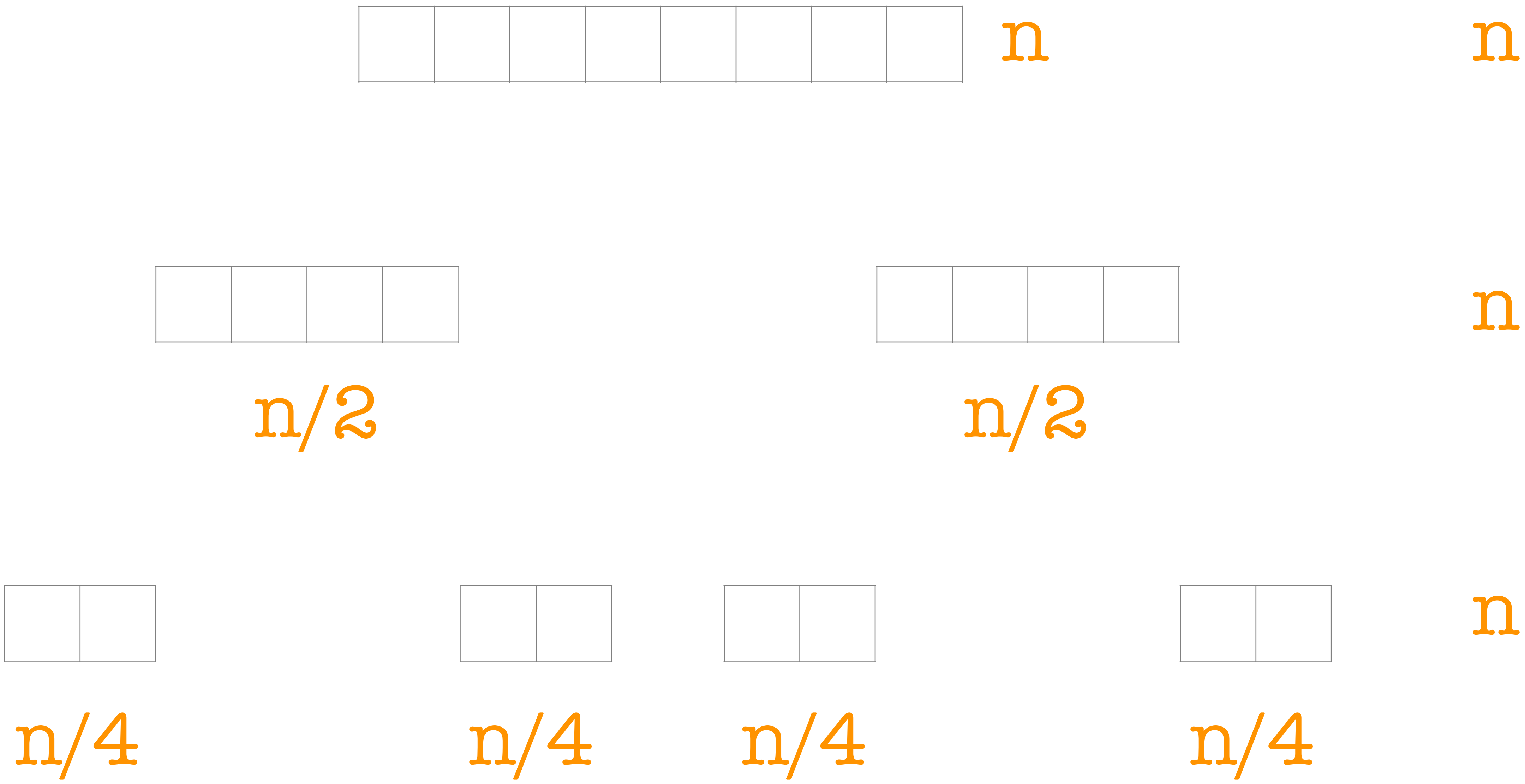
n/4

n/4

n/4

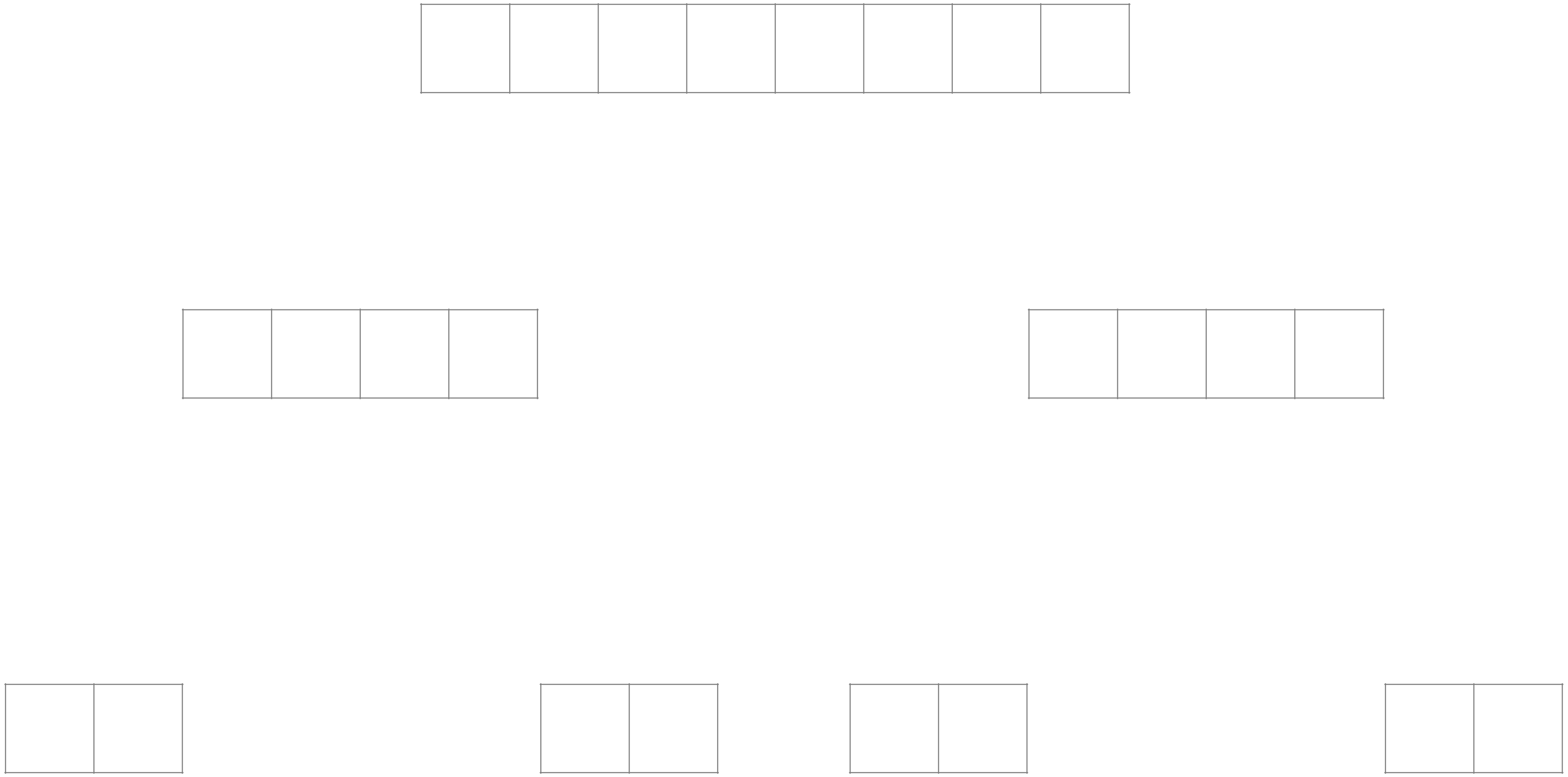
dividir para conquistar

$2T(n/2) + n$



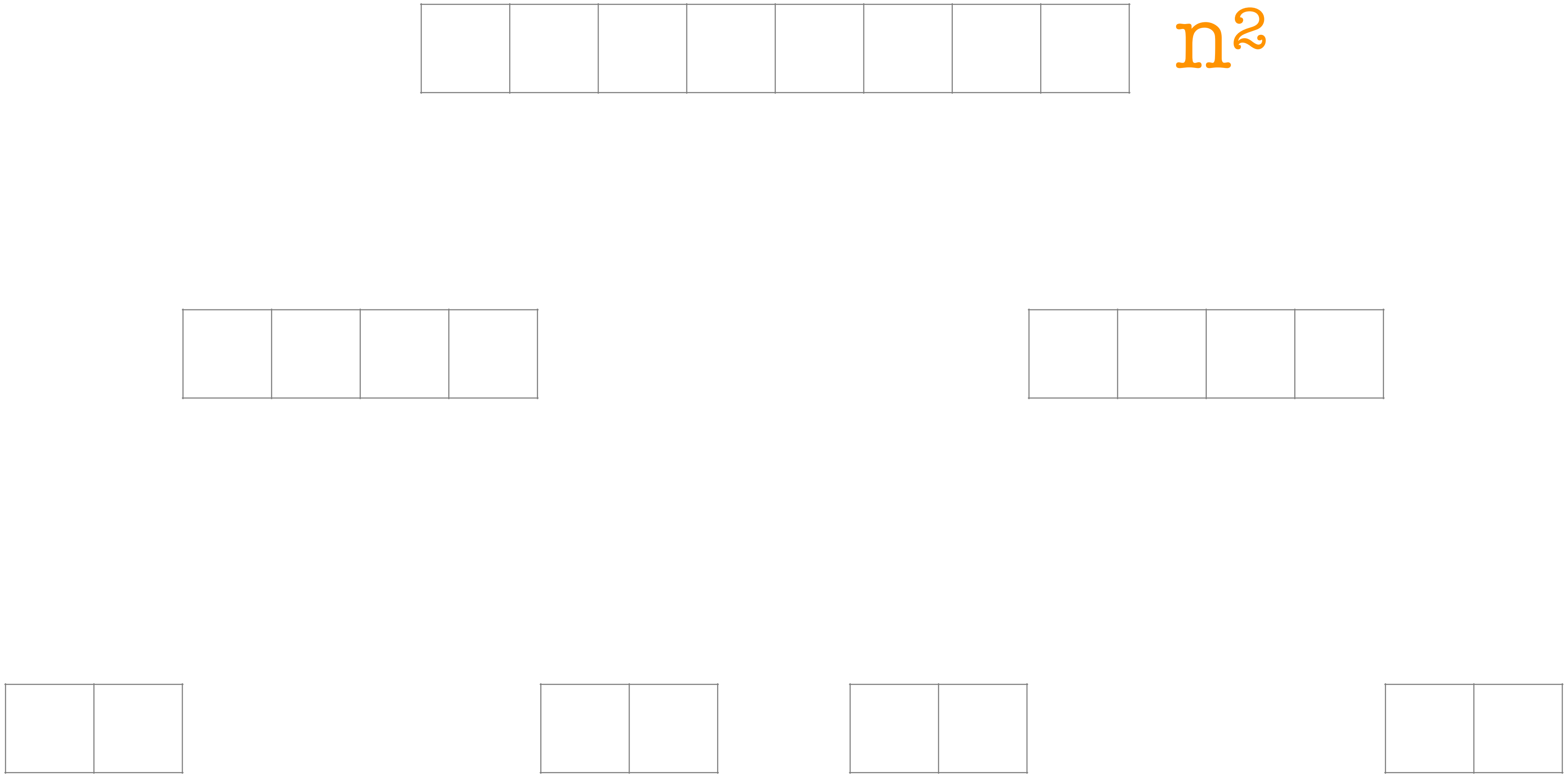
dividir para conquistar

$$2T(n/2) + n^2$$



dividir para conquistar

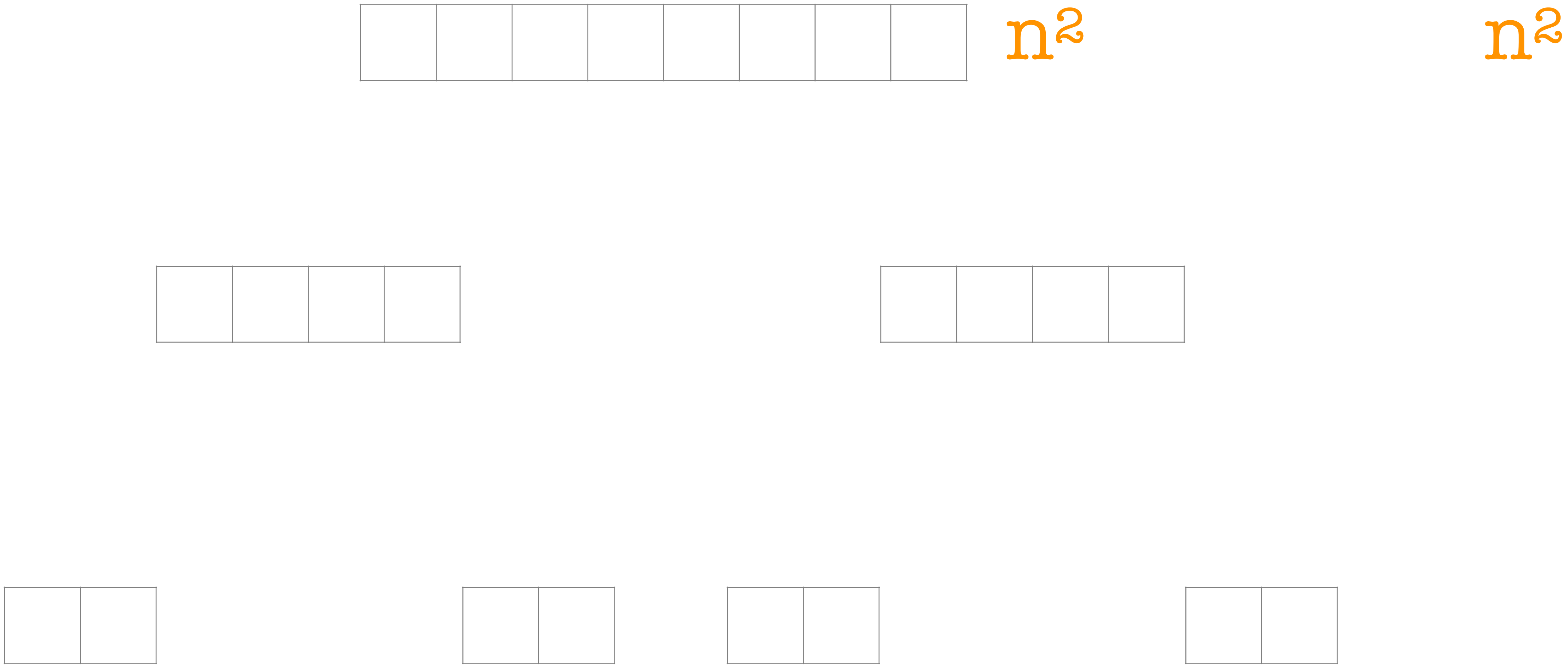
$2T(n/2) + n^2$



n^2

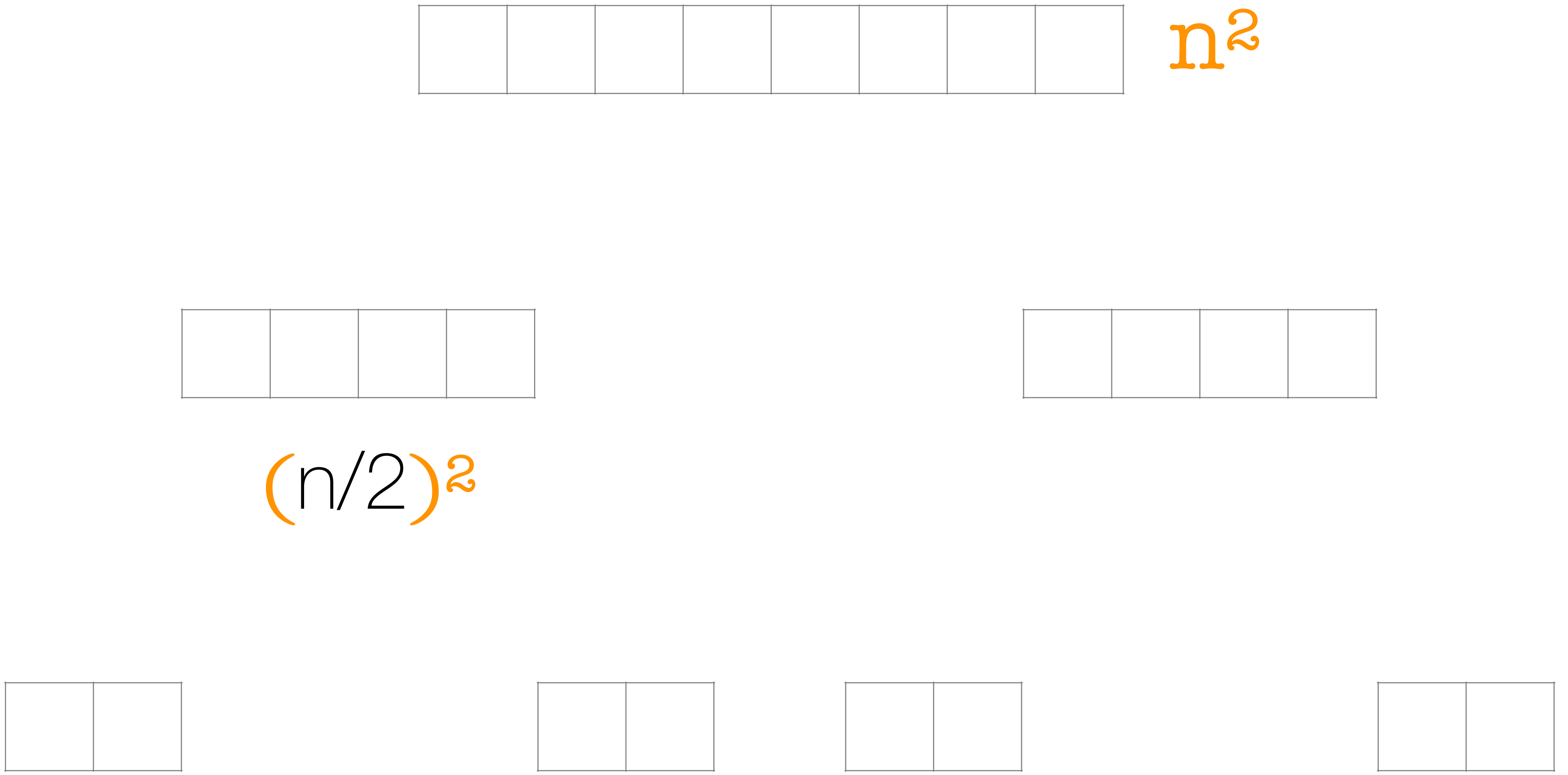
dividir para conquistar

$2T(n/2) + n^2$



dividir para conquistar

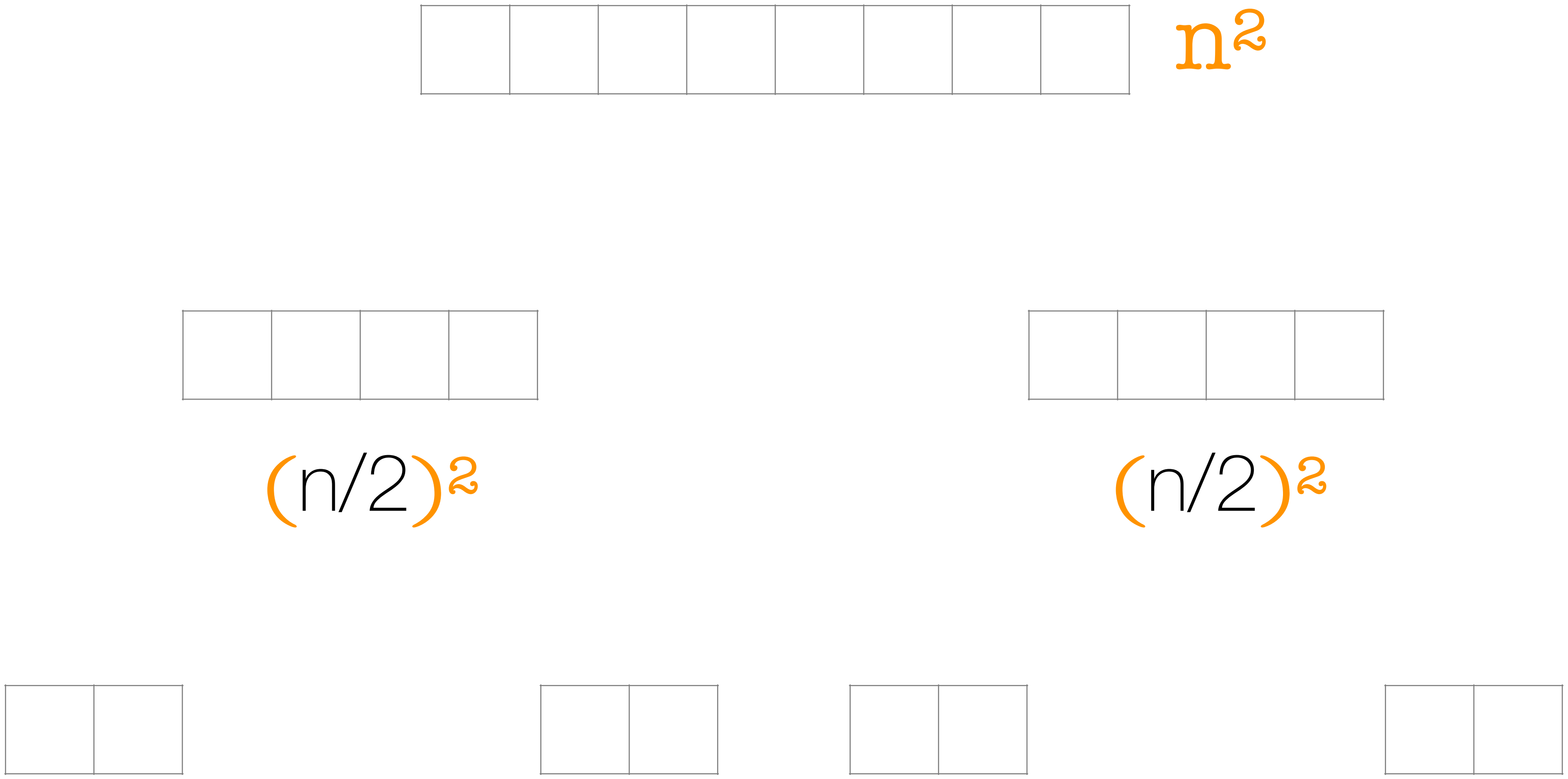
$2T(n/2) + n^2$



n^2

dividir para conquistar

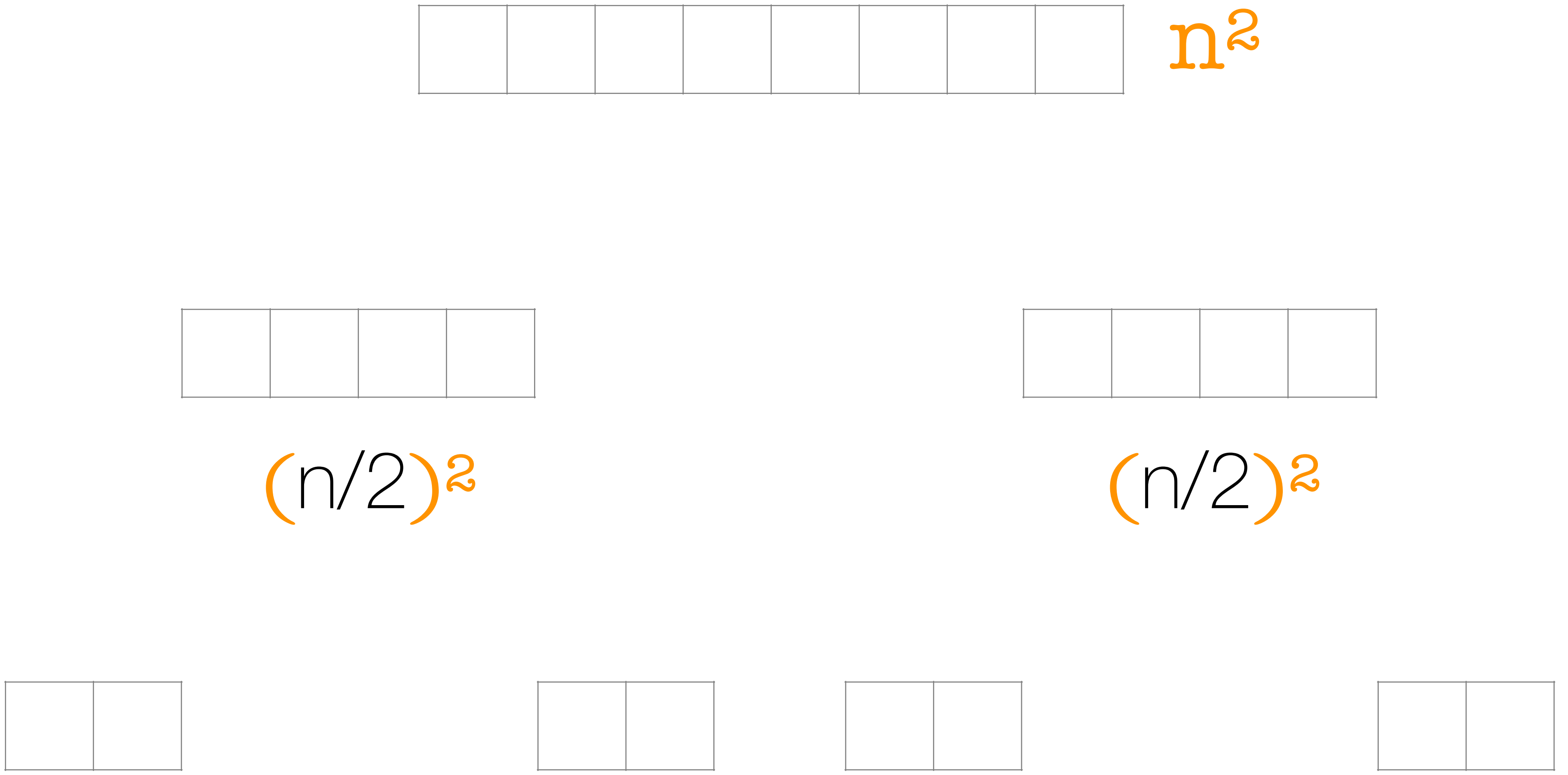
$2T(n/2) + n^2$



n^2

dividir para conquistar

$$2T(n/2) + n^2$$



n^2

$(n/2)^2$

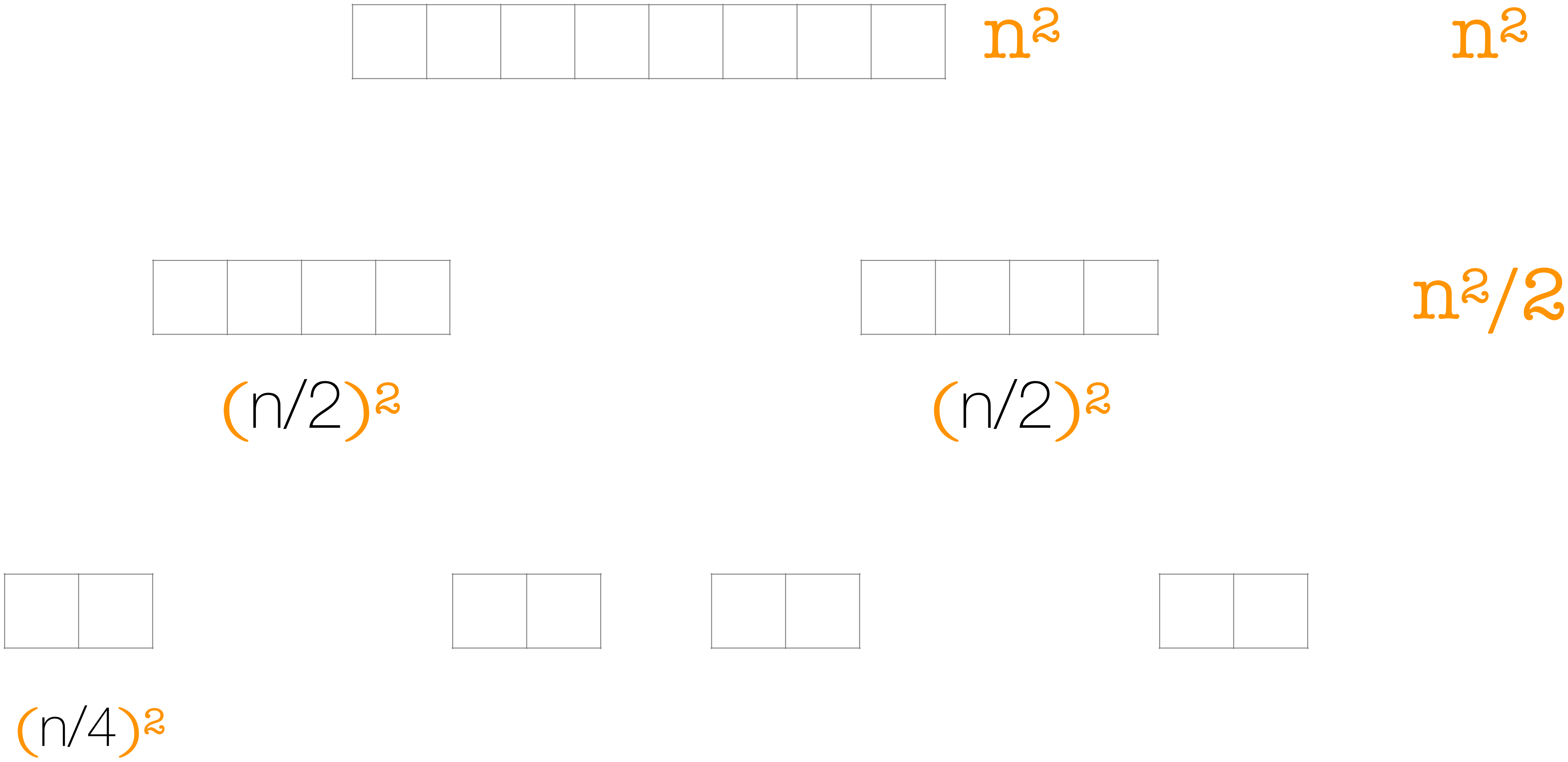
$(n/2)^2$

n^2

$n^2/2$

dividir para conquistar

$$2T(n/2) + n^2$$



n^2

n^2

$n^2/2$

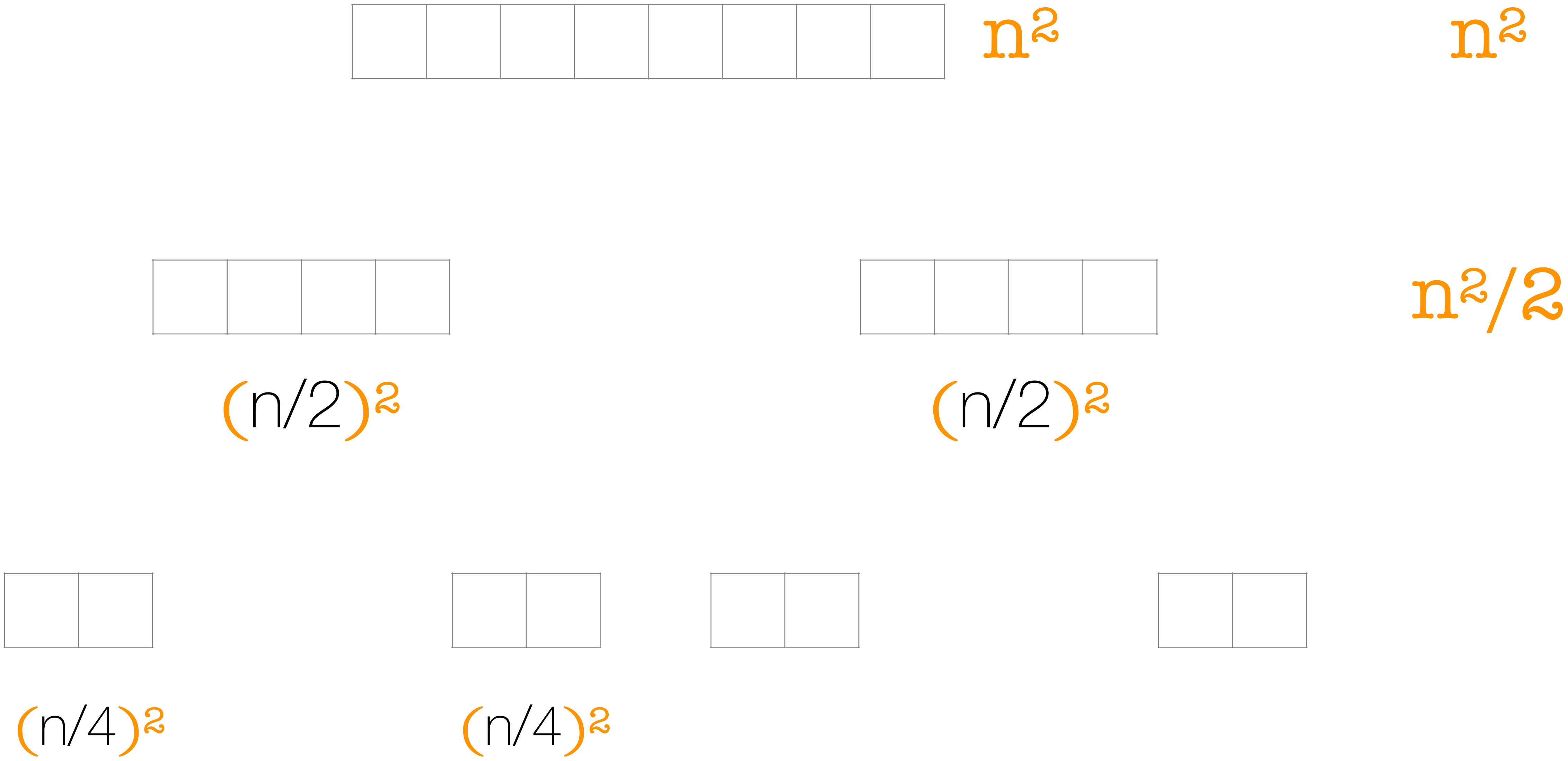
$(n/2)^2$

$(n/2)^2$

$(n/4)^2$

dividir para conquistar

$$2T(n/2) + n^2$$



n^2

n^2

$n^2/2$

$(n/2)^2$

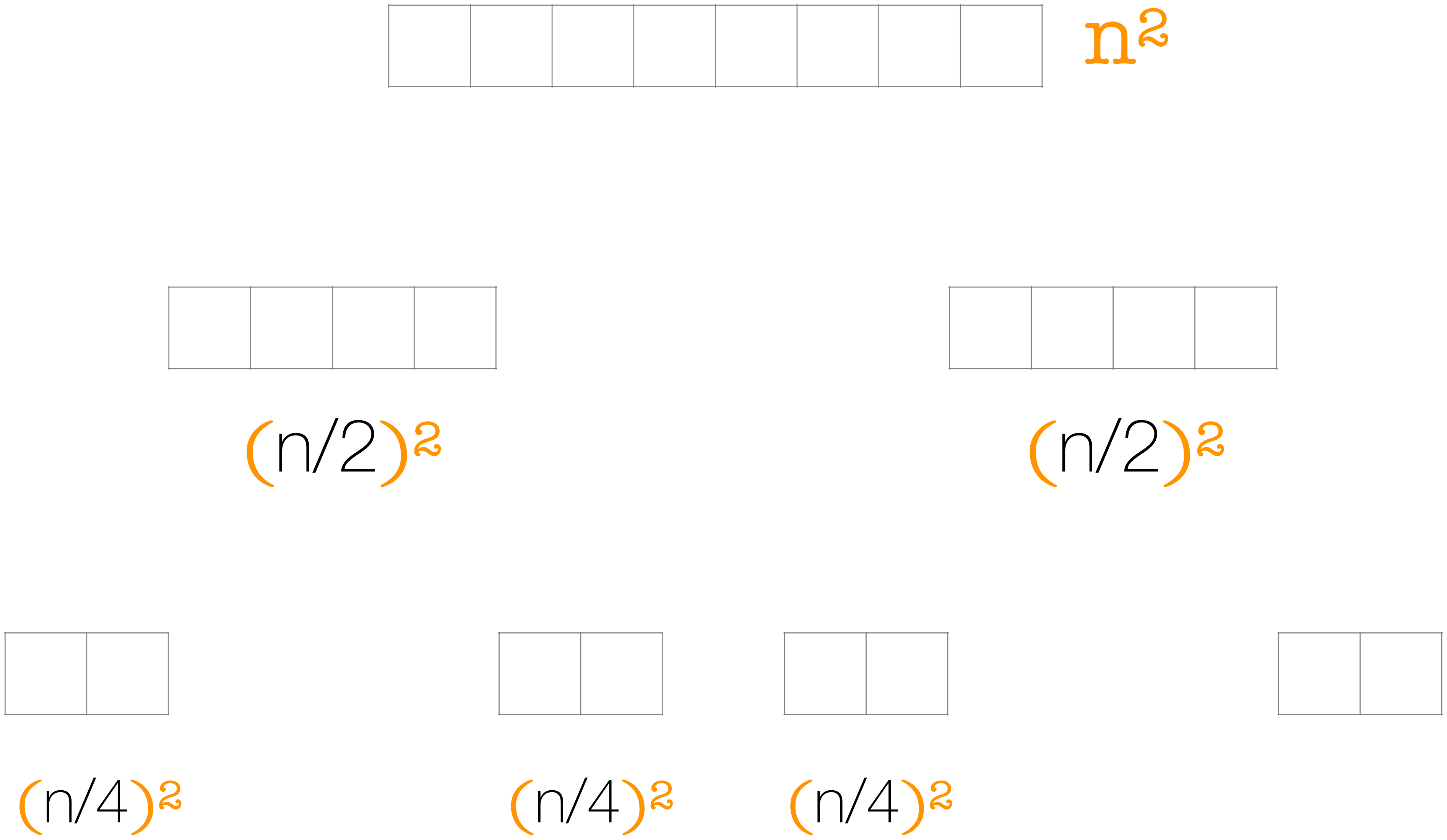
$(n/2)^2$

$(n/4)^2$

$(n/4)^2$

dividir para conquistar

$$2T(n/2) + n^2$$

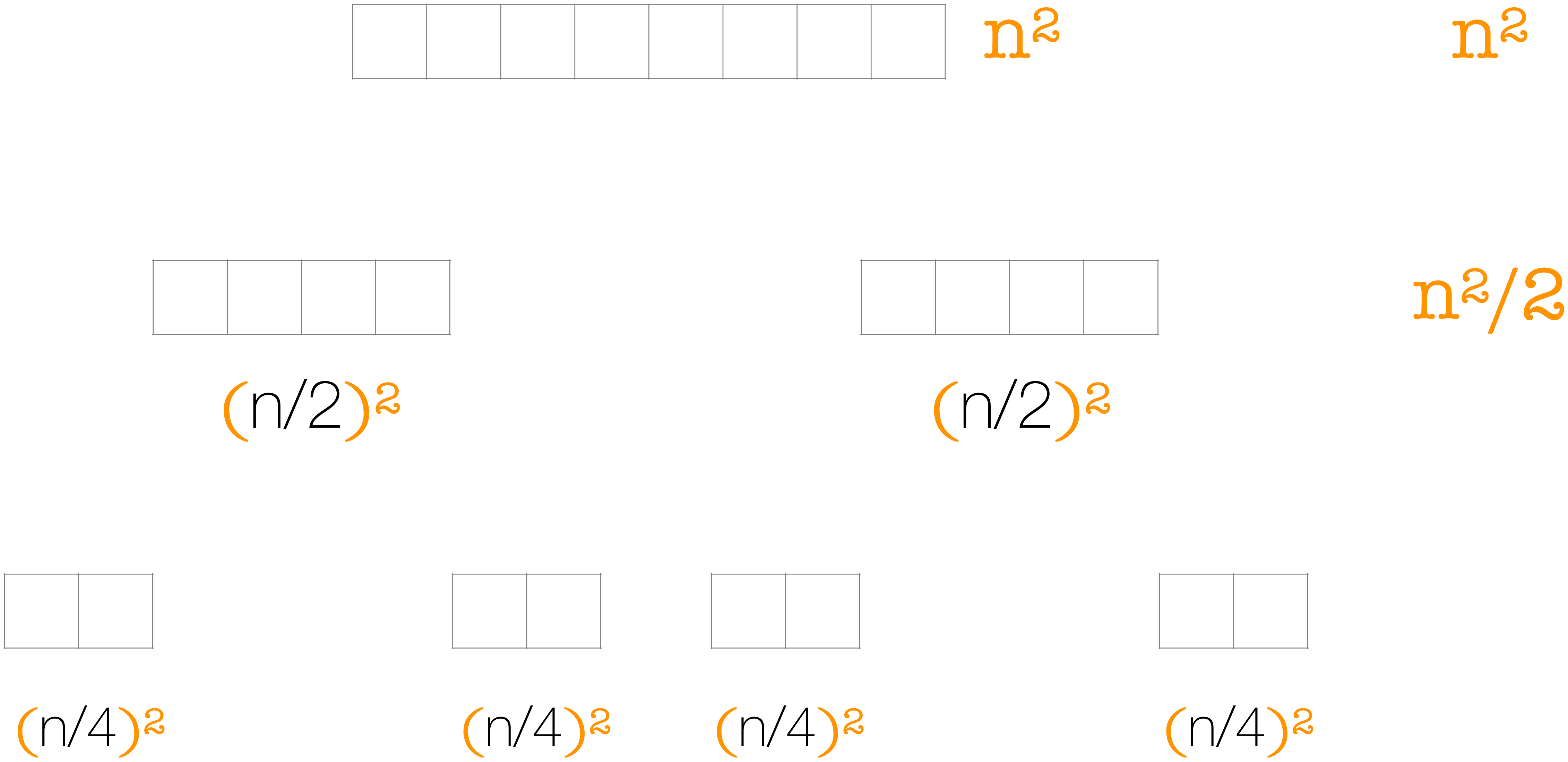


n^2

$n^2/2$

dividir para conquistar

$2T(n/2) + n^2$



n^2 n^2

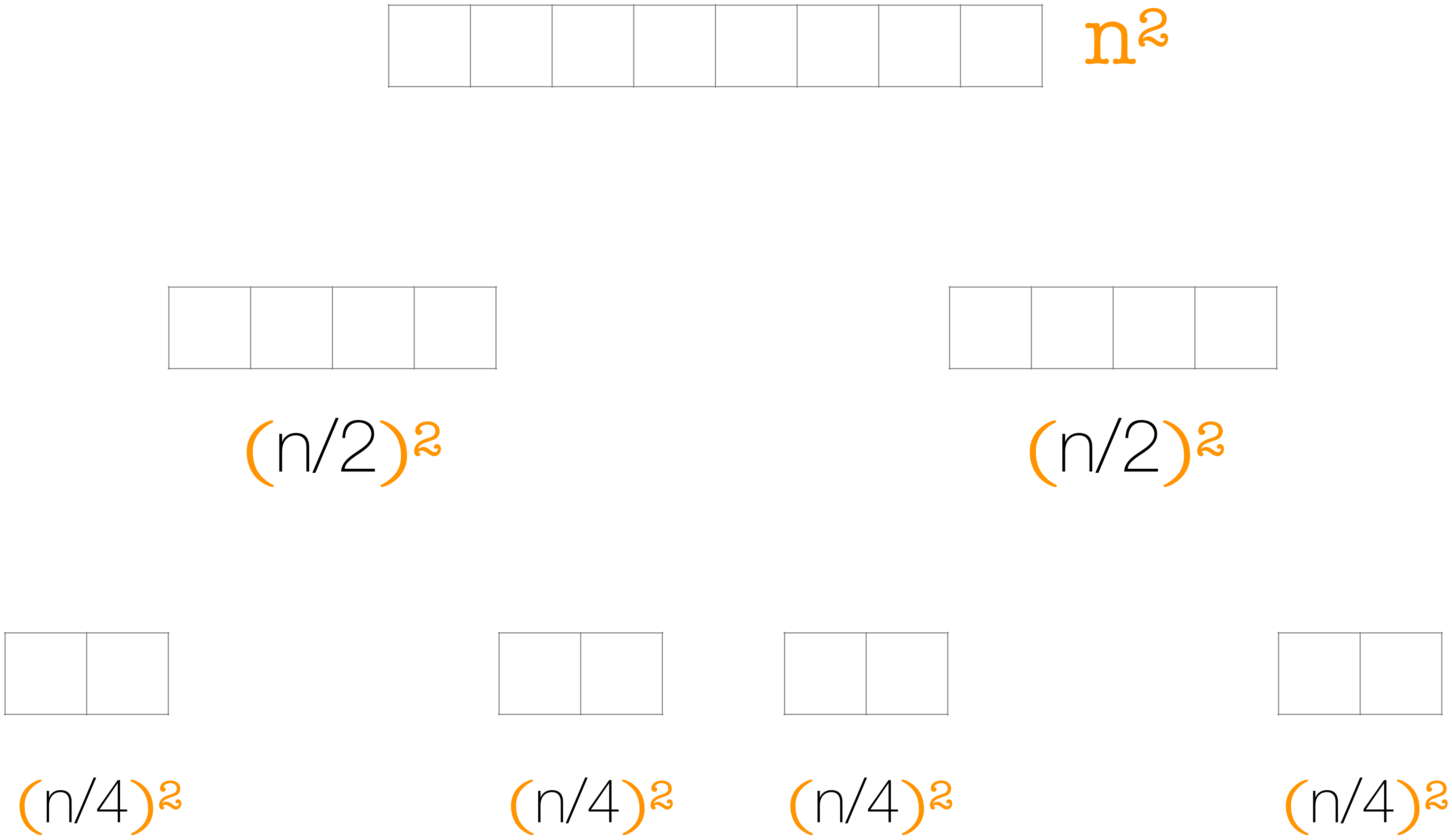
$n^2/2$

$(n/2)^2$ $(n/2)^2$

$(n/4)^2$ $(n/4)^2$ $(n/4)^2$ $(n/4)^2$

dividir para conquistar

$2T(n/2) + n^2$



n^2

$(n/2)^2$ $(n/2)^2$

$(n/4)^2$ $(n/4)^2$ $(n/4)^2$ $(n/4)^2$

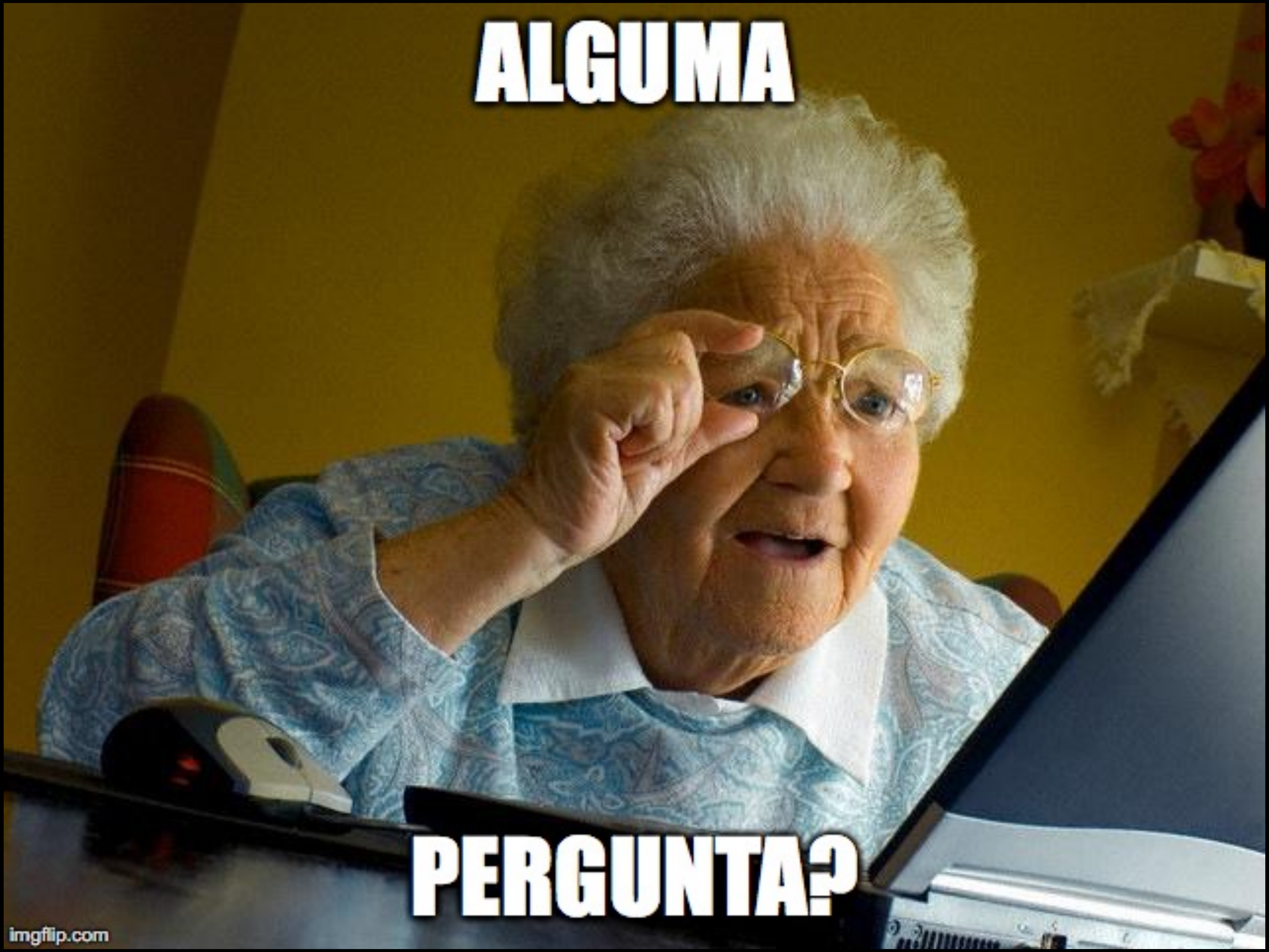
n^2

$n^2/2$

$n^2/4$

dividir para conquistar

$$T(n) = a\underbrace{T(n/b)}_{\text{dividir}} + \overbrace{f(n)}^{\text{conquistar}}$$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dividir para conquistar	intersecções entre subproblemas
recursão de cauda	demais casos

sequência de **fibonacci**

sequência de **fibonacci**

- 1 1 2 3 5 8 13 21 34 ...

sequência de **fibonacci**

- 1 1 2 3 5 8 13 21 34 ...

- fib(n) = fib(n-1) + fib(n-2)

sequência de fibonacci

- 1 1 2 3 5 8 13 21 34 ...
- fib(n) = fib(n-1) + fib(n-2)
- fib(0) = 0, fib(1) = 1

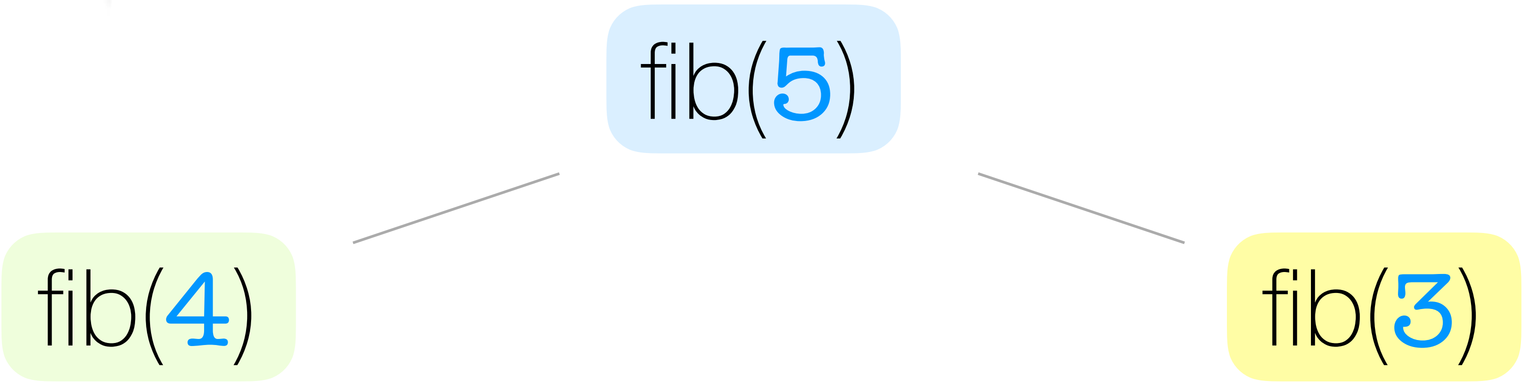
sequência de fibonacci

```
int fib_rec(int n)
{
   if (n <= 1) return n;
   return fib(n-1) + fib(n-2);
}
```

- 1 1 2 3 5 8 13 21 34 ...
- fib(n) = fib(n-1) + fib(n-2)
- fib(0) = 0, fib(1) = 1

intersecções entre subproblemas

intersecções entre subproblemas

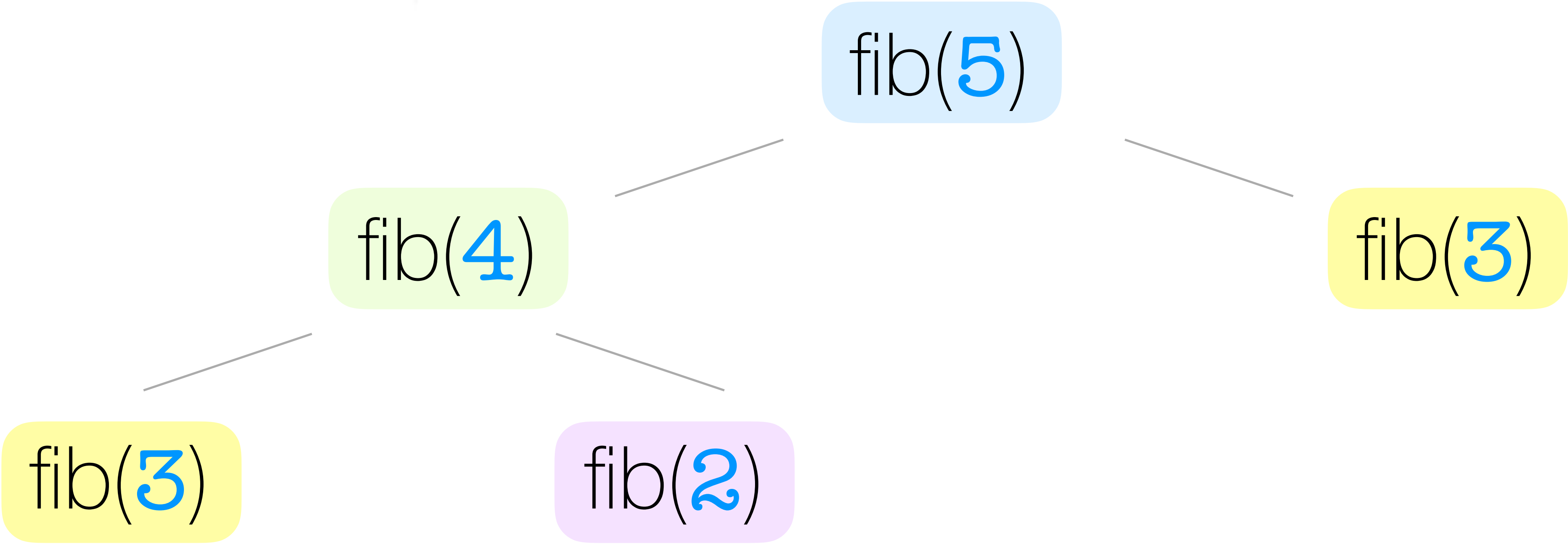


fib(5)

fib(4)

fib(3)

intersecções entre subproblemas



fib(5)

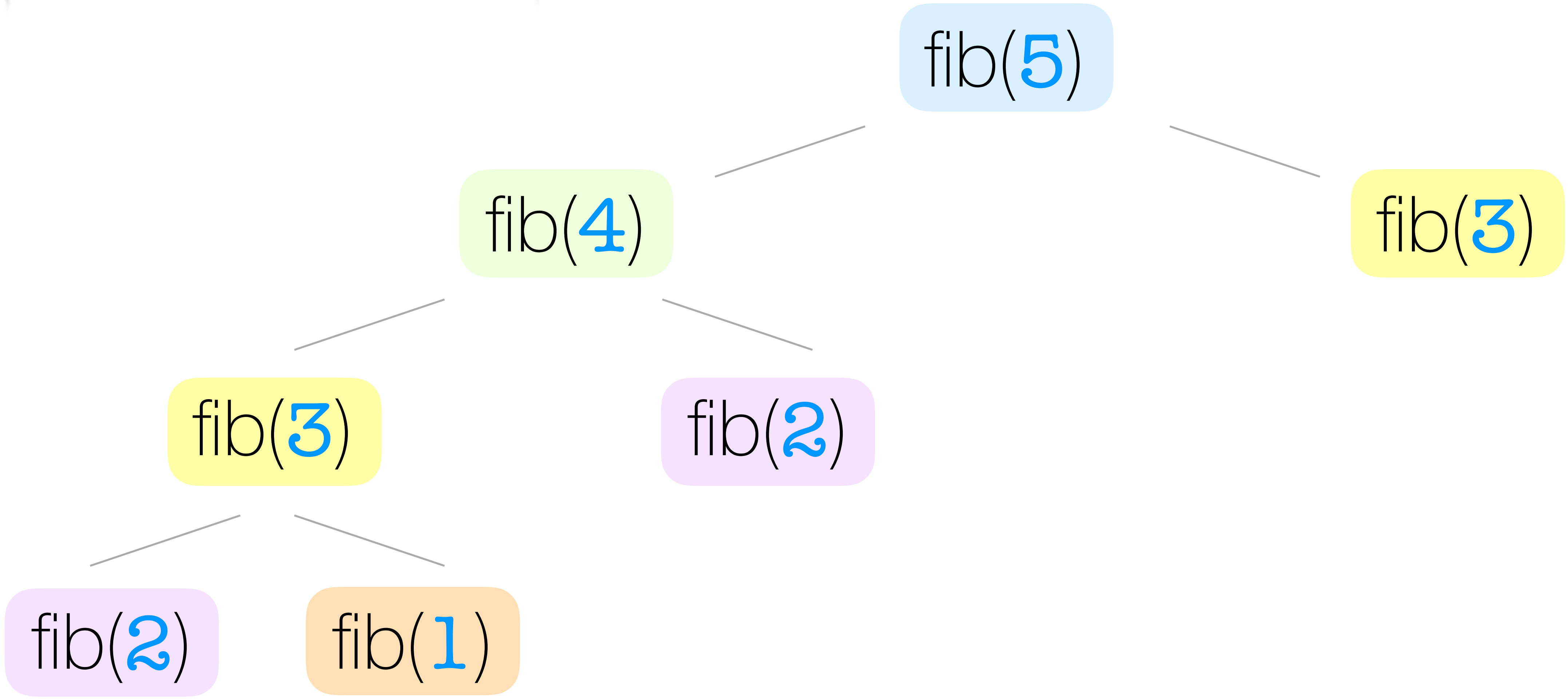
fib(4)

fib(3)

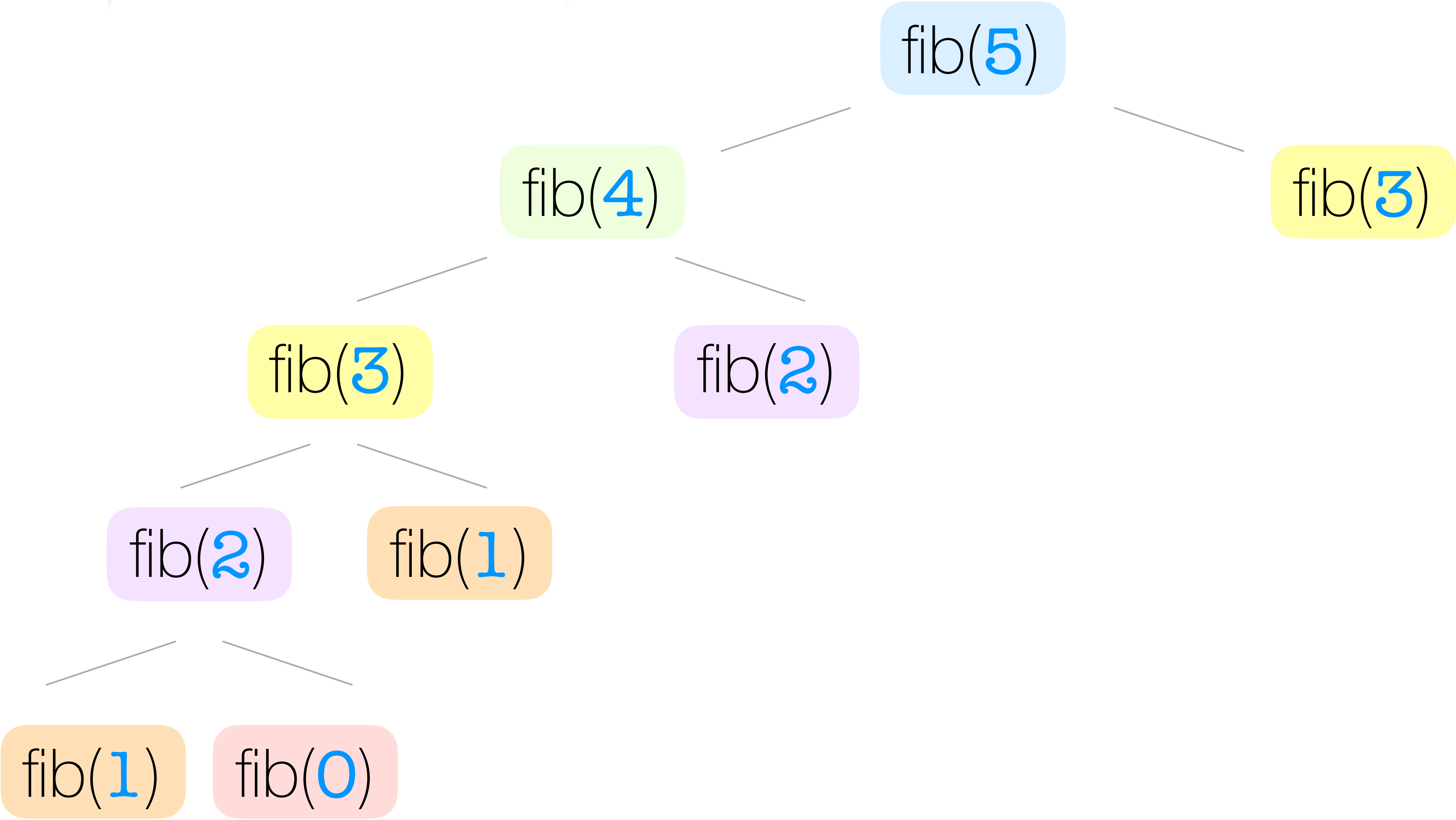
fib(3)

fib(2)

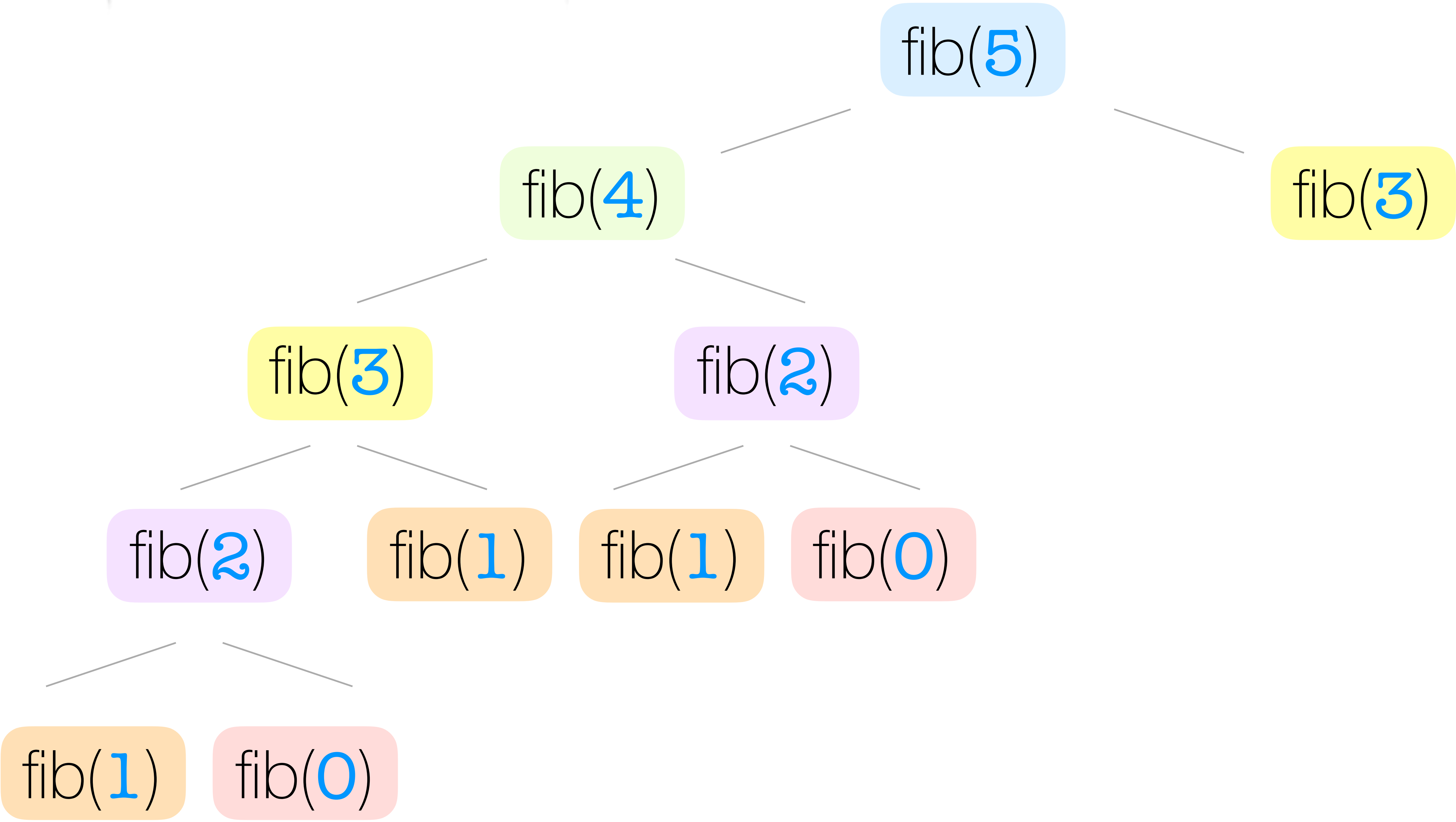
intersecções entre subproblemas



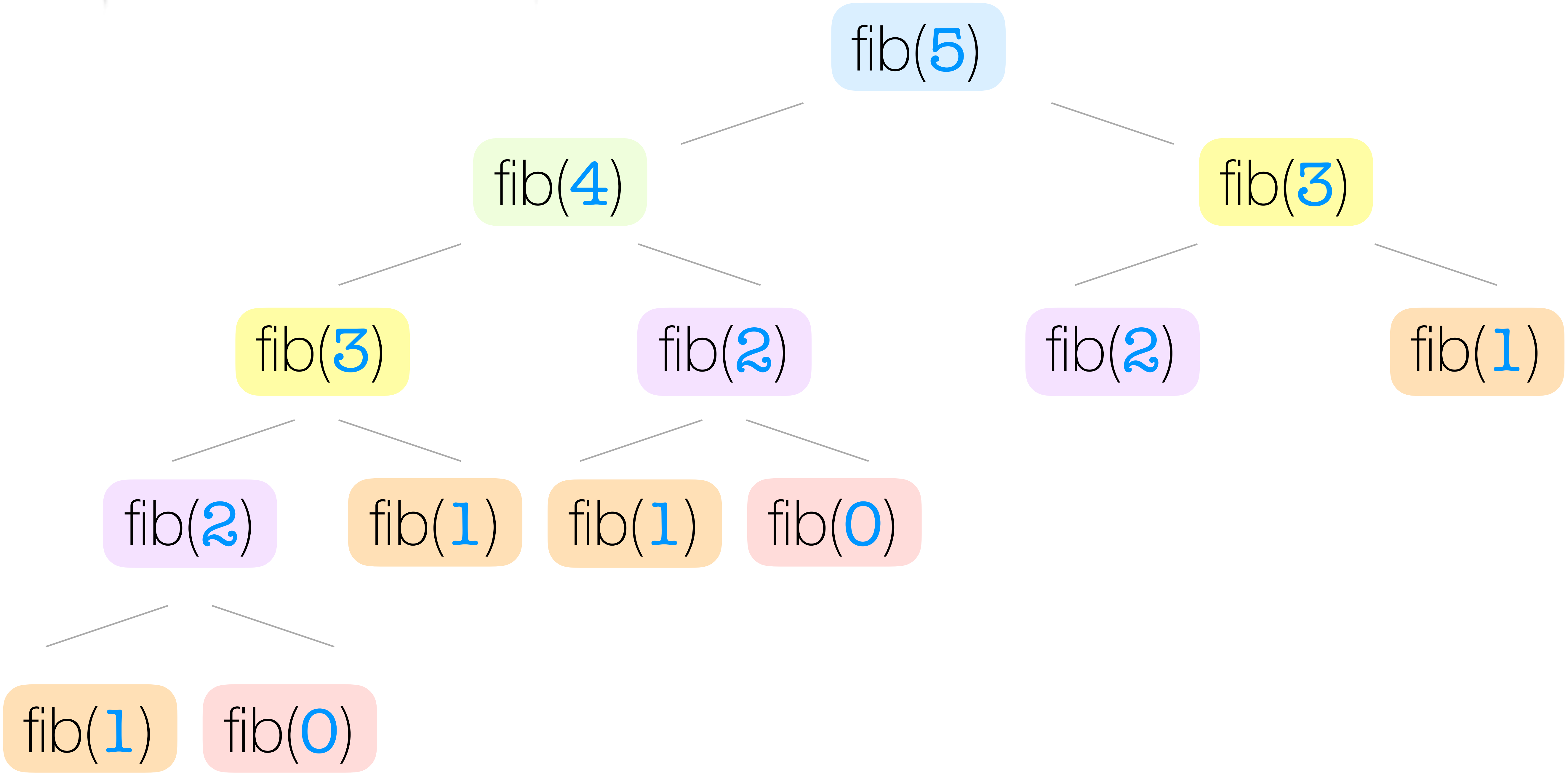
intersecções entre subproblemas



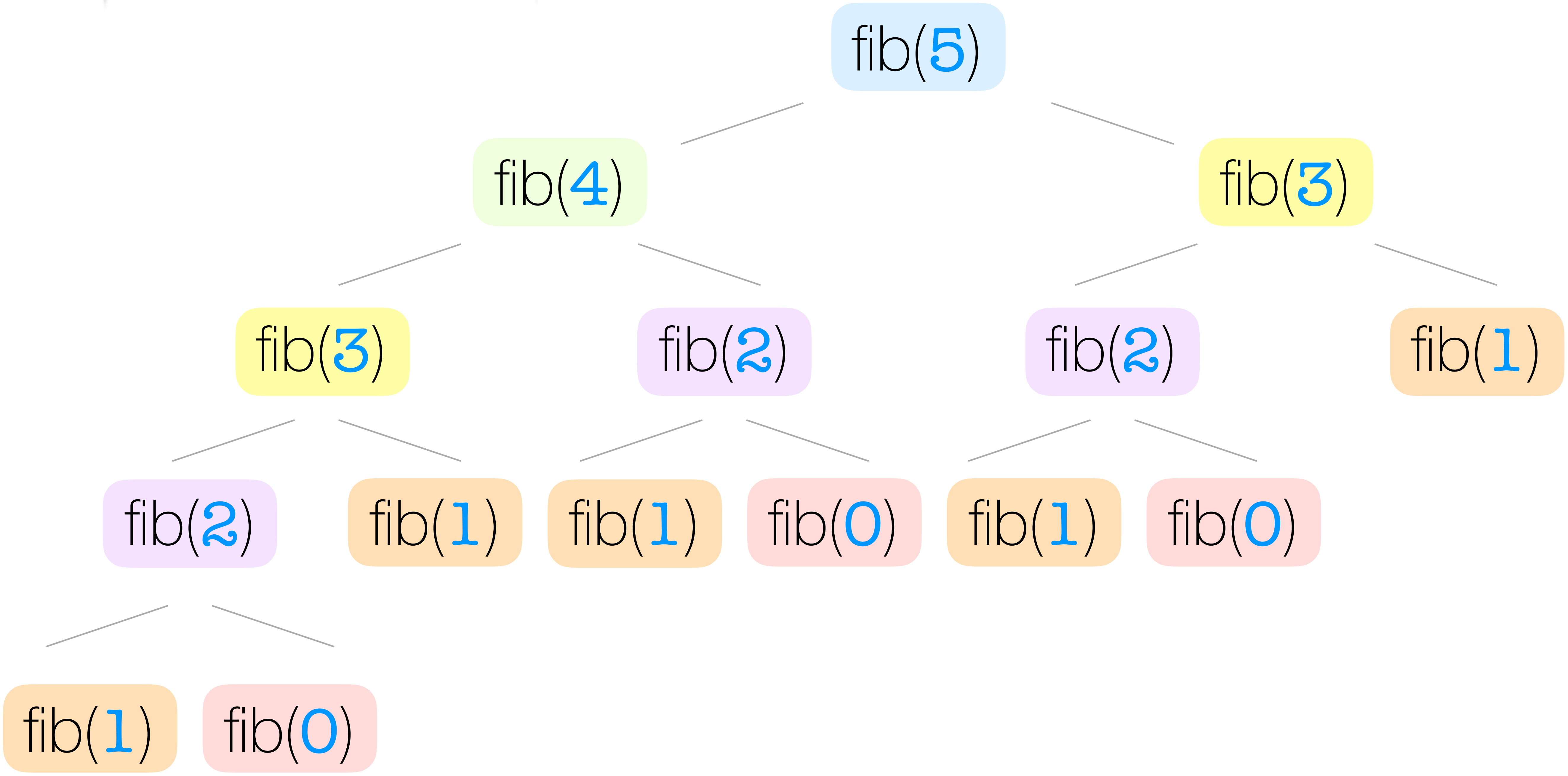
intersecções entre subproblemas



intersecções entre subproblemas



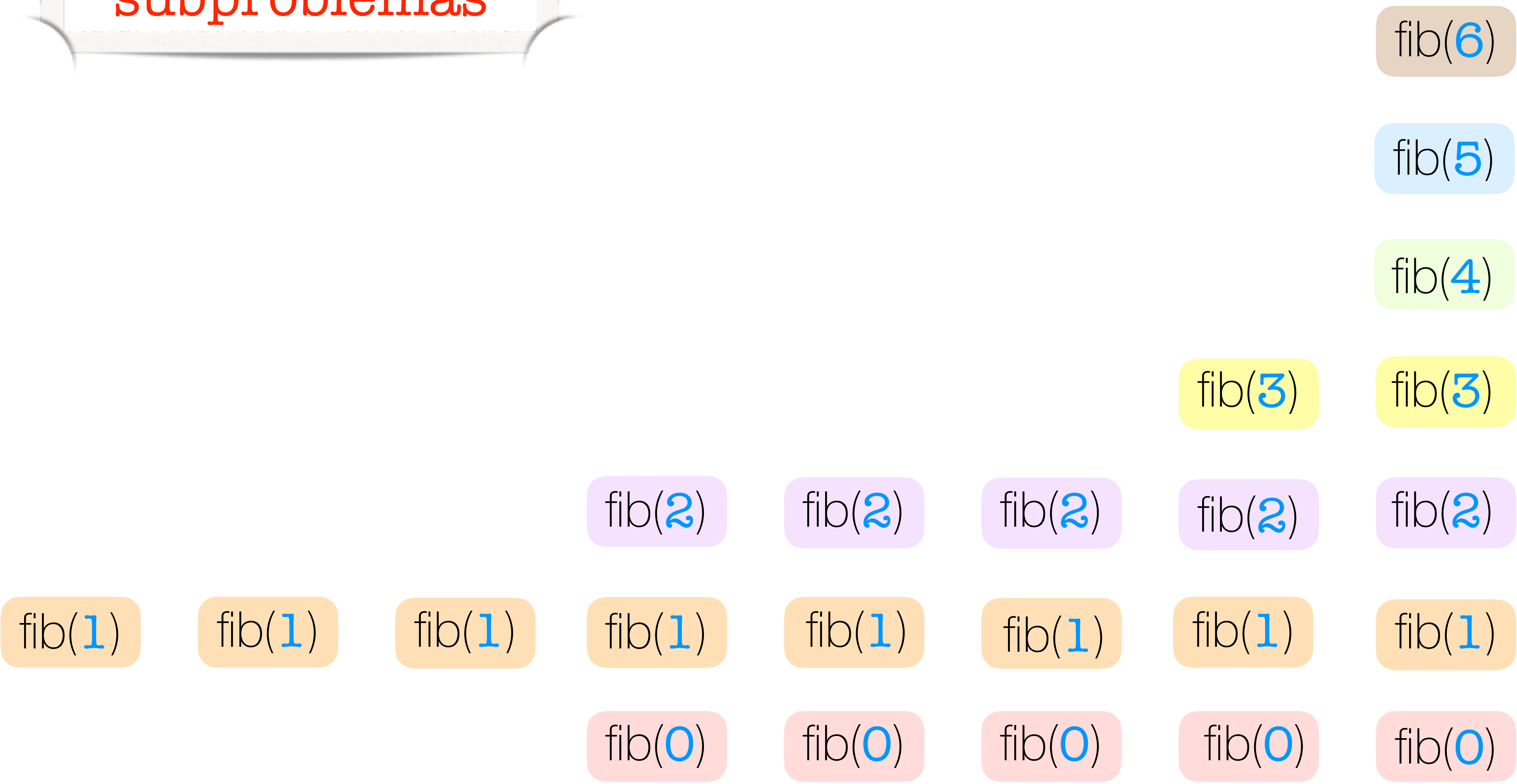
intersecções entre subproblemas



intersecções entre subproblemas

				fib(5)
			fib(4)	fib(4)
		fib(3)	fib(3)	fib(3)
		fib(2)	fib(2)	fib(2)
fib(1)	fib(1)	fib(1)	fib(1)	fib(1)
		fib(0)	fib(0)	fib(0)

intersecções entre subproblemas


fib(6)

fib(5)

fib(4)

fib(3) fib(3)

fib(2) fib(2) fib(2) fib(2) fib(2)

fib(1) fib(1) fib(1) fib(1) fib(1) fib(1) fib(1) fib(1)

fib(0) fib(0) fib(0) fib(0) fib(0)

intersecções entre subproblemas

(programação dinâmica)

intersecções entre subproblemas

fib(0) fib(1)

(programação dinâmica)

intersecções entre subproblemas

fib(0) fib(1) fib(2)

(programação dinâmica)

intersecções entre subproblemas

fib(0) fib(1) fib(2) fib(3)

(programação dinâmica)

intersecções entre subproblemas

fib(0) fib(1) fib(2) fib(3) fib(4)

(programação dinâmica)

intersecções entre subproblemas

fib(0) fib(1) fib(2) fib(3) fib(4) fib(5)

(programação dinâmica)

intersecções entre subproblemas

fib(0) fib(1) fib(2) fib(3) fib(4) fib(5) fib(6)

(programação dinâmica)

(programação dinâmica)

```
int fib_iter(int n)
```

(programação dinâmica)

```
int fib_iter(int n)
{
```

(programação dinâmica)

```
int fib_iter(int n)
{
  int fib, f1 = 1, f2 = 0;
```

(programação dinâmica)

```
int fib_iter(int n)
{
  int fib, f1 = 1, f2 = 0;
  for (int i = 2; i <= n; i++) {
```

(programação **dinâmica**)

```
int fib_iter(int n)
{
  int fib, f1 = 1, f2 = 0;
  for (int i = 2; i <= n; i++) {
     fib = f1 + f2;
```

(programação dinâmica)

```
int fib_iter(int n)
{
  int fib, f1 = 1, f2 = 0;
  for (int i = 2; i <= n; i++) {
      fib = f1 + f2;
      f2 = f1;
```

(programação dinâmica)

```
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(programação dinâmica)

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     f1 = fib;
  }
  return fib;
}
```

fib(0) fib(1)

(programação dinâmica)

```
int fib_iter(int n)
{
  int fib, f1 = 1, f2 = 0;
  for (int i = 2; i <= n; i++) {
     fib = f1 + f2;
     f2 = f1;
     f1 = fib;
  }
  return fib;
}
```

fib(0) fib(1) fib(2)

(programação dinâmica)

```
int fib_iter(int n)
{
  int fib, f1 = 1, f2 = 0;
  for (int i = 2; i <= n; i++) {
     fib = f1 + f2;
     f2 = f1;
     f1 = fib;
  }
  return fib;
}
```

fib(0) fib(1) fib(2) fib(3)

(programação dinâmica)

```
int fib_iter(int n)
{
  int fib, f1 = 1, f2 = 0;
  for (int i = 2; i <= n; i++) {
     fib = f1 + f2;
     f2 = f1;
     f1 = fib;
  }
  return fib;
}
```

fib(0) fib(1) fib(2) fib(3) fib(4)

(programação **dinâmica**)

```
int fib_iter(int n)
{
  int fib, f1 = 1, f2 = 0;
  for (int i = 2; i <= n; i++) {
     fib = f1 + f2;
     f2 = f1;
     f1 = fib;
  }
  return fib;
}
```

fib(0) fib(1) fib(2) fib(3) fib(4) fib(5)

(programação **dinâmica**)

```
int fib_iter(int n)
{
  int fib, f1 = 1, f2 = 0;
  for (int i = 2; i <= n; i++) {
     fib = f1 + f2;
     f2 = f1;
     f1 = fib;
  }
  return fib;
}
```

fib(0) fib(1) fib(2) fib(3) fib(4) fib(5) fib(6)

(programação **dinâmica**)

```
int fib_iter(int n)
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  for (int i = 2; i <= n; i++) {
     fib = f1 + f2;
     f2 = f1;
     f1 = fib;
  }
  return fib;
}
```
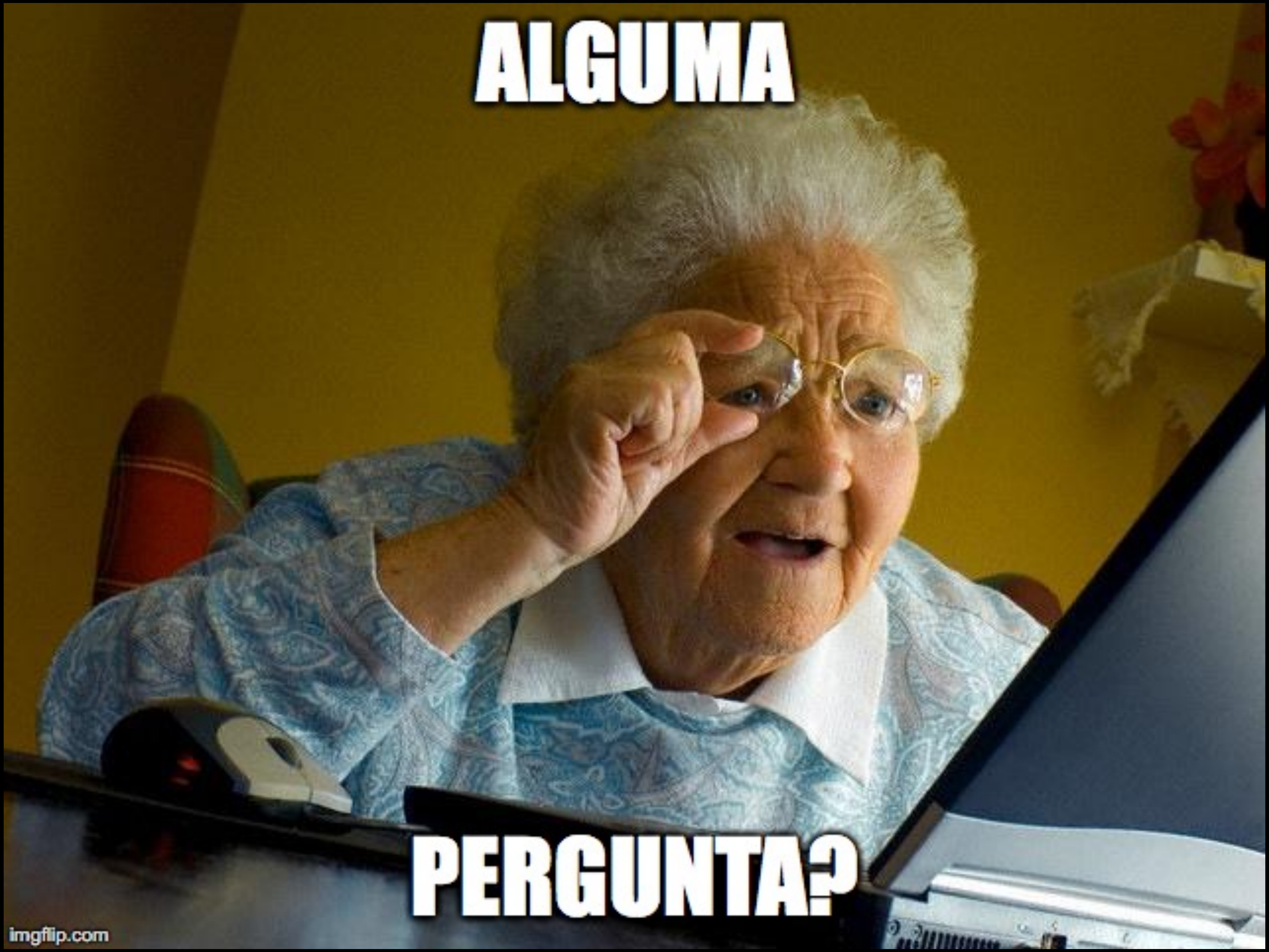

```
int fib_rec(int n)
{
   if (n <= 1) return n;
   return fib(n-1) + fib(n-2);
}
```

fib(0) fib(1) fib(2) fib(3) fib(4) fib(5) fib(6)

(programação **dinâmica**)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dividir para conquistar	intersecções entre subproblemas
recursão de cauda	demais casos

recursão de cauda

recursão de cauda

```
int fat_rec(int n)
```

recursão de cauda

```
int fat_rec(int n)
{
```

recursão de cauda

```
int fat_rec(int n)
{
  if (n <= 1) return 1;
```

recursão de cauda

```
int fat_rec(int n)
{
  if (n <= 1) return 1;
  return n * fat_rec(n-1);
```

recursão de cauda

```
int fat_rec(int n)
{
  if (n <= 1) return 1;
  return n * fat_rec(n-1);
}
```

recursão de cauda

recursão de cauda

```
int fat_cauda(int n, int f)
```

recursão de cauda

```
int fat_cauda(int n, int f)
{
```

recursão de cauda

```
int fat_cauda(int n, int f)
{
  if (n <= 1) return f;
```

recursão de cauda

```
int fat_cauda(int n, int f)
{
  if (n <= 1) return f;
  return fat_cauda(n-1, n* f);
```

recursão de cauda

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int fat_cauda(int n, int f)
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  if (n <= 1) return f;
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int fat_rec(int n)
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recursão de cauda

```
int fat_cauda(int n, int f)
{
  if (n <= 1) return f;
  return fat_cauda(n-1, n* f);
}
```

```
int fat_rec(int n)
{
```

recursão de cauda

```
int fat_cauda(int n, int f)
{
  if (n <= 1) return f;
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int fat_rec(int n)
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    return fat_cauda(n, 1);
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recursão de cauda

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    return fat_cauda(n, 1);
}
```

```
int fat(int n)
```

recursão de cauda

```
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  if (n <= 1) return f;
  return fat_cauda(n-1, n * f);
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int fat(int n)
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recursão de cauda

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int fat_cauda(int n, int f)
{
  if (n <= 1) return f;
  return fat_cauda(n-1, n * f);
}

int fat_rec(int n)
{
    return fat_cauda(n, 1);
}
```

```
int fat(int n)
{
   int f = 1;
```

recursão de cauda

```
int fat_cauda(int n, int f)
{
  if (n <= 1) return f;
  return fat_cauda(n-1, n * f);
}

int fat_rec(int n)
{
    return fat_cauda(n, 1);
}
```

```
int fat(int n)
{
   int f = 1;
   while (n > 1) {
```

recursão de cauda

```
int fat_cauda(int n, int f)
{
  if (n <= 1) return f;
  return fat_cauda(n-1, n * f);
}

int fat_rec(int n)
{
    return fat_cauda(n, 1);
}
```

```
int fat(int n)
{
   int f = 1;
   while (n > 1) {
      f = n * f;
```

recursão de cauda

```
int fat_cauda(int n, int f)
{
  if (n <= 1) return f;
  return fat_cauda(n-1, n * f);
}

int fat_rec(int n)
{
    return fat_cauda(n, 1);
}
```

```
int fat(int n)
{
   int f = 1;
   while (n > 1) {
      f = n * f;
      n = n - 1;
```

recursão de cauda

```
int fat_cauda(int n, int f)
{
  if (n <= 1) return f;
  return fat_cauda(n-1, n * f);
}

int fat_rec(int n)
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}
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recursão de cauda

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   return f;
```

recursão de cauda

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  if (n <= 1) return f;
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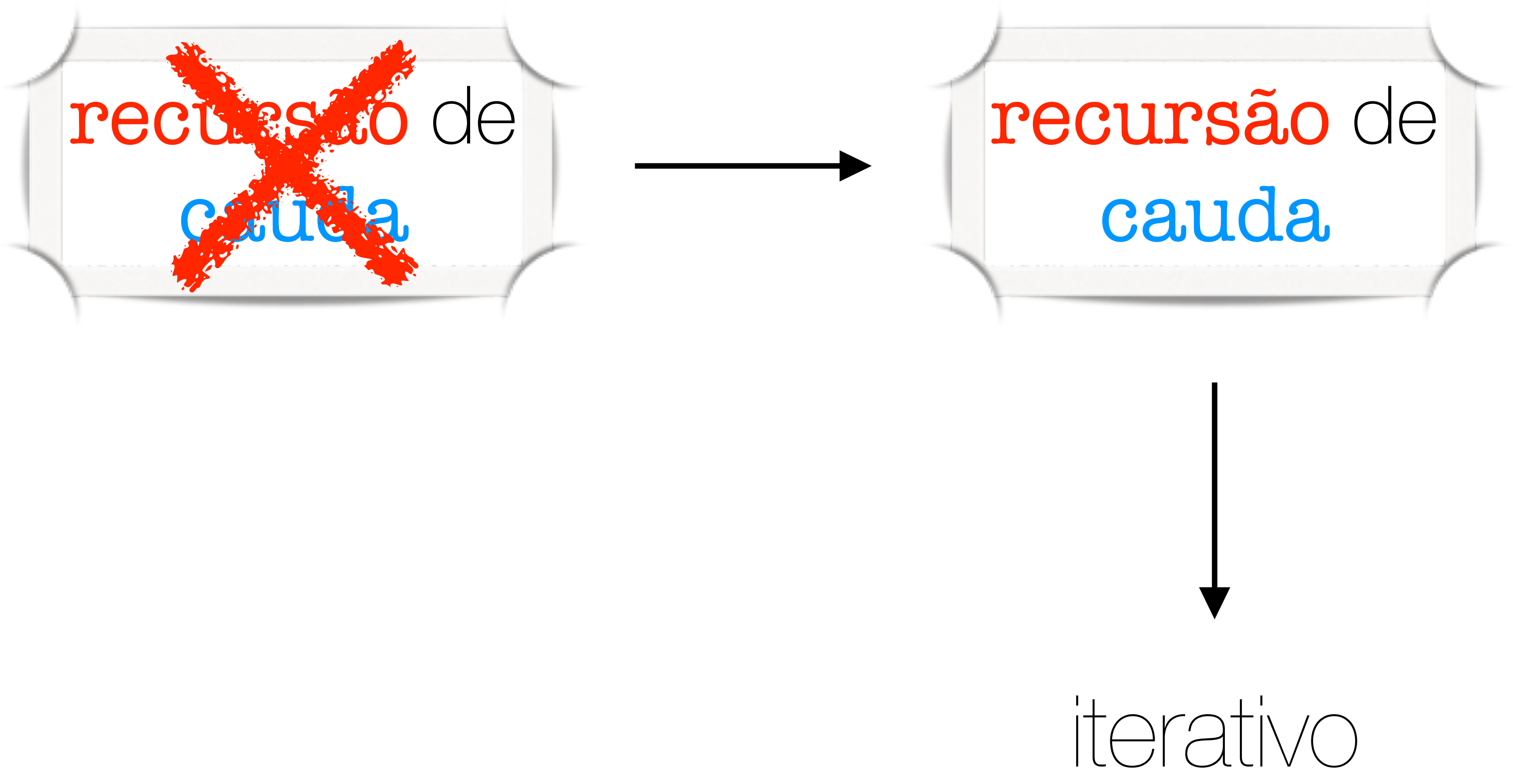
int fat(int n)
{
   int f = 1;
   while (n > 1) {
      f = n * f;
      n = n - 1;
   }
   return f;
}
```

🤔

~~recursão~~ de
~~cauda~~

~~recursão~~ de ~~cauda~~ → recursão de cauda

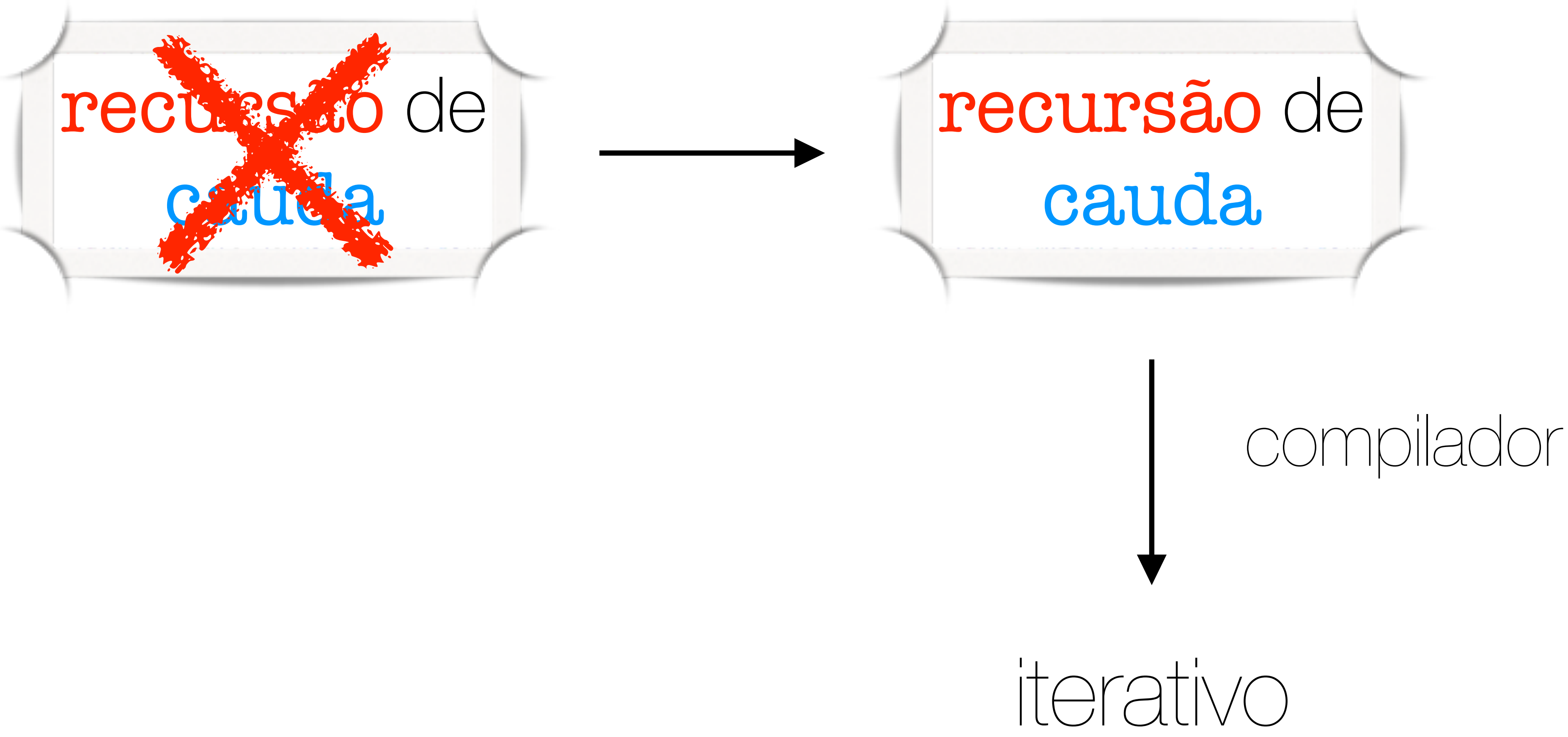
🤔



~~recursão~~ de ~~cauda~~

recursão de cauda

iterativo



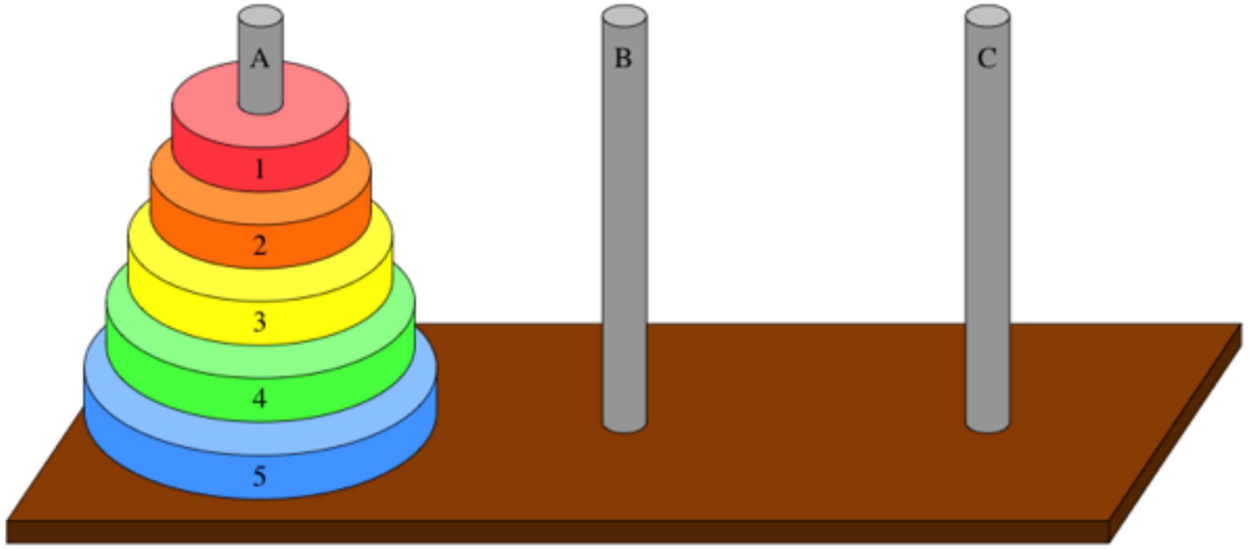
~~recursão de cauda~~

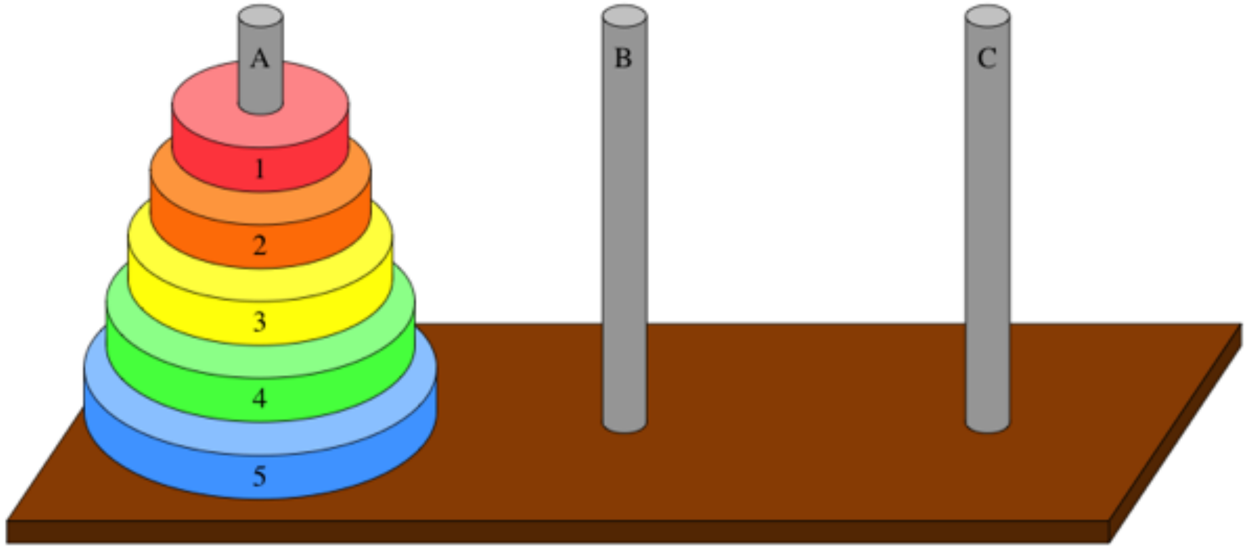
recursão de cauda

compilador

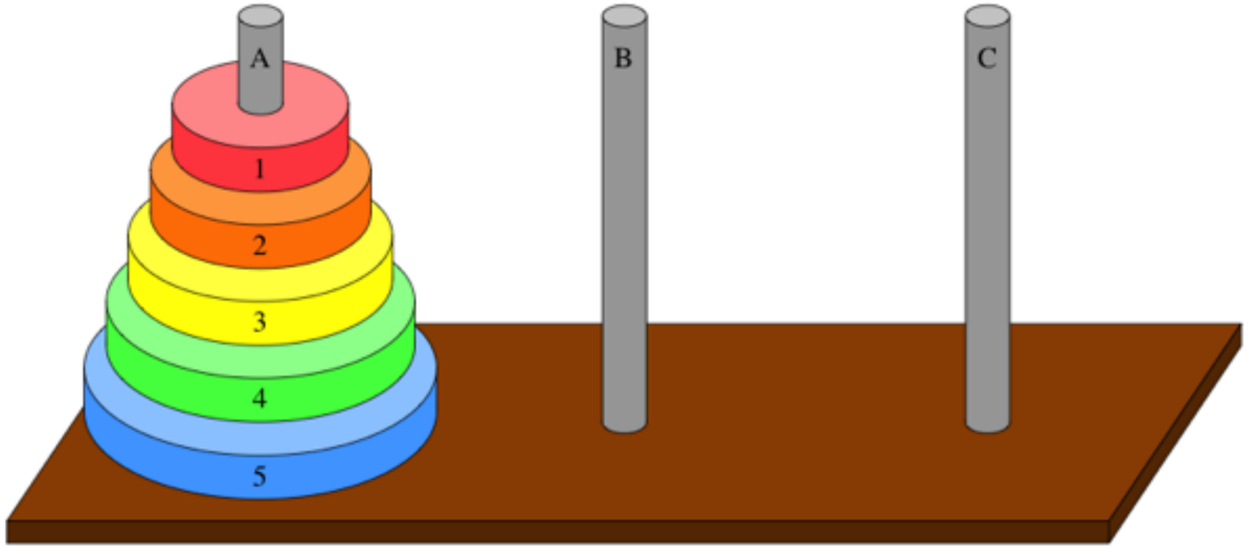
iterativo

dividir para conquistar	intersecções entre subproblemas
recursão de cauda	demais casos

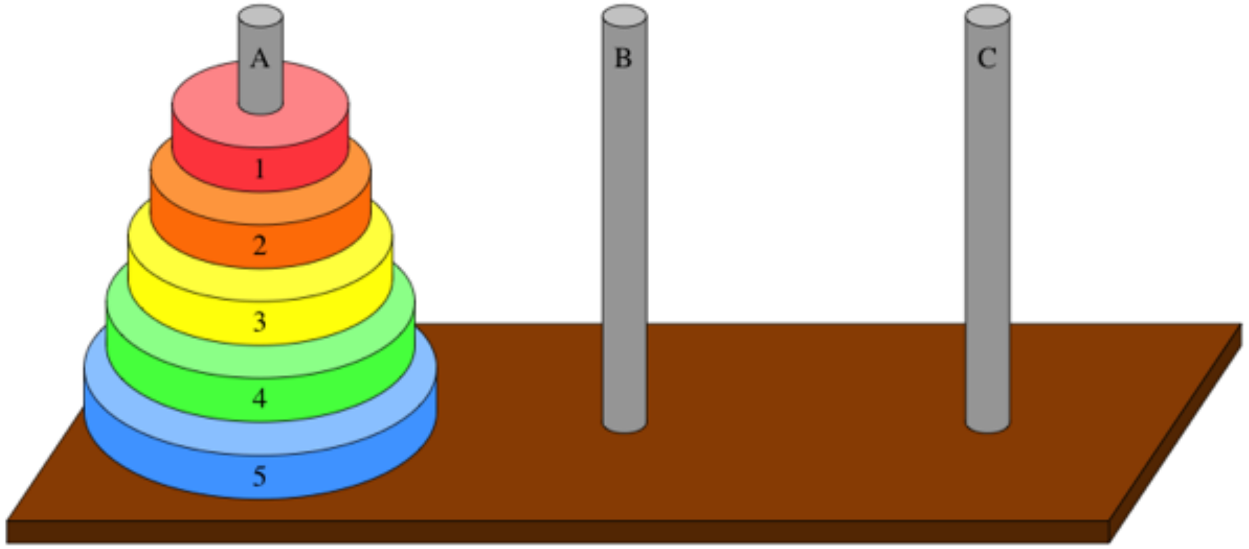

A
B
C
1
2
3
4
5



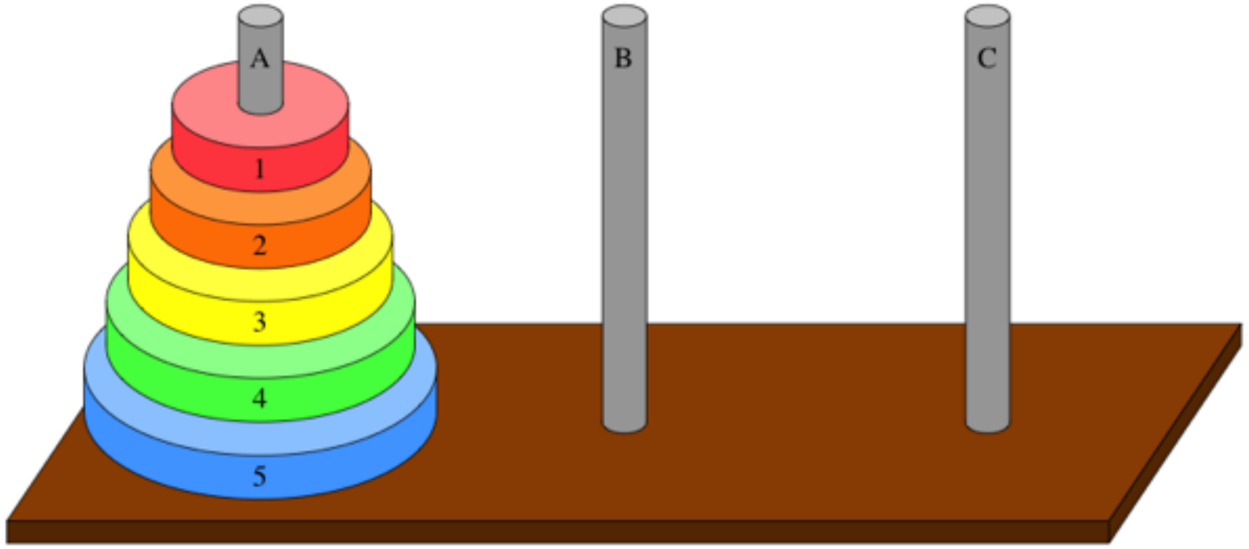
hanoi (altura, origem, destino, aux)



```
hanoi (altura, origem, destino, aux)
    if (altura == 1)
```



```
hanoi (altura, origem, destino, aux)
    if (altura == 1)
        “Movendo disco 1 da torre origem pra torre destino.”
```



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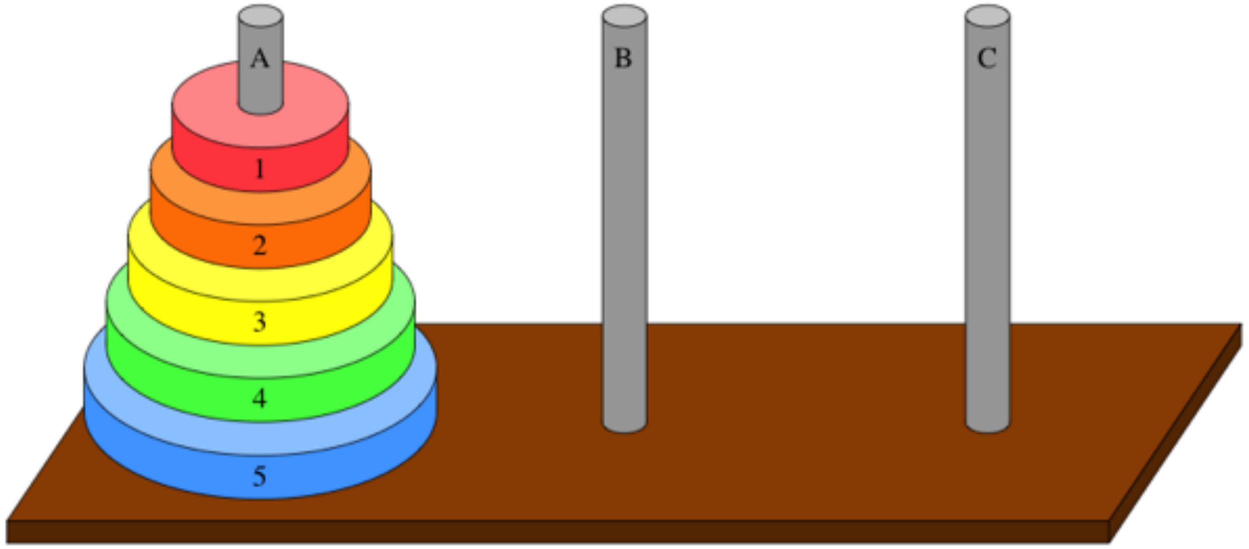

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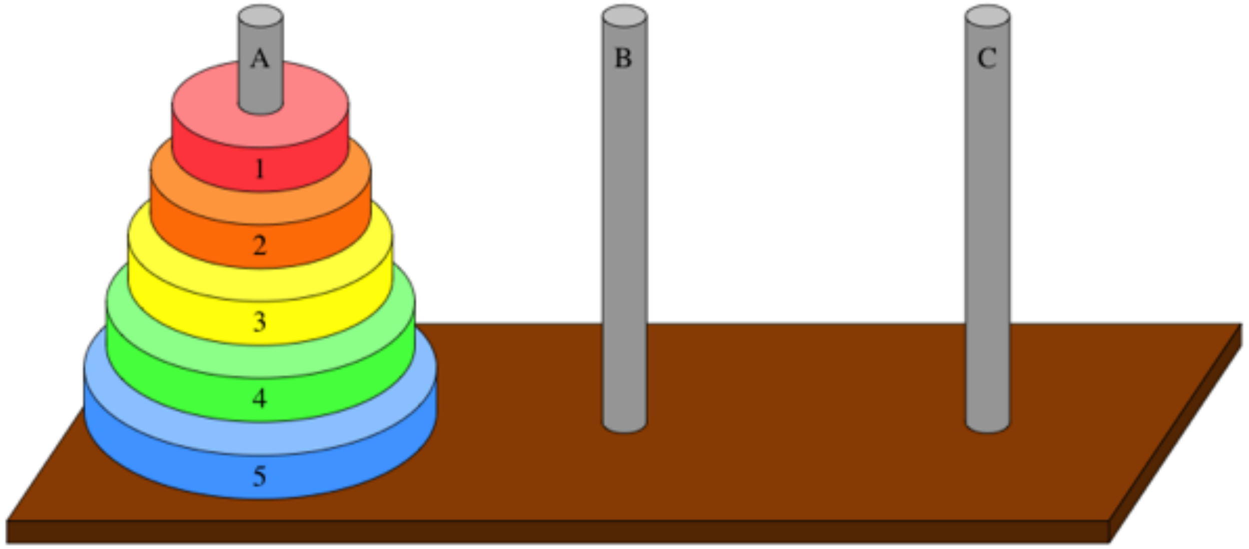
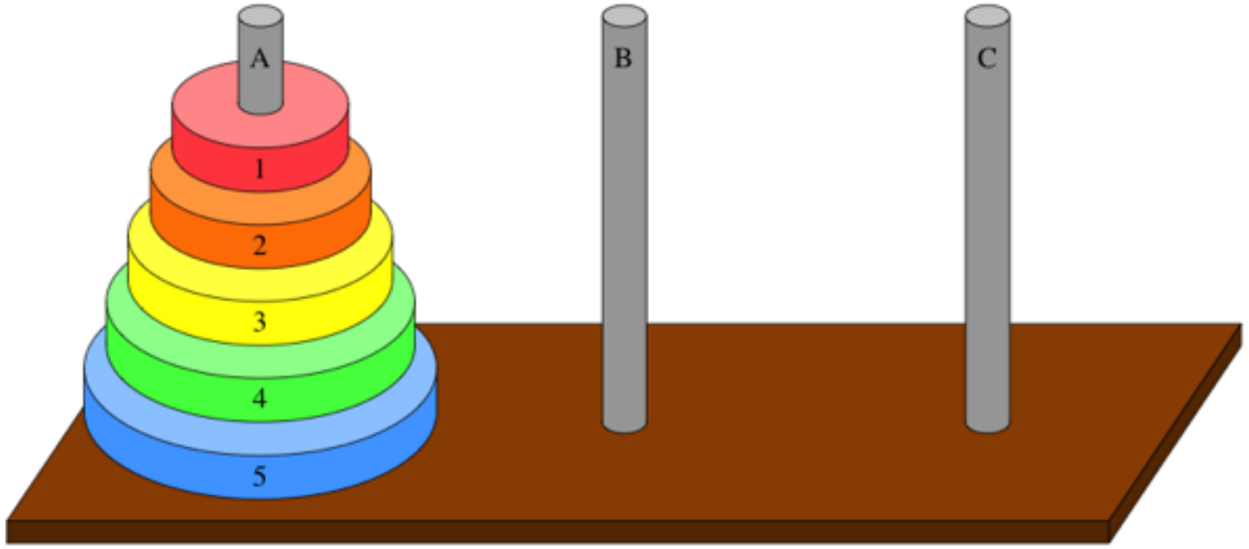


```
hanoi (altura, origem, destino, aux)
    if (altura == 1)
        "Movendo disco 1 da torre origem pra torre destino."
        return
    }
    hanoi (altura - 1, origem, aux, destino);
```



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        “Movendo disco 1 da torre origem pra torre destino.”
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    }
    hanoi (altura - 1, origem, aux, destino);
    “Movendo disco altura da torre origem pra torre destino.”
```

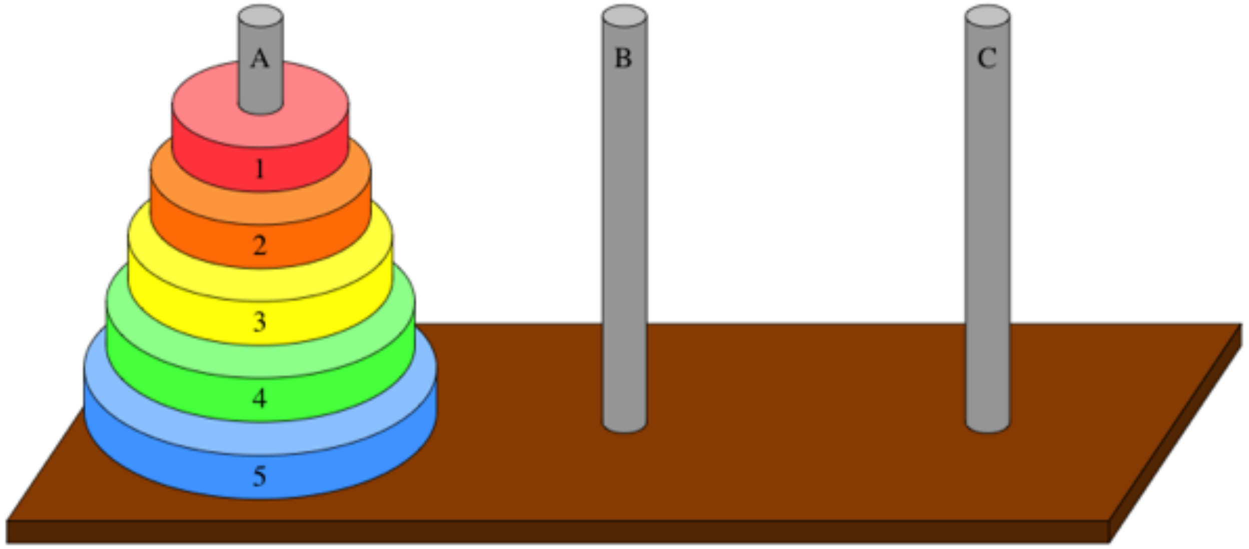

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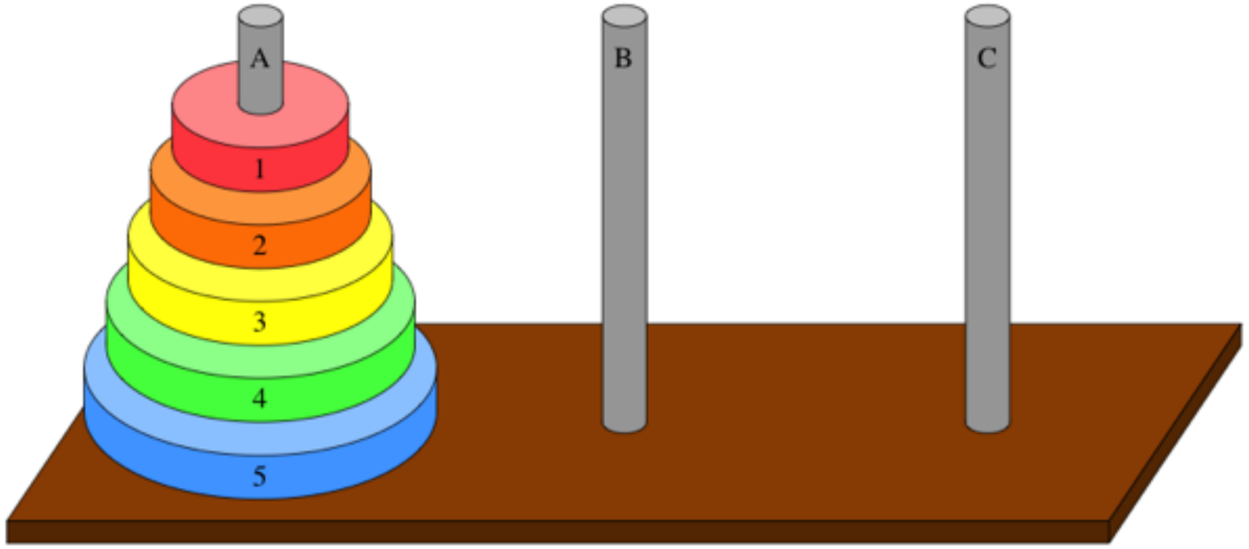
    }
    hanoi (altura - 1, origem, aux, destino);
    “Movendo disco altura da torre origem pra torre destino.”
    hanoi (altura - 1, aux, destino, origem);
```



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        return
    }
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    “Movendo disco altura da torre origem pra torre destino.”
    hanoi (altura - 1, aux, destino, origem);
}
```


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intersecções entre subproblemas	👎

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intersecções entre subproblemas	👎
recursão de cauda	👍🤔

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intersecções entre subproblemas	👎
recursão de cauda	👍🤔
demais casos	🍺