Prof. Dr. Mine Elif KARSLIGİL BLM3021 Algoritma Analizi

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Ödev #1

OĞUZHAN TOPALOĞLU
Ç19052025 – Grup 1

Bilgisayar Mühendisliği Bölümü,
Elektrik-Elektronik Fakültesi,
Yıldız Teknik Üniversitesi

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Video Linki: <bu ödevde istenilmedi>

Oguzhan Topaloglu

4)

a) $f(n) = 2^{n+1} + 3^{n-1} \Rightarrow f(n) = \Theta(3^n)$ dir.

$f(n) = \Theta(g(n)) \Leftrightarrow c_1 g(n) \geq 2^{n+1} + 3^{n-1} \geq c_2 g(n) \geq 0, \quad n \geq n_0, \quad n_0 > 0, c_1 > 0, c_2 > 0$

$\Rightarrow c_1 3^n \geq 2^{n+1} + 3^{n-1} \geq c_2 3^n \geq 0$

$\Rightarrow c_1 \geq \dfrac{2^{n+1} + 3^{n-1}}{3^n} \geq c_2 \geq 0 \Rightarrow c_1 \geq \underbrace{2\left(\frac{2}{3}\right)^n + \frac{1}{3}} \geq c_2 \geq 0$

$\max \rightarrow 2 \cdot 1 + \frac{1}{3} = 2 + \frac{1}{3} = \frac{7}{3}$

$\Rightarrow c_1 = 1000$
$c_2 = 1$ alabilir. $\Rightarrow$ $\boxed{\text{Demek ki } 2^{n+1} + 3^{n-1} \in \Theta(3^n)}$
$n_0 = 2$

b)
$f(n) = 2n \lg(n+2)^2 + (n+2)^2 \lg\left(\frac{n}{2}\right) = \underbrace{4n \lg(n+2)}_{O(n \lg n)} + \underbrace{(n+2)^2 \lg\left(\frac{n}{2}\right)}_{O(n^2 \lg n)}$

$f(n) = \Theta(n^2 \lg n)$

$\Leftrightarrow c_1 n^2 \lg n \geq 4n \lg(n+2) + (n+2)^2 \lg\left(\frac{n}{2}\right) \geq c_2 n^2 \lg n \geq 0$

$n \geq n_0, \quad n_0 > 0, \quad c_1 > 0, \quad c_2 > 0$

$\Rightarrow c_1 \lg n \geq 4 \cdot \dfrac{\lg(n+2)}{n} + \dfrac{(n+2)^2}{n^2} \cdot \lg\left(\frac{n}{2}\right) \geq c_2 \lg n \geq 0$

$\Rightarrow c_1 \geq 4 \cdot \dfrac{\lg(n+2)}{n \lg(n)} + \dfrac{(n+2)^2}{n^2} \dfrac{\lg(n/2)}{\lg(n)} \geq c_2 \geq 0$

$\Rightarrow n_0 = 5$ olsun

$\Rightarrow c_1 \geq \underbrace{4 \dfrac{\lg(7)}{5 \lg 5)} + \dfrac{7^2}{5^2} \dfrac{\lg(5/2)}{\lg(5)}}_{\approx 2.0831} \geq c_2 \geq 0$

$\Rightarrow c_2 = 1, \quad c_1 = 2000 \Rightarrow \boxed{2n \lg(n+2)^2 + (n+2)^2 \lg\left(\frac{n}{2}\right) \in \Theta(n^2 \lg n)}$

Oğuzhan Topaloğlu

1)

a) $T(n) = 9T(n/4) + n^2 \Rightarrow a=9,\ b=4,\ f(n)=n^2,\ n^{\log_b a} = n^{\log_4 9}$

b) $T(n) = 3T(n/3) + \log n \Rightarrow a=3,\ b=3,\ f(n)=\log n,\ n^{\log_b a} = n^{\log_3 3} = n^1 = n$

c) $T(n) = 3T(n/2) + n \Rightarrow a=3,\ b=2,\ f(n)=n,\ n^{\log_b a} = n^{\log_2 3}$

b ve c seçeneklerinde $f(n)$, $n^{\log_b a}$'dan daha yavaş büyüdüğü için Master Theorem'in "Case 1"ini kullanırız. a için de "Case 3" kullanırız:

$$\boxed{\begin{array}{l} a)\ T(n) = \Theta(n^2) \\ b)\ T(n) = \Theta(n) \\ c)\ T(n) = \Theta(n^{\log_2 3}) \end{array}}$$

$(n^2 \gg n^{\log_4 9})$ $(\log_4 9 = \log_2 3)$

$(\log n \ll n)$

$(n \ll n^{\log_2 3})$ $(\log_2 3 \approx 1.584)$

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2)

```
int f1(int n){
  int x=0;
  for(int i=0;i<N;i++)   --> N kez
    x++;
  return x;              --> 1
}
```

$\Rightarrow \boxed{f1(n) \in O(n)}$

```
int f2(int n){
  int x=0;
  for(int i=0;i<N;i++)        --> N kez
    for(int j=0;j<i;j++)      --> N-1 kez i'ye bağlı
      x += f1(j);             --> N kez
  return x;
}
```

$N \cdot (N-1) N = N^3 - N^2$

$\boxed{f2 \in O(N^3)}$

```
int f3(int n){
  if(n==0) return 1;
  int x=0;
  for(int i=0;i<n;i++)
    x += f3(n-1);
  return x;
}
```

$f3(n) = n \cdot f3(n-1) = n(n-1)\, f3(n-2)$

$= \dots = n(n-1)(n-2) \dots \cdot 2 \cdot 1 \cdot \underbrace{f3(0)}_{1}$

$= n!$

$\boxed{f3 \in O(n!)}$

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ÖE

2 devam)

```
int f4(int n){
  if(n==0) return 0;
  return f4(n/2)+f1(n)
     +f1(n)+f1(n)+f4(n/2);
}
```

$$f4(n) = 3\overset{n}{\overline{f1(n)}} + 2f4(n/2)$$

$$= 3n + 2f4(n/2) \quad i=1$$

$$= 3n + 2\left[3\tfrac{n}{2} + 2f4(\tfrac{n}{4})\right]$$

$$= 6n + 4f4(n/4) \quad i=2$$

ÖRÜNTÜ

$$= 6n + 4\left[3\tfrac{n}{4} + 2f4(n/8)\right]$$

$$= 9n + 8f4(n/8) \quad i=3$$

$$= 9n + 8\left[3\tfrac{n}{8} + 2f4(n/16)\right]$$

$$= 12n + 16f4(n/16) \quad i=4$$

$$f4(n) = 2^i \cdot f4\left(\frac{n}{2^i}\right) + i(3n)$$

$$\frac{n}{2^i} = 1 \rightarrow n = 2^i \rightarrow i = \log_2 n$$

$$\Rightarrow f4(n) = 2^{\log_2 n} f_4\left(\frac{n}{2^{\log_2 n}}\right) + \log_2 n \, 3n$$

$$= n\, f4(1) + 3n \log_2 n$$

$$= n[2 \cdot \underbrace{f4(0)}_{0} + \underbrace{f1(1)}_{1} \cdot 3] + \frac{3}{\log_2} n \log n$$

$$= 3n + \frac{3}{\log_2} n \log n$$

$$\boxed{f4 \in O(n \log n)}$$

6)

$$T(n) = 2n + T(n-2) \quad , i=1$$

$$= 2n + 2(n-2) + T(n-4)$$

$$= (4n-4) + T(n-4) \quad , i=2$$

Örüntü

$$= (4n-4) + 2(n-4) + T(n-6)$$

$$= 6n - 12 + T(n-6) \quad , i=3$$

$$= 6n - 12 + 2(n-6) + T(n-8)$$

$$= 8n - 24 + T(n-8) \quad , i=4$$

$$\Rightarrow T(n) = 2in + 2i(i-1) + T(n-2i)$$

$$n - 2i = 0 \Rightarrow i = n/2$$

$$\Rightarrow T(n) = 2 \cdot \frac{n}{2} \cdot n + 2 \cdot \frac{n}{2}\left(\frac{n}{2} - 1\right) + T(0)$$

Soru eksik diye 0 saydım

$$= n^2 + \frac{n^2}{2} - n + T(0)$$

$$= \frac{3}{2} n^2 - n$$

$$\Rightarrow \boxed{T(n) \in O(n^2)}$$

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5)

$$\sum_{i=1}^{n}(i+1)2^{i-1} = \sum_{i=1}^{n} i\,2^{i-1} + \sum_{i=1}^{n} 2^{i-1}$$

$$= \sum_{i=1}^{n} i\,2^{i-1} + \frac{1(2^n-1)}{1} = \sum_{i=1}^{n} i\,2^{i-1} + \overbrace{(2^n-1)}^{O(2^n)}$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} a r^{i-1} = \frac{a(r^n-1)}{r-1}$$

Formüller

$a=1 \quad r=2$

$$\sum_{i=1}^{n} i\,2^{i-1} \Rightarrow \frac{d}{dr}\left(\frac{r^n-1}{r-1}\right) = \frac{nr^{n-1}(r-1)-(r^n-1)}{(r-1)^2} = \frac{n(r^n-r^{n-1})-(r^n-1)}{(r-1)^2}$$

$$= \frac{nr^n - nr^{n-1} - r^n + 1}{(r-1)^2} = \frac{r^n(n-1) - nr^{n-1} + 1}{(r-1)^2} \rightarrow \sum_{i=1}^{n} i\,r^{i-1} \text{ için}$$

toplam formülü

$$r=2 \Rightarrow 2^n(n-1) - n2^{n-1} + 1 \longrightarrow O(n2^n)$$

$$\Rightarrow \sum_{i=1}^{n} i\,2^{i-1} \in O(n2^n) \text{ ve } \sum_{i=1}^{n} 2^{i-1} \in O(2^n) \text{ ise}$$

$$\sum_{i=1}^{n}(i+1)2^{i-1} \in O(n2^n) \text{ olur.}$$

✓

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3)

$f(n)$	$g(n)$	
n^2	n^3	O
$n\lg n$	n	Ω
1	$3+\sin(n)$	Θ
3^n	2^n	Ω
$4^{n+4}=2^{2n+8}$	2^{2n+2}	Θ
$n\lg n$	$n^{105/100}$	O
$\lg(\sqrt{10}\,n)$	$\lg(n^3)$	Θ
$n!$	$(n+1)!$	O

$f(n)=O(g(n)) \Leftrightarrow c\,g(n)\geq f(n)\geq 0\ ,\ n\geq n_0$
$n_0>0 \quad c>0$

$f(n)=\Omega(g(n)) \Leftrightarrow f(n)\geq c\,g(n)\geq 0\ ,\ n\geq n_0$
$n_0>0 \quad c>0$

$f(n)=\Theta(g(n)) \Leftrightarrow c_1 g(n)\geq f(n)\geq c_2\, g(n)\geq 0$
$n\geq n_0\ ,\ n_0>0\ ,\ c>0$

$n^2, n^3 \rightarrow$ $c\,n^{\cancel{3}}\geq \cancel{n^2}\geq 0$
$cn\geq 0 \quad n\geq n_0 \quad n_0=5\ \ c=5 \quad O$ ✓

$\cancel{n^2}^{1}\geq c\,n^{\cancel{3}}\geq 0$ yok

$n\lg n, n \rightarrow$ $c\cancel{n}\geq \cancel{n}\lg n\geq 0\ ,\ n\geq n_0$
$c\geq \lg n$ yok

$\cancel{n}\lg n\geq c\cancel{n}\geq 0$
$\lg n\geq c\geq 0 \quad c=1\ \ n_0=100 \quad \Omega$ ✓

$3^n, 2^n \rightarrow$ $c2^n\geq 3^n\geq 0$
$c\geq (3/2)^n\geq 0$ yok

$3^n\geq c2^n\geq 0$
$(3/2)^n\geq c \quad c=1\ \ n_0=1$ ✓ Ω

$2^{2n+8}, 2^{2n+2} \rightarrow$ $2^{2n+8}\geq c\,2^{2n+2}\geq 0$
$2^6\geq c\geq 0 \quad n_0=1\ \ c=2$ ✓ Ω

$\hookrightarrow$ $c\,2^{2n+2}\geq 2^{2n+8}\geq 0$
$c\geq 2^6\geq 0 \quad c=2^7\ \ n_0=100$ ✓ O

Θ

$n\lg n, n^{1.05}$

$O\rightarrow c\,n^{1.05}\geq n\lg n\geq 0$
$c\,n^{0.05}\geq \lg n\geq 0 \quad c=1\ \ n_0=10^{10}$ ✓

$\Omega\rightarrow n\lg n\geq c\,n^{1.05}\geq 0$
$\lg n\geq c\,n^{0.05}\geq 0$ YOK

$n!, (n+1)n!$ $\quad O\rightarrow c(n+1)\cancel{n!}\geq \cancel{n!}\geq 0 \quad c=1\ \ n_0=10$ ✓

$\Omega\rightarrow \cancel{n!}\geq c(n+1)\cancel{n!}\geq 0$
$1\geq c(n+1)\geq 0$ YOK

$\lg(\sqrt{10}n), \cancel{3\lg(n)}^{\lg(n^3)}$

$O\rightarrow \cancel{3}c\lg(n^3)\geq \lg(\sqrt{10}n)\geq 0 \quad c=1\ \ n_0=\sqrt{10}$ ✓

$\Omega\rightarrow \lg(\sqrt{10}n)\geq 3c\lg n\geq 0 \quad n_0=100000\ \ c=1/3$ ✓

$1,\ \sin n+3$

✓ $O\rightarrow c(\underbrace{\sin(n)+3}_{[2,4]})\geq 1\geq 0 \quad c=1\ \ n_0=10$ ✓

✓ $\Omega\rightarrow 1\geq c(\sin(n)+3)\geq 0$

$\Theta \quad c=\frac{1}{30}\rightarrow 1\geq \underbrace{\frac{3+\sin(n)}{30}}_{\left[\frac{2}{30},\frac{4}{30}\right]}\geq 0$ ✓